Essays on Business Cycles in Open Economies

by

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Abstract

This dissertation consists of three chapters about business cycles in open economies. The first chapter addresses the question of why housing investment is so volatile, especially in economies with developed mortgage markets. To this end, the chapter develops an augmented Real Business Cycle model with a housing collateral constraint. The collateral constraint creates a link between the housing market and borrowing capacity, a link that amplifies the response of housing demand to shocks and becomes stronger in economies with deeper mortgage markets. The second chapter examines an anomaly between international business cycle models and empirical evidence in cross-country employment correlation. It shows that the wealth effect on leisure plays a determining role in generating a negative employment comovement in the models, hence proposing a solution to the anomaly. The last chapter compares macroeconomic consequences of dollarized emerging countries under two alternative monetary policies: the inflation targeting rule and the fixed exchange rate regime. It shows that the floating exchange rate regime can be dominated by the fixed exchange rate regime in the role of cushioning shocks and in welfare terms.

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Dedication

For my parents

Chapter 1

Introduction

This dissertation consists of three chapters about business cycles in open economies. The first chapter addresses the question of why housing investment is so volatile, especially in economies with developed mortgage markets. The chapter begins by presenting two stylized facts for OECD countries that standard RBC models with perfect credit markets fail to explain: (i) housing investment is about five times as volatile as GDP, and (ii) housing investment is more volatile in economies with deeper mortgage markets. This chapter then develops an augmented RBC model in which these facts are reconciled with the existence of a housing collateral constraint. The collateral constraint creates a link between the housing market and borrowing capacity, a link that amplifies the response of housing demand to shocks and becomes stronger in economies with deeper mortgage markets. Meanwhile, since mortgage market innovations also offer the prospect of increased credit supply and eventually reduce the demand for collateralizable housing, this model predicts a *non-monotonic* impact of mortgage market depth on the volatility of housing investment. This chapter concludes by calibrating collateral constraint models to the U.K.

It is well known that several quantitative properties of international business cycle models are at odds with empirical data. First, the cross-country correlations are higher for consumption than for output, while in the data the opposite is true. Second, cross-country correlations of employment and investment are negative whereas in the data they are positive. In literature, while the ranking of the consumption correlation has been explained by models with incomplete financial markets, Baxter (1995) admits that: "It has proved particularly difficult to write down plausibly-parameterized models which can generate positive comovement of labor and investment across countries...Thus a major challenge to the theory is to develop a model which can explain international comovement in labor input and investment." The second chapter quantitatively shows that the wealth effect on leisure plays a determining role in generating the cross-country negative correlation of employment. As a result, a positive cross-country correlation in employment can be obtained by using preferences with zero income elasticity of leisure.

The last chapter compares macroeconomic consequences of alternative monetary policies to explore the idea that *fear of floating* can be justified as an optimal discretionary monetary policy in a dollarized emerging economy. Specifically, I consider a small open economy in which intermediate goods importers borrow in foreign currency and face a credit constraint. In this economy, exchange rate depreciation not only worsens importers' networth but also increases financing amounts in domestic currency, therefore exaggerating their borrowing finance premium. Besides, because of high exchange rate pass-through, fluctuations in the exchange rate also have strong impacts on domestic price levels. These effects, together, magnify macroeconomic consequences of the floating exchange rate policy in response to external shocks. The last chapter shows that the floating exchange rate regime can be dominated by the fixed exchange rate regime in the role of cushioning shocks and in welfare terms.

Chapter 2

Housing Investment in OECD Countries

2.1 Introduction

"Why is housing investment so volatile, especially in economies with developed mortgage markets?" The question is of interest because of several reasons. First, the current global financial meltdown has generated wide interest in the impact of recent mortgage market innovations on the housing sector and the overall business cycle, particularly the concern that these innovations may destabilize the housing market. Second, housing investment shocks account for a large share of variance in GDP in many economies and housing investment offers the best early warnings of an oncoming recession among GDP components. [1] Therefore, it is important to understand the dynamics of housing investment in order to control business cycles. Third, in the US, housing investment has been documented to be both pro-cyclical and highly volatile and while the pro-cyclicality has obtained satisfactory

[1] For example, housing demand shocks account for 20-25% of variance in GDP in the U.S and Japan (IMF, 2008) and in the past 60 years, eight out of ten recessions in the US were preceded by substantial problems in housing (Leamer, 2007).

explanations [2] the highly volatile behavior has not. [3]

Data from OECD countries first indicate that the highly volatile behavior of housing investment is not a distinguishing feature of the U.S economy. Across 17 OECD countries, housing investment is on average about five times as volatile as GDP and significantly more volatile than non-housing investment.[4] Housing investment also tends to be more volatile in economies with deeper mortgage markets like Australia, the U.K, and the U.S. In these economies, the standard deviation of housing investment is about two times as large as its non-housing counterpart and is about six to seven times as large as GDP. This positive association is interesting.[5] If housing is just a durable consumption good and consumers tend to smooth consumptions then the more developed mortgage market, which implies a broader access to credit markets, should allow households to smooth more efficiently against fluctuations. Nonetheless, consumption smoothing for housing is not supported by empirical evidence.

Standard business cycle models with perfect credit markets are at odds with these empirical findings. First, these models are unable to explain the positive correlation between housing investment volatility and mortgage market depth since the degree of mortgage market development should be

[2]The regularity that housing investment co-moves with other investments and is procyclical with GDP. The co-movement in multi-sector models is not as straightforward as it might appear, since there is a strong incentive to switch labor/production between sectors in response to sector-specific productivity shocks. See Charles Leung (2004) for further literature review and explanations.

[3]The existing literature has limited success in explaining the volatility. The exception is Davis and Heathcote (2005), which explains the high volatility from the supply side, but does not address the mortgage market. The paper will review this later.

[4]The volatility of these non-housing durable goods is already very high from the business cycle perspective; it is about four times that of GDP.

[5]The positive correlation is not limited by cross-country evidence but is also reflected by time series data. The volatility of housing investment relative to GDP has significantly risen along dramatic innovations in the mortgage market. The paper discusses more in the empirical part

immaterial under a perfect credit market assumption. Second, standard models are also at odds in reconciling the high volatility of housing investment. I shows in this chapter that a quantitative two-sector model with free borrowing fails generating a realistic volatility of housing investment.

To explain the aforementioned stylized facts, I develop an augmented Real Business Cycle (RBC) model with a housing collateral constraint. Specifically, I consider a limited obligation environment in which borrowers do not repay unless debts are secured by collateral and housing plays the collateral role for household debt.[6] The collateral constraint is inspired by the evidence that the major part of household borrowing has been in the form of collateralized debt. For example, the shares of mortgage debt in total outstanding household debt are about 80% in the US and 70% in Canada. There is also evidence of borrowers' limited obligations. For instance, when the subprime mortgage market worsened, many borrowers just walked away from their housing collateral without any further obligations. Housing collateral is rationalized by the fact that housing is a very good store of value and an important component of wealth for most households.[7]

The mechanism through which a housing collateral constraint affects the dynamics of housing investment goes as follows. The value of housing collateral is endogenously determined by the housing stock and prices, which in turn define households' borrowing capacity. As a result, the housing collateral constraint creates a link between the housing market and borrowing capacity, a link that amplifies the response of housing demand to shocks and explains the high volatility of housing investment. Intuitively, increased demand for housing in good times drives up both the housing stock and

[6] This includes, but is not limited, to mortgage debt.

[7] The value of housing structures excluding land is similar to the combined value of private non-housing structures and equipment, similar to annual GDP, and three times as large as the total stock of all other consumer durables. Moreover, the median value of a house is often much higher than the annual income of a typical household even in advanced countries, therefore, the owner usually has to access mortgage credit to purchase a house. In mortgage lending, housing naturally becomes collateral.

housing prices. These increases in turn raise the collateral value, enabling households to borrow more from capital gains to consume and further invest in housing, thereby creating a borrowing-consuming spiral. In other words, a boom in the housing market increases the collateral value, allowing households to borrow more to consume more. However, increased consumption including housing purchases in turn fuels the housing boom further, making housing investment highly volatile. Moreover, by anticipating the value of collateralizable housing in relaxing the borrowing constraint, credit constrained households rationally purchase a greater amount of housing in good times, which also accounts for the high volatility of housing investment.

The housing collateral constraint can also account for the positive correlation between housing investment volatility and the degree of mortgage market development. Up to a limit, mortgage market development enhances instability in the housing market. The underlying reason is that in economies with more flexible and developed mortgage markets, credit-constrained households can borrow a higher amount for the same value of collateral and easily withdraw equity from increased collateral for consumption.[8] As a result, more developed mortgage markets intensify the collateral role of housing, thereby encouraging credit constrained households to purchase more houses in good times. Besides, more developed markets also strengthen the link between the housing market and the consumption decisions, hence creating a stronger borrowing-consuming spiral.

But does the model imply that greater mortgage market depth leads to higher volatility in housing investment forever? The answer is "no" and the reason goes as follows. Mortgage market innovations offer the prospect of increased credit supply and a relaxation of the borrowing constraint, thereby creating the *credit effect*. In contrast to the collateral effect above, the credit effect reduces the incentive to invest in collateralizable housing for the pur-

[8]It is applied even under the case that households don't directly acquire debt or withdraw equity for consumption since by accessing more mortgage debt there would be more credit available for general consumption.

pose of relaxing the borrowing constraint. This paper shows that because the collateral effect and credit effect work against each other, mortgage market depth has a *non-monotonic* impact on the housing investment volatility. At low and medium levels of mortgage market development, the household's credit constraint is relatively severe, so the collateral effect prevails. Consequently, a marginal advance in mortgage market depth leads to relatively higher housing demand in good times, causing higher volatility in housing investment. By contrast, when the mortgage market is highly developed so that households have much broader access to credit and the credit constraint is much eased, the credit effect takes over the collateral effect and households substitute housing for other consumption goods. As a result, the volatility of housing investment declines.

This work is related to the business cycle literature that incorporates the housing sector. This literature documents regularities, distinguishes housing investment from its non-housing counterpart, and attempts to explain the co-movement between the two types of investment.[9] These authors, however, often have difficulty in accounting for the relatively high volatility of housing investment. For example, Baxter (1996) finds that consumption of durables that include housing investment is less volatile than business investment; Fisher (1997) is unable to generate household investment more volatile than business investment for all specifications. Davis and Heathcote (2005) explain the co-movement and the high volatility by building a model where housing and the other sectors all use three intermediate goods, albeit in different proportions. The high volatility mainly results from their calibration that the housing construction sector uses a relatively higher proportion of in-

[9] Greenwood and Hercowitz (1991) and Baxter (1996) assume reversibility between housing and business capital and also assume/calibrate the same or highly correlated productivity shocks between two sectors. Fisher (1997) assumes complementarity between the household and business capital in goods production. Chang (2000) argues that if there are adjustment costs in capital accumulation and substitutability between leisure time and durable goods in home production, then when households work more in periods of high productivity they also demand more durables.

termediate goods which are relatively more volatile. It is, however, unclear whether their estimate of the Solow residual of housing construction production is due to productivity shocks or the mixed equilibrium outcome of supply and demand in the housing sector. By contrast, this paper explains the high volatility from the demand side, particularly from the imperfect credit aspect of the housing sector.

The housing collateral constraint, the key ingredient of this paper, originates from the seminal work of Kiyotaki and Moore (KM) (1997) and Kocherlakota (2000). These authors show that collateral effects can be a powerful propagation mechanism by which relatively small, temporary shocks can generate large, persistent fluctuations in output and asset prices. Campbell and Hercowitz (CH) (2005) develop a one-sector real business cycle model to address the impact of credit market innovations on macroeconomic volatility. Their mechanism is through the labor supply: less tight collateral constraints weaken the connection between constrained households' housing investment and their hours worked. Iacoviello (2005) incorporates the New Keynesian monetary policy framework into the work of KM. Collateral effects enable his model to match the positive response of spending to a housing price shock. Calza et al. (2007) extend Iacoviello's work to allow production of new housing and endogenous asset price movement. They also model institutional features of the mortgage market and argue that the correlation between consumption and house prices increases with the degree of mortgage market development, and the transmission of monetary policy shocks to consumption and to housing prices is stronger in countries with more developed mortgage markets. More recently, Monacelli (2008) argues that introducing a collateral constraint into the New Keynesian framework can reconcile the co-movement of durable and non-durable spending in response to monetary shocks.

My work differs from these in many key aspects. Unlike the CH work, it develops a two-sector model and incorporates asset price movement to explore the amplification mechanism of collateral effects. In contrast with the others, which are New Keynesian models with nominal sticky prices and nominal debt, this paper is based on an RBC model with flexible prices and real debt to study the impact of the productivity shock. Moreover, the existing literature considers a closed economy model with heterogeneous agents where patient savers lend to impatient borrowers; this paper considers an open economy model in which domestic agents can access international credit markets, which captures the increasingly global credit market.[10] The paper also incorporates capital to better characterize the dynamics of the current account. Particularly, it is shown in the quantitative section that collateral effects improve the performance of the model in terms of generating the counter-cyclicality of the current account compared to models in the existing open economy literature such as Backus et al. (1992) and Mendoza (1991). Finally, the small open economy model allows the paper to have a representative agent, which makes the model simple.[11]

This chapter is organized as follows. Section 2 describes data, particularly two mortgage market depth indicators, and documents stylized facts about housing investment and its association with mortgage market depth. Section 3 explains the empirical findings using a basic model with a borrowing constrained representative household. Section 4 extends the basic model to include heterogeneous households, discusses the model's dynamics, and calibrates it for the U.K. Section 5 concludes.

[10]This is also rationalized by the fact that this paper studies 17 OECD countries, most of which can be regarded as small open economies in the global economy. Even for the U.S economy, thanks to recent dramatic financial deregulation, the major part of mortgage debts has been held by international investors.

[11]In an extended model, I also consider an economy with heterogeneous households.

2.2 Stylized Facts

This section documents major stylized facts about housing with emphasis on housing investment and the mortgage market in 17 advanced OECD countries from Q1-1980 to Q3-2007.[12]

2.2.1 Data

All time series data are quarterly, except Germany's annual and Italy's half-year house prices. House prices are mainly provided by the Bank of International Settlements, and other missing values are filled and updated via Datastream. Real house prices are then obtained by deflating nominal house prices with the consumer price index (CPI).

Housing investment or residential investment, non-housing investment, total investment, and GDP are in real values, i.e., in constant or chained prices, and obtained via Datastream and OECD Stat.[13]

I utilize two specific indicators to measure the degree of mortgage market development in these OECD countries. The first one is a synthetic mortgage market index constructed by the IMF.[14] The second measure is the ratio of total outstanding amount of mortgage debt over GDP, the mortgage-debt-to-GDP ratio or the mortgage depth, which is often used in literature.[15]

[12]The choice of 17 OECD countries is mainly based on the availability of data. They are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, New Zealand, Norway, Sweden, the U.K, and the U.S. I choose post 1980s period since most of innovations in mortgage markets of these countries began in the early 1980s.

[13]For more details about the code of each specific variable, see the Data Appendix.

[14]They are taken from Table 3.1 of Chapter 3 of IMF World Economic Outlook (WEO) April 2008: "The changing housing cycle and the implications for monetary policy".

[15]For example: Warnock and Warnock (2008) use this ratio or maximum possible of this ratio to measure mortgage market depth or market size. Some other series of OECD working papers also use this particular measure. The data for mortgage-debt-to-GDP ratio (2001-2006 average) for all countries except New Zealand is taken from the IMF. Data for New Zealand are taken from Warnock (2008).

In particular, although most of advanced OECD countries have moved toward more competitive and developed housing finance markets thanks to recent deregulation and innovations in the mortgage market, there are still significant cross-country differences in the level of mortgage market development in terms of market liberalization, legal procedures, and regulatory structures. The cross-country differences in mortgage market development are reflected through: (1) The typical ratio of a mortgage loan to property's value or loan-to-value (LTV) ratio and the standard length of mortgage loans; (2) The ability to make home equity withdrawals and to prepay mortgages without a fee; (3) Developments of secondary markets for mortgage loans. These differences then imply different households' access to housing-related financing in each country. To summarize cross-country differences in mortgage market development, a synthetic mortgage market development index is constructed.[16] The index lies between 0 and 1, with higher values indicating easier household access to mortgage credit. The IMF's mortgage market index (henceforth MMI) and the mortgage-debt-to-GDP ratio or the mortgage depth (henceforth MD) are closely positively correlated, i.e., the economies with a higher mortgage market index often have a bigger or deeper mortgage market size (Figure 2.3). Figures 2.1 and 2.2 show evidence that there are significant differences in the degree of mortgage market development and mortgage size, even among advanced OECD countries.

Since the IMF's index is a one-period time indicator, which may be able to capture precisely only the current degree of mortgage market development, I extend data for the second indicator, the mortgage-debt-to-GDP ratio, to the last 10 years in order to examine the development of the mortgage market over time.[17] Figure 2.4 suggests that the degree of mortgage market depth has been increasing for most of these countries but the rank

[16] For more detail about the construction method, see Chapter 3 of WEO 2008

[17] Sources: European Mortgage Federation, IMF, FRB release, Reserve Bank of New Zealand, OECD, and Keen (2007). Although some countries like the U.S, U.K and Australia have data before 1997, I could not find longer data for some European countries.

Figure 2.1: Mortgage Market Index

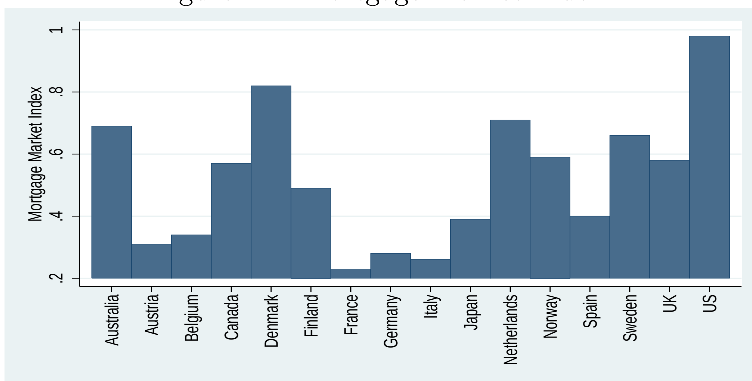


Figure 2.2: Mortgage Depth

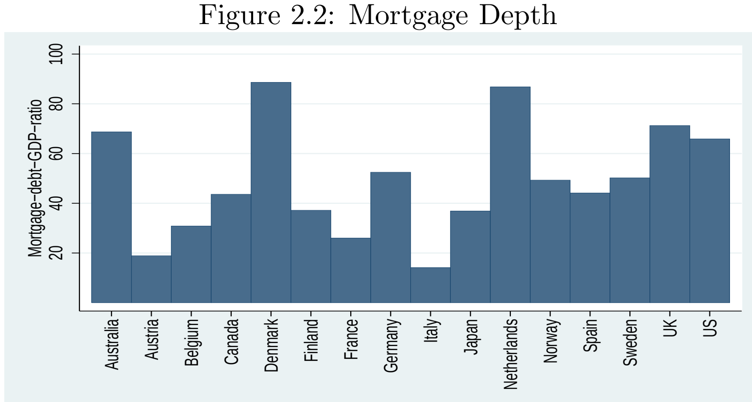
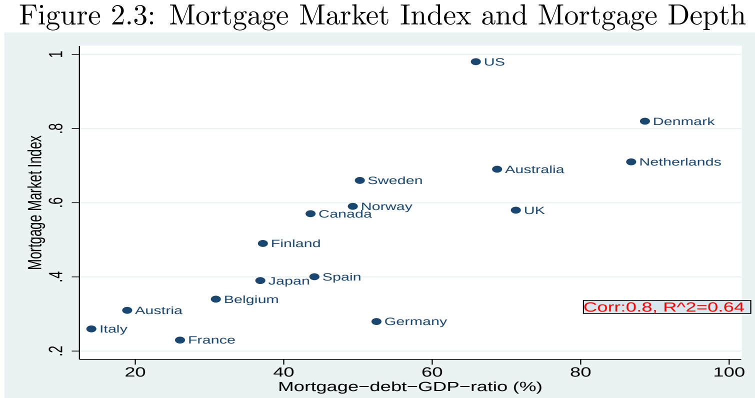


Figure 2.3: Mortgage Market Index and Mortgage Depth



remains the same, i.e., those countries that currently have deeper mortgage markets also possessed deeper ones in the 1990s. Therefore, I conclude that the IMF's index reflects the comparative degree of mortgage market development, at least from the 1990s.

2.2.2 Stylized Facts

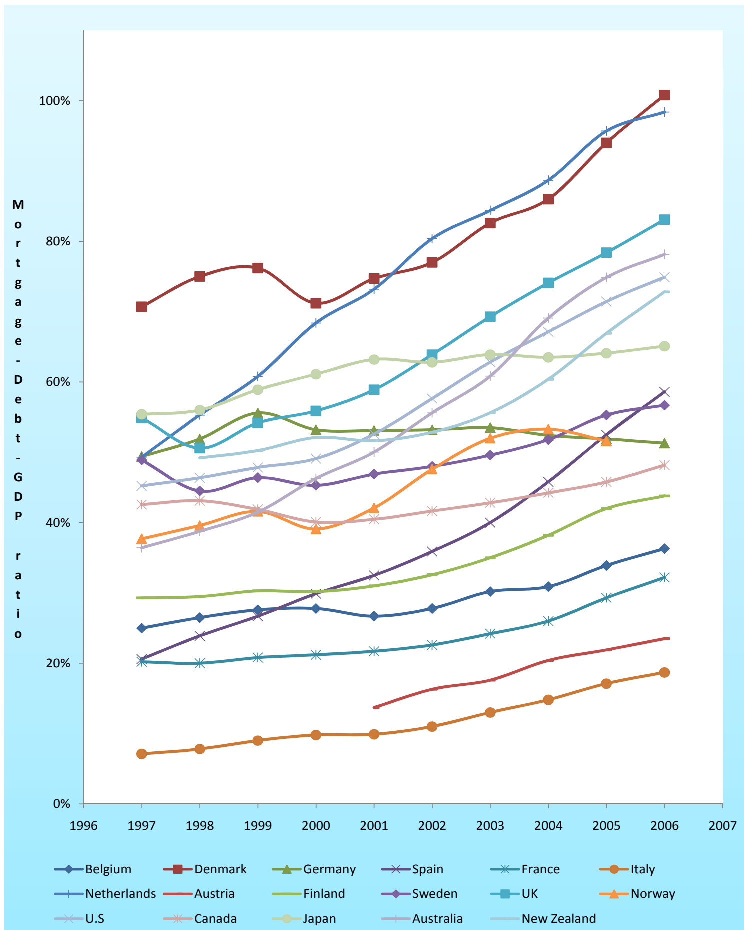
The first stylized fact about housing in OECD countries is that its real prices are significantly pro-cyclical with real GDP, which is contrary to the counter-cyclicality of non-housing investment's real prices[18] (The 2nd Column of Table 2.1).

The 3rd and 4th Column of Table 2.1 present evidence that housing investment co-moves with non-housing investment and is pro-cyclical with GDP. The co-movement property is prevalent in these advanced OECD countries and has an important implication for theoretical models that this paper will address later. Note that the pro-cyclicality of both real housing prices and housing investment makes it challenging for those models that try to explain the high volatility of housing investment from supply side, particularly the housing sector specific productivity shocks.

Compared to its non-housing counterpart, housing investment is also different in terms of volatility and cross-country dispersion. According to Table 1 (Column 10 and 11), the standard deviation of housing investment relative to GDP is not only significantly higher than that of non-housing investment but also varies widely across countries. The former ratio ranges from 2.56 in Italy to 6.67 in the U.S, whereas the latter ratio is stable at 3.8. The F-test for variances of the two groups is rejected with significant level (p-value is 4%) and the t-test for equality of the two ratios is strongly rejected (p-value=0.2%). I obtain the same conclusions when comparing

[18] As documented by Greenwood et al. (1997) and Fisher (2006), the real non-housing investment price measured by the business equipment deflator divided by consumption deflator is significantly counter-cyclical with GDP: The unconditional correlation for the U.S economy is -0.54

Figure 2.4: Mortgage Depth Development



Sources: DataStream, European Mortgage Federation, Federal Reserve Release (2008), Federal Reserve of New Zealand (2008, 2004), IMF, Keen (2007), OECD Outlook the housing investment with aggregate investment: housing investment is, on average, much more volatile and varies widely across countries than aggregate investment.

With regard to the mortgage market, Figures 2.5 and 2.6 first show significant positive correlations between the volatility of GDP and the two mortgage market indicators. More interestingly, Figures 2.7 and 2.8 present evidence that the volatility of housing investment relative to GDP is higher in economies with more developed mortgage markets, i.e., economies with higher mortgage market indices and larger mortgage market size, while there is no significant correlation between the volatility of non-housing investment and degree of mortgage market development (Figure 2.9 and 2.10). In other words, these figures show that while GDP tends to be more volatile in economies with deeper mortgage markets, housing investment is still more volatile. Therefore the volatility of housing investment to GDP significantly increases in these countries.

Finally, I explore housing investment from a historical perspective. Since most deregulation and innovation in the housing finance system in advanced OECD countries just began in the early 1980s, and it is evident that the current system has been much developed compared to that in the early stage of deregulation and innovation, I divide samples into 2 periods: prior and post Q1-1995.[19] Table 2.2, first, presents evidence of the so-called *Great Moderation* in the last decade. Particularly, the volatility of output has dropped dramatically over time across advanced OECD countries: post 1995, the average standard deviation of GDP is about two times as low as that prior to 1995. However, the volatility of housing investment has not fallen by that much so that the volatility relative to GDP has risen significantly. [20] In short, housing investment has become relatively more volatile along with

[19] I use 10 out of 17 countries that have relatively long enough observations before Q1-1995.

[20] The volatility of housing still varies widely among countries. The t-test for the equality of the two ratios of relative volatility is rejected with 10%.

Table 2.1: **Statistic I**

	Correlation with GDP				Std.dev relative to GDP		
Country	HP	RES	NRES	INV	RES	NRES	INV
Australia	0.41	0.65	0.66	0.79	6.50	5.47	4.87
Austria	0.23	-0.12	0.81	0.85	4.00	3.13	2.00
Belgium	0.38	0.58	0.40	0.58	3.89	2.89	2.78
Canada	0.52	0.48	0.66	0.73	4.09	3.57	3.07
Denmark	0.53	0.39	0.58	0.65	5.08	4.50	3.83
Finland	0.74	0.62	0.70	0.78	3.11	3.37	2.84
France	0.49	0.71	0.86	0.88	2.89	3.22	3.00
Germany	0.23	0.55	0.75	0.76	3.33	3.78	3.22
Italy	0.26	0.20	0.76	0.96	2.56	4.11	3.22
Japan	0.64	0.63	0.80	0.87	4.73	2.18	2.18
Netherlands	0.68	0.57	0.63	0.75	4.00	4.00	3.30
New Zealand	0.30	0.72	0.78	0.84	5.86	4.43	4.29
Norway	0.49	0.23	-0.08	-0.01	4.85	5.08	4.31
Spain	0.33	0.11	0.51	0.54	4.50	4.33	3.33
Sweden	0.77	-0.44	0.78	0.60	6.15	2.85	2.23
U.K	0.58	0.57	0.48	0.72	6.44	3.60	3.16
U.S	0.40	0.64	0.78	0.91	6.67	3.57	3.40
Average	0.47	0.42	0.64	0.72	4.63	3.77	3.24

Notes: HP is real house prices, RES is real housing investment, NRES is real non-housing investment, INV is real aggregate investment, GDP is real GDP. Correlations are correlation with GDP. RES/GDP, NRES/GDP, and INV/GDP denote the relative volatility of RES, NRES, and INV to that of real GDP, respectively. All series are in logs and Hodrick-Prescott filtered

Figure 2.5: GDP Volatility and MMI

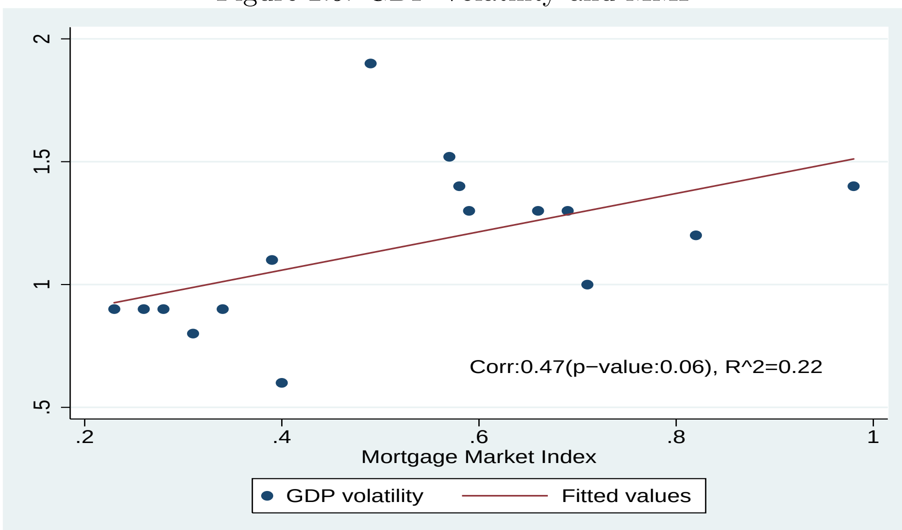


Figure 2.6: GDP Volatility and MD

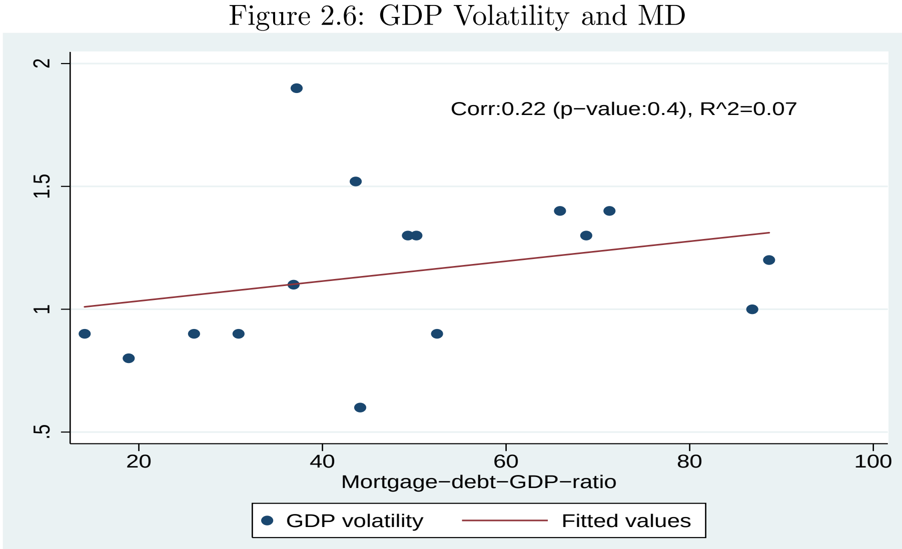


Figure 2.7: Housing Investment Volatility and MMI

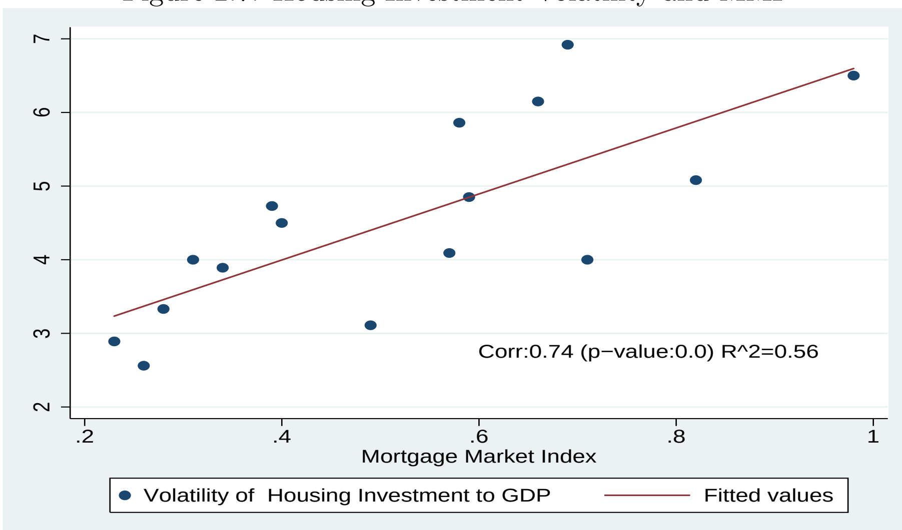


Figure 2.8: Housing Investment Volatility and MD

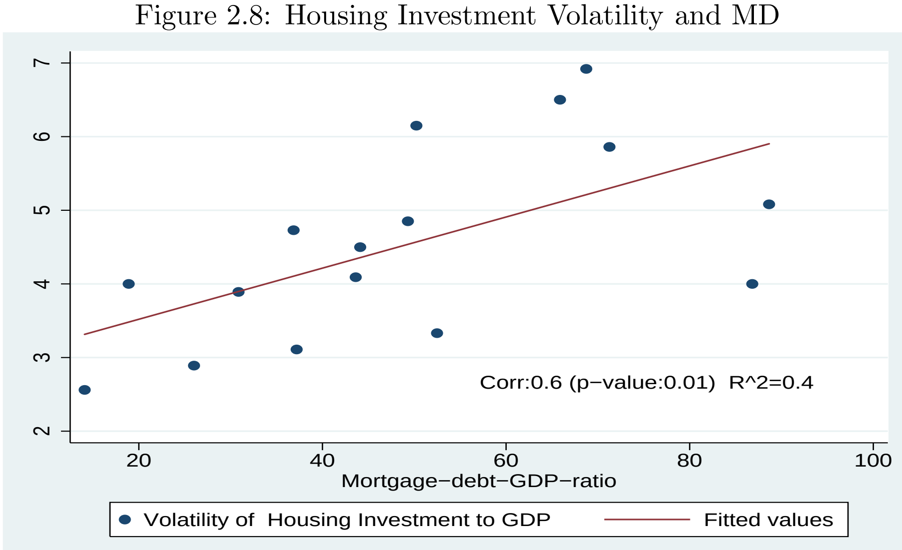


Figure 2.9: Non-Housing Investment Volatility and MMI

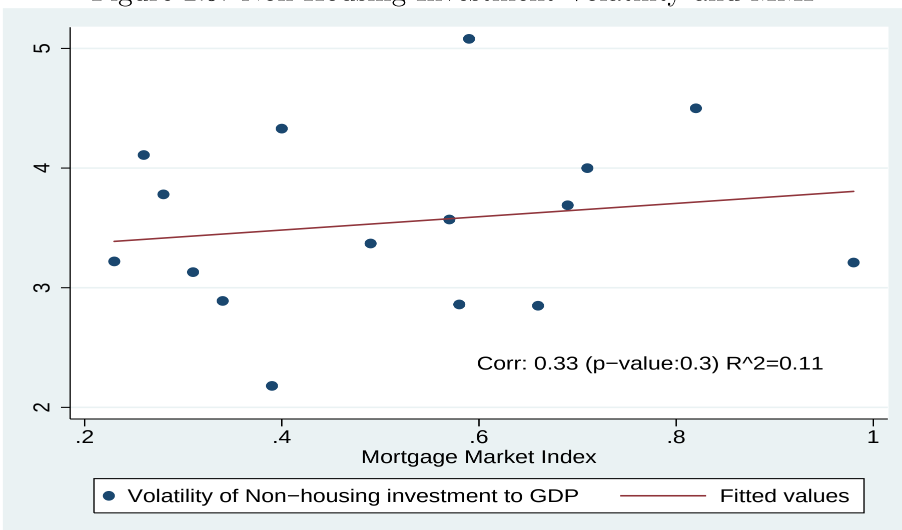


Figure 2.10: Non-Housing Investment Volatility and MD

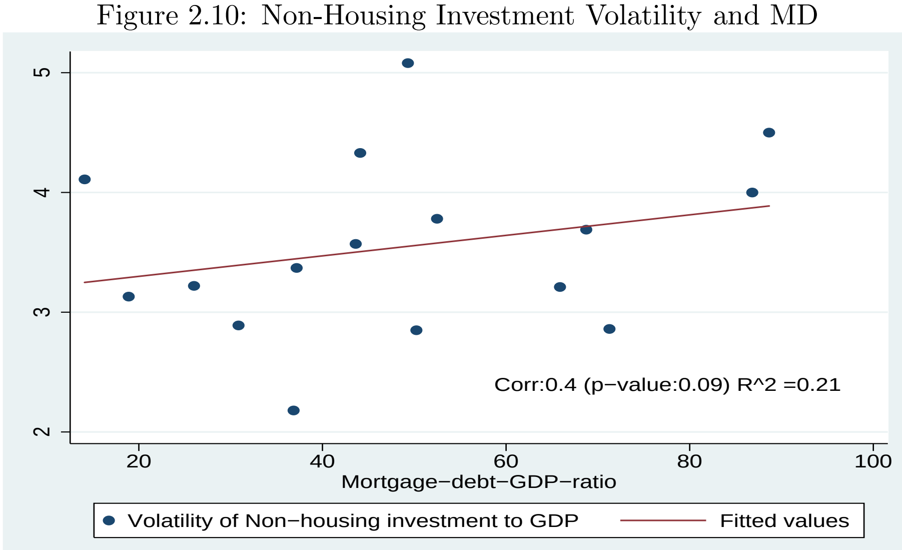


Table 2.2: **Statistic II**

	Prior 95			Post 95		
Country	RES	GDP	RES/GDP	RES	GDP	RES/GDP
Australia	9.12	1.78	5.12	8.59	0.61	14.08
Austria	3.99	0.78	5.12	2.51	0.80	3.14
Canada	7.63	1.94	3.93	4.19	0.87	4.82
Finland	5.70	2.50	2.28	6.00	1.05	5.71
France	3.15	0.97	3.25	1.99	0.74	2.69
Italy	2.39	1.04	2.30	2.16	0.76	2.84
New Zealand	8.07	1.77	4.56	8.40	1.05	8.00
Norway	7.60	1.53	4.97	5.00	0.98	5.10
U.K	9.67	1.51	6.40	3.90	0.38	10.26
U.S	10.42	1.53	6.81	5.01	0.87	5.76
Average	6.77	1.54	4.47	4.78	0.81	6.24

Notes: RES is real housing investment, GDP is real GDP. RES/GDP denotes the relative volatility of RES to that of real GDP. All series are in logs and Hodrick-Prescott filtered dramatic innovations in the mortgage market in these OECD countries.

2.3 Basic Model

To explain the aforementioned stylized facts, I construct a two-sector RBC model in which a representative household faces a borrowing constraint and housing plays the collateral role. A two-sector model is necessary to analyze housing which is a durable and non-tradable good.

2.3.1 Household

The representative household maximizes its expected lifetime utility defined over random sequences of non-durable consumption goods (c_t), housing services from the housing stock (h_t), and labor disutility (l_t):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t, h_t, l_t\big) \tag{2.3.1}$$

The budget constraint of the representative household is given by:

$$\begin{aligned} c_t + q_t[h_t - (1-\delta_h)h_{t-1}] + \frac{\phi_h}{2}\frac{(h_t - h_{t-1})^2}{h_{t-1}} + i_t^c + i_t^h + (1+r_{t-1})d_{t-1} \\ \leq w_t l_t + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + d_t \end{aligned} \tag{2.3.2}$$

$$i_t^c = k_t^c - (1-\delta_k)k_{t-1}^c + \frac{\phi_k}{2}\frac{(k_t^c - k_{t-1}^c)^2}{k_{t-1}^c} \tag{2.3.3}$$

$$i_t^h = k_t^h - (1-\delta_k)k_{t-1}^h + \frac{\phi_k}{2}\frac{(k_t^h - k_{t-1}^h)^2}{k_{t-1}^h} \tag{2.3.4}$$

Each period, the household can borrow internationally traded debt,[21] d_t, subject to a constraint described later, at an exogenous real interest rate, r_t. It supplies labor, l_t, at the real wage rate, w_t, and lends sector specific capital, k_{t-1}^c, k_{t-1}^h, to capital markets at prices r_t^c, r_t^h, where k_{t-1}^c, k_{t-1}^h are

[21] This includes, but is not limited to, mortgage debt.

capital for non-durable and durable production, respectively. The household then spreads its income on non-durable consumption goods, c_t, debt repayment, $(1+r_{t-1})d_{t-1}$, investments on two types of non-housing capitals i_t^c, i_t^h, housing investment, $q_t(h_t - (1-\delta_h)h_{t-1})$, and its adjustment costs, $\frac{\phi_h}{2}\frac{(h_t-h_{t-1})^2}{h_{t-1}}$, where q_t is real housing prices and δ_h is the depreciation rate of housing stock.

In addition to the budget constraint, the representative household faces the following collateral borrowing constraint:

$$(1+r_t)d_t \leq \phi E_t(q_{t+1}h_t) \tag{2.3.5}$$

which means that at any time the amount the household can borrow, $(1+r_t)d_t$, is limited by the expected future value of his property. As in Kiyotaki and Moore (1997) and Kocherlakota (2000), this borrowing constraint is rationalized by the borrower's limited obligations. If the household repudiates its debt obligations, the lenders can foreclose the property after paying the transaction costs, $(1-\phi)E_t(q_{t+1}h_t)$. The parameter ϕ, which presents the fraction of collateral value a household can use for borrowing, reflects market liberalization, legal procedures, and regulatory structures or institutional features prevailing in the mortgage market, therefore indicating the degree of the mortgage market flexibility and development. A higher ϕ corresponds to a higher mortgage market index and indicates a more developed and flexible mortgage market in the model.

In this paper, I specialize preferences as below:

$$U(c_t, h_t, l_t) = \frac{(x_t - \kappa\frac{l_t^\omega}{\omega})^{1-\sigma} - 1}{1-\sigma} \tag{2.3.6}$$

$$x_t = [(1-\gamma)^{\frac{1}{\eta}} c_t^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} h_t^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{2.3.7}$$

This is the GHH preference function introduced by Greenwood, Hercowitz and Huffman (1988) and is widely used in small open economy literature.[22] x_t is the composite consumption, the CES function of nondurable

[22]GHH preferences have the property that the marginal rate of substitution between

consumption, c_t, and housing services from the housing stock h_t. $\gamma > 0$ is the share of housing services in the composite consumption index. $\eta \geq 0$ is the elasticity of substitution between non-durables and housing services. σ denotes the inverse elasticity of intertemporal substitution, ω determines the elasticity of labor supply, and κ determines the amount of leisure in the steady state.

Let's denote the multiplier on the borrowing constraint at time t by λ_t then the first order conditions for the representative household read:

$$U_{ct}[1 + \phi_k(\frac{k_t^c - k_{t-1}^c}{k_{t-1}^c})] = \beta E_t\{U_{ct+1}[1 - \delta_k + r_{t+1}^c + \frac{\phi_k}{2}((\frac{k_{t+1}^c}{k_t^c})^2 - 1)]\}$$

$$U_{ct}[1 + \phi_k(\frac{k_t^h - k_{t-1}^h}{k_{t-1}^h})] = \beta E_t\{U_{ct+1}[1 - \delta_k + r_{t+1}^h + \frac{\phi_k}{2}((\frac{k_{t+1}^h}{k_t^h})^2 - 1)]\}$$

$$w_t = -\frac{U_{lt}}{U_{ct}}$$

$$U_{ct} - \lambda_t = \beta E_t\{U_{ct+1}(1 + r_t)\} \tag{2.3.8}$$

$$\begin{aligned} U_{ct}(q_t + \phi_h \frac{h_t - h_{t-1}}{h_{t-1}}) =& U_{ht} + \phi \lambda_t E_t\{q_{t+1}\} \\ &+ \beta E_t\{U_{ct+1}[q_{t+1}(1 - \delta_h) + \frac{\phi_h}{2}((\frac{h_{t+1}}{h_t})^2 - 1)]\} \end{aligned} \tag{2.3.9}$$

The first two equations are standard optimality conditions for capital with adjustment costs while the third one is a standard labor supply equation. The last two equations present distinguishing features of the borrowing constraint model. Equation (2.3.8) is a modified Euler equation and is reduced to a standard Euler equation in case of a non-binding constraint, i.e., $\lambda_t = 0$. When the constraint binds, the shadow value of borrowing is positive, $\lambda_t > 0$, so there is an intertemporal distortion in non-durable goods consumption between two different times. In other words, when $\lambda_t > 0$, this

consumption and leisure is independent of the consumption level within the period or there is no wealth effect on labor supply. GHH preferences provide a better description of consumption and the trade balance for small open economies than alternative specifications (see, for instance, Correia, Neves, and Rebelo (1995)).

equation implies that $U_{ct} > \beta E_t\{U_{ct+1}(1+r_t)\}$, which means the marginal utility of current non-durable consumption is higher than the marginal gain of shifting one unit of non-durables to the next period. A higher λ_t implies a tighter constraint, hence encouraging the household to purchase more collateralizable housing to relax the borrowing constraint, enabling it to increase current consumption.

Equation (2.3.9) is the efficiency condition for the intratemporal choice of durable housing that requires the household to equate the marginal utility of non-durable consumption, weighted by the relative housing prices and adjustment costs, to the marginal utility of housing services. The marginal utility of housing service consists of three components: (i) the direct utility gain of an additional unit of housing; (ii) the marginal gain from relaxing the collateral constraint; (iii) the expected utility derived from expanding future consumption by means of re-selling the amount of housing invested in the previous period. When the constraint doesn't bind, $\lambda_t = 0$, the distortion component $\phi\lambda_t E_t\{q_{t+1}\}$ vanishes, hence the marginal benefit of housing consists of only terms (i) and (iii), which is the standard intratemporal optimality condition.

For the sake of exposition at the moment, let's assume away adjustment costs. After integrating (2.3.9) forward, I obtain the following demand function for housing:

$$q_t U_{ct} = E_t\Big\{\sum_{j=0}^{\infty}[(1-\delta_h)\beta]^j U_{ht+j}\Big\} + E_t\Big\{\sum_{j=0}^{\infty}[(1-\delta_h)\beta]^j \phi\lambda_{t+j} q_{t+1+j}\Big\} \tag{2.3.10}$$

The first term in the RHS of (2.3.10) is the discounted stream of utility from housing services.[23] The second term is the current and expected benefits from the opportunity to increase consumption by the additional borrowing enabled by increased collateral value. This term depends on the degree of mortgage market development represented by parameter ϕ, the ex-

[23] This term is set to current marginal utility of housing U_{ht} when $\delta_h = 1$.

pected prices of housing, and the tightness of credit constraint λ_{t+j}. When the constraint doesn't bind, $\lambda_t = 0$ for all t, this term is equated to zero, hence, the weighted marginal utility of non-durable consumption in the LHS equates to the discounted stream of utility from housing services.

To explore further, I follow Manacelli (2008) to express the equation as a condition where the marginal rate of substitution between housing and non-durable goods consumption $\frac{U_{ht}}{U_{ct}}$ is equal to the user cost (Z_t) of housing, which in this case can be expressed as:

$$Z_t \equiv q_t - \frac{\phi\lambda_t}{U_{ct}} E_t\{q_{t+1}\} - (1-\delta_h)\beta E_t\{\frac{U_{ct+1}}{U_{ct}} q_{t+1}\} \tag{2.3.11}$$

When the constraint binds, $\lambda_t > 0$, the user cost of housing is determined not only by current and expected real housing prices but also by ϕ and the movement of the shadow price of borrowing, λ_t. This is one of the distinguishing features of the model. For example, suppose that λ_t rises, that is the constraint becomes tighter, then the household has more incentives to purchase more collateralizable housing to relax the borrowing constraint and increase non-durable goods consumption. However, when λ_t rises U_{ct} also tends to increase, thereby raising the (opportunity) cost of acquiring an additional unit of durable housing. Moreover, increased housing demand also often drives up housing prices, q_t, hence raising the user cost as well.

2.3.2 Firms

At time t, representative firms in the tradable non-durable sector rent previously installed capital, k^c_{t-1}, and labor, l^c_t, from the household to produce goods with the production function:

$$y_t = A_t (k^c_{t-1})^{\alpha_c} (l^c_t)^{1-\alpha_c} \tag{2.3.12}$$

Output from the tradable non-durable sector can be used as non-durable consumption c_t or investments in either type of capital goods k^c_t, k^h_t or can be exported with tb_t. Firms in the construction sector combine capital, k^h_{t-1}, with labor, l_t^h, to construct buildings (structures) for non-tradable durable housing with the following technology:

$$b_t = A_t (k_{t-1}^h)^{\alpha_h} (l_t^h)^{1-\alpha_h} \tag{2.3.13}$$

A_t is an aggregate exogenous stochastic productivity shock with law of motion:[24]

$$log(A_{t+1}) = \rho_A log(A_t) + \epsilon_{t+1} \tag{2.3.14}$$

Optimality conditions for tradable goods firms imply:

$$w_t = (1-\alpha_c)\frac{y_t}{l_t^c} = (1-\alpha_c)A_t\Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c} \tag{2.3.15}$$

$$r_t^c = \alpha_c \frac{y_t}{k_{t-1}^c} = \alpha_c A_t\Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c - 1} \tag{2.3.16}$$

Optimality conditions for the construction sector imply:

$$w_t = q_t(1-\alpha_h)\frac{b_t}{l_t^h} = q_t(1-\alpha_h)A_t\Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h} \tag{2.3.17}$$

$$r_t^h = q_t \alpha_h \frac{b_t}{k_{t-1}^h} = q_t \alpha_h A_t\Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h - 1} \tag{2.3.18}$$

2.3.3 Equilibrium

Given the interest rate, r_t, a competitive equilibrium in this economy is characterized by a sequence of allocations $\{c_t, l_t, h_t, d_t, k_t^c, k_t^h, i_t^c, i_t^h, y_t, l_t^c, l_t^h,\}$ and a sequence of prices $\{q_t, w_t, r_t^c, r_t^h, \lambda_t\}$ that satisfy the household and firms optimality conditions, the budget constraint, the binding borrowing constraint, production functions, and the following market clearing conditions.

[24] I assume that an exogenous productivity shock has the same effect on both production sectors and will consider an asymmetric case later. Notice that the symmetric productivity shock implies a perfect correlated productivity shock between two sectors, as in Greenwood and Hercowitz (1991), I do not assume reversibility between housing and business capital and housing is produced separately.

Labor market clearing:

$$l_t = l_t^c + l_t^h \tag{2.3.19}$$

Non-tradable durable housing market clearing:

$$b_t = h_t - (1 - \delta_h)h_{t-1} \tag{2.3.20}$$

Tradable non-durable goods market:

$$c_t + i_t^c + i_t^h + \frac{\phi_h}{2}\frac{(h_t - h_{t-1})^2}{h_{t-1}} + (1 + r_{t-1})d_{t-1} = y_t + d_t \tag{2.3.21}$$

The trade balance, housing investment, and aggregate output can be expressed as:

$$tb_t = y_t - c_t - i_t^c - i_t^h \tag{2.3.22}$$

$$resi_t = q_t b_t \tag{2.3.23}$$

$$Y_t = y_t + q_t b_t \tag{2.3.24}$$

2.3.4 Benchmark: Free Borrowing Economy

For comparison, I also consider a benchmark: a small open economy model augmented by the presence of the non-tradable durable housing sector with free borrowing. In this economy, the borrowing constraint does not bind so the multiplier $\lambda_t = 0\ \forall t$. Therefore, two optimal conditions for non-durables and housing can be written as:[25]

$$U_{ct}(1 - \phi_d(d_t - \bar{d})) = \beta E_t\{U_{ct+1}(1 + r_t)\} \tag{2.3.25}$$

$$q_t U_{ct} = U_{ht} + (1 - \delta_h)\beta E_t\{q_{t+1}U_{ct+1}\} \tag{2.3.26}$$

Hence, the demand function for durable housing becomes:

$$q_t U_{ct} = E_t\Big\{\sum_{j=0}^{\infty}[(1 - \delta_h)\beta]^j U_{ht+j}\Big\} \tag{2.3.27}$$

[25] All other conditions remain the same as before. The introduction of asset adjustment cost is to induce stationary dynamics in a small open frictionless economy but it does not affect the quantitative results of the model since ϕ_d is very small. For more details, see Schmitt-Grohe and Uribe (2003)

The RHS of equation (2.3.27) is the shadow value of durable housing. According to Barsky et al. (2007), there are two reasons that keep this value roughly constant against moderate-lived shocks. First, durable housing with low depreciation rates has high stock-flow ratios,[26] which implies that even relatively large changes in the production of the housing over a moderate horizon have small effects on the total housing stock, therefore, causing only minor changes in the service flows. Second, if δ_h is sufficiently low, the shadow value will be mainly affected by the marginal utilities of service flows in the distant future. Since the effects of the shock are temporary, the future terms in this equation remain close to their steady-state values. Thus, even if there were significant changes in the first few terms of the expansion, they would have a small percentage effect on the present value as a whole. The two observations together suggest that under the benchmark, demand for durable housing displays an almost infinite elasticity of intertemporal substitution: even a small rise in housing prices today relative to tomorrow would cause people to delay their housing purchases.

2.3.5 Calibration

The model period is a quarter. *Preference*: Following Schmitt-Grohe and Uribe (2003), the inverse of elasticity of substitution in consumption σ and the elasticity of labor supply ω are set to 2 and 1.6, respectively, which are in range of literature. The elasticity of substitution between non-durable goods and housing service, η, is set to unity, implying the Cobb Douglas form of the composite consumption. [27] The parameter γ is set so that the ratio of

[26] In this model, the steady-state stock-flow ratio is $1/\delta_h$

[27] There is no consensus about this elasticity of substitution yet. Piazzesi et al. (2007) argue that if η is sufficiently less than unity then the equity premium puzzle is resolved while Davis and Martin (2005) show that the value should be no less than 1.25 in order to be consistent with U.S housing stock and price data. This paper takes a neutral stance to set the value to unity. I got similar qualitative results when the value is assigned in the neighborhood of unity

private residential investment over GDP is equal to 3.5%, the average level for the U.K private residential investment over recent 20 years. Discount factor β is chosen as 0.985, which is a bit lower than the value implied by the foreign real interest rate $\frac{1}{1+0.01} = 0.9901$ to assure the binding credit constraint at steady state. The parameter κ is selected so that a fraction $\frac{40}{24\times7}$ of household's one unit time endowment is used for working in the labor market.

Technology: The share of capital in the production of non-durables and housing construction, α_c, α_h, are both set to 0.3. These parameters together with depreciation rates will determine the investment rate, which is 20% of GDP. The depreciation rate of non-housing capital is chosen at 12% per year or $\delta_k = 0.03$ whereas δ_h is set to 0.003, which implies the depreciation rate of housing is 1.2% annually.[28] The parameter of capital adjustment costs ϕ_k is chosen such that volatility of non-housing investment matches the data and that of housing investment ϕ_h is set equal to ϕ_k.

Steady state value of the real interest rate is set at 4% per year or r=0.01. The persistence coefficient ρ_A in the motion equation of the productivity shock $log(A_{t+1}) = \rho_A log(A_t) + \epsilon_{t+1}$ is set to 0.9 and the variance of the innovation is selected to match the volatility of output.

For U.K: I set borrowing constraint parameter ϕ to 0.4 compared to 0.6 of the U.S economy. The reason for assigning 0.6 to the U.S economy is as follows. First, for the first-time homebuyers, the down-payment rate is typically less than 20%, which means these households can borrow more than 80% of the housing collateral value. [29] For existing homeowners, Mian and Sufi (2009) show that these households on average extract 30 cents for every dollar increase in home equity. I take an average of these numbers, which implies a value of 0.6 for the US. Then I scale down the IMF mortgage market development index so that 0.98 is scaled to 0.6 and obtain the number 0.4 for the U.K accordingly. At the same time, the standard deviation of technology

[28] Monacelli sets it to 1% while Davis and Heathcote and others use 1.56% per year

[29] In the U.S, the Loan-to-Value ratio can reach 100% during the recent housing boom.

innovation and capital adjustment cost parameter ϕ_k are calibrated to 0.002 and 0.33, respectively in order to match the standard deviation of output and non-housing investment in the U.K over the past 30 years, 1.15% and 4.10% respectively.

2.3.6 Housing Investment Dynamics

I first fix ϕ to explore the borrowing constraint model's impulse responses to a positive aggregate productivity shock. I focus on the dynamics of housing investment and compare the credit constraint model and the benchmark. In both cases, a favored productivity shock reduces production costs, encouraging firms to hire more labor to extend production in both durable and non-durable sectors, [30] thereby raising wage rates and capital returns. Also because of the positive productivity shock, the housing supply curve shifts down to the right.

From the demand side, aggregate consumption increases due to the income effect. Since housing is normal goods, its demand also increases, leading to an upward shift in the demand curve, which applies for both free borrowing and credit constraint cases. The differences, however, lie on the interaction between the income effect, the substitution effect, and potentially the collateral effect of each demand's structure.

Figure 2.11 presents impulse responses of the free borrowing model. In a free borrowing environment, housing is just a durable consumption good so the elasticity of intertemporal substitution is almost infinite for long-lived housing.[31] As a result, increased demand from the income effect is partly offset by the high intertemporal substitution effect resulting from rising house prices. Consequently, the housing investment volatility is relatively low and unable to exceed that of non-housing investment under all reasonable parameter calibration.[32] Nonetheless, it is shown that perfect/high correlated

[30] The amount of capital was already determined from the previous period

[31] This also implies a very flat housing demand curve at any given time.

[32] Recall in the empirical section shows that housing investment is on average signifi-

productivity shocks combined with an exogenous interest rate (small open economy framework) can produce the correct co-movement of housing investment in this type of two sector model, consistent with Greenwood and Hercowitz (1991), and Baxter (1996). Intuitively, the income effect together with an increased housing supply originating from a positive productivity shock can offset the high substitution effect of the housing demand but at the same time, they are not strong enough to generate the high volatility of housing investment documented from empirics.

By contrast, Figure 2.12 presents impulse responses of the collateral constraint model in which housing plays the collateral role. There are notably two main differences from the demand side of housing. First, unlike households that smooth consumption over time in the free borrowing benchmark, borrowing-constrained households are impatient, hence tend to locate consumption toward the current period. The impatience, therefore, produces a higher demand for both durable and non-durable goods compared to the benchmark. Second, since housing plays an additional role as a collateralizable asset and rising collateral value will enable credit constrained households to expand consumption through further borrowing, households will have more incentives to invest in housing. In other words, besides the direct utility gain from housing services, households will also benefit from relaxing the borrowing constraint by an additional housing purchase. This is shown in the RHS of the demand equation (2.3.10): In addition to the standard discounted stream of utility from housing services, there is the second term presenting the current and expected benefits from the opportunity to expand consumption thanks to rising collateral value. The collateral effect implies a steeper demand curve and will shift the curve upward to a greater extent, therefore, leading to a greater response for housing investment on impact of productivity shock compared to the free borrowing benchmark. More importantly, the collateral constraint creates a borrowing-consuming spiral:

cantly more volatile than non-housing investment in the majority of OECD countries.

Figure 2.11: IRs to Productivity Shocks: Benchmark

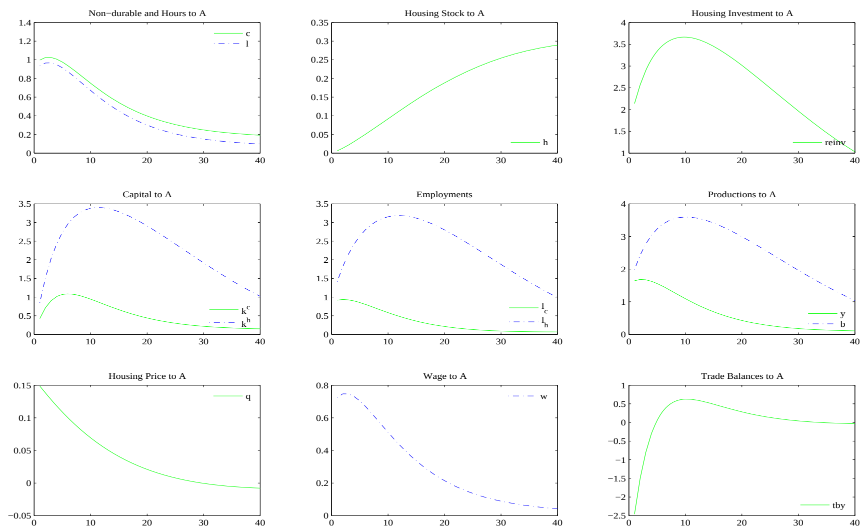
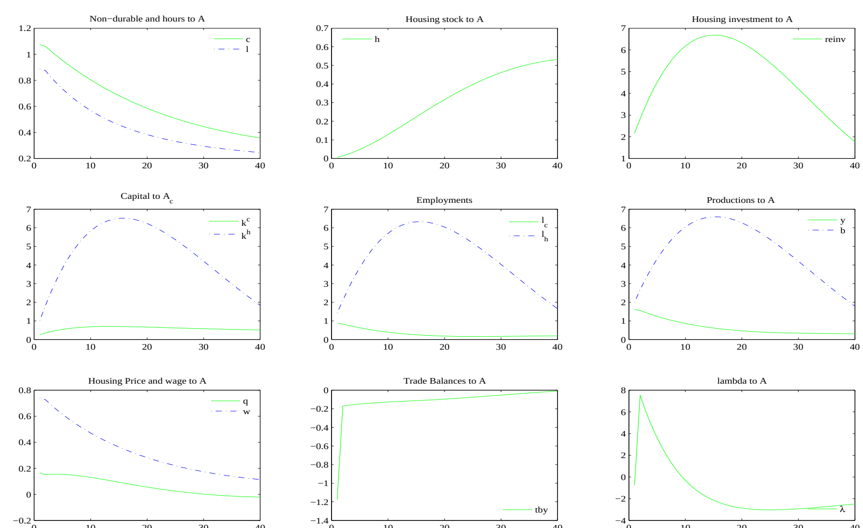
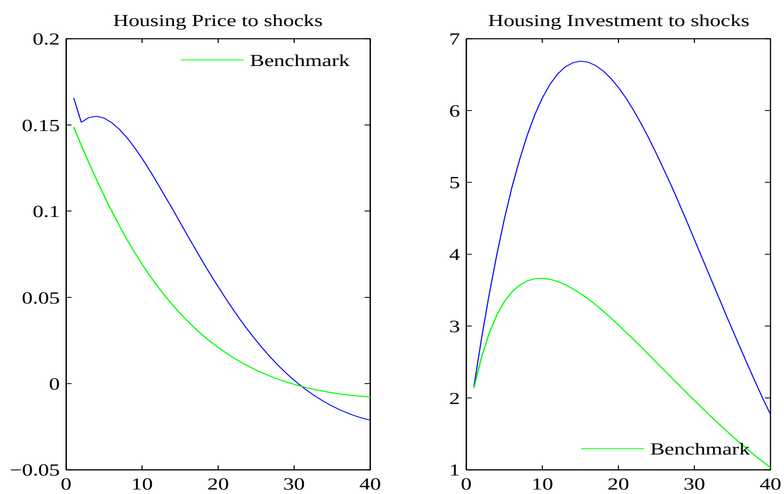


Figure 2.12: IRs to Productivity Shocks: Borrowing Constraint



initial increases in housing prices and stock raise the collateral value, enabling credit-constrained households to borrow more for consumption, which in turn will reinforce rising housing prices and housing stock. As shown in Figure 2.12, a higher demand on impact of the productivity shock and the amplification from the collateral effect can account for highly volatile housing investment. In other words, the income effect, increased housing supply, and the decisive collateral effect combined can dominate the substitution effect, hence reconciling the realistic volatility of housing investment. (Figure 2.13)

Figure 2.13: IRs of Housing Investment and Prices



2.3.7 Comparative Analysis of Mortgage Market Development

Second, I impose different values of parameter ϕ to study the impact of mortgage market development on housing investment in the borrowing constrained economy.

In deeper mortgage markets, i.e., higher ϕ, households can borrow a higher amount of debt for the same value of collateral and withdraw more equity from increased collateral value for consumption and investment.[33] This is the *collateral effect* from mortgage market development. The collateral effect increases the volatility of housing investment because a higher ϕ intensifies the collateral role of housing, thereby encouraging credit constrained households to purchase more houses in good times. A higher ϕ also strengthens the link between the housing market and consumption decisions, therefore creating a stronger borrowing-consuming spiral.

On the other hand, mortgage market innovations, by raising ϕ, also offer the prospect of increased credit supply and a relaxation of borrowing constraints, therefore creating the *credit effect* in the borrowing constrained economy. In contrast to the collateral effect, the credit effect lowers the volatility of housing investment because higher ϕ, by providing more credits to the economy, reduces the incentive to invest in collateralizable housing for the purpose of relaxing the collateral constraint. In other words, the credit effect relatively increases the user cost of housing, hence inducing the household to substitute housing with non-durable consumption. The two effects are partly reflected through the second term in the housing demand equation (2.3.10): A higher value of ϕ directly increases the value of this term but at the same time eases the tightness of the borrowing constraint, thereby endogenously decreasing the current and future shadow value of borrowing λ_{t+j}. Moreover, a higher ϕ also leads to changes in the housing demand, hence affecting future expected housing prices q_{t+1+j}, which then in turn have impacts on the second term of RHS of equation (2.3.10) as well. Therefore, the aggregate effect of a higher ϕ on housing demand is ambiguous. It turns out that at low and medium levels of mortgage market development, when the household's credit constraint is relatively tight, the collateral effect prevails. Consequently, an improvement in the mortgage market development leads to a relatively larger increase in housing

[33]It is applied even under the case where households don't directly acquire debt or withdraw equity for consumption, since by accessing more mortgage debt there would be more credit available for consumption.

demand, causing a higher housing investment volatility. By contrast, when the mortgage market is highly developed, households are much less credit constrained, so the credit effect takes over from the collateral effect, and the household starts to substitute collateralizable housing by non-durable consumption; therefore, housing investment volatility tends to decline. Figure 2.14 presents an inverse U-shape in the relative volatility of housing investment with respect to the degree of mortgage market development.

2.3.8 Asymmetric Productivity Shock

I have assumed that the aggregate productivity shock A_t has a symmetric impact on both durable and non-durable sector production as in (2.3.12) and (2.3.13). In this section, I consider an asymmetric case where the aggregate productivity shock does not have any impact on the durable housing production. By assuming the asymmetric shock, I attempt to exclude the effect of increased housing supply from a positive productivity shock and therefore be able to focus on the demand side, particularly the collateral effect. The production function of non-durable goods remains the same as in (2.3.12) but that of durable housing has the form:

$$b_t = (k_{t-1}^h)^{\alpha_h}(l_t^h)^{1-\alpha_h} \tag{2.3.28}$$

When a favored aggregate productivity shock hits the economy, since there is complete productivity spillover to nondurable production, firms in this sector will take advantage of favored productivity to hire more labor and extend production, therefore raising wage rates and the capital returns. However, since labor is freely mobile, rising wage rates will hurt the housing construction sector that does not benefit from the increased productivity. Consequently, housing production costs will increase and the housing supply curve will shift upward. This is the difference in the supply side compared to the symmetric productivity case.[34]

[34]Recall that the housing supply curve shifts downward in this case the complete spillover to housing production in (2.3.13).

Table 2.3: **Statistics: Basic Model**

	Data	Standard Model	Basic Model
Standard deviation			
output	1.15	1.14	1.12
consumption	1.18	0.83	0.88
nres	4.1	4.15	2.3
tb/y	0.54	0.73	0.16
res	7.57	3.51	6.55
sd(res)/sd(y)	6.58	3.1	5.85
hp	5.13	0.075	0.11
Correlation w/ ouput			
consumption	0.73	0.94	0.98
nres	0.48	0.55	0.85
tb/y	-0.31	-0.07	-0.63
hp	0.58	0.93	0.79
res	0.57	0.85	0.7

Notes: Data is obtained from time series for the U.K from Q1-1981 to Q3-2007. Standard model is free borrowing model. Std() is standard deviation. nres: non-housing investment, tb/y: trade-balance output ratio, res: housing investment, hp: real housing prices. All numbers are in percentage, which is the standard deviations from trend and is obtained from Hodrick-Prescott filter.

Figure 2.14: Housing Investment Volatility and ϕ

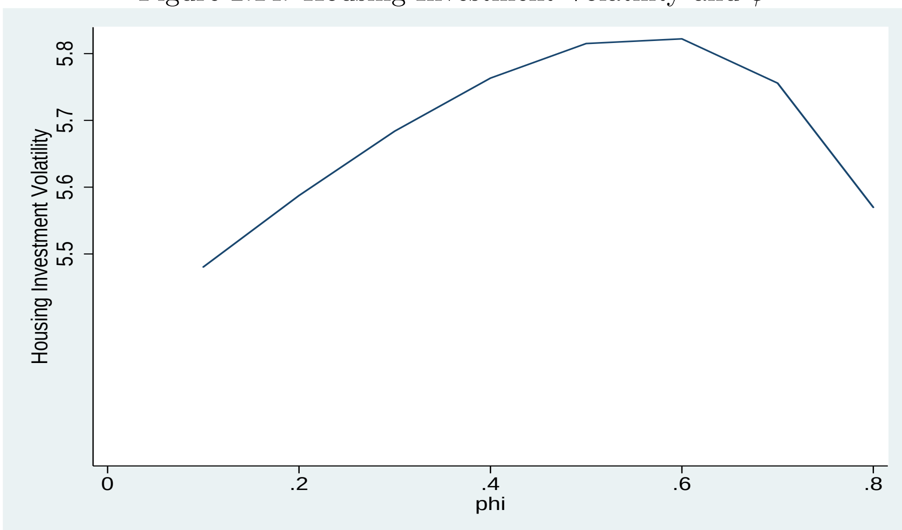


Figure 2.15: Non-Housing Investment Volatility and ϕ

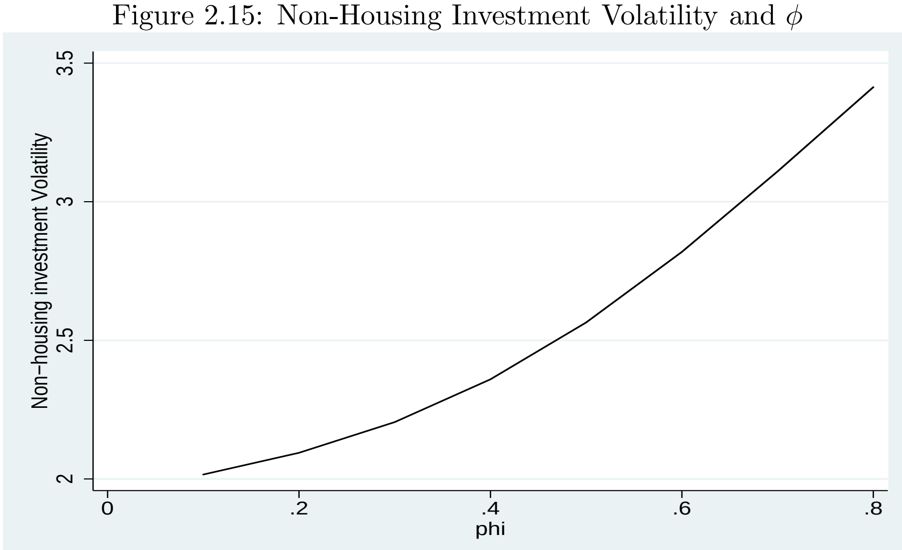


Figure 2.16: IRs: Model without Borrowing Constraint: Asymmetric Shocks

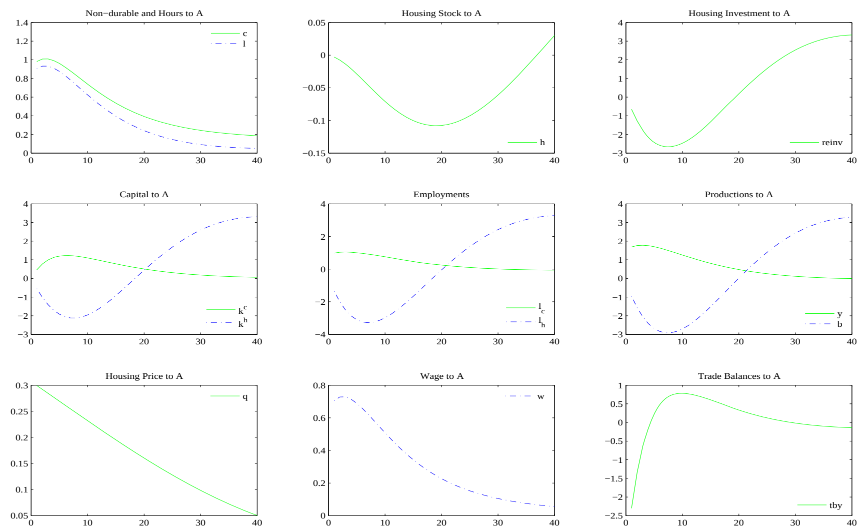
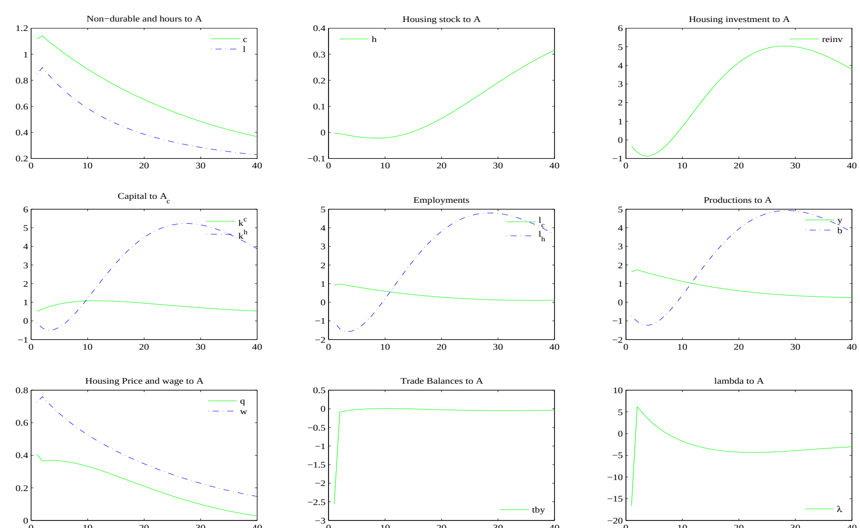


Figure 2.17: IRs: Model with Borrowing Constraint: Asymmetric Shocks



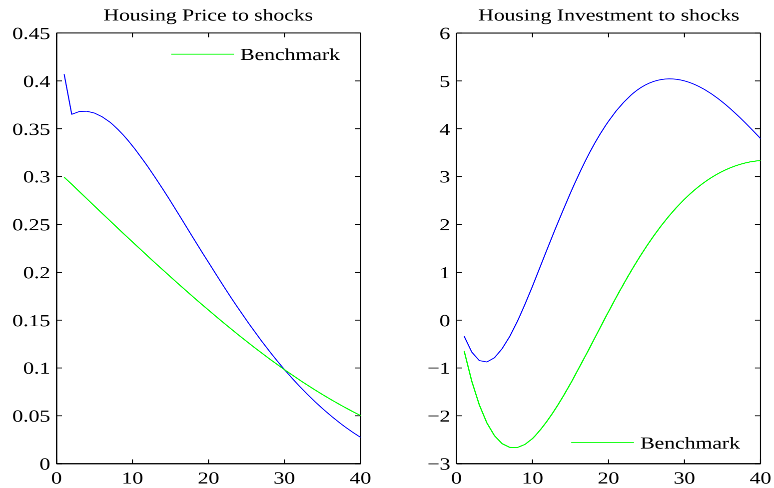
With a negative impact from the supply side, we now witness a significant difference between the free borrowing benchmark and the borrowing constraint model with the collateral effect. In the benchmark, because of the high elasticity of intertemporal substitution, households substitute housing for relatively cheap non-durable goods. As a result, housing flow/investment falls on impact of the productivity shock and keeps falling for a while before gradually increasing. Intuitively, the positive income effect on housing demand helps to mitigate the negative substitution effect at the beginning but it weakens rapidly against the latter, causing a deeper fall in housing flow/investment. However, a fall in housing investment amid rising output and non-housing investment, i.e., the so-called co-movement problem, is at odds with empirical facts. Hence, it has been shown that under an asymmetric productivity spillover to housing production sector, a standard small open economy model is unable to correct the co-movement problem. (Figure 2.16)

By contrast, Figure 2.17 presents the dynamics of the economy with the collateral constraint. It is shown that the collateral effect can help to produce a correct co-movement even without highly correlated productivity shocks. Anticipating the collateral role of housing and speculating on rising property prices, households rationally increase investing in housing in good times despite its high elasticity of intertemporal substitution. Initial increases in turn fuel the spiral, making housing investment pro-cyclical. In other words, the collateral effect together with the income effect can offset the intertemporal substitution effect, hence producing pro-cyclical housing investment as documented in the empirical work. (Figure 2.18)

2.4 Extended Model

The basic representative household model is simple but sufficient to explain the impact of the collateral effect on housing investment dynamics. This model, however, has a weakness. It implies that the volatility of non-

Figure 2.18: IRs: Housing Investment and Prices



housing/business investment increases in economies with a more developed mortgage market, at odds with empirical evidence. (Figure 2.15) The underlying reason is that in the representative agent model, business capital is also owned by those who face the borrowing constraint, hence, it is affected by their borrowing capacity. In particular, because of the credit constraint, the rate of capital return is always kept higher than the borrowing interest rate, which induces credit-constrained agents increasingly to invest in business capital when the credit constraint is relaxed and the access to credit becomes broader.

In reality, business capital is often owned by corporations or *capitalists* who have much more freedom to access financial markets than a typical credit constrained household.[35] Therefore, to separate business investment decisions from credit-constrained households, I consider an extended model

[35] For example, using data from the 1998 Survey of Consumer Finance, Diaz and Luengo-Prado document that in the U.S, households in the top 20% of the wealth distribution hold 98.9% of all financial assets while housing wealth represents 96.3% of total wealth for those in the bottom 80% of the wealth distribution.

in which there are two types of households, named capitalist and borrower with measure ϵ and $1-\epsilon$, respectively. The former groups own business capital and have free access to both domestic and international financial markets. In contrast, the latter groups don't own business capital and face the collateral borrowing constraint as in the basic model. A necessary condition for this type of heterogeneous household model is that capitalists are more patient than borrowers and at equilibrium borrowers will borrow from capitalists.[36]

2.4.1 Capitalist

The representative capitalist maximizes his expected life-time utility defined over random sequences of non-durable consumption goods (c_{1t}), housing services from housing stock (h_{1t}), and labor dis-utility (l_{1t}):

$$U = E_0 \sum_{t=0}^{\infty} \beta_1^t U\big(c_{1t}, h_{1t}, l_{1t}\big) \tag{2.4.29}$$

The budget constraint of the capitalist is given by:

$$\begin{aligned} &c_{1t} + q_t[h_{1t} - (1-\delta_h)h_{1t-1}] + \frac{\phi_h}{2}\frac{(h_{1t} - h_{1t-1})^2}{h_{1t-1}} + i_t^c + i_t^h + (1+r_{t-1})d_{ft-1} \\ &+ (1+r_{t-1}^b)d_{1t-1} + \frac{\phi_d}{2}(d_{ft} - \bar{d})^2 \leq w_t l_{1t} + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + d_{1t} + d_{ft} \end{aligned} \tag{2.4.30}$$

$$i_t^c = k_t^c - (1-\delta_k)k_{t-1}^c + \frac{\phi_k}{2}\frac{(k_t^c - k_{t-1}^c)^2}{k_{t-1}^c}$$
$$i_t^h = k_t^h - (1-\delta_k)k_{t-1}^h + \frac{\phi_k}{2}\frac{(k_t^h - k_{t-1}^h)^2}{k_{t-1}^h}$$

Each period, the capitalist can either pay adjustment cost, $\frac{\phi_d}{2}(d_{ft} - \bar{d})^2$,[37] to borrow internationally traded foreign debt at an interest rate, r_t,

[36] When $\epsilon = 1$, this extended model is reduced to a standard representative model under free borrowing. However, when $\epsilon = 0$, this model is not the same as the basic model.

[37] The introduction of adjustment costs in a small open economy framework is to induce stationarity.

which is exogenous, or access the domestic bond market, d_{1t}, at an interest rate, r_t^d. He supplies labor l_{1t} at the real wage rate, w_t, and lends capital, k_{t-1}^c, k_{t-1}^h to capital markets at prices r_t^c, r_t^h. The capitalist then spreads his income on non-durable tradable consumption goods, c_{1t}, debt payment $(1 + r_{t-1})d_{ft-1}, (1 + r_{t-1}^d)d_{1t-1}$, investments of two types of non-housing capital i_t^c, i_t^h, housing investment, $q_t[h_{1t} - (1 - \delta_h)h_{1t-1}]$, and its adjustment costs, $\frac{\phi_h}{2}\frac{(h_{1t}-h_{1t-1})^2}{h_{1t-1}}$.

The first order conditions for the capitalist, which are standard, read:

$$U_{1ct}[1 + \phi_k(\frac{k_t^c - k_{t-1}^c}{k_{t-1}^c})] = \beta_1 E_t\{U_{1ct+1}[1 - \delta_k + r_{t+1}^c + \frac{\phi_k}{2}((\frac{k_{t+1}^c}{k_t^c})^2 - 1)]\}$$

$$U_{1ct}[1 + \phi_k(\frac{k_t^h - k_{t-1}^h}{k_{t-1}^h})] = \beta_1 E_t\{U_{1ct+1}[1 - \delta_k + r_{t+1}^h + \frac{\phi_k}{2}((\frac{k_{t+1}^h}{k_t^h})^2 - 1)]\}$$

$$w_t = -\frac{U_{1lt}}{U_{1ct}}$$

$$U_{1ct}(1 - \phi_d(d_{ft} - \bar{d})) = \beta_1 E_t\{U_{1ct+1}(1 + r_t)\} \tag{2.4.31}$$

$$U_{1ct} = \beta_1 E_t\{U_{1ct+1}(1 + r_t^d)\} \tag{2.4.32}$$

$$U_{1ct}(q_t + \phi_h\frac{h_{1t} - h_{1t-1}}{h_{1t-1}}) = U_{1ht} + \beta E_t\{U_{1ct+1}[q_{t+1}(1 - \delta_h) + \frac{\phi_h}{2}((\frac{h_{1t+1}}{h_{1t}})^2 - 1)]\} \tag{2.4.33}$$

2.4.2 Borrower

The representative borrower maximizes his expected life-time utility defined over random sequences of non-durable consumption goods(c_{2t}), housing services from housing stock (h_{2t}), and labor dis-utility (l_{2t}):

$$U = E_0 \sum_{t=0}^{\infty} \beta_2^t U\big(c_{2t}, h_{2t}, l_{2t}\big) \tag{2.4.34}$$

The budget constraint of the borrower is given by:

$$\begin{aligned} c_{2t} + q_t(h_{2t} - (1 - \delta_h)h_{2t-1}) + \frac{\phi_h}{2}\frac{(h_{2t} - h_{2t-1})^2}{h_{2t-1}} \\ + (1 + r_{t-1}^b)d_{2t-1} \leq w_t l_{2t} + d_{2t} \end{aligned} \tag{2.4.35}$$

I assume that the borrower is more impatient than the capitalist or $\beta_2 < \beta_1$. The borrower does not hold capital.[38] Each period, he supplies labor l_{2t} at the real wage rate, w_t, borrows from the domestic bond market, d_{2t}, at the interest rate, r_t^d, but is subject to a borrowing constraint mentioned below. The borrower then spreads his income on non-durable tradable consumption goods, c_{2t}, debt payment $(1+r_{t-1}^d)d_{2t-1}$, and housing investment, $q_t[h_{2t} - (1-\delta_h)h_{2t-1}]$. I also assume that the borrower is not able to access to the international foreign debt.

The borrower is also subject to the following collateral borrowing constraint:

$$(1+r_t^d)d_{2t} \leq \phi E_t(q_{t+1}h_{2t}) \tag{2.4.36}$$

Let's denote the multiplier on the borrowing constraint by λ_t, the first-order conditions for the borrower read:

$$w_t = -\frac{U_{2lt}}{U_{2ct}} \tag{2.4.37}$$

$$U_{2ct} - \lambda_t = \beta_2 E_t\{U_{2ct+1}(1+r_t^d)\} \tag{2.4.38}$$

$$\begin{aligned} U_{2ct}(q_t + \phi_h \frac{h_{2t}-h_{2t-1}}{h_{2t-1}}) &= U_{2ht} + \lambda_t \phi E_t\{q_{t+1}\} \\ &+ \beta E_t\{U_{2ct+1}[q_{t+1}(1-\delta_h) + \frac{\phi_h}{2}((\frac{h_{2t+1}}{h_{2t}})^2 - 1)]\} \end{aligned} \tag{2.4.39}$$

2.4.3 Firms

Representative firms in the non-durable sector produce goods with the following technology:

$$y_t = A_t(k_{t-1}^c)^{\alpha_c}(l_t^c)^{1-\alpha_c} \tag{2.4.40}$$

Structures of the non-tradable durable housing are produced with following technology:

$$b_t = A_t(k_{t-1}^h)^{\alpha_h}(l_t^h)^{1-\alpha_h} \tag{2.4.41}$$

[38] It can be shown that because of being relatively impatient, the borrower will not hold capital.

Optimality conditions of non-durable goods firms imply:

$$w_t = (1-\alpha_c)\frac{y_t}{l_t^c} = (1-\alpha_c)A_t\Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c} \tag{2.4.42}$$

$$r_t^c = \alpha_c\frac{y_t}{k_{t-1}^c} = \alpha_c A_t\Big(\frac{k_{t-1}^c}{l_t^c}\Big)^{\alpha_c-1} \tag{2.4.43}$$

Optimality conditions of construction firms imply:

$$w_t = q_t(1-\alpha_h)\frac{b_t}{l_t^h} = q_t(1-\alpha_h)A_t\Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h} \tag{2.4.44}$$

$$r_t^h = q_t\alpha_h\frac{b_t}{k_{t-1}^h} = q_t\alpha_h A_t\Big(\frac{k_{t-1}^h}{l_t^h}\Big)^{\alpha_h-1} \tag{2.4.45}$$

2.4.4 Equilibrium

Given the interest rate, r_t, a competitive equilibrium in this small open economy is characterized by a sequence of allocations $\{c_{1t}, c_{2t}, l_{1t}, l_{2t}, h_{1t}, h_{2t}d_{1t}, d_{2t}, d_{ft}, k_t^c, k_t^h, i_t^c, i_t^h, y_t, l_t^c, l_t^h, l_{1t}, l_{2t}\}$, and a sequence of prices $\{q_t, w_t, r_t^c, r_t^h, r_t^d, \lambda_t\}$ that satisfy the household and firms optimality conditions, the borrower's budget constraint, the binding borrowing constraint, production functions, and following market clearing conditions.

Labor market clearing:

$$\epsilon l_{1t} + (1-\epsilon)l_{2t} = l_t^c + l_t^h \tag{2.4.46}$$

Non-tradable durable housing market clearing:

$$b_t = \epsilon(h_{1t} - (1-\delta_h)h_{1t-1}) + (1-\epsilon)(h_{2t} - (1-\delta_h)h_{2t-1}) \tag{2.4.47}$$

Domestic bond market:

$$\epsilon d_{1t} + (1-\epsilon)d_{2t} = 0 \tag{2.4.48}$$

Tradable goods market:

$$\epsilon c_{1t} + (1-\epsilon)c_{2t} + \frac{\phi_h}{2}\frac{(h_{1t}-h_{1t-1})^2}{h_{1t-1}} + \frac{\phi_h}{2}\frac{(h_{2t}-h_{2t-1})^2}{h_{2t-1}} \\ + i_t^c + i_t^h + (1+r_{t-1})d_{ft-1} = y_t + d_{ft} \tag{2.4.49}$$

2.4.5 Calibration

Preference: Basic parameters like σ, ω, and η, are chosen the same as those in the basic model. Both capitalists and borrowers have the same share of housing services in the composition consumption and γ_1, γ_2 are set so that the ratio of total housing investment over GDP is equal to 5%. Capitalists' discount factor β_1 is pinned down by the steady state value of the exogenous interest rate, $\frac{1}{1+r}$. Borrowers are more impatient or β_2 is set to be 0.985. κ_1, κ_2 is selected such that in steady state both capitalists and borrowers supply a fraction $\frac{40}{24\times 7}$ of household's one unit time endowment for working in the market. I set the fraction of capitalists in total population ϵ equal to 0.2, which implies that about the top 20 percent of the wealthy population in the economy own capital, have free access to both domestic and international finance markets, which is consistent with the results of Diaz and Luengo-Prado that the top 20% in the wealth distribution holds 98.9% of total financial assets.

Technology: All parameters pertaining to technology and productivity side of the model are kept the same as those in the basic model.

I calibrate the model such that steady state trade-balance-to-GDP ratio is equal to 1%, which then pins down the level of foreign debt at steady state $\bar{d}$.[39] I also follow Schmitt-Grohe et al. (2003) to set the portfolio adjustment cost ϕ_d to 0.0007.

2.4.6 Model Dynamics

When a favored productivity shock hits the economy, firms in both sectors hire more labor to extend production, driving up the wage rate and capital returns. Due to the positive income effect, the capitalist and the borrower both increase their aggregate consumption, hence raising non-durable consumption. However, there is a contrast in the housing demand between the capitalist and the borrower. For the capitalist, since housing is just a durable

[39]Although varying this ratio does not have much effect on our results.

good, his elasticity of intertemporal substitution for long-lived housing is almost infinite so even a small rise in price relative to future will lead him to delay current purchase. Facing an increase in the relative housing prices, the capitalist optimally substitutes his durable consumption with non-durable goods, therefore reducing his housing stock in the early stage after shock and then gradually accumulates it back later on. By contrast, to the borrower, housing is not just a durable good but also plays a collateral role for future borrowing, which therefore makes him increasingly invest in housing in good times. As shown in the simulation, the increase in the borrower's housing demand not only is able to absorb the sale of the capitalist's housing but also drives up the overall economy housing investment. Figure 2.19

Moreover, since business capital is owned by capitalists who are not subject to borrowing constraints, its dynamics are not affected by the development of the mortgage market. As a result, unlike housing investment whose volatility relative to GDP increases in economies with a higher mortgage market index, the volatility of non-housing investment remains almost unchanged, consistent with the empirical evidence. Figure 2.21

Finally, I calibrate the extended model for the U.K and Table 2.4.6 presents the result. For comparison, I also calibrate a standard two-sector RBC model with free borrowing and the basic model. Despite its simplicity, calibrated models' second moments match data relatively well.[40] In particular, the implied volatility of housing investment in the credit constraint model can match data quite well, whereas the volatility of housing investment in a standard free model is two times lower than the data. Although the implied volatility of housing prices from the credit constraint model is about 2-3 times higher than that in the free borrowing model, it is far below that of the data, which reflects the difficulty of business cycle models in accounting for the high volatility of asset prices. Furthermore, unlike the free borrowing benchmark, the credit constraint models also can account

[40]It is not that surprising since Mendoza (1991) uses a standard model without housing and can match data for Canada quite well.

Figure 2.19: IRs: Extended Model with Borrowing Constraint

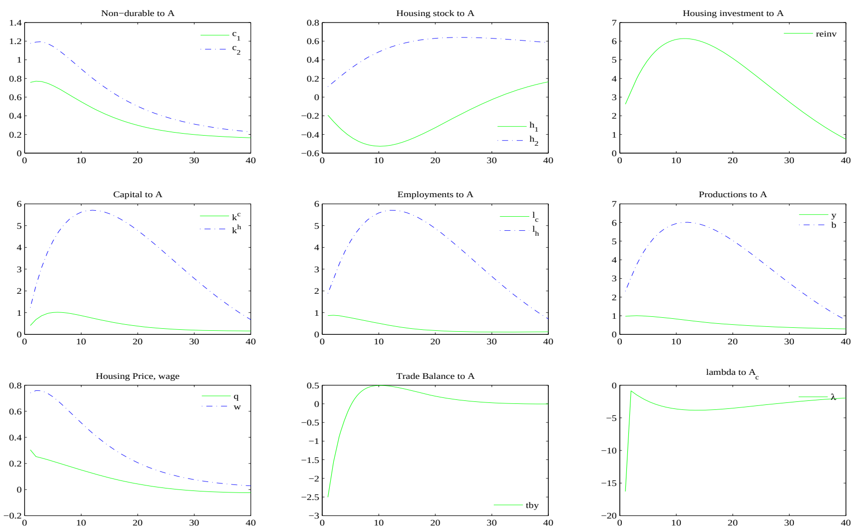


Figure 2.20: IRs of Housing Investment and Prices

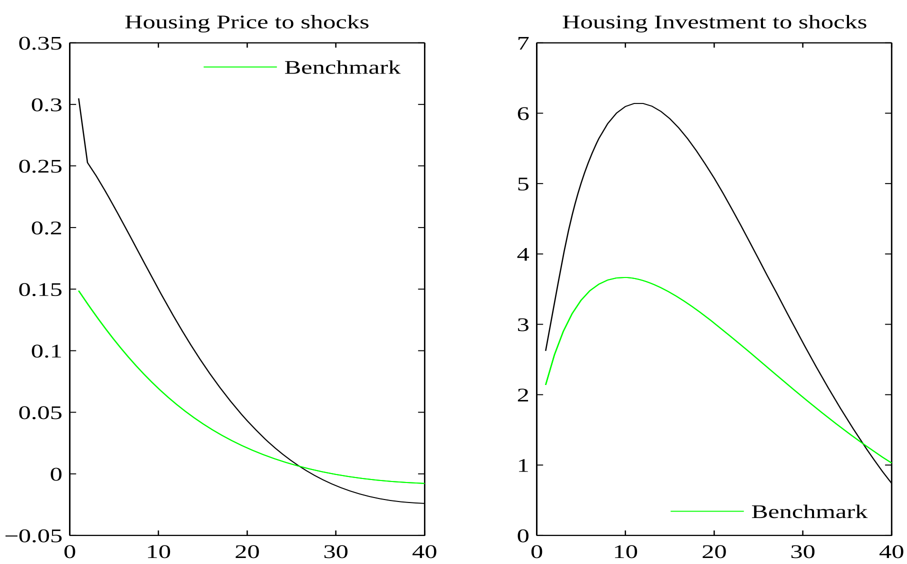


Table 2.4: **Statistics: Extended Model**

	Data	Std. Model	Basic Model	Extended Model
Standard deviation				
output	1.15	1.14	1.12	1.15
consumption	1.18	0.83	0.88	0.86
nres	4.1	4.15	2.3	4.13
tb/y	0.54	0.73	0.16	0.71
res	7.57	3.51	6.55	6.03
sd(res)/sd(y)	6.58	3.1	5.85	5.3
hp	5.13	0.075	0.11	0.18
Correlation w/ ouput				
consumption	0.73	0.94	0.98	0.97
nres	0.48	0.55	0.85	0.53
tb/y	-0.31	-0.07	-0.63	-0.2
hp	0.58	0.93	0.79	0.9
res	0.57	0.85	0.7	0.8

Notes: Data is obtained from time series for the U.K from Q1-1981 to Q3-2007. Extended model means heterogeneous household model with credit constraint, standard model is free borrowing model. Std() is standard deviation. nres: non-housing investment, tb/y: trade-balance output ratio, res: housing investment, hp: real housing prices. All numbers are in percentage, which is the standard deviations from trend and is obtained from Hodrick-Prescott filter.

Figure 2.21: Investment Volatilities and ϕ

for the significant counter-cyclicality of the current account. The reason is that since households are credit-constrained they tend to borrow more (from foreigners) to consume in good times and the borrowing-consuming spiral also reinforces borrowing as explained above.

2.5 Conclusions

This chapter begins by documenting stylized facts regarding housing investment and mortgage market depth in OECD countries. Housing investment is highly volatile, especially in economies with more developed mortgage markets. The chapter demonstrates that standard RBC models with a perfect credit market assumption are at odds with these empirical facts but the introduction of a housing collateral constraint can help reconcile the models with the facts. Collateral effects also enable the models to produce significant counter-cyclicality of the current account and the co-movement of different types of investments even without highly correlated productivity shocks. The chapter predicts a non-monotonic impact of mortgage market depth on the volatility of housing investment. In the quantitative section, calibrated models with a housing collateral constraint can match the data in the U.K quite well.

Chapter 3

International Business Cycles: A Re-Examination

3.1 Introduction

It is well known that several quantitative properties of international business cycle models are at odds with empirical data. First, the cross-country correlations are higher for consumption than for output, while in the data the opposite is true (the Backus-Kehoe-Kydland cross-country consumption correlation puzzle). Second, cross-country correlations of employment and investment are negative whereas in the data they are positive. In literature, while the ranking of the consumption correlation has been explained by models with incomplete financial markets, [41] Baxter (1995) admits that: "It has proved particularly difficult to write down plausibly-parameterized models which can generate positive comovement of labor and investment across countries...Thus a major challenge to the theory is to develop a model which can explain international comovement in labor input and investment."

An important reason for the negative comovements is that in a one-good model with the internationally mobile capital there is a strong tendency to move capital to the most productive location in response to persistent productivity shocks. The movement of capital to the more productive country leads to a rise in labor returns there accompanied by a fall in labor returns in the other country, hence inducing the negative comovement in

[41] See for example: Baxter and Crucini (1995) and Kollman (1996).

labor input in the model. As a result, employment in different countries is negatively correlated unless the cross-country correlation of the innovations to the country-specific shocks is very high. Baxter (1995) argues that although the innovations to Solow residuals are positively correlated across countries, this correlations is not strong enough to overcome the natural mechanisms leading to negative comovement.

However, a common feature of two-country international business cycle models in literature is the assumption of Cobb-Douglas preferences. This type of preferences implies a significant wealth effect on leisure even at high frequencies, which in turn plays a critical role in generating the negative comovement of employment in international business cycle models. Intuitively, when a positive productivity shock hits the foreign country, there is an increase in wealth at home because of risk sharing through financial markets. Therefore, consumers at home raise their consumption goods and *leisure* levels. The positive wealth effect, combined with the substitution effect that already helps raise leisure (reduce labor supply) because of the decline in wage rates at home, magnifies the decrease of the home country's labor supply in response to a foreign country's positive productivity shock. As a result, labor inputs are negatively correlated across country despite positive correlations in productivity innovations. This chapter quantitatively shows that without the wealth effect on leisure, relatively small positive correlations in cross-country productivity innovations, as suggested by empirical studies, are sufficient to generate significant positive comovement in employment. The result is robust with both complete financial markets with perfect risk sharing and incomplete markets with only partial risk sharing. In other words, the positive comovement in employment can be obtained even under the partial risk sharing environment, where, according to Baxter and Crucini (1995), only limited wealth is transfered between regions in response to asymmetric shocks. [42]

[42] In the final steps of completing this chapter, I became aware that Johri, Letendre, and Luo are also working on cross-country correlation anomalies. In a version of their

Devereux et al (1992) is the first to introduce preferences with a zero wealth elasticity of leisure into two-country one good models. [43] They quantitatively show that there is a realistic cross-country correlation of consumption even with complete financial markets but did not examine the possibility of the positive comovement in employment and output. My model differs from theirs in one critical ingredient. I consider a different productivity shock structure: there is persistence in productivity shocks and positive correlations in cross-country innovations, as suggested by empirical studies whereas they assume independently random productivity shocks. Under the complete market structure, I obtain a similar result for cross-country consumption correlation, hence, confirming that their quantitative result is robust to the curvature parameter of preferences. [44] At the same time, I show that by using this type of preferences, the cross-country consumption correlations can also be significantly reduced with incomplete financial markets.

working paper, they show that GHH preferences along with learning-by-doing and *incomplete markets* are sufficient to generate positive cross-country correlations of hours and investment. Although we both focus on the wealth effect on leisure, the important difference between complete and incomplete financial markets, as pointed out by Baxter and Crunini (1995), is the degree of wealth transfer across country in response to asymmetric productivity shocks. Therefore, a different financial structure will have a different impact on employment dynamics. In other words, my research shows that in order to generate a positive comovement in employment, GHH preferences are sufficient and we don't need incomplete financial markets. Moreover, as shown by Letendre (2004) and Boileau and Normandin (2008), with incomplete financial markets, the quantitative results/predictions of international business cycle is very sensitive to parameters like discount factors,degrees of persistence and spillover in productivity shocks, and more importantly the modifications to generate stationarity. I thank Kang Shi for the pointer.

[43] Roffo (2006) introduces home production through the same type of preferences to generate the fact that domestic spending is more volatile than output, hence, being able to explain the counter-cyclicality of net exports in two-country real business cycle models. Those models with Cobb-Douglas preferences fail to generate these facts.

[44] To obtain the closed form of solution for the model, Devereux et al (1992) had to assume a specific value for the curvature.

There has been a growing literature focusing on the role of credit market imperfections in explaining the positive comovements. Backus et al. (1992), Baxter and Crucini (1995) and Heathcote and Perri (2002) assume financial autarky, i.e. countries cannot trade financial claims. These papers find that extreme restrictions in the trade of financial assets, by largely reducing international capital mobility, can generate the positive cross-country correlation in output. Kehoe and Perri (2002) analyze a model where a country faces an endogenous borrowing constraint. In particular, the borrowing capacity of a country depends on the value that the country would obtain from future access to international financial markets. The borrowing constraint requires that in each period and state, allocations can be enforced only if their value is greater than it would be if the country were excluded from all further intertemporal and international trade. When the foreign country is hit by a positive shock, its output cannot increase too much otherwise the value of defaulting would become higher than the penalty of being excluded from international financial markets in the future. Therefore, the flow of capital from the home country to the foreign country is limited, which help to account for the positive cross-country correlations under relative small correlations in productivity innovations.

My work takes a different approach from the above literature in explaining the comovement of employment. It shows that the employment comovement anomaly can be reconciled with the data by simply assuming a zero wealth effect on leisure at high frequencies, hence attempting to provide insights from different point of view.

The structure of this chapter is as follows. Section 2 introduces models' setting and calibration. Section 3 discusses quantitative results. Conclusions follow in section 4.

3.2 Model

The world consists of two countries: the home country and the foreign country. Consumers in each country value leisure and consumption of the single tradable produced good while labor is internationally immobile. Firms in each country produce the single good by identical Cobb-Douglass production functions and are subject to exogenous shocks to total factor productivity.

In this model, the foreign country is distinguished from the home country by means of a star attached to all foreign-country variables. When there are no stars, the variable, parameter, or function is assumed to be identical across countries. All variables are in per capita terms.

Preferences. The representative household in each country maximizes its expected lifetime utility defined over random sequences of consumption goods (c_t) and labor disutility (l_t):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t, l_t\big), \quad \text{Home country;} \tag{3.2.1}$$

$$U^* = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t^*, l_t^*\big), \quad \text{Foreign country} \tag{3.2.2}$$

I consider two types of preferences in this paper. The first one is the Cobb-Douglas preferences, which have been commonly used in the international business cycle literature of one good two country models.[45]

$$U(c_t, l_t) = \frac{\big(c_t^{\gamma}(1-l_t)^{1-\gamma}\big)^{1-\sigma} - 1}{1-\sigma} \tag{3.2.3}$$

Parameter γ will determine the value of hours at the steady state.

The second one is non-separable GHH preferences:

$$U(c_t, l_t) = \frac{(c_t - \kappa \frac{l_t^{\omega}}{\omega})^{1-\sigma} - 1}{1-\sigma} \tag{3.2.4}$$

[45] The type of preferences was used by King, Plosser, and Rebelo, 1988

Similar to γ, parameter κ will determine the value of hours at the steady state and parameter ω determine the elasticity of labor supply. It is well known that GHH preferences imply zero elasticity of leisure to income.

Technology. Production functions are in Cobb-Douglass forms, hence exhibiting constant returns to scale; production of the single final good requires the input of both labor and capital. Capital used in production in a specific country is not necessarily owned by residents of that country; thus, k_t represents capital in place in the home country, not necessarily capital owned by residents of the home country. Labor is internationally immobile.

$$y_t = A_t(k_t)^{\alpha}(l_t)^{1-\alpha} \quad \text{Home country;} \tag{3.2.5}$$

$$y_t^* = A_t^*(k_t^*)^{\alpha}(l_t^*)^{1-\alpha} \quad \text{Foreign country.} \tag{3.2.6}$$

where A_t represents the stochastic level of productivity home country and k_t is the capital stock installed the home country at time t.

Productivity evolves according to the bivariate autoregressive process:

$$\begin{bmatrix} log(A_{t+1}) \\ log(A_{t+1}^*) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_1 \end{bmatrix} \begin{bmatrix} log(A_t) \\ log(A_t^*) \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1}^* \end{bmatrix} \tag{3.2.7}$$

where a_1 measures the persistence in productivity shocks and a_2 measures the degree of international spillovers. The variance in the innovations is denoted by σ_ϵ^2 and the correlation between ϵ_t and ϵ_t^* is σ_{12}.

I consider two types of investment technologies. The first one is a typical investment with adjustment costs. That is:

$$k_{t+1} = (1-\delta)k_t + i_t - \frac{\phi}{2}k_t\Big[\frac{i_t}{k_t} - \delta\Big]^2 \tag{3.2.8}$$

This law of motion includes investment adjustment costs governed by ϕ and is such that there are no adjustment costs in steady state.

I also follow Kydland and Prescotts (1982) to assume time-to-build investment technology. That is, in the time-to-build models:

$$i_t = \omega_1 s_{1t} + \omega_2 s_{2t} + \omega_3 s_{3t} + \omega_4 s_{4t} \tag{3.2.9}$$

where i_t is the investment at time t and s_{jt} is the volume of projects j periods away from completion at the beginning of period t and ω_j is the resource cost associated with work on a project j periods away from completion, for j = 1, 2, 3, 4. Investment projects progress according to $s_{j,t+1} = s_{j+1,t}$ for j = 1, 2, 3; and starts during period t are represented by s_{4t}. The capital stock thus evolves according to:

$$k_{t+1} = (1-\delta)k_t + s_{1t} \tag{3.2.10}$$

Notice that when $\omega_1 = 1, \omega_2 = \omega_3 = \omega_4 = 0$, we have the regular investment technology.

Market Structure. I assume that there is frictionless international trade in output, so that there is an unified world resource constraint for the single produced good:

$$(y_t - c_t - i_t) + (y_t^* - c_t^* - i_t^*) = 0 \tag{3.2.11}$$

Regarding the financial structure, I consider both complete-markets and bond economies. The difference of the two structures lies in the number of assets available to the agents. When markets are complete, the representative agents in both countries can trade a full set of contingent claims. Accordingly, the budget constraint of the home country's representative household can be expressed as:

$$c_t + i_t + \sum_{s_{t+1}} p(s_{t+1}, s_t) b(s_{t+1}) = y_t + b(s_t) \tag{3.2.12}$$

where s_t indicates the state in period t and $b(s_{t+1})$ denotes the quantity of contingent claims purchased in period t and paying off one unit of consumption the following period, conditional on the state of the world being s_{t+1} next period. $p(s_{t+1}, s_t)$ denotes the price of these contingent assets.

By contrast, in a bond economy, there is only a one-period real discount bond. Let b_{t+1} denote the per capita quantity of this discount bond purchased by the home economy, which mature in period $t+1$, and p_t^b is its price at time t.

The flow budget constraints for the bond economy are: [46]

$$c_t + i_t + p_t^b b_{t+1} + \frac{\pi_b}{2}(b_{t+1})^2 = y_t + b_t; \quad \text{home country} \tag{3.2.13}$$

$$c_t^* + i_t^* + p_t^b b_{t+1}^* + \frac{\pi_b}{2}(b_{t+1}^*)^2 = y_t^* + b_t^*; \quad \text{foreign country} \tag{3.2.14}$$

The world market clearing condition for bonds is:

$$b(s_{t+1}) + b^*(s_{t+1}) = 0; \quad \text{complete markets} \tag{3.2.15}$$

$$b_{t+1} + b_{t+1}^* = 0 \quad \text{bond economy} \tag{3.2.16}$$

Calibration. This paper follows closely calibration in Baxter and Crucini (1995), Kollmann (1996), and Kehoe and Perri (2002). See Table 3.1 for details.

In particular, parameter ω, which determines the inter-temporal elasticity of substitution in labor supply, [47] is set to 2 as a benchmark. The unit benchmark elasticity is equal to the value implied by standard preferences as in form (3.2.3). For sensitivity analysis, ω is set from 1.58 [48] to 6, which then implies the intertemporal elasticity of substitution varies from 1.7 to 0.2 accordingly. This is a range suggested by empirical studies.

Parameters κ and γ for GHH preferences and Cobb-Douglas preferences are chosen such that the hours of working in the steady state are 0.25, respectively.

Portfolio adjustment costs parameter, π_b is set to 0.0005 such that the implied volatility of the ratio of net exports to output in bond economy models is the same as in the financial complete market models.

For regular investment technology, investment adjustment cost parameter, ϕ is set such that the ratio of investment volatility to that of output match the data, which is equal to 3.24.

[46] Following Boileau et al (2008) and others, I impose quadratic portfolio adjustment costs to induce stationarity in incomplete markets. See Boileau et al (2008) for more details about other methods.

[47] The inter-temporal elasticity of substitution in labor supply is approximately $\frac{1}{\omega-1}$.

[48] 1.58 is the value used by Devereux et all (1992) in their two-country model; the value was first used by Greenwood et al (1988) in a closed-economy model.

Finally, for parameters of the productivity shock's process, which are crucial to quantitative results of international business cycle models, [49] I follow Kehoe and Perri (2002) to set $a_1 = 0.95$, $a_2 = 0$ as the benchmark. These values imply that there are medium levels of persistence but there is no direct "spillover" in productivity shocks. For sensitivity analysis, I choose high persistence ($a_1 = 0.99$) (termed HP) and low persistence ($a_1 = 0.90$) (termed LP). I also follow the original results of Backus, Kehoe, and Kydland (1992) (termed BKK) to set $a_1 = 0.906$ and $a_2 = 0.088$.

Table 3.1: Model Calibration

	Parameters
Preferences	$\beta = 0.99$, $\sigma = 2$
	hours at s.s $l = 0.25$
	$\omega = 2$ as benchmark
Technology	$\alpha = 0.3$, $\delta = 0.03$
	$\omega_1 = \omega_2 = \omega_3 = \omega_4 = 0.25$
Productivity shocks	$a_1 = 0.95$, $a_2 = 0$
	$\text{var}(\epsilon_1)=\text{var}(\epsilon_2)=0.07^2$, $\text{corr}(\epsilon_1,\epsilon_2)=0.25$
Adjustment cost	$\pi_b = 0.0005$

3.3 Quantitative Results

I consider four types of models with different financial market structures and different investment technologies: models with complete financial markets and models with restricted one period state non-contingent bonds, models with investment adjustment costs and models with time-to-build investment. I solve and simulate these models by the perturbation method [50] and Fig-

[49] Accoring to Letendre (2004) and Boileau and Normandin (2008) parameters of the productivity shock's process are particularly important for quantitative results in international business cycle models with incomplete markets.

[50] For more details, see Schmitt-Grohe and Uribe (2004)

ures 3.1-3.4 present impulse responses of these models with respect to one unit of positive shock in the foreign countries. For comparison, I combine impulse responses of models with Cobb-Douglas preferences and those with corresponding specifications but with GHH preferences in the same figure. Table 3.2 and Table 3.3 provide business cycle statistics from data and those implied by these models.

Figures 3.1-3.4 show differences between the impulse responses of models with GHH preferences and those with Cobb-Douglas preferences, particularly in consumption and labor input. In response to a positive shock in the foreign country, consumptions in both countries exhibit a relatively smooth and similar dynamic pattern for models with Cobb-Douglas preferences while for GHH preferences, the impulses are more responsive and move in opposite directions. This explains why the cross-country correlation in consumption is higher in Cobb-Douglas preferences. The impulse responses also present evidence of more risk-sharing in the complete markets, where consumption in the home country increases relatively more than it does in a bond economy.

By contrast, employment (hours) in home country decreases significantly less in models with GHH preferences. The reason is straightforward. With a zero wealth effect on leisure, consumers in the home country consume relatively less leisure in response to a positive productivity shock in the foreign country, hence reducing labor supply by a relatively smaller amount compared to a positive wealth effect case. Tables 3.2 and 3.3 show that the cross-country correlations in labor and output are positive in the models with GHH preferences whereas the correlations are negative in the models with Cobb-Douglas preferences.

Figures 3.3- 3.4 show impulse responses of models with time-to-build investment. As pointed out in Kydland et al (1982), models with time-to-build display a more persistence in all time series variables. I obtain similar quantitative results for consumption and labor input for models with time-to-build investment.[51]

Table 3.4 presents the implied business cycle statistics of the model with GHH preferences, complete financial markets, and investment with adjustment costs when I vary the intertemporal elasticity of substitution in labor supply from 1.7 to 0.2. Table 3.4 shows that when the elasticity is lower. i.e., labor supply becomes less responsive to shocks the cross-country correlation in employment improves. However, since leisure becomes less responsive to shocks, the cross-country consumption tend to move together, hence, cross-country correlation increases, which is consistent with the results of Devereux et al (1992).

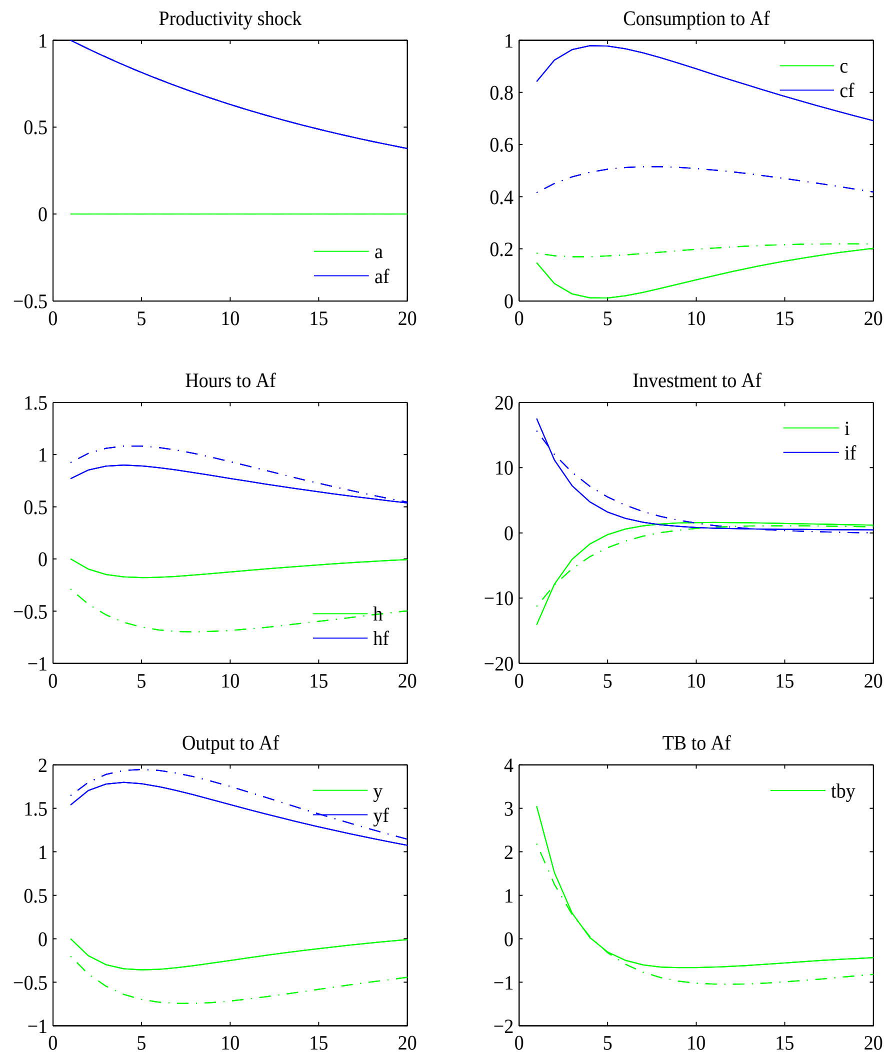
Finally, Table 3.5 shows that without wealth effects on leisure the cross-country positive correlation in employment is robust with various specifications in the productivity shock process. In particular, when there is spillover in cross-country productivity shocks as in the BKK specification, cross-country employments almost move together. However, even under the same BKK specification, cross-country employments still significantly negatively co-move (the correlation is -0.63) with Cobb-Douglas preferences. These results reconfirm that the wealth effect on leisure plays the crucial role in determining the cross-country correlation in employment.

3.4 Conclusions

This chapter quantitatively shows that the wealth effect on leisure plays a determining role in generating the cross-country negative correlation in employment. As a result, a positive cross-country correlation in employment can be obtained by simply using preferences with a zero wealth elasticity of leisure.

[51] The cross-country correlation in consumption increases in the incomplete market specification compared to that of the complete market specification because the cross-country correlation in employment is significantly improved in this case and also because of the non-separability property of GHH preferences.

Figure 3.1: **IRs: Complete Markets, Investment with Adjustment Costs**



Solid line: GHH preference; Dashed line:Cobb−Douglas preference

Figure 3.2: **IRs: Bond Economy, Investment with Adjustment Costs**

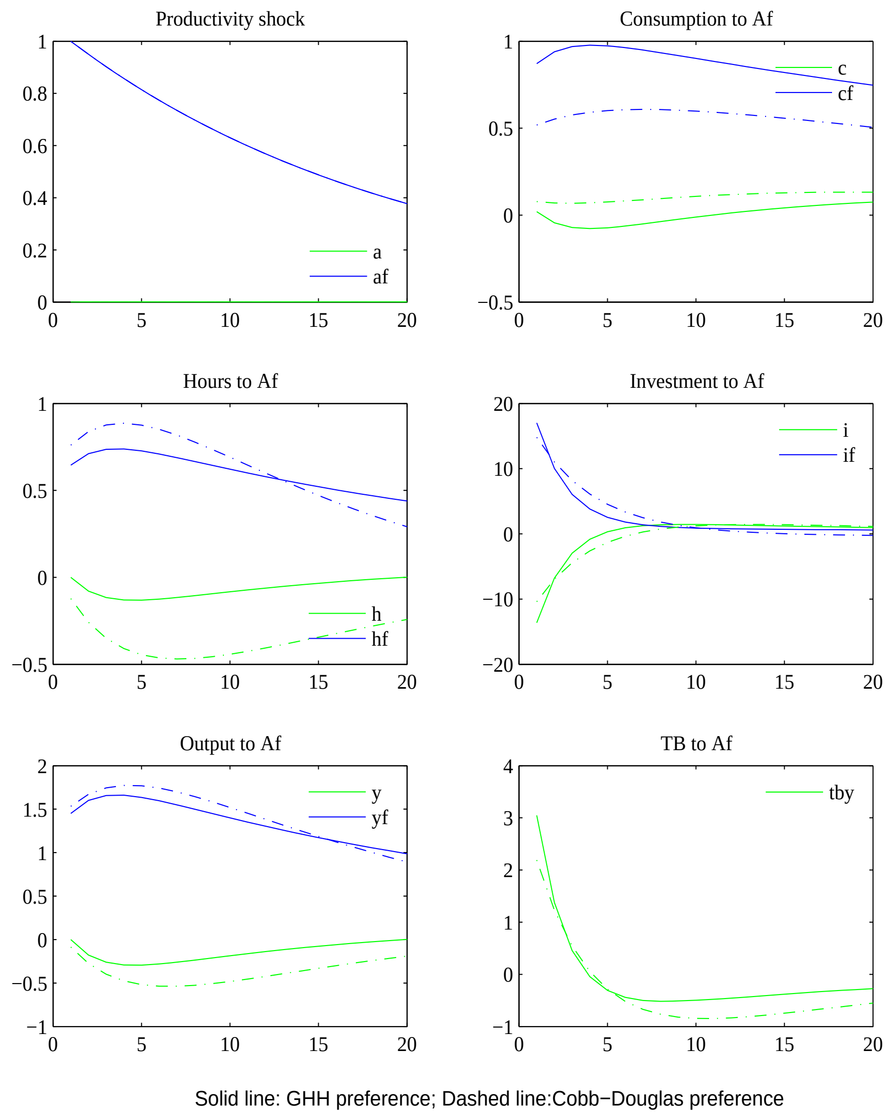


Figure 3.3: **IRs: Complete Markets, Time-to-Build Investment**

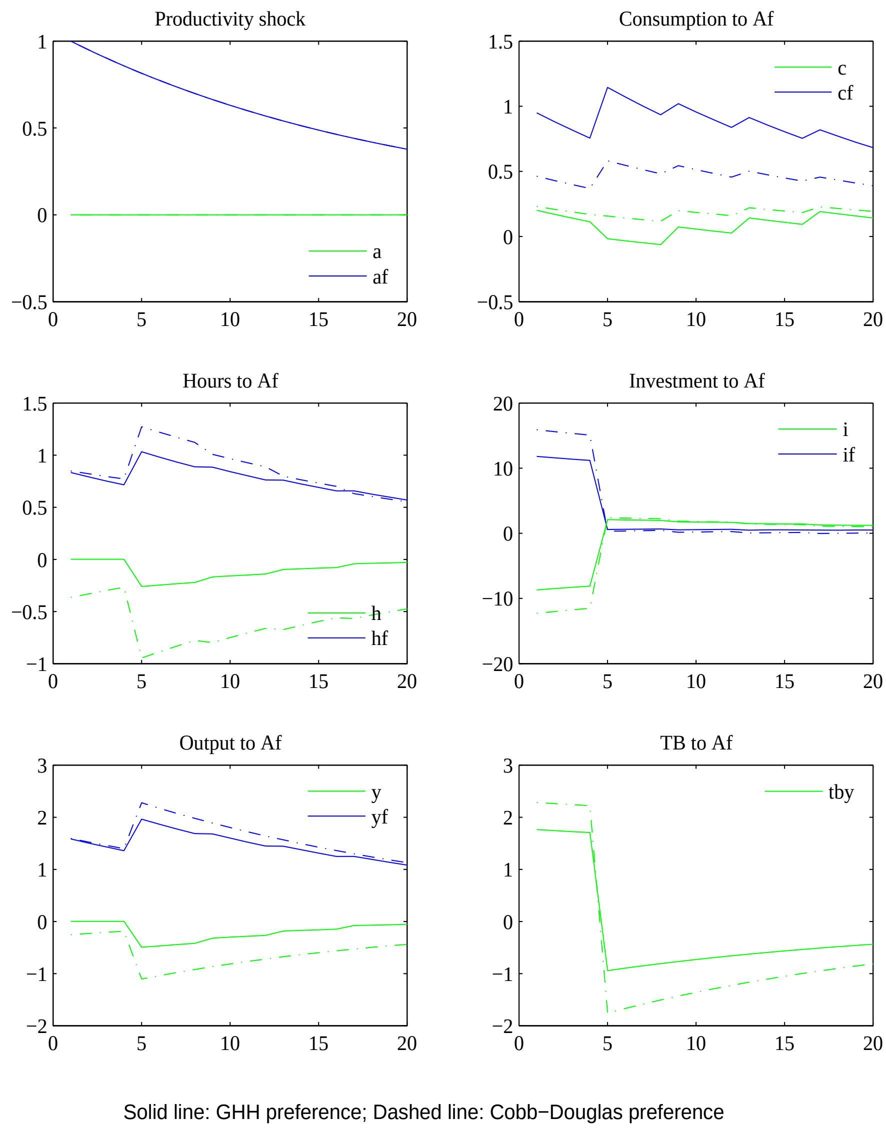
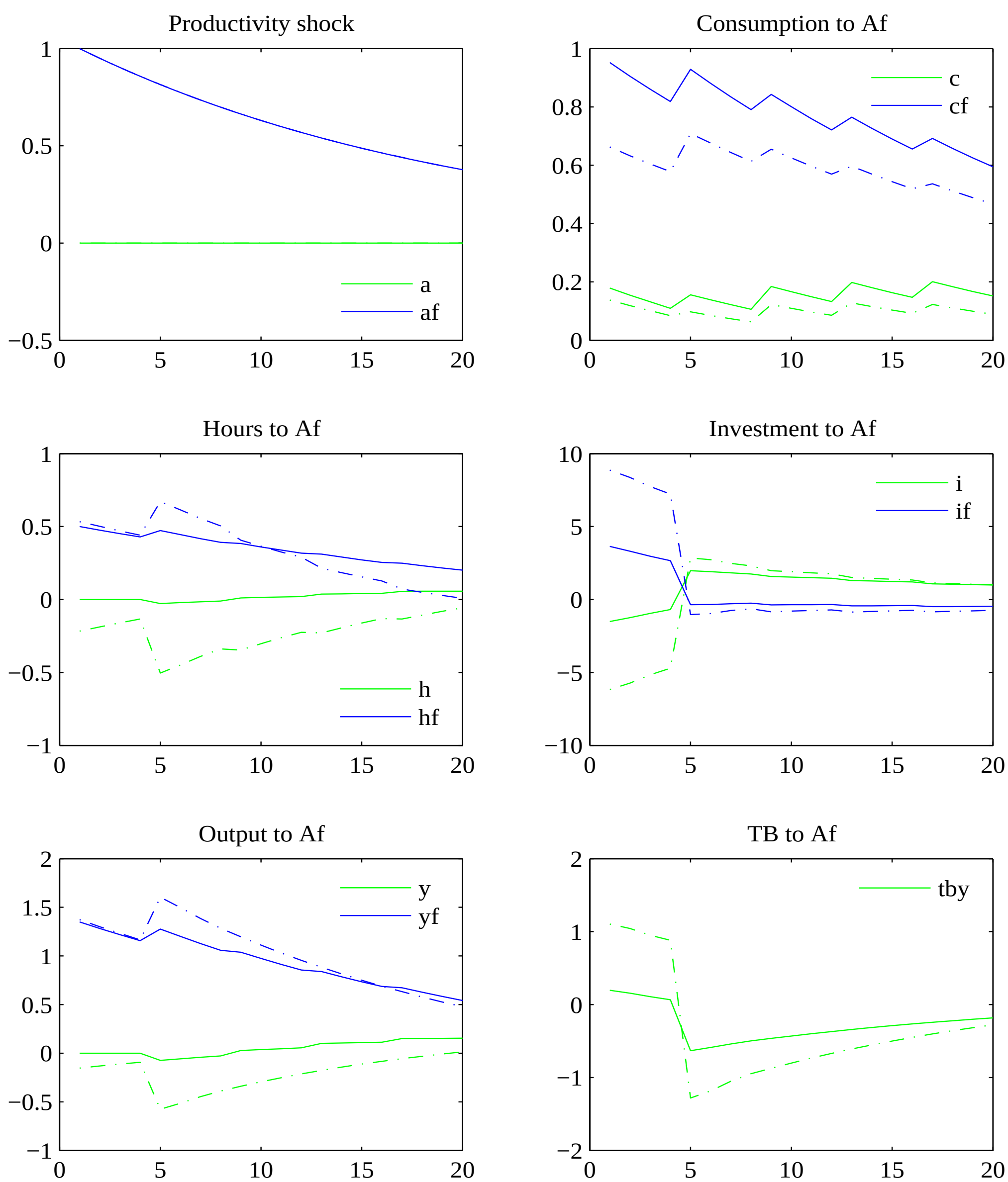


Figure 3.4: **IRs: Bond Economy, Time-to-Build Investment**



Solid line: GHH preference; Dashed line:Cobb−Douglas preference

Table 3.2: **Business Cycles Statistics: GHH Preferences**

		Economy with			
		IAC		Time-to-build	
Statistics	Data	CM	Bond	CM	Bond
Std.dev rel. to GDP					
Consumption	0.79	0.71	0.81	0.72	0.88
Investment	3.24	3.24	3.24	3.24	1.83
Employment	0.63	0.5	0.5	0.5	0.5
Net Exports/GDP	0.09	0.65	0.64	0.69	0.43
Domestic Comovement					
Corr. with GDP					
Consumption	0.87	0.96	0.96	0.96	0.91
Investment	0.93	0.5	0.5	0.43	0.7
Employment	0.86	1	1	1	1
Net Exports/GDP	-0.36	0.17	0.06	0.22	0.21
International Correlation					
Home and Foreign					
GDP	0.51	0.2	0.19	0.21	0.51
Consumption	0.32	0.68	0.28	0.69	0.72
Investment	0.29	-0.7	-0.7	-0.7	0.18
Employment	0.43	0.2	0.19	0.21	0.51

Notes: The statistics in the Data column are taken from Kehoe and Perri (2002), which are calculated from U.S. quarterly time series, 1970:1-1998:4 and an aggregate of 15 European countries. All relevant time series, except ratio of net exports to output, have been logged and HP-filtered. IAC means Investment with adjustment costs; CM indicates a complete financial market economy while bond indicates an economy where people can only trade one-period non-contingent bond. The inter-temporal elasticity of substitution in leisure is set to 1

Table 3.3: **Business Cycles Statistics: Cobb-Douglas Preferences**

		Economy with			
		IAC		Time-to-build	
Statistics	Data	CM	Bond	CM	Bond
Std.dev rel. to GDP					
Consumption	0.79	0.41	0.51	0.4	0.63
Investment	3.24	3.24	3.24	4.33	3.47
Employment	0.63	0.58	0.61	0.58	0.49
Net Exports/GDP	0.09	0.78	0.81	1.10	0.83
Domestic Comovement					
Corr. with GDP					
Consumption	0.87	0.7	0.58	0.7	0.7
Investment	0.93	0.51	0.51	0.27	0.41
Employment	0.86	0.93	0.86	0.93	0.79
Net Exports/GDP	-0.36	0.52	0.5	0.48	0.38
International Correlation					
Home and Foreign					
GDP	0.51	-0.45	-0.35	-0.5	0.04
Consumption	0.32	0.86	0.42	0.85	0.57
Investment	0.29	-0.76	-0.71	-0.88	-0.61
Employment	0.43	-0.87	-0.86	-0.9	-0.64

Note: The statistics in the Data column are taken from Kehoe and Perri (2002), which are calculated from U.S. quarterly time series, 1970:1-1998:4 and an aggregate of 15 European countries. All relevant time series, except ratio of net exports to output, have been logged and HP-filtered. AC means Investment with adjustment costs; CM indicates a complete financial market economy while bond indicates an economy where people can only trade one-period non-contingent bond.

Table 3.4: **Business Cycles Statistics: Sensitivity to Elasticity**

		ω					
Statistics	Data	1.58	2	3	4	5	6
Std.dev rel. to GDP							
Consumption	0.79	0.76	0.71	0.66	0.64	0.63	0.62
Investment	3.24	3.24	3.24	3.24	3.24	3.24	3.24
Employment	0.63	0.63	0.5	0.33	0.25	0.2	0.17
Net Exports/GDP	0.09	0.66	0.65	0.65	0.65	0.66	0.66
Domestic Comovement							
Corr. with GDP							
Consumption	0.87	0.98	0.96	0.92	0.89	0.87	0.85
Investment	0.93	0.46	0.5	0.53	0.55	0.56	0.57
Employment	0.86	1	1	1	1	1	1
Net Exports/GDP	-0.36	0.14	0.17	0.22	0.25	0.26	0.27
International Correlation							
Home and Foreign							
GDP	0.51	0.14	0.20	0.24	0.26	0.26	0.27
Consumption	0.32	0.52	0.68	0.84	0.91	0.94	0.96
Investment	0.29	-0.76	-0.7	-0.61	-0.57	-0.54	-0.53
Employment	0.43	0.14	0.20	0.24	0.26	0.26	0.27

Note: The statistics in the Data column are taken from Kehoe and Perri (2002), which are calculated from U.S. quarterly time series, 1970:1-1998:4 and an aggregate of 15 European countries. All relevant time series, except ratio of net exports to output, have been logged and HP-filtered. The model statistics are computed from an model economy with GHH preferences, complete financial markets, and investment with adjustment costs. Omega is related to the intertemporal elasticity of substitution in labor supply: the higher omega is the lower the elasticity

Table 3.5: **Business Cycles Statistics: Sensitivity Productivity Shock Process**

		Productivity Shock Process			
Statistics	Data	Benchmark	HP	LP	BKK
Std.dev rel. to GDP					
Consumption	0.79	0.71	0.79	0.67	0.92
Investment	3.24	3.24	3.06	3.24	3.24
Employment	0.63	0.5	0.5	0.5	0.5
Net Exports/GDP	0.09	0.65	0.69	0.58	0.67
Domestic Comovement					
Corr. with GDP					
Consumption	0.87	0.96	0.95	0.96	0.99
Investment	0.93	0.5	0.37	0.61	0.39
Employment	0.86	1	1	1	1
Net Exports/GDP	-0.36	0.17	0.23	0.11	0.00
International Correlation					
Home and Foreign					
GDP	0.51	0.20	0.25	0.22	0.94
Consumption	0.32	0.68	0.76	0.65	0.99
Investment	0.29	-0.7	-0.81	-0.58	-0.72
Employment	0.43	0.20	0.25	0.22	0.94

The model statistics are computed from an model economy with GHH preferences, complete financial markets, and investment with adjustment costs. HP denotes the productivity shock process with high persistence. LP denotes the productivity shock process with low persistence. BKK means the productivity shock process with Backus, Kehoe, and Kydland (1992) estimates.

Chapter 4

Liability Dollarization and Fear of Floating

4.1 Introduction

There are two distinguishing features in emerging economies' exchange rates and financial systems: (i) *fear of floating*, a phenomenon where authorities are reluctant to let their nominal exchange rates fluctuate and (ii) increasing uses of the U.S dollar in debt denomination instead of these economies' own domestic currencies or the so-called *liability dollarization*. This paper addresses the question of whether fear of floating can be justified an optimal discretionary monetary policy in a dollarized emerging economy.

Fear of floating has been seen as a *prima facie* phenomenon because most of recent exchange rate crises in emerging economies occurred in pegged exchange rate environments and the rigidity in nominal exchange rates has been perceived as one of main reasons. Calvo and Reinhart (2002), however, show that despite having experienced severe exchange rate crises, authorities in emerging economies have kept intervening to smooth exchange rate fluctuations and evidently there has not much variation in nominal exchange rates in these economies. In particular, they present evidence showing that interest rate and reserve variabilities are significantly higher in emerging market economies than in developed economies such as the U.S and Japan. The probability that the monthly variation of nominal exchange rates is in a narrow band of plus and minus 2.5% is more than 79% for all developing countries, [52] which is definitely higher than that in developed countries. Given the fact that emerging economies often experience much more volatile shocks than their developed counterpart, relatively small variation in nominal exchange rates in emerging economies is remarkable.

On the other hand, liability dollarization belongs to another broad feature that has recently obtained popularity in emerging/developing economies: dollarization. In these countries, it has become increasingly popular that governments borrow in the U.S dollar, individuals can hold U.S dollar denominated bank accounts, firms and households can borrow in the U.S dollar both domestically and internationally. In particular, to quantitatively document dollarization, Reinhart, Rogoff, and Savastano (2003) (RRS, henceforth) build a composite index of dollarization for a wide range of developing countries so are able to show that that the frequency distribution of the composite dollarization index has shifted markedly to the right between 1980-85 and 1996-2001. The shift indicates that the degree of dollarization in developing countries has risen significantly during these periods.[53] By exploring the data further, RRS are able to show that by late 90s, more than half of 143 countries in their samples have at least 10% of broad money or of domestic public debt denominated or linked to foreign currency and one third of these 143 countries have more than 10% of external debts borrowed from private sector. They also find evidence suggesting that higher level of dollarization tends to increase the exchange rate pass-through, thereby

[52] In details, the probabilities are 79%, 87%, and 92% for those who claim to have freely floating exchange rate regime, managed floating, and limited floating, respectively. The probabilities for developed countries like U.S and Japan is 59% and 61%.

[53] Concretely, RRS define a (partially) dollarized economy as one where households and firms hold a fraction of their portfolio (inclusive of money balances) in foreign currency assets and/or where the private and public sector have debts denominated in foreign currency. The composite index is defined as the (normalized) sum of bank deposits in foreign currency as a share of broad money, total external debt as a share of GNP, and domestic government debt denominated in (or linked to) a foreign currency as a share of total domestic government debt.

reinforcing the fear of floating in highly dollarized economies.

This chapter attempts to shed light on the relationship between the two aforementioned notable features, particularly the question of whether fear of floating can be justified as an optimal discretionary monetary policy in dollarized emerging economy in response to external shocks. To this end, I consider a small open economy in which intermediate goods importers borrow in foreign currencies and face credit constraints. Foreign intermediate goods are required for final goods production. In this economy, interest rates that domestic borrowers pay to foreign lenders depend on the borrowers' net-worth, which characterizes the *financial acceleration*, i.e., the higher the leverage is the higher the interest rates borrowers have to pay.

Cespedes et al. (2002) and Devereux et al (2006) (henceforth DLX) have followed Bernanke et al (1999) (henceforth BGG) to take into account credit constraints in investment financing for liability-dollarized emerging economies. In these models, exchange rate fluctuations affect firms' real net worth positions and investments through balance-sheet constraints, thereby having impacts on the macroeconomy. Despite different settings, the two papers reach quite similar conclusions: balance-sheet constraints in the presence of liability dollarization is an important propagation channel, it can magnify the effects of external shocks, leading both real and financial variables' volatility to be greater than in an economy without these constraints. However, even under financial imperfections and balance sheet constraints, the inflation targeting or the flexible exchange rate regime still dominates the fixed exchange rate regime in both the role of cushioning external shocks and in welfare terms.

Nonetheless, there is a common feature in Crespedes and DLX that limits the impact of exchange rate fluctuations on other macroeconomic variables. In these models, exchange rate fluctuations *only* affect the net worth of firms and via this channel determine the finance premium of foreign currency borrowing. Emerging economies, most of which are relatively less industrialized, have to rely heavily on imported intermediate goods for domestic production. Christiano et al (2006), for example, shows that in developing countries, more than 80% of the import is intermediate goods for domestic production. The heavy reliance on foreign intermediate goods implies a high exchange rate pass-through and high external exposure. Moreover, because of limited cross-border enforcements particularlly for emerging countries, import firms are subject to borrowing constraints. As a result, when import firms borrow in foreign currencies to finance intermediate goods, exchange rate fluctuations affects not only the borrowers' net worth but also the financing amount. This very "double-effect" from exchange rate fluctuations leads to more profound impacts on the leverage of import firms, causing much more fluctuations in finance premium than those in Crespedes and DLX 'models. The borrowing constraint imposed on import firms is the main departure from to DLX's paper.

Under aforementioned different specifications, this chapter follows DLX to re-examine the macroeconomic consequences and compare welfare of alternative monetary policies: the inflation targeting regime and the fixed exchange rate regime [54] in response to external shocks: the world interest rate and terms of trade shocks. This chapter finds that fear of floating can be justified in highly dollarized economies. The volatilities of output, consumptions, and imported goods are higher under the inflation targeting rule than under the fixed exchange rate rule. The welfare of the fixed exchange rate regime also dominates that for the inflation targeting regime in a wide range of parameter specifications.

There are several other papers addressing fear of floating. Lahiri and Vegh (2001) incorporate three key frictions into their model: an output cost of nominal exchange rate fluctuations, an output cost of higher interest rates to defend the currency, and a fixed cost of intervention. The model then

[54] We follow the setting of endogenous monetary policy as in DLX, and use the perturbation method from Schmitt-Grohe and Uribe's paper to solve the model to the second order approximation in order to calculate the welfare.

predicts a non-monotonic relationship between the nominal exchange rate and the size of the shock. For large shocks, which are identified for developing countries, the output costs resulting from exchange rate fluctuations become too large relative to the cost of intervening. Therefore, monetary authorities find it optimal to stabilize the exchange rate. My research differs with this paper in several aspects. First, I incorporate stochastic environment and financial constraints and its endogenous propagation mechanism via the financial acceleration to the macroeconomy while Lahiri and Vegh (2001) do not. Second, I address the external shocks, particularly the terms of trade shock while the paper addresses monetary shocks.

My paper shares a key aspect with the paper by Devereux and Poon (2004): Intermediate good importers in developing countries face endogenous borrowing constraints so exchange rate adjustments might become destabilizing. The difference is that Devereux and Poon (2004) assume a collateral borrowing constraint like Kiyotaki and Moore (1997). In their model, the constraint is *not always binding*; it binds only when shocks are negative and large so the model might be more suitable to address monetary policies in crises. By contrast, I follow the BGG framework in which exchange rate fluctuations always have impacts on the borrowers' leverage, hence on the financial premium, regardless of the scale and direction of shocks.

The chapter is organized as follows. Section 2 sets out the model. Section 3 discusses calibration and the solution of the model. Section 4 develops the main results including impulse responses, volatilities of macroeconomic variables, and welfare evaluation under alternative monetary policies. Some conclusions follow.

4.2 The Model

4.2.1 Model Outline

This is one sector model of a small open economy where final goods are domestically produced using labor and imported intermediate goods. Domestic agents consume only domestically produced final goods,[55] they are, however, endowed with a fixed amount of tradable goods, which can be exported to the rest of the world with exogenous prices.

The model has following characteristics: (i) rigidities in prices,[56] (ii) credit constraints in foreign currency borrowing to highlight *balance-sheet effects* of liability dollarization, (iii) imperfect substitutability between domestic value-added goods and imported intermediate goods to capture the reliance of domestic production on foreign intermediate goods.

There are four sets of domestic agents in the model: households, firms, importers, and the monetary authority, vs. one foreign "the rest of world" where foreign-currency prices of imported intermediate goods are set and lending rates of foreign fund are determined. The rest of the world also demands domestically endowed tradable goods, which domestic agents do not consume. Domestic households have access to international financial markets through two kinds of non-state-contingent bonds. Financing contracts are set up between foreign bankers and domestic importer firms who need to borrow to finance imported intermediate goods. Final goods firms hire labor from households, re-buy intermediate goods from importers, and sell goods to both domestic households and importers for consumption. Finally, the monetary authority sets domestic nominal interest rates as a monetary policy instrument.

[55] This assumption is justified by empirical evidence that suggests in the majority of developing countries less than 17% of imported goods is for consumptions and other left are intermediate goods for domestic production.

[56] To allow effective monetary policy under New-Keynesian framework

4.2.2 Households

There is a continuum of households of measure one. The representative household maximizes its expected life-time utility which is given as follows:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \eta \frac{L_t^{1+\psi}}{1+\psi}\right) \tag{4.2.1}$$

where C_t is composite consumption, and L_t is labor supply. Composite consumption is a function of only domestically produced differentiated goods $C_t(i)$, $C_t = (\int_0^1 C_t(i)^{\frac{\rho-1}{\rho}} di)^{\frac{\rho}{\rho-1}}$, with $\rho > 1$. The implied consumer price index CPI is then $P_t = (\int_0^1 P_t(i)^{1-\rho} di)^{\frac{1}{1-\rho}}$, where $P_t(i)$ is the price of differentiated good i.

Households have access to financial markets with non state-contingent bonds in the form of both domestic and foreign currency denomination. Trade in foreign currency bonds is, however, subject to small portfolio adjustment costs, $\frac{\psi_D}{2}(D_{t+1} - \bar{D})^2$, [57] where $\bar{D}$ is an exogenous steady state level of net foreign debt and D_t is the amount of foreign debts. The household can borrow directly in terms of foreign currency at a given interest rate i_t^* , or in domestic currency assets at an interest rate i_t.

Each period, the representative household's revenue comes from final goods firms' profits Π_t, the supply of labor with wages W_t, incomes from exporting endowment goods $S_t P_{Xt}^* \bar{X}$, total debts he can borrow $S_t D_{t+1} + B_{t+1}$, less debt repayment from last period $(1 + i_t^*) S_t D_t + (1 + i_t) B_t$, as well as portfolio adjustment costs. Therefore, his budget constraint can be expressed as:

$$\begin{aligned} P_t C_t =& W_t L_t + \Pi_t + S_t D_{t+1} + B_{t+1} + S_t P_{Xt}^* \bar{X} \\ &- (1 + i_t^*) S_t D_t - (1 + i_t) B_t - P_t \frac{\psi_D}{2} (D_{t+1} - \bar{D})^2 \end{aligned} \tag{4.2.2}$$

Here S_t is the nominal exchange rate, P_{Xt}^* is the price of export goods in foreign currency, D_t is the outstanding amount of foreign currency debt and

[57] As shown in Schitt-Grohe and Uribe (2003), portfolio adjustment costs induce stationarity in economy's net foreign assets.

B_t is the stock of domestic currency debt, $\bar{X}$ is the endowment amount of export goods.

The household chooses each differentiated goods to minimize expenditure conditional on total composite consumption. Demand for each differentiated goods then can be derived as follows:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\rho} C_t \tag{4.2.3}$$

The household's first order conditions can be expressed as:

$$\frac{1}{1+i^*_{t+1}}\left[1 - \frac{\psi_D P_t}{S_t}(D_{t+1} - \bar{D})\right] = \beta E_t \left\{\frac{C_t^\sigma P_t}{C_{t+1}^\sigma P_{t+1}} \frac{S_{t+1}}{S_t}\right\} \tag{4.2.4}$$

$$\frac{1}{1+i_{t+1}} = \beta E_t \left(\frac{C_t^\sigma P_t}{C_{t+1}^\sigma P_{t+1}}\right) \tag{4.2.5}$$

$$W_t = \eta L_t^\psi P_t C_t^\sigma \tag{4.2.6}$$

Equations 4.2.4 and 4.2.5 represent the Euler equations for the purchase of foreign and domestic currency bonds. Equation 4.2.6 is the labor supply equation.

4.2.3 Production Firms

Differentiated final goods $Y(i)$ is a CES function of domestically produced value added $V(i)$ and imported intermediate goods $M(i)$.

$$Y_t(i) = \left[a^{\frac{1}{\epsilon}} V_t(i)^{\frac{\epsilon-1}{\epsilon}} + (1-a)^{\frac{1}{\epsilon}} M_t(i)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{4.2.7}$$

Value added V_t is in turn produced using only labor input as follows:

$$V_t(i) = A_{vt} L_t(i) \tag{4.2.8}$$

where A_{vt} is the productivity shock.

Cost minimizing behavior of final goods firm i implies that:

$$V_t(i) = a\left(\frac{W_t}{A_{vt} MC_t(i)}\right)^{-\epsilon} Y(i) \tag{4.2.9}$$

$$M_t(i) = (1-a)\Big(\frac{Z_t}{MC_t(i)}\Big)^{-\epsilon} Y(i) \tag{4.2.10}$$

where W_t, Z_t, MC_t is the nominal wage, the domestic price of imported intermediate goods, and the marginal cost, respectively.

4.2.4 Price Setting

Firms in the final sector set their prices as monopolistic competitors. I assume that each firm bears a small direct cost of price adjustment as in Rotemberg (1982), therefore, firms will only adjust prices gradually in response to demand or the marginal cost shocks. Firms are owned by domestic households, hence firms will maximize their expected profit stream using households' discount factor. The discount factor is defined as follows:

$$\Gamma_{t+1} = \beta \frac{P_t C_t^{\sigma}}{P_{t+1} C_{t+1}^{\sigma}}. \tag{4.2.11}$$

Using this, we can define the objective function of the final goods firm i as follows:

$$E_0 \sum_{t=0}^{\infty} \Gamma_t \big[P_t(i)Y_t(i) - MC_t Y_t(i) - \frac{\psi_P}{2}(\frac{P_t(i) - P_{t-1}(i)}{P_t(i)})^2\big] \tag{4.2.12}$$

where $\Gamma_0 = 1$, and $Y_t(i) = (\frac{P_t(i)}{P_t})^{-\rho} Y_t$ represents total demand for firm i's product, and the third expression inside the parentheses are the costs of price changes.

Firm i chooses its price to maximize (4.2.12). Because all final goods firms are alike, after imposing symmetry, the optimal price setting equation can be expressed as:

$$\begin{aligned} P_t \quad = \quad & \frac{\rho}{\rho-1} MC_t - \frac{\psi_P}{\rho-1}\frac{P_t}{Y_t}\frac{P_t}{P_{t-1}}\left(\frac{P_t}{P_{t-1}} - 1\right) \\ & + \frac{\psi_P}{\rho-1} E_t \left[\Gamma_{t+1}\frac{P_{t+1}}{Y_t}\frac{P_{t+1}}{P_t}\left(\frac{P_{t+1}}{P_t} - 1\right)\right] \end{aligned} \tag{4.2.13}$$

Notice that when the parameter ψ_P is zero, the final good price is just a markup over the marginal cost. Otherwise, the price follows a dynamic adjustment process.

4.2.5 Importers

In this section, I follows closely BGG and DLX to describe credit constraints of import firms (henceforth, importers).[58] As mentioned by BGG and others, financial market imperfections make external borrowing more costly than financing project out of internal resources and the borrowing premium depends on borrower's network relative to total required borrowing.

In particular, in order to finance intermediate goods imports, importers need to borrow in foreign currency from foreign lenders. Each importer faces an idiosyncratic shock $\omega \in (0, \infty)$, drawn from a distribution $F(\omega)$, with probability density function (pdf) $f(\omega)$, and expected value $E(\omega) = 1$. Shock ω is observed by the importer, but can only be observed by the lender through monitoring that incurs extra costs. The borrowing arrangement between lenders and importers is then constrained by the presence of private information. The optimal contract is a debt contract specified by a given amount of lending and a state-dependent threshold level of shock $\bar{\omega}$. If the importer reports shock exceeding the threshold, then a fixed payment $\bar{\omega}$ times the return on the import project is made to the lender, and there is no monitoring. But if reported shock is lower than the threshold, then the lender pays monitoring costs μ times the value of the project to monitor and receives the full residual amount of the import project.

An importer j, at the end of period t, plans to import M_{t+1}^j units of intermediate goods must pay nominal price $S_t P_{Mt}^* M_{t+1}^j$ to foreigners. Here, P_{Mt}^* is the price of imported intermediate goods, which is given to him at time t. If the importer begins with nominal net worth in domestic currency given by NW_{t+1}, then he needs to borrow in foreign currency an amount given by

$$D_{Mt+1}^j = \frac{1}{S_t}(S_t P_{Mt}^* M_{t+1}^j - NW_{t+1}^j) \tag{4.2.14}$$

The total expected return on the import project is $E_t(R_{Mt+1} S_t P_{Mt}^* M_{t+1})$, where R_{Mt+1} is the return rate from importing and will be defined below.

[58] See the Appendix for further details.

The optimal contract specifies a cut-off value of the importer's shock, $\bar{\omega}_{t+1}$, and an amount of imported intermediate goods, M_{t+1}. Under this contract structure, the importer receives an expected share $A(\bar{\omega}_{t+1})$ of the total return on the import project and the lender receives a share $B(\bar{\omega}_{t+1})$. In sum, $A(\bar{\omega}_{t+1}) + B(\bar{\omega}_{t+1}) + \phi_{t+1} = 1$, where ϕ_{t+1} represents the expected cost of monitoring.[59]

As shown in the Appendix, the first order conditions for the optimal contract can be expressed by the following two equations:

$$\frac{E_t\left\{R_{Mt+1}\left[B(\bar{\omega}_{t+1})\frac{A'(\bar{\omega}_{t+1})}{B'(\bar{\omega}_{t+1})} - A(\bar{\omega}_{t+1})\right]\right\}}{E_t\left[\frac{A'(\bar{\omega}_{t+1})}{B'(\bar{\omega}_{t+1})}\frac{S_{t+1}}{S_t}\right]} = 1 + i^*_{t+1} \tag{4.2.15}$$

$$\frac{R_{Mt+1}S_t}{S_{t+1}}B(\bar{\omega}_{t+1}) = (1 + i^*_{t+1})(1 - \frac{NW_{t+1}}{S_t P^*_{Mt} M_{t+1}}) \tag{4.2.16}$$

Equation (4.2.15) represents the relationship between the expected return from the import project (LHS) and the opportunity cost of funds for lender (RHS). Without private information(hence, no monitoring costs), the expected return would equal the opportunity cost of funds for the lender. However, the presence of moral hazard in the lending environment imposes an *external finance premium*, so that the return $E_t(R_{Mt+1})$ will be greater than the opportunity cost $(1 + i^*_{t+1})E_t\frac{S_{t+1}}{S_t}$ and the extent of this premium depends on the value of $\bar{\omega}$. The key characteristic of the BGG *financial acceleration* framework is that the borrowing premium is related to the borrowing amount. This relationship is reflected through the participation constraint equation for the lender (4.2.16). The smaller is the importers net worth NW_{t+1} relative to total required amount $S_t P^*_{Mt} M_{t+1}$, the more the importer must borrow, hence the higher the share $B(\bar{\omega}_{t+1})$ for the lender.

Equations (4.2.15) and (4.2.16) may then be used to show that the external finance premium $\frac{E(R_{Mt+1})}{(1+i^*_{t+1})E\frac{S_{t+1}}{S_t}}$ is increasing in the *leverage ratio*

[59] $A(\bar{\omega})$, $B(\bar{\omega})$, and ϕ_N may be written as follows: $A(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega f(\omega)d\omega - \bar{\omega}\int_{\bar{\omega}}^{\infty} f(\omega)d\omega$, $B(\bar{\omega}) = \bar{\omega}\int_{\bar{\omega}}^{\infty} f(\omega)d\omega + (1-\mu)\int_0^{\bar{\omega}} \omega f(\omega)d\omega$, $\phi_t = \mu\int_0^{\bar{\omega}} \omega f(\omega)d\omega$. It is straightforward to show that $A'(\bar{\omega}) \leq 0$, and $B'(\bar{\omega}) \geq 0$.

$\frac{S_t P^*_{Mt} M_{t+1}}{NW_{t+1}}$.[60] A fall in the importer's net worth or an increase in the financing amount or both will directly reduce the amount of imported intermediate goods by raising the external finance premium. In other words, financial acceleration implies that the more the importer borrows or the less net-worth he has or both then importer has to bear a higher cost of borrowing. The novel feature of this paper compared to the literature, is that a nominal exchange rate depreciation leads to both a fall in importers' net-worth and a rise in the financing amount, thereby *accelerating* the finance premium more than those analyzed in literature.

Following Carlstrom and Fuerst (1997) and BGG, I design the importers so that they are always constrained by the need to borrow so that financial acceleration always takes place. This can be obtained by assuming that a fraction of the existing stock of importers randomly die each period so that importers don't build up wealth to the extent that the borrowing constraint is non-binding and at the same time a fraction of importers arrives to replace these exiting ones.

At the beginning of each period, a non-defaulting importer j receives the return on the import project $R_{Mt} S_{t-1} P^*_{Mt-1} M_t(j)(\omega_t(j) - \bar{\omega}_t)$. Importers, then, die at any time period with probability $(1-\nu)$ and consume (all their net-worth) only in the period in which they die. Therefore, at any given period, a fraction $(1-\nu)$ of the return on the import project is consumed away. Since shocks on importers are i.i.d., the functional forms here can be aggregated so that the average return on import is $R_{Mt} S_{t-1} P^*_{Mt-1} M_t A(\bar{\omega}_t)$. The consumption for the importer, therefore, can be expressed as:

$$PC^m_t = (1-\nu) R_{Mt} S_{t-1} P^*_{Mt-1} M_t A(\bar{\omega}_t) \tag{4.2.17}$$

where C^m_t is the consumption level of importers when they die. And importers' aggregate net worth is equal to:

$$NW_{t+1} = \nu R_{Mt} S_{t-1} P^*_{Mt-1} M_t A(\bar{\omega}_t) \tag{4.2.18}$$

[60]See BGG, Appendix

Using the definition of $A(\bar{\omega})$ and the lender's participation constraint equation, we re-write importer's net-worth as:

$$\begin{aligned} NW_{t+1} =& \nu(1-\phi_t)R_{Mt}S_{t-1}P^*_{Mt-1}M_t \\ & - \nu(1+i^*_t)\frac{S_t}{S_{t-1}}(S_{t-1}P^*_{Mt-1}M_t - NW_t) \end{aligned} \tag{4.2.19}$$

Notice that an depreciation of current exchange rate reduces the importer's net worth by raising the value of existing foreign currency liabilities.

To conclude this section, we define the return on the import project. Importers sell their imported intermediate goods directly to final goods firms. Therefore, the gross nominal return rate from importing is,

$$R_{Mt}S_{t-1}P^*_{Mt-1} = Z_t \tag{4.2.20}$$

4.2.6 Monetary Policy Rules

The monetary authority uses domestic interest rate as the monetary instrument. The general form of the interest rate rule used can be expressed as

$$1 + i_{t+1} = \left(\frac{P_t}{P_{t-1}}\frac{1}{\bar{\pi}}\right)^{\mu_\pi}\left(\frac{S_t}{\bar{S}}\right)^{\mu_S}(1+\bar{i}) \tag{4.2.21}$$

The parameter μ_π allows the monetary authority to control the CPI inflation rate around the desired level of $\bar{\pi}$ whereas μ_S controls the degree to which interest rates attempt to control fluctuations in the exchange rate around a target level of $\bar{S}$. I will compare the properties of alternative exchange rate regimes under two main different assumptions regarding the values of these policy coefficients.

4.2.7 Equilibrium

Every period, each final goods market must clear. After imposing the symmetry between goods we obtain:

$$Y_t = C_t + C^M_t + \frac{\psi_D}{2}(D_{t+1} - \bar{D})^2 + \frac{\psi_P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 + \frac{Z_t M_t}{P_t}\phi_t \tag{4.2.22}$$

Equation (4.2.22) means demand for final goods comes from households' consumption, importers' consumption, portfolio adjustment costs, costs of price adjustment, and costs of monitoring loans.

The aggregate balance of payments condition for this small open economy can be derived by adding the budget constraint of the household and the importer and can be expressed as follows:

$$S_t P^*_{Mt} M_{t+1} + S_t(1 + i_t{}^*)[D_t + D_{Mt}] = S_t P^*_{Xt} \bar{X} + S_t[D_{t+1} + D_{Mt+1}] \quad (4.2.23)$$

Equation 4.2.23 indicates that total expenditures, which comprise of amount of importing and debt payments, must equal total receipts, which are the amount of exporting, plus new net foreign borrowing.

4.3 Calibration and Solution

The benchmark parameter choices for the model are described in Table 1. Following literature, this paper sets the inter-temporal elasticity of substitution in consumption to 0.5 or $\sigma = 2$. ψ is set to 1, implying the unity elasticity of labor supply, which is common in empirical literature.[61]

The elasticity of substitution between varieties of final goods determines the average price-cost mark-up, hence, this paper follows standard estimates from the literature in setting a 10 percent mark-up, so that $\rho = 11$.

One important thing in this paper is that I consider relatively low substitutability between domestic value-added intermediate goods and the imported intermediate goods in the production of final goods. Since developing countries often rely on imported intermediate goods, which are essential to domestic production but they have limited resources to produce for themselves, I follow Christiano et al (2007) and others to choose the elasticity of

[61] For example, Christiano, Eichenbaum, and Evans (1997) and set elasticity of labor supply to other values different from unity does not change the paper's conclusions but the implied volatility of key macroeconomic variables.

substitution between imported intermediate goods and value added intermediate goods less than unity, $\epsilon = 0.9$. [62]

I also assume that this small open economy starts out in a steady state with zero consumption growth, therefore, the world interest rate must equal the rate of time preference. I set the world interest rate equal to 6 percent annually, an approximate number used in the macro-RBC literature, so that at the quarterly level, this implies a value of 0.985 for the discount factor. I set $\bar{D}$ so that steady state total debt [63] is 40 percent of GDP, approximately that for East Asian economies in the late 1990's. The amount of tradable endowment $\bar{X}$ is chosen such that in steady state export is equal to 40% of GDP, which is also in the range of literature.

I set parameter a in the domestic production function so that the share of imported intermediate goods in production is 40 percent, implying a is equal to 0.6. This is consistent with the estimates given for intermediate imports as a fraction of GDP in Christiano et al (2006) for Thailand.

With respect to portfolio adjustment costs, I follow the estimate of Schmitt-Grohe and Uribe (2003) to set $\psi_D = .0007$.

To calibrate the degree of nominal rigidity in the model, I set the parameter governing the cost of price adjustment, ψ_P so that, if the model were interpreted as being governed by the dynamics of the standard Calvo price, adjustment process, all prices would adjust on average after 4 quarters. To match this degree of price adjustment requires a value of $\psi_P = 120$.

I choose a steady state risk spread of 350 basis points, which is higher than DLX and BGG but might be consistent with developing countries. I follow BGG to set leverage level to 2 and bankruptcy cost parameter μ equal to 0.12. Given the other parameters chosen, the implied savings rate

[62] In another paper by Christiano et al (2004), when labor appears in production of value-added, they even allow no substitutability between value-added good and imported intermediate goods but this model does not include capital so I keep relatively high value of ϵ

[63] Which include the debt of importer

of entrepreneurs is equal to 0.93.

In this paper, I consider two types of shock as in DLX: a) shocks to the world interest rate, b) shocks to (inverse) terms of trade. In the model, a) is represented by shocks to i_t^*, b) is represented by shocks to $\frac{P_{Mt}^*}{P_{Xt}^*}$.

The general form of the interest rule (4.2.21) allows for a variety of different types of monetary policy stances. This paper focuses analysis on two types of rules. The first rule is a CPI targeting rule (CPI rule), whereby the monetary authority targets the stability of domestic consumer price index so that he sets $\mu_\pi \to \infty$. Secondly, I analyze a simple fixed exchange rate $\mu_S \to \infty$, whereby the monetary authorities adjust interest rates so as to keep the nominal exchange rate from fluctuating.

The model is, then, solved numerically using a second order approximation to the dynamic stochastic system, where the approximation is done around the non-stochastic steady state by perturbation method. Since I later proceed to compare the two alternative monetary rules in terms of welfare,[64] it is necessary to use a second order approximation. For example, as demonstrated by Kim and Kim (2002), in a simple two-agent economy, a welfare comparison based on an evaluation of the utility function using a linear/first order approximation to the policy function may yield the spurious result such that welfare is higher under autarky than under full risk sharing, which is apparently wrong. Woodford (2003) also shows that a second order accurate representation of expected utility can be obtained only through a second order representation of the underlying dynamic system, except in special cases.

4.4 Dynamics under Alternative Monetary Rules

I now examine impacts of external shocks under the two alternative monetary rules. I assume that all shocks can be described as AR(1) processes

[64] Welfare in this economy is represented by the expected utility of households and importers.

Table 4.1: Model Calibration

Parameter	Value	Description
σ	2	Inverse of elasticity of substitution in consumption
β	0.985	Discount factor (quarterly real interest rate is $\frac{1-\beta}{\beta}$)
ϵ	0.9	Elasticity of substitution between value added goods and import goods in production
ρ	11	Elasticity of substitution between varieties
η	1.0	Coefficient on labor in utility
ψ	1.0	Inverse elasticity of labor supply
a	0.6	Share on value added goods in production
ψ_P	120	Price adjustment cost
ψ_D	0.0007	Bond adjustment cost
σ_ω	0.5	Standard deviation of importers' technology shocks
μ	0.12	Coefficient of monitoring cost for lenders
ν	0.93	Aggregate saving rate of importers

and adopt the VAR results of DLX for the US interest rate, a proxy for the world interest rate, with persistence 0.46 and the standard deviation of 0.0122 and (log) term of trade shocks with persistence 0.77 and standard deviation 0.013. There is negligible correlation between innovations between the world interest rate and terms of trade.

4.4.1 Impulse Responses

Figure 4.1 presents impulse responses with respect to a negative shock in terms of trade, i.e., an increase in the imported intermediate goods price relative to the export goods price. A key difference between the CPI rule and the fixed exchange rate rule is that the former attempts to stabilize final good prices and allows the exchange rate to fluctuate whereas the latter attempts to fix the exchange rate.

In particular, under the CPI targeting rule, the monetary authority adjusts the domestic interest rate (hence, the exchange rate) so that final goods firms don't have incentives to change the price level. In other words, the monetary authority adjusts the monetary instrument so that the marginal cost of final good production stays unchanged in response to shocks. On impact of the terms of trade shock, since the cost of imported intermediate goods is already determined from the previous period, the monetary authority has to adjust the domestic interest rate so that labor costs (the wage rate) remains unchanged. Consequently, labor supply, consumption, and output remain unchanged on impact under the CPI rule. Nevertheless, the negative terms of trade shock will raise the cost of imported goods from the next period, hence having negative impacts on domestic production and consumption. Since households tend to smooth consumption, the interest rate has to be decreased significantly on impact so that households maintain the same level of consumption this period as opposed to future declines in consumption. As a result, the exchange rate relatively increases (depreciates) on impact. The depreciation of the exchange rate under the CPI rule, combined with an increase in the imported goods price, has a strong impact on the import sector by not only increasing the domestic price of imported good prices but also worsening the importers' net-worth and thereby raising the borrowing risk premium. Consequently, from the second period after the shock, the exchange rate has to appreciate a lot to offset the initial depreciation and the increase in imported goods prices. Therefore, the domestic interest rate has to be increased accordingly from the second period, which then contributes to significant drops in consumption, output, and imported goods.

By contrast, under the fixed exchange rate regime, final good prices increase to adjust and households consume less consumption goods and more leisure (the substitution effect) on impact of the shocks. The responses of other variables under the fixed exchange rate rule are straightforward. It is shown by the figure that consumption, output, and imported intermediate goods are more volatile under the CPI rule while employment is more fluctuating under the fixed exchange rate rule.

Figure 4.2 presents the impulse responses with respect to a positive shock in the world interest rate.[65] In response an increase in the world interest rate, the monetary authority raises the domestic interest rate to fight against the depreciation of the exchange rate under the fixed exchange rate regime. An increase in the interest rate leads to decreases in consumption, output, hence in imported goods. By contrast, the exchange rate depreciates on impact under the CPI rule, which makes imported intermediate goods more costly. The financial acceleration applies so that the drop in the imported goods is as profound as that in the fixed exchange rate regime. Nonetheless, the impacts of the world interest rate on real variables are small and there are not clear differences under the two alternative monetary rules.

Table 2 compares the implied standard deviations of key macroeconomic variables under the two alternative monetary rules when the model is driven by the two aforementioned shocks. It is shown that volatilities of output, consumptions, and imported intermediate goods are higher under the CPI targeting rule than that under the fixed exchange rate. However, labor input under the fixed exchange rate rule is more volatile than that under the CPI targeting rule. The reason goes as follows. Monetary policies under the CPI rule aim to stabilize the marginal cost of final good production, which consists of labor costs and imported intermediate good costs. Since the latter is determined from the previous period the monetary authority adjusts the domestic interest rate to stabilize the labor cost, which lead to a relatively stable labor market under the CPI rule. However, as explained above, exchange rate fluctuations under the CPI rule with the presence of a high exchange rate pass-through and liability dollarization have strong impacts on output, consumption, and intermediate goods. High volatility in these key macroeconomic variables may explain the stylized-fact that emerging economies are reluctant to let their exchange rates fluctuate or

[65] I scale up the IRs by 100 times.

the so-called “fear of floating”.

4.4.2 Welfare Evaluation of Alternative Monetary Policy Rules

I then proceed to compute welfare of the economy under each monetary policy regime. The solution method produces a second order accurate measure of expected utility. I follow DLX to modify the way taking into account the welfare of importers. The welfare of households, as usual, can be measured as follows:

$$E_0 \sum_t^{\infty} \beta^t U(C_t, N_t) \tag{4.4.24}$$

Since importers are risk neutral, gain utility only from final goods consumption, and consume at any time period with probability $1 - \nu$, we can express the utility of importers with unit measure in total as:

$$E_0 \sum_t^{\infty} \beta^t C_t^m \tag{4.4.25}$$

given the assumption that the monetary authority discounts the utility of future importers at the same rate that private households discount future utility.

The last column of Table 4.2 shows the implied welfare results: The welfare of economy under the fixed exchange rate regime is higher than that under the CPI targeting rule. These results are consistent with above implied volatility of key macroeconomic variables and therefore confirming the “fear of floating” phenomenon.

Figure 4.1: **IRs: Terms of Trade Shock** $\frac{P^*_{Mt}}{P^*_{Xt}}$

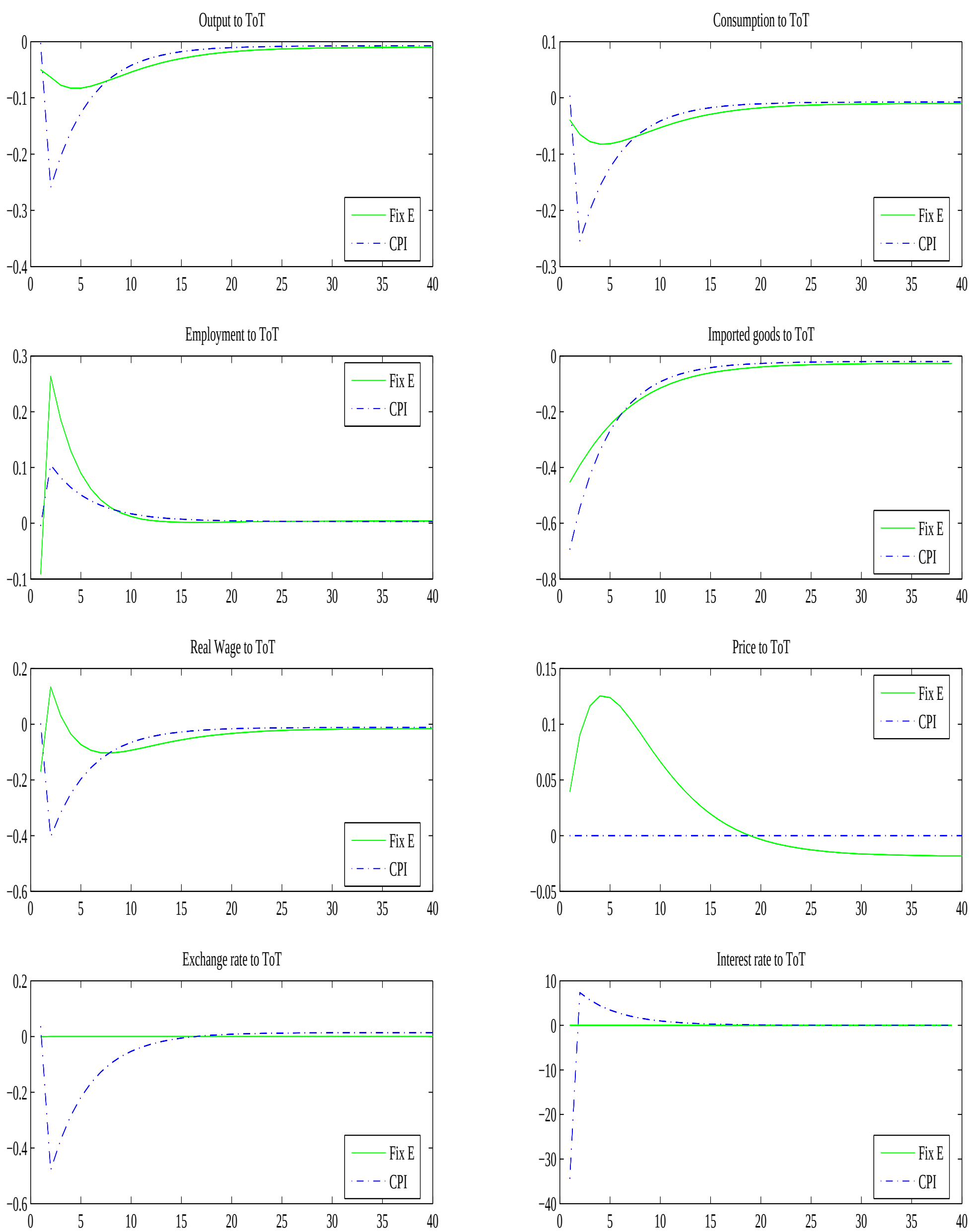


Figure 4.2: **IRs: World Interest Rate Shock i^*:**

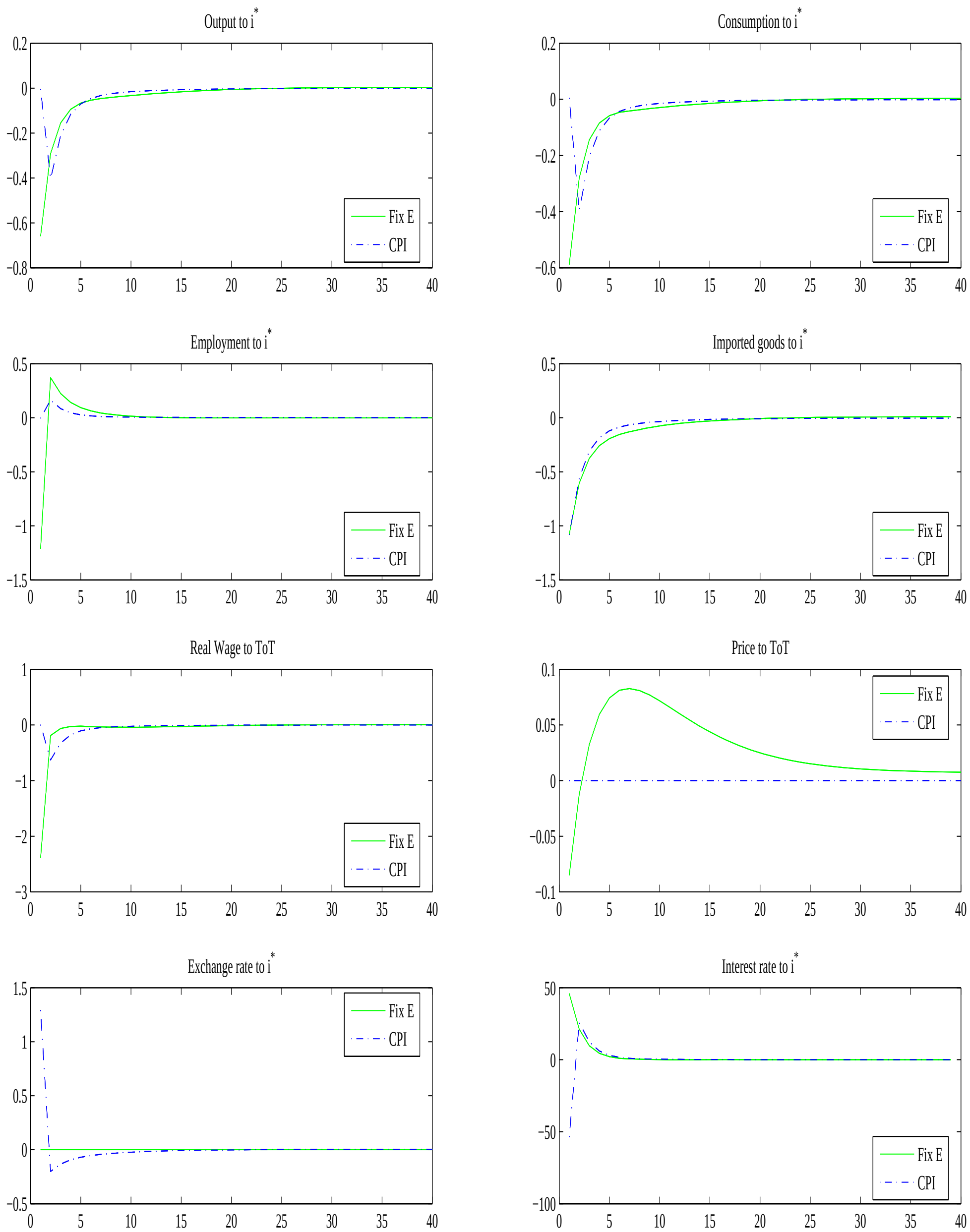


Table 4.2: Standard Deviations

	Output	Cons	Intermediate	Labor	Inflation	Nom.ER	Nom. IR	Exp. Utility
Fix. ER	0.46	0.46	1.47	0.51	0.10	0	0.01	-140.36
CPI	0.61	0.60	1.62	0.25	0	1.08	0.47	-144.96

[b] Note: CPI refers to a monetary rule which keeps the CPI inflation rate fixed, and FER refers to a monetary rule which keeps the nominal exchange rate fixed. Variables are Output, Consumption, Labor, Intermediate goods, Real Exchange Rate, Real Interest Rate,Inflation, Nominal Exchange Rate, Nominal Interest Rate, Expected Utility.

4.5 Conclusions

This chapter considers a small open highly dollarized economy borrowing in foreign currencies to import intermediate goods and facing borrowing constraints. The chapter quantitatively shows that "fear of floating" can be justified as a discretionary optimal monetary policy because floating the exchange rate leads to relatively more volatile domestic production, consumption, and import, therefore lowering welfare in response to external world shocks.

Chapter 5

Conclusions

This dissertation comprises three essays on business cycles in open economies. The first essay addresses housing investment and the impacts of mortgage market innovations. The second one sheds light on the anomalies in cross-country correlation in employment between international business cycle models and empirical data. The last one provides research on the macroeconomic consequences of two alternative monetary policies in emerging economies.

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Appendix A

Solution Method

I solve the models by the perturbation method.[66] Particularly the set of optimality conditions of the economy can be expressed as follows:

$$E_t\{F(Y_{t+1}, Y_t, X_{t+1}, X_t)\} = 0 \tag{A.0.1}$$

E_t is the mathematical expectation operator conditional on information available at time t, Y_t is the vector of non-predetermined variables, and $X_t = [x_t^1, x_t^2]'$ is the state variable vector, x_t^1 is endogenous predetermined state variables while x_t^2 is exogenous state variables. Particularly, x_t^2 follows exogenous process given as:

$$x_{t+1}^2 = \Lambda x_t^2 + \tilde{\eta}\bar{\sigma}\epsilon_{t+1} \tag{A.0.2}$$

where $\tilde{\eta}, \bar{\sigma}$ are given parameter. The solution of the optimal plan is of the form:

$$Y_t = g(X_t, \bar{\sigma}) \tag{A.0.3}$$

$$X_{t+1} = h(X_t, \bar{\sigma}) + \bar{\eta}\bar{\sigma}\epsilon_{t+1} \tag{A.0.4}$$

where $\bar{\eta} = [\emptyset, \tilde{\eta}]'$, these equations describe the policy and transition functions respectively. I compute a first order expansion of the two functions around the deterministic steady state for solutions in Chapter 2 and Chapter 3 while solutions in Chapter 4 require a second order expansion.

[66] For more details, see Schmitt-Grohe and Uribe (2004)

Appendix B

Chapter 2 Appendix

B.1 Basic Model Steady State

First, notice that the modified Euler equation at steady state which can be written as

$$U_c - \lambda = \beta U_c(1+r) \Rightarrow \lambda = U_c(1-\beta(1+r)) \tag{B.1.1}$$

Condition for binding borrowing constraint at steady state or $\lambda > 0$ requires that $\beta < 1/(1+r)$, where r is the steady state of real world interest rate.

Benchmark: Free Borrowing

$$1 = \beta(1-\delta_k+r^c) = \beta(1-\delta_k+r^h); \quad 1 = \beta(1+r) \Rightarrow r^c = r^h = r+\delta_k \tag{B.1.2}$$

$$\frac{k^c}{l^c} = (\frac{r+\delta_k}{\alpha_c})^{\frac{1}{\alpha_c-1}}; \quad \frac{k^h}{l^h} = (\frac{r+\delta}{q\alpha_h})^{\frac{1}{\alpha_h-1}} \tag{B.1.3}$$

$$w = (1-\alpha_c)(\frac{r+\delta_k}{\alpha_c})^{\frac{\alpha_c}{\alpha_c-1}} = q(1-\alpha_h)(\frac{r+\delta}{q\alpha_h})^{\frac{\alpha_h}{\alpha_h-1}} \tag{B.1.4}$$

$$q = \Big[\frac{1-\alpha_c}{1-\alpha_h}(r+\delta_k)^{\frac{\alpha_c}{\alpha_c-1}-\frac{\alpha_h}{\alpha_h-1}}\alpha_c^{-\frac{\alpha_c}{\alpha_c-1}}\alpha_h^{\frac{\alpha_h}{\alpha_h-1}}\Big]^{1-\alpha_h} \tag{B.1.5}$$

Mobile labor

$$\frac{qb}{y} = q\frac{(\frac{k^h}{l^h})^{\alpha_h} l^h}{(\frac{k^c}{l^c})^{\alpha_c} l^c} = q\frac{(1-\alpha_c)}{q(1-\alpha_h)}\frac{l^h}{l^c}; \quad \frac{l_h}{l_c} = \frac{qb}{y}\frac{(1-\alpha_h)}{(1-\alpha_h)} \tag{B.1.6}$$

$$l^c+l^h = l = \frac{40}{24*7} \Rightarrow l^c,\ l^h \Rightarrow k^c,\ k^h \Rightarrow y,\ h; \ \frac{i^c}{y} = \delta_k\frac{k^c}{y} = \delta_k\frac{\alpha_c}{r^c} = \delta_k\frac{\alpha_c}{r+\delta_k} \tag{B.1.7}$$

$$\frac{i^h}{y} = \delta_k \frac{k^h}{y} = \delta_k \frac{k^h}{qb}\frac{qb}{y} = \delta_k \frac{\alpha_h}{r+\delta_k}\frac{rein}{y} = \delta_k \frac{\alpha_h}{r+\delta_k}\frac{q\delta_h h}{y} \tag{B.1.8}$$

$$\frac{c}{y} = 1 - \frac{i^c}{y} - \frac{i^h}{y} - \frac{tb}{y} \Rightarrow c \tag{B.1.9}$$

$$q[1 - \frac{(1-\delta_h)}{1+r}] = \frac{U_h}{U_c} = \Big[\frac{\gamma c}{(1-\gamma)h}\Big]^{\frac{1}{\eta}} \Rightarrow \gamma = \Big[1 + \frac{c}{h}(\frac{1+r}{q(r+\delta_h)})^{\eta}\Big]^{-1} \tag{B.1.10}$$

Borrowing constraint

$$1 = \beta(1-\delta_k+r^c) = \beta(1-\delta_k+r^h); \quad r^c = r^h = \frac{1}{\beta} - 1 + \delta_k \equiv \bar{r} + \delta_k \tag{B.1.11}$$

$$\frac{k^c}{l^c} = (\frac{\bar{r}+\delta_k}{\alpha_c})^{\frac{1}{\alpha_c - 1}}; \quad \frac{k^h}{l^h} = (\frac{\bar{r}+\delta_k}{q\alpha_h})^{\frac{1}{\alpha_h - 1}} \tag{B.1.12}$$

$$w = (1-\alpha_c)(\frac{\bar{r}+\delta_k}{\alpha_c})^{\frac{\alpha_c}{\alpha_c-1}} = q(1-\alpha_h)(\frac{\bar{r}+\delta_k}{q\alpha_h})^{\frac{\alpha_h}{\alpha_h-1}} \tag{B.1.13}$$

$$q = \Big[\frac{1-\alpha_c}{1-\alpha_h}(\bar{r}+\delta_k)^{\frac{\alpha_c}{\alpha_c-1} - \frac{\alpha_h}{\alpha_h-1}} \alpha_c^{-\frac{\alpha_c}{\alpha_c-1}} \alpha_h^{\frac{\alpha_h}{\alpha_h-1}}\Big]^{1-\alpha_h} \tag{B.1.14}$$

Mobile labor

$$\frac{qb}{y} = q\frac{(\frac{k^h}{l^h})^{\alpha_h} l^h}{(\frac{k^c}{l^c})^{\alpha_c} l_c} = q\frac{(1-\alpha_c)}{q(1-\alpha_h)}\frac{l^h}{l^c}; \quad \frac{l^h}{l^c} = \frac{qb}{y}\frac{(1-\alpha_h)}{(1-\alpha_h)} \tag{B.1.15}$$

$$l_c + l_h = l = \frac{40}{24*7} \Rightarrow l_c \ l_h \Rightarrow k^c, \ k^h \Rightarrow y, \ h; \ \frac{i^c}{y} = \delta_k \frac{k^c}{y} = \delta_k \frac{\alpha_c}{r_c} = \delta_k \frac{\alpha_c}{\bar{r}+\delta_k} \tag{B.1.16}$$

$$\frac{i^h}{y} = \delta_k \frac{k^h}{y} = \delta_k \frac{k^h}{qb}\frac{qb}{y} = \delta_k \frac{\alpha_h}{\bar{r}+\delta_k}\frac{rein}{y} = \delta_k \frac{\alpha_h}{\bar{r}+\delta_k}\frac{q\delta_h h}{y} \tag{B.1.17}$$

$$\frac{c}{y} = 1 - \frac{i^c}{y} - \frac{i^h}{y} - \frac{tb}{y} \Rightarrow c; \quad \frac{qh}{y} = \frac{rein}{\delta_h} \Rightarrow \phi = \frac{\frac{d}{y}}{q\frac{h}{y}} \tag{B.1.18}$$

$$U_c - \lambda = \beta U_c(1+r) \Rightarrow \lambda = U_c(1-\beta(1+r)) \tag{B.1.19}$$

$$qU_c - \lambda\phi q = U_h + \beta U_c[q(1-\delta_h)] \tag{B.1.20}$$

$$\Rightarrow q[1-\beta(1-\delta_h) - \phi(1-\beta(1+r))] = \frac{U_h}{U_c} = \Big[\frac{\gamma c}{(1-\gamma)h}\Big]^{\frac{1}{\eta}} \tag{B.1.21}$$

$$\gamma = \Big[1+\frac{c}{h}(\frac{1}{q[1-\beta(1-\delta_h)-\phi(1-\beta(1+r))]})^{\eta}\Big]^{-1}; \quad w = \kappa\frac{l^{\omega-1}}{[(1-\gamma)\frac{x}{c}]^{\frac{1}{\eta}}} \Rightarrow \kappa \tag{B.1.22}$$

B.2 Extended Model Steady State

$$1 = \beta_1(1-\delta_k+r^c) = \beta_1(1-\delta_k+r^h); \quad 1 = \beta_1(1+r) \Rightarrow r^c = r^h = r+\delta_k \tag{B.2.23}$$

$$\frac{k^c}{l^c} = (\frac{r+\delta_k}{\alpha_c})^{\frac{1}{\alpha_c-1}}; \quad \frac{k^h}{l^h} = (\frac{r+\delta}{q\alpha_h})^{\frac{1}{\alpha_h-1}} \tag{B.2.24}$$

$$w = (1-\alpha_c)(\frac{r+\delta_k}{\alpha_c})^{\frac{\alpha_c}{\alpha_c-1}} = q(1-\alpha_h)(\frac{r+\delta}{q\alpha_h})^{\frac{\alpha_h}{\alpha_h-1}} \tag{B.2.25}$$

$$q = \Big[\frac{1-\alpha_c}{1-\alpha_h}(r+\delta_k)^{\frac{\alpha_c}{\alpha_c-1}-\frac{\alpha_h}{\alpha_h-1}}\alpha_c^{-\frac{\alpha_c}{\alpha_c-1}}\alpha_h^{\frac{\alpha_h}{\alpha_h-1}}\Big]^{1-\alpha_h} \tag{B.2.26}$$

Mobile labor

$$\frac{qb}{y} = q\frac{(\frac{k^h}{l^h})^{\alpha_h}l^h}{(\frac{k^c}{l^c})^{\alpha_c}l^c} = q\frac{(1-\alpha_c)}{q(1-\alpha_h)}\frac{l^h}{l^c}; \quad \frac{l_h}{l_c} = \frac{qb}{y}\frac{(1-\alpha_h)}{(1-\alpha_h)} \tag{B.2.27}$$

$$l^c + l^h = l = l_1 = l_2 = \frac{40}{24*7} \Rightarrow l^c,\ l^h \Rightarrow k^c,\ k^h \Rightarrow y,\ h \tag{B.2.28}$$

$$\frac{i^c}{y} = \delta_k\frac{k^c}{y} = \delta_k\frac{\alpha_c}{r^c} = \delta_k\frac{\alpha_c}{r+\delta_k} \tag{B.2.29}$$

$$\frac{i^h}{y} = \delta_k\frac{k^h}{y} = \delta_k\frac{k^h}{qb}\frac{qb}{y} = \delta_k\frac{\alpha_h}{r+\delta_k}\frac{rein}{y} = \delta_k\frac{\alpha_h}{r+\delta_k}\frac{q\delta_h h}{y} \tag{B.2.30}$$

$$\frac{c}{y} = 1-\frac{i^c}{y}-\frac{i^h}{y}-\frac{tb}{y} \Rightarrow c \tag{B.2.31}$$

$$\epsilon c_1 + (1-\epsilon)c_2 = c;\ \epsilon h_1 + (1-\epsilon)h_2 = h;\ c_2 + (q\delta_h + r\phi q)h_2 = wl_2 \tag{B.2.32}$$

$$\gamma_1 = \gamma_2 \Rightarrow \frac{c_1}{h_1} = \alpha\frac{c_2}{h_2} \Rightarrow h_2, c_2, h_1, c_1 \Rightarrow \gamma_1 = \gamma_2 = \gamma = \Big[1+\frac{c_1}{h_1}(\frac{1+r}{q(r+\delta_h)})^{\eta}\Big]^{-1} \tag{B.2.33}$$

B.3 Data

House Prices: Bank of International Settlements via Markus Kramer [67]. In particular, (1) File *Residential_property_prices.csv* is used for most countries from "National sources" as per detailed documentation, (2) Residential Prop prices IT.xls for Italy from Nomisma. Japanese house prices, however, are taken from Datastream with Code name JPLANDPIF.

Consumer Price Index (CPI): Seasonally Adjusted (SA). GDP: constant or chained prices, SA. Sources: Datastream, OECD Stat.[68]

Housing investment, non-residential investment, aggregate investment: real values, SA. Sources: Datastream, OECD Stats. Code means Datastream Code.

- Australia: Datastream. Housing investment: Private dwelling, code AUFXCPDWD. Aggregate Investment: Private Gross Fixed Capital Formation (GFCF), code AUPRFXCPD. Non-residential investment is obtained by subtracting housing investment from aggregate investment. Q1 1980-Q1 2008.

- Austria: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1988-Q1 2008.

- Belgium: Datastream. Housing investment: GFCF-Housing, chained prices, code BGHOUINVD. Aggregate investment: GFCF, chained prices, code BGGFCFD. Non-residential investment: GFCF-Companies, code BGBUSINVD. Q1 1995-Q1-2008

- Canada: Datastream. Housing investment: Business GFCF Residential Structures (CN100112). Aggregate Investment: Business GFCF,

[67] Email: markus.kramer@bis.org
[68] http://www.oecd.org/home/

code CNGFCFD. Non-residential investment: Business GFCF Non-Residential Structures, code CNINRSEQD. Q1 1981-Q3 2007.

- Denmark: Datastream. Housing investment: GFCF-Housing, chained prices, code DKGFCFHOD. Aggregate investment: GFCF, code DKGFCFD. Non-residential investment: GFCF subtract GFCF Housing. Q1 1992-Q1-2008.

- Finland: Datastream. Housing investment: GFCF Construction-residential building, code FNGFCRESD. Aggregate investment: GFCF, constant prices, code FNGFCFD. Non-residential investment: GFCF subtract GFCF Housing. Q1 1990-Q1-2008.

- France: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1980-Q1 2008.

- Germany: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1991-Q1 2008.

- Italy: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1981-Q1 2008.

- Japan: Datastream. Housing investment: Gross Domestic Fixed Capital Formation (GDFCF) by type: DWELLINGS, constant prices, code JPGCFBLDD. Aggregate investment: GDFCF, code JPGFCFD. Non-residential investment: GDFCF subtract GDFCF Dwellings. Q1 1994-Q1-2008.

- Netherland: Datastream. Housing investment: GFCF DWELLINGS, constant prices, code NLGFRBCVD. Aggregate investment: GFCF, constant prices, code NLGFCFD. Non-residential investment: GFCF subtract GFCF Dwellings. Q1 1993-Q1-2008.

- New Zealand: Datastream. Housing investment: GFCF Residential building constant prices, code NZGFCFRBD. Aggregate investment: GFCF, constant prices, code NZGFCFD. Non-residential investment: GFCF subtract GFCF Housing. Q2 1987-Q1-2008.

- Norway: Datastream. Housing investment: GFCF Housing investment, constant prices, code NWGFCHSID. Aggre. investment: GFCF, NWGFCFD. Non-housing investment: GFCF subtract GFCF Housing. Q1 1980-Q1-2008.

- Spain: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1995-Q1 2008.

- Sweden: From OECD Stats, national currency, chained volume estimates. Housing investment: GFCF Housing. Aggregate investment: GFCF. Non-residential investment: GFCF subtract GFCF Housing. Q1 1993-Q1 2008.

- U.K: Office of National Statistics through Datastream. Housing investment: Private sector New Dwellings excl. Land, constant prices, code UKDFEAD. Aggregate Investment: GFCF, code UKNPQTD. Non-residential investment: Fixed Capital Formation, Non-dwellings, code UKTONDWLD. Output: constant prices GDP, code UKABMID. Non-durable goods is the household final consumption excluding durable goods, constant price, code UKJSRVD). Trade balance is equal to net export of goods, constant prices, code UKBALGSVD.

House price index, UK DCLG HOUSE PRICE INDEX (MIX ADJ.), code UKNSAQHPF. Q1 1980-Q3 2007.

- U.S: Datastream. Housing investment: Residential Private Domestic Investment, constant prices, code USGPDRESD. Aggregate Investment: Private Domestic Fixed Investment, code USGFCFD. Non-residential investment: Non-residential Private Domestic Fixed Investment, code USNRSINVD. Q1 1980-Q3 2007.

Appendix C

Chapter 3 Appendix

C.1 Solving Bond Economy Model

The representative agent in Home country chooses sequences {} to solve the problem

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t, l_t\big) \tag{C.1.1}$$

subject to

$$c_t + i_t + p_t^b b_{t+1} + \frac{\pi_b}{2}(b_{t+1})^2 = A_t (k_t)^\alpha (l_t)^{1-\alpha} + b_t \tag{C.1.2}$$

$$k_{t+1} = (1-\delta)k_t + s_{1t} - \frac{\phi}{2} k_t \Big[\frac{s_{1t}}{k_t} - \delta\Big]^2 \tag{C.1.3}$$

$$i_t = \sum_{i=1}^{4} \omega_i s_{it} \tag{C.1.4}$$

$$s_{j,t+1} = s_{j+1,t} \quad \text{for j = 1, 2, 3} \tag{C.1.5}$$

The Lagrangian associated with the Home agent's optimization problem can be written as:

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(.) + \lambda_t \left[A_t k_t^\alpha l_t^{1-\alpha} + b_t - c_t - \sum_{i=1}^{4} \omega_i s_{it} - p_t^b b_{t+1} - \frac{\pi_b}{2} b_{t+1}^2 \right] \\ & + \lambda_t \nu_{2t}(s_{2t} - s_{1t+1}) + \lambda_t \nu_{3t}(s_{3t} - s_{2t+1}) + \lambda_t \nu_{4t}(s_{4t} - s_{3t+1}) \\ & + \lambda_t q_t \left[(1-\delta)k_t + s_{1t} - \frac{\phi}{2} k_t \Big[\frac{s_{1t}}{k_t} - \delta\Big]^2 - k_{t+1} \right] \Bigg\} \end{aligned} \tag{C.1.6}$$

The optimality conditions associated with the Home representative agent's problem are:

$$U_c\big(c_t, l_t\big) = \lambda_t \tag{C.1.7}$$

$$\lambda_t \left(p_t^b + \pi_b b_{t+1}\right) = \beta \lambda_{t+1} \tag{C.1.8}$$

$$U_l\big(c_t, l_t\big) = -\lambda_t (1-\alpha)\frac{y_t}{l_t} \tag{C.1.9}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left(\frac{\alpha y_{t+1}}{k_{t+1}} + q_{t+1} \left[1 - \delta_k - \frac{\phi}{2}\left(\frac{s_{1t+1}}{k_{t+1}} - \delta_k\right)^2 + \left(\frac{s_{1t+1}}{k_{t+1}} - \delta_k\right)\frac{\phi s_{1t+1}}{k_{t+1}} \right] \right) \tag{C.1.10}$$

$$-\lambda_t \nu_{2t} + \beta E_t \left\{ \lambda_{t+1} q_{t+1} \left[1 - \phi\left(\frac{s_{1t+1}}{k_{t+1}} - \delta_k\right)\right] - \omega_1 \lambda_{t+1} \right\} = 0 \tag{C.1.11}$$

$$-\lambda_t \nu_{3t} + \beta E_t \left\{ \lambda_{t+1} \nu_{2t+1} - \omega_2 \lambda_{t+1} \right\} = 0 \tag{C.1.12}$$

$$-\lambda_t \nu_{4t} + \beta E_t \left\{ \lambda_{t+1} \nu_{3t+1} - \omega_3 \lambda_{t+1} \right\} = 0 \tag{C.1.13}$$

$$-\omega_4 \lambda_t + \lambda_t \nu_{4t} = 0 \tag{C.1.14}$$

$$c_t + i_t + p_t^b b_{t+1} + \frac{\pi_b}{2}(b_{t+1})^2 = A_t (k_t)^{\alpha} (l_t)^{1-\alpha} + b_t \tag{C.1.15}$$

$$k_{t+1} = (1-\delta)k_t + s_{1t} - \frac{\phi}{2} k_t \left[\frac{s_{1t}}{k_t} - \delta\right]^2 \tag{C.1.16}$$

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha} \tag{C.1.17}$$

$$s_{j,t+1} = s_{j+1,t} \quad \text{for j = 1, 2, 3} \tag{C.1.18}$$

Similar conditions for the Foreign representative agent.

The world market clearing condition for bonds is:

$$b_{t+1} + b_{t+1}^* = 0 \quad \text{bond economy} \tag{C.1.19}$$

In solving the system, I replace the budget constraint for Foreign representative agent:

$$c_t^* + i_t^* + p_t^b b_{t+1}^* + \frac{\pi_b}{2}(b_{t+1}^*)^2 = y_t^* + b_t^*; \quad \text{foreign country} \tag{C.1.20}$$

by the unified world resource constraint for the single produced good:

$$(y_t - c_t - i_t) + (y_t^* - c_t^* - i_t^*) = 0 \tag{C.1.21}$$

C.2 Solving Complete Market Model

When financial markets are complete, the competitive equilibrium is Pareto optimal. Hence, we can derive the equilibrium system using an equal weight planner problem. The planner maximizes the sum of expected lifetime utilities

$$U = E_0 \sum_{t=0}^{\infty} \left\{ \beta^t U\big(c_t, l_t\big) + \beta^t U\big(c_t^*, l_t^*\big) \right\} \tag{C.2.22}$$

subject to:

$$c_t + i_t + c_t^* + i_t^* = A_t (k_t)^{\alpha} (l_t)^{1-\alpha} + A_t^* (k_t^*)^{\alpha} (l_t^*)^{1-\alpha} \tag{C.2.23}$$

$$k_{t+1} = (1-\delta) k_t + s_{1t} - \frac{\phi}{2} k_t \Big[\frac{s_{1t}}{k_t} - \delta \Big]^2 \tag{C.2.24}$$

$$k_{t+1}^* = (1-\delta) k_t^* + s_{1t}^* - \frac{\phi}{2} k_t^* \Big[\frac{s_{1t}^*}{k_t^*} - \delta \Big]^2 \tag{C.2.25}$$

$$i_t = \sum_{i=1}^{4} \omega_i s_{it} \tag{C.2.26}$$

$$i_t^* = \sum_{i=1}^{4} \omega_i s_{it}^* \tag{C.2.27}$$

$$s_{j,t+1} = s_{j+1,t} \quad \text{for j = 1, 2, 3} \tag{C.2.28}$$

$$s_{j,t+1}^* = s_{j+1,t}^* \quad \text{for j = 1, 2, 3} \tag{C.2.29}$$

The Lagrangian associated with the social planner's optimization problem can be set up similarly as the one in the bond economy above. The optimality conditions associated with this problem are straightforward.

Appendix D

Chapter 4 Appendix

D.1 Equilibrium

D.1.1 Households

The representative household's budget constraint is described in the text of Chapter 4. The household optimality conditions for labor supply, domestic bond demand, and foreign bond demand are as follows:

$$W_t = \eta L_t^{\psi} P_t C_t^{\sigma} \tag{D.1.1}$$

$$\frac{1}{1+i_{t+1}} = \beta E_t \left(\frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}} \right) \tag{D.1.2}$$

$$\frac{1}{1+i_{t+1}^{*}} \left[1 - \frac{\psi_D P_t}{S_t} (D_{t+1} - \bar{D}) \right] = \beta E_t \left(\frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}} \frac{S_{t+1}}{S_t} \right) \tag{D.1.3}$$

D.1.2 Production Firms

After imposing the symmetry condition, the optimality of production firms can be written as:

$$Y_t = \left[a^{\frac{1}{\epsilon}} V_t^{\frac{\epsilon-1}{\epsilon}} + (1-a)^{\frac{1}{\epsilon}} M_t^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{D.1.4}$$

$$V_t = A_{vt} L_t \tag{D.1.5}$$

$$V_t = a \Big(\frac{W_t}{A_{vt} MC_t} \Big)^{-\epsilon} Y \tag{D.1.6}$$

$$M_t = (1-a) \Big(\frac{Z_t}{MC_t} \Big)^{-\epsilon} Y \tag{D.1.7}$$

The price setting condition:

$$
\begin{aligned}
P_t &= \frac{\rho}{\rho-1}MC_t - \frac{\psi_P}{\rho-1}\frac{P_t}{Y_t}\frac{P_t}{P_{t-1}}\left(\frac{P_t}{P_{t-1}}-1\right) \\
&\quad + \frac{\psi_P}{\rho-1}E_t\left[\Gamma_{t+1}\frac{P_{t+1}}{Y_t}\frac{P_{t+1}}{P_t}\left(\frac{P_{t+1}}{P_t}-1\right)\right] \qquad \text{(D.1.8)}
\end{aligned}
$$

D.1.3 Importer

The details of the optimal contract are derived below. Here we outline the specification of one importer's behavior for the solution of the model. Each period, the importer borrows in foreign currency an amount:

$$
D_{Mt+1} = \frac{1}{S_t}(S_t P^*_{Mt} M_{t+1} - NW_{t+1}) \qquad \text{(D.1.9)}
$$

The first order conditions for the optimal contract are:

$$
\frac{E_t\left\{R_{Mt+1}\left[B(\bar{\omega}_{t+1})\frac{A'(\bar{\omega}_{t+1})}{B'(\bar{\omega}_{t+1})} - A(\bar{\omega}_{t+1})\right]\right\}}{E_t\left[\frac{A'(\bar{\omega}_{t+1})}{B'(\bar{\omega}_{t+1})}\frac{S_{t+1}}{S_t}\right]} = 1 + i^*_{t+1} \qquad \text{(D.1.10)}
$$

$$
\frac{R_{Mt+1}S_t}{S_{t+1}}B(\bar{\omega}_{t+1}) = (1+i^*_{t+1})(1 - \frac{NW_{t+1}}{S_t P^*_{Mt} M_{t+1}}) \qquad \text{(D.1.11)}
$$

$A(\cdot)$ is defined as the expected fraction of the return on capital accruing to the entrepreneur as part of the optimal contract. We may write is as:

$$
A(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega f(\omega)d\omega - \bar{\omega}\int_{\bar{\omega}}^{\infty} f(\omega)d\omega
$$

As shown later on this Appendix:

$$
A(\bar{\omega}) = \frac{1}{2}erfc\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) - \frac{\bar{\omega}}{2}erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right)
$$

where $erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt$ is the "complementary error function". Likewise the share of returns to the lender, net of monitoring costs, is

$$B(\cdot) = \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega + (1-\mu) \int_0^{\bar{\omega}} \omega f(\omega) d\omega$$

Also be shown later on:

$$B(\bar{\omega}) = \frac{\bar{\omega}}{2} erfc \left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) + (1-\mu)\frac{1}{2} \left[1 + erf \left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \right]$$

where $erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$ is the "error function".

We define ϕ_t as the fraction of the return from importing that is wasted in monitoring:

$$\phi_t = \mu \int_0^{\bar{\omega}_t} \omega f(\omega) d\omega$$

The case when ω_t^i is log-normally distributed with $E(ln\omega) = -\frac{\sigma_\omega^2}{2}$ and $Var(ln\omega) = \sigma_\omega^2$ is described in detail below.

The importer's consumption:

$$PC_t^m = (1-\nu) R_{Mt} S_{t-1} P_{Mt-1}^* M_t A(\bar{\omega}_t) \tag{D.1.12}$$

and the aggregate net-worth is:

$$\begin{aligned} NW_{t+1} =& \nu(1-\phi_t) R_{Mt} S_{t-1} P_{Mt-1}^* M_t \\ & - \nu(1+i_t^*) \frac{S_t}{S_{t-1}} (S_{t-1} P_{Mt-1}^* M_t - NW_t) \end{aligned} \tag{D.1.13}$$

Finally, the nominal return rate from importing:

$$R_{Mt} S_{t-1} P_{Mt-1}^* = Z_t \tag{D.1.14}$$

D.1.4 Monetary Policy Rules

$$1 + i_{t+1} = \left(\frac{P_t}{P_{t-1}} \frac{1}{\bar{\pi}} \right)^{\mu_\pi} \left(\frac{S_t}{\bar{S}} \right)^{\mu_S} (1 + \bar{i}) \tag{D.1.15}$$

D.1.5 Equilibrium

Final goods market must clearing conditions:

$$Y_t = C_t + C_t^M + \frac{\psi_D}{2}(D_{t+1} - \bar{D})^2 + \frac{\psi_P}{2}(\frac{P_t}{P_{t-1}} - 1)^2 + \frac{Z_t M_t}{P_t}\phi_t$$

The aggregate balance of payments condition:

$$S_t P_{Mt}^* M_{t+1} + S_t(1 + i_t{}^*)[D_t + D_{Mt}] = S_t P_{Xt}^* X + S_t[D_{t+1} + D_{Mt+1}]$$

The equilibrium of this economy is a collection of 18 sequences of allocation:

$(W_t, L_t, P_t, i_t, C_t, C_t^M, D_{t+1}, D_{Mt+1}, S_t, M_t, Y_t, MC_t, R_{Mt}, \bar{\omega}_t, Z_t, NW_{t+1}, V_t, X_t)$ satisfying the equilibrium conditions 1.1-1.18. I use perturbation method from Schmitt-Grohe and Uribe to solve this system of equations.

D.2 Derivation of External Finance Premium

In this section, I derive the external finance premium used in the text. I closely follow the model of BGG and DLX.

At the end of period t a continuum of importers indexed by j need to finance the import of $S_t P_{Mt}^* M_{t+1}^j$ that will be re-sold to domestic producers in period $t+1$. Importers are subject to idiosyncratic shocks so that if one unit of funds in terms of domestic currency is invested by importer j, then the return is given by $\omega^j R_{Mt+1}$, where R_{Mt+1} is the gross return of importer, and ω^j follows a log-normal distribution with with mean $-\frac{\sigma_\omega^2}{2}$ and variance σ_ω^2 and is distributed i.i.d. across importers and time.

The realization of ω^j can be observed by importers but not by lenders. Lenders, however, can discover the true realization at a cost ϕ times the total return from importing. Since both lenders and importers are risk neutral, standard results establish that the optimal contract between an importer and a lender is a debt contract, where the importer pays a fixed amount $\bar{\omega^j} R_{Mt+1} S_t P^*_{Mt} M^j_{t+1}$ to the lender if $\omega^j > \bar{\omega^j}$. If $\omega^j < \bar{\omega^j}$, the lender proceed to monitor the project, the importer gets nothing, and the lender receives the full amount of import net of monitoring costs. Therefore, the expected return to the importer can be expressed as:

$$R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} \left[\int_{\bar{\omega}^j_{t+1}}^{\infty} \omega^j f(\omega) d\omega - \bar{\omega}^j_{t+1} \int_{\bar{\omega}^j_{t+1}}^{\infty} f(\omega) d\omega \right] \equiv R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} A(\bar{\omega}^j_{t+1}) \quad \text{(D.2.16)}$$

The expected return to the lender is then given by:

$$R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} \left[\bar{\omega}^j_{t+1} \int_{\bar{\omega}^j_{t+1}}^{\infty} f(\omega) d\omega + (1-\mu) \int_0^{\bar{\omega}^j_t} \omega^j_{t+1} f(\omega) d\omega \right] \equiv R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} B(\bar{\omega}^j_{t+1}) \quad \text{(D.2.17)}$$

The lender should receive a return at least equal to the world opportunity cost, given by $R^*_{t+1} = 1 + i^*_{t+1}$. Therefore, the participation constraint of the lender in terms of the foreign currency can be written as:

$$\frac{R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} B(\bar{\omega}^j_{t+1})}{S_{t+1}} = \frac{R^*_{t+1} (R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} - NW^j_{t+1})}{S_t} \quad \text{(D.2.18)}$$

An optimal contract chooses the threshold value $\bar{\omega}^i_{t+1}$ and M^j_{t+1} to solve the following problem:

$$\max E_t \left(R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} A(\bar{\omega^i_{Nt+1}}) \right) \quad \text{(D.2.19)}$$

subject to the participation constraint (D.2.18).

The two first order condition implied by the contract is then:

$$E_t \left[R_{Mt+1} S_t P^*_{Mt} A(\bar{\omega}^j_{t+1}) \right] + E_t \lambda_{t+1} \left[\frac{R_{Mt+1} S_t P^*_{Mt} B(\bar{\omega}^j_{t+1})}{S_{t+1}} - \frac{R^*_{t+1} S_t P^*_{Mt}}{S_t} \right] = 0 \quad \text{(D.2.20)}$$

$$\lambda_{t+1}(\theta) = \frac{\pi(\theta) A'(\bar{\omega}^{j}_{t+1}(\theta)) S_{t+1}(\theta)}{B'(\omega^{\bar{i}}_{t+1}(\theta))} \tag{D.2.21}$$

where $\theta \in \Theta$ is a state of the world, $\pi(\theta)$ is the probability of state θ and λ_{t+1} is the Lagrange multiplier associated with the participation constraint. Substitute D.2.21 into D.2.20, we get:

$$E_t\left(R_{Mt+1}\left[\frac{A'(\omega^{\bar{j}}_{t+1})}{B'(\omega^{\bar{j}}_{t+1})} B(\omega^{\bar{j}}_{t+1}) - A(\omega^{\bar{j}}_{t+1}) \right] \right) = E_t\left[\frac{A'(\omega^{\bar{j}}_{t+1})}{B'(\omega^{\bar{j}}_{t+1})} \frac{S_{t+1}}{S_t} R^*_{t+1} \right] \tag{D.2.22}$$

Since ω^j is i.i.d across entrepreneurs, every importer actually faces the same financial contract, so we could drop the superscript j. Rearranging D.2.22 to get $(D.1.10)$ in the text.

The importers are assumed to die at any time period with probability $(1-\nu)$. Thus, at any given period, a fraction $(1-\nu)$ of importers' net-worth is consumed. So the consumption of importers is given by $D.1.12$. And the net worth NW_{t+1} is given by:

$$NW_{t+1} = \nu R_{Mt+1} S_t P^*_{Mt} M^j_{t+1} A(\bar{\omega}_t) \tag{D.2.23}$$

Use the fact that $B(\bar{\omega}) = 1 - A(\bar{\omega}) - \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega$ and imposing the participation constraint, we get D.1.13.

D.3 Derivation of $A(\cdot)$, $A'(\cdot)$, $B(\cdot)$ and $B'(\cdot)$

This derivation follows closely that on the Appendix of DLX's paper. By definitions:

$$A(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega \tag{D.3.24}$$

$$B(\bar{\omega}) = \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega + (1-\mu) \int_0^{\bar{\omega}} \omega f(\omega) d\omega \tag{D.3.25}$$

Since ω_t^i is log-normally distributed with mean $-\frac{\sigma_\omega^2}{2}$ and variance σ_ω^2, we know that

$$E(\omega) = \int_{-\infty}^{\infty} \omega f(\omega) d\omega = 1 \tag{D.3.26}$$

where the density function $f(\omega)$ is given by:

$$f(\omega) = \frac{1}{\sigma_\omega \omega \sqrt{2\pi}} \exp\left\{ -\frac{(\ln \omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} \tag{D.3.27}$$

Therefore,

$$\begin{aligned}
\int_{\bar{\omega}}^{\infty} \omega f(\omega) d\omega &= \int_{\ln \bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp\left\{ -\frac{(y + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} \exp(y) dy \\
&= \int_{\ln \bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp\left\{ -\frac{(y - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} dy \\
&= \frac{1}{\sqrt{\pi}} \int_{\ln \bar{\omega}}^{\infty} \exp\left\{ -\frac{(y - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d(\frac{y - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}) \\
&= \frac{1}{2} erfc\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \qquad \text{(D.3.28)}
\end{aligned}$$

where $erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2} dt$ is the *complementary error function*. Similarly,

$$\begin{aligned}
\bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega &= \bar{\omega} \int_{\bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \omega \sqrt{2\pi}} \exp\left\{ -\frac{(\ln \omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d\omega \\
&= \bar{\omega} \int_{\bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp\left\{ -\frac{(\ln \omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d \ln \omega \\
&= \bar{\omega} \int_{\ln \bar{\omega}}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left\{ -\frac{(\ln \omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d(\frac{\ln \omega + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}) \\
&= \frac{\bar{\omega}}{2} erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \qquad \text{(D.3.29)}
\end{aligned}$$

As results:

$$A(\bar{\omega}) = \frac{1}{2} erfc\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) - \frac{\bar{\omega}}{2} erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \tag{D.3.30}$$

At the same time,

$$\begin{aligned}\int_0^{\bar{\omega}} \omega f(\omega) d\omega &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\ln \bar{\omega}} \exp\left\{ -\frac{(y - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d(\frac{y - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}) \\ &= \frac{1}{2}\left[1 + erf\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \right] \qquad \text{(D.3.31)}\end{aligned}$$

$$B(\bar{\omega}) = \frac{\bar{\omega}}{2} erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) + (1-\mu)\frac{1}{2}\left[1 + erf\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \right] \qquad \text{(D.3.32)}$$

where $erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$ is the *error function.*

Next, since:

$$\begin{aligned}A'(\bar{\omega}) = &-\frac{1}{\sqrt{2\pi}\sigma_\omega}\left[\frac{1}{\bar{\omega}} \exp\left(-\frac{(\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right) - \exp\left(-\frac{(\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right) \right] \\ &- \frac{1}{2} erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \qquad \text{(D.3.33)}\end{aligned}$$

However,

$$\begin{aligned}\frac{1}{\bar{\omega}} \exp\left(-\frac{(\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right) &= \exp[-\ln(\bar{\omega})] \exp\left(-\frac{(\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right) \\ &= \exp\left(-\frac{(\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right) \qquad \text{(D.3.34)}\end{aligned}$$

Therefore,

$$A'(\bar{\omega}) = -\frac{1}{2} erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right) \qquad \text{(D.3.35)}$$

Note that $E(\omega) = 1$, so $B(\bar{\omega}) = 1 - A(\bar{\omega}) - \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega$, thus

$$B'(\bar{\omega}) = -A'(\bar{\omega}) - \frac{\mu}{\sqrt{2\pi}\sigma_\omega} \exp\left(-\frac{(\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right) \qquad \text{(D.3.36)}$$