THE EFFECT OF LOCAL MOTOR LOADS ON POWER SYSTEM STABILITY

by

BRUCE GEORGE PRIOR

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We accept this thesis as conforming to the
required standard

Research Supervisor ......................
Members of the Committee ..............

Head of the Department .................

Members of the Department
of Electrical Engineering

THE UNIVERSITY OF BRITISH COLUMBIA

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Department of **ELECTRICAL ENGINEERING**

The University of British Columbia
Vancouver 8, Canada

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The effect of local motor loads on power system stability is investigated. The power system consists of a synchronous generator supplying a large system through a long transmission line. The loads studied are an induction motor, a synchronous motor, and the combination of the two, although a general case of any number of local induction and synchronous motor loads can be easily formulated. Stability is determined by observing the response of the generator and the motors of the system with a fault at the transmission line. The response is calculated from the mathematical model and is also observed from tests on a dynamic power system model in the laboratory. It is found from the studies that all the local motor loads improve the stability of a power system.
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Finally, I wish to express my deepest appreciation to my wife, Jean, for her patience and understanding throughout my university career.
NOMENCLATURE

General

\( t \) \hspace{1cm} \text{time, s}

\( j \) \hspace{1cm} \text{complex operator, } \sqrt{-1}

\( \omega_0 \) \hspace{1cm} \text{synchronous angular velocity, 377 rad/s}

\( e \) \hspace{1cm} \text{base of natural logarithms, 2.71828}

\( T_x \) \hspace{1cm} \text{turns ratio of ideal transformer between motor and generator}

\( k \) \hspace{1cm} \text{superscript denoting number of particular motor in a multi-motor load}

\( p \) \hspace{1cm} \text{derivative operator, } \frac{d}{dt}

Transmission Network

\( r + jx \) \hspace{1cm} \text{series impedance, } \Omega

\( G + jB \) \hspace{1cm} \text{shunt admittance, } \text{mho}

\( V_0 \) \hspace{1cm} \text{infinite bus voltage, } \text{V/ph}

\( [V], [I] \) \hspace{1cm} \text{voltage and current vectors, } \text{V, A}

\( [Y] \) \hspace{1cm} \text{admittance matrix, } \text{mho}

\( i_B \) \hspace{1cm} \text{transmission line instantaneous current, A}

\( I_B \) \hspace{1cm} \text{transmission line phasor current, A}

\( \theta_B \) \hspace{1cm} \text{current phase angle, rad}

\( k_1, k_2, C_1, C_2 \) \hspace{1cm} \text{transmission line constants}

Synchronous Machines

\( 3 \) \hspace{1cm} \text{subscript denoting synchronous motor quantity}

\( \lambda_d, \lambda_q, \lambda_0 \) \hspace{1cm} \text{stator dq0 flux linkages, Wb-T}

\( \lambda_F \) \hspace{1cm} \text{flux linkage state variable, Wb-T}

\( e_d, e_q, e_0 \) \hspace{1cm} \text{terminal dq0 voltages, V}
\( v_t \) terminal phase voltage, V
\( V_t \) terminal phasor voltage, V
\( \alpha \) phase angle of terminal voltage, rad
\( v_r, v_m \) real and imaginary terminal voltages, V
\( e_f \) field voltage, V
\( v_F \) voltage proportional to field voltage, V
\( v_{FR} \) a defined voltage, equation (2.18), V
\( i_d, i_q, i_0 \) stator dqO currents, A
\( i_G, i_a \) instantaneous armature current, A
\( I_G \) armature phasor current, A
\( \beta \) armature current phase angle, rad
\( i_{Gr}, i_{Gm} \) real and imaginary armature currents, A
\( i_f \) field current, A
\( \sigma \) angle from stator a-phase axis to rotor direct axis, electrical radians
\( \omega \) angular velocity of rotor, rad/s
\( \delta \) angle from infinite bus reference to d-axis, electrical radians
\( s_G \) rotor slip
\( P_G, Q_G \) real and reactive power output, W and VAR
\( T_G \) electrical developed torque, N·m
\( R_a \) armature resistance, \( \Omega \)
\( R_f \) field resistance, \( \Omega \)
\( L_d, L_q, L_0 \) stator dqO inductances, H
\( L_{afM} \) peak value of mutual inductance between stator a-phase and field winding, H
\( L_{ff} \) field self-inductance, H
\( x_d, x_q \) stator dq reactances, \( \Omega \)
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<tr>
<td>( L_{d'} ), ( x_{d'} )</td>
<td>transient d-axis inductance and reactance, H and ( \Omega )</td>
</tr>
<tr>
<td>( L_{df}, x_{df} )</td>
<td>mutual inductance and reactance, H and ( \Omega )</td>
</tr>
<tr>
<td>( T_{d'} )</td>
<td>transient short circuit d-axis time constant, s</td>
</tr>
<tr>
<td>( T_{d0'} )</td>
<td>transient open circuit d-axis time constant, s</td>
</tr>
<tr>
<td>( J_G )</td>
<td>moment of inertia, J-s^2/rad</td>
</tr>
<tr>
<td>( D_G )</td>
<td>damping coefficient, J-s/rad</td>
</tr>
<tr>
<td>( T_t )</td>
<td>prime mover torque, N-m</td>
</tr>
<tr>
<td>( f_G )</td>
<td>friction torque, N-m</td>
</tr>
<tr>
<td>( POLES_G )</td>
<td>number of field poles</td>
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Voltage Regulator and Exciter

<table>
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<tr>
<td>( v_R )</td>
<td>regulator output voltage, V</td>
</tr>
<tr>
<td>( A_1, A_2 )</td>
<td>constants defining regulator output limits</td>
</tr>
<tr>
<td>( f(v_R) )</td>
<td>function defining regulator output limits</td>
</tr>
<tr>
<td>( T_E )</td>
<td>exciter time constant, s</td>
</tr>
<tr>
<td>( T_{RE} )</td>
<td>regulator time constant, s</td>
</tr>
<tr>
<td>( K_A )</td>
<td>regulator gain</td>
</tr>
<tr>
<td>( v_{ref} )</td>
<td>regulator reference voltage, V</td>
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Induction Motor

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<tr>
<td>( \theta )</td>
<td>angle used in general dq0 transformation, rad</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>angle between stator a-phase axis and rotor A-phase axis, rad</td>
</tr>
<tr>
<td>( \theta_6 )</td>
<td>angle between rotor A-phase axis and induction motor d-axis, rad</td>
</tr>
<tr>
<td>( e_{1d}, e_{1q} )</td>
<td>stator dq voltages, V</td>
</tr>
<tr>
<td>( e_{2d}, e_{2q} )</td>
<td>rotor dq voltages, V</td>
</tr>
<tr>
<td>( i_{1d}, i_{1q} )</td>
<td>stator dq currents, A</td>
</tr>
<tr>
<td>( i_{2d}, i_{2q} )</td>
<td>rotor dq currents, A</td>
</tr>
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</table>
\(i_M\)  instantaneous stator current, A

\(I_M\)  phasor stator current, A

\(\gamma\)  phase angle of stator current, rad

\(i_{Mr}, i_{Mm}\)  real and imaginary stator currents, A

\(s_M\)  rotor slip

\(T_M\)  electrical developed torque, N-m

\(\lambda_{1d}, \lambda_{1q}\)  stator dq flux linkages, Wb-T

\(\lambda_{2d}, \lambda_{2q}\)  rotor dq flux linkages, Wb-T

\(r_1\)  stator resistance, \(\Omega\)

\(r_2\)  rotor resistance, \(\Omega\)

\(L_{11}\)  stator self-inductance, H

\(L_{22}\)  rotor self-inductance, H

\(L_{aAM}\)  peak value of mutual inductance

\(L_{12}, x_{12}\)  mutual inductance and reactance, H and \(\Omega\)

\(L', x'\)  transient inductance and reactance, H and \(\Omega\)

\(C_3, C_4\)  induction motor constants

\(D_1, D_2, D_3, D_4\)  functions of \(\delta\)

\(\Delta\)  a determinant, equation (3.40)

\(J_M\)  moment of inertia, J-\(s^2/\text{rad}\)

\(D_M\)  damping coefficient, J-\(s/\text{rad}\)
1. INTRODUCTION

The stability of electric power systems has been an area of interest in research for many years. As power systems increase in size and complexity the need for maintaining system stability becomes more important. Although many problems have been solved using excitation control and optimal control of modern theory other problems remain. One of them is a more thorough study of the effect of system loads on power system stability.

In steady state stability studies loads are usually represented by constant power, constant current, constant impedance, or a combination of the three [22]. It is known that for steady state studies the constant impedance representation gives optimistic results, the constant power representation gives pessimistic results and the constant current representation gives generally satisfactory results, somewhere in between the first two. Not much work has been done on load effects in transient stability studies. Dahl [1] suggested a representation either by constant impedance or by transient load characteristics but did not say how the latter were to be obtained. Crary [2] proposed a representation by constant impedance or, alternatively, constant impedance during the fault and constant power after the fault clearance. He further recommended that if the results were significantly different using his two suggestions an attempt should be made to obtain more accurate load data. Weinbach [3] neglected the real power altogether and represented the reactive power as a constant reactance. Kimbark [4] was in favour with the constant impedance representation. Hore [5] agreed with Crary but showed qualitatively that after the elimination of the fault it would be more advisable to simulate the load as constant current. He further stated that his recommendations
were directed primarily toward system planning and that they were not adequate for the analysis of an existing system where stability was a problem. For that, he concluded, the nonlinear load characteristics representation was required. Bauman, et. al. [6] conducted tests to determine the response of parts of a power system to small changes in load, voltage and frequency. They found that an increase in voltage resulted in an increase in both real and reactive load. They also established that changes in frequency within reasonable limits would not result in appreciable changes in load provided all the bus voltages were maintained constant. Brereton, et. al. [7] discussed the various methods of representing induction motor loads during power system stability studies. They recommended that both mechanical and rotor electrical transients of an induction motor should be included in the representation if a digital computer is employed. Gevay and Schippel [8] investigated the transient stability of an isolated radial power system where the total load was kept constant but its division, synchronous motors, induction motors and static load, was varied. They found that the presence of synchronous motor load was the most effective in maintaining stability. The system was never stable if the synchronous motor load was less than 10 per cent of the total according to their study. They also found that both synchronous and induction motors would swing with relatively large amplitudes even in stable cases. Kent, et. al. [9] emphasized the need for a correct representation of electrical loads in stability studies and conducted load voltage tests on a power system. They gave methods for proportioning bus loads between constant impedance and constant power. The classification of loads was based upon kWh sales records. Robert and Robichaud [10] built a load simulator in conjunction with a micro-reseau and made comparative stability tests. They recognized that the general load problem has two
parts which may be studied separately. The first is to determine the influence of the characteristics of loads on the stability of a power system and the second part is the problem of load representation.

In this thesis the effect of local motor loads on power system stability is investigated. Like Robert and Robichaud, a power system test model is used. But, instead of simulating the loads by artificial means, actual loads, that is, induction motors and synchronous motors are employed. The motors are connected directly to the generator terminals through a transformer. For analysis the state variable equations are employed with mechanical and electrical relations described in detail. The generator and transmission line equations are developed in Chapter 2. They are nonlinear equations and many of the variables commonly neglected in stability studies are retained. The system without local motor loads with a transmission line fault is solved to provide a basis for comparison with later studies. In Chapter 3 the induction motor load is introduced and similar equations derived. Further stability studies of a transmission line fault are carried out. Similar studies are carried out in Chapter 4 but with a local synchronous motor. In Chapter 5 the most generalized formulation of local motor loads with any number of synchronous and induction motors is presented but only one induction motor and one synchronous motor are used for the study mainly because of the restriction of equipment.

A dynamic test model developed at the University of British Columbia [11,12,13,14] is used to verify the analytical results of the thesis. Since the motors used are small ones with comparatively high losses further computed results of studies using large machine parameters are included in Chapter 4 to verify the usefulness of the results from small machine studies.
2. SYNCHRONOUS GENERATOR WITHOUT LOCAL MOTOR LOAD

The state equations for a voltage regulated synchronous generator system without local motor load are derived. The mathematical model is verified by a transient stability study. The computed results are compared with the results obtained directly from laboratory tests on a power system test model.

2.1 Synchronous Generator Equations

The synchronous generator equations were originally derived by Park [15] in per unit. Lewis [16] rewrote them in MKS units as used here. The formulation is based upon the following assumptions:

1) All inductances are independent of current (saturation neglected).

2) Only the second harmonic of the permeance in the air gap in addition to the average value is considered.

3) The electric transient of the damper winding is neglected.

The positive polarities of the currents and voltages and the direction of rotation of the rotor are shown in Figure 2.1. The equations of the machine in dq0 coordinates are

\[
\begin{align*}
  e_d &= -R_a i_d - p\lambda_d - \lambda_q p\sigma \\
  e_q &= -R_a i_q - p\lambda_q + \lambda_d p\sigma \\
  e_0 &= -R_a i_0 - p\lambda_0 \\
  e_f &= R_f i_f + p\lambda_f
\end{align*}
\]
Figure 2.1 Circuit Diagram of Ideal Synchronous Machine
Figure 2.2 One-line Diagram of One Machine-Infinite Bus System

Figure 2.3 One Machine System with Equivalent-π Transmission Network
where

\[ \lambda_d = L_d i_d + \sqrt{\frac{3}{2}} L_{afM} i_f \]  

(2.5)

\[ \lambda_q = L_q i_q \]  

(2.6)

\[ \lambda_0 = L_0 i_0 \]  

(2.7)

\[ \lambda_f = L_{ff} i_f + \sqrt{\frac{3}{2}} L_{afM} i_d \]  

(2.8)

For notations, see nomenclature.

The field current \( i_f \) is usually eliminated in analysis to give

\[ e_d = -R_a i_d - p \lambda_d - \lambda_q p \sigma \]  

(2.9)

\[ e_q = -R_a i_q - p \lambda_q + \lambda_d p \sigma \]  

(2.10)

\[ \lambda_d = \frac{x_{df}}{\omega_0 R_f} \frac{e_f}{(1 + T_d' p)} + \frac{x_d}{\omega_0 (1 + T_{d0}' p)} i_d \]  

(2.11)

\[ \lambda_q = \frac{x_q}{\omega_0} i_q \]  

(2.12)

In the elimination process the following constants are introduced as

\[ x_d = \omega_0 L_d \]  

\[ x_q = \omega_0 L_q \]  

(2.13)

\[ x_d' = \omega_0 L_d' \]  

\[ L_d' = L_d - \frac{3 L_{afM}^2}{2 R_{ff}} \]  

(2.14)

\[ L_{df} = \sqrt{\frac{3}{2}} L_{afM} \]  

\[ x_{df} = \omega_0 L_{df} \]  

\[ T_{d0}' = \frac{L_{ff}}{R_f} \]  

\[ T_d' = \frac{x_d'}{x_d} T_{d0}' \]
Note that all of the parameters of (2.9) through (2.12) are measurable. The zero sequence current vanishes for balanced operation.

Equations (2.9) through (2.11) are rearranged in state variable form as

\[
p^\lambda_d = -e_d - R_a i_d - \omega_0 (1 - s_G) \lambda_q
\]

(2.15)

\[
p^\lambda_q = -e_q - R_a i_q + \omega_0 (1 - s_G) \lambda_d
\]

(2.16)

\[
p^\lambda F = v_F + v_{FR}
\]

(2.17)

where [17]

\[
v_{FR} = \frac{\omega_0 (x_d - x_d') \lambda_d}{x_d'} - \frac{x_d^\lambda F}{x_d'Td0'}
\]

(2.18)

\[
v_F = \frac{x_d e_F}{R_f}
\]

(2.19)

\[
i_d = -\frac{\lambda_F}{x_d'Td0'} + \frac{\omega_0 \lambda_d}{x_d'}
\]

(2.20)

\[
i_q = \frac{\omega_0 \lambda_q}{x_q}
\]

(2.21)

\[
\sigma = \omega_0 t + \delta
\]

(2.22)

\[
\omega = p\sigma = \omega_0 + p\delta
\]

(2.23)

\[
s_G = \frac{\omega_0 - \omega}{\omega_0}
\]

(2.24)

The electromechanical equations of the synchronous machine are

\[
p\delta = -\omega_0 s_G
\]

(2.25)
2.2 Transmission Line Equations

The transmission line is modelled with lumped parameters which are valid for fundamental frequency voltages and currents [18]. Figure 2.2 is a one-line diagram of the system under consideration. The transmission network, including local loads represented as impedances, is reduced to an equivalent-π network as shown in Figure 2.3. The infinite bus voltage is taken as the reference for all phasor quantities. The currents flowing in the system are assumed to be as follows:

From the generator: \[ i_G = \sqrt{2} I_G \cos(\omega_0 t + \beta) \] (2.27)
From the infinite bus: \[ i_B = \sqrt{2} I_B \cos(\omega_0 t + \theta_B) \] (2.28)

The generator terminal voltage (phase "a" to neutral) is

\[ v_t = \sqrt{2} V_L \cos(\omega_0 t + \alpha) \] (2.29)

When the generator current is transformed into its dq0 coordinates the resulting dq currents are constant:

\[ i_d = \sqrt{3} I_G \cos(\beta - \delta) \] (2.30)
\[ i_q = -\sqrt{3} I_G \sin(\beta - \delta) \] (2.31)

The inverse transformation for the generator current is

\[ i_a = \sqrt{\frac{2}{3}} [i_d \cos(\omega_0 t + \delta) + i_q \sin(\omega_0 t + \delta)] \] (2.32)

which may be written in phasor form as

\[ I_G e^{j(\beta - \delta)} = \frac{i_d}{\sqrt{3}} - \frac{j i_q}{\sqrt{3}} \] (2.33)
or
\[
I_G e^{j\beta} = \frac{i_d}{\sqrt{3}} e^{j\delta} - \frac{i_q}{\sqrt{3}} e^{j\delta}
\]

(2.34)

The "\(i_q\)" takes a negative sign because of the coordinates chosen. Let the left side of (2.34) be written as \(i_G + ji_{Gm}\) and separate the real and imaginary parts to obtain

\[
I_G \cos \beta = i_G = \frac{i_d}{\sqrt{3}} \cos \delta + \frac{i_q}{\sqrt{3}} \sin \delta
\]

(2.35)

\[
I_G \sin \beta = i_{Gm} = \frac{i_d}{\sqrt{3}} \sin \delta - \frac{i_q}{\sqrt{3}} \cos \delta
\]

(2.36)

A similar procedure for the voltage yields

\[
v_r = \frac{e_d}{\sqrt{3}} \cos \delta + \frac{e_q}{\sqrt{3}} \sin \delta
\]

(2.37)

\[
v_m = \frac{e_d}{\sqrt{3}} \sin \delta - \frac{e_q}{\sqrt{3}} \cos \delta
\]

(2.38)

The steady state phasor equation of the transmission system is

\[
[Y] [V] = [I]
\]

(2.39)

where \([Y]\) is the admittance matrix of the network, \([V]\) is the bus voltage vector and \([I]\) is the current vector. Expanded, this becomes [19]

\[
\begin{bmatrix}
G + jB + \frac{1}{r + jx} & - \frac{1}{r + jx} \\
- \frac{1}{r + jx} & G + jB + \frac{1}{r + jx}
\end{bmatrix}
\begin{bmatrix}
v_r + jv_m \\
v_0
\end{bmatrix} =
\begin{bmatrix}
i_G + ji_{Gm} \\
i_G + ji_{Gm}
\end{bmatrix}
\]

(2.40)
where \( v_0 \) is the infinite bus voltage phasor and \( i_{BR} + j i_{Em} \) is the phasor current from the infinite bus into the network. Expanding the first equation of (2.40) and equating real and imaginary parts yield

\[
\begin{bmatrix}
  r_G - x_B + 1 & -r_B - x_G \\
  r_B + x_G & r_G - x_B + 1
\end{bmatrix}
\begin{bmatrix}
v_r \\
v_m
\end{bmatrix}
= \begin{bmatrix}
v_0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
r & -x \\
x & r
\end{bmatrix}
\begin{bmatrix}
i_{Gr} \\
i_{Gm}
\end{bmatrix}
\]

(2.41)

Equations (2.35) through (2.38) are substituted into (2.41) and the resulting equation solved for \( e_d \) and \( e_q \). The result is

\[
\begin{bmatrix}
e_d \\
e_q
\end{bmatrix}
= \sqrt{3}
\begin{bmatrix}
k_1 & -k_2 \\
k_2 & k_1
\end{bmatrix}
\begin{bmatrix}
v_0 \cos \delta \\
v_0 \sin \delta
\end{bmatrix}
+ \begin{bmatrix}
c_1 & c_2 \\
-c_2 & c_1
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix}
\]

(2.42)

where

\[
k_1 = \frac{r_G - x_B + 1}{(r_G - x_B + 1)^2 + (r_B + x_G)^2}
\]

(2.43)

\[
k_2 = \frac{r_B + x_G}{(r_G - x_B + 1)^2 + (r_B + x_G)^2}
\]

(2.44)

\[
c_1 = k_1 r + k_2 x
\]

(2.45)

\[
c_2 = k_1 x - k_2 r
\]

(2.46)

2.3 Regulator and Exciter Equations

The block diagram of the regulator and exciter system is shown in Figure 2.4. The regulator output limit is accounted for using a hyperbolic tangent function [13]

\[
f(v_R) = A_1 \tanh (A_2 v_R)
\]

(2.47)

where \( A_1 \) and \( A_2 \) are constants determined from a least-squares criterion.
Figure 2.4 Block Diagram of Voltage Regulator and Exciter

The state equations of the block diagram are

\[ p_{ef} = \frac{1}{T_E} [f(v_R) - e_f] \quad (2.48) \]

\[ p_{v_R} = \frac{1}{T_{RE}} [K_A(v_{ref} - v_t) - v_R] \quad (2.49) \]

The terminal voltage in terms of dq voltages is

\[ v_t = \sqrt{\frac{e_d^2 + e_q^2}{3}} \quad (2.50) \]

In summary, the state equations of the complete system are (2.15) to (2.17), (2.25), (2.26), (2.48) and (2.49).

2.4 Computation and Laboratory Test Results

The system equations are verified with laboratory tests. The system disturbance for the transient stability study is as follows: a three-phase fault occurs at one of the system buses and the faulted line is isolated at 5 cycles followed by a system restoration at 30 cycles. Figure 2.2 shows the location of the fault. The parameters of the system are determined by direct measurement and are listed together with the
Table 2.1
Synchronous Generator, Voltage Regulator and Transmission Line Parameters

<table>
<thead>
<tr>
<th>Synchronous Generator Parameters</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R_a</td>
<td>0.66 Ω</td>
<td>x_q</td>
</tr>
<tr>
<td>x_d</td>
<td>16.2 Ω</td>
<td>T_d0'</td>
</tr>
<tr>
<td>x'_d</td>
<td>2.74 Ω</td>
<td>L_afM</td>
</tr>
<tr>
<td>R_f</td>
<td>4.80 Ω</td>
<td>J_G</td>
</tr>
<tr>
<td>POLES_G</td>
<td>4</td>
<td>f_G</td>
</tr>
<tr>
<td>D_G</td>
<td>0.00267 J-s/rad</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Voltage Regulator Parameters</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>K_A</td>
<td>0.152</td>
<td>A1</td>
</tr>
<tr>
<td>T_E</td>
<td>0.035 s</td>
<td>A2</td>
</tr>
<tr>
<td>T_RE</td>
<td>0.050 s</td>
<td>v_ref</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transmission Line Parameters</th>
<th>r (ohm)</th>
<th>x (ohm)</th>
<th>G (mho)</th>
<th>B (mho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault on</td>
<td>1.42</td>
<td>21.0</td>
<td>0.0097</td>
<td>-0.117</td>
</tr>
<tr>
<td>Faulted line isolated</td>
<td>1.42</td>
<td>21.0</td>
<td>0.0</td>
<td>0.0114</td>
</tr>
<tr>
<td>System restored</td>
<td>0.710</td>
<td>10.5</td>
<td>0.000033</td>
<td>0.0227</td>
</tr>
</tbody>
</table>
Table 2.2
Operating Conditions and Initial Values of State Variables

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>Initial Values of State Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_G ) 100 W/ph</td>
<td>( \lambda_d ) 0.2628 Wb-T</td>
</tr>
<tr>
<td>( V_T ) 57.6 V/ph</td>
<td>( \lambda_q ) 0.0617 Wb-T</td>
</tr>
<tr>
<td>( T_x ) 1.5</td>
<td>( \lambda_F ) 538 Wb-T</td>
</tr>
<tr>
<td>( Q_G ) 85.0 VAR/ph</td>
<td>( \delta ) 133.1 electrical degrees</td>
</tr>
<tr>
<td>( V_O ) 31.6 V/ph</td>
<td>( s_G ) 0</td>
</tr>
<tr>
<td>( T_t ) 3.73 N-m</td>
<td>( e_f ) 2.71 V</td>
</tr>
</tbody>
</table>
prefault operating conditions in Table 2.1. The computed and test results are shown in Figures 2.5 through 2.7.

Figure 2.5 shows the generator torque angle. The maximum swing from the operating point is 52 electrical degrees from computation and 50 degrees from the laboratory test which shows good agreement. The test response is somewhat slower than the computed result. This indicates that there is a larger damping effect in the machine than in the mathematical model. The terminal voltage is shown in Figure 2.6. The switching instants are easily seen. The calculated curve predicts a voltage dip to 45 volts and a rise to 60 volts. The actual test shows a dip to 43 volts and a rise to 63 volts. Voltage spikes are predicted in computation at the switching instants. They are also seen in the test results in the laboratory but do not have the same amplitudes. This is because it is difficult to realize a fault at an exact instant. Small oscillations of the voltage waveform envelope are evident in both the figures. The generator current is shown in Figure 2.7. The current rose to 5.6 amperes in the test. The predicted value was 6.2 amperes. Since the torque angle swing is most important it is concluded that the mathematical model is suitable for transient stability studies.
Fig. 2.5 Generator Torque Angle
Fig. 2.6 Generator Voltage
Fig. 2.7 Generator Current
3. SYNCHRONOUS GENERATOR WITH A LOCAL INDUCTION MOTOR LOAD

In this chapter the effect of a local induction motor load on power system stability is investigated. A one-line diagram of the system is shown in Figure 3.1.

![Figure 3.1 Synchronous Generator System With Local Induction Motor Load](image)

3.1 Induction Motor Equations

The induction motor is a round-rotor machine and the saliency effect is not present. Stanley [20] gave an analysis of the induction motor in 1938. In his equations the machine currents, voltages and flux linkages were referred to axes fixed on the stator. Saturation was neglected. The equations used here follow closely those of Fitzgerald and Kingsley [21] with axes rotating at the synchronous speed. Figure 3.2 shows the polarity of currents and voltages and the direction of rotation of the machine. The $dq0$ transformation matrix is chosen as

\[ T_{dq0} \]
Figure 3.2 Circuit Diagram of Ideal Induction Motor
where \( \theta = \omega_0 t \) for the stator transformation and \( \theta = \omega_0 t - \theta_2 \) for the rotor. Note that the numerical coefficients are different from Fitzgerald and Kingsley's. The equations in dq coordinates are

\[
\begin{bmatrix}
    a & b & c \\
    d & \sqrt{2} \cos \theta & \sqrt{2} \cos (\theta - \frac{2\pi}{3}) & \sqrt{2} \cos (\theta - \frac{4\pi}{3}) \\
    q & \sqrt{2} \sin \theta & \sqrt{2} \sin (\theta - \frac{2\pi}{3}) & \sqrt{2} \sin (\theta - \frac{4\pi}{3}) \\
    0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{bmatrix}
\]

The zero sequence currents vanish for balanced operation.

The relationship between the generator and induction motor terminal voltages is shown in Figure 3.3. The equations are

\[
e_{1d} = r_{1i1d} + p\lambda_{1d} + \omega_0 \lambda_{1q} \\
e_{1q} = r_{1i1q} + p\lambda_{1q} - \omega_0 \lambda_{1d} \\
e_{2d} = r_{2i2d} + p\lambda_{2d} + \lambda_{2q} p\theta_s \\
e_{2q} = r_{2i2q} + p\lambda_{2q} - \lambda_{2d} p\theta_s
\]

where

\[
\lambda_{1d} = L_{1i1d} + \frac{3}{2} L_{aAM} i_{2d} \\
\lambda_{1q} = L_{1i1q} + \frac{3}{2} L_{aAM} i_{2q} \\
\lambda_{2d} = L_{2i2d} + \frac{3}{2} L_{aAM} i_{1d} \\
\lambda_{2q} = L_{2i2q} + \frac{3}{2} L_{aAM} i_{1q}
\]
where $T_x$ is the turns ratio of an ideal transformer between the motor and the generator or

$$T_x = \frac{\text{motor terminal voltage}}{\text{generator terminal voltage}}$$

Following the suggestion of Brereton, et. al. [8] and Gabbard's discussion the stator flux linkages of the induction motor are assumed to remain constant giving

$$p\lambda_{1d} = p\lambda_{1q} = 0$$

Since the rotor windings are short-circuited in normal operation,

$$e_{2d} = e_{2q} = 0$$
The induction motor equations now can be written as

\[ T_x (e_d \cos \delta + e_q \sin \delta) = r_1 i_{1d} + \omega_0 L_{11} i_{1q} + \omega_0 L_{12} i_{2q} \]  
(3.11)

\[ T_x (e_q \cos \delta - e_d \sin \delta) = r_1 i_{1q} - \omega_0 L_{11} i_{1d} - \omega_0 L_{12} i_{2d} \]  
(3.12)

\[ 0 = r_2 i_{2d} + p \lambda_2 d + \lambda_2 q p \theta_s \]  
(3.13)

\[ 0 = r_2 i_{2q} + p \lambda_2 q - \lambda_2 d p \theta_s \]  
(3.14)

\[ \lambda_2 d = L_{22} i_{2d} + L_{12} i_{1d} \]  
(3.15)

\[ \lambda_2 q = L_{22} i_{2q} + L_{12} i_{1q} \]  
(3.16)

where

\[ L_{12} = \frac{3}{2} L_{aAM} \]  
(3.17)

Eliminating \( i_{2d} \) and \( i_{2q} \) from (3.11), (3.12), (3.15) and (3.16) gives

\[ i_{1d} = T_x (D_1 e_d + D_2 e_q) + C_4 \lambda_2 d + C_3 \lambda_2 q \]  
(3.18)

\[ i_{1q} = T_x (D_1 e_q - D_2 e_d) + C_3 \lambda_2 d + C_4 \lambda_2 q \]  
(3.19)

where

\[ x' = \omega_0 L' \]
\[ x_{12} = \omega_0 L_{12} \]  
(3.20)

\[ L' = L_{11} - \frac{L_{12}^2}{L_{22}} \]  
(3.21)

\[ D_1 = \frac{1}{r_1^2 + (x')^2} (r_1 \cos \delta + x' \sin \delta) \]  
(3.22)

\[ D_2 = \frac{1}{r_1^2 + (x')^2} (r_1 \sin \delta - x' \cos \delta) \]  
(3.23)

\[ C_3 = \frac{r_1 x_{12}}{L_{22}[r_1^2 + (x')^2]} \]  
(3.24)
Equations (3.13) and (3.14) may now be written in state variable form as

\[ p\lambda_{2d} = -r_2 i_{2d} - \omega_0 s_M \lambda_{2q} \]  
\[ p\lambda_{2q} = -r_2 i_{2q} + \omega_0 s_M \lambda_{2d} \]

where \( i_{2d} \) and \( i_{2q} \) are solved from (3.15) and (3.16).

The electromechanical equation of the induction motor, analogous to (2.26), is

\[ p s_M = \frac{1}{J_M} \left[ \frac{\text{POLES}_M}{2\omega_0} (T_L - T_M) + D_I (1 - s_M) \right] \]

3.2 Transmission Line Equations

The dq voltage equation (2.46) is not valid because of the addition of local motor loading. If the steady state current flowing into the motor is

\[ i_M = \sqrt{2} I_M \cos(\omega_0 t + \gamma) \]

then its dq components are

\[ i_{1d} = \sqrt{3} I_M \cos \gamma \]  
\[ i_{1q} = -\sqrt{3} I_M \sin \gamma \]

Following the procedure outlined in Chapter 2 for defining generator voltage and current, real and imaginary motor currents are defined as

\[ i_{Mr} = \frac{T_x}{\sqrt{3}} i_{1d} \]
\[ i_{Mm} = -\frac{T_x}{\sqrt{3}} i_{lq} \quad (3.33) \]

These equations are analogous to (2.35) and (2.36) but do not contain the torque angle \( \delta \) because of the dq axes chosen for the induction motor. The transmission line equation, analogous to (2.41), is

\[
\begin{bmatrix}
  rG - xB + 1 & -rB - xG \\
  rB + xG & rG - xB + 1
\end{bmatrix}
\begin{bmatrix}
  v_r \\
  v_m
\end{bmatrix} =
\begin{bmatrix}
  v_0 \\
  0
\end{bmatrix} +
\begin{bmatrix}
  r & -x \\
  x & r
\end{bmatrix}
\begin{bmatrix}
  i_{Gr} - i_{Mr} \\
  i_{Gm} - i_{Mm}
\end{bmatrix}
\]

(3.34)

Using the constants defined in (2.42) to (2.45) and also defining

\[ D_3 = C_1 \cos \delta + C_2 \sin \delta \quad (3.35) \]
\[ D_4 = C_1 \sin \delta - C_2 \cos \delta \quad (3.36) \]

an equation for the dq voltages of the synchronous generator is obtained as

\[
\begin{bmatrix}
  e_d \\
  e_q
\end{bmatrix} = \sqrt{3}
\begin{bmatrix}
  k_1 & -k_2 \\
  k_2 & k_1
\end{bmatrix}
\begin{bmatrix}
  v_0 \cos \delta \\
  v_0 \sin \delta
\end{bmatrix} +
\begin{bmatrix}
  C_1 & C_2 \\
  -C_2 & C_1
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} - T_x
\begin{bmatrix}
  D_3 & -D_4 \\
  D_4 & D_3
\end{bmatrix}
\begin{bmatrix}
  i_{ld} \\
  i_{lq}
\end{bmatrix}
\]

(3.37)

Substituting \( i_{ld} \) and \( i_{lq} \) from (3.18) and (3.19) into (3.37),

\[
\begin{bmatrix}
  e_d \\
  e_q
\end{bmatrix} = \sqrt{3}
\begin{bmatrix}
  k_1 & -k_2 \\
  k_2 & k_1
\end{bmatrix}
\begin{bmatrix}
  v_0 \cos \delta \\
  v_0 \sin \delta
\end{bmatrix} +
\begin{bmatrix}
  C_1 & C_2 \\
  -C_2 & C_1
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix} - T_x
\begin{bmatrix}
  D_3 & -D_4 \\
  D_4 & D_3
\end{bmatrix}
\begin{bmatrix}
  i_{ld} \\
  i_{lq}
\end{bmatrix}
\]

(3.37)

Substituting \( i_{ld} \) and \( i_{lq} \) from (3.18) and (3.19) into (3.37),

\[
\begin{bmatrix}
  e_d \\
  e_q
\end{bmatrix} = \sqrt{3}
\begin{bmatrix}
  k_1 & -k_2 \\
  k_2 & k_1
\end{bmatrix}
\begin{bmatrix}
  v_0 \cos \delta \\
  v_0 \sin \delta
\end{bmatrix} +
\begin{bmatrix}
  C_1 & C_2 \\
  -C_2 & C_1
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
\]

\[
\begin{bmatrix}
  D_3 & -D_4 \\
  D_4 & D_3
\end{bmatrix}
\begin{bmatrix}
  e_d \\
  e_q
\end{bmatrix} +
\begin{bmatrix}
  C_4 & -C_3 \\
  C_3 & C_4
\end{bmatrix}
\begin{bmatrix}
  \lambda_{2d} \\
  \lambda_{2q}
\end{bmatrix}
\]

(3.38)
Solving once again for \( e_d \) and \( e_q \),
\[
\begin{bmatrix}
    e_d \\ \\
    e_q
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
    T_x^2(D_1D_3 + D_2D_4) + 1 & T_x^2(D_1D_4 - D_2D_3) \\
    T_x^2(D_2D_3 - D_1D_4) & T_x^2(D_1D_3 + D_2D_4) + 1
\end{bmatrix} 
\begin{bmatrix}
    k_1 & -k_2 \\
    k_2 & k_1
\end{bmatrix} \begin{bmatrix}
    v_0 \cos \delta \\
    v_0 \sin \delta
\end{bmatrix} + \begin{bmatrix}
    C_1 & C_2 \\
    -C_2 & C_1
\end{bmatrix} \begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix}
\]
\[
-\frac{1}{\sqrt{3}} \begin{bmatrix}
    D_3 & -D_4 \\
    D_4 & D_3
\end{bmatrix} \begin{bmatrix}
    C_4 & -C_3 \\
    C_3 & C_4
\end{bmatrix} \begin{bmatrix}
    \lambda_{2d} \\
    \lambda_{2q}
\end{bmatrix}
\]
\[\text{(3.39)}\]

where
\[
\Delta = [ T_x^2(D_1D_3 + D_2D_4) + 1 ]^2 + T_x^4(D_2D_3 - D_1D_4)^2 \]  
\[\text{(3.40)}\]

In summary, the state equations of the synchronous generator system with local induction motor load are (2.15) to (2.17), (2.25), (2.26), (2.48), (2.49), (3.26), (3.27) and (3.28).

### 3.3 Computation and Laboratory Test Results

The system parameters and disturbance for the transient stability study are the same as in Chapter 2. The parameters of the induction motor are determined from tests and are listed in Table 3.1. The prefault operating conditions of the entire system are given in Table 3.2. The computed and test results are shown in Figures 3.4 through 3.8.

A comparison of Figures 2.4 and 3.4 indicates that the power system is more stable when supplying a local induction motor load. The voltage waveform of the generator with the induction motor load, Figure 3.5, is essentially the same as that in Figure 2.5 without the load. The
Table 3.1

Induction Motor Parameters

<table>
<thead>
<tr>
<th>Induction Motor Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$r_1$</td>
</tr>
<tr>
<td>$r_2$</td>
</tr>
<tr>
<td>$L_{12}$</td>
</tr>
<tr>
<td>$L_{aAM}$</td>
</tr>
<tr>
<td>POLES$_M$</td>
</tr>
<tr>
<td>$L_{11}$</td>
</tr>
<tr>
<td>$L_{22}$</td>
</tr>
<tr>
<td>$J_M$</td>
</tr>
<tr>
<td>$D_M$</td>
</tr>
</tbody>
</table>

Generator current waveform, Figure 3.6, is the same as in Figure 2.7 except that the amplitude of the oscillations after system restoration is smaller when there is a local induction motor load.

Figure 3.7 shows the computed induction motor slip increase during the fault and oscillations after system restoration. This result was also observed in the laboratory but not measured due to lack of a suitable measuring device.

The laboratory test result of the generator torque angle swing is shown in Figure 3.4(a) which in general agrees with the computed result shown in Figure 3.4(b) except for the initial decrease. The decrease in torque angle indicates a sudden increase in power output of the generator. Usually the power output of the generator decreases when a transmission line fault occurs, resulting in a torque angle increase. This is true whether or not the generator has a local motor load. Because of the sudden large change in armature current there must be a large current induced in the damper windings which results in a large damping torque for the generator. Now if the damping is larger than the accelerating effect due to the power
Table 3.2
Operating Conditions and Initial Values of State Variables

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>Initial Values of State Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P_G</strong> 100 W/ph</td>
<td><strong>λ_d</strong> 0.263 Wb-T, <strong>δ</strong> 120 electrical degrees</td>
</tr>
<tr>
<td><strong>Q_G</strong> 85 VAR/ph</td>
<td><strong>s_M</strong> 0.0617 Wb-T, <strong>s_G</strong> 0</td>
</tr>
<tr>
<td><strong>s_M</strong> 0.033</td>
<td><strong>λ_F</strong> 538 Wb-T, <strong>λ_2d</strong> 0.121 Wb-T</td>
</tr>
<tr>
<td><strong>v_0</strong> 37.6 V/ph</td>
<td><strong>λ_q</strong> 0.348 Wb-T, <strong>λ_2q</strong> 0.348 Wb-T</td>
</tr>
<tr>
<td><strong>T_t</strong> 3.73 N-m</td>
<td><strong>v_R</strong> 2.28 V, <strong>s_M</strong> 0.033</td>
</tr>
<tr>
<td><strong>v_t</strong> 57.6 V/ph</td>
<td><strong>T_x</strong> 1.5, <strong>T_L</strong> 0.371 N-m</td>
</tr>
</tbody>
</table>
Fig. 3.4 Generator Torque Angle
Fig. 3.5 Generator Voltage
Fig. 3.6 Generator Current
Fig. 3.7 Induction Motor Slip
Fig. 3.8 Induction Motor Current
output decrease, the generator torque angle decreases instead of increasing as predicted. This phenomenon will be more apparent when there is a large local load. In the analysis, however, the effect of damper windings has been neglected.

Two conclusions may be drawn from this study. First, the effect of damper windings must be included in stability studies if exact prediction of torque angle swings is required. Second, a local induction motor load increases the stability of a power system.
4. SYNCHRONOUS GENERATOR WITH A LOCAL SYNCHRONOUS MOTOR LOAD

In this chapter the local load effect of a synchronous motor on system stability is investigated. A one line diagram of the system is shown in Figure 4.1.

![Figure 4.1 Synchronous Generator System with Local Synchronous Motor Load](image)

4.1 Synchronous Motor Equations

The synchronous motor has the same equations as the synchronous generator except that the positive direction for stator currents is now defined to be into the machine. The subscript "3" is used to denote synchronous motor quantities. Analogous to state equations (2.15) to (2.17), (2.25) and (2.26) of the synchronous generator, the synchronous motor equations are

\[ p_3^d = -e_3^d + R_3 a_3^d - \omega_0 (1 - s_3) \lambda_3^q \]  
(4.1)

\[ p_3^q = -e_3^q + R_3 a_3^q + \omega_0 (1 - s_3) \lambda_3^d \]  
(4.2)

\[ p_3^F = v_3^F + v_3^{FR} \]  
(4.3)
\[ p^3 = -\omega_0 s_3 \]  

\[ ps_3 = \frac{1}{J_3} \left[ \frac{\text{POLES}_3}{2\omega_0} (T_{3L} - T_{3M}) + D_3 (1 - s_3) \right] \]  

The auxiliary equations, analogous to (2.18) to (2.24), are

\[ v_{3FR} = \frac{\omega_0 (x_{3d} - x_{3d}')}{x_{3d}'} - \frac{x_{3d}^* \lambda_{3F}}{x_{3d}' T_{3d0}'} \]  

\[ v_{3F} = \frac{x_{3df} e_{3f}}{R_{3f}} \]  

\[ i_{3d} = \frac{\lambda_{3F}}{x_{3d}' T_{3d0}'} - \frac{\omega_0 \lambda_{3d}}{x_{3d}'} \]  

\[ i_{3q} = -\frac{\omega_0 \lambda_{3q}}{x_{3q}} \]  

\[ \sigma_3 = \omega t + \delta_3 \]  

\[ \omega_3 = p\sigma_3 = \omega_0 + p\delta_3 \]  

\[ s_3 = \frac{\omega_0 - \omega_3}{\omega_0} \]  

Since the terminal voltages of the generator and the motor are common, the dq voltages of the two machines are related by
\[
\begin{bmatrix}
\frac{2}{3} \cos \sigma & \frac{2}{3} \sin \sigma & 1 \\
\frac{2}{3} \cos \left(\sigma - \frac{2\pi}{3}\right) & \frac{2}{3} \sin \left(\sigma - \frac{2\pi}{3}\right) & 1 \\
\frac{2}{3} \cos \left(\sigma - \frac{4\pi}{3}\right) & \frac{2}{3} \sin \left(\sigma - \frac{4\pi}{3}\right) & 1
\end{bmatrix}
\begin{bmatrix}
e_d \\
e_q \\
e_0
\end{bmatrix}
\]

\[
T_x = \frac{2}{3} \cos \left(\sigma - \frac{2\pi}{3}\right) \frac{2}{3} \sin \left(\sigma - \frac{2\pi}{3}\right) 1 
\]

\[
T_x = \frac{2}{3} \cos \left(\sigma - \frac{4\pi}{3}\right) \frac{2}{3} \sin \left(\sigma - \frac{4\pi}{3}\right) 1 
\]

Solving for \(e_{3d}\) and \(e_{3q}\) with \(e_0 = e_{30} = 0\),

\[
e_{3d} = T_x \left[ e_d \cos (\delta - \delta_3) + e_q \sin (\delta - \delta_3) \right] \quad (4.14)
\]

\[
e_{3q} = T_x \left[ e_q \cos (\delta - \delta_3) - e_d \sin (\delta - \delta_3) \right] \quad (4.15)
\]

4.2 Transmission Line Equations

The transmission line equation, analogous to (3.34), is

\[
\begin{bmatrix}
rG - XB + 1 & -B - xG \\
rB + xG & rG - XB + 1
\end{bmatrix}
\begin{bmatrix}
v_r \\
v_m
\end{bmatrix}
= 
\begin{bmatrix}
v_0 \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
r & -x \\
x & r
\end{bmatrix}
\begin{bmatrix}
i_{Gr} - i_{3Mr} \\
i_{Gm} - i_{3Mm}
\end{bmatrix}
\]

(4.16)

where

\[
\begin{bmatrix}
i_{3Mr} \\
i_{3Mm}
\end{bmatrix} = \frac{T_x}{\sqrt{3}} \begin{bmatrix}
\cos \delta_3 & \sin \delta_3 \\
\sin \delta_3 & -\cos \delta_3
\end{bmatrix}
\begin{bmatrix}
i_{3d} \\
i_{3q}
\end{bmatrix}
\]

(4.17)

Equation (4.17) is obtained using the same arguments as those that led to
the writing of equations (2.35) and (2.36). Substituting (2.35), (2.36),
(2.37), (2.38) and (4.17) into (4.16) and solving for $e_d$ and $e_q$,

$$
\begin{bmatrix}
  e_d \\
  e_q
\end{bmatrix} = \sqrt{3} \begin{bmatrix}
  k_1 & -k_2 \\
  k_2 & k_1
\end{bmatrix} \begin{bmatrix}
  v_0 \cos \delta \\
  v_0 \sin \delta
\end{bmatrix} + \begin{bmatrix}
  C_1 & C_2 \\
  -C_2 & C_1
\end{bmatrix} \begin{bmatrix}
  i_d \\
  i_q
\end{bmatrix}
$$

$$
-\frac{T_x}{D_5 - D_6} \begin{bmatrix}
  i_{3d} \\
  i_{3q}
\end{bmatrix}
$$

(4.18)

where $k_1$, $k_2$, $C_1$ and $C_2$ have been defined in equations (2.42) to (2.45) and

$$
D_5 = C_1 \cos (\delta - \delta_3) + C_2 \sin (\delta - \delta_3)
$$

(4.19)

$$
D_6 = C_1 \sin (\delta - \delta_3) - C_2 \cos (\delta - \delta_3)
$$

(4.20)

If equation (4.18) is compared with equation (3.39) it is seen
that the representation of the synchronous generator system with local
synchronous motor load is much less complex. The representation of the
synchronous generator system with local induction motor load is more complex
because the generator coordinates are fixed on the rotor whereas the
induction motor coordinates are rotating at the synchronous speed.

In summary, the state equations of the synchronous generator
system with local synchronous motor load are (2.15) to (2.17), (2.25),
(2.26), (2.48), (2.49) and (4.1) to (4.5).

4.3 Computation and Laboratory Test Results

The system parameters and disturbance for the transient stability
study are the same as in Chapters 2 and 3. The parameters of the
synchronous motor are determined from tests and listed in Table 4.1. The
real power input to the synchronous motor is the same as that of the
### Table 4.1
Synchronous Motor Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{3a}$</td>
<td>4.26 Ω</td>
</tr>
<tr>
<td>$x_{3d}$</td>
<td>131.7 Ω</td>
</tr>
<tr>
<td>$x_{3d}'$</td>
<td>40.4 Ω</td>
</tr>
<tr>
<td>$R_{3f}$</td>
<td>10.8 Ω</td>
</tr>
<tr>
<td>POLES$_3$</td>
<td>4</td>
</tr>
<tr>
<td>$x_{3q}$</td>
<td>124.8 Ω</td>
</tr>
<tr>
<td>$T_{3d0}'$</td>
<td>0.0619 s</td>
</tr>
<tr>
<td>$L_{3a}$</td>
<td>0.405 H</td>
</tr>
<tr>
<td>$J_3$</td>
<td>0.0121 J-s$^2$/rad</td>
</tr>
<tr>
<td>$D_3$</td>
<td>0.000995 J-s/rad</td>
</tr>
</tbody>
</table>

### Table 4.2
Operating Conditions and Initial Values of State Variables

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_G$</td>
<td>100 W/ph</td>
</tr>
<tr>
<td>$Q_G$</td>
<td>85 VAR/ph</td>
</tr>
<tr>
<td>$P_3$</td>
<td>40.0 W/ph</td>
</tr>
<tr>
<td>$Q_3$</td>
<td>19.3 VAR/ph</td>
</tr>
<tr>
<td>$v_t$</td>
<td>57.6 V/ph</td>
</tr>
<tr>
<td>$v_0$</td>
<td>32.5 V/ph</td>
</tr>
<tr>
<td>$T_x$</td>
<td>1.5</td>
</tr>
<tr>
<td>$T_t$</td>
<td>3.73 N-m</td>
</tr>
<tr>
<td>$T_{3L}$</td>
<td>0.431 N-m</td>
</tr>
<tr>
<td>$e_{3f}$</td>
<td>8.17 V</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_d$</td>
<td>0.263 Wb-T</td>
</tr>
<tr>
<td>$\lambda_q$</td>
<td>0.0617 Wb-T</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td>538 Wb-T</td>
</tr>
<tr>
<td>$e_f$</td>
<td>2.71 V</td>
</tr>
<tr>
<td>$v_R$</td>
<td>2.28 V</td>
</tr>
<tr>
<td>$\delta$</td>
<td>118 electrical degrees</td>
</tr>
<tr>
<td>$s_G$</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_{3d}$</td>
<td>0.270 Wb-T</td>
</tr>
<tr>
<td>$\lambda_{3q}$</td>
<td>-0.278 Wb-T</td>
</tr>
<tr>
<td>$\lambda_{3F}$</td>
<td>7.05 Wb-T</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>61.3 electrical degrees</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0</td>
</tr>
</tbody>
</table>
induction motor of Chapter 3. Typical induction motor slip and synchronous motor power factor values are set prior to the fault occurrence. The operating conditions of the entire system are listed in Table 4.2. The computed and test results are shown in Figures 4.2 to 4.6.

A comparison of Figures 2.4 and 4.2 indicates that the power system is more stable when supplying a local synchronous motor load. The generator current and voltage waveforms are essentially the same as in Chapter 3. All laboratory measurements agree favourably with computed results except the torque angle swing. The torque angle swing observed in the laboratory decreases initially as in the case of induction motor loading and for the same reason. The synchronous motor oscillates during the test for a short period which is also predicted by computation, Figure 4.3.

To further explore the nature of a synchronous motor load, computation is extended to include the case of steady state operation of the motor but with a sinusoidal shaft load. The operating conditions of the system for this investigation are the same as in Table 4.2 except that the shaft load is no longer constant at 0.431 N-m but varies sinusoidally as

\[ T_{3L} = 0.431 + 0.1 \cos 2\pi t \]

That is, the peak load is 0.531 N-m and the frequency of oscillation is 1.0 cycle per second. The results are shown in Figures 4.7 and 4.8. Although the generator has sustained oscillations of its torque angle the system remains stable for the case investigated.
Fig. 4.2 Generator Torque Angle
Fig. 4.3 Synchronous Motor Torque Angle
Fig. 4.4 Generator Voltage - Local Synchronous Motor Load
Fig. 4.5 Generator Current
Fig. 4.6 Synchronous Motor Current
Fig. 4.7 Generator Torque Angle
Fig. 4.8 Synchronous Motor Torque Angle
4.4 Computation Results Using Large-System Parameters

A large generator with a local synchronous motor load is investigated. The transmission line fault is at the same location and of the same duration as in the small machine studies. The system parameters and operating conditions are given in Tables 4.3, 4.4 and 4.5. The torque angle swings of the synchronous generator without and with synchronous motor load are shown in Figures 4.9 and 4.10 respectively. The maximum swing is 25.2 electrical degrees without the motor load and 20.9 electrical degrees when the motor is included.

From the results obtained in this chapter it is concluded that a local synchronous motor load will also increase the stability of a power system.

<table>
<thead>
<tr>
<th>Parameters of Large Synchronous Motor</th>
<th>( R_{3a} )</th>
<th>0.00136 ( \Omega )</th>
<th>( x_{3q} )</th>
<th>0.218 ( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{3d} )</td>
<td>0.327 ( \Omega )</td>
<td>( T_{3d0} )</td>
<td>5.3 s</td>
<td></td>
</tr>
<tr>
<td>( x_{3d} )</td>
<td>0.0819 ( \Omega )</td>
<td>( L_{3afM} )</td>
<td>0.024 H</td>
<td></td>
</tr>
<tr>
<td>( R_{3f} )</td>
<td>0.25 ( \Omega )</td>
<td>( J_{3} )</td>
<td>11100 J-s(^2)/rad</td>
<td></td>
</tr>
<tr>
<td>POLES(_3)</td>
<td>6</td>
<td>( D_{3} )</td>
<td>0 J-s/rad</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3

Parameters of Large Synchronous Motor
Table 4.4
Parameters of Large Power System

<table>
<thead>
<tr>
<th>Synchronous Generator Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a$</td>
<td>0.0 Ω</td>
</tr>
<tr>
<td>$x_d$</td>
<td>1.85 Ω</td>
</tr>
<tr>
<td>$x_d'$</td>
<td>0.361 Ω</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.182 Ω</td>
</tr>
<tr>
<td>POLES$_G$</td>
<td>48</td>
</tr>
<tr>
<td>$f_G$</td>
<td>0.0 N-m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Voltage Regulator Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_A$</td>
<td>0.925</td>
</tr>
<tr>
<td>$T_E$</td>
<td>0.003 s</td>
</tr>
<tr>
<td>$T_{RE}$</td>
<td>0.05 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transmission Line Parameters</th>
<th>$r$ (ohm)</th>
<th>$x$ (ohm)</th>
<th>$G$ (mho)</th>
<th>$B$ (mho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault on</td>
<td>-0.00145</td>
<td>0.105</td>
<td>2.22</td>
<td>-2.63</td>
</tr>
<tr>
<td>Faulted line isolated</td>
<td>-0.00244</td>
<td>0.0815</td>
<td>2.86</td>
<td>3.98</td>
</tr>
<tr>
<td>System restored</td>
<td>-0.00286</td>
<td>0.0838</td>
<td>2.96</td>
<td>3.12</td>
</tr>
</tbody>
</table>
Table 4.5
Large Power System Stability Study
Operating Conditions and Initial Values of State Variables

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>Initial Values of State Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_G$ 30 MW/ph</td>
<td>$s_G$ 0.0</td>
</tr>
<tr>
<td>$P_3$ 26.7 MW/ph</td>
<td>$\lambda_{3d}$ 5.1 Wb-T</td>
</tr>
<tr>
<td>$v_t$ 7970 V.ph</td>
<td>$\lambda_{3q}$ -7.12 Wb-T</td>
</tr>
<tr>
<td>$v_0$ 6157 V/ph</td>
<td>$\lambda_{3f}$ $1.27 \times 10^4$ Wb-T</td>
</tr>
<tr>
<td>$T_x$ 0.239</td>
<td>$e_3$ 88.6 V</td>
</tr>
<tr>
<td>$T_{3L}$ 6.33 x $10^5$ N-m</td>
<td>$\delta_3$ 17.6 electrical degrees</td>
</tr>
<tr>
<td>$Q_G$ 10 MVAR/ph</td>
<td>$s_3$ 0.0</td>
</tr>
<tr>
<td>$Q_3$ 14.3 MVAR/ph</td>
<td></td>
</tr>
<tr>
<td>$T_t$ 5.73 x $10^6$ N-m</td>
<td></td>
</tr>
<tr>
<td>$e_3$ 88.6 V</td>
<td></td>
</tr>
<tr>
<td>$X_d$ 33.7 Wb-T</td>
<td></td>
</tr>
<tr>
<td>$X_3d$ 5.1 Wb-T</td>
<td></td>
</tr>
<tr>
<td>$X_q$ 14.2 Wb-T</td>
<td></td>
</tr>
<tr>
<td>$X_3q$ -7.12 Wb-T</td>
<td></td>
</tr>
<tr>
<td>$e_f$ 136 V</td>
<td></td>
</tr>
<tr>
<td>$\delta$ 95.2 electrical degrees</td>
<td></td>
</tr>
<tr>
<td>$v_R$ 147 V</td>
<td></td>
</tr>
<tr>
<td>$s_3$ 0.0</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4.9 Large Generator Torque Angle
No Motor Loads
Fig. 4.10 Large Generator Torque Angle
Local Synchronous Motor Load
In this chapter a more general local load and its effect on power system stability is investigated. The local load may consist of any number of synchronous and induction motors as well as any other loads which may be represented by equivalent circuits. A one-line diagram of the system is shown in Figure 5.1. Note that loads represented by equivalent circuits are absorbed into the equivalent-π representation of the transmission line in the network reduction process.

Figure 5.1 Synchronous Generator System With Multiple Local Motor Loads

5.1 Machine Equations

The equations required to describe the synchronous generator system with its equivalent-π transmission network, the induction motors and the synchronous motors have already been developed in Chapters 2 through 4.
Synchronous Generator With Voltage Regulator:

\[ p\lambda_d = -e_d - R_a i_d - \omega_0 (1 - s_G) \lambda_q \]  \hspace{1cm} (5.1) \\
\[ p\lambda_q = -e_q - R_a i_q + \omega_0 (1 - s_G) \lambda_d \]  \hspace{1cm} (5.2) \\
\[ p\lambda_F = v_F + v_{FR} \]  \hspace{1cm} (5.3) \\
\[ pE_f = \frac{1}{T_E} [f(v_R) - e_f] \]  \hspace{1cm} (5.4) \\
\[ p v_R = \frac{1}{T_{RE}} [K_A(v_{ref} - v_t) - v_R] \]  \hspace{1cm} (5.5) \\
\[ p\delta = -\omega_0 s_G \]  \hspace{1cm} (5.6) \\
\[ p s_G = \frac{1}{J_G} \left[ \frac{\text{POLES}_G}{2\omega_0} (T_G + f_G - T_t) + D_G (1 - s_G) \right] \]  \hspace{1cm} (5.7) \\

Induction Motors:

\[ p\lambda_{2d}^k = -r_{2d}^k i_{2d}^k - \omega_0 s_M^k \lambda_{2q}^k \]  \hspace{1cm} (5.8) \\
\[ p\lambda_{2q}^k = -r_{2q}^k i_{2q}^k + \omega_0 s_M^k \lambda_{2d}^k \]  \hspace{1cm} (5.9) \\
\[ p s_M^k = \frac{1}{J_M^k} \left[ \frac{\text{POLES}_M^k}{2\omega_0} (T_L^k - T_M^k) + D_M^k (1 - s_M^k) \right] \]  \hspace{1cm} (5.10) \\

where for the multi-motor system the superscript \( k \) denotes the motor number, \( k = 1, 2, \ldots, m \)

Synchronous Motors:

\[ p\lambda_{3d}^k = -e_{3d}^k + R_{3a}^k i_{3d}^k - \omega_0 (1 - s_3^k) \lambda_{3q}^k \]  \hspace{1cm} (5.11)
\[ p^\lambda_{3q}^k = -e^{3q}^k + R_{3a}^{i_{3q}^k} - \omega_0 (1 - s_3^k) \lambda_{3q}^k \]  
\[ (5.12) \]

\[ p^\lambda_{3F}^k = \nu_{3F}^k + \nu_{3FR}^k \]  
\[ (5.13) \]

\[ p^\delta_3^k = -\omega_0 s_3^k \]  
\[ (5.14) \]

\[ p^s_3^k = \frac{1}{J_3^k} \left[ \text{POLES}_3^k \left( T_{3L}^k - T_{3M}^k \right) + D_3^k (1 - s_3^k) \right] \]  
\[ (5.15) \]

where

\[ k = m+1, m+2, \ldots, n \]

The order of the system is \( 7 + 3m + 5(n-m) \) where \( m \) is the number of induction motors and \( (n-m) \) is the number of synchronous motors. Motors of the same size and with similar shaft load characteristics should be lumped together as one equivalent machine in order to decrease the number of equations.

5.2 Transmission Line Equations

An equation similar to (2.40), (3.34) and (4.16) is written for the general case as

\[
\begin{bmatrix}
    rG - xB + 1 & -rB - xG \\
    rB + xG & rG - xB + 1
\end{bmatrix}
\begin{bmatrix}
    v_r \\
    v_m
\end{bmatrix}
\]

\[
= \begin{bmatrix}
    v_0 \\
    0
\end{bmatrix} + \begin{bmatrix}
    r & -x \\
    x & r
\end{bmatrix}
\begin{bmatrix}
    i_{Gr} - \Sigma_m (i_{Mr}^k) - \Sigma_n (i_{3r}^k) \\
    i_{Gm} - \Sigma_m (i_{Mm}^k) - \Sigma_n (i_{3m}^k)
\end{bmatrix}
\]  
\[ (5.16) \]
where \( \Sigma_m \) and \( \Sigma_n \) are operators defined by

\[
\Sigma_m = \sum_{k=1}^{m}
\]

\[
\Sigma_n = \sum_{k=m+1}^{n}
\]

and

\[
\Sigma_m \begin{bmatrix} i_{Mr}^k \\ i_{Mm}^k \end{bmatrix} = \frac{\Sigma_m}{\sqrt{3}} \begin{bmatrix} i_{ld}^k \\ -i_{lq}^k \end{bmatrix}
\]

(5.17)

\[
\Sigma_n \begin{bmatrix} i_{3r}^k \\ i_{3m}^k \end{bmatrix} = \frac{\Sigma_n}{\sqrt{3}} \begin{bmatrix} \cos \delta_3^k & \sin \delta_3^k \\ \sin \delta_3^k & -\cos \delta_3^k \end{bmatrix} \begin{bmatrix} i_{3d}^k \\ i_{3q}^k \end{bmatrix}
\]

(5.18)

Substituting (2.37), (2.38), (2.35), (2.36), (5.17) and (5.18) into (5.16) and solving for the dq voltages,

\[
\begin{bmatrix} e_d \\ e_q \end{bmatrix} = \sqrt{3} \begin{bmatrix} \cos \delta & \sin \delta \\ \sin \delta & -\cos \delta \end{bmatrix} \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix} \begin{bmatrix} v_0 \\ 0 \end{bmatrix}
\]

\[
+ \begin{bmatrix} \cos \delta & \sin \delta \\ \sin \delta & -\cos \delta \end{bmatrix} \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix} \begin{bmatrix} r & -x \\ x & r \end{bmatrix} \begin{bmatrix} \cos \delta & \sin \delta \\ \sin \delta & -\cos \delta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}
\]

\[
- \begin{bmatrix} \cos \delta & \sin \delta \\ \sin \delta & -\cos \delta \end{bmatrix} \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix} \begin{bmatrix} r & -x \\ x & r \end{bmatrix} \Sigma_m(T_x^k \begin{bmatrix} i_{ld}^k \\ i_{lq}^k \end{bmatrix})
\]
\[
\begin{bmatrix}
\cos \delta & \sin \delta \\
\sin \delta & -\cos \delta
\end{bmatrix}
\begin{bmatrix}
k_1 & k_2 \\
-k_2 & k_1
\end{bmatrix}
\begin{bmatrix}
r & -x
\end{bmatrix}
\Sigma_n(T_x^k)
\begin{bmatrix}
\cos \delta_3^k & \sin \delta_3^k \\
\sin \delta_3^k & -\cos \delta_3^k
\end{bmatrix}
\begin{bmatrix}
i_{3d}^k \\
i_{3q}^k
\end{bmatrix}
\]

which reduces to

\[
\begin{bmatrix}
ed \\
eq
\end{bmatrix} = \sqrt{3}
\begin{bmatrix}
k_1 & k_2 \\
-k_2 & k_1
\end{bmatrix}
\begin{bmatrix}
v_0 \cos \delta \\
v_0 \sin \delta
\end{bmatrix}
+ \begin{bmatrix}
C_1 & C_2 \\
-C_2 & C_1
\end{bmatrix}
\begin{bmatrix}
id \\
i_q
\end{bmatrix}
\]

\[
- \begin{bmatrix}
D_3 & -D_4 \\
D_4 & D_3
\end{bmatrix}
\Sigma_m(T_x^k)
\begin{bmatrix}
il_d^k \\
il_q^k
\end{bmatrix}
= \begin{bmatrix}
D_5^k & -D_6^k \\
D_6^k & D_5^k
\end{bmatrix}
\begin{bmatrix}
i_{3d}^k \\
i_{3q}^k
\end{bmatrix}
\]

where \(C_1\) and \(C_2\) are defined by (2.44) and (2.45), \(D_3\) and \(D_4\) are defined by (3.35) and (3.36) and

\[
D_5^k = C_1 \cos (\delta - \delta_3^k) + C_2 \sin (\delta - \delta_3^k)
\]

(5.21)

\[
D_6^k = C_1 \sin (\delta - \delta_3^k) - C_2 \cos (\delta - \delta_3^k)
\]

(5.22)

The induction motor stator currents are eliminated using (3.18) and (3.19) in the matrix form,

\[
\begin{bmatrix}
il_d^k \\
il_q^k
\end{bmatrix} = \begin{bmatrix}
D_1^k & D_2^k \\
-D_2^k & D_1^k
\end{bmatrix}
\begin{bmatrix}
ed \\
eq
\end{bmatrix}
+ \begin{bmatrix}
C_4^k & -C_3^k \\
C_3^k & C_4^k
\end{bmatrix}
\begin{bmatrix}
\lambda_{2d}^k \\
\lambda_{2q}^k
\end{bmatrix}
\]

(5.23)
where

\[
D_1^k = \frac{r_1^k \cos \delta + x_1^k \sin \delta}{(r_1^k)^2 + (x_1^k)^2} \quad (5.24)
\]

\[
D_2^k = \frac{r_1^k \sin \delta - x_1^k \cos \delta}{(r_1^k)^2 + (x_1^k)^2} \quad (5.25)
\]

\[
C_3^k = \frac{r_1^k x_{12}^k}{L_{22}^k [(r_1^k)^2 + (x_1^k)^2]} \quad (5.26)
\]

\[
C_4^k = \frac{-x_{12}^k x_{1k}^k}{L_{22}^k [(r_1^k)^2 + (x_1^k)^2]} \quad (5.27)
\]

Equation (5.23) is now substituted into (5.20) and the resulting equation solved for the dq voltages to obtain

\[
\begin{bmatrix}
ed \\
e_q
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
D_3 & -D_4 \\
D_4 & D_3
\end{bmatrix} \Sigma_m [(T_x^k)^2] \begin{bmatrix}
D_1^k & D_2^k \\
-D_2^k & D_1^k
\end{bmatrix}^{-1} \begin{bmatrix}
D_3 & -D_4 \\
D_4 & D_3
\end{bmatrix} \Sigma_m [(T_x^k)^2] \begin{bmatrix}
C_4^k & -C_3^k \\
C_3^k & C_4^k
\end{bmatrix} \lambda_{2d}^k
\]

\[
- \lambda_{2q}^k
\]

\[
= \begin{bmatrix}
D_5^k & -D_6^k \\
D_6^k & D_5^k
\end{bmatrix} \begin{bmatrix}
i_{3d}^k \\
i_{3q}^k
\end{bmatrix}
\]

\[
(5.28)
\]
The matrix inversion of (5.28) is 4x4 regardless of the number of motors involved and presents no computation problems. In the case of no induction motors, \( D_1^k = D_2^k = 0 \) for \( k = 1,2,\ldots,m \), and no matrix inversion is necessary.

The \( dq \) terminal voltages of any induction motor are calculated from (3.9) and (3.10) and those of any synchronous motor from (4.14) and (4.15):

\[
e_{ld}^k = T_x^k (e_d \cos \delta + e_q \sin \delta) \tag{5.29}
\]

\[
e_{lq}^k = T_x^k (e_q \cos \delta - e_d \sin \delta) \tag{5.30}
\]

\[
e_{3d}^k = T_x^k [e_d \cos (\delta - \delta_3^k) + e_q \sin (\delta - \delta_3^k)] \tag{5.31}
\]

\[
e_{3q}^k = T_x^k [e_q \cos (\delta - \delta_3^k) - e_d \sin (\delta - \delta_3^k)] \tag{5.32}
\]

In summary, the state equations of a synchronous generator system supplying multiple local induction motor and synchronous motor loads are given by (5.1) through (5.15). A general transmission line equation relating all machine terminal conditions to the transmission network is given by (5.28). As for local loads which may be represented as equivalent circuits, they can be readily incorporated into the transmission network representation.
5.3 Computation and Laboratory Test Results

Although a general formulation of a synchronous generator system with multiple synchronous and induction motor loads was given in the last section it is sufficient to investigate the stability of a synchronous generator system supplying one local induction motor and one local synchronous motor [8]. The system disturbance and generator and motor parameters are given in Tables 2.1, 3.1 and 4.1. The prefault operating conditions are given in Table 5.1. The computed and test results are shown in Figures 5.2 through 5.8.

A comparison is made of the generator torque angle of Figure 5.2 with that of Figures 3.4 and 4.2 where only single motor loads were present. It reveals that the stability of the system is increased still further with both synchronous and induction motor loads. A decrease in generator torque angle is seen immediately after the fault occurs in the laboratory test because of the omission of the damper windings in the analysis as in Chapters 3 and 4.

The synchronous motor torque angle is shown in Figure 5.3. The swing is not so severe as in the case of a single synchronous motor load alone, Figure 4.3. The induction motor slip, Figure 5.4, is almost the same as in Chapter 3, the case of the generator with the induction motor load alone.

The generator terminal voltage, Figure 5.5, and the machine currents, Figures 5.6 to 5.8, are much the same as in the previous tests of Chapters 2 to 4 except that oscillations vanish more quickly after the transmission line is restored.

It is concluded from the studies so far that a local motor load, either induction or synchronous or both, will improve the stability of a power system.
Table 5.1
Synchronous Generator System With Local Induction
And Synchronous Motor Loads
Operating Conditions and Initial Values of State Variables

<table>
<thead>
<tr>
<th>Operating Conditions</th>
<th>Initial Values of State Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_G ) 100 W/ph</td>
<td>( \lambda_d ) 0.263 Wb-T</td>
</tr>
<tr>
<td>( P_3 ) 40.0 W/ph</td>
<td>( \lambda_q ) 0.0617 Wb-T</td>
</tr>
<tr>
<td>( s_M ) 0.033</td>
<td>( \lambda_F ) 538 Wb-T</td>
</tr>
<tr>
<td>( v_0 ) 39.9 V/ph</td>
<td>( e_f ) 2.71 V</td>
</tr>
<tr>
<td>( e_{3f} ) 8.17 V</td>
<td>( v_R ) 2.28 V</td>
</tr>
<tr>
<td>( T_{3L} ) 0.431 N-m</td>
<td>( \delta ) 109 electrical degrees</td>
</tr>
<tr>
<td>( Q_G ) 85 VAR/ph</td>
<td>( \lambda_{3q} ) -0.278 Wb-T</td>
</tr>
<tr>
<td>( Q_3 ) 19.3 VAR/ph</td>
<td>( \lambda_{3F} ) 7.05 Wb-T</td>
</tr>
<tr>
<td></td>
<td>( \delta_3 ) 51.8 electrical degrees</td>
</tr>
<tr>
<td></td>
<td>( s_3 ) 0.0</td>
</tr>
<tr>
<td></td>
<td>( \lambda_{2d} ) 0.0486 Wb-T</td>
</tr>
<tr>
<td></td>
<td>( \lambda_{2q} ) 0.366 Wb-T</td>
</tr>
<tr>
<td></td>
<td>( \lambda_{3d} ) 0.270 Wb-T</td>
</tr>
<tr>
<td></td>
<td>( s_M ) 0.033</td>
</tr>
<tr>
<td></td>
<td>( s_G ) 0.0</td>
</tr>
</tbody>
</table>
Fig. 5.2 Generator Torque Angle
Fig. 5.3 Synchronous Motor Torque Angle
Fig. 5.4 Induction Motor Slip
Fig. 5.5 Generator Voltage
Fig. 5.7 Synchronous Motor Current
Fig. 5.8 Induction Motor Current
6. CONCLUSION

The effect of local motor loads on power system stability has been investigated. From analytical results of both large and small synchronous and induction motor loads and tests on a power system model using small machines the following conclusions can be drawn:

1. A local motor load, either induction or synchronous or both, will improve the stability of a power system.

2. Both synchronous and induction motors oscillate with comparatively large amplitudes during system disturbances even if stability is maintained.

3. The damper windings of both synchronous generators and synchronous motors have noticeable effect on the first swing after a fault, as revealed from the tests.

4. Motors with large periodical mechanical loads can cause oscillations to a power system.

5. The influence of local loads on power system stability can be effectively determined with a power system test model.

Conclusions 1 and 2 were also reached by Gevay and Schippel [8] for the case where the motor loads were at the receiving end of the transmission line. Conclusion 5 was also reached by Robert and Robichaud [10] using simulated constant impedance, constant power and constant current loads.

The following recommendations are made for further studies. First, the mathematical model of synchronous machines should be extended to include the damper windings if exact prediction of torque angle swings is required. Second, more extensive tests on the power system model for different power and voltage levels and different power factors of the generator and the motors should be done in order to reach more general
conclusions. Other types of loads such as lighting and arc furnaces should also be added in the analysis. Third, the variation of bus voltage frequency and magnitude should also be included. The formulation presented in this thesis can be readily incorporated into optimal stabilization and control investigations of power systems since the system modelling is a very important part of the study.
REFERENCES


