An Evaluation
of Charles Peirce's Concept of Retroduction

by

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ABSTRACT

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Peirce's theory of retroduction, or the formation of hypotheses, describes, as a form of inference, the process of reasoning by which hypotheses to explain unexpected events are arrived at. In general, retroduction consists in the suggestion of a known class of events of which the event to be explained may possibly be a particular case. On the other hand, Peirce sometimes speaks of retroduction as positing an unobserved entity to explain observed phenomena. It has been argued, however, that this second definition of retroduction constitutes a special case of the first.

The theory of retroduction is presented in two different forms, the earlier and the later, with a transition period between the two forms falling in the years from 1885 to 1900. The early theory stresses the formal structure of the retroductive form of inference, and presents retroduction and induction as parallel forms of reasoning, differing in that the former infers from observed facts other facts different from those observed, while the latter infers facts the same as those observed but of greater generality. Hypotheses and inductions are considered as of comparable stability. In the later theory retroduction originates all new ideas, in the form of suggested hypotheses, which them-
selves are little more than intelligent guesses, but qualify as forms of inference in that unlike perceptions to which they are analogous they may be subjected to criticism as to their adequacy in explaining the events under question. Induction in the later theory is the process of testing hypotheses by deduction of experiential predictions from them and comparison of observed fact with those predictions. The criterion for the acceptance of hypotheses for inductive testing is purely one of economy.

The validity of retroduction consists, not in any objective probability of its conclusions, but in the fact that only by retroduction can any new ideas be originated, and hence in the fact that together with inductive testing it constitutes the only method of arriving at true statements about the real world.
The whole series of mental performances between the notice of the wonderful phenomenon and the acceptance of the hypothesis, during which the usually docile understanding seems to hold the bit between its teeth and to have us at its mercy, the search for pertinent circumstances and the laying hold of them, sometimes without our cognizance, the scrutiny of them, the dark laboring, the bursting out of the startling conjecture, the remarking of its smooth fitting to the anomaly, as it is turned back and forth like a key in a lock, and the final estimation of its Plausibility, I reckon as composing the First Stage of Inquiry. Its characteristic formula of reasoning I term Retroduction . . . (6.469).

C. S. Peirce
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Introduction

It is hard to believe that a full century has elapsed since the birth of Charles Peirce, and that during that time the great influence which he has had upon modern thought has derived almost entirely from the crumbs of his philosophic system. To a large extent this may be accounted for by the fragmentary and cryptic nature of his writings and by the uncompromising originality of his views, but even now, in the flurry of excitement caused by the publication of the first six volumes of his collected papers, the tendency of commentators has been to skim his surface for palatable morsels, rather than to seek the core of his thought. One result of this tendency has been the overemphasis placed on the variety and disorder in Peirce's writings, to the neglect of the unified and coherent system underlying these writings.

It is the purpose of this study to follow a single concept in Peirce's system in all the detail of its presentation from its first appearance until the end of Peirce's philosophic activity. It is hoped that the results will be twofold: that not only will the full complexity of his views on that concept be revealed, but that a key may be discovered to the whole massive structure of Peirce's philosophy. The success or failure of this second aim will become apparent only in work subsequent to the present paper.

The concept to be so investigated is that of retro-
duction, or the formation of hypotheses, a concept felt worthy of investigation both for the importance accorded it by Peirce as the first stage of inquiry and the sole subject matter of pragmatism, and for its comparative neglect by other philosophers. The term retroduction is only one of the four terms used by Peirce to describe the process of hypothesis formation, and has been rather arbitrarily selected from among its three rivals, hypothesis, abduction and presumption, in accordance with Peirce's use of it in his final and in many ways most mature papers. The term hypothesis has been restricted wherever possible to denoting the conclusion of a retroduction.

In view of the aim of this study, the plan of presentation will follow the chronological order of Peirce's writings, with the first three chapters focussed upon the three main sets of papers prior to 1895 and the fourth chapter devoted to the period from 1896 to 1910. The year 1895 has been taken to represent the dividing point between Peirce's early theory of retroduction, considered in the first three chapters, and his later theory, covered in the final chapter, although, in fact, the whole period after 1883 until the turn of the century appears to have been one of transition.

The close inter-relationship between the various aspects of Peirce's thought has made it necessary to precide, to use Peirce's own term, the theory of retroduction from the many theories impinging upon it.

The sources for Peirce's writings referred to in this paper have been listed in the bibliography, with an ad-
ditional reference to Morris Cohen's excellent bibliography of the miscellaneous scientific and philosophic articles and reviews contributed to a number of periodicals, to be found in "Chance, Love and Logic". The bibliography to this paper will be found to contain also a list of commentaries on the philosophy of Charles Peirce, which, without pretending to be exhaustive, includes all the titles which have come to hand. Only a small proportion of these papers will be found relevant to the question of retroduction, the majority having been listed as an aid to the further study of Peirce.

I wish to acknowledge with gratitude the assistance and encouragement which I have received from Dr. Barnett Savery, Dr. A. P. Maslow, and Mr. Edmund Macdonald, in the preparation of this paper, as well as to express my appreciation of the influence of Dr. T. G. Henderson in first turning me toward philosophy.
An Evaluation of Charles Peirce's Concept of Retroduction

Chapter I
The Early Theory -- 1867-1877

The early theory of retroduction is set forth by Charles Peirce in papers dating from his first published work in 1867, until the mid 1890's, at which time his thought on this subject underwent the pronounced change to the later theory, first presented in the papers of 1901. Although the writings during the early period show some considerable development from the earliest to the latest, the fact that all the important papers of this period were edited by Peirce in 1893 for inclusion in a work on scientific reasoning, "The Search for a Method," without the introduction of significant changes, suggests that he found all the accounts of retroduction satisfactory. It is quite clear, however, that the account of retroduction given in the last of these important papers of the early period, "The Theory of Probable Inference" of 1883, represents Peirce's fullest and most developed description of the early theory.

In view of the pronounced variations between the main statements of the early period, it will be most convenient to present Peirce's early theory in three chapters, corresponding to the three main sets of papers written during this
period, and including the less important remarks on retroduction made in papers of the same years. This first chapter will cover the theory of retroduction as set forth in Peirce's first group of papers, published in the Proceedings of the American Academy of Arts and Sciences for 1867.

In the first of the papers of 1867 (2.461-2.516), following an exposition of the forms of deductive reasoning, Peirce shows hypothetic and inductive forms of reasoning to follow from rearrangements of the premisses of a valid syllogism in Barbara. No verbal definition of "hypothesis" or "induction" is offered until the end of the paper, and the syllogistic forms are left to speak for themselves.

Given the syllogism (2.509):

\[
\begin{align*}
\text{Any M is } & \overline{\cap}'P' \\
\text{Any S is } & M \\
\therefore\text{Any S is } & \overline{\cap}'P'
\end{align*}
\]

- where \( \overline{\cap}'P' \) denotes the conjunction of all the characters of M, we pass to the form of reasoning from definition:

\[
\begin{align*}
\text{Any M is } & \overline{\cap}'P' \\
\text{Any S is } & \overline{\cap}'P' \\
\therefore\text{Any S is } & M
\end{align*}
\]

- which constitutes what Peirce calls formal hypothesis.

The fallacy in terms of traditional logic, here exemplified, is claimed to be justified in a later passage (2.514) where, taking an identical proposition:

\[
M \text{ is } P'P''P'''
\]

it is argued that:

1. Charles Hartshorne and Paul Weiss, ed., The Collected Papers of Charles Sanders Peirce, vol. 2, para. 461-516. All citations of the collected Works will be by volume and paragraph number as above.
Whatever is $P'P''$ and $P'''$ is $P'P''P'''$
and:. whatever is $P'P''$ and $P'''$ is $M$
Hence if $S$ is $P'P''P'''$
Then $S$ is $M$.

It is difficult to see, however, how this fuller
explanation does anything more than repeat the fallacy of the
first statement, and the actual justification seems to hang
on the fact that $M$ is $P'P''P'''$ is an identical proposition by
definition: the same being true of $M$ is $\neg P'$ if the implicit
assumption that a subject is identical with the conjunction
of all its predicates be granted.

Turning again to the formal hypothesis, if, for $\neg P'$,
we substitute a group of characters of $M$, $P'P''P'''$ etc., we ar­
rive at the form of probable hypothesis or retroduction:

\[
\text{Any } M \text{ is } P'P''P''' \text{ etc.} \\
S \text{ is } P'P''P''' \text{ etc.} \\
\therefore S \text{ is probably } M.
\]

Since the major premiss is no longer identical,
and cannot therefore be validly converted without limitation,
$S$ can no longer be categorically stated to be $M$.

Peirce advances an ingenious argument to support
the probability of $M$'s predication of $S$ (2.510). Since
every proposition has a contradictory, one half of all pos­
sible propositions are true. Moreover, for every true parti­
cular proposition there is a true universal proposition, and
for every true negative proposition there is a true affirmative

1. There seems to be a clear link between this assumption
and the pragmatic principle of 1878 - 'consider what effects
that might conceivably have practical bearings we conceive
the object of our conception to have- then our conception
of these effects is the whole of our conception of the object!
(5.402).
proposition. Hence of all possible propositions of the form

\[ M \text{ is } \Pi'P \]

one half are true. In every untrue proposition of this form, some finite ratio of the P's are untrue. Hence, of all propositions of this form which are partly true, some finite ratio more than one half are wholly true. That is, since of all possible propositions

\[ M \text{ is } \Pi'P \]

one half are wholly true, of the other half -- the false half -- a finite ratio are wholly false, and a finite ratio are false with some of the characters P, true. Consequently, since retroduction deals with propositions known to be partly true -- this will be enlarged upon -- the wholly false set may be ignored, and it then follows that of all possible propositions arrived at by retroduction, more will be wholly true than false. Hence, he claims, if P' be substituted for \( \Pi'P' \) we obtain a formula of probable hypothesis which, although it gives no determinate probability to the inference, will, however weak at first, by repeated use lead us to the establishment of more and more secure hypotheses.

A further problem in the interpretation of Peirce's statement lies in the fact that although this statement is made in terms of propositions of the form

\[ M \text{ is } \Pi'P' \]

it apparently applies to the conclusion of the retroduction which is in the form

\[ S \text{ is probably } M \]
It will be convenient to restate the retroduction:

M is \( P'P''P''' \) etc.
S is \( P'P''P''' \) etc.

That is, S and M are both \( P'P''P''' \) etc., and M is by definition \( \overline{P'} \) -- although we do not know, and cannot specify every character of M. Hence, when we say that 'S is probably M, we are actually predicting that whatever characters of M can be found, will be predicable of S, that is that

S is probably \( \overline{P'} \)

Since we know from the premises that S and M have some of the characters contained in \( \overline{P'} \) in common -- namely \( P'P''P''' \) etc.-- we are justified in the statement that retroduction deals with propositions which are at least partly true. [see above].

Ingenious as this argument appears, if this interpretation is correct it would seem to be based on a confusion between logical truth and material truth, which is not apparent until Peirce's statement is fully set forth. When Peirce speaks of a true hypothesis this is presumably in terms approximating the definition: "Truth consists in the existence of a real fact corresponding to the true proposition," (2.652). This is a definition of material truth, and while it is by no means clear in what the correspondence between fact and propositions consists, it certainly appears inapplicable to 'one half of all possible propositions.'

The sense in which, of two contradictory propositions, one is true and the other is false, is quite different from
material truth and falsity, since in terms of the definition given, unless there is a fact corresponding to one of the two contradictory propositions, both must be said either to be materially false, or to be meaningless. Logical truth can be brought into harmony with material truth by saying that of two contradictory propositions, if one be true, the other must be false and vice versa.

It may be, however, that this is to misinterpret Peirce, and that by 'possible propositions' he is referring to material rather than logical possibility. If so, assuming that the remainder of the interpretation is correct, he appears to be saying that of all the statements that could be made about the world, if those which are known to be false on the basis of present knowledge are excluded, the remainder will contain slightly more true propositions than false ones. Then if any proposition be selected at random it will have a probability of being true determined by the proportion of true propositions to the whole collection, and this probability, although unknown, can be known to be greater than \( \frac{1}{2} \). This is probably closer to what Peirce meant; but without questioning the validity of this second interpretation, it must seem doubtful how much is added thereby to our understanding of the success of the experimental method. Our hypotheses are not drawn at random from the class of all possible propositions, but from a narrower class whose boundaries are set by previous scientific work, and, according to Peirce, by certain characteristics of the human
mind. The probability attaching to the general class of all possible propositions has no effect upon the probability of the truth of propositions taken from this narrower class, and it is not clear how the amount of this latter probability can be arrived at except from the relative successes and failures of the propositions selected and tested. On this basis Peirce claims that the probability of any hypothesis so selected being true is very high.

A more significant factor in the success of ampliative inference — both retroductive and inductive -- is repeatedly stressed by Peirce throughout his subsequent writings. This is the self corrective nature of experimental method, such that, however erroneous be its results at any point, "the further application of the same method must correct the error" (5.145 cf. 5.348-352). Thus the success of this method is independent of the probability of any of its particular premisses.

Toward the end (2.515) of the paper under consideration, Peirce defines hypothesis as "an argument which assumes that a term which necessarily involves a certain number of characters, which have been lighted upon as they occurred, and have not been picked out, may be predicated of any object which has all these characters."

Retroduction thus appears to be the placing of objects within previously defined classes -- or the predication, of a given object, of a class name, which name has
been defined by a sampling process. In terms of the formal
syllogism, by reason of the common characters $P'_P''P'''$ etc.
the object $S$ is placed in the class of objects $M$, and sup­
posed to share all the characters of $M$. This interpretation
is supported by another passage of the same year (2.425),
where, from similar premisses to those used above, the conclu­
sion:

$$S \text{ is all that } M \text{ is}$$

is derived, and our knowledge of $S$ is said to be potentially
increased, in that if there are any more characters common
to the class $M$, and if we discover these characters, they
will either be predicable of $S$, or will exclude $S$ from the
class $M$.

A description of retroduction given in 1868, sup­
ports the same interpretation by speaking of it as dealing
with an absence of knowledge as to whether, besides the
characters attributed to an object by the premisses, other
characters belong to it — by proceeding "as though all the
characters requisite to the determination of a certain object
or class of objects were known." In this case $M$ is spoken
of as either a class of objects or another object (5.272).
In this same paper the function of retroduction is described
as being "to substitute for a great series of predicates
forming no unity in themselves, a single one (or small number)
which involves them all, together (perhaps) with an indefinite
number of others." (2.576) This is an advance on the previous
descriptions, in that the $S$ is no longer spoken of as if already named, although in many cases it may be, but becomes a 'this', a complex of characters as yet unclassified, and the process of retroduction is extended to include the recognition of events experienced. In keeping with this restatement the form of retroduction is given as:

$$\text{if } A \text{ then } B$$

$$B$$

$$\therefore A$$

where $B$ represents a complex of characters, and might well be replaced by $P''P''P''$ etc. This form involves a logical fallacy analogous to that of the other form of retroduction discussed above, with the result that again the premisses suggest, rather than necessitate, the conclusion. This same form, although with a slightly different application, is used by Peirce to describe retroduction in the "Lectures on Pragmatism" of 1903, where (5.189):

The surprising fact, $C$, is observed;
but if $A$ were true, $C$ would be a matter of course.
Hence, there is reason to suspect that $A$ is true.

It is in this sense that retroduction is spoken of as "reasoning from consequent to antecedent" (5.276).

Five rules are given, to which retroduction must conform in order to arrive at a valid hypothesis (2.511):

1. the deductive syllogism from which the retroversive argument derives must be valid, or, as Peirce says in the course of a footnote to these rules -- "if it is granted that hypotheses are inferred . . . observed facts must follow apodictically from the hypothesis without the aid of subsidiary hypotheses."
This rule appears to be more a primary test for formed hypotheses than a guide for their formation.

2. the conclusion of the retroduction is not to be held as absolutely true and is to be rejected, if and when it is found "that P' was taken from some higher class than M" -- 'higher' presumably meaning "of broader extension."

3. following from rule 2, the predicate of the premisses must be a conjunction of predicates -- that is, of characters.

4. this aggregate must be of different qualities and not of mere names.

5. the only principle upon which the instanced predicates can be selected is that of belonging to M.

The footnote to these rules, referred to above, is a rejection of a possible sixth rule, the principle which Peirce attributes to the positivists, that "no hypothesis is admissible which is not capable of verification by direct observation." For Peirce retroduction is a form of logical inference, valid, although uncertain, because arguing from certain premisses according to a formal process of reasoning. Peirce points out that to accept the positivistic principle seriously is to rule out the whole body of accepted historic fact, all of which is dependent on retroductive reasoning, and completely out of reach of direct observation.

It must be noted that this early theory of hypothesis involves a division of all reasoning into explicative or deductive, and ampliative; and of ampliative reasoning into hypothesis and induction (2.709). These two forms of reasoning are then developed side by side, the one paralleling the other such that each statement about the one can, by the sub-
stitution of 'subject' for 'predicate' or 'S' for 'P', be made applicable to the other. Induction is described in terms of its derivation from a syllogism in Barbara — its form being

\[ \varepsilon' \quad S' \text{ is } P \\
\varepsilon' \quad S' \text{ is } M \\
\therefore \quad M \text{ is } P \]

where ε' S' denotes the sum of all classes coming under M; it is subject to the five rules and to the statement regarding probability — all made applicable by the minor adjustments described; and is finally defined as an "argument which assumes that a whole collection from which a number of instances have been taken at random has all the common characters of those instances" (2.515).

It is necessary to emphasize this parallel relationship of hypothesis and induction; the first reasoning from certain characters of an object to the total nature of the object; the second, from instances of a collection to the nature of the whole collection, in order to bring out clearly the position of retroduction in the early theory as compared with its position in the later theory.

Retroduction, in this earliest statement, has been put forward as a form of inference, starting from premisses established according to certain rules, and reaching a conclusion asserting the probable inclusion of an object in a class of objects, or the similarity of an object to another object. It has been argued that what has been called formal hypothesis rests upon the assumption that a subject is identi-
tical with the conjunction of all its predicates; and that the truth of hypothesis is possible rather than probable.
Chapter II

The Early Theory -- 1878-1882

The second main statement of the early theory of retroduction is to be found in the last of the six papers on the "Illustrations of the Logic of Science" published in the Popular Science Monthly for 1878 (2.619-644). This paper, although differing considerably from the papers considered in Chapter I, was also edited without alteration for "The Search for a Method."

It is possible, according to Peirce, to speak of every deductive syllogism in Barbara as the derivation of a result, the conclusion, from the application of a rule stated in the major premiss, to a case under that rule, stated in the minor premiss (2.620). "This definition, of course, applies . . . only to the first figure." By 'rule' Peirce refers to what he frequently calls a 'leading principle,' according to which we pass from a premiss -- the case -- to a conclusion -- the result (cf. 3.162-172). Such a leading principle is logically good if it never, or in probable inference, seldom, leads us from a true case to a false result. Thus:

All men are mortal -- Rule
Socrates is a man -- Case
\[ \therefore \] Socrates is mortal -- Result.

All three forms of inference -- deduction, induction and retroduction -- can be described and differentiated in terms

of these three classes of propositions:

1. All deduction is the application of general rules to particular cases, with a consequent result. This is not always evident, but, by reducing any deduction to Barbara, will be found always to be so (2.620).

2. All induction is the conclusion of "A rule from the observation of a result in a certain case" (2.622):

   These beans were in this bag -- Case
   These beans are white -- Result
   \Rightarrow All the beans in the bag were white -- Rule

2. All retroduction is the conclusion of a case from a rule and a result (2.623):

   All the beans from this bag are white -- Rule
   These beans are white -- Result
   \Rightarrow These beans are from this bag -- Case.

The conclusion in induction and retroduction is not, of course, claimed to follow deductively from the premisses.

An alternative formulation of retroduction and induction, given in this same paper, is based upon the derivation of syllogisms in Baroco and Bocardo by denying the conclusion and one or other of the premisses of a syllogism in Barbara (2.626).

If the conclusion and the case be denied, and the rule be admitted in:

   All men are mortal
   Socrates is a man
   \Rightarrow Socrates is mortal

the result is a Baroco syllogism:

   Socrates is not mortal
   All men are mortal
   \Rightarrow Socrates is not a man

The conclusion of this syllogism is of the nature, according to Peirce, of a very weak hypothesis -- "so timid as to lose
its amplificative character entirely" (2.630). Similarly, by denying the rule and admitting the case, the resulting Bocardo syllogism has for its conclusion a weak induction. In either case, the conclusion fails to achieve ampliative status in that it expresses merely a lack of knowledge, rather than leaping boldly into the unknown to suggest an explanation for the premisses, either in the form of a new case, or a new rule -- the first of these constituting a hypothesis, the second, an induction.

If we apply the same treatment to what Peirce calls a probable deduction in Barbara, we arrive, he claims, at a valid hypothesis or induction (2.627). Thus by denying the result and accepting the rule of the probable deduction:

Most of the beans in this bag are white
This handful of beans is from this bag
".Probably, most of this handful is white
we arrive at the form of retroduction:

Few beans of this handful are white
Most beans in this bag are white
".Probably, these beans were taken from another bag.

Similarly, by denying the result and accepting the case, of the deduction, we get the induction:

Probably, few beans in the bag are white.

A consideration of Peirce's use of probability with regard to hypotheses will be postponed until the next chapter, when the whole period during which Peirce spoke of retroduction as a form of probable inference will have been covered. It should be mentioned in passing, however, that the example of probable deduction from which he derives the examples of
reproduction and induction, above, is not of the form of simple probable deduction:

\[
\begin{align*}
\frac{1}{4} \text{ B is C} \\
\frac{1}{4} \text{ A is B} \\
\therefore \text{ Probability is } \frac{1}{4} \text{ that A is C (cf. 2.695)}
\end{align*}
\]

but is a case of statistical deduction (cf. 2.700).

A second point to be mentioned concerns the hypothesis arrived at in the example of reproduction given above:

Probably these beans were taken from another bag.

This is actually a free interpretation of the more accurate conclusion:

Probably these beans are not from this bag.

This corrected conclusion is no more an ampliative hypothesis than is the conclusion of the Baroco syllogism given above. Whether the syllogism from which the reproduction is derived be apodictic or probable, the formation of a positive hypothesis involves a step beyond the logically valid conclusion of the given premisses.

Two descriptions of the nature of reproduction are given in this paper of 1878 by Peirce (2.624):

a. Finding that two objects have a strong resemblance in certain respects, we infer that they resemble one another strongly in other respects. This is fundamentally the form taken by reproduction in Chapter I, although, with the exception of one passage (5.272), the inference is from an object to a class of objects, rather than from one object to another. This statement can, however, be
made compatible with that given in the present chapter: using the example given above, from discovering that two collections of beans resemble each other strongly as to colour, we infer that they resemble each other in having both been formerly of the same collection. The weakness of the hypothesis so described arises rather from the weakness of the example itself, than from any in-applicability of this description of hypothesis to the example.

b. Finding some curious circumstance, explainable by the supposition that it is a case of a certain general rule, we adopt that supposition. This statement is more in keeping with the viewpoint of the paper at present under investigation, although it is not wholly at odds with that of Chapter I, where retroduction is described as the inference of a minor premiss from the major premiss and the conclusion of a syllogism. The difference arises mainly from the restriction, in the first papers, of premisses to those predicating qualities, which restriction is not retained in the present paper; and from the specification of the propositions in a Barbara syllogism as rule, case and result.

A new complication is introduced, however, by the examples given by Peirce to illustrate what he means by retroduction, the clearest of which may be quoted in full:

Numberless documents and monuments refer to a conqueror called Napoleon Bonaparte. Though we have not seen the man yet we cannot explain what we have seen, namely, all these documents and monuments, without supposing that he really existed (2.625).
This is a new form of hypothesis, in which an entity or event, by its very nature immune from direct observation, is posited to explain observable evidence. As well as all historical fact, such scientific entities as 'atoms' fall into this class. This view of retroduction is formulated abstractly later in the same paper, where it is said that "by hypothesis, we conclude the existence of a fact quite different from anything observed, from which, according to known laws, something observed would necessarily result" (2.636). Retroduction consists in reasoning 'from effect to cause', as opposed to induction, which passes from particulars to a general law. The function of retroduction is to explain; of induction, to classify.

Justus Buchler in discussing Peirce's theory of retroduction, objects to the inclusion, in the above quotation, of the words 'according to known laws', on the grounds that, when, to explain a circumstance C, we posit A, because if A were true, C would follow as a matter of course, while in some cases this law may be already known, in many the law 'is actually part of the hypothesis or supposition A'. There seems at first glance justification for such criticism; the scientific hypotheses by which atoms or molecules are proposed appear to be clearly of this nature and to take the form:

given the surprising fact C -
(then there is an A of such a nature as to bring about a C and A has occurred)

- of which the whole bracketed portion is the hypothesis. What has been done is to infer a law of such a nature that if it were to be acted upon, C would result. A is nothing more than an occasion of a law. If this is to assert the primacy of laws over the events which take place according to those laws, it will be quite in accordance with Peirce's concept of the reality of law.

On the other hand, although it may be true that such a hypothetical entity A has ascribed to it, as part of its character, the laws of its activity, these laws being such that C would follow from the occurrence of A, unless the description of A's action is in terms of known laws it can hardly be claimed that C has been explained. Hence it is Peirce's conviction that "progress in science depends upon the observation of the right facts by minds furnished with appropriate ideas" (6.604, cf. 2.752). If it be claimed that necessity for previously known laws to underlie every hypothesis involves an infinite regression, this is countered by the fact that Peirce bases all retroduction ultimately on 'instinctive' knowledge of mechanical and social principles.

"It seems incontestable . . . that the mind of man is strongly adapted to the comprehension of the world . . . , that certain conceptions, highly important for such a comprehension naturally arise in his mind; and that without such a tendency, the mind could never have had any development at all" (6.417). All science is ultimately nothing but the outgrowth from these instincts (6.500).
No attempt is made by Peirce in the present paper to bring the view of retroduction as reasoning from effect to cause into harmony with the two other views given above -- of retroduction as inference of the similarity of objects in all respects, from their observed similarity in some respects, and of retroduction as inference of a case from a rule and a result.

A fourth description of retroduction -- this the most general of all -- is given toward the end of the paper under consideration (2.642), in which retroduction is spoken of as inference 'from facts of one kind to facts of another', as opposed to induction which infers 'from one set of facts another set of similar facts'. This statement suggests that Peirce may have looked upon retroduction as covering a wide number of types of ampliative inference; this would explain both the diversity of definitions and the variety of syllogistic forms given for retroduction.

Although Peirce speaks of retroduction as explaining events in terms of known laws, he warns against the idea that the validity of the hypothetic method is dependent ultimately upon an order of nature (2.633). Particular hypotheses are arrived at and strengthened as a result of our knowledge of various regularities and laws in nature, but to rest the validity of hypothetic method as a whole upon any such order constitutes an attempt to reduce ampliative reasoning to a form of deduction. Peirce considers this question more fully
with regard to induction in another paper from the same series of 1878 (6.410-413), where he argues that the validity of induction -- and the same may be said of the parallel, though weaker, retroduction -- depends on its self corrective nature, such that, if continued in long enough it will lead inevitably to truer and truer statements about real facts. The theory that ampliative reasoning depends for its validity on an order of nature, that is, upon the principle that what occurs under certain circumstances must occur under similar circumstances, leads to a number of inadequacies:

1. it fails to account for those ampliative conclusions in which the event is only stated to occur in a certain proportion of cases, under the given circumstances (6.411).

2. it forbids the drawing of ampliative conclusions with regard to any character about whose constancy we know nothing (6.412).

3. it overlooks the conditions actually necessary for valid synthetic reasoning -- namely, predesignation of the character concerned and random sampling from the whole class of objects under consideration (6.413).

4. if the validity of synthetic reasoning were dependent on the order of nature, it is conceivable that there could be a universe in which such reasoning would be invalid (5.345)

Peirce's arguments have been given very cryptically—he presents many more, and enlarges upon them considerably, but space does not allow careful consideration of this question.
Throughout the presentations of the early theory of retroduction Peirce places his emphasis upon the logical form of the reasoning involved, with only scant consideration of what is ordinarily regarded as the more important problem — that of testing hypotheses. He does devote a paragraph (2.634), however, to setting up certain principles for valid testing:

1. since it is possible, given two objects or classes of objects, to find any number of resemblances between them, 'a hypothesis should be distinctly put as a question, before making the observations which are to test its truth . . . . We must try to see what the result of predictions from the hypothesis will be'. That is, predictions must be deduced from the hypothesis itself, and the facts observed to see if such predictions are exemplified under the conditions specified in the hypothesis.

2. 'the respect in regard to which resemblances are noted must be taken at random'. Presumably by 'noted' he means 'predicted and observed', because he goes on to say: "We must not take a particular kind of predictions, for which the hypothesis is known to be good."

3. "the failures as well as the successes of the predictions must be honestly noted. The whole proceeding must be fair and unbiased." For Peirce, scientific reasoning, as opposed to deductive reasoning, is largely dependent for its validity upon the courage and high principle of the reasoner: "The logic which observational science
uses is not, like the logic that the books teach, quite independent of the motive and the spirit of the reasoner. There is an ethics indissolubly bound up with it -- an ethics of fairness and impartiality" (6.3, cf. 1.576).

In the expression of these rules Peirce is referring specifically to hypotheses concerning the similarity of objects and classes of objects -- in principle, however, these rules are applicable to all hypothetic inference.

The careful distinction maintained by Peirce between induction and retroduction has been stressed several times and constitutes one of the main characteristics of the early theory of ampliative reasoning. Peirce argues further, in opposition to persons unnamed, that not only are the processes of induction and retroduction quite distinct, but that hypotheses so arrived at, can never, since by their nature they reason from facts of one kind to facts of another, be replaced by inductions, which are limited to inference from facts to other facts of a similar kind (2.642).

The third important argument advanced to justify the distinction between retroduction and induction is based on their affinity to two physiological processes of apprehending facts (2.643). This aspect of Peirce's theory of retroduction can be briefly summarized here. Hypothesis, he claims, "produces the sensuous element of thought, and induction the habitual element." Just as in a hypothesis a single conception is substituted for a tangle of predicates,
so "when our nervous system is excited in a complicated way, there being a relation between the elements of the excitation, the result is a single harmonious disturbance which I call an emotion." This emotion which is substituted for the complex of excitation is "essentially the same thing as a hypothetic inference." Peirce draws no sharp distinction between sensation and emotion, and in the present quotations is using the term emotion to cover both.

Induction, being the inference of a rule, since "the belief of a rule is a habit" (2.643) and "a habit is a rule active in us", "is the logical formula which expresses the physiological process of formation of a habit." Deduction, it may be added, is of the nature of volition and of the concentration of attention on a fact.

A fourth merit of the distinction between induction and retroduction lies in its relevancy to the division of the sciences into the classificatory sciences, which are, according to Peirce, purely inductive; the sciences of theory, such as pure physics; and the sciences of hypothesis -- in which he includes "geology, biology, etc" (2.644).

In spite, however, of his careful bifurcation of ampliative reasoning into induction and retroduction, Peirce admits a certain mingling of the two processes in the formation of most theories. Insofar as any induction is extended quite beyond the limits of our observation, it partakes to some degree of the nature of a hypothesis, and must be supported by its efficacy in explaining observable facts (2.640).
The difference, however, between such an induction, and what Peirce would call a pure induction, limited to the ultimately observable, seems negligible as far as the reasoning involved in their formation is concerned. Any ampliative induction, as opposed to an induction based on exhaustive enumeration, is at the time of its formation a prediction about facts as yet unobserved. Whether or not such facts are ultimately observable has a great deal to do with the weight of the induction, but very little, it would appear, to do with the process of reasoning by which it was attained. Furthermore, unless Peirce is redefining retroduction as the process of reasoning involved in the formation of all statements about unobservable facts, in terms of the various definitions of retroduction given, it is not clear how the inductions referred to can in any way be said to be in part retroductions.
Chapter III

The Early Theory -- 1883-1895

In 1883 Peirce edited the Johns Hopkins "Studies in Logic", to which he contributed "A Theory of Probable Inference" (2.694-754), a paper setting out a theory of probable deduction, induction and retroduction. This paper, like those already considered, was edited in 1893 for inclusion in "The Search for a Method", without alteration of text.

An example has been given in Chapter II of what Peirce came to call simple probable deduction, and it was stated at that time that it is from the syllogism of statistical deduction rather than from this simple deduction that ampliative inference is derived. Peirce makes this point clear in the present paper, and introduces a further refinement by distinguishing between statistical deduction, and a variation which he calls statistical deduction in depth.

Statistical deduction (2.700) is of the form:

\[ \text{the proportion } r \text{ of the M's are P's} \]
\[ \text{S'S'S' etc. are a numerous set, taken at random} \]
\[ \text{from the M's} \]
\[ \text{.probably about the proportion } r \text{ of the S's are P's.} \]

Deduction in depth relates to the degree of resemblance between objects or classes of objects, in terms of a comparison of their characteristics. Statistical deduction in
depth takes the form (2.705):

every M has the numerous marks \( P'F''F''' \) etc.
S has an \( r \)-likeness to the M's
\( \therefore \) probably and approximately, S has proportion \( r \) of the marks \( P'F''F''' \) etc.

- where "\( r \)-likeness" is a measure of resemblance, with a scale of numbers from zero, denoting total dissimilarity, to unity, denoting identity (2.704).

The derivation of induction and retroduction from the probable deductive syllogism, by denial of the conclusion and one premiss, has been described in Chapter II. Peirce modifies this inversion of the statistical deduction, so that the inductive or hypothetical conclusion will be positive, rather than negative as was the case in Chapter II. Given the premiss of the statistical deduction:

- proportion \( r \) of the M's are P's
- the proportion denoted by \( r \) may be any value between 1 and 0.
- For every value of \( r \) there is a ratio \( p \), which is the logical negative of \( r \), and which admits of every value which \( r \) excludes and excludes every value of which \( r \) admits (2.720).

Then, if the major premiss and conclusion of the statistical deduction be denied and transposed, the form for induction is obtained:

- S'S"S" etc. are a numerous random sample of the M's
- proportion \( p \) of the S's are P's
- \( \therefore \) probably about proportion \( p \) of the M's are P's

Peirce points out that 'probably about' must be regarded as the modality with which the conclusion is drawn in deduction and induction, and not as a part of the proposition, in which case the deduction would be necessary, and its inversion an instance of Bocardo (2.720 n.).
In the same way, by denying and transposing the minor premiss and the conclusion, of the statistical deduction in depth, the form for retroduction is arrived at (2.721):

every M has the numerous marks \( P'P''P''' \) etc.
S has proportion \( p \) of the marks \( P'P''P''' \) etc.
\( \therefore \) probably and approximately, S has a \( p \)-likeness to the class of M's.

According to Peirce, however, it is not strictly necessary that the statistical deduction be inverted by denial of the conclusion and a premiss. This denial is necessary in the case of ordinary syllogism, which is based on a relation of containing and contained; and it has been carried over therefrom into the inversion of statistical deduction. The statistical deduction, however, involves an assumption of an approximate equality between the ratio of P's in the whole class and in the sample, and this relation of equality is convertible (2.718).

Retroduction can be, and has by other writers been spoken of as induction of characters, rather than of objects. Instead of taking a random sampling of the objects in a class, the characters of an object or class of objects are sampled, and induction proceeds from the degree of similarity discovered between the characters sampled from the two objects. The great difference between induction and retroduction depends on the difficulty of counting characters rather than things, and upon the fact that an inference from certain resemblances between objects to the overall resemblance between the two objects is far weaker than the inference of induction proper.
"There is no greater nor more frequent mistake in practical logic than to suppose that things which resemble one another strongly in some respects are any the more likely for that to be alike in others" (2.634). Hypotheses must be formed on the basis of significant resemblances and be tested by prediction of further resemblances, under the conditions of testing described in Chapter II. What Peirce means by significant resemblances is not made perfectly clear, but may in part be explained by his discussion of our knowledge of special uniformities in nature, by which we are aware of the tendency of certain types of characters to be constant throughout a class (2.743). Another possibly more important factor in this problem of the significance of certain data is Peirce's theory, briefly mentioned in Chapter II, concerning man's tendency to guess right regarding principles of mechanics and the moral sciences (2.753). Then if man has such a tendency, certain facts leading to fruitful guesses would appear more significant than other facts. Two factors thus interact in the formation of hypotheses; in addition to the "well established proposition that all knowledge is based on experience ... we have to place this other equally important truth, that all human knowledge, up to the highest flights of science, is but the development of our inborn animal instincts" (2.754).

Peirce accepts the "well established" definition of a hypothesis as "a proposition believed in because its consequences agree with experience" (2.707), as being in accordance with his own use of the term, and gives Kepler's
hypothesis as to the orbit of Mars as an example of this use. Kepler "traced out the miscellaneous consequences of the supposition that Mars moved in an ellipse, with the sun at the focus, and showed that both the longitudes and the latitudes resulting from this theory were such as agreed with observation." On the other hand, the suggestion in ordinary use of the term hypothesis, "of uncertainty, and of something to be superseded" (2.708) is quite foreign to Peirce's use of it.

In the same passage Peirce mentions Newton's use of the term, where Newton claims to give a general formula for the motions of heavenly bodies, but makes no hypotheses to explain the causes of their motions, and this again Peirce accepts as coinciding with his own usage. Now unless Peirce is taking into account Kepler's peculiar theory of mathematics as in some way of causal efficacy, it would appear that Kepler was giving a general formula for the motion of Mars, rather than suggesting the cause of such motion. Kepler appears to be doing what Peirce is referring to when he speaks of retroduction as the process of bringing a "confused concatenation of predicates" into order under "a synthetizing predicate" (2.712). This, rather than Newton's use of hypotheses as suggesting causes, seems to represent Peirce's customary use of the term.

However, in this same paper (2.713), Peirce does speak of men as looking upon Nature, from their anthropomorphic position, as continually making deductions in Barbara,
in which laws of nature constitute major premisses, and the occurrence of causes under those laws bring effects as conclusions. Then the function of retroduction is the discovery of such causes. This use of hypothesis is not unlike that in Peirce's example of our inference of the existence of Napoleon to explain certain evidence; nor is it unlike the inference of entities, such as atoms and electrons, to explain observed phenomena.

It is possible, however, that all these types of retroduction can be considered as special cases of a more general form of reasoning by which order is introduced into a confused concatenation of predicates. In the case of Kepler's retroduction an ideal orbit is suggested, and it is found that certain of the positions of a body moving in that orbit would coincide with the observed positions of Mars. In the same way retroduction might suggest a certain ideal series of events in time, following one another in terms of known laws, and culminating in an event or events coinciding with observed fact. Then it might be assumed that the ideal series of events constituted an explanation of the facts observed. What may be two unreconciled tendencies in Peirce's view on this matter are well illustrated by two statements from the two sections of his review of William James' "Principles of Psychology." In the first of these, he speaks of "the general character of scientific hypotheses" as well expressed by James' description of them as "attempts to
explain phenomenally given elements as products of deeper lying entities." In the second he says, "To explain any process not understood is simply to show that it is a special case of a wider description of process which is more intelligible." Peirce holds the theory that all valid reasoning:

consists in constructing a diagram according to a general precept, in observing certain relations between parts of that diagram not explicitly required by the precept, showing that these relations will hold for all such diagrams, and in formulating this conclusion in general terms (1.54)

Although in one passage he specifically includes the process of constructing the diagram "from the state of things asserted in the premisses" under deduction (1.66), the process appears fundamentally the same as that by which hypotheses are constructed from a collection of data. A section from a paper of 1890 seems to support this suggestion, where he speaks of "the highest kind of synthesis" as being that which "the mind is compelled to make in the interest of intelligibility . . . this it does by introducing an idea not contained in the data, which gives connections which they would not otherwise have had" (1.383).

In order to understand Peirce's theory of ampliative inference it is necessary to clarify his uses of the concept of probability in these early papers. A general distinction between necessary and probable forms of reasoning is drawn as follows:

2. Ibid., p. 33.
The difference between necessary and probable reasoning is that in the one case we conceive that such facts as are expressed by the premises are never, in the whole range of possibility, true, without another fact, related to them as our conclusion is to our premises, being true likewise; while in the other case we merely conceive that, in reasoning as we do, we are following a general maxim that will usually lead us to truth (2.696).

The distinction made in Chapter II, between simple probable deduction, of the form:

\[
\begin{align*}
\text{B is C} \\
\text{A is B} \\
\therefore \text{probability is } \frac{1}{n} \text{ that A is C,}
\end{align*}
\]

and statistical deduction:

the S's are a numerous random sample of the M's proportion \( r \) of the M's are P's

\[
\therefore \text{probably about the proportion } r \text{ of the S's are P's}
\]

illustrates a certain ambiguity in Peirce's use of the concept of probability. A qualification of the general description of probable inference given above defines the type of inference in simple probable deduction:

To say then, that a proposition has the probability \( p \) means that to infer it to be true would be to follow an argument such as would carry truth with it in the ratio of frequency \( p \) (2.697).

This, briefly, is Peirce's frequency theory of probability.

Buchler brings statistical deduction into conformity with the frequency theory by restating the above example as follows:

If the proportion \( p \) of a class \( M \) has the property \( P \), it follows with probability \( p \) that a member of a sub-class \( S \), chosen at random from \( M \) will have the property \( P \). 2.

1. cf. Buchler: Charles Peirce's Empiricism, pp. 241-54 for a detailed but confused treatment of this ambiguity.

2. Ibid., p. 245
Not only, however, does he thereby destroy the identity of statistical deduction, but, since he leaves induction in the form given by Peirce, he does away with the link between induction and deduction without resolving the equivocality in the use of the probability concept. It is more expedient to leave statistical deduction in its original form and to include it with ampliative inference as making use of a different type of probability from that found in simple probable deduction. It is in this second type of probability, excluded from the frequency theory, but in conformity with the more general description of probable inference as reasoning "following a general maxim that will usually lead us to the truth" (2.696), in which we are here interested.

The first suggestion of this equivocal sense of probability appears in a paper of 1878, "The probability of Induction," where Peirce makes the statement that "in the case of analytic inference we know the probability of our conclusion (if the premisses are true) [probability here being used in the frequency sense], but in the case of synthetic inferences we only know the degree of trustworthiness of our proceedings" (2.693). This distinction must be extended to include statistical deduction as well as ampliative inference, the significance of the term "probably" in the conclusion of each of these forms of inference referring to the tendency of the method of inference, if continued in, to yield a true proposition from true premisses.
There is, however, a further distinction along these lines, between the conclusion of a statistical deduction and that of an ampliative inference. The possible falsity of any particular conclusion from true premisses in a statistical deduction arises from the fact that "the S's constitute a numerous random sample of the M's", so that the truth of the conclusion depends on the truth of the assumption that the S's are a representative sample of the M's. If it be found that the conclusion of any particular statistical deduction from true premisses is false, further sampling will tend to vindicate the ratio stated in the conclusion (2.703 cf. 709,722). The conclusion may be false for any given set of samples, but for a truly representative sample the conclusion must hold good. As Ernest Nagel points out, in the case of any statistical deduction, either it is known that the sampling is representative, or it is hypothetized to be so, and this hypothesis is to be verified.¹

In ampliative inference the truth of the conclusion from true premisses is similarly dependent on the representativeness of the sampling, with the difference that in this case the ratio stated in the conclusion is derived from the samples taken, rather than from knowledge of the ratio in the whole collection. Here again, if the sampling was not representative of the whole collection, the conclusion will be false, but in this case further sampling will so modify the ratio as to lead to a closer approximation to that ratio true of the whole collection. "The probability of its

¹ Charles Peirce's Guesses at the Riddle", J. Philos. vol. 30, 1933, p. 382
The inductive process conclusion only consists in the fact that if the true value of the ratio sought has not been reached, an extension of the inductive process will lead to a closer approximation" (2.730).

The derivation of the ampliative forms of inference from statistical deduction gives the primary rule for the validity of induction and retroduction, which is that the deductions of which they are the inversions must be valid and strong. Even though valid, probable deductions vary in strength, this term denoting the probable error of the concluded ratio. The probable error can be arrived at by the formula:

\[ 0.477 \sqrt{\frac{2r(1-r)}{n}} \]

where \( r \) is the given ratio and \( n \) is the number of independent instances sampled. In most cases, in deduction, \( r \) will be indefinite, so that only a maximum and minimum figure can be given for the probable error. In the case of induction and hypothesis, "r being wholly indeterminate, the minimum value is zero, and the maximum is obtained by putting \( r = \frac{1}{2} \)" (2.724). It is not at all clear what Peirce means when he says that the concluded ratio \( r \) is wholly indeterminate for ampliative inference, since particularly in the case of induction a very definite ratio is obtained by sampling.

It should be noted that when Peirce speaks of retrodictions and inductions as derived from deductions
he must mean that it is the formulae, or the syllogistic forms, for these types of inference that are so derived, and not the particular inferences themselves. It is only after the ampliative inference has been made that both premisses of the deductive syllogism are known. Consequently, although Peirce himself may not have recognized the fact, it would appear that this primary rule for the validity of ampliative inferences, that the deductions of which they are the inversions must be valid and strong, is not a rule for the formation of any particular inference, but a criterion for the validity of inferences already formed.

An important rule for securing valid inferences is that the sampling of things, in induction, and of characters, in retroduction, should constitute a fair choice from the class sampled. Each sample must be drawn at random from the whole lot, and independently of all other samples taken. "The sample must be taken according to a precept or method, which, being applied over and over again indefinitely, would in the long run result in the drawing of any one set of instances as often as any other set of the same number," (2.726). Not only is strict honesty on the part of the investigator essential for good sampling, but also, in most cases, some mechanical contrivance is necessary to avoid any unconscious bias in the selection of samples.

Another indispensable precept with regard to sampling is the rule of predesignation. Before any instances
are drawn from the class under consideration, the character for which the class is being sampled must have been settled upon (2.736). That is, in any statistical deduction of the form given above, the major premiss:

proportion r of the M's are P's

must have been laid down, before the sampling specified in the minor premiss:

the S's are a numerous random sample of the M's

has been carried out. Then the inference has been made already, that the proportion r of the S's will be P's, and it is only necessary to draw the samples. If, on the other hand, the sampling is done first, before stating the major premiss, and then on the basis of the sampling the major premiss is decided upon, clearly the inference will not be a valid statistical deduction. Similarly in statistical induction in depth, the term S, of the proposition:

S has an r-likeness to the M's

must be predesignated prior to sampling.

This principle holds good for induction, just as it does for deduction. In sampling a class M, both the character P which is being sampled for and the number of samples to be drawn must be predesignated. The inference has then been made prior to sampling, that the ratio of P's in the sample will hold good for the whole class M. Samples may then be drawn and the ratio noted. If, on the other hand, the character P is not predesignated, but is decided
upon by inspection of the sample, it is possible to find, in any set of instances of a class, any number of particular regularities which will not hold good for the whole class. Such regularities, if particularly striking may suggest a question, but have no place in valid induction (2.737).

In keeping with the parallel treatment which he gives induction and retroduction in most discussions of ampliative inference, Peirce speaks of predesignation as applying in the same way to retroduction, it being the term S of the proposition:

\[ S \text{ has proportion } p \text{ of the marks } P'P''P''' \text{ etc.} \]

which must be predesignated. Since, however, it is the unexpected appearance of S, in retroduction, which initiates reasoning to explain it by the formation of a hypothesis, it is not clear where predesignation is involved. Peirce himself clarifies this matter somewhat in a later passage (2.739), where the rule of predesignation for retroduction is said to be merely a restatement of the rule "that a hypothesis can only be received upon the ground of its having been verified by a successful prediction", with the exception that "it is not at all requisite that the ratio p should be given in advance of the examination of the samples." This may be interpreted as meaning that given S to explain, and having formed a crude hypothesis that S is like M, consequences of M may be predicted and compared with the data involved in the event S, and the degree of their resemblance noted. This
is comparable with Kepler's retrodiction regarding the orbit of Mars, previously discussed.

Peirce interprets the rule that a hypothesis should be simple as meaning that the objects reasoned about, and their relationship to each other, should be familiar to the reasoner, since, in such a case the hypothesis regarding their degree of similarity is based on a knowledge of a great number of their characteristics (2.740). In addition to this aspect of familiarity there is also Peirce's theory, already mentioned, of the tendency of the human mind to make true guesses, and of the mind's affinity to those types of natural laws related to mechanics and to other living beings, so that by following the guide of familiarity and simplicity the mind is actually following its instinctive bent (2.753-4). Another aspect again of the principle of simplicity relates to the testing of hypotheses. In most cases the simpler a hypothesis is, the easier it is to test and discard if found to be false (1.68). This last aspect of simplicity is related to considerations of economy in the acceptance of hypotheses, rather than of their comparative strengths, but is none the less important.

The principle of simplicity is not to be confused with Ockham's principle -- that entities are not to be unnecessarily multiplied -- which Peirce also holds to be valid in the formation of hypotheses (4.35), this latter constituting for Peirce a purely methodological maxim without the metaphysical ramifications of the principle of simplicity.
Peirce's views on the theory of ampliative inference as dependent upon an order of nature have been discussed in Chapter II. He repeats his rejection of this theory in the present paper (2.749), but considers at the same time the effect of our knowledge of special uniformities, upon ampliative inferences. Four such types of special uniformity are listed, which may be so strong as to turn the ampliative inference into a deduction from the known uniformity, but which in most cases merely function to increase or decrease the strength of an induction or retroduction (2.743):

1. members of a class may present a greater or less general resemblance as regards a certain line of characters.

If we find a certain metal to have certain characters of certain types it is a strong induction that all other samples of that metal will show the same characters.

2. a character may have a greater or less tendency to be present or absent throughout the whole of whatever classes of certain kinds.

In biology generic and specific characters show such greater and less tendency to be present or absent throughout a genus.

3. a certain set of characters may be more or less intimately connected, so as to be probably either present or absent together in certain kinds of objects.

Then if two objects show some of these characters in common, a hypothesis concerning the similarity of these objects will be strengthened.

4. an object may have more or less tendency to possess the whole of certain sets of characters when it possesses any of them.
Both 3 and 4 clearly relate to the construction of hypotheses, while 1 and 2 relate to inductions.

This paper of 1883 is the final systematic presentation of the early theory of retroduction, although that theory must have been held by Peirce at least until 1893, when the papers for "The Search for a Method" were edited. The scattered references to retroduction, to be found in subsequent papers, included in the early period, have been considered in the present chapter, but add nothing significant to the early theory.
Chapter IV

The Later Theory -- 1896-1910

The final, and most complete, statement of Peirce's early theory of retroduction was made in the "Theory of Probable Inference", of 1883. No further writings on retroduction, of any length, appear until 1901, but since papers of the early period, written from 1867 to 1883, were edited in 1893 for a projected work, the "Search for a Method", it may be assumed that the early theory was held until that year. The first remarks on retroduction which appear to belong to the later theory occur in a manuscript of 1896, and are followed by similar statements in 1898, but no explicit renunciation or revision of the early theory is to be found prior to the papers and letters written in 1901 on Hume's refutation of miracles. ¹ Important statements appear in 1902, in the manuscript of an uncompleted work, "The Minute Logic" and in the articles on logic contributed by Peirce to Baldwin's Dictionary of Philosophy and Psychology. In 1903 the "Lectures on Pragmatism", given by Peirce under the auspices of Harvard University, restate the later theory with emphasis on the relation between pragmatism and retroduction. Nothing of great importance on the subject appears after 1903 with the exception of Peirce's "Neglected Argument for the Reality of God" which was published in the Hibbert Journal of 1908.

In view of the fact that, unlike the early theory, the later theory is presented for the most part in a large number of short passages in papers written within a few years of each other, it is possible to state it as a systematic whole, rather than in chronological order of development.

In several passages in these papers of the later period Peirce presents a rough outline of the occasion calling forth a hypothesis, the retroduction involved, and the inductive testing of the accepted hypothesis. Such an outline gives the approximate contents of the later theory. All inquiry whatsoever arises from our observation of some event, unexpected and unexplainable in terms of formulated theory, which it is the purpose of that inquiry to explain. Such inquiry Peirce divides into three stages, the first stage being retroduction, by which upon careful scrutiny of the observed facts some explanatory hypothesis is brought to mind. Since, however, such a hypothesis is merely a suggested explanation without guarantee or probability as to its truth, it must be tested by comparison with fact. The second stage of inquiry consists in the deduction from the hypothesis of a wide variety of experiential consequences which would follow from its truth. The third stage, known as induction, ascertains to what degree these predicted consequences actually accord with experience, and on the basis of these findings judges as to the truth or falsity of the hypothesis. One point with regard to Peirce's terminology
should be noted, and that is that although he terms the
second stage of inquiry deduction, and the third stage, in-
duction, on several occasions he speaks of both the second and
the third stage as induction, meaning by this use of the term,
the whole testing procedure following retroduction.

The period of transition from the early theory to
the later theory outlined above, fell as has been stated
between the years 1895 and 1901. During this time remarks
on retroduction in Peirce's writings are scarce, but show
gradual deviation from the position held in the paper of
1883. Pronounced departure from the early theory is most
evident in a manuscript of 1896, in which Peirce speaks of
retroduction as the "provisional adoption of a hypothesis
because every possible consequence of it is capable of expe-
timental verification . . . ." (1.68), and of a hypothesis as
"practically no more than a question" (1.120), of which the
best hypothesis is that "which can be most readily refuted
if it is false" (1.120). The idea of a hypothesis as an in-
telligent guess, with the emphasis in scientific method on
procedures of testing clearly belongs to the later theory, as
opposed to the early theory concept of hypotheses as the
stable conclusions of syllogistic reasoning.

The reasons for this transition are not altogether
clear, and Peirce himself makes only the barest mention of
his change of thought. One such passage occurs in a paper of
1902, wherein he speaks of a "slight positive error" in the
theory of 1883, and of a capital error on the negative side,
in that "the reasoning with which I was there dealing could not be the reasoning by which we are led to adopt a hypothesis" (2.102). A second passage appears in the Peirce-Langley correspondence of 1901, in which Peirce speaks of a new doctrine of the logic of hypothesis, following from the pragmatic principle, which doctrine he has held for the past five or six years and is now ready to publish.¹

This second statement is in keeping with A.W. Burks' suggestion that the transition from the early to the later theory may have followed from Peirce's attempt to bring the various aspects of his thought into a coherent system.² It is Burks' opinion that the theory of retroduction was amended to bring it into conformity with Peirce's pragmatism and tychism; Peirce himself suggests a link between retroduction and the doctrine of categories (2.102) but does not enlarge on the point. Another influence in the change of theory may have been the analogy which Peirce draws, from his earliest papers on, between retroduction and the act of perception. That retroduction should be the source of all new ideas, and that induction should at all times be subsequent to some retroduction are more consistent with the processes of perception and habit formation than are the doctrines of the early theory.

The changed relationship between retroduction and induction is probably the most obvious difference introduced by the later theory. In the early theory synthetic inference was bifurcated into retroduction, the reasoning from

characters of one kind to characters of another, and induction, the generalizing from facts to other facts of similar kind. In the later theory induction is a process of testing predictions from hypotheses against experienced facts; all synthetic inference falls under retroduction. "Presumption [an alternative term for retroduction] is the only kind of reasoning which supplies new ideas, the only kind which is, in this sense synthetic" (2.777). What was formerly called induction, Peirce speaks of as being "a mixture of deduction and presumption" (2.775). To some extent this change is merely one of terminology, but not only does it solve the tendency, in the early theory, for retroduction and induction to overlap in spite of the careful distinction drawn between them, but it also accords with the new emphasis, in the later theory, on the testing procedure.

Retroduction, as described in the later theory, is a process of adding to the observed facts a statement, termed a hypothesis, which explains those facts in terms of other facts and makes "them applicable . . . to other circumstances than those under which they were observed" (6.524). "Any supposed truth from which would result such facts as have been observed" (6.525) constitutes for Peirce a hypothesis explanatory of those facts. The term 'result' is used in its logical rather than material sense. That is, the relationship referred to is not specifically one of cause and effect but such that the facts observed may be seen to be particular cases of a wider class of facts, of a nature either the same
as or different from those observed. In another passage Peirce speaks of the observed facts as constituting a 'likeness' of those represented in the hypothesis, and again gives Kepler's hypothesis as to the orbit of Mars as an example, in that the observed positions of Mars constituted a likeness of those of a body moving in the orbit described in the hypothesis (2.96). This is in keeping with Peirce's concept of explanation, referred to in Chapter III, as consisting in the demonstration that the process or event to be explained is a special case of a more general and better known class of processes.

Emphasis in the early theory was thrown very heavily on the process of reasoning underlying the formation of hypotheses and inductions, to the comparative neglect of methods of testing hypotheses. In opposition to what Peirce calls the positivist position, hypotheses were regarded as in no way uncertain and "something to be superseded" (2.707), but as conclusions comparable in strength to the conclusions of induction. In the later theory, on the other hand, where testing rather than formation of hypotheses becomes the crucial process in the logic of discovery, hypotheses are no longer regarded as stable or reliable. They are accepted merely as possible explanations of the facts noted, as "practically no more than a question" (1.121). They are guesses or conjectures suggested by the premisses and "accepted as having some chance of being true" (2.786) and as having a form such as to suggest experiments which will test
the degree of their truth. The justification for the entertainment of such uncertain conjectures as having any possibility of truth at all is that "if we are ever to learn anything or to understand phenomena at all, it must be by abduction [an alternative term for retroduction] that this is to be brought about" (5.171). Only by retroduction can ideas be originated. In keeping with the extremely provisional status of hypotheses, one of the important criteria of their acceptance is that they be easily and speedily testable; "the best hypothesis . . . is the one which can be the most readily refuted if it is false" (1.120).

A third break which Peirce makes with the early theory in these later writings concerns the probability of hypotheses. In the early theory conclusions from induction and retroduction were spoken of as being probable, not in the sense of the frequency theory of probability, but in the sense of being arrived at by a method, which, if continued in long enough, must lead eventually to a true statement of the facts. Although Peirce does not declare as much, his use of the term probability for these two widely different concepts suggests that he may have considered the two forms of probability as ultimately reducible to a single form. In the later theory Peirce distinguishes these two concepts most clearly and divorces all idea of probability from retroductive conclusions. In doing so he distinguishes between four concepts frequently confused: probability, validity, likelihood, and plausibility. Probability Peirce now limits to
'objective probability' which is "the ratio of frequency of a specific to a generic event in the ordinary course of experience" (2.777). At no time, in either early or later writings, does Peirce claim this form of probability for pure hypotheses, although insofar as hypotheses are supported by deductions from known uniformities, as discussed in Chapter III, he seems in the later theory to class them as in this sense probable (cf. 6.527, 6.534). His remarks on this point are extremely brief, but clear as far as they go. "There are facts which . . . necessitate the truth, or the falsity, or the probability in some definite degree, of the hypothesis" (6.527). It would appear, however, that only a hypothesis whose character can be arrived at deductively may be spoken of as probable -- such a hypothesis, for instance, as that on a particular throw of a die a particular face would show, in which case the probability would be one sixth. This, however, is scarcely a hypothesis in Peirce's general use of that term. Apart from this special case, hypotheses, however strongly supported by known facts, can hardly be spoken of as being probable in the accepted sense of that word.

Validity, as Peirce uses it, is that characteristic of any inference by which it has "that sort of efficiency in leading to the truth, which it professes to have" (2.779). The characteristics of validity differ for each type of inference, the validity of each instance of inference depending upon its having the characteristics of its type. That charac-
teristic of the inductive method, formerly spoken of as its probability, by which if persisted in it must eventually lead to truth, is now spoken of as its validity (2.781). Similarly, a retroduction is valid if its conclusion be such "that its consequences are capable of being tested by experimentation" and "that the observed facts would follow from it as necessary conclusions" (2.781). These are the only criteria of a valid hypothesis; such a hypothesis, however, may be hopelessly weak since strength and validity are unrelated concepts.

Likelihood, as defined by Peirce, is a characteristic of theories unproved, as are in varying degrees all scientific theories, but "supported by such evidence, that if the rest of the conceivably possible evidence should turn out upon examination to be of a similar character, the theory would be conclusively proved" (2.663). Likelihood, although Peirce does not specifically relate it to retroduction, would presumably apply to hypotheses, both insofar as they have been partly proved and, it would seem, insofar as they are supported by known facts.

Plausibility, however, of all four concepts, is the one most particularly related to hypotheses. Peirce defines it as that characteristic of an untested theory which if true would so explain the given facts "as to recommend it for further examination" (2.662). Unfortunately this statement hardly describes what it is in the theory or hypothesis, apart from its affording an explanation, that recommends it for further examination. It cannot be anything of the nature of
objective facts found to support the hypothesis, since these are accounted for under probability or likelihood, but appears to be what he calls elsewhere "subjective probabilities, or likelihoods, which express nothing but the conformity of a new suggestion to our prepossessions" (2.777). It may be that plausibility is intended to cover not only the principle of the consistency of new hypotheses to accepted theories, but the principles of simplicity and familiarity as well, both of which are aspects of our prepossessions and principles of some weight in Peirce's system.

Like plausibility, the concept of the strength of hypotheses is never clearly set forth, and Peirce himself admits it to be vague (2.780). Strength appears, however, to rest on a combination of plausibility and likelihood; that is, upon the conformity of the hypothesis 'to our prepossessions' and upon the objective evidence supporting the hypothesis.

In the matter of the formation of hypotheses it has already been suggested that Peirce in the later theory discarded the systematic derivation, from the deductive syllogism, of the form of retroduction, and came to look upon the hypothesis as suggested by more or less unconscious interaction between objective facts and the mind of the observer. He distinguishes between argument and argumentation, the first being "any process of thought reasonably tending to produce a definite belief" and the second, "an Argument proceeding upon definitely formulated premisses" (6.458), and classes
retroduction as argument rather than argumentation (6.469). The process of thought in a retroduction consists in the colligation, or gathering together, of all available facts related to the subject under consideration, followed by an act of observation which in this case consists in "the deliberate yielding of ourselves" to the force of the facts, from which the hypothesis results (5.581). More explicitly, Peirce describes this process of observation as one in which, upon examining the features of an unexpected occurrence, we notice "some remarkable character or relation among them" which we at once recognize "as being characteristic of some conception" already familiar to us (2.776). This process of reasoning is not unlike that of the first statements of the early theory in which:

\[
\begin{align*}
M \text{ is } & P'P''P''' \text{ etc.} \\
S \text{ is } & P'P''P''' \text{ etc.} \\
\therefore S \text{ is like } M
\end{align*}
\]

the main difference being that Peirce now considers "syllogistic forms and the doctrine of logical extension and comprehension "as of secondary importance in the retroductive process (2.102). The gist of his meaning seems to be that while every retroduction fits, by reason of the conformity of mental activity to syllogistic patterns, into the patterns so carefully described in the early theory, the actual formation of hypotheses is not made possible or aided by the knowledge of these patterns. Retroduction, in short, is an argument, but not an argumentation.

If the syllogistic form emphasized in the early
theory is only of secondary importance, and if, as Peirce further claims, retroduction consists in mere suggestion of conjectures for which no reason can be given (5.171), can it be classed as a type of inference? R. B. Braithwaite considers that it cannot and regards Peirce's classification as verbal, on the grounds that a theory that comes to mind as a flash of insight can hardly be claimed to be the product of a reasoning process deserving the name of inference. Peirce himself appears to support this judgment when he says that "if one does not at all know how one's belief comes about, it cannot be called even by the name of inference" (6.497). Inference, for Peirce, involves by definition an element of control; for there to be control it is necessary that the reasoner be aware that he is making an inference, although he may be unaware of the actual mental processes involved. Inference proper is thus distinguished from those mental processes which Peirce, with customary perversity, calls 'unconscious inference', such inference being unconscious, not because the subject is ignorant of the processes involved, but because he is unaware that any inference has been made.

Inference is defined as "a process in which the reasoner is conscious that a judgment, the conclusion, is determined by other judgment or judgments, the premisses,"


according to a general habit of thought, which he may not be able precisely to formulate, but which he approves as conducing to true knowledge" (2.773).

If these statements on the nature of inference seem at first glance to support Braithwaite's contention that retroduction cannot be classed as inference, more careful consideration will show that they provide adequate support for Peirce's own classification. Peirce himself applies the above distinction, between inference and 'unconscious inference', to retroduction and the non-inferential process, analogous to retroduction, involved in the formation of percepts. The similarity which Peirce claims to exist between the processes of formation of hypotheses and percepts has already been outlined; the difference between them lies in the fact that percepts "are absolutely beyond criticism" (5.182), and are 'unconscious inferences'. We may, by psychological analysis, discover percepts to be the product of mental processes of the same form as retroduction, but they are differentiated from hypotheses by the fact that while the hypothesis is "something whose truth can be questioned or even denied", we cannot form the least conception of what it would be to deny the perceptual judgment" (5.186). Possibly, without distorting Peirce's meaning, it can be added that we are able to question or to deny the hypothesis because we are aware that it is the product of a mental process, (or else, that we are aware that it is the product of a mental process because we can question or deny it.) For a conclusion to be classed as an inference
we must be able to "go back and criticize the premisses and the principles that guide the drawing of the conclusion" (6.497). For such a conclusion to be admissible as a hypothesis, criticism of the reasoning involved must show "that it would account for the facts or some of them" (5.189), the form of such inference being:

The surprising fact C is observed  
But if A were true, C would be a matter of course  
Hence, there is reason to suspect that A is true (5.189).

Since this form of reasoning is invalid as deductive inference the premisses do not compel the conclusion but advance it as a suggestion and, in the face of criticism, support it as plausible. Since, however, plausibility constitutes a recommendation that the hypothesis be tested, the conclusion "should be such that definite consequences can be plenitfully deduced from it of a kind which can be checked by observation" (2.786).

The first principle of the acceptance of hypotheses -- that they account for the given facts -- is essential; on the second principle, that they suggest experiments by which they may be tested, Peirce seems to be less sure. Although in 1902 he speaks of testability as the 'principal rule' in the acceptance of hypotheses (2.786), he makes a distinction in a paper of 1898 between hypotheses upon which verifiable predictions can be based, and those which while unverifiable are held "as a mere convenient vehicle of thought -- a mere matter of form" (5.599).

Again, in 1908, he advocates acceptance of the hypothesis of God's reality, "whose ultimate test must be in its value in the self-controlled growth of man's conduct of life" (6.480), and yet, in the same paper, speaks of the necessity of testing hypotheses by prediction and verification (6.470 ff.). It may be that Peirce is drawing a distinction between those hypotheses which are to be added, if verified, to the body of existing theory, and those which are to be relied upon in the conduct of life. Judging from the 1898 statement the latter have their function even in the conduct of scientific thought, but must, like catalysts, be cleared away before the theoretic product is fully revealed. Such a distinction between theory and practice is one that becomes more and more evident in Peirce's writings as he grows older.

Peirce states the principles governing the acceptance of hypotheses for testing in another way in the "Lectures on Pragmatism" of 1903, where he claims not only that the pragmatic maxim is the only rule necessary for the admissibility of hypotheses, but that this is all that the pragmatic maxim actually is. The maxim as Peirce gives it in this particular passage is, in fact, only one of many statements which he derives from the principle of pragmatism and is as follows:

a conception can have no logical effect or impact differing from that of a second conception except insofar as, taken in connection with other conceptions and intentions, it might conceivably modify our practical conduct differently from that second conception (5.196).
If the pragmatic principle may be distinguished from the maxims derived from it, it would seem best summarized in the statement "that the possible practical consequences of a concept constitute the sum total of the concept" (5.25), in which form it is rather less cumbersome than in the form given above, and apparently more relevant to the matter in hand. The two functions of pragmatism, or "whatever the true doctrine of the Logic of Abduction may be", are first, "to give us an expeditious riddance of all ideas essentially unclear" and second, "to lend support [to] and help to render distinct, ideas essentially clear, but more or less difficult of apprehension . . . . (5.206). From these quotations it may be seen that Peirce's pragmatism is a doctrine of meaning, rather than of truth. Since "pragmatism is the doctrine that every conception is a conception of conceivable practical effects" (5.196), the pragmatic maxim legislates on the acceptability of hypotheses in terms of whether or not their conceivable practical effects account for the phenomena under consideration. On no other grounds than the adequacy of their conceivable effects to explain the phenomena, are hypotheses to be accepted for testing, or rejected as inadmissible. That this is the whole function of the pragmatic principle follows from the fact that both induction and deduction are formal processes carried on independently of their content in any particular instance. Pragmatism affects their content, in that all content is originally derived from retro-
duction but with the formal process of induction and deduction it has nothing to do.

In the event of there being a number of plausible hypotheses to explain the phenomenon under investigation, the basis upon which hypotheses are accorded priority for testing is one of economy — "Economy of money, time, thought and energy" (5.599). Peirce at one point goes so far as to call this the 'leading consideration' in retroduction (5.599).

Without attempting to supply an exhaustive or systematic set of rules, Peirce suggests a number of precepts as aids in the economical selection of hypotheses for testing.

1. If there is a large number of acceptable hypotheses, then the most economical procedure is to find, if possible, some observable result which would follow from one half of the hypotheses and not from the other half, and to test for this result. By repetition of this method a large number of hypotheses can very quickly be reduced to that small remainder which can no longer be halved and must be tested one by one (6.529).

2. Among any set of possible hypotheses, those which appear simplest to the human mind should be tested first. Two reasons exist for this rule:
   a. The simpler a hypothesis is, the easier in most cases is it to deduce its consequences and test for them. Such a hypothesis, if false, can be readily eliminated (6.533).
   b. Since it is Peirce's belief that the possibility of any scientific truth whatsoever depends on a human faculty of
guessing correctly, and since such a faculty is based on the existence of a harmony between the human mind and nature, it is his contention that the simplicity of hypotheses will in some cases be an indication of this harmony, and hence a suggestion of their possible truth (6.530-2, cf. 6.477).

3. Any hypothesis which for any reason may be more easily and more quickly tested than the other plausible hypotheses should for this reason be considered first (6.533).

4. Any hypothesis strongly supported by an objective fact, of the nature of a deduction from known principles, should be given an early trial (6.534).

5. Any hypothesis "which suggests an experiment whose different possible results appear to be, as nearly as possible, equally likely" (2.786), should always be preferred. This precept does not appear to be in harmony with that immediately preceding it, and the reason for it is not clear. Peirce does not offer any explanation, but it may be that he considers such a hypothesis more crucial than any other.

6. Ockham's razor is offered as a sound economic principle for the selection of hypotheses -- Peirce calls it "the very roadbed of science" -- but it is accorded nothing more than methodological status and does not give the simpler hypothesis any objective probability (4.1, 6.535).

7. No hypothesis should be abandoned too readily, but should be maintained until clearly unacceptable.¹

The testing of any hypothesis involves two stages

of inquiry: first, the deduction from the hypothesis of a large number of facts which would be observable under specified conditions if the hypothesis were true, and second, the inductive process of discovering whether or not such facts actually do occur as stated. The Second Stage of Inquiry, the deductive, consists in an explication of the hypothesis by logical analysis, such that it is rendered as distinct as possible, followed by a demonstration or deduction of consequences from the explicated hypothesis (6.471). The Third Stage of Inquiry, the inductive, ascertains "how far those consequents accord with Experience" (6.472) and on this basis pronounces on the truth of the hypothesis. Induction involves three sub-stages, the classificatory, "by which general Ideas are attached to objects of Experience" (6.472), the Probational, the actual testing by experiment, and the Sentential, in which the results of the probations are weighed, and judgment passed on the hypothesis.

Probation is of two forms, Crude Induction and Gradual Induction, of which the latter is again divisible into qualitative and quantitative. Crude Induction "is the only Induction which concludes a Logically Universal Proposition" (6.473); it involves an assumption that since all events of a certain class have had a certain characteristic, all future events of that class, will show the same characteristic (2.756), that since all men up to the present have been mortal, 'all men are mortal.' Such an induction is extremely weak, and is subject to refutation at any moment.
Gradual induction "makes a new estimate of the proportion of truth in the hypothesis with every new instance" (6.473), thereby gradually correcting any error in its just evaluation of the hypothesis (5.145). Quantitative gradual induction investigates the deduction from a hypothesis, that a class of objects will have a certain character, by sampling that class and noting the occurrence of the character predesignated, the assumption being that the sample is representative of the whole class (2.758). Qualitative induction, or abductory induction as Peirce sometimes calls it, consists in deducing a wide variety of characters from the hypothesis, such that the discovery of the existence of each of these characters will add weight to the hypothesis, and the discovery of all will constitute proof (2.759). Peirce gives as an example the hypothesis that a certain man is a Catholic priest, from which it would follow that upon investigation he would be found to possess certain characteristics. If the hypothesis is, in fact, correct, then examination will reveal one by one, each of these characteristics (6.526). Qualitative induction is weaker than quantitative induction, since dependant upon our estimation of the weight of the evidence gathered in support of the hypothesis, but of greater utility (2.759).

In the division of gradual induction into qualitative and quantitative induction Peirce appears to introduce equivocal meanings of the term Induction. Induction, in the later theory, is on several occasions used to describe the
whole process of testing hypotheses, which meaning may be seen to coincide with that given for qualitative induction, in spite of Peirce's classification of it as a form of Probation. As has been stated, qualitative induction involves the deduction of a number of consequences from a hypothesis, and the investigation of those consequences. This, in brief, is the general procedure of testing hypotheses (cf. 6.526). Quantitative induction, on the other hand, is a procedure by which the truth of any particular deduction from a hypothesis is tested, and does occur in the probational stage of qualitative induction. It is, in fact, the process of reasoning called induction in the early theory, but described in the later theory as "a mixture of deduction and presumption" (2.776).

It is only in terms of this classification, and by using the term induction to denote the whole testing process, that Buchler's claim with regard to the later theory, that "the conclusion of an induction ... is always some hypothesis previously abduced",¹ can be justified. This statement does not apply to 'quantitative inductions' which conclude only single deductions from hypotheses.

The details of the later theory have been compared point by point with those of the early theory, in the course of this chapter. It remains to compare the two theories in general, both as to the extent to which they actually differ, and as to the comparative merits of the two theories.

¹. Charles Peirce's Empiricism, p. 135.
Comparison of Peirce's theories of retroduction with those advanced by other philosophers is beyond the scope of this paper.

In spite of the pronounced differences to be noted between the early and later theory, it has been suggested throughout the present chapter that many of the changes introduced in Peirce's later writings go no deeper than matters of terminology and emphasis. The terms retroduction and induction are redefined in the later theory, and as a result come to bear a relationship to each other quite different from that holding in the early theory, but in spite of this change the actual sequence of processes remains much the same. Synthetic inference, bifurcated in the early theory into retroduction and induction, becomes in the later theory wholly the province of retroduction, with the inductive inference of the early theory analysed into a combination of deduction and retroduction. The term induction is carried over into the later theory to comprise the process of testing the conclusions of retroduction, a process briefly described but left unnamed in the early theory. The syllogistic form of retroduction, taken in the early theory as the actual process of reasoning involved in the formation of hypotheses, and for that reason developed and described in great detail, is recognized in the later theory as being merely a description of the reasoning, in terms of evidence, known theory and resultant hypothesis, and is consequently relegated to a position of secondary importance. Hypotheses are no longer, in the later
theory, spoken of as probable, but the change is in fact wholly one of terminology in order to avoid confusion with the frequency theory of probability. Finally, the function of retroduction, as the explanation of startling phenomena in terms of known laws remains the same throughout both theories.

This is not, however, to say that there are not differences between the two theories, nor that the change of emphasis does not in itself constitute a very real change. It is felt that in these differences the later theory represents a pronounced improvement over the early theory, in that it is thereby enabled to present a clearer and more balanced description of the logic of discovery. By his reallocation of the terms retroduction and induction, Peirce has been able to describe the sequence of reasoning from the appearance of the unexpected event which institutes inquiry, to the acceptance of a verified explanatory hypothesis, in such a manner that retroduction and induction, probation and deduction appear in inter-relation as stages in scientific procedure, rather than as independent and unrelated types of inference. The change of emphasis in the later theory, together with the abandonment of the syllogistic form as an important factor in retroduction, makes it possible to avoid the disproportionate picture given in the early theory of the relative importance of retroduction and the techniques of testing, and to consider many of the problems of verification ignored by the early theory.
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