CHILDREN'S TRANSITIVE REASONING: EFFECTS OF VISUAL-SPATIAL AND LINGUISTIC TASK CONDITIONS

by

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Abstract

This research was designed to explore the nature of reasoning. In general, three categories of theories about reasoning (the inferential rule approach, the mental models approach, and the operational constructive approach) are used to explain reasoning. In this research, a simple transitivity of length task was selected as the experimental vehicle to explore these approaches for their veracity. Each approach was assessed for spatial and linguistic conditions which might influence reasoning about transitive length relations. The length difference under consideration in the reasoning task, the order in which the premise statements about the length differences were presented and the linguistic relational term used to describe the length difference were selected as the experimental variables. Three measures of reasoning about transitive length relations were assessed: judgements, judgements-plus-justifications, and necessity understanding.

A between-within factorial, cross-sectional design was employed. The order of the premise statements (optimal/control) was manipulated as the experimental between-subjects factor. The two experimental within-subjects factors, length difference (large/small) and linguistic relational term ("longer"/"shorter"), were fully crossed and counterbalanced. Ninety-six preschool and school-age children, evenly divided by gender and age (5-6 years, 7-8 years, 9-10 years), participated in the study.

The developmental character of transitive reasoning in the age range studied was confirmed for two of the three measures of reasoning. More failures of judgement were observed when a large length difference was matched with the linguistic relational term "longer" and when a small length difference was matched with the linguistic relational term "shorter" than when

the length differences and relational terms were mismatched. The arrangement of the premise figure did <u>not</u> directly influence any measure of transitive reasoning but a large length difference in combination with the control premise figure was found to increase the frequency of transitive <u>judgements-plus-justifications</u>.

It is concluded from the analysis of the findings of this research that transitive reasoning about length is likely to result from constructive processes, rather then from application of logical rules. However, it is unclear whether the constructive processes in question are best explained in terms of cognitive <u>operations</u> or in terms of figurative <u>mental models</u>.

Table of Contents

Acknowledgment

My thanks to

Dr. Michael Chapman

May his influence

continue

to generate <u>better</u> ideas

in those of us

who knew him

briefly

Chapter 1:

Introduction

"Transitive reasoning" is reasoning about relations having the property of transitivity. By definition, a relation r is said to be transitive if the facts that A is related to B (A r B) and B is related to C (B r C) necessarily imply that A is related to C (A r C). That is, transitive reasoning is an inference of the conclusion A r C from the premises A r B and B r C. The subject of this research project was the form and the cognitive antecedents of transitive reasoning about asymmetrical relations of length. In asymmetrical relations, the relation applies in one direction. For example, if A is longer than B, and if B is longer than C, then A is longer than C. In such a case, the relation cannot simultaneously occur in the opposite direction as it would in symmetrical relations. That is, B is not longer than A, and C is not longer than B, so C cannot be longer than A. The relation is asymmetrical.

Three categories of theories about such reasoning were utilized in the design of the study and in explanation of the results. These three approaches are briefly introduced here, but are presented at length in Chapter 2. In the inferential rule approach (Braine & Rumain, 1983), reasoning is seen to be embodied in the understanding of language. Reasoning is the product of the application of mental rules of inference to a given set of propositions (i.e., the premise statements). The necessity of the conclusion so derived is guaranteed by the validity of the rules in question. According to the mental models approach (Johnson-Laird, 1983), a spatial mental model is constructed with mental tokens embued with the properties represented in the linguistic descriptions of the premises. The correct answer is read off from the mental model and represents a necessary inference to the extent that alternative possible models have been eliminated. The operational constructive approach

(Chapman & Lindenberger, in press) is described as a process by which the premise statements are understood in terms of potential operations carried out on the relevant objects and a conclusion is derived from the composition of those operations. That conclusion carries a sense of necessity to the extent that the composition of operations has a unique anticipated outcome.

In order to derive differential empirical predictions from those three theoretical approaches, the concept of a problem-solving space was employed. Newell (1980) hypothesizes that the problem space is the fundamental organizational unit of symbolic activity. Each problem space provides a way to represent a task so as to obtain a solution, each "consists of a set of symbolic structures (the states of the space) and a set of operators over the space" (p. 697). Such operators act in the transformation of the states of the space.

These problem-solving space constructs of states and operators were used by the researcher as tools to direct the analysis of the transitivity of length task. Descriptions of behaviors of interest which conform to the idea of states and operators were isolated from the transitivity of length task under each of the three approaches to reasoning. Since the problem of transitive relations of length is believed to be spatial in character (Huttenlocher, 1968; Trabasso, 1975), a spatial representation of the states of length difference is involved in the solution of the problem. A spatial problem-solving space was therefore hypothesized to be one of the organizational units involved in the representation of the problem and its solution. Further, the relation in the transitivity of length problem is linguistically described as "longer" or "shorter". Therefore, in addition to a spatial problem-solving space it was hypothesized that a linguistic problem-solving space could also be involved in the representation and solution of the problem. Such an approach is supported by Karmiloff-Smith (1979).

All three of the foregoing approaches may be said to postulate the use of a linguistic problem-solving space. That is, each approach identifies a state for the linguistic terms "longer" and "shorter"; and each specifies operators which act over the space. Table 1 summarizes how the linguistic terms "longer" and "shorter" are proposed to be states by each approach and how the operators for the linguistic problem-solving space are interpreted by each approach. The proposed states and operators of the linguistic problem-solving space are further described in Chapter 4. In addition, both the mental models and the

Table 1
Linguistic Problem Space State and Operators for each of the Theoretical Approaches to Reasoning

Theoretical Approach	Problem Space State of Relational Terms "longer" and "shorter"	Problem Space Operators
Inferential rule approach	natural language form	1) comprehending 2) calculating using rules
Mental models approach	cues	1) recognizing 2) mapping onto spatial tokens
Operational constructive approach	relations	1) operation of centering 2) operation of negating

operational constructive approaches presuppose the use of a spatial problem-solving space. Table 2 shows how each of these approaches interprets the spatial states of length and how the operators for the spatial problem-solving space are explained by each. Generally, there is a graphic-visual character to the interpretation of the spatial problem-solving space made by the mental models approach to reasoning. On the other hand, the operational constructive approach has an action orientation to the interpretation of the spatial problem-solving space. The proposed states and operators of the spatial problem-solving space are described in Chapter 3.

The experimental independent variables for this research included two manipulations involving the spatial problem-solving space, as well as one manipulation pertaining to the linguistic problem-solving space. In particular length differences were expected to facilitate reasoning in the spatial problem-solving space, insofar as large differences would be more salient in working memory (Frick, 1988). The order in which the premise statements were presented was also expected to influence reasoning through the spatial problem-solving space. An optimal premise order which supports the

Table 2
Spatial Problem Space State and Operators for the Mental Models and Operational Constructive Approaches to Reasoning

Theoretical Approach	Problem Space State of Length Differences large and small	Problem Space Operators
Mental models approach	constructed tokens which are representations	1) procedures
Operational constructive approach	constructed symbols of comprehended concepts	1) operation of comparing 2) operation of adding

construction of a link between the end terms in the transitive length series (Johnson-Laird, 1983) was expected to facilitate transitive reasoning. The relational terms used to describe the length differences were expected to influence transitive reasoning through the linguistic problem-solving space. All of the theoretical approaches support the likelihood of a cognitive bias toward better understanding of the relational term "longer". It was therefore predicted that the use of the relational term "longer" could facilitate reasoning about transitive length differences.

The foregoing considerations clearly point to the use of a main effects model for predicting the effects of the experimental variables. However, interactions are also possible. For example, emerging evidence concerning the development of linguistic spatial references has led Tanz (1980) to propose the possibility of an interaction between length differences and the linguistic relational terms used to describe them. It was suggested that when the relational term "longer" is paired with a large length difference and when the relational term "shorter" is paired with a small length difference, transitive reasoning might be enhanced. This interaction model differs from and is incompatible with the main effects model described above. The question is therefore, which model best fits the observed findings.

In summary, the overall purpose of my research was to explore spatial and linguistic conditions in the reasoning context that might influence children's transitive reasoning. The dependent variables were three levels of transitive reasoning about length. They were assessed through children's judgements, their justifications of those judgements, and their understanding of the necessity of the transitive judgement.

A between- and within-subjects cross-sectional design was employed. The order of the premise statements (optimal/control) was manipulated as the single experimental between-subjects factor. The two experimental within-subjects factors included a spatial condition in which the length difference (1.5 inches/3.81 cm & .25 inches/.635 cm) between the terms of the premise was manipulated and a linguistic condition in which the relational term descriptive of the length differences ("longer"/"shorter") between the terms was manipulated. The cross classification of the two within-subjects factors produced four task conditions. To control for order effects, the order in which those conditions were presented to subjects was completely counterbalanced. Ninety-six preschool and school-age children, evenly divided by gender and age (5-6 years, 7-8 years, 9-10 years) participated in the study.

The three approaches to reasoning are presented in Chapter 2. More complete explanations of spatial representation and spatial linguistic forms attached to each of the three approaches to reasoning are discussed in Chapters 3 and 4. An overview of the study purpose, the research questions, and the major hypotheses of the project are presented in Chapter 5. The method is detailed in Chapter 6. The results and discussion can be found in Chapters 7 and 8, respectively. In Appendix A a glossary of terms is included. Task protocols and consent forms are presented in Appendices B and C.

Chapter 2:

Reasoning and Transitive Length Relations

This chapter begins with a review of the three approaches to reasoning described in Chapter 1: the inferential rule approach, the mental models approach, and the operational constructive approach. Each approach is outlined, applied to the transitive length problem and critiqued as to its completeness as an explanation of reasoning in general. The effect of limitations the subject brings to the task and of task conditions left uncontrolled are also considered. The mechanisms for the development of better forms of reasoning are delineated for each approach. Finally, a system of strategies proposed to possibly enhance reasoning is described as a means for selecting the independent variables for this research.

Three Approaches to Reasoning

Inferential Rule Approach

In the inferential rule approach, reasoning is believed to be the deductive product of mental rules that govern relations between premises and conclusions. Piaget's (Inhelder & Piaget, 1964) explanation of transitive reasoning in terms of "the addition of asymmetric relations" is often interpreted as an example of such an approach (Brainerd, 1978; Johnson-Laird, 1983). For Piaget however, the word "addition" does not imply the presence of a rule but refers instead to the mental activity involved in transitive inference. This interpretation of Piaget's theory is discussed more thoroughly later in the section on the operational constructive approach to reasoning. Instead, the views of Braine and Rumain (1983) are presented here as a developmental version of the inferential rule approach.

The assumption that reasoning is the necessary product of the application of logical rules or inference schemas to a set of premises, provides a natural reasoning explanation for correct inferences. A theory of reasoning, however, should provide an explanation for a correct inference when it occurs, as well as for the kinds of errors commonly observed. According to Henle (1962), errors of reasoning occur because reasoners incorrectly comprehend or interpret the information provided in a problem. Braine and Rumain (1983) distinguish two kinds of comprehension. The first kind is "ordinary" comprehension. It is typical of everyday conversation and reading. Its purpose is to arrive at the meaning that a speaker/writer intends. In order to arrive at ordinary comprehension, the listener/reader uses information beyond that found in the language alone. Such information includes what is known about the speaker/writer's motives, relevant and general subject matter knowledge, and conventions about speaking and writing. The second kind of comprehension is analytic. Its purpose is to discover what the sentence means, not what the speaker means, as is the case for ordinary comprehension. The object is to contrive sentences that mean exactly what the speaker/writer intends them to mean. Such a process is involved in the writing of laws and policy statements. Aids to understanding used in ordinary comprehension are ignored and there is a reliance on natural language understanding only. Accordingly, formal reasoning from premises relies on analytic comprehension.

Under this approach, according to Braine and Rumain (1983), it is most likely that rules of inference are acquired as children learn the meanings of the natural language counterparts of logical connectives such as if . . . then, and, and or. Therefore the development of reasoning in children could be explained in terms of two complementary processes: (a) the acquisition of inference schemas, some of which may be more difficult than others and (b) developmental changes in children's comprehension from the ordinary type to the analytical type. The first process would lead to greater competence in correct reasoning, and the second to the commission of fewer errors and fallacies, with respect to the understanding of the necessity of the conclusion.

Utilizing data from an experiment on reasoning by supposition (Pieraut-Le Bonniec, 1980), Braine and Rumain (1983) speculated about the development of the concepts of possibility and necessity. At 5 or 6 years of age, children can recognize an impossible state of affairs when logical incompatibilities are noticed. Possibility and necessity understanding, which is conditional on knowledge allowed by a known specific state of affairs, develops by 7 years of age. This level of understanding of necessity may be dependent on entailment or ordinary comprehension where construal beyond the specific facts of the situation is utilized. A final level of understanding of necessity, which was not discussed by Braine and Rumain (1983), may develop by the age of 10. The understanding of necessity demonstrated at this time would reflect the use of analytic comprehension. That is, the necessity of the reasoning conclusion would be dictated by lexical entry alone.

Finally, Braine and Rumain (1983) do not consider transitive reasoning as a type of logical reasoning. According to them it depends on the empirical knowledge that the relations involved are in fact transitive. The inference schema in such a case reflects a fact about the world, rather than a syntactic relation between premises and conclusion.

A task analysis of the transitivity of length task using the inferential rule approach is pictured in Figure 1. In the left column, the components of the inference including the premise, question, rule, and conclusion are labelled in sequence. In the right column, propositional representation corresponding to the components indicated in the left column are symbolized. The premise presentations are represented as relations of length between each stick pair. The rule is a mental inference schema that is applied to the information presented in the premises to yield the correct conclusion.

Figure 1
Task Analysis of the Transitivity of Length Problem Using Inferential Rule Approach

Component	Propositional Representation[a]
1. Premise:	(A > B)
2. Premise:	(B > C)
3. Question:	(A > C) or (A < C)?
4. Rule:	(for any x, y, z if x > y and y > z then, x > z)
5. Conclusion:	(A > C)

[a] The letters A, B and C refer to the objects being compared and the symbols ">" and "<"refer to the relations "longer than" and "shorter than" respectively.

The inferential rule approach is often considered a <u>competence model</u> of reasoning insofar as the rule represents the form of reasoning of which the subject is capable. It follows that the subject is capable of only those forms of reasoning for which he or she possesses corresponding rules. Thus, successful performance of a given type of task, above and beyond that attainable through random guessing, is taken as an indication that the individual possesses the rule necessary for performing that type of task. However, the converse is not the case: unsuccessful performance cannot be taken as an unambiguous indicator of the <u>absence</u> of the requisite rule, because failure can result from numerous other factors. For example, common errors in reasoning can result from faulty comprehension or incorrect interpretation of the information provided in a problem (Braine & Rumain, 1983).

According to Johnson-Laird (1983, pp. 23-40), however, the inferential rule approach fails to account for several sources of error beyond faulty comprehension and interpretation, including: (a) the constraining action of limitations the subject brings to the task such as differences in attention and memory capacity; (b) the fact of horizontal decalage or the inability to transfer a reasoning structure learned with respect to one type of content to another type; (c) the use of extra-logical heuristics or short cuts to attain correct judgments for reasoning problems without the use of the inferential rule presumed necessary for solving that problem.

The first source of error unaccounted for in the inferential rule approach is the variety of subject limitations that constrain reasoning. Such constraints on performance have played a major role in explaining the presence or absence of transitive reasoning (Breslow, 1981). According to one argument, a major source of false negative measurement errors in assessing transitive inference is the failure to remember the premises (Bryant & Trabasso, 1971; Trabasso, 1975). Briefly, Bryant and Trabasso (1971) trained children to a criterion of eight successful trials out of ten memory trials for each premise before giving them the transitivity task. Through this method, very young children (4 years of age) were found to be capable of solving the transitive inference. Without such training, children are typically unable to solve transitivity problems until 7-8 years of age (Piaget & Inhelder, 1963; Smedslund, 1966). Bryant and Trabasso interpreted these results to mean that the inability to remember the premises was the major developmental limitation on children's transitive reasoning. This conclusion is controversial for three general reasons: it has since been shown that integration of premises can take place under specialized task conditions where specific visual or procedural cues are given (Kallio, 1982; Perner & Aebi, 1985; Chapman & Lindenberger, 1988); it has further been demonstrated that some children can reason transitively even though they cannot recall the premise statements (Halford & Galloway, 1977; Russell, 1981); and finally it has even been proposed that memory for premises and transitive reasoning are stochastically independent (Brainerd & Kingma, 1984).

The finding that children who remember premises sometimes do not reason transitively (Halford & Galloway, 1977; Russell, 1981) sparked a debate concerning the relation between memory for premises and reasoning. Brainerd and Kingma (1985) found that memory for the premises A > B and B > C were stochastically independent from the conclusion A > C, and they interpreted that finding to mean that transitive reasoning in general does not depend on memory for premises. In contrast, Chapman and Lindenberger (1988) found that the task version for which Brainerd and Kingma found independence between memory and reasoning could be solved using functional reasoning based on spatial position. That is, given a spatial array of objects that get longer towards the subject's right (or left), the correct answer can be inferred "as a function of" position without recourse to the premises A > B and B > C, the memory for which Brainerd and Kingma tested. A more complete explanation of the dependence of memory and reasoning in the transitive reasoning task was proposed by Chapman and Lindenberger (in press) who argued (a) that a sample of children solving a transitivity task can be decomposed into two subsamples, children who justify correct transitivity judgements with a composition of the premise relations A > B and B > C and children who do not, and (b) that a stochastic independence between reasoning and memory would be found only if the proportion of children in the first subsample was less than a certain threshold value. A reanalysis of the data (Chapman and Lindenberger, 1988) from previous studies supported the argument. The point made turns on the question of what constitutes demonstration of transitive reasoning. If correct judgement is sufficient to assess the presence of transitive reasoning, then memory for premises is not required. On the other hand, if transitive reasoning requires understanding of the relations of length demonstrated through justification, than transitive reasoning is dependent on memory for premises.

It is important to note that the findings of performance constraints imposed by memory on reasoning are compatible with the inferential rule approach. As indicated in the task analysis shown in Figure 1, successful reasoning depends on accurate representation of the premise relations. That is, successful reasoning would not result if the premises were not remembered or were not interpreted properly. That memory constrains reasoning by inferential rule therefore need only stimulate the development of a performance theory of memory or comprehension to predict such errors under the inferential rule approach.

The second problem for the inferential rule approach to reasoning mentioned by Johnson-Laird (1983) is the presence of horizontal decalage in reasoning. That is, within a structurally defined level of reasoning such as concrete operations, the same structure, say addition of relations, may appear at different ages for different contents (Chapman, 1988). For example, Piaget and Inhelder (1974) found that transitivity problems involving weight tended to be solved at a later age than problems involving physical quantity. Horizontal decalage is an issue which is relevant to the inferential rule approach, because a purely syntactic rule of inference once established should be immune to change in the content of the premise relations. Such need not be the case for performance models such as those established in the mental models approach or in the operational constructive approach. For those models, each problem situation is unique and therefore has an experientially established reasoning separate from any other problem.

With respect to the length-weight decalage in transitive inference, Chapman and Lindenberger (1988) found such a decalage between length and weight only when the size and weight of comparison objects were both varied in an uncorrelated manner in the weight task, but not when the objects compared were the same size. Those findings support the hypothesis that the decalage in transitivity of length and weight is the result of children's tendency to infer weight as a function of size. That is, its origin lies in the particular possibilities afforded by weight as a physical content. Such decalages, Chapman and Lindenberger (1989) called <u>content decalages</u> as opposed to <u>procedural decalages</u> which result from modifications of the procedure used in administering the tasks.

Whether Piaget's theory is best understood as an example of the inferential rule approach is a question that is considered in the sections to follow. In any case, the problem of horizontal decalage in transitive reasoning remains for the inferential rule approach, because the explanation of such decalages involves considerations beyond the kinds of syntactic structures posited in that approach.

The <u>third</u> problem left unaddressed by the inferential rule approach is the implementation of extra-logical heuristics or strategies in the solution of common reasoning problems. Such strategies contribute to correct judgements, even if no sense of necessity is achieved. In transitive reasoning about length differences, most extra-logical strategies that have been suggested are concerned with constructing a linear order from which the answer to the request for transitive inference is derived. Breslow (1981) provides a detailed and critical review of these sources of false-positive errors of measurement.

The simplest strategy to follow in answering a request for transitive inference would be a direct comparison of the terms with all three terms simultaneously perceptible. Such a solution would seem to be blocked in the standard task, in which the premises are presented singly, or in tasks in which a sufficient distance between the objects renders direct comparisons impossible (Braine, 1959; Brainerd, 1973b; Brainerd & Kingma, 1984, 1985; Bryant & Trabasso, 1971; Smedslund, 1963, 1966). However, the latter control has been found to enable children to use a form of functional reasoning called "spatial ordering" (Chapman & Lindenberger, 1988, p. 546), according to which the transitivity problem is solved by remembering that "things get bigger to the right". That is, judgements of length are inferred as a function of spatial orientation rather than from length relations themselves. Other researchers have found that performance on such tasks can be improved when salient information regarding serial order (Kallio, 1982) or graduation of relevant dimensions (Perner & Aebi, 1985) are provided.

Two other forms of extra-logical heuristics have been proposed to explain correct transitive judgement in very young children (age 4) who have been given rigorous training on the premise statements. De Boyon-Bardies and O'Regan (1973) proposed that children do not form a linear order, but use a system of labeling in which the extreme sticks are consistently labeled either long or short. The intermediate sticks are labeled through association with the extreme sticks or are not labeled at all. Request for inference is answered using secondary labeling. Concerning the question "A > C?", A is labeled long and C is labeled short, yielding a judgement that A is longer. Breslow (1981) offers a version of labeling combined with linear ordering. The sequential-contiguity model presupposes that young subjects (age 4) understand the relational terms only in a categorical way. That is, the premise terms become related because they appear together in premise pairs related through either of the linguistic relational terms, "shorter" or "longer". Only the end terms are labeled long or short and thus, only one of the comparison sticks is considered in answering the test question.

In summary, the evidence suggests that transitivity problems can often be solved through extra-logical strategies or heuristics. The question of what this implies for the inferential rule approach is addressed here. An adherent of that approach might argue simply that only versions of the transitivity task solved with the rule specified in Figure 1 are valid measures of transitive reasoning. That is, if tasks can be solved with extra-logical strategies, then they are not valid measures of such reasoning. The problem with that argument is that there is no universally agreed upon way of defining transitive reasoning much less of determining which tasks are valid measures of it. For instance Youniss and Dennison (1971) propose that seriation is a figurative aspect of transitive reasoning and that it is complementary to and dominated by operatory aspects of transitive inference. They propose that their findings cast doubt on the standard task as a developmental indicator of the presence of transitive reasoning since it does not neutralize the figurative aspect. Youniss and Furth (1973) further criticize that the training which Bryant and Trabasso (1971) utilized in their experimentation helped translate the premise information into an image of seriation which precludes the use of operatory structure in the solution of the problem. In a different vein, Trabasso (1975) points out that justification of transitive response and speed of that response is simply the "result of a more strongly associated set of linearly ordered codes rather than a result of qualitative differences in cognitive abilities" (p. 168). In such arguments, transitive reasoning is implicitly defined in terms of what is supposed to explain such reasoning. The question with respect to the inferential rule approach is whether, and how often, children actually use such a rule in solving transitivity problems. To the extent that children solve transitivity problems using extra-logical strategies, the inferential rule approach fails to provide an explanatory model of their competence.

Mental Models Approach

In the face of the foregoing problems, Johnson-Laird (1983) argued for a theory of reasoning without the application of inferential rules. The alternative he proposed was a mental models approach based on analogical reasoning (Braine & Rumain, 1983) wherein inferences regarding a certain state-of-affairs are drawn by analogy with a model of that state-of-affairs. According to Johnson-Laird (1983), inferences are drawn by means of a mental model of the problem situation. Such a mental model has several levels of representation (Johnson-Laird, 1983). The first and most basic level is perceptual. Reality is perceived through perceptual procedures and is encoded as a phonemic or graphemic representation. Further procedures encode those perceptual representations into propositional representations, which are meaningful in terms of their truth conditions: the objective conditions to which a given proposition would correspond if it were true (Russell, 1987). Finally, mental models are constructed using procedures that carry out finite searches of mental representations of the truth conditions corresponding to the premises of the inference. In this process, "elements" (p. 97) are assigned symbolic properties that correspond to our conception of the state of affairs in the premise statements, and are arranged on an "internal tableau" (p. 97).

In the case of transitive reasoning, a transitive inference can be produced by means of a mental model without a rule of transitivity. A spatial model is used to arrive at an integrated representation of the spatial descriptions in the premise statements. Either "direct comparison" (p. 136) or "digital approximations" (Johnson-Laird, 1983, p.137; Huttenlocher, 1968) can be employed in this process. Direct comparison is the representation of a premise such as (A > B) in terms of a mental model in which "A and B are lined up so that B's axis is coextensive with only part of A's axis" (p. 136). Digital approximation is the representation of a property such as length as a dimension and evaluation of the premises as values along that dimension.

The following spatial model procedures (Johnson-Laird, 1983) would be used in the solution of the transitivity of length task:

> 1) A procedure that begins the construction of a new spatial array whenever it is presented with an assertion that makes no reference to any items in an existing array. It inserts the items referred to by the assertion into an array in positions that satisfy the required spatial relation.
> 2) A procedure which, if one item referred to by an assertion is found in an existing array, inserts another item into the array in a position that satisfies the meaning of the assertion.
> 3) A procedure which, if one item in an assertion is found in one array and another item in the assertion is found in a separate array, combines the two arrays into an integrated array that satisfies the meaning of the assertions.
> 4) If both items in the assertion are found in the same array, there is a procedure that verifies whether the required spatial relation holds between them in the array.
> 5) If an assertion is true of a current array, then a procedure checks recursively whether the array can be rearranged in a way that is consistent with the previous assertions but so as to render false the current assertion.

6) If an assertion is false of a current array, then a procedure checks recursively whether the array can be rearranged in a way that is consistent with the previous assertions but so as to render the current assertion true (Johnson-Laird, 1983, p. 252-3).

The task analysis in Figure 2 demonstrates the use of the spatial procedures in the solution of the transitivity of length task. In the left column, the components corresponding to the premise, question, decision, and conclusion are listed in sequence. In the center-left column, the propositional representations corresponding to the components in the left column are symbolized. The spatial procedures employed in the construction of the mental model are given in the center-right column, and a schematic representation of the array constructed by the mental model procedures is shown in the right column.

In contrast to the inferential rule approach to transitive reasoning depicted in Figure 1, in which the movement from premises to conclusion occurred solely by means of propositional reasoning, the mental models approach implies that two distinct levels of representation are involved. The propositional representations of the premises are translated by means of the stated procedures into a mental model based on visual-spatial representation. The conclusion is then obtained by retranslating a feature of the complete

Figure 2
Task Analysis of the Transitivity of Length Problem Using a Mental Models Approach

Component	Propositional Representation	Procedure	Mental Model
1. Premise:	(A > B)	(1)	A - B
2. Premise:	(B > C)	(2,3)	A - B - C
3. Question:	(A > C) or (A < C)?		
4. Decision:		(4, 5, 6)	
5. Conclusion:	(A > C)		

mental model into propositional form. The movement from propositional representation to visual-spatial representation in the mental model and back again requires the use of a procedural semantics (Johnson-Laird, 1983). Accordingly, this procedural semantics relates language to the mental model. These semantics operate on the truth-conditions described by the discourse and represented in the propositions. In that view, the meaning of a proposition consists in the representation of the real-world conditions that would exist if the proposition were true (Russell, 1987). As described by Johnson-Laird, the mental model is a mental representation of the truth conditions of the propositions corresponding to the premises of the inference. The adequacy of truth-conditional mental representation and procedural semantics for this purpose are discussed later.

Johnson-Laird's theory of mental models is an attempt to replace the competence model of the inferential rule approach which uses natural reasoning with a performance model based on analogical reasoning. The truth conditions of the premises are represented by means of the procedures used to construct the mental models. Explicit inferences based on mental models constructed from those procedures do not appear to necessitate the postulate of a formal reasoning structure or schema in the mind. That is, the system of inferences arising from a mental model may behave in an entirely "logical" way, even though it does not employ the formal rules of inference. For Johnson-Laird (Wason & Johnson-Laird, 1972) reasoning is the extent to which the reasoner can appreciate all that follows from the premises and "remain unseduced by plausible but fallacious conclusions" (p. 2). In the transitive reasoning problem described in Figure 2, the correct transitive decision follows as an outcome of verification procedures. If procedure 6 is exhaustive (i. e., alternate models are ruled out), then the conclusion necessarily follows.

Note, however that this model is designed only to explain how necessary judgments are obtained; it does not explain how the subject <u>knows</u> the conclusion to be necessary. Interactionists (Piaget, 1971a; Campbell & Bickhard, 1986; Russell, 1987; Chapman, 1991a, 1991b) point out that a representational account of knowing is unacceptable because static representations cannot adequately reflect knowledge as action or knowledge of how to change things.

Johnson-Laird (1983), however, is a proponent of the functional approach to cognitive psychology. As such, the mind's ability to construct working models is viewed as a computational process which can be functionally described. That is, if the description matches or predicts the findings from empirical experiments on reasoning then it is judged as an adequate explanation of reasoning. The voluminous evidence presented by Johnson-Laird (1983) demonstrates that the mental models approach indeed seems capable of accounting for many of the errors and difficulties that arise in theorizing about reasoning.

The mental models approach also reportedly eliminates the need to consider the presence of "mental rules of inference" (p. 40) in the mind. On closer examination, the argument can be made that this goal was not achieved. Mental models are functionally described as effective procedures (Johnson-Laird, 1983) that can be specified and produce the functions or mappings which are the product of mental modelling. The specificity of effective procedures and their adequacy as a functional explanation of reasoning is not under question here. What is questioned is the origin and development of effective procedures.

Since they are not a product of reasoning, it is proposed here that they seemingly must be structures of reasoning which are found in the mind and are developed or pre-formed there.

In conclusion, the mental models approach is a functional model of cognition which was developed in response to the performance and theoretical inadequacies of the inferential rule approach. As such it succeeded in addressing many of the problems found in empirical studies not explained by the inferential rule approach, but it was seemingly unable to do so without positing the presence of effective procedures or structures in the mind (Johnson-Laird, 1983). A meaningful alternate approach to reasoning through the use of mental models would need to incorporate performance aspects of the mental models approach and address the issue of formal structure. Russell (1987) proposed that developmental processes be examined and that a Piagetian-derived model be explored as such an alternative. Under such a model, knowledge of abstract principles is derived from interaction of the subject with "refractory physical and social realities" (p. 45), not from rules or from procedures which represent reality. Attention to the role of interaction of the subject with reality is also supported by non-Piagetians (Kolers & Smythe, 1984; Kaufman, 1986). The operational constructive approach (Chapman & Lindenberger, in press) is such a Piagetian-derived approach. It is described and applied to the problem of transitive reasoning below.

<u>Operational Constructive Approach</u>

According to the operational constructive approach, the elements of logical reasoning include <u>figurative structures</u> and <u>operative structures</u>. Figurative structures are knowledge states and operative structures supply knowledge of transformations. Both are constructed through interaction with the physical and social environment. Figurative structures are constructed through the actions of perception, imitation, and mental imagery (Piaget, 1962). These states of knowledge are linked by the transformations of the operative structures of intelligence (Chapman, 1988). These actions of transformation are interiorized and grouped together into cohesive, complete, and comprehensive wholes called operational structures.

What does all this imply? In the simplest possible terms, the operational constructive approach addresses the construction of both the states and the operators hypothesized to be the components of a problem-solving space (Newell, 1980). Unlike the mental models approach, in which the representation or state is primary, in the operational constructive approach, the operators are primary. Operative aspects are primary, because the significance of a particular configuration of reality states and operations is believed to depend not on the the states, but on the operational transformations which modify the states.

Piaget (Inhelder & Piaget, 1964) explained transitive reasoning by means of the concrete operational structure he called primary addition of relations. He described the structure as having the following form: $(A > B) + (B > C) = (A > C)$. That is, the relation A is longer than C is operationally equivalent to an addition of the relations A is longer than B and B is longer than C. The interpretation of descriptions such as these has been controversial (Brainerd, 1978; Johnson-Laird, 1983). Generally, they have been interpreted as inference schemes, thus leading to a translation of Piaget's theory into a variant on the inferential rule approach.

In the interpretation presented here, the description of the structure in question is treated instead as the theorists' attempt to represent the operative aspects involved in transitive inference, as described by Piaget (Inhelder & Piaget, 1964). In this view, Piaget believed that relations of length differences (e.g., A is longer than B) are comprehended by children in terms of an operation of comparing lengths. The child notes that the stick A extends beyond the stick B when the other ends are aligned with each other. In other words, the operation of comparison involves aligning the objects and noting the resulting difference. As Inhelder and Piaget also described, this operation of comparing and noting length differences can be represented by the formula $a = A - B$. Where "a" represents the difference in length between sticks A and B, and "-" represents the operation of comparison. In the transitivity problem, the same operation of comparison is carried out on sticks of B and C as well (i.e., $a' = B - C$). These two first-order operations of comparison are then composed in a second-order operation of addition, which has the effect of summing the differences yielded by the two first-order operations into a total length difference b (thus, $a + a' = b$).

Figure 3 shows the perceptible differences in the three term transitivity of length task. This composition is operationally equivalent to a third operation of comparison ($b = A - C$) underlying the relation ($A > C$), which is the answer to the problem. In other words, transitive reasoning involves the following processes: Relations of length are comprehended in terms of operations of comparing length, and the composition of those operations results in a product that can be interpreted in terms of a further relation, which is the answer to the problem. The entire process is depicted schematically in Figure 3 and in terms of a sequential task analysis in Figure 4.

Figure 3
Differences Resulting from the Operational Comparison of the Terms in the Transitivity of Length Task (Three terms)

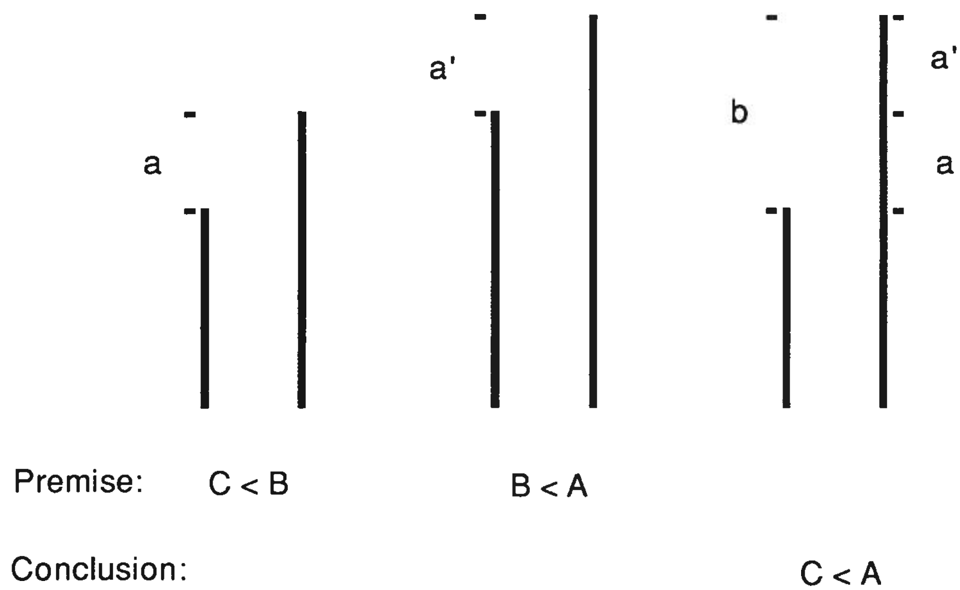


Figure 4
Operational Understanding of the Difference Relations of Length

Component	Propositional Representation	Operation
1. Premise:	(A > B)	a = B - A
2. Premise:	(B > C)	a' = C - B
3. Question:	(A > C) or (A < C)?	
4. Conclusion:	(A > C)	a + a' = b = A - C

As in the case of the mental models approach, transitive reasoning is assumed to occur on two levels. In this case, the propositional representation of the premises corresponds to certain operations, and the composition of those operations yields a result that, in return, corresponds to the propositional representation of the correct answer. An inevitable question arising from this analysis concerns the nature of the correspondence between the two levels, propositions and operations. This is the semantic dimension of the operational-constructive approach. It seems clear that the truth-conditional semantics used by Johnson-Laird (1983) are not appropriate here. For what is represented in the operations column in Figure 4 are not the conditions that would prevail if the propositions represented in the center column were true. They are not conditions at all, but potential actions or transformations. What is not clear is what alternative semantic theory is appropriate here, given that truth-conditional semantics are unacceptable.

Unfortunately, Piaget is no help in solving this problem, because his theory focused almost entirely on the level of operations. His theory of operational structures is fundamentally a theory of how operations are related to each other. About the semantic problem - how those operations are related to propositional or linguistic representations - Piaget has little or nothing to say. Recently, Chapman and Lindenberger (in press) have suggested that this problem could be solved in terms of Wittgensteinian constructivistic semantics. The basic idea is that the use of logical terms like "more than", "longer than", "the same as" and so on, is learned (and can be justified) with reference to the various operations by which they could be verified or demonstrated. Such operations thus serve as criteria for the corresponding expressions. (On Wittgenstein's concept of criteria, see Chapman, 1987.) For the present purposes, this proposal has not been sufficiently developed to be evaluated in any depth. The point is simply that, according to the operational-constructive approach, transitive reasoning involves two levels of representation and that the semantic problem of how those levels are interrelated has been neglected, but is by no means insoluble.

Note that Inhelder and Piaget's (1964) particular model of transitive reasoning as the addition of differences is only an example of the general constructive-operational approach. The essence of that approach is that the premises of an inference are comprehended in terms of operations, and the composition of those operations (by means of a second order operation) yields a result, the interpretation of which is the conclusion (Chapman & Lindenberger, in press). In effect, Inhelder and Piaget's model is one hypothesis regarding the nature of the operations in question. But other hypotheses are possible as well. For example, Chapman (1991b) suggested that the second order operation that composes the first-order comparisons might be a seriation of the objects (i.e., placing all the objects in order of increasing length on the basis of the pairwise length comparisons). That proposal would conform to the general operational-constructive approach as described here, but would differ from Inhelder and Piaget's specific model.

Necessity as a Criterion in the Operational Constructive Understanding of Transitivity of Length. The analysis of the transitivity task is intended to explain, not only how a correct answer is generated, but also how children understand that their answers are necessarily correct. Along with generativity, necessity is one of the two major issues of Piagetian epistemology (Piaget, 1983; Chapman, 1988). For Piaget, necessity was an inherent property of operational reasoning. To a child with concrete operational structures, the conclusion of a transitive inference is more than something that happens to be true; it is something that must be true. According to Piaget, "It is the feeling of necessity which constitutes evidence of the overall structures which characterize our stages" (Piaget, 1971c, p. 5).

In Piaget's later theory (Chapman, 1988; Piaget & Voyat, 1979), the development of an understanding of necessity was described to run parallel to the development of the understanding of possibilities. Possibilities are developed (using the language of problem spaces [Newell, 1980]) through the establishment of free combinations of the states and operators of the problem-solving space. In the case of transitivity of length, the problem-space states are length differences, and the problem-space operators include comparison and addition. The totality of the possible compositions of states and operators is the origin of necessity. Should states and operators be composed into a new and unfalsifiable whole, the necessary has been constructed. Three general periods in the development of possibility and necessity are identified by Piaget and Voyat (1979). The first period is one of pre-necessity or pseudo-necessity in which children recognize only a single readily predictable possibility. It is followed by a period of limited co-necessities in which certain necessities are seen to entail others. Such entailment comes about through the imagination of a limited set of concrete alternative possibilities. The final achievement is that of unlimited co-necessities in which all possibilities belonging to a system of transformation are recognized, and it is understood that certain possibilities taken together uniquely determine others. The uniqueness of that determination constitutes its necessity; it is necessary because any alternatives are not possible.

Three Approaches to Reasoning Compared

Each of the three approaches to reasoning presented above offers an explanation of reasoning which is simultaneously more and less adequate than the other two. The adequacy of each is described in turn below.

The inferential rule approach fails to account for the constraining action of memory capacity, the inability to transfer an inferential rule from one type of content to another, and the use of extra-logical heuristics in the solution of reasoning problems. On the other hand, the effect of language comprehension and its role in the acquisition of inference schemas is accounted for. Indeed, language comprehension is believed to be the sole source of inference schemas and therefore is seen as the sole source of errors in reasoning. Language comprehension is also proposed as the source of the understanding of necessity. However, Braine and Rumain (1983) do not consider transitive reasoning to be "logical". That is, according to them transitive inference results from empirical knowledge. It is important to note here that transitive inference has been judged by both children and adults to imply a sense of necessity (Miller, 1986). Therefore in the case of transitive reasoning, at least, the inferential rule approach does not distinguish between necessary inference which must be correct and probable inference which happens to be correct.

The mental models approach was designed to respond to the inadequacies of the inferential rule approach (Johnson-Laird, 1983). As such, it does account for the constraining action of limitations the subject brings to the reasoning task, for the presence of horizontal decalage across reasoning contents, and for the success of extra-logical heuristics. Necessity is accounted for in terms of an exhaustive search for alternative models. The mental models approach does not explain the presence and development of the procedures which direct the construction of the mental models. The most serious problem for the mental models approach is its reliance on truth-conditional semantics. It is implied that subjects understand a sentence by constructing a mental representation of its truth conditions (Johnson-Laird, 1983). Such representations presuppose understanding.

The operational constructive approach (Chapman & Lindenberger, in press), like the mental models approach does provide some explanation for the constraining action of limitations the subject brings to the reasoning task, for the use of extra-logical heuristics in reasoning, and for the presence of horizontal decalage across reasoning contents. There is also a beginning account of the development of necessity through the production of operative possibilities. Just what the operations are needs to be solved, possibly through research which attends to the conditions of interiorization of action. However, the operational constructive approach does not at present provide a semantic theory explaining how the representation of the problem on the level of propositions is related to the operations with which reasoning is carried out. The emerging theory of constructivist semantics (Chapman & Lindenberger, in press) may soon fill this gap.

Finally, there are differences in the overall type of approach to reasoning each of the above selected approaches represent. First, the constructive operational approach is a constructive type of reasoning approach. Within such approaches the reasoning outcomes go beyond the experiences the subject enacts through activity in the environment. For example, in the constructive operational approach, transitive reasoning is the product of the composition of the activities of comparing into a second order activity of addition. This latter activity goes beyond, so to speak, the original comparative activity with the media of the task conditions. On the other hand, both the inferential rule approach and the mental models approach are abstracted approaches where the outcomes are copied from the experience of language, in the first case, and from the truth conditional representations in the second case.

The Development of Reasoning: What are the Mechanisms that Produce New and Better Forms of Reasoning?

Since the existence of a particular form of reasoning is postulated in all three of the approaches discussed above, each is minimally obligated to account for the development of that form of reasoning in a noncircular manner. The problem is one of explaining how "better" forms of reasoning develop from "less good" ones without presupposing the existence of the better forms from the beginning. According to Pascual-Leone (1980), most common learning mechanisms only explain how a certain behavior is strengthened or selected once it is produced. They do not explain how the behavior is produced in the first place, if it is too complex or well-formed to occur solely by chance. He called this problem the "learning paradox", because the appearance of new and better forms of cognition is paradoxical from the viewpoint of typical learning mechanisms. Clearly each of the three approaches discussed in this chapter is subject to this paradox.

Braine and Rumain (1983) propose that formal rules of inference or inference schemas are acquired as children learn the meanings of the natural language counterparts of logical connectives such as "longer than" and "shorter than". That is, there are two complementary processes. One is the acquisition of rules of inference or inference schemes. The other has to do with progressing from ordinary comprehension to analytic comprehension of word meanings. The second process would seem to dictate the level and speed of aquisition of the first. The mechanism of recognizing better forms of language understanding and hence of inference schemas is not addressed by Braine and Rumain (1983).

Johnson-Laird (1983) makes no attempt to explain either the presence of or the development of effective procedures which select appropriate truth conditional propositions to re-represent in the mental model. More importantly, the mechanism of how the propositional representation is initially known is also not explained. In fact, he leans toward the belief that children reason like adults but are slower and are constrained by mental capacity. This assumption has been evaluated (Russell, 1987) as coming perilously close to advocating the default position of innateness (Fodor, 1980).

Piaget's structural theory also does not adequately answer the question of how more adequate forms of reasoning develop from less adequate forms (Piatelli-Palmarini, 1980). Briefly, the operational-constructive mechanism is conceptually based on a biologically interactive system in which new constructions of the world are assimilated to a current structure until the negative feedback from such assimilations accumulate and tip the balance in favour of accommodation to a new, more adapted and therefore better structure. One question that has been raised about this model is that of how the organism knows which accommodations are likely to be better and more adaptive (Molenaar, 1986). Such knowledge presupposes the knowledge that the process of accommodation is supposed to explain - an epistemological paradox similar to that discussed by Pascual-Leone (1980).

In a practical attempt to address the problem, Bereiter (1985) proposed that "bootstrapping" of likely mental resources might support the emergence of new and more complex cognitive adaptations without presupposing those adaptations from the beginning. Bereiter is clear that he does not perceive this process as the full solution to the problem of the learning paradox. He hopes that through the exploitation of likely mental resources which are not highly intelligent or endowed with knowledge, "processes, mechanisms, or cognitive structures can be identified that at least render mental bootstrapping more plausible and point in the direction of a theory that might give a satisfactory account of it (the learning paradox)" (p. 205).

"Piggy backing", "field facilitation", "exploiting a bias", and "optimizing mental load" are four mental resources, hypothesized in this research to contribute to the bootstrapping of reasoning about length differences. Through piggy backing the problem is framed in a familiar problem-solving space, such as the perceptual-motor space is for children. Such a strategy is possible in the transitivity of length task, to the extent that the task is posed in the visual-spatial problem-solving space. Field facilitation involves increasing or decreasing the saliency of certain states of the problem-solving space. The application of this principle to transitive reasoning about length was operationalized in this research in terms of "perceptual definition", a concept explained in the third chapter of this thesis. Exploiting a bias or a cognitive predisposition may be a source of heuristic understanding that may eventually bootstrap more complete understanding. In this research exploiting the optimal figure in the premise statements was the bias selected for examination. Further, optimizing the load of the task helps to focus sufficient capacity on its solution. In this connection, transitive reasoning has been found to be more difficult when four terms are used in the premise statements instead of only three (Chapman & Lindenberger, 1988). For this reason a three-term task was used in this research.

In conclusion, it is proposed here that through bootstrapping which

(a) utilizes a familiar problem-solving space, (b) facilitates the states in the space, (c) selects the optimal figure bias and (d) optimizes the mental load, more complete structures of understanding will develop. The specifics of the bootstrapping mechanisms introduced here are further discussed in the following two chapters. In Chapter 3 approaches to visual-spatial reasoning and representation are examined, and in Chapter 4 the linguistic aspects of transitive reasoning about length are explored.

Chapter 3:

Spatial Knowledge: Is it a state? Is it activity?

One way of distinguishing the three approaches to reasoning described in the previous chapter is in terms of the problem-solving spaces postulated by each. The term problem-solving space is defined in this report as a medium for representing all the possible states of knowledge about a problem and the operators for generating that knowledge (Newell, 1980). Two different problem-solving spaces are necessary in order to characterize the transitivity of length task within these approaches. The first would characterize the representation of the spatial aspects of the task. The second would involve the nature of the linguistic representation of the spatial aspects. As introduced in Chapter 1 and as portrayed in the task analyses presented in Chapter 2, transitive reasoning according to the inferential rule approach is carried out solely in a linguistic problem-solving space, whereas transitive reasoning according to the other two approaches utilizes both spatial and linguistic problem-solving spaces. In this chapter, the spatial problem-solving space will be described, and in Chapter 4 the role of the linguistic problem-solving space is examined.

Note that the term space is used in two senses in this dissertation. The first is the everyday meaning of the word as the three-dimensional continuum in which objects are located and potentially displaced. The second is a part of the technical term problem-solving space, just described, in which the word space specifically refers to the total of all possible representations of a given problem. To avoid confusion, the following convention is adopted: the word space occurring alone will be used exclusively in the everyday sense, and the technical sense of the word will be intended only when it is used as part of the complete phrase problem-solving space.

The Nature of Representation in the Spatial Problem-solving Space

Both in the constructive operational approach (Piaget & Inhelder, 1963) and in the mental models approach (Johnson-Laird, 1983) transitive reasoning is considered to involve a spatial problem-solving space. As depicted in the task analyses in Chapter 2, the movement from premises to conclusion occurs through the intermediary of representation. However, the nature of this intermediary is conceived of differently in each case. In the mental models approach, representation in the problem-solving space is conceived of on analogy to visual perception (Johnson-Laird, 1983) and is similar to the kinds of visual-spatial forms of representation postulated in analogue or imagery theory - for example, Baddeley's (1981) visual-spatial sketch pad or Paivio's (1983) dual coding approach. According to this conception, the main elements of representation in a problem space are the objects or figures that occupy space. In the constructive operational approach, space is treated as analogous to action, and the focus is on the system of transformations/actions that occur in space, rather than the objects or figures that are transformed or acted upon.

These two approaches to spatial representation emphasize differently the elements of the problem-solving space -- states of knowledge and operator activity. Theorists who take the position that knowledge resides in a state representation and that such a representation is adaptive, emphasize the states of knowledge element of the spatial problem-solving space. On the other hand, theorists who postulate that knowledge is constructed through action emphasize the activity of the operator elements in the spatial problem-solving space. These seemingly disparate interpretations of the role of the elements of the spatial problem-solving space in transitive reasoning will be presented below. By using the organization of the problem-solving space as a commonality, it is hoped that the tensions between the approaches can be viewed as productive, not as inflammatory.

Transitive Reasoning when Problem-space State of Knowledge is the Emphasis

As described in Chapter 2, Johnson-Laird (1983) proposed that transitive length problems are solved on the basis of a visual-spatial seriation. The series is said to be constructed through procedures involving the direct, visually-mediated length comparisons, and a conclusion is drawn by means of verification procedures in which alternative possible solutions are eliminated.

At first glance Trabasso's (1975) theory of transitive reasoning in children appears quite similar to Johnson-Laird's theory. Like Johnson-Laird, Trabasso proposed that subjects integrate the premises of a transitive inference (e.g., A > B and B > C) into an ordered representation with visual-spatial properties (e.g., A > B > C) and derive the correct conclusion (A > C) from that representation. For Trabasso, however, that conclusion is based, like the premise comparisons, in a direct, visually introspected comparison, rather than on the kinds of verification search procedures postulated by Johnson-Laird. That is, the visual-spatial series contains the property of transitivity, the test question elicits direct comparison of the end terms in that series, and the judgment obtained in that way is correct by virtue of the property of transitivity embedded in the visually-introspected series.

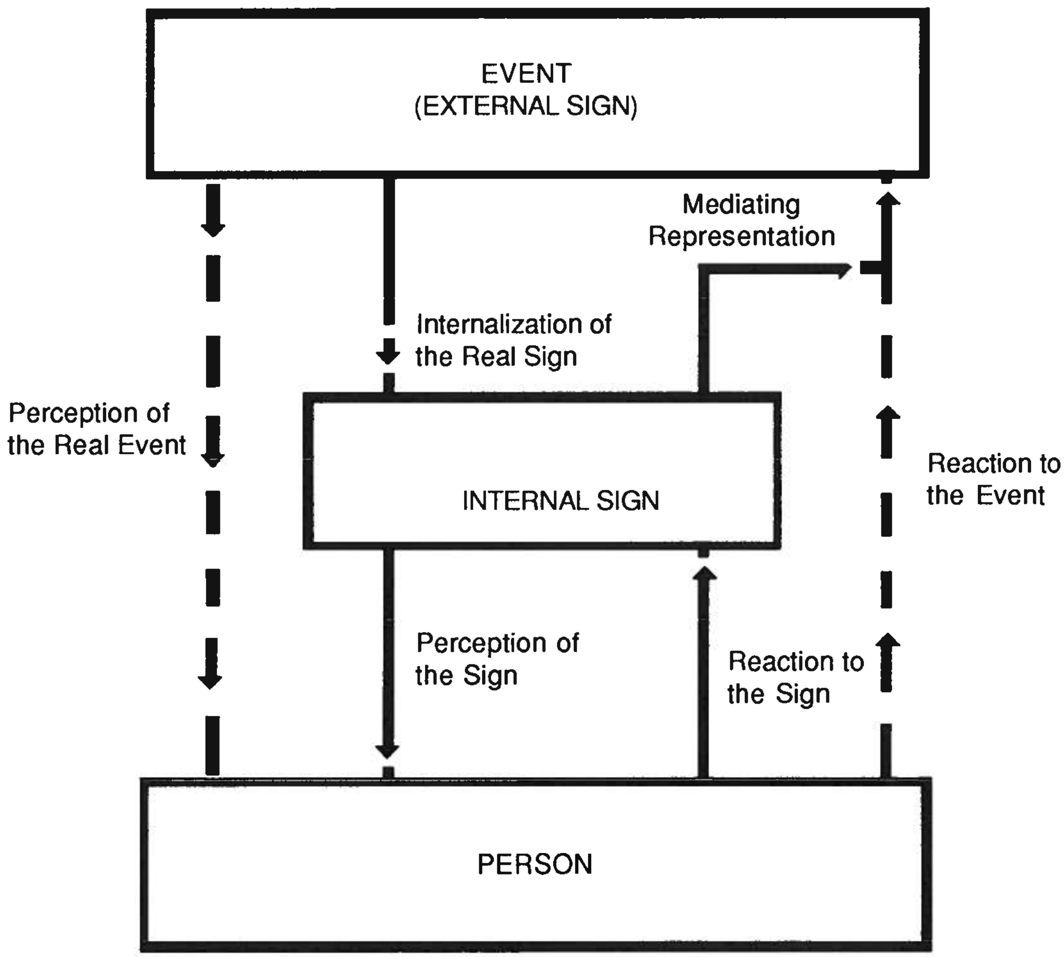
The hypothesis that transitive reasoning is based on a seriation of comparison objects is consistent with a large body of empirical research summarized by Trabasso (1975). However, the idea that the conclusion of a transitive inference is obtained merely by introspecting a visual-spatial representation of an ordered series makes it difficult to explain why anyone should consider the transitive inference to be <u>necessary</u>. There is nothing in this account to explain why children introspecting the array A > B > C should conclude that A *must* be longer than C rather than merely that A *happens to be* longer than C. Yet both children and adults have been found to recognize the necessity of A > C, given A > B and B > C (Miller, 1986). For this reason, Johnson-Laird's (1983) account of the derivation of A > C through verification procedures must be considered an advance on Trabasso's account of direct comparison.

In summary, both Johnson-Laird's (1983) and Trabasso's (1975) theories of transitive inference emphasize the state of knowledge element of the spatial problem-solving space. Trabasso makes the assumption that the image of the seriation contains the meaning of transitivity. The major problem with this is that images alone do not have meaning, they need to be interpreted (Piaget, 1971a; Campbell & Bickhard, 1986; Russell, 1987; Chapman, 1991b). Johnson-Laird, on the other hand, advances an explanation of how an image might contain meaning. He proposes that the procedures which construct and interpret the seriation model evaluate the meaning of the mental model of transitivity. Such an advancement makes the mental models approach functional. However, when such mechanisms are included in a theory, the problem of the source of and the development of the procedures remains unresolved.

Transitive Reasoning when Problem-space Operator Activity is the Emphasis

Many cognitive scientists (Bickhard & Campbell, 1989; Kaufmann, 1986; Kolers & Smythe, 1979; Kolers & Smythe, 1984; Kolers & Roediger, 1984; Russell, 1987) have advocated a conceptually different alternative to the emphasis on state of knowledge representation in the explanation of reasoning. By and large, they all have suggested that attention be directed to the role of action, skill or construction and also to the role of the context of thought in stimulating activity. Piaget and his students for many decades have advocated a theory of knowing based on action. Furth's (1969) work will be used to launch an exploration of the possibilities of Piagetian action theory as an approach which emphasizes operator activity in the problem-solving space. Bereiter's (1985) bootstrapping strategies will be used to examine the role of the context both in knowledge state representation and in stimulating operatory action in the problem-solving space.

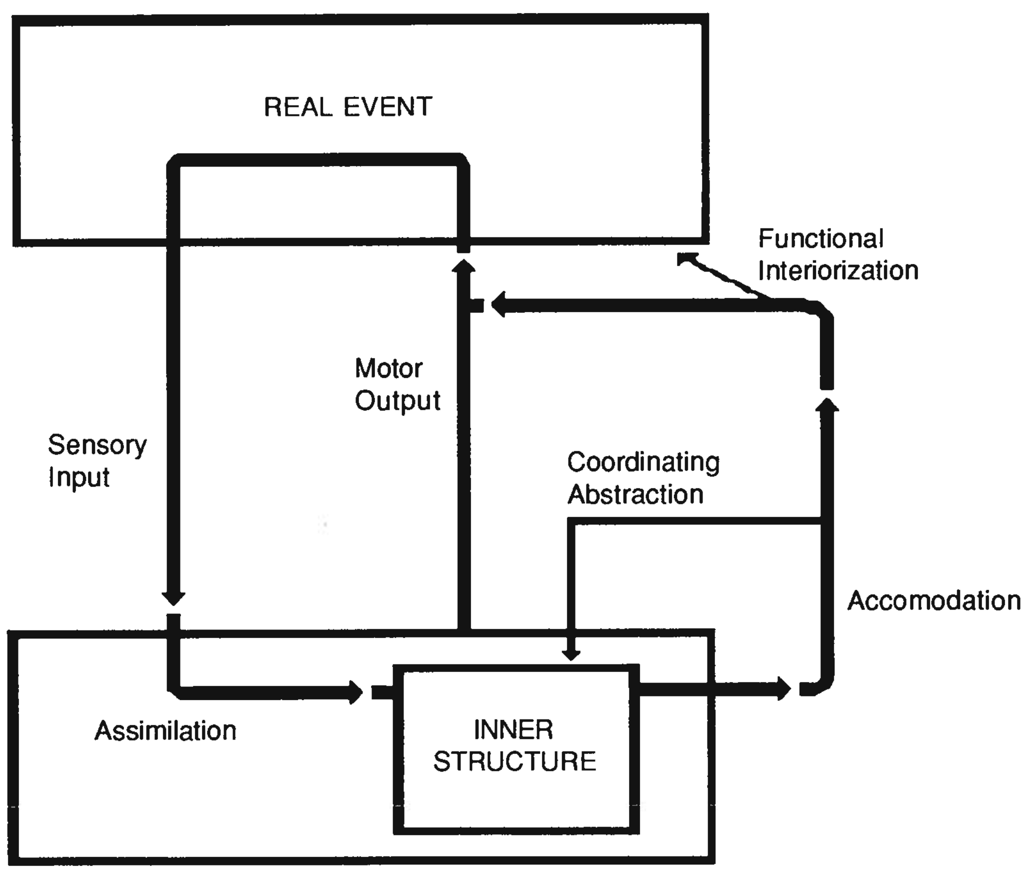
Figure 5
Reasoning Through Mediation (Furth, 1969, p.71)



According to Furth (1969), when the the state of knowledge representation is emphasized, meaning is seen to be controlled by the internal sign (see Figure 5). The internalized sign is an image or propositional representation of the external sign or event which is mapped from the external event onto the internal sign. Such representations incorporate the knowledge or meaning of the external sign or event. As such, the internal sign and the meaning embedded in it mediate subsequent reaction to the external world.

On the other hand, the essential part of the operational constructive approach is the internal operatory structure, which can be understood as the interiorized operator actions which can construct the totality of possibilities in the problem-solving space. Such actions are first assimilated to the known reality state of the action structure (see Figure 6). Accommodation of the structure to the knowing of the reality state

Figure 6
Piaget's Action Theory of Knowing (Furth, 1969, p.75)

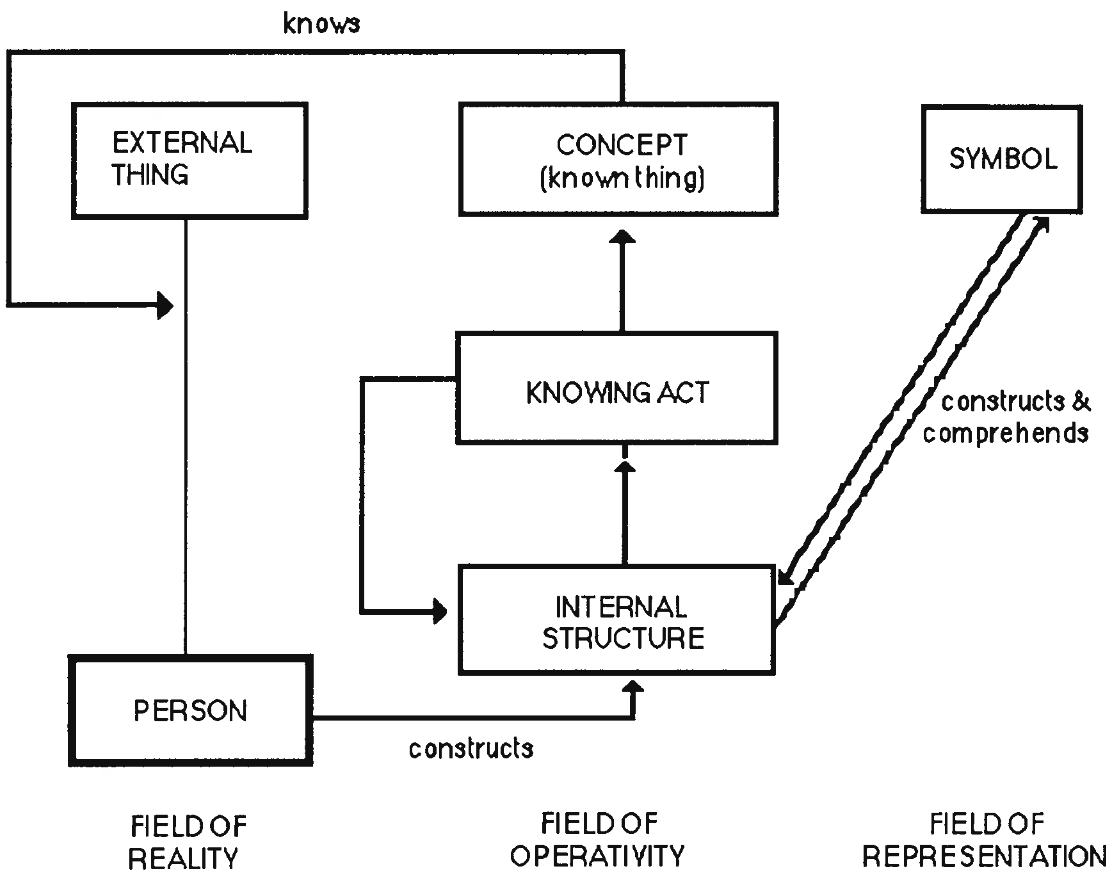


also occurs. There is a developmental shift in the balance of assimilation and accommodation over the interaction history of the child. Very young children largely assimilate and as interaction with reality states continues, a balance between assimilation and accommodation occurs. In any event, it is the structured actions that know the reality state and that subsequently produce imaginal and linguistic representation.

This differs from the state of knowledge representation emphasis proposed by the mental models approach. First, the structure does not originate externally and therefore is never in need of being mapped onto representations. Instead, in action theory, the structure is the internalized totality of possible action involving the external event. Second, the state of knowledge representation is not a copy within which meaning is embedded. In action theory, state of knowledge representation is limited to the direct product of the internalized operator activity. That is, the knowledge is in the productive activity of an active knowing structure, not in the product of that knowing, the state of knowledge representation. This approach addresses the concerns of those who advocate a new approach to reasoning that emphasizes the reasoners activity in a stimulus context.

In Figure 7, Furth (1969) further specified the role of action, in representation. He utilized three fields of "reality status" to illustrate the differences and the relationships between external reality, internal structure and representation. Basically, the knower constructs a structure through interiorizing action. The structure gives rise, through action, to a concept. That concept is not a knowledge state, it is operator activities that define the general properties of the object (Chapman, 1988). It is emphasized that it is the operations/activity, not the representation (symbol) that modifies the perceptual activity that occurs between external reality and the knower. The field of representation is separated from the field of reality by the internal structure of action. The symbol is the product of the interiorized operator or action that has produced and comprehended the reality state. Hence, from the point of view of action theory, the direct mapping of reality onto a symbol as such is impossible.

Figure 7
Three Different Fields of Reality Status in Piaget's Operative Theory of Knowledge (Furth, 1969, p. 94)



Given the spatial aspects of transitive reasoning about length (Huttenlocher, 1968; Trabasso, 1975), what are the implications of the operational constructive explanation of representation? According to this view, an internal structure of knowing activity would construct a spatial concept of length which would necessitate the selection of a spatial problem-solving space. The spatial problem-solving space would be useful in that it provides a familiar medium for representing a range of possibilities and for manipulating them in such a way that necessary spatial relations/operators can be identified. The internal structure would then construct, using spatial operations (such as comparison), a spatial symbol for the length aspects of the transitivity of length problem. Further, spatial operations (such as addition) would be utilized in the generation of possible solutions to the transitivity of length problem within the spatial problem-solving space.

The Role of the Context in Spatial Knowledge

Certain of Bereiter's (1985) bootstrapping strategies, discussed in Chapter 2, would have a facilitating effect on either mental modelling or operatory action in the spatial problem-solving space. Indeed, since the perceptual-motor space is a familiar problem-solving space for children, piggy backing on this familiar problem-solving space, as the transitivity of length task does, is itself a bootstrapping strategy. Once the problem is framed in the spatial problem-solving space, the next consideration concerns the selection of further strategies to improve visualization and manipulation in that space. Some properties of spatial representation include the relative definition and good form of the representation in question. Examples of strategies exploiting such properties are those Bereiter (1985) called field facilitation and exploiting a bias respectively. The principles of each are applied to the transitivity of length differences problem below.

Definition of the Representation: A Field Facilitation. The retrieval of the visual-spatial representations within a limited capacity visuospatial sketchpad (VSSP) (Baddeley, 1981) has been explored by Frick (1988). A video recorder/playback metaphor can be used to explain memory capacity in connection with visual-spatial representation. A video-cassette tape has a fixed amount of recording tape, it can record at different speeds. At a slow speed, each representation takes up a small portion of the tape, but at a higher speed each representation consumes more space on the tape. The playback fidelity is higher in the second case, and recovery of such representations is enhanced. In addition, retrieval of the representation is further influenced by its original perceptual clarity. As described by Frick (1988, p. 299), "when items are presented too quickly or with out sufficient luminance, the representation in the VSSP simply is less clear. Because of this reduced clarity, the items are less likely to be correctly recovered, producing errors in recall". For the context of problem solving, the likely consequence of inaccurate recall is incorrect and incomplete reasoning.

In this report, the notion of clarity in the representation was called definition. With respect to the spatial representations of the length differences in the transitivity of length problem, those differences that are more definite (i.e., those that are greater and therefore are more remarkable) should be recovered more accurately and therefore contribute less error to reasoning about the length differences. This notion is supported by the research of Kosslyn (1980) who found for both children and adults that the relative size difference of a visual property inversely affects the time required to verify it, that the dependence of verification times on size decreased with age, and that children require more time to see small objects, than do adults. The symbolic distance effect reported by Trabasso (1975) can also be interpreted as a result of relative size difference. He found that children take less time to decide about transitive relations between sticks which occur far apart in a large seriated array then they do to decide about sticks that are closer together. It may be that the spatial separation is more definite in the former than in the latter case. Definition of the representation may also lead to more processing by contributing to the distinction of the contrasts. If the contrasts stimulated by definition of the representation are distinct, then processing is more complete and recovery is more precise (Jacoby & Craik, 1979).

Increasing the definition of the representation, whether it is a state of length knowledge or a comparison operator, is therefore a field facilitation strategy that should help to bootstrap reasoning. In the research reported in the following chapters, the definition of the visual-spatial representations presumably used by children in transitive reasoning was manipulated by varying the magnitude of the differences between the elements of the length series. The assumption was that greater length differences would enhance reasoning because they would be more precisely encoded.

Good Form in the Representation: Exploiting the Bias. In the strategy Bereiter (1985) called exploiting a bias, a functional solution is supported with the hope that it would eventually bootstrap complete understanding. An example might be the "good form" that, according to Piaget (Inhelder & Piaget, 1964), promotes visualization or perceptibility of the whole. In this research, the figural effect as described by Johnson-Laird (1983) was used to operationalize the notion of good form in the context of transitive reasoning.

According to Johnson-Laird (1983), the figural effect in syllogistic reasoning consists in a bias towards certain forms of conclusion when the reasoner is given certain orders of premise presentations. An increase in the difficulty and latency of drawing correct conclusions with certain orders of premise presentations is expected. The figural effect operates when an inference has to be made, "and in particular when a direct link has to be established between the end terms -- with the middle term dropping out of the final representation of the conclusion" (Johnson-Laird, p. 111). When additional mental operations have to be made to align the middle term in the process of making it redundant, then the figure of these premise presentations is harder to integrate.

For Johnson-Laird (1983) the existence of such figural effects on syllogistic reasoning was evidence for the use of visual-spatial representation in such reasoning. If transitive reasoning about relations of length also draws on visual-spatial representation, then a similar figural effect should be observed in that case as well. Following Johnson-Laird's procedural explanation as summarized above, the premise series optimizing the transitive inference A - C would be that having the good form A - B, B - C. In contrast, if the premises were presented in the order A - B, C - B, the inference A - C should be more difficult to construct. Within the operational constructive approach, such good form in the order of the premise presentations should also promote correct judgement of the request for transitive inference. It may also be influential in justification of the transitive relation during the period of preoperational thought. However, once the concrete operational structure is established, good form should not influence either judgement or justification of transitive inference.

In this chapter, approaches which emphasis knowledge state representation of a spatial task were compared to those which emphasized the operatory action in the spatial problem-solving space. In both approaches, bootstrapping strategies for the visual-spatial problem-solving space were presented as potential aids for transitive reasoning. Such phenomena as definition in the representation and good form in the presentation of premises to be reasoned about were described as conditions that would be likely to facilitate transitive reasoning. In the following chapter, issues concerning linguistic representation of the spatial aspects of the transitivity of length task are addressed.

Chapter 4:

Linguistic Representation of Spatial Relations

Although Piaget was a pioneer in demonstrating that early sensorimotor action was a common source for language and thought, he argued that

> it suffices to look at the sensorimotor intelligence which exists prior to the acquisition of language in order to find in the infant's elementary practical coordinations the functional equivalents of the operations of combination and dissociation . . functionally comparable to the future operations of thought (Piaget, 1968, p. 94).

For Piaget, language was only one of several developmentally interdependent vehicles used by the child in the acquisition of cognitive structures. Other vehicles include symbolic play, deferred imitation, mental imagery as well as verbal utterances (Piaget, 1962; Karmiloff-Smith, 1979). Having well established his view that cognitive structures are interiorized activities, Piaget nonetheless used verbal explanation as criteria for cognitive competence (Piaget, 1950). Such a seemingly contradictory approach to the genesis and measurement of reasoning has been criticized over the years (Brainerd, 1973a; Seigel, 1978). The solution for these earlier critics was to ignore both the generative aspect of language in the formation of operatory structures and the evaluative contribution of verbal explanation to the assessment of Piagetian cognitive structures.

Likewise, Karmiloff-Smith (1979), herself a Piagetian epistemologist, argued that Piaget gave language a secondary role and thus underestimated the importance of language in the development of reasoning. She felt also that language was an object of children's cognitive attention and as such she proposed that it had its own problem space. Based on these points, she made a case for the incorporation of manipulation of language variables in cognitive tasks.

Karmiloff-Smith presupposes the presence of very general cognitive capacities that organize and seek patterns in the physical, the conceptual, the perceptual, the linguistic and the emotional aspects of the environment. Of these, Karmiloff-Smith (1979) has focused on the role of the linguistic problem-solving space in children's thought. Through experimentation, she has made a plausible case for the influence of linguistic representation in the understanding of classes (Karmiloff-Smith, 1979). Inhelder and Piaget (1964) also proposed a role for language in reasoning about classes.

However, neither Piaget nor Karmiloff-Smith articulated the role that language might play in reasoning about relations. All spoken languages, however, are said to have devices that link the word utterances with the spatial-temporal contexts within which they occur. This linkage is called deixis (Tanz, 1980). The function of the spatial deictic terms "longer" and "shorter" and their influence on the development of understanding of transitivity of length was a focus of the research presented here. In the following section the concept of deixis is discussed, and the trajectory of the development of deictic terms is outlined. The body of the chapter is devoted to a presentation of linguistic and psychological approaches to the understanding of the linguistic representation of spatial relations.

Deictic Function: What is it?

There are three usual ways in which one thing can signify another (Burks, 1949) and deictic terms combine two of these (Tanz, 1980). Words are considered to be symbols in that they are associated with the object by some conventional rule. For example, one may denote a house by using the word symbol "house". An icon, on the other hand, resembles the object in some way. In the example developed here, a blueprint is an icon which exemplifies house. Finally, a house can be signified by an index. An index is a consequence of what is being represented. For the house example, the house can be spatially indexed by pointing at it. Deictic terms, however, are intermediate between symbols and indices. That is, they combine the function of symbol and spatial index. Deictic terms therefore are indexical symbols - they name and have spatial consequences.

Classes of deictic terms and their order of acquisition, as proposed by Tanz (1980), appear below.

(1) personal pronouns (I/you/he/she)
(2) spatial deixis (in back of/in front of, shorter than/longer than)
(3) demonstratives and locatives (this/that, here/there)
(4) deictic verbs of motion (come/go, bring/take)

According to Tanz (1980) the spatial deictic terms "shorter" and "longer" can be used in a purely symbolic sense or in an indexical sense. That is, each expresses a spatial relationship with respect to the spatial attribute of length. In their symbolic sense, the relationship is defined in terms of attributes inherent in the reference object's permanent length features. In the indexical sense, the spatial relationship is defined by a comparison between two reference objects.

Through a series of studies, Tanz (1980) found that children acquire the full range of deictic signs by 8 years of age. It was also evident that personal pronouns were the first to be successfully employed. Indeed, all of the children performed at ceiling level on the pronoun task. Further, the youngest to carry out that task was 3 years old. Such apparent mastery of the personal pronouns led Tanz to propose that children learn the contextual link that characterizes deictic terms at an early age. Thus, the delay in learning the range of deictic terms cannot be because of their deictic functions, whether symbolic or indexical. It therefore must be because of their differences with respect to other properties.

Possible other properties that might influence the acquisition of and the functioning of spatial deictic terms include the possibility of a predisposition to attend to the affirmative relation of length (Piaget & Inhelder, 1971), the effect of perceptual complexity on the processing of information about length (Clark, 1970), the presence of a preformed resource which favours certain spatial conditions and spatial linguistic terms (McGonigle & Chalmers, 1985), and the possible interaction between the length difference and the linguistic relational term used to describe it (Tanz, 1980). Each of these properties and the role each plays in the explanation of the development and the functioning of spatial deixis in transitive reasoning are examined in turn in the following sections. In previous chapters, the operational constructive approach was described as more adequate in the explanation of transitive reasoning and in the explanation of representation of spatial knowledge. For this reason, an operational constructive explanation of the role of linguistic relational terms in transitive reasoning is presented first.

The Primacy of Affirmations: An Operational Constructive Approach

According to Piaget and Inhelder (1971), a balance of "affirmations" and "negations" is a property of an operational structure. Further, the primacy of affirmations over negations is seen to be a predisposition in cognitive development. The affirmative relation is described as that relation to which the child's attention apparently is first or more easily centered because affirmations are based on direct perception but negations must be constructed. Not until each affirmation is compensated through the construction of negations and those negations are further coordinated with affirmations is a cognitive structure closed. In transitive reasoning about length differences, it is proposed that the affirmative relation is longer. The relation shorter is a negation, because shorter implies that the comparison objects are not long (=short) and therefore the relation shorter is constructed.

There are four levels through which the balancing of affirmative with negative length relations should progress. The following proposal is an extension of Piaget's description of centering in a conservation of quantity task (Piaget & Inhelder, 1971) to the transitivity of length task. At level I, the child's attention is centered on the affirmative relation longer, that is, on perceived length and the relational term "longer". At level II, children remain centered on the affirmative relation longer, but are beginning to notice that the negative relation shorter also describes the same pair of sticks. At level III, children understand that longer and shorter may both characterize a pair of sticks, but not that the two relations complement one another exactly. At level IV, the child is capable of predicting in advance that if one stick is longer than another, than the second is shorter than the first by exactly the same amount. This latter level of understanding is required for reasoning about the necessity of the spatial relation which characterizes operational transitive reasoning (see Chapter 2).

If this developmental sequence in the understanding of the equality of affirmations and negations holds for the transitivity of length task as outlined, then both relational terms and perceived lengths that call attention to affirmations should enhance the natural preoperative tendency to center on affirmations, whereas relational terms and perceived lengths that call attention to negations should make the solution of the task more difficult. Children who have attained level IV should maintain reasoning unaffected by the use of either relational terms.

Embedded in the notion of developmental primacy of the affirmative relation of length (large length difference), is the view that it is primary because it is directly perceived. It follows that the effect of perceptual complexity should influence the acquisition and functioning of spatial deictic terms such as "longer" and "shorter". The linguistic feature complexity approach (Clark, 1973) considers perceptual complexity.

Linguistic Feature Complexity: An Inferential Rule Approach

In Chapter 2, the ability to reason according to an inferential rule was described as depending upon or being determined by the subject's understanding of natural language linguistic forms and expressions. Accordingly, linguistic comprehension is the main determinant of whether correct inferences are drawn. The question to be addressed here is in what the understanding of the linguistic relational terms consists?

According to the linguistic feature complexity approach, relational terms involve polar oppositions. Clark (1973) has proposed the most frequently cited explanation (Johnson-Laird, 1983; McGonigle & Chalmers, 1985) for the polarity of spatial terms. In Clark's view, preconditions in the environment (gravity) and in the human organism (asymmetry of the body, the forward facing sensory organs, and the normality of forward motion) combine to favour specific points of spatial reference and directions. The favoured points are those that are forward and vertical in direction.

Positive and negative values are assigned to the poles by a linguistic structure that is assumed to be isomorphic to, but independent from, the conceptual structure of perceived space. Comprehension of spatial terms is a function of the number of the relevant linguistic features or dimensions (Bierwisch, 1967) and of the number of rules of application entailed in the use of a term. The pair "longer"/"shorter" is unidimensional of length. All negatives are calculated to require an extra rule (Clark, 1974), therefore, "shorter" requires an additional rule application or operation (Johnson-Laird, 1983) to establish its meaning as "not longer". Therefore, "shorter" is qualified as <u>marked</u> by the need for additional rules of application whereas "longer" is <u>unmarked</u> .

Under this model, the order of acquisition of spatial terms is determined by the perceptual complexity of spatial concepts and the influence of perceptual complexity is mediated by language itself (Clark, 1974). That is, the language structure in the domain of space reflects perceptual complexity but is independent of it. The match of conceptual <u>length</u> and linguistic "<u>length</u>", each with independent sources would appear to be the weak link in Clark's position (Tanz, 1980). The comprehension and acquisition of language are hypothesized to be mediated by the facts of the language structure and not based on a conceptual structure of space. In other words, the linguistic structure and the cognitive structure have an identical source in the environmental and human organic preconditions, but each develops independently. The product of each, the relational concept and the linguistic relational term, are separate and unrelated to each other.

Clark, however, admits the possibility that, early in development, the order of acquisition of linguistic spatial terms may be governed by perceptual complexity rather than linguistic complexity. This dominance of perceptual complexity is proposed to occur when children have not fully conceived the space and before they begin to learn spatial terms. If such is the case, children's first spatial terms will relate to their first spatial concepts. This amended view would seem to support the operational constructive proposal that, as concepts are structured in the visual-spatial realm, they can be represented in the linguistic realm (see Figure 7 in Chapter 3). Thus, both the operational constructive approach and the linguistic feature complexity approach predict that "shorter" is more difficult to interpret than "longer" and that "longer" is acquired before "shorter".

If linguistic feature complexity explains the difficulty in and the aquisition of linguistic relational terms such as "longer" and "shorter", the use of unmarked spatial terms should yield more frequent correct inference earlier in development than the use of marked terms. In the context of transitive reasoning the use of the relation term "longer" should result in more correct inferences than the term "shorter". Such an outcome should diminish but not be extinguished with increasing age.

A Conditioning Framing Resource: A Mental Models Approach

According to the linguistic feature complexity approach, the aquisition of and function of spatial linguistic terms is based on the linguistic and cognitive structures which have an identical source in environmental and human organic preconditions. In a similar vein, McGonigle and Chalmers (1985) propose that an extensive range of preconditions influence transitive reasoning. According to their framing resource interpretation, it is proposed that the subject accesses a resource that pre-dates the current interaction and that it is more extensive than the input in the reasoning task context. Such a resource would affect the interpretation of linguistic input in the transitivity of length task and it would direct transitive reasoning.

Johnson-Laird's (1983) mental models approach apparently adheres to the framing resource view. In his view, words are used like cues to build a familiar mental model and linguistic comprehension is simply the recognition of the truth conditions of the statements to be understood. The pre-existing resources, in this case, are the knowledge of the representative truth conditions and the procedures involved in utilizing such knowledge. Trabasso (Trabasso & Riley, 1975) also endorsed the framing resource approach. He proposed that subjects encode premise information from a transitive reasoning task such that "end-anchors (of the series) are mapped onto a spatial dimension and the pairs are first ordered and entered into the array" (p. 394, emphasis added). The critical framing resource in that case is the spatial dimension along which the array of objects is ordered.

For both Trabasso and Johnson-Laird, a method of language comprehension as well as the framing resource are used to explain the construction of the spatial model. The question is how linguistic input interacts with the framing resource for the reasoning task being solved? McGonigle and Chalmers (1985) integrated a series of their studies (McGonigle & Chalmers, 1980; Chalmers & McGonigle, 1984: McGonigle & Chalmers, 1984) in an interpretation designed to answer this question. The inference task that was the vehicle for the investigations was the transitive reasoning task.

Well known pairs of objects were presented using words as well as pictures scaled to be of equal size. Because the framing representation was assumed to be established prior to the task, and since the task did not use informing statements (premises), the degree of mapping achieved was proposed to be between the linguistic aspects of the question and the framing representation only. That is, the absence of informing statements, eliminated the possibility that they would form the framing resource. The subjects consisted of children whose ages (ages 6 and 9 years) were previously documented to be associated with failure in such reasoning tasks (Inhelder & Piaget, 1969).

Findings from their studies led McGonigle and Chalmers (1985) to conclude that a prelinguistic and prelogical framing resource may exist. Such a resource is proposed to "play a causal role in the (later) understanding of both linguistic and logical terms of relation" (p.152) in the transitivity of length task. They described the resource as "a basic design or computational constraint on representation" (p.149) and suggested that it consisted of the following five preferences.

First, a particular form of inference request is favoured. The following list of pairings of requests for inference with the state of the objects compared, appears in descending order of success: requests for bigger when bigger is true, requests for bigger when bigger is false, requests for smaller when smaller is false and finally requests for smaller when smaller is true. Second, a symbolic distance effect is active in the proposed framing resource. That is, the time taken to judge small differences is larger than the time taken to judge large differences. Third, there is a preferred direction of working along a spatial vector from left to right . Fourth, favouring of the "big" category of relational term over the "small" category of relational term is strong. Finally, the framing resource promotes enhanced comparison when picture symbols are utilized as compared to when lexical items are used.

According to the framing resource approach, when such preferences are active, incoming linguistic information will be processed congruent to the preference in the framing resource. For Paivio (1978, p. 42), these preferences imply that "the linguistic system per se does not contain the perceptual or semantic information that corresponds to our knowledge of the world . . . Instead, the verbal system can retrieve such information only by probing the nonverbal representational system." The implication is that language is mapped into a form of pre-representation that is essentially non-linguistic before it can be understood. Given the evidence for such a framing resource (McGonigle & Chalmers, 1980; Chalmers & McGonigle, 1984: McGonigle & Chalmers, 1984), the notion that task-specific language alone may be determinant of reasoning is no longer tenable. Instead, according to this view, linguistic representations of premise information and requests for inference are comprehended within the frame of a prelinguistic resource.

In the case of reasoning about relations of length, it seems reasonable to assume that the framing resource is spatial in character (see Chapter 3). Therefore, what is required to facilitate reasoning is information about length difference, premise order presentation, and requests for inference that have "good form". That is, task conditions that conform to the spatial framing resource. Transitive reasoning therefore should be enhanced when length differences are large. Identification of length differences and requests for inferences in terms of the relational term "longer" also conform more closely to the hypothesized framing resource. These effects should decrease, but not disappear entirely with development, as children come to depend less on the framing resource alone in transitive reasoning.

Language-Context Transparency Principle

In a series of empirical studies, Tanz (1980) found that the semantic development of deixis was not fully determined by the perceptually based property of marking of spatial deictic terms. Indeed, with respect to marking effects, the incorrect predictions outweighed the correct ones for several deictic terms. Nor was the developmental hypothesis that the relational term longer is learned before the relational term shorter confirmed in Tanz' research. In order to reconcile those contradictory findings, she developed what she called the language-context transparency principle, arguing that spatial deixis involves neither language nor the physical context alone, but the relation between them. She proposed that deictic terms that most directly (most transparently) refer to the spatial context under consideration will be acquired earlier and used most efficiently.

The principle that deictic terms connected with more salient aspects of the spatial context are better matched and as such, are initially easier to decode was operationalized with respect to length in the following manner. When the adjective long is used, the article referred to will be expected to be salient with respect to length. That is, have length. In situations where short is used, length will not be salient. That is, having length would not be salient and decoding under such circumstances would be more difficult.

In the research reported in the following chapters, the relational terms "longer" and "shorter" were applied to both large and small length differences in a fully crossed design. According to the language-context transparency principle, one might expect that children should reason correctly more often when there is a match between the magnitude of length difference and the relational term used in the task. Matching should be less of a factor in the reasoning of older children since there is evidence that the full range of deictic terms is probably acquired by age 8 (Tanz, 1980).

Conclusion

In this chapter, four properties that might affect the functioning of the spatial deictic relational terms in the transitivity of length task have been examined. The four properties included (a) a developmental predisposition to attend to the relation "longer", (b) the perceptual complexity of the dimension of length which leads to "longer" being more easily processed, (c) a preformed mental resource which favours the big category of any relational pair, and (d) the match between length difference and relational term favouring the large length difference when it is paired with relational term "longer" and the small length difference when it is paired with the relational term "shorter".

Each of the first three factors were expected to facilitate correct transitive inference when premise information was communicated to children in terms of the relational term "longer", as compared to the relational term "shorter". In addition, the hypothesis of a developmental primacy of affirmations over negations predicts that use of the affirmative term "longer" should lead to a better understanding of the necessity of transitive inference.

In contrast to the other three factors, the language-context transparency principle (Tanz, 1980) applied to the transitivity of length problem predicts that reasoning will be enhanced only when there is a match between the magnitude of length differences and the relational term used in describing the premise relations. Reasoning should be better when "shorter" is paired with a small length difference and when "longer" is paired with large length difference, as compared to the unmatched pairings of relational term and length difference conditions. In other words, the language-context transparency principle predicts an interaction between linguistic relational term and the magnitude of length differences, whereas consideration of the first three factors leads only to predictions of a main effect of linguistic term.

Chapter 5:

Summary of Variables and Hypotheses

In this chapter the research purpose and the research problem are delineated. A summary discussion of the dependent variable will be presented. It is followed by statements of the hypotheses according to which the effects of the independent variables are predicted. Each hypothesis is also briefly explained.

The Research Purpose

The overall purpose of this research was to explore factors that might influence children's transitive reasoning about length. The factors studied include the following: (a) the figure of the premise statements (optimal figure vs. control figure), (b) the magnitude of the length difference between adjacent objects (1.5 inches/3.81 cm vs. .25 inches/.635 cm), and (c) the linguistic relational term ("longer" vs. "shorter"). Children's understanding of transitive reasoning was measured through (a) transitivity judgements, (b) the explanations given for the transitivity judgement, and (c) the understanding of the necessity of the transitive inference.

The Dependent Variable

Transitive Reasoning

The object of this study was the children's transitive reasoning. The assumption was that transitive reasoning is more than the ability to generate a correct answer on a transitivity task. It involves an understanding of the nature of the relation between the premises and the conclusion. According to two of the approaches to reasoning -- the inferential rule approach and the operational constructive approach -- the transitive inference also includes the element of necessity. In order to study transitive reasoning, judgements, justifications and answers to questions regarding the necessity of the relation were considered.

Over the years, there has been some disagreement concerning the role of verbal justification as a criterion for the assessment of reasoning in children (Braine, 1959; Brainerd, 1973, 1977; Gruen, 1966; Siegel, 1978). It has been claimed that verbal justification is a significant source of measurement error and as such underestimates the true cognitive abilities of children. Such a position implies that reasoning is assessed accurately by judgement only. There is a tacit assumption that cognitive processes are independent of the ability to provide verbal explanation. In that approach, verbal explanations are conceived as reconstructions of introspected cognitive processes, not as justifications of cognitive judgements.

An alternative approach was used in this research. The position taken is in agreement with Karmiloff-Smith (1979). She claims that Piaget's work seems to underestimate the role of language in the development of cognitive structures. In her approach, language itself is an object of operational constructive activity and it has its own problem-solving space. As such, verbal explanations are of interest in their own right. Further, the relation between verbal explanation and the more conceptually based judgement is also of interest.

Independent Variables

In this research a mixed between- and within-subjects design was used. There were four between-subjects variables: gender, age, the figure of the premise statements and the order of the repeated measures. Gender and age were selection variables, gender and the order of the repeated measures were controlled in the design, and the figure of the premise statements was an experimental variable which was manipulated. There were two within-subjects variables: the magnitude of length difference and the spatial deixis of the relational terms. Hypotheses which predict dependent variable outcomes are presented and summarized below.

Age: A Between-subjects Factor

H_1: Transitive reasoning will increase with age of the subject.

Age was a selection factor in the research design. Three age groups were equally sampled. Included were groups of 5-6 year-olds, 7-8 year-olds and 9-10 year-olds. Breslow (1981) pointed out that one of the major questions arising from past research on transitive reasoning concerns the age at which it emerges. The age range selected for this research reflects the variety of ages found in the literature to accompany the finding of transitive reasoning. Trabasso and others (Braine, 1959; Bryant & Trabasso, 1971; Trabasso, 1975) found that children as young as 5 years can perform transitive inferences as well as adults, provided certain preconditions of retention and understanding of premises is insured. Piaget and his colleagues (Inhelder & Piaget, 1964; Smedslund, 1963) found transitive reasoning to appear, with other concrete operational competencies at age 7 to 8 years of age. Chapman and Lindenberger (1988) found that 60 percent of third graders (mean age 9 years) could reason transitively about length relations.

Chronological age (CA) is a convenient dimension along which to measure changes in behavior with age. It is a functional component of a developmental dependent variable (DDV) (Wohlwill, 1973). A developmental function (f) is the form of the relationship between the chronological age of individuals and the changes observed to occur in their behavioral responses over the course of their development [DDV = f(CA)]. The DDV in this study was transitive reasoning. Transitive reasoning was measured through judgement, justification and the understanding of necessity. Because this project was designed as a cross-sectional study, the developmental hypothesis can involve only predictions that concern the age differences in the understanding of the transitive relation. As such, an age increase in transitive reasoning as measured by each of the measures of transitive reasoning was expected.

Premise Figure: A Between-subjects Factor

> **H2:** Under task conditions in which an optimal figure of the premise statements (A-B, B-C) is presented, transitive reasoning should be enhanced, compared to task conditions in which a control on the optimal figure of the premise statements (A-B, C-B) is presented.

According to the mental models approach (Johnson-Laird, 1983), when a direct link has to be established between end terms, as it does in a transitive reasoning problem, the figure of the premise terms operates to promote or hinder reasoning. An optimal figure, such as A-B, B-C, which aligns the middle term between the end terms, should facilitate the construction of the series A-B-C. In a condition which makes the alignment of the middle term more difficult, such as the figure A-B, C-B, additional operations are required. Johnson-Laird found that an optimal figure of the premise terms produced the predicted response A-C over 90% of the time with verbal syllogisms in adults. On the other hand, when the control condition was used, the predicted response A-C occurred only 50% of the time. The same effect was expected with transitive reasoning about length because both syllogistic and transitive reasoning involve inferences regarding the relation between two terms by means of an intermediary term.

Length Difference: A Within-subjects Factor

> **H3:** A large length difference will promote transitive reasoning relative to a smaller length difference.

With respect to the spatial representations of the length differences in the transitivity of length problem, those differences which are more definite (i.e., those which are bigger and therefore more remarkable) should be recovered from memory more accurately (Frick, 1988) and therefore should contribute to better transitive reasoning. To test this idea, each subject in this study was administered the transitivity of length task in a large length difference condition where the length comparison is that of 1.5 inches (3.81 cm) between each pair of sticks. A small length difference condition of .25 inches (.635 cm) was also administered. These length differences reflect the range of differences used in transitivity of length experiments in the past.

Relational Terms: A Within-subjects Factor

> **H_4:** Use of the relational term "longer" will promote transitive reasoning, compared to the relational term "shorter".

The linguistic feature explanation and the framing resource explanation each predict that using the relational term "longer" will facilitate making of correct inferences in the transitivity of length task, compared to the term "shorter". According to the linguistic feature view, difficulty in comprehending spatial terms is a function of the number of rules of application entailed in the use of the term. Positive and negative values are assigned to the opposite length poles by a linguistic structure. "Shorter" is determined to be negative and thus requires an extra rule to establish its meaning as "not long". According to the framing resource approach, there is a resource that pre-dates the current interaction and directs the interpretation of linguistic input. Such a spatial mental model also favors the use of the relational term "longer".

Interaction of Length Difference and Relational Term

> **H_5**: When there is a match between linguistic relational terms and length differences used in the reasoning task, children's transitive reasoning should be enhanced, compared to a mismatch between relational terms and length differences.

In the discussion of findings from a series of empirical studies Tanz (1980) proposed that the functioning of deictic terms probably is the result of what she called the principle of language-context "transparency". According to that principle, words that match with more salient aspects of the context should be initially easier to decode. In the transitivity of length task, transitive reasoning is expected to be enhanced when the relational term "shorter" is paired with small length differences and when the relational term "longer" is paired with the large length differences. In other words, the effect of the linguistic relational terms used in the transitivity of length task (i.e., the use of "longer" vs. "shorter") would be expected to interact with the magnitude of length differences (large vs. small).

Two different expectations regarding the effects of the within-subjects factors of length difference and relational term have been discussed. One predicts a main effect for both length difference and relational term. The other predicts an interaction between them. Data were expected to conform to only one of these expectations.

Controlled Factors

Gender: A Between-subjects Factor

The design was balanced for gender of the subjects. It was expected that gender would have no relation to the understanding of transitive relations. This expectation was based on the lack of such relations found in past reported research. But, language was not manipulated in past experiments.

Order of Within-subjects Conditions: A Between-subjects Factor

To control for order of testing effects of repeated measures counterbalancing was utilized.

Chapter 6:

Method

In Table 3, the independent variables described in the previous chapter are listed together with their respective levels. The three between-subjects factors were crossed, yielding the 2 (premise figure) x 3 (age group) x 2 (gender) design portrayed in Table 4. Two males and two females were tested in each of the four orders of crossed within-subjects conditions (also described in Table 3), yielding 8 subjects in each cell and 96 in the study as a whole.

Table 3
Factors

Within-Subjects Factors
- Length Difference
  - LEVEL 1 - large difference
  - LEVEL 2 - small difference
- Relational Term
  - LEVEL 1 - unmarked (longer than)
  - LEVEL 2 - marked (shorter than)

Between-Subjects Factors
- Premise Figure
  - LEVEL 1 - optimal figure (A-B, B-C)
  - LEVEL 2 - control figure (A-B, C-B)
- Age
  - LEVEL 1 - 5 & 6 years
  - LEVEL 2 - 7 & 8 years
  - LEVEL 3 - 9 & 10 years
- Gender
  - LEVEL 1 - male
  - LEVEL 2 - female

- Order of Crossed Within-subjects Factors
  - ORDER 1 - large difference with unmarked
large difference with marked
small difference with unmarked
small difference with marked
  - ORDER 2 - large difference with marked
small difference with unmarked
small difference with marked
large difference with unmarked
  - ORDER 3 - small difference with unmarked
small difference with marked
large difference with unmarked
large difference with marked
  - ORDER 4 - small difference with marked
large difference with unmarked
large difference with marked
small difference with unmarked

Table 4
Between-within Design of the Study Showing the Crossed Between-subjects Factors by Order of Crossed Within-subjects Factors

Premise Figure / Age Group / Gender	Length Difference: Large		Small		Order of Within-subjects Crossed Conditions				Total
	Relational Term								
	Unmarked	Marked	Unmarked	Marked	1	2	3	4	
Optimal									
5-6 years					2	2	2	2	8
Male					2	2	2	2	8
Female									
7-8 years					2	2	2	2	8
Male					2	2	2	2	8
Female									
9-10 year					2	2	2	2	8
Male					2	2	2	2	8
Female									
Control									
5-6 years					2	2	2	2	8
Male					2	2	2	2	8
Female									
7-8 years					2	2	2	2	8
Male					2	2	2	2	8
Female									
9-10 year					2	2	2	2	8
Male					2	2	2	2	8
Female									
Total									96

Subject Selection

The children participating in this study were recruited from three school districts in Western Canada. Children were English speakers and were assessed by the principal or the teacher as neither gifted nor involved in remedial work. Informed consents were obtained from individual parents. The contact letter and consent form is in Appendix C. All children had the procedure explained to them and were told that they could terminate the interview at any time. Verbal permission to proceed was obtained from each child before data collection commenced. Subjects were interviewed in a quiet room in their schools. Each session took 20-30 minutes and was audiotaped.

Instruments

Transitivity of Length Reasoning

A three-term "standard" version of the transitivity of length task, as described by Chapman and Lindenberger (1988) was employed in this research. It was designed to ensure that "a correct solution could be inferred only from a composition of individual premise relations" (Chapman & Lindenberger, p. 544).

Table 4
The Transitivity of Length Protocol Generic Version

Preparatory Questions
(Experimenter points at the sticks which have their ends hidden)
1) **Q: Can you name the colour of the sticks?**
A: These sticks are x, y, and z.
(Experimenter removes the sticks from sight)
Premise One
(Experimenter places sticks A and B on the table)
2) **Q: Tell me which stick is longer/shorter?**
A: The x stick is longer/shorter than the y stick.
(Experimenter hides the ends of A and B)
3) **Q: Could you remind me which stick is longer/shorter?**
(Note If answered incorrectly, show the sticks)
4) **A: The x stick is longer/shorter.**
Premise Two
(Experimenter places sticks B and C on the table)
5) **Q: Tell me which stick is longer/shorter?**
A: The y stick is longer/shorter than the z stick.
(Experimenter hides the ends of B and C)
6) **Q: Could you remind me which stick is longer/shorter?**
(Note If answered incorrectly, show the sticks)
7) **A: The y stick is longer/shorter.**

The premise comparisons and the problem were presented by showing equal portions of the comparison objects only two at a time, through a slit in a screen which was placed between the subject and the interviewer. The remaining objects were hidden out of sight. A generic version of the task used in this research can be seen in Table 4. The exact protocols are compiled in Appendix B.

Transitive reasoning was assessed through a series of questions, beginning with a request for a judgement. This was followed by a request for a justification of the judgement. The necessity of transitive reasoning was further probed through a series of questions designed by Miller (1986). The questions are set out in Table 5 below.

Table 5
Question Series Designed to Elicit the Extent of the Operational Understanding of Transitive Length Relations

Q: Which stick is longer (shorter)?
Q: How do you know?
Q: Are you really sure about your answer?
Q: Is there anything I can say to convince you that your answer is wrong?

A: No.	**A**: Yes
Q: How do you know?	**Q**: What would that be?

Q: Could your answer to this problem change tomorrow, next week, next year?

A: No.	**A**: Yes
Q: How do you know?	**Q**: What would that be?

In an attempt to explore the effect of disconfirming experience on the evaluation of necessity, a disconfirmation procedure was given to 75 of the 96 subjects following the administration of the procedures described above. For this purpose, the premises were presented in an optimal figure (Johnson-Laird, 1983) with sticks of 1 inch (2.54 cm) length difference. The exact presentation and questioning as described previously was employed. After the usual questioning the child was shown the test pair, into which the experimenter had substituted a long stick, where the shortest stick had previously been. The child was asked to explain how it could be that the long stick previously evaluated to be short turned out to be so long. The protocol can be seen in Appendix B.

Premise Figure: A Between-subjects Factor

Two distinct sets of protocols, one each for each of the between-subject factor levels, were devised. In the optimal figure condition, the premises were presented such that the middle term overlapped. That is, A-B, then B-C. This gives rise to the figure A-B-B-C which was hypothesized to facilitate transitive reasoning by making it easier for the subject to identify the two B's with each other. The control figure demands that the premises be presented A-B, then C-B. This makes for a figure A-B-C-B. According to Johnson-Laird (1983), this figure necessitates extra operations, requires more memory capacity and therefore is more difficult to integrate into a valid, necessarily correct, inference. All eight task protocols appear in Appendix B.

Length Difference and Relational Terms: Within-subjects Factors

The large length difference was defined as a 1.5 inch (3.81 cm) difference between the adjacent pairs of the three sticks in the task. The small length difference was defined as a .25 inch (.635 cm) difference in the sticks. The sticks ranged in overall length from 2.5 inch (6.35 cm) to 6 inch (15.24 cm) for the large length difference sticks and from 4.25 inch (10.80 cm) to 5 inch (12.70 cm) for the small length difference sticks. The mean length of the sticks was 4.25 inch (10.80 cm) for the large length difference sticks and 4.63 inch (11.75 cm) for the small length difference sticks. Crossing these two length differences with the two linguistic relational terms "longer" and "shorter" yielded four tasks which were given to each subject:

(a) Large Difference (1.5 inches/3.81 cm) x Unmarked Relation ("longer than")
(b) Large Difference (1.5 inches/3.81 cm) x Marked Relation ("shorter than")
(c) Small Difference (.25 inches/.635 cm) x Unmarked Relation ("longer than")
(d) Small Difference (.25 inches/.635 cm) x Marked Relation ("shorter than")

As described previously, conditions (a) and (d) represent a matched fit between length difference and relational term, and conditions (b) and (c) represent an unmatched fit between the two factors (Tanz, 1980). The four tasks were counter balanced across subjects. Four of the sixteen possible orders of crossed within-subjects conditions were chosen. The orders chosen are described in Table 3. The orders are orthogonal (Campbell & Stanley, 1963) in that each crossed within-subjects condition occurs only once in each level of the orders.

Coding

The three measures of the dependent variable, transitive reasoning, were judgements, justifications, and the understanding of necessity. In addition, a composite variable was derived from criterion judgements and justifications considered together. Each was coded according to the following categories.

1. The judgements given in response to the first test question ("Which stick is longer/shorter?) were categorized dichotomously as correct or incorrect.

2. The justifications given in response to the test question ("How do you know?") were classified into four categories. The first category contained subjects who gave no justification, were silent, or at best said "I don't know" in response to the question "How do you know?". Non-operational answers were characterized by responses which contain justifications which were irrelevant or peripheral to the problem at hand. No logical explanation for the judgment was given. An example of such a non-operational justification is, "because I saw the red one and it was about that big. And the blue one was that big". Partial operational justifications were those which did not provide all of the information required for complete understanding. Such justifications referenced only some of the relevant relations. "Because the yellow was in the middle and the red was bigger than the white" is an example of a partial operational justification.

An operational justification coordinated all of the relations into an explanatory whole. Such an example is, "Red is longer than the blue, because red is longer than yellow and yellow is longer than blue".

Forty-eight (12.5%) of the 384 completed protocols were recoded as a test on coding reliability for justification responses. One third (16) of the recoded selections came from each age group. Within each age group, half (8) were from the optimal premise figure group and half were from the control premise figure group. An equal number from this group (2) came from each paired within-subjects conditions. The interrater reliability was assessed using Cohen's Kappa (K), which ranges from zero (indicating no agreement) to 1.0 (complete agreement). Cohen's Kappa for this recoding was found to be .914.

3. Children's understanding of necessity was indexed through their judgement responses (yes/no) to the necessity questions ("Are you really sure about your answer?", "Is there anything I can say to convince you that your answer is wrong?", and "Could your answer to this problem change tomorrow, next week, next year?"). The following categorizations of necessity were compiled from the dichotomous data described above. Subjects who were classified as not certain were those who said they were unsure of the transitive relation. They could think of something that would convince them to change their minds. Subjects with no understanding of necessity were initially certain, but could be convinced that they may be wrong. Partial understanding of necessity was characterized by certainty in the present only. These subjects could not be convinced to change their mind about the present situation. They were however uncertain about necessity of the solution for the same problem in the future. A full understanding of necessity was assumed when subjects were certain, could not be convinced to change their minds, and were not influenced

by the proposed effect of the passage of time on the necessity of the understanding.

Analysis of the relations among the dependent measures (as reported in Chapter 7) revealed that judgements and justifications were well related, but of unequal difficulty. Justifications was found to be more difficult than judgements. A composite judgements plus justifications measure was calculated in order to produce a more complete estimate of transitive reasoning which reflected these findings. A pass on this variable was defined as correct judgements with either partial operational or operational justifications. A fail was assigned to all incorrect judgements, and to all no-justifications or nonoperational justifications, regardless of the corresponding judgements. This combined measure and judgements alone were analysed in order to test the hypotheses.

Likewise, the necessity criterion was dichotomized into decisions reflecting incomplete understanding of necessity and decisions reflecting full understanding of necessity. The former category was obtained by collapsing the initial codes of not certain, no understanding of necessity, and partial understanding of necessity. This measure of necessity was added to judgements and judgements-plus-justifications in the analysis. Interrater reliability was not carried out because it was based on judgements to the necessity questioning only. That is, the assessment was taken from the score sheets used by the experimenter. These sheets were used to record the judgements which were verbal and non verbal. It was believed that reliability based on such recording of data would be exact because very little or no interpretation would be needed.

The procedure in which disconfirming empirical evidence was presented to 75 of the 96 subjects was coded as evidence accepted or evidence rejected. Children who rejected the evidence claimed that the substituted stick was just that - "a different stick" or they restated their original justification. Children who accepted the evidence, had no reason, believed they had made a mistake (child error), or supposed that the experimenter had made a mistake in presentation of the task (experimenter mistake).

Chapter 7:

Results

The hypotheses of this research project were designed to examine three general questions. First, could it be reconfirmed that measures of transitive reasoning increased with age? Second, does an optimal figure of the premise presentations increase measures of transitive reasoning? Third, does the length difference utilized in the transitive reasoning task and/or the relational term used to describe the length difference influence measures of transitive reasoning? Accordingly, it was found that the transitive reasoning measures of judgements and judgements-plus-justifications increased with age, that the figure of the premise presentations did not directly influence any transitive reasoning measure and that matching of length difference with relational term (large/"longer" and small/"shorter") led to increased incidence of correct transitive reasoning judgements only. The bulk of the following presentation is focused on the main findings. In addition to these major findings, some minor findings concerning interactions of premise figure and age group with length difference and relational term are included. Finally, the exploratory analysis of the relations among the measures of transitive reasoning are included in the presentation of results.

Age Effects on Transitive Reasoning Reconfirmed and Figure of the Premise Presentation Effect Not Found

Calculation of Reasoning Success. Summed successes for each of the measures of transitive reasoning were used to examine the effect of age and figure of the premise presentations on transitive reasoning. The number of each subject's successes for each measure of transitive reasoning (judgements, judgements-plus-justifications, necessity understanding) were summed across the four between-subjects conditions of length difference (large/small) crossed with relational term ("longer"/"shorter"). Correct judgements, passing judgements-plus-justifications, and full understanding of necessity were scored as successes of transitive reasoning. The resultant summed scores ranged from 0 to 4. The particulars of these codes are discussed in Chapter 6.

The hypotheses that there would be a main effect for age and for premise figure were tested for each of the summed measures of transitive reasoning (judgement, judgement-plus-justification, and understanding of necessity) using a 2 (figure) x 3 (age group) multivariate analysis of variance. The F approximation of Wilk's lamda criterion was used as a test of significance for multivariate effects. The mean summed scores for each measure of transitive reasoning are reported by age group and premise figure in Table 7.

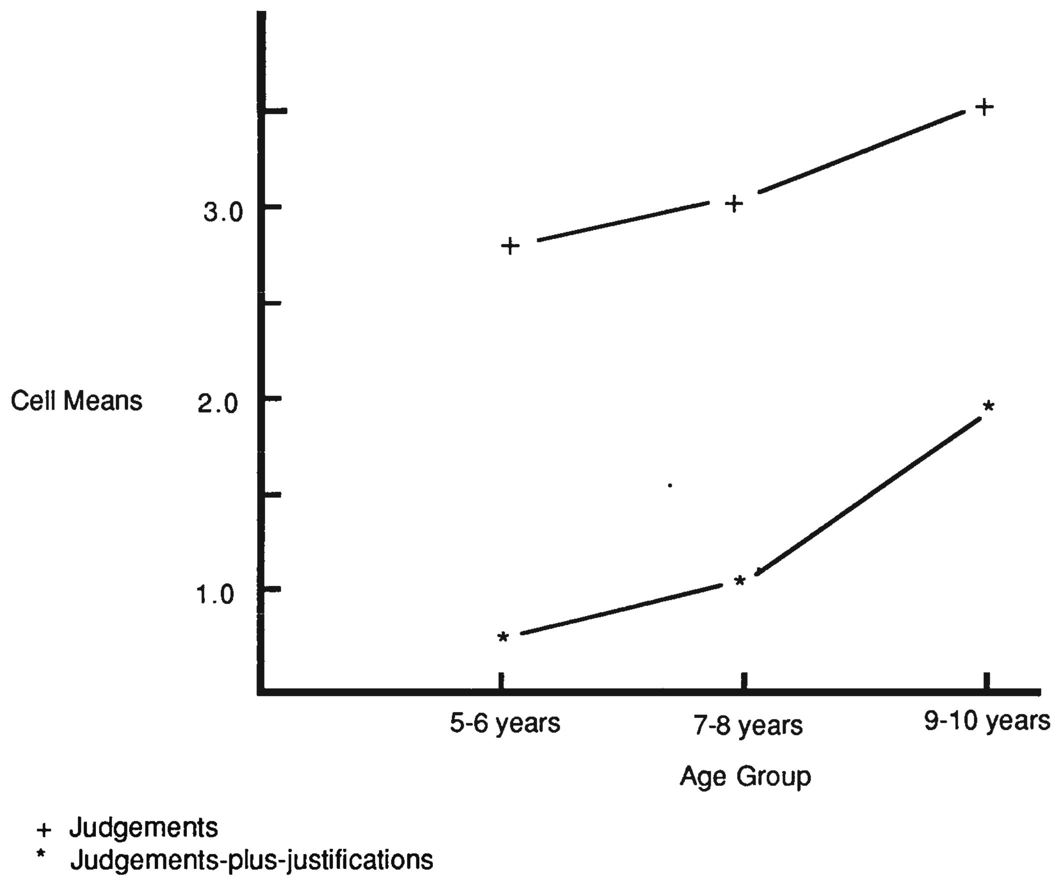
Table 7
Mean Summed Scores for each Measure of Transitive Reasoning Reported by Age Group and Premise Figure

Reasoning Measure	Age Group		
Figure	5-6	7-8	9-10
Judgements			
optimal	2.88	2.88	3.44
control	2.75	3.38	3.63
Judgements-plus-justifications			
optimal	.56	1.06	1.94
control	.94	1.00	1.94
Necessity Judgement			
optimal	1.38	1.13	2.13
control	1.25	1.81	1.81

These analyses revealed a multivariate main effect of age group $F(6) = 3.16$, $p < .006$, and univariate effects of age for judgements, $F(2,90) = 5.06$, $p <$

.008, and for judgements-plus-justifications, $F(2,90) = 7.22$, $p < .001$. The significant univariate effects are plotted in Figure 8. No univariate age effect was found for necessity understanding. No significant main effects for premise figure and no age group x premise figure interaction were found for any of the measures of transitive reasoning.

Figure 8
Mean Summed Scores of Judgement and Judgement-plus-justification Measures of Transitive Reasoning Plotted by Age Group



Length Difference and Relational Term Together Influence Transitive Judgements

The third general question of this research concerned the influence of length difference and relational term on measures of transitive reasoning. Length-difference and relational-term comparison variables were first calculated and analyses were carried out on those to determine if length difference and relational term had separate effects or if they interacted to influence transitive reasoning.

Calculation of Length-difference and Relational-term Variables. In the design the within-subjects variables of length difference and relational term were fully crossed to produce four within-subjects test conditions: (a) large length difference paired with the relational-term "longer" , (b) large length difference with the relational-term "shorter", (c) small length difference paired with the relational-term "longer" , and (d) small length difference with the relational-term "shorter". The total percentage of children passing each measure of transitive reasoning in each condition is given in Table 8.

Table 8
Total Percent Correct for Within-subjects Test Conditions (length-difference fully crossed with relational-term) for Each Measure of Transitive Reasoning

Length-difference	Term	Judgements	Judgements-plus-justifications	Necessity Judgements
large				
	"longer"	80.2	32.4	40.6
	"shorter"	78.1	30.2	43.8
small				
	"longer"	76.0	29.2	36.5
	"shorter"	81.3	32.3	37.5

Note: N = 96

In order to avoid violating the assumption of independence of observations, and to simultaneously take advantage of the strengths of within-subjects data, the within-subjects variables of length difference and relational term were converted to between-subject categorical variables. A three-level length-difference comparison variable was derived by classifying children into three categories, including (a) children who had better reasoning in the large length condition, (b) children whose reasoning was equal in each length, and (c) children who had better reasoning in the small length condition. The same procedure was followed for establishing a relational-term comparison ("longer" vs. "shorter") variable. These variables were utilized in the remaining analysis. In Table 9 the cross classification of the length-difference comparison variable

Table 9
Cross Classification of Length-difference Comparison Variable with Relational-term Comparison Variable for Each Measure of Transitive Reasoning

Reasoning Measure Length-difference Comparison	Relational Term Comparisons			
Judgements	"longer"> "shorter"	"longer"= "shorter"	"longer"< "shorter"	total
large>small	**5**	4	**13**	22*
large=small	3	48	3	54
large<small	**10**	5	**5**	20*
total	18**	57	21**	96
Judgements-plus-justification	"longer"> "shorter"	"longer"= "shorter"	"longer"< "shorter"	total
large>small	9	2	7	18*
large=small	5	55	5	65
large<small	2	6	5	13*
total	16**	63	17**	96
Necessity Judgements	"longer"> "shorter"	"longer"= "shorter"	"longer"< "shorter"	total
large>small	8	4	12	24*
large=small	5	49	3	57
large<small	4	3	8	15*
total	17**	56	23**	96

* Refers to the unequal length difference category row totals
** Refers to the unequal relational term category column totals

with the relational-term comparison variable collapsed over age group and premise-figure group are reported by measures of transitive reasoning (judgements, judgements-plus-justifications and necessity understanding).

Main effects for length difference and for relational term on each measure of transitive reasoning were tested by (a) comparing totals for the large>small category with totals for the large<small category of the length comparison (single starred in Table 9) and (b) by comparing totals for the "longer">"shorter" category with totals for the "longer"<"shorter" category of the relational-term comparisons (double starred in Table 9), using McNemar tests of correlated proportions (Siegel & Castellan, 1988). Hierarchical loglinear analysis and loglinear analyses with simple contrasts were carried out on the cross classification of length-difference comparison with relational-term comparison for each measure of transitive reasoning in order to determine if an interaction effect model explained the influence of within-subjects factors on transitive reasoning.

When the dependent variable is continuous (i.e., is normally distributed with constant variance) and the independent variable is categorical, the means of the dependent variable can be tested against the explanatory variables using analysis of variance models. When both dependent and independent variables are continuous, then relationships between them can be evaluated using regression analysis. Loglinear models were designed to deal with problems in which the dependent variable and the explanatory variables are categorical (Fienberg, 1980). They are useful for uncovering complex relationships among variables in multiway cross-classification tables. In such tables, all the variables used for classification are categorical independent variables, and the dependent variable is the number of cases (usually assigned through a dichotomous assessment) in the cells. The definition of interaction is based on cross-product ratios of expected cell values. As a result, the models are linear in the logarithms of the expected value scale. In loglinear analysis the independence model is one of many models tested. Loglinear analysis thus becomes much like the flexible model-testing approach of ANOVA or multiple regression. That is, the contribution of each variable to the estimated expected cell frequencies and the contributions of the associations among variables can be assessed precisely. The likelihood ratio test statistic (L^2) is used to evaluate the models. L^2 is equivalent to the Pearson chi-square statistic as N gets large and follows a chi-square distribution. The alternative strategy is to analyze all possible pairs of variables using a chi-square test of independence while ignoring the other variable(s) in the cross-classification table. This latter approach may not uncover important patterns of associations or interactions among the cells (Green, 1988).

No overall effect due to length difference and no overall effect due to relational term was found for any of the measures of transitive reasoning (judgements, judgements-plus-justifications or understanding of necessity). The hierarchical loglinear analysis uncovered significant interactions for all measures of transitive reasoning: for judgements, $L^2(4) = 53.81$, $p = .001$; for judgements-plus-justifications, $L^2(4) = 38.78$, $p = .001$; for necessity understanding, $L^2(4) = 49.29$, $p = .001$. However, these interactions mainly result from the large center cell in the frequency distribution. In that cell, children performed equally well in the large and small length-difference conditions as well as in the "longer" and "shorter" relational-term conditions.

In order to test the interaction represented in the corner cells only, loglinear analysis with simple contrasts was performed. This analysis tests contrasts between levels of each variable and the interactions between the contrasts pertaining to each variable. Of interest in the present context was the interaction between (a) the contrast of large>small versus large<small length differences, and (b) the contrast of "longer">"shorter" versus "longer"<"shorter" relational terms. This interaction was significant for judgements, $z = 2.12$, $p < .05$, but not for the other measures of transitive reasoning. The significant interaction is shown in Table 9 in boldface type (under judgements).

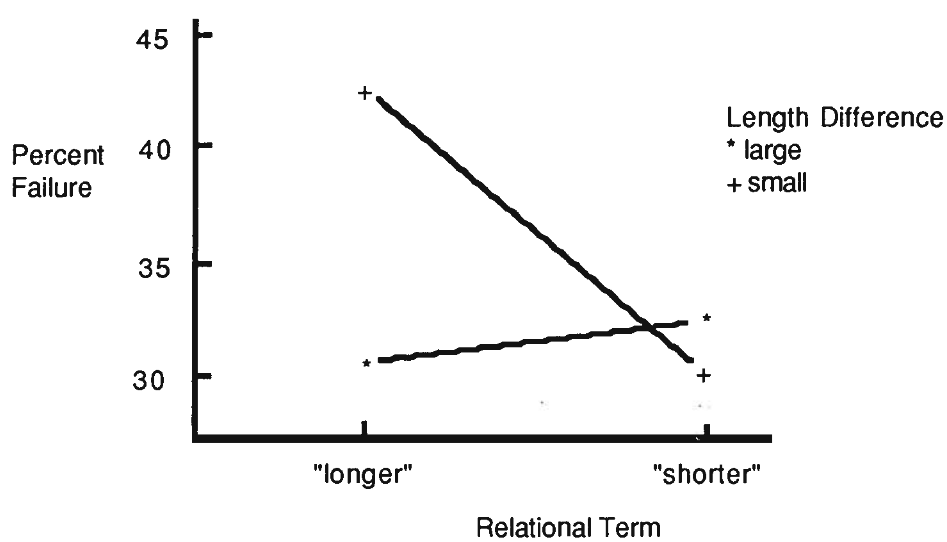
In order to interpret the foregoing interaction better, the corner cells were eroded into the percent of correct judgements in the original four within-subjects conditions: (a) large length difference paired with the relational-term "longer" , (b) large length difference with the relational-term "shorter", (c) small length difference paired with the relational-term "longer" , and (d) small length difference with the relational-term "shorter". Table 10 shows the percent of subjects from the corner cells in Table 9 with correct judgements in each of the original within-subjects conditions.

As indicated in Table 10, subjects in each cell had 0% correct judgements in the within-subjects condition which was the opposite of the within-subjects condition describing the dominant effects of the defining cell. For example, subjects who reasoned well in the large length-difference conditions and in the"longer" relational-term conditions had 0% correct judgements in the small length-difference x "shorter" relational-term condition. Accordingly, the length-term interaction shown in Figure 9 resulted because more children from the interacting cells failed the large/"shorter" (33%) and small/"longer" (42%) conditions than the large/"longer" (30%) and small/"shorter" (30%) conditions.

Table 10
Percent of Subjects (in each of the corner cells in Table 17) With Correct Judgements in Each of the Original Within-subjects Crossed Conditions

Cell ($\underline{n}$)	Within-Subjects Conditions			
	Large with 'Longer'	Large with 'Shorter'	Small with 'Longer'	Small with 'Shorter'
Large>Small x "longer">"shorter" ($\underline{n}$ = 5)	100	100	100	**0**
Large>Small x "longer"<"shorter" ($\underline{n}$ = 13)	76.9	100	**0**	76.9
Large<Small x "longer">"shorter" ($\underline{n}$ = 10)	80	**0**	100	80
Large<Small x "longer"<"shorter" ($\underline{n}$ = 5)	**0**	80	80	100
Total Percentage ($\underline{n}$ = 33)	70	67	58	70

Figure 9
Percentage of Subjects (in each of the corner cells in Table 17) with Failing Judgements in Each of the Original Within-subjects Crossed Conditions



Interactions of Length-difference and Relational-term Comparison Variables with Age and Premise Figure

An overall main effect of age for judgements and for judgements-plus-justification of transitive reasoning, and the absence of effect for figure of the premise presentation on transitive reasoning has been presented previously. The effects of age and premise figure in combination with the length-difference and relational-term comparison variables are reported here.

Interactions between length-difference comparison, age, and premise figure and those between relational-term comparison, age, and premise figure were tested with hierarchical loglinear analysis (Fienberg, 1980, p. 43). The best-fitting model was established using backward elimination (Norusis, 1990). In each case, the results of the three-way loglinear analysis are given, and the significant effects are illustrated in frequency tables.

Because of the possibility of low cell frequencies in these and all subsequent three-way loglinear analysis, (a) significant three-way interactions were interpreted with extreme caution and only after appropriate post hoc analyses, and (b) the significance of all significant two-way interactions was confirmed by recalculating the significance of the two-way interactions collapsed over the third variable. In the event, all two-way interactions remained significant when checked in this way; therefore, only the results of the original three-way analysis are given here.

The results of the analyses of the comparison variables with age group and premise figure are presented for each measure of transitive reasoning, in Table 11 for the length-difference comparison and in Table 12 for the relational-term comparison. For judgements, each comparison variable (length difference and relational term) had a significant main effect. These results merely reflect the unequal overall frequencies in the three levels of the comparison variables. That is, more children fell into the equal category (large=small and "longer"="shorter") than into either of the other two levels of the comparison variables.

Table 11
Hierarchical Loglinear Analysis of Length-difference Comparison with the Between-subjects Conditions of Age Group and Premise Figure

Dependent Variable Effects in Best-Fitting Model	Likelihood Ratio Tests	
	L^2 change[a] (df), p	L^2 fit[b] (df), p
Judgements		7.53 (15), <.941
Length	21.22 (2), <.001	
Judgements-plus -justifications		7.05 (12), < .854
Length x Figure	7.83 (2), <.05	
Necessity Understanding		12.28 (15), < .658
Length	29.27 (2), <.001	

[a] Refers to the amount that L^2 increases when the effect is dropped from the best-fitting model.
[b] Refers to the fit of the best-fitting model.

Table 12
Hierarchical Loglinear Analysis of Relational-term Comparison with the Between-subjects Conditions of Age Group and Premise Figure

Dependent Variable Effects in Best-Fitting Model	Likelihood Ratio Tests	
	L^2 change [a](df), p	L^2 fit[b] (df), p
Judgements		10.14 (15), < .81
Term	27.41 (2), <.001	
Judgements-plus -justifications		.000 (0), = 1.00
Term x Figure x Age	12.69 (4), <.05	
Necessity Understanding		4.46 (12), < .97
Term x Figure	10.17 (2), <.01	

[a] Refers to the amount that L^2 increases when the effect is dropped from the best-fitting model.
[b] Refers to the fit of the best-fitting model.

For judgement-plus-justification, an interaction between length-difference comparison and premise figure was found. The effect is shown in boldface type in Table 13. More children in the control figure group fell in the large>small length comparison category (n = 14) than children in the optimal premise-figure group (n =4), sign test, p < .015, but no such trend was found for the large<small category (n= 7 vs. n = 6, respectively, sign test n. s.). In addition, a three-way interaction between relational term, age group and premise figure was found for judgement-plus-justification. The effect is shown in boldface print in Table 14. On the whole, children in the unequal relational-term comparisons ("longer">"shorter" and "longer"<"shorter") were more likely to come from the control figure group (n = 23) then from the optimal figure group (n = 10), sign test p < .036, but this trend was more typical of the 5-6 year-olds and the 9-10 year-olds, than of the 7-8 year-olds. Because of the low cell frequencies observable in Table 14, the significant three-way interaction should be interpreted with caution.

Table 13
Cross Classification of Children by Age Group, Premise Figure and Length-difference Comparison for *Judgements-plus-justifications*

Length-difference Comparison	Premise Figure		
Age Group	Optimal	Control	Total
Large>Small			
5-6 years	0	4	4
7-8 years	3	4	7
9-10 years	1	6	7
Total	**4**	**14**	18
Large=Small			
5-6 years	15	10	25
7-8 years	11	9	20
9-10 years	12	8	20
Total	**38**	**27**	65
Large<Small			
5-6 years	1	2	3
7-8 years	2	3	5
9-10 years	3	2	5
Total	**6**	**7**	13

Note: Figures in boldface type represent the length-difference comparison x premise figure effect indicated in Table 11.

Table 14

Cross Classification of Children by Age Group, Premise Figure and Relational-term Comparison for *Judgements-plus -justifications*

Relational-term Comparison	Premise Figure		
Age Group	Optimal	Control	Total
Longer>Shorter			
5-6 years	**0**	**4**	4
7-8 years	**5**	**3**	8
9-10 years	**0**	**4**	4
Total	5	11	16
Longer=Shorter			
5-6 years	**14**	**10**	24
7-8 years	**9**	**9**	18
9-10 years	**15**	**6**	21
Total	38	25	63
Longer<Shorter			
5-6 years	**2**	**2**	4
7-8 years	**2**	**4**	6
9-10 years	**1**	**6**	7
Total	5	12	17

Note: Figures in boldface type represent the relational-term x premise-figure x age-group effect from Table 12.

Table 15

Cross Classification of Children by Age Group, Premise Figure and Relational-term Comparison for *Necessity Understanding*

Relational-term Comparison	Premise Figure		
Age Group	Optimal	Control	Total
Longer>Shorter			
5-6 years	1	3	4
7-8 years	1	4	5
9-10 years	1	7	8
Total	**3**	**14**	17
Longer=Shorter			
5-6 years	10	9	19
7-8 years	9	9	18
9-10 years	11	8	19
Total	**30**	**26**	57
Longer<Shorter			
5-6 years	5	4	9
7-8 years	6	3	9
9-10 years	4	1	5
Total	**15**	**8**	21

Note: Figures in boldface type represent the relational-term x premise-figure effect from Table 12.

As with judgements the main effect of length difference on necessity understanding reflects only the unequal overall frequencies in the three levels of the comparison variable. An interaction between relational-term comparison and premise figure was also found. Examination of the totals in boldface print in Table 15 reveals that children in the "longer">"shorter" relational-term comparison category were more likely to come from the control figure group (n = 14) than from the optimal premise-figure group (n = 3), sign test, p = .012. This trend was reversed, but nonsignificant, for the "longer"<"shorter" relational-term comparison category (n = 8 vs. n = 15, sign test, p = .28).

Relations Among the Measures of Transitive Reasoning

Are Judgement, Justification, and Necessity Understanding Related? In this research, the position was taken that language is an object of reasoning. As such it has its own problem space. Therefore verbal explanations or justifications of cognitive judgements, as well as the dichotomous judgements themselves, are important components of reasoning responses. Statistical analysis of the measures of transitive reasoning was carried out in order to establish the nature of the relation between judgements and explanations in these transitive reasoning data. Exploration of the role of necessity understanding in transitive reasoning was also made possible.

The summed successes for each measure of transitive reasoning were used to examine the relation between measures of transitive reasoning. In Tables 16, 17, and 18 the cross classification of the success scores is reported by each pair of the measures of transitive reasoning. The strength of the relation between the pairs of success scores for the dependent variables was established using Pearson correlation. The relation of each pair of reasoning variable with age partialled out was also computed.

Table 16
<u>Cross Classification of Summed Scores for *Correct Judgements* and for *Operational Justifications*</u>

Correct Judgement	Operational Justifications					
	0	1	2	3	4	Total
1	4	2	0	0	0	6
2	13	2	0	1	2	18
3	11	3	8	4	1	27
4	16	4	8	9	8	45
Total	44	11	16	14	11	96

Table 17
<u>Cross Classification of Summed Scores for *Correct Judgements* and for *Full Necessity Understanding*</u>

Correct Judgement	Full Necessity Understanding					
	0	1	2	3	4	Total
1	1	2	2	1	0	6
2	7	2	3	5	1	18
3	12	5	6	1	3	27
4	12	9	7	7	10	45
Total	32	18	18	14	14	96

Table 18
<u>Cross Classification of Summed Scores for *Operational Justifications* and for *Full Necessity Understanding*</u>

Operational Justification	Full Necessity Understanding					
	0	1	2	3	4	Total
0	17	10	8	5	4	44
1	3	3	2	0	3	11
2	7	2	3	3	1	16
3	3	2	3	2	4	14
4	2	1	2	4	2	11
Total	32	18	18	14	14	96

Between judgements and justifications, the zero-order correlation was .32, $p < .001$, and the first-order partial correlation with age partialled out was .24, $p < .009$. Between judgements and necessity understanding the zero-order correlation was .12, $p < .13$, and the first-order partial correlation with age partialled out was .06, $p < .27$. Between justifications and necessity understanding, the zero-order correlation was .23, $p < .01$, and the first-order partial correlation with age partialled out was .18, $p < .04$. In summary, judgements and justifications were found to be related, even with age partialled out. Similarly a low correlation was found between justifications and necessity understanding, which remained significant even with age partialled out.

Sign tests (Siegel & Castellan, 1988) were used to assess if the measures of transitive reasoning differed in difficulty. The number above the diagonal was compared to the number below the diagonal in each cross classification table. Those tests revealed that summed judgement scores were significantly higher than both summed justification scores $z = 8.05$, $p < .001$ and summed necessity understanding scores $z = 6.11$, $p < .001$. But no significant difference was found between justifications and necessity understanding $z = -1.43$, $p < .08$.

Since judgements and justifications were found to be related, but of differential difficulty, it was decided that a composite of these dependent variables would reflect a more complete assessment of transitive reasoning than each would on its own. Judgements and justifications were thus combined into a single variable, judgements-plus-justifications, as described in Chapter 6. All other analyses involved the three dichotomous measures of transitive reasoning (judgements, judgements-plus-justifications, and necessity understanding). Judgements were retained as a measure of transitive reasoning, in order that the view that reasoning is assessed accurately by judgement alone, be represented in the data analysis.

Are Necessity Understanding and Response to Disconfirming Evidence Related? The relationship between necessity understanding and the introduction of misleading data (Hall & Kaye, 1978; Wason & Johnson-Laird, 1972) was a minor focus of this research and an exploratory manipulation was added to the protocols in order that it could be examined. Loglinear analysis was used to analyze the data.

Cross classification of the frequency of necessity understanding and the frequency of responses to disconfirming evidence from the 75 subjects who experienced the additional manipulation appears in the starred cells of Table 19. A three-way hierarchical loglinear analysis of necessity understanding with response to disconfirming evidence and with age group revealed that the three-way interaction was the best-fitting model, L^2 change (2) = 6.39, $p < .04$. In Table 19 the cross classification of incidence of scores for necessity understanding by response to disconfirming evidence is reported by age in bold face print. The 2 x 2 cross classifications of necessity understanding with response to disconfirming evidence for each age group was analyzed using Fisher exact tests. The results were as follows: for 5-6 year-olds, $p < .28$; for 7-8 year-olds, $p < .29$; for 9-10 year-olds, $p < .008$. Thus, there was a significant relationship between necessity understanding and response to disconfirming evidence for 9-10 year-olds only. That is, most 9-10 year-olds who demonstrated complete understanding of necessity rejected disconfirming evidence (11 of 12).

Table 19
Cross Classification of Frequency of Scores for Necessity Understanding with Response to Disconfirming Empirical Evidence Reported by Age Group N=75

Necessity Understanding Age Group	Children's Response to Disconfirming Evidence	
	accept evidence	reject evidence
"incomplete"		
5-6 years	7	7
7-8 years	11	7
9-10 years	7	4
total	25*	18*
"complete"		
5-6 years	7	5
7-8 years	4	4
9-10 years	1	11
total	12*	20*

In summary, three major findings result from the data analysis. First, transitive reasoning in the form of judgements and judgement-plus-justifications increases with the age of the subject. Second, the figure of the premise presentations does not directly influence transitive reasoning. But, interactions of premise figure with length difference and with relational term influenced judgement-plus-justification and necessity understanding respectively. These latter findings were not a focus of this research project. Third, matching of length difference with relational term (large/"longer" and small/"shorter") leads to increased incidence of correct transitive reasoning judgements. Beyond these findings, exploratory data analysis of the relation among the measures of transitive reasoning were presented.

Chapter 8:

Discussion

The purpose of this research was twofold: first, to reconfirm the developmental properties of transitive reasoning, and second, to examine the influence of spatial and linguistic problem-space conditions on children's transitive reasoning. As expected, the incidence of correct transitive judgements and the incidence of operational transitive judgements-plus-justifications increased with the age-group of the subjects. Not expected, was the absence of direct influences on the measures of transitive reasoning when each spatial and linguistic problem-space condition was considered alone.

However, combinations of spatial and the linguistic problem-solving space conditions were found to influence the measures of transitive reasoning. The most important of these latter findings concerned the matching of the spatial problem-space condition, length difference, with the linguistic problem-space condition, relational term. That is, when the length difference condition and the relational term condition are matched (large length difference with relational term "longer" and small length difference with relational term "shorter") there was increased frequency of correct transitive reasoning judgements only. Other less important combinations of the spatial problem-solving space conditions with the linguistic problem-solving space condition included the following: first, a large length-difference facilitated transitive judgements-plus-justifications in the control premise-figure condition, but not in the optimal premise-figure condition, and second, the youngest and the oldest children were more likely to have unequal judgements-plus-justification success vis a vis relational term in the control premise figure condition, and third, children in the optimal premise-figure group were more likely to have necessity understanding with relational term "shorter" as compared to relational term "longer", and those in the control premise-figure condition were more likely to be successful with "longer" as compared to "shorter". Finally, findings concerning the relations between the measures of transitive reasoning (judgements, judgements-plus-justifications, and necessity understanding) were reported, since they contribute to the ongoing discussion concerning the nature and measurement of reasoning.

The measures of transitive reasoning are used to structure the discussion concerning the theoretical importance of the above findings. Summary conclusions are drawn regarding the nature of transitive reasoning. Contributions and limitations of this research are also discussed.

Judgements

In general, judgement reasoning about transitive length differences was found to be hindered when the length-difference condition and the relational-term condition were mismatched. That is, judgements were more likely to be incorrect when the large length difference was paired with the relational-term "shorter" and the small length difference was paired with the relational-term "longer". This general result was consistent with the language-context transparency principle (Tanz, 1980), as interpreted within either the operational constructive model or the mental model of transitive reasoning proposed in Chapter 2.

According to the operational constructive approach, in transitive reasoning, children must first note the extension of one stick beyond the end of the other stick when those sticks are aligned with each other in the presentation of each premise pair. This is a first-order operation called comparison. The observed length difference must then be centered on as an instance of a relation of length, either "longer" or "shorter" as directed by the linguistic relational term. Likewise, according to the mental models approach, direct comparison or digital approximation along the dimension of length is used to determine the absolute lengths of the sticks (Johnson-Laird, 1983). The accompanying linguistic term then cues the truth conditions corresponding to the proposition that one stick is "longer" (or "shorter") than the other. Anything that would facilitate the construction of those comparisons/approximations and the centering on the relation of length involved or of the recognition of the cues that correspond to the truth conditions should enhance transitive reasoning.

In general, the finding of an interaction between relational term and length difference is congruent with the language-context transparency principle (see Hypothesis 5, as stated in Chapter 5). However, on closer examination the interaction was found to be a result of the fact that the pairing of the relational-term "longer" with small length differences lowered performance relative to the other three relational-term x length-difference combinations. Such a pattern could have resulted if children's use of the term "longer" resulted in the expectation that the longest stick would be significantly longer than the comparison stick, an expectation that would be frustrated in the small length-difference condition resulting in some doubt that the longer stick was really longer. According to this finding, use of the term "shorter" would not result in a comparable expectation (that the shorter stick would be significantly shorter); therefore the actual length differences paired with "shorter" would not affect performance.

This pattern of results cannot be further illuminated by consideration of either the linguistic feature complexity approach or the framing resource approach to explanation of the function of deictic terms. The linguistic feature complexity distinction of marked and unmarked is based on a unidirectional concept of length and therefore predicts that relational-term "longer" will always be easier to interpret. Indeed the language-context transparency principle was proposed as a post hoc explanation for data analysis that did not conform to the linguistic feature complexity distinctions of marked and unmarked deictic terms (Tanz, 1980). In the prelinguistic framing resource approach requests for "bigger" or "smaller" interact with the size of the objects under consideration in the context. Requests for "bigger" are predicted to be easy whether the object is big or small. The most difficult request to comply with is when the object is small and the request is for "smaller". The results of the data analysis from this study run counter to both these explanations of deictic function.

The most likely interpretation combines features of both the constructive operational approach and Tanz's (1980) language-context transparency principle (see Chapter 4). The language-context transparency principle predicts the general interaction. The specific nature of the interaction can be described using the epistemic triangle (Chapman, 1991a) approach to constructive operation. Under this approach, cognitive activity results from interaction between the object, the active knower and elements of the social context (i.e., linguistic input). The asymmetry of primacy between the relations longer and shorter is posited on the expectation that the stick designated as "longer" would be affirmed in the spatial problem-solving space to be large and therefore easily centered on. When the small length difference is linguistically designated as "longer", difficulty is encountered because the perceptible length difference is not long. Such an interpretation must be constructed since it is a negation and once constructed, in this situation, it further competes with the direction given for centering in the relational-term "longer".

In common with the language-context transparency principle, however, this asymmetry is seen to result, not in a main effect of relational term or of length difference, but in a relational-term x length-difference interaction, such that the effect of deictic term and physical context occurs only with the affirmative term; "longer" is easier to decode when it conforms to the expectation that the stick in question is really longer than when the context runs counter to that expectation. That is, it seems that either length difference (large or small) can be termed "shorter", but only the large length difference can be termed "longer".

Judgements-plus-justifications

The concepts of good form (Inhelder & Piaget, 1964) and figural effect (Johnson-Laird, 1983) were used to predict a main effect of premise figure on transitive reasoning. Main effects were not found for any dependent variable. However, the results from further qualitative data analysis indicated that the premise-figure condition interacted with length difference (Table 11) to significantly influence judgement-plus-justification reasoning. An advantage of the large versus the small length difference was found more frequently than the reverse in the control premise-figure condition as opposed to the optimal premise-figure condition (Table 13).

The mental models approach uses judgement only as the criteria for reasoning. Therefore the lack of any influence of figure of the premise statements on judgements is a null finding against that approach. Since this finding was found for judgements-plus-justifications, it needs to be explained within the operational constructive approach which utilizes judgements-plus-justifications as a criteria for reasoning. According to Piaget (Inhelder & Piaget, 1964), "good form" promotes visualization or perceptibility of the whole.

Transitivity of length understanding based on an image having good form requires that the elements be arranged in perceptible, symmetric, repeating or graduated patterns. According to Piaget's usage, the understanding of transitivity becomes operational when it is no longer dependent on this graphic arrangement.

In addition to consideration of good form, the influence of length difference must be considered in the discussion of the finding that there is an advantage of the large over the small length difference in the control premise-figure condition as opposed to the optimal premise-figure condition. According to the operational constructive approach to transitive reasoning about length, reasoners must first carry out the first-order operation of comparison with each premise pair before combining those operations in a second-order composition. Frick (1988) found that recovery of a spatial representation is influenced by its perceptual clarity or "definition". In this study, the large length difference was assumed to be more definite and recovered more accurately than the small length difference. The better recovery of large length differences should lead to better reasoning, because it should facilitate the operation of comparison.

As described above, children in the control premise-figure group had more judgement-plus-justification reasoning success with the more easily recovered large length difference. That is, the length-difference conditions interacted in a manner predicted by Frick (1988), but only under control premise figure conditions. The effect of large length difference would seem to be washed out when the problem is posed in an optimal premise figure. Such a premise-figure effect can be explained in the operational constructive approach.

According to the operational constructive approach, transitive reasoning involves two levels of operations: first-order operations of comparison between the premise pairs and a second-order operation that integrates the first-order comparisons. The manipulation of length difference and premise figure should affect, respectively, each level of operations. Specifically, large length differences should facilitate the operation of comparison relative to small length differences, because the outcome of those comparisons is a judgment of large (or small) based on the observation of the length differences resulting from the alignment of premise pairs with each other. In contrast, the optimal premise figure should facilitate the second-order operation of integration relative to the control figure, because in the optimal figure (A>B, B>C), but not the control figure (B>C, A>B), the premise pairs are already ordered in a way that enables length differences to be added more readily ((A-B) + (B-C) = (A-C)).

The effects of length difference and premise figure would interact, if the rearrangement of premise pairs in the control premise-figure condition in some way interfered with the retention or reconstruction of the original pairwise comparisons. Such would be the case, for example, if said rearrangement of premises tapped a cognitive resource also necessary for premise recall (Case, 1985; Chapman & Lindenberger, 1989; Pascual-Leone, 1980). Significantly, in this study, the length difference x premise-figure interaction was found only for judgments-plus-justifications, a criterion that requires explicit recall of premise comparisons if a constructive operational justification is to be given by the subjects.

Thus, the observed interaction helps to differentiate the two approaches only insofar as it is present in judgements-plus-justifications only and as such can be explained post hoc under the operational constructive approach. The absence of the unpredicted interaction for judgements is a null finding under the mental models approach. The question of the nature of the spatial representation remains unresolved: Is it the length difference or the representation of absolute lengths that is affected by the interaction of length difference and premise figure?

Necessity Understanding

The finding that children reasoned better when the relational term "shorter" was paired with an optimal premise figure or when the relational term "longer" was paired with the control premise figure (see Table 16) cannot be explained by the approaches to reasoning reviewed in this thesis. The inferential rule approach (Braine & Rumain, 1983) would has predicted a main effect of language on necessity understanding. An interaction of language with the context, such as this interaction is, would only occur with immature reasoners who use ordinary comprehension of linguistic input. The analysed interaction does not include age group. As such the two-way interaction of premise figure with relational term found here is not informed by the inferential rule approach. According to the mental models approach (Wason & Johnson-Laird, 1972), functional necessity understanding can be tested by a response to misleading or disconfirming evidence. Note that exploratory data analysis found that only older children (age 9-10 years old) who were assessed to understand necessity rejected disconfirming evidence. There was no relationship between necessity understanding and rejection of disconfirming evidence among the younger children. Under the operational constructive approach (Chapman, 1988; Piaget, 1983), conditions that support the recognition of the possibilities in the problem space and that promote the identification of the necessary possibility would occur most likely when the optimal premise figure is paired with the relational-term "longer". They would least likely occur when the control premise figure is paired with the relational-term "shorter" as is found in the data displayed in Table 15. Therefore, finally, the finding that relational-term in combination with premise-figure influenced necessity understanding as described in this data analysis, does not conform to the operational constructive explanation either.

Conclusions

In this research three approaches to reasoning were examined, including the inferential rule approach, the mental models approach, and the operational constructive approach. The forms of transitive reasoning proposed in each were reviewed in Chapter 2, and specific hypotheses were drawn from the different approaches in Chapter 3 and Chapter 4, and were summarized in Chapter 5. In this conclusion, those hypotheses will be reviewed in the light of the obtained results with the goal of determining the extent to which those results do or do not provide support for the theories from which the hypotheses were generated.

1. The hypothesized age increase (H_1) in transitive reasoning was found for judgments and judgments-plus-justifications, but not for necessity understanding. These results in themselves do not provide differential support for the three approaches. All three would predict a general age increase in transitive reasoning, and none can explain why such an increase failed to materialize in the case of necessity understanding alone.

2. The prediction that performance should be higher in the optimal as opposed to the control premise-figure group (H_2) was derived from both the mental models approach and the operational approach. The failure to find such an effect thus counts against both theories. The conclusion cannot be made that mental models theory has been falsified by this result (because one cannot conclude from a null finding that the null hypothesis is true), but one can

conclude that a finding based on a prediction inferred from the theory has failed to materialize. The minor unpredicted finding of better judgements-plus-justifications reasoning when large length difference is combined with control-premise figure can be explained post hoc under the operational constructive approach.

3. The hypotheses that main effects of length difference (H_3) and relational term (H_4) on reasoning would be found was derived both from the mental models approach (on the assumption that larger length differences would be better processed and thus more easily recoverable from a mental model and that use of "longer" would cue the length relation more easily) and from the operational constructive approach (on the assumption that the addition of length differences described by Inhelder and Piaget [1964] would be more easily carried out with more obvious length differences and that the relational-term "longer" would always promote centering on the length relation). Thus, the failure of this hypothesis counts against both theories.

The interaction of language and operations (H_5) follows from Chapman's (1990) concept of the "epistemic triangle" as well as from Tanz's (1980) transparency principle. However, the specifics of the length-difference x relational-term interaction was best explained according to the constructive operational approach on the assumption that linguistic meanings play a role in the second-order integration of first-order operations (Chapman, 1990; Piaget, 1968, Chapter 3). It is unclear how this finding could be explained even post hoc in the inferential rule approach or in the mental models approach.

On balance, it is concluded that the results favor the operational constructive approach, but not overwhelmingly. This judgement rests chiefly on the length x term interaction for judgements, the specifics of which were predicted post hoc only in that approach. However, the absence of a main effect of length and a main effect for premise figure, also predicted from the operational constructive approach, suggests that some modification of that approach might be necessary. This need to modify the theory is also supported by the post hoc theoretical explanation of the length-difference x premise-figure interaction for judgements-plus-justification. In this context it is important to note that Inhelder and Piaget's (1964) addition of length differences is only one candidate for the second-order operation that serves to integrate the first-order premise comparisons. Another possibility, that a second-order operation of seriation might serve this function instead, was suggested by Chapman (1991b). Perhaps significantly, the predicted main effect of length differences was derived from Inhelder and Piaget's model, and the failure of that prediction could be interpreted as indicative of the need for an alternative.

In summary, the results reported in this dissertation do not fit in any of the three theoretical approaches without some accommodation of the theories in question. But the amount of accommodation is perhaps less in the operational constructive approach than in the other two.

Limitations

Several issues of limitation constrain the results and theoretical inferences made possible by the results of this study. They arise from the deductive theorizing, the selected methodologies and the empirical aspects of this study. First, a deductive test of the approaches to reasoning failed to materialize. Further, significant post hoc discussion was required in the explanation of the findings which were important. Second, limited power in the design, the usual problems surrounding measurement of transitive reasoning, and selection of the levels of the experimental manipulations tested, need to be calculated into consideration of the results and their interpretation. Finally, the scale of the measures and the match of that with the data analysis is an issue of limitation which also needs to be counted.

This research failed in its attempt to discriminate the approaches to reasoning. I began this project, by planning a direct test of the three approaches to reasoning. The pilot of this first test, failed miserably, for a variety of reasons. One of which was that the test was too large in its scope. This second project was designed as a more subtle test, perhaps even an elegant test. Through examination of length difference and linguistic aspects common to each approach, discrimination among them was expected. As it turned out however the complementary aspects of the approaches were uncovered, and neither approach was discriminated from the others as the most relevant explanation of transitive reasoning.

The need for post hoc explanation of both major findings from this research is also a limitation of this study. Judgement reasoning about transitive length differences was found to be hindered when the length-difference condition and the relational-term condition were mismatched. In general this finding was predicted by the deductive hypothesis that when there is a match between linguistic relational terms and length differences used in the reasoning task, children's transitive reasoning should be enhanced, compared to a mismatch between relational terms and length differences. However, the significance of this general finding was based on the result that the pairing of the relational-term "longer" with small length differences lowered performance relative to the other three relational-term x length-difference combinations. Post hoc discussions involving theoretical elements of the constructive operational approach were required to explain the specifics of the finding. This cannot be counted as empirically derived support for that approach. The second major finding that there was an advantage of the large over the small length difference in the control premise-figure condition as opposed to the optimal premise-figure condition was not predicted in the deductive theorizing. Further significant post hoc discussion involving the constructive operational approach was required to explain the finding. Therefore, in the end, neither approach was supported through the deductive tests in this research and only the operational constructive approach was adequate alone in explaining the findings post hoc.

Methodological problems constitute the major threats to the findings of this study. Since this study is exploratory in nature, type I and type II errors are a concern insofar as an accounting needs to be made of their presence. With respect to type I errors, the alpha was set at the conventional level of .05 which is considered adequate protection in most studies of this sort.

Type II errors are concerned with power. Power is enhanced primarily when there is a large sample size, when there are strong manipulations of the experimental variables, and when the error variance is reduced. In this study the use of many variables resulted in a small cell size (see Table 4). Balancing the less critical variables, order of the within-subjects conditions and gender, was not a remedy for the inadequacy of the sample size, but it constituted a rationale for collapsing their levels and thus creating adequate cell sizes for the planned analyses.

As for strength of the manipulations, in the case of premise figure there was good reason to expect that the manipulation would be strong enough to produce results (Johnson-Laird, 1983). As a result, it was placed between subjects in the design. Length difference was a manipulation not used in previous research and it therefore was placed within subjects to maximize its effect. Relational term had mixed findings in past research (Tanz, 1980) and it was also placed within subjects to maximize and clean up its effect. The fact that the predicted effect did not materialize in the case of premise figure, indicates either that the premise figure manipulation was weaker than expected and/or the design may have been too complex for the effect to emerge. Perhaps in future studies fewer variables should be tested within subjects only.

A final method of increasing power is to decrease variance in the dependent measures. In the qualitative analysis, all four trials were collapsed to give rise to a categorical comparison variable. Such a strategy should have decrease the variance in the analyses in which it was used and thus increased the power of the results.

Aspects of measuring the dependent variable also limit the results and interpretations of the study. First, only one task, the standard task, was utilized, thus perhaps limiting the validity of the results by eliminating the opportunity of a calculation of criterion validity. Further, collapsing the codes for justification and necessity understanding into dichotomous readings of those measures of transitive reasoning may place the construct validity at risk. The same limitation applies to the combination of judgement with the dichotomous assessment of justification to calculate a more complete measure of transitive reasoning. Although, a theoretical and an empirical case was made for this latter action. Beyond all of this, but included in the issue of validity, is the usual concern with the reliability of the measures of judgement and of justification in reasoning tasks. It is to be remembered that: (1) the use of judgement only to measure reasoning has been criticized for false positive errors of measurement and (2) the use of justification has been criticized for false negative errors of measurement (Brainerd, 1973a). More recently, a meaningful case has been made for the use of both (Chapman, 1987).

The selection of the levels of the spatial problem-solving space conditions, figure of the premise presentations and length difference, may also have limited the findings of this research. The optimal and control figure of the premise presentations were chosen to optimize the differential in difficulty between them (Johnson-Laird, 1983). Other, premise combinations are possible and may have produced different results. As for the length differences selected, they were chosen to be representative of length differences used in transitive reasoning studies to date. The nature of the concept of length was not considered in their selection. The results of the study begin to demonstrate that the conceptualization of length is relative to the context of the task. Perhaps length difference even has a perceptual threshold which interacts with the task. These points were not tapped adequately by the selected length differences. They and other possible speculations need to be considered should the research be repeated.

Finally, one needs to be aware of the level of the measure used in this study and of the power of the statistical methods utilized to analyse the data. All of the transitive reasoning measures were categorical. Only by assuming equality of each reasoning success, was the interval style reasoning success measure possible and the use of the more powerful analysis of variance methods defensible. The remaining data analysis using loglinear analysis was correctly applied to the categorical between subjects data, but as a statistical analysis method it has less power than the analysis of variance techniques. These trade-offs of measurement scale and data analysis technique are limitations which would need to be attended to in further research projects.

Contributions

Theoretical, empirical, and methodological contributions were made by this work to the psychology of reasoning about transitive length relations. First, a balance in deductive theorizing about reasoning was attempted, and it was somewhat successfully achieved. Second, deductive tests of some obvious spatial and linguistic conditions of the transitive reasoning task were carried out. Third, the methodology allowed the more powerful within-subjects data to be collected and to be analysed using appropriate cross-classification methods of data analysis that demand independence of observations.

For myself, one of the most useful phases of this work was the process of achieving a balance in the tension between theoretical approaches to reasoning about transitive length relations. The use of the concept of problem space (Newell, 1980) as an advance organizer for the theoretical analysis encouraged consideration of both the static and the active elements of reasoning. This balance of consideration led to each approach being most realistically represented and perhaps to the better understanding of reasoning overall. It allowed for the similarities of the approaches to be highlighted. The conflict between the approaches was therefore reduced and consideration of the operational constructive approach (Chapman, 1990; Piaget, 1968) as a candidate among contemporary cognitive views was thus encouraged.

A second accomplishment of this study was that it attended to experimental conditions which had never been previously tested in the transitive reasoning task. Both theory and common sense suggests that spatial and linguistic problem-space conditions such as length difference (Frick, 1988; Jacoby & Craik, 1979), figure of the premise presentations (Johnson-Laird, 1983; Inhelder & Piaget, 1964), and relational term (Clark, 1970; McGonigle &

Chalmers, 1985; Tanz, 1980) should influence reasoning about transitive length relations. An extensive review of the literature failed to unearth tests of these conditions within transitive reasoning tasks. That these conditions did not influence transitive reasoning in this research as predicted implies that they are more complex then originally assumed or that the methodological limitations of the study were too important or too many to allow the predicted results to emerge. Hence, the results from this research make only a beginning contribution to understanding their influence.

Finally, aspects of the methodology of this research contribute to the general experience of testing the effects of development. It is the case, when studying change/development, that longitudinal designs are the ideal since they are based on evidence of individual change. Results from longitudinal designs can therefore tell us something about the shape of the change function. However, for economic reasons it is most usual to find research into change to be carried out on cross-sectional sample panels. Such data can only tell us something about frequency of phenomena. The collection of within-subjects data in cross-sectional designs strengthens the quality of the results because there is repeated measures from individuals. However, frequency data, in order to be cross-classified and analysed with chi square or with modeling techniques such as log linear analysis, should fulfill the requirement of independence of observation. That is, each subject must only be counted once in a cross-classified table. In this research, a technique was used where within-subjects data was transformed into categories such that each subject was categorized only once, thus making it possible to model the incidence data appropriately. I think that this is an advance over the common practice of assuming independence when analyzing such data.

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Appendix A: Glossary

<u>action theory</u>: This is a position taken with respect to representation. From this perspective representation is a product of a structure of knowing constructed through activity.

<u>analogical reasoning</u>: Reasoning based on the assumption that if things have some similar attributes, other attributes also will be similar.

<u>attention span</u>: The number of items subjects successfully apprehend or reproduce through immediate ordered recall

<u>bootstrapping</u>: Strategies which, in combination with current mental structures and with companion strategies, are hypothesized to bring about new more complex cognitive structures (Bereiter, 1985).

<u>definition</u>: Any characteristic which increases its clarity of representation.

<u>deixis</u>: Linguistic devices which link word utterances with the spatial-temporal contexts in which they occur.

<u>functional solution</u>: A correct answer in the transitivity task, which has been inferred as a function of some perceptual dimension, for example, spatial position.

<u>inferential rule approach</u>: A deductive approach to reasoning, where reasoning is believed to be the product of mental rules which govern the relations between premises and conclusions.

<u>language-context transparency principle</u>: When the relaton between the concept and the words attached to it is easily interpreted there is transparency and the language-contect unit is more easy to understand.

<u>learning paradox</u>: The issue of defining how more adequate forms of reasoning arise from less adequate forms.

mediation theory: The position that knowledge resides in or is coexistent with a mental representation.

mental models approach: A constructive approach to reasoning, wherein models are constructed with mental tokens embued with the properties represented in the linguistic descriptions of the premises.

natural reasoning: Reasoning that is embodied in the understanding of natural language.

operational constructive approach: A constructive process by which the premise statements are understood in terms of otential operations carried out on the relevant objects and a conclusion is derived from the composition of those operations.

operations: Interiorized actions which are coordinated within a total system.

performance conditions: A variety of personal states that the subject brings to the test situation and which contribute to error.

problem-solving space: A way to represent a problem or task which includes all the possible states of knowledge about the problem and the operations for generating and manipulating that knowledge (Newell, 1980).

structure: The totality of possible transformations formed by the intercoordination of interiorized actions.

test conditions: Conditions that can be manipulated in an experiment or left uncontrolled. Such conditions may contribute to Tupe I errors of measurement.

truth conditions: The objective conditions to which a given proposition would correspond if it were true.

The Transitivity of Length Protocol:
Minimum Comparison (0.25 inches/.635 cm) X
Unmarked Descriptor (longer than)
Optimal Figure (A-B-B-C)
(three stick version)

Materials

Three coloured sticks of graduated length will be used. (5.0 inches/12.7 cm, 4.75 inches/12.065 cm, 4.5 inches/11.43 cm)
Note Colours will be different from all other protocals. (Blue, White, Red)

Protocol

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)

1) **Q: Can you name the colour of the sticks?**
A: These sticks are blue, white and red.

(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks A and B on the table-**long-**blue on the right)

2) **Q: Tell me which stick is longer?**
A: The blue stick is longer than the white stick.

(Experimenter hides the ends of A and B)

3) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

4) **The blue stick is longer.**

Premise Two-

(Experimenter places sticks B and C on the table - **long**-white on the right)

5) **Q: Tell me which stick is longer?**
A: The white stick is longer than the red stick.
Which stick is longer?

(Experimenter hides the ends of B and C)

6) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

7) **The white stick is longer.**

Premise Memory Check

(Examiner hides the ends of A and B - **long**-blue on the right)

8) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of B and C - **long**-white on the right)

9) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

Test Questions

Three term problem

(Experimenter hides the ends of A and C (blue and red)- **long**-blue on the right)

10) **Which stick is longer?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Protocol: Minimum Comparison (0.25 inches/.635 cm) X Marked Descriptor (shorter than) Optimal Figure (C-B-B-A) (three stick version)

Materials

Three coloured sticks of graduated length will be used. (4.75 inches/12.065 cm, 4.5 inches/11.43 cm, 4.25 inches/10.795 cm)

Note Colours will be different from all other protocals. (Red, Yellow, White)

Protocol

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)

1) **Q: Can you name the colour of the sticks?**
A: These sticks are white, yellow, and red.

(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks C and B on the table-**short**-white on the right)

2) **Q: Tell me which stick is shorter?**
A: The white stick is shorter than the yellow stick.

(Experimenter hides the ends of C and B)

3) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

4) **The white stick is shorter.**

Premise Two-

(Experimenter places sticks B and A on the table-**short**-yellow on the right)

5) **Q: Tell me which stick is shorter?**
A: The yellow stick is shorter than the red stick.

(Experimenter hides the ends of B and A)

6) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

7) **The yellow stick is shorter.**

Premise Memory Check

(Examiner hides the ends of C and B - **short**-white on the right)

8) **Could you remind me again, which stick is shorter?**

(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of B and A - **short**-yellow on the right)

9) **Could you remind me again, which stick is shorter?**

(Note If answered incorrectly, do **not** correct)

Test Questions

Three term problem

(Experimenter hides the ends of C and A (white and red) - **short**-white on the right)

10) **Which stick is shorter?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Protocol:
Maximum Comparison (1.5 inches/3.81 cm) X
Unmarked Descriptor (longer than)
Optimal Figure (A-B-B-C)
(three stick version)

Materials

Three coloured sticks of graduated length will be used. (6 inches/15.24 cm, 4.5 inches/11.43 cm, 3 inches/7.62 cm)
Note Colours will be different from all other protocols. (Yellow, Blue, Red)

Protocol

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)
1) **Q: Can you name the colour of the sticks?**
Q: These sticks are yellow, blue and red.
(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks A and B on the table-**long-**yellow on the right)
2) **Q: Tell me which stick is longer?**
A: The yellow stick is longer than the blue stick.

(Experimenter hides the ends of A and B)
3) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)
4) **The yellow stick is longer.**

Premise Two-

(Experimenter places sticks B and C on the table-**long**-blue on the right)
5) **Q: Tell me which stick is longer?**
A: The blue stick is longer than the red stick.

(Experimenter hides the ends of B and C)
6) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)
7) **The blue stick is longer.**

Premise Memory Check

(Examiner hides the ends of A and B - **long**-yellow on the right)
8) **Could you remind me again, which stick is longer?**
(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of B and C - **long**-blue on the right)

9) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

Test Questions

Three term problem

(Experimenter hides the ends of A and C (yellow and red) - **long**-yellow on the right)

10) **Which stick is longer?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Protocol: Maximum Comparison (1.5 inches/3.81 cm) X Marked Descriptor (shorter than) Optimal Figure (C-B-B-A) (three stick version)

Materials

Three coloured sticks of graduated length will be used. (5.5 inches/13.97 cm, 4 inches/10.16 cm, 2.5 inches/6.35 cm)

Note Colours will be different from all other protocols. (Red, Yellow, Blue)

Protocol

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)

1) **These sticks are blue, yellow, and red.**
Can you name the colour of the sticks?

(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks C and B on the table-**short**-blue on the right)

2) **Q: Tell me which stick is shorter?**
A: The blue stick is shorter than the yellow stick.

(Experimenter hides the ends of C and B)

3) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

4) **The blue stick is shorter.**

Premise Two

(Experimenter places sticks B and A on the table-**short-yellow** on the right)

5) **Q: Tell me which stick is shorter?**
A: The yellow stick is shorter than the red stick.

(Experimenter hides the ends of B and A)

6) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

7) **The yellow stick is shorter.**

Premise Memory Check

(Examiner hides the ends of C and B - **short**-blue on the right)

8) **Could you remind me again, which stick is shorter?**

(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of B and A - **short**-yellow on the right)

9) **Could you remind me again, which stick is shorter?**

(Note If answered incorrectly, do **not** correct)

Test Questions

Three term problem

(Experimenter hides the ends of C and A (blue and red) - **short**-blue on the right)

10) **Which stick is shorter?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Protocol:
Minimum Comparison (0.25") X Unmarked Descriptor (longer than)
Controlled Figure (A-B-C-B)
(three stick version)

Materials

Three coloured sticks of graduated length will be used. (5.0", 4.75", 4.5")

Note Colours will be different from all other protocals. (Blue, White, Red)

Protocal

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)

1) **Q: Tell me the colours of these sticks?**

A: These sticks are blue, white and red.

(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks A and B on the table-**long-**blue on the right)

2) **Q: Tell me which stick is longer?**

A: The blue stick is longer than the white stick.

(Experimenter hides the ends of A and B)

3) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

4) **The blue stick is longer.**

Premise Two

(Experimenter places sticks B and C on the table - **short**-red on the right)

5) **Q: Tell me which stick is longer?**

A: The white stick is longer than the red stick.

(Experimenter hides the ends of B and C)

6) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

7) **The white stick is longer.**

Premise Memory Check

(Examiner hides the ends of A and B - **long**-blue on the right)

8) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of B and C - **short**-red on the right)

9) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

Test Questions

Three term problem

(Experimenter hides the ends of A and C (blue and red)- **long**-blue on the right)

10) **Which stick is longer?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Protocol:
Minimum Comparison (0.25") X Marked Descriptor (shorter than)
Controlled Figure (A-B-C-B)
(three stick version)

Materials

Three coloured sticks of graduated length will be used. (5.75", 4.5", 4.25")

Note Colours will be different from all other protocals. (Red, Yellow, White)

Protocal

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)

1) **Q: Can you name the colour of the sticks?**
   A: These sticks are red, yellow, and white.

(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks A and B on the table-**long**-red on the right)

2) **Q: Tell me which stick is shorter?**
   A: The yellow stick is shorter than the red stick.

(Experimenter hides the ends of A and B)

3) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

4) **The yellow stick is shorter.**

Premise Two-

(Experimenter places sticks C and B on the table-**short**-white on the right)

5) **Q: Tell me which stick is shorter?**
   A: The white stick is shorter than the yellow stick.

(Experimenter hides the ends of C and B)

6) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

7) **The white stick is shorter.**

Premise Memory Check

(Examiner hides the ends of B and A - **long**-red on the right)

8) **Could you remind me again, which stick is shorter?**

(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of C and B - **short**-white on the right)

9) **Could you remind me again, which stick is shorter?**

(Note If answered incorrectly, do **not** correct)

Test Questions

Three term problem

(Experimenter hides the ends of A and C (white and red) - **long**-red on the right)

10) **Which stick is shorter?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Protocol:
Maximum Comparison (1.5") X Unmarked Descriptor (longer than)
Controlled Figure (A-B-C-B)
(three stick version)

<u>Materials</u>

Three coloured sticks of graduated length will be used. (6", 4.5", 3")

Note Colours will be different from all other protocols. (Yellow, Blue, Red)

<u>Protocol</u>

<u>Preparatory Questions</u>

(Experimenter points at the sticks which have their ends hidden)

1) **Q: Can you name the colour of the sticks?**
   A: These sticks are yellow, blue and red.

(Experimenter removes the sticks from sight)

<u>Premise One</u>

(Experimenter places sticks A and B on the table-**long-**yellow on the right)

2) **Q: Tell me which stick is longer?**
   A: The yellow stick is longer than the blue stick.

(Experimenter hides the ends of A and B)

3) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

4) **The yellow stick is longer.**

<u>Premise Two</u>-

(Experimenter places sticks B and C on the table-**short**-red on the right)

5) **Q: Tell me which stick is longer?**
   A: The blue stick is longer than the red stick.

(Experimenter hides the ends of B and C)

6) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

7) **The blue stick is longer.**

Premise Memory Check

(Examiner hides the ends of A and B - **long**-yellow on the right)

8) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of B and C - **short**-red on the right)

9) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

Test Questions

Three term problem

(Experimenter hides the ends of A and C (yellow and red) - **long**-yellow on the right)

10) **Which stick is longer?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Protocol: Maximum Comparison (1.5") X Marked Descriptor (shorter than) Controlled Figure (A-B-C-B) (three stick version)

Materials

Three coloured sticks of graduated length will be used. (5.5", 4", 2.5")
Note Colours will be different from all other protocols. (Red, Yellow, Blue)

Protocol

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)

1) **Q: Can you name the colour of the sticks?**
A: These sticks are red, yellow, and blue.

(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks A and B on the table-**long**-red on the right)

2) **Q: Tell me which stick is shorter?**
A: The yellow stick is shorter than the redstick.

(Experimenter hides the ends of A and B)

3) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

4) **The yellow stick is shorter.**

Premise Two

(Experimenter places sticks C and B on the table-**short**-blue on the right)

5) **Q: Tell me which stick is shorter?**
A: The blue stick is shorter than the yellow stick.

(Experimenter hides the ends of C and B)

6) **Could you remind me which stick is shorter?**

(Note If answered incorrectly, show the sticks)

7) **The blue stick is shorter.**

Premise Memory Check
(Examiner hides the ends of B and A - **long**-red on the right)
8) **Could you remind me again, which stick is shorter?**
(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of C and B - **short**-blue on the right)
9) **Could you remind me again, which stick is shorter?**
(Note If answered incorrectly, do **not** correct)

Test Questions
Three term problem
(Experimenter hides the ends of C and A (blue and red) - **long**-red on the right)
10) **Which stick is longer?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

The Transitivity of Length Disconfirming Evidence Protocol: Comparison (1.00 inches/2.54 cm) X Unmarked Descriptor (longer than) Optimal Figure (A-B-B-C) (three stick version)

Materials

Three coloured sticks of graduated length will be used. (3.0 inches/7.62 cm & 6.0 inches/15.24 cm, 5.0 inches/12.7 cm, 4.0 inches/10.16 cm)

Note Colours will be different from all other protocals. (Blue & Blue*, Red, Yellow)

Protocol

Preparatory Questions

(Experimenter points at the sticks which have their ends hidden)

1) **Q: Can you name the colour of the sticks?**
A: These sticks are blue, yellow and red.

(Experimenter removes the sticks from sight)

Premise One

(Experimenter places sticks A and B on the table-**long-**red on the right)

2) **Q: Tell me which stick is longer?**
A: The red stick is longer than the yellow stick.

(Experimenter hides the ends of A and B)

3) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

4) **The red stick is longer.**

Premise Two-

(Experimenter places sticks B and C on the table - **long**-yellow on the right)

5) **Q: Tell me which stick is longer?**
A: The yellow stick is longer than the blue stick. Which stick is longer?

(Experimenter hides the ends of B and C)

6) **Could you remind me which stick is longer?**

(Note If answered incorrectly, show the sticks)

7) **The yellow stick is longer.**

Premise Memory Check

(Examiner hides the ends of A and B - **long**-red on the right)

8) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

(Examiner hides the ends of B and C - **long**-yellow on the right)

9) **Could you remind me again, which stick is longer?**

(Note If answered incorrectly, do **not** correct)

<u>Test Questions</u>

<u>Three term problem</u>

(Experimenter hides the ends of A and C (blue* and red)- -red on the right)

10) **Which stick is longer?**

11) **How do you know?**

12) **Are you really sure about your answer?**

13) **Q: Is there anything I can say to convince you that your answer is wrong?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

14) **Could your answer to this problem change tomorrow, next week, next year?**

A: No.	**A: Yes**
Q: How do you know?	**Q: What would that be?**

15) **What if I showed you the sticks?**
(show the sticks)

16) **How can it be that the blue* stick is longer than the yellow one?**

17) **Probe??**

Appendix C: Consent

Fall 1989

Dear Parent

I am a graduate student at the University of British Columbia and I have been granted permission by your school board to carry out research in the school your child attends. The project studies human inventiveness. I am interested in how people acquire new ideas and adapt to new problem situations. Like adults, children are capable of producing solutions that are new to them in problem-situations they have never previously encountered. In this project we plan to study inventiveness in children by posing them simple problems that they are unlikley to have experienced before and then asking them how they did it. In this way, I intend to study the origins of inventiveness in children, and from its origins I hope to learn something about its nature. I looking for children in the early school years, between the ages of five and ten years, to take part in this study.

Children generally enjoy participating in the study because the problems they are asked to solve are not hard and are conducted in a game-like atmosphere. Toy-like material is used to pose puzzles to the children. Word quizzes are also used in the project. Of course, your child, should he or she participate will be free to stop at any time during the session. The administration of the puzzles and quizzes should keep your child out of he class room for 30 minutes and your children's answers will be audio taped. The data gathered from your child will be kept strictly confidential. In fact, no permanent record of the names of participants will be kept. Participants will be identified only by number.

I am a doctoral student in developmental psychology and I myself will be administering the puzzles and quizzes. If you have questions before giving consent, please call me at **683-6539**. Dr. Chapman is the research advisor for this project and he would also be happy to answer any of your questions. He can be contacted at **228-2229**.

I thank you for your time and consideration. I remain

Yours Sincerely

Jane Drummond
Doctoral Student
Developmental Psychology
University of British Columbia

Parental Consent Form - page 2

Project: **Invention**

I consent to the participation of my child ____________________in the study described on the previous page.

Name:____

Date: ____

Signed:____

Age of Child:____

I do not consent to the participation of my child _______________in the study described on the previous page.

Name:____

Date:____

Signed:____

Please retain the front page of this form for your future reference. Please return this page to your child's school. Thank you for your time and consideration.