THE DEVELOPMENT
OF THE CONCEPT OF NUMBER
IN THE MENTALLY DEFICIENT

by

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We accept this thesis as conforming to the
required standard

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Date April 30, 1965
In this study an attempt was made to validate, in the case of mentally deficient children, Piaget's hypothesis of a necessary sequence of three stages in the development of the concept of number. The test procedures used were derived from a similar investigation of normal children by Elkind, (1960a) The subjects were 90 children between the ages of 9 and 17 with a range of mental ages from 2:3 to 10:2 on the Peabody Picture Vocabulary Test. These included 60 resident at The Woodlands School and 30 living at home, both groups evenly divided between boys and girls.

Scalogram analysis (by Green's method) of the data supported Piaget's hypothesis. The 18 questions of the test procedure were found to have the necessary sequence of a true scale and this sequence was reflected with great regularity in the subjects' response patterns. The characteristics of the raw data suggested that Piaget's three stages were adequately represented by the scale of questions.

$X^2$ test of institutionalized and non-institutionalized groups of both sexes showed no significant difference in their responses to the test procedures. From this it was inferred that neither sex nor length of institutionalization influence the development of the concept of number.

A significant positive correlation between raw scores on the Peabody Picture Vocabulary Test and correct responses to the test procedures was found. The existence of this relationship between language and level of ability to understand number was dealt with speculatively in terms of the role played by language in the development of intelligence.

The chief finding of interest in this study was that Piaget's
hypothesized stages appear to have validity for mentally deficient children. This suggests that Piaget's model of intellectual functioning may be useful in understanding the mentally deficient. The potential value of using a Piagetian analysis of stage of developmental process instead of a standardized I.Q. measurement for intellectual investigations with this group was discussed.
ACKNOWLEDGEMENTS

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CHAPTER I
INTRODUCTION

The search for new models to describe intellectual functioning is on. More and more, the popular concept of the "I.Q." is being labelled as "not useful" (the term applied by theoreticians to ideas that do not give a true picture of the facts), and discarded. Along with it is going the acceptibility of the notion of "intelligence" as an entity, and instead terms that imply process, like "intellectual functioning" and "cognition" are coming into favour.

Early measures of intelligence such as the Binet tests (Terman, 1916), appear to have been based only on the assumption that some children are more able than others, rather than on any formal model of intelligence. However, the standardization technique by which Binet and his successors constructed scales to measure relative ability and thus give it scientific stature as "I.Q.", produced a model as an artifact: that is, that intelligence is a global function distributed on a normal curve throughout the human race. It is this model's implication that intelligence is a something of which all of us have more or less, that is now found to be "not useful".

The present search for new models to describe intellectual functioning is, historically, the second wave of intelligence test reform. About twenty years ago, the Wechsler intelligence scales (Wechsler, 1944) were constructed as an application of a model which broke down the global function concept of intelligence into verbal and performance aspects. When validation studies did not support this dualistic hypothesis, the global intelligence model seemed vindicated.

Subsequently, in the construction of intelligence tests for atypical
human beings such as the blind, deaf, or cerebral palsied, Binet test results became the criterion of intelligence against which the validity of the new scales was evaluated. Typical are such tests as the Peabody Picture Vocabulary Test (Dunn, 1959) and the Columbia Mental Maturity Scale (Burgomeister et al, 1954) both constructed for people who cannot speak. These give intelligence quotients that are significantly correlated with those obtained on the Binet tests, and are therefore assumed to tap an underlying global intellectual ability. Despite their ingenious methods of obtaining standardized responses from people who communicate with great difficulty, these tests still describe intellectual functioning in quantitative terms only because they follow, at one remove, the Binet test which is based on a model that does not deal with intellectual processes.

Current theoretical criticism of the static global intelligence model finds it "not useful" as a concept because it does not account for observed qualitative variations in intellectual behaviour. As an hypothesis, it is unrelated to any theoretical framework in fields of study which encompass intelligence, such as biology, psychology or logic. Finally, it is unproductive of further hypotheses for scientific investigation.

Clinicians also often find the I.Q. tests "not useful" in a practical sense because such instruments do not consistently measure the intellectual behaviour of people who are not like the standardization group. The very young or the very old, the exceptionally gifted or the mentally deficient are, like the blind, deaf and cerebral palsied mentioned earlier, inadequately measured by I.Q. tests. An even more serious deficiency of the I.Q. tests as clinical tools results from the
fact that they ignore the qualitative differences in intellectual functioning of such individuals.

Recently, research activity concerned with intelligence has followed two divergent trends, both of which turn their backs on the I.Q. concept and instead, focus on intellectual process. For one group of theorists (e.g. Myklebust, 1964 and Griffiths, 1954) the synthesizing principle introduced in lieu of global function, is that of semi-autonomous intellectual systems. These systems may be investigated by tests which are ingeniously related to a model of brain function, as in the Reitan battery (Wheeler, 1963), or by procedures evolved from a theory of perceptual organization, as in the Frostig Test (Frostig, 1961), or by examinations derived from hypotheses concerning language processes, as in the Illinois Test of Psycholinguistic Abilities (McCarthy & Kirk, 1961).

The other current trend in research on intelligence stems from the sophisticated analysis of the development of intelligence by Piaget (Piaget, 1952). On the basis of intensive observations of a few growing children and the fastidious application of logic in theory building, Piaget has produced a model that describes the development of intelligence as an orderly progression through a fixed sequence of stages which mark integrative achievements in the process. Although he began his work over forty years ago, his rationale meets the challenge of contemporary criticism:

"The psychologist, for his part, welcomes the qualitative characteristic of logic, since it facilitates the analysis of the actual structures underlying intellectual operations, as contrasted with the quantitative treatment of their behavioural outcome. Most 'tests' of intelligence measure the latter, but our real problem is to discover the actual operational mechanisms which govern such behaviour and not merely to measure it. The
algebra of logic can therefore help the psychologist by
giving him a precise method of specifying the structures
which emerge in the analysis of the operational mechanisms
of thought." (Piaget, 1957, p. xviii)

Of the many parameters of intellectual development described
by Piaget, the development of the concept of number has probably
been most frequently chosen for validational investigation by other
workers. In Piaget's analysis, the understanding of the concept of
quantity is the operational mechanism basic to the development of con­
cept of number.

Piaget's model of the development of intellectual processes looks
promising and many validation studies have been published. Similarly,
the particular procedures he has described to define the stages of the
development of the understanding of quantity and number have been
replicated in studies by many investigators. However, most such
studies, whether concerned specifically with the development of the
concept of number or with other parameters of intellectual development,
have been made with normal children. Partly for this reason, their
results have not been entirely conclusive about such questions as, "Is
this the way all children grow?", "Must children in all cultures follow
this discrete set of stages in achieving full-blown intelligence?", "What effects might blindness or deafness have on intellectual progress?", and "Does this model explain the intellectual development of the mentally
deficient?"

The present study is concerned primarily with the last of the
foregoing questions, seeking data to reveal specifically whether
mentally deficient children show the sequence of stages which Piaget
defines as generally characteristic of the development of the concept
of number. To obtain such data, procedures used by Elkind (1961a) in
a study of number concept development in normal children, have been applied to 90 mentally deficient children. The results thus produced may not only help to clarify how the mentally deficient progress towards an understanding of quantity and number, but also they may contribute to the testing of Piaget's general hypothesis that there is a fixed, orderly sequence of stages through which intellectual development always proceeds. In addition, secondary attention has been given in this study to the relation between performance on tasks assessing number concept development and the variables of institutionalization, sex and vocabulary level, because there has been some suggestion in the literature that such variables may exert some influence on the achievement of arithmetical skills.
CHAPTER II
SURVEY OF INVESTIGATIONS OF THE DEVELOPMENT
OF THE CONCEPT OF NUMBER

Generally speaking, the dissemination of Piaget's theory of the development of the concept of number appears to have inhibited other promising lines of inquiry in this area. A review of the literature suggests that, as Piaget's work became known, psychologists dropped investigative theory-building and took up validation of Piaget's provocative hypotheses.

A year before the translation of Piaget's "The Child's Concept of Number" (1952) was published, Martin (1951) presented a summary of other types of investigations of the concept of number. He characterized these investigations as lacking "any systematic attempt to examine the process by which children arrive at the answers." (Martin, 1951, p. 155) Nonetheless, these studies are of value as well as interest because they provide acute observations of children's responses to numerical problems, and because they make some attempt to organize developmental scales from a behavioural point of view. For example, Judd (1927) analysed the growth of numerical discrimination abilities beginning with the child's primitive perception of quantity in the recognition of familiar groups, and concluding with discrimination of individual items at first ego-centrically and then as a result of social considerations. He observed that a child enumerates by naming before he enumerates by counting. Judd's analysis supports the hypothesis that number systems are imposed socially and do not arise directly through a process of maturation. A similar order of development was described by McLaughlin (1935). His analysis focussed
on perceptual-conceptual development beginning with the pre-school perception of the simple spatial forms of aggregates. Following this, four subsequent stages of numerical abilities were distinguished:-(1) counting single unities; (2) analysis by recognition of small numbers like two or three; (3) analysis by recognition of small groups; and, finally, (4) prompt recognition and naming of aggregates as groups or cardinal numbers. McLaughlin pointed out that, "the awareness of cardinal numbers involves a higher stage of abstraction than is the case in serial counting." (McLaughlin, 1935, p. 351)

Werner (1940) provided a physiological-functional-social explanation of developmental progress in attaining a concept of number, basing his analysis on observation of the qualitative responses made by children to numerical problems. He defined four levels of development in the attainment of a concept of number:

1. the level of qualitative configurations in which sensations are substitutes for number - for example the motor-rhythmical recognition of the number of steps in a stairway or the optical configuration of the number in a familiar family group,

2. the level of concrete number where configurations are organized by perceptions such as the rhythm in rote counting or the geometrical configuration of number forms,

3. the level of concrete schematic numbers such as body number schemes like the five fingers, or optical schemas such as tallys in a row,

4. the level of abstract number concepts.

Although this analysis is tied to cognitive processes, Werner does not attack the problem of what is going on, subjectively, in the
child's mind. Techniques based on Werner's findings for teaching arithmetic to mentally deficient children (Carrison and Werner, 1943), seem still to lack the "thorough knowledge of the normal development of number concept" which these authors, themselves, consider a prerequisite to teaching. (Carrison and Werner, 1943, p. 309)

In contrast to the multi-faceted developmental process described by Werner, Reiss (1943) summarized the development of the concept of number in terms of four stages of logical operations: pre-numerical, naming, ordering and grouping. He pointed out the difference between enumeration by naming and by counting and suggested that "only when the last number named becomes the identification of the group of objects being named in order, can the child be said really to count." (Reiss, 1943, p. 161)

This emphasis on cardination as a numerical achievement is reminiscent of Judd and has been noted by Werner as well.

A summary of the findings of American research into the development of number concept by Russell gives a picture of this field before the influence of Piaget had been felt. On the basis of evidence found in a variety of sources, he compiled the following list of characteristics of children's numerical abilities as the chief products of the research in that area up to that date.

i. The child's first use of number has no "numerical" significance. Number is used like any other descriptive name.

ii. The child's first numerical concept is one of "manyness" or a differentiation between one or two and more.

iii. Rote counting ability far outstrips the concept of number and is superior to object counting (naming).

iv. Children often learn numbers in serial order.
v. At four and five years most children use terms like "most," "both," and "biggest," but words like "same" and "equal" are not clearly understood.

vi. Listed in order from easiest to most difficult, numerical abilities seem to follow the order "number finding, distinguishing numbers, producing numbers, and number naming."

vii. Few numerical figures are recognized until a child is about six years of age.

viii. At six a child is capable of simple addition and simpler subtraction, especially in concrete problems.

ix. At six there is a beginning knowledge of simple fractions.

x. Girls are somewhat superior to boys in their knowledge of number concepts but not in their use of quantitative expression." (Russell, 1956, p. 128)

With these isolated kernels, constituting the whole meagre harvest of their investigations, it is not surprising that psychologists should greet the newly-translated, dynamic theory of the development of the concept of number presented by Piaget after 1952 with both hope and skepticism. That these mixed feelings still actuate investigations in this field is apparent in reviewing the literature of number concept development during the past ten years.

Piaget's theory of the development of the concept of number is an important integral part of his developmental theory of intelligence and must be understood in relation to the whole theory. Briefly, Piaget hypothesizes a series of stages and substages of intellectual development which are achieved in a fixed order by every child in the course of his intellectual growth. Because Piaget developed his definitions of these stages gradually over a long period of time, there
is some minor inconsistency between descriptions in earlier and later works. The following summary is derived from the outline given in Flavell (1963):

**Stage I:** (birth to two years) The period of sensori-motor intelligence during which the child gradually builds up notions of himself and objects around him in pre-symbolic, self-centred experiential ways. Piaget distinguishes six substages in this period.

**Stage II:** (two to eleven years) The period of preparation for and organization of concrete operations which commences with the first crude symbolization and concludes with the beginnings of formal thought in early adolescence. Piaget distinguishes three substages in this period:

1. The period of pre-conceptual thought (two to four years) in which the child’s thought is tied to his sensori-motor activity and, although he can deal with the idea of an object, he cannot grasp its relation to other objects.
2. The period of intuitive thought (four to seven years) when, although the child can perceive relationships, he can understand them only while he experiences them.
3. The period of concrete operations (seven to eleven years) when the child begins to deal with concepts symbolically, but only in the solution of practical problems in concrete situations.

**Stage III:** (eleven to fifteen years) The period of formal operations when the child achieves the capacity for abstract thought through "decentering" of his concepts and the development of "reversibility" in his logic. Now he can deal effectively,
not only with the reality before him, but also with the world of pure possibility.

Piaget has identified this series of stages (though not with rigid consistency) along many parameters of intelligence such as the ability to deal with the concepts of space, of causality, of number etc. The investigations described in "The Child's Conception of Number" indicate clearly that Piaget is looking for the development of the underlying logic of number in the child.

Flavell points out:

"He wanted to prove and diagnose for developing number-relevant capabilities considerably more subtle and basic than those involved in the familiar elementary operations of counting, of rote addition, subtraction, etc." (Flavell, 1963, p. 310)

He goes on later to say:

"According to Piaget, number is essentially a fusion or synthesis of two logical entities: class and assymetrical relation...One enumerates a set of objects and thereby arrives at its cardinal number value...treating the objects as if they were all alike...a common class...Is number simply a class then? No...one has to order the objects: count this object first, then the next, then the next and so on...so as not to count the same object twice. This ordination process partakes not of class but of relation operations...Numerical units have therefore, a peculiar status: they appear to be both class elements and assymetrical relation elements at one and the same time" (Flavell, 1963, pp. 310-311)

Piaget says that the ability to do all this "number work" rests on the child's discovery that if nothing is added to or subtracted from a quantity, it remains the same, despite all appearances to the contrary. In achieving the idea of this "conservation of quantity" fundamental to the development of a concept of number, the child traverses three substages. These correspond to the three previously described developmental periods in Piaget's theoretical Stage II - "the preparation for and organization of concrete operations".
1. Children in the period of pre-conceptual thought perceive a quantity not as a constant but as a variable, characterized by the single immediate qualitative perception of the moment: e.g. a quantity is "higher" or "smaller" depending on how it is presented. This perception of quantity is variously described as "gross" or "global".

2. Children in the period of intuitive thought are capable of understanding that a quantity remains constant, even though qualities change: e.g. liquid poured from one glass into two is "the same", but these children become uncertain if it is poured into many containers. At this period of development the child begins to "compose" qualities two by two, to realize that if the quantity remains constant, every increase in height is compensated for by a decrease in width, etc. However, the child is still subject to uncertainty when his perceptions contradict this logical insight. This linkage of logic to experience in the perception of quantity is termed "intuitive" or "intensive".

3. Children in the third substage, the period of concrete operations, perceive quantity as unit relations between objects: e.g. "X is twice Y; Y is half of X," etc. Now quantity is always logically "conserved" despite appearances to the contrary. The perception of quantity as remaining the same despite changes in its perceived qualities is called "operational" or "extensive".

Piaget acknowledges some discrepancies in the ages he has assigned to these substages in the development of the quantity-number concept and lays stress on the notion of orderly, logic-linked
intellectual development in an unbroken sequence, rather than on age norms. He feels that his hypothesized sequence will remain unaltered no matter what population is investigated:

"We may recall how Margaret Meade, who proposed repeating in New Guinea certain of our intelligence tests, was in agreement with the theory according to which the stages of reactions to these tests might be the same as regards the order of succession, but might be very different as regards average ages or even the non-attainment of higher levels." (Piaget, 1957, p. 86)

A number of studies have sought to validate age norms suggested or implied in Piaget's work by evaluating statistically the association between stage of development and age. Estes (1956) reported results which confirm her skepticism about the age designations of particular intellectual operations, and Lovell (1959) also queried the link between age and stage. Braine (1959) similarly rejected age levels on the basis of the appearance in his subjects of some operations two or three years earlier and others six months later than reported by Piaget. On the other hand later studies by Lovell and Ogilvie (1960, 1961) and Lovell and Slater (1960) produced results which they interpreted as confirming Piaget's hypothesis. Dodwell (1960 and 1962) came to a similar conclusion, and Elkind (1961a, 1961b, 1962) found strong support for age-linked stages in all three of his reported studies. While these results have, on the whole, supported Piaget, efforts to tie age to stage of development seem rather beside the point in consideration of Piaget's stated point of view.

Without digressing further on the validation of the general aspects of Piaget's theory by individual studies, mention should be made of two comprehensive series of investigations. Six studies produced in Sweden by Smedslund (1961) have sought to identify the kind of learning experience necessary for a child to achieve (among
other things) the ability to use Piaget's 'conservation' in the period of concrete operations. Although these reports are not available in full translation, it appears that Smedslund has chosen a very creative method of validating Piaget, and that his results are, on the whole, consistent with the Piagetian hypothesis. Mention should also be made of the extensive series of studies undertaken under the direction of Father Pinard at the University of Montreal with a view to developing a comprehensive standardized intelligence test based on the work of Piaget. This has not yet been published.

Turning now to studies concerned particularly with validating Piaget's theory of the development of the quantity-number concept, consideration will be given first to findings about normal children and then to findings about the mentally deficient. The responses of normal children have confirmed Piaget's theory in general, if not in every particular. Playing games designed to give experience in comparing quantities was found to alter the Piagetian logical structures used by four children, (Churchill, 1957). Somali, Arab, and European children showed developmental trends like those described by Piaget, but variations were found between socio-economic and cultural groups (Hyde, 1959). Significant correlations between age or I.Q. and stage of development were found by Fiegenbaum (1961). The negative findings obtained by Estes (1956) have been mentioned previously, and positive results obtained by Dodwell (1960, 1962) and Elkind (1961a) will be discussed at greater length in the following pages.

Three English authors, - Woodward (1961), Hood (1962) and Mannix (1959) - have devoted their attention to the development of the quantity-number concept in the mentally deficient. They all report findings lending support to Piaget's theory. Like Dodwell (1960, 1962)
and Elkind (1961a) whose work was done with normal children, they all chose for their area of investigation the three substages * posited by Piaget in the development of conservation of quantity which are also the subject of this paper. For this reason, a brief comparative review of the methods and findings of these five investigators will be given.

Woodward (1961) questioned 94 mentally deficient subjects individually in four Piagetian experimental situations but was not able to classify the answers to one set of experimental questions into all three stages. She presented age and stage of development in a table where a positive relationship was apparent, and found that I.Q.'s of 50 or higher occurred significantly more often among individuals who gave one or more "stage III" (i.e. operational) responses. Hood, (1962) compared 120 normal boys and girls with 40 subnormal children on eight Piagetian tasks. He, too, used an interview method, but did not mention difficulty in classifying his subjects' responses into stages. Again, using tabular presentations, he like Woodward, enunciated a positive relationship between mental age or chronological age and stage of development of the quantity-number concept. Mannix (1959) also made use of eight of Piaget's experiments, many of them the same as those used by the other investigators. He, too, used an open-ended interview technique, and, like Woodward, experienced difficulty in assigning a stage to some of his subjects' responses. Using only the first and third stages for scalogram analysis, he obtained statistical confirmation of the Piagetian sequence of stages in his subjects' responses. He made the reservation that "the developmental stage is not necessarily constant for the same child from test to test."

* Since no other stages will be mentioned subsequently, the prefix "sub" will be omitted for the rest of the paper in the interest of simplicity.
Dodwell's 1960 study corresponded closely to the three described above, except that his 250 subjects were of normal intelligence. His procedure made use of individual structured interviews consisting of fifty-four questions based on five of the Piagetian experiments designed to illustrate the acquisition of the conservation of quantity. He experienced little difficulty in assigning a stage of development to each subject and found significant correlations between age and stage of development and between I.Q. and stage of development. In this study Dodwell also examined the consistency of stage of development of each subject's responses to three different experimental tasks in the light of probability figures based on the theoretical Piagetian sequence. He concluded that his data did "not yield unequivocal support for Piaget's theory of cognitive development." (Dodwell, 1960, p. 205) In a subsequent study designed to evaluate an individual test of number concept, Dodwell (1961) found by scalogram analysis that the Piagetian order of the test did not meet the criteria of a true scale, and concluded that, "the pattern of development of number concepts does not follow the sequence described by Piaget with great regularity." (Dodwell, 1961, p. 36)

Elkind (1961a), in his paper on the Development of Quantitative Thinking provides comparable information in the same area, again about children of normal intelligence. He used a structured interview technique to question 80 children about three Piagetian tasks. He did not attempt to assign an overall stage of development to each child, but instead, analysed the variance of the three age levels in his population to infer support for Piaget's hierarchical, age-related stages of development. He also found that liquids were significantly
more difficult for the children to compare than other materials although there were significant positive correlations between responses to all three of the types of material he used. Finally, like several other investigators cited above, he found a positive correlation between intelligence test scores and competence in the judgement of quantity.

It is often stated that one sex is superior to the other in arithmetical ability at various ages, and that institutionalized children lag behind others in intellectual development. However, few reports have been published on investigations of number competence or the development of a concept of number taking into consideration the subject variables of sex and institutionalization. Three or four published reports bear obliquely on the question of whether living in an institution for a long time affects the development of a concept of number. Cutts and Lane, (1947) found that subjects institutionalized for less than one year had significantly higher achievement scores on the WAIS Arithmetic sub-test than those institutionalized for more than seven years. Bensburg (1953) did not find any significant difference between institutionalized and non-institutionalized boys or girls on arithmetic achievement scores. Lyle, (1960) and Tizard, (1961) suggest that speech development is hindered by institutionalization.

Two studies bear on the question of difference in ability related to sex. Russell, (1956) found girls somewhat superior to boys in knowledge of number concept. Bensburg, (1953) found significant differences in favour of girls in the arithmetic achievement of mildly retarded young people over nineteen years of age, but he attributed this difference to social factors.
To summarize, in this chapter the findings of investigations into the development of the concept of number before translation into English of Piaget's *The Child's Conception of Number* were contrasted with subsequent validational studies of the hypotheses stated in this book. Particular emphasis was given to descriptions of studies validating the Piagetian stages of the concept of number with mentally deficient children and to techniques for making use of Piaget's experiments in replicative studies. No similarly pertinent papers on the effects of sex or length of institutionalization were discovered, but the findings of several published investigations in this general area of investigation were reviewed.
The present study is concerned primarily with testing the validity of Piaget's hypothesis - that all children follow the same sequence of three developmental stages to achieve a concept of number - in the particular case of mentally deficient children. For this purpose it has made use of essentially the same test procedure as that used by Elkind (1961a) in studying number concept development in normal children, but with a different statistical analysis of the resultant data. As well as in the type of subject and kind of statistic used, this study differs from Elkind in its interest in the variables of sex, length of institutionalization and level of verbal understanding, as they may be related to the development of the concept of number.

Subjects

A total of 90 children ranging in age from 9:0 to 16:7 years were used as subjects in this study. All were classed as mentally deficient and * the range of mental ages on the Peabody Picture Vocabulary Test was 2:3 to 10:2.

* The Peabody was chosen as a measure to control for intelligence for three reasons:

1. Although all I.Q. tests have questionable validity for the mentally deficient, the Peabody has a reputation as a useful short form for gross estimates of intelligence.

2. It does not make use of arithmetic or the number concept in any of its discriminating tasks, as do the Cattell, WISC, Binet and Canadian Intelligence Examination.

3. It provides data on level of verbal understanding.
In order to make it possible to assess the relationship between sex, length of institutionalization and number concept development, as well as to control for age and intelligence, the sample was selected so that all relevant variables were evenly represented. The 90 subjects were equally distributed among three age groups, i.e. 30 children between the ages of 9:0 and 12:12; 30 between the ages of 13:0 and 14:12; and 30 between the ages of 15:0 and 16:12. In each of these groups there were 10 boys and 10 girls who had lived at The Woodlands School for four or more years, and 5 boys and 5 girls who had not been in the institution for more than six months *, and who were attending school in the community. Table 1 summarizes these characteristics of the children who were subjects in this study.

Table 1.
Characteristics of Subjects

<table>
<thead>
<tr>
<th>Age</th>
<th>Sex</th>
<th>Length of Institutionalization</th>
<th>Number</th>
<th>Mental Age Range</th>
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<tbody>
<tr>
<td>9:0-12:12</td>
<td>M</td>
<td>+4 yrs.</td>
<td>10</td>
<td>2:4 - 5:11</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-6 mos.</td>
<td>5</td>
<td>3:10 - 6:10</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>+4 yrs.</td>
<td>10</td>
<td>3:3 - 7:10</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>-6 mos.</td>
<td>5</td>
<td>2:3 - 4:5</td>
</tr>
<tr>
<td>13:0-14:12</td>
<td>M</td>
<td>+4 yrs.</td>
<td>10</td>
<td>3:11 - 8:11</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-6 mos.</td>
<td>5</td>
<td>2:9 - 10:0</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>+4 yrs.</td>
<td>10</td>
<td>2:3 - 8:1</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>-6 mos.</td>
<td>5</td>
<td>3:6 - 7:8</td>
</tr>
<tr>
<td>15:0-16:12</td>
<td>M</td>
<td>+4 yrs.</td>
<td>10</td>
<td>2:5 - 7:10</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-6 mos.</td>
<td>5</td>
<td>3:2 - 10:2</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>+4 yrs.</td>
<td>10</td>
<td>3:2 - 6:8</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>-6 mos.</td>
<td>5</td>
<td>2:5 - 6:8</td>
</tr>
</tbody>
</table>

* A number of these children had been institutionalized for short periods for assessment or family relief etc.
Procedures for Obtaining Data

This study makes use of three experiments that have been described and used by Piaget to define the three stages of the quantity-number concept. These are:

A. The liquid test, in which coloured water is poured into containers of various shapes and sizes and which measures the ability to understand what Piaget terms, "continuous quantity".

B. The beads test, in which big wooden beads are used in the same containers as the coloured water. This test measures the ability to understand what Piaget terms "discontinuous quantity".

C. The sticks test, in which small identical sticks are grouped in various ways to measure the ability to understand quantity in the circumstance called "unprovoked correspondence" by Piaget.

These are the same three Piagetian experiments upon which Elkind (1961a) based the test procedures of his study, and Mannix (1959) and Dodwell (1960) also made use of them in their batteries. Woodward (1961) and Hood (1962) used two of them (continuous quantity and unprovoked correspondence) in their investigations.

Despite this similarity of the tests used, there are marked differences in results obtained from them. These differences are apparently attributable at least in part, to variations in the methods by which the tasks were presented to subjects. Further, these authors report two chief difficulties in dealing with the results obtained from Piagetian tests. These are inconsistancy in the stage of development of the responses given by any one child from test to test, and
difficulty in assigning a stage of development to the responses of some children.

In the present study, some attempt has been made to reduce both of the types of difficulties which the foregoing workers in this field have experienced and reported. For one thing, the opportunity for variability between tests has been minimized by limiting the number of Piagetian investigations to the above three. In the matter of assigning a stage of development to each response, the experience of the authors cited above suggests that the best method is by means of a structured interview such as that used by Elkind (1961a). In his procedure every subject is asked a total of 18 categorical questions, 6 about each of the 3 Piagetian tasks or tests he uses in his study. The questions asked on each task are designed to assess different levels of understanding of quantity. That is, for each task, there are 2 questions to which a correct answer will show understanding of quantity at the first stage (global comparison of quantity), two questions to which a correct answer will show understanding of quantity at the second stage (intuitive comparison of quantity), and two questions to which a correct answer will show understanding of quantity at the third stage (operational comparison of quantity). For each task, two questions are used to assess each developmental level or stage in order to preserve the searching and repeated questioning procedure which Piaget regarded as an essential feature of his unstructured, non-standardized inquiry. Thus, as will be described in detail below, Elkind after presenting the original problem to the child, systematically varied the arrangement of the materials in each experimental task and then repeated the inquiry.

The following describes the three experimental tasks as they were
administered to each subject in the present study.

A. Liquid test.

Two sixteen ounce and two eight ounce drinking glasses are placed on a table beside a tall narrow glass and a pitcher of orange-coloured water. The experimenter (E) says to the child, "Let's pretend we are at a party and this coloured water is orangeade. Your mother pours this much for you." E then fills one sixteen ounce glass three-quarters full, and says, "and this much for me". E then fills the other sixteen ounce glass three-quarters full and asks, "Do we both have the same amount of orangeade to drink — do I have just as much as you?" (Stage 1 Question - global comparison of gross quantity.)

Then while the child watches, E pours the "orangeade" from E's glass into the two eight ounce glasses so that the levels in the two are equal, but no more than the level in the sixteen ounce glass. E then says, "Now my drink is in these two glasses, and yours is in this glass (pointing). Do I have the same amount to drink as you?" (Stage 3 Question - comparison of extensive quantity)

Next, E pours all the "orangeade" back into the pitcher and takes one sixteen ounce glass and the narrow glass, leaving the other containers aside. E fills the sixteen ounce and the narrow glass to the same level and says, "My drink is in here," pointing to the narrow glass, "and your drink is in here", pointing to the sixteen ounce glass. "Do we both have the same amount to drink?" (Stage 2 Question - intuitive comparison of intensive quantity)

E then repeats the entire procedure, but the second time uses the eight ounce glass in the comparison with the narrow glass.

B. Beads Test.

The procedures and containers used in this test are the same as in
the test with liquids. However, large wooden beads instead of coloured water are used as the material. The diameter of the beads is such that only one can fit into the diameter of the narrow glass at a time, making a single column like an upturned necklace. The questions are phrased "Do we have the same amount of beads - do we have the same number of beads?" etc. Questions are asked to define all three stages, just as in the liquid test, and, the entire procedure is repeated in similar fashion.

C. Sticks Test.

Twenty-four wooden sticks, $\frac{1}{4}$ inch square by $1\frac{1}{2}$ inches long, are placed on the table. E says to the child, "Let's pretend these sticks are candies. Your mother gives this many candies to me," E takes six and puts them in a row at one inch intervals, and then says, "You take just as many candies as I have; take the same number of candies as me." (Stage 2 question - intuitive comparison of intensive quantity.)

After the child takes his "candies", E shortens his row by pushing the sticks closer together and asks the child to do the same. If the child has taken more or less than six, shortening the rows will make this immediately apparent, and E asks him to "Make them the same." When the two rows are the same length, E asks, "Do we both have the same number of candies?" (Stage 1 question - global comparison of gross quantity.)

E then spreads his "candies" apart again, but leaves the child's together so that E's row is much longer but still equal in number to the child's row. E then asks, "Do we both have the same number of candies?" (Stage 3 question - operational comparison of extensive quantity.)
The entire test is then repeated. The second time E arranges the sticks lengthwise and asks the child to pretend the sticks are railroad cars and that they are making trains. The questions are phrased accordingly, "Do we both have the same number of cars?" etc.

In scoring the tests, a categorical 1 or 0 was given for each of the 18 questions so that the highest possible score was 18. Responses were scored on a separate form for each child in a tabular arrangement in which the stage of development of each response, could readily be seen. An example is shown in Figure 1.

Figure 1
Sample Scoring Form

Name: John White

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid i</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Liquid ii</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Beads i</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Beads ii</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sticks i</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Sticks ii</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

This hypothetical, but typical, protocol shows at a glance that John White understands gross quantity well at a Stage 1 level, is almost sure of the concept at Stage 2, but has not yet achieved the operational level of Stage 3. He answered 11 questions correctly out of the possible 18.

Apart from the innovation of this scoring form, the only refinement added to Elkind's procedure in this study was that of administering
the tests in a counter-balanced order, i.e. the first subject was given
the tests in the order above, Liquid, Beads, Sticks; the second child
in the order, Beads, Sticks, Liquid; the third child in the order
Sticks, Liquid, Beads, etc.

Analysis of Data

In the studies reviewed in the literature the usual methods of
testing for Piaget's sequence of stages have been based on the following
line of reasoning. Since chronological ages proceed in sequence by
definition and since Piaget has mentioned age levels in his descriptions
of stages of development, his theory of sequence may be evaluated
statistically by seeing how closely stage is correlated with age
(Dodwell, 1960; Fiegenbaum, 1961) or by seeing how distinct in stage
characteristics various age groups are (Elkind, 1961a). However,
the overall result of these studies has been inconclusive, and Piaget
himself, has specifically emphasized that it is invariable sequence
and not age of achievement which is at issue. (Piaget, 1957)

Therefore, in order to test Piaget's hypothesis about the invariability
of the order of number concept development, this study makes use of
the technique of scalogram analysis (Torgerson, 1958) which, rather
than relating stages to ages, is a direct test of sequence of stages.

Scalogram analysis may be applied when a subject makes cate-
gorical responses to stimuli. It assumes that "the systematic
variability of responses of subjects to stimuli is ascribed to
variability both in the subjects and in the stimuli...both can be
assigned scale values." (Torgerson, 1958 p. 298) Two scales can
thus be constructed: - a scale of subjects in order of some subject
variable of interest (such as age), and another scale of subjects in order of some stimulus variable of interest, (such as the amount of noise tolerated). A perfect scale exists if the position of each subject is the same on both scales. If this is so, the position of a subject on one scale may be reproduced from knowledge of his position on the other. However, such perfection is unlikely in a world of chance, and a technique for evaluating closeness of fit or reproducibility of imperfect scales has been developed in scalogram analysis.

In the present study Piagetian stage of development might have been chosen as the stimulus variable of interest for constructing the stimulus scale against which to evaluate the subjects' responses. Mannix (1959) used two Piagetian stages for this purpose, and Fens and Lodwick also are reported to have analysed responses against this criterion (Peel, 1959). However, use of Piagetian stage as the stimulus variable in the present study would have necessitated categorically assigning a stage of development to each child whether or not all of his responses were at the same level. In order to avoid this difficulty, the stimulus scale has been arranged in order of item popularity as suggested by Green, (1956). That is, the number of correct responses to each of the 18 questions was totalled for the whole group of subjects and the resultant popularity of each item determined its order in the stimulus scale.

The next step in the procedure was to post this scale, one question-item to a column, along the top row of a grid. Each subject's responses were entered on a separate row beneath according to the column headings. Thus each subject's responses were formed into a scale for comparison with the stimulus scale.

Now, if all subject responses conformed to the order of
popularity on which the stimulus scale was constructed, popular questions would always be passed before unpopular questions. The pattern of success and failure, looking across the row, would show no reversals as would happen if a more popular item were failed and a less popular one passed. The latter pattern constitutes an error in the reproducibility of the scale. First order errors involve a displacement of one scale place, and second order errors a displacement of two. The reproducibility of a scale is calculated by totalling the first and second order errors and comparing this, in a ratio, to the number of possible errors. *

In the procedure followed in this study, first and second order errors were totalled for all subjects and the data entered into the following formula for the calculation of reproducibility.

\[
\text{Rep} = 1 - \frac{1}{N_n} \sum_{i=1}^{n-1} \frac{N_{i+1,i} - 1}{N_n} - \sum_{i=2}^{n-2} \frac{N_{i+2,i+1,i,i-1}}{N_n}
\]

where \( N \) = number of subjects
\( n \) = number of items
\( \sum_{i=1}^{n-1} N_{i+1,i} = \text{total number of first order errors}(+-) \)
\( \sum_{i=2}^{n-2} N_{i+2,i+1,i,i-1} = \text{total number of second order errors}(++) \)

(Torgerson, 1958, p. 327)

The final step in scalogram analysis is to make a correction for

* Reproducibility arrived at in this way "differs from the true sample reproducibility for items in popularity order only in that the infrequent third order and higher order errors are ignored." (Torgerson, 1958, p. 327)
chance reproducibility. If a subject passes or fails every item, the pattern of his responses will show no reversals and will tend to increase the reproducibility figure. Similarly, response patterns which are nearly all passes or failures tend to spuriously inflate reproducibility. The originator of scalogram analysis, Guttman (1944) "handles this problem by recourse to a number of somewhat arbitrary intuitive supplementary criteria" for the evaluation of the chance factor. (Wohlwill, 1960, p. 358) Green's Index of Consistency, "I", (Torgerson, 1958) takes chance reproducibility into account statistically, and for that reason was chosen for application to the data of this study. Following his method, "I" was obtained by finding the reproducibility to be expected by chance if there were complete independence between responses to individual items (by equation #8 in Torgerson, 1958, p. 327) and subtracting it from the previously obtained reproducibility by the following formula:

\[ I = \frac{\text{Rep} - \text{Rep}_1}{1 - \text{Rep}_1} \]

where \( \text{Rep} \) = the reproducibility obtained on the basis of the 1st and 2nd order errors

\( \text{Rep}_1 \) = the reproducibility obtained on the basis of complete independence between responses to individual items

(Torgerson, 1958 p. 328)

For the reasons explained above, Green's method of scalogram analysis was chosen as most suitable to analyse the data of this study. This choice is also supported both in that the number of subjects in this study, 90, approaches closely the 100 suggested by Green as desirable for application of the scalogram technique; and in that the
number of items, eighteen, comfortably exceeds the 10 that he feels are requisite. (Torgerson, 1958 p. 326)

The hypotheses of interest and the types of analysis used to investigate them in this study may be summarized as follows:

a. To test Piaget's hypothesis that the development of a concept of number proceeds through a fixed sequence of three stages in mentally deficient as in normal children, a direct test of sequence was used, scalogram analysis by Green's method, as described in the foregoing pages.

b. To test the hypothesis that sex and length of institutionalization affect significantly the development of the quantity-number concept, $X^2$ analysis (Edwards, 1959) of the differences in the responses of four groups of children, boys and girls, both institutionalized and living at home, was done.

c. To test the relationship between verbal understanding and the ability to understand the quantity-number concept, a correlation coefficient using a raw score formula (Edwards, 1959) between raw scores on the Peabody Picture Vocabulary Test and scores obtained in response to the test procedures derived from Piaget was calculated. A statistical test of significance (Hammond and Householder, 1962) was used to evaluate the resultant "r".
CHAPTER IV
RESULTS AND DISCUSSION

In his theory of intellectual development, Piaget postulates a sequence of integrative stages that mark successive achievements in the child's ability to understand his world and think logically about it. Piaget identifies these stages along many lines of cognitive progress, including the development of the concept of number. The major specific hypothesis tested in this study is that mentally deficient children follow the same sequence of three stages postulated by Piaget for normal children in achieving the ability to make logical comparisons of quantity, this ability being the foundation of the number concept.

Results of the scalogram analysis used to test this hypothesis suggest that the data confirm Piaget's sequence of stages. Certainly the sequence of test questions used meets the requirements of a true scale. The individual scale patterns of each of the 90 children reproduce at a high level the pattern of the order of popularity of the 18 questions used. Of the 1620 answers, only 121 were 1st or 2nd order errors.* The Rep obtained, .926, compares favourably with perfect reproducibility, which would be 1. Correction for chance factors by Green's method yields an Index of Consistency of .57, which is more than the .5 considered by Green to indicate satisfactory scalability in a series of items. In sum, a sequence of test questions has been established, together with the fact that the subject follow it with great regularity.

The problem remains, however, of whether the sequence of test questions

* Third and higher order errors were a negligible 3.
adequately represents Piaget's order of three stages. Inspection of a graph illustrating the scale of questions arranged in order of popularity suggests that this scale does conform to the three stages of Piaget's hypothesis. (Figure 2)

Figure 2 indicates clearly how the questions, arranged in a scale according to the number of correct responses each has received, do not show an even decrease in number of correct responses from one to the next, but rather, fall into three general levels corresponding to the three types of comparison by which the three Piagetian stages were defined in the test procedure.

A similar impression is obtained by inspection of the raw data on each child's answer form. Of the 90, only 12 children show reversals in the stage of development attained for any one type of material. These 12 made, in all, 26 such errors. That is to say, when each child's comparisons of liquid or comparison of beads or comparisons of sticks are looked at separately, only 26 answers were successful at a higher stage where a lower stage had been unsuccessful. The remainder (121-26=95) of the first and second order errors found in the scalogram analysis, must, therefore, be attributed to variability introduced by pooling the responses to all three types of material. Most of the failure of the scalogram to give absolute support to Piaget's three stage theory would therefore appear to be attributable to the statistical method used rather than to lack of confirmation in the raw data.

Variability in overall stage of response was shown by most of the children - a finding common to all the previous studies cited. In the present case, apart from the 11 children who received 100% pass or fail answers, only 5 were at exactly the same level for all three types of material.
Figure 2

Histogram Showing Grouping By Stages Of Questions Arranged In Order Of Popularity

* In the figure, the scale of questions is shown across the bottom of the histogram. It reads: LiG (comparison of gross quantity of liquid - first question), BiIG (comparison of gross quantity of beads - second question), LiIG (comparison of gross quantity of liquid - second question) etc.
Even so, a further illustration that the data fit the Piagetian hypothesis is provided by the shape of the distribution of scores. It will be recalled that in the test procedure each child was asked six questions for each of the three successive stages of development of the quantity-number concept - a total of 18 in all. With each of the questions scored 1 or 0, score values could vary from 0 to 18. Now, if the children tested do progress in their attainment of the quantity-number concept through these three stages, it might be expected that their total scores would tend to cluster at points which show that a stage of development has been achieved. Accordingly the present data should show four clusters at the following points in the distribution of scores: "0" for children who have no concept, "6" for children who have achieved the ability to make gross comparisons of quantities (Stage 1), "12" for children who have achieved the ability to make intuitive comparisons of quantities (Stage 2), and "18" for children who have achieved the ability to make operational comparisons of quantities (Stage 3). In figure 3

Figure 3
Graph Showing The Distribution Of Total Scores On The Comparison Of Quantity Tests
despite the fact that few children produced protocols which showed them to be at the same stage for all three types of material, the distribution of scores does show a pattern of clustering at or near the predicted four points. The only dissonant frequency is that of the eight children who received a score of 14. Apart from this, the shape of the distribution of scores supports Piaget's hypothesis that achievement of the quantity-number concept does not develop evenly, but by three stages.

Results of the $X^2$ analysis used to test the hypothesis that sex and length of institutionalization do not significantly affect the development of the quantity-number concept, were as follows. The four groups compared, girls who had been institutionalized for four or more years, girls who had not been institutionalized for more than six months, boys who had been institutionalized for more than four years, and boys who had not been institutionalized for more than six months, did not differ significantly in the number of responses they gave above and below the population median. $X^2$ was equal to 2, and had a probability of between 50% and 75%. Evidently, for this group of subjects, neither sex nor length of institutionalization affects the development of the quantity-number concept.

The result of the correlation calculated between raw scores on the Peabody Picture Vocabulary Test and the number of successful quantity comparisons made by each child, was .6170. This was significant at the 1% level. From this it appears that there is a relationship between the understanding of single words and the level
of ability to compare quantities correctly.

What does this positive relationship between successful comparison of quantity and Peabody Picture Vocabulary scores mean? For the child, the Peabody is the association of words spoken by the examiner with pictured objects and actions. Success depends upon understanding of language, not expressive vocabulary. Inability to understand language was often cited by non-Piagetian investigators as a factor deterrent to achievement on tasks measuring number skills. Russell, (1956) summarizes their observations that at four and five years most children use terms like 'most', 'both', 'biggest', but words like 'same' and 'equal' are not clearly understood. Whether a child understands such terms is pertinent to the evaluation of the results of this study also. In the procedure used, correct answers depend upon understanding of such questions as, "Do I have the same amount as you?" or "Do I have as much to drink as you?"

Standardized tests of intellectual abilities pin-point the understanding of such questions in this age range. For example, the similarities sub-test on the WISC equates five year old competence with the correct understanding and expression of the ways in which a plum and a peach are alike. (Wechsler, 1949) The 1960 revision of the Stanford Binet test standardized similar tasks, concerned with comparisons of pictures at IV-6 and V years in the first and second "Pictorial Similarities and Differences" tasks. By the years VI and VII, the average child in the Binet standardization group was able to use the concept expressively in the "Differences and Similarities: two things" task, and by VIII he had achieved facility and Piagetian "reversibility" in performing a similar task, "Similarities and Differences." (Terman and Merrill, 1960)
Experimenters working with Piaget's questions have handled the problem of whether the child understands the terminology in the question variously. Dodwell acknowledged that "unfamiliarity and verbal confusion may be important determinants of the child's responses." (Dodwell, 1962, p. 155) He also took pains to ensure that there was no verbal confusion "by pointing out the meanings of the words to the child, if it was not quite sure of them, and by allowing and encouraging the manipulation of the objects." (Woodward, 1961, p. 251) found that even upon repeated presentation "some experiments were not understood by some younger children and adults with very low I.Q.'s."

In the present study, it was noted that about twenty children showed the primitive language traits of perseveration and or echolalia. Their answer patterns tended to have many reversals and, it seems highly likely that they did not understand the terms used in the test. The built-in repetition of the test procedure did not appear to help the understanding of these children. Six of them used spontaneous counting which did not define either order or cardination.

Possibly it was such children that Estes had in mind in making the criticism that: "It may be that perceptual 'more' and 'numerical equal' simply co-exist without adequate differentiation in the cognition of children at a certain level, and that an experimenter can bring either one to the foreground by appropriate questioning." (Estes, 1956, p. 221). Similarly, Berko and Brown (1960, pp. 536-537) suggest that in achieving a concept of number a child is only learning various applications of semantic rules. Flavell summarizes such criticisms, saying that to a number of investigators Piaget's developmental studies
"are merely vocabulary growth studies in disguise. His tasks variously assess the child's understanding of terms like... 'amount', 'long', 'all', 'some' and so on... Initially as might be expected, the child shows an imperfect or incomplete grasp of such terms. And as he grows older, he gradually approaches the adult's understanding and usage of them... We have known for a long time that vocabulary acquisition is a developmental fact... there are no grounds for assuming that the vocabulary change points to anything important other than itself - no grounds for assuming, for instance, that it reflects any kind of qualitative change or alteration in cognitive structure..." (Flavell, 1963, p. 434)

In the opinion of the present writer, this type of criticism implies a naive view of language development: that is, that meanings are independent of subjective experience, and that a child knows what something means but does not know or understand the word for it - like an adult learning a foreign language.

This is not the finding of such developmentalists as Griffiths, (1954) and Gesell, (1949) who point out that children babble "words" before they attach meaning to them in the reinforcing social situations of babyhood. Werner, (1957) speaking of the early development of concepts insists on the "fundamental dependence of the function of representation * on concrete activity." (Werner, 1957, p. 251)

Piaget himself has made a detailed analysis of speech beginnings. A brief summary of his theory will help to advance the argument. Piaget says that in the sensori-motor period (birth to approximately two years) which immediately precedes the three stages with which our study is concerned, a child experiences meaning only in such activities as the manipulation of objects. "Before language appears, the small child can only perform motor actions, without thought activity." (Piaget, 1957, p. 9) By the end of this period, the

* "the capacity for communicating a cognition by symbolic formulation (gesture, sound, writing, drawing)" (Werner, 1957, p. 250)
child has achieved great facility with some kinds of motor activity and, in remembering this, employs a synoptic "motor representation" as a crude symbol for a well-learned activity. For example a two year old, looking at the cat outside the window, will make unself-conscious clutching movements with his hands. Such motor representations are the meaningful condensations of experience which become associated by reinforcement to the fairly large repertoire of "words" and "word" groups which a two year old has learned to produce. In his daily communication with a reinforcing adult, through activities which come roughly under the heading of "speech stimulation", the child attaches a global meaning to language activity at first and later arrives at specific definitions by trial and error. Such specific meaning grows up from the beginning of speech through to adulthood in more and more complex activities which help to define experience, and such activities are in turn, guided by the evolving language-defined concepts.

In applying this theory to the development of the concept of number, Flavell says:

"in the case of conservation of number, for example, it is undeniably true that the development process Piaget described does entail a changed understanding in use of expressions like 'same number', 'just as many' etc. The crucial question, however, is not whether vocabulary growth takes place, but whether anything else also takes place, and what the relation is between this vocabulary growth and this something else. We tend to believe that, in most of Piaget's studies, whatever vocabulary change occurs is in a large measure a consequence, reflection or sympathetic expression of an underlying and more fundamental cognitive change." (Flavell, 1963, p. 435)

Flavell suggests that, while "you cannot teach 'red' to a blind man", at the same time "it may be that a stable and generalizable conception...just cannot exist without symbolic vehicles and anchors
Hood says much the same thing about the responses of children in his study:

"While it is certainly true that there is an element of linguistic confusion among Stage I children because their understanding of the words in question is incomplete and limited, it is the eventual appearance of the concept and operation, the internalized action becoming reversible, that completes the verbal understanding and removes the limitations." (Hood, 1962, p. 281)

In discussing the arithmetical achievement of mentally deficient subjects, Carrison and Werner (1943) observe that in this group also, verbal achievement does not necessarily imply understanding: "a child may perform a mechanically advanced operation on the verbal level without understanding its concrete basis." (Carrison and Werner, 1943, p. 310)

Piaget summarizes this view in a context in which "logic" means the ability to use correctly such terms as "equal" and "same":

"Our hypothesis is that the construction of number goes hand in hand with the development of logic, and that a pre-numerical period corresponds to the pre-logical level...logical and arithmetical operations, therefore, constitute a single system that is psychologically natural..." (Piaget, 1952, p. viii)

Smedslund (1961) infers from his experiments that it is not training by reinforced practice of arithmetical procedures or counting, but cognitive conflict which makes possible the change from non-conservation to conservation of quantity. To relate this to the present study, the resolution of the paradox that what looks the same is not always, in fact equal, and vice-versa, may be the kind of cognitively conflictual situation which defines experience for a child and gives it specific meaning.

Children who have mastered such situations are those who score
highly in the Piagetian comparisons of quantity posed by the procedures of this study. According to Piaget, having acquired this logical ability, they will have concurrently acquired language to symbolize it. Herein may lie the functional relationship which shows up in the correlation found in this study between vocabulary and the successful conversation of quantity.

It would be very interesting to investigate further what kinds of experience foster this growth. Smedslund's experiment might pertinently be repeated and elaborated on other populations. Investigations of the cognitive operations of non-verbal children and adults might throw light on the role played by language in the development of intelligence.

Without further speculation, consideration will now be given to the implications for intelligence test construction and use, and for research in educational procedures, of the results of this study.

The finding of this study that Piaget's three stage theory is valid for the development of the number concept in the group of mentally deficient tested, lends support to the argument in favour of using Piagetian analysis for standardized descriptions of intelligence. Flavell is enthusiastic about these possibilities stating that "the advent of his (Piaget's) system may, for the first time, make possible a psychometric which is anchored to a cognitive-developmental theory..." (Flavell, 1963, p. 417) and "positive findings now make one optimistic about the test-construction possibilities of his research". (Flavell, 1963, p. 364)
Certainly, the common developmental sequence for normal and defective children, in which each stage is necessary to the next, is potentially a powerful diagnostic tool. It is out of the need for just such a tool that many psychologists use mental age norms as if they might validly be assumed to correspond to stages of development. Woodward says much the same thing: "a theory which stresses the order in which developmental stages occur, rather than the age at which they develop, is immediately applicable to any group, however slow the rate of development; the complications arising from age comparisons of extremely deviant with normal populations are avoided." (Woodward, 1961, p. 249) Flavell also points out that, diagnostically, the establishment of a sequence of Piagetian stages may enable psychologists to make "specific predictions about the cognitive performance of individuals whose current level of functioning is lower than it once was." (Flavell, 1963, p. 417) This opens a much wider frame of reference for the diagnostic concept of regression.

A final benefit which might be looked for if stage of development rather than general global functioning were used for interpersonal comparisons, is in the area of experimental psychology. Here the way would be opened for more accurate matching of subjects in the analysis of many facets of behaviour. Possibly developmental levels would be more culture-free than normative measures which are standardized on large social groups.

In terms of the actual items which might be selected for a test of stages of development, our data suggests first that, in general, it is probably not meaningful to try to divorce measures of the effective use of language from measures of intelligence.
Second, the first presentation of the series of three questions about the comparisons of quantities of liquid, appears to have excellent discriminating power: the gross comparison of quantities of liquid (Stage 1) was found to be the easiest comparison by our population; the operational comparison of quantities of liquid (Stage 3) was found to be the most difficult comparison; the intuitive comparison of quantities of liquid (Stage 2) was well to the middle of the distribution of item popularity. This is an interesting finding in view of Elkind's conclusion, reached on the examination of total scores, that liquids are the most difficult to compare. (Elkind, 1961a)

In the field of education, Piaget's theory has implications both for the diagnosis of capacity and achievement, and the formulation of education methods. A Piagetian analysis might contribute to solution of the problem voiced by Werner and Strauss, (1939):

"measuring the child's capacity by means of standardized achievement tests has proved to be a most necessary and successful approach in the field of normal child psychology as well as mental deficiency. A true insight into the laws of development and retardation, nevertheless, cannot be gained by this approach alone."..."With the growth of achievement, the underlying functional patterns may change fundamentally from step to step." (Werner and Strauss, 1939, p. 7)

Woodward, (1961, 1962) whose investigations of Piagetian stages with mentally retarded children have been undertaken with a view to developing educational techniques, says

"to apply the results of Piaget's work to the education of the subnormal...we need to know at what ages they achieve the different types of thinking. Then those activities of training centres which have the object of encouraging
"the maximum development of intelligence can be graded progressively in accordance with the development of children." (Woodward, 1962, p. 18)

She concludes,

"the direct investigation of number concepts is more useful than an attempt to estimate from an intelligence test result whether an individual can learn arithmetical operations with understanding." (Woodward, 1961, p. 258)

In a similar vein, Lunzer (1957) acknowledges "in the sphere of immediate educational policy, Piaget's work shows the futility of teaching mechanical formulae when the child is not ready to understand the operations involved in such techniques." (Lunzer, 1957, p. 27)

As well as lending validity to the massive and obvious support Piaget's theory gives to the educational desiderata of readiness in the child and appropriateness of level in the instruction, our findings give some hint of hypotheses about what may be the most effective form of instruction. In the discussion of the relation of language development to the development of the number concept in an earlier section, Smedslund's (1961) inference that cognitive conflict is necessary for intellectual growth was discussed. Churchill (1957) reports a similar finding.

Children who were given "play experience designed to increase the understanding of number relations and operations" (not counting and arithmetic) were found subsequently to be significantly better than a control group in the performance of a series of tests involving the number operations. "A detailed analysis of each individual response to determine its place in Piaget's stages showed that the change represented a development in structured understanding." (Churchill, 1957, p. 67)
In the light of our hypothesis about language development, and in view of the experience of Smedslund (1961) and Churchill, (1957), it may be hypothesized that the most efficient way to inculcate number concept would be to assess the child's stage of development, and then have him participate in active physical problem-solving games a little beyond that level while using words which define the concept. According to our findings, this hypothesis should be equally appropriate for mentally deficient as for normally intelligent children.

To recapitulate, the main findings and inferences from the present study are that:

1. Mentally deficient children follow the same sequence of three stages in the development of a concept of number that Piaget describes for normal children. This gives some support to the validity of Piaget's theory of the development of intelligence, of which the three stages are an integral part.

2. Neither sex nor institutionalization appear to be related to differences in the level of development of the concept of number in mentally deficient children.

3. Level of verbal understanding, on the other hand, does appear to have a positive relation to the level of the development of the concept of number in mentally deficient children. Observations of the most linguistically primitive children in this study and the experience of other investigators on this point may be seen as supporting Piaget's hypothesis that the
development of an understanding of number and the development of a vocabulary to describe it correctly are mutually reciprocal processes.

All of these results are of interest, not only for what they reveal about the intellectual functioning of mentally deficient children and the process of the development of the number concept, but also because they carry theoretical implications and practical suggestions pertinent to some of the concerns of psychology and education. Among these are the construction of intelligence tests, the teaching of arithmetic, the assessment of school readiness, and research into educational methods for special groups.
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