HOUSING DEMAND AND TAXATION

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Abstract

This dissertation has two primary objectives: (1) to develop and test empirically a model of housing demand complete with tenure choice and moving costs, and (2) to demonstrate how this model can be used to evaluate alternative housing-related tax changes.

The proposal evaluated here is the introduction of mortage interest deductibility in Canada. Similar income tax deductions have long been in effect in the United States and Great Britain. However, this model rejects the proposal on both equity and efficiency grounds.

The housing demand model was tested empirically using a sample of households from the Toronto metropolitan area. These results confirm that transactions costs and other barriers to residential mobility are a vital component of the households' decision-making process.

This key empirical result is not only important in the context of the demand for housing. It also impinges on the equity and efficiency of proposed tax changes. It is shown in this thesis that the deadweight loss attributable to mortgage interest subsidies are not as severe as has sometimes been claimed, particularly in the short run. The reason is that subsidies are effectively transformed into lump sum grants when residential immobility is high. And there is no deadweight loss due to lump sum grants.

The main findings of this thesis may therefore be summarized as follows: (1) the housing demand decision is best

understood when barriers to residential mobility are modelled explicitly, and (2) the presence of these barriers must be taken into consideration when calculating the short run welfare implications of proposed housing-related tax changes.

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I. Introduction

A. Overview

This dissertation has two primary objectives: (1) to develop and test empirically a model of housing demand complete with tenure choice and moving costs, and (2) to demonstrate how this model can be used to evaluate alternative housing related tax reforms. In pursuit of these goals, the work presented here extends and synthesizes two distinct streams in the housing economics literature.

The first is represented by work in the tradition of Laidler (1969), Aaron (1972), Rosen (1979), and King (1980,1981,1983). Each of these authors developed a model of housing demand which they used to analyze the implications of a price subsidy to owner-occupiers. This subsidy is rooted in the American and British income tax provisions which allow households to deduct mortgage interest payments and other selected housing expenditures from taxable income.

The second stream of literature focuses on moving costs, and is best represented by work in the tradition of Hanushek and Quigley (1978), Weinburg, Friedman, and Mayo (1981), and Venti and Wise (1984). The housing demand models developed by these authors express current period housing consumption in terms of a stock adjustment process, where moving costs may impede that adjustment.

By merging these two separate streams, the beneficial aspects of each are pooled. Incorporating moving costs results in a more realistic and robust model of housing demand, particularly when demographic factors are instrumental in determining household mobility. And this in turn leads to better estimates of the welfare and efficiency effects of a price subsidy to owner-occupiers.

The next two sections of this chapter discuss in more detail how this work builds upon and extends the contributions of earlier authors. They are followed by the final section, which provides a brief synopsis of each of the remaining chapters.

B. Housing Demand and Mortgage Interest Deductibility

A branch of the housing taxation literature which is closely related to this study began with Laidler's (1969) classic paper on the welfare losses associated with 'hidden' federal income tax subsidies to homeowner-occupiers. In the United States, homeowners are allowed to deduct mortgage interest payments and state and local property taxes from household taxable income, yet they are not required to claim the imputed rental income which they derive from homeownership.

Working with aggregate housing demand data, Laidler estimated the aggregate excess burden associated with these subsidies. Aaron (1972) used micro data to examine the impact that eliminating the tax subsidy would have on homeowners and to analyze how the resulting changes in tax liabilities would vary

with income levels. Neither Laidler (1969) nor Aaron (1972) examined the interaction between rental and homeownership markets.

It remained for Rosen (1979) to develop a framework for analyzing household responses to tax changes where households are free to switch tenures as well as to change the quantity of housing demanded. He then examines the distribution of excess burden among gross income groups. Like Lee and Trost (1978), Rosen examines the tenure choice and conditional demand decisions within a likelihood framework.

Three papers by M.A. King (1980,1981,1983) taken together present the most sophisticated approach to date in measuring the welfare costs associated with these tax subsidies for owner-occupiers. In the first of these papers, King (1980) develops an econometric model of housing demand and tenure choice in the United Kingdom. His is the first model in which both the tenure choice and the conditional demand decisions are explicitly derived from one set of preferences.

He uses this model in the two subsequent papers to evaluate the distribution of gains and losses associated with the introduction of a tax on the imputed rental income from owner-occupation. These distributions are assessed either by an inequality index (King,1981), or directly through a social welfare function (King, 1983).

This dissertation differs from these earlier contributions in several respects. Most importantly, the model developed here recognizes explicitly that moving costs will persuade many

households to remain stationary despite changes in relative prices and income.

The models discussed above implicitly assume that households adjust their level of housing consumption effortlessly from one period to the next in the face of changing prices and expenditures. Whereas in this model, the empirical evidence suggests that demographic variables are the chief determinants of whether a household relocates, and only then does it look to traditional economic variables as it determines how much accommodation to consume.

It is useful in itself that the model presented here gives us a better representation of how the housing decision is made. But more importantly, introducing this feature into the model results in a greatly reduced estimate of the short run deadweight loss resulting from the introduction of the price subsidy.

When the disutility of moving is high, price subsidies are less likely to induce households to readjust their level of housing consumption. When this happens, the price subsidies are effectively transformed into lump sum grants. And there is no deadweight loss associated with lump sum grants.

This state of affairs is likely to hold only over the short run. Over the longer run, more and more households will relocate for reasons unrelated to the subsidy. However, once they do move, the results presented here suggest that they are likely to be influenced by the subsidy when deciding how much housing to consume.

The tenure choice decision is complicated somewhat by this extension. Instead of two possible tenures there are three; households may choose to (o) move into owner-occupied housing, (r) move into rental accommodation, or (s) remain stationary in any given period. Demographic variables play an important part in this move-stay decision.

In order to make a direct comparison of a household's potential utility under each tenure, the model is restricted to functional forms for which both the indirect and the direct utility functions may be written explicitly. This rules out the translog forms favored by King and by Rosen.

Finally, the empirical application of this model is set in a Canadian context. Earlier studies with a similar research focus have been applied to the British and American housing markets, where mortgage interest deductibility has long been part of their respective tax systems. In Canada, there is no mortgage interest or property tax deductibility for homeowners. The tax policy simulated here, therefore, is the introduction rather than the removal of the tax reform.

C. Residential Mobility

Most models of housing demand, including those developed by Rosen (1979) and King (1980) do not incorporate moving costs explicitly. The standard model of housing demand is often nothing more than a single equation describing housing consumption as a log-linear function of housing prices and income.[1] In some cases these models recognize the potential for

disequilibrium in the housing market by including only recent movers in the sample.

A unique experiment conducted in the United States, the Housing Allowance Demand Experiment (HADE)[2], has served as impetus for a number of studies on moving costs and housing demand. These studies all focus on the move-stay decision faced by low income renters in Pittsburg and Phoenix.

Hanushek and Quigley (1978) analyze housing disequilibrium in terms of a stock adjustment process. They use probit analysis to estimate the probability of a move; where that probability increases monotonically with the gap between desired and actual housing consumption.

Weinburg, Friedman, and Mayo (1981) use the same data set, but cast their moving costs explicitly in terms of household utility maximizing behavior. That is, households move only if the gain in utility from readjusting housing consumption outweighs the disutility of incurring transactions costs. They determine whether this is the case by comparing the change in estimated Marshallian consumer surplus with the estimated transactions costs resulting from a move.

Venti and Wise (1984) present a very thorough analysis based on this same HADE data set. They model the move-stay decision and the conditional housing demand decision interactively, rather than in a two stage procedure. Their focus is on the effectiveness of various rental subsidy programs conducted within HADE. They demonstrate that high moving costs overwhelm the programs' incentives to increase housing

consumption, thereby limiting the effectiveness of those programs.

The research undertaken for this thesis is very much in the spirit of these authors, but differs in a number of ways. For example, the analysis here is not confined to renters alone; thus, tenure choice is introduced. The HADE studies model a bimodal move-stay decision. This contrasts with the bimodal own-rent decision modelled by Rosen (1979) and King (1980), as discussed in the preceding section. This research integrates both aspects, thus yielding the trimodal decision; move/own, move/rent, or stay.

Similarly, the empirical focus for this research is less restricted. The data used here[3] comprise a sample of households from the Toronto Census Metropolitan Area (CMA), representing all tenures and income groups. The resulting estimates are therefore of wider applicability than those from the HADE studies.

Finally, this model of housing demand is applied to examine the welfare, tenure, mobility, and efficiency impacts of price subsidies to owner-occupiers. Such was not the intention of the HADE studies, which examine the efficacy (but not the welfare or tenure effects) of rental subsidies.

D. Chapter Synopses

This thesis comprises seven chapters. Chapter two formulates the theoretical model of housing demand. In any given period the household faces three basic options or tenures; it may move into ownership or rental accommodation or it may remain stationary. Each of these options is associated with an indirect 'subutility' index defined over the household's prices, net income, and demographic status. The household chooses that option from which; given prices, income, and moving costs; it derives the greatest utility.

Chapter three specifies a likelihood function which is jointly concerned with the tenure decision and the quantity of housing services demanded conditional upon tenure choice. Both decisions are modelled within the framework of a linear expenditure system.

Chapter four describes the data and variables used in estimating the model's parameter values. It employs a relatively new data set, the Social Change in Canada: Quality of Life Survey. It contains panel data on many pertinent financial and housing variables which may be readily incorporated into our model.

The estimation procedures and results are described in chapter five. The results from the maximum likelihood estimation confirm the importance of including moving costs in the model. And demographic variables are shown to have a major influence on the likelihood of a move.

These results have their counterpart in chapter six, which

applies the model's findings to the evaluation of income tax subsidies to homeowners. Using the standard model, where there are no moving costs, the subsidy results in substantial deadweight loss and in a less equitable distribution of household welfare. In contrast, the transactions model shows an almost negligible short run impact, due to the paucity of movers.

Chapter seven summarizes the research findings and outlines possibilities for further research.

NOTES TO CHAPTER ONE

1. For a good review of these standard models, see Mayo (1981).

2. The Housing Allowance Demand Experiment was funded by the United States Department of Housing and Urban Development. Much of the HADE research reported on here was conducted under the auspices of Abt Associates Inc.

3. The data used in this thesis was generated by the Social Change in Canada Project directed by Tom Atkinson, Bernard Blishen, Michael Ornstein, and H. Michael Stevenson of York University, Toronto. The research was supported by the Social Sciences and Humanities Research Council of Canada (Grant # S75-0332). Responsibility for the interpretations presented here is mine alone.

II. Modelling Housing Demand

A. The Basic Model

At the beginning of each period a household has three basic housing options, denoted by subscript j. It may move into ownership accommodation (j=o), move into rental accommodation (j=r), or remain in its present location (j=s). The household selects that option which it expects will yield the greatest utility over the forthcoming period.[1]

In making this decision, the household reviews economic factors, demographic attributes, and intangible considerations of residential attachment. Price (p_j), expenditure levels (e_j) on housing and other consumables, and moving costs associated with each tenure (j=o,r,s) are the main economic factors. Demographic attributes (d) include age and marital status of household head and family size.

Three indirect subutility indexes (V_j) correspond to the three basic tenure choices. For the two options which entail relocation (m=o,r), the indirect utility for household h[2] is given by

$$V_m^h = V_m(p_m, e_m)^h = \max_x \; U(x_m, x_c)^h - \kappa(d)^h \qquad (2.1)$$

$$\text{subject to} \quad p_m x_m^h + x_c^h \leq e_m^h$$

where x_m^h is the quantity of accommodation services consumed by household h in tenure m, x_c^h is the quantity of other goods consumed, and U is the direct utility function net of relocation.

The expression $\kappa(d)$ gives the disutility associated with residential relocation. The standard model of tenure choice as developed by Lee and Trost (1978), Rosen (1979), and King (1980) is obtained by setting $\kappa(d) = 0$ for all h. In the model described here, the transactions term $\kappa(d)$ expresses the total disutility of a move[3] as a function of household demographic characteristics[4].

If the household does not move, then its housing consumption is fixed at x_s^h,[5] and the remainder of its expenditures for the period go to other consumables. Thus, the indirect utility associated with the stay option is defined by

$$V_s = V_s(p_s, e_s)^h = U[x_s, (e_s - p_s x_s)]^h \tag{2.2}$$

The indirect utility function for the household is the supremum of these indirect subutility indexes. That is,

$$V^h = V(p,e)^h = \max\,[V_o, V_r, V_s]^h \tag{2.3}$$

This view of housing demand bears a certain kinship to Muth's (1974). In his model, the decision to move is an exogenous shock based largely on demographic changes. But expected price and expenditure paths over the expected length of tenure determine the quantity of accommodation sought.

B. Alternative Representations of Moving Costs

In the model just described, moving costs are measured in terms of foregone utility, $\kappa(d)$. Alternatively, one could express moving costs in terms of foregone expenditures, TC(d). Specifically, let TC(d) be defined as the monetary transactions costs which are equivalent to $\kappa(d)$, so that[6]

$$V_m(p_m, e_m - TC) + \kappa = V_m(p_m, e_m) \tag{2.4a}$$

The relationship between these two measures can also be expressed in terms of the expenditure function;

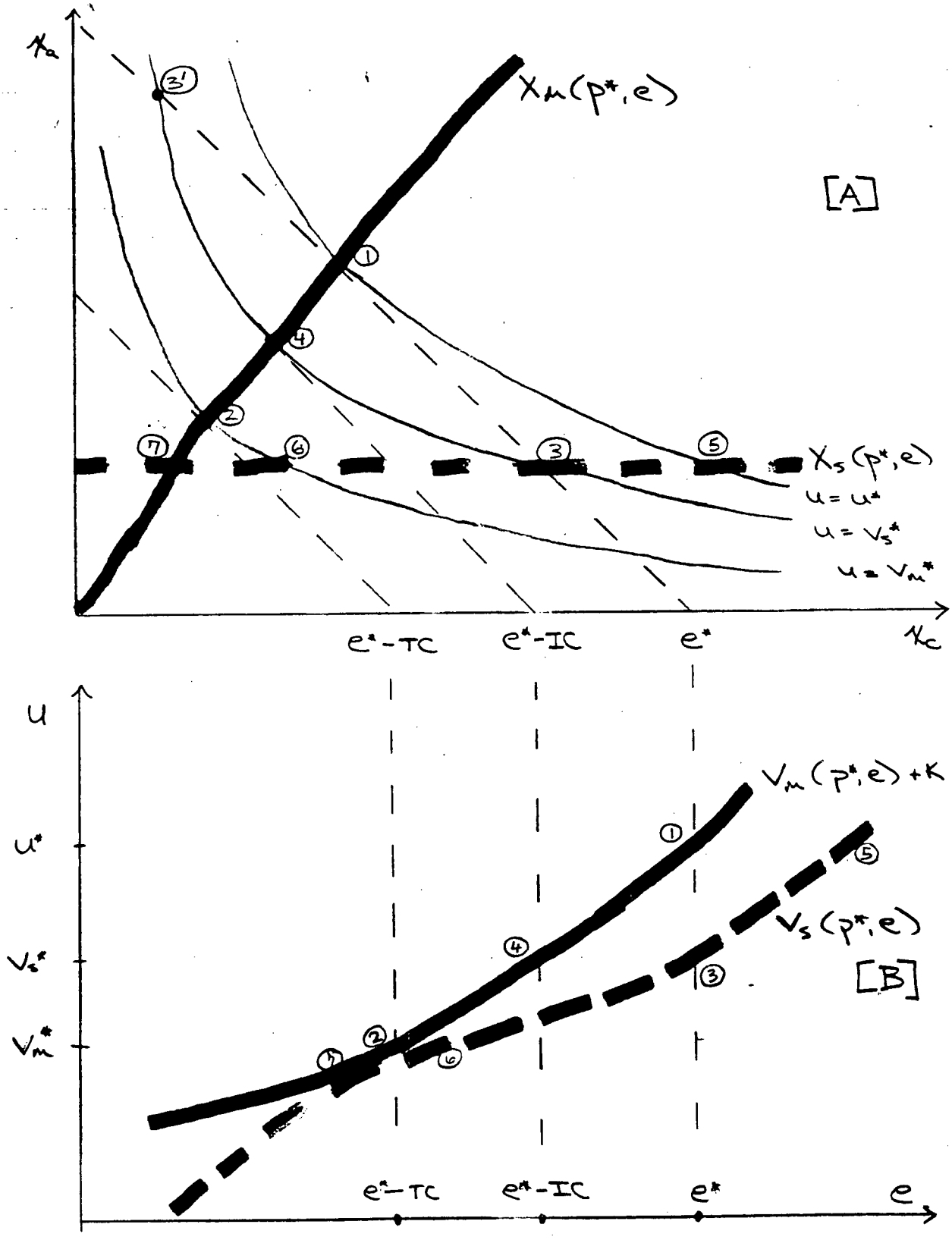
$$TC = e_m(p_m, V_m) - e_m(p_m, V_m - \kappa) \tag{2.4b}$$

These alternative representations are illustrated in figures 1a and 1b.

U* denotes the maximum level of utility attainable in a world without moving costs, given prices p* and expenditure e*. This corresponds to point (1), the mover's choice, in figure 1a. But in our model, because of transactions disutilities,

Figure 1

Different Measures of Transactions Costs



the mover's choice yields a utility of only $V_m^* = U^*-\kappa$. $U^*-\kappa$ is also the maximum utility which a household could attain with expenditures of e^*-TC in a world with no moving costs, as represented by point (2).

The utility and expenditure level (u,e) for each point in 1a has its counterpart in 1b.[7] Both points (1) and (2) lie on the movers' Engel curve, $X_m(p^*,e)$, in 1a, and so are found on its counterpart, $V_m(p^*,e)+\kappa$, in 1b. TC and κ are represented respectively by the horizontal and vertical distances between these two points in figure 1b.

Figures 1a and 1b may also be used to illustrate the difference between our measure of moving costs and those used by other authors. In this regard, it is useful to observe that (3) in figure 1a is the consumption bundle which the household chooses if it wishes to remain in its current premises given p* and e*. In this representation the household will indeed remain stationary because $V_s^* > V_m^*$.

Weinberg, Friedman, and Mayo (1981) define the income compensation IC for the household (using our notation) by

$$V_s(p^*,e^*) = V_m(p^*,e^*-IC)+\kappa \qquad (2.5)$$

That is, IC is the maximum amount which the household could spend on transactions costs and still be as well off moving as

not.[8] It is represented in figures 1a and 1b by the horizontal distance between points (1) and (4).

IC is a measure of loss for those households who remain stationary, relative to a world without moving costs. TC is a measure of loss for those households who choose to move and suffer the resulting transactions disutilities, relative to the same benchmark.[9] Then, in accordance with Weinberg et al (1981), a household moves only if IC > TC. When $p_j = p^*$ and $e_j = e^*$ for all tenures (j), their condition is equivalent to our condition for a move; namely that $V_m^* > V_s^*$.[10]

There are any number of other measures which could be used to evaluate the relative costs of moving and staying. For example, instead of the income compensation used by Weinberg et al (1981), one could define an analogous equivalent compensation EC as follows:

$$V_s(p^*, e^* + EC) = U^* \qquad (2.6)$$

This is the additional income which would just compensate the household for remaining stationary, were there no moving costs. It measures the distance between U^* and V_s^*, but along the stayers' Engel curve, $X_s(p^*, e^*)$, rather than along $X_m(p^*, e^*)$. The solution to 2.6 corresponds to point (5) in figures 1a and 1b.

The analogous definition for transactions costs is given by TC':

$$V_s(p^*, e^* + EC - TC') = V^*_m \qquad (2.7)$$

This measures the distance between U* and V_m^*, along $X_s(p^*, e^*)$. The solution to 2.7 corresponds to point (6) in figures 1a and 1b. And as before, the household will move if EC > TC'.

To summarize, we have examined three sets of measures related to the move-stay decision. The corresponding conditions for a move are

Our model:	$V_m^* > V_s^*$
Income Compensation:	IC > TC
-->	$U^* - V_s^* > U^* - V_m^*$
Equivalent Compensation:	EC > TC'
-->	$U^* - V_s^* > U^* - V_m^*$

These conditions are equivalent; but the first is expressed explicitly in terms of a direct utility comparison between the possible tenure choices.

C. Moving Costs and the Demand for Housing

The presence of moving costs leads to discontinuities in consumers' housing demand functions. Figure 2 depicts the demand for accommodation x_a^{11} for a typical household currently consuming a certain quantity x_s. In this section the corresponding price, p_a, applies equally to movers and stayers. This allows the role of moving costs on the demand for housing to be viewed independently of the effects of price differentials between tenures.

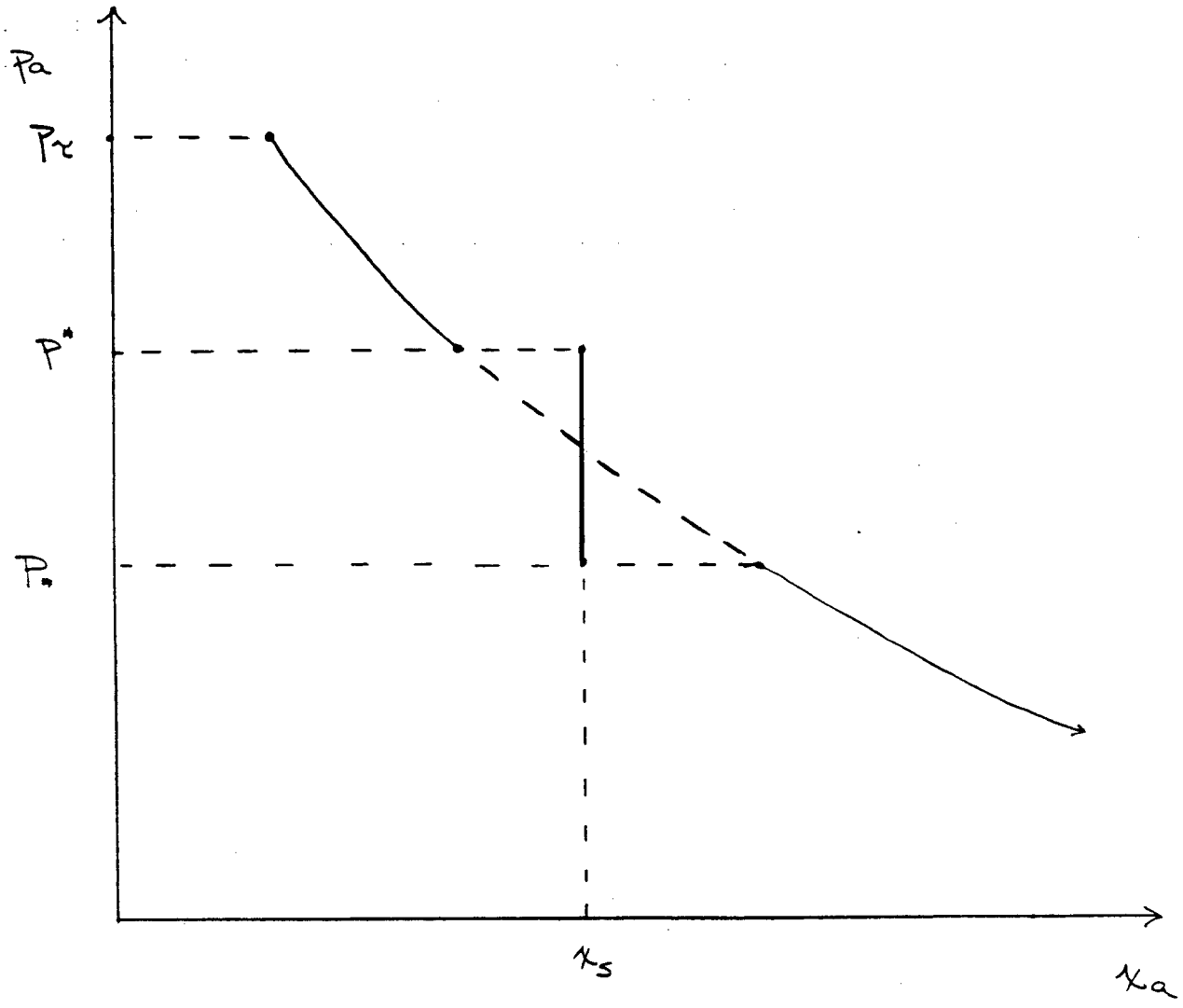
Three discontinuities distinguish the demand curve in figure 2 from the neoclassical version typically employed in discussions of housing demand. Two of these discontinuities, occuring at p_* and at p* (defined forthwith), are associated with moving costs which may inhibit the ease with which households adjust their accommodation levels in response to changes in the economic environment.

The third discontinuity occurs at p_τ (also defined below) and is related to tenure switching. The inherent properties of discrete choice regarding the tenure of a household's primary residence are such that we must model a switching point or threshold rather than a smooth transition between tenures.

Define S as the set of price vectors $p=(p_o, p_r, p_s)$ for which the household would prefer to remain with its current

Figure 2

Discontinuities in Housing Demand



accommodation bundle (x_s) rather than bear the transactions costs involved in moving, given household expenditure (e) and demographic characteristics (d). That is,

$$S = [p \mid V_s(p,e) > V_m(p,e)] \tag{2.8}$$

A subset of S which is of interest for the purposes of this illustration is defined by

$$S_a = [p \mid p=(p_a, p_a, p_a) \text{ and } V_s(p_a, e) > V_m(p_a, e)] \tag{2.9}$$

Then p* and p_* are defined as the maximum and minimum values associated with the set S_a:

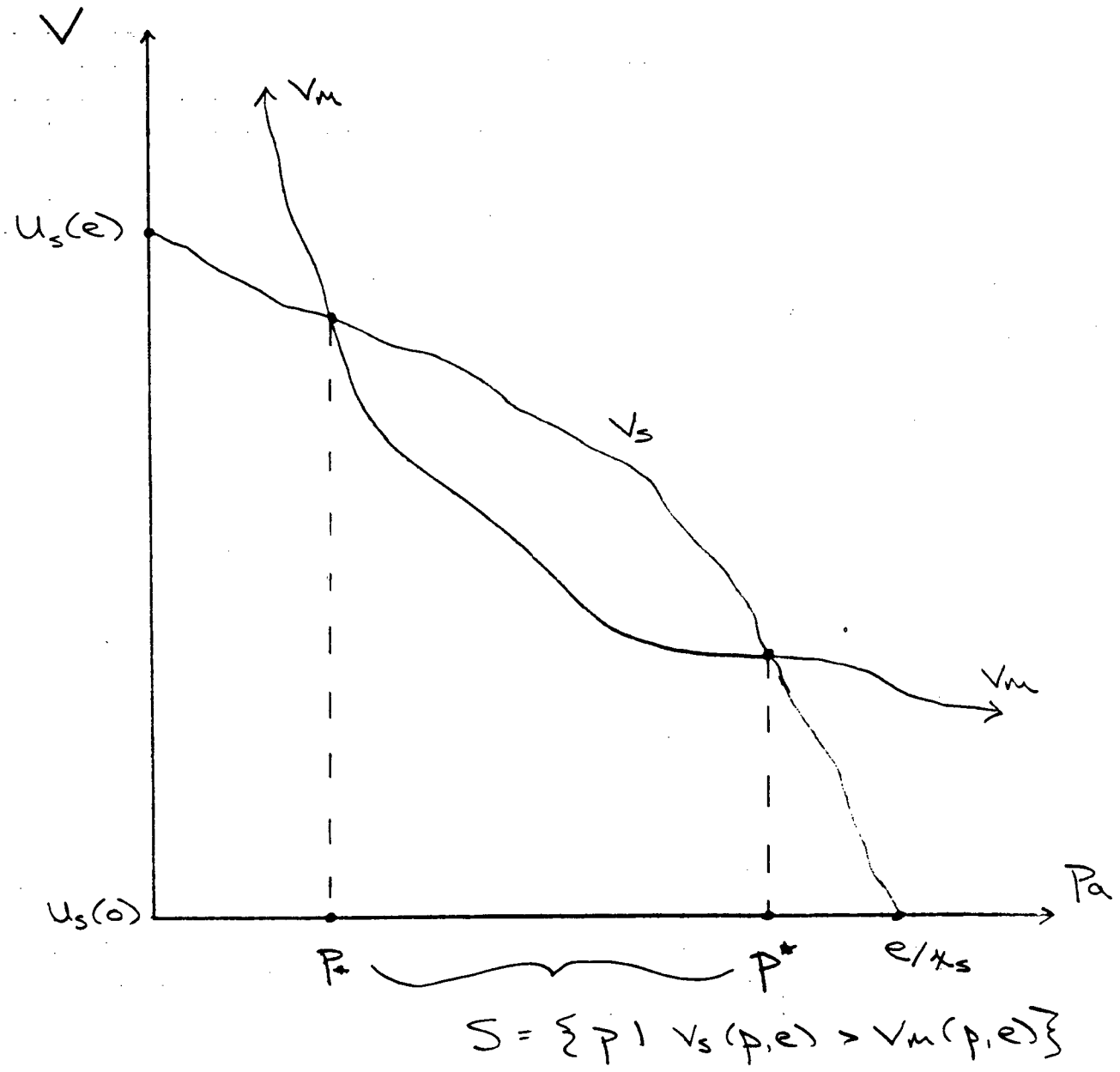
$$p^* = \max [p_a \mid p \in S_a]$$

$$p_* = \min [p_a \mid p \in S_a]$$

the sub-utility functions V_m and V_s are both decreasing in p_a and for a typical case may be drawn as in figure 3, in which the set S consists of the closed interval $[p_*, p^*]$. Over this range of prices, the household prefers to remain in its present place of residence.[12]

Figure 3

Staying Preferred to Moving



The existence of p_* and p^* in a typical case seems likely given sufficiently large transactions costs κ and considering the following. As the price of accommodation falls to zero, we know from equation 2.2 that

$$V_s(0,e) = U(x_s,e) = U_s(e) \tag{2.10}$$

And as p_a rises, $V_s(p_a,e)$[13] declines until $p_a = e/x_s$, whereupon

$$V_s(e/x_s,e) = U(x_s,0) = U_s(0) \tag{2.11}$$

Thus V_s is somewhat 'pegged in' at its maximum and minimum values, as shown in figure 3.

This is in contrast to $V_m(p_a,e)$. As p_a becomes very small we expect that V_m, while not unbounded, obtains very substantial heights, since the household can indulge in unlimited housing consumption. Similarly, as p_a becomes large, V_m affords more flexibility in reducing housing consumption so as to have residual funds for food, clothing and other essential non-housing commodities. Thus in general one expects that V_m will be more convex from the origin than is V_s. Indeed the latter is quite conceivably concave from the origin since it likely drops off rather quickly as it nears its endpoints.

Finally note from equation 2.1 that V_m decreases with κ while 2.2 shows that V_s is invariant to changes in moving costs. Thus the composition of the set S may depend critically upon the size of the transactions costs involved, as illustrated in figure 4.

Demand curves corresponding to different values of κ in figure 4 are depicted in figures 5a through 5c. As one would expect, the measure of the set S is non-decreasing in κ and so the discontinuity at x_s tends to become more severe as κ increases. This says nothing more than that households are less likely to relocate when the costs of doing so are higher.

Stated another way, one expects that

$$\partial\,[p^* - p_*]/\partial\kappa \geq 0 \qquad (2.12)$$

This is so by reason of two givens

(1) $\partial\, Vm/\,\partial\kappa < 0$ for all prices and therefore the V_m curves in figure 4 do not intersect, and

(2) $\partial\, Vs/\,\partial p < 0$ and so V_s in figure 4 is downward sloping.

Thus the equality in 2.12 holds only when there are no transactions costs discontinuities. This point is illustrated in appendix A, with reference to the Linear Expenditure System used in the empirical chapters of this dissertation.

Figure 4

Effect of Differing Values of Kappa on S

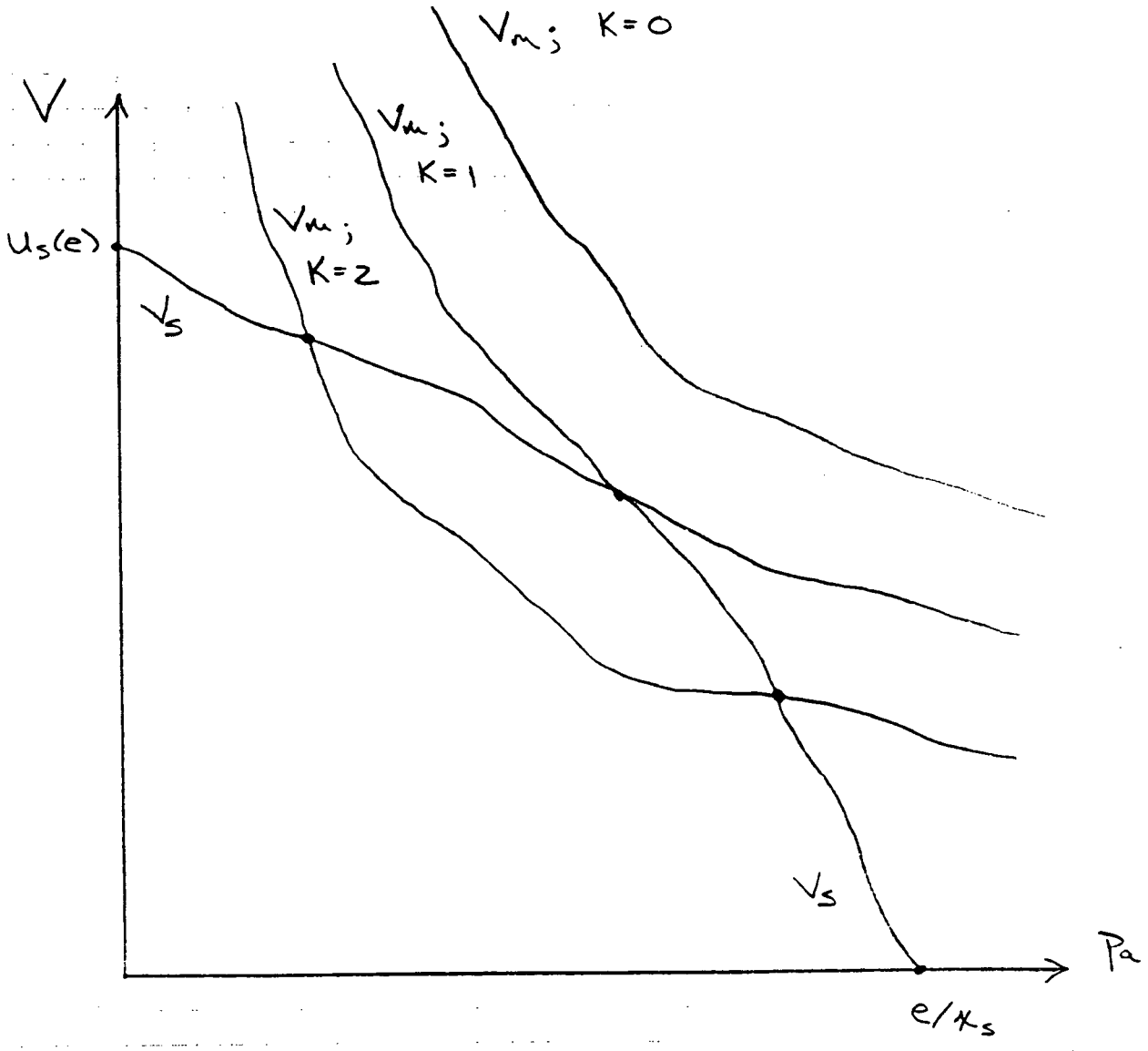


Figure 5

Effect of Kappa on Discontinuities

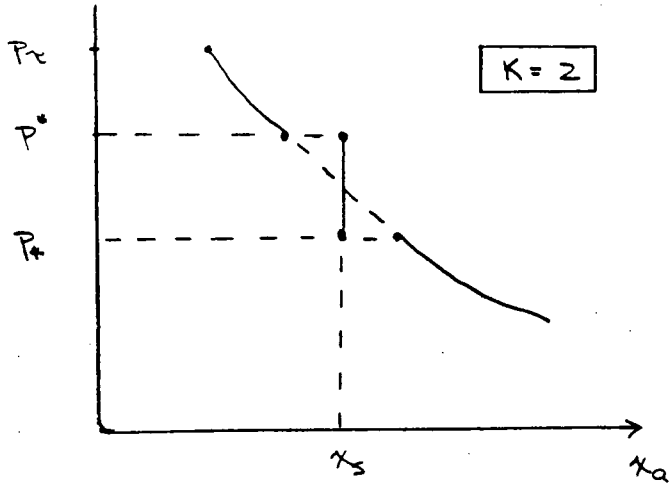


Figure 2.5a

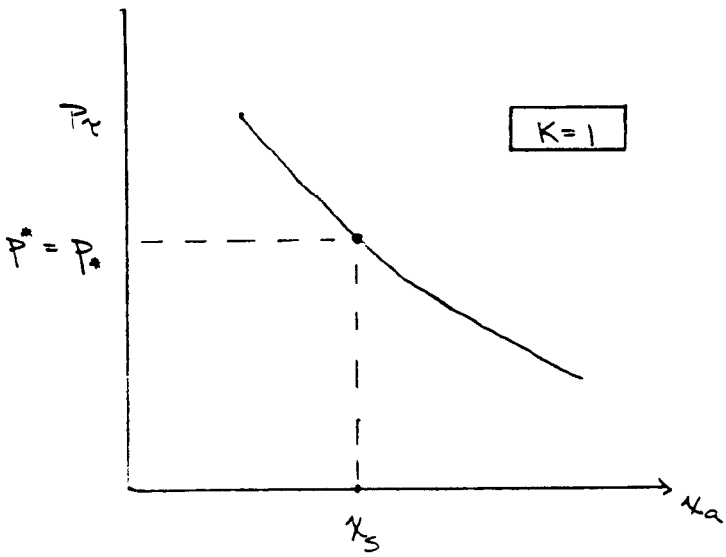


Figure 2.5b

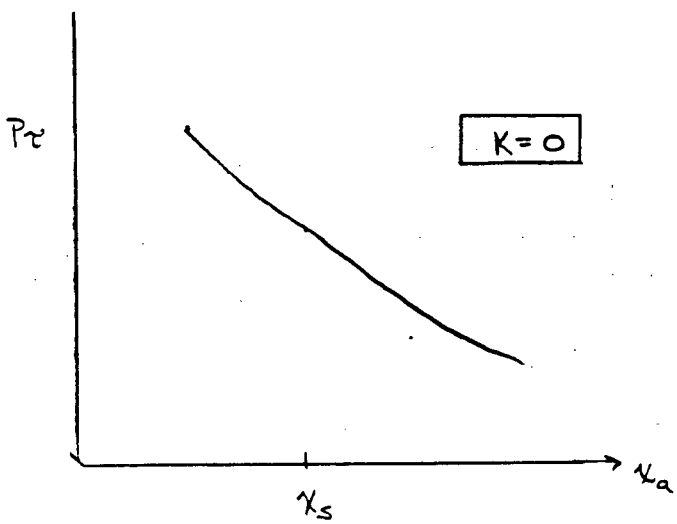


Figure 2.5c

D. Tenure Choice and the Demand for Housing

Transactions costs as discussed above constitute one source of discontinuity in the demand for home ownership. But even in the absence of moving costs, discontinuities may arise in the demand for housing due to the discrete nature of tenure choice.

The nature of these tenure choice discontinuities is better understood with reference to the following distinct sets:

$$O = \{p \mid V_o(p,e) > V_r(p,e) \text{ and } V_o(p,e) > V_s(p,e)\}$$

and

$$R = \{p \mid V_r(p,e) > V_o(p,e) \text{ and } V_r(p,e) > V_s(p,e)\}$$

The decomposition of the $[p_o, p_r]$ plane into these sets as illustrated in figure 6 is implicit in work on tenure choice done by King(1980) and Rosen(1979) and others. Figure 6 also illustrates that the tenure switching price p_τ for one tenure depends upon the prevailing price in the other.

Figure 6 however is a specific case of the more general decomposition shown in figure 7 which shows six distinct sets:

$$ORS = \{p \mid V_o(p,e) > V_r(p,e) > V_s(p,e)\}$$

$$OSR = \{p \mid V_o(p,e) > V_s(p,e) > V_r(p,e)\}$$

Figure 6

Owning versus Renting

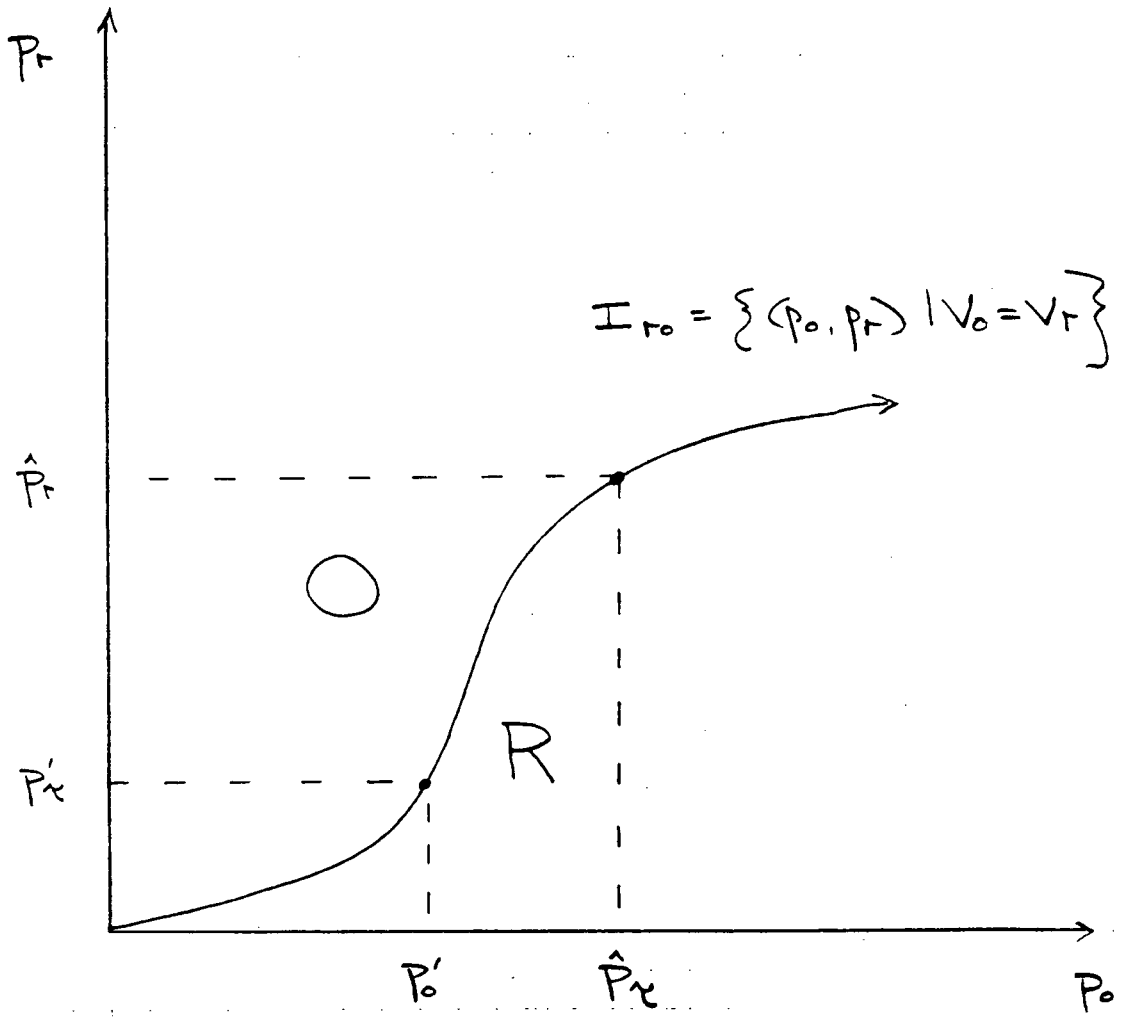
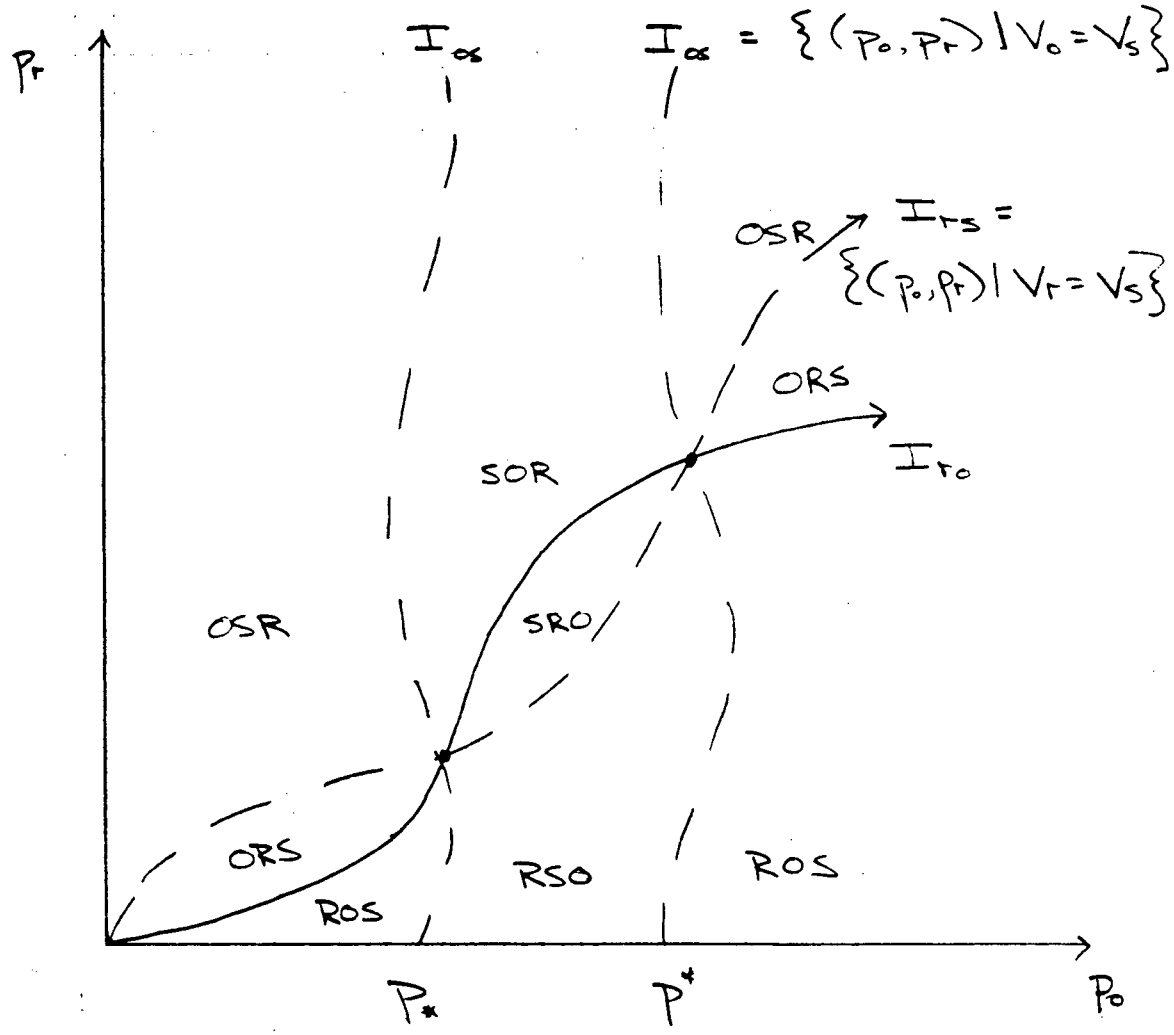


Figure 7

Owning versus Renting versus Staying



$$ROS = \{p|\ V_r(p,e) > V_o(p,e) > V_s(p,e)\}$$

$$RSO = \{p|\ V_r(p,e) > V_s(p,e) > V_o(p,e)\}$$

$$SOR = \{p|\ V_s(p,e) > V_o(p,e) > V_r(p,e)\}$$

$$SRO = \{p|\ V_s(p,e) > V_r(p,e) > V_o(p,e)\}$$

Note that

$$O = ORS \vee OSR$$

$$R = ROS \vee RSO$$

and

$$S = SOR \vee SRO$$

where V denotes the intersection of two sets.

A study of housing demand which ignores transactions costs is valid only in ORS and ROS, and one which also neglects the option to rent is valid only for ORS.

Tenure choice is modelled in terms of the probability of a household's being associated with any of the six sets above, while the quantity of accommodation services demanded is conditional upon this tenure decision. Both aspects of the household's housing decision are modelled jointly within a maximum likelihood framework. The econometric specification for this model is discussed in chapter three.

NOTES TO CHAPTER TWO

1. This model implicitly assumes weak intertemporal separability, so each period's decision is viewed in isolation from that of future periods. This characteristic is shared by each of the models discussed in chapter one. One consequence of this simplifying assumption is that it does not allow the household to decide in the current period whether it would prefer to move in this period or the next.

2. The superscript 'h' has the following "distributive property":

$$V_m(p_m, e_m^h) = V_m(p_m^h, e_m^h)$$

and likewise for $U(x_m, x_c^h)$ and other multiple-argument functions.

3. Dynarski (1986) makes a distinction between transactions costs (including intangibles) and residential attachment, where the former enters the utility function as a negative, while the latter enters as a positive to household utility. This distinction is of no consequence here.

4. The specification of $\kappa(d)$ is spelled out in more detail in chapters four and five.

5. The option of renovating is not allowed here, although depreciation and maintanence expenditures are incorporated explicitly in the price of home ownership.

6. For notational ease, we have dropped explicit references to the demographic variables (d) in TC and κ, and to the superscript 'h' wherever the context allows.

7. The correspondence between points in figures 1a and 1b is not unique, except for the Engel curve $X_m(p^*,e)$ in figure 1a and its counterpart $V_m(p^*,e)+\kappa$ in 1b. For example, both (3) and (3') in figure 1a are mapped into the same point in figure 1b because they are associated with the same expenditure and utility levels.

8. Actually, Weinberg et al (1981) state their definition of income compensation in two different ways. On the one hand that state that it is

> (a) "the maximum amount of money that the household could spend on transactions costs . . . and be as well off after the move as before".

On the other hand they define it as

> (b) "the amount of money IC that if subtracted from a household's income would leave the household as well off with $[x_s]$ as it would if it were to consume the optimal amount $[x_m^*]$ at a rent of $[p_m^* x_m^*]$.

These two definitions are not equivalent. In the text, I use their first definition because it leaves point (4) on the Engel curve.

9. Our definition of TC is a relatively broad one in that it incorporates pecuniary and psychic transactions costs, whereas Weinberg et al (1981) incorporate only the former.

10. To see that these conditions are equivalent when $p_j = p^*$ and $e_j = e^*$ for all tenures, note from 2.4 and 2.5 that

$$IC > TC$$

$$\rightarrow \quad V_m(p^*,e^*) > V_m(p^*,e^*-IC)+\kappa(d) = V_s(p^*,e^*).$$

Put another way, the condition IC > TC is equivalent to the condition that $(U^*-V_s^*) > (U^*-V_m)$, and so of course $V_m^* > V_s^*$.

11. The terms x_a and p_a are "tenure neutral". This exposition applies equally therefore to owner occupied and rental accomodation.

12. The arguments presented in this section, and illustrated in figures 3 through 5 are made with reference to the subset S_a. Similar arguments apply to the rest of the set S, but attempting to illustrate those arguments in two dimensions would likely result in more confusion than insight.

13. Here, we take the liberty of writing "$V(p_a,e)$" as shorthand for "$V(p,e)$ where $p = (p_a,p_a,p_a)$".

III. Econometric Specification

A. General Formulation

This chapter develops the econometric specification corresponding to the model of housing demand presented in chapter two. The key feature of this model is the choice which each household has from among three tenure alternatives: moving into ownership accommodation (o), moving into rental accommodation (r), or remaining stationary (s). In addition, the household must decide upon the actual quantity of accommodation services consumed.

Econometrically, the challenge is to specify where the error terms arise and how they are applied in the estimation process. In this model, the joint tenure choice and housing demand decisions are modelled sequentially.

First, the household makes a tenure decision based on its expected utility under the three options available. That is, the stochastic tenure choice is marginal on the actual opportunity sets. Secondly, the household determines the quantity of housing it wishes to consume conditional upon the choice set selected in the initial (tenure) decision.

To explain further, consider the outcome vector associated with household h (ϵ [1,H]) denoted by

$$v^h = (x_o d_o, x_r d_r, x_s d_s)^h \qquad (3.1)$$

where d_j (j=o,r,s) is either 0 or 1 depending on tenure choice and x_j is the quantity of housing services demanded in tenure j conditional upon tenure j being chosen. Mixed tenures are ruled out, so $\sum_j d_j = 1$.

Denoting $f(v^h)$ as the density of an observed outcome vector for household h we may write (where the h subscripts are suppressed on the right hand side for notational convenience)

$$f(v^h) = [P_o * f_o(.)]^{d_o} * [P_r * f_r(.)]^{d_r} * (P_s)^{d_s} \qquad (3.2)$$

where P_j is the probability of tenure j being selected and $f_j(.)$ is the density demand function describing the distribution of x_j about its conditional mean. By definition, the only dwelling unit the household can stay in is the one in which they have been residing, so when tenure s is selected the actual level of demand is uniquely determined. Thus,

$$\text{Prob}[x=x_s \mid d_s=1] = 1.$$

The likelihood of observing a given random sample of H household outcomes is given by

$$e^L = \prod_h f(v^h) \qquad h \in [1,H] \qquad (3.3)$$

Substituting from (3.2) for $f(v^h)$ and taking logs of both sides yields the log-likelihood function which serves as the object maximand:

$$L = [\sum_h \sum_j d_j * \log P_j]$$

$$+ [\sum_h (d_o * \log f_o(.) + d_r * \log f_r(.)]$$

$$= Lt + Ld. \qquad (3.4)$$

Equation 3.4 has been grouped into two distinct terms; the first, Lt, is the log-likelihood associated with tenure choice and is the general formulation for quantal response models found, for example, in Judge et al (1980) or Hausman and Wise (1978). The second component, Ld, represents the log-likelihood of observing the conditional demand levels x_o and x_r.

These two aspects of housing demand are often considered in isolation from one another. Equation 3.4 allows both to be jointly determined within a unified framework; the same household preferences which underlie the tenure decision also underlie the subsequent conditional demand decision. There are numerous ways to specify equation 3.4; this chapter explores issues arising in the choice between competing specifications.

The purpose of developing an econometric model is to allow the household preference parameters to be estimated directly. Sections B and C of this chapter establish the mechanisms by which these household preference parameters are linked, respectively, to the tenure selection (Lt) and conditional demand (Ld) components of the likelihood function. Section D then evaluates alternative specifications of functional form for household preferences, bearing in mind the particular requirements of the model in question. Section E discusses some

tradeoffs between logit and probit models as a basis for modelling tenure selection. This is followed in section F by a discussion of alternative likelihood function specifications, assumptions regarding covariance between conditional demand and tenure choice error terms, and starter values for the likelihood iteration process. Section G closes the chapter with a complete specification of the base case likelihood function, based on discussion from the preceding sections.

B. Linking Tenure Probabilities to Preference Parameters

The component of the log-likelihood function related to tenure choice is given in the previous section as

$$Lt = \sum_h \sum_j d_j * \log P_j \tag{3.5}$$

In order to estimate the parameters (Ψ) underlying the household choice of tenure it is necessary to formulate a relationship linking those parameters to the probabilities denoted by P_j. In particular, let

$$\begin{aligned} P_o &= \text{Prob}[V_o > V_r \text{ and } V_o > V_s] \\ P_r &= \text{Prob}[V_r > V_o \text{ and } V_r > V_s] \\ P_s &= \text{Prob}[V_s > V_o \text{ and } V_s > V_r] \end{aligned} \tag{3.6}$$

where the $V_j(\Psi)$ are the indirect subutility functions from chapter two.

The expressions in 3.6 are developed further by assuming that the V_j may be expressed as the sum of two distinct components:

$$V_j(\Psi) = \Lambda_j(\Psi) + \lambda_j \tag{3.7}$$

where $\Lambda_j(\Psi)$ is an index of "expected" or "representative" utility and λ_j is a random error term with $E(\lambda_j) = 0$. Equations 3.6 may then be rewritten:

$$\begin{aligned}
P_o &= \text{Prob}[(\lambda_r - \lambda_o) < (\Lambda_o - \Lambda_r) \text{ and } (\lambda_s - \lambda_o) < (\Lambda_o - \Lambda_s)] \\
P_r &= \text{Prob}[(\lambda_o - \lambda_r) < (\Lambda_r - \Lambda_o) \text{ and } (\lambda_s - \lambda_r) < (\Lambda_r - \Lambda_s)] \\
P_s &= \text{Prob}[(\lambda_o - \lambda_s) < (\Lambda_s - \Lambda_o) \text{ and } (\lambda_r - \lambda_s) < (\Lambda_s - \Lambda_r)]
\end{aligned} \tag{3.8}$$

Defining λ_{ij} as the random variable $\lambda_i - \lambda_j$, let $G_j(\lambda_{ij}, \lambda_{kj})$ [j=o,r,s] be the set of cumulative joint distribution functions for these random differences. Then the expression for the probabilities in equations 3.8 may be further simplified; for example

$$\begin{aligned}
P_o &= G_o(\Lambda_o - \Lambda_r, \Lambda_o - \Lambda_s) \\
P_r &= G_r(\Lambda_r - \Lambda_o, \Lambda_r - \Lambda_s) \\
P_s &= G_s(\Lambda_s - \Lambda_o, \Lambda_s - \Lambda_r).
\end{aligned} \tag{3.9}$$

The two standard models of quantal response, logit and probit, arise from alternative specifications of the stochastic elements λ_j in equation 3.7. In particular, if λ_i and λ_j are normally distributed, so too is λ_{ij}, in which event the probit model applies:

$$P_j = G_j(\Lambda_j - \Lambda_i, \Lambda_j - \Lambda_k) = \Phi_j(\Lambda_j - \Lambda_i, \Lambda_j - \Lambda_k) \qquad (3.10)$$

where Φ_j is the standardized joint normal distribution function corresponding to tenure j.

The other assumption typically made with regards to the stochastic components λ_j is that they follow a Weibull, or extreme value distribution. Because the difference, λ_{ij}, between two Weibull-distributed random variables itself follows a logistic distribution, G_j in this event becomes the standardized cumulative joint logistic functon and so the logit model applies:

$$P_j = G_j(\Lambda_j - \Lambda_i, \Lambda_j - \Lambda_k) = \exp(\Lambda_j) \,/\, \Sigma_i \exp(\Lambda_i) \qquad (3.11)$$

With either 3.10 or 3.11, the tenure selection probablities found in the first component, Lt, of the likelihood function are directly linked to the household preference parameters Ψ which are found in the subutility index $\Lambda_j(\Psi)$.

C. Linking Conditional Demand to Preference Parameters

The second channel linking the likelihood function to underlying household preference mappings is through the conditional demand functions for tenures o and r. The observed conditional demands by households within each tenure group are assumed to be drawn from underlying conditional probability density functions centered on the corresponding conditional means for the entire population, X_m. That is, (for m=o,r)

$$x_m = X_m(\Psi) + \epsilon_m \tag{3.12}$$

where the behavior of ϵ_m is described by the probability density function $f_m(.)$. It is this term which appears in the second component of the log likelihood function;

$$Ld = [\Sigma_h (d_o * \log f_o(.) + d_r * \log f_r(.)]$$

For empirical purposes it is necessary to specify the conditional mean $X_m(\Psi)$ explicitly. It is assumed here that it may be derived immediately from the corresponding non-stochastic indirect utility index Λ_m using Roy's Identity';

$$X_m(\Psi) = -(\partial\Lambda_m/\partial p_m) / (\partial\Lambda_m/\partial e) \tag{3.13}$$

Doing so establishes $X_m(\Psi)$, and hence ϵ_m, as a function of the

parameters appearing in $\Lambda_m(\Psi)$.

Thus the expression for the density function $f_m(\epsilon_m)$ which appears in the conditional demand component, Ld, of the likelihood function will serve to forge an additional link between household preference parameters (Ψ) and the likelihood (Ld) of observing a particular outcome vector of conditional demands.

D. Specifying Functional Form of Household Preferences

There are several constraints or considerations pertinent to choice of functional form in this model. The foremost of these is the requirement that the indirect utility function be explicitly derivable from the direct utility function (or vice versa). This is necessary for purposes of direct comparison of the levels of utility derived from each tenure alternative in the model.

Recall from chapter two that each household has a subutility index for each tenure; these indices assess the level of welfare which the household would enjoy were it to choose that particular tenure. The household then chooses the tenure which corresponds to the highest index value, and of course this necessitates a direct comparison of these subutility indexes, as is reflected in the repeated occurrance of the expression $\lambda_{ij} = (\Lambda_i - \Lambda_j)$ in the preceding discussion (see section B) on tenure choice models.

The two subutility indexes for the moving options (j=o,r) are in the form of indirect utility functions while the index for the stay option (j=s) is of a direct utility form. The indirect form is not appropriate for the stay option because the level of housing demand for j=s was determined in some prior period and therefore cannot be interpreted as the utility maximizing alternative prevailing under current period prices due to the housing constraint, $x_a = x_s$.

Exact comparison of the indirect and direct utility indexes does not pose a problem from the perspective of the underlying theoretical constructs; both indexes are expressed in the same units of measure and are therefore immediately comparable in that sense. However it is not always possible to derive explicitly the dual form for a given direct or indirect utility functional form.

Requiring that we be able to do so rules out several common functional forms, notably the translog form, either as employed by King (1980) for the indirect utility function or by Rosen (1979) for the demand functions themselves. It is not possible with the translog or most other flexible functional forms to derive explicitly the dual counterparts. For King and Rosen this was not a concern, but as discussed above, direct comparisons are an integral part of the model developed here.

Practical considerations strongly favor an additional restriction on functional form; namely, that it be linear in parameters for both the direct and indirect specifications of

the utility function. Domencich and McFadden (1975, p.54) argue that having an index which is linear in parameters "greatly facilitates its estimation and statistical interpretation", and indeed virtually all applications of logit or probit models employ this assumption (see, for example, Judge et al (1980), Lee and Trost (1978) McFadden (1973) or Schmidt and Strauss (1975)).

This additional restriction precludes several more functional forms. For example, the quadratic specification discussed by Diewert (1975) and Diewert and Wales (1986) is linear in parameters in its direct form, but this characteristic is shared by the inverse of the dual rather than by the dual itself. As another example, the translated CES forms discussed by Pollak (1970) are self-dual; both the direct and indirect specifications of the utility function share the same translated CES forms, but neither are linear in parameters.

One functional form which does satisfy the prerequisites mentioned above is the Cobb-Douglas (C-D) utility function. Its linear-in-parameters form appears in both its direct and indirect manifestations. However, the constant shares implied by the C-D form are restrictive.

It would appear that no functional form satisfies all our criteria. But consider the Linear Expenditure System (LES), described by Pollak and Wales (1969,1978). Applied to our model, the LES takes on the following form for the indirect and direct subutility indexes.

$$\Lambda_m^h = b(\beta) + \ln(e_m - p_m\gamma_a - \gamma_c) - \beta\ln p_m - \kappa(d) \qquad (3.14a)$$

for m=o,r

$$\Lambda_s^h = \beta\ln(x_s - \gamma_a) + (1-\beta)\ln(x_c - \gamma_c) \qquad (3.14b)$$

Here, β, $\kappa(d)$, and the γs are parameters to be estimated, and $b(\beta) = \beta\ln\beta + (1-\beta)\ln(1-\beta)$. The γs are translation parameters which, when not equal to zero, distinguish the model from its Cobb-Douglas roots. The γs are useful as well for removing the assumption of homotheticity through the origin (see Pollak (1971))[2].

The LES clearly satisfies our first criterion, namely, that the direct and indirect specifications be mutually derivable. But examination of equations 3.14 indicates that this specification is not linear in parameters for the utility indexes. However, if γ_a and γ_c are held constant, the linear in parameters criterion is satisfied. If estimation should prove to be too difficult where the γs are estimated freely, one can estimate the model while holding the the γs fixed at various values.

Thus the LES specification appears to offer a reasonable compromise between variable expenditure shares and a linear in parameters form. It is therefore selected for the purposes of this applied research.

E. Logit vs Probit

In choosing between the logit and probit models several factors must be considered: (1) computational tractability, (2) possible covariance between the random variables in the tenure choice decision in equation 3.7, and (3) the formulation of good initial parameter estimates for the maximum likelihood iteration process. The discussion in this section focuses on the first two factors; the role of starter estimates is reserved for section F where it is discussed within the context of the likelihood function specification.

There is little dispute regarding the advantages in terms of computational practicality of the logit versus the probit model. These advantages can be readily noted by contrasting equation 3.10 with 3.11. Explicit calculation by the probit method of the probabilities P_j which lie at the heart of the likelihood function requires repeated numerical evaluation of the double integral

$$\Phi_j = \iint \phi_j(i,k)\, di\, dk \qquad (3.15)$$

where $\phi_j(\ ,\ )$ is the joint normal density function and the limits of integration are from minus infinity to Λ_{ji} and Λ_{jk} respectively. These calculations are required at each iteration of the algorithm which seeks the solution parameters maximizing the likelihood function.

In contrast, the logit model in equation 3.11 expresses P_j in closed form; no numerical integration is required. The consequent savings in computation time are evident in a study by Albright et al (1977) who report that the probit CPU requirements per iteration exceeded the logit requirements by a factor of 15.

However, the advantages of computational ease enjoyed by the logit form, particularly in its simplest manifestation, are obtained at the expense of restrictions imposed upon the covariance matrix for the random terms λ_j in equation 3.7. Thus for example, it follows from equation 3.11 that

$$\log(P_j/P_k) = \Lambda_{jk} \qquad (3.16)$$

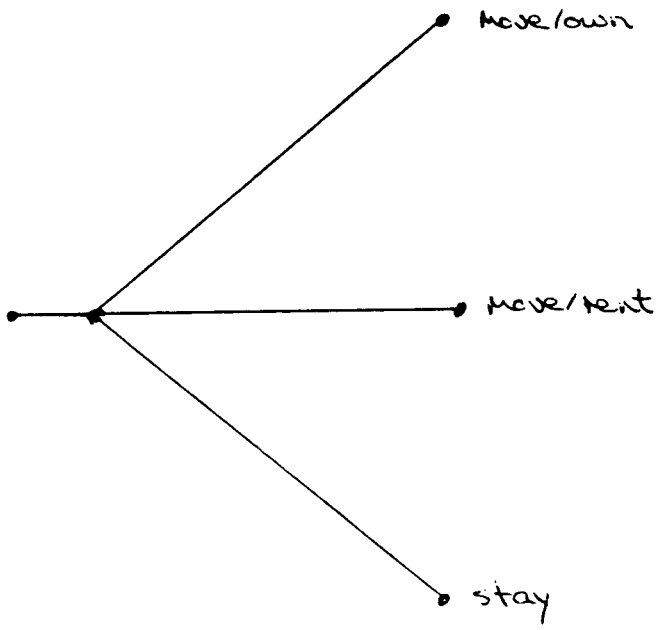
This indicates that the odds of choosing tenure j over k in the logit model are independent of the presence of other alternatives.

Independence of the error terms in equation 3.7 is a dubious assumption when the alternatives in question are likely to be perceived as being similar. For example, one could argue that households make a primary distinction between moving and not moving, so that tenures o and r would be perceived as being "lumped together" as alternatives requiring relocation. In this event one would expect a positive correlation between λ_r and λ_o; a willingness or aversion to relocate to rental accommodation indicates some willingness or aversion to relocate to self-owned

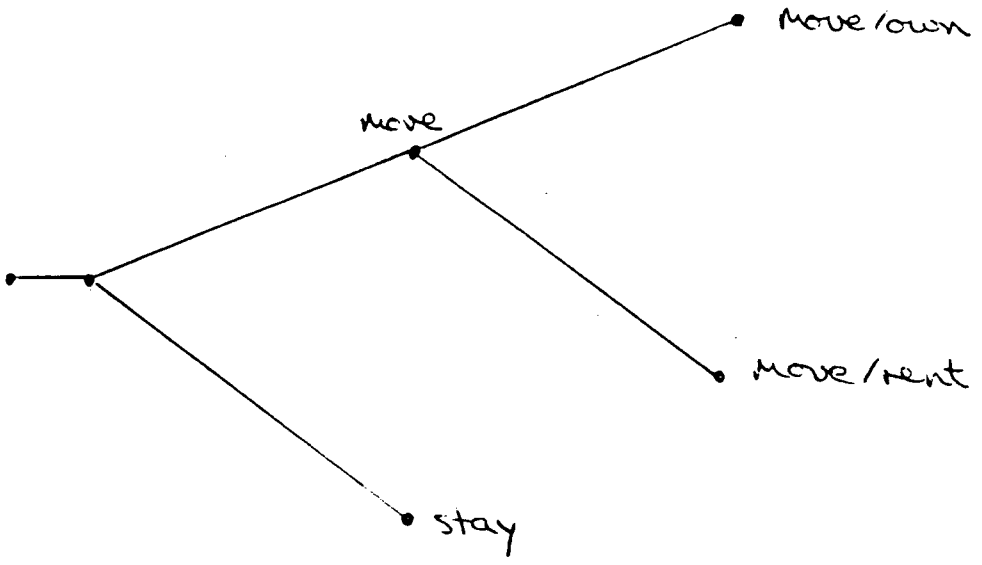
Figure 8

Structure of Tenure Decision

[A]



[B]



accommodation.

Were this the case, the household decision structure would be better represented by figure 8b than by 8a. In figure 8a the three tenure alternatives are perceived to be independent of one another and so the "independent logit model" (Amemiya (1981)) applies. Whereas in figure 8b, household decision making is best modelled in terms of a sequential choice structure. For example the household may first make a binary decision between tenures o and r conditioned upon a decision to move. This is followed by a move/stay binary decision where the comparison is between Vs and max[Vr,Vo]. Domencich and McFadden (1975) and McFadden (1973) argue, in the context of modelling urban travel demand, that sequential modelling of the decision process is a practical and sensible means of mitigating the potential estimation biases resulting from similarly perceived alternatives.

Another method of allowing for more flexible treatment of the error structure in 3.7 is also due to McFadden (1981), and is termed the nonindependent logit by Amemiya (1981), or the generalized extreme value (GEV) model by Maddala (1983). To continue with the move/stay example, we may assume that λ_r and λ_o jointly follow a bivariate extreme value distribution while λ_s adheres to a univariate extreme value distribution. Then we have the probability of staying given as

$$P_s = \exp\Lambda_s / (\exp\Lambda_s + M) \tag{3.17}$$

where

$$M = [\exp(\Lambda_o/1-\sigma) + [\exp(\Lambda_r/1-\sigma)]^{1-\sigma} \tag{3.18}$$

is a "weighted average" of $\exp(\Lambda_o)$ and $\exp(\Lambda_r)$, and σ is a coefficient varying between 0 and 1 as λ_o and λ_r are entirely uncorrelated or perfectly correlated, respectively. Then

$$P_o = (1-P_s)\,[(\exp\Lambda_o)/M]^{1/(1-\sigma)} \tag{3.19}$$

and

$$P_r = (1-P_s)(1-P_o) \tag{3.20}$$

This nonindependent logit is attractive in that the choice between the two similar alternatives (those requiring relocation) is made according to a dichotomous logit model while the move/stay decision sets a certain kind of weighted average of the relocation alternatives against the stay alternative within a dichotomous logit framework. Note that the independent logit model from 3.11 is a special case, corresponding to σ=0. McFadden (1981) provides a detailed consideration of alternative specifications for the weighting parameter σ.

The sequential decision framework need not be confined to the logit model. Amemiya (1981) discusses a sequential probit where, using our terminology:

$$P_s = \Phi_s(\Lambda_s)$$

$$P_o = (1-P_s)\,\Phi_o(\Lambda_o) \qquad (3.21)$$

$$P_r = (1-P_s)\,(1-P_r)$$

This sequencing structure helps to ease the computational burden posed by the unrestricted probit model. However it is not clear that the sequential probit has any advantages over the sequential or nonindependent logit, since they each rely on an essentially binary decision structure. And as Hausman and Wise (1978) attest, the properties of the binary probit and binary logit models are rather similar due to similarity (except in the extreme tails) of the distributions upon which they are based. This suggests that the advantages of the probit model come into full force only where decisions are based on n-way comparisons for n>2.

The third factor to consider when weighing the probit and logit alternatives, in addition to computational tractability and possible covariance between the tenure choice error terms, is the formulation of good initial maximum likelihood parameter estimates. Lee and Trost (1978) argue that this is particularly important when searching for the global maximum of a likelihood function which is highly nonlinear in parameters, as in our case.

Although the availability of good starter value estimates for the likelihood iteration procedure may have a bearing on the decision between the probit and logit models, the discussion of

starter values is best placed in the context of the maximum likelihood function specification. It is to this discussion that we now turn.

F. Likelihood Function Specification

The likelihood function set out in section A is of a gendre proposed by King (1980):

$$L = \prod_h \prod_j [P_j f_j(\epsilon_j)]^{d_j} \qquad (3.22)$$

While well disposed to handling multiple tenures, this specification also implicitly restricts the error terms for the discrete and continuous decisions to be uncorrelated with one another. This is shown by delineation of the decision structure into separate and distinct tenure (P_j) and conditional demand (f_j) components. Put another way, 3.22 implicitly constrains $cov(\lambda_{ij}, \epsilon_j) = 0$ for all (i,j). If in fact $cov(\lambda_{ij}, \epsilon_j)$ differs from zero, then the implicit assumption to the contrary in 3.22 may lead to sample selectivity bias in the parameter estimates.

To explain further, consider three sets H_j [j=o,r,s] where a household h belongs to H_j if and only if $d_j^h = 1$. In general, the knowledge that household h has chosen tenure j may influence our expectation of the quantity of housing he would select in tenure j. From 3.12:

$$E(x_j^h \mid h \epsilon H_j) = X_j(\Psi) + E(\epsilon_j^h \mid h \epsilon H_j)$$

$$= X_j(\Psi) + E(\epsilon_j^h \mid \lambda_{ij}^h < \Lambda_{ji} \text{ and } \lambda_{kj}^h < \Lambda_{jk})$$

$$\neq X_j(\Psi) = E(x_j^h) \tag{3.23}$$

since, in general, $\mathrm{cov}(\lambda_{ij}, \epsilon_j) \neq 0$.

A more fully general model would seek to incorporate these covariances more explicitly. This is done by allowing the λ_{ij} and ϵ_j to be jointly distributed by a common density function, as shown here:

$$L = \Pi_h \left[\int_{-\infty}^{\Lambda_{or}} \int_{-\infty}^{\Lambda_{os}} g_o(\epsilon_o, \lambda_{so}, \lambda_{ro})\, d\lambda_{so}\, d\lambda_{ro}\right]^{d_o}$$

$$\left[\int_{-\infty}^{\Lambda_{os}} \int_{-\infty}^{\Lambda_{rs}} g_r(\epsilon_r, \lambda_{sr}, \lambda_{or})\, d\lambda_{sr}\, d\lambda_{or}\right]^{d_r}$$

$$\left[\int_{-\infty}^{\Lambda_{so}} \int_{-\infty}^{\Lambda_{sr}} g_s(\lambda_{so}, \lambda_{sr})\, d\lambda_{os}\, d\lambda_{rs}\right]^{d_s} \tag{3.24}$$

where, as before, L is to be maximized subject to the parameters defining Λ_{ji} and ϵ_j.[3]

The expression for L in 3.24 has some interesting special cases. For example, reducing this to a two tenure problem for all households allows one to rewrite L in simpler form:

$$L = \Pi_h \left[\int_{-\infty}^{\Lambda_{or}} g_o(\epsilon_o, \lambda_{ro})\, d\lambda_{ro}\right]^{d_o}$$

$$\left[\int_{\Lambda_{or}}^{\infty} g_r(\epsilon_r, \lambda_{ro})\, d\lambda_{ro}\right]^{d_r} \tag{3.25}$$

This likelihood function allows for only two tenures, owning and renting; no distinction is made between movers and stayers. When g_o and g_r are joint normal density functions, 3.25 corresponds to the model set out by Lee and Trost (1978). Their model is developed for the purposes of presenting a two stage procedure for producing consistent parameter estimates which may in turn serve as starter values for the likelihood iteration routine.

They point out that attempts to use OLS to estimate the parameters Ψ for the demand functions from 3.12

$$x_j^h = X_j(\Psi) + \epsilon_j^h \qquad (3.12)$$

based on the sample set H_j may result in biased and inconsistent estimates since, in general, $E(\epsilon_j^h | h \epsilon H_j) \neq 0$, in violation of the assumptions underlying the OLS procedure. When the likelihood function is highly nonlinear in parameters, there is a danger that these OLS estimates will not prove to be sufficiently close to the true parameter values to serve as effective starter values.

The remedial procedure which they propose, similar to methods outlined by Heckman (1976), involves obtaining direct estimates of $E(\epsilon_j^h | h \epsilon H_j)$ and using these estimates to adjust the non-zero mean for the error term in 3.12 for the selected sample H_j. Then OLS estimates for the adjusted demand functions

$$(x_j^h | h\epsilon H_j) = X_j(\Psi) + E(\epsilon_j^h | h\epsilon H_j) + \eta_j \tag{3.26}$$

are shown by Lee and Trost and by Heckman to yield consistent estimates of Ψ. For the new error terms introduced in 3.26 we have $E(\eta_j) = 0$.

If one were inclined to adopt this two stage method there are further arguments to be made in favor of the probit model versus the logit model. The reason is that the expression for $E(\epsilon_j^h | h\epsilon H_j)$ has been shown by these previous authors to be relatively forthcoming when the joint density functions $g_j(\epsilon_j, \lambda_{ij})$ in 3.25 are normal. For in that event

$$\begin{aligned} E(\epsilon_j^h | h\epsilon H_j) &= \mathrm{cov}(\epsilon_j, \lambda_{ij})\, E(\lambda_{ij}^h | \lambda_{ij}^h < \Lambda_{ji}) \\ &= \mathrm{cov}(\epsilon_j, \lambda_{ij})\, [-\phi_j(\Lambda_{ji})/\Phi_j(\Lambda_{ji})] \end{aligned} \tag{3.27}$$

since by definition (assuming the λ_{ij} are normally distributed),

$$\begin{aligned} E(\lambda_{ij}^h | \lambda_{ij}^h < \Lambda_{ji}) &= \int_{-\infty}^{\Lambda_{ji}} z\phi_j(z)/\Phi_j(\Lambda_{ji})\, dz \\ &= -\int_{-\infty}^{\Lambda_{ji}} \phi_j'(z)/\Phi_j(\Lambda_{ji})\, dz \\ &= -\phi_j(\Lambda_{ji})/\Phi_j(\Lambda_{ji}) \end{aligned} \tag{3.28}$$

This latter expression is the inverse of Mill's ratio (see Heckman, 1979).

The Lee and Trost (1978) procedure then entails use of the probit model

$$P_j = \Phi_j(\Lambda_{ji}) \tag{3.29}$$

to retrieve consistent estimates of Λ_{ji}, and thus of $E(\lambda_{ij}^h | \lambda_{ij}^h < \Lambda_{ji})$. When these latter estimates are added as independent variables to equation 3.12 for the purposes of OLS regression, two results ensue. First, the OLS estimates of the parameters Ψ are now free of sample selectivity bias since the nonzero means of the error terms for the subsample H_j have been adjusted for. Second, by virtue of equation 3.26, the OLS estimate for the recently added regressor $E(\lambda_{ij}^h | \lambda_{ij}^h < \Lambda_{ji})$ will itself be a consistent estimator of $\mathrm{cov}(\epsilon_j, \lambda_{ij})$.

We may modify the procedure advocated by Lee and Trost (1978) for the bivariate normal case to accommodate other distributions. For example, if one wished to retain a logit framework for the tenure choice model while allowing the error terms ϵ_j for the conditional demand functions to be normally distributed, the joint density function $g_j(\epsilon_j, \lambda_{ij})$ from 3.25 would have to be a normal-logistic density function.

In this case, the expression analogous to 3.28 is

$$E(\epsilon_j^h | h\epsilon H_j) = cov(\epsilon_j^h, \lambda_{ij}) E(\lambda_{ij} | \lambda_{ij} < \Lambda_{ji})$$

$$= cov(\epsilon_j^h, \lambda_{ij}) I(\Lambda_{ji}) \tag{3.30}$$

where

$$I(\Lambda_{ji}) = [1-F(\Lambda_{ji})]\Lambda_{ji} - F(\Lambda_{ji})*\ln[F(\Lambda_{ji})] \tag{3.31}$$

and

$$F(\Lambda_{ji}) = 1 + \exp(-\Lambda_{ji}) \tag{3.32}$$

Appendix B contains a detailed derivation of the expression in 3.31. When a binomial logit model is applied to the tenure choice decision this expression replaces the inverse of Mill's ratio found in 3.27 and 3.28.

Thus, a method for obtaining starter values when a logit model governs the tenure decision and when the conditional demand error terms ϵ_j are normally distributed is as follows. First, use a logit model to estimate the index Λ_{ji}. Second, substitute these estimates into 3.31 and use the estimated values for $I(\Lambda_{ji})$ thus obtained as additional regressors for the conditional demand function in 3.12. Doing so adjusts for the nonzero mean of ϵ_j for the sample set H_j in a manner similar to that proposed by Lee and Trost (1978) but allowing for λ_{ij} to be

logistically distributed.

To recapitulate briefly, the likelihood function in 3.22 has a potential drawback in that it does not allow for covariance of the error terms for the discrete and continuous decision functions. If these error terms are in fact correlated, then OLS estimates for the demand functions will be biased and inconsistent and will therefore be less attractive as starter values for the likelihood iteration procedure.

Lee and Trost (1978) have proposed a two stage procedure which corrects for this sample selection bias, but their method assumes that the joint density functions in the likelihood function in 3.25 are binormal, and so does not allow for a logit framework to govern the tenure decision. The modification to the Lee-Trost method proposed above removes this restriction and so allows one to enjoy the advantages of the logit model as discussed in section E while deriving satisfactory starter values for the likelihood search algorith despite the presence of sample selection bias.

Notwithstanding the above, the distinction between the likelihood function in 3.22 which does not allow for covariance of ϵ_j and λ_{ij} and the likelihood functions in 3.24 and 3.25 which do may be more apparent than real. The empirical evidence to date does not suggest a strong degree of simultaneity bias in housing demand functions. For example Lee and Trost (1978) themselves conclude that "simultaneity does occur, albeit the evidence is weak". Rosen (1979) applies the same method and

finds that the resulting estimate of the covariance of errors in the housing demand and probit equations does not differ significantly from zero. According to Rosen, "omission of the 'missing variable' has essentially no impact on the estimated coefficients."[4]

This evidence suggests that the theoretical advantages of the more fully general likelihood function in 3.24 may not be required from the practical empirical perspective. Therefore, a course of action which suggests itself is to assume for estimation purposes that $\mathrm{cov}(\epsilon_j, \lambda_{ij}) = 0$. Using this assumption we may rewrite the likelihood function in 3.24 as follows:

$$L = \prod_h [f_o(\epsilon_o) \quad g_o(\lambda_{so}, \lambda_{ro})\, d\lambda_{so}\, d\lambda_{ro}]^{d_o}$$

$$[f_r(\epsilon_r) \quad g_r(\lambda_{sr}, \lambda_{or})\, d\lambda_{sr}\, d\lambda_{or}]^{d_r}$$

$$[\quad g_s(\lambda_{os}, \lambda_{rs})\, d\lambda_{os}\, d\lambda_{rs}]^{d_s}$$

$$= \prod_h [f_o(\epsilon_o) P_o]^{d_o} [f_r(\epsilon_r) P_r]^{d_r} P_s^{d_s} \qquad (3.33)$$

This is nothing other than the likelihood function discussed in section A and which reappears in equation 3.22. The discussion in this section suggests that this is the appropriate likelihood function to proceed with, using logit to model the tenure decision. If in the course of the estimation it appears that

troubles are arising which may be rooted in sample selection bias, then the modified Lee-Trost procedure outlined in this section may be called upon.

G. Specification of Base Model

The full model may now be specified, beginning with the log-likelihood function;

$$\log L = [\sum_h \sum_j d_j * \log P_j^h] + [\sum_h \sum_m d_m * \log f_m^h(\epsilon_m)]$$

$$= Lt + Ld. \qquad (3.34)$$

$h=(1,H)$

$j=o,r,s$

$m=o,r$

This likelihood function describes the probability of observing the pattern of tenure choices and conditional housing demands which actually appear in our sample.

The tenure choice probability for household h (P_j^h) depends upon the expected utility for tenure j (Λ_j), as given by a generalized extreme value (nonindependent logit) model;

$$P_m^h = \exp[\Lambda_m/(1-\sigma)]*M^{\sigma}/D \qquad (3.35a)$$

$$P_s^h = \exp[\Lambda_s]/D \qquad (3.35b)$$

where

$$M^h = \exp[\Lambda_o/(1-\sigma)] + \exp[\Lambda_r/(1-\sigma)] \tag{3.35c}$$

$$D^h = \exp[\Lambda_s] + M^{1-\sigma} \tag{3.35d}$$

The expected utility is described by an LES model, so that

$$\Lambda_m^h = b(\beta) + \ln(e_m - p_m\gamma_a - \gamma_c) - \beta\ln p_m - \kappa \tag{3.36a}$$

for m=o,r

$$\Lambda_s^h = \beta\ln(x_s - \gamma_a) + (1-\beta)\ln(x_c - \gamma_c) \tag{3.36b}$$

The generalized logit model embodied in Lt is complemented in 3.34 by Ld, which incorporates density functions (f_m^h) for the households' conditional share equations ($s_m = p_m x_m$); so that

$$f_m^h = (2\pi\sigma_m^2)^{-1/2} \exp[-\epsilon_m^2/2\sigma_m^2] \tag{3.37a}$$

where

$$(\epsilon_m)^h = E(s_m) - s_m$$

$$= [p_m(1-\beta)\gamma_a + \beta(e_m - \gamma_c)]/e - s_m \tag{3.37b}$$

The likelihood function in 3.34 is maximized with respect to six basic parameters; $\beta, \upsilon, \gamma_a, \gamma_c, \kappa^5$, and σ. The results of

this estimation procedure are detailed in chapter five, following a discussion of data and variables in chapter four.

NOTES TO CHAPTER THREE

1. Roy's identity cannot be imposed upon the stochastic component of household utility in this context without introducing an unwanted interdependence between ϵ_m and λ_m.

2. The translation parameters (γ_i) in the LES specification are sometimes refered to in the Stone-Geary sense as "committed bundles". However, that interpretation is not applicable here, where we have no prior expectations regarding the sign of γ_i.

3. For the general likelihood function specified in 3.24 it is assumed that the housing decision is sequential. That is, households first select their tenure marginal on the actual opportunity sets. Secondly, the household determines the quantity of housing it wishes to consume conditional upon that tenure.

4. In contrast, Gillingham and Hagemann (1983) do find significant evidence of simultaneity bias. They attribute their different findings to differences in the data set used by both Rosen (1979) and Lee and Trost (1978), as well as to differences in the specification of housing expenditures for owner-occupiers.

5. As discussed in chapter five, $\kappa(d)$ is itself a function of demographic variables.

IV. Data and Variables

A. Introduction

This chapter addresses some of the practical data issues arising from the econometric model outlined in chapter three. That model assumes that observations for each household in some data set are available for each of the following variables:

(1) x_s --This variable is a measure of the flow of accommodation services derived from the dwelling unit in which the household resided during the period prior to that in which the housing decision is being modelled.

(2) $d_o x_o$ --If a household moves to a owner occupied dwelling unit during the current period (ie, if $d_o = 1$), then observations are required for the flow of accommodation services, x_o, derived therefrom.

(3) $d_r x_r$ --A similar requirement applies when the household chooses to move to rental accommodation.

(4) x_c --Observations are required for the consumption of goods unrelated to housing.

(5) p_o --This is the user cost of housing capital for households which own the dwelling unit in which they reside. Note that this is a flow price rather than a stock price.

(6) p_r --This is the flow price of accommodation for households who reside in rental units. This measure of gross rent includes utilities.

(7) p_s --Current period price of the household's accommodation, x_s, from the previous period. This may differ from p_o or p_r.

(8) p_c --Current period price of other consumables, normalized at 1.

(9) e_o --Income after taxes for the household, were it to choose to relocate to owner occupied accommodation. Based on average

income for 1977, 1979, and 1981.

(10) e_r --Income after taxes for the household, were it to choose to relocate to rental accommodation. Based on average income for 1977, 1979, and 1981.

(11) e_s --Income after taxes for the household, as defined above for y_o or y_r, were it to choose to remain in its present dwelling unit. Based on average income for 1977, 1979, and 1981.

Many of these variables require careful definition and construction.

B. The Data

This research employs a relatively new data set. The Social Change in Canada: Quality of Life Survey (QOL) is based on a nationwide sample of households. The QOL survey is designed, as the name suggests, to provide information on the quality of life in Canada. In doing so it touches upon many pertinent financial and housing data which may be readily incorporated into our model.

The surveys are conducted in three waves; 1977, 1979 and 1981; where many households are interviewed in all three waves. This satisfies an important criterion for our examination of households' move-stay decisions. We require information on both the dwelling units households move to and the dwelling units they move from. This essential requirement excludes many data sets which might otherwise be eligible, for example the 1981 Census of Canada. The 1979 survey year will serve as the base year for the empirical work undertaken here. This allows information from the 1977 survey to be drawn upon in reference to the 'previous period', particulary as regards x_s, the quantity of housing services consumed in the previous period. Using 1979 as the base year also allows the 1981 and 1977 surveys to play a role in defining permanent income or housing price trends.

Other data sets were also considered before the QOL set was selected. The CMHC Survey of Housing Units (SHU) provides excellent detail on dwelling structure and financing attributes,

but suffers from lack of information on neighborhood attributes and intra-CMA location. In addition, the SHU data is rapidly losing its applicability to today's housing markets because many changes have taken place since the 1971 and 1974 survey years.

The Michigan Panel Survey of Income Dynamics (PSID) was also considered. However the focus of that survey is on income, and the housing information is poor. Also, the sample is geographically dispersed over the United States so that difficulties would arise in trying to provide data on local housing-related taxes and or grants.

It would be too much to expect to find a data set with variables corresponding precisely to those indicated in section A of this chapter. Instead, one must convert the set of variables provided by the data set into a new set of variables. These transformations to the QOL data set are described in the remaining sections of this chapter.

C. Measuring Accomodation

1. Hedonic index of housing quantity

Two issues arise in developing a viable measure of the accommodation services actually consumed by a household. First, there is a question of the appropriate dimension and form for the regression equations which seek to translate observed attributes into a meaningful index of housing quantity. Secondly, one must find a suitable means of reconciling the quantity indexes for the ownership and rental sectors. These issues are discussed in turn, where discussion focuses first on the ownership sector.

The use of hedonic indexes to measure the flow of housing services is quite common (see, for example, Kain and Quigley (1970) or Rosen (1974)). Its purpose is to develop estimates, x_o, of the quantity of owner occupied accommodation services consumed based on a (nx1) vector, z_o, of attributes for owner occupied dwelling units. In particular, we have the linear relationship:

$$m(x_o) = \omega_o' z_o, \quad \text{or} \tag{4.1a}$$

$$x_o = m^{-1}(\omega_o' z_o) \tag{4.1b}$$

where ω_o' is a (1xn) vector of coefficients or weights and m( ) is some monotonic transformation of x_o.

Equations 4.1 describe the consumption of housing services as a monotonic transformation of some weighted average of observed housing attributes. The relevant questions are: (1) what vector of characteristics (or transformations thereof) ought to be included, and (2) which set of weights is appropriate?

It is reasonable to expect an index of housing consumption to satisfy two criteria:

1. Any two dwelling units with identical vectors of observed characteristics should also have identical values of the measured index. This rules out the use of v_o, the owner's evaluation of what the home would sell for, as a proxy index.
2. The weights ought to convey a sense of the market's valuation of the relative importance of specific characteristics. Thus if "floorspace" is observed to be relatively important in determining the value of a dwelling unit, it should also play a prominent role in establishing the index value.

The first criterion is satisfied by any vector ω_o from equation 4.1 which is constant across dwelling units. The second criterion may be met by regressing $m(v_o)$ on z_o for the

hedonic equation:

$$m(v_o) = a_o' z_o + \epsilon \tag{4.2}$$

The resultant OLS estimates, $\hat{a}_o'$ may then be used as the weights ω_o in 4.1 to generate values for x_o. Thus, x_o is the value of v_o which would appear to be consistent with the observed characteristics z_o of the dwelling unit. That is,

$$m(x_o) = E[m(v_o)|z_o] \tag{4.3}$$

And so we have

$$m(x_o) = \hat{a}_o' z_o \tag{4.4}$$

Although many specifications of the hedonic equation are consistent with 4.4, a semilog form, whereby one defines

$$m(x_o)=\log x_o ,$$

is found most frequently in the housing literature. Butler (1980), for example, uses the semilog form and bases his choice on four considerations:

(1) the ease of interpreting results;

(2) comparability with other studies;

(3) the high computational cost of the sometimes preferred alternatives based upon the quadratic Box-Cox functional form, and;

(4) the extreme lack of consensus on the subject at present.

Estimation results for this hedonic form are reported in table I. The attributes z_o are defined in appendix C, and are based on dwelling unit and neighborhood characteristics as supplied by the Quality of Life Survey.[1] These include dummy variables indicating in which census electoral district the dwelling unit is situated, thereby allowing location to enter the hedonic equation directly.

To avoid problems of multicollinearity principal components were extracted from the data for z_o. Then $m(v_o)$ was regressed on the principal components in an OLS regression. The econometrics program used here (see White (1978)) is able to transform the OLS coefficients back to correspond to the original variables as reported in table I. The method used is described in Judge et al (1985).

TABLE I
Estimation Results for
Hedonic Regressions

VARIABLE*	COEFFS (T-STATS)
STREETS	-0.800E-02 (-1.17)
HOMES	-0.196E-01 (-2.48)
PARKS	-0.255E-01 (-4.23)
SCHOOLS	-0.221E-01 (-3.09)
SHOPPING	-0.999E-02 (-1.45)
SATISFCN	+0.202E-01 (+4.04)
BATHROOM	+0.103E+00 (+12.5)
BEDRM2	-0.138E+00 (-6.48)
BEDRM3	-0.408E-01 (-2.67)
BEDRM4	+0.721E-01 (+3.65)
BEDRM5	+0.207E+00 (+6.11)
BEDRM6	+0.288E+00 (+5.41)
BEDRM7	-0.156E+00 (-1.19)
SEMID	-0.531E-01 (-2.73)
ATTACH	-0.157E+00 (-2.52)
FLAT	+0.380E+00 (+3.91)
APTLT6	-0.186E+00 (-1.40)
APTGT6	-0.196E+00 (-3.17)
PARKING	+0.137E+00 (+5.34)
ZED506	-0.164E+00 (-1.63)
ZED507	+0.826E-01 (+1.11)
ZED508	-0.197E-01 (-0.48)
ZED511	-0.678E-01 (-2.52)
ZED515	+0.167E-01 (+2.31)
ZED518	-0.847E-01 (-1.07)
ZED525	+0.290E-01 (+0.37)
ZED531	-0.838E-01 (-1.30)
ZED544	-0.223E+00 (-3.39)
ZED551	-0.164E+00 (-3.44)
ZED553	-0.220E+00 (-3.35)
ZED554	-0.407E-01 (-0.91)
ZED560	+0.335E+00 (+10.6)
ZED564	+0.604E+00 (+6.62)
ZED565	+0.149E+00 (+0.79)
ZED566	-0.437E-01 (-0.88)
ZED574	-0.701E-01 (-1.18)
ZED582	+0.187E+00 (+2.37)
ZED583	+0.245E+00 (+2.19)
ZED584	+0.185E+00 (+2.40)
ZED585	+0.137E-01 (+0.49)
ZED586	+0.386E-01 (+0.74)
ZED587	+0.268E-01 (+0.40)
ZED588	+0.486E-01 (+1.03)
INTERCEPT	+0.409E+01 (+59.4)

Dependent variable: LOGVALUE
Adjusted R-square = .5715 F=15.382
249 observations; t-ratios reported with 225 degrees of freedom
*See appendix C for further definition of attributes

2. Owner occupied vs. rental accommodation

Another characteristic to consider when calculating the quantity of accommodation consumed by the household is whether the dwelling unit is owner occupied or rental. This distinction may be perceived by households to be an important one and is therefore worth testing for.

Several attempts were made to estimate hedonic equations for the rental sector, but these did not produce usable results. Rent was used as the dependent variable in a regression similar to those run for the owner occupied sector. The relatively poor performance of the hedonic equations for the rental sector may well be due to institutional restrictions on rent; namely rent controls and/or the possible tendency for rents to depend upon length of tenure. While this poses a bit of a problem, other methods may be employed to provide estimates of x_r.[2]

One method which suggests itself is to use the coefficients a derived from hedonic regressions for the owner occupied sector together with observed characteristics from the rental sector, z_r, in order to produce estimates, x_a, of the quantity of accommodation services which the vector z_r would represent were it in the owner-occupied sector. That is,

$$x_a = m^{-1} (\hat{a}' z_r)$$

Another parameter, v, may then be introduced, the purpose of which is to effect the translation from the rental sector to the

owner occupied sector.

Three possible specifications for the relationship between x_a, x_r and υ are discussed here. They are

$$x_r = \upsilon x_a \tag{4.5a}$$

$$\log x_r = \upsilon \log x_a \tag{4.5b}$$

$$x_r = x_a - \upsilon \tag{4.5c}$$

Once x_a has been calculated, the corresponding flow price, p_a, may be derived directly since we have observations on rent; that is,

$$p_a = R/x_a \tag{4.6}$$

Similarly, we know that

$$p_r = R/x_r \tag{4.7}$$

Thus, using equations 4.5 thru 4.7 we may derive the following relationships between p_r and p_a:

$$p_r = p_a/\upsilon \tag{4.8a}$$

$$p_r = x_a^{1-\upsilon} p_a \tag{4.8b}$$

$$p_r = p_a x_a/(x_a - \upsilon) \tag{4.8c}$$

respectively.

While the demand equation for x_r may be derived in a straightforward fashion from the indirect utility function, the resultant expression is stated in terms of x_r and p_r, neither of which is observed.

It is necessary therefore to substitute for x_r and p_r from equations 4.5 and 4.8 in order to translate the demand equation

$$x_r = (1-\beta)\gamma_a + \beta(e-\gamma_c)/p_r \tag{4.9}$$

into new equations which are expressed in terms of the observed variables x_a and p_a and in terms of the new parameter υ. Doing so yields the following;

$$x_a = (1-\beta)\gamma_a/\upsilon + \beta(e-\gamma_c)/p_a \tag{4.10a}$$

$$x_a = (1-\beta)\gamma_a x_a^{(1-\upsilon)} + \beta(e-\gamma_c)/p_a \tag{4.10b}$$

$$x_a^2 - [\upsilon+(1-\beta)\gamma_a+\beta(e-\gamma_c)/p_a]x_a + \beta(e-\gamma_c)\upsilon/p_a = 0 \tag{4.10c}$$

The first of these equations, corresponding to the specification $x_r = \upsilon x_a$, has the most desirable properties. It is quite similar to the demand function in 4.9, with x_a and p_a substituted for x_r and p_r respectively

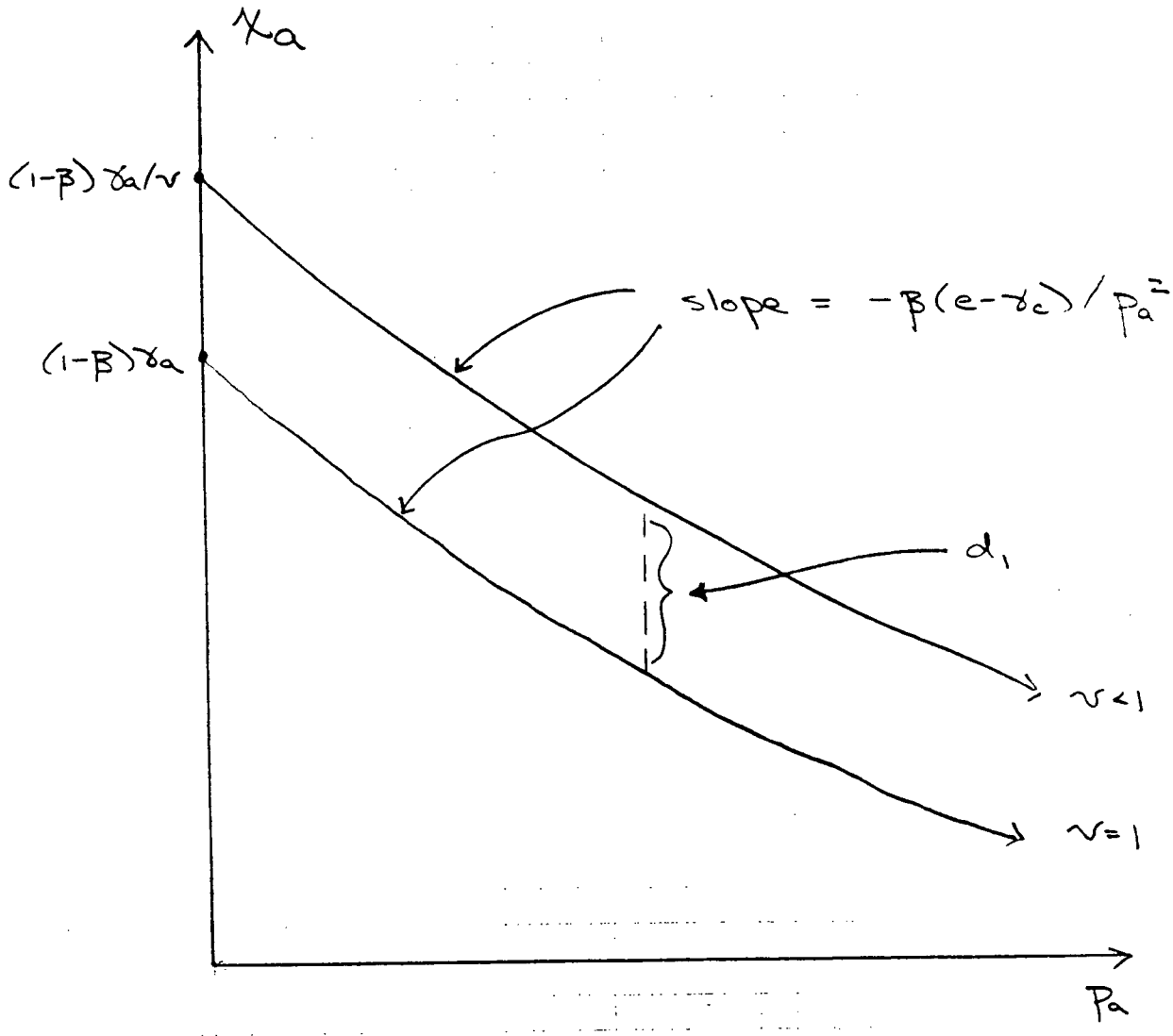
If $\upsilon < 1$, then rental accommodation is a "lesser good" than owner occupied accommodation, even where they otherwise share similar observable characteristics. In this case, the household is compelled to consume more housing in the rental sector than in the owner sector in order to derive a specified level of utility from housing, given its consumption of non-housing goods. And so $\upsilon < 1$ serves to shift the demand curve upwards by an amount $d_1 = (1-\beta)\gamma_a(1-\upsilon)/\upsilon$, as illustrated in figure 9.

The second demand equation in 4.10, corresponding to the specification $\log(x_r) = \upsilon\log(x_a)$, is intractable. It cannot be solved explicitly for x_a unless $\gamma_a=0$ or $\beta=1$.

The third equation corresponds to the specification $x_r = x_a - \upsilon$. The demand function here is in the form of a quadratic equation in x_a, and so it is possible to solve explicitly for two distinct roots. However, these solutions are most unwieldly and offer little or no hope of retrieving the original parameters.

Figure 9

Theta Shifting Demand Curve



On balance therefore the first specification, $x_r = \upsilon x_a$, is the best choice among the three alternatives. It allows us to test whether households perceive a given set of housing characteristics in the rental sector in the same light as they would in the ownership sector. That is, we can test whether $\upsilon=1$.

D. The Price of Accomodation

A great deal of controversy has arisen over the proper measurement and definition of p_o, the flow price of owner-occupied housing. Yet there is a surprising degree of concensus among housing researchers as to the appropriate definition of p_o. Namely,

$$p_o = v_o [w+r+\lambda_g - \pi^e] \tag{4.11}$$

where v_o is the dwelling unit specific stock price for one hedonic unit of owner occupied housing, w is the effective mill rate, r is the household's opportunity cost of capital, λ_g is the gross rate of physical depreciation and π^e is the expected rate of capital gains for that dwelling unit.

Dougherty and Van Order (1982) observe that this flow price may be derived in two ways. As a "user cost", p_o is the amount necessary to induce a household to give up one unit of housing,

and is derived from the first order conditions for the household's utility maximization problem. Alternatively, p_o may be viewed as an "implicit rent", the rents which would prevail in a competitive market. This rental price may be derived from the first order conditions for the landlord's profit maximization problem, and in equilibrium, is equal to the consumer's user cost.

While the recent literature appears to be largely in agreement regarding at least the appropriate theoretical construction of p_o, available data samples have not yet wholly conformed to this view. For example, as noted by Diamond (1980), the U.S. CPI places undue emphasis on the stock price of new dwelling units and on nominal interest rates, and this can lead to serious misperceptions as to the true cost of housing. Another alternative measure of the flow value of housing services, proposed by DeLeeuw (1971), is the sum of expenditures on maintanence, taxes, utilities and mortgages. As noted by Jones (1984), this measure is seriously flawed because it ignores the opportunity cost of capital on the owner's housing equity.

The DeLeeuw measure is in fact provided by the QOL data. Specifically, the total monthly cost (TMC) variable for owner-occupiers provided by the QOL survey may be expressed in our notation as follows:

$$TMC = (1/12)v_o x_o [(1-\Theta)r_m + w + \lambda_m] \qquad (4.12)$$

while the opportunity cost of capital for homeowners is given by

$$r = (1-\Theta)r_m + \Theta(1-t)r_b \qquad (4.13)$$

where

$(1-\Theta)$ is the proportion of the dwelling unit's value which is mortgaged,

r_m is the mortgage rate of interest for the household,

$r_b(1-t)$ is the after tax rate of return the household could expect to receive from bonds or other investment income,

λ_m is the value of maintanence expenditures expressed as a proportion of dwelling unit value, and

t is the household head's marginal income tax rate.

Using equations 4.11 through 4.13, the relationship between p_o and TMC can be summarized by

$$p_o = 12*TMC/x_o + v_o[\Theta(1-t)r_b + \lambda_n - \pi^e] \qquad (4.14)$$

where $\lambda_n = \lambda_g - \lambda_m$.

In other words, the variable TMC when expressed on an annualized per unit basis is a suitable proxy for p_o, once it has been adjusted for the opportunity cost of capital on homeowner equity as well as for net physical depreciation and anticipated capital gains.

The QOL data provide explicit information on household mortgage indebtedness for 1977, the first year of the panel study. With this one may develop suitable estimates of Θ for 1979, the base year using Tobit regressions.

An adjustment is also required for the depreciation variable implicit in the QOL variable TMC from equation 4.12. As noted by Chinloy (1980), the gross depreciation rate (λ_g) applied to the dwelling unit value is the appropriate measure for total depreciation costs faced by households. Gross depreciation is net depreciation plus expenditures on maintanence and repair, where these are all converted to rates by dividing through by dwelling unit value, yielding the identity $\lambda_g = \lambda_n + \lambda_m$.

The QOL variable TMC encompasses maintanence and repair expenditures but not net depreciation, which must therefore be included explicitly, as in equation 4.14. It may be possible to do so by noting that

$$(\Delta v/v)^e = \pi^e - \lambda_n \qquad (4.15)$$

since π^e refers to capital gains net of any physical depreciation effects while movement in the stock price of a dwelling unit would reflect both capital gains and physical depreciation effects. Thus 4.14 may be rewritten:

$$p_o = 12*TMC/x_o + v_o[\Theta(1-t)r_b - (\Delta v/v)^e] \qquad (4.16)$$

It remains to develop suitable estimates for $(\Delta v/v)^e$. This is perhaps the most difficult variable to pin down empirically because it entails all the measurement problems with which expectations are fraught. In particular, short of asking homeowners outright what their expectations are regarding the future value of their dwelling units (which was not done in the QOL survey), one has little firm ground upon which to base estimates.[3]

This problem has been faced by other housing researchers as well. For example, Rosen (1979) refers to the potentially significant role played by capital gains expectations in determining perceived user costs but states that in the absence of a reliable method of estimating each homeowner's expected capital gains, he is forced to ignore them.

King (1980), too, consciously exludes capital gains from both his income and price variables. He derives an expression for the effective rental rate for homeowners similar to our own, but cites formidable empirical difficulties in proceeding along these lines and so follows Rosen (1979) and Laidler (1969) by invoking an assumption which serves to remove the capital gains term from the effective rental price expression.

Dougherty and Van Order (1982) suggest, and this researcher agrees, that the appropriate rate for $(\Delta v/v)^e$ is the annualized expected rate of return over the relevant holding period. The longer the holding period, the less likely is this expectations term to be influenced by cyclical or speculative fluctuations in

housing stock prices. If we choose a relatively distant expectations horizon, therefore, the resulting user costs may differ from those perceived by speculators who may operate within the confines of relatively short time horizons. Thus, to the extent that the households in our sample are behaving as "dwellers" rather than as "speculators", a longer expectations time horizon is appropriate.

Capozza (1983) argues that, in general, inflation rates may be a good indicator of expected appreciation rates for houses since over long periods of time houses have tended to appreciate only slightly more quickly than the inflation rate. He argues further that housing is not particularly advantageous as a speculative vehicle even in volatile real estate markets such as those found in Vancouver in the early 1980s. This is due to the illiquidity of housing stock as well as the high transactions costs and tax considerations related to housing.

Thus, by applying a longer time horizon, we may not be grossly misrepresenting the outlook of most households in our sample. This suggests using some measure of expected inflation rates over a five or six year period as a proxy for anticipated capital gains on housing. This may be modified by actual changes in dwelling unit values as reported by respondents to the QOL survey.

Institutional constraints on borrowing may further modify our definition of p_o. Jones (1984) distinguishes between several classes of homeowners, based on their debt/equity

ratios. One of these is the prototype 'first time homebuyer', whose net worth is a relatively small proportion of the capital cost of the house being acquired. He argues that borrowing constraints for this class of owner are most likely to be the determinants of the amounts borrowed. This may have a bearing on observed relationships between the price of owner-occupied housing services and the quantity demanded, as well as influencing the tenure choice itself.

For example, where housing stock values are widely anticipated to appreciate more rapidly than the general rate of inflation, p_o can become quite low or even negative. Institutional constraints on borrowing would help to explain the behavior of households who rent in the face of seemingly negligible costs of owner-occupied housing services.

Dougherty and Van Order (1982) observe that, similar to Kearl's (1979) cash flow constraint, a constraint on total borrowing causes the expression for the user cost of housing capital to contain an additional term:

$$p_o = v_o [w+r+\lambda_g - \pi^e + c] \qquad (4.17)$$

where c is the ratio of the shadow price of the borrowing constraint to the marginal utility of x_c, the nonhousing good. (Note that c drops out when the constraint is nonbinding.)

Intuitively, this revised expression suggests a "counterbalance" to the tendency for π^e to pull the flow price

of owner occupied housing down "too far". If Jones' hypothesis is correct, we would expect our results to improve if we add a term 'c' to user costs for first time home buyers, ie, those with little net worth relative to housing stock values. This applies particularly to highly inflationary housing markets.

The discussion thus far has focused on the price of accommodation as pertains to owner occupied dwelling units. The price of rental units is considerably more straightforward to determine. In particular, for each household we have

$$p_r = 12*R/x_r \qquad (4.18)$$

where R is the monthly cost of residing in the respondent's dwelling unit, as reported by the QOL survey. Considerations of capital gains, depreciation and the like are the concern of the landlords, who are not modelled here, rather than of the tenants of rental accommodation, who are.

E. Defining Expenditure

Three issues arise regarding the use of household expenditure data from the QOL survey. They are (1) the use of permanent versus current income measures, (2) the role of savings, and (3) implicit expenditures on owner occupied housing.

Most authors agree that current period expenditures alone are not an adequate measure of expenditure or income when viewed in the context of the demand for housing or other major durables.[4] Theoretically, one should use anticipated household

earnings over the expected length of tenure, as outlined in Muth (1974). However, anticipated earnings are not reported in the QOL survey. As a proxy therefore this research uses average household income after taxes[5] reported in 1977, 1979, and 1981.[6]

The second issue concerns the role of savings. The linear expenditure system used in this model is a complete system of demand equations. That is, it describes the household's allocation of total expenditure between two alternatives, housing (x_a) and other consumables (x_c). By definition therefore, total expenditure (e) satisfies

$$e = p_o x_o + x_c \tag{4.19}$$

As well, total expenditure may be defined as net income (y) less savings;

$$e = y - s \tag{4.20}$$

A slight problem arises in that the panel data used here do not provide explicit data on either s, x_c, or e; although information on net income (y) is provided.

We are compelled therefore to use net income as a proxy for total expenditure. This is equivalent to viewing 'savings' as just another good, distinguished only by the period in which the expenditure is actually made[7] since, by 4.19 and 4.20, we have

$$y = p_o x_o + (x_c + s) \tag{4.21}$$

This is less of a problem when expenditure is defined as an average over several years, as is the case here. For savings in one year will tend to offset dissavings in another.

A final adjustment needs to be made for implicit expenditures on housing. Recall from the previous section that the price (p_o) of owner occupied housing has an explicit $(p_o)^e$ and an implicit $(p_o)^i$ component. The former includes cash disbursements on mortgage payments, taxes, and maintanence. The latter includes implicit costs such as the opportunity cost of capital on homeowner equity and (the negative of) capital gains on the home. Thus, 4.19 may be rewritten[8] for homeowner as

$$e_o = y = [(p_o)^e + (p_o)^i]*x_o + x_c \tag{4.22}$$

However, net income (y_Q) from the QOL survey is given by

$$y_Q = (p_o)^e x_o + x_c \tag{4.23}$$

It is necessary therefore to add implicit housing expenditures $(p_o)^i]*x_o$ to net income reported in the QOL survey before it can be used as a proxy for expenditures in the manner described above.[9]

NOTES TO CHAPTER FOUR

1. Several hedonic regressions were run using several sets of attributes. The results reported table I are those which were judged by the author to yield the best combination of overall fit and sensible interpretation.

2. Rent controls were implemented in Ontario effective July 1975, and so had been in effect for four years prior to our base year.

3. Even then, one must reconcile these owner expectations with the views of prospective buyers.

4. For example, most of the papers reviewed by Mayo (1981) use an average of reported incomes over several years as a proxy for "permanent income". That is similar to the approach taken here.

5. Canadian income tax forms for the appropriate years were used to calculate the relevant deductions and tax rates.

6. If observations are lacking for one of these years, the estimate of household income is based on the remaining years. Children's income is not included.

7. This is consistent with the implicit assumption of weak intertemporal separability discussed in chapter two.

8. Setting savings (s) equal to zero.

9. Actually, these net implicit expenditures are typically negative due to capital gains.

V. Estimation and Results

A. Summary of the Basic Model

For ease of reference, the full model from chapter three is summarized by equations 5.1 through 5.4, beginning with the log-likelihood function

$$\log L = [\sum_h \sum_j d_j * \log P_j^h] + [\sum_h \sum_m d_m * \log f_m^h(\epsilon_m)]$$

$$= Lt + Ld. \qquad (5.1)$$

$h=(1,H)$

$j=o,r,s$

$m=o,r$

The first term, Lt, embodies a generalized extreme value (GEV) model. It describes the probability (P_j^h) of household h selecting tenure j as a function of its expected utility in that tenure (Λ_j)^h relative to the alternatives. That is;[1]

$$P_m^h = \exp[\Lambda_m/(1-\sigma)]*M^{\sigma}/D \qquad (5.2a)$$

$$P_s^h = \exp[\Lambda_s]/D \qquad (5.2b)$$

where

$$M^h = \exp[\Lambda_o/(1-\sigma)] + \exp[\Lambda_r/(1-\sigma)] \tag{5.2c}$$

$$D^h = \exp[\Lambda_s] + M^{1-\sigma} \tag{5.2d}$$

As described in chapter three, these utility levels are based on an LES specification. Thus,

$$\Lambda_m^h = b(\beta) + \ln(e_m - p_m\gamma_a - \gamma_c) - \beta\ln p_m - \kappa \tag{5.3a}$$

for m=o,r

$$\Lambda_s^h = \beta\ln(x_s - \gamma_a) + (1-\beta)\ln(x_c - \gamma_c) \tag{5.3b}$$

The second expression in 5.1 comprises the density functions for the error terms from the households' conditional demand share ($s_m = p_m x_m$) equations;

$$f_m^h = (2\pi\sigma_m^2)^{-1/2} \exp[-\epsilon_m^2/2\sigma_m^2] \tag{5.4a}$$

where

$$(\epsilon_m)^h = E(s_m) - s_m$$

$$= [p_m(1-\beta)\gamma_a + \beta(e_m - \gamma_c)]/e - s_m \tag{5.4b}$$

Equation 5.1 is maximized with respect to the six basic parameters appearing in equations 5.2 through 5.4. They are

β -- the marginal share of household expenditures which goes to housing,

υ -- a perceptual parameter which translates rental accommodation quantities and prices into owner-occupied measurement units,

γ_a, γ_c -- parameters from the LES model which serve to translate the axes for accommodation services and other goods, respectively,

κ -- a measure of the disutility of residential relocation,

σ -- a parameter from the GEV model which allows for correlation between error terms for the two moving options (m=o,r).

B. Nonlinear Estimation

The likelihood function summarized by equations 5.1 through 5.4 is clearly nonlinear in parameters, and so calls for nonlinear estimation techniques. This research employs a quasi-Newton method[2] (FLETCH) based on programs described by Fletcher(1970,1972). The documentation for FLETCH is found in Vaessen(1984).[3] The user may employ either numeric or analytic derivatives. For this research, numeric derivatives were used.[4]

C. Demographic Variables

1. Demographic scaling and demographic translating

There are many ways to introduce demographic variables into complete demand systems, a number of which are discussed by Pollak and Wales (1981) and by Deaton and Muellbauer (1980, ch.8). Of these procedures, demographic scaling and demographic translating are of particular interest[5].

Demographic scaling modifies the original demand system by replacing all quantities x_i from the original utility function U^* by their respective scaled values, x_i/m_i. That is,

$$U(x) = U^*(x/m) \tag{5.5a}$$

In order to satisfy the budget constraint, prices must be scaled in an inverse manner;

$$V(p,e) = V^*(pm,e) \tag{5.5b}$$

with the corresponding demand functions given by

$$x_i(p,e) = m_i x_i^*(pm,e) \tag{5.5c}$$

These scaled quantities and prices were first proposed by Barten (1964) as a vehicle for introducing demographic variables. To illustrate, if m is household size, then the Barten scaling method effectively transforms the demand system into one based on per capita consumption levels. However, m is typically a more general function of the pertinent demographic variables, and may differ for each good.

Demographic translating, first applied by Pollak and Wales (1978), employs translation parameters (t_i) to shift the axes. Thus,

$$U(x) = U^*(x-t) \tag{5.6a}$$

$$V(p,e) = V^*(p,e-\Sigma_i p_i t_i) \tag{5.6b}$$

and

$$x_i(p,e) = x^*_i(p,e-\Sigma_i p_i t_i) + t_i \tag{5.6c}$$

where the t_i are some functions of household characteristics.

The demographic translating parameters (t_i) are quite similar to the translating parameters (γ_i) found in the LES specification discussed in chapter three. Indeed, demographic

translating is applied to the LES model by simply replacing γ_i with $\gamma_i + t_i$ in equations 5.3 [6]. By virtue of this technique the translation parameters in the LES model become functions of household characteristics.

For the LES model, it can be shown that the demographic scaling and translating procedures are equivalent [7]. However, for reasons explained in the following pages, neither procedure is employed here to obtain the final parameter estimates.

2. Initial estimation attempts

A good deal of effort was expended in early attempts to obtain convergence with the basic model, both with and without demographic translating. Throughout this period, there were no hints of success. All attempts to run the nonlinear estimation procedure either stalled at meaningless values [8] without converging, or bombed due to violation of regularity conditions.

A frequent source of problems were the LES translation parameters γ_i (i=a,c). These parameters enter the LES model embedded within logarithms, implying the following parameter restrictions (see equations 5.3):

$$e_m - p_m \gamma_a - \gamma_c > 0 \qquad (5.7a)$$

$$x_s - \gamma_a > 0 \qquad (5.7b)$$

$$x_c - \gamma_c > 0 \qquad (5.7c)$$

The estimation routine frequently produced estimates of γ_c and γ_a which violated these restrictions, thereby causing the estimation process to abort. This problem was exacerbated by efforts to introduce demographic translating.

One strategy for dealing with this problem is to use linear approximations of the expressions in 5.7, so that the direct and indirect subutility indexes become

$$\Lambda_m^h = b(\beta) + \ln(e) - (p_m \gamma_a + \gamma_c)/e_m - \beta \ln p_m - \kappa \qquad (5.8a)$$

and

$$\Lambda_s^h = \beta(\ln x_s - \gamma_a / x_s) \qquad (5.8b)$$

This formulation places no restrictions on the γ's.

Estimation of 5.8 (logit only) uncovered two distinct convergent solutions with almost identical likelihood values. These solutions have intriguingly different interpretations. The first has parameter estimates which accord reasonably with <u>a priori</u> expectations, although the estimate for β is quite high at 0.64. The second solution suggests a model of "demographic determinism", where β=0 and the tenure choice selection is driven purely by demographic considerations. These two solutions are briefly reviewed in appendix D.

While this second strategy provided welcome relief in the form of convergent solutions, it is not entirely satisfactory. In particular, 5.8a and 5.8b are no longer dual to one another.

Instead, they are approximations of the true dual forms in 5.3. Moreover, substituting the solution values from the linear form into 5.3 causes a majority of households to violate the regularity conditions specified in 5.7. Thus, the approximation procedure does not generate useful starter values for the LES model. Finally, the likelihood values for the linear solutions fall short of those reported later in this chapter.

It was therefore necessary to place a ceiling on the estimated values for γ_a and γ_c. The ceilings used here (γ_a*=0 and γ_c*=-1000) were arrived at through a simple grid search procedure, varying γ_a by increments of 1 and γ_c by increments of 1000. The model was then re-estimated using

$$\gamma_i = \gamma_i^* - \exp(\tau_i) \qquad i=c,h \qquad (5.9)$$

where τ_i is the new parameter to be estimated. In all instances, the values for τ_i were large and negative so that $\exp(\tau_i)$ became indistinct from zero. Therefore, the γ_i were fixed at γ_i* for the final estimation.[9]

3. Demographic variables and tenure choice

The use of demographic translating within an LES framework focuses upon the LES translation parameters, γ_i. This has worked well for Pollak and Wales (1978) and others in the context of complete demand systems for non-durable consmer goods. These earlier efforts did not seek to model the tenure decision. Some adaptations may therefore be in order for respecifying the manner in which demographic variables are introduced in the context of tenure choice.

A straightforward means is offered by allowing κ, the parameter which measures the disutility of relocation, to be modified by demographic variables. This specification recognizes that households may have various sociodemographic reasons for wanting (or not wanting) to relocate. Simple crosstabulations of tenure choice versus demographic attributes are summarized in table II. They confirm that age of household head, family size, and marital status are correlated with a willingness to relocate.

For example there appears to be a demarcation in this sample between household heads below versus above forty-three years of age. Of the 31 movers in the sample, 26 were younger than forty-three, while only 42 of the 107 stayers were younger than forty-three. Likewise, of the 17 married movers, 16 had been married for less than ten years. In contrast, only 13 of the 71 married stayers had been married for less than ten years. It would appear as well that moving is more disruptive for

TABLE II

Crosstabulations of Residential Mobility and Selected Demographic Attributes

	Movers	Stayers
Age of Head		
< 43 years	26	42
≥ 43 years	5	65
How Long Married		
< 10 years	16	13
≥ 10 years	1	58
Family Size		
< 4	22	64
≥ 4	9	43

larger families. Of the 52 families with four or more members, only 9 were movers.[10]

Based on these findings, κ was replaced in 5.3a by

$$\kappa(d) = \delta_k + \delta_{kl} d_l + \delta_{kw} d_w + \delta_{kn} d_n \qquad (5.10)$$

where

$d_l = 1$ if age of household head is less than 43,

otherwise, $d_l = 0$

$d_w = 1$ if the household head is married and has

been for less than 10 years, otherwise, $d_w = 0$; and

d_n is household size.

When κ is replaced by $\kappa(d)$ in 5.3a, the demographic variables d_i influence the tenure decision but not the conditional demand decision. However, it is not unreasonable to expect that the marginal share of expenditures toward accommodation might be influenced by family size.

To test for this possibility directly, β is modified as well;

$$\beta(d) = \delta_b + \delta_{bn} d_n \qquad (5.11)$$

Estimation results reported below cannot reject the null hypothesis that $\delta_{bn} = 0$. This may be partly explained by the

fact that x_a is a hedonic measure of accommodation services consumed. A number of tradeoffs may be made by households behind this hedonic veil which are not captured by the data. For example, a larger family may choose to reside in a less prestigous location in favor of a dwelling unit with more bedrooms.

D. Results

1. Disutility of Relocation (κ)

Table III presents the estimation results for the model described above. This solution satisfies the second order conditions for a maximum point of the likelihood function, and is robust to arbitrary changes in the starter values used during the estimation process.

There are several interesting features of these results. Perhaps most central to this model is the finding that relocation generally results in significant household disutility due to transactions costs and uprooting effects. Moreover, the extent of this utility loss is clearly a function of household demographic status. This is shown by the values for the δ_k reported in table III, which are all significant at 95 percent confidence levels.[11] Perceived transactions costs are higher for larger families, but are lower for younger families and newlyweds. This has a direct effect on tenure choice probabilities, as will be seen shortly.

TABLE III

Maximum Likelihood Parameter Estimates[1] for the Full Model

Parameter	Estimated Value	Asymptotic t-stat
β	.19084	10.49
δ_k	2.5494	3.61
δ_{kl}	-2.5473	-3.57
δ_{kw}	-1.5479	-2.78
δ_{kn}	0.5699	2.66
log L	-47.6583	

1. The following parameter values were held fixed during this estimation; $\upsilon = 1.0$, $\gamma_a = 0$, $\gamma_c = -1000$, $\sigma = 0$. See text for details.

As described in chapter two, one can use the expenditure function to calculate moving costs in terms of foregone expenditures or equivalent transactions costs (TC). That is,

$$TC = e_m(p_m, \Lambda_m) - e(p_m, \Lambda_m - \kappa) = e_m - e(p_m, \Lambda_m - \kappa) \qquad (5.12)$$

For the LES model outlined in 5.3a, the utility for a mover is given by

$$\Lambda_m = b(\beta) + \ln(e_m - p_m \gamma_a - \gamma_c) - \beta \ln p_m - \kappa$$

So by definition,

$$e_m = e_m(p_m, \Lambda_m) = \exp(G+\kappa)+g \qquad (5.13)$$

where

$$G = \Lambda_m + \beta \ln(p_m) - b(\beta)$$

and

$$g = p_m \gamma_a + \gamma_c .$$

Combining 5.12 and 5.13 it is now possible to solve for the transactions terms explicity;

$$\begin{aligned} TC &= \exp(G+\kappa) - \exp(G) \\ &= \exp(G+\kappa)*[1-\exp(-\kappa)] \\ &= (e_m - g)*[1-\exp(-\kappa)] \qquad (5.14) \end{aligned}$$

As shown in table IV, the values for the expression [1-exp(-κ)] vary widely by household type. Of the eight prototypes considered here, the least likely to relocate is a family of four with a household head aged forty-three or more who has been married for at least ten years. In contrast, a young newlywed

TABLE IV

Transcations Costs (TC) as a Proportion of Residual Household Income ($e_m - g$) for Eight Household Prototypes

Household Characteristics*	$TC/(e_m - g)$
LE42 = 0 NWED = 0 NHSD = 2	0.975
LE42 = 1 NWED = 0 NHSD = 2	0.681
LE42 = 0 NWED = 1 NHSD = 2	0.882
LE42 = 1 NWED = 1 NHSD = 2	-0.501
LE42 = 0 NWED = 0 NHSD = 4	0.992
LE42 = 1 NWED = 0 NHSD = 4	0.898
LE42 = 0 NWED = 1 NHSD = 4	0.962
LE42 = 1 NWED = 1 NHSD = 4	0.520

*Definitions:

LE42=1 if head is younger than 42, LE42=0 otherwise
NWED=1 if married less than 10 years, NWED=0 otherwise
NHSD equals size of household

couple without children is positively inclined to relocate. These results are consistent with the crosstabulation results reported earlier.

The values of TC implied by table IV are quite large relative to the expenditure levels observed in our sample. This constitutes strong evidence that demographic variables are dominant in the move-stay decision.[12]

A more meaningful context for interpreting the effect of demographic variables on the disutility of moving is based on tenure choice probabilities. Table V presents these probabilities for the same eight household types considered in table IV. It shows that the young newlywed couple has less than a 25 percent probability of staying in their current residence from one (two-year) period to the next. In contrast, the well-established family of four has a probability of not relocating in excess of 98 percent.[13] These results are consistent with the findings of Steele (1979, p16) who, in her study of housing demand in Canada, reports that "greater age is associated with a lower probability of relocation.

TABLE V

COMPARATIVE HOUSEHOLD CHARACTERISTICS AND TENURE PROBABILITIES
AT MEAN PRICE AND EXPENDITURE VALUES

HSHD CHARS	OWNERS PROB THAT TENURE = M/OWN	M/RENT	STAY	RENTERS PROB THAT TENURE = M/OWN	M/RENT	STAY	ALL HSHDS PROB THAT TENURE = M/OWN	M/RENT	STAY
LE42 = 0 NWED = 0 NHSD = 2	0.025	0.022	0.953	0.026	0.024	0.950	0.025	0.023	0.952
LE42 = 1 NWED = 0 NHSD = 2	0.202	0.183	0.614	0.211	0.191	0.598	0.205	0.186	0.609
LE42 = 0 NWED = 1 NHSD = 2	0.099	0.089	0.812	0.104	0.094	0.802	0.100	0.091	0.809
LE42 = 1 NWED = 1 NHSD = 2	0.392	0.355	0.253	0.399	0.361	0.240	0.394	0.357	0.249
LE42 = 0 NWED = 0 NHSD = 4	0.008	0.007	0.985	0.009	0.008	0.983	0.008	0.008	0.984
LE42 = 1 NWED = 0 NHSD = 4	0.088	0.079	0.833	0.093	0.084	0.823	0.089	0.081	0.830
LE42 = 0 NWED = 1 NHSD = 4	0.036	0.033	0.931	0.039	0.035	0.927	0.037	0.033	0.930
LE42 = 1 NWED = 1 NHSD = 4	0.255	0.231	0.515	0.264	0.239	0.497	0.258	0.233	0.509

2. Expenditure and Price Elasticities

The expenditure and price elasticities for the LES model outlined above are given respectively by

$$\eta(e) = \beta e \,/\, [p_a(1-\beta)\gamma_a + \beta(e-\gamma_c)] \tag{5.15a}$$

$$\eta(p) = -\beta(e-\gamma_c) \,/\, [p_a(1-\beta)\gamma_a + \beta(e-\gamma_c)] \tag{5.15b}$$

As discussed earlier in this chapter, it was necessary to place ceilings on the estimated values for γ_a and γ_c, so that

$$\gamma_a = \gamma_a^* - \exp(\tau_a) \tag{5.16a}$$

$$\gamma_c = \gamma_c^* - \exp(\tau_c) \tag{5.16b}$$

By substituting for γ_a and γ_c from 5.16, the elasticities in 5.15 may be rewritten as

$$\eta(e) = \beta e \,/\, [p_a(1-\beta)(\gamma_a^* - \exp\tau_a) + \beta(e-\gamma_c^* + \exp\tau_c)] \tag{5.17a}$$

$$\eta(p) = \beta(e-\gamma_c + \exp\tau_c) \,/\, [p_a(1-\beta)(\gamma_a^* - \exp\tau_a) + \beta(e-\gamma_c^* + \exp\tau_c)] \tag{5.17b}$$

It is clear from inspection that the restrictions imposed by 5.16 do not impose *a priori* any restrictions on the signs or magnitudes of the expenditure and price elasticities in 5.17.

However, as reported earlier, both restrictions in 5.16 held with equality, so that

$$\gamma_a = \gamma_a^* = 0 \tag{5.18a}$$

$$\gamma_c = \gamma_c^* = -1000 \tag{5.18b}$$

As a consequence, the elasticities in 5.15 now become

$$\eta(e) = e / (e+1000) \tag{5.19a}$$

$$\eta(p) = -1 \tag{5.19b}$$

The unitary price elasticity is particularly striking, and is a direct result of the finding that $\gamma_a=0$. That is, $\gamma_a=0$ implies $\eta(p)=-1$, regardless of the values of p_a, e, β, or γ_c. Thus, since we are unable to reject the null hypothesis that $\exp(\tau_a)=0$, we are also unable to reject the hypothesis of unitary price elasticity.

And given $\gamma_a=0$, $\eta(e)$ becomes $e/(e-\gamma_c)$, and so neither β nor p_a has any effect on the estimated value of the expenditure elasticity. Both e and γ_c are reported in units of annual expenditure dollars, so $\gamma_c=-1000$ in 5.18b ensures that the expenditure elasticity lies between 0.9 and unity for households

with annual expenditures in excess of $9000.

These results are consistent with the evidence on housing demand elasticities as summarized by DeLeeuw(1971), Quigley(1979), and Mayo(1981). To the extent that a consensus has been reached, it is generally agreed that the demand for housing is slightly inelastic with respect to both expenditure and price.

However, most such studies estimate their elasticities using a log-linear housing demand equation;

$$x_a(p_a, e) = \eta_e \ln e + \eta_p \ln p_a + \epsilon \qquad (5.20)$$

where η_e and η_p are the expenditure and price elasticities. While their results are similar to those reported here, this model provides useful confirmation of those earlier estimates because its distinguishing features.

In particular, this model examines (a) the simultaneous nature of tenure choice and conditional demand within a consistently defined framework of household utility maximizing behavior, and (b) the key role played by transactions disutilities and demographic variables in delineating movers from stayers. Each of these features may impact upon the reported elasticity estimates.

First, in this model the parameters are consigned to double duty. They play a key role in both the tenure choice and conditional demand decisions. Those studies which examine the

latter decision in isolation, therefore, may be producing biased parameter and elasticity estimates, as suggested by Lee and Trost(1978).

The second major feature distinguishing this model from the bulk of earlier studies on housing demand elasticities is the role played by transactions costs. It is argued in chapter two that models by King(1980), Rosen(1979) and others which overlook this barrier to residential relocation will tend to understate the true elasticities. The reason is that small changes in prices or income will not likely result in relocation for a typical household, thus giving the appearance of an inelastic demand response [14].

While most studies of housing demand do not consider relocation barriers explicitly, some do limit their sample to households which have recently relocated. For example, both Polinsky and Ellwood (1979) and Goodman and Kawai (1982) confined their sample to recent mover/owners.

By doing so, current housing consumption levels within their sample are more likely to reflect current expectations regarding future prices, income and demographic variables than would otherwise be the case. A disadvantage in their method lies in the risk of sample selectivity bias. A good deal of information is discarded when one considers only recent movers, and this body of information may not be faithfully reflected in the limited sample.

This contrasts with the method used in this study, which bases its parameter estimates on movers and nonmovers alike, using transactions costs as modified by household demographic attributes to highlight the differences between these two groups.

3. Rental versus Owned Accomodation (υ)

As described in chapter four, the parameter υ measures perceived differences in accommodation services flowing from identical units in different sectors. That is, υ measures the extent to which a bundle of housing characteristics in the rental sector is equivalent, in the eyes of the consumer, to a similar bundle of characteristics found in the owner-occupied sector;

$$x_r = \upsilon x_a \tag{5.21}$$

If υ equals one, then the consumer evaluates accommodation services on the basis of the stock characteristics themselves, and not on the sector in which they reside. In this case, the decision to own or rent must be explained with reference to price, income, and demographic considerations.

Testing of the model shows that the likelihood value is quite insensitive to the value of υ. In particular, earlier sets of results produced estimates of υ as high as 5.2, but none were significantly different from zero, let alone from unity. Thus, for the estimates reported in table III, υ was fixed at

unity, with no significant reduction in the model's explanatory power.

4. Red House--Blue House (σ)

Chapter three describes the role played by σ, which allows for correlation between error terms for the two moving options (m=o,r). The maximum likelihood estimate of σ is 0.5004, but the corresponding t-statistic of 0.66 suggests the likelihood value is very insensitive to changes in the value of σ.

The parameters were therefore re-estimated with σ fixed at zero, as reported in table III. Estimates of the remaining parameters were also insensitive to σ.

These results suggest that any perceived similarities between the two moving options have little or no bearing on household tenure choice. This may underline the importance of demographic variables--by default, they assume a major role.

E. Is the Full Model Necessary?

1. Tenure Choice Alone

Having constructed and estimated a model which jointly considers transactions costs, tenure choice, and the conditional demand for housing, it is useful to explore whether other researchers should be urged to do likewise. In particular, are there special cases of this model which are easier to estimate and which yield similar parameter estimates?

One method which suggests itself is to use a tenure choice model with transactions disutilities, but which does not also incorporate the share equations into the likelihood calculation. This is done by replacing the likelihood maximand in 5.1 with an abbreviated version:

$$\log L = [\ \sum_h \sum_j d_j * \log P_j\ ] = Lt \qquad (5.22)$$

Equations 5.2 and 5.3 are maintained as described earlier; only equation 5.4 is dropped from the model. Consequently, this logit model incorporates the same set of parameters which appear in the full model. Estimation results from the "logit only" model are summarized in table VI.

It is interesting to compare tables 5.6 and 5.2. The parameter estimates for the two are quite similar except for one striking difference, and that is the estimated value for β, the marginal share of expenditures devoted to accommodation services. In the logit model, β is twice the size of its

counterpart in the full model (.38895 versus .19084), yet the asymptotic t-statistic is much smaller (1.72 versus 10.49). These differing results are not too surprising, in light of the active role which β plays in both the tenure choice and the conditional demand decisions. In contrast κ, the transactions disutility parameter, does not appear in the conditional demand function (equation 5.4b).

To test whether the logit parameters in table VI are acceptable proxies for the full model parameters, they were evaluated by the full likelihood function in equation 5.1. The resulting log likelihood value of -70.2647 does not compare favorably to the maximum value of -47.6583 which was attained using the full model.[15] More specifically, the logit model is rejected on the basis of the likelihood ratio test with five degrees of freedom.

This suggests that the full model provides a significantly improved ability to explain the joint tenure choice and conditional demand decisions over the solitary logit model. The contrast is particularly clear regarding the estimate of β, the marginal share of expenditure going to housing. It is only after the tenure decision is made that β comes into active play in determing the quantity of housing consumed.[16]

TABLE VI

Maximum Likelihood Parameter Estimates[1] for Logit Model Only

Parameter	Estimated Value	Asymptotic t-stat
β	.38895	1.72
δ_k	2.6094	3.55
δ_{kl}	-2.4827	-3.49
δ_{kw}	-1.4843	-2.66
δ_{kn}	0.5577	2.63
log L^2	-70.2647	

1. The following parameter values were held fixed during this estimation; $\upsilon = 1.0$, $\gamma_a = 0$, $\gamma_c = -1000$, $\sigma = 0$. See text for details.

2. This is the value of the full likelihood function evaluated at the parameter values which maximized the logit likelihood function.

2. Conditional Share Equations Alone

An analogous procedure to that described in the preceding subsection is to drop the tenure choice term from equation 5.1. In this event the likelihood maximand becomes

$$\log L = \sum_h \sum_m d_m * \log f_m(\epsilon_m) = Ld \qquad (5.23)$$

where f_m is as described in 5.4. This strategy is equivalent to estimation of the share equations using maximum likelihood estimation.

An immediate problem with this alternative is that it does not offer a means of estimating the four transactions disutility parameters (δ_k, δ_{kl}, δ_{kw}, and δ_{kn}), because they do not appear in the share equation. It is clear therefore that estimates of the full model cannot be retrieved from the share equations alone.

3. Concluding Remarks

The estimation procedure described in this chapter yields parameter estimates (table III) which are statistically viable and which satisfy the precepts of common sense. They suggest that demographic variables dominate the tenure choice decision, but that traditional economic variables (price and expenditure) determine the level of accommodation services consumed conditional upon tenure choice. The results confirm that both of these decisions can be modelled within a mutually consistent utility maximizing framework. Moreover, the full model is

preferable to one which limits itself to either the tenure choice or conditional demand decisions alone.

NOTES TO CHAPTER FIVE

1. In the interests of legibility, the 'h' superscripts have been suppressed from the right hand side of equations 5.2 through 5.4. They are understood to belong with all price, expenditure, and quantity terms.

2. As described in Vaessen (1984, p. 142), FLETCH is a variation of the Variable Metric Method, which is a "robust implementation of th classical quasi-Newton approach."

3. The basic input to this routine is a fortran program XDFUNC(X,N) which gives the value of the log-likelihood function for any vector X(N) of parameter values in the function's domain.

4. The analytic derivatives for L are quite lengthy, due to the nestled logrithmic and exponential functions. Using numeric derivatives therefore saves a great deal of time and reduces the danger of programming errors.

5. Other procedures for incorporating demographic variables reviewed by Pollak and Wales (1981) include the Gorman or reverse Gorman specifications (which are composites of the scaling and translating procedures) and the modified Prais-Houthakker specification (which may yield a theoretically plausible demand system only if the original system corresponds to an additive direct utility function).

6. Demographic translating parameters are introduced only for accomodation services.

7. In our model, demographic scaling and demographic translating yield equivalent preference orderings iff

$$\gamma_i m_i = \gamma_i + d_i$$

8. For example, β often stalled at values of .03 or less, suggesting that less than 3 percent of household expenditure increases go towards accomodation services. The likelihood values at these stalled estimates were not at all competitive with those reported later in this chapter.

9. The gradients of the likelihood function with respect to the γ^* were nonzero. This suggests that the LES specification may be somewhat restrictive.

10. Further details on these exploratory crosstabs are provided in appendix E.

11. The units for κ, and hence for the δ in table III are 'utils'. But as discussed later in the text, κ is more meaningfully presented in terms of the impact which it has on tenure choice probabilities.

12. The values of $[1-\exp(-\kappa)]$ reported in table IV are almost unity in some cases, suggesting that households would rather forego almost their entire income in order to avoid a move. While this is highly dubious, it brings to mind the survey results of the Roskill study (1970), where eight percent of the sample stated that "no sum of money" could compensate them for having to move (reported in Dynarski (1986)).

13. Recall that this sample excludes households who may have relocated from one city to another.

14. As Kent (1983) points out, "there is not one income elasticity or one price elasticity of housing demand. Rather, there is a series of elasticities." These correspond to the various stages of decision making in the housing process, such as tenure choice and conditional demand. Because different studies focus on differing combinations of these housing decisions, their results are not directly comparable.

15. These models are nested in the sense that (a) they have the same dependent variables, and (b) one is estimated freely while the other is evaluated with all parameter values restricted to the logit-only values reported in table VI.

16. An additional test was performed to determine whether the value of of β entering the conditional housing demand equation differed from that entering the tenure choice equations. There was no significant difference. This result provided confirmation of the consistency hypothesis; namely, that the same set of preferences underlie both aspects of the household's housing choice. This contrasts favorably with results reported in King (1980).

VI. Evaluating Tax Reforms: An Example

A. Mortgage Interest and Property Tax Deductibility

As discussed in the introductory chapter, this model has been developed with a view to evaluating alternative taxation policies as they relate to housing. A change in the effective rate of housing taxation will typically impact on prices or expenditures for each household. The preference parameter values estimated in chapter five will determine the extent to which these changes are translated into welfare gains or losses for each household.

The tax reform[1] which is evaluated in this chapter is the deductibility of mortgage interest payments and property taxes from taxable income. While not allowable in Canada, these deductions are an integral part of the income tax systems in both the United States and Great Britain. Homeowners in those two countries are allowed to deduct mortgage interest payments and other housing expenses from gross income, yet are not required to declare the imputed landlord income which they receive from themselves.

This implicit subsidy to owner-occupiers violates the precept of a comprehensive income tax which states that all income (imputed or otherwise) should be taxed similarly. Under a comprehensive income tax, a landlord's tax is given by

$$T_1 = tN = t(G-C) \qquad (6.1)$$

where

N -- net rent

G -- gross rent

C -- user cost of capital plus operating expenditures

t -- landlord's marginal tax rate[2]

Compare this to the housing related income taxes T_h paid by the homeowner who does not declare gross rent, but is nonetheless allowed to deduct a portion, s, of his user costs from other income;

$$T_h = -t(sC) \tag{6.2}$$

In a competitive market for housing accommodation, these tax differentials will be translated into differences in rent. In both cases, gross rents will be just large enough to cover all costs:[3]

$$G_l = C \tag{6.3a}$$

$$G_h = C(1-ts) \tag{6.3b}$$

Thus, the subsidized homeowner receives $1 worth of housing services at the reduced rate of $(1-ts).

In the United States, where mortgage interest payments and state and local taxes are deductible from taxable income, the value for s in equation 6.2 is quite large. Laidler (1969) uses s=0.682 for his estimation purposes, and Rosen (1979) uses comparable figures. In the United Kingdom mortgage interest payments are deductible, and King (1980) takes a similar approach in computing s.

The tax reform evaluated by these authors is the removal of the implicit subsidy. This is implemented by setting s=0 in the preceding equations, thereby increasing the effective rental price faced by owner-occupiers.

The Canadian tax treatment of owner occupied housing is quite different from that in the United States or in the United Kingdom. Neither mortgage payments nor local taxes are deductible from taxable income, so we begin with s=0.

However, the federal Progressive Conservative party announced on September 21, 1978 that they would introduce such deductions were they to become the governing power in the next federal election. While this policy was not implemented[4] it was subject to considerable debate,[5] and it is clear that the proposal has a certain political appeal in Canada despite its obvious shortcomings on the grounds of economic theory.[6]

More specifically, the reform evaluated here is

$$(p_o^h)_2 = (p_o^h)_1 * (1-ts)$$

where the subscripts '1' and '2' denote the pre-reform and post-reform states, respectively. Here, s is the explicit price of

owner occupied housing expressed as a share of the total owner-occupied price, p_o. That is,

$$s = (w+\delta+(1-\upsilon)r_m)/p_o$$

and, as described in chapter 4,

w -- true mill rate for local property taxes

δ -- rate of depreciation

r_m -- mortgage interest rate

$1-\upsilon$ -- share of dwelling unit value subject to mortgage.

Household expenditures are not affected by this reform; thus, $e_1 = e_2 = e$.

Using this particular tax reform for the example in this chapter thus has several advantages. First, it is a direct application of the housing demand model developed earlier. Secondly, it allows for more direct comparability between this research and related work by earlier authors. Finally, it sheds some light on a recurring issue in the context of Canadian housing policy.

B. Efficiency

1. Equivalent Gain of a Tax Reform

One of the most persistent and serious grounds for criticizing the implicit subsidy to owner-occupiers has been that of efficiency. A subsidy is inefficient because the benefits which households derive from it do not fully reflect its cost to the government.

The efficiency aspects of this tax reform may be evaluated by first calculating the associated equivalent gain (EG_h) for each household. This is defined (following King(1981)) as the additional income which, if it were disposed of by the household in the pre-reform state (p_1, e_1), would make the household just as well off as it is in the post reform state (p_2, e_2).[7] Once the reform has been fully specified, the post-reform level of utility (v_2) for any household is readily calculated:

$$v_2 = V(p_2, e) = \max\,[V_o(p_2, e), V_r(p_2, e), V_s(p_2, e)] \qquad (6.4)$$

More formally, EG is defined implicitly for each household by

$$V(p_1, e_1 + EG) = v_2 \qquad (6.5)$$

There are three measures of equivalent gain which need to be distinguished; these correspond to the three tenure choices which may apply to the left hand side (LHS) of equation 6.5.

Thus, we have

$$V_o(p_1, e+EG_o) = v_2 \tag{6.6a}$$

$$V_r(p_1, e+EG_r) = v_2 \tag{6.6b}$$

$$V_s(p_1, e+EG_s) = v_2 \tag{6.6c}$$

These can be solved explicitly as follows, based on the LES model described earlier:

$$EG_o = \exp[v_2 + \kappa(d) - b(\beta)] * (p_o)^{\beta} - (e - p_o \gamma_a - \gamma_c) \tag{6.7a}$$

$$EG_r = \exp[v_2 + \kappa(d) - b(\beta)] * (p_r)^{\beta} - (e - p_r \gamma_a - \gamma_c) \tag{6.7b}$$

$$EG_s = -(e - p_s x_s - \gamma_c) + [(x_s - \gamma_a)^{-\beta} \exp(v_2)]^{1/(1-\beta)} \tag{6.7c}$$

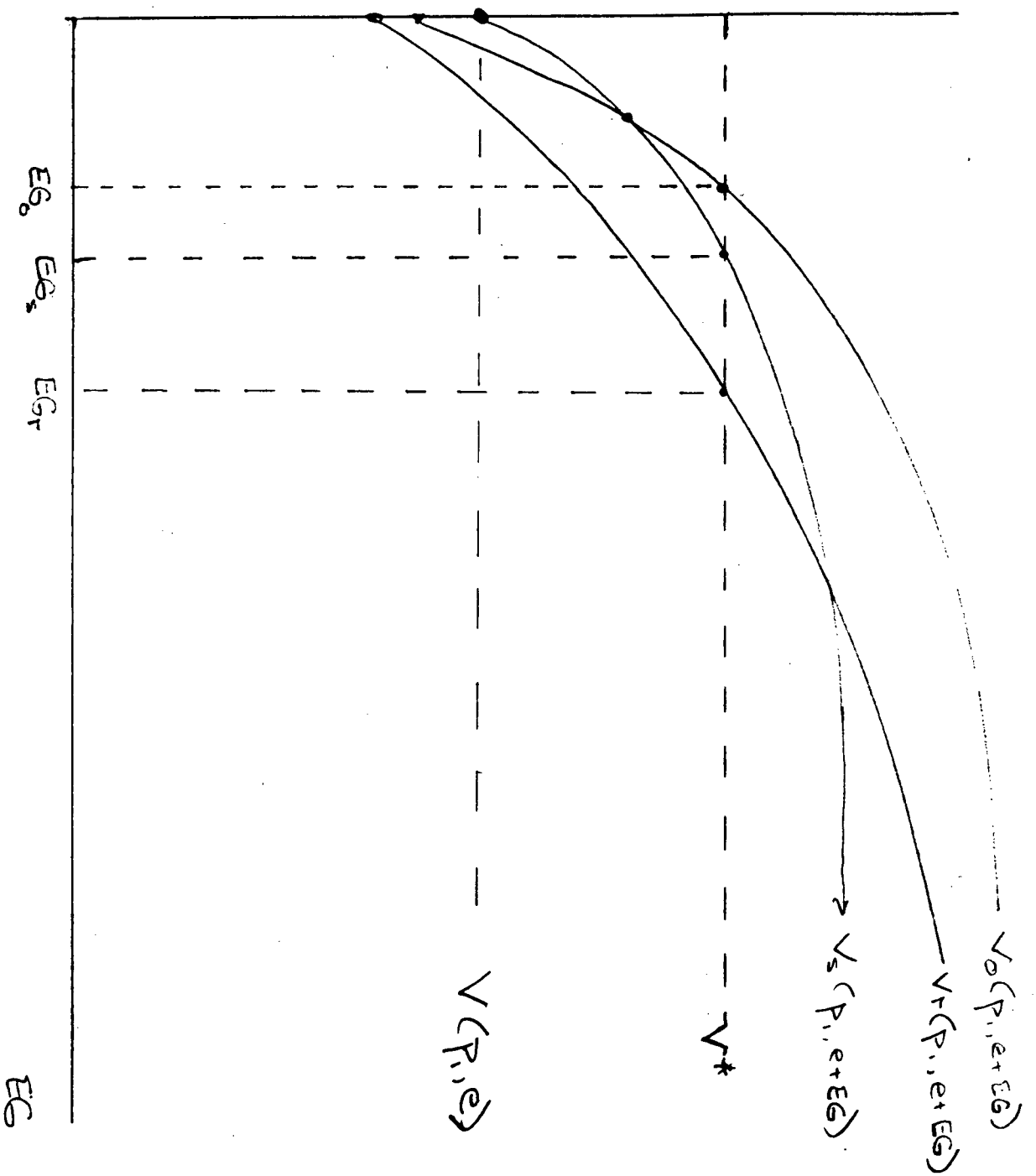
The true measure of equivalent gain is given by the minimun value among these. That is,

$$EG = \min [EG_o, EG_r, EG_s]$$

This point is illustrated in figure 10.

Figure 10

Choosing the Correct Measure of Equivalent Gain



It graphs $V_j(p_1, e+EG)$ for j=o,r,s. In this example, $EG=EG_o$ because

$$
\begin{aligned}
v_2 &= V_o(p_1, e+EG_o) \\
&= \max[V_j(p_1, e+EG_o) \mid j=o,r,s] \\
&= V(p_1, e+EG_o) \qquad (6.8)
\end{aligned}
$$

This means that the household could, by choosing to move into owner-occupied accommodation, realize a level of utility equal to v_2 in the state $(p_1, e+EG_o)$.

As shown in figure 10, this need not imply that the household chooses to move into owner-occupied accommodation in the pre-reform state.[8] In this example, increasing the value of EG from zero in figure 10 leads to a tenure switching decision before EG reaches EG_o.

2. Deadweight Loss of a Price Subsidy

As noted earlier, a tax reform is inefficient to the extent that the benefits derived therefrom do not fully reflect its cost to the government. This prompts the following definition of the deadweight loss of a reform

$$DL = TL - EG \tag{6.9}$$

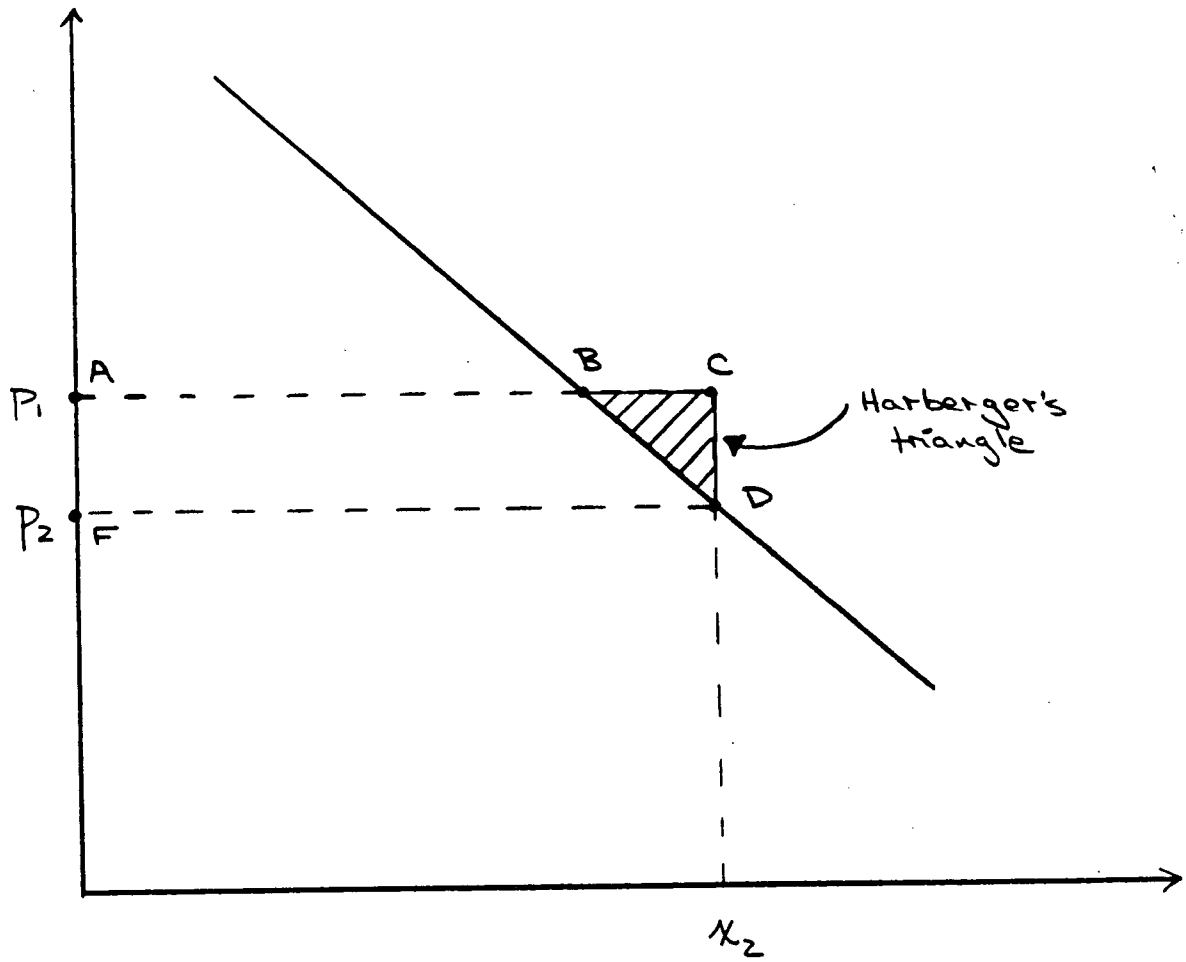
where TL is the tax loss or foregone tax revenues due to the reform. Estimating this (or some related) measure of efficiency loss has been a focal point in the debate about mortgage interest deductibility.

The standard argument is most easily outlined with reference to figure 11. In the pre-subsidy state, homeowners face a pure rental price of p_1 and consume an amount x_1. A subsidy lowers the cost to $p_2 = p_1(1-ts)$ and encourages homeowners to consume a greater amount, x_2. The annual cost to the government of this program is represented by the rectangle ACDF, while the increase in marshallian consumer surplus for the household is given by ABDF. The deadweight loss is therefore given by BCD, Harberger's triangle.

This standard argument applies to a world in which there are no transactions disutilities and in which households adjust their housing stock effortlessly from one period to the next in the face of changing prices and incomes.

Figure 11

Deadweight Loss of a Price Subsidy



In our model, the analogous argument can be made as follows. The tax loss resulting from the implicit subsidy is given by

$$TL = d_2 * (p_1 - p_2) * x_2(p_2, e) \tag{6.10}$$

where $d_2 = 1$ if the household is an owner-occupier in the post-reform state, $d_2 = 0$ otherwise.[9] It remains to specify the measure of efficiency gain (EG_m) which corresponds most closely to that implied by the standard model, where households are assumed to relocate each period (t=1,2). This is derived by using the indirect utility form for both sides of the equation which implicitly defines equivalent gain:

$$V(p_1, e+EG) = V(p_2, e) \tag{6.11}$$

For the LES specification, this can be readily solved for EG_m:

$$EG_m = (p_1/p_2)^{\beta} (e - p_2\gamma_a - \gamma_c) - (e - p_1\gamma_a - \gamma_c) \tag{6.12}$$

The deadweight loss (DL_m) for a homeowner in a "move-move" world is therefore given by

$$DL_m = (p_1 - p_2)\,[(1-\beta)\gamma_a + \beta(e - \gamma_c)/p_2]$$

$$- (p_1/p_2)^{\beta} (e - p_2\gamma_a - \gamma_c) + (e - p_1\gamma_a - \gamma_c) \tag{6.13}$$

It is clear that the deadweight loss is zero for $p_2 = p_1$ (that is, when there is no subsidy). And appendix F shows that the expression for deadweight loss is generally positive and increasing as the level of subsidy $(p_1 - p_2)$ is increased. A possible exception to this rule of thumb arises when the "income effect", represented here by the expression $(p_1/p_2)^{\beta}$, becomes too large.

It is instructive to contrast this result with the expression for deadweight loss (DL_s) in a "stay-stay" world. This is obtained by using the direct utility form on both sides of equation 6.11, yielding $EG = xcnet_2 - xcnet_1$.

Thus,

$$
\begin{aligned}
DL &= (p_1 - p_2)x_2 - xcnet_2 + xcnet_1 \\
&= (p_1 - p_2)x_2 - [e - p_2(x_s)_2 - \gamma_c] + [e - p_1(x_s)_1 - \gamma_c] \\
&= (p_1 - p_2)x_2 + p_2(x_s)_2 - p_1(x_s)_1 \\
&= 0 \qquad (6.14)
\end{aligned}
$$

since $x_2 = (x_s)_2 = (x_s)_1$ when households fail to relocate.

This result is not surprising. It states that a housing price subsidy is equivalent to a lump sum transfer when the quantity of housing services consumed is fixed; and there is no deadweight loss with a lump sum transfer.

It is clear therefore that the standard model, which implicitly assumes a 'move-move' world, will systematically overestimate the deadweight loss of a price subsidy. The next section examines the empirical evidence in this regard.

3. Empirical Evidence on Efficiency

The model of housing demand developed in previous chapters enables us to simulate the proposed tax reform and to estimate its impact on tenure choice and deadweight loss. This is done twice; once with the standard model[10] and once with the transactions model, where household utilities are reduced as a result of residential relocation. The contrasting sets of results are compared in tables VII and VIII.

Not surprisingly, the simulation with the standard model leads to a substantially larger number of households which relocate in either period.[11] And the effect of the price subsidy on tenure choice is much more apparent. Introducing the subsidy nearly doubles the number of mover/owners to 94, out of a possible 137.

This is in marked contrast to the transactions model, where the price subsidy has a negligible effect on tenure choice in the post-reform period. Most households remain stationary, but happily accept the windfall gain which befalls them.

This contrast between the two models reappears in table VIII, which reports the mean tax loss, equivalent gain, and deadweight loss of the price subsidy for owner-occupied housing.

TABLE VII

Effect of Price Subsidy on Tenure Choice

	Standard Model	Transactions Model
# of pre-reform mover/owners	50	5
# of post-reform mover/owners	94	6
# of pre-reform mover/renters	8	3
# of post-reform mover/renters	4	1
# of pre-reform stayers	79	129
# of post-reform stayers	39	130

sample size: 137

TABLE VIII

Deadweight Loss Due to Price Subsidy
(in constant 1979 dollars)

	Standard Model ($)	Transactions Model ($)
mean tax loss	2973	675
mean equivalent gain	1716	615
mean deadweight loss	1257	60
deadweight loss as a percentage of tax loss	42.3%	8.9%

sample size: 137

It shows that the mean tax loss from the subsidy is much higher ($2973 versus $675) where there are no transactions disutilities. This is due in part to those original owner occupiers who now must be subsidized in place. And it is due in part to the new owner occupiers, each of whom has been induced by the subsidy to consume a relatively large quantity of accommodation services.

And, as is consistent with the standard argument, the mean equivalent gain falls considerably short of the tax loss. Indeed, over forty percent of the subsidy yields nothing but deadweight loss. This figure is comparable to the findings of earlier authors,[12] and is an indication of why mortgage interest deductibility continues to be such a controversial tax reform proposal.

In the transactions model, the mean equivalent gain very nearly matches the corresponding tax loss. Only 8.9 percent of the subsidy outlay translates into deadweight loss. In this sense, the tax reform is a reasonably efficient one.[13]

Thus it would appear that failure to incorporate transactions disutilities into previous models has resulted in unjustifiably alarming conclusions regarding the inefficiency of price subsidies such as mortgage interest deductibility for owner-occupied housing.

However, the truth is likely to lie somewhere between the two models presented here. The reason is that the simulation captures only the period immediately preceeding and immediately following the reform. Over time, the cumulative number of

households who relocate will slowly increase. And when any household does move, it is likely to be influenced in its consumption decision by the favorable price of owner-occupied housing.

The result of this process is a parallel cumulative increase in the annual deadweight loss attributable to the subsidy. So what is in effect a lump sum grant to owner-occupiers in the short term steadily evolves into deadweight loss, due to the subsidy's ongoing and open-ended nature.

The pace of this evolution depends upon the demographic profile of the population under consideration, as well as other influences not modelled here, which may govern the timing of the actual moves.

C. Distributional Evaluation

1. Equivalent Expenditure

The preceding analysis requires only an ordinal representation of household utilities. For our purposes, however, further restrictions on the household utility functions are required. In particular, we assume that household equivalent expenditures, as defined below, are translation-scale measurable and fully comparable, as defined by Blackorby, Donaldson, and Weymark (1984). This assumes that all utility numbers have complete numerical significance, and allows for

explicit comparisons between households.

The utility measures are normalized through a money or expenditure metric.[14] In particular, we define the equivalent expenditure for the t'th state (E_t) as that sum of money which, if spent by the household subject to some reference price vector (p^*), would yield the same level of utility enjoyed by that household in state t. That is,

$$V(p^*,E_t) = V(p_t,e_t) = v_t \qquad (6.15)$$

King (1981) suggests that a natural reference price vector is the average set of prices prevailing in the pre reform period.

The value v_t denotes a particular indifference curve; that which corresponds to the maximum utility the household can attain subject to the state vector (p_t,e_t). And E_t is simply the minimum cost of attaining v_t subject to the reference prices, p^*.

This concept of equivalent expenditure is closely related to the measure of equivalent gain used in the previous section. In particular, if the pre-reform prices are used as reference prices we have

$$V(p^*,E_2) = V(p_2,e_2)$$

$$\rightarrow V(p_1,E_2) = v_2$$

$$\rightarrow E_2 = e_1 + EG$$

And, of course, $E_1 = e_1$ when $p^* = p_1$.

As was the case with equivalent gain, three measures of equivalent expenditure can be derived, corresponding to the three tenures which may apply to the left hand side of 6.15. For the LES specification used here we have:

$$(E_o)_t = \exp[v_t + \kappa(d) - b(\beta)]\,(p_o^*)^{\beta} + (p_o^*)\gamma_a + \gamma_c \qquad (6.16a)$$

$$(E_r)_t = \exp[v_t + \kappa(d) - b(\beta)]\,(p_r^*)^{\beta} + (p_r^*)\gamma_a + \gamma_c \qquad (6.16b)$$

$$(E_s)_t = [\exp(v_t)*(x_s - \gamma_c)^{-\beta}]^{1/(1-\beta)} + (p_s^*)x_s + \gamma_c \qquad (6.16c)$$

and then, analogously to the earlier derivation:

$$E_t = \min\,[(E_o)_t, (E_r)_t, (E_s)_t] \qquad (6.17)$$

2. Social Welfare and Proportional Social Gain

Having selected a fully comparable and translation-scale measurable index of household utility, we now turn our attention to measuring aggregate welfare. This research uses an ethically flexible social welfare function developed by Atkinson (1970):

$$W_t(E_t) = \Sigma_h [(E_t)_h - (t-1)*TL/H]^{(1-\epsilon)}/(1-\epsilon) \qquad \epsilon \text{ not}=1$$

$$W_t(E_t) = \Sigma_h \ln[(E_t)_h - (t-1)*TL/H] \qquad \epsilon=1 \tag{6.18}$$

where the expression (t-1)*TL/H reduces each household's equivalent gain in the post-reform period (t=2) by an amount equal to the average value of the tax loss resulting from the reform.[15]

The parameter ϵ measures the curvature of the social welfare indifference loci. Three values are of particular interest:

$\epsilon=0$ a simple utilitarian measure

$\epsilon=1$ a proportional utilitarian measure

$\epsilon=\infty$ a Rawlsian welfare measure

A useful summary statistic for evaluating the distributional effect of the tax reform is the proportional social gain (σ) defined implicitly by

$$W_1(\sigma E_1) = W_2(E_2) \tag{6.19}$$

That is, σ gives the proportion by which each household's initial equivalent income would need to be increased in order to yield the same aggregate level of welfare found in the post reform state. This can be solved explicitly for σ with reference to the Atkinson measure described above:

$$\begin{aligned} \sigma &= [W_2/W_1]^{1/(1-\epsilon)} \qquad \epsilon \text{ not}=1 \\ \sigma &= \exp[(W_2 - W_1)/H] \qquad \epsilon=1 \end{aligned} \tag{6.20}$$

where H is the number of households. The amount by which σ exceeds one indicates the degree to which aggregate social welfare has been increased as a result of the reform, after adjusting for increased taxes.

3. Empirical Evidence on Aggregate Welfare

Table IX gives estimates of pre-reform and post-reform social welfare and of the proportional social gain resulting from the tax reform for various values of ϵ. These results are presented for both the standard and the transactions models.

The most striking result from table IX is the contrast once again between the two models. The standard model without relocation disutilities shows a strong aversion to the price

TABLE IX

Effect of Price Subsisy on Social Welfare

STANDARD MODEL

(ϵ)	Pre-reform value of SW index [$W_1(\epsilon)$]	Post-reform value of SW index [$W_2(\epsilon)$]	Proportional social gain of reform [$\sigma(\epsilon)$]
0.0	2.743*E+6	2.531*E+6	0.9227
0.1	1.121*E+6	1.038*E+6	0.9176
0.5	3.769*E+4	3.564*E+4	0.8939
1.0	1.341*E+3	1.319*E+3	0.8536
1.5	-2.128	-2.383	0.7977
2.0	-8.91*E-3	-1.23*E-2	0.7268

TRANSACTIONS MODEL

0.0	2.700*E+6	2.696*E+6	0.9985
0.1	1.105*E+6	1.102*E+6	0.9975
0.5	3.734*E+4	3.722*E+4	0.9936
1.0	1.337*E+3	1.336*E+3	0.9886
1.5	-2.168	-2.185	0.9847
2.0	-9.44*E-3	-9.59*E-3	0.9846

subsidy modelled here. Even with the simple utilitarian version (ϵ=0) one could reduce each household's pre-reform equivalent expenditure by 7.7 percent and still leave society as well off as it would be if the reform were introduced. The results are even stronger as the social welfare function takes on a greater inequality aversion (ie, as ϵ increases).

In sharp contrast, the transactions model is largely undisturbed by the price subsidy. For the simple utilitarian version, the net effect on social welfare of introducing the subsidy is equivalent to that obtained by reducing each household's pre-reform equivalent expenditure by 0.15 percent. Even with a strong built-in inequality aversion (at ϵ=2.0), the transactions model shows σ=98.5. This is in contrast to a corresponding figure of σ=72.7 for the standard model.

These divergent results are in large measure attributable to the significant presence of deadweight loss in the standard model. Any tax reform which hopes to overcome this disadvantage will require particularly favourable distributional impacts. That is not the case here. In fact, as just discussed, the reform rates much more poorly as heavier emphasis is placed on its distributional consequences in the standard model.

Note that the level of deadweight loss does not vary with changes in ϵ. Thus, we may conclude that those households who are most inclined (or best able) to take advantage of the subsidy in the standard model are those who least require assistance. And that explains why $\sigma'(\epsilon) < 0$.

The transactions model takes a largely neutral view of the distributional consequences of the subsidy. This result is somewhat surprising. As discussed in the previous section, a price subsidy in the transactions model works very much like a lump sum grant to homeowners financed in equal measure by all households. And because homeowner are by and large better off than renters[16], one might speculate a priori that such a reform would have negative distributional implications.

That this is not the case suggests that most of the inequality introduced by the reform in the standard model results from households who adjust their consumption in response. Those households who are subsidized in place do not seriously aggrevate the reform's impact on aggregate welfare.

These same basic findings reappeared when the model was used to test a similar subsidy applied at half the rate proposed earlier. As shown in table X, the values of σ are all close to unity for the selected values of ϵ in the transactions model, as was the case for the full subsidy.

And not surprisingly, the standard model shows a less serious loss in social welfare when the subsidy is applied at half the previous rate, although the negative correspondence between ϵ and σ remains intact.

TABLE X

Comparative Effects of Half versus Full Subsidy

STANDARD MODEL

	Proportional social gain [$\sigma(\epsilon)$]	
(ϵ)	Half Subsidy	Full Subsidy
0.0	0.9600	0.9227
0.1	0.9580	0.9176
0.5	0.9488	0.8939
1.0	0.9345	0.8536
1.5	0.9160	0.7977
0.1	0.8934	0.7268

TRANSACTIONS MODEL

0.0	1.0007	0.9985
0.1	1.0003	0.9975
0.5	0.9993	0.9936
1.0	0.9990	0.9886
1.5	1.0014	0.9847
0.1	1.0093	0.9846

D. An Assessment of Price Subsidies for Homeowners

There appear to be neither efficiency grounds nor equity grounds for introducing a price subsidy for homeowners. None of the results presented in this chapter would favour such a move.

However, the vehemenence with which one opposes that particular tax reform may depend upon one's view of how households respond to it. The standard model predicts a wholesale response by households; consequently a great deal of deadweight loss arises, and that is compounded by distinctly unfavourable distributional effects.

In contrast, the transactions model predicts that very little happens as a direct result of the subsidy. Few households bother to relocate in order to take advantage of the reduced price of owner-occupied housing. However, those who do move for unrelated reasons will likely be influence by the new price structure when making their housing decision. They are the ones who benefit most from the subsidy.

The aggregate impact on social welfare resulting from the transfers to owner-occupiers is slight in the transactions model. However, for practical reasons it may be desirable to rule out subsidies due to remortgaging on the part of existing homeowners, and this could lead to numerous horizontal inequities.

The perspective of the transactions model is more applicable in the short run. Over time, more and more households will relocate for largely demographic reasons. As

they do so, the cumulative impact of the price subsidy will come to resemble more closely that predicted by the standard model.

The proposed reform is therefore unambigously rejected by this housing demand model on both equity and efficiency grounds, particularly in light of its long term effects.[17]

NOTES TO CHAPTER SIX

1. The term "tax reform" is used here in a neutral sense. A reform may be a "good" one or a "bad" one.

2. The landlord's marginal tax rate is assumed here to be constant over the relevant range, and equal to the marginal tax rate of the homeowner.

3. However, housing markets are not always competitive, and so the non-taxation of net imputed rental income becomes an issue as well. For example, the Royal Commission on Taxation in 1966 concluded that the exclusion of imputed rent from taxation constituted "a substantial preference for homeownership".

4. The Progressive Conservatives did in fact win the next election, but did not retain office long enough to implement their proposed reform.

5. See, for example, "A Hot Debate" in the Toronto Globe & Mail, April 19, 1979, p.B2.

6. A useful review of these shortcomings is provided by Shaffner (1979).

7. In this section, the vector (p_t, e) represents the relevant price vector and expenditure level for period t. The pre-reform state is denoted by t=1, the post-reform state by t=2. Where there is a need to specify the precise tenure in question, the text uses $(p_j)_t$, where j denotes tenure.

8. This can be seen by noting that

$$V_o(p_1, e+EG) = \max [V_j(p_1, e+EG) \mid j=o,r,s]$$

evaluated at the intercept (ie, when EG=0).

9. This definition of tax loss applies to both the standard model and the transactions model.

10. The second simulation is effected by setting $\kappa(d) = 0$ for all households.

11. Note that some households still choose not to relocate even when there are no transactions disutilities in the model. The reason is that these households enjoy anomolous price advantages in their present locations.

12. King (1981), for example, reports that the equivalent gain for owner-occupiers in his sample is just over fifty percent of the calculated tax loss. While this is of similar magnitude to the results reported here, a strict comparison is of dubious value because King evaluates his reform in the context of a different housing market with different proportions of owners versus renters.

13. However, another possible source of inefficiency is the incentive for stayer/owners to increase their debt-equity ratio by remortgaging in order to realize more benefits from the subsidy. This type of capital market incentive compatibility problem is not examined in these simulations.

14. While commonly employed, the money metric does have some drawbacks. For example, Blackorby and Donaldson (1986) demonstrate that money metrics are not concave for arbitrary reference price vectors unless household preferences are homothetic.

15. It is conceivable that some households' demand for housing will be affected by the reduction in equivalent expenditure brought about by the new tax burden. In that case, total tax revenues will not be precisely equal to the total subsidy cost. The resulting fiscal imbalance is likely to be small, particularly in the transactions model, and so is tolerated.

16. Average owner income is $21,115, compared with $16,854 for renters.

17. The preceding discussion implicitly assumes that the supply of housing is perfectly elastic. As discussed in the concluding chapter, the introduction of a housing supply model is a priority for future research. Until that research is undertaken, policy prescriptions emerging from this research must be seen as tentative.

VII. Summary and Agenda for Future Research

A. Summary

The object of this dissertation has been to develop and test empirically a model of housing demand which can in turn be used to evaluate housing related tax reforms. The hypothetical tax reform evaluated here is the introduction of mortgage interest deductibility.

Of particular interest throughout this research is the reluctance shown by most households to adjust their housing consumption in light of current prices and expenditures by relocating from one dwelling to another. This research confirms that a household's willingness to relocate is influenced primarily by its demographic status, and very little by traditional economic variables. The implications of this finding are manifested both in the housing demand equation and in the efficiency and equity of specific tax reforms.

In estimating the demand for housing, it becomes important to distinguish between the conditional elasticity of demand and the actual elasticity. The conditional elasticities produced by this research fall well within the range reported by earlier works. Actual elasticities, however, are considerably lower since the probability of relocation is small, in the order of 20 percent.

Household immobility, in effect, can transform a price subsidy into a lump sum grant. The deadweight loss associated with such subsidies is therefore relatively small (in the order of 9 percent of the tax loss, compared to 42 percent when there are no relocation disutilities), particularly in the short run. The findings reported here also suggest that the short run distributional implications of mortgage interest deductibility are much less consequential than would be the case were households perfectly mobile.

Ideas for Future Research

The model presented here can be built on or applied in a number of ways. The most obvious is to evaluate other possible housing-related tax reforms. Now that the model is in place, it is relatively easy to evaluate and compare several reform proposals on both equity and efficiency grounds.

For example, one may wish to evaluate the introduction of a homeowner grant, similar to that found in British Columbia, in the context of metropolitan Toronto. Similarly, this model could (with some revisions) be applied to examine the effects of a tax on the capital gains realized on the sale of housing assets. A more ambitious goal is to specify optimal tax reforms subject to institutional constraints.

Whatever the reform, its evaluation is rooted in household preferences as revealed through their behavior in the market for

accommodation services, and as summarized by the model for housing demand.

Another useful line of enquiry would be to test this model on another data set. Doing so would allow one to gain a better sense of how critically the parameter estimates depend on such changes. There are a number of reasons that one might expect some changes in the estimates.

First, the nature of housing and housing prices can vary quite dramatically from one urban setting to another.[1] Secondly, the array of variables one has for constructing hedonic indexes of housing quantity will vary from one data set to the next. Note however that a change in some parameter estimates need not alter the fundamental conclusions one arrives at concerning, for example, the desirability of mortgage interest deductibility.

Several restrictions were in place throughout the empirical portions of this research. For example, the Linear Expenditure System may unduly restrict the estimates of housing demand elasticity in chapter five. Similarly, the Atkinson social welfare function, together with the assumption of cardinality in chapter six, may impinge upon the resulting measures of social gain. Further research may therefore be useful in determining how serious these restrictions actually are.

The important role of demographic attributes in determining a household's propensity to relocate suggests another line of research. That is to investigate more thoroughly the relationship between household characteristics and tenure choice. This would be particularly useful if it were modelled

in terms of life cycle consumption and investment patterns. A more explicit modelling of the role played by wealth in the demand for housing is also desirable, given a data set which is sufficiently comprehensive.

This research focuses exclusively on the demand for housing, and in so doing is consistent with numerous precedents.[2] Nonetheless, it would be useful to develop this demand model in conjunction with an explicit short and long run model of the supply of housing. This is particularly true where the model is to be used to evaluate specific policy proposals. White and White (1977), for example, demonstrate that supply considerations can significantly impinge upon the welfare and efficiency effects of a housing related tax reform.

Finally, the applicability of this work need not be confined to housing. Labour markets exhibit many characteristics found in this model. This includes a demonstrable employee reluctance to move from one employer to the next in order to adjust to current salary differentials.

One suspects that demographic variables play a significant role in the labour market setting, just as they do in the housing market. Indeed, the relationship between the housing and labour markets may well provide a fruitful line for further enquiry; one which may be pursued by building upon the foundations developed here.

NOTES TO CHAPTER SEVEN

1. For example, the housing market will be quite different between two similar urban areas, where one area is experiencing significantly higher growth than the other. For an explanation of how urban growth translates into land prices, see Capozza, Helsley, and Mills (1986).

2. For example; Venti and Wise (1984), King (1983), Rosen (1979), and Laidler (1969) all focus exclusively on housing demand. King (1981), however, does introduce finite supply elasticities.

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Appendix A

Illustration Using LES

This appendix illustrates that

$$\partial\,[p^* - p_*]/\partial\kappa \geq 0 \qquad (1)$$

with reference to the Linear Expenditure System used in the empirical chapters of this dissertation.

All variables used here are as defined in the text. In particular, p^* and p_* are the maximum and minimum points, respectively, of the set Sa, defined by

$$S_a = [p|\ p=(p_a, p_a, p_a) \text{ and } V_s(p_a, e) > V_m(p_a, e)] \qquad (2)$$

The linear expenditure system introduced in chapter three is defined by

$$V_s(p_a, e) = \beta\ln(x_s - \gamma_a) + (1-\beta)\ln(e - p_a x_s - \gamma_c) \qquad (3)$$

$$V_m(p_a, e) = b(\beta) + \ln(e - p_a\gamma_a - \gamma_c) - \beta\ln p_a - \kappa(d) \qquad (4)$$

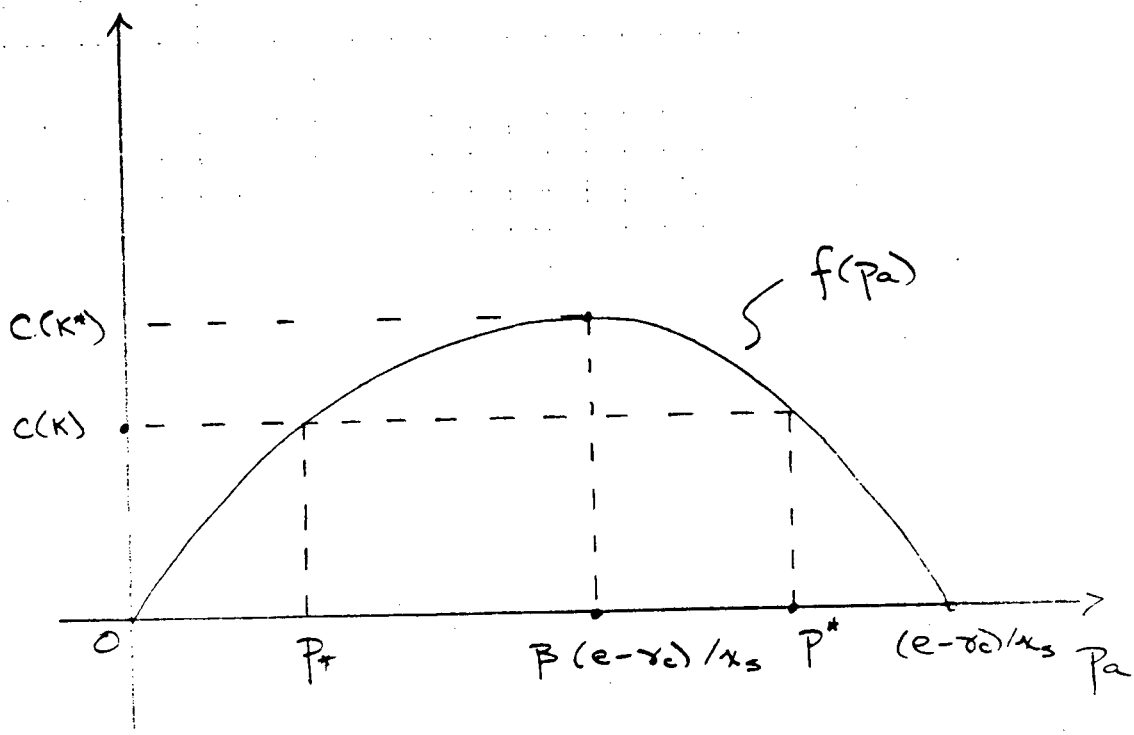
where β, γ_a, γ_c, and $\kappa(d)$ are parameters, and $b(\beta) = \beta\ln\beta + (1-\beta)\ln(1-\beta)$.

From (3) and (4) we know that $V_s(p_a, e) > V_m(p_a, e)$ implies

$$(e - p_a x_s - \gamma_c)^{1-\beta}\, p_a^{\beta} > (e - p_a\gamma_a - \gamma_c)*\exp[b(\beta) - \beta\ln(x_s - \gamma_a) - \kappa(d)] \qquad (5)$$

Figure 12

Relationship Between c( ) and f( )



To simplify matters, we assume here that $\gamma_a = 0$. This assumption is consistent with the empirical results reported in chapter five, and allows us to rewrite (5) as

$$f(p_a) > c(\kappa(d)) \tag{6}$$

where we define

$$f(p_a) = (e - p_a x_s - \gamma_c)^{1-\beta} p_a^{\beta} \qquad p_a \epsilon (0, [e-\gamma_c]/x_s) \tag{7}$$

and

$$c(\kappa(d)) = (e-\gamma_c)*\exp[b(\beta)-\beta\ln(x_s-\gamma_a)-\kappa(d)] \tag{8}$$

The relationship in (6) is illustrated in figure 12.

It can be shown readily that $f(p_a)$ is a concave function of p_a and that its maximum value is at $p_a = \beta(e-\gamma_c)/x_s$. And it is immediately clear from (8) that $c'(\kappa(d)) < 0$.

Now define $\kappa(d)*$ by

$$f[\beta(e-\gamma_c)/x_s] = c(\kappa(d)*) \tag{9}$$

Then, for values of $\kappa(d) > \kappa(d)*$, it is clear from figure 12 that an increase in transactions costs causes $c(\kappa(d))$ to decrease, and the distance $(p^* - p_*)$ to increase, thereby confirming the proposition in (1).

Appendix B

Derivation of $I(\Lambda_{ji})$

From equation 3.30 in the text, $I(\Lambda_{ji})$ is defined by

$$I(\Lambda_{ji}) = E(\lambda_{ij} | \lambda_{ij} < \Lambda_{ji}) \tag{1}$$

where λ_{ij} is a random variable which adheres to a binomial logistic distribution.

The cumulative distribution function and probability density function for λ_{ij} respectively are given by

$$G(z) = 1/F(z) = 1/[1+\exp(-z)] \tag{2}$$

$$g(z) = G'(z) = \exp(-z)/F^2(z) \tag{3}$$

Therefore, the expected value of λ_{ij}, given $\lambda_{ij} < \Lambda_{ji}$, is

$$\begin{aligned} I(\Lambda_{ji}) &= E(\lambda_{ij})/G(\Lambda_{ji}) \\ &= F(\Lambda_{ji}) \int_{-\infty}^{\Lambda_{ji}} z \exp(-z)\, G^2(z)\, dz \end{aligned} \tag{4}$$

To solve for this integral explicitly, make the following substitutions:

$$r = F(z)$$

$$r-1 = \exp(-z)$$

$$-\ln(r-1) = z$$

$$-dr/(r-1) = dz$$

Then 4 can be rewritten as

$$I(\Lambda_{ji}) = F(\Lambda_{ji}) \int_{\infty}^{F(\Lambda_{ji})} r^{-2} \ln(r-1)dr \qquad (5)$$

Integrating by parts, we get

$$I(\Lambda_{ji}) = [1-F(\Lambda_{ji})]\Lambda_{ji} - F(\Lambda_{ji})*\ln[F(\Lambda_{ji})] \qquad (6)$$

as in the text.

Appendix C

Variables Used in Hedonic Regression

Table I in the text reports the results of the hedonic regression used to calculate our measure of housing services. This appendix describes the variables which are employed in that regression.

The dependent variable is LOGVALUE, the log of the homeowner's estimate (in $'000) of his dwelling unit's market value in 1979.

The first five variables in table I provide the homeowner's subjective evaluation of the neighbourhood environment regarding condition of streets, condition of other homes, number of parks and playgrounds, quality of schools, and quality of shopping facilities in the area. These variables are rated by the homeowner on a scale of 1 to 5, with 1=excellent and 5=poor.

Similarly, the variable SATISFCN describes the respondent's overall satisfaction with the neighbourhood on a scale of 1 to 11, with 1=completely dissatisfied and 11=completely satisfied.

Next, we have variables which focus on the dwelling unit itself. For example, BATHROOM is the number of bathrooms in the dwelling unit. And BEDRMX is a dummy variable, with BEDRMX=1 if the dwelling unit has X bedrooms, BEDRMX=0 otherwise, for X=2,3,..,7.

The next five variables describe the kind of structure in which the dwelling unit is located. These are dummy variables corresponding to semidetached housing, attached townhomes, flats, and apartment buildings of more than or less than six stories. Also, there is a dummy variable to indicate whether parking facilities are available on the premises.

The final set of dummy variables correspond to the electoral district in which the unit is located. For example, ZED506=1 if the respondent resides in electoral district #506, ZED506=0 otherwise.

Appendix D

Two Solutions from the Linear Model

As discussed in the text, one strategy for locating solution values for the nonlinear model entails the use of linear approximations for the direct and indirect utility indexes. Thus equations 5.3:

$$\Lambda_m^h = b(\beta) + \ln(e_m - p_m \gamma_a - \gamma_c) - \beta \ln p_m - \kappa \qquad (5.3a)$$

for m=o,r

$$\Lambda_s^h = \beta \ln(x_s - \gamma_a) + (1-\beta)\ln(x_c - \gamma_c) \qquad (5.3b)$$

are replaced with equations 5.8:

$$\Lambda_m^h = b(\beta) + \ln(e) - (p_m \gamma_a + \gamma_c)/e_m - \beta \ln p_m \qquad (5.8a)$$

and

$$\Lambda_s^h = \beta(\ln x_s - \gamma_a / x_s) \qquad (5.8b)$$

Domencich and McFadden (1975) have shown that use of linear indexes in a logit model results in a convex likelihood function and so guarantees a unique solution. However, our model imposes additional restrictions on the parameters appearing in each index, and so a unique solution is not guaranteed. These

restrictions stem from our interpretation of the indexes in equations 5.8 as (linear approximations of) the direct and indirect utility functions.

Estimation of the linearized model leads to two distinct solutions as characterized in table XI. It is quite interesting to observe that these two solutions have almost identical likelihood values despite having markedly different parameter values and interpretations.

The first set of values (column A in table XI) is suggestive of highly elastic household responses to changes in traditional economic variables such as price and expenditure. Note for example that $\beta = 0.64$ indicates that 64 percent of additional household expenditures are directed towards accomodation services.

This contrasts sharply with the second set of results (column B), which suggests a model of demographic determinism whereby β approaches zero, and each household's housing decisions do not contribute to household utility but are dictated by that household's demographic status.

As discussed in the text, these solutions are only of secondary interest in the context of this research because (1) they are not entirely satisfactory from a theoretical perspective, (2) the parameter values do not satisfy the regularity conditions of the nonlinear model, and (3) the likelihood values thus obtained are not competitive with the likelihood value reported in table III in the text.

TABLE XI

Two Sets of Parameter
Estimates for the Linear Model

Parameter	Solution "A"	Solution "B"
β	.64229	.01405
υ	.72861	1.1780
γ_a	-287.86	810.80
γ_c	41120	1475
δ_k	.95917	1.3521
σ_k	0.0(F)	0.0(F)
δ_{kl}	343.77	-692.9
δ_{kw}	99.261	-808.5
δ_{kn}	-82.547	394.60
-log L	75.057	74.114

Appendix E

Summary of Crosstabulations

The following demographic variables play a key role in the model described in chapter five, particularly with regards to the tenure choice decision:

Dl=1 if age of household head is less than 43, otherwise, Dl=0;
Dw=1 if age of household head is married and has been for less than 10 years, otherwise, Dw=0; and
Dn=household size.

These variables were selected on the basis of exploratory crosstabulations of residential mobility by demographic status. The major findings of these crosstabs are summarized briefly here.

Age of Household Head (xhage)

* For 14 of the 19 mover/renters in our sample, xhage<36
* For 11 of the 12 mover/owners in our sample, xhage<43
* For 65 of the 107 stayers in our sample, xhage>42.

Marital Status

* 16 of the 17 movers who were married have been married for less than or equal to 9 years
* All of the 12 mover/owners have been married at least once, and none had been separated
* 4 the 31 movers had recently been divorced.
* All recently widowed household heads (male or female) are stayers.

Household Size (xnhh)

* For all of the 19 mover/renters, xnhh<5
* For all of the 12 mover/owners, xnhh<7
* 20 of the 31 movers had no children over 7 years of age.

Previous Tenure

* Of the 19 mover/renters, 10 were previously renters, 2 were owner occupiers, and 7 were neither (and so presumably are newly formed households)
* For 7 of the 12 mover/owners, the previous tenure was owner occupied accomodation

Appendix F

Deadweight Loss

This appendix identifies the conditions under which the expression for deadweight loss

$$DL(p_2) = TL(p_2) - EG(p_2) \qquad (1)$$

is non-negative, and is a decreasing function of the post-reform price p_2, for $p_2 \leq p_1$, where p_1 and e are given.

Equivalent gain (EG) is defined implicitly by

$$V(p_1, e+EG) = V(p_2, e) = v_2 \qquad (2)$$

For expositional ease, we define the following partial derivatives:

$$V_2'(p) = \partial V(p_2, e)/\partial p_2$$

$$V_2'(e) = \partial V(p_2, e)/\partial e$$

$$V_1'(EG) = \partial V(p_1, e+EG)/\partial(e+EG)$$

Thus, taking the partial derivative of both sides of (2) with respect to p_2 yields

$$V_1'(EG) * EG'(p_2) = V_2'(p)$$

$$\rightarrow \quad EG'(p_2) = [V_2'(p)/V_2'(e)] * [V_2'(e)/V_1'(EG)]$$

$$= -x_2 * [V_2'(e)/V_1'(EG)] \qquad (3)$$

Similarly, tax loss is defined by

$$TL = (p_1 - p_2)x_2 \qquad (4)$$

so

$$TL'(p_2) = (p_1 - p_2)*x_2'(p_2) - x_2 \qquad (5)$$

It is clear from equations 1, 2, and 4 that DL=0 when $p_2 = p_1$. As p_2 falls (ie, as the rate of subsidy increases), the change in deadweight loss is given by

$$DL'(p_2) = TL'(p_2) - EG'(p_2)$$

$$= (p_1 - p_2)*x_2'(p_2) + x_2 * [V_2'(e)/V_1'(EG) - 1] \qquad (6)$$

Looking at equation 6 it is difficult not to sympathize with Marshall in his assumption that the marginal utility of money is best represented by a constant. For in that event $V_2'(e) = V_1'(EG)$, and we could declare unambiguously that $DL'(p_2)<0$ for $p_2>p_1$ provided that the marshallian demand curve is downward sloping.

As it is, however, the term on the right hand side of equation 6 is positive. To see this, note from 1 that both partial derivatives are evaluated at $V(\ ) = v_2$. Thus we know that $V_2'(e) > V_1'(EG)$ since a little extra income will go further in the post reform subsidized state. The most we can conclude therefore is that the deadweight loss is likely to increase with the size of the subsidy, but will not do so if the "income effect" (ie, the ratio $[V_2'(e)/V_1'(EG)]$) becomes too large.