THE LOCATION OF FLASHOVERS

ON

TRANSMISSION LINES

by

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Abstract

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SUMMARY

The object of this thesis is to find a method for locating transient as well as permanent faults on transmission lines. Transient faults are those lasting for a fraction of a second or so which do not cause serious enough damage to necessitate immediate repairs before the line may be re-energized. However, transient faults such as insulator flashovers may cause enough damage to be a potential permanent outage. It is thus desirable to be able to locate the position of the fault, and to inspect the line and insulators so that they may be repaired if necessary when the line can be conveniently removed from service.

The method that seemed most desirable was based on the echo-ranging principle such as is used in radar. This method has the advantages of accuracy and ease of interpretation. A damped sine wave pulse is generated at short intervals and fed onto the transmission line by means of a coupling capacitor. This pulse travels along the line and is partially reflected from any discontinuity such as a flashover to ground. The transmitted pulse, and pulses reflected from the end of the line and the fault are shown on a viewing tube; the distance to the fault being found by proportion.

The line is pulsed only on the occurrence of a

fault; thus any interference with radio is eliminated. The pulse generator is tripped by zero-sequence current or from the surge created by the fault itself.

The pulses were to be recorded on a skiatron or memory tube which holds the trace on the tube until it is erased at will by the operator. This eliminates the necessity of photographic equipment and the disadvantages of delay and inconvenience of developing the film.

The work accomplished on the project included the theory of wave propagation along transmission lines and the reflection to be expected for arcing ground faults. A pulse generator was built to produce either a damped sine wave or a sharp-fronted wave with exponential decay. Experiments were carried out on coaxial cable with carbon and oil arcs as the fault, but no experiments were carried out on actual transmission lines as no line was available. The results of these experiments and the theory indicate that the method should be satisfactory on transmission lines.

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1 INTRODUCTION

The purpose of this investigation is to develop a method for locating transient as well as permanent faults on transmission lines. Methods now exist for finding permanent faults accurately. However, transient faults comprise approximately ninety percent of all faults, and are important because a damaged line or insulator may result in a permanent outage at some future time.

If the faulty insulator can be located accurately, then the time and money spent patrolling the line can be considerably reduced. Even a small reduction in the amount of patrolling would justify the initial cost of the equipment. Thus it would seem that problem is economically worthwhile.

The method developed uses the echo-ranging principle. This principle is by no means new, and is being used at present to locate non-transient faults on transmission lines and cables. In the equipment developed, a pulse generator is to be tripped from the initial surge of the flashover itself, or from the zero-sequence current in a transformer bank neutral. The pulses are reflected from the arc and returned to the sending end where they are shown along with the transmitted pulse on a viewing tube.

The pulses are sent out for a time interval exceeding the arc duration in order to ensure a clear reflection from the end of the line. The distance to the fault can then be found by proportion.

11 REVIEW OF LITERATURE

Several methods for locating transient faults are in use, and have met with various degrees of success. Before these methods can be judged it is necessary to state the desirable qualities of a fault locator. These are as follows: (1) accuracy; (2) ease of interpretation; (3) simple recording; (4) low cost; (5) safety.

There are various fault-locating devices which do not depend on echo-ranging principles. One of these devices is adapted to measure and record variations in current and potential at one or more points in the length of the line by means of magnetic links[20]. The fault is found approximately by calculations dependent on the degree of magnetism of the links. This method, however, lacks the qualities of accuracy and ease of interpretation. Another system uses an annunciator ammeter. Precision is limited here also because the current is measured in steps, and because of variation in tower footing resistance. A method using automatic oscillographs has been developed to locate faults[10]. The distance is found by analysis of the recorded currents and voltages. The accuracy of this system is of the order of ten per cent. Thus the main disadvantage of these and other such devices is lack of accuracy.

Fault-locating methods utilizing travelling waves come closest to fulfilling the desirable qualities as listed previously. These systems can be classified under three main headings.

20 For references see Bibliography.

(1) Fault-Generated surge method[29]. This method has just recently been developed, and from it two types of fault locator have been produced. In one of these the surge from the fault travels to the station end of the line where it starts a timing device. It is then reflected from the bus back to the fault where it is returned once more to the end of the line. The time interval between the two pulses is proportional to the distance from the fault. The accuracy of this system depends to a large extent on the steepness of the wave front and the length of the time base employed. The investigators hope that with refinements a precision of plus or minus 0.1 mile will be obtainable. The limitations of this fault locator are: operation on spurious pulses; the necessity of photography; and the training required for interpretation of the results.

The second method makes use of both of the surges which originate at the fault and travel down the line in opposite directions. One arrives at the near end of the line and triggers a timing device. The other arrives at the far end and causes a radio transmitter to send out a pulse which stops the timing device. The time interval is used to find the distance to the fault. An electronic time interval counter records the distance directly in miles. This system has all the advantages listed except low cost. It is thus economically practical only where a radio link is already in existence.

(2) Frequency Modulation Method. A high frequency voltage

with linear variation of frequency is impressed on the end of the line. The waves travel to the fault and back, and the difference in frequency at the sending end is thus proportional to the distance to the fault. This system has not the accuracy desirable.

(3) Echo-Ranging Methods. The Hydro-Electric Power Commission of Ontario have a transient fault locator under development[21]. A high frequency burst is impressed through a coupling capacitor onto the transmission line. The line is monitored this way continuously, and the pulses are recorded by means of an automatic photographic attachment fitted to the cathode-ray tube. This method is accurate and easily interpreted, but has the disadvantage of requiring photographic equipment. Also the pulses must not have a frequency that will interfere with carrier-current devices or radio waves. Experiments performed by the Commission indicate that radiation from their pulses was in the limits allowable.

111 INVESTIGATION

A. Choice of Method

A review of the existing methods indicates that one of the main disadvantages is the necessity for photography which entails a delay before any information is available. To eliminate this disadvantage it was decided to use a long memory tube which retains the trace until it is examined and erased by the operator. An echo-ranging method was chosen since it is suitable for this type of recording, and has the additional qualifications of accuracy, ease of interpretation, and reasonable cost.

To eliminate any interference with radio, since pulses of broadcast frequency may be used, the pulses are sent out only for a fraction of a second when the fault occurs. It has been suggested that the high voltage pulses originating from the fault and their successive reflections may obscure the relatively weak transmitted pulses. However, the reflected fault-generated pulses are soon attenuated, have a different shape from the artificial pulse, and are not synchronized with the sweep. On the other hand, the transmitted pulses should show up clearly because of the many retraces. The results will depend largely on the tube characteristics. If the fault surges give more trouble than anticipated, the artificial pulses may be delayed until these surges have attenuated greatly, but not until the power arc is extinguished.

B. Characteristics of Power Arcs.

Something of the characteristics of power arcs must be known before the type of reflection to be expected from an arcing ground can be calculated. Interest is thus mainly concentrated on the resistance of the arc path. An average voltage gradient at current peak for 60-cycle arcs in air with currents ranging from less than 100 to over 20,000 amperes peak current is 34 volts per inch[30]. Thus at peak current for say a 7-foot arc, the voltage would be 2850 volts. The corresponding current depends on the various impedances to the fault, but would probably be over 500 rms amperes or 707 peak amperes. The resulting arc resistance is 4 ohms at peak current. For other points in the half cycle the resistance increases slightly, and becomes much higher near the zero current point.

The arc resistance would be the only consideration for a line-to-line fault. However, for arcing ground faults which comprise about 80 per cent of all faults[4], the resistance to ground depends also on the tower footing resistance. This resistance may vary from below an ohm to thousands of ohms, but in general will probably be below 25 ohms.

C. Theory of Wave Propagation.

It is desirable to be able to calculate the modifications that a pulse undergoes as it travels along the transmission line, and to know what magnitude and shape of reflection is to be expected from a given discontinuity. As an approximation, the losses are sometimes neglected as this results in great simplification in mathematics.

1. No-Loss Lines.

(a) Single-circuit lines.

The fundamental differential equations for waves on single-circuit lines will now be derived. The derivation follows that given in reference 2, Chapter 6.

For a short length of line δx let

r = resistance in ohms per unit length

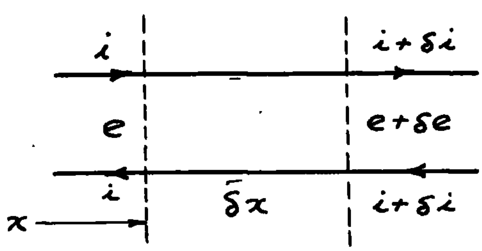
l = inductance in henries per unit length

c = capacitance between conductors in farads per unit length

g = leakage conductance between conductors in mhos per unit length

e = voltage between lines at the beginning of δx

i = current in lines at the beginning of δx



The change in voltage along δx = $\delta e = -ri\,\delta x - l\frac{\partial i}{\partial t}\delta x$

The change in current along $\delta x = \delta i = -eg\,\delta x - c\frac{\partial e}{\partial t}\delta x$

As $\delta x \to 0$ the equations become

$$-\frac{\partial e}{\partial x} = ri + lpi = (r+lp)i \qquad 1$$

$$-\frac{\partial i}{\partial x} = (g+cp)e \qquad 2$$

where $p \equiv \frac{\partial}{\partial t}$

Differentiating (1) with respect to x and substituting in (2)

$$-\frac{\partial^2 e}{\partial x^2} = (r+lp)\frac{\partial i}{\partial x}$$

$$\therefore \quad \frac{\partial^2 e}{\partial x^2} = (r+lp)(g+cp)e \qquad 3$$

Differentiating (2) with respect to x and substituting in (1)

$$\frac{\partial^2 i}{\partial x^2} = (r+lp)(g+cp)\,i \qquad 4$$

let

$$k = \sqrt{(r+lp)(g+cp)} = \sqrt{lc}\sqrt{\left(\frac{r}{l}+p\right)\left(\frac{g}{c}+p\right)}$$

$$v = \frac{1}{\sqrt{lc}}$$

$$\alpha = \frac{r}{2l}+\frac{g}{2c} \qquad \beta = \frac{r}{2l}-\frac{g}{2c}$$

Then

$$k = \frac{1}{v}\sqrt{(\alpha+\beta+p)(\alpha-\beta+p)} = \frac{1}{v}\sqrt{(p+\alpha)^2-\beta^2}$$

From (3) and (4)

$$\frac{\partial^2 e}{\partial x^2} = k^2 e \qquad \frac{\partial^2 i}{\partial x^2} = k^2 i$$

Treating these as ordinary linear differential equations the solutions are

$$e = \epsilon^{-kx}A + \epsilon^{kx}B \qquad 5$$

$$i = \epsilon^{-kx}C + \epsilon^{kx}D \qquad 6$$

where A, B, C, D are functions of t only. From equations (2), (5), and (6)

$$-\frac{\partial i}{\partial x} = (g+cp)e = \left(\epsilon^{-kx}A + \epsilon^{kx}B\right)(g+cp)$$

$$= k\epsilon^{-kx}C - k\epsilon^{kx}D$$

Since the equation must be true for all values of x, it must be an identity. Equating coefficients

$$C = \frac{1}{k}(g+cp)A' \qquad D = -\frac{1}{k}(g+cp)B$$

$$C = \sqrt{\frac{g+cp}{r+lp}}\,A \qquad D = -\sqrt{\frac{g+cp}{r+lp}}\,B$$

Let $$\mathfrak{z} = \sqrt{\frac{r+lp}{g+cp}} = \sqrt{\frac{l}{c}}\sqrt{\frac{\frac{r}{l}+p}{\frac{g}{c}+p}} = \sqrt{\frac{l}{c}}\sqrt{\frac{\alpha+\beta+p}{\alpha-\beta+p}}$$

and let $Z = \sqrt{\frac{l}{c}}$ $\qquad \therefore\ \mathfrak{z} = Z\sqrt{\frac{\alpha+\beta+p}{\alpha-\beta+p}}$

Thus (5) and (6) become

$$e = \epsilon^{-kx}A + \epsilon^{kx}B \tag{7}$$

$$i = \epsilon^{-kx}C + \epsilon^{kx}D = \frac{1}{\mathfrak{z}}\epsilon^{-kx}A - \frac{1}{\mathfrak{z}}\epsilon^{kx}B = \frac{1}{\mathfrak{z}}\left[\epsilon^{-kx}A - \epsilon^{kx}B\right] \tag{8}$$

For a line extending to infinity, B in equations (7) and (8) must be zero since e and i must be finite as $x \to \infty$

$$\therefore\ e = \epsilon^{-kx}A \tag{9}$$

$$i = \frac{1}{\mathfrak{z}}\epsilon^{-kx}A \tag{10}$$

Thus $\frac{e}{i} = \mathfrak{z}$ where $\mathfrak{z}$ is called the surge impedance operator. For very steep wave fronts the effect of the operator p is predominant and $\mathfrak{z} \to Z = \sqrt{\frac{l}{c}}$. Z is called the surge impedance.

If the applied voltage is given as a function of t

$$e_{x=0} = E(t)$$

Thus from (9) $$A = E(t)$$

$$\therefore\ e = \epsilon^{-kx}E(t) \tag{11}$$

$$i = \frac{1}{\mathfrak{z}}\epsilon^{-kx}E(t) \tag{12}$$

For the loss-free line $\quad r = g = 0$

Thus $k = \frac{p}{v}$ and $\mathfrak{z} = Z$

$$\therefore \quad i = \frac{1}{Z} \epsilon^{-\frac{px}{v}} E(t) \qquad 13$$

But by Taylor's theorem

$$f(t+a) = f(t) + a f'(t) + \frac{a^2}{\underline{|2}} f''(t) + \cdots$$

$$= \left(1 + ap + \frac{a^2}{\underline{|2}} p^2 + \cdots\right) f(t) = \epsilon^{ap} f(t)$$

$$\therefore \quad i = \frac{1}{Z} E\left(t - \frac{x}{v}\right) \qquad 14$$

This equation shows that for an ideal line the wave travels with no distortion or attenuation at a velocity

$$v = \frac{1}{\sqrt{lc}}$$

(b) Multi-conductor Systems.

The previous section was concerned with a single-phase line with either a line or perfectly conducting earth return. When practical power systems are considered, the above theory must be modified because waves are induced in the conductors not carrying the main waves. A knowledge of the magnitude of these induced waves is necessary for the calculation of the reflection expected from various types of faults. These induced waves also cause a modification in the type and rate of attenuation and distortion of the main waves.

If losses are neglected, it can be shown that the currents and voltages on the lines of the system are related by a set of linear equations. The following treatment follows that given in reference 2, Chapter 6.

Fig. 1 shows the system of m parallel fixed conductors to be considered. The dimensions are

ρ = radius of conductor r

h = height of conductor r above the zero potential plane

a = distance between r and the image of s

b = distance between r and s

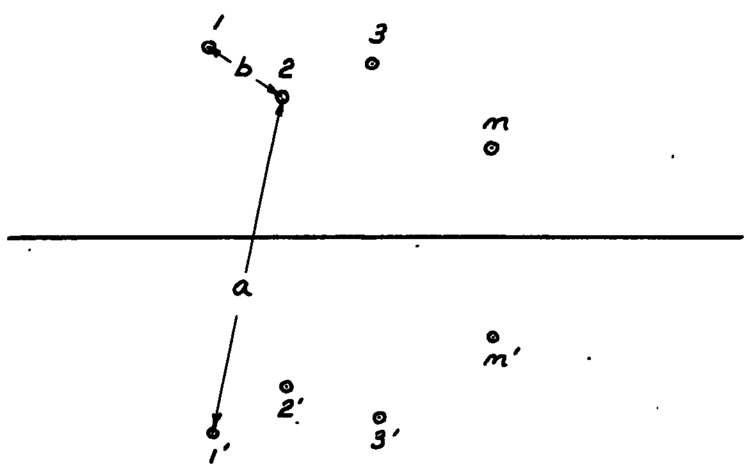


Fig. 1. System of conductors and their images.

From electrostatics the following equations may be written:

$$\left.\begin{aligned} e_1 &= p_{11}Q_1 + p_{21}Q_2 + \cdots + p_{n1}Q_m \\ e_2 &= p_{12}Q_1 + p_{22}Q_2 + \cdots + p_{m2}Q_m \\ &\cdots\cdots\cdots\cdots\cdots \\ e_n &= p_{1n}Q_1 + p_{2n}Q_2 + \cdots + p_{nn}Q_m \end{aligned}\right\} \tag{15}$$

where e_k is the potential of conductor k, Q_k the charge per unit length, and p_{rr} and p_{rm} are the reciprocal of capacities and can be calculated accordingly. The general equations are:

$$P_{rr} = 2 \ln \frac{2h}{\rho} \times 9 \times 10^{11} \quad (\text{farads})^{-1} \text{ per cm}$$
$$P_{sr} = P_{rs} = 2 \ln \frac{a}{b} \times 9 \times 10^{11} \quad (\text{farads})^{-1} \text{ per cm} \qquad 16$$

Equations (15) may be written

$$\left.\begin{aligned} q_1 &= K_{11} e_1 + K_{12} e_2 + \cdots + K_{1m} e_m \\ q_2 &= K_{21} e_1 + K_{22} e_2 + \cdots + K_{2m} e_m \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ q_m &= K_{m1} e_1 + K_{m2} e_2 + \cdots + K_{mm} e_m \end{aligned}\right\} \qquad 17$$

where $K_{rs} = (-1)^{r+s} \dfrac{[\text{minor of } D \text{ for which cofactor is } P_{rs}]}{D}$ 18

and $$D = \begin{vmatrix} P_{11} & P_{21} & \cdots & P_{n1} \\ P_{12} & P_{22} & \cdots & P_{n2} \\ \cdot & \cdot & \cdot & \cdot \\ P_{1n} & P_{2n} & \cdots & P_{nn} \end{vmatrix} \qquad 19$$

It can be seen that P_{rr}, P_{rs}, K_{rr} are positive, and that K_{rs} is negative if $r \neq s$.

The flux linking conductor r due to its own current is $L_{rr} i_r$, and the flux linking it due to the current in conductor s is $L_{rs} i_s$ where L_{rr} = self-inductance coefficient of conductor r, and

L_{rs} = mutual-inductance coefficient between r and s.

Thus the total flux linkages for the conductors are

$$\left.\begin{aligned} \varphi_1 &= L_{11} i_1 + L_{12} i_2 + \cdots + L_{1m} i_m \\ \varphi_2 &= L_{21} i_1 + L_{22} i_2 + \cdots + L_{2m} i_m \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \varphi_m &= L_{m1} i_1 + L_{m2} i_2 + \cdots + L_{mm} i_m \end{aligned}\right\} \qquad 20$$

The inductances are given by

$$\left.\begin{aligned} L_{rr} &= \left(\tfrac{1}{2} + 2 \ln \frac{2h}{\rho}\right) 10^{-9} \text{ henry per cm} \\ L_{rs} &= \left(2 \ln \frac{a}{b}\right) 10^{-9} \text{ henry per cm} \end{aligned}\right\} \qquad 21$$

The general differential equations of the traveling waves will now be obtained. The circuit constants invelved are shown in Fig. 2 and designated as follows:

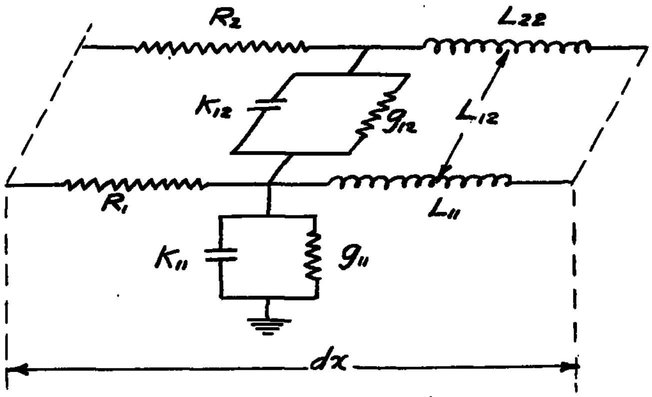


Fig. 2. Circuit constants for mutually coupled circuits.

K_{rr} = self-capacitance coefficient of conductor r

K_{rs} = mutual-capacitance coefficient between r and s

R_r = series resistance of conductor r

g_{rr} = leakage conductance to ground of conductor r

g_{rs} = leakage conductance between r and s

also let

$$G_{rr} = g_{r1} + g_{r2} + g_{r3} + \cdots + g_{rm}$$
$$G_{rs} = G_{sr} = -g_{rs} = -g_{sr}$$
$$Z_{rr} = R_r + pL_{rr}$$
$$Z_{rs} = pL_{rs}$$
$$Y_{rr} = G_{rr} + pK_{rr}$$
$$Y_{rs} = G_{rs} + pK_{rs}$$
$$p = \frac{\partial}{\partial t}$$

The leakage currents are

$$\left.\begin{aligned} i_1' &= g_{11} e_1 + g_{12}(e_1 - e_2) + \cdots + g_{1m}(e_1 - e_m) \\ &= G_{11} e_1 + G_{12} e_2 + \cdots + G_{1m} e_m \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ i_m' &= g_{m1}(e_m - e_1) + g_{m2}(e_m - e_2) + \cdots + g_{mm} e_m \\ &= G_{m1} e_1 + G_{m2} e_2 + \cdots + G_{mm} e_m \end{aligned}\right\} \qquad 22$$

The differential equations of the first conductor are

$$-\frac{\partial e_1}{\partial x} = \frac{\partial \Phi_1}{\partial t} + R_1 i_1 = Z_{11} i_1 + Z_{12} i_2 + \cdots + Z_{1m} i_m \qquad 23$$

$$-\frac{\partial i_1}{\partial x} = \frac{\partial Q_1}{\partial t} + i_1' = Y_{11} e_1 + Y_{12} e_2 + \cdots + Y_{1m} e_m \qquad 24$$

Differentiating equation (23) with respect to x and substituting equation (24) there is

$$\begin{aligned} \frac{\partial^2 e_1}{\partial x^2} &= (Z_{11} Y_{11} + Z_{12} Y_{21} + \cdots + Z_{1m} Y_{m1}) e_1 \\ &\quad + (Z_{11} Y_{12} + Z_{12} Y_{22} + \cdots + Z_{1m} Y_{m2}) e_2 \\ &\quad \cdots\cdots\cdots\cdots\cdots\cdots \\ &\quad + (Z_{11} Y_{m1} + Z_{12} Y_{m2} + \cdots + Z_{1m} Y_{mm}) e_m \\ &= J_{11} e_1 + J_{12} e_2 + \cdots + J_{1m} e_m \end{aligned} \qquad 25$$

where $J_{rs} = Z_{1r} Y_{1s} + Z_{2r} Y_{2s} + Z_{3r} Y_{3s} + \cdots + Z_{mr} Y_{ms}$ 26

Let $A_{rr} = \left(J_{rr} - \frac{\partial^2}{\partial x^2}\right)$ 27

The complete set of differential equations for the m conductors are then

$$\left.\begin{aligned} 0 &= A_{11} e_1 + J_{12} e_2 + \cdots + J_{1m} e_m \\ 0 &= J_{21} e_1 + A_{22} e_2 + \cdots + J_{2m} e_m \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ 0 &= J_{m1} e_1 + J_{m2} e_2 + \cdots + A_{mm} e_m \end{aligned}\right\} \qquad 28$$

where the J's are operators in the time derivative $p = \frac{\partial}{\partial t}$, and the A's are operators in both time and space derivatives. For any e to have a finite value the determinant of the coefficients must be zero.

Thus

$$\begin{vmatrix} A_{11} & J_{12} & \cdots & J_{1m} \\ J_{21} & A_{22} & \cdots & J_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ J_{m1} & J_{m2} & \cdots & A_{mm} \end{vmatrix} e = 0 \qquad 29$$

It can be seen that equation 29 will lead to a polynomial of degree m in $\frac{\partial^2}{\partial x^2}$ and degree m in $p^2 = \frac{\partial^2}{\partial t^2}$. This partial differential equation will give the most general solution to the problem.

If there are no losses the equations are simplified considerably since $R = G = 0$ and

$$ZY = p^2 LK$$

Thus equation 26 becomes

$$J_{rs} = p^2 \left(L_{1r} K_{1s} + L_{2r} K_{2s} + \cdots + L_{mr} K_{ms} \right) = p^2 D_{rs} \qquad 30$$

and equation 27 becomes $A_{rr} = \left(p^2 D_{rr} - \frac{\partial^2}{\partial x^2} \right)$ 31

If e is assumed to be a traveling wave

$$e = f(x + vt) \qquad 32$$

Thus

$$\left. \begin{aligned} J_{rs} e &= v^2 D_{rs} f''(x + vt) \\ A_{rr} e &= (v^2 D_{rr} - 1) f''(x + vt) = v^2 B_{rr} f''(x + vt) \end{aligned} \right\} \qquad 33$$

Where f'' denotes the second derivative with respect to x. Equation 29 now becomes

$$\begin{vmatrix} v^2 B_{11} & v^2 D_{12} & \cdots & v^2 D_{1m} \\ v^2 D_{21} & v^2 B_{22} & \cdots & v^2 D_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ v^2 D_{m1} & v^2 D_{m2} & \cdots & v^2 B_{mm} \end{vmatrix} f^{2m}(x + vt) = 0 \qquad 34$$

On dividing by $v^{2m} f^{2m}(x + vt)$, equation 34 gives

$$\begin{vmatrix} B_{11} & D_{12} & \cdots & D_{1m} \\ D_{21} & B_{22} & \cdots & D_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ D_{m1} & D_{m2} & \cdots & B_{mm} \end{vmatrix} = 0 \qquad 35$$

The velocity v must satisfy this equation of degree m in v^{-2}. Thus there are $2m$ values for the velocity of propogation (m positive and m negative). On each conductor there then can exist m pairs of waves of velocities $v_1, v_2, \cdots v_m$, each pair consisting of a forward and backward wave. Therefore

$$e_1 = f_{11}(x - v_1 t) + F_{11}(x + v_1 t) + \cdots + f_{1m}(x - v_m t) + F_{1m}(x + v_m t) \qquad 36$$

Integrating equation (24) partially with respect to x and remembering that $Y = pK$ for no losses

$$i_1 = -\frac{\partial}{\partial t}\int (K_{11} e_1 + K_{12} e_2 + \cdots + K_{1m} e_m)\, dx$$

$$= \sum v_r \left[K_{11}(f_{1r} - F_{1r}) + K_{12}(f_{2r} - F_{2r}) + \cdots + K_{1m}(f_{mr} - F_{mr}) \right] \qquad 37$$

For high frequency waves there is no internal magnetic field and equations (21) become

$$L_{rr} = 2 \ln \frac{2h}{\rho} \times 10^{-9} = \frac{P_{rr}}{c^2} \text{ henrys per cm} \qquad 38$$

$$L_{rs} = 2 \ln \frac{a}{b} \times 10^{-9} = \frac{P_{rs}}{c^2} \text{ henrys per cm} \qquad 39$$

where $c = 3 \times 10^{10}$ cm per sec

From equation (30)

$$D_{rs} = c^{-2}(p_{1r} K_{1s} + p_{2r} K_{2s} + \cdots + p_{mr} K_{ms}) \qquad 40$$

By reference to equations (18) and (19) it can be shown that

$$I_{rs} = \begin{cases} 0 & \text{if } r \neq s \\ c^{-2} & \text{if } r = s \end{cases} \qquad 41$$

Thus $B_{rr} = (c^{-2} - v^{-2}) = B$ 42

and determinate (35) reduces to $B^m = (c^{-2} - v^{-2}) = 0$ 43

Therefore $v = \pm c$ and all waves have the velocity of light.

Equations (37) thus become

$$\left.\begin{aligned} i_1 &= y_{11}(f_1-F_1) + y_{12}(f_2-F_2) + \cdots + y_{1m}(f_m-F_m) \\ i_2 &= y_{21}(f_1-F_1) + y_{22}(f_2-F_2) + \cdots + y_{2m}(f_m-F_m) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ i_m &= y_{n1}(f_1-F_1) + y_{n2}(f_2-F_2) + \cdots + y_{nn}(f_n-F_m) \end{aligned}\right\} \qquad 44$$

where $y_{rr} = cK_{rr}$ = self surge admittance
$y_{rs} = cK_{rs}$ = mutual surge admittance. } 45

If the current is defined by equations similar to (36) the following equations are obtained:

$$\left.\begin{aligned} e_1 &= z_{11}(g_1-G_1) + z_{21}(g_2-G_2) + \cdots + z_{n1}(g_n-G_n) \\ e_2 &= z_{12}(g_1-G_1) + z_{22}(g_2-G_2) + \cdots + z_{n2}(g_n-G_m) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ e_n &= z_{1n}(g_1-G_1) + z_{2n}(g_2-G_2) + \cdots + z_{nn}(g_n-G_n) \end{aligned}\right\} \qquad 46$$

where $z_{rr} = \dfrac{P_{rr}}{c} = 60 \ln \dfrac{2h}{\rho}$ = self surge impedance
$z_{rs} = \dfrac{P_{rs}}{c} = 60 \ln \dfrac{a}{b}$ = mutual surge impedance } 47

$g = g(x - vt)$ = forward current wave
$G = G(x + vt)$ = reverse current wave } 48

The z's and y's are related as follows:

$$y_{rs} = \frac{(-1)^{r+s}(\text{minor of } D \text{ for which cofactor is } z_{rs})}{D} \qquad 49$$

$$z_{rs} = \frac{(-1)^{r+s}(\text{minor of } D' \text{ for which cofactor is } y_{rs})}{D'} \qquad 50$$

where

$$D = \begin{vmatrix} z_{11} & z_{21} & \cdots & z_{n1} \\ \cdot & \cdot & \cdots & \cdot \\ z_{1n} & z_{2n} & \cdots & z_{nn} \end{vmatrix} \qquad 51$$

$$D' = \begin{vmatrix} y_{11} & y_{21} & \cdots & y_{n1} \\ \cdot & \cdot & \cdots & \cdot \\ y_{1n} & y_{2n} & \cdots & y_{nn} \end{vmatrix} \qquad 52$$

For waves in one direction only

$$\left.\begin{array}{l} \pm e_1 = z_{11} i_1 + \cdots + z_{n1} i_n \\ \cdots\cdots\cdots\cdots\cdots \\ \pm e_n = z_{1n} i_1 + \cdots + z_{nn} i_n \end{array}\right\} \qquad 53$$

$$\left.\begin{array}{l} \pm i_1 = y_{11} e_1 + \cdots + y_{n1} e_n \\ \cdots\cdots\cdots\cdots\cdots \\ \pm i_n = y_{1n} e_1 + \cdots + y_{nn} e_n \end{array}\right\} \qquad 54$$

where the plus sign is for waves traveling in the forward direction, and the minus sign for waves traveling in the reverse direction. These linear equations will be used later to determine the behaviour of traveling waves at transition points.

It is possible to resolve traveling waves into components, each having its own velocity and surge impedance. The advantage of this resolution is that it may be used to simplify the calculation of the behaviour of waves at transition points[1], and it may also be used to simplify the calculation of attenuation and distortion[17]. After the wave components have been found at the point of application, each component may be treated separately as a wave traveling according to its kind. Thus the shape of the main wave is found by addition of the attenuated and distorted component waves.

The following method for calculating the wave components was developed by Bekker[10]. He showed that the waves may be resolved into two components for three-phase lines; one consisting of waves equal on all three lines, and the other consisting of waves adding up to zero on the three lines. Satoh extended the method to find component waves for a double-circuit three-phase system[26].

The theory is developed for a single-circuit three-phase system with completely transposed lines. Thus the inductances and capacitances are the same for all three lines and losses are neglected. Let l and l_1 be the self and mutual inductances per unit length. Then:

$$\left.\begin{aligned} -\frac{\partial e_1}{\partial x} &= l\frac{\partial i_1}{\partial t} + l_1\frac{\partial i_2}{\partial t} + l_1\frac{\partial i_3}{\partial t} \\ -\frac{\partial e_2}{\partial x} &= l_1\frac{\partial i_1}{\partial t} + l\frac{\partial i_2}{\partial t} + l_1\frac{\partial i_3}{\partial t} \\ -\frac{\partial e_3}{\partial x} &= l_1\frac{\partial i_1}{\partial t} + l_1\frac{\partial i_2}{\partial t} + l\frac{\partial i_3}{\partial t} \end{aligned}\right\} \qquad 55$$

From equation (17)

$$Q_1 = Ce_1 + C'e_2 + C'e_3 \qquad 56$$

where $K_{12} = K_{13} = K_{23} = C'$ and $K_{11} = K_{22} = K_{33} = C$

since $P_{12} = P_{23} = P_{31}$ and $P_{11} = P_{22} = P_{33}$

in equation (18).

Since $\frac{\partial i_1}{\partial x} = -\frac{\partial Q_1}{\partial t}$ equation (56) becomes

$$\left.\begin{aligned} -\frac{\partial i_1}{\partial x} &= C\frac{\partial e_1}{\partial t} + C_1\frac{\partial e_2}{\partial t} + C_1\frac{\partial e_3}{\partial t} \\ \text{similarly} \quad -\frac{\partial i_2}{\partial x} &= C_1\frac{\partial e_1}{\partial t} + C\frac{\partial e_2}{\partial t} + C_1\frac{\partial e_3}{\partial t} \\ -\frac{\partial i_3}{\partial x} &= C_1\frac{\partial e_1}{\partial t} + C_1\frac{\partial e_2}{\partial t} + C\frac{\partial e_3}{\partial t} \end{aligned}\right\} \qquad 57$$

Now the waves are resolved into components so that

$$\begin{aligned} e_1 &= e + e_a & \qquad i_1 &= i + i_a \\ e_2 &= e + e_b & \qquad i_2 &= i + i_b \\ e_3 &= e + e_c & \qquad i_3 &= i + i_c \end{aligned} \quad \Bigg\} \qquad 58$$

$$e_a + e_b + e_c = 0 \qquad\qquad i_a + i_b + i_c = 0 \qquad 59$$

Equations 58 are now substituted into equations 55 and 57 to give six new equations. These together with equations 59 give

$$\left.\begin{aligned} -\frac{\partial e}{\partial x} &= (l + 2l_1)\frac{\partial i}{\partial t} \\ -\frac{\partial i}{\partial x} &= (c + 2c_1)\frac{\partial e}{\partial t} \end{aligned}\right\} \qquad 60$$

$$\left.\begin{aligned} -\frac{\partial e_a}{\partial x} &= (l - l_1)\frac{\partial i_a}{\partial t} \\ -\frac{\partial i_a}{\partial x} &= (c - c_1)\frac{\partial e_a}{\partial t} \end{aligned}\right\} \qquad 61$$

$$\left.\begin{aligned} -\frac{\partial e_b}{\partial x} &= (l - l_1)\frac{\partial i_b}{\partial t} \\ -\frac{\partial i_b}{\partial x} &= (c - c_1)\frac{\partial e_b}{\partial t} \end{aligned}\right\} \qquad 62$$

$$\left.\begin{aligned} -\frac{\partial e_c}{\partial x} &= (l - l_1)\frac{\partial i_c}{\partial t} \\ -\frac{\partial i_c}{\partial x} &= (c - c_1)\frac{\partial e_c}{\partial t} \end{aligned}\right\} \qquad 63$$

These are the well known wave equations, the solutions of which are two traveling waves propagating in opposite directions. Thus the equations for waves of the first kind such that the voltages and currents are the same on all lines are

$$i = F + G \qquad\qquad e = Z_1 F - Z_1 G \qquad 64$$

where F is an arbitrary function of $(x - v_1 t)$ and G is an arbitrary function of $(x + v_1 t)$

The velocity is given by $$v_1 = \frac{1}{\sqrt{(l+2l_1)(c+2c_1)}} \qquad 65$$

and the surge impedance by $$Z_1 = \sqrt{\frac{l+2l_1}{c+2c_1}} \qquad 66$$

Waves of the second kind such that the voltages and currents on the three lines add up to zero are given by $$i_a = F_1 + G_1 \qquad e_a = Z_2 F_1 - Z_2 G_1 \qquad 67$$

where F_1 is an arbitrary function of $(x - v_2 t)$ and G_1 is an arbitrary function of $(x + v_2 t)$. The velocity and surge impedance are given by

$$Z_2 = \sqrt{\frac{l-l_1}{c-c_1}} \qquad v_2 = \frac{1}{\sqrt{(l-l_1)(c-c_1)}} \qquad 68$$

Similarly

$$i_b = F_2 + G_2 \qquad e_b = Z_2 F_2 - Z_2 G_2 \qquad 69$$

$$i_c = F_3 + G_3 \qquad e_c = Z_2 F_3 - Z_2 G_3 \qquad 70$$

Now the main waves may be expressed in terms of these arbitrary functions with the aid of equations 58 and 59

$$\left.\begin{aligned} e_2 &= e + e_b = Z_1 (F - G) + Z_2 (F_2 - G_2) \\ e_1 &= e + e_a = Z_1 (F - G) + Z_2 (F_1 - G_1) \\ e_3 &= e + e_c = Z_1 (F - G) + Z_2 (F_3 - G_3) \end{aligned}\right\} \qquad 71$$

$$\left.\begin{aligned} i_1 &= i + i_a = F + G + F_1 + G_1 \\ i_2 &= i + i_b = F + G + F_2 + G_2 \\ i_3 &= i + i_c = F + G + F_3 + G_3 \end{aligned}\right\} \qquad 72$$

Also since $i_a + i_b + i_c = 0$ and $e_a + e_b + e_c = 0$,

then $F_1 + F_2 + F_3 = 0$ and $G_1 + G_2 + G_3 = 0$. 73

Equations (71), (72) and (73) permit the calculation of the magnitudes of the component waves and induced waves when any of the conductors are given a surge. For example, assume line 1 is given a surge e_1 and consider only the positively traveling waves. Since there is no current in lines 3 and 2

$$i_2 = F + F_2 = 0 \qquad i_3 = F + F_3 = 0$$

But $F_1 + F_2 + F_3 = 0$ and thus $F = -F_2 = -F_3 = \frac{F_1}{2}$

$$\therefore e_1 = Z_1 F + Z_2 F_1 = Z_1 F + 2 F Z_2$$

$$F = \frac{e_1}{Z_1 + 2Z_2} \qquad F_1 = 2F = \frac{2e_1}{Z_1 + 2Z_2}$$

$$F_2 = F_3 = -\frac{e_1}{Z_1 + 2Z_2}$$

The voltage waves on lines 2 and 3 are then

$$e_2 = e_3 = Z_1 F + Z_2 F_2 = Z_1 F + Z_2 F_3$$

$$= \frac{e_1 [Z_1 - Z_2]}{Z_1 + 2Z_2}$$

The component waves are

$$e_a = \frac{2Z_2 e_1}{Z_1 + 2Z_2} \qquad e_b = -\frac{Z_2 e_1}{Z_1 + 2Z_2}$$

$$e_c = -\frac{Z_2 e_3}{Z_1 + 2Z_2} \qquad e = \frac{Z_1 e_1}{Z_1 + 2Z_2}$$

So far all the theory has been developed for a three-phase system with no ground wires. If ground wires are present, the capacity and inductance coefficients C, C_1, ℓ, ℓ_1 of equations (55) and (57) must be modified. One way to calculate the new values is to start with equations (53). Suppose, for example, that calculations are to be made on a three-phase line with two ground wires. Then

$$\left.\begin{aligned}
e_1 &= z_{11} i_1 + z_{21} i_2 + z_{31} i_3 + z_{41} i_4 + z_{51} i_5 \\
e_2 &= z_{12} i_1 + z_{22} i_2 + z_{32} i_3 + z_{42} i_4 + z_{52} i_5 \\
e_3 &= z_{13} i_1 + z_{23} i_2 + z_{33} i_3 + z_{43} i_4 + z_{53} i_5 \\
0 &= z_{14} i_1 + z_{24} i_2 + z_{34} i_3 + z_{44} i_4 + z_{54} i_5 \\
0 &= z_{15} i_1 + z_{25} i_2 + z_{35} i_3 + z_{45} i_4 + z_{55} i_5
\end{aligned}\right\} \quad 74$$

Lines 4 and 5 are grounded and thus $e_4 = e_5 = 0$. The currents i_4 and i_5 are now found from the last two equations and substituted into the first three equations to give

$$\left.\begin{aligned}
e_1 &= i_1 z'_{11} + i_2 z'_{21} + i_3 z'_{31} \\
e_2 &= i_1 z'_{12} + i_2 z'_{22} + i_3 z'_{32} \\
e_3 &= i_1 z'_{13} + i_2 z'_{23} + i_3 z'_{33}
\end{aligned}\right\} \quad 75$$

where

$$\left.\begin{aligned}
z'_{11} &= z_{11} - \frac{z_{41}(z_{14} z_{55} - z_{54} z_{15}) + z_{51}(z_{44} z_{15} - z_{14} z_{45})}{z_{44} z_{55} - z_{45}^2} \\
z'_{12} &= z_{12} - \frac{z_{41}(z_{24} z_{55} - z_{54} z_{25}) + z_{51}(z_{44} z_{25} - z_{24} z_{45})}{z_{44} z_{55} - z_{45}^2} \\
z'_{13} &= z_{13} - \frac{z_{41}(z_{34} z_{55} - z_{54} z_{35}) + z_{51}(z_{44} z_{35} - z_{34} z_{45})}{z_{44} z_{55} - z_{45}^2}
\end{aligned}\right\} \quad 76$$

$$Z_{22}' = Z_{22} - \frac{Z_{24}(Z_{24}Z_{55} - Z_{54}Z_{25}) + Z_{25}(Z_{44}Z_{25} - Z_{24}Z_{45})}{Z_{44}Z_{55} - Z_{45}^2}$$

$$Z_{23}' = Z_{23} - \frac{Z_{24}(Z_{34}Z_{55} - Z_{54}Z_{35}) + Z_{25}(Z_{44}Z_{35} - Z_{34}Z_{45})}{Z_{44}Z_{55} - Z_{45}^2}$$

$$Z_{33}' = Z_{33} - \frac{Z_{43}(Z_{34}Z_{55} - Z_{54}Z_{35}) + Z_{35}(Z_{44}Z_{35} - Z_{34}Z_{45})}{Z_{44}Z_{55} - Z_{45}^2}$$

These surge impedances may be found in terms of the line dimensions by means of equations (47), and the results used to obtain the modified capacitance and inductance coefficients - For example,

$$\ell_{11}' = \frac{2Z_{11}'}{60} \times 10^{-9} \text{ henry per cm}$$

$$P_{11}' = \frac{1}{C_{11}'} = \frac{2Z_{11}'}{60} \times 9\times 10^{11} (\text{farad})^{-1} \text{ per cm}$$

The equations for the modified surge impedances simplify considerably if the lines are completely transposed.

It is seen that the effect of the ground wires is to decrease the effective surge impedance of the ungrounded conductor.

2. Attenuation and Distortion.

(a) Single-circuit Lines.

As a pulse travels along a transmission line its maximum amplitude will be decreased, its irregularities will be smoothed out, and the voltage and current waves will cease to be similar. The distortion will be eliminated if the relationship $RC = GL$ holds. This equation never holds for transmission lines, but is approached for loaded telephone lines. Attenuation is always present as it is caused by conductor resistance as modified by skin effect, leakage to ground, and dielectric losses.

The attenuation and distortion may be found from the solution of equations (11) and (12). An accurate solution of these equations is difficult, and thus an approximation is made as follows. In equations (11) and (12)

$$k = \frac{1}{v}\sqrt{(p+\alpha)^2 - \beta^2} = \frac{1}{v}(p+\alpha)\sqrt{1 - \frac{\beta^2}{(p+\alpha)^2}}$$

$$= \frac{1}{v}(p+\alpha)\left[1 - \frac{1}{2}\left(\frac{\beta}{p+\alpha}\right)^2 + \frac{\frac{1}{2}(-\frac{1}{2})}{2}\left(\frac{\beta}{p+\alpha}\right)^4 + \cdots\right]$$

Here $\beta = \frac{r}{2l} - \frac{g}{2c}$ and thus the expression $\left(\frac{\beta}{p+\alpha}\right)^2$ is small for large values of p or high rates of change of voltage and current. Thus

$$k \cong \frac{1}{v}(p+\alpha) \qquad 77$$

and

$$\mathfrak{z} \cong \frac{Z}{1 - \frac{\beta}{p}} \qquad 78$$

Equations (11) and (12) now become

$$e = \epsilon^{-\frac{x}{v}(p+\alpha)} E(t) = \epsilon^{-\frac{\alpha x}{v}} E\left(t - \frac{x}{v}\right) \qquad 79$$

$$i = \frac{1}{Z}\epsilon^{-\frac{\alpha x}{v}}\left[E\left(t - \frac{x}{v}\right) - \beta\int_{\frac{x}{v}}^{t} E\left(t - \frac{x}{v}\right) dt\right] \qquad 80$$

These equations show that the voltage wave is propagated without distortion and subject to attenuation of $\epsilon^{-\frac{\alpha}{v}}$ per unit length. The current wave travels at the same velocity, but is distorted as can be seen from the integral term of equation (80).

If a more accurate solution is desired, it is necessary to solve equations (11) and (12) for the pulse used. Thus for a damped sine wave voltage applied, the equations become

$$e = E\epsilon^{-kx}\epsilon^{-at}\sin\omega t \qquad 81$$

$$i = \frac{E}{Z}\epsilon^{-kx}\epsilon^{-at}\sin\omega t \qquad 82$$

which give $$e = E\epsilon^{-\frac{x}{v}\sqrt{(p+\alpha)^2-\beta^2}}\epsilon^{-at}\sin\omega t \qquad 83$$

$$i = \frac{E}{Z}\sqrt{\frac{\alpha-\beta+p}{\alpha+\beta+p}}\,\epsilon^{-\frac{x}{v}\sqrt{(p+\alpha)^2-\beta^2}}\epsilon^{-at}\sin\omega t \qquad 84$$

The solution of these equations is not attempted here.

The foregoing theory is worked out for values of r, l, c, and g constant with regard to frequency. The theoretical results for this assumption do not agree entirely with practical results. Thus modifications are made. The resistance increases with respect to frequency owing to skin effect and eddy current losses in neighbouring conductors. For earth returns the resistance increases owing to the change in distribution of currents in the earth. For two fairly widely spaced conductors, the inductance falls slightly as frequency increases owing to the concentration of current at the surface. The reduction occurs in the constant term $\frac{1}{2}$ in the ordinary formula. For earth return the inductance varies due to redistribution of currents in the earth as frequency changes, and is always greater than the inductance calculated by the method of images which assumes $r=0$. Both capacitance and leakage may be considered independent of frequency.

To account for these new factors a modified differential equation is derived. (See reference 2). If g is assumed to be zero, then from equation (3)

$$\frac{\partial^2 e}{\partial x^2} = (r + lp)\, cp \cdot e \tag{85}$$

where r and l are functions of p . The inductance l may be written as $l = l_i + l_e$ where l_i is the part due to internal linkages and l_e is the part due to flux outside the wire and thus is independent of frequency. Now it may be shown that the relation between the effective resistance r, the effective internal inductance l_i , and the d-c resistance r_o may be expressed by the equation

$$\frac{r + pl_i}{r_o} = q\sqrt{p} \qquad \text{where } q = a\sqrt{\frac{\pi}{\rho}}\,10^{-9} \tag{86}$$

and $a =$ conductor radius. Thus from equation (85)

$$\frac{\partial^2 e}{\partial x^2} = \frac{p^2}{v^2}\left(1 + \frac{q r_o}{l_e\sqrt{p}}\right) e \tag{87}$$

where $v = \dfrac{1}{\sqrt{l_e C}}$

For a circuit with earth return the differential equation can be shown to be

$$\frac{\partial^2 e}{\partial x^2} = \frac{p^2}{v^2}\left(1 + \frac{g' r_o'}{l_e' \sqrt{p}}\right) e \qquad 88$$

where

$$v = \frac{1}{\sqrt{l_e' C'}} \qquad g' = a\sqrt{\frac{\pi}{\rho} 10^{-9}}\left(1 + \frac{a}{h}\sqrt{\frac{\rho'}{\rho}}\right)$$

$$l_e' = 2 \ln\left(\frac{2h-a}{a}\right) \times 10^{-9}$$

ρ' = specific earth resistance in ohms per cu.cm.

r_o' = d-c resistance of the wire.

Thus the equation to be solved is

$$\frac{\partial^2 e}{\partial x^2} = \frac{p^2}{v^2}\left(1 + \frac{g r_o}{l_e \sqrt{p}}\right) e$$

Following the procedure used in obtaining equation (11), the voltage may be written

$$e = \epsilon^{-\frac{px}{v}\sqrt{1 + \frac{g r_o}{l_e \sqrt{p}}}} E(t) \qquad 89$$

Expanding the index by the binomial theorem and neglecting all but the first two terms, since the rate of change of voltage is large, the equation becomes

$$e = \epsilon^{-\frac{px}{v}} \epsilon^{-\sigma x \sqrt{p}} E(t) \qquad 90$$

where

$$\sigma = \frac{g r_o}{2 l_e v}$$

(b) Multiconductor Systems.

The attenuation and distortion on multiconductor systems may be found from the solution of equation (29). However, this solution would be difficult, and it is only approximate since the variation of resistance and inductance with frequency is neglected.

Another method is to break up the traveling waves into two kinds as done on pages 19 to 22, and then to apply differential equation (90) to each kind of wave. The value of σ must of course be calculated for each kind of wave. The attenuation and distortion of each kind of wave is then calculated, and the results added in the proper manner to obtain the actual wave shapes on the lines. This method is simpler, and also takes variation of resistance and inductance into account. For the damped sine wave voltage applied, the equation to be solved would be

$$e = \epsilon^{-\frac{px}{v}} \epsilon^{-\sigma x/\sqrt{p}} \left[\epsilon^{-at} \sin \omega t \right] \qquad 91$$

This equation is not solved here. However, equation (90) has been solved for a unit function applied voltage[2], and a sharp fronted wave with an exponential decay[15, 17].

If the pulse is applied to one line, the attenuation may be found approximately by means of equation (79). The attenuation factor is thus $\epsilon^{-\frac{\alpha x}{v}}$ where $\alpha = \frac{r}{2l}$. The resistance r is increased by skin effect above its d-c value. For a frequency of 500 kc the effective resistance is about 40 times its d-c value. For a typical 220 kv transmission line this results in an attenuation to one-half the original pulse in about 130 miles.

3. <u>Reflection of Waves</u>.

(a) General equations.

The general equations for the behaviour of traveling waves on a multi-conductor system at a transition point have been developed by L.V. Bewley[1]. The network used is shown in Fig. 3.

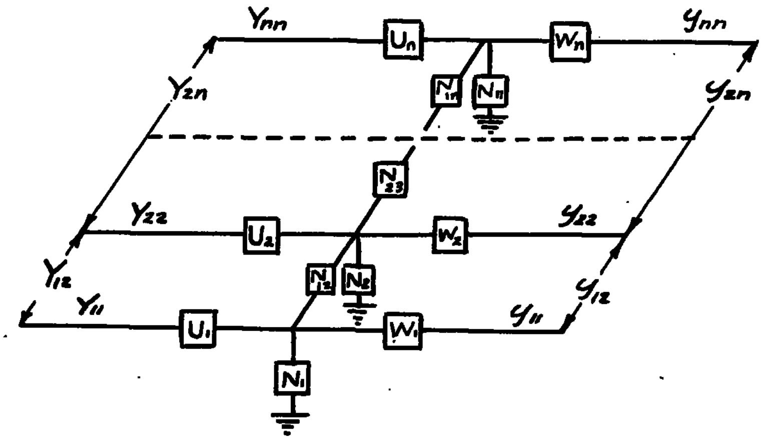


Fig. 3. General Multi-conductor System.

The symbols are defined below:

$Y_{11}, Y_{22}, \cdots Y_{nn}$ = self surge admittances of lines on the left.

Y_{12}, Y_{13}, etc. = mutual surge admittances of lines on the left.

$y_{11}, y_{22}, \cdots y_{nn}$ = self surge admittances of lines on the right.

y_{12}, y_{13}, etc. = mutual surge admittances of lines on the right.

$U_1, U_2, \cdots U_n$ = series impedance network on the left.

$W_1, W_2, \cdots W_n$ = series impedance network on the right.

$N_1, N_2, \cdots N_n$ = admittances to ground.

N_{12}, N_{23}, etc. = admittances from junction to junction.

e, i = potential and current incident waves.

e', i' = potential and current reflected waves.

e'', i'' = potential and current transmitted waves.

The general equations are

$$\left[Y_{r1} + Y_{r1} U_r (N_r + N_{1r} + \cdots + N_{mr}) - Y_{11} N_{1r} U_1 - \cdots - Y_{m1} N_{mr} U_m \right] (e_1 - e_1')$$
$$\cdots\cdots$$
$$+ \left[Y_{rm} + Y_{rm} U_r (N_r + N_{1r} + \cdots + N_{mr}) - Y_{1n} N_{1r} U_1 - \cdots - Y_{mm} N_{mr} U_n \right] (e_n - e_n')$$
$$+ N_{1r}(e_1 + e_1') + \cdots + N_{mr}(e_n + e_n') - (N_r + N_{1r} + \cdots + N_{mr})(e_r + e_r')$$
$$= (y_{r1} e_1'' + \cdots + y_{rn} e_n'') \qquad 92$$

$$(e_r + e_r') - U_r \left[Y_{r1}(e_1 - e_1') + \cdots + Y_{rm}(e_n - e_n') \right]$$
$$= e_r'' + W_r (y_{r1} e_1'' + \cdots + y_{rn} e_n'') \qquad 93$$

For an n-wire system n equations of type (92) and n equations of type (93) can be written, and these equations solved for the 2n unknowns

$$e_1' \cdots e_n' , e_1'' \cdots e_n''$$

If the mutual connecting networks are removed then N_{12}, N_{23}, etc., are all zero and equations (92) and (93) reduce to

$$(1 + N_r U_r) \left[Y_{r1}(e_1 - e_1') + \cdots + Y_{rn}(e_n - e_n') \right] - N_r (e_r + e_r')$$
$$= y_{r1} e_1'' + \cdots + y_{rn} e_n'' \qquad 94$$

$$(e_r + e_r') - U_r \left[Y_{r1}(e_1 - e_1') + \cdots + Y_{rn}(e_n - e_n') \right]$$
$$= e_r'' + W_r (y_{r1} e_1'' + \cdots + y_{rn} e_n'') \qquad 95$$

(b) Reflections from arcs.

As under Heading 111 B the resistance of the arc varies throughout the half cycle of the 60-cycle wave. The magnitude of the reflections to be expected may be calculated from the general equations. Thus if the fault is on line 2, equations (94) and (95) give

$$\left.\begin{aligned} U_r = W_r = 0 \qquad & Y_{rr} = y_{rr} \qquad Y_{rm} = y_{rm} \\ N_1 = N_3 = 0 \qquad & N_2 = G \end{aligned}\right\} \qquad 96$$

Therefore

$$\left.\begin{aligned} & y_{11}(e_1 - e_1') + y_{12}(e_2 - e_2') + y_{13}(e_3 - e_3') \\ & \qquad = y_{11} e_1'' + y_{12} e_2'' + y_{13} e_3'' \\ & y_{21}(e_1 - e_1') + y_{22}(e_2 - e_2') + y_{23}(e_3 - e_3') - G(e_2 + e_2') \\ & \qquad = y_{21} e_1'' + y_{22} e_2'' + y_{23} e_3'' \\ & y_{31}(e_1 - e_1') + y_{32}(e_2 - e_2') + y_{33}(e_3 - e_3') \\ & \qquad = y_{31} e_1'' + y_{32} e_2'' + y_{33} e_3'' \end{aligned}\right\} \qquad 97$$

$$\left.\begin{aligned} e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \\ e_3 + e_3' &= e_3'' \end{aligned}\right\} \qquad 98$$

The transmitted and reflected waves are found by substituting (98) in (97) and solving the resulting equations.

Thus

$$\left.\begin{aligned} e_1' &= \frac{\frac{G}{2} e_2 \left[y_{12} y_{33} - y_{13} y_{32} \right]}{D} \\ e_2' &= \frac{\frac{G}{2} e_2 \left[y_{13}^2 - y_{11} y_{33} \right]}{D} \\ e_3' &= \frac{\frac{G}{2} e_2 \left[y_{11} y_{32} - y_{12} y_{31} \right]}{D} \end{aligned}\right\} \qquad 99$$

where

$$\begin{aligned} D = {} & y_{11}\left[y_{33}\left(y_{22} + \tfrac{G}{2}\right) - y_{23}^2 \right] - y_{12}\left[y_{21} y_{33} - y_{31} y_{23} \right] \\ & + y_{13}\left[y_{21} y_{32} - y_{31}\left(y_{22} + \tfrac{G}{2}\right) \right] \end{aligned} \qquad 100$$

As an approximation it is assumed that the lines are completely transposed, and thus average surge impedances are used in calculating the surge admittances. Equations (99) thus become

$$\left.\begin{aligned} e_1' = e_3' &= \frac{G}{2} e_2\, y'\left[y - y'\right] \frac{1}{D} \qquad (a)\\ e_2' &= \frac{G}{2} e_2 \left[y'^2 - y^2\right] \frac{1}{D} \qquad (b)\end{aligned}\right\} \qquad 100$$

If the pulse is applied to one line only, it will be of interest to calculate the reflection to be expected when the fault is on the pulsed line, and when on one of the other lines. From equations (100) it is seen that the reflected voltage depends only on the incident voltage on the faulted line. This is known directly if the pulse is applied to that line. If the pulse is applied to one of the other lines the incident voltage on the faulted line is found by means of equations (53) where $i_2 = i_3 = 0$ if the pulse is applied to line 1.

Thus $$e_2 = e_1 \frac{Z_{12}}{Z_{11}} \qquad e_3 = e_1 \frac{Z_{13}}{Z_{11}} \qquad 101$$

For transposed lines

$$e_2 = e_3 = \frac{Z'}{Z} e_2 \qquad 102$$

To obtain an approximation of the magnitude of reflection to be expected on an actual line, an example problem is worked out. According to a report on 220 kv transmission lines in the United States and Canada as given by an A.I.E.E. Committee[9], about 67 percent of 220 kv lines have two ground wires. Thus the modifications to be expected due to the presence of these ground wires should be calculated.

The dimensions of an average three-phase line with two ground wires are given in Fig. 4.

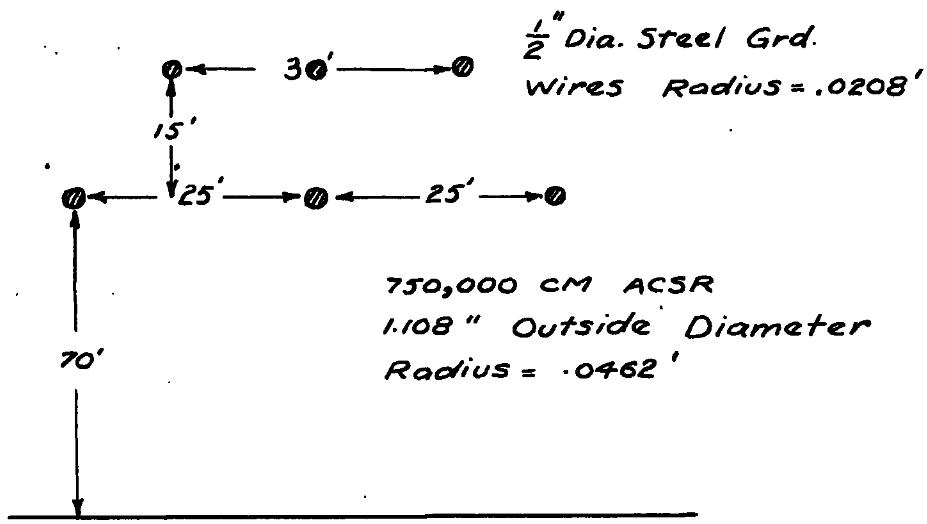


Fig. 4. Configuration of the Transmission Line.

The linear relationships between voltages and currents on the line are given by equations (75) where the modified surge impedances are defined by equations (76) . The actual surge impedances as given by equations (47) are

$$\mathfrak{Z}_{11} = 60 \ln \frac{140}{.0462} = 481 \qquad \mathfrak{Z}_{22} = 481 \qquad \mathfrak{Z}_{34} = 79.3$$

$$\mathfrak{Z}_{12} = 60 \ln \frac{142.5}{25} = 104.4 \qquad \mathfrak{Z}_{23} = 104.4 \qquad \mathfrak{Z}_{35} = 129.4$$

$$\mathfrak{Z}_{13} = 60 \ln \frac{148.7}{50} = 65.3 \qquad \mathfrak{Z}_{24} = 109.5 \qquad \mathfrak{Z}_{44} = 541$$

$$\mathfrak{Z}_{14} = 60 \ln \frac{155.5}{18} = 129.4 \qquad \mathfrak{Z}_{25} = 109.5 \qquad \mathfrak{Z}_{45} = 105$$

$$\mathfrak{Z}_{15} = 60 \ln \frac{160}{42.7} = 79.3 \qquad \mathfrak{Z}_{33} = 481 \qquad \mathfrak{Z}_{55} = 541$$

From these impedances the modified surge impedances are calculated from equations (76) .

$$\mathfrak{Z}'_{11} = 481 - 36.6 = 444.4 \qquad \mathfrak{Z}'_{22} = 481 - 37.3 = 443.7$$

$$\mathfrak{Z}'_{12} = 104.4 - 35.4 = 69.0 \qquad \mathfrak{Z}'_{23} = 104.4 - 35.4 = 69.0$$

$$\mathfrak{Z}'_{13} = 65.3 - 30.8 = 34.5 \qquad \mathfrak{Z}'_{33} = 481 - 36.6 = 444.4$$

It will be assumed that the lines under consideration are completely transposed, and thus average surge impedances will be used for calculating the surge admittances. Let

$\mathfrak{Z}$ = self surge impedance = 444.2

$\mathfrak{Z}'$ = mutual surge impedance = 57.5

These two values are substituted into equation (49) to give

y = self surge admittance = 2.32×10^{-3}

y' = mutual surge admittance = -2.66×10^{-4}

If the fault is on the pulsed line, equation (100) gives the reflected voltage as $e_2' = \dfrac{\frac{G}{2} e_2 \left[y_1^2 - y^2 \right]}{D}$. After substitution, the equation becomes

$$e_2' = - \frac{G e_2}{G + 4.50 \times 10^{-3}}$$

If the fault is not on the pulsed line the reflection is given by equation (100) as

$$e_1' = e_3' = \frac{\frac{G}{2} e_2 y' \left[y - y' \right]}{D} = \frac{e_2'}{7.72} = - \frac{G e_2}{7.72 G + .0348}$$

These equations give the maximum value of the reflection to be expected for a resistance to ground of $R = \frac{1}{G}$. A table is given below to show how the magnitude of the reflected voltage varies with grounding resistance. Fig. 5 is a graph of the results. Due to the varying arc resistance, the reflected wave will be shaded in as shown approximately in Fig. 5.

Tower Grounding Resistance	Minimum Arc Resistance	Total Resistance to Ground = R = 1/G	Reflected voltage wave	
			Fault on Pulsed line	Fault on other line
1	2	3	-0.987e	-0.1278e
25	2	27	-0.892e	-0.1155e
50	2	52	-0.811e	-0.1050e
75	2	77	-0.743e	-0.0962e
100	2	102	-0.686e	-0.0888e

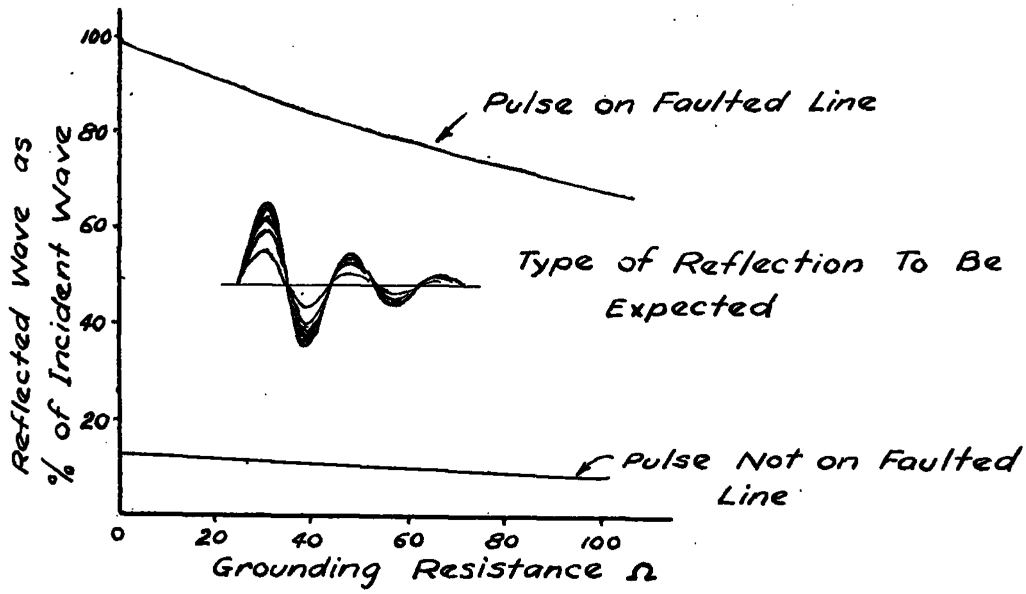


Fig. 5. Graph Showing How the Reflected Voltage Varies with Grounding Resistance.

It would be expected where the pulse is applied to one line only, that the reflection would not be affected by adjacent lines as they are not carrying current. Thus from single line theory the magnitude of the reflection would be

$$-\frac{3}{2R+3}e = -\frac{Ge}{\frac{2}{3}+G}$$

This expression may be shown to be equivalent to equation (18b) by means of equation (50) .

(c) Reflections from Terminations.

For the majority of lines the termination consists of transformers. As an approximation, the transformer will appear as a capacitance to a high-frequency pulse[2]. The effective capacitance C_f may be calculated from the expression

$C_f = \sqrt{C_\omega C_g}$ (approx.) where C_ω = capacitance measured from end to end of the winding in farads and C_g = capacitance to earth of the complete winding in farads. To obtain some idea of these capacitances the values of $C_g = 9 \times 10^{-10}$ farads and $C_\omega = 9 \times 10^{-12}$ farads are taken. (See p. 234 reference 2). Thus $C_f = \sqrt{C_\omega C_g} = 9 \times 10^{-11}$ farads. Some idea of the effect of this capacity on the applied pulse may be obtained by calculation of the impedance at pulse frequency. For frequencies of 300 kc and 500 kc the impedances are respectively 8850 Ω and 3540 Ω . This indicates that the reflection will be approximately that of an open circuit.

According to Bewley[1] the effective capacitance may vary from 0.0002 μf to 0.001 μf for ordinary transformers. These values are considerably higher than that given above, and the approximation of an open circuit is no longer valid.

The size and shape of the reflection could be found by calculation. However, the reflection magnitude will be decreased from that for an open circuit.

(d) Reflections from Transpositions.

A reflection would be expected from a transposition in a line since it represents a discontinuity. If the pulse is applied between two lines, a reflection would be expected wherever the spacing between those two lines is varied at a transposition. However, if the pulse is applied between one line and ground, a reflection would be expected only when the self surge impedance of that line is changed. From single-line theory the magnitude of the reflection is given by the expression

$$e' = \frac{Z_2 - Z_1}{Z_2 + Z_1} e \qquad 104$$

For the example line under consideration the self surge impedances of the two outside lines are equal. Thus a reflection will be produced in only two of three transpositions. However, for these two transpositions the reflection will be negligible as is seen by substituting the values of Z_1 and Z_2.

$$e' = \pm \frac{444.4 - 443.7}{444.4 + 443.7} e_1 = 0.0008\, e_2$$

Thus one advantage in applying the pulse from line to ground is that echos from transpositions will not as likely be mistaken for faults, as is possible if the pulse were applied from line to line.

However, echos from transpositions have some value in fixing points on the line.

(e) Successive Reflections.

When a pulse is reflected from a discontinuity it will be reflected again from the beginning of the line, the magnitude of the reflection depending on the terminal impedance. A convenient method for keeping track of these reflections is by means of a lattice diagram[1].

Suppose, for example, that the pulse is applied to the line with the short circuit. The lattice diagram to be used for this line is shown in Fig. 6. As the fault is not a complete short circuit, waves will propagate past the fault as shown by the dotted lines. The magnitudes of the reflections depend on the impedances at the points of discontinuity. The waves will be distorted and attenuated as they travel along the line, and if the defining functions are known they may be included in the diagram.

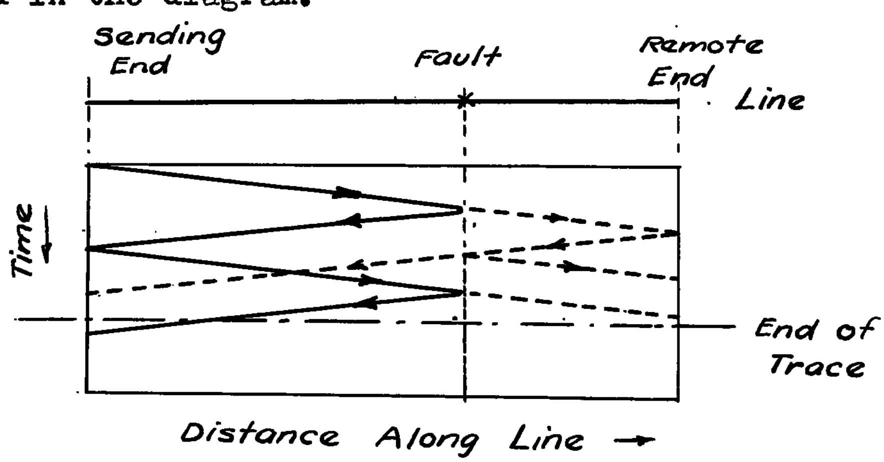


Fig. 6. Lattice Showing Successive Reflections.

For the example shown the trace on the recording tube will consist of a reflection from the fault and one from the far end of the line. The many successive reflections reaching the tube after the end of the trace will show up only very faintly on the tube as they are smaller and will occur at a different point on the screen for each pulse.

If the fault is close to the sending end, then successive reflections will show up on the trace as shown by Fig. 7. These multiple echos can be removed if the output impedance of the pulse-sending apparatus is made equal to the characteristic impedance of the line. However, it is not necessary to remove these multiple echoes as they are easily recognized as such.

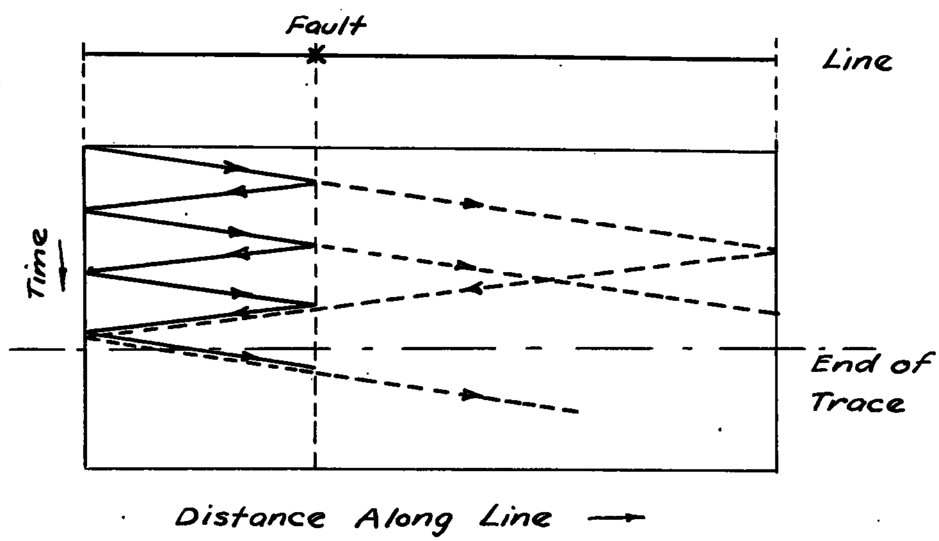


Fig. 7. Lattice Showing Successive Reflections for Fault Close to Sending End.

(f) Effect of Insulator Capacitance.

As an insulator may be represented by an impedance to ground, a reflection would be expected from each tower. However, the insulator capacity is small, and as a result the reflection is very small for wave packets of the frequency being used. If the capacity of an insulator is taken as 10^{-11} farads[2], then the reactance to ground for a frequency of 500 kc is about 30000 Ω, which is large compared to the line surge impedances of the order of 500 Ω. Thus no visible reflection would be expected.

D. The Equipment.

1. General Description

Before any equipment could be built it was necessary to decide upon a suitable pulse shape. The pulse must be short enough to allow an echo to be recognized from a reasonably close fault; it must not have any interference with radio waves or carrier equipment; and must be of such a character that it can pass through a coupling capacitor onto a transmission line with little effect on its magnitude and shape.

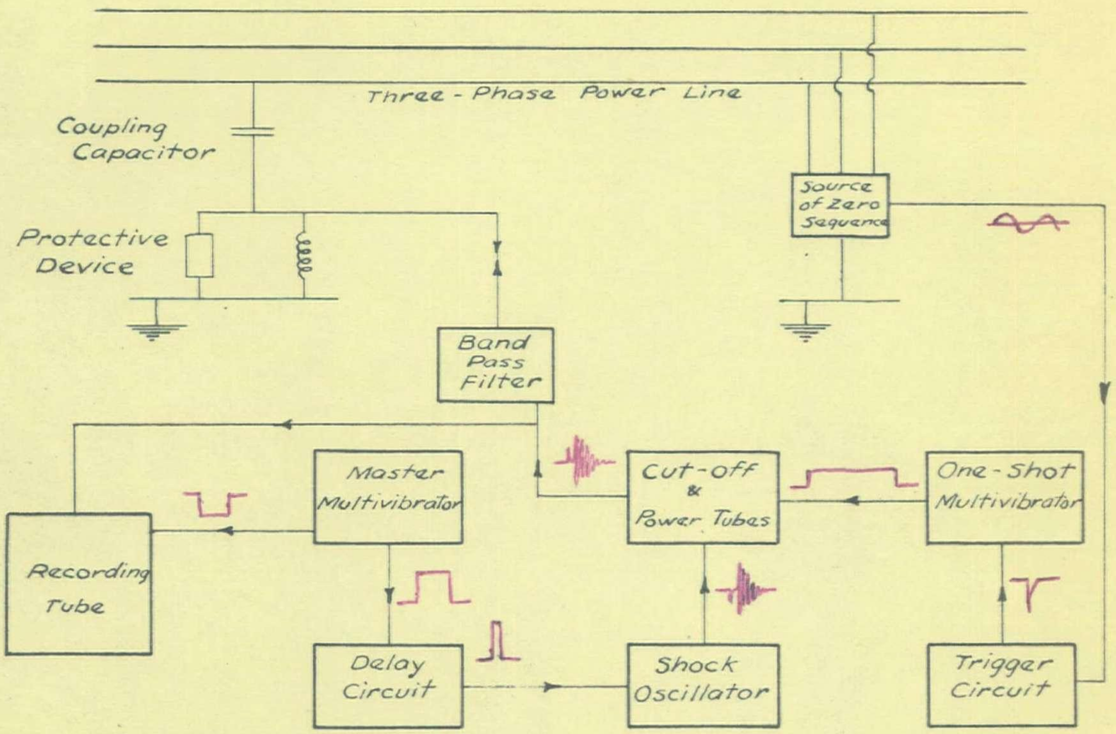
This latter consideration prohibits the use of fairly long duration unidirectional pulses. For example, if a 0.003 μf coupling capacitor were used on a 60-kv line with a surge impedance of 500 ohms, then the time constant would be 1.5 μ-sec. Thus if a 10 μ-sec. rectangular wave were impressed on the coupling capacitor, the line voltage would consist of the initial sharp rise of the pulse which would then die off exponentially. The trailing edge of the rectangular pulse would produce a similar pip in the opposite direction. On higher voltage lines the coupling capacitor has a smaller capacity, and an even shorter time constant results. The only unidirectional pulse that could be used would be a very narrow one which is subject to comparatively high attenuation and distortion. However, this short pulse would have the advantage of accuracy, and is a possibility for short lines.

The above considerations led to the choice of a high-frequency wave burst in the form of a damped sine wave. Since the pulses are to be sent out for a short interval only, the interference with radio may be neglected. Interference with carrier equipment can easily be avoided by an appropriate choice of pulse frequency and the use of filters to prevent mutual interference between the fault locator and carrier equipment. The reactance drop in the coupling capacitor may be tuned out with a suitable tuning coil. The pulse frequency is not critical. It is limited in the upper direction by increasing attenuation due to radiation and skin effect, and in the lower direction by increased length of pulse and the resulting difficulty in detecting near echos.

Fig. 8 is a block diagram of the fault locator. The fault locator is tripped by means of zero-sequence current or voltage initiated by the flashoever. A source of zero-sequence is usually available for relaying, and may be used to fire a trigger circuit which in turn trips a one-shot multivibrator. This multivibrator keys a tube in the output of the pulse generator, and allows the pulses to be sent out on the line for a period that is somewhat longer than the interrupting time of the circuit breakers feeding the line. Thus after the arc is extinguished a clear echo will be received from the end of the line. The pulse generator starts the sweep of the recording tube a few microseconds before the pulse

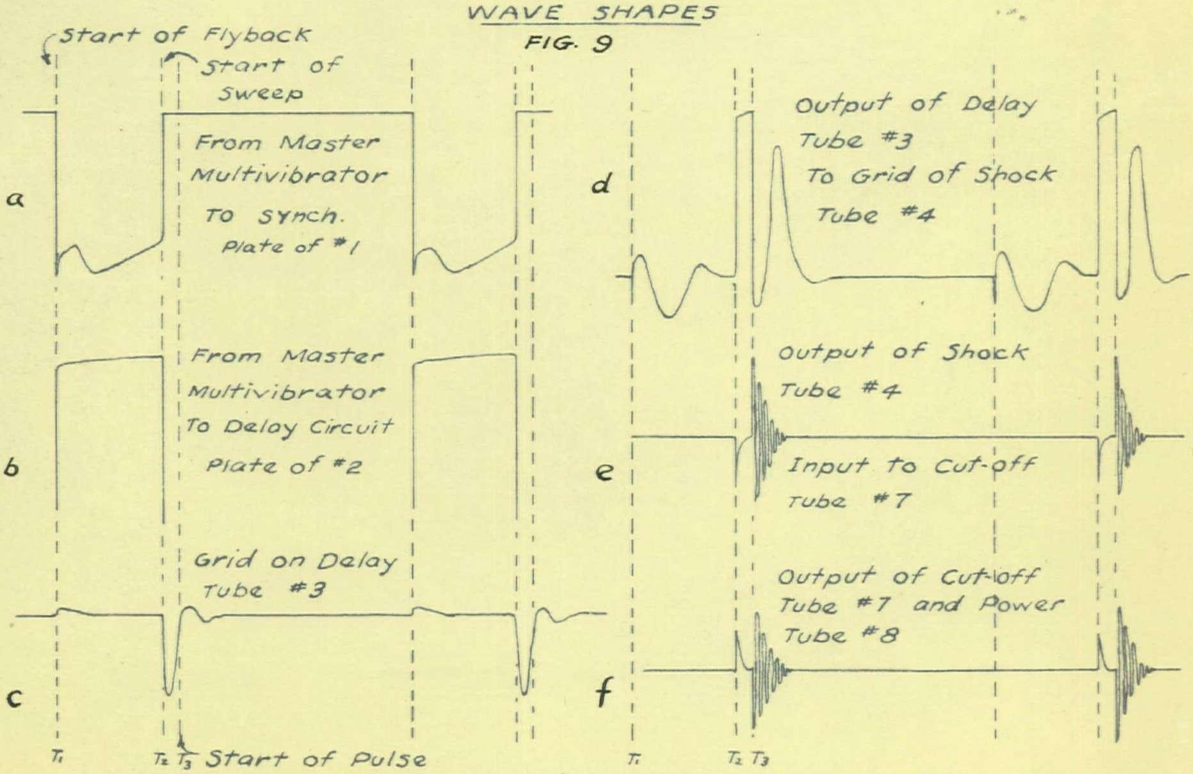
BLOCK DIAGRAM

FIG. 8



WAVE SHAPES

FIG. 9



is produced; thus allowing the transmitted pulse to be seen on the tube.

2. Pulse Generator

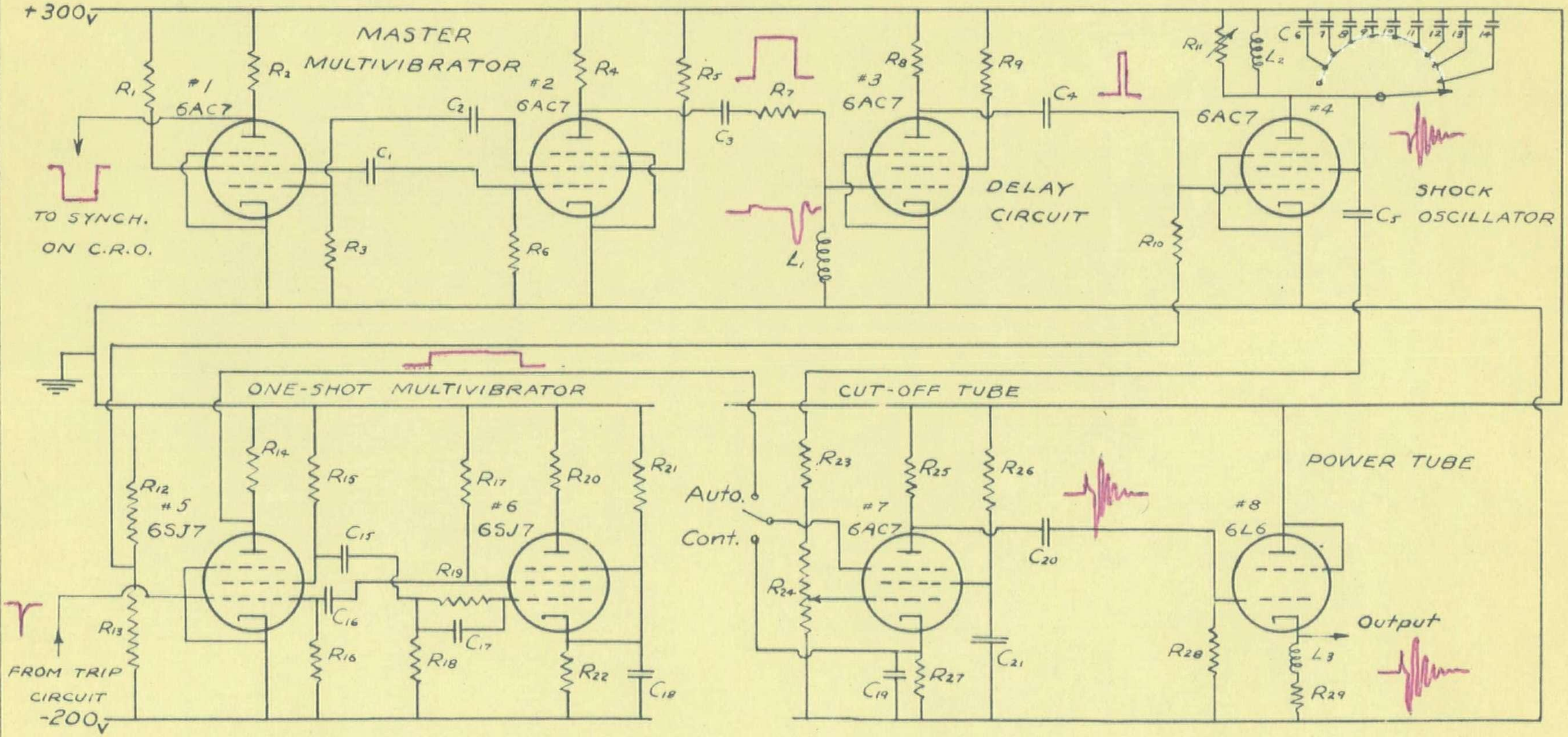
The complete circuit diagram of the pulse generator is shown in Fig. 10. Tubes 1 and 2 comprise a multivibrator which controls the time interval between pulses. The time interval should be a little longer than the time necessary for a pulse to travel to the end of the line and back which is approximately equal to $\frac{2\ell}{1000}$ μ-sec., where ℓ is the length of the line in feet.

The frequency stability of multivibrators is poor. However, this is not a particular disadvantage since the sweep of the cathode ray tube is synchronized with the outgoing pulse which thus appears at the same point on the trace each time. Pentode tubes are used in this circuit, and coupling is between the screen and control grids. Consequently, the plates have no extra loading as in the triode type of multivibrator and steeper sides are obtained. The approximate wave shapes are shown in Fig. 9 (a),(b).

The negative rectangular wave from the plate of tube 1 is fed to the "synch". terminal of the Cossor oscilloscope which is built in such a way that the negative-going side starts the flyback of the trace (time T_1 on Fig. 9). After the flyback is complete, the beam is ready for the main trace. However, this trace cannot begin until the "synch." terminal becomes positive

CIRCUIT DIAGRAM OF PULSE GENERATOR

FIG. 10



LIST OF PARTS

$R_1 = 25000\ \Omega$	$R_9 = 50000$	$R_{17} = 20000$	$R_{25} = 5000$	$C_1 = 0.001$ uf	$C_9 = 0.00004$	$C_{17} = 0.00004$
$R_2 = 5000$	$R_{10} = 1M$	$R_{18} = 20M$	$R_{26} = 40000$	$C_2 = 0.001$	$C_{10} = 0.000075$	$C_{18} = 50$
$R_3 = 75000$	$R_{11} =$	$R_{19} = .5M$	$R_{27} = 100$	$C_3 = 0.01$	$C_{11} = 0.000107$	$C_{19} = 0.1$
$R_4 = 5000$	$R_{12} = 50000$	$R_{20} = 30000$	$R_{28} = 39000$	$C_4 = 0.001$	$C_{12} = 0.000160$	$C_{20} = 0.01$
$R_5 = 25000$	$R_{13} = 100000$	$R_{21} = 20000$	$R_{29} = 400$	$C_5 = 0.05$	$C_{13} = 0.000295$	$C_{21} = 0.01$
R_6	$R_{14} = 30000$	$R_{22} = 4000$		$C_6 = 0.000004$	$C_{14} = 0.000450$	
$R_7 = 15000$	$R_{15} = 20000$	$R_{23} = 1M$		$C_7 = 0.00001$	$C_{15} = 0.5$	
$R_8 = 8300$	$R_{16} = 20M$	$R_{24} =$ [illegible]		$C_8 = 0.000025$	$C_{16} = 0.5$	

$L_1 = 30$ mh
$L_2 = 2.5$ mh
$L_3 = 2.5$ mh

again at time T_2. The width of this negative pulse depends on the time taken for the completion of the flyback which in turn increases as the speed of the main sweep decreases. Thus the width of this negative wave is determined by the slowest sweep to be used, and is found by experiment.

The transmitted pulse must be delayed for a few microseconds after the start of the sweep in order that it will show on the trace. The first step in producing the delay is to apply the positive rectangular wave from tube 2 through condenser C_3 and resistor R_7 to a choke coil L_1, as shown in Fig. 10. The resulting wave form at the grid of delay tube 3 is shown in Fig. 9 (c). The small rise in grid voltage at time T_1 is caused by potentiometer action of the resistor R_7 in series with the low effective resistance of the grid to cathode of tube 3. After the positive rise of the applied wave is completed, the grid voltage returns exponentially to zero because of the inductance L_1. At time T_2 the negative-going side of the wave from tube 2 suddenly cuts off tube 3 and causes shock oscillations in the inductance L_1. These oscillations are damped out after the first negative half cycle owing to the grid drawing current on the following positive half cycle. The resulting plate voltage is a positive-going rectangular wave having a width which depends on the natural frequency of the inductance L_1 (see Fig. 9 (d)). The accompanying irregularities do not affect the next tube because of the large negative bias on its grid.

This positive rectangular wave is applied to the control grid of tube 4, a 6AC7 connected as a triode. The grid is biased past cut-off so that the tube conducts only for the period of the réctangular wave. The plate circuit consists of a variable damping resistor, an air-core inductance, and a choice of various condensers. When the grid is driven positive, the tube draws a very high current as the plate is connected to the power supply through the small resistance of the inductance. After about four μ-secs., the grid is suddenly driven negative, and the plate current stops very quickly and shocks the inductance into oscillation. The frequency of oscillation depends on which condenser is placed in parallel with the inductance, and the number of oscillations is controlled by the variable damping resistor R_{11} shunting the L-C circuit. A triode connection is used in this tube to obtain the low plate resistance necessary to damp out oscillations that tend to occur when the grid is driven positive.

A variable part of the output of the shock tube determined by the resistor R_{23} and potentiometer R_{24} is fed into the grid of the cut-off tube. This tube is connected as an ordinary amplifier except that for normal operation the switch S_1 is on "automatic", and consequently the suppressor grid is negative with respect to the cathode.

Thus there is no output from the plate. When it is desired to send pulses out on the line, the suppressor is temporarily connected to ground, and allows the pulses to be reproduced on the plate. If the apparatus is to be used to find a permanent fault where continuous pulsing is desired, the suppressor grid may be connected directly to the cathode by changing switch S_1 over to "continuous". This modification allows a line to be surveyed for faults before the breaker is reclosed.

The necessary power to feed the low-impedance transmission line is obtained from a 6L6 tube (8) connected as a triode cathode-follower fed from the plate of the cut-off tube. A choke-coupled cathode follower is used to obtain high pulse output with low distortion. A 400 ohm resistance is placed in series with the choke to bias the tube at the desired point.

Thus when suitably tripped the pulse generator produces pulses to be applied to the transmission line and synchronizes them with the sweep of the recording tube.

3. <u>Tripping Circuits.</u>

As explained previously, the pulse generator is to be tripped either from zero-sequence current, or from the initial surge of the flashover. The latter method has the disadvantage of being susceptible to a greater number of false operations due to such things as switching surges and the sudden release of bound charges due to lightning. For either method the pulses must be suddenly connected to the line and then removed a fraction of a second after the arc has been extinguished to obtain a reflection from the far end of the line. This is accomplished with the aid of the suppressor grid of the cut-off tube used in conjunction with a long time-constant one-shot electron-coupled or pentode multivibrator. Pentodes are used in preference to triodes to obtain a sharp front necessary for sudden application of the pulses. The rise time does not have to be as fast as that of the master multivibrator, however, and thus lower power tubes (6SJ7) are used.

The action of this circuit is similar to that of the master multivibrator except that one tube is biased past cutoff to prevent continuous oscillation. This is accomplished for tube 6 by the voltage divider consisting of resistors R_{21} and R_{22} which produce a bias of 33 volts on the grid. Thus when the transmission line is operating normally, tube 5 is conducting and the voltage drop across its plate resistor (R_{14}) is impressed on the suppressor grid of the cut-off tube. As the suppressor must be at least 120 v below the cathode, the plate supply for the one-shot multivibrator is taken between ground and 200 v negative.

The circuit can be tripped by a negative pulse applied to the control grid of tube 5, or a positive pulse on the control grid of tube 6. If, for example, the grid of tube 5 were driven negative, the screen current would be reduced and the screen voltage would rise. Owing to the coupling between the screen of tube 5 and the control grid of tube 6, tube 6 starts to conduct, and consequently its screen voltage falls because of the voltage drop in R_{17}. However, the screen is connected through C_{16} to the control grid of tube 5 which is thus driven more negative. The resulting cumulative action drives tube 5 to cut-off and tube 6 into conduction. The plate of tube 5 changes from a potential of nearly 200 v negative to ground potential, and, being directly connected to the suppressor grid of the cut-off tube, allows the pulses to be sent out on the line.

The period of conduction of tube 6 depends mostly on the resistances R_{15}, R_{18}, and R_{19} and the condenser C_{15}. When tube 5 is cut off, the grid of tube 6 is connected to the power supply through R_{15}, R_{19}, and C_{15}; and, being positive with respect to the cathode, begins to draw current from the power supply to charge up C_{15}. Condenser C_{15} will also charge up through R_{18} but its resistance is so large compared to R_{19} that it may be neglected. This charging process through R_{19} will continue until the grid on tube 6 is slightly below cathode potential, when charging must continue more slowly through R_{18}. The screen current of tube 6 now begins to drop, and the screen voltage to rise. However, cumulative action cannot begin until the grid of tube 5 reaches cut-off. This depends to some degree on the time constant $R_{16}C_{16}$

since it determines the residual charge left on C_{16} which must be removed by the increasing screen voltage of tube 6. As soon as the grid reaches cut-off, cumulative action begins and tube 5 is driven to conduction while tube 6 stops conducting. Now the plate voltage of tube 6 drops again, and the pulse generator is disconnected from the line. It is thus apparent that the pulse width of the multivibrator is determined principally by the resistance R_{19} which can be varied to give the desired length of signal. The small condenser C_{17} shunting this resistor increases the speed of action by allowing the inter-electrode capacities to be charged directly instead of through R_{19}.

The one-shot multivibrator must be tripped on the occurrence of a fault. If zero-sequence current is to be used to start the pulses, the thyratron trip circuit of Fig. 11 is suitable. With no signal applied, the tube is not conducting and the voltage across the tube and condenser is equal to the supply voltage. On application of a large enough signal to the grid, the thyratron

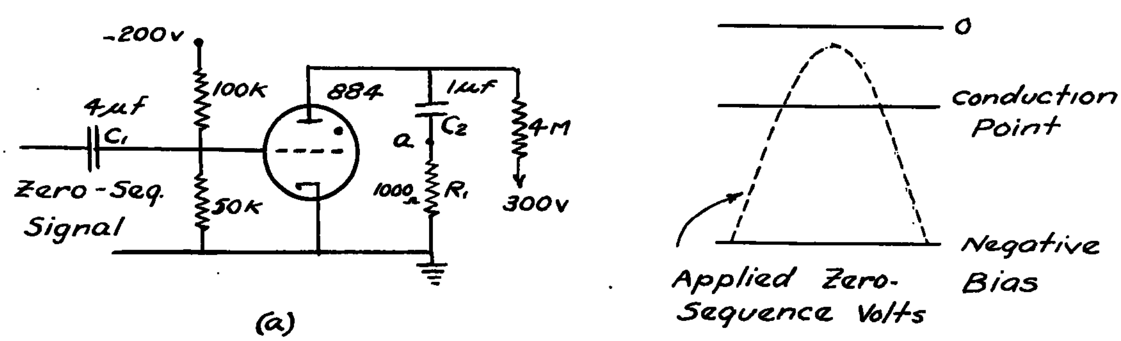


Fig. 11. The Thyratron Tripping Circuit.

conducts suddenly, and the condenser C_2 discharges through the resistor R_1 until the plate voltage falls below that necessary for conduction. The wave form at point "a" is thus a sudden negative voltage which dries away exponentially as shown in Fig. 11. This pulse is applied to the control grid of tube 5 and trips the one-shot multivibrator. The thyratron is prevented from continued conduction by the four megohm resistor in series with the power supply. This resistor is large enough to reduce the current from the power supply below that necessary to keep the tube in conduction.

Reference to the characteristics of the 884 thyratron tube show that with 300 v on the plate, the tube conducts for a definite grid voltage. The grid is thus biased beyond this point; the amount of additional bias depending on the minimum zero-sequence voltage desired to trip the equipment. This is a valuable feature because zero-sequence current produced by switching may possibly be discounted with a large enough bias.

A disadvantage of this tripping circuit is that it trips only on the positive half of the zero-sequence wave. Thus if the first half cycle were negative, this time would be lost as far as the fault-locating apparatus is concerned. To overcome this difficulty two thyratrons may be used, and the grids fed from a transformer with a center-tapped secondary as shown in Fig. 12.

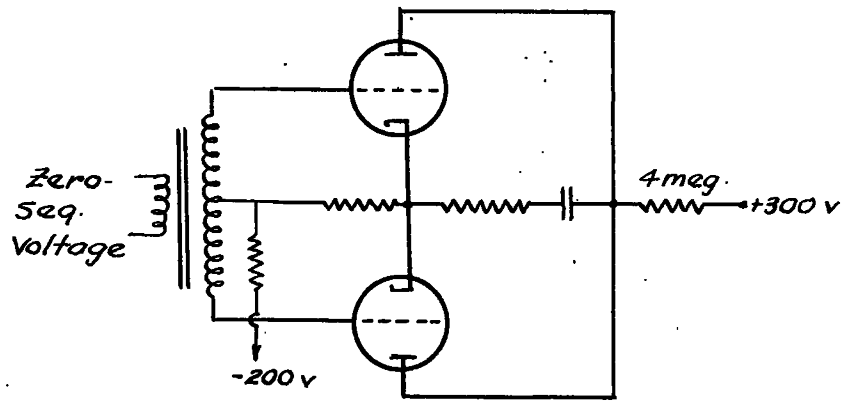


Fig. 12. Two Tube Thyratron Trip Circuit.

4. Recording the Information.

After a survey of different ways of recording the information obtained during the flashover, it was decided that a trace of the pulses transmitted and received would be most desirable. This method has the advantages of accuracy and ease of interpretation of distance, as well as giving information as to the type of fault or multiple faults. It also shows reference points such as transpositions and junctions which may help to locate the fault.

The accuracy of this method depends largely on the length of sweep used to represent the transmission line. For example, if a 30-mile line with say 10 supports per mile is to be used; then with a linear sweep on a five-inch tube, the flashover could probably be located within two towers. For longer lines the absolute accuracy would decrease proportionately. To offset this, however, it is possible to use circular or spiral sweeps and timing pips.

The distance to the fault is easily obtained by proportion because the trace on the screen represents a map of the transmission line. The disadvantage of any non-linearity of the sweep is easily overcome by a timing wave in the form of pips or brightness modulation.

Flashovers will always give the characteristic echo of a short circuit. If, however, the equipment were used to locate a permanent fault, additional information about the character of the fault could be obtained from the size and shape of the echo. Although multiple faults seldom occur, this method of recording will detect them; the second fault showing up as another echo on the trace.

The "Skiatron"*, a memory tube, was investigated as an alternative to the use of fluorescent tubes and photographic equipment[28]. This tube is similar to the ordinary cathode ray tube except that the fluorescent screen is replaced by a screen consisting of alkali-halide crystals which become intensely coloured under electron bombardment. The length of time that the trace remains on the screen depends on the intensity of electron bombardment. It is possible to obtain a trace which will remain on the screen long enough for examination in detail at leisure. The trace may be inspected under ordinary lighting, and after the desired information has been obtained it may be erased by the application of heat and light to the screen.

* Skiatron is a trade name registered in the United Kingdom in the name of Scophony Ltd. under number 640179.

These properties of the tube make it suitable for the recording of the pulses. A record may be obtained by means of a tracing, or by photography with a simple camera. It may even be satisfactory to just note the required measurements from the tube.

This method has the advantage of not requiring the complication and expense of a high-speed camera and film. Also the results are immediately available since there is no film to be developed. A disadvantage of the Skiatron tube is that special equipment is necessary to supply the heat and light for removal of the trace. Also during the time interval needed to obtain the information from the tube and to remove the trace, there is a slight possibility of another transient fault occurring. This difficulty could be overcome by the use of two tubes mounted such that one tube is always in operation.

Further information regarding the availability and cost of the tubes was not obtainable from Scophony Ltd., and owing to lack of time, further investigation on these tubes had to be postponed.

5. Line Coupling Equipment.

The line coupling equipment consists principally of a high-voltage coupling capacitor in series with a power-frequency drain coil. The generated pulse is applied between the capacitor and the coil. The coil must of course have enough inductance to present a high impedance

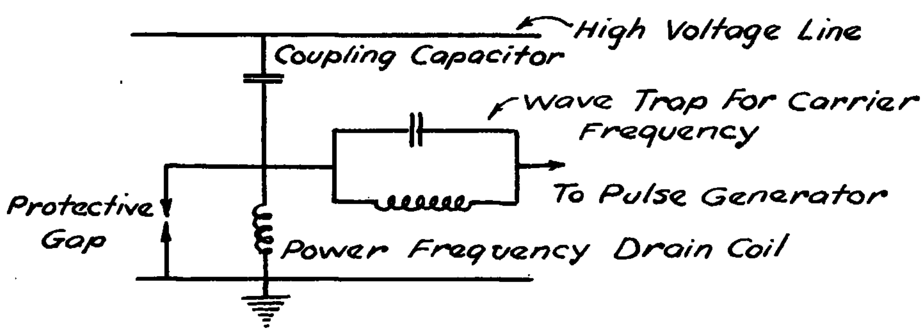


Fig. 13. Line Coupling Apparatus.

to the pulse frequency and carrier-current frequency, and yet be practically a short circuit to power frequency. Under steady-state conditions this equipment would be adequate. However, if the line is struck by lightning, a high voltage will be incident on the drain coil with resulting danger to personnel and equipment. This difficulty is overcome by means of a protective air gap placed in parallel with the drain coil. A thyrite lightning arrester is not necessary because the power arc is extinguished by the high impedance of the coupling capacitor to power frequency. The gap may be set for a voltage slightly higher than the maximum generated pulse voltage.

The line coupling equipment must present a high impedance to any carrier-current on the line. This may be accomplished by the insertion of a wave trap in series with the pulse output as shown in Fig. 13. The wave trap consists of an inductance L_1 in parallel with a condenser C_1, the two forming a resonant circuit at the carrier frequency. The pulse frequency will probably be higher than the carrier frequency, and will thus pass through the condenser C_1 and the coupling condenser onto the transmission line. The reactance drop in these condensers will decrease the pulse amplitude on the transmission line somewhat, but not too seriously for the sizes of coupling condensers used in practice. This drop in the condensers may be tuned out with a series inductance. However, the inductance has the undesirable effect of slowing down the initial amplitude of the damped sine wave. Because of lack of time, no experimental data was obtained on the effect that the tuning has on the pulse shape.

E. Experimental Results.

No experiments on transmission lines could be carried out as no lines were available. As a result, all the equipment was tested out on coaxial cable. To obtain some information on the type of reflection to be

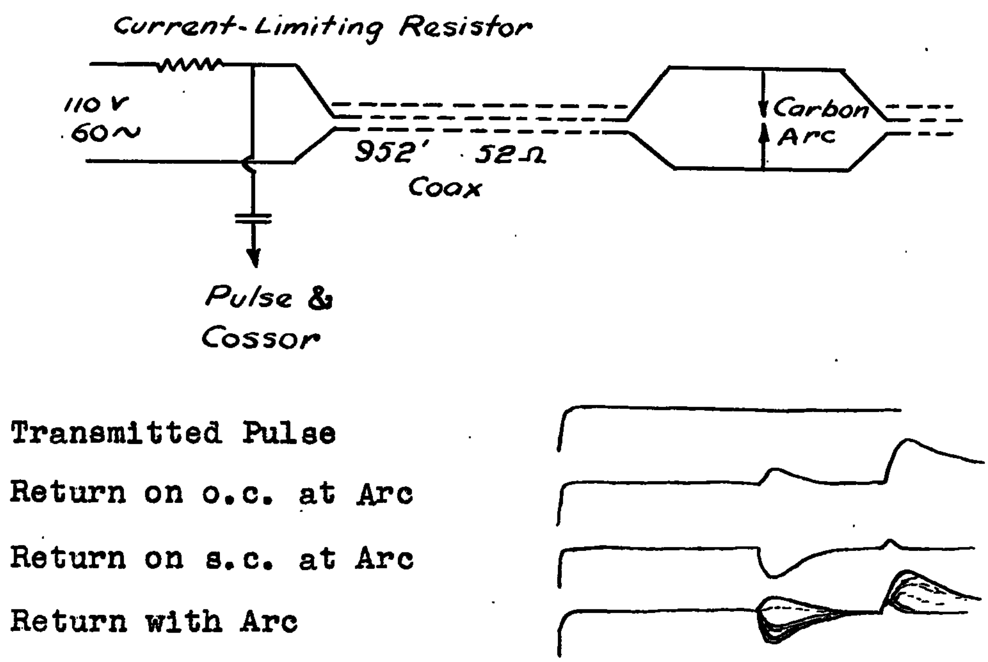


Fig. 14. Experiment with Carbon Arc.

expected from arcs, experiments were performed with carbon arcs and arcs through a small gap in oil.

The equipment used for the carbon arc is shown in fig. 14 with the resulting wave forms for a step wave input. It is seen that the resistance of the arc varies between that of an open circuit and a short circuit, but is more often nearer that of a short circuit as indicated by the darker reflection shown on the negative pulse. The carbon arc sputtered considerably during its initiation, and resulted in appreciable background disturbance. However, the reflection from the arc was still easily visible. After the sputtering had died out, the background disturbance disappeared entirely and the reflection showed up clearly. It is noted that a somewhat weaker echo is obtained from the end of the line due to pulses striking the arc when its resistance is high. The pulses are then reflected from the far end of the line and pass again through the arc to the sending end.

The apparatus used for the experiment with the arc in oil is shown in Fig. 15. The voltage was raised with the voltage regulator until the gap flashed over, and the echoes from the transmitted pulses were noted on the oscilloscope. The clearness of the reflection varied greatly with the impedance of the current limiter and the power fed into the fault. With a 1200-ohm current-limiting resistor the reflection was not as clear as for a lower impedance reactor, and best results were obtained with no current limiter at all. This is probably due to lowering of the arc resistance as well as increased arcing time. The current fed to the fault in these experiments was limited by the low kva rating (1 kva) of the transformer used.

A considerable amount of high-frequency high-voltage interference was noticed on the oscilloscope especially when current-limiting impedances were used. It was of high enough voltage to flashover the telephone-type

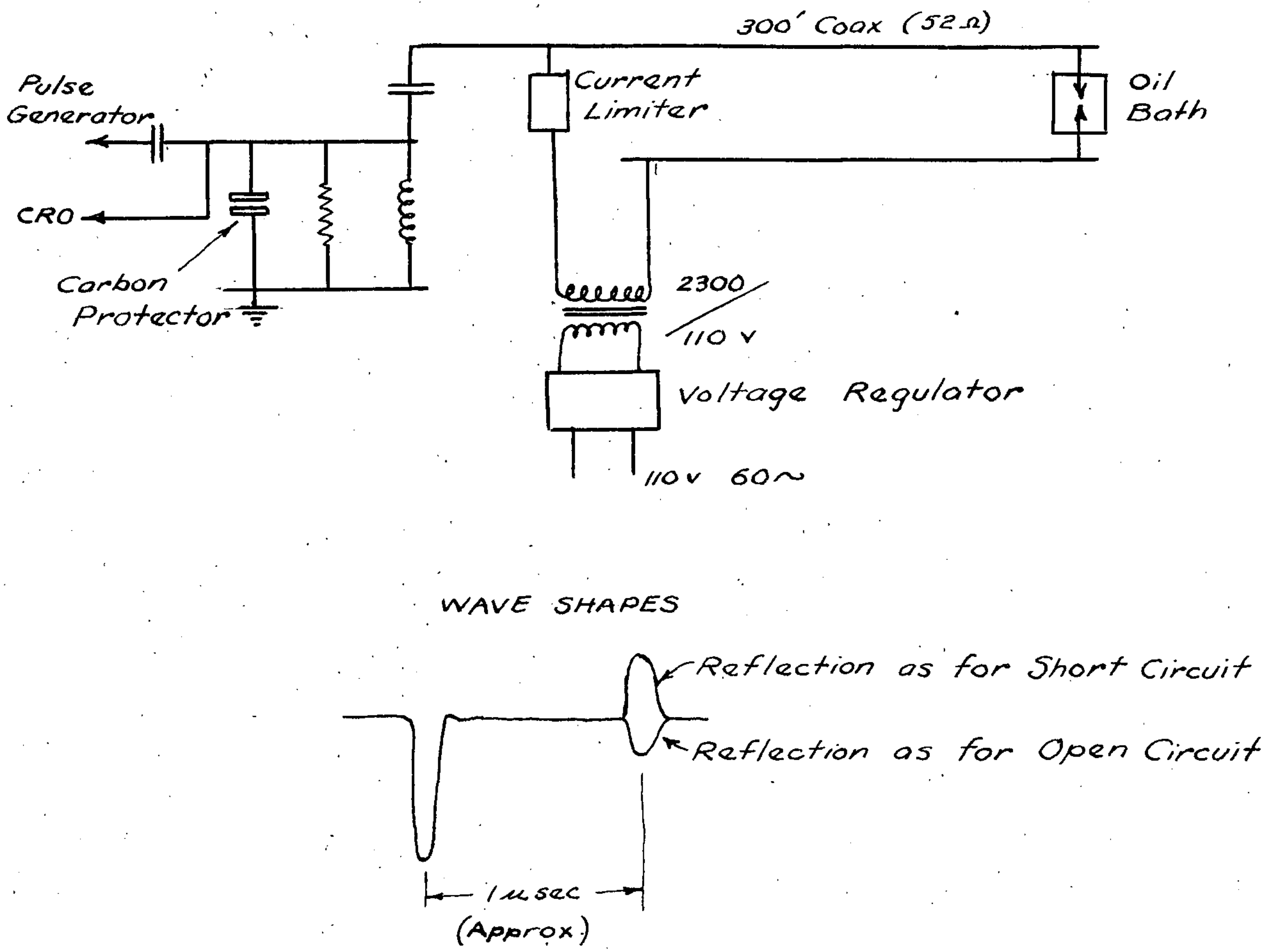


Fig. 15. Experiment With An Arc In Oil

carbon protecting blocks, and is caused by the low capacity of the transformer which is unable to sustain the arc.

From these results it is concluded that the more current fed into the fault, the lower the resistance and the more nearly the reflection approaches that of a short circuit. Thus it seems probable that a good reflection may be expected from the high-current power arcs on transmission lines.

The pulse used for the latter experiment was approximately one-tenth of a microsecond wide, and was obtained by means of the circuit shown in Fig. 16. The positive rectangular wave input pulse is obtained from

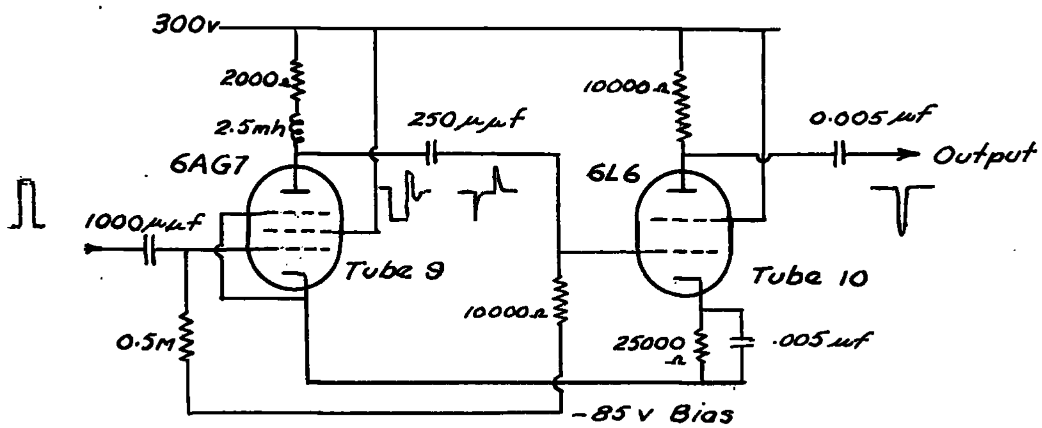


Fig. 16. Pulse Generator Used for Carbon Arc Experiment.

the delay tube (No.3) and applied to the grid of tube 9. This is a 6AG7 biased past cut-off to produce a negative rectangular wave with steep sides on the plate. The plate circuit contains a choke coil which tends to decrease the steepness of the leading edge of the negative wave, but results in a steeper trailing edge. This is a desirable effect because the trailing edge only is used by the next tube.

The output of tube 9 is differentiated by the capacitor C_1 and resistance R_1 with a resulting input to the grid of tube 10 (6L6) of a negative pulse followed by a positive pulse as shown in Fig. 16. Tube 10 is also biased past cut-off to produce an output of a negative-going rectangular pulse with a sharp leading edge and a somewhat slower trailing edge. A very narrow pulse can be obtained if the output is differentiated to remove the low-frequency components. The two tubes used to obtain this pulse were built into the set for convenience.

IV DISCUSSION

Although the equipment has not been used on a transmission line, it is expected from the theoretical and practical work done on the problem that the locator will be successful. The example three-phase line worked out in the theory indicates that for a grounding resistance of 25Ω, a reflected voltage wave of approximately 90 percent of the incident wave is obtained if the pulse is applied to the faulted line. If the pulse is not applied to the faulted line, the reflected voltage wave is only about 11 percent of the incident wave.

The pulse is also attenuated in its travel along the line. The approximate attenuation formula shows that the pulse will attenuate to about 58 percent of its original value after traveling 100 miles. Thus for a fault 100 miles away, the reflected pulse magnitude is calculated to be 30 percent of the impressed pulse for the fault on the pulsed line, and 3.7 percent for the fault on one of the other lines. This latter figure shows that amplifiers will be desirable.

The equipment may be used to locate both transient and permanent faults. For transient faults the grounding resistance of the tower will usually be low enough to give an easily detected reflection. The brightness of the echo will depend on the number of retraces made before the breakers open and the arc is extinguished. Even with the fastest breakers a few retraces will occur before the line is opened.

For permanent faults the limiting factor is the grounding resistance of the fault. The smallest reflection that can be recognized depends on the amount of background disturbance, and must be obtained by experiment. The grounding resistance for a given magnitude of echo may be calculated by means of equation (103). For example, a reflection efficiency of 1 percent is obtained for a grounding resistance of about 20000 Ω for the line considered. It is expected that the locator will also indicate a conductor fallen to the ground but not actually grounded.

One disadvantage of the present method is that the faulted line is not indicated by the locator. The size of the reflection would probably tell whether the fault was on the pulsed line or one of the other two lines, but in general some other information would be necessary. One difficulty with the pulse now used is that echoes from near faults will be confused with the transmitted pulse. This drawback may be remedied somewhat on short lines by an increase in pulse frequency and a decrease in pulse width, or by a change to a sharp-fronted pulse with an exponential decay.

The accuracy of the method depends to a great extent on the length of sweep and the distortion of the wave shape. The sweep length may be increased by the use of circular or spiral sweeps. However, with a Skiatron tube the limitation may be in sweep speed, as information from Scophony Limited gives the writing speed of these tubes as approximately 200 meters per second.

As time was not available to complete the project, certain theoretical and practical problems had to be left unsolved. One of the most interesting points in the theory, the accurate calculation of distortion and attenuation, had to be left uncompleted. This solution would be of value in deciding which points on the wave should be used to measure the distances accurately, and would also be useful in determining the best wave shape to use. An accurate solution of the reflection to be expected from the end of the line would also be useful. The shape will be determined by the buses and transformers.

Some interesting experimental work should also be carried out. The equipment should be tried out on a transmission line to check the calculations in the theory. The pulse magnitude must be checked to see if it is satisfactory, and the frequency and width of the pulse should be varied to find the one best fitting the requirements of narrow width, low attenuation and distortion, negligible interference with carrier current, low modification from the coupling capacitor, and a desirable shape for accuracy. The exponentially decaying wave should also be tried out to see if it would be useful for short lines.

A further investigation into the characteristics of the Skiatron tube or other memory tubes is warranted. The tube must have a clearly defined image and a high enough writing speed to give the required accuracy. One difficulty with these tubes is that the trace has to be erased before another can be shown over it. However, it would not be desirable to have the tube inoperative during a storm since another fault may occur at any time. One solution to the problem would be to shift the bias on the tube slightly after an operation so that the next trace would be spaced from the previous one.

Some work will be necessary on line coupling equipment. The effectiveness of the scheme suggested must be tested. If the equipment is to be installed inside a substation then a connecting line of coax could be used. This introduces problems in matching, since the characteristic impedance of coaxial cable is usually between 50 and 100 Ω.

Another unknown that can be found only by experiment is the effect of the surge created by the flashover itself on the pulse-sending apparatus.

V CONCLUSIONS

A fault locator has been developed for the location of transient as well as permanent faults. Theory and experiment indicate that the equipment should be capable of locating flashovers of insulators since the arc resistance is of the order of two ohms, and the tower grounding resistance will usually be small enough to give an easily detected reflection.

The distortion of the applied wave in its travel along the line has not been calculated. By an approximate method the attenuation has been found to decrease the pulse to one half of its initial magnitude in a distance of about 120 miles on a typical 220-kv line.

An echo may be received from the transpositions of a three-phase line. For a typical three-phase line with two ground wires, however, the magnitude of this echo will be of the order of 0.08 percent of the incident wave, and will be theoretically zero for a three-phase line with flat spacing and no ground wires. If the lines were spaced vertically, then definite echoes would be obtained from the transpositions. The echo from the terminal transformers and bus work can be used to mark the end of the line. If the fault is less than half way to the far end of the line, successive reflections will occur on the trace unless precaution is taken to match the output impedance to the characteristic impedance of the line.

A pulse generator capable of producing a damped sine wave or sharp-fronted wave with exponential decay has been constructed. The generator has been used on coaxial cable with carbon arcs and arcs in oil, and satisfactory reflections have been obtained. These experiments indicate a possibility of locating faults in power cables by superposition of pulses on a high-voltage power-frequency or d-c voltage used to produce an arc at the fault.

The pulse generator is to be tripped by means of zero-sequence current in a transformer bank neutral, or by means of the initial surge of the fault. The first method has the disadvantage of indicating only line-to-ground faults. Also false indications may be obtained from ordinary switching operations if the breaker does not close the three phases simultaneously. The second method has the disadvantage of being subject to tripping from spurious surges such as those caused by switching and the release of bound charges due to lightning.

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