A STUDY OF THE RELATIONSHIP BETWEEN THE ABILITY TO COMPUTE WITH DECIMAL FRACTIONS AND THE UNDERSTANDING OF THE BASIC PROCESSES INVOLVED IN THE USE OF DECIMAL FRACTIONS

by

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ABSTRACT

A STUDY OF THE RELATIONSHIP BETWEEN THE ABILITY TO COMPUTE WITH DECIMAL FRACTIONS AND THE UNDERSTANDING OF THE BASIC PROCESSES INVOLVED IN THE USE OF DECIMAL FRACTIONS

Modern theory of arithmetic instruction supports the idea that the development of understandings of basic mathematical principles produces a desirable type of learning. This is a reaction against the traditional method of instruction which places emphasis upon mechanical drill procedures, devoid of meanings. This study is an attempt to determine what relationship, if any, exists between computational ability and understanding of fundamental processes. The investigation has been limited to the area of decimal fractions.

Two tests were developed for the purpose of the investigation. The test in computation was constructed and validated using pupils of the junior high school level as testees. Student-teachers constituted the personnel for the construction and validation of the test in understandings.

The investigation of relationship was performed using 236 Normal School students as testees. The tests, which had been constructed for use in the study, were administered at the beginning of the school term.

The data obtained from the investigation were analyzed and the following conclusions were formulated:

1. There is a positive correlation of considerable magnitude between the scores on the test in computation and the scores on the test in understandings. ($r = .640$). This is an indication that there is a tendency for the scores to vary in the same direction.

2. When the factor of intelligence is held constant, there is a net correlation of marked magnitude which is somewhat less than the apparent coefficient. This indicates that the common factor of intelligence has an influence upon the relationship between the two variables.

3. The magnitude of the relationship between scores in understanding and intelligence test scores is an indication of common elements in both these tests.

4. The relationship between the scores in computation and the intelligence test scores is not high. A high intelligence does not appear to be a prerequisite for high achievement in computation.

5. There is evidence that ability in computation is not essential for high achievement in understandings and vice versa, nor do high scores in one of these factors guarantee high scores in the other.

6. Although a study of the scatter diagram suggests that success in computation is more probable if it is accompanied by a high degree of understanding, it cannot be inferred from the data that one variable is the cause or the effect of the other.

ACKNOWLEDGMENTS

The assistance and cooperation of many students and teachers at various educational levels were necessary in this investigation. The writer is most appreciative of the help he has received from these sources. It is impossible to name all the principals and teachers who have contributed in some way to the development of this study. Without the testees, obtained through the permission of the Chief Inspector of Schools of the Greater Victoria School District and the Principals of the Provincial Normal Schools, this thesis could not have been written.

Dr. C. B. Conway of the British Columbia Department of Education gave much needed advice in the early stages of the preparation of the tests. His help is acknowledged with thanks.

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CHAPTER I

INTRODUCTION

Introductory Statement

During the past two or three decades, the theory of arithmetic instruction has been subjected to a close scrutiny because of fairly general dissatisfaction with the achievement of the graduates of our schools. As a result, there has emerged a method of instruction known as the meaning theory, which stresses the desirability of developing understandings of processes in contrast to the teaching of the mechanical manipulation of numbers, devoid of meanings. The advocates of this theory include such authorities as Brownell, Morton, Wheat, Brueckner, Grossnickle, Spitzer, Buckingham, and many others. However, in spite of the weighty opinions of these experts, much teaching continues to be of the more traditional type--based upon meaningless drill, repetition and rote memory. If teachers are to pay more than lip-service to the meaning theory in classroom practice, doubtless it will be necessary to demonstrate conclusively, time and again, that learning proceeds best, and is more permanent, when a high degree of understanding is present. Not until they have been convinced of the efficacy of the meaningful approach, by the evidence of sound statistical studies, are teachers likely to be concerned about objectives in the arithmetic programme other than those that are purely mechanical.

Failure to produce a strong case in support of the meaning theory may well result in a continuation of the status quo as set forth by

Wingo[1] in the following:

> With few, if any, exceptions, the investigators have found grounds for dissatisfaction with the present status of arithmetic instruction. Administrators, supervisors, and teachers cannot dismiss the criticism lightly. It is founded on sober, and sometimes alarming fact. It is directed at the most important aspect of any teaching problem: the problem of method.

Statement of the Problem

How adept at manipulating numbers may students become and yet possess little or no understanding of the underlying principles involved in the computations? Will they be more successful in the operation of numbers if meanings of basic concepts are clear to them? Weaver[2] suggests that:

> (1) It is quite possible that a person may possess considerable skill in arithmetic computation but have little or no understanding of why he does things in a manner which has become habituated.
> (2) It is equally possible that a person may have a thorough understanding of the mathematical bases for the algorisms which he uses but operate at a relatively low level of computational efficiency. Neither ability is prerequisite for the other. The attainment of either ability does not guarantee attainment of the other.

The present study will attempt to examine what relationship, if any, exists between the ability to manipulate numbers on a mechanical level and the understanding of the processes which underly the number operations in one phase of arithmetic, namely, decimal fractions.

A major aspect of this study will be the construction of

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[1] G. Max Wingo, "The Organization and Administration of the Arithmetic Program in the Elementary School", Arithmetic 1948, p. 69. Supplementary Educational Monographs, No. 66. Chicago: University of Chicago Press, 1948.

[2] J. Fred Weaver, "Some Areas of Misunderstanding About Meaning in Arithmetic", The Elementary School Journal, LI (September, 1950), p.36.

appropriate tests for the investigation. This will entail testing, observation and analysis extending over a period of more than a year. This task in itself, while subordinate to the main investigation, will constitute a study of considerable magnitude. It is hoped that the resulting tests will provide evaluative instruments which will have further usefulness.

Plan of the Study

The background of the study and a statement of the problem are presented in the introductory chapter. This will be followed by a description of the development and analysis of a test in computation with decimal fractions. Next, the construction and validation of a test in understandings of basic processes involving decimal fractions will be described. With the use of these tests, an investigation will be made to determine the degree of relationship, if any, that exists between mechanics and meanings. The results will then be analyzed and the conclusions formulated.

Materials of the Study

The investigator decided to work in the area of decimal fractions because of the essential nature of this subject matter and its extensive use both in and out of school, and also because of fairly general criticism of the lack of competence demonstrated in the application of this phase of arithmetic. When considering the subjects to be used in the study, he was guided by the availability of sufficient numbers for the purpose. It was decided to perform the investigation with Normal School students because, in this case, teachers-in-training provided a convenient group with which to work. Also, it was thought that, if the possession of understandings is a desirable outcome of arithmetic instruction for the pupils, surely, at the student-teacher level, it must be an even more essential aspect of learning. In the opinion of Wren:[3]

> It should be a trite remark to say that, unless the teacher himself can tread over hill and dale through the fields and forests of arithmetic with confidence and assurance that he knows where he is going and how he is going to get there, he certainly cannot render a great deal of assistance to his pupils.

As Normal School students were to provide the ultimate group to be used in the investigation, it was deemed necessary to use similar subjects for the purpose of validating the test on understandings. However, it was felt that the test on computation could be prepared by using any groups of unselected students who had completed the work on decimal fractions, since the basic material remains the same. By working with pupils at the Grade 7-8-9 level in a city school system, the investigator had available a large number of subjects, thus obtaining more scope for the construction of the test. It was thought that the resulting test could be used equally well with groups at higher levels and that it would provide a satisfactory testing instrument for the purpose of the proposed investigation.

Background of the Problem

A popular conception exists that the schools are not adequately preparing the pupils to handle the basic skills of arithmetic.

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[3] F. Lynwood Wren, "The Professional Preparation of Teachers of Arithmetic", Arithmetic 1948, p. 82. Supplementary Educational Monographs, No. 66. Chicago: University of Chicago Press, 1948.

Grossnickle[4] reports that:

> A frequent criticism directed towards public and elementary schools concerns the failure of their students to demonstrate adequate preparation in the fundamental subjects. This particularly holds true in the field of arithmetic.

As a result of the concern over the plight of arithmetic in the elementary schools, there has been extensive research conducted in this area during the past forty years. Psychological study on the process of learning has directed attention to <u>how</u> the child learns and has developed the view that one of the objectives of arithmetic instruction should be the development of understandings. According to Brownell:[5]

> From all this research and from experimentally oriented teaching emerged the notion that one ingredient in a functional program in arithmetic is provision for meaningful learning.

This theory of meaning or understanding impregnates the philosophy of practically all the authorities in the field of arithmetic today. While there are minor differences in interpretation of this theory, most of the experts base their philosophy upon the basic ideas propounded by Brownell[6] in the following:

> The "meaning" theory conceives of arithmetic as a closely knit system of understandable ideas, principles, and processes. According to this theory, the test of learning is not mere mechanical "figuring". The true test is an intelligent grasp upon number relations and the ability to deal with the arithmetical situations with proper comprehension of their mathematical as well as their practical significance.

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[4] Foster E. Grossnickle, "Dilemmas Confronting the Teachers of Arithmetic", <u>The Arithmetic Teacher</u>, I (February, 1954), p. 12

[5] William A. Brownell, "The Revolution in Arithmetic", <u>The Arithmetic Teacher</u>, I (February, 1954), pp. 3-4.

[6] William A. Brownell, "Psychological Considerations in the Learning and the Teaching of Arithmetic", <u>The Teaching of Arithmetic</u>, p. 19, Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Teachers' College, Columbia University, 1935.

Brownell goes on to explain that the meaning theory has been designed to encourage the understanding of arithmetic and the most frequent question put to the child should be, "Why did you do that?"

This study is concerned with that aspect of the meaning theory which relates to the understanding of basic skills--to the ability to rationalize processes--to that mathematical phase of arithmetic which provides the "why" for the algorism.

There is a convincing body of opinion in support of the idea that the development of meanings and understandings is an essential aspect of the kind of instruction that will produce better results in arithmetic. Brueckner and Grossnickle[7] state:

> Today there is almost universal acceptance of the view that children learn arithmetic more easily if they understand what they are learning and if it is mathematically meaningful to them.

Further support for the efficacy of the meaning theory is provided by Wren[8] in the following:

> Appreciation of significant meanings cannot be overemphasized either for the pupil or for the teacher .... Meanings not only form a basis for more intelligent use of computational skills, but they also give a background for a better appreciation of arithmetic as part of our cultural heritage.

Because of opinions such as these, impetus has been given to the pursuance of a method of instruction designed to develop understandings. The emergence of this new approach to arithmetic instruction forms the background of the present study.

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[7] Leo J. Brueckner and Foster E. Grossnickle, <u>Making Arithmetic Meaningful</u>, p. iii. Philadelphia: The John C. Winston Company, 1953.

[8] Wren, <u>op. cit.</u>, p. 85.

The Measurement of Understandings

If the aforementioned gospel of the proponents of the meaning theory of arithmetic instruction is to spread, convincing evidence of the superiority of this method must be produced. To do this requires, firstly, some method of evaluating the existence and growth of basic mathematical understandings.

The immediate problem becomes one of determining just what constitutes a body of understandings in arithmetic. One aspect of the study of arithmetic during the past twenty years has been an attempt to identify and isolate the ideas, principles, relationships and generalizations which constitute the understandings of the mathematical phase of the subject. As yet, research has produced little to guide the investigator in this field and, to some extent, he is forced to grope in the dark.

For the purposes of this study, it is assumed that a person's understanding of a process may be revealed by his ability to rationalize the procedure and that his insight into number operations may become apparent by his grasp of the "why" behind the performance of the algorism. This will form the basis of the measurement of understandings in the present investigation.

Criteria for Measurement of Understandings

The attempt to measure understandings of arithmetic processes is rendered very difficult by the lack of criteria for this purpose. The setting up of suitable criterion measures by which to evaluate understandings suggests an area for further research. In lieu of existing criteria, the investigator must determine arbitrarily those concepts that should be included in a measuring instrument designed to evaluate understandings of any phase of arithmetic. (Most researchers have been forced to adopt this procedure.)

The mechanics of the test on understandings will be discussed in a later chapter. It is considered that an understanding of the processes involved in computation with decimal fractions should include intelligent control of the following concepts:

1. Meaning of decimal fractions
2. Reading and writing decimals
3. Value of decimal fractions
4. Comparison of decimal fractions
5. Function of zero
6. Rounding off numbers
7. Accuracy of measurement
8. Effect of moving the decimal point
9. Location of the decimal point in addition and subtraction
10. Location of the decimal point in multiplication
11. Location of the decimal point in division
12. Changing common fractions to decimals
13. Changing decimals to common fractions
14. Relative value of digits

Related Studies

While the literature is replete with material supporting the meaning theory of instruction, more studies are needed to show the result of its application; and there seems to have been little attempt made to evaluate the mathematical understandings acquired by pupils. Little research is available to lend support to the opinion that the development of understandings produces a higher level of achievement.

The following studies are illustrative of the research that has been conducted in this area:

1. Glennon[9] conducted a frontier research study designed to discover the degree of mathematical understanding possessed by representative groups on different educational levels. For this purpose he was obliged to construct a special test. His study revealed that some progress is being made in the field of evaluation in arithmetic. However, he reports that his findings "do not offer a favorable picture of our present practices in teaching meanings and understandings in arithmetic". He was forced to the conclusion that teachers are not succeeding in developing an understanding of mathematical principles.

2. Kilgour[10] used Glennon's test to conduct a study on the development of understandings in a one-year teacher-training programme. She found that small, but significant, gains were made by the students in their understandings of basic concepts.

3. Orleans and Wundt[11] conducted a study to ascertain the extent of understandings possessed by teachers and student-teachers. They designed

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[9] Vincent J. Glennon, "Testing Meanings in Arithmetic", Arithmetic 1949, pp. 64-74. Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949.

[10] Alma Jean Kilgour, The Effect of a Year's Teacher-Training Course on the Vancouver Normal School Students' Understanding of Arithmetic. Unpublished Master's thesis in education. University of British Columbia, 1953.

[11] Jacob S. Orleans and Edwin Wandt, "The Understanding of Arithmetic Possessed by Teachers", The Elementary School Journal, LIII (May, 1953), pp. 501-507.

their own test for the investigations. They concluded:

> There are apparently few processes, concepts, or relationships in arithmetic which are understood by a large per cent of teachers.

4. Brownell[12] reports a study conducted by Spainhour to determine the relationship between arithmetic understanding and ability in problem solving and computation. He found correlations of .665 and .751 in grade four and .751 and .756 in grade 6.

5. Taylor[13] tested college freshmen on mathematical meanings and revealed a woeful lack of understanding of basic concepts. His findings lead him to the conclusion that "students entering teachers colleges are deficient in both the mechanics and the understanding of arithmetic".

These studies reveal, in the opinion of the writer, that little research has been done in the area of evaluation of arithmetical understandings; that, generally speaking, students possess very little insight in respect to basic number operations and that only slight progress has been made in developing meanings in arithmetic; that growth in understandings may be an outcome of arithmetic instruction; and that there is a significant positive correlation between understanding of processes and computational ability. Obviously, there is a definite need for further research in this field of arithmetic.

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12 William A. Brownell, "The Evolution of Learning in Arithmetic", Arithmetic in General Education, p. 229. Sixteenth Yearbook of the National Council of Teachers of Mathematics. New York: Teachers' College, Columbia University, 1941.

13 E. H. Taylor, "Mathematics for a Four-Year Course for Teachers in the Elementary School", School Science and Mathematics, XXXVIII (May, 1938), pp. 499-503.

Summary

The "new" theory of arithmetic instruction is based upon the psychologists' knowledge of how the child learns. While much has been written about the meaning approach to the teaching of arithmetic, more evidence is desirable to give support to the opinion that this theory of instruction is likely to produce more worthwhile results. The purpose of the present study is to investigate the relationship, if any, between a student's achievement in a test in computation with decimal fractions and a test in understandings of the processes used in the computations. It is an attempt to supply data which will help to answer the question: "Does instruction designed to develop meanings result in higher achievement than a method of teaching which stresses mechanical operation of numbers only?"

The investigation necessitates the construction and validation of suitable tests for the purpose. The construction of a test on understandings is made difficult because of relatively little research in this area, because of the difficulty of defining understandings, and because of the lack of adequate criteria for the purpose of validation.

The study will be conducted with the aid of Normal School students as subjects. An attempt will be made to analyze the results and to arrive at conclusions which may have significance in terms of the method of arithmetic instruction which is most desirable.

CHAPTER II

CONSTRUCTION AND ANALYSIS OF A TEST IN COMPUTATION WITH DECIMAL FRACTIONS

Introduction

As discussed in the previous chapter, it was decided that a necessary part of the investigation must be the construction and validation of a test on the basic skills involved in computation with decimal fractions. The material of such a test should consist of items based upon the subject-matter in the curriculum of the elementary schools of British Columbia. As the basic essentials of computation with decimal fractions are re-taught and used throughout the school years beyond the elementary grades, a test on this material should be suitable for the evaluation of pupils of any higher grade level. Pupils of the Junior High School grades should possess a high degree of proficiency in computation with decimal fractions, and it is likely that this degree of skill will be maintained or raised during the following years. Thus, it is felt that the construction of a test based upon the achievement of subjects at the grade 7-8-9 level should provide a valid testing instrument which could be used for evaluating proficiency at a more advanced level, including high school graduation and teacher-training. A description of the preparation and validation of the test on computation with decimal fractions will be presented in this chapter.

Characteristics of a Good Test

In the construction of a test there are a number of factors which must receive due consideration. Greene, Jorgensen and Gerberich[1] include the following distinguishing characteristics of good examinations in their specifications: validity, reliability, objectivity, administrability, comparability, economy, utility.

In a study in which he examined the worth of a teacher-made test, Carlile[2] considered the following criteria to be significant: validity, reliability, discrimination, level of difficulty, objectivity, ease of administration and ease of scoring.

In the construction of the present tests attention will be paid to the aforementioned factors in an endeavour to produce statistically sound testing instruments.

Curricular Validity

An examination of the course of study and the current text books was made and a list of the major skills and concepts involving computation with decimal fractions was compiled. Items, designed to test these concepts, were constructed and entered on individual cards. These items were then put together indiscriminately and mimeographed copies were submitted to a number of teachers for criticism and suggestion. In this manner material which fitted the curriculum was selected for the test and an attempt was made to assure curricular validity.

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1 Harry A. Greene, Albert N. Jorgensen, J. Raymond Gerberich, Measurement and Evaluation in the Secondary School. New York: Longmans, Green and Co., 1943.

2 A. B. Carlile, "An Examination of a Teacher-made Test", Educational Administration and Supervision, 40 (April, 1954), pp. 212-218.

Experimental Form

On the bases of suggestions received and some individual testing which was carried out, changes were made and the items were put into an experimental form. This form of the test was administered to 135 pupils in grades 7-8-9 in different types of schools in the Greater Victoria School District. The writer personally administered some of the tests, made careful observations to determine "face" validity and interest, and recorded the time factor.

The results of the first experimental run were carefully studied and discussed with classroom teachers. Certain weaknesses in form, content and wording were immediately apparent and further revision ensued. A crude analysis of the degree of difficulty was made and the test items were rearranged in what appeared to be ascending order of difficulty. The test contained 32 items at this stage of development. After consultation with an expert in the field of testing, the material was put into a form suitable for a trial run.

The time factor appeared to be satisfactory. Thirty minutes provided ample time for most pupils to finish the work, which meant that the test could be administered comfortably in a normal forty-minute period.

As the population upon which the final study was to be made consisted of teachers-in-training, it was deemed wise to obtain some indication of the performance of such a group on the test. Therefore, the test was administered to 165 students of the Victoria Normal School. Although these students had been studying the topic of decimal fractions recently, their scores were distributed over a fairly wide range of achievement.

Analysis of this performance led to further revision to clear up ambiguities and to improve objectivity. The opportunity of closely observing reaction to the items at various stages in the development of the test made it possible to revise and restate the items in such a way as to ensure a high degree of objectivity. At this stage it was decided that a few more items could be added to advantage, bringing the total up to thirty-five.

Preliminary Form

The test was now organized into a Preliminary Form, containing 35 items, with a time limit of 30 minutes. It was administered to over 300 pupils in grades 7-8-9 in various types of schools throughout the Greater Victoria area. A brief list of instructions for administering the test was issued for the purpose of ensuring uniformity of procedure. The papers were returned and scored and were made ready for analysis. As the nature of the scoring was highly objective, it was possible to enlist the assistance of student-teachers for the task.

Method of Item Analysis Used in This Study

An accepted method of determining the discriminating power of a test item is to compare the performance of the best section of the group with that of the poorest section. Although of obscure origin, the technique of comparing portions of the group is widely used and is described in some detail by Long and Sandiford[3]. They explain:

> The idea underlying all the Upper and Lower Methods is that the good item is one which the good pupils do well, and the poor pupils do poorly.

[3] J. A. Long and P. Sandiford, The Validation of Test Items. Bulletin No. 3 of the Department of Educational Research (Toronto, Ontario: The Department of Educational Research, University of Toronto, 1935), p. 31.

While various fractions of the distribution may be used, Long and Sandiford[4] state unequivocally that the Upper and Lower Thirds technique gives the best results.

> Experiments show that Upper and Lower Halves, which use all the data at the disposal of the examiner, is not so good a method as Upper and Lower Thirds or the Upper and Lower 27 per cent.

They[5] go on to point out that the Upper and Lower Thirds technique tends to discriminate in favour of items of 50% difficulty.

> Taking both effectiveness and ease of computation into consideration, of those techniques which tend to select 50%, or balanced difficulty, the Upper versus Lower Thirds may be adopted as the preferred technique.

The procedure used in this study is outlined below:

1. A group of 300 papers was selected for analysis and arranged in score order.

2. The papers were divided into three groups of 100 each—an Upper Third--a Lower Third--a Middle Third. The number of papers employed facilitated the use of per cents.

3. Large sheets of squared paper were prepared and the correct responses to each item on each test paper were tabulated.

4. Per cents of correct responses on each item were calculated for the upper Third and for the Lower Third.

5. Discriminatory value or item validity was found by comparing the performance of the Upper Third group with that of the Lower Third group. Validity of each item was expressed as a per cent and determined by finding the difference between the per cent correct in the Upper Third and that in the Lower Third.

6. Per cent difficulty was calculated on the basis of the total item-errors in the Upper and Lower Thirds combined.

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[4] *ibid.*, p. 32

[5] *ibid.*, p. 118

7. Per cent difficulty and per cent validity[6] were plotted on graph paper. An arbitrary curve was drawn. An item falling unduly far below the curve was considered unsuitable or in need of revision. Items falling above the curve were considered satisfactory and were retained in that form.

An example or two will serve to illustrate the procedure. An item with a validity of one hundred per cent would be answered correctly by all in the Upper Third and incorrectly by all in the Lower Third; its difficulty would thus be fifty per cent. A situation of this kind would be quite unlikely to occur. (Per cent of difficulty refers to the per cent of incorrect responses to an item.) An item with fifty per cent difficulty might be answered correctly by eighty per cent of the upper group and by twenty per cent of the lower group, which would give it a validity of sixty per cent. This item would lie well above the curve and would be considered a satisfactory item to retain. An item answered correctly by forty-five per cent of the top group and by thirty-five per cent of the bottom group would have a sixty per cent difficulty (forty per cent ease) but its power to discriminate would be only ten per cent. It would fall well below the curve and would contribute little to the test. In order to place items along the full range of difficulty, which is desirable, there will, of course, be some with small value from the point of view of discriminatory power. This cannot be avoided. Any item showing negative validity, however, would be discarded immediately.

Following the procedure as outlined, the papers were grouped and analyzed. Per cents of validity and difficulty were computed using the

6 This graphical technique was devised and is used by Dr. C. B. Conway, Director, Division of Tests, Standards and Research, Department of Education, Victoria, B. C.

Upper Third-Lower Third technique. The results of this procedure appear in Table I.

Using a prepared grid, with the vertical axis representing the per cent of validity and the horizontal axis the per cent of difficulty, the efficiency of the items was demonstrated graphically. An arbitrary curve was drawn and items appearing above the curve were considered satisfactory for inclusion in the test, while those falling below appeared to be in need of revision or deletion. As will be seen by the graph, shown in Figure I, no item falls seriously out of line and it was decided that all might be retained for the final form of the test. Per cents of validity range from 14% to 70% and all are positive. Thus, each item has some power to discriminate between the good and the poor pupils. Per cents of difficulty for the items extend from 9% to 81% with an average difficulty of 44%. This is in line with the findings of Hawkes, Lindquist and Mann[7] who report that in general test authorities "..... _are agreed that there should be a range of difficulty from about 5 to 20 per cent to 80 to 95 per cent, and that the average difficulty of all items should be about 50 per cent._"

Reliability

The reliability of a test must be estimated statistically but it is not dependent upon an external criterion. Thus, it does not present the same difficulties as are encountered in establishing the validity of

[7] Herbert E. Hawkes, E. F. Lindquist, C. R. Mann, _The Construction and Use of Achievement Examinations_, p. 32. Boston: Houghton Mifflin Company, 1936.

Table I

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT ** OF THE ITEMS OF THE PRELIMINARY FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS

Item	Per cent of Validity	Per cent of Difficulty
1	47	29
2	24	17
3	20	15
4	21	15
5	35	19
6	49	68
7	43	30
8	18	12
9	22	23
10	52	41
11	28	25
12	36	29
13	62	53
14	63	50
15	37	31
16	54	41
17	26	38

** While it is recognized that the term "per cent of validity" is one of current usage, it is realized that it is somewhat of a misnomer and, as pointed out previously, it represents the difference between the per cent of correct responses obtained by the top third of the group and the bottom third of the group.

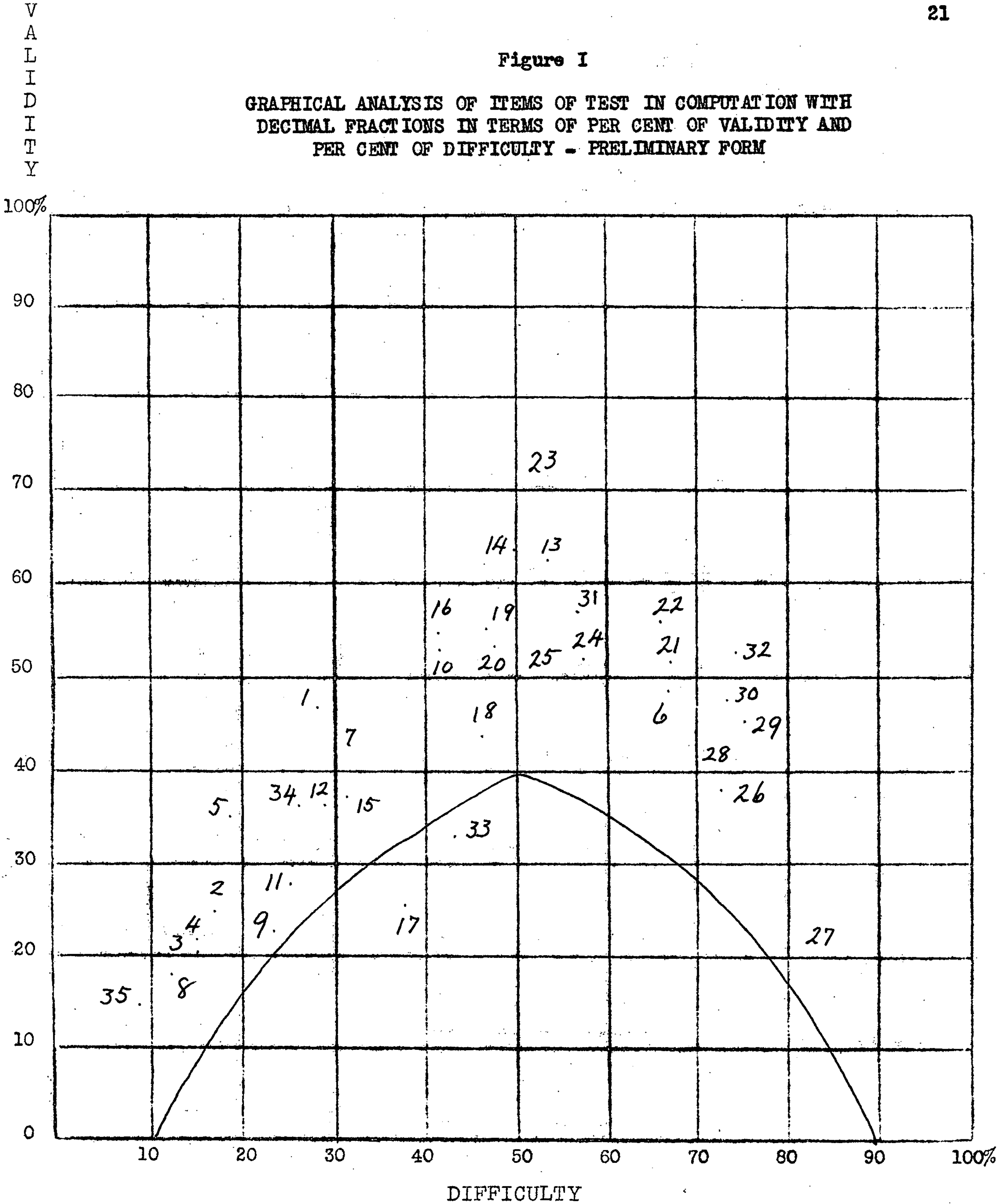
Table I (continued)

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT OF THE ITEMS OF THE PRELIMINARY FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS

Item	Per Cent of Validity	Per cent of Difficulty
18	43	46
19	55	46
20	53	47
21	51	67
22	56	66
23	70	52
24	51	58
25	50	52
26	38	72
27	20	81
28	40	70
29	45	75
30	48	72
31	57	57
32	52	73
33	32	42
34	36	26
35	14	9

Figure I

GRAPHICAL ANALYSIS OF ITEMS OF TEST IN COMPUTATION WITH DECIMAL FRACTIONS IN TERMS OF PER CENT OF VALIDITY AND PER CENT OF DIFFICULTY - PRELIMINARY FORM



a test. For the purpose of determining the reliability of this test, all of the 300 papers were included in the calculations. The odd-even split-halves technique was used and the result was corrected by the Spearman-Brown Prophecy Formula. Greene, Jorgensen and Gerberich[8] point out that, while the coefficient obtained by this method is likely to be spuriously high, "... this is one of the most feasible methods for use with informal objective examinations for which ordinarily no second or alternate form is available."

The result of these calculations appears in Table II.

Table II

COEFFICIENT OF RELIABILITY OF PRELIMINARY FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS DETERMINED BY THE ODD-EVEN SPLIT-HALVES TECHNIQUE

Form	Coefficient	Corrected Coefficient
Preliminary	.780	.876

Although the coefficient of correlation appeared to be reasonably satisfactory, it was decided to estimate the reliability coefficient by a second method. Kuder and Richardson have devised a simple formula for determining the reliability of a test. The only data used by this method

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[8] Greene, Jorgensen and Gerberich, op. cit., p. 63

are the number of test items, the standard deviation, and the arithmetic mean. It is based upon a number of assumptions which are hard to meet but, nevertheless, Remmers[9] believes that: "..... it is probable that the quick estimate afforded by this formula is good enough for all practical purposes." This technique usually gives an estimate lower than that obtained by the split-half method used with the Spearman-Brown Formula. The result obtained by the Kuder-Richardson method is given in Table III and, as is to be expected, is slightly lower than the previously determined coefficient. In terms of reliability the test now seemed to be acceptable and ready for use in final form.

Table III

COEFFICIENT OF RELIABILITY OF THE PRELIMINARY FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS DETERMINED BY THE KUDER-RICHARDSON FORMULA

Form	Reliability Coefficient
Preliminary	.820

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9 H. H. Remmers and N. L. Gage, Educational Measurement and Evaluation, p. 205. New York: Harper & Brothers, 1943.

Final Form

The only revision undertaken for the Final Form of the test was to arrange the items in order of difficulty. With this, the test was ready to be administered for a final run.

The test was now given to over 300 grade 7-8-9 pupils in a single junior high school. The same procedure was used as in the Preliminary Form. The papers were checked objectively, arranged in rank order, and divided into three sections of 100 each.

Correct item responses were tallied and per cent of validity and per cent of difficulty were calculated as before. Table IV shows the results. As before, the material is presented in graphical form in Figure II. It may be seen that only one item falls slightly below the arbitrary curve. The validity of all items is positive and ranges from 7% to 79%. The per cent of difficulty of items runs from 9% to 87% with an average difficulty of 31.51%. It will be noted that the test appeared to be less difficult for this group than for the group tested in the preliminary run. This may be accounted for, in part, by the fact that only one school was used in this run and by the fact that the test was administered later in the school year. It may also be observed that the order of difficulty of the items, although not identical with that found in the first run, is approximately the same.

Item Validity Based on Flanagan's Tables

Although the method above appeared to provide a satisfactory estimate of the internal consistency of the items of the test, it was decided to apply a further check of the item validity indices. According

Table IV

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT OF THE ITEMS OF THE FINAL FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS

Item	Per cent of Validity	Per cent of Difficulty
1	7	11
2	9	9
3	7	10
4	13	13
5	20	13
6	27	15
7	19	17
8	22	15
9	15	10
10	30	19
11	21	14
12	29	17
13	22	21
14	29	28
15	36	20
16	51	29
17	14	24

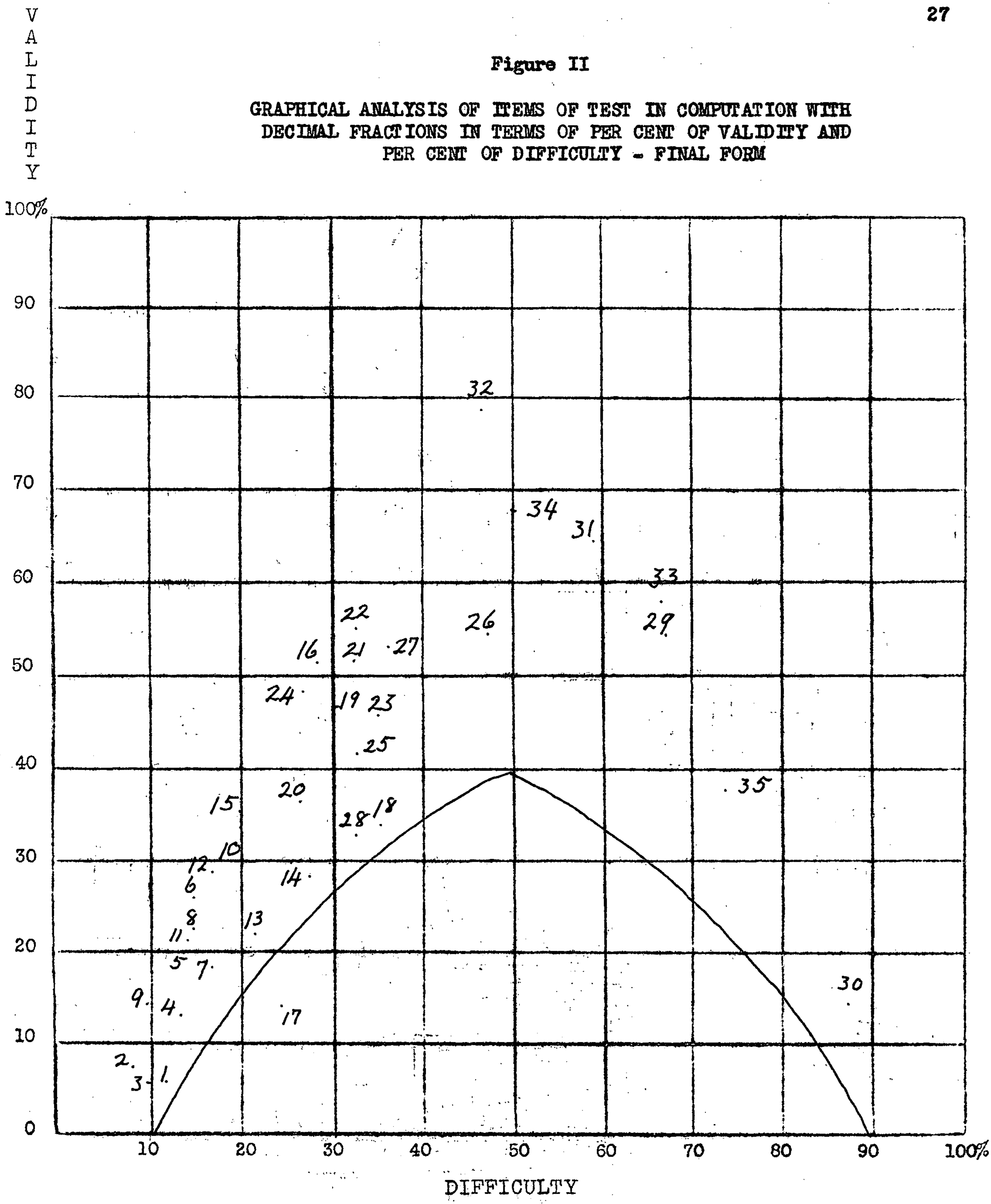
Table IV (continued)

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT OF THE ITEMS OF THE FINAL FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS

Item	Per cent of Validity	Per cent of Difficulty
18	34	35
19	48	30
20	37	27
21	51	32
22	56	32
23	47	35
24	49	27
25	41	32
26	55	48
27	54	36
28	33	32
29	55	68
30	15	87
31	65	59
32	79	47
33	58	67
34	68	50
35	38	73

Figure II

GRAPHICAL ANALYSIS OF ITEMS OF TEST IN COMPUTATION WITH DECIMAL FRACTIONS IN TERMS OF PER CENT OF VALIDITY AND PER CENT OF DIFFICULTY - FINAL FORM



to Thorndike[10], who favours working with the top 27 per cent and the bottom 27 per cent of the total group:

> The most satisfactory item validity index based on the upper and lower 27 per cent is the estimate of the coefficient of correlation between item and test obtainable from tables prepared by Flanagan.

Using the data obtained from the final administration of the test, per cents succeeding in the upper 27 per cent and the lower 27 per cent were calculated. Estimates of product-moment coefficients of correlation were found by reference to tables prepared by Flanagan[11]. The results appear in Table V. It will be seen that the coefficients of correlation range from .120 to .810 and that all are positive. Items 1 and 2 are not significantly greater than zero at the 1% level and are barely significant at the 5% level. The rest of the items are significantly greater than zero.

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10 Robert L. Thorndike, Personnel Selection, p. 242. New York: John Wiley & Sons, Inc., 1949.

11 *ibid.*, pp. 348-351.

Table V

INTERNAL CONSISTENCY OF THE FINAL FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS, BASED ON FLANAGAN'S ESTIMATES OF THE CORRELATION BETWEEN INDIVIDUAL ITEMS AND THE TEST AS A WHOLE

Item	Coefficient
1	.120
2	.140
3	.215
4	.243
5	.400
6	.700
7	.280
8	.420
9	.370
10	.580
11	.505
12	.520
13	.320
14	.375
15	.560
16	.665
17	.245
18	.400
19	.640
20	.495
21	.638
22	.680
23	.510
24	.795
25	.533
26	.600
27	.605
28	.460
29	.600
30	.345
31	.680
32	.810
33	.715
34	.690
35	.470

Reliability

Reliability of the Final Form of the test was determined by using the same methods as those used in the Preliminary Form. The findings are given in Table VI. While the reliability coefficients have dropped slightly, the figures are reasonably close to those obtained for the earlier form.

Table VI

COEFFICIENTS OF RELIABILITY OF THE FINAL FORM OF THE TEST IN COMPUTATION WITH DECIMAL FRACTIONS

Form	Method	Reliability Coefficient
Final	Split-halves	.853 (corrected)
"	Kuder-Richardson	.812

Summary

In the opinion of the writer, the analysis has demonstrated that the Final Form of the test is a satisfactory instrument for use in this study. The test is almost self-administering; it can be given in a normal classroom period; it is easily scored and is highly objective; it is economical in terms of cost; the material of the test is based entirely upon the curriculum; in construction of the test, logical and "face" validity have received consideration; the items meet the requirements of validity and difficulty and succeed in discriminating between the good and the poor pupils; on the basis of two statistical techniques the test is found to possess a fairly high degree of reliability. In the light of the foregoing arguments, the test is considered suitable for its intended purpose and will be used in the investigation.

The Final Form of the Test In Computation with Decimal Fractions appears in Appendix A.

CHAPTER III

CONSTRUCTION OF A TEST IN UNDERSTANDINGS OF PROCESSES INVOLVING DECIMAL FRACTIONS

Introduction

The construction of a test in understandings of processes involves not only the same problems as those encountered in the preparation of the test in computation, but also, additional difficulties due to the nature of such a test. A discussion of understandings and meanings was presented in an earlier chapter. The problem now arising is one of securing a test designed to evaluate a pupil's understanding of the basic processes inherent in computation with decimal fractions. That little research has been carried out in this area is indicated by Glennon[1] in the following:

> The paucity of research studies in the area of testing for meanings justifies the conclusion that this is one of the most neglected educational problems of the day.

An examination of Glennon's test in mathematical understandings revealed that, by its nature, it would not suit the purpose of this study. In addition, no data relative to its validity were available. As no other tests which would fit the requirements of this study could be located, it was decided that it would be necessary to construct a test in understandings of basic processes involved in the use of decimal fractions.

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[1] Glennon, *op. cit.*, p. 68

Factors Involved in the Construction of Test Items

In constructing the test items the test-maker had to be conscious of a number of factors that were not relevant to the computational test. The items must not involve computation, otherwise they would duplicate the function of the other test. The material must be based upon the material of the companion test in computation--it must try to evaluate the subject's understanding of his use of the mechanics which involve the same basic concepts. Verbalism must be minimized so that the test does not become an evaluation of reading comprehension. Wording of the items must receive careful consideration so that ambiguity may be avoided. A suitable form of item must be selected to ensure a high degree of objectivity.

With an awareness of these requirements in mind, the writer set about to prepare the material for the test. The lack of a criterion against which to validate the test made it imperative to construct it with the greatest care so that it might be efficient in terms of curricular or analytical validity. Both Carlile[2] and Glennon[3] depended upon this aspect of validation in their tests.

The questions on the computational test were taken as a basis on which to work and an attempt was made to design items to test understandings of the processes used in these computations. To obtain some idea of the manner in which students are likely to describe their thought processes when performing computations, some subjective questions were given inform-

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[2] Carlile, _op. cit._, p. 214

[3] Glennon, _op. cit._, p. 70

ally to several groups. A series of questions was then prepared on individual cards.

The form of item decided upon was the multiple-choice. Discussing the use of this style of question, Ross[4] has this to say:

> The multiple-choice type of item is usually regarded as the most valuable and most generally applicable of all test forms. Lee regards it as "one of the best means for testing judgment that is available". Lindquist asserts that it is "definitely superior to other types" for measuring such educational objectives as "inferential reasoning, reasoned understanding, or sound judgment and discrimination on the part of the pupils".

Remmers[5] and others support this opinion. In addition, this style of item ensures objectivity of scoring, which is an essential characteristic for this test.

Experimental Form

An exploratory group of twenty-seven items was prepared and the material was submitted to a number of competent educationalists for criticism and suggestion. The combined judgments of these people ensured some degree of validity and made it possible to prepare an experimental form of the test.

The test was administered to over one hundred grade 7-8-9 pupils in the Greater Victoria School District. Some of the testing was done personally by the writer to observe pupil reaction and to calculate the time factor. In addition, some testing of individual subjects was performed. Results were carefully analyzed and a crude comparison was made

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[4] C. C. Ross, <u>Measurement in Today's Schools</u>, p. 145. New York: Prentice-Hall, Inc., 1941.

[5] Remmers, <u>op. cit.</u>, p. 167

with the performance of the same pupils on the test in computation. At this point it was decided that there was little to be gained by validating the test at this grade level because reading ability appeared to be such an important factor. As the study was to be performed ultimately with a group of student-teachers, it appeared that further testing should be conducted with a similar group.

The Experimental Form was next administered to a group of first year college students under the personal supervision of the writer. In all stages of preparation of the test comments and suggestions were invited from the students so that revision might be made with a view to securing greater "face" validity. A careful analysis of individual working times was made and it was found that the test could be completed with ease in fifteen to twenty minutes. The results of this group were carefully analyzed in terms of alternatives selected and a crude comparison was made with their general achievement standing in mathematics. As a result of these observations, it was now possible to revise the test and prepare a Preliminary Form.

Preliminary Form

The Preliminary Form of the test was made up of twenty-seven multiple choice items of four or five alternatives. The time factor was set at fifteen minutes or until a large per cent of the group had completed the paper. Simple directions were prepared and a marking key was provided. The paper was administered to over three hundred teachers-in-training who were invited to comment upon their reaction to the various items. The items were easily and objectively scored by competent students.

Three hundred completed papers were selected and arranged in score order. They were then divided into three piles of one hundred each, thus providing an upper and a lower third. Item responses were tabulated on large sheets of squared paper and, using the Upper Third-Lower Third technique, per cent of validity and per cent of difficulty were calculated. The analysis is given in Table VII.

As in the test in computation, the per cents of validity and of difficulty were plotted upon a grid and an arbitrary curve was drawn. This is illustrated in Figure III. It will be seen that only one item falls below the curve to any appreciable degree. All items show a positive validity ranging from 10% to 52% with only a few much below 20%. Thus they meet the requirement set up by Carlile[6] in his study. He reported:

> Items which did not show a positive discrimination of as much as twenty per cent were considered lacking in the power of discrimination.

At the same time, he pointed out that there is a tendency to include some easy and some difficult items which will have little discriminative value.

Per cent of difficulty of the items ranges from 7% to 69% with an average difficulty of 33%. The test, as a whole, is easier than desirable but perhaps that is inevitable in a test of this type. At the same time, it must be remembered that the test was administered near the end of a year's consideration of the content.

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6 Carlile, _op. cit._, p. 215

Table VII

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT OF THE ITEMS OF THE PRELIMINARY FORM OF THE TEST IN UNDERSTANDING OF PROCESSES

Item	Per cent of Validity	Per cent of Difficulty
1	25	17
2	13	33
3	32	34
4	43	30
5	19	79
6	25	19
7	18	10
8	31	31
9	10	7
10	30	35
11	33	36
12	18	10
13	19	11
14	25	17
15	42	33

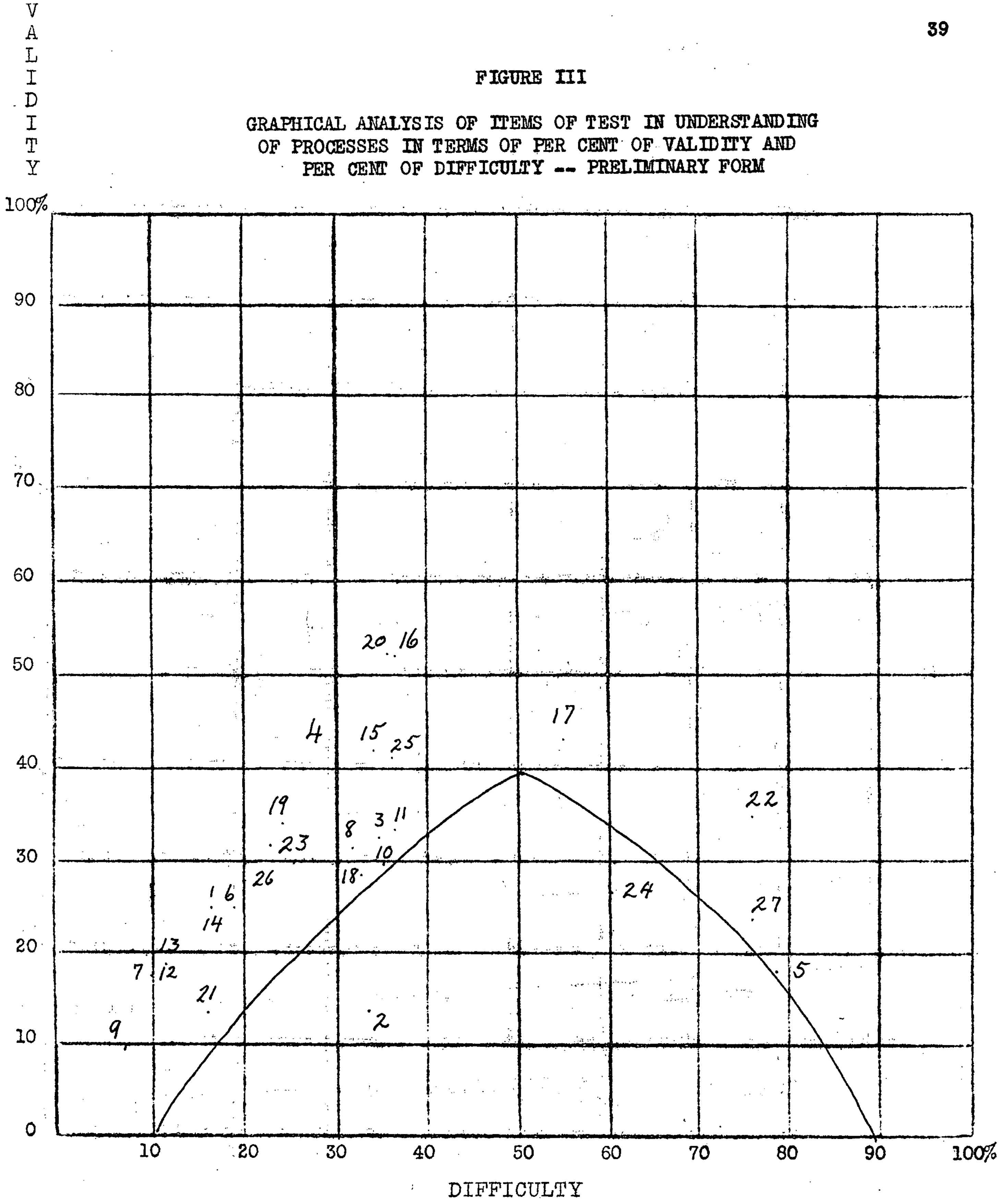
Table VII (continued)

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT OF THE ITEMS OF THE PRELIMINARY FORM OF THE TEST IN UNDERSTANDING OF PROCESSES

Item	Per cent of Validity	Per cent of Difficulty
16	52	37
17	44	55
18	29	32
19	34	24
20	52	36
21	14	17
22	35	76
23	31	22
24	27	60
25	41	36
26	28	20
27	24	76

FIGURE III

GRAPHICAL ANALYSIS OF ITEMS OF TEST IN UNDERSTANDING OF PROCESSES IN TERMS OF PER CENT OF VALIDITY AND PER CENT OF DIFFICULTY -- PRELIMINARY FORM



Reliability

The coefficient of reliability was calculated by dividing the test into chance-halves, using the odd and even scores. The Pearson Product-Moment method was employed to find the coefficient of the half test. The result was corrected by the Spearman-Brown Prophecy Formula and appears in Table VIII.

Table VIII

COEFFICIENT OF RELIABILITY OF THE PRELIMINARY FORM OF THE TEST IN UNDERSTANDING OF PROCESSES DETERMINED BY ODD-EVEN SPLIT-HALVES TECHNIQUE

Form	Coefficient	Corrected Coefficient
Preliminary	.566	.723

The Kuder-Richardson Formula was then applied with the result shown in Table IX.

Table IX

COEFFICIENT OF RELIABILITY OF THE PRELIMINARY FORM OF THE TEST IN UNDERSTANDING OF PROCESSES DETERMINED BY THE KUDER-RICHARDSON FORMULA

Form	Reliability Coefficient
Preliminary	.574

The reliability coefficients obtained for the Preliminary Form of the test were disappointingly low.

The Final Form

The results of the Preliminary Form were subjected to a most careful scrutiny and a number of changes in the wording of the items were made. The questions were then arranged in approximate order of difficulty. Three additional items were added, bringing the total up to thirty. The test was now prepared in final form.

In Chapter I there was set forth a list of the concepts which, it is believed, forms the basis of the understandings involved in the use of decimal fractions. It is the function of the present test to evaluate the students' understanding of these concepts. Following is an analysis of the test which indicates the items that are designed to measure the various concepts:

	Concept		Item
1.	Meaning of decimal fractions	-	27
2.	Reading and writing of decimals	-	7
3.	Value of decimal fractions	-	5, 10, 21, 28
4.	Comparison of decimal fractions	-	4
5.	Function of zero	-	3, 8, 9
6.	Rounding off numbers	-	21, 22, 28
7.	Accuracy of measurement	-	18, 30
8.	Effect of moving decimal point	-	6, 8, 14, 19, 23, 25
9.	Location of point in addition	-	1
10.	Location of point in subtraction		
11.	Location of point in multiplication	-	16, 25, 26
12.	Location of point in division	-	12, 13, 14, 17, 23, 24
13.	Changing common fractions to decimals	-	2, 29
14.	Changing decimals to common fractions	-	15
15.	Relative value of digits	-	11, 20

The revised test was now administered to 150 teachers-in-training. It was made up of thirty multiple-choice items and the time factor was set at twenty minutes. The test was completed by nearly all students in the allotted time. Interest appeared to be fairly high and ambiguity had been largely eliminated. The test was readily scored and was completely objective.

The results were tabulated and analyzed as in the Preliminary Form. Table X gives the per cent of validity and difficulty of the items. In Figure IV the results are depicted graphically. Per cents of validity run from 8% to 72%. Difficulty of the items ranges from 4% to 77% with an average difficulty of 33%, the same as for the Preliminary Form. The graph shows that only a few items fall below the curve but none so far as to cause any real concern. The three additional items proved to be quite satisfactory, having the following ratings:

No.	Per cent of Validity	Per cent of Difficulty
5	66	59
7	26	55
14	68	48

Table X

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT OF THE ITEMS OF THE FINAL FORM OF THE TEST IN UNDERSTANDING OF PROCESSES

Item	Per cent of Validity	Per cent of Difficulty
1	10	5
2	8	4
3	14	9
4	12	8
5	66	59
6	40	34
7	26	55
8	34	19
9	16	14
10	30	67
11	22	13
12	26	13
13	32	16
14	68	48
15	16	24

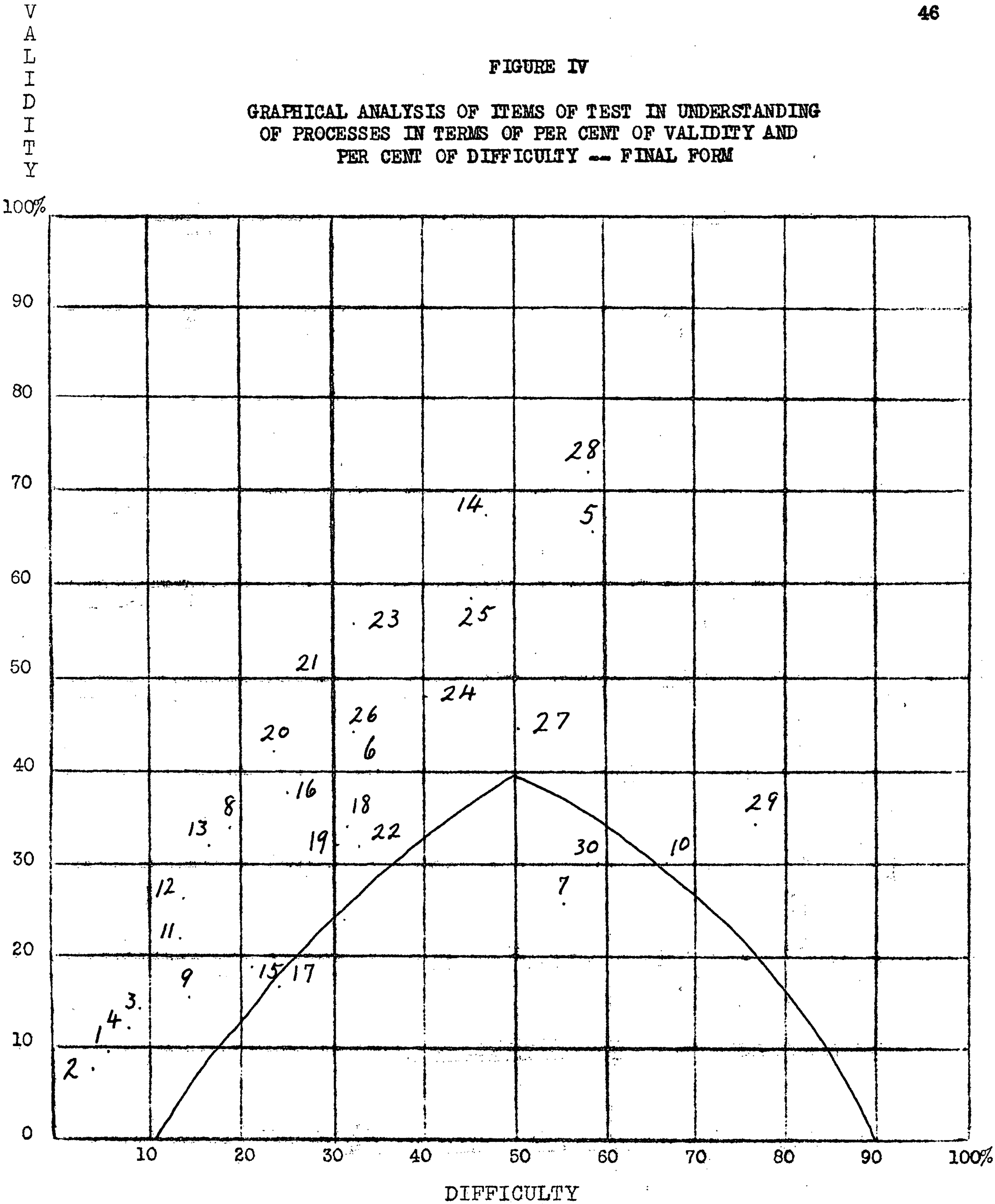
Table X (continued)

THE VALIDITIES AND DIFFICULTIES IN TERMS OF PER CENT OF THE ITEMS OF THE FINAL FORM OF THE TEST IN UNDERSTANDING OF PROCESSES

Item	Per cent of Validity	Per cent of Difficulty
16	38	25
17	18	29
18	34	31
19	32	30
20	42	23
21	50	29
22	32	32
23	56	32
24	48	40
25	58	45
26	44	32
27	44	50
28	72	58
29	34	77
30	30	59

FIGURE IV

GRAPHICAL ANALYSIS OF ITEMS OF TEST IN UNDERSTANDING OF PROCESSES IN TERMS OF PER CENT OF VALIDITY AND PER CENT OF DIFFICULTY -- FINAL FORM



Item Validity Indices Based on Flanagan's Tables

Following the same procedure as that used in the test on computation, validity coefficients were determined on the basis of the upper and lower twenty-seven per cent of the group tested. Reference to Flanagan's tables gives values as shown in Table XI. All coefficients are positive and range from .150 to .820. The coefficients of all items, with the exception of one, are significantly greater than zero. The coefficient of item No. 15 is too small to be indicative of any real correlation.

Table XI

INTERNAL CONSISTENCY OF THE FINAL FORM OF THE TEST IN UNDERSTANDING OF PROCESSES, BASED ON FLANAGAN'S ESTIMATES OF CORRELATION BETWEEN INDIVIDUAL ITEMS AND THE TEST AS A WHOLE

Item	Coefficient
1	.430
2	.500
3	.415
4	.415
5	.700
6	.485
7	.280
8	.600
9	.380
10	.280
11	.505
12	.655
13	.700
14	.725
15	.150
16	.550
17	.205
18	.415
19	.540
20	.780
21	.645
22	.415
23	.805
24	.665
25	.730
26	.560
27	.450
28	.820
29	.645
30	.720

Reliability

The reliability coefficient of the Final Form of the test was calculated by the same methods as were used previously and the results are shown in Table XII. The coefficient of reliability was estimated to be considerably higher than that found for the Preliminary Form and was considered to be satisfactory.

Table XII

COEFFICIENTS OF RELIABILITY OF THE FINAL FORM OF THE TEST IN UNDERSTANDINGS OF PROCESSES

Form	Method	Reliability Coefficient
Final	Split-halves	.809 (corrected)
"	Kuder-Richardson	.717

Relationship Between Scores on Test on Understandings and Intelligence Test Scores

In an endeavour to discover more clearly the nature of the test on understandings, the scores on this test were compared with the scores obtained by the same students on the Otis Test of Mental Ability. The coefficient of correlation was determined by the Pearson Product-Moment

Method with the result shown in Table XIII. This is somewhat higher than the usual 4 = .45 (approx.) found between intelligence test scores and achievement scores at this level. A correlation of this magnitude is classified by most writers as denoting substantial or marked relationship. We may conclude that a fairly strong relationship exists between the two sets of measures. Based upon logical considerations, this may suggest the influence of such factors as reading comprehension or the existence of similarities in the two tests, but the cause-and-effect relationship cannot be determined from the data.

Table XIII

RELATIONSHIP BETWEEN SCORES ON TEST IN UNDERSTANDINGS OF PROCESSES AND OTIS TEST OF MENTAL ABILITY OBTAINED BY 150 NORMAL SCHOOL STUDENTS

$r = .585$

Summary

After considering the foregoing factors, i.e., curricular and "face" validity, discriminatory value, degree of difficulty, objectivity, practicality, etc., it was decided that all the items in the test should be retained and that the test might be considered an acceptable instrument of evaluation to be used in the present study. The Test in Understandings of Basic Processes Involved in the Use of Decimal Fractions will be found in Appendix B.

CHAPTER IV

INVESTIGATION OF THE RELATIONSHIP BETWEEN COMPUTATION AND UNDERSTANDINGS IN THE USE OF DECIMAL FRACTIONS

The Subjects

The subjects selected for the investigation of the relationship between computational ability and understanding of basic processes involved in the use of decimal fractions consisted of students enrolled in the one-year teacher-training course at the Victoria Normal School. All of these students possessed the basic qualifications required for admission into the Normal Schools of the Province of British Columbia, i.e., graduation from high school on the University Entrance Programme (or its equivalent). Many had acquired additional credits extending from first year university standing through to the Bachelor of Arts Degree. While the majority of the students were graduates of the high schools of British Columbia, some had acquired their basic education in other parts of Canada and Europe. The range of I.Q. scores of the group measured by the Otis Test of Mental Ability, extended from 91 to 137, with a standard deviation of 8.7. The ages of the testees ranged from 17 to over 40. While the students formed a diversified group, they were typical of the people who teach school in British Columbia. It may be assumed that, having fulfilled the requirements for entrance into Normal School, all had received basic instruction in the fundamentals of computation with decimal fractions. However, it is likely that there would be considerable variation in the methods by which they had received their instruction.

Administration of the Tests

The tests which have been described in the previous chapters, and which appear in Appendix A and Appendix B, were used for the investigation of relationship. Shortly after the students assembled for the fall term, and before any instruction had taken place, the tests were administered to the entire student-body, consisting of two hundred forty students. Thus, the testees received no benefit from the course of instruction at the Normal School in which the meaning theory is emphasized. The students answered the papers, equipped with the knowledge and understanding acquired from their previous education and experience.

The test in computation with decimal fractions was administered first and a time-limit of thirty minutes was allowed. This proved to be ample time for most students to finish with ease. The test in understanding of processes was given immediately following the first test and a time-limit of twenty minutes provided long enough for the majority to complete all the items. This order of presenting the tests appeared to be the logical one as the intention was to discover the subjects' understanding of the processes employed in the computation. Had the reverse order been used, it is possible that the items in understandings might have provided clues to the solution of some of the questions in computation.

Analysis of the Results

The scoring of the papers, under the supervision of the writer, was completely objective. In the test in computation the scores ranged from 23% to 100% correct with a mean of 74% correct. In the test in understandings the scores were distributed from 18% correct to 100% correct with a mean of 61% correct. There is a wide range in achievement shown in the results of both tests--an outcome which might be anticipated from the range in intelligent quotient scores previously stated.

Two hundred thirty-six papers were selected and the Pearson Product-Moment method was employed to calculate the coefficient of correlation between the results of the test in computation and the test in understandings. The result is shown in Table XIV. The data indicates the existence of a positive relationship of considerable magnitude between the scores obtained on the tests in computation and understandings.

Table XIV

RELATIONSHIP BETWEEN SCORES OBTAINED ON TESTS IN COMPUTATION AND UNDERSTANDING OF PROCESSES INVOLVED IN THE USE OF DECIMAL FRACTIONS BY 236 NORMAL SCHOOL STUDENTS

$$r = .670$$

A study of the scatter diagram reveals the following points of interest:

1. High scores in understandings tend to be accompanied by high scores in computation.

2. Low scores in computation are generally accompanied by low scores in understandings.

However,

3. High scores in computation are found throughout a fairly wide range of scores in understandings,

and

4. There is a fairly wide range in scores in computation accompanying low scores in understandings.

Two examples of extreme scores may be cited. Case A succeeded in answering 86% of the items in computation correctly while possessing only 21% of the understandings of the other test. This individual had a relatively low I. Q. rating. Case B scored 54% in the understandings but only 28% in computation. These cases, however, were isolated and not indicative of the general trend of relationship.

In a further attempt to analyze the results and to secure more data on the inter-relationships of the tests, coefficients of correlation between the results of the tests and intelligence test scores were determined. The relationship between I.Q. and scores on the test in understanding was found to be $r = .547$. (This is reasonably close to the correlation of $r = .585$ referred to in the last chapter.[1]) The relationship between computation and I. Q. was calculated to be $r = .481$. A composite statement of these correlations is given in Table XV.

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1 See p. 50

Table XV

COEFFICIENTS OF CORRELATION BETWEEN TESTS

	I. Q.	Computation	Understandings
I. Q.		.481	.547
Computation	.481		.670
Understandings	.547	.670	

Partial Correlation

The influence of the factor of intelligence, which has a common relationship to the variables of computation and understandings, tends to obscure the true results. The differences among individuals, introduced by the factor of intelligence, can be eliminated by using the method of partial correlation. Using the technique described by Garrett,[2] the net correlation between computation and understandings, with intelligence "partialled out", was calculated. The result obtained was:

$$r = .554$$

[2] Henry E. Garrett, Statistics in Psychology and Education, pp. 378-403. New York: Longmans, Green and Company, 1953.

Summary

The tests in computation and understandings in the field of decimal fractions, were administered to a group of student-teachers. While the testees formed a diversified group, it was assumed that they had a common background in the area covered by the tests. No instruction preceded the testing programme. The results of the investigation indicated the existence of a positive relationship of considerable magnitude between computation and understandings. The coefficient of correlation was found to be $r = .670$. When differences in intelligence had been allowed for, the net correlation was found to be somewhat less than the apparent relationship, i.e. $r = .554$.

CHAPTER V

SUMMARY AND CONCLUSIONS

Summary

Purpose of the study:

This study was undertaken in an attempt to discover what relationship, if any, exists between a subject's ability to perform mechanical computations and his understanding of the inherent mathematical principles, in the area of decimal fractions. Modern theory of arithmetic instruction places great emphasis upon the acquisition of mathematical meanings in the learning process. The writer's purpose in conducting this study was to investigate the validity of the claims set forth by the proponents of the meaning theory and to add some evidence to the slowly accumulating body of knowledge concerning the place of understandings in arithmetic instruction.

Materials of the study:

The topic of decimal fractions was chosen as the area of investigation because of the universality of its content and the essential nature of its material in our society. The subjects chosen for the investigation were student-teachers, because the topic of the investigation has particular significance for teachers and also because Normal School students provided a convenient group for the conduct of the study.

Procedure of the study:

The pursuance of the investigation depended upon the use of suitable testing instruments. As no tests which met the rather rigid requirements of the study could be obtained, it became necessary to undertake the construction and validation of original materials suitable for the specific purpose. The preparation of the tests in computation and understandings became a major phase of the study.

Much preliminary testing took place and the tests were revised several times in an effort to meet the specifications of sound, acceptable testing instruments. Pupils of the grade 7-8-9 level provided the subjects for the establishment of the validity and reliability of the test in computation. The test in understandings was developed using student-teachers as testees.

The internal consistency of test items was determined by using a technique based upon the discriminating power and the degree of difficulty of the items, using the upper and lower thirds of the groups. A further check of internal consistency was made by reference to tables prepared by Flanagan, using the upper and lower twenty-seven per cent of the groups. Reliability was estimated by the split-halves technique and a further estimate was made using the Kuder-Richardson method. These statistical results indicated that the final forms of the tests could be used for the purpose of the investigation with some confidence in their efficiency. The finished tests should have further usefulness beyond the purpose of this study.

Results of the investigation:

The investigation was conducted using student-teachers as subjects. The group tested was diversified in respect to age, background, academic status, and intelligence. All had in common some knowledge of the use of decimal fractions. No instruction or explanation preceded the administration of the tests. The test in computation (Appendix A) was first given and was immediately followed by the test in understandings (Appendix B). Two hundred thirty-five papers were used to estimate the degree of relationship between computational ability and understandings of mathematical principles. The coefficient of correlation was found by the Pearson Product-Moment method to be: $r = .640$. Further correlation coefficients were computed which produced the following data:

1. Understandings - Intelligence: $r = .547$
2. Computation - Intelligence: $r = .481$

The common factor of intelligence was "partialled out" and the net correlation between computation and understandings was found to be: $r = .554$.

Conclusions

The data obtained from the investigation leads to the following inferences and conclusions:

1. There is a positive coefficient of correlation between scores on the test in computation and scores on the test in understandings in decimal fractions. This indicates that there is a tendency for the scores to vary in the same direction. High scores in one tend to accompany high scores in the other, while low scores in one are usually found along with low scores in the other. The size of the correlation coefficient ($r = .640$) is of substantial magnitude.

2. The common factor of intelligence has an influence upon the relationship between the two variables. When intelligence is held constant, the partial coefficient is less than the apparent coefficient, which indicates that relationship is due, in part, to the common dependence of both variables upon the intelligence factor. The net correlation is of marked magnitude ($r = .554$).
3. The magnitude of the relationship between understandings and intelligence is indicative of common elements in both ($r = .547$). However, a high I. Q. is not a guarantee of a high level of understanding.
4. The relationship between intelligence and computational ability is positive but not high ($r = .481$). Computational competence in decimal fractions seems to be possible with a relatively low I. Q. (in terms of the group used in this investigation).
5. While the trend is that increase or decrease in one variable is accompanied by increase or decrease in the other, there is considerable evidence that neither is essential for the other, and that high scores in one do not guarantee high scores in the other.
6. Although it appears, from a study of the scatter diagram, that one who is aware of the mathematical principles involved in the use of decimal fractions has a greater likelihood of success in computation, the suggestion of causal influence must be rejected. It cannot be inferred from the data that the concomitance is an indication that understandings insure better computation, or vice versa.

Suggestions for Further Study

As the study progressed, lack of sufficient research in certain areas became evident. The following points are suggested as fields for further investigation:

1. It immediately became apparent that there is need for a clearly defined statement concerning what constitutes a body of understandings in arithmetic. While volumes have been written on understandings, there seems to be reason for Van Engen's[1] statement that:

> Judged by its crucial importance in determining methods of instruction, curriculum content, and supervisory practices, the precise nature of meaning has received relatively little attention in the educational literature dealing with the outstanding problems of arithmetic in the elementary schools. Failure to make more precise the nature of meaning in arithmetic has resulted in confusion and controversy.

This lack of a specific statement of the nature of understandings presented a difficulty to the writer and points the way to needed studies.

2. After deciding upon the nature of the investigation, it became necessary to secure suitable testing instruments. It became apparent at once that satisfactory tests were not available and would have to be constructed. While there are many good tests in computation with decimal fractions, none could be found that entirely met the specifications demanded by the proposed study. It was found that, while a small beginning has been made in the measurement of understandings, there is need for much more research in this area of evaluation.

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[1] H. Van Engen, "An Analysis of Meaning in Arithmetic. I", The Elementary School Journal, XLIX (February, 1949), p. 321.

3. Having embarked upon the task of constructing a test on understandings, the writer was confronted with the problem posed by lack of adequate and valid criterion measures. Thus, it was necessary to employ internal consistency techniques in validating the test items. It is obvious that there is great need for research in this area.

4. Although much has been written about the desirability of developing mathematical understandings, more studies are needed to reveal the results which accrue from the use of the meaning theory of instruction. More investigations, based upon the evaluation of the outcomes of different methods of instruction, are needed to indicate the results of meaningful instruction.

5. Many more studies, similar to the present one, should be conducted in other areas of arithmetic and at other educational levels to provide data on the relationship of computation and understandings.

Only by diligent application to these relevant problems, can data be accumulated to add to the body of knowledge concerning arithmetic instruction and the place in it of understandings.

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APPENDIX A

TEST IN COMPUTATION WITH DECIMAL FRACTIONS

APPENDIX A

DECIMAL FRACTIONS

Time: 30 minutes

SCORE

SCHOOL ______________________ NAME ______________________

GRADE ______________ DATE ______________

Place answers in spaces at the right.

1. .09 + .06 + .07 — 1. ________

2. .4 x .2 — 2. ________

3. \$1.67 + \$4 + \$ .03 + \$12.00 + \$2.05 — 3. ________

4. \$ .78 x 56 — 4. ________

5. 1000 x 19.62 — 5. ________

6. Which is largest:
   1.01; 101; 11.001; .010; 100.1 — 6. ________

7. .012 x 2.45 — 7. ________

8. $.25\overline{)\ 6.5}$ — 8. ________

9. .001 x 1.01 — 9. ________

10. .001 ÷ 1 — 10. ________

11. $4\overline{)\ .0104}$ — 11. ________

12. Subtract \$15.67 from \$4671 — 12. ________

13. From 9674.196 take 362.8074 — 13. ________

14. 69.7 + 145.962 + .0346 + 1.002 + 18.11

14. ________

15. $.6\overline{)\ 300}$ — 15. ________

16. Write as a decimal fraction:

fifteen hundredths — 16. ________

17. $.6\overline{)\ .0126}$ — 17. ________

18. .7 + .9 — 18. ________

19. Find 2/3 of \$3.15 — 19. ________

20. Write in words: (as common fraction)

.09 — 20. ________

21. Express as a mixed number:

8.031 — 21. ________

22. Find the difference between .044 and .22 — 22. ________

23. Divide to find the value of:

$$\frac{3.69}{12.3}$$

23. ________

24. Express as a common fraction: .0017 — 24. ________

25. Write as a mixed decimal:

six hundred thirty and seventeen thousandths

25. ________

26. Find the average of: (give answer to the) (nearest hundredth)

95; 103; 90.5; $105\frac{3}{4}$; 100

26. ________________

27. Divide 9.02 by 1000 (answer in decimal form)

27. ________________

28. Write in words: (as mixed number)

107.029 28. ________________________________

29. Which is largest:

.1764; .2; .199; .003; .21

29. ________________

30. $45\overline{)\ 98\ }$ (Correct to 2 places of decimals)

30. ________________

31. What will be the cost of $6\frac{1}{2}$ gallons of gasoline at 40.4¢ per gallon? (nearest cent)

31. ________________

32. Express to the nearest hundredth: (as a decimal)

0.106

32. ________________

33. Express as a decimal fraction correct to the nearest hundredth:

6/7

33. ________________

34. Express to the nearest thousandth(as a decimal)

1706.17428

34. ________________

35. \$57.00 ÷ 60¢

35. ________________

APPENDIX B

TEST IN UNDERSTANDING OF PROCESSES WITH DECIMAL FRACTIONS

APPENDIX B

SCORE

UNDERSTANDING OF PROCESSES WITH DECIMAL FRACTIONS

SCHOOL ______________________ NAME ______________________

DATE ______________________

CHOOSE THE MOST SUITABLE ANSWER FOR EACH QUESTION.

1. In addition of mixed decimal fractions it is important to arrange the numbers so that: ______

   A. the last figures of all numbers are in the same column
   B. all figures with the same place value are in the same column
   C. the first figures of all numbers are in the same column
   D. none of these

2. To change a common fraction to a decimal fraction one must know that a common fraction indicates: ______

   A. multiplication
   B. enumeration
   C. addition
   D. division
   E. subtraction

3. Adding a zero to the end of a decimal fraction: ______

   A. makes the value 10 times as much
   B. makes the value 1/10 as much
   C. makes the value 10 more
   D. does not change the value

4. The largest of several decimal fractions will be the one with the largest figure in: ______

   A. tenths place
   B. hundredths place
   C. thousandths place
   D. any place

5. The number: 6.00 has a value of: ______

   A. 6 hundreds
   B. 600 hundreds
   C. 6 hundredths
   D. 600 hundredths

6. If a decimal point is moved two places to the left the number becomes: ______

   A. one-tenth as large
   B. ten times as large
   C. one-hundredth as large
   D. one hundred times as large

7. The number .0170 should be read: ______

   A. seventeen hundredths
   B. One hundred seventy ten-thousandths
   C. one hundred seventy thousandths
   D. seventeen thousandths

8. Changing .645 to .0645: ______

   A. does not change the value
   B. makes value 10 times as much
   C. makes value 1/10 as much
   D. makes value 1/100 as much

9. If the number 42.56 is changed to 42.056, by inserting a zero after the decimal point, the value becomes: ________

   A. unchanged
   B. less
   C. greater
   D. ten times greater
   E. one-tenth as much

10. The value of a decimal fraction is determined by: ________

   A. the size of the first digit after the decimal point
   B. the position of the last digit after the decimal point
   C. the position of the largest digit after the decimal point
   D. the position of the first digit, not including zeros, after the decimal point.

11. Which of the following numbers has the figure "6" in the thousandths place: ________

   A. 4695.5417
   B. 6495.1724
   C. 4325.2163
   D. 4175.6000

12. In the question $\begin{array}{r} 32 \\ .5\overline{)\,16} \end{array}$ the answer is larger than the number divided because: ________

   A. 16 is more than .5
   B. it is the same as multiplying by $\frac{1}{2}$
   C. dividing a number always gives an answer larger than the number
   D. it is the same as finding how many $\frac{1}{2}$'s in 16

13. If a decimal fraction is divided by 1000 the decimal point is moved three places to the left because: ________

   A. the number becomes 1000 times as large
   B. the number is increased by 1000
   C. the number becomes 1/1000 as large
   D. the number is decreased by 1000

14. In the question: $1.6\overline{)\,620.54}$ if the decimal point is moved one place to the right in the divisor and one place to the left in the dividend, the answer will be: ________

   A. one hundred times as great
   B. ten times as great
   C. one hundredth as great
   D. one tenth as great
   E. unchanged

15. When a decimal fraction is changed to a common fraction (not reduced) the denominator will have one zero for: ________

   A. every figure to the right of the decimal point
   B. every figure, except zeros, to the right of the point
   C. every zero to the right of the point
   D. none of these

16. Multiplying a decimal by 1000 moves the decimal point ________

   A. two places to the right
   B. three places to the left
   C. two places to the left
   D. three places to the right

17. In division with decimals the divisor may be made a whole number before dividing because: ________

   A. you can't divide by a decimal
   B. moving the point does not change the value of a number
   C. it is more convenient
   D. the point in the quotient must be directly above the point in the dividend
   E. the value of a fraction is unchanged when both terms are multiplied by the same quantity.

18. The measurement 1.050 inches is accurate to the nearest: ________

A. tenth of an inch
B. hundredth of an inch
C. thousandth of an inch
D. ten thousandth of an inch

19. Moving a decimal point two places to the right has the same effect as: ________

A. multiplying the number by 10
B. multiplying the number by 1000
C. dividing the number by 100
D. none of these

20. In the number: 5 <u>5</u> 5.<u>5</u> 5 (a) (b) ________

A. Digit (a) is 100 times digit (b)
B. Digit (a) is 10 times digit (b)
C. Digit (a) is 1/10 of digit (b)
D. Digit (a) is 1/100 of digit (b)

21. The number .6925 has a value of about: ________

A. .69 hundredths
B. 2 hundredths
C. 9 hundredths
D. 69 hundredths
E. 692 hundredths

22. If a number is to be expressed accurately to the nearest hundredth it must be found to at least: ________

A. one place after the decimal point
B. two places after the decimal point
C. three places after the decimal point
D. four places after the decimal point

23. In the question: $1.25\overline{)\ 642.3}$ if the decimal point were located <u>one place to the right</u> in both numbers the answer would be: ________

A. ten times as large
B. one-tenth as large
C. one hundred times as large
D. one-hundredth as large
E. unchanged

24. If no zeros are added to the dividend, the answer to the question: $4.2\overline{)\ 69.735}$ will be a two-place decimal because: ________

A. thousandths divided by tenths is hundredths
B. there are two figures in the divisor
C. tenths times tenths is hundredths
D. there are two places before the point in the dividend

25. In the question: 6.42 x 15.7 if the decimal point were located <u>one place to the right</u> in the first number and <u>two places to the left</u> in the second number the answer would be: ________

A. ten times as large
B. one tenth as large
C. one hundred times as large
D. one hundredth as large

26. In the question: 6.92 x 74.3 = 514.156 the decimal point is located thus in the answer because: ________

A. one and two are three
B. hundredths times tenths is thousandths
C. tens times hundreds is thousands
D. there are three places to the left of the point in the numbers multiplied.

27. A "decimal" is a fraction with an unwritten, but understood, denominator which will always be: ________

A. one
B. ten
C. any multiple of ten
D. any power of ten
E. none of these

28. The number: 2.134 has a value of about: ________

A. 1 tenth
B. 13 tenths
C. 21 tenths
D. 213 tenths
E. 2.1 tenths

29. To change a fraction, such as 3/4, to a two-place decimal we divide the numerator by the denominator and we must think of the numerator as: ________

A. 3 hundreds
B. 3 hundredths
C. 300 hundredths
D. 30 hundredths
E. none of these

30. The sum of: 16.17", 459.4", 142.167", 2.130" will be accurate to the nearest: ________

A. inch
B. tenth inch
C. hundredth inch
D. thousandth inch