AN INVESTIGATION OF THE EFFECTS OF THE GEOMETRIC SUPPOSER SOFTWARE ON GEOMETRIC PROOF WRITING AT THE GRADE 10 LEVEL

By

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ABSTRACT

The purpose of this study was to determine if the use of a computer program called the Geometric Supposer would result in improved proof writing by grade 10 geometry students.

The researcher studied 44 students enrolled in a grade 10 geometry course. The students were divided into two classes; one class used the Geometric Supposer computer program while the other class did not. Both classes were taught at the same time every day and both classes covered the same content. The researcher kept in close contact with the teacher of the noncomputer group regarding the content, the assignments, and the overall progress of the students.

Both classes were given two tests (an introductory geometry test and the van Hiele geometry test) at the beginning of the course. At the end of the course (one semester in length) three tests were given to both classes--the same van Hiele geometry test (measures geometric thought levels), a proof test, and an attitude test. Weekly interviews were conducted with each of five students from the computer group. Two students from the noncomputer group were each interviewed twice near the end of the course. These students were chosen based on their van Hiele levels. The interviews provided the researcher with a better understanding of how some students approach and write geometric proofs.

The data gathered from the introductory geometry test, the proof test, and the attitude test were each analyzed using the independent *t*-test. The median test was applied to the pre van Hiele geometry

test results and to the post van Hiele test results. The sign test was used to analyze the pre and post van Hiele data. A chi square test of association was also applied to the van Hiele levels and tests. A .05 level of significance was used in each of these tests.

The results indicate that the group of students using the computer program, Geometric Supposer, performed significantly better on the proof test than the group of students who did not use the computers. The pre van Hiele geometry test results indicate that more than 50% of students entering the grade 10 geometry course are at a 0 or 1 level. This level is too low to begin the study of geometric proof writing. The post van Hiele geometry test results indicate that, after a semester of geometry, students do move up in the van Hiele levels, with or without the use of computer programs like the Geometric Supposer. The results from the attitude test indicate that there was no difference between the two groups of students. Both classes value the study of mathematics in general, and geometry in particular.

In summary, the computer, with appropriate software and teacher commitment, can contribute to reducing the difficulty generally experienced by students in mastering the writing of geometric proofs.

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CHAPTER 1

INTRODUCTION TO THE PROBLEM

It is the glory of geometry that from so few principles, fetched from without, it is able to accomplish so much. Newton

Despite Newton's admiration for this branch of mathematics, the role of geometry in the mathematics curriculum has been the subject of many controversial discussions and diverging opinions. "It is easy to find fault with the traditional course in geometry, but sound advice on how to remedy these difficulties is hard to come by." (Allendoerfer, 1969, p. 165) Allendoerfer also stated, "In geometry . . . there is not even agreement as to what the subject is about." (p. 165) In 1970 the United States Comprehensive School Mathematics Project (CSMP) sponsored a conference on the teaching of geometry and reported, "Of all the decisions one must make in a curriculum development project with respect to choice of content, usually the most controversial and least defensible is the decision about geometry." (Morris, 1986, p. 9)

Geometry and the van Hieles

The Soviet Union had not only identified the "geometry problem" much earlier but in the 1960's, revised the geometry section of their mathematics curriculum. This revision was based on a theory developed by a Dutch husband and wife team who were mathematics teachers--the van Hieles. Although not widely known by teachers in North America today, the work of the van Hieles plays a major part in this investigation.

The International Commission on Mathematical Instruction held an international seminar in Kuwait in 1986. Their discussions centered on the mathematical curriculum for the 1990's and again, geometry appeared as the controversial topic. "No particular mathematical area within the school curriculum arouses so much concern amongst mathematicians as does geometry, . . . "(Howson & Wilson, 1986, p. 58).

The literature contains credible reasons for geometry continuing to be a major topic in the mathematics curriculum. There is, however, some general debate as to whether geometry should be integrated throughout the mathematics curriculum or should be taught as a one year course, usually at the grade ten level. A more specific issue arises from the role of proofs and deductions.

Many students in high school geometry have difficulty with deduction and proof. "They don't understand the role or meaning of an axiomatic system. Despite our best efforts to teach them, even the most capable algebra students may struggle and get through geometry by sheer willpower and memorization but with little understanding of the logical system we have been developing all year." (Shaughnessy & Burger, 1985, p. 419) We should therefore not be surprised by the fact that many students tend to dislike geometry--in particular, writing proofs (Farrell, 1986; Hoffer, 1981; Senk, 1985; Usiskin, 1980).

The van Hieles also experienced frustrations while teaching geometry. They were familiar with the work of Piaget and from this, Pierre van Hiele developed his system of thought levels in geometry. To help students raise their thought levels, the van Hiele system

specified a sequence of phases that moved from very direct instruction to the students becoming independent from their teachers (Hoffer, 1983).

Mayberry (1981) summarized two consequences of these levels:

- a student cannot function adequately on a given level unless he has passed through and learned to think intuitively on each preceding level.

- If instruction, that is, the language of the instructor, problems in the textbook, or pedagogical techniques assume the student to be on one level while in fact the student is on a lower level, there will be serious communication problems between the instructor and the student because their geometric knowledge is organized differently (p. 6).

Further, van Hiele in his 1959 article, stated, "The bad results of the teaching of geometry must almost entirely be attributed to the disregard of the levels. The learning process in geometry, as we have seen, covers many levels, but appreciation of this has still so little penetrated into the teaching world that one even encounters teaching methods in which beginners are confronted with modes of reasoning based on symbols of the third (formal deduction) level." (p. 21)

Wirszup (1976) also stated, "The majority of our high school students are at the <u>first</u> level of development in geometry, while the course they take demands the <u>fourth</u> level of thought. It is no wonder that high school graduates have hardly any knowledge of geometry, and that this irreparable deficiency haunts them continually later on." (p. 96)

<u>Geometry and the Microcomputer</u>

Geometry is clearly a visual subject, yet much of the student's time is spent writing. If students are to have an opportunity to

think intuitively they need a faster, less cumbersome medium in which to experience it. The microcomputer has these characteristics but has only been used in a limited way in North American geometry classes.

Can the microcomputer be used to reduce the difficulties that students have with proofs? This will be possible only if appropriate software is used. The researcher initially considered using the programming language LOGO to teach geometric concepts. This notion was discarded when the Geometric Supposer software was brought to the researcher's attention. The advantages of the Geometric Supposer over LOGO are listed in Table 1.1.

Table 1.1

Comparison between LOGO and the Geometric Supposer

LOGO	GEOMETRIC SUPPOSER
1. Need to learn language before studying geometric concepts.	Easy-to-use menu driven programs.
2. Need to develop experiments/ exercises that demonstrate geometric concepts.	Built on geometric shapes and relationships.
3. Need to develop all support materials.	Some teaching and student learning material available.

This study investigated the effects on a grade ten geometry class of including the use of microcomputers and the Geometric Supposer software in the course. Further, the class was compared to a second class which was taught the same course content in the traditional, proof writing manner.

Statement of the Problem

Because of criticism of high school geometry, Gearhart (1975) conducted a nationwide survey of secondary school mathematics teachers. "Proof was regarded as an important topic by nearly all teachers." (p. 490) However, the teachers "also indicated that many students do in fact have trouble with the material and do not like it" (p. 490). More recently, Suydam's (1985) report on the NCTM (1981) survey indicated that mathematics teachers preferred the geometry curriculum be kept intact with the focus on Euclidean geometry.

When students were asked what they disliked most about their geometry course, "There is only one strong answer: proof." (Usiskin, 1980, p. 419) Despite the emphasis on proofs in tenth grade geometry, students seem to emerge from this course with only a limited ability to generate proofs and not much understanding about the nature of proof.

Senk (1985) reported the results of the Cognitive Development and Achievement in Secondary School Geometry Project--approximately 30% of the students in geometry courses in which proof is taught reach a 75% mastery level in proof writing, and about 25% of the students have no competence in writing proofs.

Thus, there seems to be a discrepancy between the intentions of the geometry curriculum in high school and what students actually learn.

Craine (1985), in an attempt to improve the geometry course, made several assumptions. His first assumption, "Students entering this course have not necessarily had the informal geometric experiences

that should ideally occur in the middle grades." (p. 120) His second assumption, "Many students entering this course are below the third van Hiele level, the minimum level at which one can fully appreciate definitions and relations of class inclusion. Students who have not reached this level cannot be expected to succeed in writing proofs." (p. 120)

"According to the van Hieles, the learner, assisted by appropriate instruction, passes through five levels of thinking. The learner cannot achieve one level without having passed through the previous levels." (Fuys, 1985, p. 449)

The following is a brief description of the van Hiele levels:

Level 0 (recognition) - Students recognize figures by appearance alone. They can say triangle, square, etc., but cannot identify properties of the figures.

Level 1 (analysis) - Students reason about geometric properties of figures, ie. diagonals of a rectangle are equal, but do not interrelate the figures or properties.

Level 2 (abstraction or informal deduction) - Students relate figures and their properties, ie. every square is a rectangle, but do not understand the role and significance of deduction.

Level 3 (formal deduction) - Students reason formally, can construct proofs, understand the role of axioms, postulates, theorems, and definitions.

Level 4 (rigor) - Students can compare systems based on different axioms and can study various geometries in the

absence of concrete models. "Few students are exposed to, or reach this level." (Crowley, 1987, p. 2)

Other studies have found that students were ill prepared (had low van Hiele levels of geometric thinking) for their geometry 10 course. Were the students in this study in the same position?

"The van Hiele model reveals an alarming lack of harmony in the teaching and learning of mathematics." (Hoffer, 1983, p. 218)

In an attempt to "bridge the gap" this investigation endeavoured to answer the following questions:

1) Will the students who use the Geometric Supposer software be better able to write formal proofs than students who are taught by more traditional methods?
2) What changes in the students' van Hiele levels take place after a semester of geometry?
3) Will the students who receive the treatment have a more positive attitude towards geometry than the students in the traditional group?

Significance of the Study

The primary significance of this study was to integrate the Geometric Supposer software into the geometry curriculum to provide a bridge between the spatial-visual aspects of geometry and the deductive aspects in order to increase students' ability to write proofs. Second, the change in students' van Hiele levels between the beginning of the semester and the end was measured. Third, this study presented information regarding the attitudes of students from two groups towards geometry at the end of the course. One group of

students used the computer software throughout the course while the other group did not.

The results of this study provide some helpful suggestions for the teaching of geometric proofs to grade ten students.

CHAPTER 2

REVIEW OF THE LITERATURE

This chapter contains a review of the literature describing high school geometry and its apparent shortcomings, the van Hiele theories, and the Geometric Supposer computer programs.

Literature Relating to High School Geometry

Geometry in the high school has been a very controversial topic with opinions ranging from Dieudonne's famous slogan, "Down with Euclid." (Grünbaum, 1981, p. 235) to "Teach them a rigorous Euclidean geometry." The Euclidean camp has dominated despite numerous suggestions for changes in the high school geometry course.

Proofs are central

Proof is the cornerstone for teaching Euclidean geometry as, "It enables us to test the implication of ideas thus establishing the relationship of the ideas and leading to the discovery of new knowledge." (Smith & Henderson, 1959, p. 178) The purpose then of teaching proof is to move students from a subjective point of view to an objective one.

What is this term, proof? According to Smith and Henderson:

> *Proof* is a common word in our vocabularies with various shades of meaning in its daily usage, but it has a very special and precise meaning in mathematics. As a mature concept, proof in mathematics is a sequence of related statements directed toward establishing the validity of a conclusion (p. 111).

In high school geometry each statement or step in the proof must be justified. The justifications can be drawn either from given information, definitions, postulates, or previously proven theorems. Most often the geometric proof is written in two-column form with statements on the left and a reason for each statement on the right.

In order to put the subject of proof in perspective it is necessary to look at the historical development. The synthetic methods of Euclid existed from approximately 325 B. C. until the seventeenth century when Descartes first used numbers in the study of geometry. This new approach became known as analytic geometry. Some flaws were noted in Euclid's axioms but were corrected. As these corrections were beyond the comprehension of the average secondary school student, the geometric postulates were modified to make them more understandable. Thus, this modified form of Euclidean geometry became the basis of the current geometry course.

Other non-Euclidean geometries have been developed, some of which are "affine, projective, hyperbolic, elliptic, combinatorial, absolute, analytic, differential, algebraic, Minkowskian, integral, transformation, vector, linear, topological, conformal, relativistic, optical, and so forth" (Fehr, 1972, p. 152). Despite these additions, the high school geometry course is still mainly Euclidean. "The treatment of geometry in the high schools today is remarkably similar to the Euclidean model set down more than twenty-three centuries ago." (Eccles, 1972, p. 103) Brumfiel (1973) concurred with this statement but he gave the reason being that "no one has found better proofs" (p. 95).

Many students enrolled in geometry courses have had difficulty with the concept of proof and have ended up disliking geometry as a whole. In the eyes of the student "geometry" has become synonymous with "proof" which is understandable when students spend so much time in grade ten geometry doing two-column proofs, many of which are self-evident. Because of the preoccupation with rigor, students are forced to write down every step along with an associated reason. The average student gets lost in the myriad of symbols and steps. An example of such a textbook proof appears in Table 2.1 (Usiskin, 1980, p. 421).

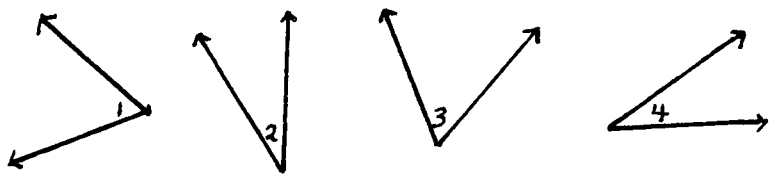
Table 2.1

The Complementary Angle Theorem

Complements of congruent angles are congruent.

GIVEN: $\angle 1$ is a complement of $\angle 2$;
$\angle 3$ is a complement of $\angle 4$;
$\angle 2 \cong \angle 4$

PROVE: $\angle 1 \cong \angle 3$



PROOF:

Statements	Reasons
1. $\angle 1$ is a complement of $\angle 2$; $\angle 3$ is a complement of $\angle 4$.	1. Given
2. $m\angle 1 + m\angle 2 = 90$; $m\angle 3 + m\angle 4 = 90$	2. Def. of comp. angles [1]
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Substitution princ. [2]
4. $\angle 2 \cong \angle 4$	4. Given
5. $m\angle 2 = m\angle 4$	5. Def. of $\cong$ angles [4]
6. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	6. Substitution princ. [3,5]
7. $m\angle 1 = m\angle 3$	7. Add. prop. of equality [6]
8. $\angle 1 \cong \angle 3$	8. Def. of $\cong$ angles [7]

Students in a secondary school geometry class have to be able "to hypothesize, reason deductively, understand the role of mathematical models, and understand the difference between defining and deducing" (Farrell, 1987, p. 239). These cognitive abilities are characteristic of Piaget's formal operational stage. However, the results from various tests measuring cognitive development indicate that a minimum of 30% of these students reason at the concrete operational level while another 30 - 40% of the students are transitional reasoners (Farrell, 1987).

Carpenter, Lindquist, Matthews, & Silver (1983) found in their report of the results from the third mathematics assessment of the National Assessment of Educational Progress for 13 and 17-year olds, that students are failing when mathematical reasoning and understanding are required. "The problem is particularly critical in high school geometry, where success depends on propositional thinking and deductive reasoning about geometric properties and relations." (Olive & Lankenau, 1986, p. 78)

Throughout geometry courses, learning to write proofs has been an important objective of the curriculum. However, proof writing has been perceived to be one of the most difficult topics for students to learn. Usiskin (1980) suggested that the amount of time spent on proofs be reduced and that many theorems of lesser importance be deleted. These suggestions have been ignored. "The concept of proof in mathematics will always be important whatever may be the nature of the curriculum." (Lovell, 1971, p. 66) Given that this prediction is true, the basic problem of how to increase student mastery of writing geometric proofs remains critical.

Proofs become less rigorous

The amount of detail in the proof illustrated in Table 2.1 tended to overwhelm the majority of students who then 'turned off'. Textbook authors are attempting to keep symbols and technical vocabulary to a minimum. A similar proof but from a more recent textbook (Jurgensen, Brown, & Jurgensen, 1985, p. 41) appears in Table 2.2.

Table 2.2

The Supplementary Angle Theorem (a)

It two angles are supplements of congruent angles (or of the same angle), then the two angles are congruent.

GIVEN: $\angle 1$ and $\angle 2$ are supplementary;
$\angle 3$ and $\angle 4$ are supplementary;
$\angle 2 \cong \angle 4$ (or $m\angle 2 = m\angle 4$)

PROVE: $\angle 1 \cong \angle 3$ (or $m\angle 1 = m\angle 3$)

PROOF:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary; $\angle 3$ and $\angle 4$ are supplementary.	1. Given
2. $m\angle 1 + m\angle 2 = 180$; $m\angle 3 + m\angle 4 = 180$	2. Def. of supp. angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Substitution prop.
4. $m\angle 2 = m\angle 4$	4. Given
5. $m\angle 1 = m\angle 3$	5. Subtraction prop. of =

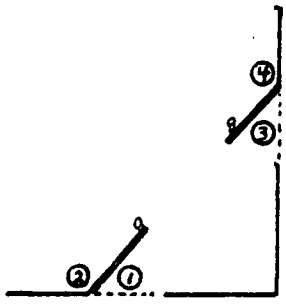
Table 2.3 contains an even more recent textbook (Kelly, Alexander, & Atkinson, 1987, p. 354) example. The examples in Tables 2.2 and 2.3 show proofs of exactly the same theorem.

Table 2.3

The Supplementary Angle Theorem (b)

If two angles are equal, their supplements are equal.

Suppose there are two doors in the room. Suppose also that each door is opened the same amount, that is $\angle 1 = \angle 3$. Then, we might predict that $\angle 2$ and $\angle 4$ are equal. We can use deductive reasoning to explain why $\angle 2 = \angle 4$.



Since $\angle 1$ and $\angle 2$ are supplementary:

$$\angle 1 + \angle 2 = 180^0$$

$$\angle 2 = 180^0 - \angle 1 \quad . . . [1]$$

Since $\angle 3$ and $\angle 4$ are supplementary:

$$\angle 3 + \angle 4 = 180^0$$

$$\angle 4 = 180^0 - \angle 3 \quad . . . [2]$$

Comparing [1] and [2], we see that the expressions on the right side are equal, since it is given that $\angle 1 = \angle 3$. Therefore, $\angle 2 = \angle 4$.

In spite of proofs becoming less rigorous, as can be seen from Tables 2.1, 2.2, and 2.3, students still have difficulty grasping the concept of deducing a chain of steps.

Euclidean geometry questioned

Various educators have questioned the value of traditional Euclidean based geometry. Fehr (1972) felt that Euclid's geometry played a very minor role in accomplishing the goals of geometric instruction. He advocated that geometry "be conceived of as a *study of spaces*" (p. 152) integrated into the curriculum and taught every year from grade seven to grade twelve.

In 1973 Brumfiel reported on a study he did in 1954 and repeated again in 1973. He was curious about students' understanding of the axiomatic structure after they left high school. "Students of 1954 who studied an old-fashioned hodgepodge geometry had no conception of geometric structure. Students of today who have studied a tight axiomatic treatment also have no conception of geometric structure." (p. 102) Is the emphasis on axioms in school geometry a waste of time?

Usiskin (1980) also noted that the reason given for studying Euclidean geometry was that it "provides an example of a mathematical system. It is the place where the student is asked to do what mathematicians presumably do, that is, prove theorems." (p. 419) But mathematicians do a fair amount of exploration prior to their writing of a proof. "In contrast, geometry students seldom explore and almost always are told what they should prove." (p. 420)

Hoffer (1981) criticized the high school geometry course for putting too great an emphasis on developing the skill of writing

proofs. "When this occurs, precious class time is taken from providing students with experiences in other, possibly more practical, skills of a geometric nature." (p. 14)

Grünbaum (1981) felt that there was only a pretense to teach the "classical" Euclidean geometry when, in fact, the geometry being taught was "rather misleading" (p. 235).

According to Driscoll (1982) " . . . proof has been touted as a means to discipline the mind to think in an orderly fashion, as a vehicle for improving logical thinking, and as a stimulus toward the kind of responsible, critical and reflective thinking that should be the mainstay of democratic life." (p. 155) But does proof really promote deductive thinking?

Senk (1985) questioned the value of teaching the traditional geometry course. Is it preparing high school students to meet the challenges of the future?

In an attempt to use geometry as the vehicle to illustrate mathematics as an axiomatic system, students come to the conclusion that axioms, theorems, and proofs solely belong to this area. Geometry textbooks contain lists of postulates and theorems. New ones are even being created, for example, Pasch's axiom. Niven (1987) posed the questions, "Are we not in danger that the students will see geometry as just so much nitpicking? Why should the first course in geometry carry the special burden of illustrating and exemplifying the foundations of mathematics?" (p. 39).

Teachers recommend that Euclid stays

In 1973 Gearhart surveyed a random sample of 999 secondary school mathematics teachers from across the United States to find out

their thoughts on the geometry course. Over half of the teachers disagreed that the course should be more informal and less rigorous; 76% agreed that learning to write proofs was important for high school students; and over half agreed that the course should be based on Euclid's development as found in standard textbooks. Thus, this survey indicated support for the status quo in the geometry course.

Similarly, in 1981 the National Council of Teachers of Mathematics also conducted a survey. The results indicated that geometry should be taught for the following reasons:

- to develop logical thinking abilities;
- to develop spatial intuitions about the real world;
- to impart the knowledge needed to study more mathematics; and
- to teach the reading and interpretation of mathematical arguments (Suydam, 1985, p. 481).

Some explanations for the difficulty with proofs and suggestions for overcoming them

Lester (1975) was convinced of the importance for students to develop an ability to write proofs correctly. If students were to be properly prepared for this task, Lester felt that they should be introduced to various aspects of proof as early as possible. In an attempt to determine the appropriate time for students to be introduced to proofs, Lester (1975) conducted his study. In his research of the literature he found inconsistent evidence regarding developing the ability to perform certain formal operations. On the one hand, theories seem to support the suggestion that there is no relationship between age and logical reasoning and on the other hand,

that logical reasoning improves with age. For his study Lester selected four groups of subjects, 20 in each group. The groups consisted of students from grades 1-3, 4-6, 7-9, and 10-12. His subjects all interacted with a computer terminal in a game setting where they were asked to supply proofs of "theorems". The resulting data from Lester's research indicated that "certain aspects of mathematical proof can be understood by children nine years old or younger. Perhaps children may be able to deal with formal operations at an earlier age than proposed by Piaget." (p. 23)

First-year students at the University of Oregon are asked to list their favorite and least favorite high school subjects. Hoffer (1981) reported that "the subject that was almost universally disliked was geometry" (p. 11). He suggested that formal proofs should not be introduced early in the course as students may not have reached the formal operational level of development. He recommended spending a good portion of time "exploring geometric concepts informally, without requiring proofs. This enables students to study what they call 'fun things' while preparing for more formal aspects in the second half of the course." (p. 18)

Prior to 1980 research had been limited in this area, consequently little was known about the actual nature of the difficulties that students experienced in writing proofs. Thus, in 1981 the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project commenced to organize research in this area. The project was designed to address a variety of questions but Senk (1985) reported specifically on the question, "To what extent do secondary school geometry students in the United States write proofs

similar to the theorems or exercises in commonly used geometry texts?" (p. 448). A total of seventy-four geometry classes from eleven schools in five states were involved. A proof test consisting of six items was administered during the regular class period. The conclusions reached from the data were:

- about 70% of students can do simple proofs requiring only one deduction.
- achievement is considerably lower on proofs requiring auxiliary lines or more than one deduction.
- only 30% master proofs similar to the theorems and exercises in standard textbooks.

These data indicate a low level of achievement in writing proofs but perhaps the reason for this is lack of practice. Most mathematical skills that are acquired have been practised for a length of time at various grade levels. "In contrast, the typical high school mathematics program provides virtually no opportunity for students to practice writing proofs in any context outside the geometry course." (Senk, 1985, p. 454)

To overcome this weakness, Senk proposed that more effective ways must be identified for teaching proof. She specifically recommended:

- more attention be given to teaching students how to start a chain of deductive reasoning.
- greater emphasis be placed on the meaning of proof.
- the need to teach students how, why, and when they can transform a diagram in a proof (p. 455).

Brown (1982) noted that students entering geometry find the subject quite different from other mathematics courses which they have taken. "There are no elaborate arithmetic problems, no polynomials to factor, and few equations to solve. And most different of all is geometric proof, where the solution is not a neat number or algebraic expression that can be underlined and labeled 'answer' " (p. 442). At the same time the student is having to adjust to these differences; s/he is expected to "invent a chain of deductions" (p. 442) in order to arrive at some conclusion for which the student can see no purpose.

Brown suggested that students should be encouraged to experiment, guess, generalize, and deduce the various formulas and theorems themselves.

Criticism has been levelled at the elementary school for not teaching sufficient informal geometry to better prepare students for writing proofs. The CDASSG Project "confirms the need for systematic geometry instruction before high school if we desire greater geometry knowledge and proof-writing success among our students" (Usiskin, 1982, p. 89). However, the mathematics curriculum has not been changed and in 1987 Usiskin still bemoaned the fact that there was no geometry curriculum in the elementary school to prepare students entering high school for Euclidean geometry.

To combat some of the difficulties encountered by students in the high school geometry course, Prevost (1985) suggested that geometry in the junior high should be an integral part of mathematics rather than a single chapter in a whole year's study. He also championed the cause for a manipulative approach to geometry. He

criticized teachers for using too few devices that allow students to do geometry rather than merely watching it.

Craine (1985) admitted to a preference of a unified approach to secondary mathematics where geometry, algebra, and analysis would be integrated throughout the entire curriculum. However, failing this integration, he proposed to reorganize the geometry course. He recommended using informal methods to introduce the basic concepts of geometry followed by an inductive discovery of the properties. Deductive reasoning would gradually be introduced. Similar to Hoffer, Craine saw the last part of the course being devoted to writing proofs.

The logical arguments that form the basis of Euclidean geometry cause students difficulty. Students are unable to organize their thoughts to construct a deductive sequence of steps. "To deal directly and explicitly with the organization of students' thought patterns and their construction of logical arguments" (p. 47), Dreyfus and Hadas (1987) developed a set of methods and new curriculum materials. To test the effectiveness of this course, twenty-two experimental classes from fifteen different schools and ten control classes from other schools were selected. The results indicated that the students using this new geometry course increased their ability to reason logically within a geometric context somewhat more than the students using the traditional approach.

Because of a history of poor achievement, only about one-half of the secondary population enrolls in the geometry course and of these only about one-third really understand it (Usiskin, 1987).

Consequently, approximately one-sixth of high school students

are proficient in writing proofs. Various suggestions have been given about how to improve the situation, including the factor of readiness. "In fact, research has suggested that many students at the age when formal geometry is usually studied are incapable of the formal and abstract thinking required. As a result, they stumble through the yearlong course by mimicking the teacher's two-column proofs, and they emerge at the end with a few facts, a vague sense of the difference between axioms, theorems, and definitions." (Fey, 1984, p. 32) This memorization has prevented students from achieving the major objectives of the geometry course: to develop the ability to reason deductively and to appreciate the role of deduction in mathematics.

In summary, the literature contains two major criticisms of the current way that geometry is taught and organized: 1) students are not "ready" for geometry and, 2) the method of instruction with its heavy dependency on writing proofs does not allow students to discover geometric relationships upon which to base their deductive reasoning.

In the present study, the researcher depended upon the van Hiele theories to assess readiness and progress in geometric thinking, and the Geometric Supposer software to offer students some discovery experiences in geometry.

Literature Relating to the van Hiele Theories

Background

Two Dutch mathematics teachers, Pierre van Hiele and his late wife, Dina van Hiele-Geldof, became troubled about their students'

difficulties in learning geometry. From their concerns they developed a theory in 1957 involving levels of thinking in geometry. They surmised that these levels could be used to explain why students have difficulties in geometry. "They believed that high school geometry involves thinking at a relatively 'high' level and that many students have not had sufficient experiences in thinking at prerequisite 'lower' levels." (Fuys, 1985, p. 448)

Between 1960 and 1964 the Soviet Academy of Pedagogical Sciences verified the validity of the theory of the van Hieles and revised their own mathematics curriculum accordingly. In 1973 Professor Hans Freudenthal wrote about Dina van Hiele-Geldof's experiments. Wirszup, an American, became acquainted with the work of the van Hieles and the way the Soviets had applied it. Wirszup was the first to introduce the van Hiele theory to the United States in 1974 when he presented a paper at the Annual NCTM (National Council of Teachers of Mathematics) Meeting. Despite this early introduction, it is only recently that the theory has gained more popularity, possibly because English translations of their original work are now appearing.

Description

The van Hiele model actually consists of three components: the thought levels, the properties of the levels, and the phases of learning. Table 2.4 illustrates how these components are interrelated.

The thought levels

Five levels of geometric thinking were proposed by the van Hieles. Each level describes certain characteristics of the thinking process. "These levels are inherent in the development of the

thought processes. The development which leads to a higher geometric level proceeds basically under the influence of learning and therefore depends on the content and methods of instruction. However, no method not even a perfect one, allows the skipping of levels." (Wirszup, 1976, p. 79) The van Hieles began with the basic level, level 0, and ended with level 4. (Different numbering systems may be found in the literature.) "According to the van Hieles, two major factors that determine a student's level are ability and prior geometry experiences." (Fuys, Geddes, & Tischler, 1988, p. 12)

Table 2.4

The van Hiele Model

PROPERTIES OF THE LEVELS	THOUGHT LEVELS		PHASES OF LEARNING*
Sequential	0	- Recognition	Inquiry
Advancement	1	- Analysis	Directed Orientation
Adjacency	2	- Informal Deduction	Explanation
Linguistics	3	- Formal Deduction	Free Orientation
Mismatch	4	- Rigor	Integration

*Note: van Hiele (1984) suggested that students move through the phases each time they advance a level.

The following is a description of each of the thought levels:

Level 0 (recognition). At this level students perceive the geometric figure in its totality. They "do not see the parts of the figure, nor do they perceive the relationship among components of the figure and among the figures themselves." (Wirszup, 1976, p. 77) For example, squares and rectangles would be recognized as different kinds of figures.

Level 1 (analysis). On this level students "become aware of the properties of geometric figures by a variety of activities such as observation, measuring, cutting, and folding" (Mayberry, 1981, p. 4). An example would be that the diagonals of a rectangle are equal. "Relationships between properties, however, cannot yet be explained by students at this level, interrelationships between figures are still not seen, and definitions are not yet understood." (Crowley, 1987, p. 2)

Level 2 (abstraction or informal deduction). Students at this level can "establish relations among the properties of a figure and among the figures themselves. At this level there occurs a logical ordering of the properties of a figure and of classes of figures. The pupil is now able to discern the possibility of one property following from another, and the role of definition is clarified." (Wirszup, 1976, p. 78) Proof is not understood at this level.

Level 3 (formal deduction). "Students develop sequences of statements to deduce one statement from another, such as showing how the parallel postulate implies that the angle sum of a triangle is

equal to 180^0. However, they do not recognize the need for rigor nor do they understand relationships between other deductive systems" (Hoffer, 1983, p. 207).

Level 4 (rigor). "Students grasp the significance of deduction as a means of constructing and developing all geometric theory." (Wirszup, 1976, p. 78)

The properties of the levels

An important aspect of the literature related to the van Hiele work is the properties of the system of levels. These properties not only describe the relationships between levels but also how a student is affected by his/her placement and movement in the levels. Teachers can use this information to actually plan lessons.

Property 1 (sequential). In order to understand geometry, the student must progress through the levels in order. "A student cannot be at van Hiele level n without having gone through level n-1." (Usiskin, 1982, p. 5)

Property 2 (advancement). The content and the instructional methods can affect the progress of a student from level to level. "No method of instruction allows a student to skip a level, some methods enhance progress, whereas others retard or even prevent movement between levels." (Crowley, 1987, p. 4)

Property 3 (adjacency). At each level what appears as extrinsic had become intrinsic in the preceding level. In other words, a student at the recognition level perceives figures as is regardless of their properties.

Property 4 (linguistics). "Each level has its own language, its own set of symbols and its own network of relations uniting these symbols." (Wirszup, 1976, p. 82) For example, a student at level 1 does not realize that a figure can have more than one name--a rectangle is a parallelogram.

Property 5 (mismatch). "If the student is at one level and instruction is at a different level, the desired learning and progress may not occur." (Crowley, 1987, p. 4)

The phases of learning

The van Hieles (1984) stated that the method and organization of instruction influenced the progress (or lack of) of a student from level to level. They have identified a five phase cycle which they consider as a necessary sequence for students as they progress through the levels. Age or maturation are viewed as minor factors. The five phases of learning are described below:

Phase 1 (inquiry). Here the teacher and students discuss the objects of study for this level. The teacher discovers what the students already know about the topic and the students become acquainted with the topic to be studied.

Phase 2 (directed orientation). The material, consisting of short tasks where manipulation is prominent, is carefully sequenced by the teacher. The teacher is looking for specific responses from the students.

Phase 3 (explanation). At this phase, the students express the results of their manipulations in words. The figures take on geometric properties and the role of the teacher is to introduce the correct terminology.

Phase 4 (free orientation). "The student learns, by doing more complex tasks, to find his/her own way in the network of relations (e.g. knowing properties of one kind of shape, investigates these properties for a new shape, such as kites)." (Fuys et al. 1988, p. 7)

Phase 5 (integration). The students now take their newly acquired knowledge and form an overview. "The objects and relations are unified and internalized into a new domain of thought." (Hoffer, 1983, p. 208)

When the fifth phase has been completed, students have reached a new level of thought. "The new domain of thinking replaces the old, and students are ready to repeat the phases of learning at the next level." (Crowley, 1987 p. 6)

At first glance it may appear that the van Hiele model simply states the obvious--students need to learn in an organized progression. However, the U.S.S.R. did make major changes in their mathematics curriculum based on this work.

Research based on the van Hiele theories

Despite its wide acceptance by the U.S.S.R. in the 1960's, in North America only a limited amount of research on the van Hiele model has been done. It was Wirszup's speech in 1974 followed by his article in 1976 that prompted the American educators to do some investigations of the van Hiele levels. Three major projects have received U. S. federal funding to carry out research on the model. A description of each one along with their results follows.

The Chicago project: van Hiele levels and achievement in secondary school geometry

The purpose of this three year project (1979-82) was to address various questions about student achievement in grade ten geometry and how this relates to the van Hiele theory. Approximately 2700 students in high school geometry courses from five different states were included in the study. (Senk, 1985; Usiskin, 1982) Two tests were administered near the beginning of the school year (September, 1980). One of these tests consisted of multiple-choice questions dealing with prerequisite geometry knowledge. The second test, also multiple-choice, was intended to indicate the van Hiele level of each student. Near the end of the school year (May, 1981) these students sat the van Hiele tests again. They also took a standardized multiple-choice test that measured geometry achievement plus a third test, dealing with their proof-writing ability.

The results show that:

- a van Hiele level can be assigned to most students.
- these levels are good indicators of student performance both in proof-writing and standard geometry content.
- application of the van Hiele theory "explains why many students have trouble learning and performing in the geometry classroom" (Usiskin, 1982, p. 89). The van Hiele levels were low for many students entering grade ten geometry.
- over half of the students who enroll in geometry courses which emphasize proof, experience little or no success in

writing proofs.

- one-third of the students rose one level, one-third rose two or more levels, and one-third stayed at the same level.
- "The geometry course is not working for large numbers of students. At the end of their year of study of geometry many students do not possess even trivial information regarding geometry terminology and measurement." (p. 89)

<u>The Brooklyn project: the van Hiele model of thinking in geometry among adolescents</u>

This three year research project (1980-83) focused on four objectives (Fuys, 1985; Fuys et al. 1988):

- to translate the van Hiele writings into English, then develop and implement working modules based on the level and phases of the van Hieles.
- to determine whether the van Hiele model describes how sixth and ninth graders learn geometry.
- to determine if teachers of these grades can be trained to identify the van Hiele levels of students and of geometry curriculum materials.
- to analyze levels of thinking of the geometric content of several major textbook series.

Three modules were developed based on the experiments in Dina van Hiele-Geldof's thesis. These modules were used in the clinical interviews involving 16 sixth graders and 16 ninth graders and were intended to facilitate the students' movement through the lower levels. The students were videotaped as they individually worked

through the modules in six to eight 45-minute sessions. This one-on-one contact provided the researchers with information on changes in a student's thinking within a level or to a higher level.

The results of this study first verified the existence of each of the properties of the van Hiele model. Next, their results indicated that a range in levels of thinking existed among the sixth and ninth graders (level 0 to level 2). "Findings in this study show that geometry was a neglected part of the school mathematics experiences of many students, and what was taught was often taught rotely or required minimal student explanation." (Fuys et al. 1988, p. 188)

The researchers found that students in these grades do have the potential for level 1 and level 2 thinking. However, various factors were found which limited a student's progress within a level or to a higher level. These factors included:

- lack of prerequisite knowledge
- poor vocabulary/lack of precision of language
- unresponsiveness to directives and given signals
- lack of realization of what was expected of them
- lack of experience in reasoning/explaining
- insufficient time to assimilate new concepts and experiences
- rote learning attitude
- not reflective about their own thinking (p. 139).

This study also found that "preservice and inservice teachers can learn to identify van Hiele levels of thinking in student responses and in text materials" (p. 154) and that such training should be included in teacher preparation programs.

In their analysis of current K-8 mathematics textbooks, the investigators found them to be written at level 0. "Students will presumably encounter difficulty with a secondary school geometry course at level 2 if they can successfully complete grade 8 with level 0 thinking." (p. 169)

The Oregon project: Assessing children's Intellectual Growth in Geometry

The project was sponsored from September, 1979 to February, 1982. "The purpose of the study was to investigate the extent to which van Hiele levels serve as a model to access student understanding of geometry." (Hoffer, 1983, p. 212) Forty-five students from grades K-12 and college mathematics majors were selected from three states. The candidates were audio-taped during two 45-minute interviews involving tasks with quadrilaterals and triangles. The tasks were designed to reflect the van Hiele levels and to combine some ideas from Dina van Hiele-Geldof's research with her students.

The findings from this project were:

- the hierarchical nature of the van Hiele levels was confirmed.
- the difficulty of assigning some students a van Hiele level. These students may be in transition from one level to the next.
- the movement from one level to the next is not discrete. "Students may move back and forth between levels quite a few times while they are in transition from one level to the next." (Burger & Shaughnessy, 1986, p. 45)

- that the use of formal deduction (level 3 thinking) among secondary and post-secondary students was nearly absent.
- the teachers and students, while trying to communicate, may be at different levels.
- that students may be at a geometric level quite different from what their teacher assumes they are.

The three projects described above helped to raise the awareness level of the van Hiele theory. This in turn began to answer some of the questions about poor performance in writing proofs. The interest stirred by these projects has resulted in several articles and dissertations.

Mayberry's (1981) dissertation centered on preparing tasks which would be used to place preservice elementary teachers on the van Hiele scale. The Chicago project had developed a 25 question multiple-choice test for this purpose. She prepared 62 tasks which were used while interviewing 19 preservice teachers. Mayberry's results showed that "the general van Hiele level of the preservice elementary teachers in the study was rather low" (Mayberry, 1983, p. 102). Using Mayberry's tasks, Denis (1987) assessed Puerto Rican high school students who had already taken the Euclidean geometry course. He found that nearly three-quarters of the high school students were not at a level sufficient to deal with a traditional Euclidean geometry course.

Mayberry also tested the hierarchical nature of the van Hiele levels. Her results verified that a student at level n could answer all questions at and below level n but none of the questions above that level. Denis also concurred with the hierarchical structure.

Senk's (1983) dissertation used the same data as the CDASSG project. One of the issues that she addressed was readiness--were students prepared for the proof writing course? She found that the higher the student's van Hiele level was at the beginning of the geometry course, the greater the prospect for success in writing proofs. However, a high van Hiele level does not guarantee success in writing proofs. "Instruction plays a large part in determining which of the students with the basic prerequisite knowledge will eventually be successful on a given topic. For this reason, teachers, curriculum developers, and researchers need to share materials and methods found to be effective in teaching proofs." (Senk, 1985, p. 455)

Following on this theme of instruction, Prevost (1985) wrote appealing to teachers to integrate their geometry curriculum into the van Hiele model. Also, in keeping with the theory, he urged teachers in junior high to develop the geometric ideas over time rather than in a concentrated unit.

To provide a more effective learning experience for his students in grade ten geometry, Craine (1985) developed an alternative course based on the van Hiele model. He used an informal approach to introduce the basic concepts of geometry gradually building in the appropriate vocabulary. Proofs were developed near the end of the second semester.

In response to the charge that students do not have the necessary prerequisite experiences to succeed in writing proofs, Scally (1987) proposed a LOGO learning environment as a means to provide these experiences at the grade nine level. Students' van Hiele levels were

obtained by using interview items and tasks based on those developed by Burger and Shaughnessy. A group of ninth grade LOGO students and a group of ninth grade non-LOGO students were interviewed individually at the beginning and end of each semester. "The vast majority of student responses on both pre- and post-interviews were at the first and second van Hiele levels." (Scally, 1987, p. 51) Overall, the LOGO students made more gains in performing the various tasks at the end of the semester. Yet to be tested is whether the LOGO experience will in fact enhance the students' thought processes in grade ten geometry.

Similarly, Battista and Clements (1988) recommended the introduction of LOGO into the junior and senior high school geometry classes as they believed "that the Logo environments can be used to help students progress within this hierarchy" (p. 166). They cautioned teachers not to "expect that merely working with Logo automatically moves students into high levels of geometric thought" (p. 167). There needs to be correlations between LOGO activities and curriculum content. They summarized by stating that, "It is imperative, therefore, that we help students attain high levels of geometric thought *before* they begin a proof-oriented study of geometry." (p. 166)

A source of support for teachers to create a "discovery" atmosphere is the microcomputer. "In particular, the microcomputer could prove to be the best bridge yet between the spatial-visual aspects of geometry and the logico-deductive aspects" (Driscoll, 1982, p. 149). The computer language, LOGO, has been used by others as a vehicle to prepare students for geometry and as an instructional

aid while teaching the subject. Rather than LOGO, the investigator used the Geometric Supposer program for this purpose in the present study.

Literature Relating to the Geometric Supposer Software

The Geometric Supposer is a series of educational software programs, published by Sunburst Communications in 1985, which allow the user to carry out many different geometric constructions and measurements. A more detailed description from the manual, The Geometric Supposer: Triangles, follows:

> The GEOMETRIC SUPPOSER is a microcomputer program that allows the user to carry out with ease constructions that are possible using straightedge and compass. These include the construction of triangles as well as the drawing of segments, medians, altitudes, parallels, perpendiculars, perpendicular bisectors, angle bisectors, and inscribed and circumscribed circles. In addition, the user can measure lengths, angles, areas and distances as well as arithmetic combinations of these measures, such as the sum of two angles, the product of two lengths, or the ratio of two areas (p. 2).

"Part of the rationale behind the SUPPOSER was to provide a tool that could help students understand that a picture is a special case and that examining one picture is part of a larger process that includes viewing many special cases and not one static example." (Yerushalmy & Chazan, 1987, p. 58) One of the problems in teaching proof writing is that students view the diagrams as fixed, immobile

objects. "The Supposers provide an exploratory environment where students can experience and develop an intuitive understanding of geometric concepts." (Mathis, 1986, p. 45)

The researcher's survey has found very few articles and only one study involving the Geometric Supposer software. Reference was first made to this software in Aieta's (1985) article. He referred to the Geometric Supposer as being a "powerful and accessible" (p. 473) package that teachers could consider as a new approach to geometry.

A review of these programs appeared in The Computing Teacher in June, 1986. "The software encourages the higher level thinking skills involved in formulating and testing hypotheses." (p. 45)

The use of the Geometric Supposer was paralleled to that of a science class where the students collect data, conjecture, and generalize. According to Yerushalmy and Houde (1986), using the Geometric Supposer "encourages students to behave like geometers because it offers a wealth of visual and numerical data and because conjectures about relationships observed within the data can be quickly tested" (p. 418). For any conjecture that the student makes, this software allows the experiment to be repeated on similar figures very quickly. Students would not have the time nor the inclination to manually construct counter examples. "Our experience demonstrates that students brought a high degree of enthusiasm to their work and demonstrated an ability to create geometry that we never thought possible." (p. 422)

Two of the Geometric Supposer programs were chosen as being in the top six for 1987 by Classroom Computer Learning magazine. "This program is truly a discovery tool that helps the user become an

active participant in the quest for mathematical knowledge." (p. 20)

In the summer of 1986 fifty expert high school geometry teachers were brought together in New Jersey for the purpose of looking at new materials and methods related to the field of geometry. Various materials were developed for distribution. One of the topics dealt with the Geometric Supposer. A series of investigations on triangles and quadrilaterals, based on Polya's model, were produced for teacher and student use. The authors noted, "Do not assume that using the Geometric Supposer will allow you to cover the course material any more quickly. This cannot be guaranteed. It will, however, allow you to teach a much richer course in which students glean a better understanding of what mathematics is all about." (Birt & Koss, 1986, p. 48)

Yerushalmy conducted a yearlong research project in 1984-85 on inductive reasoning in geometry and the use of the Geometric Supposer. Three geometry classes (83 students) at different sites used this computer software. At each site a comparison class was taught mainly from the textbook using the traditional approach. The goal of this project was to provide students with "an opportunity to experiment with geometric shapes and elements, to move from the particular to the general, and to make conjectures before grappling with proofs. This approach to geometry is absent from the 'formal' secondary geometry curriculum." (Yerushalmy, Chazan, & Gordon, 1987, p. 6) The instructional approach used is referred to as guided inquiry. This approach emphasizes a combination of laboratory work and class discussion.

From their comments, the teachers involved were not positive

about the Geometric Supposer. They expressed misgivings, disappointment in student progress, and concern about the time-consuming nature of laboratory work and follow-up. They did note some improvement in students' ability to organize data and felt that the Geometric Supposer had potential as a diagnostic tool. They also felt "that these students did get more out of their Geometry class than they would have done in a traditional class" (p. 40) and that most students had achieved an understanding of the need for a proof.

The students, on the other hand, were generally positive about the computer experience. They indicated that the Geometric Supposer was easy to use, added to their understanding, and provided enjoyment when they were successful. The negative aspect for the students was making conjectures. "Knowing what to conjecture about, discerning patterns and relationships, and generating conjectures were all hard work." (p. 42)

Yerushalmy et al. (1987) concluded that students from both groups (Geometric Supposer and comparison) learned equal amounts of geometry. However, the experimental group "significantly outperformed the comparison group in their ability to develop generalizations, and they were equal to and/or somewhat better than the comparison group in their ability to devise informal arguments and traditional formal proofs." (p. 68)

In summary, the literature indicates that geometry and the proof writing activity associated with it will continue to be viewed as an important and critical part of the high school mathematics curriculum. Despite the well documented fact that students dislike

geometry and that only a minority gain the kind of understanding their instructors hope to instill, writing proofs is considered to be a necessary part of their education.

The van Hiele model appears to have potential as a way of understanding the problems with proof writing and of designing solutions for these problems. The Geometric Supposer, while less well documented, has been identified as a specific instrument for allowing a discovery method to make inroads into the traditional proof writing method used in teaching geometry.

CHAPTER 3

PROCEDURES

The procedures used to investigate students' ability to write proofs using the computer program, Geometric Supposer, are reviewed in this chapter. A description of the subjects, the steps taken in the study, and the data collection instruments is given.

The Subjects

The community in which this study was carried out is a relatively isolated, northern Canadian town of approximately 4000 residents. The nearest major center is 240 kilometres (150 miles) to the south. However, as this link is by a well-paved highway, isolation is not considered to be a critical factor in the study.

The community is a service center for a large portion of the northern part of the province. Three small airlines are based there as well as a hospital, a community college, and some government administrative offices. The community services the tourism industry and more recently, considerable mining exploration. On the other hand, some residents, particularly Treaty Indians, still earn their living by trapping and fishing. Thus, despite its small size, the community has a wide range of socioeconomic levels.

The educational system in the community consists of three schools: a K-5 elementary school, a K-8 Treaty school, and a grade 6-12 school. The latter has a population of 500 students and was the site of the study. The population of this school reflects the social

makeup of the community which is roughly 55% Native persons and 45% Europeans.

As an overview, the mathematics curriculum in this province is prescribed until grade nine. In grade nine, generally, students have a choice of regular mathematics or general mathematics. Most students opt for the regular mathematics. To graduate from high school, students must have a mathematics credit at the grade ten level. Students can obtain this credit by taking one or more of algebra, geometry, mathematics, or general mathematics. Similarly, in grade eleven the same choices are available. In grade twelve the mathematics courses offered are algebra, geometry, and mathematics.

In the school where this study took place, the students in grade nine had to choose between algebra or general mathematics. Geometry was not an option. Algebra, geometry, and general mathematics were offered in grade ten. In grade eleven an algebra course and a geometry course were included in the timetable choices. This was also the case for grade twelve. Thus, after grade ten, a student could take from zero to four senior mathematics classes.

The subjects in this study had opted to take the grade ten geometry course which was scheduled in the second semester of the school year 1987-88 from February until June. Of the 62 grade ten students in the school, 41 (66%) chose to take the geometry course. Three students from grade 11 also elected to take geometry. The 44 students were divided into two classes by the school administration on an ad hoc basis with the researcher having no input. The class assigned to the researcher used the Geometric Supposer software throughout the course and will be referred to as the computer group

in this study. In the computer group there were 12 males and 10 females, 9 students were of native ancestry. The second class did not use the computer and will be referred to as the traditional group. The traditional group consisted of 13 males and 9 females, 8 students were of native ancestry. The ages of all the students ranged from 15 to 18 with the majority being either 15 or 16 years old.

As described above, the students in this study had had no geometry exposure since grade eight when they studied it as one of the chapters in their textbook.

During the semester one student from the computer group withdrew from school and one student was added, thus this group remained at 22 students. In the traditional group three students withdrew from school and two others discontinued the geometry course. This left 17 students in the traditional group.

Permission was obtained from the parents for the participation of their children in the study (Appendix A). One student in the traditional group was not given permission to participate in the study. This student obtained the highest final mark in that class, thus her absence could have affected the balance of the two groups.

Design of the Study

A quasi-experimental design was used. The subjects were assigned to two groups based on their preassigned homerooms. Formalized random selection was not possible in the school setting. Classical pretest, posttest, and experimental group, control group methodology was followed. In this section a description is given of the design

and the tests used. A description is also included of the open-ended clinical interviews that were carried out.

Quasi-experiment

This study was a quasi-experimental investigation. The researcher gathered data from two groups of students who were enrolled in the grade 10 geometry course. The curriculum content, as prescribed by the provincial Department of Education, was the same for both groups. The difference between the two groups was that one class (computer group) used a computer program, the Geometric Supposer, throughout the course while the second class (traditional group) did not use the computer.

The geometry classes were both scheduled first period (9:00 a.m. - 10:00 a.m.) daily. As the administration was unable to re-schedule, it was impossible for the researcher to teach both groups. The researcher taught the computer group while another member from the mathematics department taught the traditional group. The researcher had a B.A. degree, a Professional A Teaching Certificate, and 22 years teaching experience. The teacher of the traditional class had a B.Ed. degree, a B.Sc. (Honors in Geology) degree, eight years experience in exploration and mining plus four years teaching experience. Both teachers had taught the geometry course previously and kept constant contact throughout the course regarding the curriculum content and expectations of the students.

The Geometric Supposer

The computer program, the Geometric Supposer, was used as the treatment in this study. The Geometric Supposer is a series of software programs especially designed to provide a "playground" in

which students can experiment with geometric figures and form conjectures. The two Supposer programs used in this study were Triangles and Quadrilaterals.

Description

Each program is contained on a 5 1/4" disk and takes approximately 30 seconds to load into a 64K Apple computer. When the Triangle program has been loaded into memory, the screen will look as shown in Figure 1.

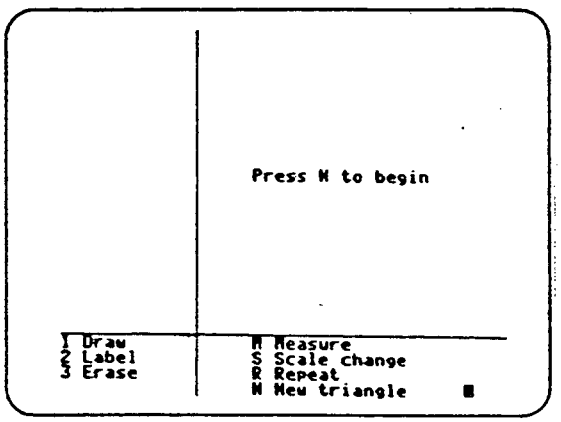


Figure 1. The opening screen of the Geometric Supposer Triangle program

Each screen of the Supposer is divided into three parts--the left column is for data recording, the right side is the area for constructions, and the region below the horizontal line is for menus and prompts.

After N (New triangle) is pressed the user is presented with a new menu:

	1 Right	4 Isosceles
	2 Acute	5 Equilateral
	3 Obtuse	6 Your own

Depending on the type of triangle the user wishes to experiment on,

s/he would select accordingly. If #1 was chosen the screen as shown in Figure 2 would appear.

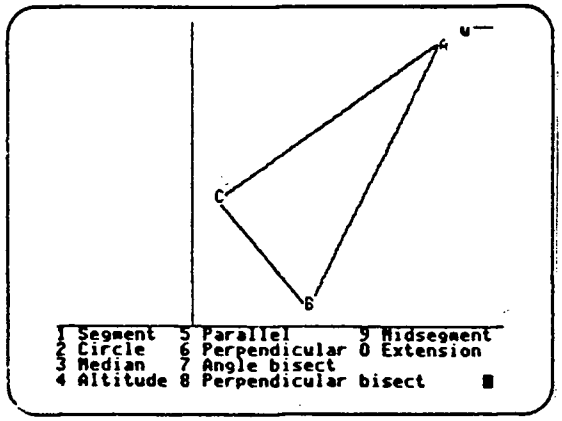


Figure 2. The Right Triangle

The triangles do not appear exactly the same each time selected.

Assuming a student had been assigned to investigate altitudes in triangles, s/he would have made menu selections to display such diagrams as illustrated in Figure 3 or Figure 4.

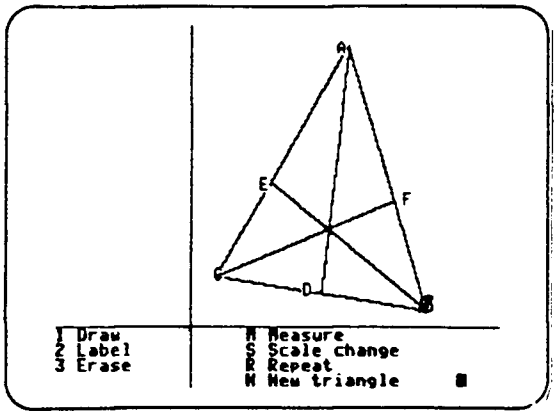


Figure 3. An acute triangle with altitudes.

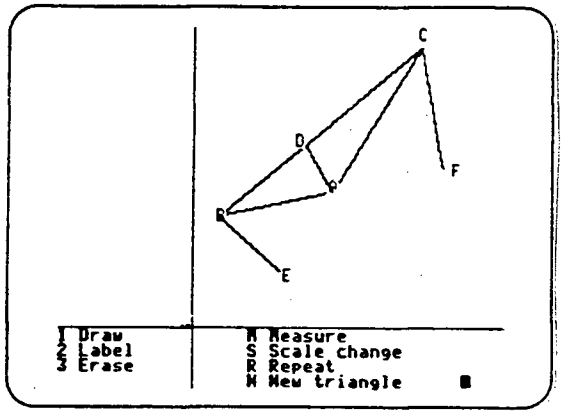


Figure 4. An obtuse triangle with altitudes.

Then using the measurement function (M), the program offers a choice of:

1 Length
2 Perimeter
3 Area
4 Angle
5 Distance Point-line
6 Distance Line-line
7 Adjustable element(s)

At this point, the user may decide to find out what relationship exists between angles and altitudes. Option #4 allows any angle on the screen to be measured. The program requests the name of the angle, using three letters. In the case of the acute triangle (Figure 3), if ADB was entered, the program would respond with ∠ADB = 90 as shown in Figure 5. The user would continue to "measure" until satisfied.

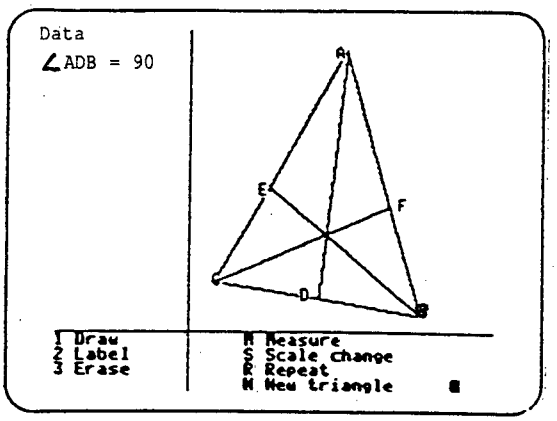


Figure 5. Information recorded on angle measurement.

The above description provides an overview of some of the capabilities of this program. The constructions and measurements are performed quickly by the program giving almost instantaneous feedback to the user.

Use in the study

The researcher used the Supposer software as a method of developing geometric concepts. The program was first demonstrated to

the class by using it as an electronic chalkboard to develop definitions for the different types of triangles. Following this group exposure, the students worked in pairs on specific assignments.

In a typical period in which the Geometric Supposer was used, the instructional period would be divided into four sections. First, the objective would be defined. For example, the task might be to explore the relationship among the interior angles in different types of triangles. Secondly, the students would carry out a pencil, paper, protractor exercise on this topic. Thirdly, in pairs, the students would work on the computers using the software to further explore interior angles in all types of triangles. As part of this assignment, they would record their observations and after discussions with their partner, write out their conjectures. Fourthly, the class would recongregate and discuss their findings. Depending on the nature of the objectives, a theorem or definition would emerge.

Without this software, students would have had to use paper and pencil constructions exclusively to explore the various triangles. This process would be tedious, time-consuming, and result in frustration and/or boredom for the students.

The class used the Geometric Supposer software in the way described above approximately twice a week. The average session on the computer was about 15 minutes.

Pretests

Both groups were given two tests on consecutive days within the first week of the semester. The tests were an introductory geometry

test and a test to measure geometric thought levels of students--the van Hiele geometry test.

Introductory geometry test

(Note: The province in which this study was carried out refers to its geometry course as "geo trig." This "geo trig" course contains the same content and uses similar textbooks as other provinces and states in their first year geometry courses. Hence, the test referred to in this chapter as the introductory geometry test, appears in Appendix B under the title, Introductory Geo Trig 10 Test.)

The first test given, the introductory geometry test (Appendix B), was created by the researcher. The test was based on the geometry chapter from the grade eight textbook (Fleenor, Eicholz, & O'Daffer, 1974) which these students had previously used. As an assessment of face validity, the introductory geometry test was examined by a grade eight mathematics teacher who approved the reasonableness of the content. However, he felt that due to the lapse of nearly two years (three years for the grade elevens) since they had studied this material, the students would not do well. After a preliminary analysis, one question was removed from the test as only one of the forty students answered it correctly. This reduced the test to 24 questions from the original 25. The reliability of this test is discussed later in this chapter.

van Hiele geometry test

Based on writings of the van Hieles, Usiskin (1982), in the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project, developed, piloted, and modified test items to be

used to determine geometric thought levels of students. These levels were referred to as the van Hiele levels of the students in the CDASSG Project. Permission was granted by Professor Zalman Usiskin, Department of Education, University of Chicago, to use the van Hiele geometry test for this study (Appendix C).

Posttests

Three weeks prior to the end of the five month semester, two tests, the van Hiele geometry test and a proof test, were given to both geometry classes on consecutive days at the same time. One week later an attitude test was administered to both groups, again, at the same time.

van Hiele geometry test

This test was identical to the one given at the beginning of the semester.

Proof test

Sharon Senk of the CDASSG Project developed three tests on proof. Some of the items on these tests were not suitable for the present study as they were either unfamiliar to the students or were identical to what had been covered in class. However, the researcher followed the format of Senk's tests to create her own proof test (Appendix D).

Attitude test

Similarly, Aiken (1963, 1974) developed attitude tests related to mathematics. Since Aiken's test items did not deal specifically with geometry, a new test (Appendix E) had to be designed by the researcher.

Interviews

The researcher carried out individual interviews with students from the computer group. Eight interviews were scheduled with each of four students (two male students and two female students). Permission (Appendix F) was obtained from the students involved. Each interview lasted 20-30 minutes and took place in the morning before school started, at lunch hour or after school, depending on which time was convenient for the student.

Two students from the traditional group (one male, one female) were each interviewed twice in the latter half of the semester.

Experimental Controls

In an attempt to reduce uncontrolled factors in this study, all tests were administered at exactly the same time to both groups. This was done to avoid students obtaining any prior knowledge of the test questions. The teacher of the traditional group was very supportive of the study. He was keen to cooperate and was kept informed of the study's progress. He acknowledged the need to keep the kinds of questions assigned, the theorems to be emphasized, and the frequency of quizzes and tests generally equal.

The two classes were taught in rooms some distance from each other. The traditional group was thus unaware of when the computer group was actually using the computer lab.

In this study the teacher of the traditional group had taught the majority of his students the previous semester in algebra. Thus, he was aware of their mathematical strengths, weaknesses, and personality traits. The researcher had been away from this school on secondment for two years and did not know the students. Therefore,

the researcher was not biased by previous knowledge about the computer group.

Originally, the researcher had proposed to make formal observations during class time using comment cards, checklists, and/or rating scales. The researcher did try this on several occasions but found it was not feasible to combine these activities with teaching responsibilities.

Data Collection

This section provides detailed descriptions of each of the tests employed to obtain data about the subjects' geometric knowledge at the beginning and end of the course. A detailed description of the interviews is also provided.

Tests

A total of five tests were administered to both groups of geometry students--two at the beginning of the semester and three at the end. One of the tests, the van Hiele geometry test, was given both at the beginning and at the end.

Introductory geometry test (Appendix B)

The primary purpose of the introductory geometry test was to measure the geometry knowledge of the students entering the grade ten course. The students' previous contact with geometry had been in grade eight. The questions for this test were based on the geometric material they covered in their grade eight textbook. The researcher developed the test to cover only the main ideas with little emphasis on details.

A second purpose for the test was to establish whether any

significant difference existed between the mean scores of the two groups.

The test originally consisted of 25 multiple-choice questions but was subsequently reduced to 24 questions when only 3% of the students were able to answer question #7. Thus, a student could achieve a maximum score of 24 on this test. Thirty-five minutes were allocated for the test but the majority of students in both classes were finished within half an hour.

The results of an item analysis of the introductory geometry test can be found in Appendix B. The reliability measure was calculated using the answers from all students who wrote the test. The Hoyt estimate of reliability was .73 (SPSSX package).

<u>van Hiele geometry test</u> (Appendix C)

The purpose for giving the van Hiele geometry test was to measure the geometric thought levels of the students at the beginning of the course and again at the end. A comparison of the results should enable the following questions to be answered:

- What changes in the students' van Hiele levels take place after a semester of geometry?
- Did the change in the van Hiele pre and postlevels of the computer group differ significantly from those of the traditional group?

This test, developed by the CDASSG Project, consists of 25 multiple-choice questions divided into five levels, with five questions at each level. To test for reliability, the CDASSG Project used the Kuder-Richardson formula 20 (.77) and Horst's modification (.79) (Usiskin, 1982, p. 29).

The CDASSG Project developed two methods for calculating a student's van Hiele level: 3 out of 5 criterion and 4 out of 5 criterion. In this study the 3 out of 5 criterion was used. This requires at least 3 out of the 5 questions correct at a level in order to be assigned that level. For example, if a student had at least 3 out of the 5 first questions (items 1-5) correct and at least 3 out of the 5 questions (items 6-10) correct and less than 3 questions correct in each of the remaining categories, s/he would be assigned a van Hiele level 2 (informal deduction). If, on the other hand, a student had met the criterion (a minimum of 3 out of 5 correct) for items 1-5, 6-10, and 21-25, this student would also be assigned a van Hiele level 2. The reason for the same van Hiele level is that according to Property 1 of the van Hiele levels, a student at level n must have met the criterion at levels below n but not above n. In this case, the student has satisfied the criterion for levels 1 and 2, then jumped to 5. Although the van Hiele test may be "a rather crude device for classifying students" (p. 30) into levels, it "still may be useful for analyzing behavior and treating students" (p. 34).

In order to assign more students a van Hiele level, the CDASSG Project developed a schematic description of the 32 possibilities which could exist that do not comply with Property 1. The various possibilities are each given a forced van Hiele level. Forced van Hiele levels were used in this study in order to make comparisons in movements of students from one level to another.

The 4 out of 5 criterion was not used because a greater percentage of students would have been placed at level 0 (22% as

opposed to 6% on the 3 out of 5 criterion). One of the purposes of this study was to observe the students' movement between levels. "If weaker mastery, say 80%, is expected of a student operating at a given level, then it is absolutely necessary to use the 3 of 5 criterion, for Type II errors with the stricter criterion are much too frequent." (p. 24)

The CDASSG Project also used two theories in assigning levels: the classical theory and the modified theory. Here again, the classical theory employs a more rigid method of assigning levels. In the example above, the student satisfying the criterion for levels 1, 2, and 5 would have been assigned a van Hiele level of zero. The researcher used the modified theory since it resulted in more of the students being assigned a van Hiele level greater than zero. For a more detailed description of assigning van Hiele levels, the reader should refer to Usiskin, 1982.

The CDASSG Project found that the van Hiele geometry test could be used to measure changes in students after a year of geometry. They also found that the "van Hiele levels are a very good indicator when it comes to predicting success on proof" (p. 51).

Proof test (Appendix D)

The purpose for giving the proof test was to measure the students' ability to write proofs and to compare the achievement of the two groups in this area.

The test contained six questions resembling those found in the students' geometry textbook (Jurgensen et al. 1985) and used a similar format to that developed by Sharon Senk of the CDASSG Project. The first question required the students to fill in the

missing statements or reasons of the proof. The second, fourth, fifth, and sixth questions required the students to write full proofs. The third question required a translation from an English statement to an appropriate "given," "to prove," and a "diagram." The students were then required to write a proof.

The teacher of the traditional group examined the test to ensure that none of the problems had previously been attempted in class. Thirty-five minutes (the same time as the CDASSG Project) were allocated for the test.

The questions on the test were each scored on a 0-to-4 scale. The criteria used for grading the proofs was as follows:

0 - Student writes nothing, writes only the "given," or writes only invalid or useless deductions.

1 - Student writes at least one valid deduction and gives reason.

2 - Student shows evidence of using a chain of reasoning, either by deducing about half the proof and stopping or by writing a sequence of statements that is invalid because it is based on faulty reasoning early in the steps.

3 - Student writes a proof in which all steps follow logically but in which errors occur in notation, vocabulary, or names of theorems.

4 - Student writes a valid proof with at most one error in notation (Senk, 1985, p. 449).

The researcher marked all the proofs using this scale. Each test had a cover sheet with the student's name. Prior to any marking taking

place, both groups of tests had their cover sheets turned. After this, the tests were marked. Hence, all marking took place while the researcher was unaware of whose test she was actually marking. In summary, the proof tests were blindly scored.

Since questions #1 and #3 were of a different type than the other four questions, two measures of overall achievement are given. The Hoyt estimate of reliability on all six questions was .80 and an item analysis appears in Appendix D. The maximum possible score was 24. The second measure pertains to the four questions (2, 4, 5, 6) which were strictly proof writing exercises and the maximum score was 16. The Hoyt estimate of reliability on this section was .74.

A similar six-item proof test was given to 1520 students in the CDASSG Project and formed the basis of conclusions about students' readiness for geometry.

Attitude test (Appendix E)

The attitude test was given to measure the affective objectives of mathematics instruction such as attitude, value, and enjoyment. The purpose for giving this test was to analyze any significant difference in attitude between the two groups at the end of the semester.

This test (28 questions) was constructed using Likert's method of summated ratings. Half of the items on the test are worded in the direction of a favorable attitude and the remaining half are in the direction of an unfavorable attitude. The test can be divided into three sections:

(1) questions 1 - 10 relate specifically to attitudes towards geometry.

(2) questions 11 - 20 relate to the enjoyment of mathematics.

(3) questions 21 - 28 relate to the value or importance of mathematics.

To encourage students to be as honest as possible no names were required on this test. The second part of the test requested written "likes" and "dislikes" of the geometry course. The purpose of these general questions was to obtain "gut" reactions about what stood out in the students' minds as positive and negative factors.

On the first 28 questions the students were asked to respond with "Strongly Agree" (SA), "Agree" (A), "Undecided" (U), "Disagree" (D), or "Strongly Disagree (SD). For the attitude items which were stated positively, the responses to each item were coded as 5, 4, 3, 2, or 1 respectively. For the attitude items stated negatively, the responses were coded as 1, 2, 3, 4, or 5 respectively. An item analysis for each group is in Appendix E.

The Hoyt estimate of reliability for the 28 questions was .93 with the estimates for each of the three sections being .91, .94, and .74.

Interviews

The purpose for carrying out interviews was to increase the researcher's understanding and appreciation of the difficulties students were experiencing with proof writing. The interviews also provided additional insights to the researcher about misconceptions held by students that were not apparent in their classroom work.

Four students were selected from the computer group. The interviewees consisted of two males and two females. One male (Scm1) had a van Hiele level 1, the second male (Scm2) was a level 3, one

female (Scf1) was level 0, and the second female (Scf2) was level 1. The researcher had also derived the van Hiele levels for each student using the 4 out of 5 criterion. These levels were compared with their levels from the 3 out of 5 criterion. The levels for Scm2 were 3 on the latter criterion and 0 on the stricter criterion. The levels for this student varied the most in both groups (computer and traditional) and hence, the researcher selected him for the interviews in order to resolve this quandary.

The researcher met individually with each student for approximately 20-30 minutes on a biweekly basis. The sessions were held in an office which provided privacy. Each session was audio-taped and later transcribed. At first, the students were nervous with the tape recorder. The beginning sessions mainly dealt with homework problems. In the later sessions each student solved the same geometry proofs.

One of the male students (Scm1) was not consistent in attending the sessions, consequently, only six interviews were held with him before the end of semester.

After four sessions it was obvious that Scm2 had a sound grasp of the geometric concepts involved and was able to quickly analyze and write a proof. Hence, sessions with him were unproductive in terms of the study. At this point, another male student (Scm3) with a van Hiele level 1 was selected, and he attended four interviews. To obtain some in-depth information regarding proof writing from the traditional group, two students were selected--a female (Stf3) with van Hiele level 0 and a male (Stm4) with level 1. These two students were each interviewed twice and worked through some of the same

proofs that the computer group had had. A summary of the number of interviews held with each student follows:

Scm1 - 6 sessions	Scf1 - 8 sessions
Scm2 - 4 sessions	Scf2 - 8 sessions
Scm3 - 4 sessions	Stf3 - 2 sessions
Stm4 - 2 sessions	

These interviews were all held out of class time.

Given the control of the various extraneous factors identified in this chapter, the quasi-experimental model does appear to be appropriate for this investigation.

CHAPTER 4

DATA ANALYSIS

The results obtained from the five tests and the interviews are presented in this chapter. The following questions were raised:

1) Will the students who use the Geometric Supposer software be better able to write formal proofs than students who are taught by more traditional methods?
2) What changes in the students' van Hiele levels take place after a semester of geometry?
3) Will the students who receive the treatment have a more positive attitude towards geometry than the students in the traditional group?

Assessment of the Groups

Did the geometric knowledge of the two groups (computer and traditional) differ at the beginning of this study? The instrument used to answer this question was the introductory geometry pretest (Appendix B). An independent *t*-test was used to analyze the data involving the two independent groups. The test is available in the SPSSX package on the U. B. C. computer system.

The findings, as shown in Table 4.1, indicate that there was no statistically significant difference between the two group means at the .05 level of statistical significance in terms of geometric knowledge. The independent *t*-test involved the data from all forty students who originally wrote the introductory geometry test. The

mean scores were 14.95 and 13.16 for the computer and traditional groups respectively. The marks on this 24 item test ranged from 5 to 24. An item analysis of this test for each group is presented in Appendix B.

During the semester four students left the traditional group, thus a second *t*-test was run without the scores of those four students. Again, the result was similar in that no statistically significant difference was revealed ($t = 1.51$; $p = .141$). Thus, the two groups were accepted as being from the same population and appropriate for this study.

Table 4.1

Means, Standard Deviations, and Statistical Comparison of Groups: Introductory Geometry Pretest

Groups	n	Mean	SD	t-value	p
Computer	21	14.95	4.73		
				1.40	.169
Traditional	19	13.16	3.10		

Geometric Thought Levels Prior to Treatment

While the first data analysis was concerned with the actual performance in geometry of the two groups, the second data analysis was concerned with the geometric thought levels of the subjects. An ordinal scale was used to represent the van Hiele levels (geometric thought levels) of the students. These levels represent rankings and cannot be used as scores. Therefore, the median test, a

nonparametric test, was appropriate. The median test was used to test the hypothesis that the two groups came from populations that have the same median. This test was applied to the data from the van Hiele pretest. Again, the two groups appear to be homogeneous as can be seen in Table 4.2. A significance level of .05 was used.

Table 4.2

Median Test: van Hiele Pretest

	Computer Group	Traditional Group	Combined
Number greater than median	11	4	15
Less than or equal to median	10	11	21
Total	21	15	36
(p = .230)			

Changes in Geometric Thought Levels

What changes in the students' van Hiele levels take place after a semester of geometry? The instruments used to answer this question were the pre and post van Hiele geometry tests. The sign test, another nonparametric test which is used to measure the same sample on two different occasions when it is suspected that changes are taking place, was applied to the data. The results, as shown in Table 4.3, indicate that there was a statistically significant difference at the .05 level between the pre and post van Hiele levels for students in both groups. Therefore, the null hypothesis, there was no significant difference in the rankings of the geometric thought levels between the pre and post test results within each

group, was rejected. The geometric thought levels for both groups did improve after a semester of geometry. A median test was applied to the data from the van Hiele posttest. The result of this test was that no significant difference in the rankings between the computer group and the traditional group existed at the .05 level of significance ($p = .569$).

Table 4.3

Sign Test: Pre and Post van Hiele Test Data

	Computer Group	Traditional Group	Combined
No change in van Hiele level	6	6	12
Decrease in van Hiele level	1	1	2
Increase in van Hiele level	14	8	22
Total	21	15	36
	($p = .0010$)	($p = .0391$)	

In the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project it was found that after a year's study of geometry approximately, "a third go up one level; a third exhibit 'great growth', increasing two or more levels; the final third exhibits 'no growth', staying the same or decreasing their level." (Usiskin, 1982, p. 38)

The researcher found a similar pattern. After a semester of geometry, in the computer group one-third stayed the same or decreased a level, 43% increased one level, and 24% increased two levels. In the traditional group 47% stayed the same or decreased a level, 20% increased one level, and one-third increased two or more

levels. Hence, these findings are generally consistent with the CDASSG experience.

Written Proofs

The written proof test was analyzed using the independent *t*-test. There was a significant difference, at the .05 level, between the means of the two groups as reported in Table 4.4. A second *t*-test was done involving only the questions which required writing proofs--four questions (#2, 4, 5, 6). Again, there was a significant difference between the means of the two groups (Table 4.4). Therefore, the null hypothesis, there was no significant difference between the means of the written proof tests of students in the computer group and those in the traditional group, was rejected.

Table 4.4

Means, Standard Deviations, and Statistical Comparison of Groups: Proof Test

Groups	n	Mean	SD	t-value	p
PART A*:					
Computer	22	11.00	5.46		
				2.89	.006
Traditional	16	6.00	4.97		
PART B* (Proof Writing):					
Computer	22	7.73	4.04		
				3.84	< .001
Traditional	16	2.94	3.44		

*PART A: 6 questions (24 marks)
*PART B: 4 questions (16 marks)

Out of a possible 24 marks on the six questions, the computer group's results ranged from 2 to 23 while the traditional group ranged from 1 to 20. On the fill-in question (Appendix D, question #1) 43% of the computer group and 31% of the traditional group could identify the alternate interior angle. On the diagram drawing question (Appendix D, question #3) 57% of the computer group and 38% of the traditional group could draw the diagram but were unable to proceed any further.

An item analysis for both groups on the proof writing test is in Appendix D. Half of the computer group and one-fifth of the traditional group were able to apply the converse of the Isosceles Triangle Theorem in question #2. Two deductions were required in question #4. Forty-one percent of the computer group and 12% of the traditional group achieved half or more of the marks on this question. Question #5 required the addition of an auxiliary line segment. This question was answered correctly by 59% of the computer group and by 19% of the traditional group. The last question required more than two deductions. No student completed this proof correctly. However, 41% of the computer group and 25% of the traditional group got half or more of the required answer.

Considering the four questions that required proof writing collectively, one student (5%) in the computer group received no marks while six students (31%) in the traditional group had the same result.

Attitudes

The attitude test was analyzed using the independent *t*-test. The statistics, as reported in Table 4.5, support the null hypothesis at the .05 level of significance. Thus, the null hypothesis of no significant difference in the means between the attitudes of the two groups was accepted. This result is consistent whether the whole test is used or if the subsets (geometry, mathematics in general, or the value of mathematics) are used. An item analysis for each group along with the test items are included in Appendix E. Also in Appendix E is a summary comparing the mean score of each item between the computer group, the traditional group, and the total sample.

Table 4.5

Means, Standard Deviations, and Statistical Comparison of Groups: Attitude Test

Test		Computer Group (n=21)	Traditional Group (n=17)	t-value	p
A. Geometry:	Mean	31.62	29.06		
	SD	7.68	9.46	0.92	.363
B. Mathematics:	Mean	34.00	32.35		
	SD	8.50	9.41	0.57	.575
C. Value of Mathematics:	Mean	32.67	33.24		
	SD	4.27	4.06	-.42	.679
D. Total Test:	Mean	98.29	94.65		
	SD	17.34	17.42	0.64	.525

The students were asked for their written comments about the course. No prompting was given. Their spontaneous remarks are summarized in Table 4.6.

Table 4.6

Written Comments

	Computer group	Traditional group
Likes:		
- Computer use	33%	n/a
- Constructions	29%	29%
- Proofs	10%	18%
- Project	10%	6%
- Trigonometry	5%	6%
Dislikes:		
- Memorizing	5%	35%
- Computer	5%	n/a
- Constructions	5%	0
- Proofs	24%	35%
- Pythagorean Theorem	19%	0

Student suggestions:

- Computer group - "It would be faster and easier to just tell us."
  - "More computer use."
- Traditional group - "I hope that computers are used more and more in the classroom." - two students made this comment.

The majority of the written comments were concerned with classroom management functions and added nothing to this study.

Interview Data

This section contains a condensed summary of the activities carried out during the interview sessions with the students. Responses from the students are also included.

Sessions 1 - 3

The first three sessions with students from the computer group were used to establish rapport and to set the stage for the sessions focussed on solving proofs. A variety of geometrical concepts, definitions, postulates, and theorems were reviewed along with class work and tests.

In reporting the findings of the student interviews the researcher used a code to identify each student. The code consisted of four characters. The first two were either Sc or St to denote the computer group or the traditional group respectively. The third character indicated gender (M or F). The last character is numerical to differentiate among the students.

The following are some unexpected student responses from the first three sessions.

After three weeks in the geometry class Scf1 asked, "Does a triangle have three sides?" This student was also unable to describe parallel lines and had difficulty with the concept of straight angle. In each of the three sessions the exercise in Figure 6 was reviewed. She approached the problem in the same way each time--with complete naivety. She had no idea that a line represents an angle of 180 degrees even though we had used the protractor to measure it. It also never occurred to her to use the protractor herself.

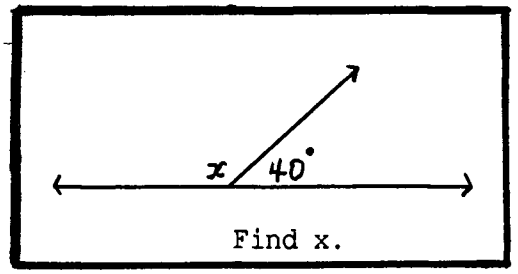


Figure 6. Supplementary angles

Scm1 had difficulty with any question involving equations. In Figure 7 he could identify that $\angle 1 \cong \angle 2$ but was unable to proceed to the next step.

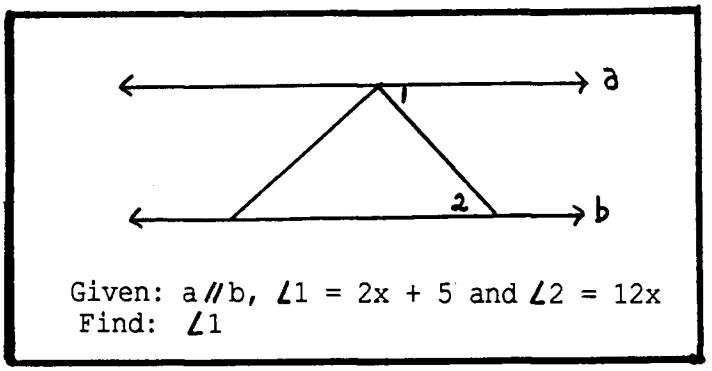


Figure 7. Parallel lines with alternate interior angles.

Scm2 liked questions with numbers. When given the example in Figure 8 he replied, "You can't do it if it has letters."

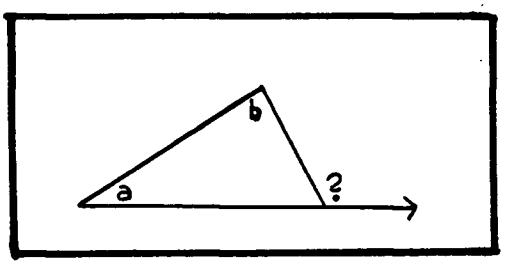


Figure 8. Relationship between the exterior angle and the remote interior angles.

Session 4

The fourth session was spent reviewing proof exercises from the classroom. The two girls were unable to make any deductions. They would write the "given" and come to a standstill. With a constant flow of directed questions from the researcher, they would eventually solve the proof. When looking back and reviewing the steps, Scf1 would say, "It all makes sense but I could never do that on my own." Perhaps she was operating in the zone of proximal development, a theory proposed by Vygotsky. The zone of proximal development is "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance" (Vygotsky, 1978, p. 86).

Scm1 could make simple deductions. Scm2 was selected for the interviews because of his inconsistent van Hiele level. He was able to quickly analyze the problem and move forward to writing up the proof. He sometimes assumed a problem to be the same as a previous one such as in Figure 9.

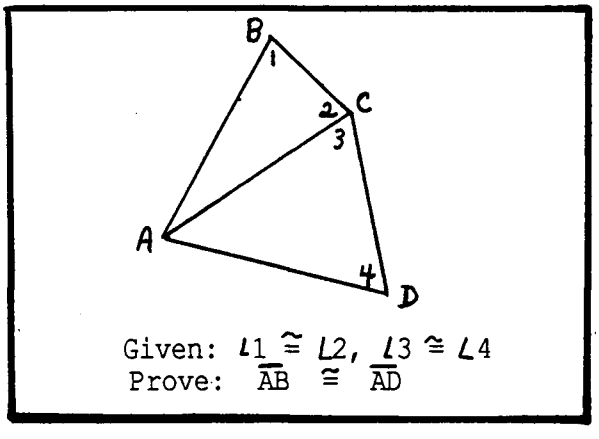


Figure 9. Proving two segments congruent.

He immediately wrote down the "given" followed by:

$\angle BAC \cong \angle DAC$ because each triangle has 180^0

$AC = AC$ because of the reflexive property

$\Delta ABC \cong \Delta ADC$ because of ASA

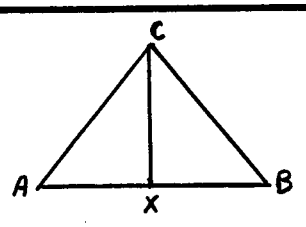
$\overline{AB} \cong \overline{AD}$ because corresponding parts of congruent triangles are congruent.

When he was asked to check the "given" he quickly realized his mistake and rewrote the proof. The researcher felt that this student had a van Hiele level much closer to 3 than to 0 and having solved the quandary ceased interviewing him.

Another male (Scm3, van Hiele level 1) who was having difficulty with proofs in class was selected from the computer group to replace Scm2.

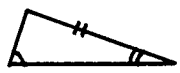
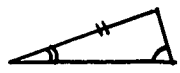
Session 5

The four students from the computer group and two additional students from the traditional group were each given the same two proofs to complete during this interview session. The first proof is in Figure 10 followed by a summary of the students' responses. The second exercise is shown in Figure 11 and a similar summary of the students' replies follows.



Given: $\overline{CX} \perp \overline{AB}$, $\overline{AC} \cong \overline{BC}$
Prove: $\Delta ACX \cong \Delta BCX$

Figure 10. Proof #1

All six students immediately wrote the "given" without reading the whole question. Three students each marked their diagrams with what was given. Two students were able to make a correct deduction from the fact that the segments were perpendicular but the other four (Scf1, Scm1, Stf3 and Stm4) had difficulty with this concept. None of the six students were able to make a deduction from the fact that two sides of the triangle were congruent. They also had difficulty stating why the triangles were congruent. When given pairs of premarked figures such as   students were able to give the correct reasons for these triangles being congruent. However, in a proof writing situation they were unable to relate the written work to their diagram and then draw a conclusion.

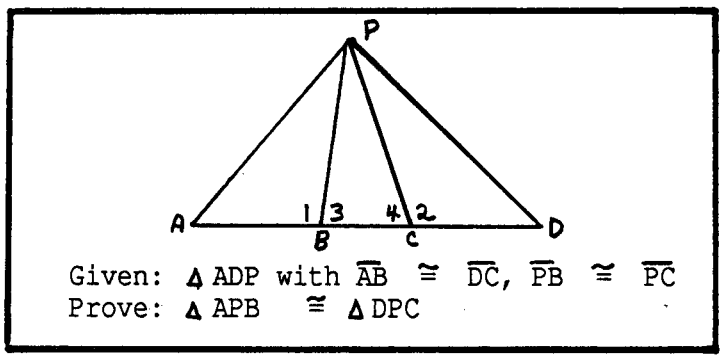


Figure 11. Proof #2

In proof #2, one student (Scm1) correctly used the Isosceles Triangle Theorem to obtain $\angle 3 \cong \angle 4$. The other students made no connection with Proof #1 which contained similar information. Scm3 correctly identified the supplementary angle relationships and quickly completed the proof. Here again, the majority of these students required many probing questions in order for them to make deductions from what the question had given and to achieve what was to be proved.

Session 6

The same six students were each given two additional proofs to complete during this session. Proof #3 appears in Figure 12 and Proof #4 appears in Figure 13. A summary of the students' responses follows each figure.

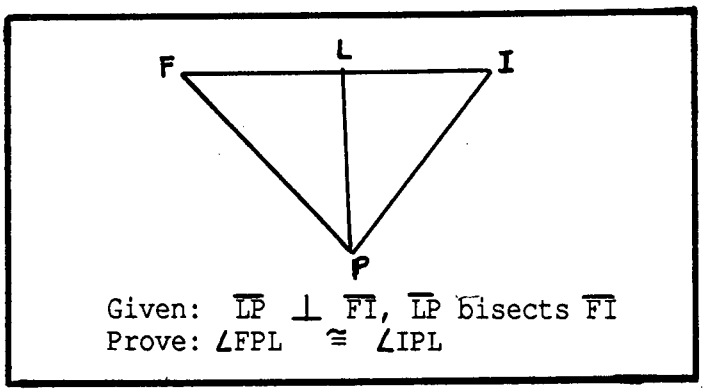


Figure 12. Proof #3

On proof #3 Scm1 successfully completed this proof with no assistance. Three students said that $\angle F \cong \angle I$ because $\overline{LP}$ bisects $\overline{FI}$. One student said that $\angle FLP \cong \angle ILP$ because $\overline{LP}$ bisects $\overline{FI}$. These four students all defined bisect as meaning "to cut in half." One student was adamant about $\angle F$ being congruent to $\angle I$ until she, without suggestion from the interviewer, turned the page around. Stm4, despite having sufficient information to prove the triangles congruent, kept coming back to $\angle F$ and $\angle I$. He was determined that they should be equal.

Scf1 asked if there was a short way to write "bisects" (like a symbol for perpendicular). She was unable to write any steps after the "given" without being specifically directed.

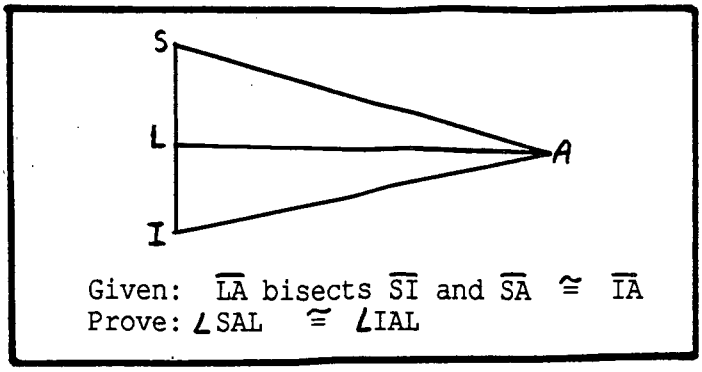


Figure 13. Proof #4

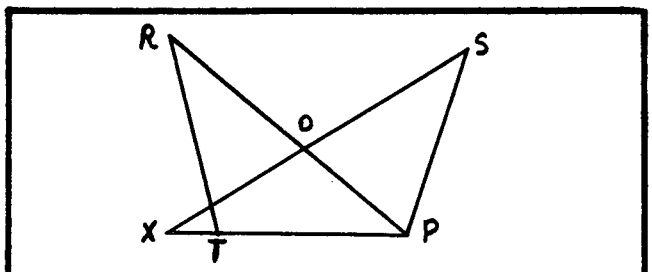
On proof #4 all six students assumed $\angle ALS \cong \angle ALI$ because $\overline{LA}$ bisects $\overline{SI}$. Scm1 reread the "given" and erased his markings on the diagram, then proceeded to correctly complete the proof. Once the questions, "What does bisect mean?" and "What is being bisected?" were asked, the five students then corrected their work. Three students used SAS (Side-angle-side) to prove the triangles congruent and the other half used SSS (Side-side-side).

In summary, the students all understood that proving the triangles congruent was a critical step in proofs #3 and #4 prior to the last statement. The students had to be encouraged to mark their diagrams. Once they did this, they found it easier to determine the next step. There were no further interviews with the two students from the traditional group.

Sessions 7 & 8

The four students in the computer group were interviewed on two more occasions. Their responses generally mirrored their progress in class. In the interview situation they experienced the same kind of difficulty as they did in class. As an example of this, the following section contains a report of an interview with each of the

students that focussed on the concept of overlapping triangles as illustrated in Figure 14.



Given: $\angle RTP \cong \angle XPS$, $\overline{PT} \cong \overline{SP}$,
$\angle PSO \cong \angle TPO$
Prove: $\overline{RT} \cong \overline{XP}$

Figure 14. Overlapping Triangle Proof

The students methodically recorded what was given then attempted to mark the diagram. Scm3 was confused between the angles $\angle XPS$, $\angle PSO$, $\angle TPO$, and $\angle SPO$. Once he had the angles sorted out, he was able to quickly visualize the triangles being congruent by ASA (Angle-side-angle). They all had difficulty matching the appropriate vertices of the triangle for the congruence statement. None of them attempted to redraw the diagram.

The interviews provided the researcher with an opportunity to collect data regarding student approaches to proof writing and student misconceptions about geometric concepts.

Additional Data

In order to investigate the effects of the Supposer programs on proof writing, the students' geometric thought levels were analyzed. Table 4.7 indicates the percent of all the geometry 10 students with van Hiele levels at the beginning of the course and again at the end. This data was obtained from the pre and post van Hiele geometry

tests. The results are similar to the findings of Usiskin's 1982 study. He found that at the beginning of the geometry course "over half of students classifiable into a van Hiele level are at levels 0 or 1." (p. 81) He found that at the end of his study when the geometry course was completed, more students were at level 3 than at any other level.

A chi square test of association showed that the two variables (the van Hiele levels and the tests) were related at the .05 level.

Table 4.7

Students' van Hiele Levels

	Percentage of students			
Level	Pretest	(n = 36)	Posttest	(n = 36)
0	6% . .	2	3% . .	1
1	53% . .	19	17% . .	6
2	22% . .	8	28% . .	10
3	14% . .	5	39% . .	14
4	6% . .	2	14% . .	5

The pre and post van Hiele levels for each of the students interviewed are shown in Table 4.8. The results from the proof writing test (possible total = 16) are also shown in the table.

Table 4.8

Interviewees' Pre and Post van Hiele Levels and their Proof Test Scores

Student	van Hiele Levels Pre	van Hiele Levels Post	Proof Test Scores
Scf1	0	1	0
Scf2	1	2	11
Scm1	1	2	14
Scm2	3	3	11
Scm3	1	2	10
Stf3	0	2	1
Stm4	1	2	5

Data Summary

In this chapter the results from the five tests given to the students have been presented. Appropriate statistical tests were used to analyze the data gathered with each of these instruments. The findings indicate rejection of the first null hypothesis (There was no significant difference between the means of the written proof tests of students in the computer group and those in the traditional group.) and rejection of the second null hypothesis (There was no significant difference in the rankings of the geometric thought levels between the pre and post van Hiele test results.). The findings indicate support for the third null hypothesis (There was no significant difference in the means between the attitudes of students in the computer group and those in the traditional group.). The data indicates that teaching geometry, with or without computer programs, does improve students' van Hiele levels.

CHAPTER 5

SUMMARY AND DISCUSSION

The purpose of this study was to investigate how the computer program, Geometric Supposer, would affect a grade 10 geometry class's ability to write proofs. As part of this investigation, tests were administered to determine the geometric thought levels, the geometric knowledge, and the attitudes of the students. Data was collected from two groups of students--those using microcomputers and those learning geometry the traditional way.

Summary of the Problem, Methodology, and Results

Many high school students who take the grade 10 geometry course experience difficulty with the section on writing geometric proofs. This study was an attempt to investigate the effectiveness of a computer program, the Geometric Supposer, in increasing the performance level of students in writing proofs. The subjects in this study were all the students enrolled in the grade 10 geometry course in one particular high school. One class of these students used the computer program and the other class did not.

The van Hiele test, which measures geometric thought levels, was administered at the beginning and the end of the geometry course. An introductory geometry test was also administered at the beginning of the course to measure the geometric knowledge of the students prior to the study. At the end of the course the students wrote a proof test as well as an attitude test.

A series of interviews were carried out with five students from the computer group and two students from the noncomputer group. This was done in order to gain some insight into the methods the students were actually using to write geometric proofs and to identify changes in their approaches.

The van Hiele test was developed and tested in Chicago by the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project. The Hoyt measure of internal consistency was used as an estimate of reliability for the other tests, which were developed by the researcher.

The results showed that there was no statistically significant mean difference in the geometric knowledge of the two groups (computer and traditional) at the beginning of the study. As was shown in Table 4.7 the van Hiele levels of the two groups both improved after one semester of geometry. There was no significant mean difference between the attitudes of the two groups towards geometry at the end of the course. However, the computer group scored significantly higher than the traditional group on the final proof test.

Interpretation of the Findings

The introductory geometry test

The findings indicate that there was no significant difference in the geometric knowledge of the two groups at the beginning of the study.

The students in this study had received some geometry instruction in grade 8 but no geometry content in grade 9. The study covers the

geometry they took in the second semester of their grade 10 year. The introductory geometry test was given at the beginning of this semester to assess the level of geometry knowledge of the students entering the grade 10 course. The averages on this test were 62% for the computer group and 55% for the traditional group (Table 4.1). Further, the results of this test are comparable to the results of the entering geometry test given in the CDASSG Project. The mean percentage correct in their study was 54% (Usiskin, 1982, p. 68).

The students in both groups performed poorly on the questions involving the identification of obtuse angles (23% correct) and the calculation of the area of an obtuse triangle (25% correct). However, 85% of the students could calculate the area of a rectangle, 83% could define an equilateral triangle, 93% could identify a reflection point, and 90% could find the volume of a rectangular solid (Appendix B).

The van Hiele geometry pretest

The van Hiele geometry test was given to assess the geometric thought levels of the students entering the grade 10 geometry course. The results of this test not only supported the hypothesis that there was no statistically significant difference between the rankings of the geometric thought levels of the students in the two groups (Table 4.2), but also shed some light on the overall poor performance of both groups on the introductory geometry test. The results from this test indicate that 59% of the students entering the geometry course were at level 0 or 1 (Table 4.7). This data is consistent with data of the CDASSG Project. Using the same criterion, their results indicated 54% (p. 100) of the students entering grade 10 geometry

were at van Hiele levels 0 or 1. As indicated earlier, various researchers (Battista & Clements, 1988; Craine, 1985; Scally, 1987; Senk, 1983; Usiskin, 1982; Wirszup, 1976) have all discussed the need for students to be at level 3 in order to cope successfully with the abstract concepts of proof writing.

In other words, over half the students entering the grade 10 geometry course had not achieved a sufficient geometric thought level to deal with the section on writing proofs. More simply, they were not ready.

The van Hiele geometry posttest

At the end of the semester the students again wrote the van Hiele geometry test. The results of this test indicated that there was a significant difference in scores between the beginning of the semester and the end of the semester (Table 4.3 and Table 4.7) and thus the null hypothesis, there was no significant difference in the rankings of the geometric thought levels between the pre and post test results within each group, was rejected. One would hope that students would perform better on a test of geometric thought levels after a semester of geometry. Nevertheless, one third of the students exhibited no change in their geometric thought level. This is also consistent with the CDASSG Project results. Even after a semester of geometry 20% of the students in this study were still at a van Hiele level of 0 or 1 (Table 4.7).

Were the geometric fundamentals of these students so weak that there was little to build on? Was the content of the geometry 10 course inappropriate for these students and thus were they denied the opportunity to improve their thought levels? Were the teaching

methods inappropriate for these students? Probably all three factors contributed to the students' lack of growth during this semester. Perhaps, more individualized instruction needs to be incorporated into the classroom directed at students with van Hiele levels of 0 or 1. More attention also must be directed at geometry instruction in the elementary and middle year's curriculum.

The proof test

The null hypothesis, there was no significant difference between the means of the written proof tests of students in the computer group and those in the traditional group, was rejected. The computer group performed significantly better than the traditional group.

The proof test, written at the end of the semester, resulted in overall averages of 46% (computer group) and 25% (traditional group) --Table 4.4. However, if the analysis is limited to only those questions on the test involving proof writing, the spread between the groups increases. The computer group had an average of 48% while the traditional group had an average of 18%.

The use of the Geometric Supposer computer programs appears to have contributed to the students' consolidation and understanding of geometric concepts. Using these programs throughout the semester seemed to make it easier for students to make deductions. They had become accustomed to looking for relationships, testing their ideas, and making conjectures. The students looked forward to using the software. On the whole, the students were able to write simple proofs more easily than previous classes the researcher has taught. However, when the proofs became more complex the students experienced the kind of difficulty typical of grade 10 classes.

Usiskin (1982) stated that "a student who enters geometry at van Hiele levels 0 or 1 has an almost even chance of failure at proof" (p. 57). Sixty-five percent of the students in this study who were unsuccessful (obtained below 50%) in the proof test had beginning van Hiele levels of 0 or 1 while 40% of the successful (obtained 50% or more) students had levels of 0 or 1. In the CDASSG Project 71% of students unsuccessful at proofs had beginning van Hiele levels of 0 or 1 as compared to 37% who were successful. "Thus students unsuccessful at proof are about twice as likely as the more successful others to have these low van Hiele levels." (p. 61) The above results were in the same direction as the results from the CDASSG Project. However, a chi square goodness-of-fit test (.05 level) indicated that the researcher's results were not statistically significant. This may have occurred because of the relatively small sample size.

As long as more than 50% of the students entering the grade 10 geometry course have van Hiele levels of 0 or 1, either the proof section of the course should be removed or more emphasis should be placed on the type of instruction employed in the course.

The attitude test

The results were obtained from an attitude test given to both groups at the end of the semester. Overall both groups had positive attitudes towards the study of geometry in particular, the study of mathematics in general, and the value of mathematics as a whole.

Despite the relative ease with which the computer group approached proofs, 24% (Table 4.6) specifically mentioned a dislike for proof writing as compared to 35% in the traditional group. Proof

writing may have had such a negative reputation that the positive experience with the computer software was insufficient to overcome this negative valence.

The interviews

During the interviews the researcher discovered why many students were experiencing difficulty in geometry. They lacked the basics. The fact that a student can get all the way to grade 10 without understanding what a triangle is seems incredible but does occur (Scf1). Without the constant one-on-one situation of the interview, the gap in this student's knowledge may not have been discovered.

It was also surprising to discover the general confusion that existed regarding a segment bisecting another segment in a triangle. Despite the fact that "bisect" was used frequently in class without any apparent difficulty, it was only during the interviews that the researcher realized these students had failed to understand the generalized concept of "bisect."

The researcher benefited from the contact during the interviews and felt that the students did likewise, especially Scm1, Scm3, and Scf2.

Scm1 approached proofs in a methodical fashion--carefully reading, formulating a plan, and carrying out the plan. He said that he enjoyed solving the proofs. At first, Scm3 would "flit" from step to step, reason to reason. Once he was focussed, he could analyze the situation and foresee what had to be done. It was sheer hard work for Scf2 as she tried to find a rule for every situation, rather than analyzing and dealing with what was given in the problem. These

three students all achieved well on the proof test (Table 4.8).

The researcher recommends one-on-one contact with all geometry students who have van Hiele levels 0 or 1. This one-on-one contact should begin as near to the beginning of the course as possible. Obvious weaknesses could be worked on so that the student is better prepared to cope with geometry concepts as they are presented in the general classroom. This type of remedial help could also be provided through appropriately designed Computer Assisted Instruction.

Limitations of the Study

The main limitations of the study were the length of the treatment period, the different instructors, and the effects of the interviews.

Limited time for exploration

The treatment period of this study was one semester. Hence, the opportunity for students to assimilate computer experiences into their repertoire of problem solving skills was limited. Also, there was a prescribed curriculum to be covered within this time. This limited the amount of time for computer exploration.

Different instructors

The computer group and the traditional group were taught by two different teachers. However, both teachers were qualified mathematics teachers and both had taught the grade 10 geometry course previously. The teachers communicated frequently as to the content being covered and various approaches used. They undertook to set similar standards for class performance and homework. The fact that there was no significant difference between the attitudes of the two

groups towards geometry suggests that the effects of different instructors may have been minimal.

Effects of the interviews

The interview sessions tended to be tutorial--correcting misconceptions and reviewing concepts which had not been understood in class. The same proofs were given to all the students and the same format of questioning was followed. Through this interview experience, some students (Scf2, Scm1, Scm2, Scm3) could have gained additional geometrical knowledge and, thus obtained higher scores on the proof test than if they had not been interviewed. These students may also have been influenced by the Hawthorne effect.

In retrospect, the same number of students, with similar van Hiele levels, from the traditional group should have been interviewed for the same length of time as from the computer group.

Suggestions for Further Research

Given that this study indicates the Geometric Supposer software has value in the geometry class, further research studying its effect over a longer period of time appears warranted.

Another possible research suggestion would be the use of other computer programs (i.e., LOGO, LOGOWRITER) in the grade 10 geometry course and their effect on proof writing.

If proof writing is to remain in the grade 10 geometry course then research should be undertaken with respect to methods of incorporating geometric content into the elementary and middle grade levels. Research could center on the development and/or use of manipulatives and computer software at those levels.

The current practice in the province in which this study took place is to have proof writing in the grade 10 geometry course. An alternative would be to include it in the grade 11 geometry course. Hence, more time could be spent on informal geometry at the grade 10 level. Research in this area could be valuable for designing future curriculum.

Designing appropriate experiences to help students achieve at least a van Hiele level 2 prior to undertaking the writing of proofs could also prove fruitful.

Implications

Process rather than product

One of the current trends in education is to give students opportunities to be actively involved in knowledge construction. The Geometric Supposer programs provide such an opportunity. This software is designed to promote experimentation--the process. It has no product requirements built in. The usefulness and power of the program lies in the context of the task or problem given to the student.

The teacher is the key factor in directing this process. S/he defines the student-software interaction. The role of the teacher shifts from the traditional one of being the sole source of knowledge to one of supporting and integrating student inquiry. The teacher needs to teach and model such skills as collecting data, analyzing, making conjectures, testing, and generalizing. With this software, lesson planning will consist mainly of defining tasks and developing objectives without giving away the outcomes.

The teacher must encourage students to take diversified approaches to solving problems. Students should work on the computer in pairs. Class discussions should follow hands-on sessions.

More time involved

The use of the Geometric Supposer programs involves a greater investment of time than noncomputer instruction. The teacher must be flexible in order to respond to unexpected discoveries by students in the class. Similarly, assessment strategies and techniques will need to be adaptable and flexible.

Hardware accessibility

Access to the hardware is the main factor. Having the computers right in the room is the most appropriate arrangement as it provides opportunities for spontaneous investigation. Ideally, students should have access to the program during free periods or after school.

Geometry prior to proof writing

The overall averages of the two groups (computer and traditional) on the introductory geometry test were relatively low. This result can be attributed in part to the one year gap in geometric instruction. In order for students to be ready for proof writing, they should be exposed to geometry in the previous grade. Hopefully, studying geometry in the previous grade would help students to gain at least a van Hiele level 2 before attempting to write proofs.

Conclusions

The van Hiele geometry test is a useful aid for grade 10 geometry teachers to better identify and appreciate the geometric thought levels of their students. It allows teachers to plan, prepare, and have realistic expectations when teaching the process of writing proofs.

The Geometric Supposer computer programs have potential as an instructional aid in the geometry classroom. They do, however, require preparation by and guidance from the teacher.

This study has shown that using the Geometric Supposer software can assist students in being able to better write geometric proofs at the grade 10 level.

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Appendix A

PERMISSION LETTER SENT TO PARENTS/GUARDIANS

February 2, 1988

Dear Parent/Guardian of Geo Trig 10 Students:

Your son or daughter is enrolled in one of our Geo Trig 10 classes. These two classes are participating in a study which I will be doing under the supervision of the University of British Columbia. The purpose of the study is to determine if computers can be used to improve the way students learn to write proofs. Over the years I have found that most students have difficulty with writing geometric proofs.

One class, which I will be teaching, will use computers and computer software as part of their course. The other class will use the standard method of learning geometry.

I will need to test both classes at the beginning of the semester and again after the proof writing section. Each test will require approximately 45 minutes to complete. The specific tests to be administered are:

- a geometry test based on the work covered in grade 8.
- a test to determine the geometric thought levels of the students. This test will be administered twice - beginning of semester and after proof writing.
- a final proof test.
- an attitude test.

In addition, I want to carry out regular interviews with four students which I will select from my class. These interviews will be done to find out how the students link the computer work to proof writing. The eight interviews will be one-half hour each, one every two weeks.

All information collected in this project is for research purposes only. To assure confidentiality, no family names will be used in any

report or release of the information. No personal, family or other sensitive information is being sought.

The parent or student may withdraw from this project at any time by a statement orally or in writing. Refusal to cooperate will have no consequences for the student.

I will appreciate very much the cooperation of the parents and students in this project. I will be happy to answer any questions you have regarding the project. I can be contacted through the school office. Please return the form at the bottom.

Thank you.

Jo Worster

I have read the above description of the research project entitled AN INVESTIGATION TO DETERMINE THE EFFECTS OF THE GEOMETRIC SUPPOSER SOFTWARE ON GEOMETRIC PROOF WRITING AT THE GRADE 10 LEVEL to be carried out by Mrs. Worster.

[ ] I consent [ ] I do not consent
to my child writing the written tests of the project.

[ ] I consent [ ] I do not consent
to my child being involved in the individual interviews to be conducted by Mrs. Worster.

_____ _____

Signature (Parent/Guardian) Student's name

Appendix B

INTRODUCTORY GEOMETRY TEST

AND

ITEM ANALYSIS

INTRODUCTORY GEO TRIG 10 TEST

Directions

Do not open this test until your are told to do so.

Please write your name on the line below.

_____

This test contains 25 questions. It is not expected that you will remember everything on this test.

When you are told to begin:

1) Read each question carefully.

2) There is only one correct answer to each question. Print neatly the letter of your choice on the line to the right of each question.

3) You will have 35 minutes for this test.

INTRODUCTORY GEO TRIG 10 TEST

1. The area of a rectangle with length 4 cm and width 11 cm is:
   a) 30 sq. cm
   b) 19 sq. cm
   c) 44 sq. cm
   d) 15 sq. cm
   e) 26 sq. cm _____

2. How many lines of symmetry does a square have?
   a) 2 only
   b) 4 only
   c) 6 only
   d) 8 only
   e) infinite _____

3. The measure of an acute angle is:
   a) 90^0
   b) between 45^0 and 90^0
   c) less than 90^0
   d) between 90^0 and 180^0
   e) more than 180^0 _____

4. Perpendicular lines:
   a) do not intersect.
   b) are two intersecting lines that form right angles.
   c) intersect to form three acute angles and one obtuse angle.
   d) intersect to form four acute angles.
   e) none of the above. _____

5. An equilateral triangle has:

a) all sides the same length.

b) all sides with different lengths.

c) two sides only with the same length.

d) all angles with different measures.

e) two acute angles and one obtuse angle. _____

6. If $\triangle BAD$ is similar to $\triangle RSV$, then $\angle A$ is congruent to which angle in $\triangle RSV$?

a) R only

b) S only

c) V only

d) SRV only

e) none of these _____

7. Given right $\triangle ABC$, sin A equals:

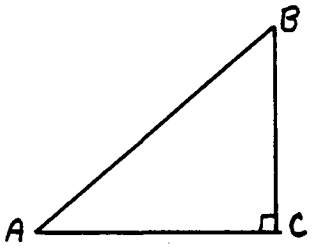
a) $\frac{BC}{AC}$

b) $\frac{AC}{BC}$

c) $\frac{AC}{AB}$

d) $\frac{AB}{AC}$

e) $\frac{BC}{AB}$



_____

8. If P is the center of the circle, segment $\overline{PC}$ is called the:

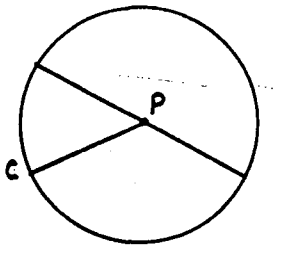
a) chord of the circle.

b) diameter of the circle.

c) segment of the circle.

d) radius of the circle.

e) minor arc of the circle.



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9. What is the reason that the two triangles below are congruent?

a) AAA (Angle-Angle-Angle Theorem)
b) AAS (Angle-Angle-Side Theorem)
c) SAS (Side-Angle-Side Theorem)
d) ASA (Angle-Side-Angle Theorem)
e) SSS (Side-Side-Side Theorem)

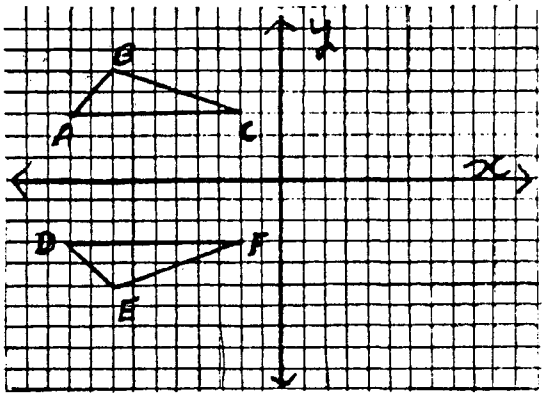
_____

10. In every circle, what is the ratio of the circumference to the diameter?

a) 22/7 (π)
b) 7/22 ($1/\pi$)
c) $(22/7)^2$
d) $(7/22)^2$
e) there is no constant ratio

_____

11. If Δ DEF is the reflection of Δ ABC in line x, what is the image of point B?



a) point D
b) point E
c) point F
d) no image point
e) point B

_____

12. The measure of the third angle in the triangle below is:

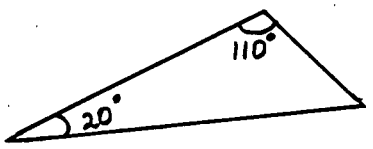
a) 50^0

b) 130^0

c) 20^0

d) 40^0

e) 60^0



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13. The length of the third side in the right triangle below is:

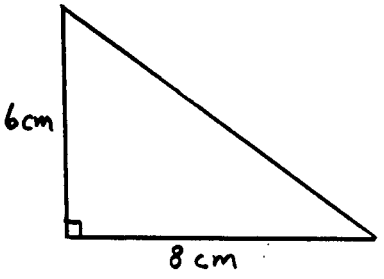
a) 8 cm

b) 10 cm

c) 12 cm

d) 14 cm

e) 16 cm



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14. The volume of the box shown is:

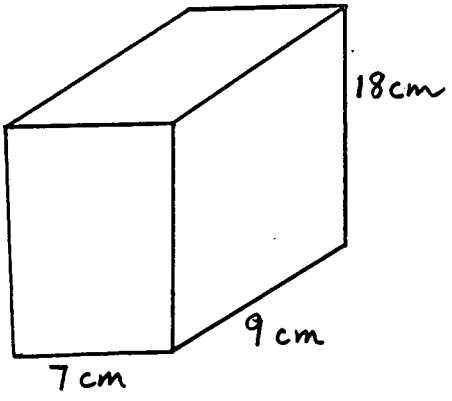
a) 63 cm^3

b) 126 cm^3

c) 162 cm^3

d) 1134 cm^3

e) 2268 cm^3



_____

15. The horizontal lines are parallel. The length of x is:

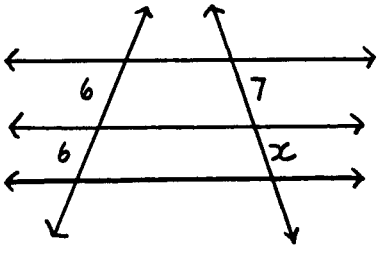
a) 12

b) 14

c) cannot be calculated

d) 6

e) 7



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16. The perimeter of the parallelogram below is:

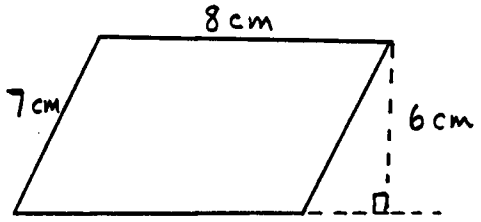
a) 48 cm

b) 56 cm

c) 30 cm

d) 29 cm

e) 28 cm



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17. ∠ABC is a right angle. ∠DBC measures 15^0. The measure of ∠ABD is:

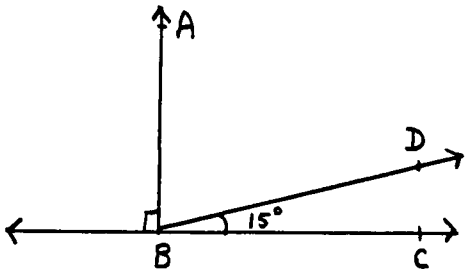
a) cannot be calculated

b) 105^0

c) 165^0

d) 75^0

e) 65^0



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18. A cube has how many edges?

a) 4

b) 8

c) 12

d) 16

e) 20

_____

19. Lines a and b are parallel. The measure of angle y is:

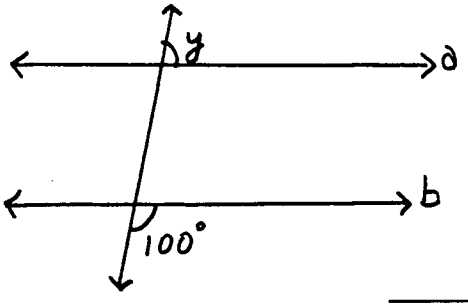
a) 100^0

b) 80^0

c) cannot be calculated

d) 90^0

e) 70^0



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20. Given the number line below, which statement is true?

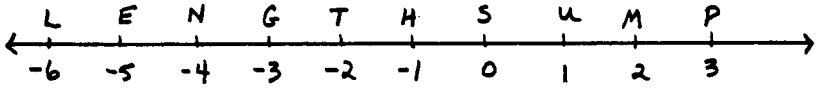
a) $\overline{US} \cong \overline{TE}$

b) $\overline{TG} \cong \overline{TS}$

c) $\overline{HT} \cong \overline{UP}$

d) $\overline{LG} \cong \overline{GU}$

e) $\overline{HG} \cong \overline{UH}$



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21. In the figure shown, the obtuse angles are:

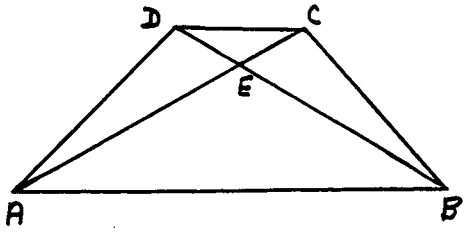
a) ∠ADC, ∠DCB, ∠DEC, ∠AEB

b) ∠ADC, ∠DEB, ∠DCB, ∠DEC

c) ∠AEB, ∠ABC, ∠BAD, ∠DEC

d) ∠AEB, ∠EAB, ∠EBA, ∠DEC

e) ∠AED, ∠BEC, ∠EDC, ∠ECD



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22. In parallelogram ABCD, point O is the midpoint on $\overline{AC}$. Using a 180^0 rotation (1/2 turn) around point O, the rotation image of $\overline{AB}$ is:

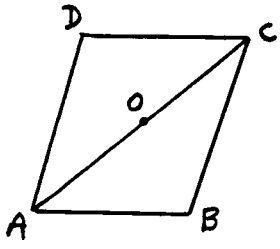
a) $\overline{BC}$

b) $\overline{CD}$

c) $\overline{AD}$

d) $\overline{AO}$

e) $\overline{AB}$



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23. The area of the triangle below is:

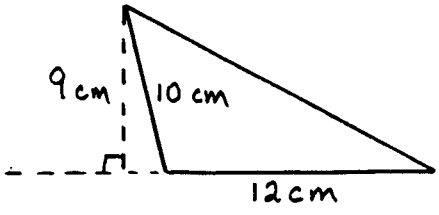
a) 120 cm^2

b) 108 cm^2

c) 60 cm^2

d) 54 cm^2

e) 31 cm^2



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24. The formula to find the volume of a right circular cylinder is $V = \pi r^2 h$. The volume of the figure below is:

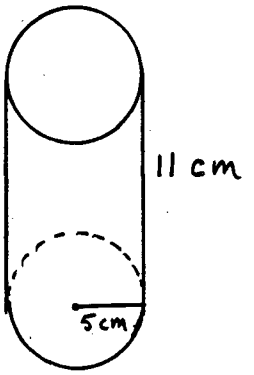
a) 605π cm^3

b) 55π cm^3

c) 275π cm^3

d) 27.5π cm^3

e) 3025π cm^3



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25. Angles a and b are:

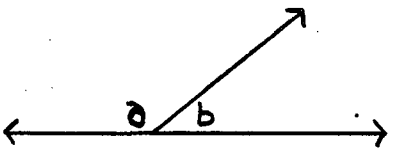
a) interior

b) exterior

c) vertical

d) complementary

e) supplementary



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Introductory Geometry Test

Item Analysis

Computer Group

Item Number	Percentage with choice A	B	C	D	E	Blank	% Correct
1	14	0	86	0	0	0	86
2	14	81	0	0	5	0	81
3	24	14	57	0	5	0	57
4	5	86	0	10	0	0	86
5	81	0	0	5	14	0	81
6	5	71	5	0	19	0	71
7							
8	0	0	10	90	0	0	90
9	5	23	19	48	5	0	48
10	52	10	5	0	33	0	52
11	0	90	0	5	5	0	90
12	48	14	14	19	5	0	48
13	5	43	10	33	10	0	43
14	10	0	0	90	0	0	90
15	0	0	33	10	57	0	57
16	10	14	57	19	0	0	57
17	19	0	0	67	14	0	67
18	5	43	48	0	5	0	48
19	24	33	29	10	5	0	33
20	0	5	10	19	62	5	62
21	29	10	19	14	29	0	29
22	10	62	5	10	5	10	62
23	24	19	10	38	10	0	38
24	5	19	57	5	10	5	57
25	5	0	5	24	57	10	57

Introductory Geometry Test

Item Analysis

Traditional Group

Item Number	Percentage with choice A	B	C	D	E	Blank	% Correct
1	16	0	84	0	0	0	84
2	21	47	0	11	21	0	47
3	26	11	47	11	5	0	47
4	26	47	0	0	26	0	47
5	84	0	11	0	5	0	84
6	0	84	5	11	0	0	84
7							
8	5	5	27	63	0	0	63
9	0	16	32	42	11	0	42
10	42	16	16	0	26	0	42
11	0	95	0	5	0	0	95
12	42	21	0	26	11	0	42
13	0	37	26	32	5	0	37
14	0	5	5	90	0	0	90
15	0	11	21	0	68	0	68
16	11	21	47	11	11	0	47
17	0	11	5	84	0	0	84
18	5	42	47	5	0	0	47
19	21	32	26	0	21	0	32
20	5	0	0	21	74	0	74
21	16	11	42	11	16	5	16
22	11	53	26	5	5	0	53
23	15	32	32	11	11	0	11
24	26	11	47	11	5	0	47
25	0	21	11	42	26	0	26

Appendix C

PERMISSION LETTER

AND

VAN HIELE GEOMETRY TEST

VAN HIELE GEOMETRY TEST

Directions

Do not open this test until your are told to do so.

Please write your name on the line below.

_____

This test contains 25 questions. It is not expected that you know everything on this test.

When you are told to begin:

1) Read each question carefully.

2) There is only one correct answer to each question. Place the letter of your choice on the line to the right of each question.

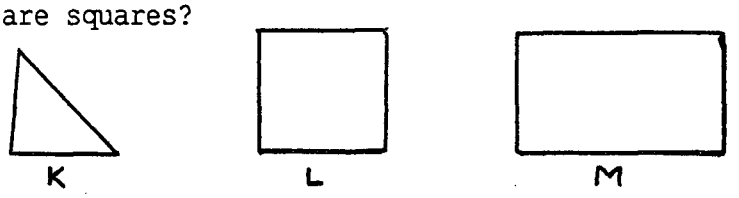
3) You will have 35 minutes for this test.

This test is based on the work of P.M. van Hiele.

Permission has been granted by Professor Zalman Usiskin, Director of the CDASSG Project at the University of Chicago to use this test.

VAN HIELE GEOMETRY TEST

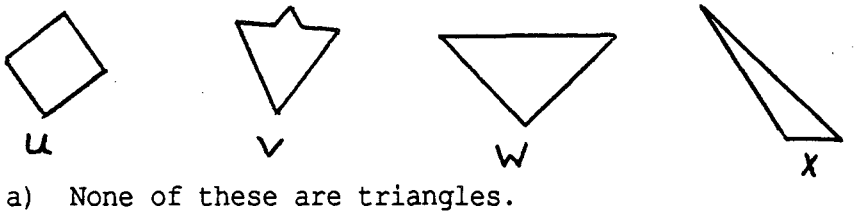
1. Which of these are squares?



a) K only
b) L only
c) M only
d) L and M only
e) All are squares.

_____

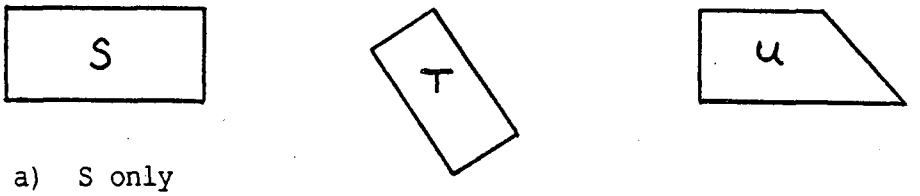
2. Which of these are triangles?



a) None of these are triangles.
b) V only
c) W only
d) W and X only
e) V and W only

_____

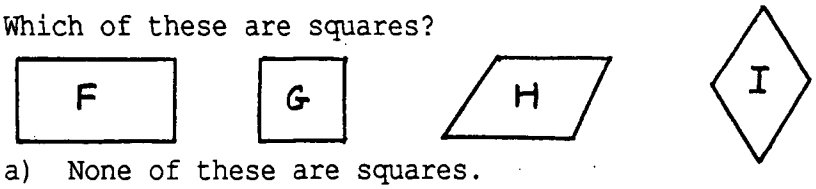
3. Which of these are rectangles?



a) S only
b) T only
c) S and T only
d) S and U only
e) All are rectangles.

_____

4. Which of these are squares?



a) None of these are squares.

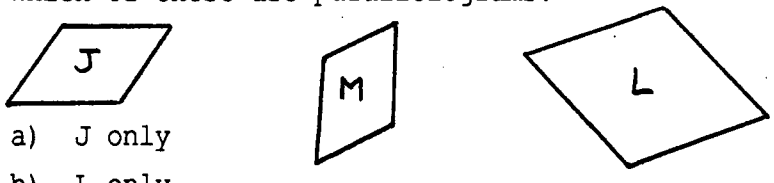
b) G only

c) F and G only

d) G and I only

e) All are squares. _____

5. Which of these are parallelograms?



a) J only

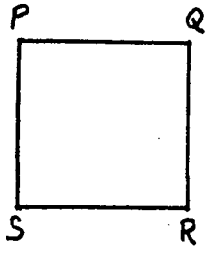
b) L only

c) J and M only

d) None of these are parallelograms.

e) All are parallelograms. _____

6. PQRS is a square.



Which relationship is true in all squares?

a) $\overline{PR}$ and $\overline{RS}$ have the same length.

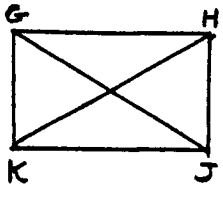
b) $\overline{QS}$ and $\overline{PR}$ are perpendicular.

c) $\overline{PS}$ and $\overline{QR}$ are perpendicular.

d) $\overline{PS}$ and $\overline{QS}$ have the same length.

e) Angle Q is larger than angle R. _____

7. In a rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the <u>diagonals</u>.



Which of (a) to (d) is <u>not</u> true in <u>every</u> rectangle?

a) There are four right angles.

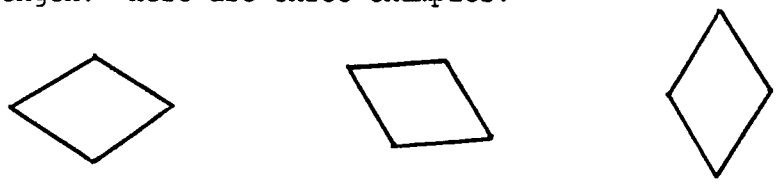
b) There are four sides.

c) The diagonals have the same length.

d) The opposite sides have the same length.

e) All of (a) to (d) are true in every rectangle. _____

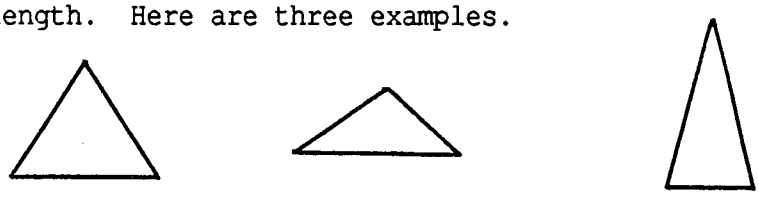
8. A <u>rhombus</u> is a 4-sided figure with all sides of the same length. Here are three examples.



Which of (a) to (d) is <u>not</u> true in every rhombus?

a) The two diagonals have the same length.
b) Each diagonal bisects two angles of the rhombus.
c) The two diagonals are perpendicular.
d) The opposite angles have the same measure.
e) All of (a) to (d) are true in every rhombus. _____

9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.

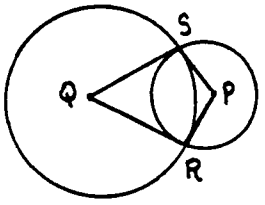


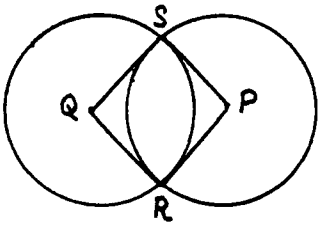
Which of (a) to (d) is true in every isosceles triangle?

a) The three sides must have the same length.
b) One side must have twice the length of another side.
c) There must be at least two angles with the same measure.
d) The three angles must have the same measure.
e) None of (a) to (d) is true in every isosceles triangle.

_____

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.





Which of (a) to (d) is <u>not</u> always true?

a) PRQS will have two pairs of sides of equal length.

b) PRQS will have at least two angles of equal measure.

c) The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.

d) Angles P and Q will have the same measure.

e) All of (a) to (d) are true. _____

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

a) If 1 is true, then 2 is true.

b) If 1 is false, then 2 is true.

c) 1 and 2 cannot both be true.

d) 1 and 2 cannot both be false.

e) None of (a) to (d) is correct. _____

12. Here are two statements.

Statement S: Δ ABC has three sides of the same length.

Statement T: In ΔABC, $\angle$B and $\angle$C have the same measure.

Which is correct?

a) Statements S and T cannot both be true.

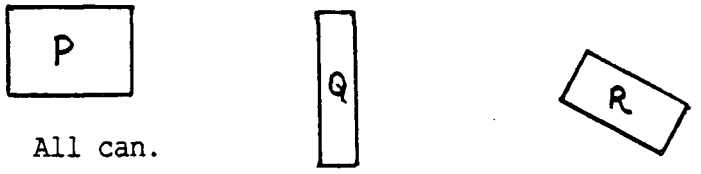
b) If S is true, then T is true.

c) If T is true, then S is true.

d) If S is false, then T is false.

e) None of (a) to (d) is correct. _____

13. Which of these can be called rectangles?



a) All can.
b) Q only.
c) R only.
d) P and Q only.
e) Q and R only. _____

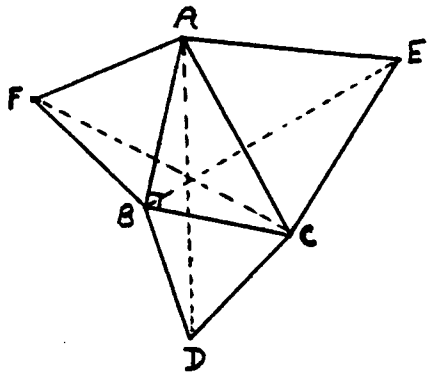
14. Which is true?

a) All properties of rectangles are properties of all squares.
b) All properties of squares are properties of all rectangles.
c) All properties of rectangles are properties of all parallelograms.
d) All properties of squares are properties of all parallelograms.
e) None of (a) to (d) is true. _____

15. What do all rectangles have that some parallelograms do not have?

a) opposite sides equal
b) diagonals equal
c) opposite sides parallel
d) opposite angles equal
e) none of (a) to (d) _____

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

a) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common.
b) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
c) In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
d) In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
e) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common. _____

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

a) D implies S which implies R.
b) D implies R which implies S.
c) S implies R which implies D.
d) R implies D which implies S.
e) R implies S which implies D. _____

18. Here are two statements.

I. If a figure is a rectangle, then its diagonals bisect each other.

II. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

a) To prove **I** is true, it is enough to prove that **II** is true.
b) To prove **II** is true, it is enough to prove that **I** is true.
c) To prove **II** is true, it is enough to find one rectangle whose diagonals bisect each other.
d) To prove **II** is false, it is enough to find one non-rectangle whose diagonals bisect each other.
e) None of (a) to (d) is correct. _____

19. In geometry:

a) Every term can be defined and every true statement can be proved true.
b) Every term can be defined but it is necessary to assume that certain statements are true.
c) Some terms must be left undefined but every true statement can be proved true.
d) Some terms must be left undefined and it is necessary to have some statements which are assumed true.
e) None of (a) to (d) is correct. _____

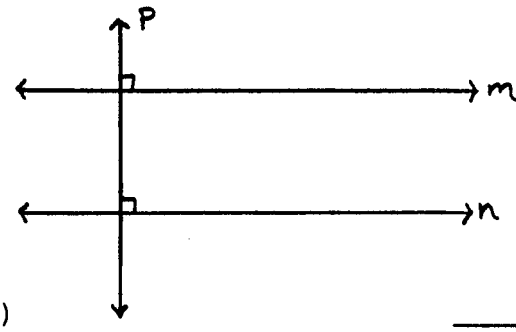
20. Examine these three sentences.

i) Two lines perpendicular to the same line are parallel.

ii) A line that is perpendicular to one of two parallel lines is perpendicular to the other.

iii) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?



a) (i) only

b) (ii) only

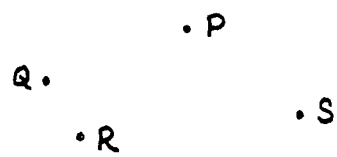
c) (iii) only

d) Either (i) or (ii)

e) Either (ii) or (iii)

_____

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are {P,Q}, {P,R}, {P,S}, {Q,R}, {Q,S}, and {R,S}.



Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

a) {P,R} and {Q,S} intersect.

b) {P,R} and {Q,S} are parallel.

c) {Q,R} and {R,S} are parallel.

d) {P,S} and {Q,R} intersect.

e) None of (a) to (d) is correct.

_____

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

a) In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
b) In general, it is impossible to trisect angles using only a compass and a marked ruler.
c) In general, it is impossible to trisect angles using any drawing instruments.
d) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
e) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

_____

23. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180^0.

Which is correct?

a) J made a mistake in measuring the angles of the triangle.
b) J made a mistake in logical reasoning.
c) J has a wrong idea of what is meant by "true."
d) J started with different assumptions than those in the usual geometry.
e) None of (a) to (d) is correct.

_____

24. Two geometry books define the word rectangle in different ways.

Which is true?

a) One of the books has an error.
b) One of the definitions is wrong. There cannot be two different definitions for rectangle.
c) The rectangles in one of the books must have different properties from those in the other book.
d) The rectangles in one of the books must have the same properties as those in the other book.
e) The properties of rectangles in the two books might be different.

25. Suppose you have proved statements **I** and **II**.

I. If p, then q.
II. If s, then not q.

Which statement follows from statements **I** and **II**?

a) If p, then s.
b) If not p, then not q.
c) If p or q, then s.
d) If s, then not p.
e) If not s, then p.

Appendix D

PROOF TEST

AND

ITEM ANALYSIS

GEO TRIG 10 PROOF TEST

Name_____

Date today_____

Your birthdate_____ , _____ , _____

Month Day Year

Directions:

- You will have 35 minutes to complete this test.
- All answers should be written on these pages.
- Partial credit will be given so do the best you can on each question.

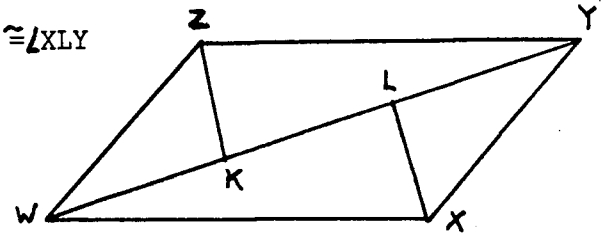
GEO TRIG 10 PROOF TEST

1. Complete the following proof:

GIVEN: $\overline{WZ} \parallel \overline{XY}$, XL = ZK, $\angle ZKW \cong \angle XLY$

PROVE: $\angle WZK \cong \angle YXL$

PROOF:



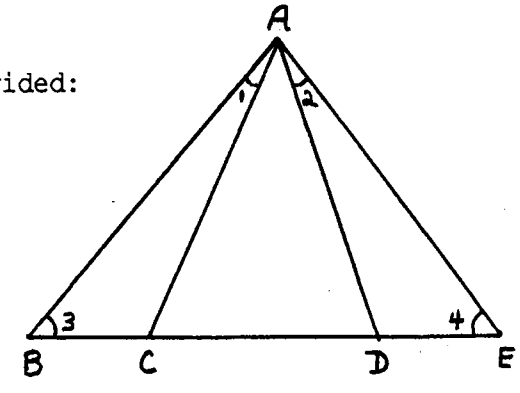
Statements	Reasons
a) $\overline{WZ} \parallel \overline{XY}$, XL = ZK, $\angle ZKW \cong \angle XLY$	_____
b) _____	If parallel lines, then alternate interior angles congruent.
c) $\Delta WKZ \cong \Delta YLX$	_____
d) _____	_____

2. Write the proof in the space provided:

GIVEN: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE: $\Delta ABC \cong \Delta AED$

PROOF:



3. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

To prove the above statement:

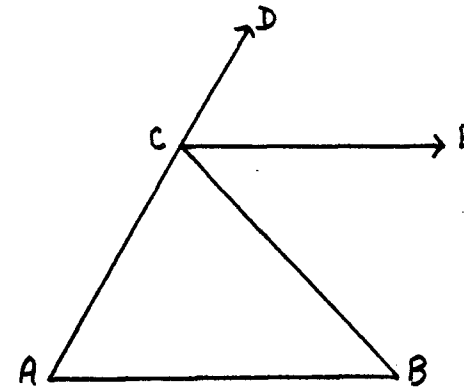
a) Draw and label a diagram.
b) Write what is given and what is to be proved in terms of your diagram.
c) Write the proof.

4. Write the proof for the following:

GIVEN: AC = BC, $\angle DCE \cong \angle B$

PROVE: $\overrightarrow{CE} \parallel \overline{AB}$

PROOF:

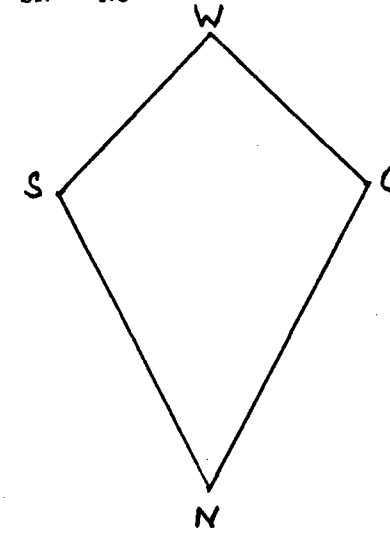


5. Write this proof below:

GIVEN: Quadrilateral SNOW with SW = WO, SN = NO

PROVE: $\angle S \cong \angle O$

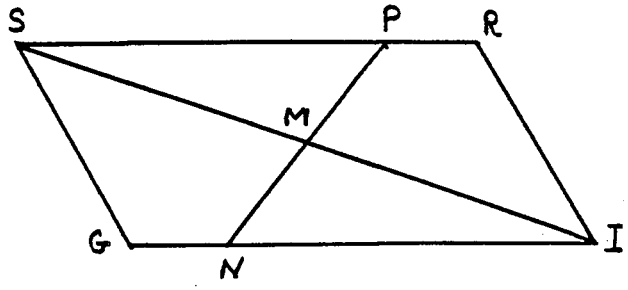
PROOF:



6. Write this proof in the space provided below:

GIVEN: Quadrilateral SRIG with SR = GI, SG = RI, $\overline{PN}$ bisects $\overline{SI}$ at M.

PROVE: PM = MN



PROOF:

Proof Test

Item Analysis

Computer Group

Question Number	Percent of students receiving this score 0	1	2	3	4
1	0	14	45	5	36
2	32	14	9	14	32
3	68	14	9	5	5
4	41	18	9	9	23
5	5	9	18	9	59
6	36	23	36	5	0

Item Analysis

Traditional Group

Question Number	Percent of students receiving this score 0	1	2	3	4
1	0	6	63	13	19
2	63	25	6	0	6
3	56	38	0	0	6
4	88	0	6	6	0
5	63	0	13	6	19
6	50	25	19	6	0

Appendix E

ATTITUDE TEST, ITEM ANALYSIS

AND

SUMMARY OF ITEM STATISTICS

GEO TRIG 10

June 1988

Draw a circle around the letter(s) that show(s) how closely you agree with each statement: SD (Strongly Disagree), D (Disagree), U (Undecided), A (Agree), SA (Strongly Agree).

1. I am always under a terrible strain in GeoTrig.	SD	D	U	A	SA
2. Geo Trig is very interesting to me, and I enjoyed this course.	SD	D	U	A	SA
3. Geo Trig is fascinating and fun.	SD	D	U	A	SA
4. Geo Trig makes me feel secure, and at the same time it is stimulating.	SD	D	U	A	SA
5. My mind goes blank, and I am unable to think clearly when working in Geo Trig.	SD	D	U	A	SA
6. I feel a sense of insecurity when attempting Geo Trig.	SD	D	U	A	SA
7. Geo Trig makes me feel uncomfortable, restless, irritable, and impatient.	SD	D	U	A	SA
8. The feeling that I have toward Geo Trig is a good feeling.	SD	D	U	A	SA
9. Geo Trig makes me feel as though I'm lost in a jungle of information and can't find my way out.	SD	D	U	A	SA
10. Geo Trig is something which I enjoy a great deal.	SD	D	U	A	SA

11. When I hear the word math, I have a feeling of dislike.	SD	D	U	A	SA
12. I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.	SD	D	U	A	SA
13. I really like mathematics.	SD	D	U	A	SA
14. Mathematics is a course in school which I have always enjoyed studying.	SD	D	U	A	SA
15. It makes me nervous to even think about having to do a math problem.	SD	D	U	A	SA
16. I have never liked math, and it is my most dreaded subject.	SD	D	U	A	SA
17. I am happier in a math class than in any other class.	SD	D	U	A	SA
18. I feel at ease in mathematics, and I like it very much.	SD	D	U	A	SA
19. I feel a definite positive reaction to mathematics; it's enjoyable.	SD	D	U	A	SA
20. I do not like mathematics, and it scares me to have to take it.	SD	D	U	A	SA
21. Mathematics has contributed greatly to science and other fields of knowledge.	SD	D	U	A	SA
22. Mathematics is less important to people than art or literature.	SD	D	U	A	SA

23. Mathematics is not important for the advance of civilization and society.	SD	D	U	A	SA
24. Mathematics is a very worthwile and necessary subject.	SD	D	U	A	SA
25. Mathematics is not important in everyday life.	SD	D	U	A	SA
26. Mathematics is needed in designing practically everything.	SD	D	U	A	SA
27. Mathematics is needed in order to keep the world running.	SD	D	U	A	SA
28. There is nothing creative about mathematics; it's just memorizing formulas and things.	SD	D	U	A	SA

Please write or print your reactions to the following questions:

1. What I liked most about this course was:

2. What I disliked most about this course was:

3. I would like to make the following suggestions:

Attitude Test

Item Analysis

Computer Group

Item Number	Percentage with choice SD	D	U	A	SA
1	10	52	29	10	0
2	5	24	14	43	14
3	10	19	43	24	5
4	10	48	19	19	5
5	14	38	19	24	5
6	14	38	19	14	14
7	10	29	29	24	10
8	0	14	52	29	5
9	0	57	5	33	5
10	5	33	19	38	5
11	14	52	10	10	14
12	19	43	19	14	5
13	0	14	38	33	14
14	14	19	24	33	10
15	14	71	5	10	0
16	38	43	10	5	5
17	19	29	29	19	5
18	10	33	29	24	5
19	0	24	33	38	5
20	24	52	10	14	0
21	0	0	10	43	48
22	10	48	14	19	5
23	52	38	0	5	5
24	0	0	0	57	43
25	43	48	0	5	5
26	0	5	0	52	43
27	0	0	10	57	33
28	19	43	14	10	14

Attitude Test

Item Analysis

Traditional Group

Item Number	Percentage with choice SD	D	U	A	SA
1	6	29	29	29	6
2	24	24	12	35	6
3	18	18	29	29	6
4	18	29	41	12	0
5	12	35	6	41	6
6	12	41	18	18	12
7	12	18	18	47	6
8	24	24	18	29	6
9	18	35	12	18	18
10	12	29	18	35	6
11	12	59	6	6	18
12	12	59	0	12	18
13	18	6	24	47	6
14	12	18	24	35	12
15	6	65	24	6	0
16	12	65	6	12	6
17	24	41	35	0	0
18	18	18	29	29	6
19	24	18	12	41	6
20	35	41	12	12	0
21	0	0	24	24	53
22	29	35	18	18	0
23	53	47	0	0	0
24	0	0	18	35	47
25	47	47	6	0	0
26	0	0	6	59	35
27	0	0	18	47	35
28	18	41	18	18	6

Attitude Test

Summary of Item Statistics

Item	Total		Computer Group		Traditional Group	
Number	Mean	SD	Mean	SD	Mean	SD
1	3.34	0.97	3.62	0.81	3.00	1.06
2	3.11	1.27	3.38	1.16	2.77	1.35
3	2.92	1.10	2.95	1.02	2.88	1.22
4	2.55	1.01	2.62	1.07	2.47	0.94
5	3.21	1.19	3.33	1.16	3.06	1.25
6	3.24	1.26	3.24	1.30	3.24	1.25
7	2.95	1.16	3.05	1.16	2.82	1.19
8	3.00	1.07	3.24	0.77	2.71	1.31
9	3.16	1.22	3.14	1.06	3.18	1.43
10	3.00	1.12	3.05	1.07	2.94	1.20
11	3.42	1.29	3.43	1.29	3.41	1.33
12	3.47	1.22	3.57	1.12	3.35	1.37
13	3.34	1.07	3.48	0.93	3.18	1.24
14	3.11	1.23	3.05	1.24	3.18	1.24
15	3.82	0.73	3.91	0.77	3.71	0.69
16	3.87	1.07	4.05	1.07	3.65	1.06
17	2.40	1.03	2.62	1.16	2.12	0.78
18	2.84	1.13	2.81	1.08	2.88	1.22
19	3.08	1.12	3.24	0.89	2.88	1.36
20	3.92	0.97	3.86	0.96	4.00	1.00
21	4.34	0.75	4.38	0.67	4.29	0.85
22	3.55	1.08	3.38	1.07	3.77	1.09
23	4.40	0.86	4.29	1.06	4.53	0.51
24	4.37	0.63	4.43	0.51	4.29	0.77
25	4.29	0.87	4.19	1.03	4.41	0.62
26	4.32	0.66	4.33	0.73	4.29	0.59
27	4.21	0.66	4.24	0.63	4.18	0.73
28	3.45	1.25	3.43	1.33	3.47	1.18

Appendix F

PERMISSION LETTER FOR INTERVIEWS

February 2, 1988

Dear Geo Trig 10 Student:

During this semester I will be conducting a study in Geo Trig 10 under the supervision of the University of British Columbia. The purpose of the study is to determine if computers can be used to improve the way students learn to write proofs. Over the years I have found that most students have difficulty with writing geometric proofs.

Besides administering certain tests to the class, I would like to interview you every two weeks for one-half hour. There will be a total of eight interviews. The purpose of these interviews is to find out more specifically how you relate the computer work to proof writing. The interviews will be tape-recorded so that I may analyze them further at a later time.

All information collected in this project is for research purposes only. To assure confidentiality, no family names will be used in any report or release of the information. No personal, family or other sensitive information is being sought. You may withdraw from this project at any time by a statement orally or in writing. Refusal to cooperate will have no consequences for you.

If you wish any further information please ask me and I will be happy to answer any questions you have regarding the project.

If you agree to being interviewed, please check the most convenient time for your interview on the form on the next page and return to me.

Thank you.

Mrs. J. Worster

I have read the above description of the research project entitled AN INVESTIGATION TO DETERMINE THE EFFECTS OF THE GEOMETRIC SUPPOSER SOFTWARE ON GEOMETRIC PROOF WRITING AT THE GRADE 10 LEVEL to be carried out by Mrs. Worster.

[ ] I consent [ ] I do not consent
to being interviewed every two weeks for one-half hour during second semester.

The best time for my interview is:
[ ] 8:15 A.M. - 8:45 A.M. [ ] 12:15 P.M. - 12:45 P.M.

[ ] 3:45 P.M. - 4:15 P.M.

Signature (Student)