YIELD AND VOLUME TABLES FOR ASPEN IN CENTRAL AND NORTHERN ALBERTA

by

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ABSTRACT

Even-aged well stocked aspen stands in Northern Alberta were sampled in order to derive "normal" yields per acre for number and size of trees, basal area, and various measures of volume. The stands occur chiefly on three qualities of site which have been classified into Fair, Medium and Good by the average height of dominant trees at 80 years. Exceptionally high mortality per decade is characteristic in fully stocked young aspen stands and the relation of number of trees per acre to average diameter is markedly different from the trends found by investigators for other species. A table of stand density units was constructed to permit the rapid calculation of stand-density index when average diameter and number of trees per acre have been determined.

The mean annual cubic volume growth on medium sites is maximum at age 40. For merchantable cubic and board feet, growth reaches a peak at 85 and 130 years respectively. Data from two other regions show that the amount of cull is high in aspen trees above 80 years of age, this indicates that the wood should be utilized before a stand-age of 130 years is reached if maximum return in board foot volume is desired. By 140 years aspen stands show signs of breaking up.

The hardwood stands measured were composed mainly of aspen but both white birch and black poplar occurred on many of the plots. The growth of the black poplar is only slightly less than that for the aspen. It has consistently lower volumes per tree which permits the use of aspen volume tables when correction factors are applied.

ACKNOWLEDGMENT

The author would like to express his appreciation to the Department of Resources and Development, Forestry Branch, Ottawa, for permission to use the data employed in the development of the tables presented.

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INTRODUCTION

Aspen[1] (Populus tremuloides Michx.) is the most abundant tree of the foothill and plains region of Alberta, but at present it is of general value only as a cover crop. Locally, it is important for fuelwood, fenceposts, wagon stock, rough lumber, and logs for log cabin construction. This situation exists for many reasons, the most significant ones being, (1) the range of aspen is so extensive that outside markets have adequate supplies, (2) the more valuable coniferous woods on this continent are still relatively plentiful, (3) the prevalence of heart rot in aspen makes it unsuitable to an industry which is geared to the production of lumber only.

The future possibilities for this species are therefore subject to speculation, but the ever increasing value of wood and its by-products strongly indicates that it is only a question of time before aspen will be important in the provincial forest economy.

It was in anticipation of an increased interest in and utilization of aspen that this study was undertaken.

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[1]Other names in use are: white poplar, poplar, popple, asp, aspen poplar, quaking aspen, and smooth-barked poplar.

THE POPLAR FOREST SAMPLED

The area for which the present yield tables were constructed is contained within the limits of the B 18 region described by Halliday (5). The bulk of the sample plots measured, however, are concentrated in the vicinity of the Lesser Slave Lake sub-drainage where poplar attains optimum development. Additional data were collected west of Edmonton in locations shown in red in the sketch map, Figure 1.

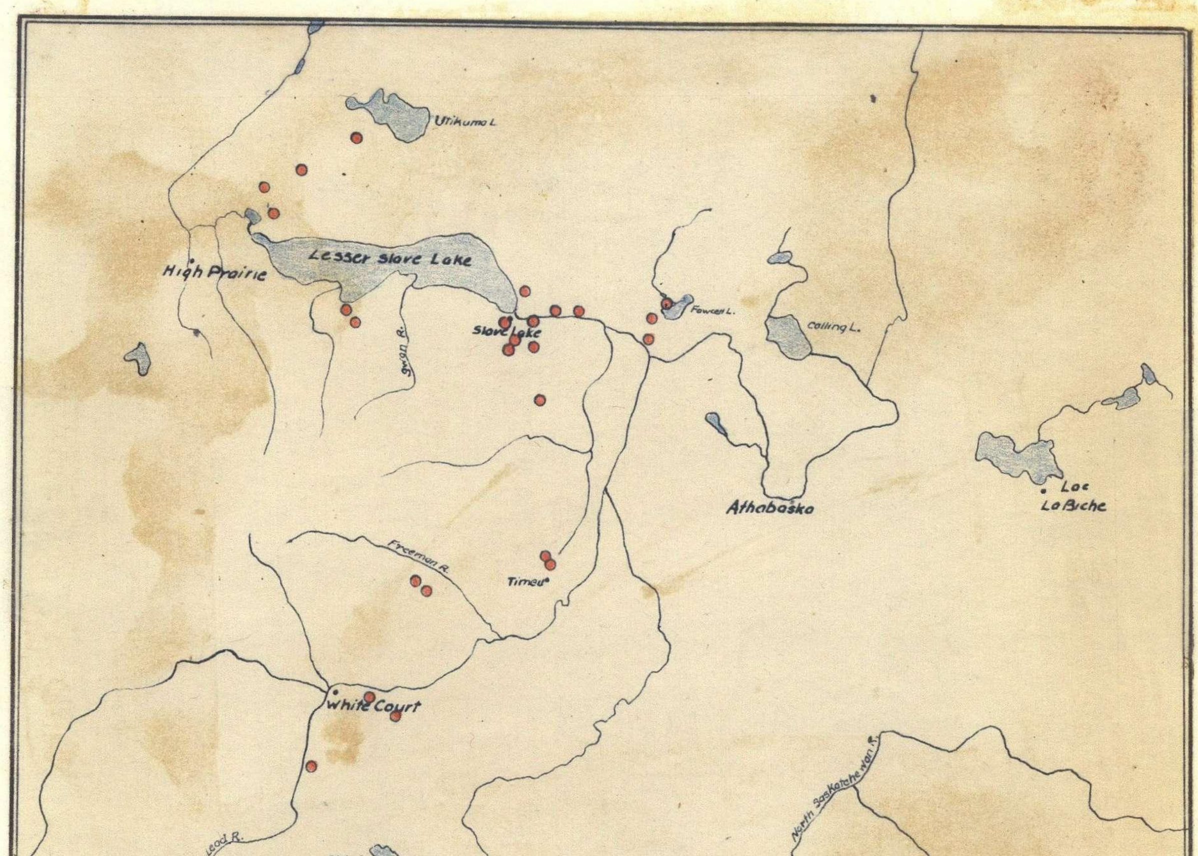

FIGURE 1 - Map showing location of sample plots; each dot marks a locality where one or more plots were taken.

Only a few of the rivers and streams which wind through the entire region are shown and, although a gently rolling countryside is a feature of the topography, the larger streams, centres of rather deep U-shaped valleys, provide abrupt relief to an otherwise regular landscape.

The forest cover in general is of a patchy nature due in part to the destruction caused by past fires and to the heterogeneous mixtures of aspen with white spruce (*Picea glauca* (Moench.) Voss), balsam fir (*Abies balsamea* (L.) Mills), lodgepole pine (*Pinus contorta* Dougl.) and jack pine (*Pinus Banksiana* Lamb.). The many swamps of black spruce (*Picea mariana* (Mill.) B.S.P.) but add to this apparent patchiness. Of the hardwoods black poplar[1] (*Populus balsamifera* L.) and white birch (*Betula papyrifera* Marsh.) are seldom entirely absent. The former is the more common particularly in moist habitats, existing in pure stands on the alluvial soils of the river flats, or as large individuals along creek margins. The black poplar competes successfully with aspen as a general rule but the degree of its success depends upon soil and moisture characteristics. White birch may be found growing in almost pure stands of high marketable value in a few localities but such occurrences are not very common. It most frequently exists in the mixtures as a twisted spindly tree of relatively little commercial importance. The average percentage composition and frequency of occurrence of these deciduous species included in the plots measured are shown in Table 1. These figures should not be taken as representative of the entire region but of the population sampled.

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[1] Other names in use are: balsam poplar, balm, balm of Gilead, poplar, tacamahac, and rough-barked poplar.

TABLE 1

STAND COMPOSITION AND FREQUENCY OF OCCURRENCE OF SPECIES ON SAMPLE PLOTS

Common name	Scientific name	Composition by basal area	Frequency of occurrence
		%	%
Aspen	Populus tremuloides Michx.	93.92	100.0
Black poplar	Populus balsamifera L.	5.16	71.8
White birch	Betula papyrifera Marsh.	0.92	28.2

ASPEN YIELD TABLES

The yield tables presented are derived from stands estimated to be utilizing fully the growing capacity of the land. In such stands the aspen crowns form a fairly complete canopy except in the case of those under 30 years of age. Here the crowns become narrower and an apparent overlapping is characteristic. Yields obtained from such fully stocked stands are commonly referred to as "normal" yields. There are a number of obvious objections to both the basis of sampling and the application of the resulting yield tables but it will be generally agreed that normal yield tables have been and are useful during early phases of forestry development.

Tables 2 to 6 inclusive, show total height of the average dominants, number of trees, average size of tree, basal area, and volume of wood per acre for different ages and qualities of site. They are the result of a project begun and completed during the summer of 1950. In this study yields of fully stocked stands containing even-aged aspen between 10 and 140 years were measured on 89 small temporary sample areas

TABLE 2

TOTAL HEIGHT OF AVERAGE DOMINANT ASPEN

Total age	Total ht. by site index[1]					Total age	Total ht. by site index				
	50	60	70	80	90		50	60	70	80	90
years	ft.	ft.	ft.	ft.	ft.	years	ft.	ft.	ft.	ft.	ft.
10	10	14	17	20	23	70	47	56	66	75	85
20	20	26	31	36	42	80	50	60	70	80	90
30	28	35	41	48	55	90	52	63	73	84	94
40	34	42	49	57	65	100	54	65	76	86	97
50	39	47	56	64	73	110	56	67	78	89	90
60	43	52	61	70	79	120	58	69	80	91	102

1 total height of average dominant aspen of 80 years

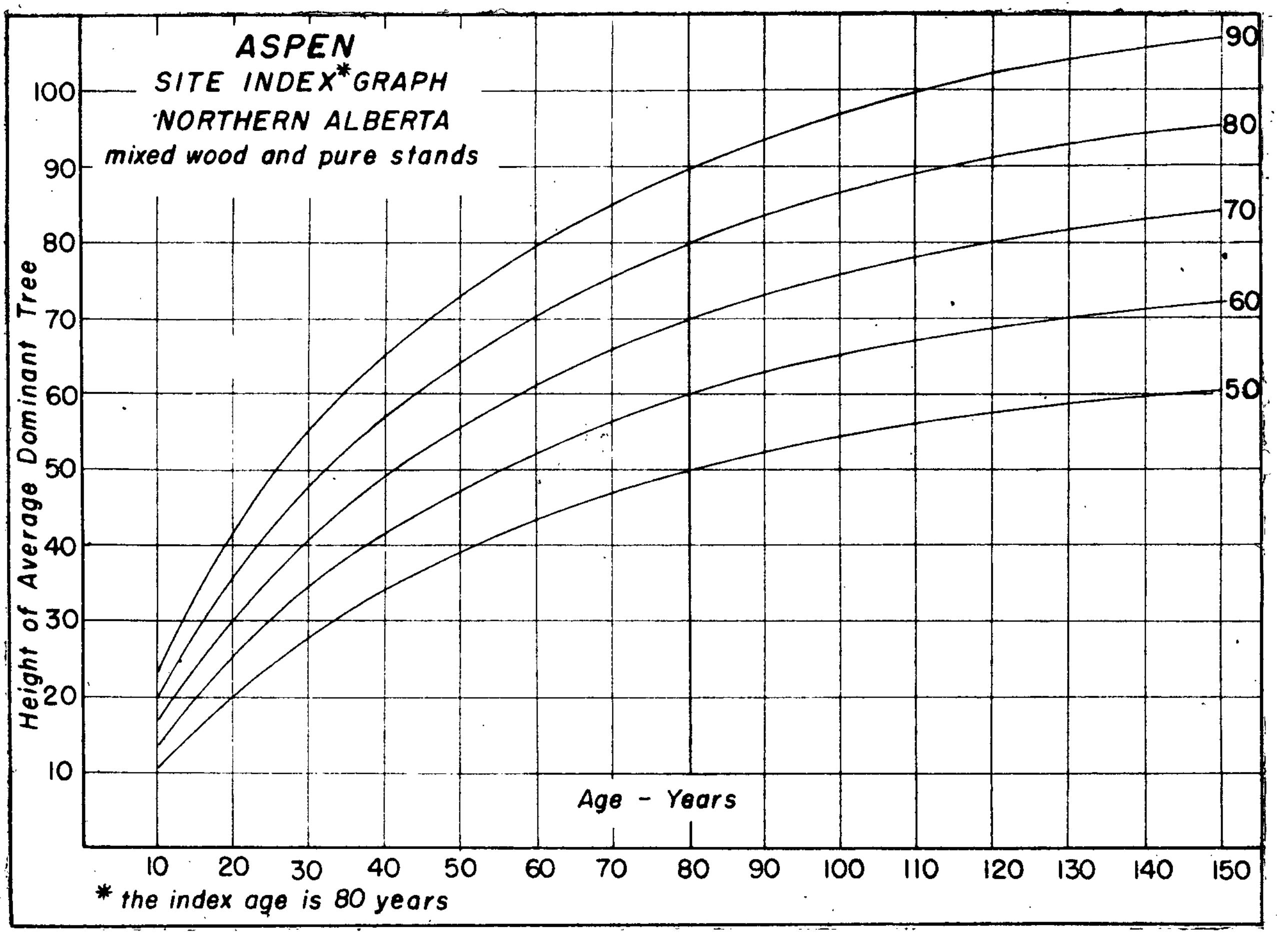

FIGURE 2 - Height curves used for site classification.

TABLE 3

TOTAL NUMBER OF TREES PER ACRE
0.6-INCH D.B.H. AND LARGER

Total age	Trees per acre by site index:				
	50	60	70	80	90
(years)	Number				
10	10,000 +	8500	7700	6700	6050
20	6900	5800	5000	4400	3750
30	4700	3900	3300	2800	2500
40	3250	2650	2240	1900	1680
50	2200	1840	1550	1350	1180
60	1550	1300	1100	960	850
70	1100	930	800	705	630
80	790	690	600	525	470
90	610	535	465	410	370
100	495	435	380	330	295
110	420	365	320	280	245
120	370	315	276	240	210

TABLE 4

DIAMETER OF THE AVERAGE TREE BY AGE
CLASS AND SITE INDEX

Total age	Diameter at breast height by site index:				
	50	60	70	80	90
(years)	Inches				
10	0.6	0.7	0.8	1.0	1.3
20	1.3	1.5	1.7	2.0	2.3
30	1.9	2.2	2.5	2.9	3.2
40	2.5	2.9	3.3	3.8	4.2
50	3.2	3.6	4.1	4.6	5.1
60	3.8	4.4	4.9	5.5	6.1
70	4.6	5.2	5.9	6.5	7.1
80	5.4	6.1	6.8	7.6	8.3
90	6.2	6.9	7.8	8.6	9.4
100	6.9	7.7	8.6	9.6	10.5
110	7.5	8.4	9.4	10.4	11.5
120	8.0	9.0	10.1	11.2	12.4

TABLE 5

TOTAL BASAL AREA PER ACRE INCLUDING ALL TREES 0.6-INCH D.B.H., AND LARGER

Total age	Basal area per acre by site index:				
	50	60	70	80	90
(years)	Square feet				
10	14	26	38	50	61
20	66	78	91	103	114
30	94	106	119	130	141
40	111	122	136	147	158
50	118	130	143	155	168
60	122	134	148	160	172
70	124	137	150	162	176
80	126	139	152	164	177
90	128	140	154	165	178
100	128	140	154	166	178
110	128	140	154	166	178
120	128	140	154	166	178

TABLE 6

YIELD PER ACRE IN CUBIC FEET, EXCLUDING BARK, FOR ALL TREES 0.6-INCH D.B.H., AND LARGER

Total age	Yield per acre by site index:				
	50	60	70	80	90
(years)	Cubic feet				
10	80	180	280	420	570
20	620	855	1120	1415	1715
30	1145	1480	1890	2315	2775
40	1605	2030	2575	3145	3775
50	1970	2480	3115	3800	4555
60	2270	2850	3560	4345	5155
70	2525	3180	3925	4790	5685
80	2735	3440	4240	5160	6115
90	2910	3650	4510	5460	6475
100	3050	3805	4730	5700	6760
110	3165	3920	4900	5900	6980
120	3255	4030	5035	6080	7170

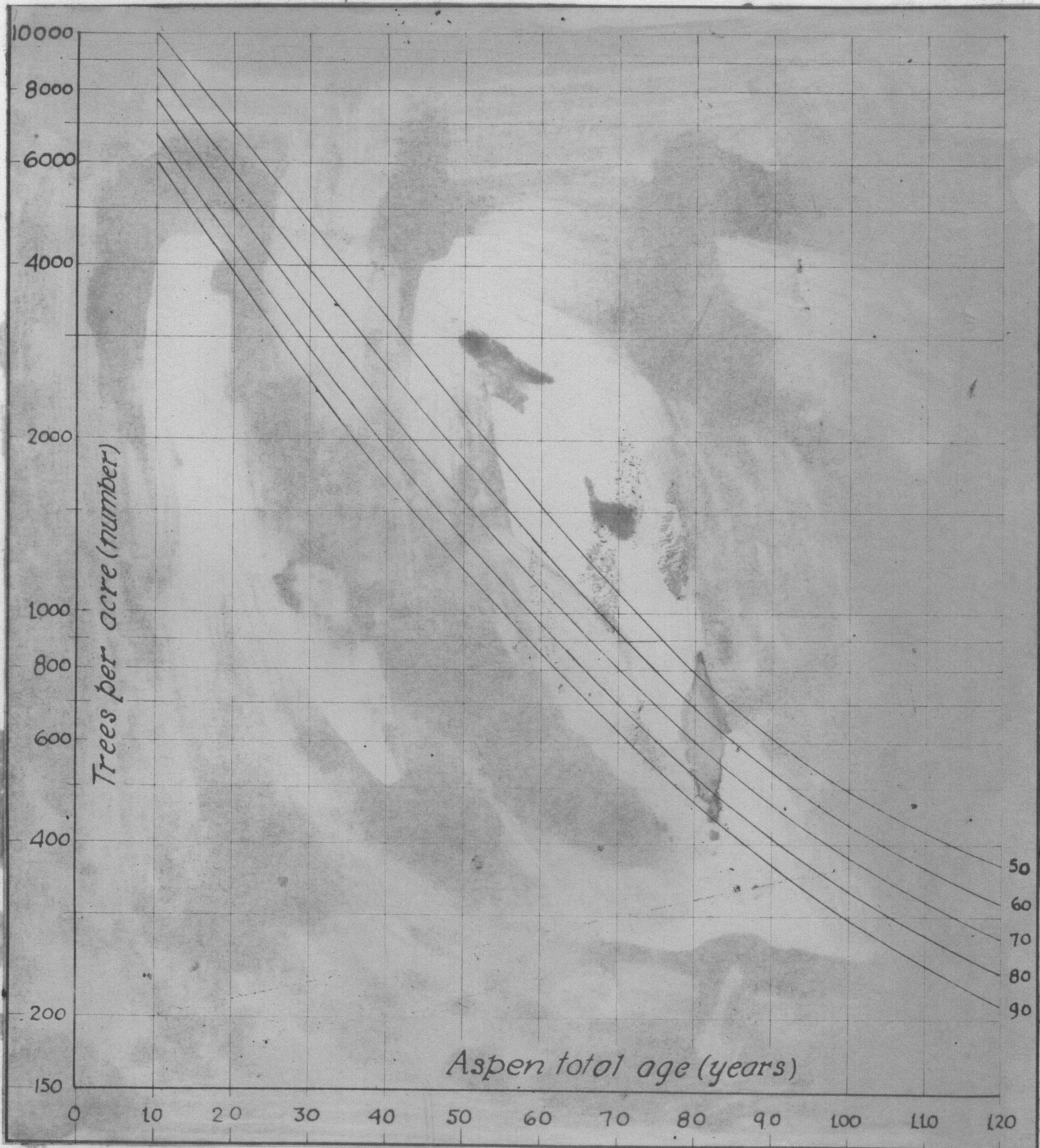

FIGURE 3 - Number of trees per acre showing trends with age and site index.

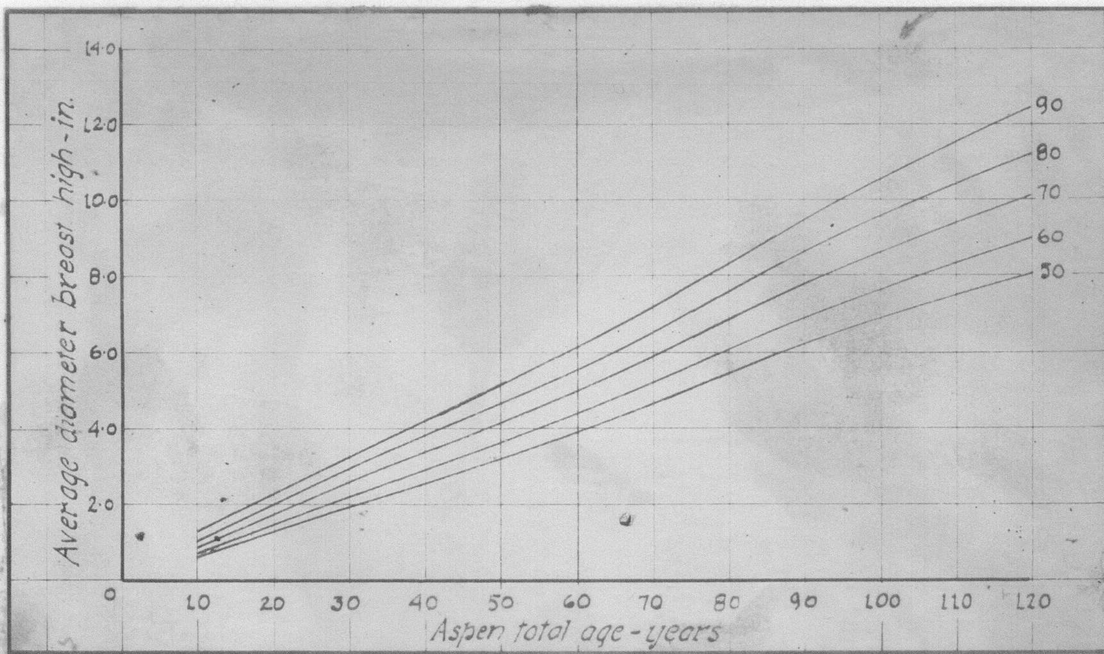

FIGURE 4 - Diameter of tree of average basal area at breast height, showing trends with age and site index.

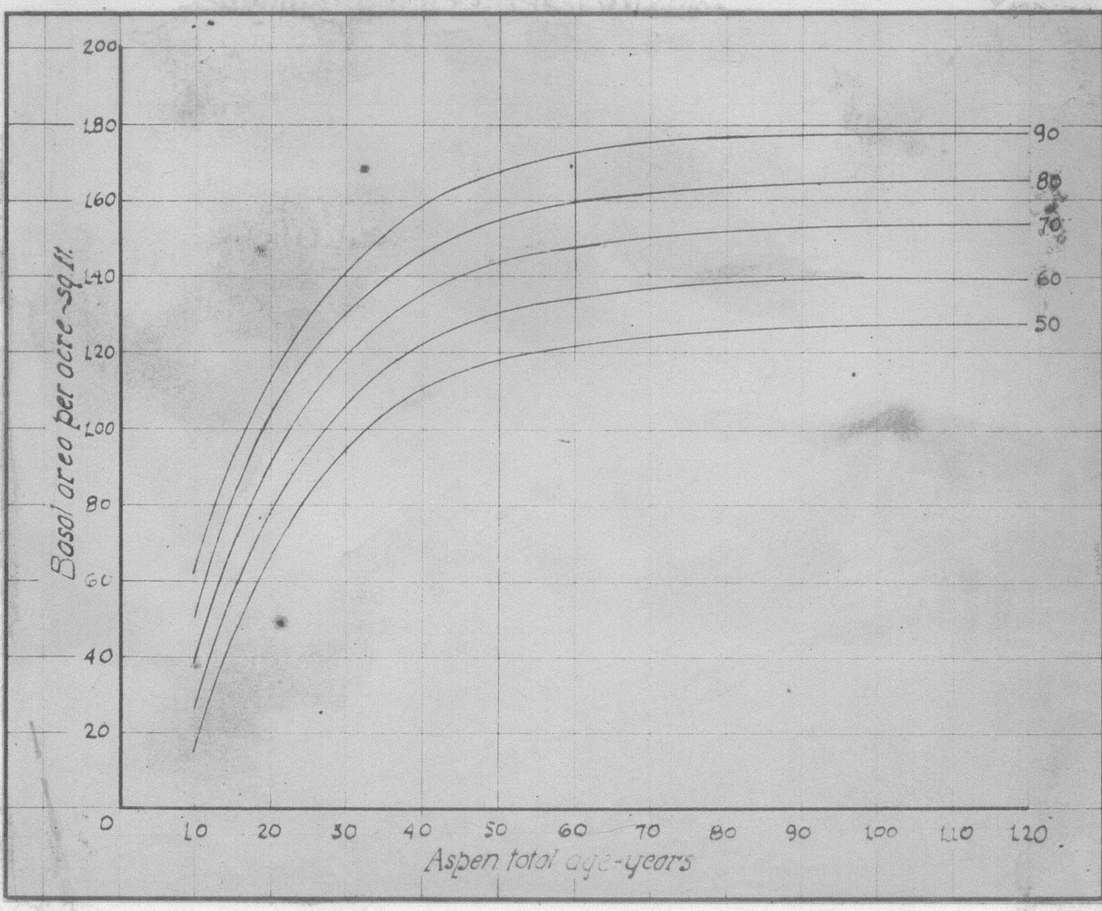

FIGURE 5 - Total basal area per acre for trees over 0.6-inch d.b.h., showing trends with age and site index.

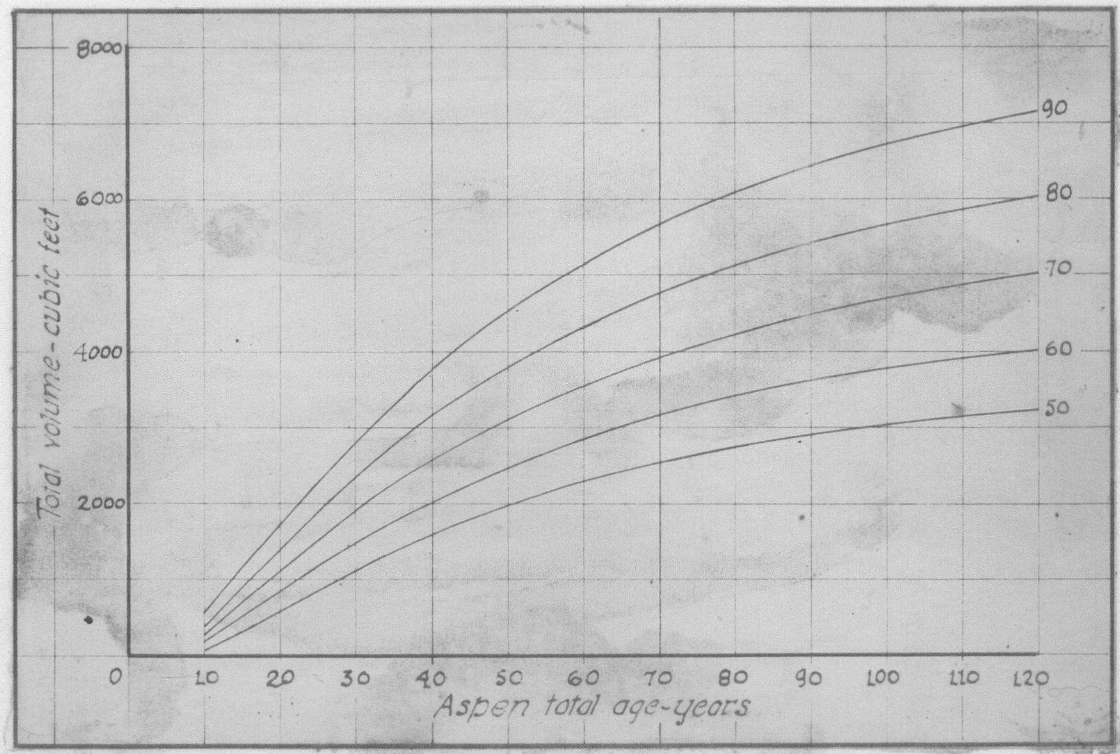

FIGURE 6 - Yield per acre in cubic feet excluding bark, showing trends with age and site index.

scattered throughout the region shown in Figure 1. For each plot age, site quality, number and size of trees and quantity of wood were carefully determined. Full details of the analysis and methods of measurement are described in the Appendix. The variables shown in the tables were analysed and related to age and site from which yields per acre for each of the age and site classes shown were obtained.

These yield tables are strictly only applicable to the area sampled but it is believed they may be used with some assurance over the region included by Grande Prairie, Peace River, Athabaska and Lac La Biche. South of Edmonton the poplar has a scrubby appearance and from the few measurements available the height-diameter relationships have a different shape. This evidence is inconclusive and does not permit limiting the area of applicability. It does indicate however, that poplar growth in other parts of Alberta may not be comparable with the yield table values. For stands with a different degree of stocking from that shown in the tables, predicted values must be corrected according to standard procedures. The amount of this correction will be described later under "density."

Height Growth and Site Index

The height attained by the average dominant at 80 years was used as the index of site quality. This age was chosen because aspen stands by this time have attained maturity, and because this age has been used to develop site curves for white spruce (6) which is commonly associated with the poplar.

The following illustrates the rather striking similarities between plot frequency occurrence percent for the three principal site classes.

Site classes (height at 80 years)	Frequency of plot occurrence aspen (percent)	white spruce (percent)
70	42	44
70-80	71	72
60-70-80	87	93

Figure 2 portrays the trend of average dominant height with age from which the values in Table 2 were derived. The use of this information to estimate site quality of an area follows standard procedures when age and average dominant height are known. In stands above 100 years of age, the average height of the upper canopy may be used since dominants are difficult to recognize. For younger ages the average height of from 4 to 8 dominants is adequate, particulary if the stand has a uniform appearance. For varied types of stands, larger samples should be taken. In this study, the diameters of the dominants were averaged for each plot and used to obtain dominant height from the height-diameter curve. The difference between this estimate and the average of actual heights is small and will likely be unimportant in practical work. The mechanics of estimating site index may best be illustrated by an example.

> If the cruise information shows the age of the stand to be 60 years and the average height of the dominants to be 65 feet, what is the site index?

Referring to Figure 1, opposite an age of 60 on the horizontal axis, a height of 65 feet falls between site class 70 and 80, which may be interpolated by eye as 74. The same result may be obtained from the use of Table 2. A height of 65 feet falls between the 70 and 80 site index class, which calculated precisely is, 70 + (4/9 x 10) or 74.4. In practice usually only broad site index classes are used, so for convenience a descriptive title has been given to each.

	Site Index
Very best or excellent	90
Good	80
Medium	70
Fair	60
Poor	50

Number and Size of Trees

Figure 3, from which Table 3 was derived, depicts the total number of trees 0.6-inch d.b.h., and larger by age classes. From 10 to 50 years on medium sites the number of trees per acre decreases from 7700 to 1500, an average mortality per decade of 1500 trees. From ages 50 to 100 years the mortality per decade decreases to 234 trees.. For Douglas fir in the Pacific Northwest by comparison, the mortality is only 360 and 60 per decade respectively for the same periods. Similar high mortality for aspen has also been found in the Lake States; Zehngraff (20) refers to "the exceptionally high mortality rate of the species" (aspen) as one of the underlying reasons earlier Lake States estimates of volume yields were found to be too high.

The large number of trees per acre in the young age classes are reflected in the small average stand diameters,

(Figure 4 and Table 4). This situation leads to an unexpected effect on diameter increment per decade since from 50 to 100 years it is slightly greater than from ages 10 to 50 years.

Basal Area Yields

The trend of basal area with age for five sites is shown in Figure 5. The average values for each site and age class are shown in Table 5. On medium sites, age 50, 143 square feet of basal area are produced; this increases to 154 by age 100.

Volume Yields

Total Cubic Feet

Figure 6 portrays the total wood produced with increasing age, including stump and top, in all trees 0.6-inch d.b.h., and larger. Volumes for each site and age class are given in Table 6.

Merchantable Cubic and Board Feet

Yields in merchantable cubic and board foot volume, by the Scribner log rule, for portions of stands on the different sites, are presented in Tables 7 to 11 inclusive. Figures 7 and 8 show the yields at various ages by site classes in cubic feet for trees 4-inches d.b.h. and larger, and in board feet for trees 7-inches d.b.h. and over. The merchantable tables were computed from total cubic yield values by means of factors obtained from the ratio of merchantable volume to total cubic volume, plotted over average stand diameter. Figures 9 and 10 illustrate the ratio trends for cubic feet in trees 4-inches d.b.h.

and larger, and for board feet in trees 7-inches d.b.h., and above, respectively.

The ratios are of value for rapid conversion into the desired units of measure. After the average diameter and total cubic volume, have been computed other volume measures may be obtained readily. The accuracy of these converted volumes will depend on the degree of stocking and the distribution of the stem diameters. If the former is average and the latter more or less regular, no serious inaccuracies should result. The conversion units used are presented in Tables 12 to 15 inclusive.

TABLE 7

YIELD PER ACRE IN CUBIC FEET OF MERCHANTABLE STEM, EXCLUDING BARK, FROM A 1-FOOT STUMP TO A 4-INCH TOP INSIDE BARK; FOR ALL TREES 4-INCHES D.B.H. AND LARGER

Total age	Yield per acre by site index:				
	50	60	70	80	90
(years)	Merchantable cubic feet				
10	-	-	-	-	-
20	-	-	-	45	140
30	20	95	220	440	695
40	185	385	710	1230	1790
50	495	860	1415	2070	2825
60	890	1455	2105	2900	3725
70	1375	2010	2770	3600	4480
80	1795	2485	3270	4185	5110
90	2125	2835	3695	4620	5620
100	2370	3100	4000	4975	6045
110	2555	3290	4255	5255	6370
120	2690	3455	4455	5520	6655

TABLE 8

YIELDS PER ACRE IN CUBIC FEET OF MERCHANTABLE STEM, EXCLUDING BARK, FROM A 1-FOOT STUMP TO A 4-INCH TOP INSIDE BARK; FOR TREES 4-11 INCHES D.B.H. INCLUSIVE

Total age	Yield per acre by site index:				
	50	60	70	80	90
(years)	Merchantable cubic feet				
10	-	-	-	-	-
20	-	-	-	45	140
30	20	95	220	440	695
40	185	385	710	1230	1790
50	495	860	1415	2070	2825
60	890	1455	2105	2900	3725
70	1375	2010	2770	3600	4420
80	1795	2485	3270	3925	4425
90	2125	2835	3395	3855	4170
100	2370	2880	3340	3560	3525
110	2370	2815	3155	3145	2585
120	2345	2725	2875	2545	1550

TABLE 9

YIELDS PER ACRE IN CUBIC FEET OF MERCHANTABLE STEM, EXCLUDING BARK, FROM A 1-FOOT STUMP TO A 4-INCH TOP INSIDE BARK; FOR TREES 12 INCHES D.B.H. AND LARGER

Total age	Yields per acre by site index:				
	50	60	70	80	90
(Years)	Merchantable cubic feet				
10	-	-	-	-	-
20	-	-	-	-	-
30	-	-	-	-	-
40	-	-	-	-	-
50	-	-	-	-	-
60	-	-	-	-	-
70	-	-	-	-	60
80	-	-	-	260	685
90	-	-	300	765	1450
100	-	220	660	1415	2520
110	135	475	1100	2110	3785
120	275	730	1580	2975	5105

TABLE 10

YIELDS PER ACRE IN BOARD FEET, SCRIBNER LOG RULE, FROM A 1-FOOT STUMP TO A 6-INCH TOP INSIDE BARK; FOR ALL TREES 7-INCHES D.B.H. AND LARGER

Total age	Yield per acre by site index:				
	50	60	70	80	90
(years)	Board feet				
10	-	-	-	-	-
20	-	-	-	-	-
30	-	-	-	-	55
40	-	-	90	375	755
50	40	200	560	1160	2070
60	270	725	1405	2630	4510
70	770	1560	3060	5270	8640
80	1545	3010	5470	9905	14980
90	2705	4965	9380	14525	20395
100	4150	7610	12580	18640	25215
110	5825	9880	15435	21770	28550
120	7260	11725	17825	24320	31045

TABLE 11

YIELDS PER ACRE IN BOARD FEET, SCRIBNER LOG RULE, FROM A 1-FOOT STUMP TO A 6-INCH TOP INSIDE BARK; FOR ALL TREES 12-INCHES D.B.H. AND LARGER

Total age	Yield per acre by site index:				
	50	60	70	80	90
(years)	Board feet				
10	-	-	-	-	-
20	-	-	-	-	-
30	-	-	-	-	-
40	-	-	-	-	-
50	-	-	-	-	-
60	-	-	-	-	-
70	-	-	-	190	655
80	-	-	320	1060	2295
90	45	330	1130	2565	5635
100	275	855	2225	5815	11965
110	585	1590	4265	9910	18845
120	960	2540	7150	14650	25310

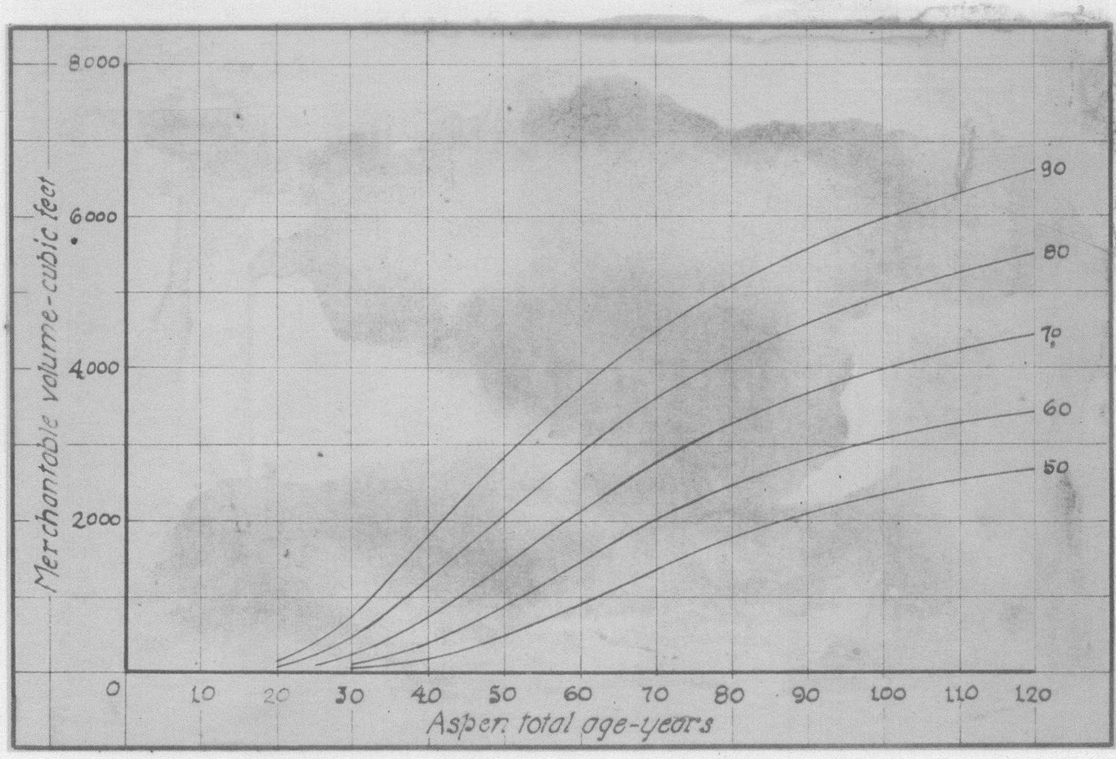

FIGURE 7 - Yield per acre in cubic feet of merchantable stem, excluding bark (to a 4-inch top inside bark), showing trends with age and site index.

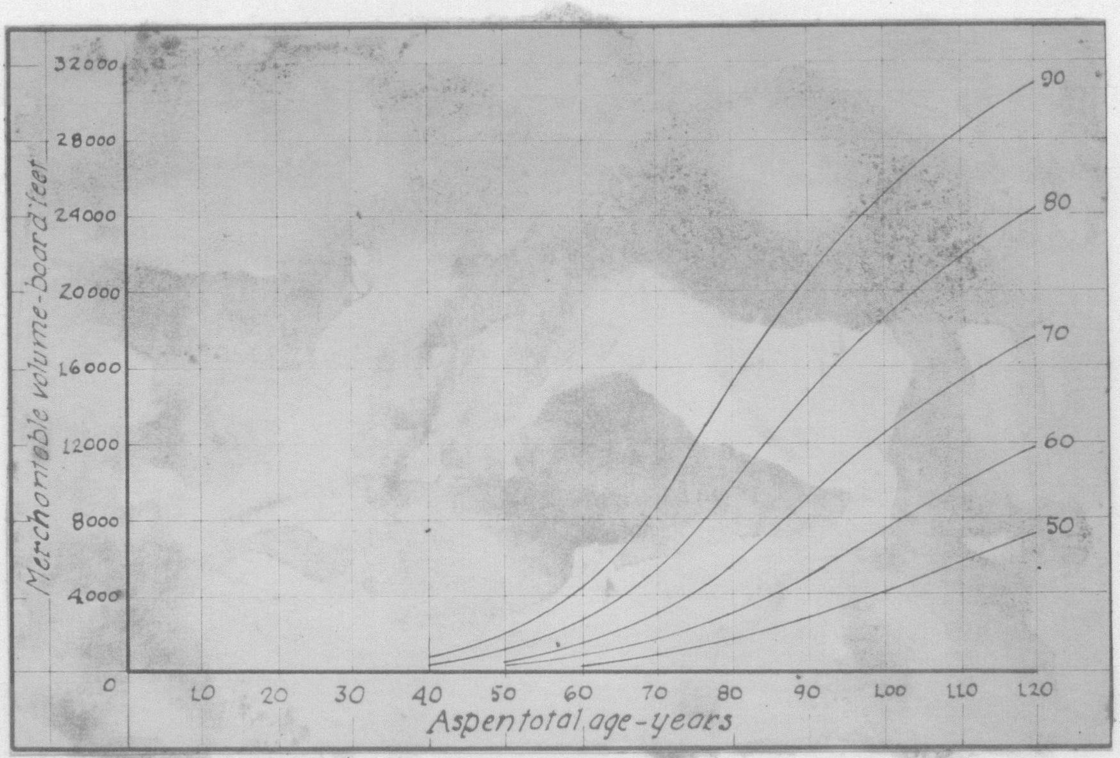

FIGURE 8 - Yield per acre in board feet, Scribner rule (to a 6-inch top inside bark), showing trends with age and site index.

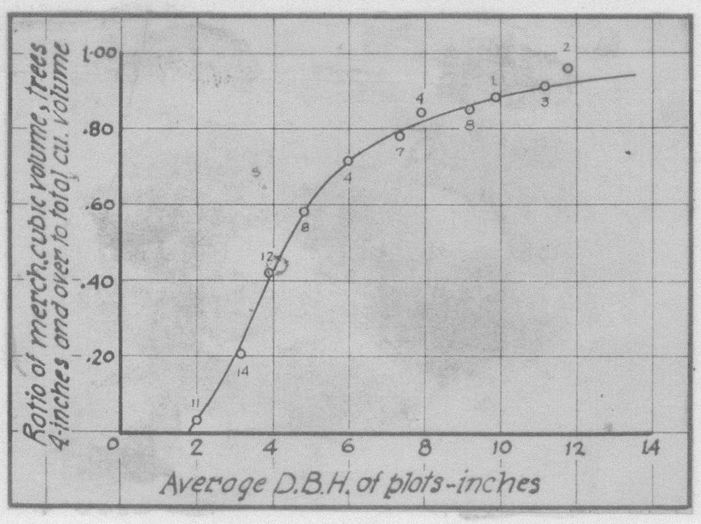

FIGURE 9 - The relationship between the merchantable cubic foot-total cubic foot volume ratio, to average stand d.b.h., where the merchantable stand includes trees 4-inches and larger.

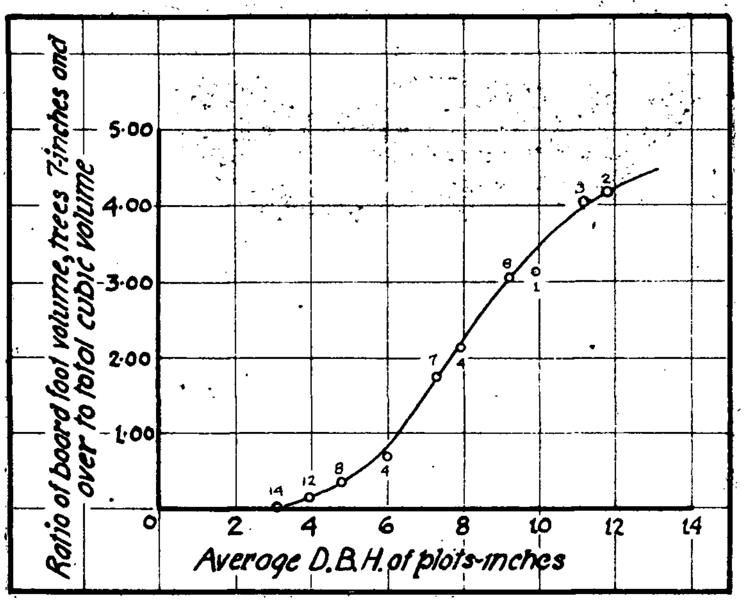

FIGURE 10 - The relationship between board foot-total cubic foot volume ratio, to average stand d.b.h., where the merchantable stand includes trees 7-inches d.b.h. and larger.

TABLE 12

CONVERSION UNITS APPLIED TO TOTAL CUBIC VOLUME OF WELL STOCKED ASPEN STANDS TO OBTAIN MERCHANTABLE CUBIC VOLUME OF ALL TREES 4-INCHES D.B.H. AND LARGER

Average stand d.b.h.	Average stand diameter, tenths of an inch									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Conversion units, total cubic to mer. cubic vol., trees 4"+									
Inches										
1.0	-	-	-	-	-	-	-	-	-	.017
2.0	.033	.049	.065	.082	.099	.116	.134	.152	.170	.190
3.0	.210	.230	.251	.275	.298	.322	.346	.369	.392	.414
4.0	.435	.455	.474	.493	.511	.528	.545	.560	.575	.591
5.0	.606	.620	.633	.645	.656	.667	.677	.688	.697	.706
6.0	.715	.723	.731	.738	.745	.752	.759	.765	.771	.777
7.0	.783	.788	.794	.799	.803	.807	.811	.815	.819	.822
8.0	.826	.830	.833	.836	.839	.843	.846	.849	.852	.854
9.0	.857	.860	.862	.865	.868	.870	.873	.875	.878	.880
10.0	.883	.885	.887	.889	.891	.894	.896	.898	.900	.902
11.0	.904	.906	.908	.910	.911	.913	.915	.917	.918	.920
12.0	.922	.924	.925	.926	.928	.930	.932	.934	.936	.937

TABLE 13

CONVERSION UNITS APPLIED TO TOTAL CUBIC VOLUME OF WELL STOCKED ASPEN STANDS TO OBTAIN MERCHANTABLE CUBIC VOLUME OF ALL TREES 12-INCHES D.B.H.AND LARGER

Average stand d.b.h.	Average stand diameter,tenths of an inch									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Conversion units,total cubic to mer.cubic vol.trees 12"+									
(inches)										
6.0	-	-	-	-	-	-	-	-	-	-
7.0	.004	.011	.019	.026	.034	.042	.050	.058	.066	.075
8.0	.084	.094	.103	.112	.121	.131	.140	.150	.160	.170
9.0	.818	.191	.202	.213	.224	.236	.248	.260	.273	.286
10.0	.300	.314	.328	.343	.358	.373	.389	.405	.421	.437
11.0	.454	.472	.489	.506	.524	.542	.561	.580	.599	.618
12.0	.637	.656	.675	.694	.712	.729	.747	.764	.780	.796

TABLE 14

CONVERSION UNITS APPLIED TO TOTAL CUBIC VOLUME OF WELL STOCKED ASPEN STANDS TO OBTAIN MERCHANTABLE BOARD FOOT VOLUMES SCRIBNER RULE, OF ALL TREES 7-INCHES D.B.H. AND LARGER

Average stand d.b.h.	Average stand diameter,tenths of an inch									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Conversion units, total cubic vol. to board ft.,trees 7"+									
(inches)										
3.0	-	-	.02	.04	.05	.06	.08	.10	.12	.14
4.0	.16	.18	.20	.23	.26	.28	.30	.34	.36	.40
5.0	.42	.46	.49	.53	.56	.60	.64	.69	.73	.78
6.0	.82	.88	.93	.98	1.04	1.10	1.16	1.22	1.29	1.36
7.0	1.44	1.52	1.60	1.68	1.76	1.84	1.92	2.00	2.08	2.16
8.0	2.23	2.31	2.38	2.45	2.52	2.59	2.66	2.72	2.79	2.85
9.0	2.91	2.97	3.03	3.09	3.15	3.21	3.27	3.32	3.38	3.43
10.0	3.49	3.54	3.59	3.64	3.69	3.73	3.78	3.82	3.86	3.90
11.0	3.93	3.97	4.00	4.03	4.06	4.09	4.12	4.15	4.18	4.21
12.0	4.23	4.26	4.29	4.31	4.33	4.35	4.38	4.40	4.42	4.44

TABLE 15

CONVERSION UNITS APPLIED TO TOTAL CUBIC VOLUME OF WELL STOCKED ASPEN STANDS TO OBTAIN MERCHANTABLE BOARD FOOT VOLUMES, SCRIBNER RULE, OF ALL TREES 12-INCHES D.B.H., AND LARGER

Average stand d.b.h. (inches)	Average stand diameter, tenths of an inch									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Conversion units, total cubic vol. to board ft., trees 12"+									
6.0	-	0.01	0.02	0.02	0.03	0.04	0.05	0.06	0.08	0.09
7.0	0.10	0.12	0.13	0.15	0.16	0.18	0.20	0.22	0.25	0.27
8.0	0.30	0.32	0.35	0.38	0.40	0.44	0.47	0.40	0.54	0.58
9.0	0.63	0.68	0.74	0.80	0.87	0.95	1.02	1.10	1.18	1.26
10.0	1.34	1.42	1.50	1.59	1.68	1.77	1.86	1.95	2.04	2.14
11.0	2.23	2.32	2.41	2.51	2.61	2.70	2.80	2.89	2.99	3.08
12.0	3.17	3.26	3.35	3.44	3.53	3.61	3.70	3.78	3.86	3.94

TABLE 16

MEAN AND PERIODIC ANNUAL ASPEN GROWTH PER ACRE IN CUBIC FEET, ENTIRE STEM, EXCLUDING BARK, ALL TREES 0.6-INCH D.B.H., AND LARGER

(years)	Mean and periodic annual growth per acre by site classes:					
Total	Fair 60		Medium 70		Good 80	
age	m.a.i.	p.a.i.	m.a.i.	p.a.i.	m.a.i.	p.a.i.
	Total cubic feet					
10	18.0	-	28.0	-	42.0	-
20	42.8	65.5	56.0	80.5	70.8	94.0
30	49.3	59.0	63.0	73.5	77.2	87.0
40	50.8	50.0	64.4	62.0	78.6	75.0
50	49.6	41.0	62.3	49.0	76.0	59.5
60	47.5	35.0	59.3	39.5	72.4	49.0
70	45.4	29.5	56.1	34.0	68.4	40.5
80	43.0	23.0	53.0	29.0	64.5	33.5
90	40.6	18.0	50.1	24.5	60.7	26.5
100	38.0	13.5	47.3	20.0	57.0	22.0
110	35.6	11.0	44.5	15.0	53.6	18.5

Increment and rotation age

Tables 16, 17 and 18 show rates of growth on Fair, Medium and Good sites in terms of periodic and mean annual increment. Tables 16 and 17 show these growth values in cubic feet for the entire stand and for the merchantable portion. The values in Table 18 are expressed in board feet for the trees 7-inches and over.

The rotation age is usually referred to as the age at which periodic and mean annual growth are the same. In practice, the number of years that will be required to obtain regeneration after the final cut, should be included. Since aspen produces seed frequently and abundantly and is capable of prolific coppice growth it may safely be assumed that a new crop will come in immediately and no additional years need be added. A stand cut at the rotation age will yield the maximum volume return per year of growth. Rate of growth is only one of a number of factors to be evaluated in fixing the proper age at which to cut a given stand. The type of product desired, the financial aspects of forest management and the silvicultural features of the forest must all be taken into account. Nevertheless, volume production alone is an important item. It will be noted from the tables that the peak of mean annual growth is influenced by site and measure of volume. The rotation ages indicated by the data collected in this study are shown in Table 19.

For maximum production of wood per acre, short rotations of 40 years are indicated. Since most of the trees in stands of this age are below 4-inches in diameter, a 40-year rotation period is of rather theoretical interest at present. This low rotation age, however, it is interesting to note, is the same as that suggested by MacLeod (6) for aspen in mixed-wood stands growing in the same general area. When only trees 4-inches d.b.h. and larger are considered, the rotation period is lengthened to 85 years. For board foot measure, trees 7-inches and above, a rotation interval of 130 years is indicated.

TABLE 19

ROTATION AGE TO THE NEAREST HALF DECADE FOR MAXIMUM WOOD PRODUCTION, ACCORDING TO PORTION OF STAND AND UNIT OF VOLUME CONSIDERED, FOR THREE SITE CLASSES

When unit of volume and part of stand is	Site index		
	Fair 60	Med. 70	Good 80
	Total age in yrs.		
Total cubic volume, all trees 1" + included	40	40	35
Merchantable cubic feet, all trees 4" + included	90	85	80
Merchantable board feet, all trees 7" + included (Scrib. rule)	-	(130)	(125)

( ) - bracketed figures are extrapolated.

Cull

Unfortunately there is no information on the amount of wood rot per tree for the area nor were any data taken in this study. Some consideration must be given to this very important factor, however, which in other areas is the greatest single cause for cull in aspen. Although it is obviously unwise to apply cull percentages derived from studies made in other regions, some of the dangers in doing so may be more apparent than real, particularly when one pathogen, *Fomes igniarius*, is the most important rot causing agent throughout the entire range of poplar.

From Black's data (1), for aspen in Northern Ontario, figures on percentage of cull related to diameter were used to estimate total rot in percent for each plot measured in Northern Alberta. The values shown in Table 20 were read from a graph where these percentages were correlated with age.

TABLE 20

CULL[1] PERCENT, FOR TREES 4-INCHES D.B.H. AND LARGER, RELATED TO ASPEN TOTAL AGE FOR MEDIUM SITES

Total age (years)	Cull (percent)	Total age (years)	Cull (percent)
30	1.0	80	9.0
40	1.0	90	13.0
50	2.0	100	18.0
60	4.0	110	22.0
70	6.0	120	27.0

Meineke (8) studied the pathology of aspen in Utah in 1929. The cull percentages shown in his publication are similar, when correlated with age, to the values in Table 20. The percentages derived in both regions are given below for comparison:

Age	Cull as a percentage of merchantable volume for: Northern Ontario	Utah
30	1.0%	1.0%
60	4.0%	3.5%
90	13.0%	9.0%
120	27.0%	30.0%

It should be noted that the cull percentages shown for the two regions do not have a strictly comparable basis. In Black's

[1]All portions of the merchantable stem having a rot diameter greater than 1-inch; based on Black's data (1) for aspen in Northern Ontario.

study, merchantable length included that portion of the stem from a 1-foot stump to a 3.5-inch top; any portion of the stem with a rot diameter greater than 1-inch was culled. The volume of this culled portion was then expressed as a percentage of the total volume of each tree. Meinecke included as merchantable stem that portion between a 1-foot stump and a 2-inch top. He expressed the volume of the mass of decay as a percentage of the merchantable volume of each tree. Black considered only trees above 6-inches d.b.h., and although Meineke measured merchantable trees, the lower diameter limit employed is not clear. For large samples these differences in method are small, increasing the possibility that the similarity of the percentages shown above are not due to chance alone. Although these cull percentages are not strictly applicable to Northern Alberta aspen, it is likely that the error incurred will not be great if they are applied as a correction to volume estimates for that region.

The cull figures indicate also that for stands older than 80 years, an allowance for rot is necessary if volume estimates are to be of much value, and that it may be better to employ the estimated deductions than to disregard them entirely.

Application of the cull percentages given in Table 20 to volume yields, changes the rotation ages previously suggested very little for both total and merchantable cubic feet. For stands 120 years and older, amount of cull is high which should result in lowering the rotation period given for board feet. Since these ages are beyond the range covered by the data an adjusted rotation age cannot be determined.

Density

Basal area as a measure of density in the application of normal yield tables was found by Meyer (10) to be better than any other yield table measure. Basal area can be obtained from field data and compared to the tabular value (interpolated from the yield tables according to age and site index) in order to obtain a stocking value for a forest. The ratio is usually expressed in percent. Thus, in the case of aspen, if the basal area of a stand of site quality 80 and age 60 has been computed as 130 square feet per acre, it is compared with a density of 160 square feet (from Table 5). The stocking therefore, is 130/160 x 100 or 81 percent. For future predictions of growth it is usual to assume that this figure will remain the same; thus at age 100 the basal area will be 166 x .81 or 134 square feet. Growth information from permanent plots indicate that a trend towards normality takes place in overstocked and understocked stands. Use of the above assumption, therefore, is to give conservative future growth estimates for understocked stands (the most common) and optimistic forecasts for stands which are overstocked. The extent of this error will depend upon stocking, age of the stand and growth characteristics of the particular species in question.

Although basal area has been suggested here as the measure to be used when estimating density in conjunction with the yield tables, number of trees per acre has been receiving greater attention by foresters when related to the diameter of the tree of average basal area. Reineke (16) has shown that for a number of species this relationship assumed a straight line form when plotted on double logarithmic paper. Out of the 14 species that were tested the slopes for 12 of these were the same. Employing an average d.b.h. of 10 inches as his index diameter and a sheaf of lines parallel to the main guide line, he was able to ascribe a density number to any stand where the number of trees per acre and the average stand diameter were known. This stand density index is therefore the number of trees per acre at an index diameter of 10 inches. Mulloy (12) (13) has brought the stand density index concept to the attention of Canadian foresters and Spurr (19) also has explored its possibilities. Although stand density indices have not been employed to develop any portion of the yield tables, they are presented here to facilitate comparison with aspen growing in other regions where this measure of stocking is used. It will also be shown that the slope of the regression line of number of trees on average diameter (on double logarithmic paper) may be quite different for aspen than the one derived by Reineke and used by Mulloy for species in Eastern Canada.

To determine the regression line, the average of the number of trees per acre in each 2-inch average diameter class was plotted over average d.b.h. on log paper. These points are shown in Figure 11, and a straight line obviously fits the data best. The equation was determined as $\log N = -1.767 \log D + 4.257$, where N equals the number of trees per acre and D the basal area of the average tree in inches. When the plot data were all converted to logarithmic values and fitted by the method of least squares, a slope of -1.646 was obtained, a

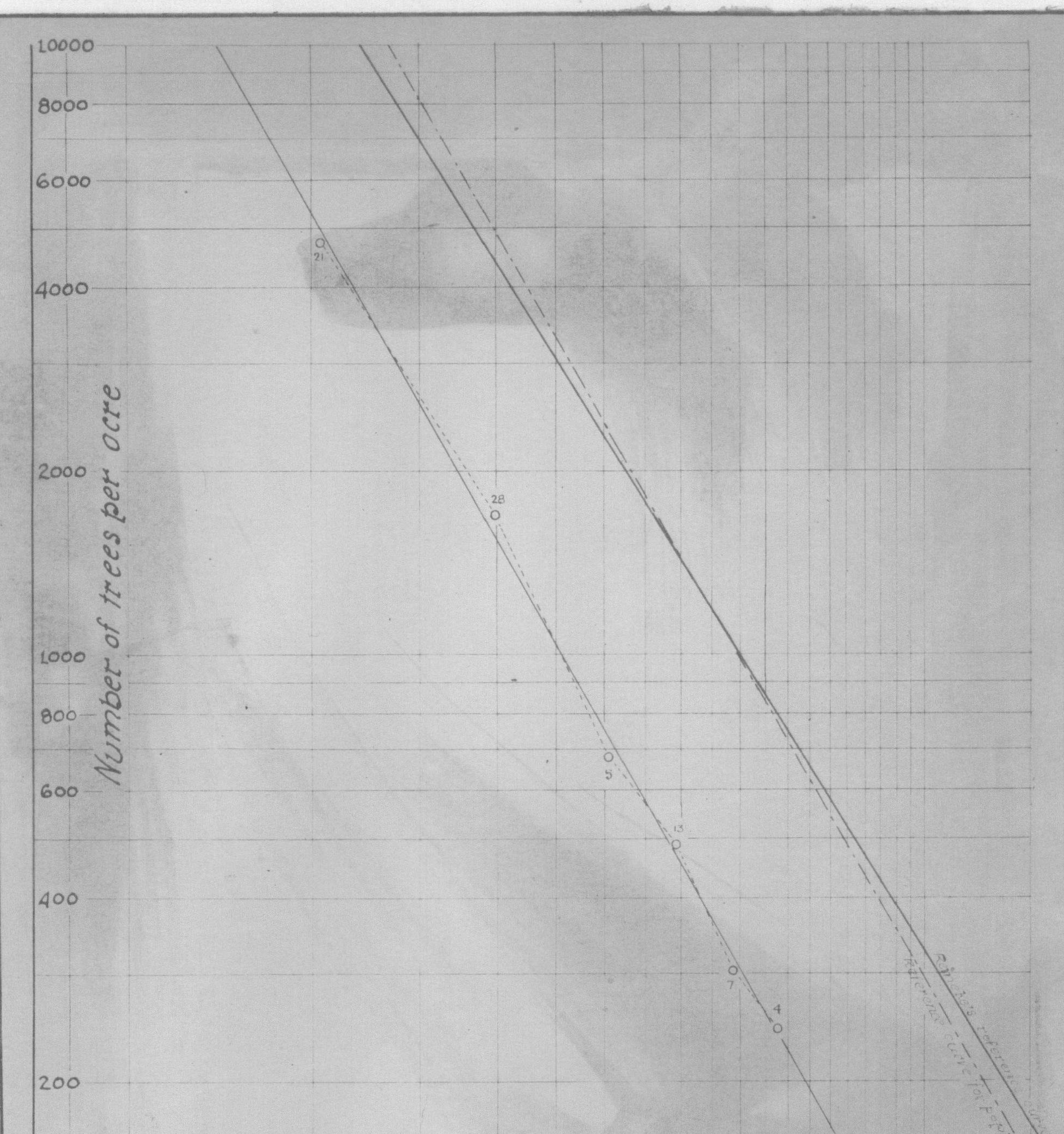

FIGURE 11 - The relation of total number of trees per acre to average stand d.b.h., for 78 fully stocked aspen sample plots. The slope of the fitted regression line is compared with that obtained by Reineke for a number of other species.

TABLE 21

STAND DENSITY UNITS FOR ASPEN CORRESPONDING TO
AVERAGE STAND DIAMETERS IN INCHES

Average stand d.b.h. (inches)	Average stand d.b.h.: tenths of an inch									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	Stand density units									
1.0	0.017	0.020	0.023	0.027	0.031	0.035	0.039	0.043	0.048	0.053
2.0	0.058	0.063	0.068	0.074	0.080	0.086	0.092	0.098	0.105	0.112
3.0	0.119	0.126	0.133	0.140	0.148	0.156	0.164	0.172	0.180	0.189
4.0	0.198	0.207	0.216	0.225	0.234	0.244	0.254	0.264	0.274	0.284
5.0	0.284	0.305	0.316	0.327	0.338	0.349	0.360	0.371	0.382	0.394
6.0	0.406	0.418	0.430	0.442	0.454	0.467	0.480	0.493	0.506	0.519
7.0	0.532	0.545	0.559	0.573	0.587	0.601	0.615	0.628	0.643	0.658
8.0	0.673	0.688	0.703	0.718	0.734	0.750	0.766	0.782	0.798	0.814
9.0	0.830	0.847	0.863	0.880	0.897	0.914	0.931	0.948	0.965	0.982
10.0	1.000	1.018	1.036	1.056	1.074	1.092	1.110	1.128	1.146	1.164
11.0	1.183	1.202	1.221	1.240	1.259	1.278	1.298	1.318	1.338	1.358
12.0	1.378	1.498	1.418	1.439	1.460	1.481	1.502	1.523	1.544	1.565
13.0	1.587	1.609	1.631	1.653	1.675	1.697	1.719	1.742	1.765	1.788
14.0	1.811	1.834	1.857	1.880	1.903	1.926	1.949	1.972	1.996	2.020
15.0	2.044	2.068	2.092	2.116	2.140	2.164	2.188	2.213	2.238	2.263
16.0	2.288	2.313	2.388	2.363	2.389	2.415	2.441	2.467	2.493	2.519

somewhat lower slope than derived by freehand methods. The reason for this difference has been explained by H. A. Meyer (9) as due to errors incurred by the use of logarithms in the place of natural numbers. This error is a systematic one and may be corrected by use of the standard error. The line fitted by the method of least squares did not conform to the plotted averages of the data, but corrections were awkward to make when the standard error decreased with an increase in average stand diameter. The line illustrated in Figure 11, fitted by freehand methods, was therefore assumed to be the best fit and employed in the computations described later. It is not clear whether Reineke obtained his curve by fitting the data by least squares or whether the wealth of data analyzed revealed the slope of -1.605 presented in his paper by standard procedures of balancing data by freehand methods. Schnur (18) for Upland Oak found the slope to be somewhat flatter than the slope obtained by Reineke, while for aspen the slope appears much steeper (refer to Figure 11). If this last is true the explanation must lie in the rather excessive mortality of the aspen in the early years.

To be consistent with the stand density index concept outlined by Reineke, the regression $\log N = -1.767 \log D + 4.257$ was converted to the 1000 tree level at an average d.b.h. of 10-inches. The resulting formula is expressed by, $\text{Log } N = 1.767 \log D + 4.767$.

In order that stand density indices may be calculated rapidly, a table of stand density units (Table 21) was constructed for aspen similar to the one derived by Mulloy using Reineke's formula. The procedure adopted to derive these tables is as follows.

The required average diameter values were converted to logarithmic numbers and the number of trees per acre calculated from the formula given above, $\log N = -1.767 \log D + 4.767$. The stand density units were obtained by dividing the values for N into 1000. From these the stand density index may be computed quickly. If the average diameter has been determined as 6.1 with 200 trees per acre, the stand density index will be 0.418 x 200 or 84. It is admitted that the regression of number of trees over average diameter may be too steep either as a result of bias in the sample or as a result of below-normal stocking of the older stands. Since fire is a major problem in the area, it is conceivable that older stands in the larger diameters have been subjected to fire at some period in the past.

BLACK POPLAR, PERCENTAGE COMPOSITION CHANGE

Plots were chosen in pure hardwood stands composed mainly of aspen, and although black poplar occurred on 71.8 percent of them, it never exceeded 30 percent of the stand composition. Since no restrictions were imposed to govern the number of stems allowable, significant changes in the amount of black poplar throughout the age classes can be mainly credited to differences in growth between this species and aspen. It is conceivable, however, that the colour and texture of the black poplar bark, which is almost black in older stands, would have some effect on the choice of plots made. The results which will be presented may therefore be influenced by personal bias.

Only those plots containing black poplar were used to determine the regression of percentage composition and age by the method of least squares. The average percentage within each age class and the regression line are shown in Figure 12. An analysis of variance was made to test the reliability of the relationship. The figures obtained are given below:

Source	Degrees of Freedom	Sum Squares	Mean Squares	Variance Ratio
Variation due to regression	1	221.71	221.71	4.55
Residual variation	59	2877.25	48.76	
Total variation	60	3098.96		

From tables shown by Fisher and Yates (3), the variance ratio at the 5 percent level is 3.97 compared with the 4.55 value determined. The regression, therefore of the two variables employed is significant at the 5 percent probability level.

The percentage composition of black poplar at each 20-year age class is given in Table 22. The average change is small (0.65 percent per decade) and will not be introduced as correction in the prediction of black poplar growth from the yield tables.

The relationship shown in Figure 12 indicates the similarity in basal area growth of aspen and black poplar where they exist in close association. This justifies, somewhat, treating all species in the hardwood mixture as one unit. This procedure has been followed in the development of poplar yields described in this thesis.

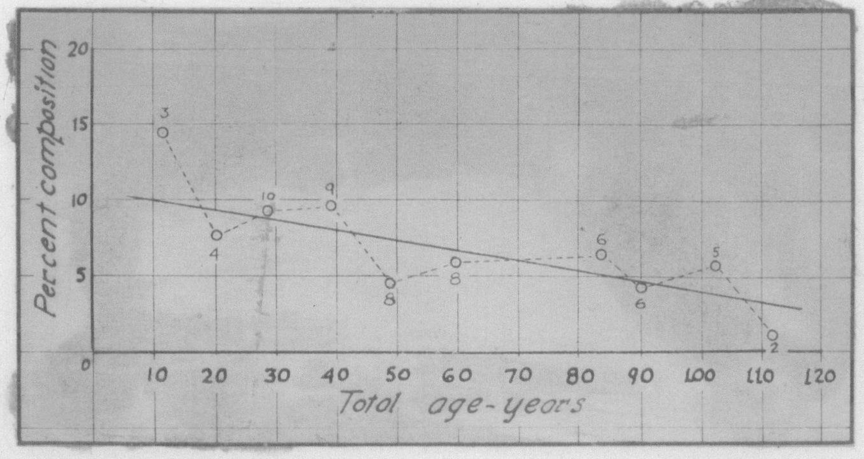

FIGURE 12 - The relation of percentage composition, by basal area, to total age for black poplar on 61 sample plots.

TABLE 22

PERCENTAGE COMPOSITION OF BLACK POPLAR RELATED TO AGE

Total age	Composition by basal area
(years)	(percent)
20	9.3
40	8.1
60	6.7
80	5.4
100	4.1

VOLUME TABLES FOR ASPEN AND BLACK POPLAR

Measurements and Computations

Aspen and black poplar trees were felled and measured during yield studies conducted by the writer for mixedwood and pure poplar stands growing in Northern Alberta. For each tree, diameters and bark thicknesses were recorded at the following points: at a stump height of 1-foot, at 4.5 feet above ground and at the end of each equal-lengthed 1/10-section between breast height and the tip of the tree. Lengths of stem were accumulated from the base and this accumulated height at each point measured was plotted over the corresponding diameter inside bark. By joining the points with a smoothed line, a graph for each tree was obtained from which it was possible to read measurements for the utilization desired. The volume measures used in Alberta were total cubic feet,merchantable cubic feet (1-foot stump to a 4-inch top inside bark) and board feet (1-foot stump to a 6-inch top inside bark,Scribner rule).

Total cubic foot volume inside bark for each tree was calculated using Smalian's formula; the portion between ground and 1-foot stump height was treated as a cylinder. The merchantable volume was obtained by subtracting the volume of stump and top. From the graphs of each tree, 16.3-foot logs were scaled in board foot volume. The top log could be 8.3, 10.3, 12.3, 14.3, or 16.3 feet long depending on the length of stem which remained from the last 16.3-foot log.

For analysis of volume, the measurements used for each aspen and black poplar tree were:

1. Diameter at breast height in inches outside bark,
2. Total height in feet,
3. Volume in
    (a) total cubic feet,
    (b) merchantable cubic feet,
    (c) and merchantable board feet, Scribner rule.

Total Cubic Feet

The total cubic volume for each aspen tree was compared with volumes given in a table constructed for this species in eastern Canada[1]. The table gave values which were consistently high. Adjusted volumes were obtained from a graph derived by plotting actual volumes over tabular volumes on double logarithmic paper (Table 23).

From a comparison made previously by the writer, it was found that black poplar had a characteristically thicker bark and poorer form than the aspen. The volumes obtained therefore, when the black poplar trees were interpolated in Table 23, were too high. A correction factor of 0.842 was determined and applied. The estimate of the actual volumes which resulted proved to be satisfactory. The aggregate difference was small and the average deviation did not increase beyond that obtained for the aspen.

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[1] Dominion Form Class Volume Tables, page 182, 1948.

Merchantable Cubic Feet

No aspen volume tables applicable to the merchantable limits used in Alberta were located. It was necessary therefore, to construct a new table. The method employed in the analysis was essentially that suggested by Dwight (2). This method is an improvement on standard graphical techniques because the relationship of height-to-diameter within each height class is taken into account. The group of curves in this step (Figure 13) are fitted concurrently with those obtained by plotting volume over diameter by height classes. The volumes and heights read for each 2-inch class are next used to derive the relationship of volume to height by diameter classes. For the aspen data these curves proved to be a family of straight lines, most of them radiating from a common origin. Although the curves may be harmonized in this part of the analysis, volume was replotted over diameter by height classes in order to facilitate reading of volume for 1-inch diameter intervals. From these curves the values shown in Table 24 were obtained.

The black poplar tree data for merchantable cubic feet were now compared with volumes interpolated for diameter and height in Table 24. The tabular values obtained were again too high. Multiplying by a correction factor of 0.817 resulted in a much improved estimate with but a slight increase in the average deviation percent over that obtained for the aspen.

Merchantable Board Feet, Scribner Rule

A similar analysis for diameter, height and board foot volume was carried out for the aspen up to and including the step of plotting volume over height by d.b.h. classes. Figures 13 and 14 illustrate the first two steps (the number of trees used to weight each point on the graphs are omitted). Figure 15 shows the preliminary group of straight lines fitted to the data. The spacing and slopes of these lines for each diameter class are harmonized by an adaptation of a method outlined by Meyer (11).

Since volume in board feet for any one diameter class is directly related to total height, it may be expressed by the following formula:

$$V = a + bH$$

where V = board foot volume,

a = intercept,

b = slope,

H = total height in feet.

The slope of the line for each diameter class was determined. These values plotted over diameter were fitted by a freehand curve (Figure 16). This relationship is a straight line for 13-inch diameters and larger, with a slope of $b'' = -5.65 + 0.612D$. Below 13-inches the relation of slope to diameter is curved, and a different procedure is required. For the present it may be noted that individual slope values may be read from the curve in Figure 16 for diameters between 7 and 12-inches.

In the next step, an adjusted volume for a standard height class of 80-feet was computed for each 2-inch diameter, where the adjusted volume = the average volume for the d.b.h. class - b" (average height for the d.b.h. class) - 80, and b" = -5.65 + 0.612D. The average volumes and heights were obtained by calculating weighted averages for each 2-inch class. Adjusted volumes were then plotted over diameter and a curve fitted as shown in Figure 17. Again the regression proved to be a straight line for diameters 13-inches and larger, the resulting regression is expressed by the equation,

$$\text{adj. vol.} = -236 + 28.5D.$$

For diameters below 13-inches, it may be noted that adjusted volume may be read for any d.b.h. from the curve.

From the slope equation and adjusted volume equation for a standard height of 80-feet the following is true:

1. Adj. volume (V) = -236.0 + 28.5D
2. -bH = -5.65+ 0.612D) 80 or 452.0 - 49.0D

---

by adding (1) and (2)

V-bH = + 216.0 - 20.5D

a = + 216.0 - 20.5D

since V = a + bH and a = V - bH.

For estimating volumes of trees with diameters 13-inches and above, there are now three equations;

A	General expression	V = a + bH
B	The slope expression	b = - 5.65 + 0.612D
C	The intercept expression	a = + 216.0 - 20.5D

Since slope "b" and intercept "a" are changing in a curvilinear manner for diameters below 13-inches, equations B and C are not applicable to the smaller diameters. While slope values may be obtained from Figure 16, corresponding figures for the intercept are still required in order to derive individual equations for each 1-inch diameter class. This difficulty was overcome by calculating values of 'a' and plotting these over diameter as shown in Figure 18. The computations for 'a' were made from the expression,

$$a = \text{volume at chosen height} - b \times (\text{chosen height})$$

where volume may be read from Figure 17 and slope values from Figure 16. An individual equation therefore, was determined for the diameters required. From the equations shown below the values for the board foot volumes shown in Table 25 were obtained.

Group I - Diameters at breast height from 7-12 inches inclusive

7"	-	$V = -8.8 + 0.22H$
8"	-	$V = -13.6 + 0.50H$
9"	-	$V = -18.5 + 0.79H$
10"	-	$V = -24.1 + 1.12H$
11"	-	$V = -31.0 + 1.46H$
12"	-	$V = -40.0 + 1.87H$

Group II - Diameters at breast height for trees 13-inches and larger.

General expression	$V = a + bH$
The slope expression	$b = -5.65 + 0.612D$
The intercept expression	$a = +216.0 - 20.5D$

Examples of application

For clarity, volumes in board feet are calculated for examples occurring in Group I and Group II.

1. Wanted: to determine the volume in board feet for a 10-inch tree of height 63 feet.

From Group I the equation is:

$$V = -24.1 + 1.12H$$

Substituting for H

$$V = -24.1 + 1.12 \times 63$$
$$V = 46.5 \text{ board feet.}$$

2. Wanted: to determine the board foot volume for a 19-inch tree of height 94 feet.

From Group II the general equation is:

$$V = a + bH$$

to determine the slope substitute 19 for D in $b = -5.65 + 0.612D$

$$b = -5.65 + 0.612 \times 19$$
$$= +5.98$$

to determine the intercept substitute 19 for D in $a = +216.0 - 20.5D$

$$a = +216.0 - 20.5 \times 19$$
$$= -173.5$$

substituting for b, a, and H in the general equation

$$V = -173.5 + 5.98 \times 94$$
$$= 388.6 \text{ board feet.}$$

To apply the values in Table 25 to estimate black poplar volumes the tabular figures were reduced by multiplying them by 0.787.

The method employed above to harmonize a family of straight lines was originally suggested by W.H. Meyer to develop volume tables in board feet where the ratio of volume/diameter plotted over diameter by height classes, form a series of straight lines. The aspen data plotted this way indicated curvilinearity for diameters above 20-inches. The explanation may be that larger diameters are found mainly in understocked stands and are usually trees of poorer form. It was felt therefore, that Meyer's method could not be justifiably employed.

Adapting the method described, offers little advantage to those who are familiar with the systems of crosscurving, but for those who are not, the introduction of this semi-mathematical system may facilitate the harmonization of both slope and spacing. Another feature of the method is that it may be employed for other relationships where a system of straight lines is applicable. In certain cases, some advantage may be gained by a mathematical control of tabular and intermediate values.

The weakness lies in the manner original averages have been obscured by previous curving. This would seem to be no more serious however, than the weaknesses involved in any multicurvilinear method.

VOLUME TABLES FOR WHITE BIRCH

Total Cubic Feet

Height measurements for 93 birch trees were taken by MacLeod (6) as they occurred in mixedwood stands throughout the same region as this study. The total cubic volumes shown in Table 26 are taken from his report.

Merchantable Cubic and Board Feet

To derive merchantable tables, aspen volumes in cubic and board feet were interpolated for the diameters and heights given in Table 26. To the volume shown for each d.b.h. class, the ratio of aspen total cubic volume to that for birch was applied as a correction.

To assume aspen-birch total volume ratios were the same for merchantable measures provided the only basis possible under the circumstances since no birch trees were measured for volume. The amount of birch present in the stands measured was small however, and any errors that might arise from incorrect volume tables for this species cannot seriously affect the results.

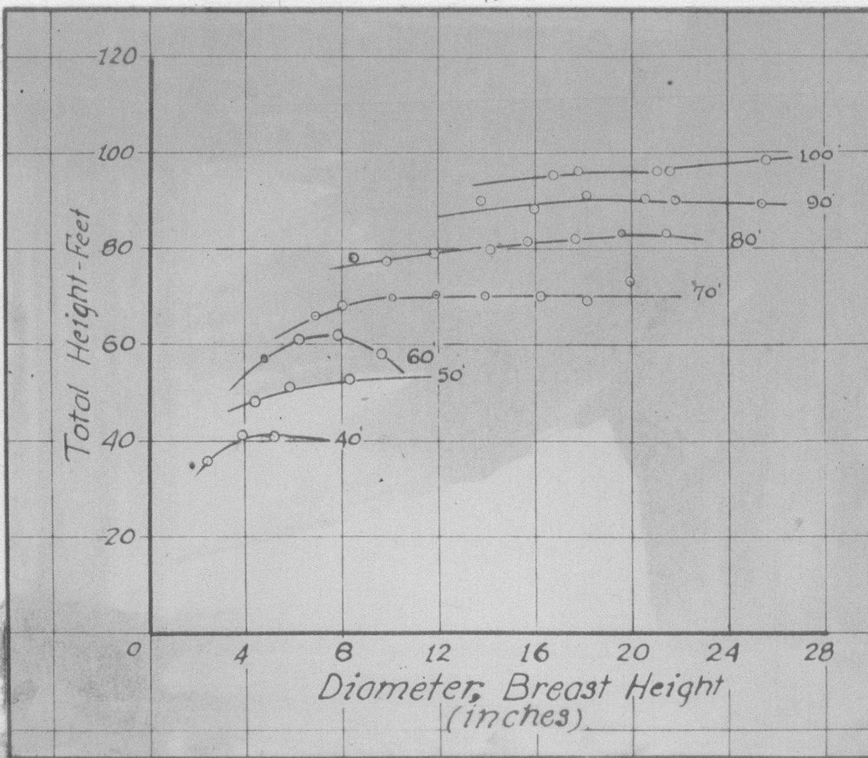

FIGURE 13 - The trend of height on d.b.h. within each 10-foot height class.

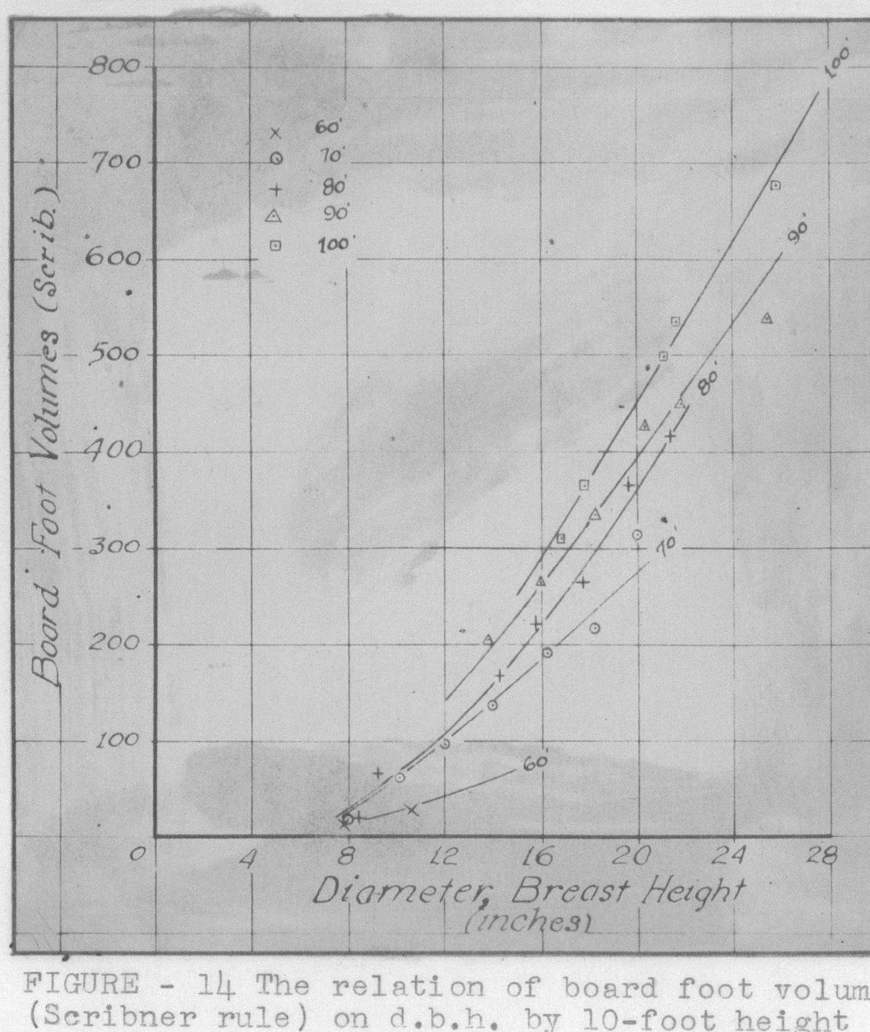

FIGURE - 14 The relation of board foot volume (Scribner rule) on d.b.h. by 10-foot height classes.

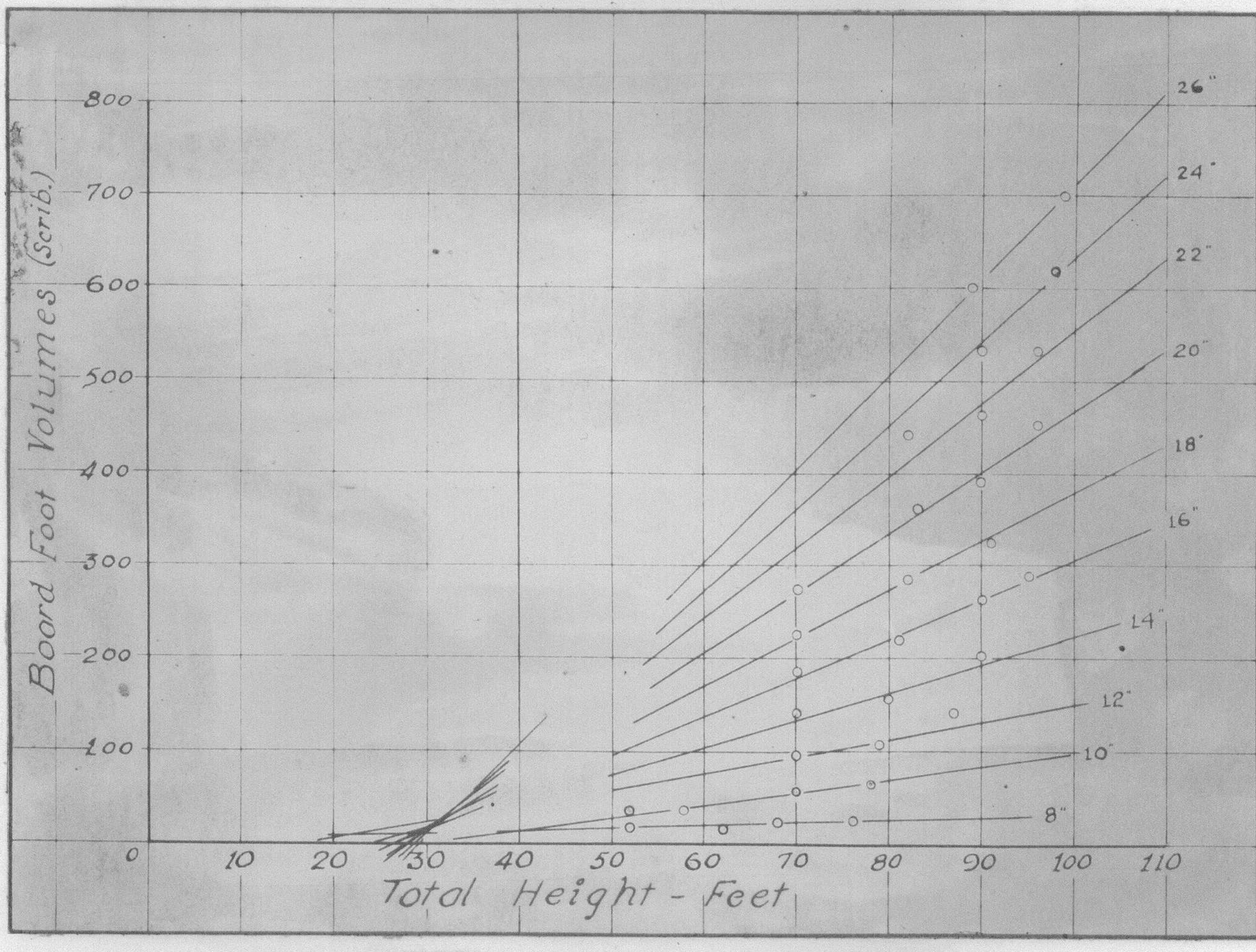

FIGURE 15 - The relation of board foot volume (Scribner rule) on total height by 2-inch diameter classes.

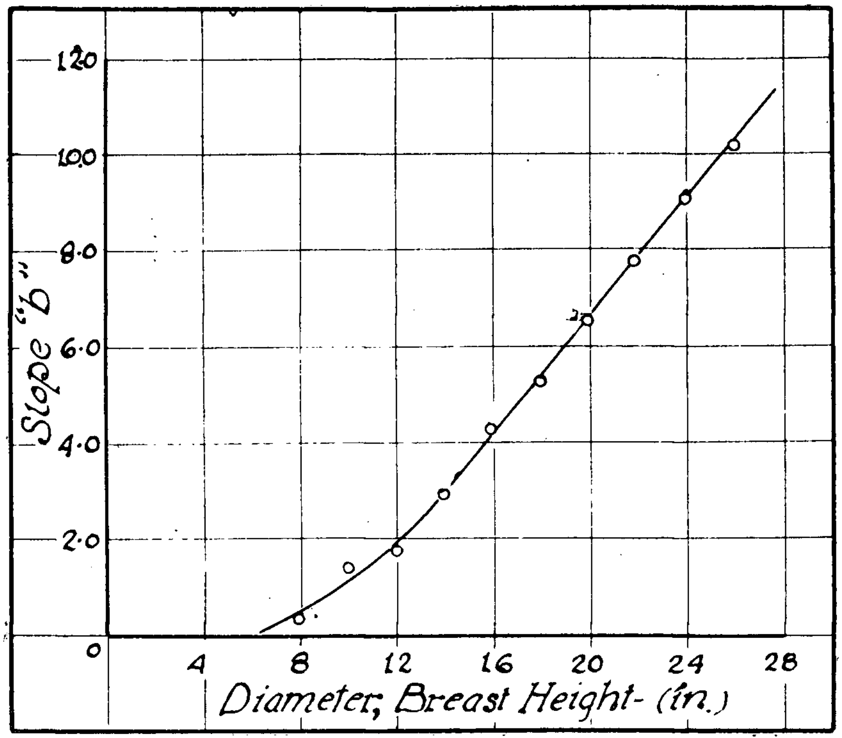

FIGURE 16 - Slope coefficients "b" for trend of Scribner board foot volume on total height, plotted on diameter at breast height.

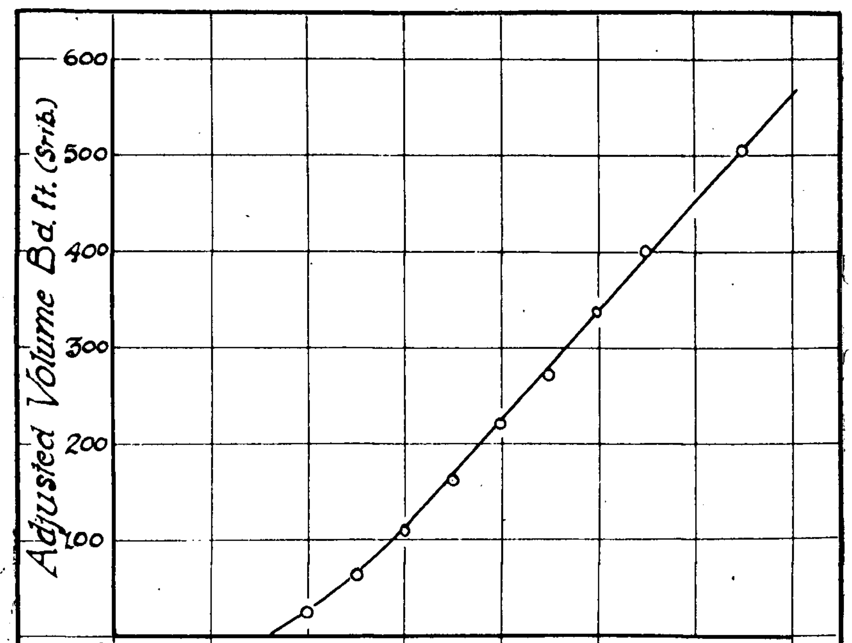

FIGURE 17 - Adjusted volume ratios (basis = 80 feet) plotted over diameter at breast height.

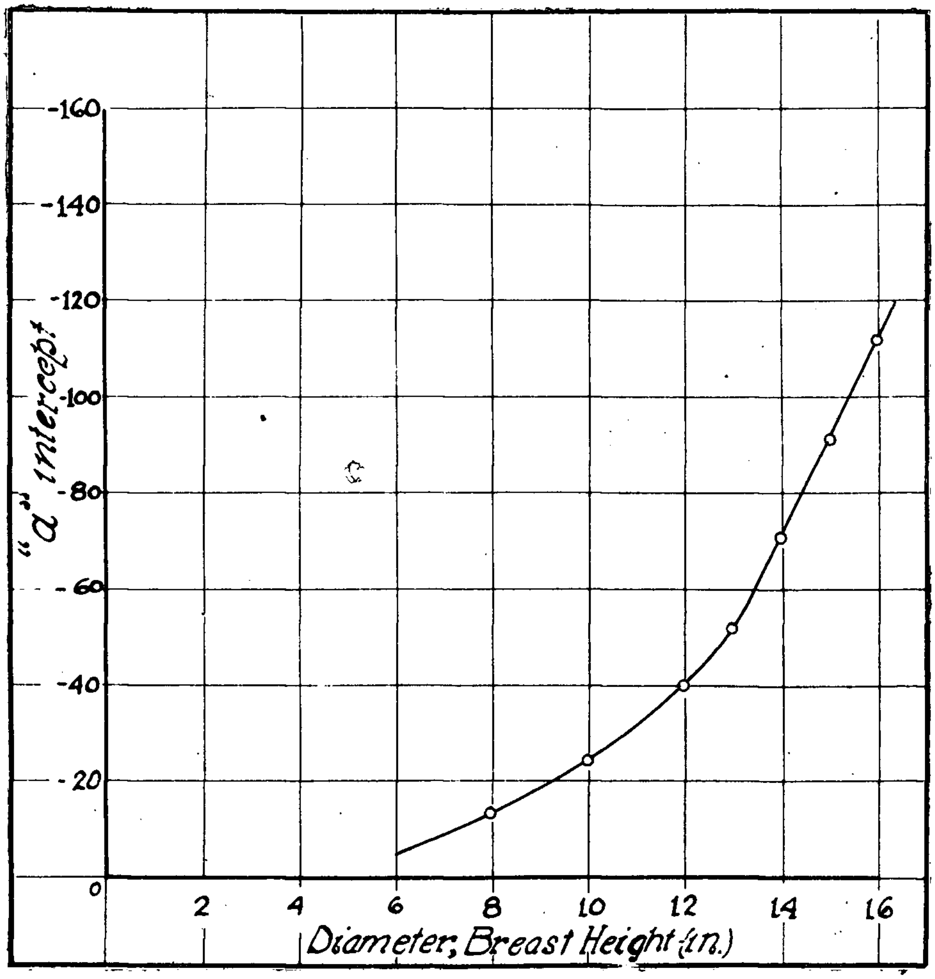

FIGURE 18 - Intercept coefficients "a" for trend of Scribner board foot volume on total height, plotted on diameter at breast height.

TABLE 23

VOLUME TABLE

Aspen (P. tremuloides Michx.) and Black Poplar[1] (P. balsamifera L.) Total Cubic Foot Volume Central and Northern Alberta

d.b.h. in.	Total Height - feet: 10	20	30	40	50	60	70	80	90	100	110	Basis number of trees
	Total peeled volume in cubic feet											
1	0.04	0.06	0.07									2
2	0.13	0.21	0.29	0.37	0.45							4
3	0.28	0.47	0.66	0.85	1.04							9
4		0.85	1.19	1.53	1.87	2.21						13
5		1.36	1.90	2.44	2.98	3.52						13
6			2.75	3.55	4.35	5.15	5.95					6
7				4.86	5.97	7.09	8.20	9.31				9
8				6.38	7.84	9.30	10.76	12.22	13.7			10
9				8.10	10.00	11.8	13.6	15.5	17.3			5
10					12.3	14.5	16.7	18.9	21.1			6
11					14.8	17.4	20.0	22.6	25.2			8
12					17.5	20.5	23.6	26.7	29.8	32.9		5
13					20.4	24.0	27.6	31.2	34.8	38.4		7
14					23.5	27.8	31.9	36.0	40.1	44.2		8
15					26.9	31.6	36.3	41.1	45.8	50.5		7
16					30.3	35.6	40.9	46.3	51.6	56.9	62.2	11
17						40.1	46.1	52.2	58.2	64.2	70.2	11
18						44.6	51.3	58.1	64.8	71.5	78.2	8
19						49.5	56.9	64.4	71.9	79.3	86.7	5
20						54.7	62.9	71.0	79.2	87.3	95.5	4
21							68.8	77.6	86.5	95.3	104.2	6
22							74.7	84.2	93.8	103.3	112.9	4
23								90.8	101.1	111.3	121.6	1
24								97.4	108.4	119.3	130.3	-
25								104.0	115.7	127.3	139.0	2
26								110.6	123.0	135.3	147.7	1
27								117.2	130.3	143.3	156.4	-
Basis number of trees	5	5	8	15	18	16	29	39	28	7	-	165

[1] for Black Poplar multiply tabular volumes by 0.842 - basis, 46 trees.

Heavy line indicates the extent of the original data.

Volumes include entire stem inside bark.

Table was prepared by adjusting aspen volume table, page 196, Dominion Form Class Volume Tables, 1948.

Aspen	: Aggregate difference, table	0.267 percent high
	: Average deviation	± 6.15 percent
Black Poplar	: Aggregate difference, table	0.023 percent low
	: Average deviation	± 6.09 percent

TABLE 24

VOLUME TABLE

Aspen (P.tremuloides Michx.) and Black Poplar[1]
(P.balsamifera L.) Merchantable Cubic Feet
Central and Northern Alberta

d.b.h. in.	Total Height Ft. 30	40	50	60	70	80	90	100	110	Basis number of trees
	Merchantable volume in cubic feet									
4		0.15	0.45	0.75						10
5		1.05	1.50	1.95	2.40					13
6		2.40	3.05	3.70	4.35					6
7		3.90	4.80	5.70	6.60	7.50				9
8		5.50	6.70	7.80	9.00	10.1	11.2			10
9		6.95	8.60	10.25	11.90	13.55	15.2			5
10		8.20	10.50	12.9	15.2	17.5	19.8			6
11			12.5	15.4	18.4	21.3	24.3			8
12			14.6	18.2	21.9	25.5	29.4			5
13			16.6	21.1	25.6	30.1	34.7	39.2		7
14			18.6	24.1	29.6	35.1	40.6	46.1		8
15				27.1	33.5	39.9	46.3	52.7		7
16				30.5	37.7	44.9	52.1	59.3	66.5	11
17				33.6	41.8	50.0	58.2	66.4	74.6	11
18				37.9	46.0	55.1	64.2	73.3	82.4	8
19				40.2	50.2	60.2	70.2	80.2	90.2	5
20				43.9	54.7	65.5	76.2	87.0	97.8	4
21					58.8	70.5	82.3	94.0	105.7	6
22					63.0	75.6	88.2	100.7	113.3	4
23						80.6	94.2	107.7	121.3	1
24						85.6	100.2	114.8	129.4	-
25						90.6	106.2	121.7	137.3	2
26						95.6	112.2	128.7	145.2	1
27							118.2	135.7	153.2	
28								142.7	161.2	
Basis, number of trees -		10	18	16	29	39	28	7	-	147

[1] for Black Poplar multiply tabular volumes by 0.817 - basis, number of trees 46.

Heavy line indicates the extent of the original data.
Volumes include stem from 1-foot stump to a 4-inch top, inside bark.
Table was prepared for aspen by mult-curvilinear methods.
Correction term 0.817 was applied to table values for estimating Black Poplar volumes.

Aspen	: Aggregate difference,	table 0.462 percent low.
	: Average deviation	± 6.18 percent
Black Poplar	: Aggregate difference,	table 0.114 percent high
	: Average deviation	± 6.49 percent.

TABLE 25

VOLUME TABLE

Aspen (P. tremuloides Michx.) and Black Poplar[1]
(P. balsamifera L.) Merchantable Board
(Scrib.) inside bark-Central and
Northern Alberta

d.b.h. in.	Total Height Feet 40	50	60	70	80	90	100	110	Basis Number of trees
	Merchantable volume in board feet (Scrib.)								
7		2	4	7	9				3
8		11	16	21	26	31			9
9		21	29	37	45	53			5
10		32	43	54	66	77			6
11		42	57	71	86	100			8
12			72	91	109	128			5
13			88	111	134	157	180		7
14			104	133	163	192	221		8
15			120	156	191	226	262		7
16			136	178	219	261	302		11
17			152	200	248	295	342	390	11
18			169	223	277	330	384	438	8
19			185	245	305	365	424	484	5
20			201	267	333	399	465	531	4
21				290	362	434	506	578	6
22				312	390	468	546	624	4
23					419	503	588	672	1
24					447	538	628	718	-
25					476	572	669	765	2
26					504	606	709	812	1
27						641	750	858	-
28							791	906	-
Basis number of trees	-	-	9	28	39	28	7	-	111

[1]for Black Poplar multiply tabular volumes by 0.787 - basis 46 trees.
Heavy line indicates the extent of the original data.
Volumes include stem from 1-foot stump to a 6-inch top inside bark.
Table prepared for aspen by multi-curvilinear methods.
Correction term 0.787 was applied to tabular values to estimate Black Poplar volumes.

Aspen	: Aggregate difference, table	0.126 percent high
	: Average deviation	± 12.65 percent
Black Poplar	: Aggregate difference, table	0.029 percent low
	: Average deviation	± 14.84 percent

TABLE 26

VOLUME TABLES

White Birch (B. Papyrifera Marsh.)
Central and Northern Alberta

d.b.h. (inches)	Total Height (feet)	Volumes inside bark: Total cubic feet	Merch. cubic feet	Merch. Board feet (Scrib.)
1	13	0.07	-	-
2	22	0.35	-	-
3	30	0.88	-	-
4	38	1.76	0.11	-
5	45	3.00	1.46	-
6	51	4.61	3.25	-
7	56	6.63	5.33	3
8	61	9.12	7.64	15
9	65	11.9	10.4	31
10	69	15.1	13.7	49
11	73	18.9	17.5	68
12	76	22.9	21.6	92
13	78	27.1	25.9	115
14	80	31.7	30.9	143

Based on a height /d.b.h. curve for 93 trees.

Total cubic volume interpolated from volume table for white birch, in Quebec (Dominion Form Class Volume Tables, page 182,1948).

Volumes in merchantable cubic and merchantable board feet (Scrib.) obtained by adjusting corresponding aspen volumes for these measures by a ratio equal to: total cubic volume for aspen divided by corresponding white birch total cubic volume.

Merchantable cubic volumes include stem from 1-foot stump to a 4-inch top inside bark.

Merchantable board foot volumes include stem from 1-foot stump to a 6-inch top inside bark.

APPENDIX

Basic Data

It is difficult to obtain measurements in forests for every age and site quality, but considering the small size of the sample a fair distribution among the possible combinations of age and site quality was obtained. This is illustrated in Table 27. The field method followed was to select fully stocked or normal plots from the best, the medium and the poor portions of stands of each age class. This procedure was satisfactory where a range in site existed. Some stands however, were very uniform and in such cases two plots representing the average condition were measured. Forests of other site qualities then, had to be sought out during the progress of the study. An average guide curve of dominant height over age previously prepared by the writer for the aspen in mixedwood stands offered some assistance in keeping a record of the approximate distribution of the site qualities sampled.

TABLE 27

PLOT DISTRIBUTION BY AGE AND SITE INDEX CLASSES

Aspen total age	Site index classes:					Totals
	46-55	56-65	66-75	76-85	86-95	
(years)			Number of plots			
6-15		1	3	1		5
16-25	2			5	1	8
26-35		1	4	4	2	11
36-45			3	4	2	9
46-55		2	6	1		9
56-65		3	5	1		9
66-75						-
76-85		2	5			7
86-95		2	2	4		8
96-105	1	2	3	1	2	9
106-115			1	2		3
Totals	3	13	32	23	7	78

Field Work

For the most part, aspen in Northern Alberta does not naturally occur in uniform well stocked stands extending over large areas but rather in small patches. Individual plots therefore had to be selected rather than composite plots such as taken by McArdle (7).

The plot boundaries were surveyed with a compass and steel tape. Most plots were rectangular but a few were taken with acute angles never less than 60 degrees. A minimum of 100 trees in the main stand was required on each plot. Size therefore varied from less than 1/10 acre to over 1/2 acre from young to old stands. All living trees were calipered and recorded by inch classes according to species. Small trees of the understory which were found in some stands above 80 years of age were recorded by inch classes. Those measuring less than 0.6 of an inch were recorded in two height groups, 6 inches to 3 feet and 3 feet plus. These smaller trees were not included in the yield computations.

For each plot, total height was measured with steel tape and Abney level except in dense young stands where trees were felled and measured directly. Two heights for each 1-inch diameter class were taken for stands below 80 years and for each 2-inch diameter class in older stands. Diameters and heights of dominants were recorded separately; they varied in number from 4 to 8 trees per plot.

Age determinations were made by counting the annual rings. It was found that consistent estimates could only be obtained by counting the rings on the butt of the tree after felling. It was necessary to make a very sloping cut with a sharp axe in order that minute rings could be discerned. Though the average age of the dominants was taken as plot age, the procedure adopted for each stand was first to make age counts throughout to ascertain if all trees could be considered even-aged. Such a condition was found to be generally true for this species. Since age determinations were usually made at several feet above the ground level, it was necessary to add the ages obtained at these heights the number of years required to reach the height of the stump. The values in Table 28 based on some 300 measurements of dominant aspen seedlings were used to convert the ring count to total age.

TABLE 28

CORRECTION IN YEARS TO BE ADDED TO AGE DETERMINED FROM INCREMENT BORINGS OR STUMP COUNTS, TO OBTAIN ASPEN TOTAL AGE

Distance from average ground level to boring or top of stump	Age Correction	Distance from average ground level to boring or top of stump	Age Correction
(inches)	(years)	(inches)	(years)
1	1	10	2
2	1	12	3
3	1	18	3
4	1	24	4
5	1	30	4
6	2	36	5
7	2	42	5
8	2	48	5
9	2	54	5

Office Computations

The basal area per plot for each diameter class was now computed and a height diameter curve drawn. The average height of each d.b.h. class and average dominant were obtained. In this last case, the height was read from the curve corresponding to the average of the dominant diameters recorded. This average was calculated by basal area. Total cubic, merchantable cubic and Scribner board foot volumes for each diameter class were now computed with the aid of volume tables (Tables 23 to 26 inclusive). Totals were then taken for each of five items (number of trees, basal area and the three volumes) and converted to an acre basis.

The basal area of the average trees for each plot was calculated by dividing the total basal area by the total number of trees. A ratio (total volume divided by basal area) adopted to provide a cross-check between volume and basal area in the later analysis was also computed.

Method of Analysis

The method of analysis employed was essentially that described by Osborne and Schumacher (15). To obtain curves of average dominant height over age by site index classes, the standard deviation and coefficient of variation within each age class was calculated first. These statistics plotted on age, Figure 19, provided the basis with which to construct the sheaf of curves illustrated earlier in Figure 2. By means of this graph a site index value could be given to each plot

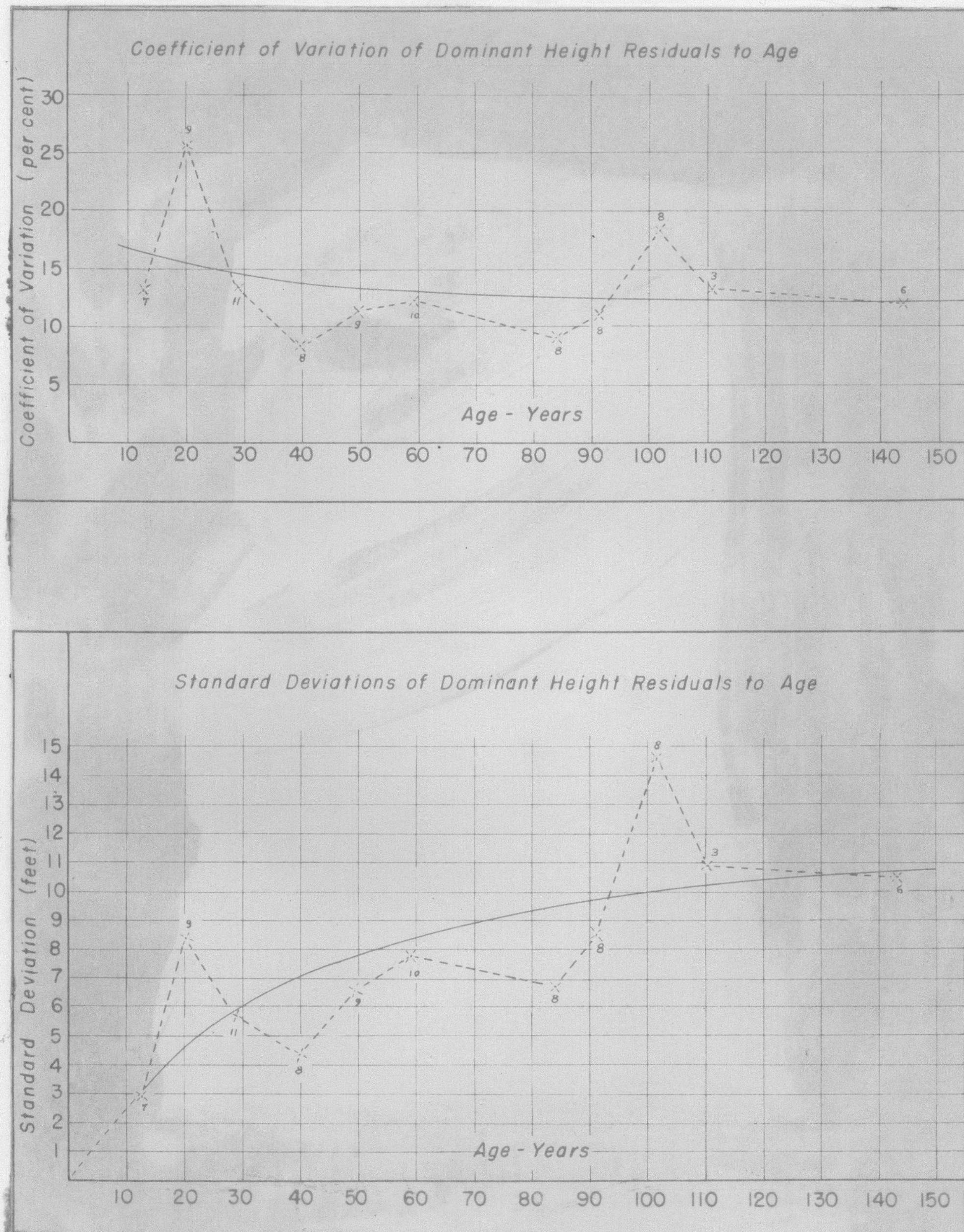

FIGURE 19 - The relation of the coefficients of variation and standard deviations of dominant height residuals to total age.

according to its age and dominant height.

Next, it was found necessary to reject 6 plots in the 140 year age group. This rather arbitrary action was considered expedient from the result of field observations. The aspen stands found at this age could not be considered truly fully stocked and in many cases evidence of breakup was obviously apparent. Inclusion of such unsatisfactory plots to develop normal yields was therefore considered to be of little value, particularly when no stands in the 120 and the 130 year age classes were located.

To ensure uniformity of density in the remaining plots the relation of number of trees to average diameter was used to test the data. The number of trees per acre was converted to logarithms, grouped into 2-inch classes and plotted over average stand diameter. The curve fitted to the plotted points is illustrated in Figure 20.

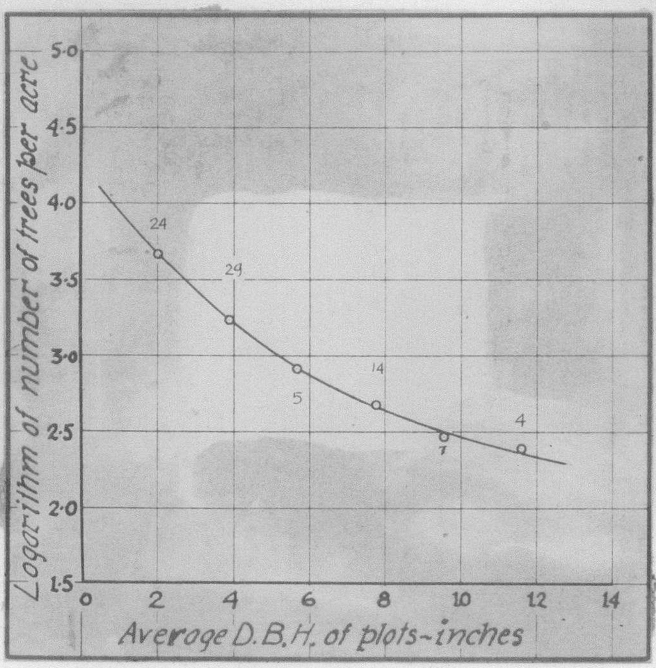

FIGURE 20 - The relation of the logarithm of number of trees to average d.b.h.

To test for normality the frequency distribution of the residuals from the curve was first obtained. These were converted to cumulative frequency percentages for each of which, appropriate probit values were extracted from tables given by Fisher and Yates (3). The distribution was now checked by plotting the upper class limits shown in the first column of Table 29 over the probit values given in the last column. The slightly sigmoidal curve indicated, revealed kurtosis, but not strongly enough to affect the balance of the procedure to any extent.

TABLE 29

FREQUENCY DISTRIBUTION OF RESIDUALS AND THE PROBITS CORRESPONDING TO THEIR CUMULATIVE FREQUENCY PERCENT

Class interval	Plots	Cumulative frequency	Probits
logarithms	number	percent	units
+0.20-0.22	1	1.2	2.74
+0.17-0.19	1	2.4	3.02
+0.14-0.16	1	3.6	3.20
+0.11-0.13	5	9.6	3.70
+0.08-0.10	5	15.7	3.99
+0.05-0.07	8	25.3	4.33
+0.02-0.04	14	42.2	4.80
-0.01-0.01	15	60.2	5.26
-0.04-0.02	11	73.5	5.63
-0.07-0.05	7	82.0	5.92
-0.10-0.08	5	87.9	6.17
-0.13-0.11	6	95.2	6.66
-0.16-0.14	1	96.3	6.79
-0.19-0.17	1	97.6	6.98
-0.22-0.20	1	98.8	7.26
-0.34-0.32	1	100.0	8.72 +
Total	83	-	-

The standard deviation of 0.089 log number of trees was calculated and following the accepted procedure, all plots falling outside of two standard deviations (2 x 0.089) were discarded. This meant a total of 5 plots were rejected, 4 understocked and 1 overstocked. The remaining 78 were employed in the analysis described below.

To ensure that the independent variables, age and site were not correlated, their interrelation was tested by means of a scatter diagram, Figure 21, and the "Corner test of Association" described by Olmstead and Tukey (14). No significant correlation between them was found.

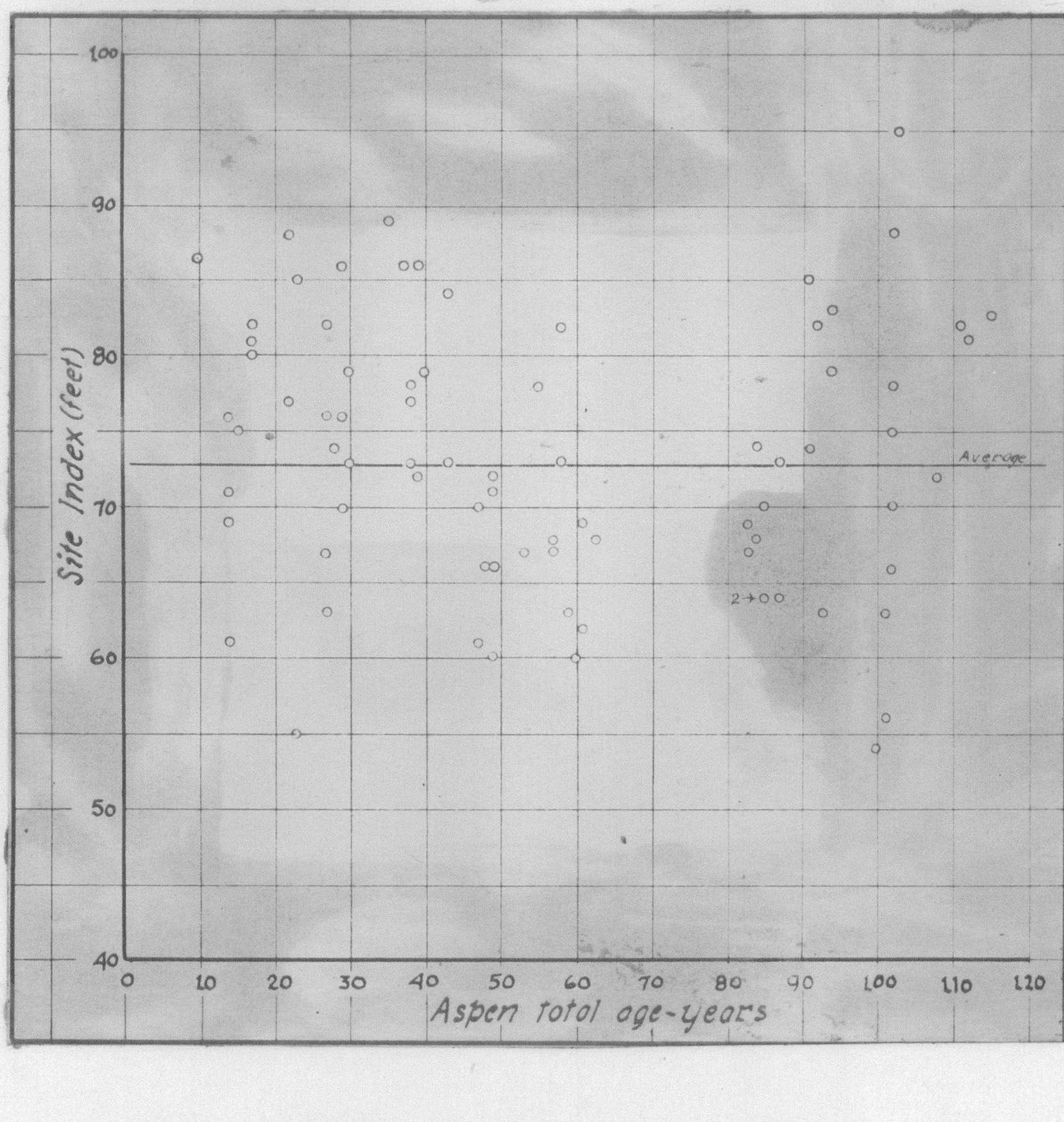

FIGURE 21 - The relation of site index to age; no correlation is shown.

The data from each plot were now sorted into 10-year age classes. The average values plotted on age formed the basic guide curves for number of trees, average diameter[1], basal area, total cubic volume/basal area and total cubic volume. A method of cross-checking is conveniently provided by this system of curves. The basal area per acre read from the mean curve for a given age, should equal the product of the corresponding number of trees per acre and the average basal area per tree. Also the product of this basal area and the volume/basal area ratio will give the corresponding total cubic volume per acre. The average guide curves were all brought into agreement by means of this cross-checking process.

To obtain curves of site index the residuals from these average curves were obtained and the standard deviation and coefficient of variation were computed for each age class. From the relation of these two measures with age a basis was provided to derive other curves of site quality about the average. The curves were brought into coincidence as before by the cross-checking of interrelated values. Figures 22 and 23 are shown to illustrate the trends of standard deviation and coefficient of variation with age, and standard units with site index, obtained when deriving total cubic volume yield for different age and site classes. Similar curves were

---

[1] average diameter = average basal for each age class divided by the corresponding average number of trees. These values did not coincide with the mean of the average diameters themselves. Therefore the corresponding values were plotted on double log paper and a line fitted. This regression was used as a basis to adjust the average diameter (sq.ft.) for each plot.

constructed for each of the five variables described above.

The yield tables derived from these curves were checked against the data from which they were constructed. The statistics obtained are given in Table 30. It will be noted that only for the number of trees item does the variation accounted for become smaller when the affect of site is taken into account. The reason for this was found to exist in stands 30 years and younger. In these, other influences overweigh the effect of site. When the plots were divided into two age groups, 10-30 and 40-110, the variation accounted for by site now increased by 9.1 percent in the older age class. The results of the computations are given below:

Item	Age only	Age and Site
Number of trees - age class 10-30 years		
Variation accounted for, percent	67.2	53.5
Number of trees - age class 40-110 years		
Variation accounted for, percent	80.3	89.4

In Table 30 the figures show that basal area is the least sensitive and total volume the most sensitive to the influence of site. The results are in general, comparable with those obtained by investigators who have presented yield tables for other species.

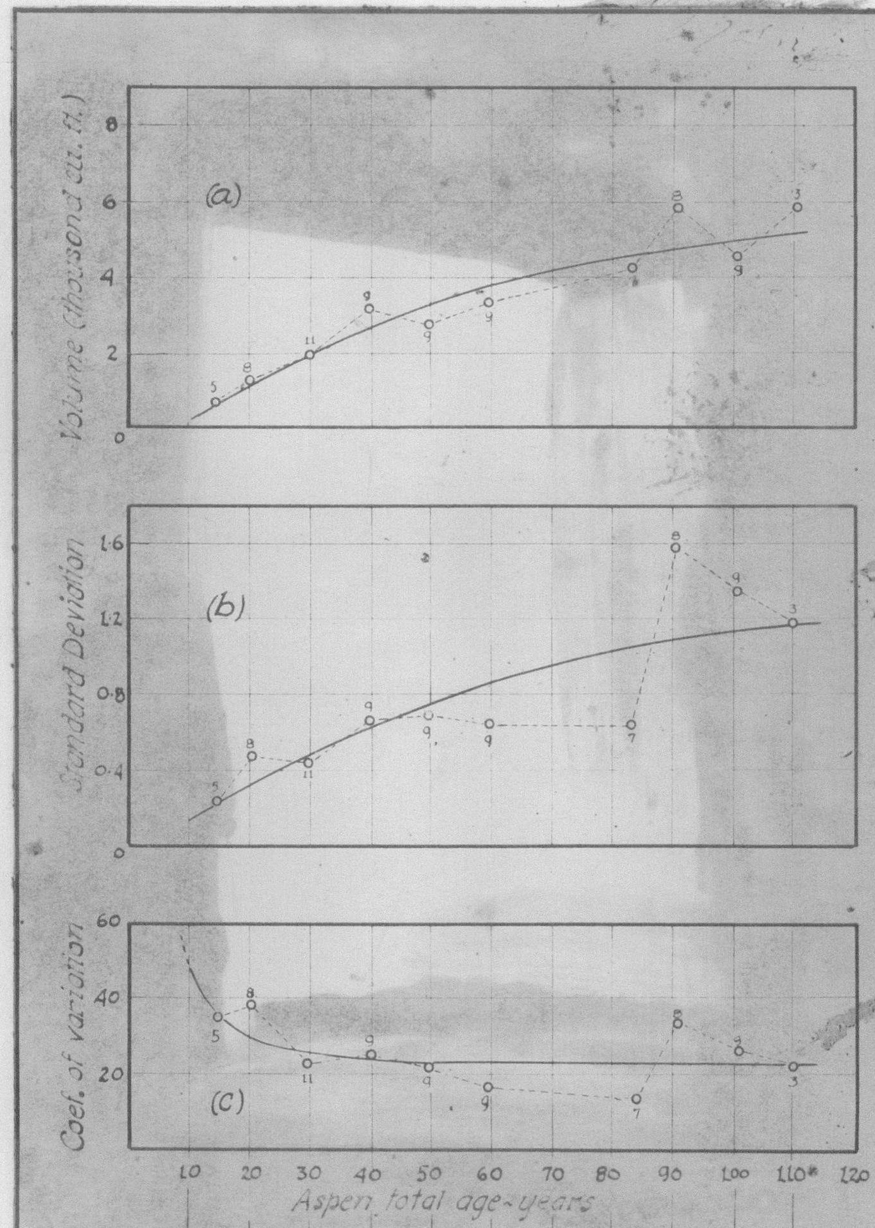

FIGURE 22 - Freehand curves showing in (a) the average relation of volume to age; in (b) the relation of the standard deviation of volume to age; and in (c) the coefficient of variation of volume to age.

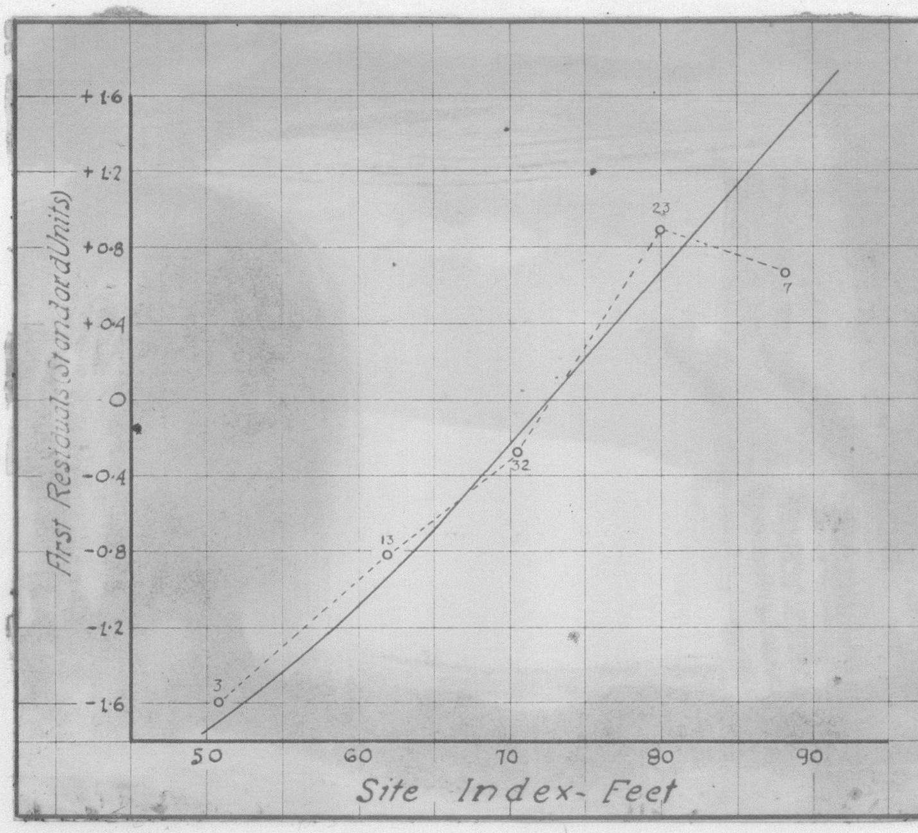

FIGURE 23 - The relation between the first residuals of total cubic volume (expressed in standard units for its age) and site index.

TABLE 30

STATISTICS ILLUSTRATING RELIABILITY OF YIELD TABLES, AND EFFECT OF AGE, AND SITE AND AGE

Item	Age only	Age and Site
NUMBER OF TREES		
Aggregate difference, percent	+0.776	+ 0.953
Standard error, number	691	785
Variation accounted for, percent	88.3	84.6
AVERAGE DIAMETER (inches)		
Aggregate difference, percent	+0.563	+ 0.687
Standard error, inches	3.7	2.8
Variation accounted for, percent	81.7	93.7
BASAL AREA (Square feet)		
Aggregate difference, percent	-0.311	- 0.725
Standard error, square feet	23	20
Variation accounted for, percent	69.4	76.0
TOTAL CUBIC VOLUME		
Aggregate difference, percent	-0.043	+ 0.139
Standard error, cubic feet	819	514
Variation accounted for, percent	76.0	94.4

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