AN ASSIGNMENT MODEL OF URBAN HOUSING DEMAND

by

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A B S T R A C T

A critical view of the literature on land rent and residential location is undertaken with special emphasis on the journey to work hypothesis.

A housing demand model is constructed based upon the new demand theory advanced by K. J. Lancaster and an assignment model of housing developed by W. F. Smith.

The model that is presented is a simple integer program that attempts to analyze housing demand given the assumption that both household and houses have unique and separable characteristics. These attributes of both the product and consumer are thought to affect the demand for different parts of the housing stock.

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INTRODUCTION

Housing is the most complex good any consumer has to purchase. This essay deals with the economics of housing demand. Nothing is attempted with respect to house finance, macro housing policy for a nation, or the economics of housing supply.

Chapter I begins by reviewing the history of land rent from Ricardo through Von Thunen and the land economists of the 1930's. The work of recent authors such as Lowdon Wingo, Jr., William Alonso and J. F. Kain are surveyed in detail and then criticized. The main weakness that my model attempts to overcome is the lack of analysis that deals with housing as a complex of characteristics. Most of the writing has paid lip service to the existence of technically separate attributes that affect the demand for housing; however, very little has been attempted in restructuring the theory.

Chapter II lays the basic theory for my model. The work of K. J. Lancaster is reviewed. He was the first to restate demand theory using the assumption that it was the characteristics of a good which are demanded and not the good itself. The pure theory presented by Lancaster is unsuitable for the analysis of consumer durables and recourse must be made to an integer programming framework. At this point the work of W. F. Smith is introduced. Recently Smith has formulated a simple model of housing demand using the assignment approach. His model is a significant advance in that the households in a community are pictured as examining each element of the housing stock and placing a bid on each house. Then with an assignment algorithm, each household is placed in one and only one house so as to maximize the aggregate rent of the community. The rationale for rent maximization is shown to be sound within the context of this model. A defect in his theory is that the basis of the bid formulations is very vague. By using the conclusions from Lancaster's theory a more secure basis for the formation of rent offers of bid can be made.

The last chapter presents the model and a test run using completely imaginary data and bid function matrices. At this point the model is exceptionally unrealistic and the concluding part of the chapter is spent in examining some statistical methods that are "quick and dirty" so that the model may be made operational in a short time. Aside from the statistical problems, there are some questions of the manner in which the model may best be made dynamic. As it stands, the model is simply a static one-shot assignment. Nothing is said about the supply or financing of housing and this certainly is a major defect. In addition, nothing is mentioned about the interaction of demand and supply, since one of the assumptions of the assignment solution is that the number of assigned households must equal the number of houses to which these households are assigned.

Considerably more effort must be made in these last areas before the model can become a useful planning tool.

CHAPTER I

SURVEY OF THE LITERATURE ON LAND RENT AND THE ECONOMICS OF HOUSING

In this chapter, the history of land rent is briefly traced from its origins in the 19th Century to the present day.

The original thinker on the matter of why economic activity locates where it does and why land prices are what they are was, of course, David Ricardo.[1] Ricardo showed that the most productive land was the first to be cultivated. As the demand for farm produce grew with population, less productive land was used. Since the land already in use yielded a higher return, the competitive process resulted in differential prices of the land. The difference between the price of a particular plot and the price of the least fertile or marginal plot was called the economic rent. Ricardo gives little consideration to the other costs in agriculture such as transportation costs to and from the market place and most often he assumed that these costs were equal. As a result, he came to no conclusions concerning the exact location of various types of farming or recourse extracting activity.

Later, an economist in Germany explicitly treated the problem of transport costs. J. Von Thunen assumed that fertility differentials were non-existent.[2] Like Ricardo, the competitive market process results

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[1]David Ricardo, *On The Principles of Political Economy and Taxation,* 1817.

[2]Johann H. Von Thunen, *The Isolated State,* 1826.

in the highest land prices being paid for the most desirable plots of land. In this case, however, the attractiveness of land is based upon the savings due to locating as close to the market as possible. The rent the land earns is the transport cost saving from locating closer to the market place. This theory underlies virtually all the analysis of economic location theory. Once the costs of production are also included, the rent on a plot of land is the value of the product less the costs of transport and production.

The theory remained in this relatively crude state apart from some embellishments by Alfred Marshall until the early 20th Century.[3] Marshall made the distinction between the situation or site value and the agricultural rent. The site value or the price of urban land is the price it would obtain as farm land plus the sum total of monetary advantages it possesses by its location in the city. Much later, writers were to emphasize the role that external economies of scale play in conditioning the value of the land.

The next contribution to land rent theory was made by R. M. Hurd[4] who outlined a theory of urban land values parallel to Von Thunen's.

> "Since value depends upon economic rent, and rent on location and location on convenience, and convenience on nearness, we may eliminate the intermediate steps and say that value depends upon nearness." 5

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[3] Alfred Marshall, *Principles of Economics* (London: Macmillan and Co., 1917), especially Appendix G.

[4] R. M. Hurd, *Principles of City Land Values* (New York: The Record and Guide, 1903).

[5] *Ibid.*, p. 11.

The next step in the evolution of the theory was the formalization of Hurd's analysis into a canon. R. M. Haig first advanced the proposition that there was a complementarity of rent and transportation costs.[6] In other words, the rent on any site was equal to the transport costs not paid. Until very recently, this "law" formed the basis of land rent theory and residential location theory.

One of the most recent and well known contributions is *Transportation and Urban Land* by Lowdon Wingo, Jr.[7] The assumption made by Wingo is common to most of the work in the field since Hurd. A featureless plain with no geographical or institutional barriers to movement is assumed. Wingo assumes that there is perfect competition in the labour markets and the workers in the city have completely homogeneous tastes and incomes. Also assumed is the complementarity of transport costs and rent. Transport costs are composed of financial costs which include the expenditure dependent upon the distance travelled and terminal costs which are a function of the congestion in the city. In addition there are the opportunity costs of the time travelled which cannot be computed directly. These opportunity costs are thought to be an extension of the working day. Since the labour market is assumed to be perfect the worker can simply shift them back to the employer by demanding an increment to the pure wage or the wages of those who live at the job site.

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[6] R. M. Haig, "Toward An Understanding of Metropolis," *Quarterly Journal of Economics,* Vol. 35, No. 2 (May, 1926).

[7] Lowdon Wingo, Jr., *Transportation and Urban Land* (Washington, D.C.: Resources for the Future, Inc., 1961).

Three equations form the basis of Wingo's theoretical system. First, there is the demand for centrality or location:

$$pq - k(t_o) = k(t_m) \tag{1}$$

where p is the price per square foot of land; q the quantity of land consumed; k(t) the transport costs function with t_o the point of settlement and t_m the distance from the centre of the city to the fringe. This equation states that the transport costs plus the rent for a site is equal for all workers in the city and is equal to the total commutting costs from the centre of the city to the fringe.

Second, there is the demand for space which is a simple parametric expression:

$$q = (a/p)^b \tag{2}$$

where q and p are as before and a and b are parameters. At any given location equation (1) gives the amount spent on land while equation (2) indicates the amount of land consumed by the worker.

Third, if the availability of land is given by a simple expression such as $S = \pi t^2$ where π is the conventional expression and t the radius of the city , then Wingo calculates the margin of settlement according to the following equation:

$$n = 2\pi \int_o^{t_m} tq(t)dt \tag{3}$$

where n is the population of the city; 1/q(t) is the density of settlement and the other variables are as defined as before. The only unknown in this formulation is tm which is found simultaneously with equations (1) and (2).

One of the important conclusions Wingo arrives at is that the transport technology will be reflected by the land prices in the city. If the cost of movement is high, then competition for the central sites will be keen and thus the land prices for the residential sites close to the job will command a premium. The actual level of land price is a function of the numbers of workers in the city. This is elementary but important.

Recently, there has been considerable empirical work done on the importance of the journey to work as a determinant of residential location.[8] J. F. Kain is perhaps the most known of researchers in this area. His theoretical base is similar to Wingo and the land economists, in that the complementarity of transport costs and rent is assumed.

The transport costs of the household are broken into three categories:

1. The costs of travelling to and from service obtainable within the residential area.

2. The costs of travelling to and from work.

3. The costs of travelling to and from those services only available outside the residential area.

Kain is vague on the definition of area and appears to use the word interchangeably with ring. He presents some preliminary statistical evidence to show the importance of the journey to work. For example, 43.9

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[8] J. F. Kain, *The Journey to Work as a Determinant of Residential Location*, Papers and Proceedings of the Regional Science Association, IX, 1962.

per cent of all trips undertaken by households sampled from 39 major cities are journeys to work, while 21.4 per cent are social and recreational journeys. Since many of the recreational and social centers in cities are close to employment centers, the social destination trips are likely to reinforce the journey to work.

Kain takes the conclusions of the land economists in developing his hypotheses that are to be tested:

1. Transport costs increase with distance from the workplace.

2. The price of land decreases as distance from the job site increases.

3. The workplace of the individual is fixed.

4. The household maximizes utility.

5. Housing is a normal good.

The assumption about the complementarity of the rent and transport costs is retained. Location rent is the saving possible per unit of land consumed the household may achieve by moving farther from the place of work. If rents per unit area decrease as the household moves from the place of work, then the absolute savings depend upon the amount of space consumed. Kain describes this situation by isospace or bid price curves (BPC) which show the decline in location rent for each amount of land consumed as the household moves away from the job site.

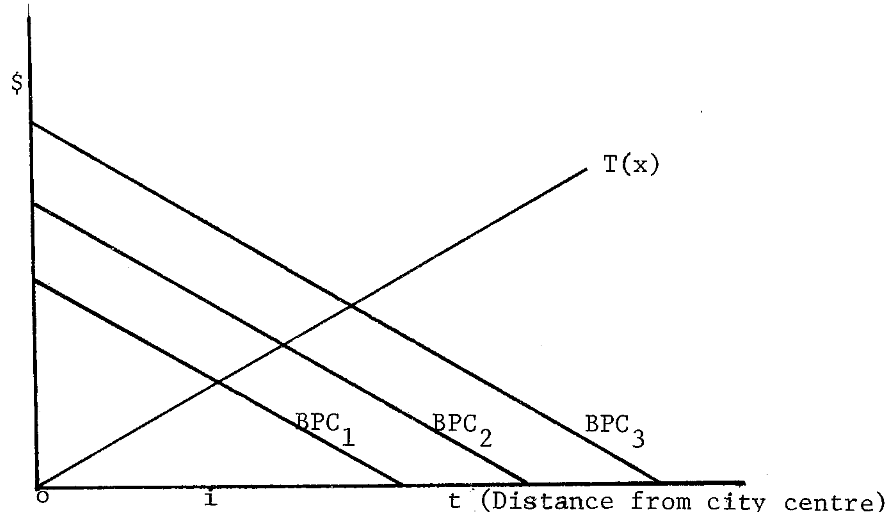

Figure 1

While the location rent declines with movement away from the employment site, the transport costs increase. T(x) is the transport cost function and the total transport costs paid in living at any one location is the area under the curve from the place of employment to the residential site. Similarly, the transport cost savings or location rent is the area under the isospace curve that applies to the amount of land being consumed by the household. The minimum costs location is simply at the intersection of the two curves.

The locations which minimize the location costs of the households are now known. The total location cost divided by the space consumed is the price the household must pay for residential space. Given the price of all other goods and services, the consumption of residential space is determined. Once the amount of space consumed is known, then the residential location of each household is determined.

From this analysis, the author concluded that for households of different types, depending upon ethnic origin, income, age composition and family size, there will be different propensities to undertake a journey to work of given length. Also, residential location is a function of the job site.

The empirical test consists of stratifying Detroit into six rings. The first finding is not surprising. Residences as a proportion of land area increases as distance from the inner ring increase. The inner ring is, of course, the prime employment area in any urban area.

It was discovered that most of the journeys to work are from the outer to inner rings. This supports the theory in that the only way to reduce the location rent paid is by moving toward the perimeter of the city. As the edge of the city is reached, the location rent curve flattens and the constraint on space consumption eases.

Other findings of interest are that workers in the CBD make considerably longer journeys to work. In addition, the length of the journey to work is a function of income. The rich appear to have a high preference for space and can afford the transport costs to get it. Very small households and large households tend to make the shortest journeys to work with middle sized households undertaking longer commuting journeys.

At first blush, the empirics seem to substantiate the claims of the land economists. The household does appear to substitute savings in land costs for transport expenditures. Before considering objections to the theory, I will now consider the work of W. Alonso.

The work of Alonso, *Location and Land Use*, is a theoretical improvement over that of Wingo and the other land economists in that where Wingo postulates separate demands for space and location,[9] Alonso integrates

[9] William Alonso, *Location and Land Use* (Cambridge, Mass.: Harvard University Press, 1961).

them into a utility maximizing framework. Both space and location enter the utility and budget equations of the household. Previous work was content with asking only where the household will locate; or if the amount of space consumed is investigated, and arbitrary demand for space equation with little basis in reality is employed.

The basic assumptions used are common to most work in land economics: a featureless plain with no geographical barriers or features which distinguish one area from another. No institutional barriers to the transfer of property between landlords are assumed to exist. Similarly, transportation is uninhibited by natural barriers and there are no unusual costs to overcoming the friction of space. Perfect knowledge abounds with the firm maximizing profits and the consumer maximizing utility.

Price as used by Alonso refers to the amount the household or firm pays for the right to use one unit of land. Under these terms are subsumed the costs of ownership, contract rent and sales price in the long term, which given perfect competition tend to equality (in terms of discounted present value).

The study commences by assuming that all economic (commercial, retail, industrial, etc.) activity takes place at the core of the city. Therefore the household always faces the center of the city when attempting to fulfill various demands. Once commercial and agricultural users are explicitly accounted for this assumption is lent some plausibility.

Since the consumer is asked to make the dual decision of how much to buy and where to settle this must be included in the income and utility functions. The income equation consists of the direct costs of site control, the costs of commuting to and from the site, plus the costs of all other goods and services. To prevent the study from becoming too complex, all goods aside from land are aggregated into a composite commodity — z.

The budget equation appears as follows:

$$y = p_z z + p(t) q + k(t) \tag{4}$$

where p_z is the price of the composite good; z is the composite good; p(t) is the price of land at distance t from the center of the city; q the amount of land consumed; and k(t) the costs of commutting associated with that particular site.

The utility function is simple and straightforward:

$$U = u(z, q, t) \tag{5}$$

Subjecting these expressions to the usual tools of differential calculus, Alonso obtains the following solutions:

$$u_a/u_z = p(t)/p_z \tag{6}$$

$$u_t/u_z = (qdp/dt + dk/dt)/p_z \tag{7}$$

The interpretation of these expressions is simple. The first [equation (6)] states the marginal rate of substitution between land and all other goods is equal to the ratio of their respective prices. In a similar vein the second equation [equation (7)] states that the marginal rate of substitution between distance and all other goods is again equal to their price ratios. The price of distance is equal to the total amount spent on land at point t (qdp/dt), and the costs of commutting to point t(dk/dt). The numerator in the second expression [equation (7)] is important since it indicates that cost of a marginal movement is equal to the change in the amount paid for land plus the change in commuting costs. Since consumption of the composite good is pleasurable, therefore U_z is positive and since commuting has disutility attached to it, the U_t is negative. Commuting costs are related to the distance travelled and even if there were no direct costs to movement there would always be the opportunity costs, this makes dk/dt negative. Since q is positive, this implies that the price of land declines as one moves away from the center of the city.

The conclusion that Alonso derives from this analysis is that the consumer of urban housing settles at the point where the costs of commutting are just greater than the savings from consuming cheaper land. In other words, the point of equilibrium for the resident is the point at which the costs of commutting incurred by moving incrementally from the centre are exactly balanced by the savings in the price of land by such a move.

Alonso proceeds to examine the actions of commercial and agricultural users of land. Instead of maximizing utility, they maximize profits. A revenue and cost function are substituted for the utility and budget constraint. Since the details are not germane to the analysis, I will skip them and proceed to consider the nature of the bid price function which forms a crucial link in Alonson's work. It also is an important step in the construction of the simulation model and thus needs to be examined carefully.

THE RESIDENTIAL BID PRICE CURVE

As was stated in the introduction, the bid price curve for an individual is defined as "the set of land prices an individual would pay at various distances and still derive a constant level of satisfaction."[10] Several points need to be stressed. First, the curves for different households could and most likely would be very different. Secondly, a particular bid price curves refers to a given level of satisfaction and as a result each household has many bid price curves corresponding to different levels of satisfaction. Third, the bid price curve is in no way related to the price that is eventually paid. There is no consideration of supply factors and therefore in a sense this is a very unrealistic concept. This point will arise later in the essay. Once the bid prices are established for commercial, residential and agricultural users, Alonso employs a game theoretic approach where the users compete for the land with the sale going to the highest bidder. Since commercial and agricultural users are extraneous to the essay I will pass over this point.

The bid price function is derived very simply. By definition a bid price is the amount the household pays for a given combination of location and utility. Therefore both the utility and distance are treated initially as given with utility set at u_o and location set at t_o. The utility function now appears as follows:

$$U_o = u(z, q, t_o) \tag{8}$$

[10] Alonso, *op. cit.*, p. 59.

and the budget constraint with y_o, p_z, $k(t)$ all given:

$$y_o = p_z z + p(t_o)_q + k(t_o) \tag{9}$$

The signs of the partials remain the same as before. If the problem is cast in a Lagrangean framework the following set of equations result,[11]

$$U = u(z, q, t_o) - \lambda[y - (p_z z + p(t_o) q + k(t_o)] \tag{10}$$

$$\frac{\partial u}{\partial q} = u_z + \lambda p_z = 0 \tag{11}$$

$$\frac{\partial u}{\partial q} = u_q - \lambda p(d_o) = 0 \tag{12}$$

$$\frac{\partial u}{\partial p(d_o)} = \lambda_q = 0 \tag{13}$$

and from (11) and (12):

$$\frac{u_p}{u_z} = \frac{-p(d_o)}{p_2} \tag{14}$$

Equations (8), (9) and (14) now form a system of three equations with u, t, p_z, x, y and $k(t_o)$ all given and three unknowns [z, q, $p(t_o)$]. As with any system of linear equations, it can be made parametric simply by choosing a "given" and denoting it a parameter. In this case, if t is chosen as the parameter, then the price of land p(t) can be solved for various different locations and becomes the bid price curve, which is a function of distance t.

Some important corollaries are proved by Alonso:[12]

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[11]Alonso uses the total differential instead of the Lagrangean; however, the results are the same.

[12]Alonso, *op. cit.*, Appendix H.

1. The bid price curve is single valued, implying that for any given utility function at any specified location there is only one bid price for the household.

2. Lower bid prices imply greater utility since they signify that the bids for land in the community are lower.

3. Bid price curves for the same household do not cross.

The equilibrium of the household can be found by superimposing the price of land upon the mapping of the bid price curves as shown in Figure 1. Incidentally, it can be shown that the bid price curves slope downward. In addition, Alonso shows that the price of land declines less rapidly as the distance from the centre of the city increases as is also shown in Figure 2. The point becomes clear when the next author's work is considered and as will be demonstrated has to do with the transport technology of the city.

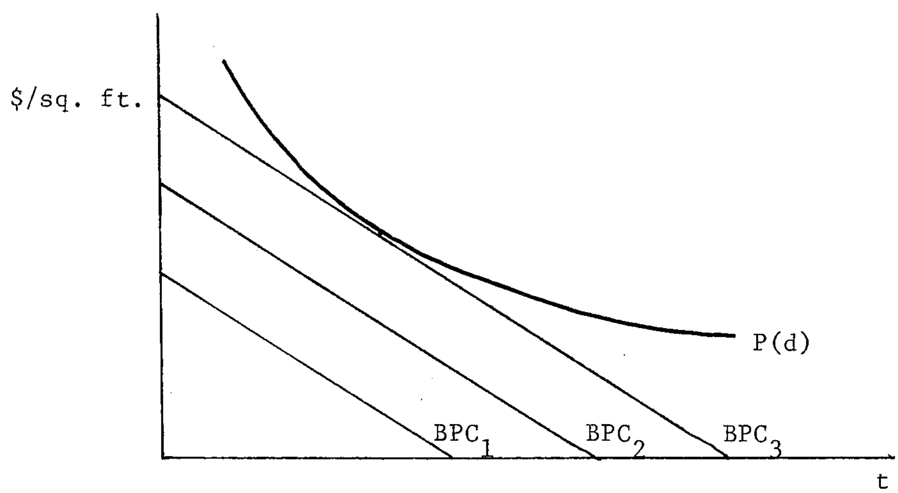

Figure 2

A CRITIQUE OF THE JOURNEY TO WORK

The literature just surveyed suffers from several serious defects that stem from the assumptions employed by some or all of the researchers.

The first issue involves the assumption of perfection in either the labour or land markets. The writers named above all view the consumer or worker operating in an environment of perfect competition. Wingo's assumption of perfect labour markets is not directly related to land and housing economics but the assumption used by Wingo, Alonso and Kain that there are no impediments to the entry and exist in the housing or land market is very restrictive.

The requirements for competition in a market are well known and are fulfilled by the following conditions.

1. Buyers and sellers must be numerous.
2. The transactions of any one economic unit must be small enough not to have any effect on the prices or quantities offered in the market.
3. There is no collusion.
4. Entry and exit is free and unimpeded for both buyers and sellers.
5. All participants have complete and costless information.
6. There are no institutional barriers to transaction.
7. The product is homogeneous.

These points can be summarized by three conditions that there be homogeneous goods, many buyers and sellers and the costs of information and acquisition are nil. Taking these points in turn, it becomes apparent that the housing market may by definition be imperfect. Virtually all goods are differentiated: even simple commodities such as cement and wheat are differentiated to the informed buyer normally in these markets. Housing is perhaps the most complex of consumer goods. In addition most housing possesses a fixed location which automatically acts to give each house a uniqueness. To judge a market as imperfect simply due to the very nature of the good appears to be misguided.

It can be argued that housing is one economic good that information is easily obtained. Classified ads and real estate companies act to disseminate this information in an efficient manner. Information in a market does not merely consist of easy knowledge of what goods are presently available but what goods will be demanded and in supply in the future. Due to the nature of housing, there tends to be a long production period. In addition, housing is durable. Durability, long production periods and fixed location are the common reasons that are given as to why the housing market should by definition be imperfect.

Surely the most important consideration when examining the perfection of any particular market must be the relative number of buyers and sellers. If single family units are considered alone then no violation to reality is done if the perfect competition assumption is used. If the market consists of multiple dwelling units, then it is likely that the numbers of owners or sellers is very much less than the number of buyers.

Since there is little empirical evidence on the structure of the housing market, all that can be said is that *a priori* housing is an imperfect market.

A second objection to the literature surveyed is the treatment of the location decision of the household. At best the household is viewed considering only the distance to the job site and the amount of land consumed. Some evidence in the form of the frequency of the trips to various destinations about the city was given by Kain which showed that almost 50 per cent of all trips were work-oriented. This is not enough to enable it to be stated categorically that the journey to work is the sole determinant of residential location. None of the locational theorists surveyed maintained that this was the case; however, little work has been done in establishing other locational motivations to the residential decision.

Some recent empirical work was attempted by J. Wolforth who examined Kain's hypothesis concerning the substitution of journey to work expenditures for site expenditures.[13] The alternate hypothesis advanced by Wolforth is that the costs of commutting are not sufficient to affect the location of the residences. The consumer lives where it can be afforded and meets the costs of commutting as best as possible. While this does not directly contradict Kain's hypothesis, however, its verification would indicate that the journey to work was not such an important factor in residential location that other motivations can be ignored.

The proposition was tested similarly to Kain. Vancouver was divided into six rings and the labour force was classified into six occupational classes. The percentage of each occupational group in each ring was

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[13] J. Wolforth, *Residential Location and Place of Work in Vancouver*, (Vancouver: Tantalus Press, 1965).

compared to the mean income of the ring. Wolforth discovered that lower income workers tend to locate closer to the CBD than do more affluent workers.

A second test was tried in which the city was divided into areas assumed to have homogeneous housing costs. The median value of housing in each census tract was assumed to be the cost of housing in a particular tract. Spearman correlation coefficients were computed for the occupational groups ranked according to percentage in each tract and occupation groups ranked by income. The coefficients were significant and positive.

The conclusion of Wolforth's study is that there is considerably more variation in residential location patterns than would be expected if proximity to work was the only motivation to choosing a house. Unfortunately, little more can be said from his study.

The difference between Wolforth's and Kain's study can largely be attributed to differences in the cities studied. Those cities that have been established for a long period of time (more than 100 years), grew using a transport technology that was costly to individuals. As a result, a premium was placed upon centrality and the placement of industrial and commercial activity was located at the core. Thus these cities, such as New York, Detroit, Montreal, etc., conditioned the inhabitants into accepting certain location patterns. Admittedly, these constraints on residential location are weakening as the cities expand; however, compared to Vancouver which grew using more individual and less costly transportation (the car) they still likely place more constraint upon the consumption alternatives of the worker.

The point that must be made here is the household in all likelihood is responsive to other trips. The propensity to travel to various destinations such as shopping centres, schools, recreational centres, nightclubs, etc. varies with the structure of the household. The number of children and their ages are important factors in where to locate for those families. Similarly for single person households, proximity to night life is important and reflected in the amount the household would be willing to pay to live in an area close to such facilities.

The households responds to many locational pulls. The theory surveyed while paying lip service to this has assumed that these trips were insignificant and no damage to the realism of the results was made if the journey to work was assumed to be the only locational motivation. I propose a model that views the house as a collection of attributes, not all locational, that households of different structure and income value differently. In the next chapter the theory underlying the model is outlined. Two separate strands -- one from operations research, the other from modern demand theory -- are united to form the base of the model.

CHAPTER II

PROGRAMMING THEORY AND THE ECONOMICS OF HOUSING

In the previous chapter the literature on residential location was surveyed and found deficient in its assumption that the household decides on the basis of only two characteristics -- space and location to work. The reason for this is that traditional economic theory is quite constrained in the analysis of consumer goods. In this chapter I outline a theory of demand first developed by K. J. Lancaster. He employs a linear activity analysis to study the behaviour of consumers when goods are considered to have characteristics which differentiate them from one another. Once this theory is outlined, it becomes apparent that for consumer durables such as housing, the analysis given by Lancaster needs to be amended to an integer programming framework or assignment model. Once this is established, the work of W. F. Smith is reviewed. Smith has used an assignment approach to housing studies. From here it is a short step to my model of housing demand.

THE NEW THEORY OF DEMAND

Traditional economic theory has the consumer sliding up and down a smooth utility curve choosing between two goods, often totally unrelated and pinned to reality only by a budget constraint. The consumer is pictured as making rational choices between shoes and cars or guns and butter with the trade-off varying (in two dimensions) from a straight line for perfect substitutes to a right angle for perfect complements. An undercurrent of economic theory has always argued that the choice is more ordered. Cars are traded off with commuter service of various types, butter with margarine and shoes with other clothes, implying that consumers choose among characteristics rather than goods.

K. J. Lancaster has formalized this view into a fairly rigorous theory and postulates three assumptions as the departure from conventional thinking on the matter.[14]

> 1. It is the characteristics inherent in the good and not the good itself which yields utility.
>
> 2. In general, goods possess more than one characteristic.
>
> 3. Goods in combination, or complements may give rise to more than one characteristic and different characteristics than goods singly.

Actually, the application of linear activity analysis to consumption theory is merely the reverse of production theory. In production theory an activity involves the combination of several inputs in the creation of one product, while in consumption theory the act of consuming involves one input or good and several joint outputs or characteristics.

Therefore, associated with each good there is a vector of characteristics. If b^j is this vector and b_{ij} is the amount of its characteristics possessed by good j, then all the vectors of characteristics may be represented by a matrix B which Lancaster refers to as the consumption

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[14] K. J. Lancaster, *Mathematical Economics* (New York: Macmillan, 1968); "The New Theory of Consumer Demand," *Journal of Political Economy*, Vol. 74, No. 4 (July, 1966).

technology matrix. In general, the numbers of characteristics and goods will not be equal and Lancaster postulates that for advanced economies the number of goods is greater than the number of characteristics.

If the entire array of characteristics is represented by z and the array of goods by x then the fundamental relationship of the theory follows:

$$z = Bx. \tag{15}$$

Assumed in the simply theory is that the characteristics are normal or no satiation of characteristics is possible. Also, if the matrix B is square, and can be decomposed into a diagonal matrix, this new theory is nothing more than a restatement of the old theory.

Since the consumer operates in characteristic space and not goods space, the utility function is of the following form:

$$U = U(z) \tag{16}$$

This is maximized subject to the following constraints:

$$z = Bx \tag{17}$$

$$px \leq k \tag{18}$$

$$x \geq o \tag{19}$$

where p is a vector of goods prices and k is the income of the household. The program as it stands is non-linear but can be transformed simply by substituting Bx for z in the utility function.

As long as all the elements of B are positive and that x is non-negative, then the "attainable characteristics set" is in the positive quadrant. Given are two characteristics and four goods. Goods are represented by rays which indicate the mixture of characteristics each possesses. The attainable characteristics set is shown by the shaded area. The quantities of any one good that can be purchased by spending one's entire income on it is reflected by the length of the ray in question. Some goods (as good 3 in Figure 3) may not be considered at all by any consumer. The personal choice is shown by the indifference curves $I_1 - I_3$.

An optimal solution is one which minimizes the expenditure of the household within the constraints set by the attainable characteristics set.

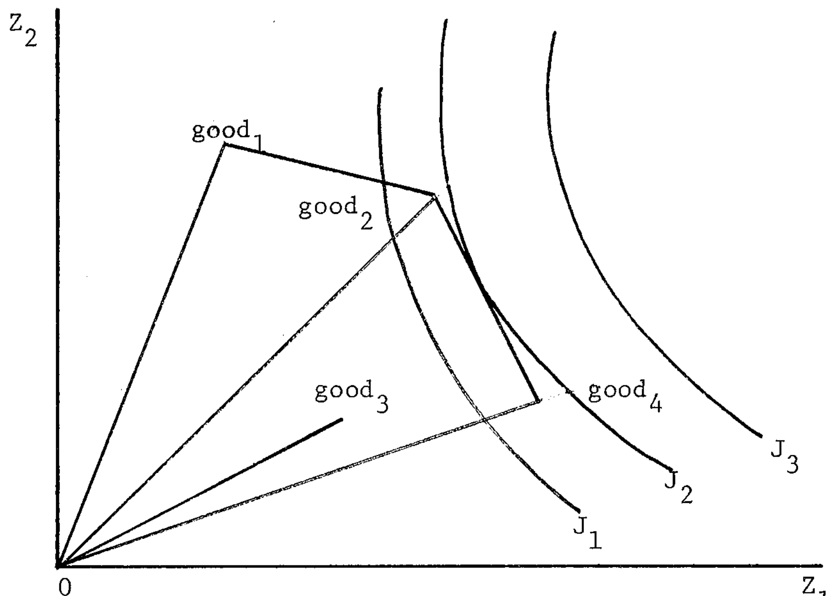

Figure 3

There are two substitution effects in operation here called the efficiency substitution and the convention substitution, shown in Figures 4 and 5.

Suppose the price of good two rises. This has the effect of moving point X2 along the ray 2 (Figure 4). As soon as #X1, X2, X3 form a

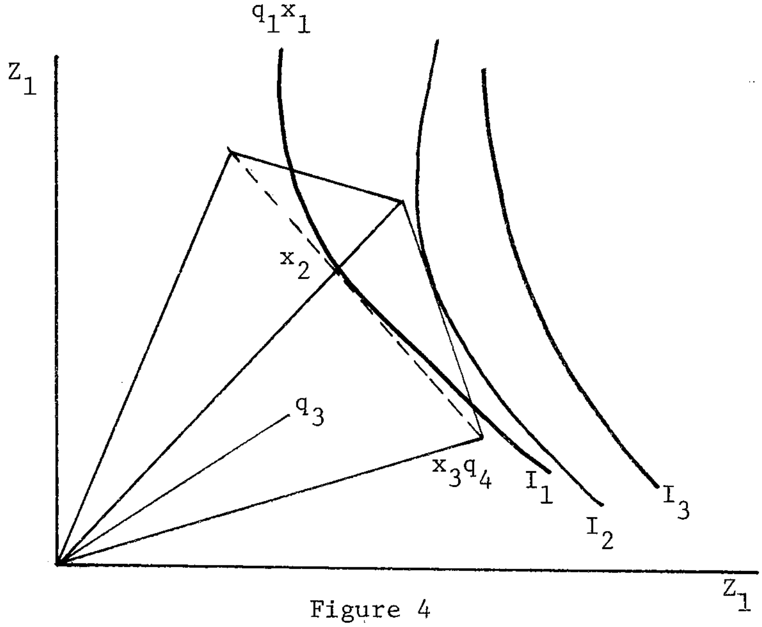

Figure 4

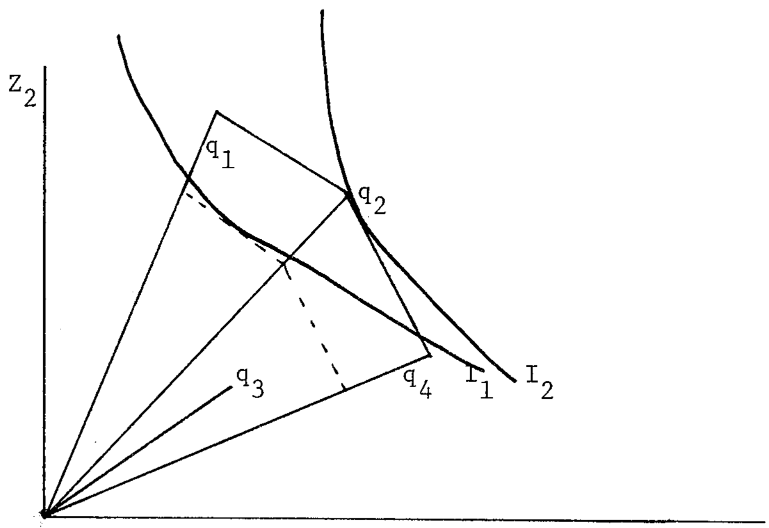

Figure 5

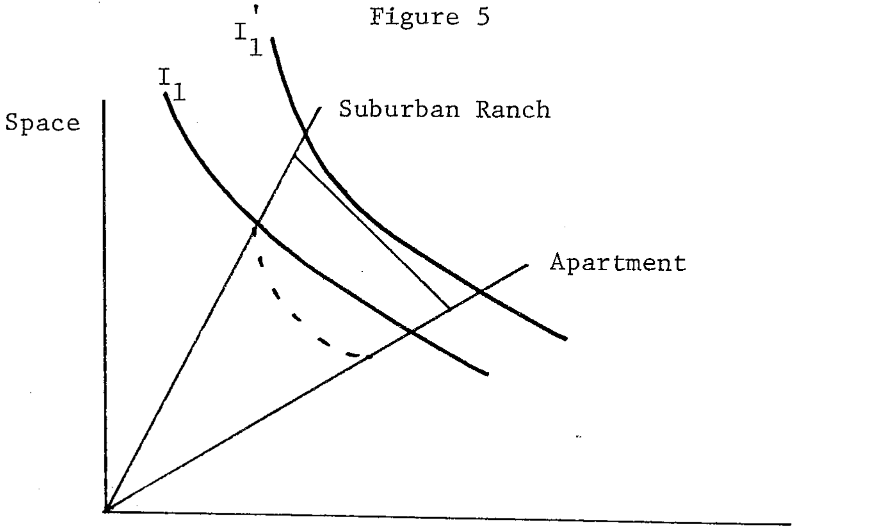

Figure 6

straight line the consumer will maximize his welfare by switching to a combination of goods 1 and 3. This is the efficiency substitution. Conventional substitution occurs when the entire constraint function shifts inwards and the indifference map governs the switch in portfolio of goods held as in Figure 5.

APPLICATIONS OF THE THEORY TO HOUSING

Consider a straightforward approach to housing with no modification of the theory. Assume a residential bundle with a well defined set of characteristics. These include such factors as proximity to work, play, the schools, the noise level of the area, the ethnic mix of the neighbourhood, etc.

Graphic portrayal of the situation is shown in Figure 6 where two characteristics and two housetypes are shown. The theory implies that given the type of utility function in the figure, then the rational household would hold two types of houses. However, the formulation ignores the problem of the costs of multiple dwelling ownership. It is very likely that these are very high. Representation of this situation becomes very difficult since if the feasible frontier made non-convex by having the price line bow in as in the dotted line this would indeed force the consumer to hold only one house, however the economic meaning of such geometry is dubious. In effect, this formulation implies that there is some smooth functional relationship between the type of house and the transaction costs. All that can be really stated unequivocably is that the feasible region remains non-convex for most consumers of housing with incomes below a certain level simply because the holding of multiple dwelling units for pure consumption purposes is very rare.

A second important issue revolves around the actual representation of houses in this framework. By using a continuous vector, what is implied is that the consumer can freely adjust the amount of housing that is consumed. In other words, the price line may fall anywhere along a given ray and there would be a house that would exist with the exact mix of characteristics specified. In reality this is unlikely. Large consumer goods such as houses need to be represented by discrete points in characteristics space. Once this is recognized then the problem becomes an integer programming problem with all the activities and constraints in the form of integers. No fractional solutions are permitted.

The next step in the evolution of the housing model is to outline an assignment model of housing demand developed by W. F. Smith. The assignment problem is one form of integer programming problem that has received considerable refinement in recent years.

W. F. SMITH AND A MATRIX ANALYSIS OF NEIGHBOURHOOD CHANGE

The last chink in the theoretical atmosphere is now to be filled. To recapitulate, housing has been conceived of as a bundle of characteristics roughly divided into those resulting from the spatial situation of the house and those intrinsic to the good itself. Of course, quality is not independent of distance from various urban activities; however, the conception of such interdependence let alone the measurement is beyond me. I also view the urban landscape as the result of a competitive bidding process where potential users for specific plots of land compete with one another and the property goest to the highest bidder. This will be formalized shortly in the context of the "optimal assignment model."

Smith's[15] theory or urban residential structure is a direct descendent of the sector theory formulated by Homer Hoyt[16] in the 1930's. At that time the controversy was over the concept of "filtering." Basically, the proposition was that filtering is the process whereby the demand for durable goods -- in particular housing -- was met for low income groups through a process of "hand-me-downs" or filtering. New residential construction is inhabited by the rich who "bequest" their old homes for the next lower status group. As an armchair empirical fact there appears to be little to dispute, however, controversy existed as to whether enlightened social policy consisted of building high quality housing to induce the rich to move and thereby increase the supply of housing for poor people, or whether it was preferable to construct low income housing projects. Even today housing policy is very much divided on this matter.

The sector theory formulated by Hoyt seemed to indicate that the succession of houses from the rich to the poor was a natural fact of urban ecology. As the city matures the rich areas were hypothesized to move outward in rays resembling pie slices. The exodus of the rich leaves behind housing that is quickly converted to multiple occupancy. The middle income groups cluster about the rich forming an insulation between the income extremes. Hoyt gave several rules of migration. Generally the rich move to the high ground, along transportation routes and tend to avoid

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[15] W. F. Smith, *Filtering and Neighborhood Change* (Berkeley: Center for Real Estate and Urban Economics, 1965).

[16] Hoyt, cited in Smith, *op. cit.*, p. 9.

situations where subsequent outward movement is impeded. At the base of Hoyt's theory is a recognition that the rate of change in population is an important determinant of the success of the rich migrating outward without being encroached upon by the poor. However, with a great influx of low-income households and a sluggish supply response to new housing demand, the distinctions between the sectors may very well become blurred.

The sector theory is not sufficient to predict or even adequately explain the existing spatial structure of cities. In particular, the actions of the middle income portion of the population were never accounted for in detail. Certainly considerable portions of the new housing stock was aimed at the middle income groups simply because the rich did not form a sufficient part of the population to allow the lower income groups to inherit enough houses. Smith's model of housing is very simple and is based upon the optimal assignment model from the theory of integer programming. It is a process whereby the existing population is assigned according to some predetermined rule to the existing housing stock. Households are differentiated with respect to income while houses are characterized according to "value" or price.

Several questions are asked of the model:

1. What is the pattern of urban residential structure produced by the purely competitive market?

2. What is the impact within the constraints of the model given a change in the income distribution of the community?

3. If the population of the model city is invariant, what new housing will be constructed in the event of replacement construction?

4. Given the addition of such new stock, how will the pattern of occupancy change?

5. If there is an increase in the housing stock and population, what new housing will be constructed?

Several important assumptions are made which limit the applicability of the model considerably. Only five families are assumed to inhabit this city and only five houses of different price or quality are available. Price is defined similarly to Alonso's definition. A very crucial point in the construction of the model is the creation of rent offers for various house types. Each household makes an offer on each house: an offer which varies according to the desirability of the house and income of the family.

The effects of various income levels and house quality on the rent offers of families can be shown simply as a matrix:

HOUSEHOLDS	HOUSES				
	A	B	C	D	E
1	L	+5	+10	+15	+20
2	+10	+15	+20	+25	+30
3	+20	+25	+30	+35	+40
4	+30	+35	+40	+45	+50
5	+40	+45	+50	+55	+60

Assumed is that the ranking of the houses as to desirability is the same for all families. L is the lowest rent offered by any household for any house; it represents the basic demand for shelter. What is important is not the amount L but the differences from L that will be offered for various houses by various households.

Smith then examined the implications of this matrix and concluded that it is not realistic to assume that the income elasticity of demand for quality is zero. Smith argues that an increase in quality would be worth something to a higher income household and that it would be willing to pay a premium for quality. In other words, each increment up in both income and quality results in an increase in the rent offer of one dollar. There is no reason for choosing this figure since the analysis is unchanged as long as the income and quality effects are positive.

HOUSEHOLDS	HOUSES				
	A	B	C	D	E
1					
2		+1	+2	+3	+4
3		+2	+4	+6	+8
4		+3	+6	+9	+12
5		+4	+8	+12	+16

The two matrices are now simply added together and result in a rent offer which depicts the bid made by each household for each house. The problem is now to assign households to house according to some rule. If the houses and households are ranked according to some objective criterion such as sales price and income, then theory from linear programming indicates that an optimal solution exists when houses are matched to households along the main diagonal as in the following:

HOUSEHOLDS	HOUSES				
	A	B	C	D	E
1	L				
2		+16			
3			+34		
4				+54	
5					+76

It is this assignment that results in a maximization of the rent offers of the community. Smith uses the results from the theory of pure competition to justify the relation of a Pareto optimal solution and rent maximizing program.[17] Note that like the bid price of Alonso, these rent offers have no relation to the price actually paid. Before examining some of the experiments that Smith subjects his model to, I might just point out that any other assignment other than the one shown results in a lower aggregate rent offer on the part of the community.

One of the assumptions that was made at the outset was that the supply of housing was already fixed, or in other words, housing is used without regard to the costs of production. If household five leaves the community and then a family of income level one moves in there is a reshuffling with households 1 and 2 occupying houses A and B and households 3, 4 and 5 moving to houses C, D and E. The aggregate rent offer now dips to L + 130 which reflects the loss of high income family. The fact that the aggregate rent offer has declined in no way has any implication upon the standard of housing in this simple model. What is notable is that families 3, 4 and 5 now inhabit "better" housing.

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[17]See Appendix 1 for a simple explanation.

Smith also points out that predictions as to what type of housing is required to meet anticipated demand can be made using this framework. The basic technique is to calculate the "economic value" of a particular type of house (i.e., its rent offer in the optimal assignment) and then compare this with the construction cost. This is done by considering the original matrix and examining the change in aggregate rent offers when different types of houses are added. For example, the change in rent offers when houses of type A, B, C, D, or E area added are as follows:

HOUSE ADDED	CHANGE IN AGGREGATE VALUE
A	+ 0
B	+ 5
C	+ 11
D	+ 18
E	+ 26

If a house of type D were added, then households 1, 2, 3 and 4 would filter up with households 1 occupying house type B, household 2 occupying house type C, etc. If the value curve is plotted and compared with costs curves, i.e., the costs of building each type of house as in the figure no new construction would be justified unless the economic value equals or exceeds the construction costs. Here house type C or better is warranted.

The housing model that I present is based directly on the work of Smith and extends it in several directions. In the first place, the concept of bidfunction is amplified to include bids by households for not just "valuable" houses. One of the weaknesses in Smith's theory is that the concept of value is very slippery. It assumes that value is an objective category, i.e., what is valued by one person will also be valued by another. Secondly, and related to the first point, is the assumption that the basis for such judgements is upon income alone.

Once this assumption is questioned and rejected on the grounds that seem intuitively apparent, another more complex premise is required. I postulate that for different types of households different valuations of what is desirable will be made. Thus the desirability of a particular house will vary with not only income but the household size, ethnic origin, class origin and other idiosyncratic details.

In the next chapter I present a model which divides the bid for various houses according to what different households can be expected to offer for various housing attributes. At this stage with very little empirical work done in the area of land values and the determination, the functions imputed to different households are pure conjecture. I also assume, very heroically, that the bids for different characteristics are additive and form a bid-function matrix which reflects the bid by each household for each house type. Once this matrix is obtained, then an algorithm based upon the optimal assignment problem is used to assign each household to a house according to a rule of maximizing the aggregate rent of the community.

CHAPTER THREE

AN ASSIGNMENT MODEL OF URBAN HOUSING DEMAND

In this section of the essay I outline an extension of the work of Smith, which incorporates some of the aspects of modern demand theory. As mentioned in the previous chapter, one of the weaknesses of the model presented by Smith was the vague use of the notion of quality of housing. The model I present attempts to overcome this deficiency by disaggregating housing into its characteristics and making these the basis for the desirability of particular houses by the households in the community. As a result, it is no longer possible to *a priori* rank the housing stock by classes which have different quality.

With this modification made, it is no longer possible to have a ranked bid function matrix simply have the optimal solution pop out as does Smith. A complex algorithm is needed to find the optimal solution. The second modification then is merely to use such an algorithm to do the assignment.

Before examining the structure of the bid function matrix as I employ it, some examination of the properties of the assignment problem should be made. It is a sub-class of linear programming problems in that the same assumption about optimizing with linear objective functions and constraints is needed; however, in this case, no fractional answers are permitted. It was evolved by operations researchers to solve the problem that arises when a job has to be assigned to a specific facility and only to that facility.

THE ASSIGNMENT PROBLEM

The assignment problem involves several factors whose productivity can be measured to several jobs in such a manner as to maximize the aggregate return.[18] An example is the matching of employees to tasks. The crucial requirement is that one and only one factor be matched to one job.

Mathematically, the problem can be stated as follows.

Given an n x n matrix, A_{ij} (the rating matrix) with $a_{ij} \geq 0$ for all i, j,

find an n x n matrix X_{ij} such that

$$\sum_i X_{ij} = 1$$

$$\sum_j X_{ij} = 1$$

$$\sum_i \sum_j a_{ij} X_{ij} = \max$$

The first two conditions ensure that the value of X will be 1 if facility i is assigned to job j and will be 0 otherwise. Each column and row contain only one entry of unit value with all the others zero. The third condition specifies that the elements chosen from the rating matrix will maximize the product.

With a few amendments, the assignment problem can be transformed into one which has a very simple and straightforward solution. The housing

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[18] See Appendix 2 for more details on the algorithm.

market that Smith has solved by the assignment process can be described as follows:[19]

> 1. Households can be ranked by income and houses can be ranked by quality.
>
> 2. Each household offers a rent for each house.
>
> 3. Rent offers increase with income and quality.
>
> 4. A premium is offered by each household for increases in quality.

The situation is stated mathematically by Smith as follows:

> "Let a_{ij} be the rent offer of the ith family for the jth house, then there is a matrix of rent offers in which i + 1 is a higher income level than i and j + 1 is higher quality dwelling than j such that,
>
> $$a(ij) < a(i+1, j)$$
>
> $$a(ij) < a(i, j+1)$$
>
> $$[a(i, j+1) - a(ij)] < [a(i+1, j+1) - a(i+1, j)]$$
>
> If the rent offer (bid function matrix) is set up with households and houses ranked, then assigning the ith to the jth house with j = i results in a rent maximizing assignment." 20

With the simple structure of Smith's model, it is simple to prove that any assignment other than that which assigns the ith household to the jth house with j=i results in a maximization of the aggregate rent of the matrix.[21]

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[19] Smith, *op. cit.*, Appendix 1.

[20] *Ibid.*, p. 68.

[21] *Ibid.*, pp. 67-70 for proofs.

In fact, there is no need to rank the bid function matrix. The purpose of my reformulation is to stress the basis of the quality of the various classes of housing. Once the housing stock is disaggregated into economic goods with characteristics which each possess in different degrees and the households are not simply characterized by income but ethnicity, household size, choice of lifestyle, etc., the ability to rank the columns and rows is impossible.

I now turn and examine the basis for a more extended and complete bid function matrix. For this I must repeat, at the risk of overstating, the bid price curve as formulated by Alonso.

THE BID FUNCTION MATRIX

This section develops the notion of bid function matrix which is very close to the bid price curve employed by Alonso. To repeat the usage of Alonso:

> "A bid price curve of a resident is the set of land prices the individual could pay at various locations while deriving a constant level of satisfaction; that is to say, if land prices were to vary in the manner described by the bid price curve, then the individual would be indifferent among locations."[22]

Three important points that were stressed were:

1. The bid price curve refers to the individual household.

2. The curves vary from individual to individual.

3. There is no relation between the bid price curve and the price that is actually paid for land.

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[22]Alonso, *op. cit.*, p. 58.

The extension of Alonso's work I wish to make involves developing bid price curves not only for land but some of the objective characteristics of housing such as the space available in the house, the distance to work, shopping, schools and other destinations. I am assuming that the household when in the market for a house has a clear idea what exactly it wants and can state with a fair degree of precision those characteristics which it values and those attributes which are unimportant. Thus not only is there a bid price curve for land but bid price curves for each of the distance parameters mentioned, various amenities associated with the neighbourhood and the compatibility of the house design with the chosen lifestyle of the household. Attempting to cast the problem in a framework similar to that employed by Alonso becomes very difficult and cumbersome. For the situation that Alonso was considering it was fairly reasonable to picture the housing consumer as moving to and from the centre of the city until the optimum combination of land and distance was discovered; however, I demand that the consumer not only find an equilibrium between land, and distance from the city centre, but an equilibrium among all the characteristics of houses.

A start can be made, however, if the same variables as used by Alonso are retained except for a second distance or variable-distance from shopping areas. The problem now appears as follows.

The urban housing consumer is pictured as separating each prospective dwelling into its constituent characteristics upon which it places a value. The bid that is offered upon the house is the sum of the bids offered upon each characteristic. This is the crucial assumption of the model and is certainly the most suspect and unrealistic. Implied is that the utility function for these characteristics is separable, and this is most certainly wrong. For example, the ease of driving to work may be entirely negated by the lack of parking or the proximity to shopping areas could not be a factor simply because that particular household does all its shopping to and from work. The interdependence of characteristics is a very serious qualification of Lancaster's theory of demand. The assumption of addivity seems to me to be the only workable empirical hypothesis. In fact, virtually all land values investigations implicitly make this assumption. Until some method of sifting the impacts of characteristics from one another is derived, it appears that this assumption needs to be retained.

A MODEL OF URBAN HOUSING DEMAND

The model is in two parts. The first takes various characteristics of households and houses and formulates a bid function matrix while the second part of the model is an algorithm which allocates the housing stock to the households in such a way as to maximize the aggregate rent offered by the community and such that no household can be reallocated without making any one other household worse off.

The functions that are employed have no basis in empirical work since little work has been done which could shed insight into the way in which different households value different characteristics of housing. Arbitrarily, I chose 15 house types and 15 household types. The households are characterized by the number of people in the household to a maximum of three, and income of which there are five classes, ranging from \$500 per year to \$15,000. All the numbers used are admittedly naive and have little basis in reality.

The attributes of space and location to the downtown are weighted or measured by an index number and how this number is obtained will be discussed later. I make the assumption that given almost complete ignorance of the relation between the valuation, the demand for urban space and family size and income, the relation is linear and takes the form:*

Bidspace (ij) = f[Income i, Space j, Pers i]

In this manner a matrix with house type along the rows and household type along the column is constructed which shows the bid by each household for each house. This I term the bid space matrix.

The bid location matrix is built in a similar manner. I assume that the importance of a "central" location increases as does the income. The function that is being used at the moment takes the form:

Bidloc (ij) = f[Income i, Loc j]

---

*where

Space j is the space in house j;

Income i is the income of household i;

Pers i is the number of people in household i;

Loc j is the index of centrality for house j.

Recalling the tenuous assumption about addivity of the characteristics in the utility function, the bid location matrix is simply added to the bid space matrix to form the bid function matrix which is the same matrix that was employed by Smith in his study except that it was somewhat more laboriously derived.

At this point the households are assigned to houses by an algorithm which is explained along with a program listing in Appendices 2 and 3.

Table 1 shows the bid location matrix while Table 2 is the bid space matrix. The final solution with aggregate bid offered by the community is shown in Tables 3 and 4. The tables are presented in Appendix 4.

The model as it now stands is very unrealistic and contains many simplifications which result in a direct contravention of what is already known about residential location. If the final solution is examined closely, it is indicated that low income families locate in the most remote housing while high income families are clustered about the core of the city. Every study on residential structure and even casual empirical observations contradict this result. The problem lies in an incomplete specification of the bid function matrix. A more complete analysis would no doubt add several more variables and have more sophisticated behavioural functions. More will be said about this later.

However, perhaps the most glaring point at this moment is the omission of any consideration of the budget constraints faced by the urban housing consumer. In a way this is included in the behavioural functions; nevertheless, some explicit supply factors need to be incorporated if the model is to approach some realism.

The strength of this style of thinking is that the consumer of residential housing is viewed not as simply choosing a good called housing, but in fact discriminating between types of house. From here the study of various location characteristics and not just the journey to work. Little is known about the importance of journeys other than that of work-oriented journeys in the demand for housing and therefore until the empirical work has been completed, nothing other than conjecture can be advanced. The same must be said about other characteristics than space. One certainly could say that elements such as design (ranch, apartment, townhouse or split level) have for some households an importance that is reflected in the price these households are willing to offer to live there.

In the next section I consider a statistical method whereby the relation of house and household characteristics and the demand for urban residences could be discovered. In addition, the way in which the model could become a dynamic model is discussed as is the problem of integrating a supply side to housing allocation. I conclude the essay with a brief examination of the political bias of the model -- namely, is the model relevant for income levels?

THE MODEL: AN EVALUATION

As it stands now, the model that I have presented is very naive and restricted. In the first place, the bid functions and behavioural equations that I have used are very simplistic and have no basis in reality. Secondly, the model is not dynamic.

SOME STATISTICAL CONSIDERATIONS

The problem quite simply is to discover how the bid for houses of different types varies with the structure of the household. *A priori*, the premium placed on space and privacy will be a function of the number of persons in the household and the age structure of the family. Similarly, the effect of industrial centres will be less for those households whose members possess skills normally used in the CBD, such as professionals.

Two methods suggest themselves immediately. First, the households could be asked directly what aspects and attributes of their present house they like or dislike. This type of information would then be correlated with data on the family structure. Aside from the cost of such a survey, the chance that accurate responses could be obtained is slight.

An alternate method would be to use land values as a trap for the advantages and disadvantages of various locations and types of house. The relation between the sales value of a house and the characteristics of a house such as space, indices of privacy, indices of environmental quality, and indices of proximity to work, shopping, schools, etc. could be measured using regression techniques.

The problem with the last method is that no indication of how bids vary with the structure of the household is possible. In all likelihood, the market is too sluggish to be very sensitive to changes of family structure for a particular house. For two houses of the same quality (exactly the same attributes), the sales price likely would not reflect any difference in the structure of the two families living in these houses. Other considerations such as bargaining skills of the various buyers and sellers would be important if home ownership were the case.

The only way to resolve this difficulty appears to be to use both methods to gain some idea of how consumer demand varies, then to construct bid function curves that reflect the results of the statistical tests but are not directly related to the parameters discovered. Thus a combination of questionnaire and regression should indicate which variables or attributes of a house and household are the most important.

PROBLEMS OF DEMAND AND SUPPLY

At the moment, no attempt is made to consider a situation in which the numbers of households of houses in a particular class are greater than one. Certainly if the model is to be realistic, this must be corrected. At the moment, only simplistic solutions come to mind.

These classes of household or houses which are in excess supply could simply be ignored.

A better solution would be to have the model assign the various classes optimally. A cell with an oversupply of households would have some of its members assigned to the next class down. If one were to start at the top and move through the entire solution, if there was an oversupply of households, the households at the bottom of the scale would be forced out of the market. At this point, this is pure conjecture and nothing conclusive can be said until a firmer ground is constructed.

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APPENDICES

APPENDIX 1

A NORMAL COMPETITIVE MARKET AND RENT MAXIMIZATION

In David Gale's book, *The Theory of Linear Models*, some effort is spent on the theory of competitive markets and resource allocation. Gale gives an example of price equilibrium using the housing market. The basic difference between the housing market described by Gale and the market and the one used by Smith and myself is that suppliers are used by Gale. The use of assignment techniques in the analysis is valid. The housing stock is varied and households differ in both their ability to bid for various houses and their tastes. Because of long production lags the solution that flows from such analysis can be used for fairly long periods. Changes in the stock of houses is also influenced by the condition and numbers of the present housing stock.

The market used by Gale has n individuals interested in buying n houses. A value matrix a_{ij} is constructed which shows the worth of each house to each household. Also, the suppliers have set sales prices p_i on each house. Naturally a household would not be interested in purchasing unless its valuation of that house were higher than the sales price. Gale uses programming theory to show that such a market results in that impossible dream of the "greatest good for the greatest numbers." He shows that the profits of the producers is matched by the surpluses of the consumers and the assignment problem yields a value maximizing arrangement of house settlement.

From the theory of competitive markets it can be said that these markets result in a value maximizing arrangement of the stock. The market that both Smith and I use are purely competitive in nature since the housing stock and since the household bids can be ranked an optimal assignment where no arbittrage or arrangement other than the solution could improve the position of any member is possible and is consistent with the theory of pure competition.[1]

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[1]See D. Gale, *Theory of Linear Economic Models* (Toronto: McGraw Hill Co. Inc., 1961).

APPENDIX 2

AN ALGORITH TO SOLVE THE TRANSPORTATION PROBLEM

The algorithm that is used in my housing model is a variation of a routine designed by L. R. Ford and D. R. Fulkerson[1] to solve the Hithcock transportation problem. The problem can be stated mathematically as follows:

Find an m x n array of numbers $x = (x_{ij})$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ that minimizes $\sum_{ij} c_{ij} x_{ij}$ subject to the constraints

$$\sum_i x_{ij} = a_i$$

$$\sum_j x_{ij} = b_j$$

$$x_{ij} \geq 0$$

where a_i, b_j, c_{ij} are non-negative integers and the sum of the vector a = the sum of the vector b. If m = n and a and b are equal to 1 this becomes the optimal assignment problem.

Usually the c_{ij} matrix is a tableau of unit shipping costs from point i to point j; a_i is a vector that indicates the supply of goods at point i and b_j reflects the demand at point j. The purpose of the algorithm designed by Ford and Fulkerson is to allocate the movement of goods between supply and demand points so that transportation costs are at a minimum.

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[1] L. R. Ford and D. R. Fulkerson, "Solving the Assignment Problem," *Management Science* (July, 1956).

The algorithm is modified to search for a maximum value merely by scanning the cost matrix, finding the maximum value in the entire matrix and subtracting this value from each element of the matrix. Using this augmented matrix within the cost minimizing framework produces the solution for a cost maximum.

A discussion of the algorithm requires detail and development that would be outside the scope of this paper. All that really need be said is that the method is a variation of the simplex method that is so widely used in linear programming. Because of the unique feature of this problem, such as integer values, square value matrix and no surpluses or shortages for any of the row or column entires of the value matrix, several shortcuts can be used to arrive at the solution faster.

Proofs and a description of the method can be found in Ford and Fulkerson, and in H. W. Kuhn.[2]

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[2]Kuhn, H. W. "The Hungarian Method for the Assignment Problem," *Naval Logistics Quarterly*, Vol. IV, No. 1 (1955).

APPENDIX 3

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      C ASSIGNMENT SOLUTION FOR HOUSING MODEL MARK 1
      C BASED UPON THE HITCHCOCK TRANSPORTATION PROBELM,MODIFIED
      C BY CONSTRAINING THE SURPLUSES AND SUPPLIES TO 1.
      C
      C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      C THIS PROGRAM IS INPUT FOR THE ASSIGNMENT MODEL
      C IT READS IN DATA ON HOUSEHOLD AND HOUSE CHARACTERISTICS
      C AND CREATES COMPLETELY IMAGINARY BEHAVIOURAL FUNCTIONS
      C TO BE USED IN THE CREATION OF THE BIDFUNCTION MATRIX
            REAL*8 A,B,C,D,E,INCOM(15),PERS(15),SPA(15),LOC(15)
            REAL*8 BIDLOC(15,15),BIDSPA(15,15),AA,BB,CC,DD
            DATA A,B,C,D,E/6H      ,6H    **,6H     *,6H******,1H*/
            DIMENSION K(100,100),L(100,100)
            DIMENSION IA(100),IW(100),IS(100),IC(100),JR(100),KR(100)
            DIMENSION JB(100),JW(100),JS(100),IR(100),JC(100)
            DIMENSION HEAD(20)
        202 FORMAT(' ','HOUSEHOLDS **',17(I4,2X,1H*)/(11X,'**',17(I4,2X,'*')))
      203   FORMAT (1X,2A6,17(A1,A6))
      204   FORMAT (1X,I7,2X,3H **,17(I4,2X,1H*)/(11X,2H**,17(I4,2X,1H*)))
      2041  FORMAT (1X,2A6,17(1X,A6))
      306   FORMAT (13H SUB-COSTS **,17(I5,1X,1H*)/(11X,2H**,17(I5,1X,1H*)))
      C     -----------------------------------------------------------------------
            READ(5,701)M,N,AA,BB,CC,DD
        701 FORMAT(2I5,4F5.0)
            READ(5,123)(SPA(J),LOC(J),J=1,N)
        123 FORMAT(2F3.2)
            READ(5,77)(INCOM(I),PERS(I),I=1,M)
         77 FORMAT(F6.0,F3.0)
            WRITE(6,1112)M,N,AA,BB,CC,DD
       1112 FORMAT(' ','INPUTS',2I5,4F5.0)
            DO 533 I=1,M
            DO 534 J=1,N
            BIDSPA(I,J)=(INCOM(I)*SPA(J)+INCOM(I)*PERS(I)/10.)*1.0/BB
            BIDLOC(I,J)=AA/INCOM(I)*LOC(J)*INCOM(I)/3.0
            K(I,J)=BIDSPA(I,J)+BIDLOC(I,J)
        534 CONTINUE
        533 CONTINUE
            LA=1
        900 WRITE(6,400)((I),I=1,M)
        400 FORMAT('1','     HOUSETYPE',15(5X,I2))
        402 FORMAT(' ','LOCATION',10X,F3.2,14(4X,F3.2))
            WRITE(6,401)(SPA(J),J=1,N)
        401 FORMAT(' ','  SPACE',11X,F3.2,14(4X,F3.2))
            WRITE(6,402)(LOC(J),J=1,N)
            WRITE(6,405)
        405 FORMAT(' ',' * * * * * * * * * * * * * * * * * * * * * * * * * * *
           * * * * * * * * * * * * * * * * * * * * * * * * * * * * *')
            WRITE(6,406)
        406 FORMAT(' ','HOUSEHOLD')
            WRITE(6,407)
        407 FORMAT(' ','INCOME PERSONS')
            I=1
            IF(LA-2) 500,501,502
        500 WRITE(6,408) I
        408 FORMAT(' ',7X,I2)
```

```
            WRITE(6,409) INCOM(I),PERS(I),(BIDSPA(I,J),J=1,N)
        409 FORMAT(' ',F6.0,1X,F3.0,5X,15(F6.0,1X))
            I=I+1
            IF(I.GT.M) GO TO 410
            GO TO 500
        501 WRITE(6,408) I
            WRITE(6,409) INCOM(I),PERS(I),(BIDLOC(I,J),J=1,N)
            I=I+1
            IF(I.GT.M) GO TO 410
            GO TO 501
        502 WRITE(6,408) I
            WRITE(6,740) INCOM(I),PERS(I),(K(I,J),J=1,N)
        740 FORMAT(' ',F6.0,1X,F3.0,5X,15(I6,1X))
            I=I+1
            IF(I.GT.M) GO TO 410
            GO TO 502
        410 LA=LA+1
            IF(LA.GT.3) GO TO 700
            GO TO 900
        700 DO 102 I=1,M
        102 IA(I)=I
            DO 103 J=1,N
        103 JB(J)=J
        105 MN=M*N
      C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      C     DATA PRINT OUT
            WRITE(6,200)
        200 FORMAT('1',30X,'BID FUNCTION MATRIX')
             WRITE(6,201)
        201 FORMAT(' ',9X,'HOUSES',6X,A2,14(5X,A2))
            WRITE(6,202)(JB(J),J=1,N)
            DO 170 JJ = 1,2
            IF (N .GT. 17) GO TO 151
            WRITE (6,203) D,D,(E,D,II = 1,N)
            IF (JJ .EQ. 2) GO TO 160
            GO TO 170
      151   WRITE (6,203) D,D,(E,D,II = 1,17)
      170   CONTINUE
      160   DO 3060I=1,M
            WRITE(6,204) IA(I),(K(I,J),J=1,N)
            IF (N .GT. 17) GO TO 150
            WRITE (6,2041) A,B,(C,II = 1,N)
            GO TO 3060
      150   WRITE (6,2041) A,B,(C,II = 1,17)
      3060  CONTINUE
            DO 171 JJ = 1,2
            IF (N .GT. 17) GO TO 153
            WRITE (6,203) D,D,(E,D,II = 1,N)
            IF(JJ.EQ.2) GO TO 162
            GO TO 171
      153   WRITE (6,203) D,D,(E,D,II = 1,17)
      171   CONTINUE
      C CONVERSION TO MAX PROBLEM
        162 MIN=K(1,1)
            DO 600 I=1,M
```

```
          DO 640 J=1,N
          IF(K(I,J).GT.MIN) MIN=K(I,J)
      640 CONTINUE
      600 CONTINUE
          WRITE(6,741) MIN
      741 FORMAT(' ',I7)
          DO 602 I=1,M
          DO 603 J=1,N
          K(I,J)=(K(I,J)-MIN)*(-1)
      603 CONTINUE
      602 CONTINUE
          WRITE(6,702)((K(I,J),J=1,N),I=1,M)
      702 FORMAT(' ',15I5)
          DO 608 J=1,N
      608 JB(J)=1
          DO 607 I=1,M
      607 IA(I)=1
          WRITE(6,1111)(JB(J),J=1,N)
          WRITE(6,1111)(IA(I),I=1,M)
     1111 FORMAT(' ',15I5)
C THIS COMPLETES THE PROBELM CONVERSION TO MAXIMUM
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C   GETTING STARTED
  705   DO 1 I=1,M
        IS(I)=IA(I)
        JIG=K(I,1)
        DO 2 J=1,N
        JS(J)=JB(J)
        L(I,J)=-1
        IF (JIG-K(I,J))2,2,3
  3     JIG=K(I,J)
  2     CONTINUE
        IW(I)=-JIG
        DO 4 J=1,N
        IF (JIG-K(I,J))4,5,4
  5     L(I,J)=0
  4     CONTINUE
  1     CONTINUE
        DO 6 J=1,N
        DO 7 I=1,M
        IF (L(I,J))7,8,7
  8     JW(J)=0
        GO TO 6
  7     CONTINUE
        JIG=K(1,J)+IW(1)
        DO 11 I=1,M
        KR(I)=K(I,J)+IW(I)
        IF (JIG-KR(I))11,11,10
  10    JIG=KR(I)
  11    CONTINUE
        JW(J)=-JIG
        DO 46 I=1,M
        IF (JIG-KR(I))46,95,46
  95    L(I,J)=0
  46    CONTINUE
```

```
0149      6     CONTINUE
          C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          C     DETERMINATION OF INITIAL ALLOCATIONS
0150            DO 12 I=1,M
0151            DO 13 J=1,N
0152            IF (L(I,J))13,14,13
0153     14     IF (IS(I)-JS(J))16,15,15
          C     JS LESS THAN IS
0154     15     L(I,J)=JS(J)
0155            IS(I)=IS(I)-JS(J)
0156            JS(J)=0
0157            GO TO 13
          C     IS LESS THAN JS
0158     16     L(I,J)=IS(I)
0159            JS(J)=JS(J)-IS(I)
0160            IS(I)=0
0161     13     CONTINUE
0162     12     CONTINUE
0163            GO TO 51
          C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          C     ITERATIVE PROCEDURE
          C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          C     LABELING PROCEDURE
0164     57     DO 19 J=1,N
0165            JC(J)=-1
0166     19     IR(J)=-1
0167            DO 20 I=1,M
0168            IC(I)=-1
0169            JR(I)=-1
0170            IF (IS(I))20,20,21
0171     21     IC(I)=IS(I)
0172            JR(I)=0
0173     20     CONTINUE
0174     36     IND=0
          C     LABEL ROWS
0175            DO 22 I=1,M
0176            IF (IC(I))22,22,23
0177     23     DO 24 J=1,N
0178            IF (L(I,J))24,25,25
0179     25     IF (IR(J))26,24,24
0180     26     IR(J)=IC(I)
0181            JC(J)=I
0182            IND=1
0183            IF (JS(J))24,24,27
0184     24     CONTINUE
0185     22     CONTINUE
          C     LABEL COLUMNS
0186            DO 28 J=1,N
0187            IF (IR(J))28,28,29
0188     29     DO 30 I=1,M
0189            IF (L(I,J))30,30,31
0190     31     IF (IC(I))32,30,30
0191     32     JR(I)=J
0192            IF (L(I,J)-IR(J))33,34,34
0193     33     IC(I)=L(I,J)
```

```
0194            IND=-1
0195            GO TO 30
0196      34    IC(I)=IR(J)
0197            IND=-1
0198      30    CONTINUE
0199      28    CONTINUE
0200            IF (IND)36,35,36
          C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          C     BREAKTHROUGH  PROCEDURE
0201      27    IF (JS(J)-IR(J))37,38,38
0202      38    LH=IR(J)
0203            GO TO 39
0204      37    LH=JS(J)
0205      39    JS(J)=JS(J)-LH
0206            L(I,J)=L(I,J)+LH
0207            I1=I
0208      42    IF (JR(I1))18,50,18
0209      18    J1=JR(I1)
0210            L(I1,J1)=L(I1,J1)-LH
0211            I1=JC(J1)
0212            L(I1,J1)=L(I1,J1)+LH
0213            GO TO 42
0214      50    IS(I1)=IS(I1)-LH
          C     ARE ALL SHORTAGES SATISFIED
0215      51    DO 80 J=1,N
0216            IF (JS(J))80,80,57
0217      80    CONTINUE
0218            GO TO 44
          C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          C     NONBREAKTHROUGH PROCEDURE
0219      35    LK=99999
0220            DO 590I=1,M
0221            IF (IC(I))590,590,60
0222      60    DO 59 J=1,N
0223            IF (IR(J))61,59,59
0224      61    LY=K(I,J)+IW(I)+JW(J)
0225            IF (LY-LK)62,59,59
0226      62    LK=LY
0227      59    CONTINUE
0228      590   CONTINUE
0229            DO 580I=1,M
0230            IF (IC(I))580,580,63
0231      63    DO 58 J=1,N
0232            IF (IR(J))72,58,58
0233      72    IF (K(I,J)+IW(I)+JW(J)-LK)58,64,58
0234      64    L(I,J)=0
0235      58    CONTINUE
0236      580   CONTINUE
0237            DO 65 J=1,N
0238            IF (IR(J))65,66,66
0239      66    JW(J)=JW(J)+LK
0240            DO 90 I=1,M
0241            IF (L(I,J))90,70,90
0242      70    IF (IC(I))91,90,90
0243      91    L(I,J)=-1
```

```
   90   CONTINUE
   65   CONTINUE
        DO 67 I=1,M
        IF (IC(I))67,69,69
   69   IW(I)=IW(I)-LK
   67   CONTINUE
        GO TO 57
C       * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       TERMINATION PROCEDURE
   44   LC=0
        DO 205 J=1,N
        KR(J)=0
        DO 305 I=1,M
        IF (L(I,J))210,305,206
  206   LY=L(I,J)*K(I,J)
        LC=LC+LY
        KR(J)=KR(J)+LY
        GO TO 305
  210   L(I,J)=0
  305   CONTINUE
  205   CONTINUE
        DO 611 I=1,M
    611 IA(I)=I
        DO 612 J=1,N
    612 JB(J)=J
        WRITE(6,207)
  207   FORMAT (1H1,30X,8HSOLUTION/29X,12H************ /)
        WRITE(6,208)LC
  208   FORMAT(1H ,12HTOTAL COST= ,I10)
        WRITE(6,201)
        WRITE(6,202)(JB(J),J=1,N)
        DO 172 JJ = 1,2
        IF (N .GT. 17) GO TO 152
        WRITE (6,203) D,D,(E,D,II = 1,N)
        IF (JJ .EQ. 2) GO TO 161
        GO TO 172
  152   WRITE (6,203) D,D,(E,D,II = 1,17)
  172   CONTINUE
  161   DO 307 I=1,M
        WRITE(6,204) IA(I),(L(I,J),J=1,N)
        IF (N .GT. 17) GO TO 155
        WRITE (6,2041) A,B,(C,II = 1,N)
        GO TO 307
  155   WRITE (6,2041) A,B,(C,II = 1,17)
  307   CONTINUE
        DO 173 JJ = 1,2
        IF (N .GT. 17) GO TO 154
        WRITE (6,203) D,D,(E,D,II = 1,N)
        IF (JJ .EQ. 2) GO TO 163
        GO TO 173
  154   WRITE (6,203) D,D,(E,D,II = 1,17)
  173   CONTINUE
  163   WRITE(6,306)(KR(J),J=1,N)
        WRITE(6,959)
  959   FORMAT(1H1)
```

```
0297            DO 747 J=1,N
0298            DO 743 I=1,M
0299            IF(L(I,J).EQ.1) GO TO 744
0300        743 CONTINUE
0301        744 WRITE(6,745) I,J
0302        745 FORMAT(' ','HOUSEHOLD',I2,' LOCATED IN HOUSE',I2)
0303            WRITE(6,746) INCOM(I),PERS(I),SPA(J),LOC(J)
0304        746 FORMAT(' ','INCOME=',F6.0,2X,'FAM SIZE=',F3.0,5X,'SPACE COEF=',F3.
               *2,'  LOCATION COEF=',F3.2)
0305        747 CONTINUE
0306            STOP
0307            END

 TOTAL MEMORY REQUIREMENTS 017EC2 BYTES

 COMPILE TIME =      17.4  SECONDS
```

A P P E N D I X 4

TABLE 1

BIDSPACE MATRIX

HOUSETYPE		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
SPACE		.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62
LOCATION		.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62
HOUSEHOLD INCOME $	PERSONS															
500	1	97	98	100	102	103	105	107	108	110	112	113	115	117	118	120
500	2	113	115	117	118	120	122	123	125	127	128	130	132	133	135	137
500	3	130	132	133	135	137	138	140	142	143	145	147	148	150	152	153
1,000	1	193	197	200	203	207	210	213	217	220	223	227	230	233	237	240
1,000	2	227	230	233	237	240	243	247	250	253	257	260	263	267	270	273
1,000	3	260	263	267	270	273	277	280	283	287	290	293	297	300	303	307
5,000	1	967	983	1000	1017	1033	1050	1067	1083	1100	1117	1133	1150	1167	1183	1200
5,000	2	1133	1150	1167	1183	1200	1217	1233	1250	1267	1283	1300	1317	1333	1350	1367
5,000	3	1300	1317	1333	1350	1367	1383	1400	1417	1433	1450	1467	1483	1500	1517	1533
10,000	1	1933	1967	2000	2033	2067	2100	2133	2167	2200	2233	2267	2300	2333	2367	2400
10,000	2	2267	2300	2333	2367	2400	2433	2467	2500	2533	2567	2600	2633	2667	2700	2733
10,000	3	2600	2633	2667	2700	2733	2767	2800	2833	2867	2900	2933	2967	3000	3033	2067
15,000	1	2900	2950	3000	3050	3100	3150	3200	3250	3300	3350	3400	3450	3500	3550	3600
15,000	2	3400	3450	3500	3550	3600	3650	3700	3750	3800	3850	3900	3950	4000	4050	4100
15,000	3	3900	3950	4000	4050	4100	4150	4200	4250	4300	4350	4400	4450	4500	4550	4600

TABLE 2

BIDLOC MATRIX

HOUSETYPE		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
SPACE		.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62
LOCATION		.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62
HOUSEHOLD INCOME $	PERSONS															
500	1	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
500	2	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
500	3	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
1,000	1	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
1,000	2	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
1,000	3	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
5,000	1	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
5,000	2	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
5,000	3	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
10,000	1	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
10,000	2	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
10,000	3	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
15,000	1	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
15,000	2	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207
15,000	3	160	163	167	170	173	177	180	183	187	190	193	197	200	203	207

TABLE 3*

BIDFUNCT MATRIX

HOUSEHOLD		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
SPACE		.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62
LOCATION		.48	.49	.50	.51	.52	.53	.54	.55	.56	.57	.58	.59	.60	.61	.62
HOUSEHOLD INCOME $	PERSONS															
500	1	256	261	266	271	276	281	286	291	296	301	306	311	316	321	327
500	2	273	278	283	288	293	298	303	308	313	318	323	328	333	338	343
500	3	289	294	299	304	309	314	319	324	329	334	339	344	349	354	359
1,000	1	353	359	366	373	379	386	393	399	406	413	419	426	433	439	446
1,000	2	386	393	399	406	413	419	426	433	439	446	453	459	466	473	479
1,000	3	419	426	433	439	446	453	459	466	473	479	486	493	499	506	513
5,000	1	1126	1146	1166	1186	1206	1226	1246	1266	1286	1306	1326	1346	1366	1386	1406
5,000	2	1293	1313	1333	1353	1373	1393	1413	1433	1453	1473	1493	1513	1533	1553	1573
5,000	3	1459	1479	1499	1519	1539	1559	1579	1599	1619	1639	1659	1679	1699	1719	1739
10,000	1	2093	2129	2166	2203	2239	2276	2313	2349	2386	2423	2459	2496	2533	2569	2606
10,000	2	2426	2463	2499	2536	2573	2609	2646	2683	2719	2756	2793	2829	2866	2903	2939
10,000	3	2759	2796	2833	2869	2906	2943	2979	3016	3053	3089	3126	3163	3199	3236	3273
15,000	1	3059	3113	3166	3219	3273	3326	3379	3433	3486	3539	3593	3646	3699	3753	3806
15,000	2	3559	3613	3666	3719	3773	3826	3879	3933	3986	4039	4093	4146	4199	4253	4306
15,000	3	4059	4113	4166	4219	4273	4326	4379	4433	4486	4539	4593	4646	4699	4753	4806

*Due to a truncation error this table may not be the exact sum of Tables 1 and 2.

TABLE 4

ASSIGNMENT SOLUTION

TOTAL COST =

HOUSES

HOUSEHOLDS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
9	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
15	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
SUB-COSTS	4550	4528	4507	4433	4393	4353	3227	3373	3520	2383	2013	1643	107	553	1000