A COMPARISON OF THE INDUCTIVE AND THE DEDUCTIVE METHODS IN TEACHING TWO UNITS OF SEQUENTIAL MATHEMATICS IN HETEROGENEOUS CLASSES OF THE SENIOR HIGH SCHOOL

by

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ABSTRACT

Problem: Does the inductive method offer advantages over the deductive for heterogeneous classes in Senior High School mathematics?

A proposal is made that all students in such classes start together with practical applications and that each proceed as far into theory as he is able. There is some question, however, as to whether the inductive order and style of presentation would result in loss of learning, especially in the theoretical aspects, as compared with the deductive method.

To help answer this question a controlled experiment was conducted in which two classes, equated by mean and standard deviation on the bases of I.Q. and previous mathematics marks, worked during eight 40 minute periods on elementary trigonometry and during seven similar periods on chords in a circle. This subject matter, the same for both classes, formed part of their regular course in Grade XI mathematics. The inductive group began with practical applications and proceeded to theory while the deductive group followed the reverse order; both classes were held to the same length of time for each type of work, however. Mimeographed sheets were provided to pupils for each lesson. The groups were reversed as to method for the second unit. Teacher-made tests were employed for measuring learning gain.

The first unit of the experiment was later carried on with sample classes in two other schools.

Results showed no statistically significant differences in general learning gain between the two methods.

Results in the first unit by the original sample indicated no loss in the theoretical aspects under the inductive method. Information concerning this feature was not available from the other groups or from the second unit.

In general, the evidence favoured the null hypothesis.

TABLE OF CONTENTS

Page

CHAPTER I BACKGROUND OF THE PROBLEM

Transition and Trends in Secondary Mathematics........ 1
Limitations of the Double or Multiple Track System.... 5
A Review of Attempts to Provide for Individual Differences.... 6
Organization of Subject Matter for Heterogeneous Classes.... 11
The Inductive vs the Deductive Approach............... 13
Summary... 15
Statement of the Problem in the Present Investigation. 16

CHAPTER II PLAN, SETTING, AND LIMITATIONS OF THE STUDY

General Plan.. 17
Setting for the Experiment............................ 17
Design of the Experimental Study...................... 19
Subject Matter.. 20
Measurement and Comparison............................ 21
Limitations of the Study.............................. 21

CHAPTER III PROCEDURE

Preparation of Material............................... 23
Classroom Procedure................................... 29
Measurement... 30
Administration.. 30

CHAPTER IV ANALYSIS OF RESULTS

Equality of Groups.................................... 34
General Achievement of Groups......................... 36
Achievement in Practical and in Theoretical Work...... 39
Correlation Between Ability and Achievement........... 41

Page

CHAPTER V SUMMARY AND CONCLUSIONS

Summary of the Problem and Its Background............. 43
The Experiment.................................... 45
Equality of Groups................................ 46
Measurement....................................... 46
Summary of Results................................ 47
Interpretation of Results......................... 48
Summary of Conclusions............................ 51
Critical Note and Suggestions for Further Study....... 52

BIBLIOGRAPHY.. 55

APPENDIX A MIMEOGRAPHED DETAIL OF LESSONS.............. 59

APPENDIX B TESTS....................................... 61

APPENDIX C SAMPLE OF CALCULATIONS...................... 62

LIST OF TABLES

TABLE		Page
I	Outline of Lesson Sequences - Elementary Trigonometry Unit........	24
II	Outline of Lesson Sequences - Chords in a Circle Unit............	25
III	Preliminary Equating of Kamloops Groups............	31
IV	Final Equating of Kamloops Groups.................	34
V	Equating of Chilliwack Groups.....................	35
VI	Equating of Langley Groups........................	35
VII	General Achievement of Kamloops Groups in Unit I...	36
VIII	General Achievement of Kamloops Groups in Unit II..	37
IX	Test and Gain Differences - Unit II................	37
X	General Achievement of Chilliwack Groups in Unit I.	38
XI	General Achievement of Langley Groups in Unit I....	38
XII	Comparison of Mean Gains in General Learning.......	39
XIII	Results in Theoretical and Practical Work Shown Separately.....	40
XIV	Mean Score Differences Between Practical and Theoretical....	40
XV	Correlation Between I.Q. and Test Score Gain.......	41
XVI	Correlation Between I.Q. and Test Marks - Practical and Theoretical - Unit I - Kamloops Groups.......	42

LIST OF ILLUSTRATIONS

I	Lesson 1, Sequence A............................	26
II	Lesson 1, Sequence B............................	27
III	Lesson 5, Sequence A............................	28

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CHAPTER I

BACKGROUND OF THE PROBLEM

Transition and Trends in Secondary Mathematics

The retreat of traditional mathematics before the advance of modern psychology, the emphasis upon actual needs which forms part of our philosophy of education to-day, and the tremendous increase in attendance at High Schools as they seek to provide for all youth have combined to bring about a great change in secondary mathematics during the past several decades. And the movement is not yet complete. Our whole secondary school system appears to be in a transitional stage and the mathematics phase of it remains in process of reorganization. Current educational literature contains many expressions of opinion by leaders in the mathematics teaching field; the majority of these point towards common goals but remain consistently general. In development of programs and rebuilding of mathematics curricula trends have become fairly well marked, but detailed surveys are few and construction is almost erratic.

In a fairly recent article E. R. Breslich[1] has traced the outstanding changes of the past fifty years, noting the development of general mathematics with emphasis upon applications to daily life, general correlation, the increase

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1. Breslich, E. R., How Movements of Improvement Have Affected Present Day Teaching of Mathematics, Sch. Sci. & Math., Feb. 1951: 131-141.

of emphasis on meaning and understanding, greater stress on concrete materials, use of multi-sensory aids, and the project and laboratory methods to provide for individual differences. H. F. Fehr[1] in discussing a modern program compares the older aim of college preparation with that of mathematics for all members of society, yet stresses in the latter respect that all are not alike. P. S. Jones[2] attempts to summarize such trends as teaching for meaning and understanding (both social and mathematical), emphasis upon logic, development of several types of course, enrichment materials, laboratory methods, use of instruments and teaching aids, source units, and utilization of applications. He notes that little has been done to improve the sequential courses. W. D. Reeve[3] outlines such significant trends as stress on meaning, general mathematics, multi-sensory aids, omissions and changed emphases in particular features of subject matter, and recognition of individual differences; although he questions whether much more than lip service has been paid to the last named. Wm A. Gager[4] reports that a Florida workshop group studying improvement of

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1. Fehr, H. F., A Proposal for a Modern Program in Mathematical Education in the Secondary Schools, Sch. Sci. & Math., Dec. 1949: 723-730.

2. Jones, P. S. and others, Report on Progress in Mathematics Education, Sch. Sci. & Math., June 1949: 465-474.

3. Reeve, W. D., Significant Trends in Secondary Mathematics, Sch. Sci. & Math., Mch. 1949: 229-236.

4. Gager, Wm A., Functional Mathematics, Math. Tch., May 1951: 297-301.

mathematics curricula favoured functional mathematics as a constant in grades 7 to 10 and elective in grades 11 and 12, with sequential algebra, geometry, and trigonometry from grades 9 to 12 also elective. While this was advocated, reported indications were that few schools had come close to the plan in actual practice.

The Commission on Post War Plans of the National Council of Teachers of Mathematics[1] conducted an extensive survey. In their second report in 1944 they set forth the responsibility of the High School as twofold: to provide sound mathematical training for future leaders in science, mathematics, and related fields, and to ensure mathematical competence in ordinary affairs of life for all. To meet this responsibility they recommended a unified program of general mathematics for grades 7 and 8, followed by a "double track" of sequential mathematics for those of higher ability and of general mathematics for the remainder. R. Schorling,[2] who was a member of the commission, later published an interpretation of some of their data, summarizing that two-thirds of the schools which had reported did offer the double track in grade 9, and one-half carried it on through grade 10, but a relatively small number provided any alternative to the single track of sequential mathematics courses in grades 11 or 12.

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1. Commission on Post War Plans - National Council of Teachers of Mathematics, Second Report, Math. Tch., May 1945: 195-221.

2. Schorling, R., What's Going On in Your School, Math. Tch., Apr. 1948: 147-153.

Quite recently, a report on a special research project, sponsored by the Southern Section of the California Mathematics Council but carried on over thirty-five states, indicated favour for a three track or a multiple track program.[1] This report did not give a specific plan, and there does not appear to have been agreement upon organization details. It was recommended that formal mathematics courses be strengthened and that there be "sequential ungraded courses in non-traditional mathematics." Beyond these appeared to be simply the aim to expand general mathematics. It is noteworthy that a considerable number of the personnel engaged on this project had also been leading members of the Commission on Post War Plans.

All these reports show considerable general accord, and from them the writer attempts to summarize presently established trends as follows:

1. Emphasis on meaning and understanding:
    a. more concrete illustrations and applications
    b. closer relation to real life situations
    c. use of multi-sensory aids

2. Re-emphasis on critical thinking:
    a. inclusion of non-mathematical subject matter
    b. teaching for transfer

3. General mathematics:
    a. decompartmentalization of arithmetic, algebra, and geometry
    b. social value topics
    c. correlation with other subjects

4. Recognition of individual differences

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1. Irvin, Lee, The Organization of Instruction in Arithmetic and Basic Mathematics in Selected Secondary Schools, Math. Tch., Apr. 1953: 235-240.

5. Curriculum reorganization:
   a. general mathematics - compulsory for grades 7 and 8
   b. double track (expanding to multiple track) - for grades 9 to 12, one or two years compulsory.

Limitations of the Double or Multiple Track System

Looking more particularly at the Senior High School or at grades 9 to 12, while the double or multiple track seems a working attempt to meet the needs of our greatly increased and varied population, it appears to have limitations. The Commission on Post War Plans[1] noted in their survey that more than two-thirds of all High Schools had less than 200 students and eight teachers. To meet their situation, it was recommended that two courses be handled simultaneously by one teacher, that cycling of courses be carried on, and that use be made of correspondence courses. Schorling[2] later pointed out, however, that the response to the enquiry had been very weak from small High Schools. There seems reason to think, then, that the Commission's recommendations for them were not as adequately considered as was the general question. In order to provide a single track many small schools have used the practices recommended by the Commission. In these situations, where a teacher previously handled two courses simultaneously, the double or multiple track would require him to direct a variety of interests at once.

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1. Commission on Post War Plans, op. cit.

2. Schorling, R., op. cit.

Even in the large schools which can offer a number of elective courses, guidance services are not perfect and social pressures exist, hence the composition of many classes appears likely to be quite heterogeneous for some time to come.

There remains, them, a considerable problem of making reasonably adequate provision for the varied abilities, needs, and interests which occur within single classes.

<u>A Review of Attempts to Provide for Individual Differences</u>

A number of methods of providing for individual differences have been considered in the past. An investigation of supervised study by Minnick as early as 1913 has been reviewed by Reed[1] together with later ones by Jones and Douglass. Another by Johnson combined supervised study with a project method and socialized presentation. Stokes achieved unusual success with a low I.Q. class by individual instruction. Reed's general summary of all these indicates some value in homogeneous grouping, supervised study, project method, differentiated assignments, individual instruction, and special teaching for slow pupils.

The University of Chicago High School[2] developed over some years a general pattern of supervised study with

1. Reed, Homer B., Psychology and Teaching of Secondary School Subjects, New York, Prentice-Hall, 1939.

2. University of Chicago, Mathematics Instruction in the University High School, Pub. No. 8, Nov. 1940.

added instruction for the weak and enrichment for students of high ability; they also found sectional grouping worthwhile. Brownman[1] in a controlled study compared lecture-demonstration with the individual-laboratory method for teaching experimental geometry, and he found the latter significantly superior in test scores and in experimental concepts. Durell[2] considered three stages of mastery and advocated gradation of exercises; this plan is used in a number of modern text-books. Lane[3] graded original exercises in plane geometry on three levels; in an experimental class students were allowed to make their own selection so that those completing difficult exercises did not have to attempt the easier ones, while in the control class students simply were assigned a certain number of exercises per day. According to her report, comparison of test results gave indications of superiority for the experimental method except in the case of students of low ability. She also reported that the majority of those who had choice of exercises appeared to select intelligently rather than lazily.

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1. Brownman, David E., Measurable Outcomes of Two Methods of Teaching Experimental Geometry, Jnl. Exp. Ed., Sept. 1938: 31-34.

2. Durell, Fletcher, Mathematical Adventures, Boston, Bruce Humphries, 1938: 60-75.

3. Lane, Ruth, The Use of Graded Originals in Plane Geometry, Math. Tch., Nov. 1940: 291-300.

Several more recent studies differ in their treatment of individual differences. Albers and Seagoe[1] in a ninth grade algebra class allowed fifteen minutes of each daily period to students whose I.Q. was 125 or above for explorative enrichment on a more or less voluntary basis; a small library of enrichment material was provided and extra voluntary homework allowed. In the control class the corresponding students carried on only regular work. In a final test on algebra achievement the experimental group showed progress equal to that of the control group, and in a further test on the enrichment material they showed good results. The conclusions were that superior students can afford time for enrichment, that such work is self-motivating, and also that the procedure is administratively possible in small schools unable to use homogeneous grouping. But the findings of this study are limited to students of superior ability. Lee[2] described a plan used by a large High School where several general or functional courses, each including some theoretical work, were carried on simultaneously with more formal offerings in algebra and geometry. All these were organized on a semester basis, and a student showing interest and ability could move

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1. Albers, M. E. and Seagoe, M. V., Enrichment for Superior Students in Algebra Classes, Jnl. Ed. Rsch., Mch. 1947: 481-495.

2. Lee, Wm, Provision for Individual Differences in High School Mathematics Courses, Math. Tch., Oct. 1947: 294-297.

from the general to the formal or vice versa at the beginning of any term. While the plan appears complicated and, in the form described, limited to large schools, its mosaic pattern strikes the writer as uniquely apt for any heterogeneous group of developing youth.

In a project by Fowler,[1] several features were combined in an experimental procedure for teaching geometry. A mimeographed syllabus was prepared containing definitions, axioms, postulates, constructions, and theorems, grouped around eight main topics. Class sets of two texts were provided and a small library of supplementary material. Basic concepts were developed through discussion with some teacher demonstration, apparently much like the pattern described by Fawcett[2] in his classic. Formal proof was approached through exercises and cooperative work with "the instructor prodding and questioning." Formal proof of only 24 out of 133 theorems was required of the students, the remainder being either informally demonstrated or discovered; the ideas were learned but the weight of proof was eliminated in favour of practice and application. Homework "was of a standing variety." Four groups of students were employed in the investigation, each under a different teacher; one followed the above plan for a whole year, two others carried on the usual practice with text

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1. Fowler, Wynette, An Experiment in the Teaching of Geometry, Math. Tch., Feb. 1947: 84-88.

2. Fawcett, H. P., The Nature of Proof, Thirteenth Yearbook, National Council of Teachers of Mathematics.

and homework during the first term but changed to the experimental procedure for the second term, and the fourth followed the routine of text and homework over the whole year. All groups were tested by standardized plane geometry tests given periodically throughout the year and the results showed superior achievement by those in the experimental situation. While the multiplicity of variables would seem to render valid conclusions somewhat questionable in this study, provision for individual differences appears to be inherent in the experimental procedure. Modification of subject matter requirements coupled with the placing of an outline syllabus in the hands of each student and standing homework could provide for considerable differentiation. There even seems to be some resemblance, although vague, between this procedure and Lee's plan.

Through these attempts to provide for student differences there appears to be a certain progression of development. Individual instruction is the dominant if not the only feature of the early studies but, while it remains common, its limitations in the group situation have become recognized. Homogeneous grouping within classes, with added instruction for the weak and enrichment for the more able students, is aided by the grading of exercises on a three-level basis. More recent attempts lean towards provision for greater variation in ability and achievement, both in amount and in type, with the student participating to some extent in selection of material, and finding his own level. This plan for meeting individual differences within a single class seems a promising attempt to achieve the advantages of the multiple track plan for groups.

Organization of Subject Matter for Heterogeneous Classes

The foregoing trends in the teaching of secondary mathematics are by no means unique to that field but are part and parcel of general developments in education. They are consistent with a general tendency to adjust all subject matter to the needs, abilities, and interests of the student. Such an arrangement for a single heterogeneous class is well illustrated by the "differentiated unit" advocated by Billett,[1] in which certain minimum essentials are expected of all, but variation in both amount and type of further activity and achievement is regarded as a natural occurrence to be provided for by flexibility in subject matter. Durell's proposal for heterogeneous classes in mathematics, that all pupils start at the same place but that some proceed further than others, may originally have dealt with only three levels, but applied to a continuum it could provide the basis for a type of differentiated unit.

Certain problems suggest themselves, however. What is to be the common starting place? Upon what basis are exercises to be graded? Modern psychology has substantiated the

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1. Billett, Roy O., Fundamentals of Secondary-School Teaching, Cambridge, The Riverside Press, 1940, esp. Chap. XVII.

principle of leading from the concrete to the abstract, noted levels of learning, and pointed out that some pupils require longer periods of concrete work and are more limited in their ability to generalize or deal with abstractions. Organization of subject matter in line with this psychological approach now is carried generally from Elementary School arithmetic through Junior High to some of the general mathematics courses of the Senior High School, but little change has been made in the sequential courses which tend to continue at the upper levels in the traditional style. In this connection Wren[1] asks: "How do we know that the traditional sequence and the traditional treatment of subject matter is the most significant possible in the perspective of problems of modern education?" Perhaps organization and presentation along a progression from the concrete to the abstract, from the practical application to the theoretical background, can provide the common starting place and the basis of gradation even for the material of sequential courses in the Senior High School.

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1. Wren, F. Lynwood, What about the Structure of the Mathematics Curriculum, Math. Tch., Mch. 1951: 166-167.

The Inductive vs the Deductive Approach

Some studies of this inductive type of approach have been made in addition to those previously mentioned as attempts to provide for individual differences. Luchins[1] tried building the concept of areas through the use of concrete materials, proceeding to diagrams, and leading to deductive geometry. He reported a clearer grasp and good retention of the deductive proof as well as proper and wide application of formulas, but this was only a subjective view. Michael,[2] with fifteen classes in ninth grade algebra, compared an inductive method, in which the class discovered rules through numerous exercises built around familiar situations, with a deductive method where the teacher gave rules without reasons followed by extensive practice. He found the deductive group significantly better in generalizations, but otherwise no evidence to support preference for either method. It seems important to note, however, that with the inductive group no attempt was made to state verbally the discovered rules -- that these pupils had not had practice in expressing generalizations.

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1. Luchins, A. S. & E. H., A Structural Approach to the Teaching of the Concept of Areas in Intuitive Geometry, Jnl. Ed. Rsch., Mch. 1947: 528-533.

2. Michael, R. E., The Relative Effectiveness of Two Methods of Teaching Certain Topics in Ninth Grade Algebra, Math. Tch., Feb. 1949: 83-87.

Dodes,[1] in a review of experimental studies, has noted that so far there is no strong evidence in favour of either method.

Theoretically, then, the inductive approach offers a basis of progression which could be used to advantage with heterogeneous classes whose study includes formal mathematics, and the evidence thus far indicates that no general learning loss would result. But the amount of that evidence is relatively small; the existing total of systematically gathered data concerning pupil achievement under the inductive method as compared with the deductive is insufficient to justify any conclusion. Moreover, the inductive approach might be suspected of emphasizing the concrete, and a loss in comprehension of theory coupled with a gain in practical achievement could appear as no loss in general learning. Serious consideration should be given, therefore, to the two aspects viewed separately.

With the aim of securing further information as to the relative merits of each method, the writer proposes to undertake an experimental study, comparing results under progression from the concrete or the practical application to the underlying theory with those where students proceed in the traditional style from theorems or rules to applications.

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1. Dodes, Irving Allen, The Science of Teaching Mathematics, Math. Tch., Mch. 1953: 159.

Summary

Secondary school mathematics is in a state of transition. Emphasis on meaning and understanding together with applications to everyday life situations have become well marked trends. "General mathematics," compulsory through the Junior High School, is organized around these principles.

In the Senior High School, to assist in providing for the greatly increased and varied population, social utility mathematics courses have been added to the traditional offerings of formal geometry and algebra, and all of these made elective. This "double (or multiple) track" can hardly be carried on in small High Schools, however, and in the larger institutions many students of mediocre ability continue to attempt the formal courses. Since heterogeneous groups are common, there is need for some means of applying the multiple track principle within these single classes; of providing both theoretical and practical mathematics with the amount of each varied according to student ability.

Durell's proposal, that all pupils start at the same place and some proceed further than others, offers guidance towards organization. Modern psychology and the trend of common practice in the lower grades suggests progression from the concrete or the practical application to the abstract or the theoretical, but the amount of scientific data concerning the advantages or disadvantages of such an inductive approach is decidedly limited.

Statement of the Problem in the Present Investigation

Search for a means of providing for heterogeneous groups has led to the question of the relative efficiency of two teaching methods, and the problem for investigation is now stated as follows:

General Problem: In the teaching of formal or sequential mathematics in Senior High School, does an inductive method in which progression is from the concrete or practical application to the underlying theory (hereinafter referred to as "the inductive method" or Method A) offer advantages over a deductive method in which progression is from theory to application (hereinafter referred to as "the deductive method" or Method B) when applied to heterogeneous classes?

Specific Problems:

1. Will there be statistically significant differences between the mean gains in general learning resulting under Method A as compared with Method B?

2. Will there be statistically significant differences between the mean gains resulting under Method A and Method B:

   (a) in the theoretical aspects?

   (b) in the practical aspects?

3. Will there be a higher correlation between ability and learning gain under Method A than under Method B?

CHAPTER II

PLAN, SETTING, AND LIMITATIONS OF THE STUDY

General Plan

In order to investigate the problem, it was planned to conduct a controlled experiment using for the sample two equated groups of students, one taught under Method A, the other under Method B. The subject matter, time, and working conditions would be the same for both groups, while the method of presentation would form the variable. Evidence as to the advantages or disadvantages of the inductive method would be sought in comparison both of the learning gains in general and of the theoretical and the practical aspects considered separately. Since the problem concerned a specific type of situation, it was planned to draw the sample from a common heterogeneous population and the subject matter from the normal material in a sequential type course.

Setting for the Experiment

Two classes in the same mathematics course were being taught by the writer at Kamloops, British Columbia. This course, known as Mathematics 30 in the British Columbia Programme of Studies, formed the second year's work on the sequential line of a double track program and was compulsory for students seeking entrance to university in the province. It contained selected topics in geometry and algebra, graphs, elementary trigonometry, and logarithms. Although considerable stress was placed upon applications and upon critical thinking, in line with modern trends, organization of the subject matter tended to remain traditional in style with the deductive approach generally dominant.[1]

One of these classes contained 17 girls and 10 boys of approximately 16 to 18 years of age whose I.Q.'s ranged from 90 to 125, the other had 16 girls and 10 boys similarly aged from 16 to 18 years with I.Q.'s from 93 to 125. These were in a Junior-Senior High School of composite type having a total enrolment of about 900 pupils. In the senior grades this school provided a variety of academic courses including some specialization in languages, science, mathematics, and social studies. It also offered a fair program in commercial subjects, home economics, industrial arts, music, and art. The majority of the students in the two mathematics classes were attempting university entrance but, as there was a considerable diversity of courses within the entrance program, no common pattern predominated among the members of either group.

The community served by the school is a rapidly growing city and suburban village of close to 10,000 people, and the surrounding country within a radius of approximately 30 miles. One-third or more of the pupils are transported by school bus. The city is a railroad divisional point and a

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1. British Columbia Department of Education, Division of Curriculum, Mathematics 1950.

commercial distribution centre for a wide area; it also has resident a large number of government service employees. In the surrounding country served directly by the school, intensive fruit and vegetable growing, cattle ranching, and lumbering are industries of considerable importance. Perhaps one-quarter of the adults in the school district are of foreign birth. Thus, the occupations of the parents and the social and economic backgrounds of the pupils varied considerably and this diversity was common through both mathematics classes.

In this situation, the two classes in a sequential course, similarly heterogeneous as to background and ability, appeared to satisfy reasonably well the requirements of the plan for a sample, and they were selected for the experiment.

Design of the Experimental Study

Examination of the two classes revealed that matched pairs were not obtainable in any quantity, and it was decided to equate the groups by mean and standard deviation on the bases of both I.Q. and previous achievement in mathematics.

To compensate for the smallness of the sample, it was planned to attempt a series of short experiments as the equivalent of several pairs of groups. Here an opportunity arose to strengthen conditions for equality of factors other than the variable, and the experimental design now was structured so as to alternate the two methods with each class.

The idea of conducting the experiment in several schools was considered. While this would provide a larger sample, the conditions of the experiment would be much more difficult to control. The plan was not discarded, however, but left in abeyance as a possible addition later.

The short unit experiments allowed for more rigid control of working conditions. Originally four of these were considered but, because the length of time in operation would reintroduce problems of control and because preparation of four complete sets of exercises and tests would be necessary, the final plan employed only two units.

Subject Matter

From the material regularly prescribed for the Mathematics 30 course two sections were chosen: one on elementary trigonometry dealing with the theory of simple trigonometric ratios and their application to indirect measurement, the other on plane geometry dealing with both theoretical and practical calculation aspects of chords in a circle. While this material was selected arbitrarily, an attempt was made to choose portions of the course that would lend themselves to inductive treatment neither more nor less readily than others.

Measurement and Comparison

Since the experiment was designed as a series of short units with alternation of method between the two groups, it was planned to measure the learning gains by unit tests. The mean and standard deviation would be employed, and comparisons of the unit means under each method viewed over the whole series.

Limitations of the Study

This study seeks evidence as to whether the inductive method offers advantages over the deductive method. The general problem restricts its scope to the teaching of sequential mathematics in heterogeneous classes of the Senior High School. The sample groups appear heterogeneous, and statistical procedures will determine the extent to which their results may be applicable to a large population, yet these procedures can not establish that such population is the great mass of Senior High School students in sequential mathematics classes.

The subject matter for the experiment consists of two units from a specific course and, as both course and units were chosen subjectively, it may or may not be truly representative of all formal or sequential mathematics.

Any conclusions drawn from this experiment, therefore, must be applied cautiously to the teaching of mathematics generally or even to heterogeneous classes and sequential mathematics in general. Yet results may be viewed together with those from other objective studies as bits of evidence contributing to knowledge of a general picture.

CHAPTER III

PROCEDURE

Since method was to constitute the single variable, the plan required that subject matter, time, and working conditions should be equalized under rigid control.

Preparation of Material

To ensure that subject matter would be a constant, a definite selection was made at the beginning from the content prescribed for each unit. This material was then divided into lesson sections, each containing basically either theoretical or practical work. Generally one section fitted a single class period of forty minutes, but some required two periods. The aim of this division was to provide that the time devoted to each type of material as well as the total time would be the same for each group; it also paved the way for the next step.

Order of presentation formed a main feature of the difference in method. Accordingly, the lesson sections were arranged in two sequences, one for Method A had practical exercises first and theory last, the other for Method B had the same material in reverse order. For example, in the first unit Sequence A began with indirect measurement of real objects while the first lesson of Sequence B dealt with the theory of trigonometric ratios. The latter material occurs in Sequence A, however, in Lesson 5. Outlines of both sequences showing the general subject matter content are given in Tables I and II.

TABLE I

Outline of Lesson Sequences - Elementary Trigonometry Unit

Sequence A Lesson No.	Subject Matter	Sequence B Lesson No.
1	Indirect measurement of real objects using tangent	6
2	Indirect measurement using sin & cos	7
3 & 4	Calculation problems as above but from given data	4 & 5
5	Theory - ratios constant for same angle	1
6	Theory - ratios vary as angle changes	2
7	Construction of angle from given function	3

TABLE II

Outline of Lesson Sequences - Chords in a Circle Unit

Sequence A Lesson No.	Subject Matter	Sequence B Lesson No.
1 & 2	Calculation exercises - chord, distance from centre, and radius	5 & 6
3	Construction exercises - circle through three points, etc.	4
4	Two theorems on chord and perpendicular	1
5 & 6	Third theorem and theoretical exercises	2 & 3

The individual lessons then were organized for presentation. Those for each sequence were prepared separately since method, the variable, frequently required differences in explanation of the same subject matter to accord with the order of presentation. As a means of control over the teacher factor, explanations and work for each lesson were set down in specific detail, and to guard against departure from the pattern during class operation, the material as finally arranged was mimeographed for distribution to students; these lesson sheets formed a combination text and work-book.

Three lessons from the first unit are given on the next pages for illustration. Comparison of Lesson 1 of Sequence A with Lesson 1 of Sequence B shows the difference in approach between the two methods. Comparison of the latter

Elementary Trigonometry Unit.   Group A.

Period 1.

Introduction: Certain dimensions which are difficult or impossible to measure directly, such as the height of a tree, a building, or a room, can often be calculated if we can measure one related distance and one related angle.

Demonstration Example: (The working of this will be shown by the teacher, one step at a time, with students following and carrying out the operations step by step).

What is the height of this classroom?

Using a sighting protractor and level placed on a desk, sight the intersection of wall and ceiling and read the angle of elevation. Measure the distance along the floor from point under the observer's eye to the vertical wall. Record these two measurements: Angle of elevation __________; horizontal distance _______

Make a diagram in the space at the right; mark the angle, base distance, and unknown to be found on it.

In a right-angled triangle, the ratio of vertical side to base is called the tangent of the lower angle; we can find the value of this from a table on page 512 of the text-book. Write: tangent of ____° is _______

We then write an equation ____________________________ =

and solve it

Table Practice: Find tangent of 7°; 16°; 30°; 53°; 72°; 80°.

Practice Exercises: Students work in pairs; sight angle and measure distance together, but each work out calculations and check result with each other.

1. Find height of a tree immediately outside school.
2. Find height of a pole " " "
3. Find height of school building.
4. Find height of any point on classroom wall (in case weather does not allow outside work)
5. Find height of electric light in classroom.

Illustration I - Lesson 1, Sequence A.

Elementary Trigonometry Unit. (GROUP B.) SEQUENCE B

Period 1.

Introduction: **Recently we proved the theorem: "If two triangles are equiangular their corresponding sides are proportional." The ratios of sides of equiangular right-angled triangles are of great importance in mathematics and are widely used.**

Exercises: The following are to be read and worked or completed by each student. As these are demonstration and study examples, explanations will be given and results checked as the work proceeds.

1. BAC and EDF, shown immediately below, are equiangular right-angled triangles

Complete: (i) $\frac{BC}{AB}$ = ___ (ii) $\frac{AC}{AB}$ = ___ (iii) $\frac{BC}{AC}$ = ___

2. Construct a right-angled triangle lettered like the sample BAC above but having sides: a = 3 cm; b = 4 cm.
(Use the left side space below)

With protractor, measure angle A and write it's size here ________ degrees.
Calculate the length of side "c" (Pythagoras theorem) ____; check by measuring.
Write in figures, first as common fractions, then as decimals, the ratios:

(i) $\frac{a}{c}$ = ___ = . (ii) $\frac{b}{c}$ = ___ = . (iii) $\frac{a}{b}$ = ___ = .

3. In the right hand space above, construct another right-angled triangle, lettered the same, but having side "b" 6 cm; make angle A the same size as in No. 2 by using your protractor. Measure the other two sides after the triangle is drawn and write the ratios, first as common fractions, then as decimals:

(i) $\frac{a}{c}$ = ___ = . (ii) $\frac{b}{c}$ = ___ = . (iii) $\frac{a}{b}$ = ___ = .

Since angle A remained constant, would you expect these ratios to be the same for both triangles?

4. If side b is 10 ft. and angle A the same, calculate side a ________

5. If side c is thirty miles and angle A the same, calculate sides a _____ and b ____

Definitions: Because triangles can be lettered in many ways, a standard means of naming sides has been adopted to avoid confusion. One of the acute angles is taken as a reference point and the sides are spoken of as:

the hypotenuse
the side opposite to the angle
the side adjacent to the angle

The ratios have been given the following names:

$\frac{\text{side opposite to angle}}{\text{hypotenuse}}$ is called SINE (Sin) of the angle.

$\frac{\text{side adjacent to angle}}{\text{hypotenuse}}$ is called COSINE (Cosin; Cos) of the angle.

$\frac{\text{side opposite to angle}}{\text{side adjacent to angle}}$ is called TANGENT (Tan) of the angle.

Exercise: Identify the ratios Sin A, Cos A, and Tan A of No. 2 above.
Check their values with those given for angle A in tables at back of text-book.

Elementary Trigonometry Unit. (Group A.) SEQUENCE A

Period 5.

What are these ratios: Sine, Cosine, and Tangent?

A standard method of naming the sides of right angled triangles has been adopted. One of the acute angles is taken as a reference point and the sides are called:

the hypotenuse
the side opposite to the angle
the side adjacent to the angle

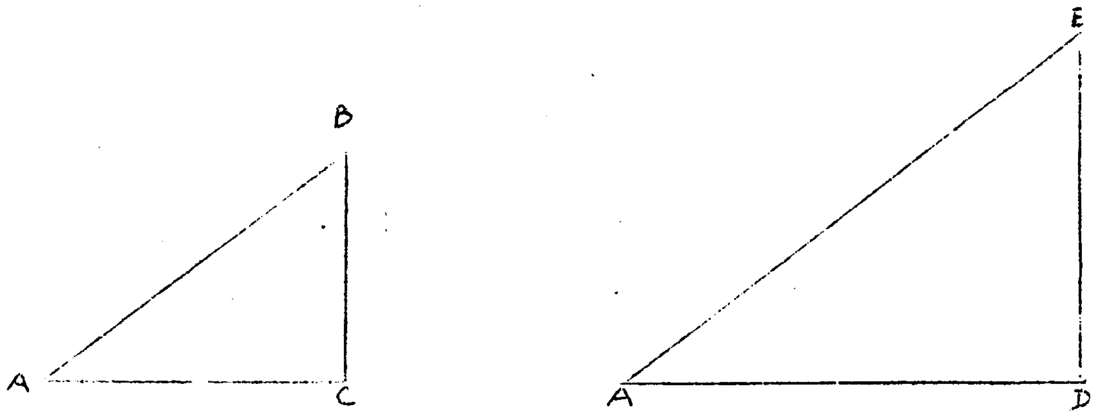
The SINE (Sin) of the angle is always $\frac{\text{side opposite to angle}}{\text{hypotenuse}}$

The COSINE (Cosin, Cos) of the angle is always $\frac{\text{side adjacent to angle}}{\text{hypotenuse}}$

The TANGENT (Tan) of the angle is always $\frac{\text{side opposite to angle}}{\text{side adjacent to angle}}$

What happens to these ratios when triangles differ in length of sides but angles remain constant?

In the figures immediately below, BAC and EAD are equiangular right-angled triangles; side BC is 3 cm., side AC is 4 cm., side ED is $4\frac{1}{2}$ cm., side AD is 6 cm.



(In the exercises below, the length of the hypotenuse may be found by calculation or measurement

1. Write: tangent of angle A is $\frac{\text{side -}\_\_\_\_\_\_\_\_}{\text{side -}}$

   For triangle BAC, tangent A = _____ For triangle EAD, tangent A = _____

2. Write: sine of angle A is side ____________

   For triangle BAC, sine A = _____ For triangle EAD, sine A = _____

3. Write: cosine of angle A is side ____________

   For triangle BAC, cosin A = _____ For triangle EAD, cos A = _____

4. Reduce each of the above ratios to its lowest terms and complete this statement: If an angle remains constant then the sine, cosine, and tangent each ____________ ____________ no matter how large the triangle.

5. Reduce each of the above ratios to a decimal:

   Sin A

   Cos A

   Tan A

6. Measure angle A with protractor, find its sin, cos, and tan from tables and check your values of No. 5.

7. Measure angle B (note that A + B must total 90°) and find from table the values of sin B _______, cos B _______ and tan B _______

8. From the triangle BAC above write the values of sin B, cos B, and tan B from the lengths of the sides. Reduce each to decimal and compare with No. 7.

with Lesson 5 of Sequence A indicates the difference in treatment of the same subject matter. A copy of the mimeographed detail for all lessons is attached as Appendix A.

Classroom Procedure

As a further control of the time factor, all work during the experiment was confined to the regular class periods, no homework being assigned. The mimeographed sheet for the lesson was distributed at the beginning of each period together with all previous pages but none in advance. Students' work was written on these sheets or on foolscap and all papers were taken from them at the end of the period to be returned on the following day.

Since these features of the procedure were foreign to normal routine and the students would realize that some unusual type of test was occurring, it was felt that a more stable situation would prevail if they were taken into confidence. Accordingly, both groups were informed that a special piece of work was to be conducted which required no homework, that the course was being tested rather than themselves, and that they could contribute to the success of the project by working as normally as possible.

The lessons of the first unit occupied seven ordinary class periods and those of the second unit six. In each case one additional period was taken for review, and the final test was administered on the following day.

Measurement

Teacher-made tests were used to measure results. For the unit on elementary trigonometry, the subject matter was considered to be entirely new material hence only a final test was given and the marks on this were treated as gain. Items of this test were balanced between practical and theoretical types of work so as to provide a basis for considering the question of whether Method A would result in higher practical achievement at the expense of the theoretical.

The unit on chords in a circle included material previously covered so both a pretest and a final test were employed here. In constructing these, pairs of similar items were made ready and one of each pair allotted to the pretest or to the final test by tossing a coin. Learning gain was measured as the difference between the final and pretest marks of each student. An attempt was made to balance these tests between theoretical and practical but was abandoned as unsatisfactory. However, a theoretical proof was added to the final test.

A copy of each of the three tests is attached as Appendix B.

Administration

The first part of the experiment was conducted at Kamloops early in 1952 with Group A taught under Method A and Group B under Method B. For the second unit, carried on about six weeks later, the planned reversal was made; Method A was used for Group B and Method B for Group A. Both classes met in the mornings, one immediately after the other, and all periods were forty minutes in length. Individual attendance was recorded throughout so that absentees might be eliminated as subjects or, as an alternative, the groups equalized in this respect.

Before class work was begun an intelligence test, the Otis Quick-Scoring Gamma, was given to all students. Groups were equated by mean and standard deviation using both I.Q. and first term achievement in mathematics. Four students who were repeating the course as well as two chronically irregular attendants were not considered, and two transfers were made to secure a better balance. This gave prospective groups for the sample as shown in Table III.

TABLE III

Preliminary Equating of Kamloops Groups

	Number in Group	I.Q. Mean	I.Q. S.D.	1st Term Marks Mean	1st Term Marks S.D.
Group A	23	110.0	8.4	66.6	14.5
Group B	23	109.0	8.5	66.9	12.8

Final selection of personnel was postponed, however, until the record of attendance had become available. It was then found that exclusion of all absentees, while ideal, would greatly reduce the size of the sample and seriously disturb the balance between the groups. Yet students having any appreciable number of absences could hardly be considered as participants. In a rather arbitrarily determined compromise, all those absent for any test or during more than two periods of either unit were eliminated, and two others dropped in equating. The result of this procedure is shown in Table IV in the next chapter.

In November of the same year, the work of the first unit was repeated with classes at Chilliwack and at Langley in the lower Fraser Valley of British Columbia. This area is a rich delta where dairying and the growing and processing of small fruits and vegetables are basic industries. A fair amount of lumbering also is carried on. Both towns are commercial centres, and each has a large composite High School with approximately half the pupils urban resident and the others conveyed by bus from rural territory.

In each of these schools the experimental work was conducted by the regular teacher of two groups who used the two sequences of mimeographed lesson sheets and the achievement test under the direction of the writer. As available mathematics marks for the previous year were in letter grade form, equating was possible only on the basis of I.Q.'s as regularly obtained and used in each school.

At Chilliwack, Group A originally contained 34 pupils and Group B 38, each in one class. In several cases I.Q.'s were unavailable; these pupils were eliminated along with absentees, and two were dropped in equating. The resulting groups are shown in Table V of the next chapter. In this school, because 55 minute periods were customary, the final review period was omitted.

At Langley, two small classes of 14 and 15 pupils respectively were considered Group A, while Group B had 33 pupils in one class. Eliminations as before gave equated groups as shown in Table VI of the next chapter. Here the seven 40 minute periods plus one for review were employed.

CHAPTER IV

ANALYSIS OF RESULTS

Equality of Groups

Each pair of groups was equated by the mean and standard deviation of I.Q.'s. Those at Kamloops were equated also on the basis of mathematics marks for the previous term, the mean and standard deviation again being employed.

Table IV shows a comparison of the two Kamloops groups as finally equated.

TABLE IV

Final Equating of Kamloops Groups

	Number in Group			I. Q.		1st Term Marks	
	Boys	Girls	Total	Mean	S.D.	Mean	S.D.
Group A	7	11	18	109.3	8.8	68.5	13.9
Group B	6	12	18	109.2	8.7	69.1	11.1

The difference between the means for I.Q. is 0.1, and for first term marks 0.6. In the latter case the standard error of the difference has been computed as 4.3, using the formula $SE_D = \sqrt{\frac{\Sigma x_1^2 + \Sigma x_2^2}{N_1 - 1 + N_2 - 1}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$, and the critical ratio, $t = \frac{M_1 - M_2}{SE_D}$, as 0.14. The difference is not significant statistically. Calculations are shown in Appendix C.

A comparison of the Chilliwack groups is shown in Table V, and of the Langley groups in Table VI.

TABLE V

Equating of Chilliwack Groups

	Number in Group	Mean I.Q.	S.D. I.Q.
Group A	25	110.2	10.7
Group B	32	108.4	10.3

The difference between these means is 1.8. The standard error of the difference and the critical ratio have been computed as before at 2.88 and 0.63 respectively. The difference is not significant statistically.

TABLE VI

Equating of Langley Groups

	Number in Group	Mean I.Q.	S.D. I.Q.
Group A	22	108.0	9.4
Group B	28	108.0	10.9

There is no difference between the calculated means. Group A, however, was composed of two small classes whereas Group B was a single class.

General Achievement of Groups

The learning gains of students taught under both methods were measured by the teacher-made tests previously described in Chapter III. Comparison is made by the mean and standard deviation of the raw test scores for each group. For Unit I these are the final test marks while for Unit II the difference between the final and the pretest marks has been taken for each student.

For each pair of groups the standard error of the difference between means and the critical ratio "t" have been calculated using the formulas previously given on page 34. In order to determine the significance of any difference, (N - 1) degrees of freedom for the combined sample have been considered in each case.

The general achievement of the Kamloops groups is shown in Tables VII and VIII.

TABLE VII

General Achievement of Kamloops Groups in Unit I

	Number in Group	Mean Test Score	S.D.
Group A	18	22.9	4.4
Group B	18	22.3	4.4

The difference between the two means is 0.6. The standard error of the difference is 1.5 and the critical ratio is 0.40. For N - 1 = 35 degrees of freedom the difference is not significant statistically.

TABLE VIII

General Achievement of Kamloops Groups in Unit II

	Number in Group	Final Test		Pretest		Gain	
		Mean	S.D.	Mean	S.D.	Mean	S.D.
Group A	18	27.3	5.6	6.8	4.3	20.5	5.3
Group B	18	27.9	5.0	8.2	5.9	19.7	5.2

The difference in mean gains and also those between the means of final test and pretest scores are shown, together with the standard error and critical ratio for each, in Table IX.

TABLE IX

Test and Gain Differences - Unit II

	Final Test	Pretest	Gain
Difference in Means	0.6	1.4	0.8
S.E. Difference	1.8	1.8	1.8
Critical Ratio	0.33	0.78	0.44

No difference above is significant statistically.

Tables X and XI show the general achievement of the Chilliwack and Langley groups in Unit I.

TABLE X

General Achievement of Chilliwack Groups in Unit I

	Number in Group	Mean Test Score	S.D.
Group A	25	26.4	2.4
Group B	32	26.6	2.0

The difference in means is 0.2, the standard error of the difference 0.59, and the critical ratio 0.34. N - 1 for the combined sample is 56. The difference is not significant statistically.

TABLE XI

General Achievement of Langley Groups in Unit I

	Number in Group	Mean Test Score	S.D.
Group A	22	24.6	3.1
Group B	28	23.6	3.9

The difference in means is 1.0, the standard error 1.04, and the critical ratio 0.96. N - 1 is 49. The difference is not significant statistically.

The foregoing data on mean gains in general learning are summarized in Table XII.

TABLE XII

Comparison of Mean Gains in General Learning

	Kamloops Unit I	Kamloops Unit II	Chilliwack Unit I	Langley Unit I
Method A, Mean	22.9	19.7	26.4	24.6
Method B, Mean	22.3	20.5	26.6	23.6
Difference (A - B)	+ 0.6	- 0.8	- 0.2	+ 1.0
S.E. Difference	1.46	1.79	0.59	1.04
Critical Ratio	0.40	0.44	0.34	0.96
Degrees of Freedom	35	35	56	49
Significance	nil	nil	nil	nil

Achievement in Practical and in Theoretical Work

The attempt to measure learning gains in practical and in theoretical work separately was confined to Unit I as the Unit II tests were considered unsatisfactory for this purpose. Because of faulty communication with the other two schools, results became available for the Kamloops groups only. These results are shown in Table XIII.

TABLE XIII

Results in Theoretical and Practical Work Shown Separately

	Theoretical		Practical	
	Mean	S.D.	Mean	S.D.
Method A	6.3	2.4	8.3	1.8
Method B	6.4	2.7	8.1	1.4
Difference (A - B)	- 0.1		+ 0.2	

As the differences between means for the two methods appear negligible in each case, no calculations of standard error and critical ratio have been made. For either method, however, the mean score on practical items was larger than that on theoretical items. A summary of these differences is shown in Table XIV.

TABLE XIV

Mean Score Differences Between Practical and Theoretical

	Method A	Method B
Mean Score - Practical	8.3	8.1
Mean Score - Theoretical	6.3	6.4
Difference (Prac. - Theor.)	2.0	1.7
S.E. Difference	0.74	0.74
Critical Ratio	2.70	2.30
Level of Significance	.01	.05

Correlation Between Ability and Achievement

Coefficients of correlation between I.Q. and learning gain under each method as measured by test scores have been computed using the formula $r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$.[1] Those which include general or total learning gain are shown in Table XV.

TABLE XV

Correlation Between I.Q. and Test Score Gain

	Number in Group		Corr. Coefficient	
	Method A	Method B	Method A	Method B
Unit I				
Kamloops	18	18	.63	.55
Chilliwack	25	32	- .19	.17
Langley	22	28	.13	.13
Unit II				
Kamloops	18	18	.06	.14

Use of the critical ratio, $t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}}$, shows that for N - 2 degrees of freedom the coefficients of .63 and .55 are significant at the .02 level while the others have no statistical significance. It should be noted, however, that

1. This formula was used because the components were already available from previous calculations.

Kamloops I.Q.'s were from one recent test but that the origin of the others is not definitely known. Also that Unit I scores were from one final test while Unit II scores were the differences between marks on two tests.

Coefficients have been computed also of correlation between I.Q. and achievement in the theoretical aspects, and between I.Q. and achievement in the practical aspects of the work of Unit I as evidenced by test marks for the Kamloops groups. These are shown in Table XVI.

TABLE XVI

Correlation Between I.Q. and Test Marks

Practical and Theoretical - Unit I - Kamloops Groups

	Number in Group		Corr. Coefficient	
	Method A	Method B	Method A	Method B
I.Q. - Theor.	18	18	.54	.45
I.Q. - Practical	18	18	.37	.35

Use of the critical ratio as before shows that for N - 2 degrees of freedom the coefficient of .54 is significant at the .02 level and that of .45 at the .10 level.

CHAPTER V

SUMMARY AND CONCLUSIONS

Summary of the Problem and Its Background

This study considers an inductive approach to the sequential mathematics of the Senior High School as a method whereby better provision may be made for the differences in ability and interests of students in heterogeneous classes.

The secondary school of to-day attempts to serve a greatly increased and varied population and to this end offers as diversified a program as facilities will permit. In the mathematics of the higher grades the double or multiple track system provides formal mathematics for some pupils and vocational or social utility arithmetic for others. But, since these are organized in separate courses, the system appears limited to the larger High Schools and its success in these dependent on adequate guidance. In the many small High Schools and to a considerable extent in the larger ones heterogeneous classes are common, and the means of providing for individual differences within these groups remains a problem.

Modern psychology and the trend of common practice in lower grades suggests that all pupils in such classes might start together with practical applications and each progress as far into theoretical work as he is able. But a long held view concerning the logical sequence of formal mathematics seems to conflict with this idea, hence it is necessary to consider whether or not such an inductive approach would result either in loss of learning generally or in loss of achievement in the theoretical aspects of mathematics.

Studies made thus far comparing the inductive and deductive methods of teaching mathematics indicate little preference for either, but the amount of such research is limited.

In an attempt to add to existing knowledge a controlled experiment was undertaken, based on the problem stated as follows:

General Problem: In the teaching of formal or sequential mathematics in Senior High School, does an inductive method in which progression is from the concrete or practical application to the underlying theory (hereinafter referred to as "the inductive method" or Method A) offer advantages over a deductive method in which progression is from theory to application (hereinafter referred to as "the deductive method" or Method B) when applied to heterogeneous classes?

Specific Problems:

1. Will there be statistically significant differences between the mean gains in general learning resulting under Method A as compared with Method B?

2. Will there be statistically significant differences between the mean gains resulting under Method A and Method B:

(a) in the theoretical aspects?

(b) in the practical aspects?

3. Will there be a higher correlation between ability and learning gain under Method A than under Method B?

The Experiment

The project was designed in the form of a series of short unit experiments in each of which two equated groups would carry on a section of their regular mathematics course, one of these classes being taught by Method A and the other by Method B. The method was to be alternated between the groups for succeeding units.

A series of two units, each occupying eight 40 minute periods, was carried out with sample groups at Kamloops, B. C. Later the first unit was repeated with two classes at Chilliwack and with two others at Langley in the same province.

The subject matter consisted of two sections from a course in the B. C. curriculum designated Mathematics 30, in which all students participating had enrolled. The material selected for each unit was first divided into a definite number of lessons which then were organized in two sequences according to method. Subject matter, time, and to some extent emphasis upon each type of work were thus controlled as constants. Elimination of homework also aided in control of time. Additional restraint of emphasis upon either type of work and regulation of the teacher factor were provided by mimeographing the complete detail and presenting it to the classes lesson by lesson.

Equality of Groups

Kamloops groups were equated by the mean and standard deviation on the bases of previous marks in mathematics and of I.Q.'s obtained from a standardized test given shortly before the experimental work began. The others were equated only on the basis of I.Q.'s as on file in their respective schools. In all cases the difference between the means was slight and not significant statistically as shown in Tables IV, V, and VI. In each group there was a considerable range and a sizable deviation indicating that it was heterogeneous as to ability. Economic and social background of students appeared to be similarly heterogeneous.

In general it may be fairly claimed that in each of the three samples the groups were well equated but, because of variation in the means of obtaining I.Q.'s, there is some question about combining the three pairs into a single sample of two groups.

Measurement

All tests employed to measure learning gains were constructed by the author. For Unit I only a final test was used, as trigonometry was entirely new to the students, and this test contained sections on theory and on practical work of equal score value. For the plane geometry of Unit II, however, the difference between a final test and a pretest formed the score counted as gain. An attempt in these to measure theory and practical work separately was abandoned.

Summary of Results

Results of the experiment stated as direct answers to the specific questions of the problem are as follows:

1. There were no statistically significant differences between the mean gains in general learning resulting under Method A as compared with Method B.

2. There were no statistically significant differences between the mean gains resulting under Method A and Method B in either the theoretical or the practical aspects. (Information here came from one sample only).

3. In one case the correlation between I.Q. and test score gain under Method A was slightly higher than that under Method B. In the other three cases all coefficients were negligible.

Two additional results were obtained from a single pair of groups, information on the same features not being available from the others. These results, numbered with reference to those above, are as follows:

2A. Under both methods the scores for practical work were significantly higher than those for theoretical work.

3A. The coefficients of correlation between I.Q. and scores in theoretical work were higher than those between I.Q. and scores in practical work, with some significance, and of the former, that for Method A was slightly higher than that for Method B.

Interpretation of Results

On the question of advantage in general learning, the results of this experiment do not favour either method. This finding is in accord with that of Michael.[1] It differs from views favouring the inductive method by Luchins[2] and by McCreery[3] but these were only subjective opinions. In this connection it is worthy of note that at least two teachers who participated in the experiment likewise expressed preference for the inductive approach, yet the mean test scores of their classes did not show significant differences. To refer back to Dodes' summary,[4] there is still no strong evidence in favour of either method.

But the fact that significant differences did not appear is not to be accepted as conclusive evidence that there were none. It is possible that differences in learning gain existed which the tests failed to detect, or that differences due to method were counteracted by other factors.

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1. Michael, op. cit. (See page 13).
2. Luchins, op. cit. (See page 13).
3. McCreery, Gene S., Mathematics for All the Students in High School, Math. Tch., Nov. 1948: 302-308.
4. Dodes, op. cit. (See page 14).

In this experiment the results were from three small but well equated samples. The amount of subject matter was limited. Time and working conditions were reasonably well controlled. But the validity of the tests from which results were obtained is open to some question. However, in the case of one sample where I.Q.'s were taken from a standardized test, recently administered, the relatively high correlation between I.Q. and test scores gives some support to the validity of the latter. This may appear to be contradicted by the low coefficients in the other cases, but due to circumstances already noted these can not be given as much weight.[1]

Taking all factors into consideration, the writer concludes that, with respect to advantage in general learning between the two methods, the results of this experiment give a definite indication in favour of the null hypothesis.

The question of differences in learning gain in the theoretical and the practical aspects considered separately was originally raised because of a seeming possibility that the inductive method might emphasize the latter at the expense of the former. No such disadvantage for the inductive method is indicated from this experiment, although once again the fact that differences were not significant statistically does not certify that there were none. The higher correlation of I.Q. to theoretical than to practical, shown in Table XVI,

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1. See Table XV and following comment; also pages 31-33.

coupled with the significant and relatively high coefficients between I.Q. and total score by the same sample, lends some reinforcement to the distinction made subjectively by the writer in composing the test, since I.Q. is accepted as our best present measure of ability in handling abstractions. But the test items covering each aspect of the work were few in number. Moreover, results were available only for one of the three samples.

The writer concludes that from incomplete measurement there is no evidence of learning loss in the theoretical aspects under the inductive method. This conclusion is at variance with that of Michael[1] who found the deductive group significantly better in generalizations.

The questionable validity of the test and the peculiarity of the single source of information affect also any interpretation to be placed on the result that under both methods the mean scores for practical work were significantly higher than those for theoretical work.[2] Subject to this qualification, the writer would conclude that the practical or more concrete aspects form an area of better achievement for heterogeneous classes. From the viewpoint of modern psychology that success causes satisfaction and encouragement to proceed, this suggests a possible advantage for the inductive method.

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1. Michael, op. cit.

2. See Table XIV.

The third specific question of the problem dealt with correlations between I.Q. and test score gain under each method. By comparison of these it was sought to discover whether greater efficiency in matching achievement to ability would occur under the inductive method than under the deductive. The resulting correlation coefficients, shown in Table XV, have already been discussed. From such conflicting results no conclusion can be drawn.

Summary of Conclusions

1. The results of this experiment show neither advantage nor disadvantage in general learning for the inductive method as compared with the deductive. There is a definite indication in favour of the null hypothesis.

2. From incomplete measurement there is no evidence of advantage in the theoretical aspects under either method.

3. From incomplete measurement there is an indication that the practical aspects form an area of better achievement for heterogeneous classes.

4. Because of conflicting evidence no conclusion can be drawn as to whether either method results in a higher correlation between ability and achievement.

Critical Note and Suggestions for Further Study

The experiment described herein was undertaken to secure information on the relative merits of two methods of teaching mathematics, especially as applied to heterogeneous classes in Senior High Schools. The question seems of considerable importance yet, although subjective opinions are common, the number of objective studies is small and the total of their findings inconclusive.

For an acceptably complete study of such a problem under statistical procedure, random sampling of both students and subject matter would be essential. In the writer's situation the former particularly had to be selected from those readily available, hence the potential accomplishment was a very slight addition to existing knowledge. As the resources of the average investigator similarly limit the contribution from any single experimental study of this problem, there seems need for a considerable number of these to be undertaken.

In the selection of students for the sample in this experiment, the question arose of whether to restrict it to classes taught by the writer or to employ also groups under other teachers in schools at some distance. Participation of classes under a number of teachers is normal to mathematics instruction generally and provides a larger sample, but unless these are accessible for close supervision it is difficult or even impossible for the investigator to observe adequate control of the experimental procedure. It may be that the difference in correlation results between the Kamloops and the other groups was due to such lack of control. Incomplete measuring of the practical and the theoretical gains separately must be charged to neglect on the part of the writer, but difficulty of access to the other groups and their teachers provided the setting for that neglect.

Inadequacy of measurement generally is probably the most serious adverse criticism which can be made of this experiment. Employment of normally obtainable standardized tests was not feasible as these cover a much broader range of subject matter than was dealt with here. The validity of teacher constructed tests as a measure of achievement in any single feature might have been established, but to do this for theoretical aspects, practical aspects, and general learning together would have been a tremendous if not impossible undertaking. An obvious conclusion is that too many questions were attempted in one project.

The basic question from which this experiment was conceived was whether or not use of the inductive approach would be advantageous in heterogeneous classes. It was suggested that all pupils in such groups might start together with practical applications and each proceed into theory according to ability. The experiment attempted to test only part of the question, i.e. whether learning loss would result from the inductive order and style of presentation.

Individual differences in both rate and extent of progression into theory were avoided. Further study involving this vital feature of the originally suggested inductive approach should be worthwhile.

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APPENDIX A

MIMEOGRAPHED DETAIL OF LESSONS

Elementary Trigonometry Unit. (Group A) SEQUENCE A

Period 1.

Introduction: Certain dimensions which are difficult or impossible to measure directly, such as the height of a tree, a building, or a room, can often be calculated if we can measure one related distance and one related angle.

Demonstration Example: (The working of this will be shown by the teacher, one step at a time, with students following and carrying out the operations step by step).

What is the height of this classroom?

Using a sighting protractor and level placed on a desk, sight the intersection of wall and ceiling and read the angle of elevation. Measure the distance along the floor from point under the observer's eye to the vertical wall. Record these two measurements: Angle of elevation __________; horizontal distance _______

Make a diagram in the space at the right; mark the angle, base distance, and unknown to be found on it.

In a right-angled triangle, the ratio of vertical side to base is called the tangent of the lower angle; we can find the value of this from a table on page 512 of the text-book. Write: tangent of ____° is _______

We then write an equation ____________________________ =

and solve it

Table Practice: Find tangent of 7°; 16°; 30°; 53°; 72°; 80°.

Practice Exercises: Students work in pairs; sight angle and measure distance together, but each work out calculations and check result with each other.

1. Find height of a tree immediately outside school.
2. Find height of a pole " " "
3. Find height of school building.
4. Find height of any point on classroom wall (in case weather does not allow outside work)
5. Find height of electric light in classroom.

Period 2.

Review Exercise: Find height of a tree, viewed through the classroom window. Three students will measure outside distance (horizontal) while three others measure angle of elevation. Whole class then works problem from their data.

Demonstration Exercise: Worked by teacher and students together as in that of period 1.

(a) The distance up the slope of a hill can be measured directly; the angle of elevation of its top can be determined by sighting from the foot. Find the vertical height of the hill.

(A cross-section diagram of the hill, drawn on blackboard, will be used)

Measure and record slope distance _____

Measure and record angle of elevation ____

Make right-angled triangle diagram in space at right, marking data.

In this case we use the ratio of vertical side to hypotenuse; it is called the sine of the angle.

Write equation as before (obtaining value of sine from table)

Solve equation

(b) Calculate the horizontal distance from foot of hill to a point directly under its top.

This is worked in a similar manner, but the ratio of the base of the triangle to the hypotenuse is called the cosine of the angle.

Practice Exercises: (Working in pairs as in period 1).

Measure the angle of ascent of a flight of steps; measure the sloping distance along the steps from bottom to top. Calculate the vertical rise.

Measure thelength and angle of elevation of a sloping board. Calculate the height of its upper end and also the horizontal distance it covers.

Review of differences between tangent, sine, and cosine.

Elementary Trigonometry Unit. Group A.

Periods 3 & 4.

Brief review of sine, cosine, andtangent from blackboard diagram.
Brief review of system of writing equation from diagram with data, and solution.

Practice Exercises: Each student is to work at his or her own rate. Individual help will be given by teacher as requested. Papers will be collected at end of period 3 and reissued for period 4. The last half of period 4 will be devoted to class checking of answers and corrections. Work on foolscap.

and BC = ____________ and BC = ______________

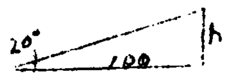
2. In the above diagram, if angle A is exactly 27 degrees and AB is 200 yds, find the length of AC using trigonometric function value from table.

$\frac{AC}{AB}$ is ________ of angle A; that is, $\frac{AC}{200}$ is ________ 27° = .

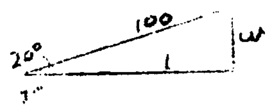
Then AC = . x 200 =

Practice Exercises: Each student is to work at his or her own rate. Individual help will be given by teacher as requested. Papers will be collected at end of period 4 and reissued at beginning of period 5. Answers to the first five exercises will be given at the beginning of period 5. All work is to be handed in again at the end of period 5. Work on foolscap.

1. Find "h"

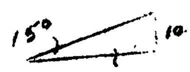


2. Find "w"



3. Find "l"

4. Find "a"



5. Find "l"

6. From a point on level ground 250 feet from the foot of a tree, the angle of elevation of its top is 12 degrees. How tall is the tree?

7. A highway slopes upward on a steady climb at an angle of 7 degrees for a distance of one-half mile. How many feet is the top higher than the foot of the hill?

8. A fire-truck ladder is raised until it is at an angle of 52 degrees from horizontal, and is extended until its length is 85 feet. How high is the upper end of the ladder above its base? How far out horizontally does it extend?

9. If the fire-truck ladder is extended to a length of 100 feet and raised until its upper end is 75 feet above the base, what is the angle of elevation?

10. An observer on a bridge 160 feet above the water sees a boat downstream at an angle of depression of 15 degrees. How far is the boat downstream from the bridge?

11. An ordinary ladder is considered safest when it is placed at an angle of 75 degrees from horizontal. How far from the foot of a vertical wall should a 60 foot ladder be placed if this ruleis followed?

12. Two towns A and B are 350 miles apart in a straight line. Town B is 37 degrees west of due north from town A
   (a) By how many miles is B further North than A?
   (b) By how many miles is B further West than A?

13. Problem 1, page 319 of text-book
14. Problem 3, " " "
15. Problem 4, " " "

Elementary Trigonometry Unit. Group A.

Period 5.

What are these ratios: Sine, Cosine, and Tangent?

A standard method of naming the sides of right angled triangles has been adopted. One of the acute angles is taken as a reference point and the sides are called:

the hypotenuse
the side opposite to the angle
the side adjacent to the angle

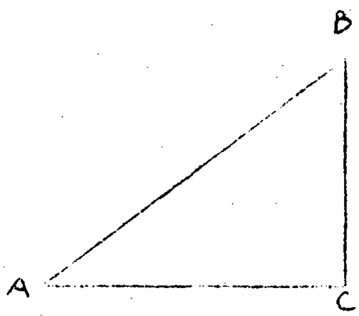
The SINE (Sin) of the angle is always $\frac{\text{side opposite to angle}}{\text{hypotenuse}}$

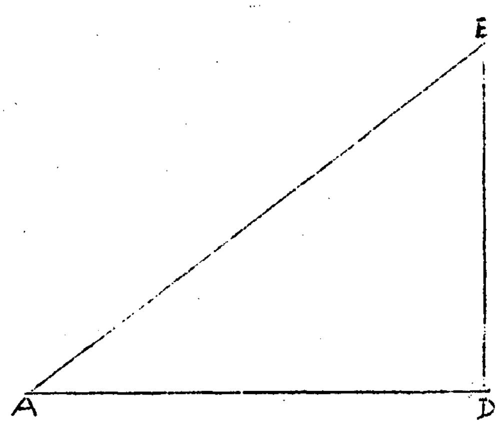
The COSINE (Cosin, Cos) of the angle is always $\frac{\text{side adjacent to angle}}{\text{hypotenuse}}$

The TANGENT (Tan) of the angle is always $\frac{\text{side opposite to angle}}{\text{side adjacent to angle}}$

What happens to these ratios when triangles differ in length of sides but angles remain constant?

In the figures immediately below, BAC and EAD are equiangular right-angled triangles; side BC is 3 cm., side AC is 4 cm., side ED is $4\frac{1}{2}$ cm., side AD is 6 cm.





(In the exercises below, the length of the hypotenuse may be found by calculation or measurement

1. Write: tangent of angle A is $\frac{\text{side -}\_\_\_\_\_\_\_\_}{\text{side -}}$

For triangle BAC, tangent A = ______ For triangle EAD, tangent A = ______

2. Write: sine of angle A is side____________

For triangle BAC, sine A = ______ For triangle EAD, sine A = ______

3. Write: cosine of angle A is side____________

For triangle BAC, cosin A = ______ For triangle EAD, cos A = ______

4. Reduce each of the above ratios to its lowest terms and complete this statement: If an angle remains constant then the sine, cosine, and tangent each ____________ ____________ no matter how large the triangle.

5. Reduce each of the above ratios to a decimal:

Sin A

Cos A

Tan A

6. Measure angle A with protractor, find its sin, cos, and tan from tables and check your values of No. 5.

7. Measure angle B (note that A + B must total 90°) and find from table the values of sin B ______, cos B ______ and tan B ______

8. From the triangle BAC above write the values of sin B, cos B, and tan B from the lengths of the sides. Reduce each to decimal and compare with No. 7.

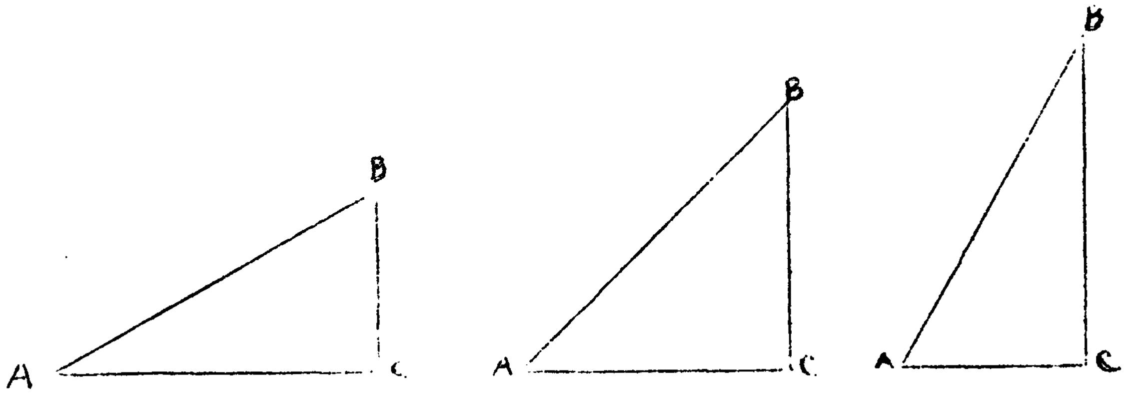
Elementary Trigonometry Unit. Group A.

Period 6.

Review: Study from previous page followed by oral drill on:
(1) Definitions of Sine, Cosine, and Tangent.
(2) If angle A remains constant, what happens to these ratios for different sized right-angled triangles?

Class Exercises: The following are to be worked by each student with explanations and checking at frequent intervals as for those of period 1:

A. In each of the three right-angled triangles below, the hypotenuse AB is 5 cm. Measure the other two sides of each triangle and write, first as common fractions, then as decimals, the values of Sin A; Cos A; Tan A.



Sin A ____________ ____________ ____________

Cos A ____________ ____________ ____________

Tan A ____________ ____________ ____________

Complete the statement: As angle A increases, Sin A becomes ____________, Cosin A becomes ____________, Tangent A becomes ____________.

Check your statement from the values given of these functions for various sized angles in the tables at the back of the text-book.

From these same tables and the ratios worked out above, find, to the nearest degree the size of each angle A in the above figures.

B. 1. Construct an angle whose tangent is 5/8; that is, draw a right-angled triangle which has a base of 8 units and an altitude of 5 units. Mark the angle.

2. Reduce 5/8 to a decimal and find from table what size the angle should be. Check your construction by measuring the angle with protractor.

3. Calculate (Pythagoras) the length of the hypotenuse of this triangle to the nearest tenth.

4. Write the value of the sine of the angle, using hypotenuse just calculated, reduce it to a decimal and compare with the value given in table.

5. Write the value of the cosine of the angle, using hypotenuse calculated, reduce it to a decimal and compare with table.

C. Write from memory the definitions of sine, cosine, and tangent.

Elementary Trigonometry Unit. Group A.

Period 7.

Practice Exercises: Work on this sheet or use additional paper as necessary. Each student is to proceed at his or her own rate. Help may be gained by restudy of previous page; individual assistance will be given also by teacher as requested. All work is to be handed in at the close of the period.

1. Construct an angle whose sine is 3/7. Measure with protractor and check with table

2. Construct an angle whose cosine is 4/9. Measure and check.

3. Construct an angle whose tangent is 2/3. Measure and check.

4. If the tangent of an angle is 5/12, calculate:
   - (a) its sine
   - (b) its cosine

5. If the cosine of an angle is 4/9, calculate:
   - (a) its sine
   - (b) its tangent.

6. Construct an angle whose sine is .3

7. Construct an angle whose tangent is $2\frac{1}{2}$.

8. Exercises 5, 6, and 12 on page 316 of text-book.

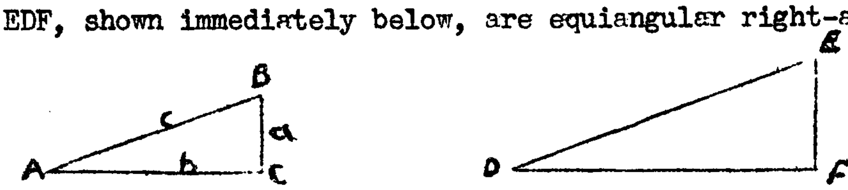
Elementary Trigonometry Unit. (GROUP B.) SEQUENCE B

Period 1.

Introduction: **Recently we proved the theorem: "If two triangles are equiangular their corresponding sides are proportional." The ratios of sides of equiangular right-angled triangles are of great importance in mathematics and are widely used.**

Exercises: The following are to be read and worked or completed by each student. As these are demonstration and study examples, explanations will be given and results checked as the work proceeds.

1. BAC and EDF, shown immediately below, are equiangular right-angled triangles



Complete: (i) $\frac{BC}{AB}$ = ____ (ii) $\frac{AC}{AB}$ = ____ (iii) $\frac{BC}{AC}$ = ____

2. Construct a right-angled triangle lettered like the sample BAC above but having sides: a = 3 cm; b = 4 cm.
(Use the left side space below)

With protractor, measure angle A and write it's size here ________ degrees.
Calculate the length of side "c" (Pythagoras theorem) ____; check by measuring.
Write in figures, first as common fractions, then as decimals, the ratios:

(i) $\frac{a}{c}$ = ____ = . (ii) $\frac{b}{c}$ = ____ = . (iii) $\frac{a}{b}$ = ____ = .

3. In the right hand space above, construct another right-angled triangle, lettered the same, but having side "b" 6 cm; make angle A the same size as in No. 2 by using your protractor. Measure the other two sides after the triangle is drawn and write the ratios, first as common fractions, then as decimals:

(i) $\frac{a}{c}$ = ____ = . (ii) $\frac{b}{c}$ = ____ = . (iii) $\frac{a}{b}$ = ____ = .

Since angle A remained constant, would you expect these ratios to be the same for both triangles?

4. If side b is 10 ft. and angle A the same, calculate side a ________

5. If side c is thirty miles and angle A the same, calculate sides a _____ and b ____

Definitions: Because triangles can be lettered in many ways, a standard means of naming sides has been adopted to avoid confusion. One of the acute angles is taken as a reference point and the sides are spoken of as:

the hypotenuse
the side opposite to the angle
the side adjacent to the angle

The ratios have been given the following names:

$\frac{\text{side opposite to angle}}{\text{hypotenuse}}$ is called SINE (Sin) of the angle.

$\frac{\text{side adjacent to angle}}{\text{hypotenuse}}$ is called COSINE (Cosin; Cos) of the angle.

$\frac{\text{side opposite to angle}}{\text{side adjacent to angle}}$ is called TANGENT (Tan) of the angle.

Exercise: Identify the ratios Sin A, Cos A, and Tan A of No. 2 above.
Check their values with those given for angle A in tables at back of text-book.

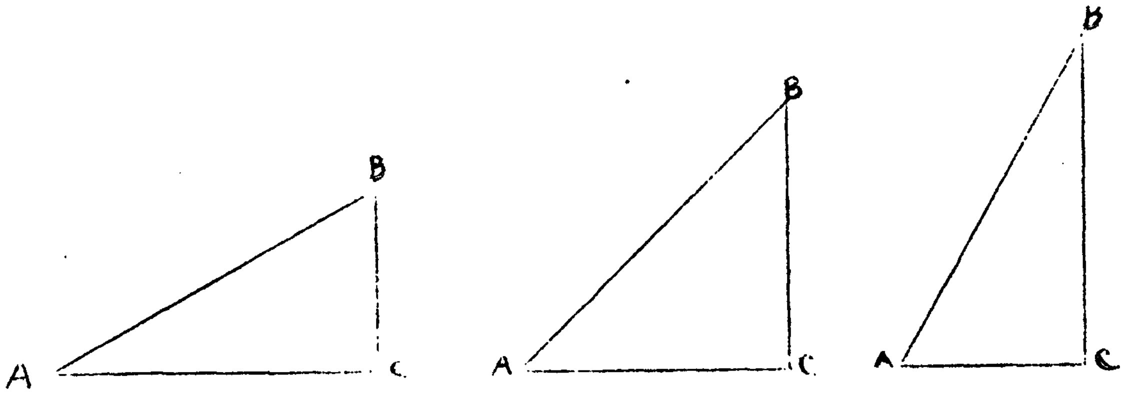
Elementary Trigonometry Unit. Group B.

Period 2.

Review: Study from previous page followed by oral drill on:
(1) Definitions of Sine, Cosine, and Tangent.
(2) If angle A remains constant, what happens to these ratios for different sized right-angled triangles?

Class Exercises: The following are to be worked by each student with explanations and checking at frequent intervals as for those of period 1:

A. In each of the three right-angled triangles below, the hypotenuse AB is 5 cm. Measure the other two sides of each triangle and write, first as common fractions, then as decimals, the values of Sin A; Cos A; Tan A.



Sin A ______________ ______________ ______________

Cos A ______________ ______________ ______________

Tan A ______________ ______________ ______________

Complete the statement: As angle A increases, Sin A becomes ______________, Cosin A becomes ______________, Tangent A becomes ______________.

Check your statement from the values given of these functions for various sized angles in the tables at the back of the text-book.

From these same tables and the ratios worked out above, find, to the nearest degree the size of each angle A in the above figures.

B. 1. Construct an angle whose tangent is 5/8; that is, draw a right-angled triangle which has a base of 8 units and an altitude of 5 units. Mark the angle.

2. Reduce 5/8 to a decimal and find from table what size the angle should be. Check your construction by measuring the angle with protractor.

3. Calculate (Pythagoras) the length of the hypotenuse of this triangle to the nearest tenth.

4. Write the value of the sine of the angle, using hypotenuse just calculated, reduce it to a decimal and compare with the value given in table.

5. Write the value of the cosine of the angle, using hypotenuse calculated, reduce it to a decimal and compare with table.

C. Write from memory the definitions of sine, cosine, and tangent.

Elementary Trigonometry Unit. Group B.

Period 3.

Practice Exercises: Work on this sheet or use additional paper as necessary. Each student is to proceed at his or her own rate. Help may be gained by restudy of previous page; individual assistance will be given also by teacher as requested. All work is to be handed in at the close of the period.

1. Construct an angle whose sine is 3/7. Measure with protractor and check with table

2. Construct an angle whose cosine is 4/9. Measure and check.

3. Construct an angle whose tangent is 2/3. Measure and check.

4. If the tangent of an angle is 5/12, calculate:
   (a) its sine
   (b) its cosine

5. If the cosine of an angle is 4/9, calculate:
   (a) its sine
   (b) its tangent.

6. Construct an angle whose sine is .3

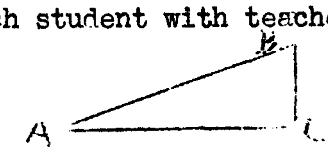
7. Construct an angle whose tangent is $2\frac{1}{2}$.

8. Exercises 5, 6, and 12 on page 316 of text-book.

Elementary Trigonometry Unit. Group B.

Periods 4 & 5

Demonstration Exercises: These are to be worked by each student with teacher showing steps and method on blackboard as necessary.



1. In the accompanying right-angled triangle BAC,

$\frac{BC}{AB}$ or ________ of angle A $= \frac{9}{20} = .45$

If AB actually is 100 yds, then:

$\frac{BC}{100} = \frac{9}{20}$ or, using decimal, $\frac{BC}{100} = .45$

and BC = ____________ and BC = ________________

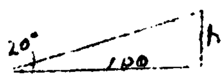
2. In the above diagram, if angle A is exactly 27 degrees and AB is 200 yds, find the length of AC using trigonometric function value from table.

$\frac{AC}{AB}$ is __________ of angle A; that is, $\frac{AC}{200}$ is __________ $27^{\circ} =$.

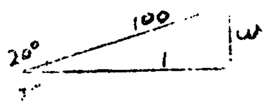
Then AC = . x 200 =

Practice Exercises: Each student is to work at his or her own rate. Individual help will be given by teacher as requested. Papers will be collected at end of period 4 and reissued at beginning of period 5. Answers to the first five exercises will be given at the beginning of period 5. All work is to be handed in again at the end of period 5. Work on foolscap.

1. Find "h"



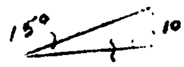
2. Find "w"



3. Find "l"

4. Find "a"

5. Find "l"



6. From a point on level ground 250 feet from the foot of a tree, the angle of elevation of its top is 12 degrees. How tall is the tree?

7. A highway slopes upward on a steady climb at an angle of 7 degrees for a distance of one-half mile. How many feet is the top higher than the foot of the hill?

8. A fire-truck ladder is raised until it is at an angle of 52 degrees from horizontal, and is extended until its length is 85 feet. How high is the upper end of the ladder above its base? How far out horizontally does it extend?

9. If the fire-truck ladder is extended to a length of 100 feet and raised until its upper end is 75 feet above the base, what is the angle of elevation?

10. An observer on a bridge 160 feet above the water sees a boat downstream at an angle of depression of 15 degrees. How far is the boat downstream from the bridge?

11. An ordinary ladder is considered safest when it is placed at an angle of 75 degrees from horizontal. How far from the foot of a vertical wall should a 60 foot ladder be placed if this ruleis followed?

12. Two towns A and B are 350 miles apart in a straight line. Town B is 37 degrees west of due north from town A
   (a) By how many miles is B further North than A?
   (b) By how many miles is B further West than A?

13. Problem 1, page 319 of text-book
14. Problem 3, " " "
15. Problem 4, " " "

Elementary Trigonometry Unit. Group 8

Period 6.

Check answers and make corrections to exercises of last two periods (15 minutes)

Demonstration Example: (This will be shown by the teacher with students using his observed data to complete the solution).

What is the height of this classroom?

Using a sighting protractor and level placed on a desk, sight the intersection of wall and ceiling and read the angle of elevation. Measure the distance along the floor from point under the observer's eye to the vertical wall. Record these two measurements: Angle of elevation ________; horizontal distance __________

Working on this paper, make a diagram, mark data, and work out the height.

Practice Exercises: Students work in pairs; sight angle and measure distance together, but each work out calculations and check result with each other.

1. Find height of a tree immediately outside school
2. Find height of a pole " " "
3. Find height of school building.
4. Find height of any point on classroom wall (in case of unsuitable weather for outside work)
5. Find height of electric light in classroom.

Period 7: Completion of unfinished work of period 6 (5 to 10 minutes)

Demonstration Exercise: Teacher demonstrates and students work from his data as before. In this case one or two students may make the actual measurements for the class.

The distance up the slope of a hill can be measured directly; the angle of elevation of its top can be determined by sighting from the foot.

(A cross-section of the hill, drawn on the blackboard, will be used)

(a) Find the vertical height of the hill.

(b) Find the horizontal distance from foot of hill to a point directly under its top.

Practice Exercises: (Working in pairs as in period 6).

Measure the angle of ascent of a flight of stairs; measure the sloping distance along the steps from bottom to top. Calculate the vertical rise.

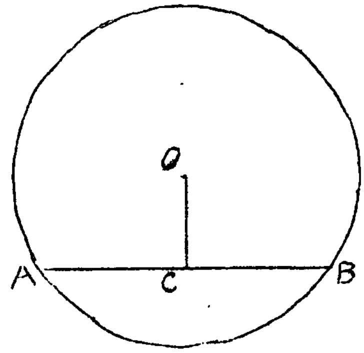
Measure length and angle of elevation of a sloping board. Calculate the height of its upper end and also the horizontal distance it covers.

Chords in a Circle Unit. (Group A.) SEQUENCE B

Period 1.

Introduction: Any straight line joining two points on the circumference of a circle is called a chord. In studying these, the line joining the mid-point of any chord to the centre of the circle is a key line. We can prove certain features important enough to be classed as theorems.

Theorem: The straight line joining the centre of a circle to the mid-point of a chord is at right angles to the chord.



Data: Let AB be any chord with C as its mid-point, and O be the centre of the circle.

Aim: To prove that OC is perpendicular to AB.

Construction: Join OA and ________

Proof: In the triangles OCA and _______
(to be completed with help)

Brief review questioning on main features of theorem and proof:
What two things are given about the line OC?
What third feature is to be proved?
What is the general method of proof?
What final step is necessary after proving triangles congruent?

Write again on foolscap the complete proof of this theorem.

Theorem: The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

What two things are given in this theorem?
What is to be proved?
On foolscap, write the general enunciation, draw the figure, write down in proper form the data and aim. (Upon completion these will be checked with blackboard sample)
Can this theorem probably be proved by congruent triangles?
What construction is necessary?
Students will attempt to complete this proof. ~~Th~~ Sample for checking will then be provided on blackboard.

Chords in a Circle Unit. Group A

Periods 2 & 3

Theorem: The perpendicular bisector of a chord passes through the centre of the circle.

On foolscap, draw a suitable figure, and write down in good form the data and sim. (To be checked with sample on blackboard before proceeding further.

Demonstration of proof will be given. Students then write out proof for themselves.

Exercises:

[illegible]

1. Prove that the distance of any chord from the centre of a circle is equal to the square root of (the square of the radius minus the square of half the chord).

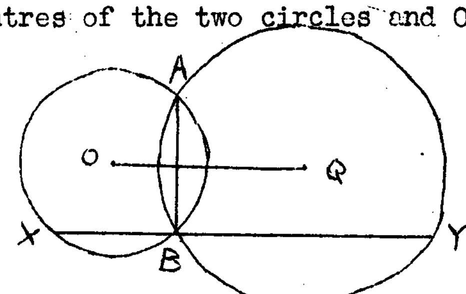
2. If two equal chords are drawn in a circle prove that they are equidistant from the centre.

3. State and prove the converse of exercise 2.

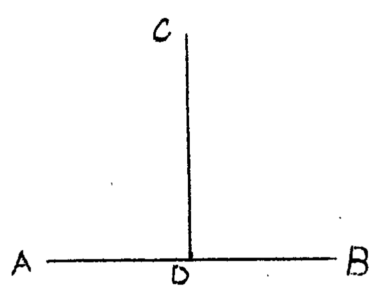
4. AB and CD are two chords of a circle. AB is longer than CD. Prove that AB is closer to the centre than CD.

5. Two circles whose centres are O and Q respectively intersect at two points, A and B. The straight line AB is called the common chord of the two circles.
Draw two such circles with their common chord and mark its mid-point C. Join OC and QC, and prove that OCQ forms one straight line. (This line is called the line of centres.

6. In the accompanying diagram, O and Q are centres of the two circles and OQ the line of centres. AB is the common chord.
XBY is a straight line perpendicular to AB.
Prove that XY equals twice the line of centres.

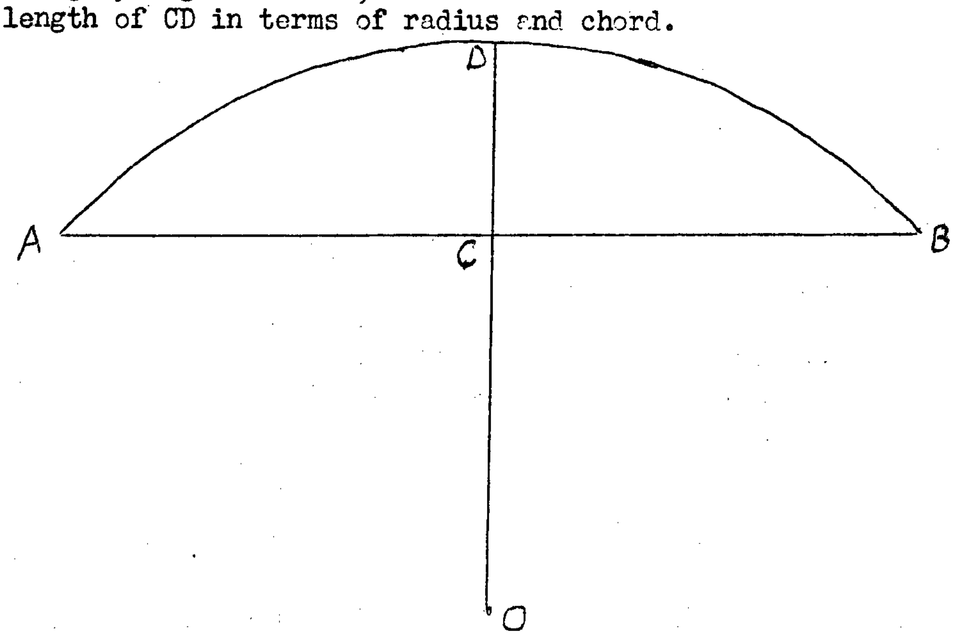


7.



In the diagram at the right, AB is any straight line and CD is the perpendicular bisector of AB. Quote a proven reference to show that ~~thm~~ if the circumference of any circle passes through points A and B, its centre must lie on the line CD.

8. In the diagram ~~at the right~~ below, O is the centre of the circle of which only an arc is shown. AB is a chord and OCD the perpendicular bisector.
Using Pythagoras' rule, work out a formula for the length of CD in terms of radius and chord.



Chords in a Circle Unit. Group A

Period 4

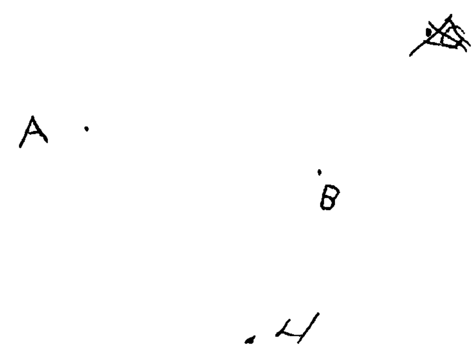
Construction Exercises: (Geometrical construction using ruler and compasses only)

1. A circle is to be drawn through two given points, A, and B.

    (i) Join AB
    (ii) Construct the perpendicular bisector of AB
    (iii) Taking any point, O, on this perpendicular bisector as centre, with radius OA, draw the circle.

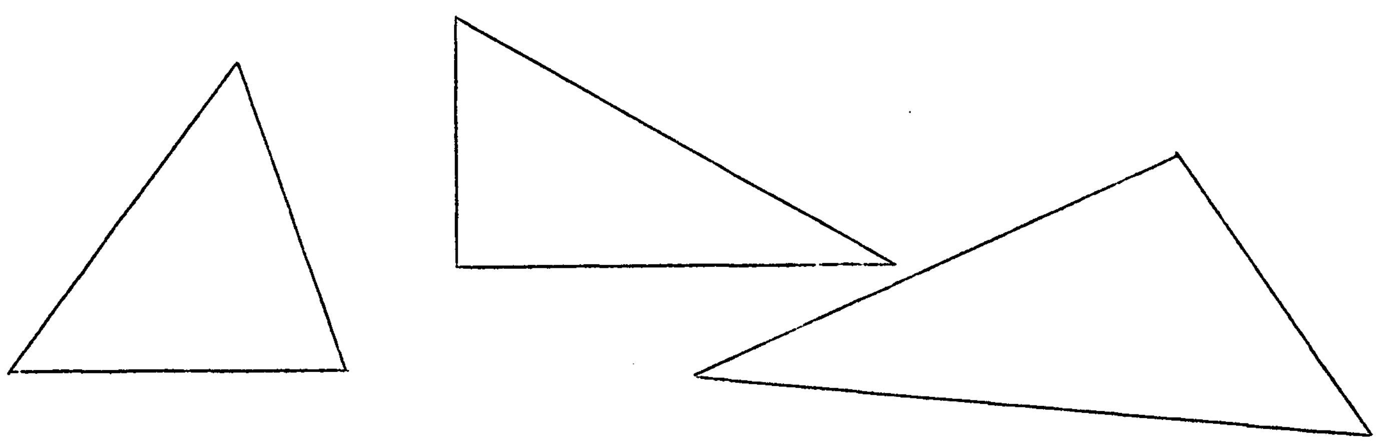
    How many such circles can be drawn?

2. A circle is to be drawn through three given points, A, B, and C, which are not in one straight line.
    (i) Join AB and construct the perpendicular bisector.
    (ii) Join BC and construct the perpendicular bisector.
    (iii) With O, the point where these lines intersect, as centre and radius OA, draw the circle.



3. Given an arc of a circle, locate the centre and complete the circle.

4. A circle is to be drawn through the three vertices of a given triangle:
(a) Acute angled triangle. (b) Right angled triangle. (c) Obtuse angled triangle.

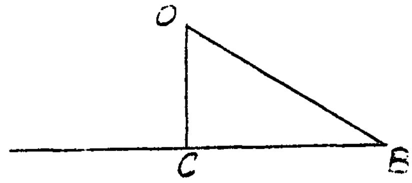


Chords in a Circle Unit. Group A

Periods 5 & 6

Review Exercise of Right-angled Triangle Calculations: This is to be worked by all students immediately. Work will be checked and explanations given before next section is begun.

OCB is a right-angled triangle having the right angle at C.

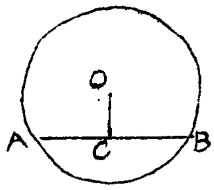


(a) If OC is 4 cm and CB is 7 cm, calculate the length of OB.

(b) If OC is 5 cm and OB is 8 cm, calculate the length of CB.

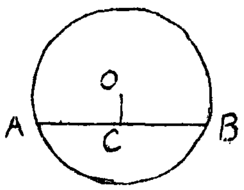
Calculation Exercises Involving Chords: (Work on foolscap)

1. In the figure below, O is the centre of the circle and OC the perpendicular bisector of the chord AB. Join OB and name the right-angled triangle.



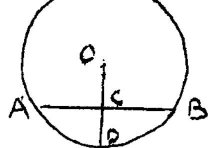
If OC is 4 cm and chord AB is 14 cm:
(a) How long is CB?
(b) Calculate the length of the radius OB.

2. In the figure below, O is the centre of the circle and OC is the perpendicular bisector of the chord AB. Complete a right-angled triangle and name it.



If OC is 5 cm and the radius is 8 cm:
(a) Calculate the length of CB.
(b) How long is the chord AB?

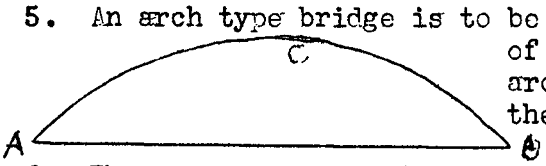
3. In the figure below, O is the centre of the circle and OCD the perpendicular bisector. If CD is 5 inches and the radius 9 inches:



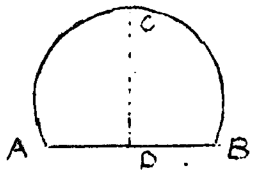
(a) Calculate the length of OC.
(b) Calculate the length of CB and then of AB.

4. A certain circle has a 16 foot chord placed so that its greatest distance from the circumference is 3 feet. Calculate the radius of the circle.

5. An arch type bridge is to be built over a canyon. The diagram shows AB, the span of the bridge which is 240 feet, and the arch ACB, an arc of a circle whose radius is 135 feet. Calculate the height of the middle point of the arch above AB.

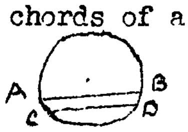


6. The cross-section of a tunnel, circular except for a flat bottom, is shown in the accompanying diagram. If the chord AB is 10 feet, and the diameter of the circle is 20 feet, calculate the height CD.



7. A chord of a circle is 24 inches long. The radius of the circle is 15 inches. How far is the chord from the centre of the circle?

8. Two chords of a circle, AB and CD as shown in the diagram, are 4 inches apart. Chord AB is 24 inches and CD is 16 inches long. Calculate the radius of the circle.



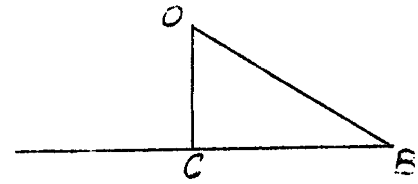
Additional problems will be found on pages 456-7 of text-book: Education Through Mathematics.

Chords in a Circle Unit. (Group B) SEQUENCE A

Periods /42

Review Exercise of Right-angled Triangle Calculations: This is to be worked by all students immediately. Work will be checked and explanations given before next section is begun.

OCB is a right-angled triangle having the right angle at C.

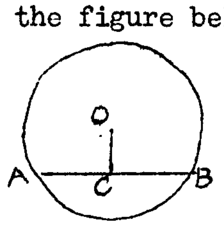


(a) If OC is 4 cm and CB is 7 cm, calculate the length of OB.

(b) If OC is 5 cm and OB is 8 cm, calculate the length of CB.

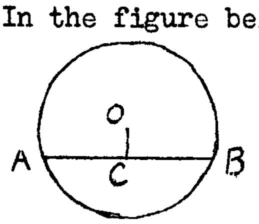
Calculation Exercises Involving Chords: (Work on foolscap)

1. In the figure below, O is the centre of the circle and OC the perpendicular bisector of the chord AB. Join OB and name the right-angled triangle.



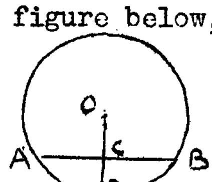
If OC is 4 cm and chord AB is 14 cm:
   (a) How long is CB?
   (b) Calculate the length of the radius OB.

2. In the figure below, O is the centre of the circle and OC is the perpendicular bisector of the chord AB. Complete a right-angled triangle and name it.



If OC is 5 cm and the radius is 8 cm:
   (a) Calculate the length of CB.
   (b) How long is the chord AB?

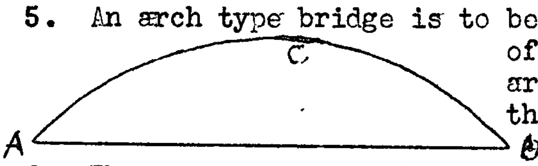
3. In the figure below, O is the centre of the circle and OCD the perpendicular bisector. If CD is 5 inches and the radius 9 inches:



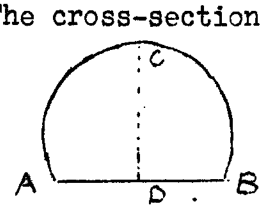
   (a) Calculate the length of OC.
   (b) Calculate the length of CB and then of AB.

4. A certain circle has a 16 foot chord placed so that its greatest distance from the circumference is 3 feet. Calculate the radius of the circle.

5. An arch type bridge is to be built over a canyon. The diagram shows AB, the span of the bridge which is 240 feet, and the arch ACB, an arc of a circle whose radius is 135 feet. Calculate the height of the middle point of the arch above AB.

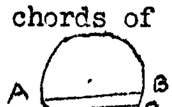


6. The cross-section of a tunnel, circular except for a flat bottom, is shown in the accompanying diagram. If the chord AB is 10 feet, and the diameter of the circle is 20 feet, calculate the height CD.



7. A chord of a circle is 24 inches long. The radius of the circle is 15 inches. How far is the chord from the centre of the circle?

8. Two chords of a circle, AB and CD as shown in the diagram, are 4 inches apart. Chord AB is 24 inches and CD is 16 inches long. Calculate the radius of the circle.



Additional problems will be found on pages 456-7 of text-book: Education Through Mathematics.

Chords in a Circle Unit. Group B

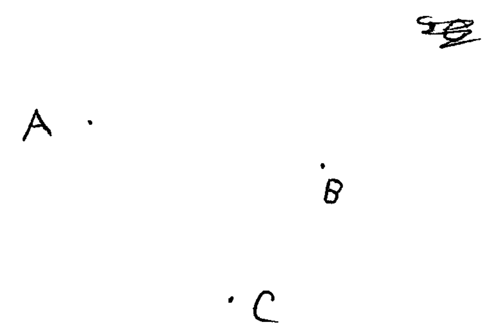
Period 3

Construction Exercises: (Geometrical construction using ruler and compasses only)

1. A circle is to be drawn through two given points, A, and B.

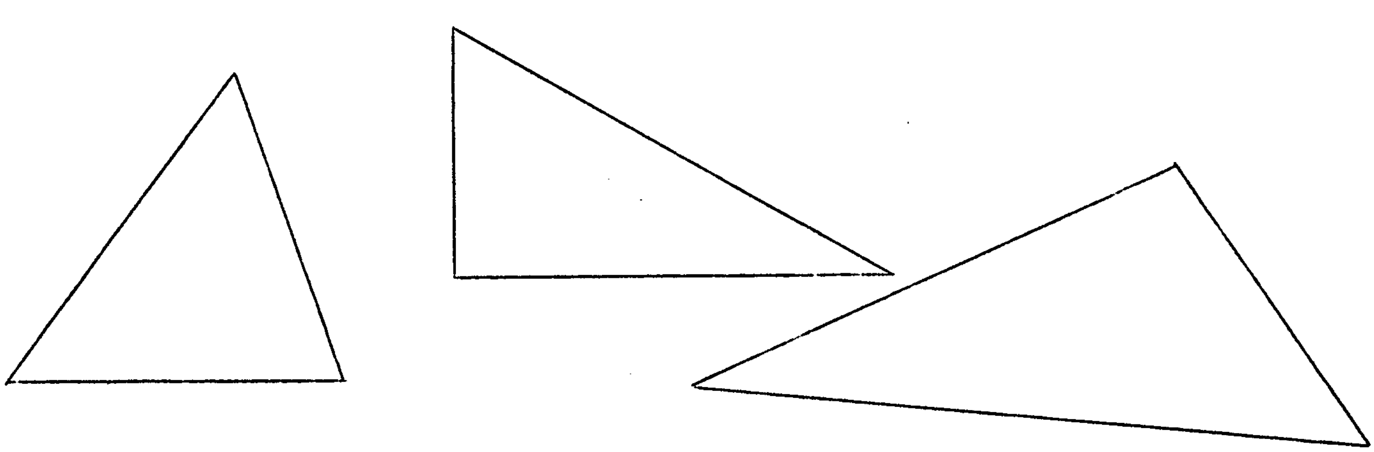
   (i) Join AB
   (ii) Construct the perpendicular bisector of AB
   (iii) Taking any point, O, on this perpendicular bisector as centre, with radius OA, draw the circle.
   How many such circles can be drawn?

2. A circle is to be drawn through three given points, A, B, and C, which are not in one straight line.
   (i) Join AB and construct the perpendicular bisector.
   (ii) Join BC and construct the perpendicular bisector.
   (iii) With O, the point where these lines intersect, as centre and radius OA, draw the circle.

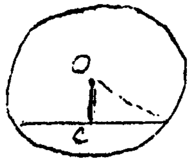


3. Given an arc of a circle, locate the centre and complete the circle.

4. A circle is to be drawn through the three vertices of a given triangle:
   (a) Acute angled triangle. (b) Right angled triangle. (c) Obtuse angled triangle.



Period 4.

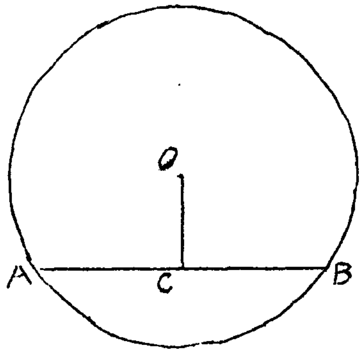


Introduction: In the calculation and construction exercises we have just completed, a line similar to OC in the accompanying figure apparently:

(i) passes through the centre of the circle,
(ii) passes through the mid-point of the chord,
(iii) is perpendicular to the chord.

We have assumed that it does all these three at once, and while high probability assumptions have to be used as the basis of action in many features of life, yet if proof is possible then we have a much stronger basis. For example, a construction company spending many thousands of dollars on an arch type bridge such as in exercise 5 of page 2 would appreciate such a proof before investing their money.

Theorem: The straight line joining the centre of a circle to the mid-point of a chord is at right angles to the chord.



Data: Let AB be any chord with C as its mid-point, and O be the centre of the circle.

Aim: To prove that OC is perpendicular to AB.

Construction: Join OA and ________

Proof: In the triangles OCA and _______
(to be completed with help)

Brief review questioning on main features of theorem and proof:
What two things are given about the line OC?
What third feature is to be proved?
What is the general method of proof?
What final step is necessary after proving triangles congruent?

Write again on foolscap the complete proof of this theorem.

Theorem: The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

What two things are given in this theorem?
What is to be proved?
On foolscap, write the general enunciation, draw the figure, write down in proper form the data and aim. (Upon completion these will be checked with blackboard sample)
Can this theorem probably be proved by congruent triangles?
What construction is necessary?
Students will attempt to complete this proof. Sample for checking will then be provided on blackboard.

Chords in a Circle Unit. Group B

Periods 5 & 6.

Theorem: The perpendicular bisector of a chord passes through the centre of the circle.

On foolscap, draw a suitable figure, and write down in good form the data and aim. (To be checked with sample on blackboard before proceeding further.

Demonstration of proof will be given. Students then write out proof for themselves.

Exercises:

[illegible]

1. Prove that the distance of any chord from the centre of a circle is equal to the square root of (the square of the radius minus the square of half the chord).

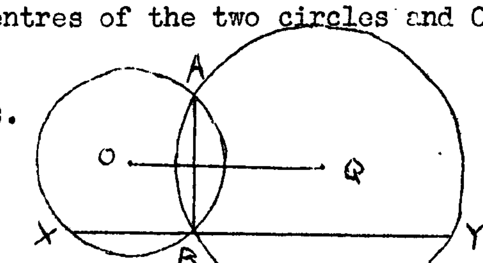
2. If two equal chords are drawn in a circle prove that they are equidistant from the centre.

3. State and prove the converse of exercise 2.

4. AB and CD are two chords of a circle. AB is longer than CD. Prove that AB is closer to the centre than CD.

5. Two circles whose centres are O and Q respectively intersect at two points, A and B. The straight line AB is called the common chord of the two circles.
Draw two such circles with their common chord and mark its mid-point C. Join OC and QC, and prove that OCQ forms one straight line. (This line is called the line of centres.

6. In the accompanying diagram, O and Q are centres of the two circles and OQ the line of centres. AB is the common chord.
XBY is a straight line perpendicular to AB.
Prove that XY equals twice the line of centres.



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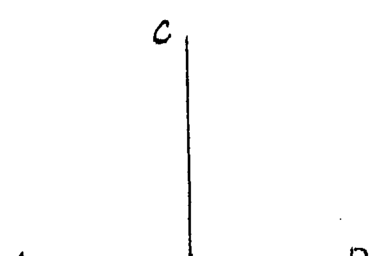
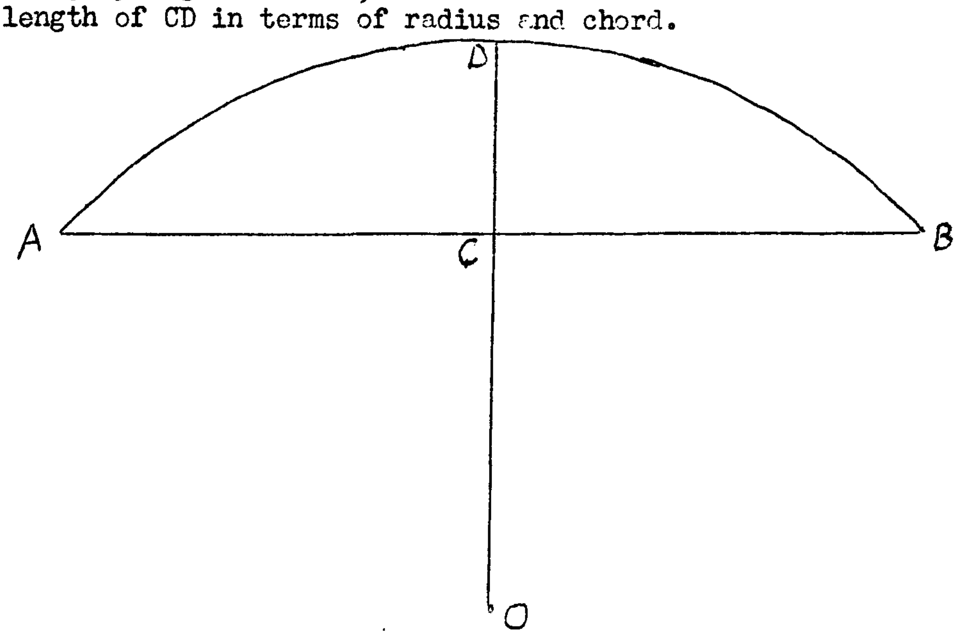


In the diagram at the right, AB is any straight line and CD is the perpendicular bisector of AB. Quote a proven reference to show that ~~thm~~ if the circumference of any circle passes through points A and B, its centre must lie on the line CD.

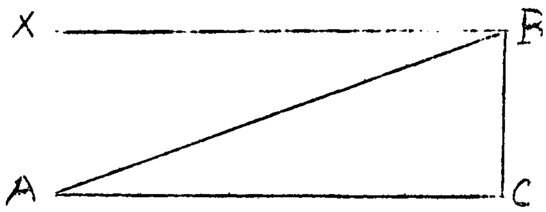
8. In the diagram ~~at the right~~ below, O is the centre of the circle of which only an arc is shown. AB is a chord and OCD the perpendicular bisector.
Using Pythagoras' rule, work out a formula for the length of CD in terms of radius and chord.



APPENDIX B

TESTS

Elementary Trigonometry Unit. TEST. Student's name ____________



Part A. (Values, 1 each)
The accompanying triangle BAC is right-angled at C. XB and AC are parallel horizontal lines. Underline the correct answer to each of the following:

1. The angle of elevation of point B is: ABX; ABC; BAC; XBA.

2. The side opposite to angle A is: AC; BC; AB; XB.

3. The side adjacent to angle B is: AC; BC; AB; XB.

4. The tangent of angle A is: $\frac{BC}{AC}$ $\frac{BC}{AB}$ $\frac{AC}{AB}$ $\frac{AC}{BC}$

5. The ~~co~~sine of angle A is: $\frac{BC}{AB}$ $\frac{AB}{AC}$ $\frac{AC}{AB}$ $\frac{AC}{BC}$

6. The ~~co~~sine of angle B is: $\frac{AB}{AC}$ $\frac{AB}{BC}$ $\frac{AC}{BC}$ $\frac{AC}{AB}$

7. The cosine of angle A is: $\frac{AB}{AC}$ $\frac{AB}{BC}$ $\frac{AC}{BC}$ $\frac{AC}{AB}$

8. If AB is 7 and BC is 3, then AC is: 4; 40; $\sqrt{40}$; $\sqrt{58}$.

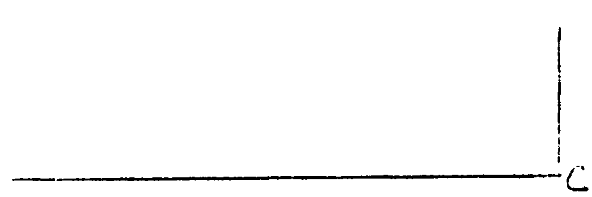
9. If $\frac{AC}{AB} = \frac{5}{6}$ and BA is 15 units, then AC is: 10; $12\frac{1}{2}$; 14; 18.

10. If $\frac{BC}{AB} = .3652$ and AB is 200 units, then BC is: 18.26; 182.6; 70; 73.04

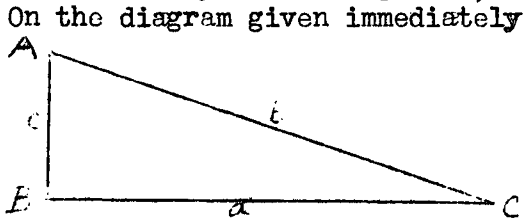
Part B. Work as directed in spaces provided. (Value of each is given in margin at right).

Values

2 1. Using the given framework of straight lines in which the angle C is a right angle, construct accurately an angle whose sine is 3/5.



2. Wishing to determine the height of a tall building, an observer, from a position 500 feet away on level ground, sights the angle of elevation of its top as 16 degrees. On the diagram given immediately below:



/ (a) Which is the angle of elevation? ____

/ (b) Which side represents the building? ____

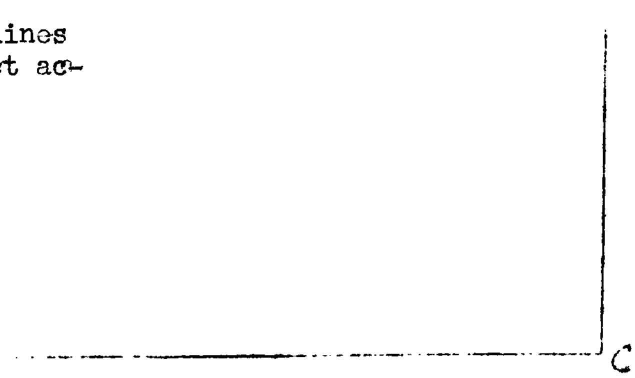
/ (c) What function of the angle would you use to solve the problem?....... ____

(Do not work any further on this question)

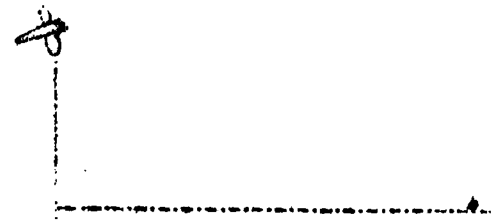
3. If sine A is 2/7, calculate cosine A (leaving answer in surd form)

2

2 4. Using the given framework of straight lines which has angle C a right angle, construct accurately an angle whose tangent is .4



5. An observer in an aeroplane, known to be 5000 feet above level ground, sights a town ahead at an angle of depression of 40 degrees. Complete the given diagram and:



1 (a) Mark the angle of depression.

2 (b) To solve this problem you should use the (tell which function) ______ of ______ degrees.

(No other work is required on this problem)

6. If tangent A is 5/6, calculate sin A (leaving answer in surd form)

2

7. Sighted from the bottom of a hill, the angle of elevation of its top is 24 degrees. The measured distance up the even slope is 1200 feet. What is the vertical height of the hill? (Work this problem using one of the given function values: sin 24 is .4067; cos 24 is .9135; tan 24 is .4452)

4

8. Complete the following: As an angle becomes larger, its tangent ________________ and its ________________________ decreases.

2

Exp. Unit II - Chords in a Circle. PRETEST. Name ____________________

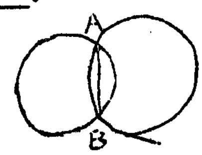
(Items 1 to 6 are valued at 1 mark each; items 7 to 10 at 6 marks each).

1. Complete this statement: The straight line drawn from the centre of a circle to bisect a chord is also ______________________________

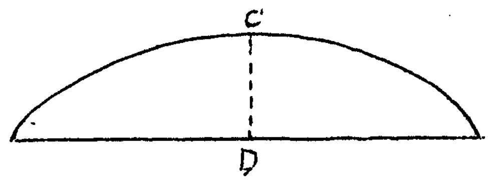
2. Complete the statement: Of two unequal chords in a circle, the one farther from the centre is ______________________________.

3. Complete the statement: If two chords of a circle are the same distance from the centre, they ______________________________.

4. In the accompanying figure, the line AB is called the ______________________.

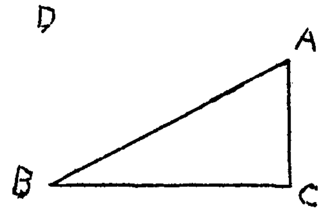


5. The figure at the right shows an arc of a circle and a chord. If the chord is 9 inches from the centre of the circle, and the radius is 15 inches, what is the greatest height of the arc at CD? __________.

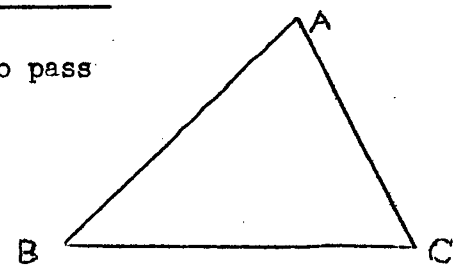


6. In the accompanying right-angled triangle, AB is 9 units and AC is 5 units. Calculate BC, leaving answer in surd form.

BC = __________

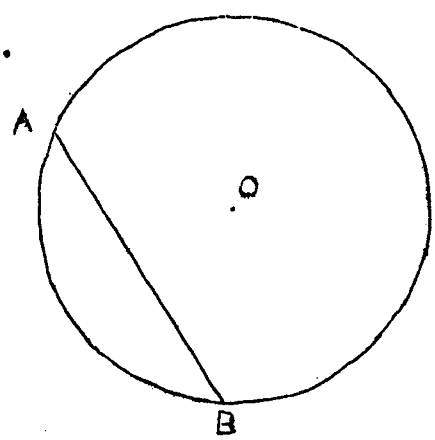


7. Construct a circle (geometrical construction) to pass through the vertices of the given triangle ABC.



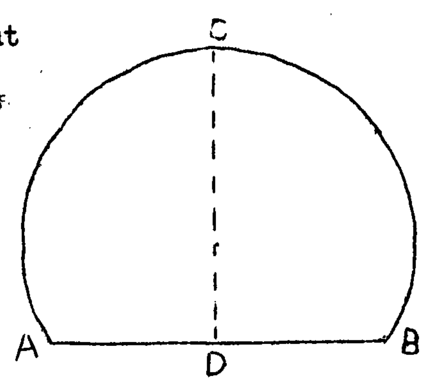
8. On the reverse side of this page, prove that: The perpendicular drawn to a chord from the centre of a circle bisects the chord.

9.



In the accompanying diagram, O is centre of the circle whose radius is 12 feet. If chord AB is 9 feet from O, calculate the length of AB.

10. The tunnel ACB is a circle whose lower part has been cut off by the chord AB. If CD, the greatest height of the tunnel, is 20 ft., and AB is 16 ft., calculate the radius of the circle.



Exp. Unit II.- Chords in a Circle. Final TEST. Name ______________________________

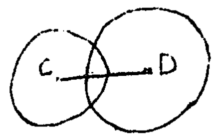
(Items 1 to 6 are valued at 1 mark each; items 7 to 10 at 6 markd each).

1. Complete the statement: The straight line drawn from the centre of a circle perpendicular to a chord also ______________________________.

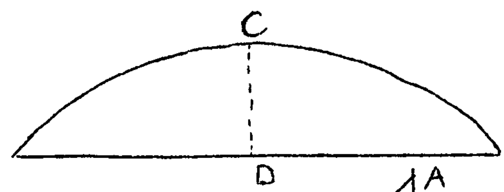
2. Complete the statement: Of two unequal chords of a circle, the one nearer the centre is ______________________________.

3. Complete the statement: If two chords of a circle are equal, they are ____________ ______________________________ the centre.

4. In the accompanying figure, the line CD is called the ______________________.

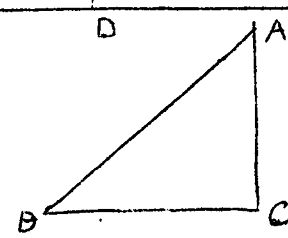


5. The figure at the right shows an arc of a circle and a chord. If the greatest height of the arc at CD is 5 inches, and the radius is 13 inches, how far is the chord from the centre of the circle? ________

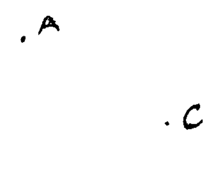


6. In the accompanying right-angled triangle, AB is 7 units and BC is 4 units. Calculate AC, leaving answer in surd form.

AC = ____________

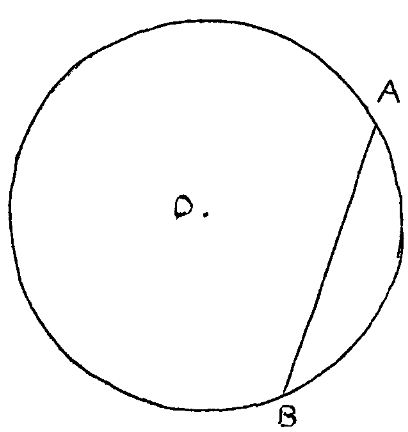


7. Using ruler and compasses only, construct a circle that will pass through the three given points A, B, and C.



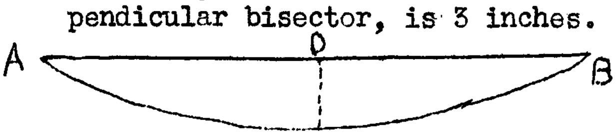
8. On the reverse side of this page, prove that: The straight line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

9.



In the accompanying diagram, O is centre of the circle whose radius is 12 feet. If chord AB is 16 feet long, calculate the distance from O to AB.

10. ACB represents a curved mirror whose chord AB is 20 inches long, while CD, the perpendicular bisector, is 3 inches. Calculate the radius of the arc.



(over)

11. AB and CD are two chords in a circle whose centre is O. X is the mid point of AB and Y is the mid point of CD.

Prove: $OX^2 + XB^2 = OY^2 + YD^2$

APPENDIX C

SAMPLE OF CALCULATIONS

Equating of Kamloops Groups - I.Q.

Pupil No.	Group A			Group B		
	X	x'	$(x')^2$	X	x'	$(x')^2$
1	125	+16	256	125	+16	256
2	122	+13	169	122	+13	169
3	121	+12	144	118	+ 9	81
4	119	+10	100	117	+ 8	64
5	116	+ 7	49	116	+ 7	49
6	116	+ 7	49	115	+ 6	36
7	115	+ 6	36	112	+ 3	9
8	110	+ 1	1	111	+ 2	4
9	108	- 1	1	109	0	0
10	108	- 1	1	108	- 1	1
11	104	- 5	25	106	- 3	9
12	103	-6	36	106	- 3	9
13	103	- 6	36	105	- 4	16
14	102	- 7	49	104	- 5	25
15	101	- 8	64	104	- 5	25
16	99	-10	100	102	- 7	49
17	98	-11	121	93	-16	256
18	97	-12	144	92	-17	289
	1967	+ 5	1381	1965	+ 3	1347

$\bar{X}$ = 109.3 - - - - - - - - - - - - - 109.2

Assumed $\bar{X}$ = 109 - - - - - - - - - - - - - 109

$$S = \sqrt{\frac{\Sigma(x')^2}{N} - \left(\frac{\Sigma x'}{N}\right)^2}$$

$$S = \sqrt{\frac{1381}{18} - \left(\frac{5}{18}\right)^2} \qquad\qquad S = \sqrt{\frac{1347}{18} - \left(\frac{3}{18}\right)^2}$$

$$= 8.8 \qquad\qquad = 8.7$$

Equating of Kamloops Groups - 1st Term Marks

Pupil No.	Group A			Group B		
	X	x'	(x')^2	X	x'	(x')^2
1	81	+12	144	93	+24	576
2	64	- 5	25	84	+15	225
3	81	+12	144	68	- 1	1
4	77	+ 8	64	79	+10	100
5	82	+13	169	82	+13	169
6	75	+ 6	36	72	+ 3	9
7	63	- 6	36	56	-13	169
8	78	+ 9	81	62	- 7	49
9	80	+11	121	73	+ 4	16
10	42	-27	729	76	+ 7	49
11	87	+18	324	72	+ 3	9
12	56	-13	169	66	- 3	9
13	51	-18	324	71	+ 2	4
14	68	- 1	1	58	-11	121
15	86	+17	289	53	-16	256
16	44	-25	625	51	-18	324
17	60	- 9	81	70	+ 1	1
18	58	-11	121	58	-11	121
	1233	+ 9	3483	1244	+ 2	2208

$\overline{X}$ = 68.5 - - - - - - - - - - - - - 69.1

Assumed $\overline{X}$ = 69 - - - - - - - - - - - - - 69

$$S = \sqrt{\frac{3483}{18} - \left(\frac{9}{18}\right)^2} \qquad S = \sqrt{\frac{2208}{18} - \left(\frac{2}{18}\right)^2}$$

$$= 13.9 \qquad = 11.1$$

Equating of Kamloops Groups - 1st Term Marks (continued)

Estimated Standard Deviation of a population from the combined information of two samples:

(N.B. Since the difference between the assumed mean and the actual mean is very slight in each case, the $\Sigma x'$ and the $\Sigma (x')^2$, immediately available from the previous page, have been used for Σx and Σx^2 in the formula below).

$$\tilde{\sigma} = \sqrt{\frac{\Sigma x_1^2 + \Sigma x_2^2}{N_1 + N_2 - 2}}$$

$$= \sqrt{\frac{3783 + 2208}{34}} = 12.9$$

Standard Error of the Difference between the two samples when assumed to be drawn from the same population:

$$SE_D = \tilde{\sigma} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

$$= 12.9 \sqrt{\frac{18+18}{18 \times 18}} = 12.9 \times \frac{1}{3} = 4.3$$

Critical Ratio:

$$t = \frac{M_1 - M_2}{SE_D} =$$

$$= \frac{69.1 - 68.5}{4.3} = 0.14$$

General Achievement of Kamloops Groups in Unit I

Pupil No.	Group A			Group B		
	X	x'	$(x')^2$	X	x'	$(x')^2$
1	27	+ 4	16	24	+ 2	4
2	21	- 2	4	25	+ 3	9
3	28	+ 5	25	27	+ 5	25
4	29	+ 6	36	24	+ 2	4
5	29	+ 6	36	29	+ 7	49
6	25	+ 2	4	26	+ 4	16
7	27	+ 4	16	22	0	0
8	24	+ 1	1	12	-10	100
9	23	0	0	28	+ 6	36
10	16	- 7	49	25	+ 3	9
11	25	+ 2	4	21	- 1	1
12	17	- 6	36	24	+ 2	4
13	19	- 4	16	21	- 1	1
14	23	0	0	20	- 2	4
15	24	+ 1	1	24	+ 2	4
16	15	- 8	64	15	- 7	49
17	23	0	0	17	- 5	25
18	17	- 6	36	18	- 4	16
	412	- 2	344	402	+ 6	356

$\overline{X}$ = 22.9 - - - - - - - - - - - - - 22.3

Assumed $\overline{X}$ = 23 - - - - - - - - - - - - - - 22

$$S = \sqrt{\frac{344}{18} - \left(\frac{-2}{18}\right)^2} \qquad S = \sqrt{\frac{356}{18} - \left(\frac{6}{18}\right)^2}$$

$$= \underline{\underline{4.4}} \qquad = \underline{\underline{4.4}}$$

$$SE_D = \sqrt{\frac{344+356}{34}}\ \sqrt{\frac{18+18}{18 \times 18}}$$

$$= 4.5 \times \frac{1}{3} = \underline{\underline{1.5}}$$

Correlation Between I.Q. and Unit I Test Scores
(Kamloops Group A)

Pupil No.	(I.Q. dev. from page 62) x	(Test Score dev. from page 65) y	xy
1	+16	+ 4	+64
2	+13	- 2	-26
3	+12	+ 5	+60
4	+10	+ 6	+60
5	+ 7	+ 6	+42
6	+ 7	+ 2	+14
7	+ 6	+ 4	+24
8	+ 1	+ 1	+ 1
9	- 1	0	0
10	-1	- 7	+ 7
11	- 5	+ 2	-10
12	- 6	- 6	+36
13	- 6	- 4	+24
14	- 7	0	0
15	- 8	+ 1	- 8
16	-10	- 8	+80
17	- 11	0	0
18	-12	- 6	+72
			+ 440

From page 62, $\Sigma x^2 = 1381$ From page 65, $\Sigma x^2 = 344$

(N.B. As noted on page 64, here again the difference between assumed mean and actual mean is so slight in each case that x' has been used for x).

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$$

$$= \frac{440}{\sqrt{1381 \times 344}} = .64$$