Travel Time Reliability Based Bridge Network Maintenance Optimization under Budget Constraint

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ABSTRACT: This study develops a decision model to assist bridge authorities in determining a preferred maintenance prioritization schedule for a degraded bridge network in a community that optimizes the performance of transportation systems within budgetary constraints at a regional scale. The study utilizes network analysis methods, structural reliability principles and meta-heuristic optimization algorithms to integrate individual descriptive parameters such as bridge capacity rating, condition rating, traffic demand, and location of the bridge, into global objective functions that define the overall network performance and maintenance cost. The performance of the network is measured in terms of travel time reliability between all possible origin/destination (OD) pairs. In addition to the global budgetary constraint, the optimization is also conditioned on local constraints imposed on traffic flow by insufficient load carrying capacity of deficient bridges. Uncertainties in traffic demands, vehicle weights and maintenance costs are also considered in the problem formulation. This decision model is illustrated with a hypothetical bridge network.

1. INTRODUCTION

Highway bridges deteriorate in service as a result of a wide variety of events (e.g. floods, heavy truck traffic, aggressive environmental conditions, industrial action, and inadequate maintenance), making bridges the vulnerable nodes in transportation networks. Resources allocated to the maintenance of transportation infrastructure in the United States (and societies worldwide) typically are limited, and seldom are sufficient to maintain in-service performance levels required for the infrastructure. As stated in the 2013 ASCE Infrastructure Report Card, every year over $12 billion has been spent on the maintenance and rehabilitation of the nation's bridges, while the annual investment that would be necessary to improve the current condition of existing highway bridges has been estimated to be $20 billion. Bridge managers are facing ever-increasing challenges in prioritizing expenditures to maintain safety and functionality of deteriorating bridge systems. A decision-making framework that maximizes the functionality of a regional transportation system while ensuring that individual bridges conform to the minimum safety requirements stipulated by Association of State Highway and Transportation Officials (AASHTO) (2012) is essential.

2. BACKGROUNDS

Bridge maintenance programs in the past usually have been developed to optimize the life-cycle cost (LCC) of individual bridges without considering the interaction between the bridges in making the transportation system functional. For instance, the condition of one bridge, e.g. deterioration, failure, maintenance priority and the timing of expenditure, may very well affect the performance and maintenance scheduling of the neighboring bridges. Finite resources for renewal/replacement of bridges within a transportation system must be distributed among the bridges to optimize the performance of the system as a whole.

In the past decade, several research studies have investigated bridge maintenance strategies considering bridges collectively as integral parts of a network. Frangopol and his co-researches have made number of important contributions to the bridge network maintenance planning. Liu and Frangopol (2005a; 2005b; 2006a) introduced a bridge reliability importance factor that relates individual bridge reliability to the reliability of the bridge network, and proposed a comprehensive mathematical model for evaluating the overall performance of a bridge network based on probabilistic analyses of network connectivity, user satisfaction, and structural reliability of the critical bridges in the network. Their later study (Liu and Frangopol, 2006b) optimized bridge network maintenance based on a multi-objective approach using genetic algorithms. A thorough review of their work can be found in Frangopol (2011). Orcesi and Cremona (2010) proposed a bridge network management approach using visual inspection data and Markov chains. The above studies have based maintenance and project prioritization decisions on bridge LCC analysis of their projected service life.

Rather than using LCC as a criterion for decision, several studies have attempted to obtain optimal bridge maintenance strategies by maximizing the operational performance of a transportation network. Connectivity reliability and travel time reliability often have been proposed to assess the transportation network performance. Previous studies on connectivity reliability (e.g. Bocchini and Frangopol, 2011) and travel time reliability (e.g. Asakura and Kashiwadani, 1995) are useful for providing a basis to analyze network traffic equilibrium, but in these studies the links (bridges) of the network were modeled as either fully functional or completely closed. In reality, however, many structurally deficient bridges are in neither status; rather, they continue to operate with a reduced load capacity imposed by lane closures or posting limits.

In this paper, we develop a methodology for bridge network management and project prioritization that maximizes the operational performance of a transportation system measured in terms of travel-time reliability under budgetary constraints. The study utilizes network analysis methods, structural reliability principles and meta-heuristic optimization algorithms to integrate individual descriptive parameters such as bridge capacity rating, condition rating, traffic demand, and location of the bridge, into global objective functions that define the overall network performance and maintenance cost. The performance of the network is measured in terms of travel time reliability between all possible origin/destination (OD) pairs. In addition to the global budgetary constraint, the optimization is also conditioned on local constraints imposed on traffic flow by insufficient load carrying capacity of deficient bridges. Uncertainties in traffic demands, vehicle weights and maintenance costs are also considered in the problem formulation.

3. BRIDGE NETWORK SAFETY AND FUNCTIONALITY

3.1. Safety Criteria for Individual Bridges-Bridge Condition Rating and Capacity Rating

Safety is the first priority among all bridge performance objectives. In current engineering practice, bridge condition rating (on the scale of 0 to 9), assigned according to National Bridge Inspection Standard (NBIS), is widely used in bridge condition assessment in the US and is an overall measure of bridge's condition. Bridge engineers assign condition ratings to existing bridges based on inspection data, traffic survey and highway types. The NBIS stipulates that a bridge with a condition rating less or equal to 4 must be repaired, replaced or closed to operation.

Bridges in good physical condition could still pose a threat to the functioning of the transportation system if they were designed according to archaic standards because they may not have adequate load carrying capacity for modern traffic demands. Therefore, in addition to condition rating, the AASHTO Manual for Bridge Evaluation (2011) utilizes the bridge live load capacity rating factor (RF), calculated using bridge design strength minus dead load effect, divided by the live load effect, to reflect the bridge's live load capacity with respect to its traffic demand. If the resulting RF ≥1.0, the bridge is deemed to have adequate load carrying capacity, while if RF < 1, the bridge is required to be strengthened, replaced or posted. If the bridge is posted, vehicles that are heavier than the weight limit suggested by the bridge's RF are restricted from passing the bridge.

Accordingly, either a low condition rating or a low capacity rating factor can trigger maintenance activities for a given bridge. If closure of a bridge due to either severe deterioration (i.e. condition rating ≤4) or posting (i.e. RF<1) creates intolerable adverse impacts on the performance of the transportation system, the bridge should be scheduled for renewal/replacement. The priority of the renewal/replacement project should be based on the level of impact that the bridge's operational status (e.g. fully functional, posting, or closure) has on the efficiency of the transportation system as a whole, which is reflected in the network travel time reliability discussed in the next section.

3.2. Bridge Network Functionality Measure - Travel Time Reliability

In addition to the safety concerns regarding individual bridges discussed in section 3.1, it is important that appropriate system performance measures are introduced in order to improve the performance efficiency of the transportation network as a whole. In this study we use travel time reliability to evaluate network performance for degraded transportation networks. We define the travel time reliability as the probability that the total travel time of all vehicles in the network between all possible origin-destination (O/D) pairs is less than a prescribed threshold. The travel time reliability therefore is a function of traffic supply/demand in the network, distance between O/D pairs, the shortest time paths between each O/D pair for vehicles of different weight, and other network properties such as its topology, speed limit on links, and deterioration condition and load capacities of the bridges in the network.

Let ϕ denote the service metric, computed by summing up all the fastest paths travel time for all vehicles traveling between network O/D pairs. The network travel time reliability, R_T, is then defined as:

$$R_T = \mathrm{P}[\frac{\phi}{\phi_p} < \alpha] \tag{1}$$

in which $P[\cdot]$ = probability of the statement in the bracket; ϕ_p = the minimum travel time (or service value) for an existing bridge network in its designed ("new") condition; α = the threshold for acceptable service value, which is set larger than 1. If a network cannot transfer vehicles from their origins to destinations, ϕ approaches to infinity and R_T to 0.

4. MATHEMATICAL FORMULATION AND SOLUTION PROCEDURE

4.1. Travel Time Optimization

We define the network topology $G = (N, A)$ as a set of nodes N and set of arcs A. The nodes represent origins, destinations, and transition nodes. The set of arcs, each denoted by a distinct node pair (i, j) where $i, j \in N$ for $i \neq j$, represent all existing roadways and bridges in the transportation system. Let K denote the total number of vehicles in the network and $t \in K$ identify each vehicle. Let W denote the set of all O/D pairs in G. $T^t_{O(t)D(t)}$ is the total travel time of vehicle t from its origin, $O(t)$, to its destination $D(t)$, where $(O(t), D(t)) \in W$, and can be obtained by modifying the classical shortest path method (Ahuja et al, 1993). The network service value ϕ represents the sum of all vehicle travel times:

$$\phi(G) = \sum_{t \in K} \sum_{(O(t),D(t)) \in W} T^t_{O(t)D(t)} \tag{2}$$

Let d_{ij} and v_{ij} denote the distance and speed limit on arc $(i,j) \in A$, respectively, $B \subseteq A$ denote the set of network bridges, l_{ij} denote the posted load limit of bridge $(i,j) \in B$ on arc (i,j). Let r_i^t denote the traffic supply/demand at node i of vehicle t (i.e. $r_i^t = 1$ if i is the origin of vehicle t; $r_i^t = -1$ if i is the destination of vehicle t; and $r_i^t = 0$ if i is a transition node of vehicle t). Flow variable x_{ij}^t denotes the flow of vehicle t on arc (i,j). $x_{ij}^t = 1$ if arc (i,j) is on the fastest path of vehicle t from $O(t)$ to $D(t)$, and $x_{ij}^t = 0$ if otherwise. The shortest travel time of all vehicles between all O/D pairs can be computed as the following:

$$\phi(G) = \min \sum_{t\in K} \sum_{(i,j)\in A} \frac{d_{ij}}{v_{ij}} x_{ij}^t \tag{3}$$

$$\text{s.t.} \ \sum_{\{j:(i,j)\in A\}} x_{ij}^t - \sum_{\{j:(j,i)\in A\}} x_{ji}^t = \begin{cases} 1, & i = O(t) \\ -1, & i = D(t) \forall i \in N, \forall t \in K \\ 0, & otherwise \end{cases} \tag{4}$$

$$0 \le x_{ij}^t \le 1, \forall (i,j) \in A \ \ \forall t \in K \tag{5}$$

Eq. (4) ensures that the inflow and outflow satisfy the traffic supply/demand at node i of vehicle t.

A posted load limit of structurally deficient bridges will certainly affect the path choice of many heavy trucks which, in turn, will impact the travel time of those trucks from their origin to destination. These effects are integrated in the calculation of the travel time by prohibiting vehicle t from passing bridge (i,j) if the vehicle weight, ω_t, exceeds the posting limit of that bridge, l_{ij}, where $l_{ij} = f_{ij} \cdot z_{ij} \cdot l_{ij}^p$; l_{ij}^p = design (as "new") load capacity of bridge (i,j); z_{ij} = capacity rating factor (RF) of the bridge (i,j); and $f_{ij} = 0$ when bridge condition rating is less or equal to 4 and $f_{ij} = 1$ if otherwise. It is assumed that the load capacities of roads (arcs without bridges) are much larger than the weight of any vehicle. It is further assumed that the load capacity of any bridge that is renewed or replaced will be brought to l_{ij}^p (as "new" condition). Let y_{ij} denote the binary maintenance decision variables, where $y_{ij} = 1$ if bridge (i,j) is selected for renewal/replacement and $y_{ij} = 0$ otherwise. The local constraints imposed on traffic flow by posting limits of deficient bridges are:

$$\omega_t x_{ij}^t \le \max\left(l_{ij}, y_{ij} l_{ij}^p\right), \forall (i,j) \in B, \forall t \in K \tag{6}$$

$$y_{ij} = \{0,1\}, \forall (i,j) \in B \tag{7}$$

Let c_{ij} denote the maintenance cost for bridge $(i,j) \in B$, which is assumed to be a function of both posting limit and deck area of the bridge and let Θ denote the total annual budget available for maintenance of the entire network. The cost constraint can then be expressed as:

$$\sum_{(i,j)\in B} c_{ij} y_{ij} \le \Theta \tag{8}$$

The optimal maintenance strategy and project prioritization will be obtained by minimizing the network travel time [as calculated using Eqs. (2)-(5)] considering local constraints imposed by the reduced load capacity of deficient bridges [as expressed by Eqs. (6)-(7)] within a prescribed (global) budget limit [as expressed by Eq. (8)].

In this study we use Binary Particle Swarm Optimization (BPSO) to provide optimal solutions. BPSO was designed to solve discrete optimization problems (Eberhart and Kennedy 1995) and has been proved to perform better than Genetic Algorithms in several research studies (Elbeltagi, et al 2005, Hassan et al. 2005, Tudu et al. 2011, Chiu et al. 2012). In BPSO, each state variable is modeled as a particle; all possible positions of particles define the feasible solution space; BPSO iteratively updates the positions of all "particles" to search for a better solution according to its mathematical formulations. For our problem, each "particle" is a vector of maintenance decision variables. The maintenance cost and the network parameters (i.e. traffic demand and vehicle weight) are considered as random variables in this study, making the travel time reliability [as defined in Eq.(1) and computed in Eqs. (2)-(8))] stochastic in nature. Monte Carlo Simulation (MCS) coupled with Latin Hypercube Sampling (LHS) (Iman and Conover, 1980) is employed to account for the uncertainties in these variables in the optimization process.

4.2. Project Priority Indices

Due to budget consideration and limited available resources, often only a portion of deficient bridges can be scheduled for renewal or replacement. Bridges that have a larger impact on the overall network performance should be prioritized for maintenance activities. In this section, we introduce two priority indices for individual bridge project - static priority index (SPI) and dynamic priority index (DPI).

The static priority index, SPI, is defined as a function of the difference in network travel time reliability between block running (with reduced load carrying capacity before repair) and smooth running (design-level load carrying capacity after repair) of the bridge considered, and can be calculated as:

$$\mathrm{SPI}_{ij} = \frac{E[\phi_0]-E[\phi_{ij}]}{E[\phi_0]-\min\{E[\phi_{ij}]\}} \tag{9}$$

where $E[\cdot]$ is the mean operator; ϕ_0 = service value of the network without any renewal/replacement activities (i.e. the network is in its as-is condition); ϕ_{ij} = service value of the network when only bridge$(i,j) \in B$ is selected for renewal/replacement; $\min\left\{\phi_{ij}\right\}$ = the minimum of all ϕ_{ij}. The priority index defined by Eq (9) reflects the net (or "absolute") impact of the renewal of bridge(i,j) on the network efficiency. SPI_{ij} will take on values between 0 to 1, with 1 being the top priority. We denoted this measure the *static* priority index because it can be viewed as an "absolute" importance measure of the bridge within the network.

In contrast, the dynamic priority index, DPI, is defined as a function of the likelihood of a bridge being selected for repair for a given overall maintenance budget when the uncertainties in the transportation network are considered. It can be calculated as:

$$DPI_{ij|\Theta} = \{\frac{\sum_{s=1}^{S} y_{ij|s}\phi_s}{\sum_{s=1}^{S} \phi_{ij|s}}|\Theta\} \tag{10}$$

where S is the total sample size used in the LHS for MC simulation; s denotes the sample ID. $y_{ij|s}$ are binary variables, where $y_{ij|s} = 1$ if bridge (i,j) is selected for renewal/replacement in the optimization with the sample set s and the fixed budget Θ and $y_{ij|s} = 0$ otherwise; ϕ_s = service value of the network resulting from the optimization with the sample set s and the fixed budget Θ. In contrast to the SPI, DPI depends not only on the characteristics of the network, but also on the available budget and on which other bridges are selected under this budget; in this sense, it is a "dynamic" bridge importance measure which ensures that the selected bridges collectively maximize the network performance for a given budget. When decision makers have a clearly determined budget limit to work with, the DPI is likely to result in bridge selections and prioritizations that will improve in the network performance beyond what SPI would provide. Comparison of these two ranking mechanisms will be further illustrated in Section 5 through an example.

5. NUMERICAL APPLICATION

A hypothetical bridge network is used in this section to illustrate the application of the proposed methodology and priority ranking measures.

5.1. Hypothetical Bridge Network

A hypothetical bridge network is generated, in which the nodes in the network represent origins, destinations, and transition nodes. Links between nodes are roads, with or without bridges. There are 600 roads in the network; 160 of them have one bridge on their road path. We divide the network into nine equal-area regions and randomly select one node from each region to represent the regional business hub with assumed mean traffic supply/demand as listed in Table 1.

A capacity rating factor less than 1.0 is assigned to 22% of the bridges randomly selected in the network; these bridges would be posted if not renewed or replaced. In addition, a condition rating less than or equal to 4 is assigned to another 3% of the bridges which, if no maintenance activities were performed, would be closed due to severe deterioration. Accordingly, 40 out of the total of 160 network bridges require repair if the budget for network maintenance is sufficient. The weight of vehicles in the highway network is modeled using a normal distribution (Nowak, 1999). The renewal cost for each individual bridge is also modeled with a normal distribution, with the mean assumed to be positively proportional to its capacity rating factor and its deck area (Fragkakis and Lambropoulos 2004). The statistics of the network variables used in the analysis are summarized in Table 2.

Table 1: Mean traffic demand/supply at O/D nodes.

Node (City)	Traffic supply (O)	Traffic demand (D)
3	2000	1400
11	1700	3000
19	1000	1500
28	600	900
30	4500	3600
35	800	400
37	1000	700
41	1200	1800
49	1600	1100
Total	**14400**	**14400**

Table 2: Statistics of the network variables.

Parameter	Symbol	Distribution	Mean	COV
Capacity rating factor	RF	Discrete [0.4, 0.5, 0.6, 0.7, 0.8,. 0.9]	0.65	0.20
Speed limit	v_{ij}	Uniform [50,60,70,80] mph	65	0.13
Traffic supply/demand	r_i	Normal	Table 3	0.06
Vehicle weight	w	Normal	65 kips	0.17
Renewal cost	c_{ij}	Normal	16.6 units*	0.15

*The mean cost assumed to be a function of the deck area and RF of the bridge. The unit is hypothetical for ranking purpose.

5.2. Project Selection and Prioritization

We apply BPSO to this hypothetical network, using LHS with 30 samples to account for the uncertainties in maintenance cost, vehicle weight and traffic supply/demand for each O/D pair. It was found that the mean service value for the "as-designed" network (all bridges are "new" without deterioration or weight posting), ϕ_p, and that for the "as-is" network (degraded network without maintenance), ϕ_0, are 123,968 and 297,095 hours, respectively. In other words, the network travel time in its current "as-is" condition, ϕ_0, is 2.39 ϕ_p. Assuming the total maintenance budget limit Θ is 150 units, the mean total travel time of the network after the optimized repair projects, ϕ, will improve to 1.63 ϕ_p. The COV of ϕ is 1%, which is quite small because in each sample the algorithm searches the optimal solution and ϕ is always the minimum travel time.

To further investigate the relation between the budget limit and the service metric, multiple budget values are tested with the same network. Figure 1 indicates that the normalized service value decreases as the available budget for maintenance increases. As the budget increases to a certain level, approximately 400 unit in this case, the service metric of the network became 1.0 ϕ_p, meaning that the network performance recovers to its "as-new" condition. At this point, the budget is no longer a constraint for network maintenance activities and all deficient bridges can be scheduled for renewal/replacement.

The number of selected bridges for repair will increase with the growth in the budget, but this increases is not simply due to adding more bridges to the selected group associated with a lower budget. As shown in Figure 2, the likelihood of some bridges being selected, such as bridge (31,42), increases as Θ increases. Other bridges that are critical to network performance are always selected, regardless of the budget limit, such as bridge (23,31). Finally, for bridges such as bridge (4,7), the likelihood of being selected decreases initially because a neighboring bridge on an alternative road path might become a more effective candidate for improving the network performance as budget limit increases; however, when the budget become essentially sufficient, they would be selected again. The DPI, as a function of the likelihood of being selected (as defined in Eq. (10)), can reflect such dynamics of the selection process encapsulated in the formulation of the optimization. As an example, Figure 3 shows the comparison between DPI and SPI for selected bridges for a fixed budget $\Theta = 150$. Obviously the DPI and SPI result in different project selections and priority rankings. While the maintenance strategy using the SPI resulted in a mean normalized travel time of 1.946, the strategy using the DPI led to a normalized mean travel time of 1.630, representing a 32% improvement over the SPI for Θ equal to 150.

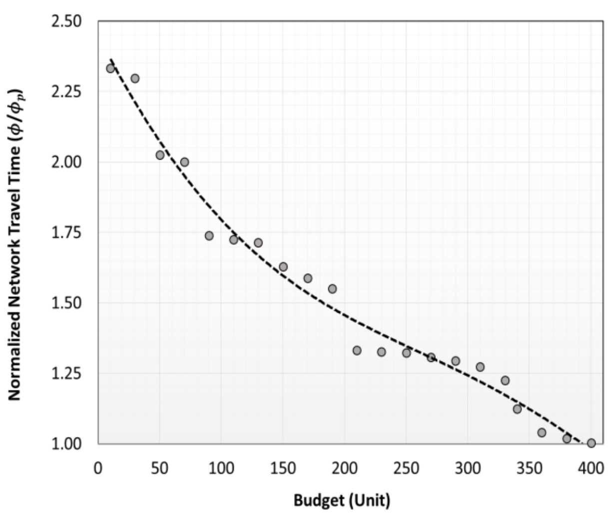

Figure 1: Normalized mean network travel time as a function of budget.

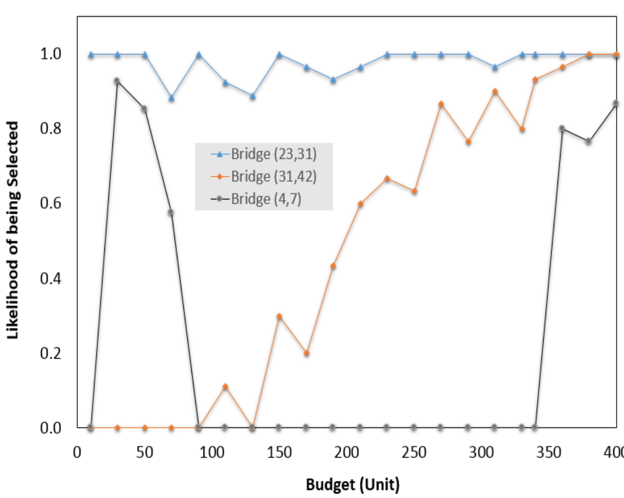

Figure 2: The likelihood of bridge being selected for renewal as budget increases.

6. CONCLUSIONS

This paper presented a framework for optimizing bridge maintenance decisions under budget constraints, which integrates uncertain traffic demand within the network, bridge condition ratings, bridge capacity ratings, and network characteristics (e.g. topology, vehicle speed limit, etc.). It was found that weight posting of deficient bridges collectively will impact the operational performance of the transportation network significantly and must be taken into account in making maintenance decisions to maximize the network performance; binary particle swarm optimization is efficient for solving mixed binary programming problems involved in optimizing maintenance schedules of large transportation networks; and the dynamic priority index (DPI) is a more effective ranking measure than the static priority index (SPI) when the available budget for network maintenance is fixed.

Static Ranking	Bridge ID	Dynamic Ranking	Bridge ID
1.000	(0,50)	1.000	(23,31)
0.962	(39,42)	0.934	(26,39)
0.952	(7,46)	0.933	(33,43)
0.950	(33,43)	0.932	(0,29)
0.895	(4,8)	0.867	(3,6)
0.865	(40,42)	0.866	(32,49)
0.854	(3,14)	0.865	(31,44)
0.850	(32,49)	0.833	(28,39)
0.828	(1,7)	0.832	(17,19)
0.795	(21,31)	0.799	(27,32)
0.788	(27,32)	0.767	(21,23)
0.743	(2,7)	0.534	(0,50)
0.710	(39,50)	0.435	(39,50)
0.665	(2,33)	0.434	(2,33)
Total Budget	150		150
ϕ/ϕ_p	1.946		1.635

Figure 3: Comparison of SPI and DPI (Θ = 150 units)

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