AN EXPERIMENTAL STUDY TO DETERMINE THE EFFECTIVENESS
OF GROUP INSTRUCTION USE OF CERTAIN MANIPULATIVE
MATERIALS IN CONTRIBUTING TO AN UNDERSTANDING
OF DECIMAL CONCEPTS

by

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ABSTRACT

AN EXPERIMENTAL STUDY TO DETERMINE THE EFFECTIVENESS OF GROUP INSTRUCTION USE OF CERTAIN MANIPULATIVE MATERIALS IN CONTRIBUTING TO AN UNDERSTANDING OF DECIMAL CONCEPTS

The increasing emphasis on teaching arithmetic meaningfully intensifies the search for materials of instruction which can effectively communicate arithmetical understandings to children. Though manipulative aids are widely approved as effective teaching media for achieving this purpose, most of the endorsements are subjective opinions rather than objective evaluations based on experimentation.

This study is an attempt to determine the effectiveness of group instruction use of certain manipulative aids in teaching decimal fraction concepts to Grade VII pupils. The effectiveness was determined by comparing the achievement of two unselected groups, randomly assigned, on a test of understanding of the processes involved in decimal fractions.

The two groups were given teaching treatments identical except in so far as the materials of instruction were concerned. One group used manipulative aids; the other used static representations of these aids. These materials were intended to differ only with respect to the characteristic of manipulability. Since manipulability of concepts is the most essential property of manipulative aids, it was
isolated as the experimental variable.

Because the groups were randomly assigned, analysis of covariance was selected to control statistically the initial differences between groups in the four variables considered likely to influence achievement on the criterion test: initial understanding of the processes involved in decimal fractions, computational ability in decimal fractions, mental ability, and reading ability.

The data obtained from the investigation were analyzed and the following conclusions reached.

1. The pupils taught by means of group instruction with the manipulative aids used in this investigation did not acquire a significantly better understanding of decimal fractions than did the pupils taught with static representations of these aids. In other words, the manipulation of the concepts, performed by using the manipulative aids in group demonstrations, was not effective in contributing to the pupils' understanding of these concepts.

2. A study of the correlations for both treatment groups between achievement on the criterion variable and achievement on each of the independent variables indicates that the manipulative aids proved to be neither more nor less effective than the static representations as media for conveying an understanding of decimal fractions to pupils of any particular ability in the areas represented by the independent variables.

3. It must not be inferred that any generalization concerning the effectiveness of these specific materials of instruction, used
exclusively by the teacher for group demonstration purposes, would be applicable also to similar materials if they were used in a teaching procedure in which the pupils themselves participated individually in the manipulative activity.

It must not be inferred that any generalization concerning the effectiveness of these specific materials of instruction, which were used in a brief teaching assignment devoted exclusively to the rationalization of processes, would be applicable also to the same materials if they were used in a teaching assignment of longer duration, and/or a teaching assignment in which the emphasis on the WHY of the processes was taught concurrently with, or preceded, the emphasis on the HOW of the processes.

5. Independently of treatment groups, the achievement on the initial test of understanding of the processes involved in decimal fractions was the variable most predictive of achievement on the final test of understanding. Computational ability in decimal fractions and mental ability each shared approximately one-half the predictive capacity of the initial test of understanding. Reading ability was a negligible predictor of achievement on the final test of understanding.
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Department of Education

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These contributions are acknowledged with sincere appreciation.
CHAPTER I

INTRODUCTION

I. THE PURPOSE OF THE STUDY

The purpose of this study is to determine experimentally the effectiveness of the group instruction use of certain manipulative materials in contributing to an understanding of particular decimal concepts.

Stated in other words, the purpose is to ascertain whether there is any significant difference in the achievement on a criterion measure of two unselected groups of Grade VII pupils. One group was taught by group demonstration with the use of instructional materials which are concrete and movable. The other group was taught by group demonstration with the use of instructional materials which are merely static representations of the manipulative devices.

The study seeks to discover whether the characteristic of manipulability actually contributes to the pupils' understanding of decimal fractions when the particular materials are used in a prescribed manner.

The primary concern of this investigation is with the practicality, not the essentiality, of the meaningful approach to teaching arithmetic. No matter how valid the Meaning Theory
may be, its worth as a trend in arithmetic pedagogy depends upon the discovery of ways of transmitting theory into effective and economical teaching practices.

During the past several years manipulative aids have been acclaimed by many competent authorities as effective means of making arithmetic meaningful. This study is an examination of one small area of the foundation for these claims.

Its objective is to add something to the search for materials of instruction that facilitate the communication of arithmetical meanings to children. The interest in teaching materials, it may be emphasized, is only a means to the end of securing better learning.

II. DESCRIPTION OF THE PROBLEM

This study is an attempt to determine the effectiveness of certain manipulative materials in contributing to the pupils' understanding of specific decimal fraction concepts when the manipulative materials are used exclusively by the teacher in class demonstrations. The manipulative materials are designated: (1) place value charts, (2) place value cards, (3) wall rule with movable indicator, (4) flannel board.

The effectiveness of these materials is determined by comparing the achievement on a selected criterion measure of an experimental group composed of 59 subjects in two classes located in different schools, and a control group composed of 88 subjects.
in three classes also located in different schools.

The criterion measure, administered at the end of the experiment, is Farquhar's Test of the Understanding of Processes with Decimal Fractions, which formed a minor part of an unpublished Master of Arts Thesis in Education. Since no way has yet been devised to identify the composition of a body of understandings in arithmetic, Farquhar assumed that "a person's understanding of a process may be revealed by his ability to rationalize the procedure and his insight may become apparent by his grasp of the 'why' behind the performance of the algorism." The specific decimal fraction concepts represented in the test were, according to the author, arbitrarily chosen.

The participating classes were selected in accordance with definite criteria. After being matched on the basis of size, they were assigned at random to the experimental or control groups.

Suitable tests were administered at the beginning of the experiment to determine the status of the classes in four relevant areas: initial understanding of concepts of decimal fractions, computational ability in decimal fractions, mental ability, and reading ability. By applying analysis of covariance to the results, the initial differences between the groups in these areas were held constant.

Both groups were subjected to teaching treatments intended to be identical in all details except insofar as the instructional materials are concerned. The experimental group was taught with
specified manipulative aids; the control group was taught with static representations of these aids commonly referred to as visualization materials.

Further, these two types of instructional materials are intended to differ only in their capacity to represent concepts in movable forms. Manipulability, the basic characteristic of manipulative aids, constitutes the experimental variable.

The extent to which the intended identity in teaching methods and materials used by the two treatment groups actually exists may be judged by examining the lessons contained in Appendix B, and the representations of the materials shown in the Figures on pages 39 to 42, inclusive.

The teaching programme was designed to impose rigid controls in the conduct of the experiment, while the analysis of covariance technique was selected to impose statistical controls over the concomitant influences affecting the pupils' responses to the criterion measure.

The imposition of these controls enables any difference between the treatment groups in achievement on the criterion measure to be attributed to the experimental variable.

The hypothesis to be tested is that there is no difference in the performance of the two groups on the criterion measure which is attributable to the treatments involved.
III. DELIMITATION OF THE PROBLEM

Date and locale of the experiment. The administration of the experiment took place in May, 1957, and involved Grade VII classes located in five elementary schools in School District No. 36 (Surrey).

Instructional materials. Subsection II of Chapter III contains a description of all the teaching aids employed in this study.

The judging of the identity of the aids used by the two treatment groups, which was suggested on the previous page, may be facilitated by referring to Table II on page 36. This table shows the corresponding aids employed by the two groups for the teaching of the particular objectives in each lesson. The extent of the identity in the corresponding aids used for each objective may then be judged by referring to the Figures on pages 39 to 42, inclusive.

No instructional materials of a special nature were used for those objectives which, though essential to the continuity of the lessons, did not refer to concepts included in the Farquhar test.

Teaching programme. Subsection III of Chapter III contains a description of the lessons employed in this study.

Eleven lessons comprise the total teaching programme. Of these, eight were devoted to the presentation of material new to the experiment, while three were reserved to review material
previously taught during the experiment. The schedule of lessons is contained in Appendix A.

The duration of each lesson is one hour. This includes thirty minutes of group instruction, fifteen to twenty minutes of seatwork, and ten to fifteen minutes for the marking of this seatwork by the class.

The entire set of lessons is contained in Appendix B. The lessons for the experimental group are on pink sheets, while those for the control group are on yellow sheets.

**Testing programme.** Subsection VII of Chapter III contains a brief outline of the testing programme, while subsections II to V, inclusive, of Chapter IV contain a complete description and evaluation of the four tests used.

These tests were administered at the beginning of experiment to measure the independent or concomitant variables in the four areas previously referred to. The results obtained were, obviously, unaffected by the treatments.

The Farquhar test, one of the four, performed a dual function in the study. It was administered at the beginning of the experiment to measure one of the independent variables, and, in addition, it was readministered at the close of the experiment to measure the dependent or criterion variable.
IV. JUSTIFICATION OF THE PROBLEM

The desirability of teaching arithmetic according to the Meaning Theory is now widely accepted in educational psychology. The best ways to teach meanings, however, are still a matter of uncertainty.

To the question "How are meanings best developed?", Brownell ventured the following statement as part of his answer in an article written ten years ago:

The problem (or group of problems) epitomized in the question above arises from the recognition of the fact that concepts, generalizations, etc., if they are to be of real use, must be more than pat verbalizations. They must be ways of thinking meaningfully about arithmetic relationships. As yet we have little exact knowledge with regard to ways and means for developing those meaningful thought processes. But we must find out.

The problem of finding ways and means for developing meaningful thought processes in arithmetic is almost as unsolved, and certainly as urgent, today as it was then.

One noticeable development that has taken place since the Meaning Theory gained prominence in educational psychology has been the increased importance placed on the use of teaching aids supplementary to the text. Among these aids, manipulative materials rank high in the approval of those who espouse the Meaning Theory.

In an article written in 1950, Buswell interpreted the Meaning Theory to include the judicious use of manipulative devices. In it he stated:
We are only beginning to realize the important place that manipulative aids can play in learning. We have thought of them, usually, as devices to help pupils to get their answers. A more important use is to show the thinking which lies back of the answers that they got. Used with intelligence and insight, manipulative aids may contribute much to superior thinking.\(^6\)

In an article written in 1952 on the subject "A Few Recommendations for the Improvement of the Teaching of Mathematics", Lazar said:

Let an abacus, or its equivalent, serve in the mathematics classroom in the same role as the demonstration models do in the science room - a constant source for the discovery of new laws and for the confirmation of hunches.\(^7\)

In an article written in 1953 on the subject "How to Make Arithmetic Meaningful in the Junior High School", Stein commented:

...arithmetic can be made meaningful in the Junior High School by utilizing concrete situations and by moving gradually from the concrete to the abstract and symbolic. ... It is just as reasonable for Junior High School pupils to use markers, pegs, or an abacus to gain insight into the meanings of the operations as it is for them to study plants and animals objectively rather than from pictures in a book. Junior High School teachers should not consider it beneath their dignity to utilize concrete materials to develop abstract processes.\(^8\)

Current educational periodicals show that a wide variety of manipulative aids are being used in the classrooms for the teaching of arithmetic.\(^9\) Judging by the reports contained in these periodicals, it appears that these aids are being used mainly to develop in pupils an understanding of arithmetical processes, although in some cases they are being used merely for computation purposes.

The justification of this study lies in the fact that while manipulative aids are becoming increasingly prominent in arithmetic
teaching, and obviously for the purpose of giving insight into number operations, there is practically no experimental evidence to prove that these highly recommended instruments of instruction are as effective for this purpose as they are claimed to be.

The need for research in this area is all the more important because of certain opinions which suggest that some manipulative aids are used injudiciously. One such view is that expressed in 1953 by Van Engen, who is an advocate of manipulative activity in arithmetic teaching:

Many of the manipulatory activities now "going the rounds" in the world of mathematics instruction do not include those manipulatory activities which develop the concept, or concepts, for which they were intended.10

Previous studies which have in any way involved the use of manipulative aids have not been primarily concerned with investigating their effectiveness in contributing to teaching objectives. Instead, the purpose has been to investigate certain aspects of teaching, such as the computational and problem solving effects of teaching with varying degrees of emphasis on meaning, in which manipulative materials have been included only incidentally.

As a consequence, the findings of even the most relevant of these studies are not very helpful in evaluating the effectiveness of these aids in any phase of arithmetic instruction.

The present study, which is designed exclusively to determine the effectiveness of certain manipulative aids when used in a prescribed manner and for a definite purpose, is intended to provide an answer to one small aspect of the still pertinent question posed
ten years ago by Brownell: "How are meanings best developed?"

V. FACTORS DETERMINING THE CHOICE OF SUBJECT MATTER

One factor which determined the choice of decimal fractions was the availability of a test designed to measure meanings and understandings. The difficulty of evaluating the development of understandings, which is admitted in the literature, is reflected in the scarcity of suitable tests to measure this type of learning. The Farquhar test is considered by the present investigator to be the most suitable of the tests which purport to measure understanding in arithmetic. This consideration, therefore, was an important factor determining the choice of subject matter.

Apart from the influence which the suitability of Farquhar's test had upon the choice of decimal fractions as the material for study in the present investigation, there was one other consideration which reinforced the wisdom of this choice. Decimal fractions is a teaching topic which offers many opportunities for the effective use of both manipulative and visual materials to clarify and extend meanings. References contained in "Teachers' Guide for Thinking with Numbers" indicate pages in the text where manipulative and visual materials may be used. In comparison to other topics of instruction in the text, the chapter on decimal fractions contains many concepts for the teaching of which these aids are recommended. Decimal fractions, therefore, seems to be a curriculum
area where the materials of instruction used in this investigation would be subjected to a fair test.

VI. FACTORS DETERMINING THE GRADE PLACEMENT
OF THE EXPERIMENT

The instructional programme in this study consists of reteaching decimal fractions with exclusive emphasis on conveying to the pupils an understanding of the concepts involved.

The factor which determines the approximate level at which a study of this nature should be conducted is the consensus of opinion that the junior high school years are an appropriate time to reteach by using a meaningful approach those concepts previously taught in the elementary grades, often before pupils are mature enough to understand their significance.

One such opinion is that expressed by Morton who, after referring to the desirability of reteaching arithmetical concepts in the junior high school grades, states:

... there should be a carefully planned reteaching program covering what has previously been taught. The term "reteaching" means more than a mere review. It means teaching again, at a higher and more mature level, and more rapidly, what has been taught before.12

A similar opinion is held by Stein, who writes the following in an article which emphasizes the desirability of introducing a deliberate effort to teach arithmetic meaningfully in the junior high school grades:
... the junior high school teacher, by using a meaningful approach, can help students to improve their computational skill and to orient their thinking about arithmetic processes by (a) clarifying anew the nature of the number system and (b) teaching the rationale of the arithmetic processes as a basis for review and practice.13

The factor which determines the exact level at which this study should be conducted is the prevailing opinion that Grade VII is the most appropriate stage to reteach the concepts of decimal fractions with full emphasis on a meaningful approach.

For example, Morton advises in another work: "In general, it should not be necessary to reteach decimals in Grade VIII".14 On this opinion there is general agreement to the extent of saying that Grade VII is the last grade at which it should be necessary to reteach the entire field of decimal fractions to all the pupils.

To facilitate further the effectiveness of the lessons, the experiment was conducted as late as possible in the school year without encountering the usual end-of-the-term classroom interruptions. This ensured that all classes had the maximum opportunity to benefit from normal teaching procedures before being confronted with the concentrated teaching for understanding which took place during the experiment.

VII. LIMITATIONS OF THE STUDY

Before using the evidence provided by this study for formulating generalizations respecting the contribution which these
particular manipulative aids make to the development of meanings and understandings in arithmetic instruction, the limitations of the study must be borne in mind.

1. The manipulation of the materials was performed exclusively by the teacher as group demonstrations before the class.

Most advocates of the use of manipulative aids would insist that their maximum effectiveness in contributing to the pupils' understanding of the concepts taught would depend upon involving the pupils individually in the acts of manipulation.

A considerable amount of recently published literature emphasizes the relevance of mental activity to direct motor reaction. This suggests that manipulative aids may not be as effective in conveying the meaning of a concept when the learner's activity is confined to observing another person perform the manipulation. A review and discussion of this literature is contained in Chapter III.

Because of the importance of this point of view, it would be a fallacy to formulate from the evidence presented in this study any generalization concerning the effectiveness of these particular aids in situations where the pupils individually participated in the manipulation.

2. The manipulative materials were used in a limited number of lessons which were devoted exclusively to teaching the rationalization of processes, after the method of performing the processes had been taught.
As with the previous limitation, it would be a fallacy to formulate from the evidence presented in this study any generalization concerning the effectiveness of these manipulative aids in situations where the period of instruction is of longer duration and where it permits emphasis on the rationalization of the concept to be interpolated with emphasis on the actual performance of the algorism appropriate to the concept.

This limitation assumes some importance in view of the so-called HOW-WHY versus WHY-HOW controversy. This involves the question of whether the teaching of HOW the algorism is performed should precede or succeed the teaching of the WHY behind the performance of the algorism.

The present study follows the HOW-WHY sequence. The effectiveness of the aids used in this experiment may well have been different either if the sequence had been reversed or if the rationalization had been presented concurrently with the drill performed in teaching the algorism.

Further reference to this controversy is found in Chapter III, which contains also a statement explaining why these two limitations were imposed on the study.

VIII. ORGANIZATION OF THE REMAINDER OF THE THESIS

Chapter II is composed of two subsections. The first deals with the trends in arithmetic pedagogy leading to the present
popularity of the Meaning Theory. The search for materials of instruction which effectively communicate arithmetic meanings is obviously a worthwhile pursuit only if the need to make arithmetic meaningful is considered important. The second subsection indicates the research involving manipulative materials which has already been undertaken.

Chapter III deals with the planning, organization, and administration of the experiment. This account serves principally to show the extent to which the teaching treatments are the same for both the experimental and control groups, except with respect to the experimental variable.

In view of the fact that the purpose of the experiment is to discover whether the characteristic of manipulability of certain teaching aids actually contributes to the pupils' understanding of decimal fractions, and thereby to determine the effectiveness of these aids in that regard, it was necessary to ensure that this characteristic would emerge as the one experimental variable. In other words, the objective of this chapter is to provide assurance that, as far as the treatments themselves are concerned, any difference in the performance of the two groups on the criterion measure may be attributed to this variable.

The purpose of Chapter IV is to extend the objective of the previous chapter in order to provide assurance that any difference in the performance of the two groups on the criterion measure may first of all be attributed to the treatments involved, rather than
to any concomitant influences. Unmatched initial differences in four areas of capacity and achievement are considered to comprise the total of these influences.

The first subsection of Chapter IV presents an overview of the analysis of covariance technique which was employed to control these influences statistically. Subsequent subsections of the chapter, which contain a description and analysis of the various tests, are intended to serve as bases for evaluating the adequacy of the derived raw data which was subjected to the statistical analysis.

Chapter V is devoted to this statistical analysis of the data. Several steps are involved in the entire analysis. These steps may be grouped into major categories. The first category includes making an analysis of variance of the criterion variable and each of the independent variables. The second category includes examining the nature of the correlations among the variables to ascertain whether the analysis of covariance will increase appreciably the test of significance. Since the nature of these correlations indicate that it will, the analysis was continued. The third category includes calculating the sums of squares of residuals and subjecting these residuals to an analysis of covariance, in which the F value is obtained and the final test of significance applied. The fourth category of steps involved in the entire analysis includes ascertaining that the assumptions underlying the application of the analysis of covariance have been satisfied.
Finally, the last subsection of Chapter V contains a brief account of the manner in which the raw data obtained in this experiment was processed through the electronic binary computer to result in the automatic performance, within approximately five minutes, of all the essential calculations relevant to the analysis of covariance technique.

Chapter VI contains a summary of the experiment, the conclusions reached on the basis of the statistical evidence, and some suggestions for further study.

2 Ibid, p. 7.

3 Infra, p. 48

4 Infra, p. 54


9 For example, the November and December, 1956, issues of "The Arithmetic Teacher" contain four accounts of the classroom use of various manipulative aids.


13 Stein, op. cit., p. 680

CHAPTER II

REVIEW OF RELATED LITERATURE

Two purposes underlie the review of the literature relevant to this study. The first purpose is to show the background of the problem by tracing the developments in arithmetic psychology which have led to the present emphasis on meaning and understanding in the teaching of arithmetic. The second purpose is to reveal the exact research which has already been performed in connection with the use of manipulative materials in the teaching of this subject.

I. CHANGING CONCEPTS IN ARITHMETIC PSYCHOLOGY

The present experiment is an effort to determine the effectiveness of group instruction use of certain manipulative aids in contributing to an understanding of decimal fraction concepts. Confined to specific manipulative aids used in specified situations, this study deals with only a small portion of one avenue in the search for practical materials which will be helpful to teachers in making arithmetic meaningful to their pupils.

To be fruitful this search in general, as well as all parts of it, must be motivated by a knowledge of the trends in arithmetic psychology leading to the present emphasis on meaning,
and also an awareness of why each successive stage has yielded to another in the process of this development. This section of the review deals briefly with these two matters.

Compendiums of opinion found in various yearbooks and other publications during the past quarter of a century reflect the changing concepts in arithmetic psychology. During this period three stages are evident, though they are not necessarily consecutive.

One stage is that marked by the popularity in teaching practices of the drill theory. In the 1930 Yearbook of the National Society for the Study of Education, a chapter written by F. B. Knight proposed methods of teaching arithmetic which clearly shows the application of the prevailing stimulus-response psychology. The exclusive purpose of the methods suggested by this author is to present number stimuli repetitively to the pupils in order to facilitate their ability to make correct responses. Drill was considered to be the prime factor in teaching; and the accumulation of a repertoire of specific responses was believed to be the major end of learning. There is no question that this was the prevailing psychology for many years prior to 1930.

A second stage is that marked by the popularity of the Incidental Learning Theory in the psychology, if not in the practice, of teaching arithmetic. In the 1935 Yearbook of the National Council of Teachers of Mathematics there is a chapter entitled "The New Psychology of Learning" by R. H. Wheeler. The Gestalt psychology was an important influence upon the Incidental Learning
Theory which Wheeler sought to implement in his recommended teaching procedures. He wrote: "The whole purpose of arithmetic is to discover relationships and to be able to reason with numbers."  

This desirable principle, which shows the application of the Gestalt psychology, had the unfortunate consequence of leading to the Incidental Learning Theory and to the implication that it was necessary to forget drill and to concentrate instead on projects designed to simulate functional situations. Wheeler expressed the idea thus: "Do not try to teach arithmetic; teach discovery, life, nature, through arithmetic". The proponents of this theory believed that if a situation involving quantity happened to arise during a project the child would be motivated to grasp, and then to use, the number ideas.

A third stage is that marked by the popularity, at least in educational literature, of the Meaning Theory. It is interesting to note that the first articulate presentation of the Meaning Theory is also contained in the same Yearbook in a chapter entitled "Psychological Considerations in the Learning and Teaching of Arithmetic" by W. A. Brownell.

Brownell's criticism of the Drill Theory is implicit in these words:

The teacher need give little time to instructing the pupils in the meaning of what he is learning; the ideas and skills involved are either so simple as to be obvious even to the beginner, or else they are so abstruse as to suggest the postponement of explanations until the child is older and is better able to grasp the meaning.
His criticism of the Incidental Learning Theory is explicit in these words:

Incidental learning, whether through "units" or through unrestricted experience, is slow and time consuming. ... Such arithmetic ability as may be developed in these circumstances is apt to be fragmentary, superficial, and mechanical. 7

Brownell's own position is between these two extremes. The Meaning Theory stands in marked contrast to the theory which placed such reliance on the drill of isolated number facts. At the same time it stands opposed to the feasibility of giving up all organized learning experience in arithmetic because a prevalent method had been formalistic and mechanical. In his own words Brownell explains that position:

This name (the Meaning Theory) is selected for the reason that, more than any other, this theory makes meaning - the fact that children shall see sense in what they learn - the central issue in arithmetic instruction. ... Within the "meaning" theory the virtues of drill are frankly recognized. There is no hesitation to recommend drill when those virtues are the ones needed in instruction. 9

In the 1941 Yearbook of the National Council of Teachers of Mathematics, published only six years after the Yearbook which contained the two chapters containing the extremely different approaches to the teaching of arithmetic, there is unmistakable evidence that the Meaning Theory was growing in favor. A chapter by T. R. McConnell reaffirmed the place of organization in learning and the concept that learning arithmetic is a meaningful process. 9

By the time the 1951 Yearbook of the National Society for the Study of Education was published (an issue devoted to the
teaching of arithmetic) the Meaning Theory was so generally accepted by educationists that Horn wrote: "They (the members of the Yearbook Committee) favor the meaning theory, involving the active processes on the part of the pupils of discovering relationships, of utilizing concrete experiences, and of generalization".\(^\text{10}\)

A final authoritative statement by Dawson and Ruddell in 1955 brings the development up to date:

Evidence supporting the meaning theory approach to arithmetic is not complete but it is impressive when it is noted that no such evidence is being accumulated to support other theories of instruction.\(^\text{11}\)

Therefore, the place has been reached where the primary question no longer is: "Should we teach meanings?". The important issues now, certainly from the standpoint of research, are suggested by the questions: "What constitutes the basic arithmetic understandings?" and "What are the most effective materials for, and methods of, instructing pupils in these understandings?".

The fact that the issues suggested by these two questions are not new is indicated by Brownell in an article written almost twenty years after his first presentation of the Meaning Theory:

It is not too much to say that one of the major developments in the past twenty years or so has been the attempt to discover just what this concept of meaningful learning implies for the arithmetic program. One aspect of the development has been the effort to identify the meanings — ideas, principles, relationships, generalizations — that are essential to arithmetic learning. Another aspect of the movement toward meaningful learning is revealed in the search for more effective learning materials and methods of instruction.\(^\text{12}\)

The growing acceptance of the Meaning Theory, which has introduced more serious attempts to implement the theory of meaningful
learning into the practice of meaningful teaching, is the circum-
stance which makes these two questions of major current importance.

The findings of the present study provide some information,
positive or negative, with respect to the general area of
investigation suggested by part of the second question: "What are
the most effective materials for instructing pupils in arithmetic
understandings?"

Since this experiment is an effort to determine the effective-
ness of a specific group of these materials, namely, those which
are manipulative in character, the next section of this review
contains an account of the reported research which in any way
involves the use of manipulative materials in the teaching of arith-
metic.

II. REPORTED RESEARCH INVOLVING THE USE OF MANIPULATIVE
MATERIALS IN THE TEACHING OF ARITHMETIC

The 1951 Yearbook of the National Society for the Study of
Education contains a chapter entitled, "Proposals for Research on
Problems of Teaching and of Learning in Arithmetic". Foster E.
Grossnickle's contribution to this chapter contains a proposal for
research dealing with the use of manipulative materials in the
teaching of arithmetic. Research in this area was represented at
that time to be of urgent importance.

Since that time, however, the use of manipulative materials
has been subjected to extremely little experimentation.

One experiment somewhat related was reported by Dawson and Ruddell. The purpose was:

... to compare the relative effectiveness of common textbook practices in the introduction of the division of whole numbers with an experimental procedure based on a subtractive approach and a greatly expanded use of visualization devices.

One of the several questions to which answers was sought was:

Will achievement be affected adversely if practice through object manipulation and visualization of process replaces much of the paper and pencil drill?

The experimental group used counting discs, spool boards, and place value charts. Dawson and Ruddell stated:

The data may be interpreted to advocate a teaching procedure which utilized manipulation of representative materials. Higher achievement, greater retention, and an increased ability to solve examples in a new situation were found in the experimental group which devoted time to the development of meanings, principles, and generalizations, through the use of manipulative materials and visualization materials.

Another study also somewhat related was reported by Martha Norman. The purpose was:

... to investigate the effects of three methods of teaching certain basic division facts to third grade children. The three teaching methods were named the textbook, the conventional, and the developmental. Each method was designed to vary in degree of emphasis on meaning.

The findings of this study are relevant to the present investigation insofar as the developmental method, which possessed the greatest degree of emphasis on meaning, involved the use of such
manipulative materials as the number line, counters, and number charts. As the names suggest, the other methods involved the use of various non-manipulative materials.

Data obtained from one test, which was used as a pre-test, an immediate recall test, and a delayed recall test, were analyzed to compare the effects of the teaching methods used in the 8 forty minute lessons which comprised the instructional programme. Provision was contained in the test to measure pupil achievement in both facts taught and facts not taught.

Among the various conclusions reached, those which are of interest in the present study may be summarized as follows. First, there are no significant differences among the three teaching method groups in the immediate recall of taught facts. Second, in the delayed recall of taught facts there is a difference, significant between the .05 and .01 levels, among the three teaching method groups. That is, while the developmental and conventional methods are each more effective than the textbook method in the delayed recall of taught facts, there is no significant difference between the effectiveness of the first two mentioned methods.

On the basis of all the conclusions reached, this final statement is reported:

This finding implies that conventional procedures may be more effective in ordinary classroom situations when teachers are given specific directions and when pupils are motivated to learn. Developmental procedures profitably may be used particularly in the early stages of presenting division to third grade children. However, research is needed to refine the principles and teaching procedures which are outgrowths of the meaning theory.19
Several other studies reported in the literature involve the use of manipulative materials in one way or another. Except in minor details, such as grade level, duration of the experiment, number of treatment groups, and so forth, these studies are similar to the ones which have just been discussed. It must be noted that this area of experimentation is not entirely relevant to the present study. Important differences exist.

The experimental purpose is the first way in which the two studies described differ from the present one. These two studies, which parallel each other very closely, were designed for the purpose of testing the role of meaning in teaching to attain certain objectives. This was done by comparing a meaningful method with a control method either in which recognizably conventional procedures were followed or in which drill exercises were emphasized. Manipulative materials were only incidentally introduced into the experimental method because they were regarded by the authors to be the most effective materials by which to teach meaningfully the specified objectives.

The present study, on the other hand, was designed for the purpose of testing, deliberately and exclusively, the effectiveness of the role of manipulative aids in teaching to attain certain objectives.

The teaching objectives is the second way in which the two studies described differ from the present one. The objective of the teaching in the former is the pupils' attainment of skill in
certain arithmetical operations, while the objective of the
teaching in the present study is the pupils' attainment of an
understanding of certain arithmetical concepts.

It is seen, therefore, that there are important differences
between this experiment and previous experiments which have in
any way involved the use of manipulative aids in the teaching
of arithmetic.

Previous experiments of that nature have been mainly
concerned with the investigation of the computational and problem
solving effects of teaching with varying degrees of emphasis on
meaning. Manipulative materials have been included quite
incidentally in the teaching aids used in these experiments, and,
as a consequence, the problem of determining their effectiveness
in contributing to the teaching objectives has been in each case
a very subordinate part of the total investigation.

The growing acceptance of the Meaning Theory should produce
in the future more experimentation designed exclusively to
ascertain the effectiveness of teaching materials, especially
manipulative aids, in conveying to pupils an understanding of
arithmetical concepts.
FOOTNOTES


3 Ibid, p. 247

4 Ibid, p. 243


6 Ibid, p. 2

7 Ibid, p. 17

8 Ibid, p. 19


15 Ibid, p. 6

16 Ibid

17 Ibid, p. 8


19 Ibid
CHAPTER III

THE PLAN AND ADMINISTRATION OF THE EXPERIMENT

This chapter contains an account of the planning involved in organizing the experiment, a description of the administrative arrangements undertaken in connection with its performance, and, finally, a statement of the limitations, and reasons for the limitations, imposed upon the study.

I. STEPS IN PLANNING THE EXPERIMENT

The first step in planning the experiment was to select a suitable teaching unit. It was considered essential to confine the subject matter to one homogeneous unit, and thereby to restrict the duration of the teaching assignment, in order to maintain adequate controls in the performance of the experiment and to reduce to a minimum the number of materials of instruction required.

Subsection V of Chapter I contains an account of the two factors which determined the choice of decimal fractions as the unit on which to test the effectiveness of the particular manipulative aids used in this study.

The second step in planning the experiment was to decide on the grade level most appropriate for the purpose of communicating
to pupils an understanding of decimal fractions. Subsection VI of Chapter I contains an account of the factors which determined the grade placement of the experiment.

The third step in planning the experiment was to delineate the lesson areas. As a basis for establishing a plan, Farquhar's test was analyzed question by question. Lesson topics were then formulated, first, in accordance with the existence of Farquhar questions to evaluate each lesson, and, second, in accordance with the teaching practices currently recommended in the published materials which were consulted in preparation for writing the lessons. These published materials, which are listed in the first section of the bibliography, include the most recent arithmetic texts, books dealing with the teaching of arithmetic, and brochures advertising commercially prepared materials of instruction.

The plan of the lessons which evolved from this procedure is presented in Table I.

The fourth step in planning the experiment was to subdivide each lesson area into component lesson objectives. Appendix A contains a summary of the lesson objectives in the entire set of eight lessons. In most cases each objective formed a separate entity within the lesson area, although in a few cases more than one objective could be grouped because they lent themselves to common teaching treatment.

For the teaching of each objective, or group of objectives, a time allotment was decided upon. The amount of time assigned
<table>
<thead>
<tr>
<th>Lesson Number</th>
<th>Farquhar Question Number</th>
<th>Lesson Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>nil</td>
<td>Introductory Lesson</td>
</tr>
<tr>
<td>II</td>
<td>4, 11, 20</td>
<td>Identification and Meaning of Place Names in mixed decimal fractions</td>
</tr>
<tr>
<td>III</td>
<td>5, 7, 10, 15, 27</td>
<td>Reduction of Decimals to common fractions</td>
</tr>
<tr>
<td>IV</td>
<td>3, 8, 9</td>
<td>The use of zero as a place holder</td>
</tr>
<tr>
<td>V</td>
<td>6, 13, 16, 19</td>
<td>Changing the location of the decimal point: its effect on the value of the expression</td>
</tr>
<tr>
<td>VI</td>
<td>18, 21, 22, 28, 30</td>
<td>Rounding decimal fractions</td>
</tr>
<tr>
<td>VII</td>
<td>12, 14, 17, 23, 24</td>
<td>Division involving decimal fractions</td>
</tr>
<tr>
<td>VIII</td>
<td>1, 2, 25, 26, 29</td>
<td>Miscellaneous concepts involving decimal fractions (changing common fraction to decimal fraction, addition and multiplication)</td>
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</tbody>
</table>
depended upon the number of Farquhar questions devoted to the teaching of each objective, or group of objectives, and also upon the evident complexity of the teaching task involved. In the case of Lesson I, which is an introductory lesson area for general orientation purposes, the first mentioned factor was not a consideration because there are no Farquhar questions to evaluate this topic.

The first three columns of Table II contain a summary of the planning involved up to this point.

The fifth, and last, step in planning the experiment was to select the materials of instruction considered most effective for the teaching of each objective in the entire eight lessons. The materials selected are designated in the last two columns of Table II. These materials may be identified by referring to the representations of the materials shown in Figures on pages 39 to 42.

The foremost consideration in selecting these materials was the necessity to ensure that the aids used by the two treatment groups embodied as far as possible the same characteristics except the capacity to be manipulated.

A classification of arithmetic teaching aids contained in "Teachers' Guide for Thinking with Numbers" by Brueckner, Grossnickle, and Merton, one of the source materials, proved valuable in making the selection. These authors classify the aids into four groups, each of which possesses the characteristics briefly described.
### TABLE II

**NAMES OF INSTRUCTIONAL MATERIALS USED BY EXPERIMENTAL AND CONTROL GROUPS, AND THE TIME ALLOWED, FOR TEACHING THE OBJECTIVES OF EACH LESSON**

<table>
<thead>
<tr>
<th>Number and Title of Lesson</th>
<th>Number of Objective</th>
<th>Time Allotment (Minutes)</th>
<th>Instructional Materials Used by Experimental Group</th>
<th>Control Group (No. of Card)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson I</strong></td>
<td>Objectives 1 &amp; 2</td>
<td>12</td>
<td>nil</td>
<td>nil</td>
</tr>
<tr>
<td></td>
<td>Objective 3</td>
<td>18</td>
<td>P.V. Charts</td>
<td>1 to 6</td>
</tr>
<tr>
<td><strong>Lesson II</strong></td>
<td>Objectives 1, 2 &amp; 3</td>
<td>15</td>
<td>P.V. Charts</td>
<td>7, 8, 9</td>
</tr>
<tr>
<td></td>
<td>Objective 4</td>
<td>15</td>
<td>P.V. Cards</td>
<td>10</td>
</tr>
<tr>
<td><strong>Lesson III</strong></td>
<td>Objective 1</td>
<td>8</td>
<td>nil</td>
<td>nil</td>
</tr>
<tr>
<td></td>
<td>Objective 2</td>
<td>12</td>
<td>P.V. Carts</td>
<td>7, 8, 9</td>
</tr>
<tr>
<td></td>
<td>Objective 3</td>
<td>10</td>
<td>nil</td>
<td>nil</td>
</tr>
<tr>
<td><strong>Lesson IV</strong></td>
<td>Objective 1</td>
<td>20</td>
<td>P.V. Charts</td>
<td>9, 10</td>
</tr>
<tr>
<td></td>
<td>Objective 2</td>
<td>10</td>
<td>P.V. Cards</td>
<td>10</td>
</tr>
<tr>
<td><strong>Lesson V</strong></td>
<td>Objective 1</td>
<td>15</td>
<td>P.V. Charts</td>
<td>9, 10</td>
</tr>
<tr>
<td></td>
<td>Objective 2</td>
<td>15</td>
<td>P.V. Cards</td>
<td>Same</td>
</tr>
<tr>
<td><strong>Lesson VI</strong></td>
<td>Objective 1</td>
<td>15</td>
<td>Wall rule</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Objective 2</td>
<td>8</td>
<td>Wall rule</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Objective 3</td>
<td>7</td>
<td>nil</td>
<td>nil</td>
</tr>
<tr>
<td><strong>Lesson VII</strong></td>
<td>Objective 1</td>
<td>8</td>
<td>nil</td>
<td>nil</td>
</tr>
<tr>
<td></td>
<td>Objective 2</td>
<td>22</td>
<td>Flannel Board</td>
<td>12</td>
</tr>
<tr>
<td><strong>Lesson VIII</strong></td>
<td>Objective 1</td>
<td>12</td>
<td>Flannel Board</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Objective 2</td>
<td>12</td>
<td>Wall rule</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Objective 3</td>
<td>6</td>
<td>nil</td>
<td>13</td>
</tr>
</tbody>
</table>
1. Manipulative materials. These materials provide the highest level of concreteness in the presentation of an arithmetical idea. The idea is represented in an actual object, capable of being manipulated.

2. Visualization materials. These materials provide the second highest level of concreteness. The idea is not shown in the form of a concrete and movable object; rather, it is in the form of a representation of the object, drawn on a chart, with arrows to indicate the movement or the thinking necessary to arrive at an answer.

3. Illustration materials. These materials provide the third highest level of concreteness. The component mental processes necessary to formulate the answer are not shown. It is merely the answer which is illustrated.

4. Abstract symbolism. These materials provide the lowest level of concreteness. The symbol is not in any way anchored to its referrent, except in so far as the learner is capable of providing this link in his own imagination.

In order, therefore, to make the instructional materials used by the experimental and control groups as nearly alike as possible in all characteristics except the capacity to be manipulated, it was necessary to choose from the first two of the above mentioned categories of materials.

The selected materials utilize instructional ideas and principles commonly referred to, though in some cases under different
names, in the various sources of reference consulted.

These materials, which were constructed by the experimenter, are described in the following subsection.

II. DESCRIPTION OF THE INSTRUCTIONAL MATERIALS

The first four materials described are the instructional aids used by the experimental group. These manipulative devices, with the exception of the flannel board, are represented in Figure 1. The next set of materials described are the instructional aids used by the control group. These visualization materials, which are designated by number in an entirely arbitrary manner, are represented in Figures 2, 3, and 4.

**Place value charts.** Made of $\frac{4}{8}$ inch plywood, these charts are each one foot square. The decimal point is on a chart which is one foot by six inches in size. Seven charts represent place values extending from thousands to thousandths.

The ONES' chart, occupying the central place in our system of notation, is painted red, while the decimal point is a red dot on a white background. This colour arrangement was chosen to emphasize that the primary function of the decimal point is to designate the location of the ONES' digit. In order to present the visual symmetry of the different place values around the ONES' place, the decimal point was, in the actual teaching process, placed in front of the ONES' board and towards the right edge,
Figure 1. Illustrations of Manipulative Materials
Figure 2. Illustrations of Visualization Materials
### Visualization Card No. 5

<table>
<thead>
<tr>
<th>HUNDREDS</th>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Visualization Card No. 6

<table>
<thead>
<tr>
<th>HUNDREDS</th>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Visualization Card No. 7

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Visualization Card No. 8

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Illustrations of Visualization Materials
### Visualization Card No. 9

<table>
<thead>
<tr>
<th>1000's</th>
<th>100's</th>
<th>10's</th>
<th>ONES</th>
<th>10ths</th>
<th>100ths</th>
<th>1000ths</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>.2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### Visualization Card No. 10

![Image of Visualization Card No. 10](image)

### Visualization Card No. 11

![Image of Visualization Card No. 11](image)

### Visualization Card No. 12

| .1 | .1 | .1 | .1 | .1 | .1 | .1 | .1 | .1 | .1 |

### Visualization Card No. 13

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 4. Illustrations of Visualization Materials
rather than entirely to the right as shown in the Figure 1.

To emphasize further the symmetry of the number system, the corresponding place values on either side of the ONES' place have the same colours. This colour arrangement - blue for TENS and TENTHS, green for HUNDREDS and HUNDREDTHS, and yellow for THOUSANDS and THOUSANDTHS - was followed consistently in the preparation of all the materials.

Each chart holds 30 hooks. Cardboard tickets, 1\(\frac{1}{2}\) inches by 3 inches, were supplied in the same colours as the charts.

Place value cards. Made of \(\frac{1}{4}\) inch plywood, these cards are each one foot square. They employ the same decimal point arrangement and colour scheme used in the place value charts.

These materials are designed to present the actual relationship in size of the positional values extending from ONES to THOUSANDTHS. The wider range of place values from THOUSANDS to THOUSANDTHS, besides being difficult to present within the limitations of a reasonable amount of materials, was considered unnecessary. The idea of the relationship in size of the positions to the left of ONE is adequately conveyed by the previously described place value charts, where a bundle of ten tickets represents ten, a bundle of one hundred tickets represents one hundred, and so on. Since this procedure could not be used to the right of ONES' place, the place value cards had to be used to present the actual relationship in size of the positional values extending in this direction.
Rule with movable indicator. Made from a board 4 feet long, 3 1/2 inches wide, and 3/8 inch thick, this aid includes a movable indicator.

The rule involves the same principle found in such pupil materials as the decimal fraction cards and number line, and differs mainly in that it is designed for group instruction.

The entire length of the rule represents one unit (that is, from the integer marked "1" to the integer marked "2"). Placing the integer "1" at the beginning of the measurement is to facilitate the explanation of rounding to the nearest whole number.

The integers which designate whole numbers appear in red, while the subdivisions into TENTHS appear in blue, and the further subdivisions into HUNDREDTHS are indicated by green markings.

Flannel board. Since this is a conventional and widely used type of teaching aid, it is not illustrated in Figure I. The dimensions of the flannel boards used in the experiment are 4 feet by 2 feet. The manipulative materials attached to the boards are made of lightweight paper. Illustrations of the various materials are shown in the appropriate sections of Lessons VII and VIII of the experimental group.

Visualization materials. These materials, used by the control group, are identified merely as Visualization Cards 1 to 13. Heavyweight paper was used for all thirteen cards. The cards designated as 1 to 8, inclusive, 12, and 13, are each 36 inches by 24 inches in size; Card 9 is 36 inches by 12 inches; Card 10
is 36 inches by 18 inches; and Card 11 is 36 inches by 6 inches.

As mentioned previously, the characteristic of the visualization materials is that the arithmetical idea is presented in the form of a representation of a concrete object, drawn on a chart, with arrows to indicate the movement or the thinking necessary to arrive at an answer. By suspending these materials from the moulding at the top of the blackboard, the necessary arrows could be drawn on the blackboard in the presence of the class.

III. DESCRIPTION OF THE LESSONS

In subsection I it was stated that the third, fourth and fifth, steps in planning the experiment were, respectively, to delineate the lesson areas, to subdivide each lesson area into component lesson objectives, and to select the materials of instruction. The selected materials, described in subsection II, were then constructed.

When these steps were accomplished the next undertaking was to prepare the lessons.

The construction of the eight lessons involved, which are contained in Appendix B, proved to be a major part of the work entailed in organizing the experiment. It was imperative to include in the lessons only carefully planned procedures to which the aids used by both treatment groups are adaptable, and in which both types of aids are provided with opportunities, intended to be
as equal as possible, of contributing to the pupils' understanding of decimal fractions. Furthermore, since it is the capacity of the teaching aids to be manipulated which constitutes the single experimental variable, the lessons had to be equalized for the two treatment groups in every detail except those related to this variable.

The lessons for the experimental group are on pink sheets, while those for the control group are on yellow sheets. The first three lessons are accompanied by introductory material which is common to both groups. This material, which is on white sheets, is intended primarily to provide the teachers with a common background knowledge of the Hindu-Arabic system of notation.

The format of the lessons, as well as the general instructions and the number of steps involved in the presentation, is identical for the experimental and control groups.

In addition to the regular purple lettering, three special colours are employed consistently throughout all the lessons for the following purposes: RED lettering indicates the statement of each objective, and the teaching time allowed to achieve it; GREEN lettering indicates instructional directions or summaries of a more general nature than is contained in the detailed steps of the lessons; BLACK lettering indicates the generalizations which the pupils are expected to formulate in their own words after the présentation of the whole lesson or part of it.
As mentioned in subsection III (Chapter I), the entire time for each lesson is one hour. This includes thirty minutes of group instruction, fifteen to twenty minutes of seatwork, and ten to fifteen minutes for the marking of the seatwork by the class.

The general psychology of the teaching procedure is to present the various concepts at the lowest level of abstraction permitted by the particular teaching aids used. Emphasis on the meaningful relationship of these ideas is developed through a process of induction which leads to the concluding direction for each objective. This concluding direction indicates that the teachers are to "draw" from the pupils the generalizations which they have formulated, not by pat verbalizations but by their own insight and understanding. These generalizations are the concepts to the understanding of which the instructional aids are expected to contribute. There is no question on the Farquhar test which is not covered by a suitable generalization. In cases where there was no Farquhar question to test a concept considered essential to the development of the whole lesson, the concept was taught without the use of the teaching aids.

In the schedule of lessons contained in Appendix A, it will be observed that arrangements were made for three review lessons: one following the third lesson, another following the sixth lesson, and the third review following the eighth lesson. To maintain adequate controls over the use of the teaching aids, these lessons were confined to a recapitulation of the lesson worksheets (contained in Appendix C) which accompanied each lesson. The incidental review of concepts previously taught was conducted without
using the aids. No formal outline was provided for the teaching of these review lessons.

IV. SELECTION OF CLASSES TO PARTICIPATE IN THE EXPERIMENT

In School District No. 36 (Surrey) there were thirty-seven elementary schools at the time the classes were being selected (January, 1957). The mean number of divisions in each school was six. Since there were no junior high schools, all the Grade VII and Grade VIII classes were located in elementary schools, although not all the elementary schools had Grade VII and Grade VIII classes. In order of enrollment the six largest schools had nineteen, eighteen, thirteen, twelve, ten, and ten, divisions.

There were twenty-eight schools which had Grade VII classes. In nineteen of these schools the Grade VII pupils were either grouped with pupils of another class, or they were divided on a homogeneous grouping basis into two or more classes.

Excluding these nineteen schools, which were obviously unsuitable to participate in the study, there were nine schools with completely unselected Grade VII classes.

The three main criteria used for the selection of the five classes considered necessary to participate in the study were: (1) the teacher's interest in, and aptitude for, taking part in an educational experiment, (2) the teacher's experience and ability in classroom management, (3) the teacher's normal adherence to
reasonably conventional teaching methods.¹

The five schools selected are located within a three mile radius centering on Whalley. Comparatively homogeneous socio-economic conditions exist within the area.

Appendix A contains the first communication concerning this study which the experimenter had with the teachers who were to participate. Although two details noted in this letter were later changed, the original choice of teachers and classes remained. The five teachers who took part are male.

V. ASSIGNMENT OF CLASSES TO THE TREATMENT GROUPS

The analysis of covariance statistical design used in this experiment is valid only if certain conditions surrounding the conduct of the experiment are satisfied. One condition, which Lindquist says "has perhaps most often been violated with serious consequences" ² in educational research, concerns the manner of selecting the treatment groups.

In a controlled experiment, if one is safely to conclude from a significant F that the experimental and control treatments have been responsible for producing different results, then it is necessary, in Lindquist's words, to assume that "the subjects in each treatment group were originally drawn either (a) at random from the same parent population, or (b) selected from the same parent population on the basis of their X-measures only ..." ³
In this study, the classes were assigned to the treatment groups in a manner intended to satisfy the first of these alternate assumptions, and at the same time to take into one other consideration.

This consideration was the size of the classes. It is understandable that the effectiveness of the instructional aids would be related to the size of the classes in which they were used. For example, in the case of observing a group demonstration, the pupils at the back of a large class would be at a disadvantage in comparison with the pupils at the back of a small class. The size of the classes, therefore, is a concomitant variable which could not be controlled except through the procedure of pairing the classes.

As the first step in this procedure, the classes with approximately equal enrollments, as they were immediately prior to the experiment, were considered as a unit. Three smaller classes (in General Montgomery, Hjorth Road, and Simon Cunningham Schools) formed one such unit, referred to as Unit A; two larger classes (in Prince Charles and Fleetwood Schools) formed another such unit, referred to as Unit B.

As the second step in this procedure, the classes for the experimental and control groups were selected from each of these units by the random method of tossing a coin. It was previously decided that two classes should be in the experimental group and three classes in the control group. From the first unit containing the three smaller classes, one was selected at random for the experimental group, thus leaving two classes for the control group
From the second unit containing the two larger classes, one was selected at random for the experimental group, thus leaving one class for the control group.

Table III shows the composition of the treatment groups, together with the class enrollments.

**TABLE III**

**NUMBER OF PUPILS IN THE CLASSES ASSIGNED TO EACH TREATMENT GROUP**

<table>
<thead>
<tr>
<th>School</th>
<th>Treatment Group</th>
<th>Enrollment prior to Experiment</th>
<th>Net Number Studied After Eliminations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Montgomery</td>
<td>Experimental</td>
<td>25</td>
<td>23</td>
</tr>
<tr>
<td>Hjorth Road</td>
<td>Control</td>
<td>27</td>
<td>25</td>
</tr>
<tr>
<td>Simon Cunningham</td>
<td>Control</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td><strong>Unit B</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prince Charles</td>
<td>Experimental</td>
<td>39</td>
<td>36</td>
</tr>
<tr>
<td>Fleetwood</td>
<td>Control</td>
<td>42</td>
<td>37</td>
</tr>
<tr>
<td><strong>Total in experimental group</strong></td>
<td></td>
<td>64</td>
<td>59</td>
</tr>
<tr>
<td><strong>Total in control group</strong></td>
<td></td>
<td>96</td>
<td>88</td>
</tr>
<tr>
<td><strong>Total in both groups</strong></td>
<td></td>
<td>160</td>
<td>147</td>
</tr>
</tbody>
</table>
Pupils who were absent for any of the eleven prescribed lessons, or who missed any of the tests, were eliminated from the study. This accounts for the withdrawal of the thirteen pupils noted in Table III.

VI. ACCOUNT OF ORIENTATION AND EVALUATION MEETINGS

The selection of the teachers to participate in this study was made early in January, 1957. Originally scheduled for February, the experiment was eventually held in May. During this time the teachers had an opportunity to orient themselves in a general way to the experimental idea, which was discussed with them by the experimenter during this time.

After the selection of the classes for each treatment group, separate orientation meetings were held for the teachers of each group. The experimental group teachers had two pre-instructional meetings, and the control group teachers had two similar meetings.

At the first of these two meetings with each group, the experimenter distributed the materials for the first four lessons. This pair of meetings (one for the teachers of the experimental group, the other for the teachers of the control group) was held on consecutive days immediately prior to the commencement of the experiment. The distributed materials included: the teaching aids, the introductory material to the first three lessons (white sheets), lessons I to IV (pink and yellow sheets), and the worksheets I to IV.
The experimenter demonstrated the teaching of each lesson to the teachers of each group, and provided an opportunity for a full discussion of any issues that were raised.

At the second of the two meetings with each group, the experimenter distributed the materials for the last four lessons. This pair of meetings (one for the teachers of the experimental group, the other for the teachers of the control group) was held on consecutive days during the course of the teaching of the first four lessons. The procedure noted above, and relevant to lessons V to VIII, was followed.

During the two week period from Monday, May 13th to Tuesday, May 28th, when the experiment was in progress, the experimenter visited the teachers at least twice each week, and on the remaining days he contacted them by telephone. They were invited, and encouraged, to communicate with him by telephone in the event of any problem arising.

Shortly after the close of the experiment, on Thursday, May 30th, an evaluation meeting was held with all five participating teachers. At the outset of the experiment, the teachers had been asked to make preparations for the concluding meeting by fulfilling two requests: (1) to keep a diary of their experiences in the teaching of the lessons and (2) to complete an evaluation form which was distributed at the second pair of pre-instructional meetings. This form is contained in Appendix A.
This information was used in considering the implications of the conclusions reported in Chapter VI.

VII. ADMINISTRATION OF THE TESTING PROGRAMME

Immediately prior to the commencement of the experiment, the following four tests were administered personally by the experimenter: (1) Farquhar's Test of the Understanding of Processes with Decimal Fractions, (2) A Decimal Fraction Computation Test, (3) Otis Self-Administering Test of Mental Ability, Intermediate Examination, Form A, and (4) The Stanford Achievement Test, Advanced Reading, Form E.

The administrations took place, in the absence of the classroom teachers, during the week from Monday, May 6th to Friday, May 10th. Two sittings were held in each school to ensure that conditions of fatigue would be equalized among the classes and reduced to a minimum.

By using in the analysis of covariance the results of these tests, which provide measures of the four independent variables considered relevant to the problem, the initial unmatched differences between the treatment groups were controlled statistically.

Immediately following the experiment, the Farquhar test was re-administered personally by the experimenter. To ensure an equalization of testing conditions, the re-administrations to the five classes involved were held during the mornings only, on
Wednesday, May 29th, and Thursday, May 30th.

By evaluating the results of this re-administration of the Farquhar test, which provides a measure of the criterion variable, the effectiveness of the teaching aids was judged. Complete details concerning these four tests are given in Chapter IV.

In this testing programme about 160 pupils were tested. The marking of approximately 800 papers was undertaken by the experimenter, assisted by his wife.

VIII. PSYCHOLOGICAL SIGNIFICANCE OF THE LIMITATIONS IMPOSED ON THE EXPERIMENT

In Chapter I mention was made of two limitations of this study. The first limitation is that the pupils themselves were given no opportunity of manipulating the instructional materials. The second limitation is that the nature of the experiment dictated a very rigid sequence of instruction in which the pupils experienced a short, intensive encounter with meanings sometime after they had learned the actual performance of the algorithms involved. This latter learning had taken place prior to the experiment during the course of normal classroom instruction. The experiment thus allowed no opportunity to reverse the procedure so as to teach the rationalization of a process before teaching the method of performing the process. Neither did it allow an opportunity to present the two emphases by teaching rationalization concurrently with method.
These details could pass unnoticed, were it not for the fact that they represent important issues about which a considerable amount of psychological and educational literature has been written. An evaluation of the conclusions reached in Chapter VI requires an awareness of this literature.

The issue surrounding the first limitation is that of the learner's own involvement in the manipulative activity. The reports that this involvement facilitates learning is found mainly in psychological literature.

Heidbreder, for example, has investigated the manner in which concepts are learned. Her experimentation with adults furnishes some evidence that the ease of attaining a concept, in the case of adults at least, "seems more highly correlated with manipulability than with perceptibility". 4

In a chapter entitled "The Formation of Concepts", contained in a recent yearbook, Van Engen cites an impressive list of authorities to support his conviction that, in the case of children as well as adults, manipulability, or relevance to direct motor reaction, is an important factor in the learning of arithmetical concepts. 5

He quotes from Gesell:

All mental life has at its roots the actions or manipulations performed in a learning situation. ... It is probable that all mental life has a motor basis and a motor origin. ... This behaviour (of motor priority) is so fundamental that virtually all behaviour ontogenetically has a motor origin and aspect. 6
He quotes from Werner:

To conceive and define things in terms of concrete activity is in complete accordance with the world of action characteristic of the child.  

Finally, he quotes from Piaget, whose work "seems to be resting in an undeserved obscurity":

... it (childish thought) is nearer to action than ours, and consists simply of mentally pictured manual operations...

In view of the literature which testifies to the importance of action or manipulability in the child's thought processes, it would seem that the learning outcomes resulting from the instruction offered in the experimental group were likely curtailed as a consequence of the fact that the pupils in that group were not afforded an opportunity to manipulate the instructional materials themselves.

The issue surrounding the second limitation concerns the place in the instructional sequence where emphasis should be laid on rationalization or understanding. This may be referred to as the "HOW-WHY versus WHY-HOW controversy".

Though the views on this controversy of the two authors quoted below are not exactly opposed to each other, they serve to show the shades of opinion expressed in the literature.

Commenting on one aspect of the controversy, Johnson writes:

... a rationalization of a process in arithmetic is meaningless unless the HOW to do that process is understood first. Let is be understood that I do not minimize the importance of the role played by rationalization. When rationalization of a process is understood, the process is better appreciated.
... But what I am trying to say here is that since rationalization of a process is not understood until the HOW of the process is understood, and the HOW is not understood on first presentation by all students, and since it takes a greater maturity of mind to understand the rationalization than to understand the HOW of a process, many teachers err in trying to rationalize every process upon first presentation before the HOW of the process is known.9 Later in the same article he states:

What could be a better program of teaching than to bring in rationalization of newer and higher orders as the process is reviewed in later grades? The review would then not be a rehash only, but a true review with the process seen in a new light. Research would have to lead the way showing at what mental age the various arithmetic processes could be rationalized.10

Commenting on another aspect of the controversy, Weaver writes:

There are numerous persons who advocate exclusive adherence to a HOW-WHY sequence: the HOW of a computational process or skill must precede the WHY. The present writer is not at all certain that the HOW of a process or skill must necessarily precede the WHY. No contention has been made or implied that it is always feasible for WHY to lead to HOW. In some instructional situations it may seem virtually necessary to present the algorismic form of certain computational skills on the basis of a HOW-WHY sequence. In such instances, when HOW-WHY is selected as the course to be taken, let us be certain that the WHY is coupled with the HOW just as soon as possible or feasible. There is grave danger that WHY may follow HOW at such temporal distance that ultimate rationalization is minimized in effectiveness.11

In view of the prominence of these views expressed in the literature, it is necessary to be aware of the fact that the effectiveness of teaching rationalization of arithmetical concepts and processes may be affected, not only by the instructional aids and the other factors that have been taken into account in this
study, but also by the particular temporal sequence employed in the HOW-WHY teaching relationship. The generalizations resulting from this experiment must be drawn with recognition of this fact.

IX. REASONS FOR IMPOSING THE LIMITATIONS ON THE STUDY

The first limitation could have been removed by supplying suitable forms of the manipulative aids in sufficient quantities to permit the pupils to use them either individually or in small groups.

Difficulties were evident in this plan. In the first place, the manipulative materials would have had the added advantage of providing increased motivation through allowing pupils the opportunity of self-participation. Within the design of this experiment it would have been difficult to equalize this opportunity for the pupils of the other treatment group because visualization materials do not lend themselves to the same degree of pupil participation. The experimental variable would not, therefore, be confined to the characteristic of manipulability.

In the second place, the difficulty of establishing uniformity between the two treatment groups in such things as teaching procedure and teacher competence to manage individual pupil activity would inevitably have been increased.

The second limitation could not have been entirely removed. It would always be necessary to make some choice between the HOW-WHY and WHY-HOW sequences.
However, an experiment could have been designed to provide a compromise whereby the HOW would precede the WHY in the teaching of some decimal concepts and processes, and follow it in the teaching of others, with the intervening temporal distance between the two emphases reduced as much as possible for each concept or process.

In fact, such a design would have afforded a more likely usage to which manipulative materials would be put in normal classroom practice.

One major difficulty, as usual, presented itself with this idea. It would have necessitated extending the area of the experiment to include the teaching of decimal fractions in their entirety - the HOW as well as the WHY. Since this assignment comprises a large part of the arithmetic programme normally undertaken in Grades VI and VII, one or other of two major problems would have been encountered.

On the one hand, there would have been an enormous problem of maintaining adequate controls in an experiment which extended over the long period of time during which decimal fractions are ordinarily taught. On the other hand, there would have been an awkward problem of arranging to shorten this long period of time by providing for the teaching of the entire area of decimal fractions, uninterrupted by the teaching of other units in the arithmetic syllabus.

The nature of these difficulties, as well as the desirability of using manipulative aids in the type of "carefully planned
re teaching program" recommended by Morton, encouraged the experimenter to proceed with the present design, notwithstanding the two limitations involved.

X. SUMMARY

The purpose of this study is to determine the effectiveness of the group instruction use of certain manipulative aids in contributing to an understanding of decimal concepts. The essence of the proposition involved is to determine the effectiveness which results, specifically, from the capacity of these aids to be manipulated, rather than from their capacity, for example, to influence motivation or to be prominently displayed.

Chapter III contains a description of the instructional aids and lessons used in the experiment, and an account of the planning and administration undertaken, to ensure that the manipulative characteristic of the aids would emerge as the experimental variable.

This chapter also contains a discussion of the limitations imposed upon the use of the particular manipulative materials used in the study. The psychological nature of these limitations, as revealed by the literature on the subject, emphasizes the need to proceed with caution in forming generalizations respecting the effectiveness of the particular aids used.

While this chapter contains a description of the actual controls
exercised in the conduct of the experiment, Chapter V contains, in addition to the test of significance of the achievement of the two treatment groups on the criterion variable, an account of the statistical controls imposed upon the independent variables.

Complete descriptions and evaluations of the tests used to measure all these variables are contained in Chapter IV.
The third criterion is important in order to avoid the inadvertent introduction of systematic differences into the experiment, even though the subjects were originally drawn at random from the same normally distributed and homogeneous population. From the standpoint of satisfying one of the basic assumptions underlying the analysis of covariance, it is necessary to adhere to this criterion. A full discussion is contained in Chapter V, pages 151 and 152.


2 Ibid, p. 323


9 Ibid, p. 366

Supra, p. 11
CHAPTER IV

THE STATISTICAL DESIGN OF THE EXPERIMENT
AND DESCRIPTION OF THE MEASURES USED

I. STATISTICAL DESIGN OF THE EXPERIMENT

The General Nature and Purpose of the Statistical Method

In Chapter III it was stated that the classes were matched on the basis of size only. This matching resulted in the formation of two so-called units, referred to in Table III on page 51 as Unit A and Unit B. From Unit A one class was selected at random for the Experimental Group, leaving two classes for the Control Group; and from Unit B one class was selected at random for the Experimental Group, leaving one class for the Control Group.

Since size was the only factor taken into account in establishing the equivalence of the classes, there were obviously many unmatched individual differences in the classes assigned to the two treatment groups. The relative response of each group to the criterion could conceivably be influenced by these differences.

It is apparent that if these unavoidable concomitant influences were not controlled, any differences between the Experimental and Control Groups on the criterion could not specifically be attributed to the treatments being tested.
To provide statistical control over these unmatched individual differences in the Experimental and Control Groups, analysis of covariance was selected as the statistical design to be applied to the data derived from the experiment.

The following statements respecting the analysis of covariance technique indicate in general terms its suitability for the present study. Further statements, referring to somewhat more technical aspects of its appropriateness, are contained in Chapter V.

Edwards writes:

The analysis of covariance is applicable to any experiment in which a source of variation, which it may not be possible to equalize between the various experimental groups prior to the experiment proper, can be measured. An adjustment is then made for this source of variation in the analysis of the outcomes of the experiment.¹

Wert, Neidt, and Ahmann write:

To provide the investigator with a means of attaining a measure of control of individual differences, the statistical technique known as analysis of covariance was developed. Analysis of covariance incorporates elements of the analysis of variance and of regression. In general, it will provide tests of significance for the comparison groups whose members may have been stratified and whose members have been measured with regard to one or more variable characteristics other than the criterion.²

Analysis of Covariance has really two purposes. First, by providing for the correlation between the criterion and control scores, it makes it possible to determine the relative weight with which each independent variable "enters in" or contributes to the criterion independently of the other variables. Thus, depending on the nature of correlations, the precision of the test of significance may be increased
considerably, even though extremely small differences exist between the means of the treatment groups in the various independent variables. Second, by making allowances for the differences that exist, analysis of covariance makes it possible to exercise a degree of statistical control over these independent variables which permits the treatment effect to be evaluated with as much accuracy as if the variables had been experimentally controlled by actually matching the groups with respect to these variables.

Limitations, as well as possibilities, accompany the use of the covariance technique. It is not a magic formula capable of eliminating all differences, without reservations, between the means of the treatment groups in the independent X variables. Still less capable is it of eliminating the effects of systematic differences originally existing between the groups in certain characteristics which are independent of the X variables employed.

Subsections IX and X of Chapter V contain an account of all the limitations imposed by the assumptions underlying the use of analysis of covariance. The same subsections contain also the necessary statistical tests to ensure that the analysis is appropriate to this specific problem.

In the present study the concomitant influences are considered to exist, primarily, in four areas, namely: initial understanding of concepts of decimal fractions, computational ability in decimal fractions, mental ability, and reading ability.

Table IV shows the names of the tests selected to measure performance in these areas. It also indicates the instrument used to
measure the criterion. These tests are contained in Appendix D.

The contribution which each of these tests made to the prediction of the criterion is eventually reported in Chapter V (Table XLI, page 155.) Judged by this information, it is unlikely that additional measurable influences would have an appreciable effect upon the performance of the treatment groups on the criterion.

It may be said, therefore, that the application of analysis of covariance to the data removed the possible bias introduced by unmatched individual differences between the Experimental and Control Groups. This is true, at least, to the extent that the four areas referred to represent the differences in question, and, further, to the extent that the differences in these areas are adequately controlled by the tests administered for that purpose.

With the removal of this bias, and the imposition of necessary controls in the plan and administration of the experiment, any significant statistical difference in the criterion measures of the experimental and control groups is assumed to be attributable to the treatments used in each group. These treatments, it may be emphasized again, are intended to be identical in every respect except in the use of the materials of instruction, which differ only in the characteristic of manipulability.

Statement of the Hypothesis

The hypothesis to be tested is that there is no significant difference in the criterion achievement of the two treatment groups which
<table>
<thead>
<tr>
<th>Classification of Variables</th>
<th>Names of Variables</th>
<th>Tests selected to measure Each Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent or Concomitant Variables</td>
<td>1. Initial Understanding of Concepts of Decimal Fractions</td>
<td>Farquhar's Test of the Understanding of Processes with Decimal Fractions (First Administration)</td>
</tr>
<tr>
<td></td>
<td>2. Computational Ability in Decimal Fractions</td>
<td>Adapted from Unit Test &quot;Making Sure of Decimals&quot; contained in Silver Burdett Text &quot;Making Sure of Arithmetic&quot;</td>
</tr>
<tr>
<td></td>
<td>3. Mental Ability</td>
<td>Otis Self-Administering Test of Mental Ability (Intermediate Examinations) Form A</td>
</tr>
<tr>
<td></td>
<td>4. Reading Ability</td>
<td>Stanford Achievement Test (Advanced Reading Test: Form E for Grades 7-9)</td>
</tr>
<tr>
<td>Dependent or Criterion Variable</td>
<td>1. Final Understanding of Concepts of Decimal Fractions</td>
<td>Farquhar's Test of the Understanding of Processes with Decimal Fractions (Second Administration)</td>
</tr>
</tbody>
</table>
is attributable to the treatments involved. Stated in other words, this hypothesis is that the pupils who are taught with the use of manipulative aids in the manner prescribed in this experiment achieve an understanding of decimal fractions which is not significantly different, after the bias due to unmatched individual differences in each group has been removed, from the understanding achieved by pupils who are taught with the use of "visualization" materials which bear characteristics identical to those of manipulative materials in all details except that of manipulability.

II. DESCRIPTION OF THE FARQUHAR TEST OF UNDERSTANDING OF PROCESSES WITH DECIMAL FRACTIONS

The Farquhar Test, shown in Appendix D, performed a dual function in this study. Immediately prior to the assignment, it was used to measure one of the independent variables shown in Table IV. Immediately following the teaching assignment it was used, in a second administration, to measure the dependent or criterion variable. These two functions confer upon the Farquhar Test an importance which necessitates thorough investigation of its efficiency for these purposes.

This necessity is all the greater in view of the fact that the Farquhar test was validated in relation to groups of teachers-in-training. The fact that this validation took place against educationally more advanced subjects than those participating in this experiment provided the major source of apprehension concerning the suitability of the test for the present investigation.
Data derived from a trial administration of the Farquhar Test

To provide further data on which to determine its suitability for the present study, the Farquhar Test was administered on April 4th, 1957 by the experimenter to a group of forty representative pupils selected from 2\(\frac{1}{2}\) classes of unselected Grade VII pupils in White Rock Elementary School, located outside the proposed experimental area. It was assumed that the results obtained from this trial administration would be substantially the same as those which could be expected from the initial administration of the same test to the classes participating in the experiment.

Tables V to X, inclusive, contain data derived from the trial administration. Part of this information was obtained from an item analysis of the test undertaken to indicate the effectiveness of individual test items. The items were evaluated on the bases of two internal criteria, namely, their difficulty and their discriminating value or validity.

Method of Item Analysis

The method employed is based on the simplified item analysis procedure devised by Stanley. Page 308, Appendix E contains the recording sheets, page 311 contains the calculation sheets, both of which were used in the present analysis. The technique deals with the top and bottom 27% of the group.

Rows (a), (b), and (c) of the calculation sheets merely show the data obtained from the recording sheet.
TABLE V

FREQUENCY OF SCORES IN THE TRIAL ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL
(Maximum: 30 Items)

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
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<td>0</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>7</td>
<td>7</td>
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<tr>
<td>15</td>
<td>1</td>
<td>6</td>
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<td>5</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE VI

SUMMARY OF STATISTICAL DETAIL RESULTING FROM TRIAL ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL

<table>
<thead>
<tr>
<th>Mean Item Difficulty</th>
<th>Median</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Corrected for chance</th>
<th>Not Corrected for chance</th>
<th>Range of Item</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>9.7</td>
<td>10.175</td>
<td>3.382</td>
<td>85.6%</td>
<td>64.9%</td>
<td>-18% to 64%</td>
<td>.549</td>
</tr>
</tbody>
</table>
### TABLE VII

THE DIFFICULTIES AND VALIDITIES OF ITEMS RESULTING FROM TRIAL
ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII
PUPILS IN WHITE ROCK ELEMENTARY SCHOOL

<table>
<thead>
<tr>
<th>Item</th>
<th>Per cent of Difficulty</th>
<th>Per cent of Validity</th>
<th>Validity Coefficient</th>
<th>WL - WH Discrimination</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Corrected for chance</td>
<td>Uncorrected for chance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30</td>
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</tr>
<tr>
<td>2</td>
<td>62</td>
<td>50</td>
<td>9</td>
<td>.10</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>27</td>
<td>55</td>
<td>.75</td>
</tr>
<tr>
<td>4</td>
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</tr>
<tr>
<td>5</td>
<td>127</td>
<td>95</td>
<td>-9</td>
<td>-.38</td>
</tr>
<tr>
<td>6</td>
<td>97</td>
<td>73</td>
<td>36</td>
<td>.46</td>
</tr>
<tr>
<td>7</td>
<td>97</td>
<td>73</td>
<td>36</td>
<td>.46</td>
</tr>
<tr>
<td>8</td>
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<td>50</td>
<td>64</td>
<td>.63</td>
</tr>
<tr>
<td>9</td>
<td>68</td>
<td>55</td>
<td>18</td>
<td>.20</td>
</tr>
<tr>
<td>10</td>
<td>115</td>
<td>86</td>
<td>27</td>
<td>.60</td>
</tr>
<tr>
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<td>55</td>
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</tr>
<tr>
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<td>91</td>
<td>18</td>
<td>.51</td>
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<td>73</td>
<td>55</td>
<td>36</td>
<td>.38</td>
</tr>
<tr>
<td>Item</td>
<td>Per cent of Difficulty</td>
<td>Per cent of Validity</td>
<td>Validity Coefficient (Flanagan's)</td>
<td>$W_L - W_H$ Discrimination</td>
</tr>
<tr>
<td>------</td>
<td>------------------------</td>
<td>----------------------</td>
<td>----------------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
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<td>Corrected for chance</td>
<td>Uncorrected for chance</td>
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<td></td>
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<td>18</td>
<td>.20</td>
</tr>
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<td>82</td>
<td>0</td>
<td>.00</td>
</tr>
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<td>82</td>
<td>0</td>
<td>.00</td>
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<tr>
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<td>-.23</td>
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<td>.66</td>
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<tr>
<td>22</td>
<td>115</td>
<td>86</td>
<td>9</td>
<td>.18</td>
</tr>
<tr>
<td>23</td>
<td>85</td>
<td>86</td>
<td>64</td>
<td>.78</td>
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<tr>
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<td>30</td>
<td>121</td>
<td>91</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
### TABLE VIII

**FREQUENCY OF ITEMS AT THE VARIOUS PER CENT LEVELS OF DIFFICULTY RESULTING FROM TRIAL ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL**

<table>
<thead>
<tr>
<th>Per Cent Range of Difficulty</th>
<th>Frequency of Items</th>
<th>Per Cent Range of Difficulty</th>
<th>Frequency of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% -</td>
<td></td>
<td>46% - 50%</td>
<td>2</td>
</tr>
<tr>
<td>1% - 5%</td>
<td></td>
<td>51% - 55%</td>
<td>2</td>
</tr>
<tr>
<td>6% - 10%</td>
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<td>2</td>
</tr>
<tr>
<td>11% - 15%</td>
<td></td>
<td>61% - 65%</td>
<td>2</td>
</tr>
<tr>
<td>16% - 20%</td>
<td></td>
<td>66% - 70%</td>
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</tr>
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<td>71% - 75%</td>
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<td>26% - 30%</td>
<td>1</td>
<td>76% - 80%</td>
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<td>31% - 35%</td>
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<td>81% - 85%</td>
<td>4</td>
</tr>
<tr>
<td>36% - 40%</td>
<td>1</td>
<td>86% - 90%</td>
<td>2</td>
</tr>
<tr>
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<td>1</td>
<td>91% - 95%</td>
<td>4</td>
</tr>
</tbody>
</table>

### TABLE XIX

**FREQUENCY OF ITEMS AT THE VARIOUS PER CENT LEVELS OF VALIDITY RESULTING FROM TRIAL ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL**

<table>
<thead>
<tr>
<th>Per Cent Range of Validity</th>
<th>Frequency of Items</th>
<th>Per Cent Range of Validity</th>
<th>Frequency of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% - or less</td>
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<td>46% - 50%</td>
<td>1</td>
</tr>
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<td>1% - 5%</td>
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<td>6% - 10%</td>
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<td>2</td>
</tr>
<tr>
<td>11% - 15%</td>
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<td>61% - 65%</td>
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</tr>
<tr>
<td>16% - 20%</td>
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<td>66% - 70%</td>
<td>4</td>
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<tr>
<td>21% - 25%</td>
<td></td>
<td>71% - 75%</td>
<td>4</td>
</tr>
<tr>
<td>26% - 30%</td>
<td>5</td>
<td>76% - 80%</td>
<td>2</td>
</tr>
<tr>
<td>31% - 35%</td>
<td></td>
<td>81% - 85%</td>
<td>4</td>
</tr>
<tr>
<td>36% - 40%</td>
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<td>86% - 90%</td>
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<tr>
<td>41% - 45%</td>
<td>1</td>
<td>91% - 95%</td>
<td>4</td>
</tr>
</tbody>
</table>
TABLE X

FREQUENCY OF ITEMS IN THE VARIOUS VALIDITY COEFFICIENT RANGES RESULTING FROM TRIAL ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL

(Based on Flanagan's Estimates of Correlation between Individual Items and the Test as a Whole)

<table>
<thead>
<tr>
<th>Validity Coefficient Range</th>
<th>Frequency</th>
<th>Validity Coefficient Range</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>2</td>
</tr>
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<td>0.61 - 0.65</td>
<td>1</td>
</tr>
<tr>
<td>0.16 - 0.20</td>
<td>4</td>
<td>0.66 - 0.70</td>
<td>3</td>
</tr>
<tr>
<td>0.21 - 0.25</td>
<td>0</td>
<td>0.71 - 0.75</td>
<td>2</td>
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<td>0.26 - 0.30</td>
<td>1</td>
<td>0.76 - 0.80</td>
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<tr>
<td>0.31 - 0.35</td>
<td>1</td>
<td>0.81 - 0.85</td>
<td>0</td>
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<tr>
<td>0.36 - 0.40</td>
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<td>0.86 - 0.90</td>
<td>0</td>
</tr>
<tr>
<td>0.41 - 0.45</td>
<td>0</td>
<td>0.91 - 0.95</td>
<td>0</td>
</tr>
</tbody>
</table>
Row (d) indicates the per cent difficulty of each item, uncorrected for chance. It is the ratio, converted to per cent, of the total number of incorrect or omitted responses to the total number of possible responses in the top and bottom 27% sections. Expressed algebraically it is \( \frac{100c}{2n} \) where "c" is the total obtained in Row (c) and "n" is the total number of possible responses in 27% of the entire group.

Rows (e), (f), and (g) deal with the per cent difficulty, corrected for chance. To obtain this, the total number of wrong or omitted responses recorded in row (c) is multiplied by the correction factor shown in row (f). Expressed algebraically the correction factor is \( \frac{100 "0"}{2n \left( "0"-1 \right)} \), where "0" is the number of options in each question and "n" is the total number of possible responses in 27% of the entire group.

Rows (h) and (i) deal with the item discriminating value or validity. The discrimination of each item is found by subtracting row (b) from row (a). This value may be converted to a per cent ratio by dividing it by the maximum discrimination possible and multiplying by 100. Expressed algebraically the per cent of the discrimination or validity of each item is \( \frac{100h}{n} \), where "h" is the discrimination recorded in row (h) and "n" retains the representation indicated above.

To find the Flanagan validity coefficient it is necessary, in addition to the foregoing procedure, to compute from rows (a) and (b) the per cent of correct responses in the bottom and top sections,
respectively. The validity index is obtained for each item by entering Flanagan's Table with these per cent computations.

**Interpretation of data obtained from the trial administration**

Two internal criteria are available by which to evaluate the effectiveness of the individual test items.

One of these criteria is that of item difficulty. On this subject Ross and Stanley write:

Difficulty alone, therefore, is not a dependable measure of discrimination ... Test experts have usually found, however, that the average difficulty of the items in a test is related to the adequacy of the test as a whole. The rule suggested for the construction of tests to discriminate best among all the members of a group is to make every item of 50 per cent difficulty when corrected for chance, so far as possible. This will mean that virtually all the items of 0-15 per cent and 85-100 per cent difficulty when corrected for chance will be omitted from the revised form of the test, unless they can be rewritten to make them closer to the 50 per cent difficulty level.4

An examination of Table VII shows that compliance with this suggestion would result in the omission of 19 of the 30 test items. In fact, the difficulty, corrected for chance, of 10 of these items exceeds 100 per cent. This means that fewer pupils answered these items correctly than would be expected on the basis of chance alone.

However, as Ross and Stanley say, "Quite a few test experts do not favor correcting item difficulty indexes for 'chance'".5

If the correction for chance is not made, the mean per cent difficulty of the items is reduced from 85.6 to 64.9 (Table VI), and, as seen in Table VIII, only six items have a per cent difficulty greater than 85. In addition, it will be observed that the distribution
of scores is a satisfactory one (Table V), in which the mean slightly exceeds the median, and the standard deviation indicates a reasonable, though small, variability. (As samples, these three calculations are shown in Appendix E.)

Nevertheless, the results of the trial administration indicated that Farquhar's test would likely be rather difficult when used as a measure of one of the independent variables at the beginning of the experiment. Despite this realization, the experimenter believed that the difficulty of the test would not be entirely a disadvantage. After an intensive period of instruction on the subject matter covered by the test, it was to be used a second time in the even more important role of measuring the criterion performance. It was anticipated, and hoped, that in this capacity the level of difficulty of the Farquhar items would enable the test to meet the ideal statistical requirements.

The second of the criteria by which to evaluate the effectiveness of the individual test items is that of item discrimination or validity. One commonly used standard of validity is that items must show a positive discrimination of as much as 20 per cent. A reference to Table IX reveals that 12 items fell at, or below, the 20 per cent level. Another commonly used standard of validity is that items must show a positive validity coefficient, based on Flanagan's Table, of more than .25. A reference to Table X reveals that 10 items fell below the .25 coefficient level.

One particularly undesirable feature which resulted from this administration is that 3 items have a zero validity and 2 have a
negative validity. In 4 of these 5 items the per cent of difficulty, corrected for chance, exceeded 100 per cent. In the case of the fifth, question 19, the per cent was 97 (Table VII).

In view of the difficulty of the test for this sample of pupils, the degree of validity was to be expected. At the 50 per cent level of difficulty an item has the maximum opportunity to discriminate between the top and bottom 27 per cent sections.

The results of the trial administration indicated, therefore, that Farquhar's test, as well as being rather difficult, would likely also be rather low in discrimination value when used as a measure of one of the independent variables at the beginning of the experiment.

Yet the general levels of difficulty and discrimination, when used for this purpose, were not considered likely to be sufficiently extreme to make the test unsuitable.

Furthermore, when used a second time as a measure of the criterion variable at the close of the experiment, it was believed that the decrease in difficulty anticipated in nearly all the items would be just about the right amount to increase quite substantially the discriminating value of these items.

While this desire to find a test which, from the standpoint of item effectiveness, would be a satisfactory measure of one of the independent variables and also of the criterion variable was a main consideration affecting the experimenter's decision to select the Farquhar test, there were the following additional considerations, though not necessarily in this order of importance.
The first of these considerations was the reliability of the test which, as reported in Table VI, was .549. This reliability was computed by applying the Hoyt modification of the Kuder-Richardson Formula to the data obtained from the trial administration. The Hoyt Formula and the calculations involved are shown in Appendix E.

While there are obvious difficulties in the interpretation of test reliability, certain minimal requirements have been suggested for the reliability coefficients of tests which serve various purposes. Reference is made to this suggestion by Ross and Stanley, who write: "".50 (reliability coefficient needed) for determining the status of a group in some subject or group of subjects." 6 They note also, of course, that considerably higher reliability coefficients are required where the purpose of the test is to differentiate the achievement or status of individuals, rather than that of a group.

Since the Farquhar test was to be used for the latter of these two purposes, it appeared that the reliability coefficient of .549, obtained from the trial administration, was deservedly a consideration in favor of selecting the test for use in the experiment.

The second of these considerations was the nature of the concepts covered by the test. Concerning the method of selecting these concepts, Farquhar wrote:

The attempt to measure understanding of arithmetic processes is rendered very difficult by the lack of criteria for this purpose. The investigator must determine arbitrarily those concepts that should be included in a measuring instrument designed to evaluate understanding of any phase of arithmetic.
Farquhar listed fifteen of these arbitrarily chosen concepts in the field of decimal fractions. They proved to be specific concepts around which it was convenient to plan the topics of the eight lessons involved in the teaching procedure. The lesson planning is described in Chapter III.

The curricular validity of the test, when used as a measure of one of the independent variables at the beginning of the experiment, seems assured by the fact that the concepts measured by the test are identical to the concepts emphasized for teaching in the Grade VII text currently prescribed by the British Columbia Department of Education. The curricular validity of the test, when used as a measure of the criterion variable at the end of the experiment, is more definitely assured by the fact that the lessons were carefully planned according to the specific concepts measured by the test.

After taking account of the foregoing considerations, that is, the effectiveness of the individual test items when evaluated on the bases of their difficulty and discrimination value, the test reliability, and the specific nature and curricular validity of the concepts measured, it was decided that Farquhar's test would be suitable for the present study.

Interpretation of data obtained from the final administration

It has been stated that Farquhar's test was used immediately prior to the experiment to measure one of the independent variables, and immediately following the experiment to measure the criterion variable.
The interpretation given above referred to the data obtained from the trial administration of the test to a sample group outside the experimental area. A similar study of the data obtained from the initial administration has not been undertaken since it is assumed that these results are substantially the same as those obtained from the trial administration.

Such an assumption, however, could not be made regarding the results obtained from the final administration. Therefore, these results have been subjected to an analysis similar to that which was undertaken in connection with the trial administration.

Tables XI to XVI, inclusive, which correspond respectively to Tables V to X, inclusive, contain data derived from the final administration. To facilitate making comparisons, a summary of comparative data from the two administrations is presented in Table XVII.

An examination of this latter table shows that the level of difficulty, which was the most serious criticism of the test when it was used with the trial group, was considerably reduced when it was used as a measure of the criterion variable. Whereas 6 of the 30 items lay beyond the suggested levels of difficulty (uncorrected for chance) in the trial administration, only 1 item was in this position in the final administration. It is interesting to note that this 1 item (No. 4) tended to be too easy (15%).

The general extent of the reduction in difficulty is shown by the fact that 25 of the 30 items were easier for the pupils in the
TABLE XI
FREQUENCY OF SCORES IN FARQUHAR'S TEST ADMINISTERED AT THE CLOSE OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS
(Maximum: 30 Items)

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
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</thead>
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</tr>
<tr>
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</tr>
<tr>
<td>24</td>
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<td>15</td>
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<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
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<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
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TABLE XII
SUMMARY OF STATISTICAL DETAIL IN FARQUHAR'S TEST ADMINISTERED AT THE CLOSE OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

<table>
<thead>
<tr>
<th>Mean Item Difficulty</th>
<th>Median</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Corrected for chance</th>
<th>Not corrected for chance</th>
<th>Range of Reliability</th>
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<tr>
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<td>13.77</td>
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<td>5.019</td>
<td>66.8%</td>
<td>51.1%</td>
<td>8 to 65%</td>
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<td></td>
<td></td>
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<td>.541</td>
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</tbody>
</table>
### TABLE XIII

**THE DIFFICULTIES AND VALIDITIES OF ITEMS IN FARQUHAR'S TEST**
**ADMINISTERED AT THE CLOSE OF THE EXPERIMENT**
**TO THE 147 PARTICIPATING SUBJECTS**

<table>
<thead>
<tr>
<th>Item</th>
<th>Per cent of Difficulty Corrected for chance</th>
<th>Uncorrected for chance</th>
<th>Per cent of Validity</th>
<th>Validity Coefficient (Flanagan's)</th>
<th>W_L</th>
<th>W_H</th>
<th>Discrimination</th>
</tr>
</thead>
<tbody>
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<td>28</td>
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<td>11</td>
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<td>.32</td>
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</tr>
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<td>Item</td>
<td>Corrected for chance</td>
<td>Uncorrected for chance</td>
<td>Per cent of Validity</td>
<td>Validity Coefficient (Flanagan's)</td>
<td>WL - WH Discrimination</td>
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<tr>
<td>------</td>
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<td>.34</td>
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<td>70</td>
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<td>43</td>
<td>.44</td>
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<td>25</td>
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<td>.51</td>
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<td>14</td>
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TABLE XIV

FREQUENCY OF ITEMS AT THE VARIOUS PER CENT LEVELS OF DIFFICULTY RESULTING FROM ADMINISTRATION OF FARQUHAR'S TEST AT THE CLOSE OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

<table>
<thead>
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<th>Per Cent Range of Difficulty</th>
<th>Frequency of Items</th>
<th>Per Cent Range of Difficulty</th>
<th>Frequency of Items</th>
</tr>
</thead>
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<tr>
<td>0% -</td>
<td></td>
<td>46% - 50%</td>
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<td>1% - 5%</td>
<td></td>
<td>51% - 55%</td>
<td>4</td>
</tr>
<tr>
<td>6% - 10%</td>
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<td>56% - 60%</td>
<td>3</td>
</tr>
<tr>
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<td>1</td>
<td>61% - 65%</td>
<td>4</td>
</tr>
<tr>
<td>16% - 20%</td>
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<td>66% - 70%</td>
<td>3</td>
</tr>
<tr>
<td>21% - 25%</td>
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<td>71% - 75%</td>
<td></td>
</tr>
<tr>
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<td>76% - 80%</td>
<td>1</td>
</tr>
<tr>
<td>31% - 35%</td>
<td>3</td>
<td>81% - 85%</td>
<td>1</td>
</tr>
<tr>
<td>36% - 40%</td>
<td></td>
<td>86% - 90%</td>
<td></td>
</tr>
<tr>
<td>41% - 45%</td>
<td>3</td>
<td>91% - 95%</td>
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</tr>
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TABLE XV

FREQUENCY OF ITEMS AT THE VARIOUS PER CENT LEVELS OF VALIDITY RESULTING FROM ADMINISTRATION OF FARQUHAR'S TEST AT THE CLOSE OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

<table>
<thead>
<tr>
<th>Per Cent Range of Validity</th>
<th>Frequency of Items</th>
<th>Per Cent Range of Validity</th>
<th>Frequency of Items</th>
</tr>
</thead>
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<td>46% - 50%</td>
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</tr>
<tr>
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<td>51% - 55%</td>
<td>1</td>
</tr>
<tr>
<td>6% - 10%</td>
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<td>2</td>
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<tr>
<td>11% - 15%</td>
<td>2</td>
<td>61% - 65%</td>
<td>3</td>
</tr>
<tr>
<td>16% - 20%</td>
<td></td>
<td>66% - 70%</td>
<td></td>
</tr>
<tr>
<td>21% - 25%</td>
<td>1</td>
<td>71% - 75%</td>
<td></td>
</tr>
<tr>
<td>26% - 30%</td>
<td>1</td>
<td>76% - 80%</td>
<td></td>
</tr>
<tr>
<td>31% - 35%</td>
<td>9</td>
<td>81% - 85%</td>
<td></td>
</tr>
<tr>
<td>36% - 40%</td>
<td>2</td>
<td>86% - 90%</td>
<td></td>
</tr>
<tr>
<td>41% - 45%</td>
<td>6</td>
<td>91% - 95%</td>
<td></td>
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TABLE XVI

FREQUENCY OF ITEMS IN THE VARIOUS VALIDITY COEFFICIENT RANGES RESULTING FROM ADMINISTRATION OF FARQUHAR'S TEST AT THE CLOSE OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

(Based on Flanagan's Estimates of Correlation between Individual Items and the Test as a Whole)

<table>
<thead>
<tr>
<th>Validity Coefficient Range</th>
<th>Frequency</th>
<th>Validity Coefficient Range</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>.06 - .10</td>
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<td>.51 - .55</td>
<td>4</td>
</tr>
<tr>
<td>.11 - .15</td>
<td>1</td>
<td>.61 - .65</td>
<td>1</td>
</tr>
<tr>
<td>.16 - .20</td>
<td></td>
<td>.66 - .70</td>
<td></td>
</tr>
<tr>
<td>.21 - .25</td>
<td></td>
<td>.71 - .75</td>
<td>1</td>
</tr>
<tr>
<td>.26 - .30</td>
<td></td>
<td>.76 - .80</td>
<td>1</td>
</tr>
<tr>
<td>.31 - .35</td>
<td>5</td>
<td>.81 - .85</td>
<td></td>
</tr>
<tr>
<td>.36 - .40</td>
<td>6</td>
<td>.86 - .90</td>
<td></td>
</tr>
<tr>
<td>.41 - .45</td>
<td>6</td>
<td>.91 - .95</td>
<td></td>
</tr>
<tr>
<td>Criteria</td>
<td>Correction for chance (Difficulty)</td>
<td>Trial Administration</td>
<td>Final Administration</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>------------------------------------</td>
<td>-----------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td><strong>Item Difficulty</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No. Items below desirable</strong></td>
<td>Yes</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>minimum difficulty</td>
<td>No</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><em>(15% or below)</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No. Items above desirable</strong></td>
<td>Yes</td>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td>maximum difficulty</td>
<td>No</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td><em>(over 85%)</em></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total No. Items beyond desire</strong></td>
<td>Yes</td>
<td>19</td>
<td>5</td>
</tr>
<tr>
<td>able difficulty</td>
<td>No</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td><strong>No. Items over 100%</strong></td>
<td></td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Difficulty when corrected for chance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean Per Cent Difficulty</strong></td>
<td>Yes</td>
<td>85.6</td>
<td>66.8</td>
</tr>
<tr>
<td><strong>No</strong></td>
<td></td>
<td>64.9</td>
<td>51.1</td>
</tr>
<tr>
<td><strong>Item Discrimination</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No. Items below desirable</strong></td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>minimum discrimination</td>
<td>(below 20%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No. Items below desirable</strong></td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of Validity*(Planagan Coefficient over .25)*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No. Items with negative</strong></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>discrimination</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
final administration, and 5 items (No's. 3, 12, 20, 23, 27) were more difficult for this group than they were for the pupils in the trial group, who did not have any special instruction in the concepts involved.

The extent of the reduction in difficulty is shown also by the fact that the mean per cent level of difficulty (uncorrected for chance) fell to 51.1%.

As the test proved to be generally easier, though not too easy, for the pupils in the final administration, so also it proved to be more discriminating. Table XVII shows that only one item fell below the desirable minimum discrimination. This item (No. 10) is one that remained quite difficult (76%). However, two other items were at the minimum acceptable level (20%): one of these is item 14, which also remained quite difficult (83%), and the other is item 4, referred to above, which tended to become too easy (15%). The decrease in difficulty which was anticipated in nearly all the items proved to be just about the right amount to increase quite substantially the discriminating value of these items. In the case of three (No's. 3, 12, 20) of the five questions which for some reason proved more difficult for the group in the final administration than for the trial group, the increase in difficulty actually resulted in an increase in per cent of validity. The validity of the other two items, though decreased in the final administration, remained satisfactory (40% in No. 23; 32% in No. 27).

When used to measure the criterion variable, the Farquhar test proved to be a suitable measuring instrument in other respects beside
item difficulty and discrimination, which have just been discussed.

The distribution of scores (Table XI) is a very satisfactory one, in which the mean slightly exceeds the median, and the standard deviation (Table XII) indicates greater variability than existed in the results of the trial administration.

The reliability of the test in this situation is .541, approximately the same as in the previous analysis (.549).

The assurance of curricular validity, when the test was used in its final role, has already been discussed. 8

Concluding Comments about the Farquhar Test

In the planning of this experiment it was considered necessary to use the same test to measure the independent variable concerned with the pupils' initial understanding of decimal fractions, and to measure the criterion variable also. A test suitable for these two purposes was difficult to find.

Although the results of the trial administration to the 40 Grade VII pupils in White Rock Elementary School indicate that Farquhar's test was probably somewhat more difficult than was desirable when used as a measure of one of the independent variables, it proved to be an almost ideal instrument by which to measure the criterion variable.

The capacity of the test to perform these two functions in this manner indicates its suitability for the present study.
III. DESCRIPTION OF THE DECIMAL FRACTION COMPUTATION TEST

The Decimal Fraction Computation Test, shown in Appendix D, was used to measure one of the independent variables shown in Table IV, page 69. It is the second of the battery of four tests which was administered by the experimenter immediately prior to the commencement of the experiment.

The test was constructed by the experimenter, although it is to some extent an adaptation of a diagnostic unit test entitled "Making Sure of Decimals", which is contained in the Silver Burdett Text "Making Sure of Arithmetic". Tables XVIII to XXIII, inclusive, which correspond to the two previous sets of tables, contain data derived from the results of the test which was administered at the beginning of the experiment. This analysis was undertaken to ensure that the test had been a satisfactory instrument by which to measure the pupils' computational ability in decimal fractions. As in the two previous cases, an item analysis was made to indicate the effectiveness of individual test items.

Interpretation of data obtained from the administration of the decimal fraction computation test.

The effectiveness of the items is the first consideration determining its suitability for the present study. Item difficulty is one of the two internal criteria used to evaluate item effectiveness.
TABLE XVIII
FREQUENCY OF SCORES IN THE DECIMAL COMPUTATION TEST ADMINISTERED AT THE BEGINNING OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS
(Maximum: 25 Items)

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>24</td>
<td>6</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>23</td>
<td>6</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>22</td>
<td>9</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>21</td>
<td>15</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>13</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>14</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>18</td>
<td>10</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>17</td>
<td>11</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>8</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>8</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE XIX
SUMMARY OF STATISTICAL DETAIL IN THE DECIMAL COMPUTATION TEST ADMINISTERED AT THE BEGINNING OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

<table>
<thead>
<tr>
<th>Median</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Mean Item Difficulty</th>
<th>Range of Item Validity</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.75</td>
<td>16.98</td>
<td>4.804</td>
<td>34.28%</td>
<td>10 to 80%</td>
<td>.821</td>
</tr>
</tbody>
</table>
### TABLE XX

THE DIFFICULTIES AND VALIDITIES OF ITEMS IN THE DECIMAL COMPUTATION TEST
ADMINISTERED AT THE BEGINNING OF THE EXPERIMENT
TO THE 147 PARTICIPATING SUBJECTS

<table>
<thead>
<tr>
<th>Item</th>
<th>Per cent of Difficulty</th>
<th>Per cent of Validity</th>
<th>Validity Coefficient (Flanagan's)</th>
<th>( W_L - W_H ) Discrimination</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>40</td>
<td>45</td>
<td>.48</td>
<td>18</td>
</tr>
<tr>
<td>(b)</td>
<td>31</td>
<td>22</td>
<td>.26</td>
<td>9</td>
</tr>
<tr>
<td>(c)</td>
<td>31</td>
<td>42</td>
<td>.50</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>32</td>
<td>.49</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>23</td>
<td>.51</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>22</td>
<td>.40</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>33</td>
<td>.46</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>10</td>
<td>.18</td>
<td>4</td>
</tr>
<tr>
<td>7(a)</td>
<td>11</td>
<td>22</td>
<td>.55</td>
<td>9</td>
</tr>
<tr>
<td>(b)</td>
<td>26</td>
<td>42</td>
<td>.57</td>
<td>17</td>
</tr>
<tr>
<td>(c)</td>
<td>23</td>
<td>35</td>
<td>.51</td>
<td>14</td>
</tr>
<tr>
<td>(d)</td>
<td>51</td>
<td>52</td>
<td>.52</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>34</td>
<td>43</td>
<td>.49</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>48</td>
<td>60</td>
<td>.60</td>
<td>23</td>
</tr>
<tr>
<td>10</td>
<td>45</td>
<td>70</td>
<td>.70</td>
<td>28</td>
</tr>
<tr>
<td>Item</td>
<td>Per cent of Difficulty</td>
<td>Per cent of Validity</td>
<td>Validity Coefficient (Flanagan's)</td>
<td>$W_L - W_H$ Discrimination</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------</td>
<td>----------------------</td>
<td>----------------------------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>11(a)</td>
<td>29</td>
<td>37</td>
<td>.46</td>
<td>15</td>
</tr>
<tr>
<td>(b)</td>
<td>38</td>
<td>55</td>
<td>.59</td>
<td>22</td>
</tr>
<tr>
<td>(c)</td>
<td>45</td>
<td>75</td>
<td>.75</td>
<td>30</td>
</tr>
<tr>
<td>(d)</td>
<td>20</td>
<td>35</td>
<td>.60</td>
<td>14</td>
</tr>
<tr>
<td>(e)</td>
<td>41</td>
<td>68</td>
<td>.70</td>
<td>27</td>
</tr>
<tr>
<td>(f)</td>
<td>40</td>
<td>75</td>
<td>.80</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>53</td>
<td>80</td>
<td>.77</td>
<td>32</td>
</tr>
<tr>
<td>13</td>
<td>60</td>
<td>70</td>
<td>.73</td>
<td>28</td>
</tr>
<tr>
<td>14</td>
<td>53</td>
<td>60</td>
<td>.60</td>
<td>24</td>
</tr>
<tr>
<td>15</td>
<td>48</td>
<td>55</td>
<td>.55</td>
<td>22</td>
</tr>
</tbody>
</table>
TABLE XXI

FREQUENCY OF ITEMS AT THE VARIOUS PER CENT LEVELS OF DIFFICULTY RESULTING FROM ADMINISTRATION OF THE DECIMAL COMPUTATION TEST AT THE BEGINNING OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

<table>
<thead>
<tr>
<th>Per Cent Range of Difficulty</th>
<th>Frequency of Items</th>
<th>Per Cent Range of Difficulty</th>
<th>Frequency of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td></td>
<td>46% - 50%</td>
<td>2</td>
</tr>
<tr>
<td>1% - 5%</td>
<td></td>
<td>51% - 55%</td>
<td>3</td>
</tr>
<tr>
<td>6% - 10%</td>
<td></td>
<td>56% - 60%</td>
<td>1</td>
</tr>
<tr>
<td>11% - 15%</td>
<td>3</td>
<td>61% - 65%</td>
<td></td>
</tr>
<tr>
<td>16% - 20%</td>
<td>2</td>
<td>66% - 70%</td>
<td></td>
</tr>
<tr>
<td>21% - 25%</td>
<td>3</td>
<td>71% - 75%</td>
<td></td>
</tr>
<tr>
<td>26% - 30%</td>
<td>2</td>
<td>76% - 80%</td>
<td></td>
</tr>
<tr>
<td>31% - 35%</td>
<td>3</td>
<td>81% - 85%</td>
<td></td>
</tr>
<tr>
<td>36% - 40%</td>
<td>3</td>
<td>86% - 90%</td>
<td></td>
</tr>
<tr>
<td>41% - 45%</td>
<td>3</td>
<td>91% - 95%</td>
<td></td>
</tr>
</tbody>
</table>

TABLE XXII

FREQUENCY OF ITEMS AT THE VARIOUS PER CENT LEVELS OF VALIDITY RESULTING FROM ADMINISTRATION OF THE DECIMAL COMPUTATION TEST AT THE BEGINNING OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

<table>
<thead>
<tr>
<th>Per Cent Range of Validity</th>
<th>Frequency of Items</th>
<th>Per Cent Range of Validity</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td></td>
<td>46% - 50%</td>
<td>3</td>
</tr>
<tr>
<td>1% - 5%</td>
<td></td>
<td>51% - 55%</td>
<td>3</td>
</tr>
<tr>
<td>6% - 10%</td>
<td>1</td>
<td>56% - 60%</td>
<td>2</td>
</tr>
<tr>
<td>11% - 15%</td>
<td></td>
<td>61% - 65%</td>
<td></td>
</tr>
<tr>
<td>16% - 20%</td>
<td></td>
<td>66% - 70%</td>
<td>3</td>
</tr>
<tr>
<td>21% - 25%</td>
<td>4</td>
<td>71% - 75%</td>
<td>2</td>
</tr>
<tr>
<td>26% - 30%</td>
<td></td>
<td>76% - 80%</td>
<td>1</td>
</tr>
<tr>
<td>31% - 35%</td>
<td>4</td>
<td>81% - 85%</td>
<td></td>
</tr>
<tr>
<td>36% - 40%</td>
<td>1</td>
<td>86% - 90%</td>
<td></td>
</tr>
<tr>
<td>41% - 45%</td>
<td>4</td>
<td>91% - 95%</td>
<td></td>
</tr>
</tbody>
</table>
TABLE XXIII

FREQUENCY OF ITEMS IN THE VARIOUS VALIDITY COEFFICIENT RANGES RESULTING FROM ADMINISTRATION OF THE DECIMAL COMPUTATION TEST AT THE BEGINNING OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS

(Based on Flanagan's Estimates of Correlation between Individual Items and the Test as a Whole)

<table>
<thead>
<tr>
<th>Validity Coefficient Range</th>
<th>Frequency</th>
<th>Validity Coefficient Range</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.46 - .50</td>
<td>.51 - .55</td>
<td>6</td>
</tr>
<tr>
<td>.01 - .05</td>
<td>.56 - .60</td>
<td>.66 - .70</td>
<td>5</td>
</tr>
<tr>
<td>.06 - .10</td>
<td>.51 - .55</td>
<td>.66 - .70</td>
<td>5</td>
</tr>
<tr>
<td>.11 - .15</td>
<td>.61 - .65</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>.16 - .20</td>
<td>1</td>
<td>.71 - .75</td>
<td>2</td>
</tr>
<tr>
<td>.21 - .25</td>
<td></td>
<td>.76 - .80</td>
<td>2</td>
</tr>
<tr>
<td>.26 - .30</td>
<td>1</td>
<td>.81 - .85</td>
<td>2</td>
</tr>
<tr>
<td>.31 - .35</td>
<td></td>
<td>.86 - .90</td>
<td>2</td>
</tr>
<tr>
<td>.36 - .40</td>
<td>1</td>
<td>.91 - .95</td>
<td>2</td>
</tr>
<tr>
<td>.41 - .45</td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>
The suggestion of Ross and Stanley, referred to on page 78, is that virtually all items of 0-15% and 85-100% difficulty should be omitted to ensure adequate discrimination for the test as a whole. Three items - No. 3 (14%), No. 6 (15%), and No. 7(a) (11%) - fall into the former category, while none fall into the latter. There are other indications that the test was somewhat easier than was desirable: the mean difficulty of all 25 items is 34.28 per cent (Table XIX), and, further, there is a frequency of 3 scores at the maximum (Table XVIII). However, an examination of this table shows that there is a satisfactory range of scores. Likewise, the median, mean, and standard deviation, reported in Table XIX, suggest that the test could not be considered unduly easy for the group to which it was administered.

Item discrimination, the second criterion used to evaluate item effectiveness, should be at least 20 per cent, according to one of the standards of discrimination previously accepted in dealing with the Farquhar test. Table XXII shows that only one item (No. 6 - 10%) fell short of this desirable minimum. According to another of the standards of discrimination previously accepted, the Flanagan coefficient of validity of an item should exceed .25. Table XXIII shows that only one item (No. 6, once again) fell into this category. The coefficient of this item is .18. It will be seen that two of the three items which have already been regarded as unsatisfactory because they were too easy, nevertheless retained an acceptable discrimination value. Item 6, alone, remains unsatisfactory with respect to both difficulty and discrimination value.
The effectiveness of the items on the computation test, judged on the bases of difficulty and discrimination, is considered satisfactory with the exception of this item.

The reliability of the test is the second consideration determining its suitability for the present study. The reliability, calculated by means of the Hoyt Formula, is .821. According to the standard referred to previously (page 81), this coefficient indicates that the test was satisfactory from the point of view of reliability.

The curricular validity of the test is the third consideration determining its suitability for the present study. The questions contained in the test dealt with the four fundamental processes and with the conversion of common fractions into decimal fractions. These areas of computation are of primary importance in the unit dealing with decimal fractions in the Grade VII Arithmetic Course of Studies for British Columbia.

After taking account of the foregoing considerations, that is; the effectiveness of the individual test items when evaluated on the bases of their difficulty and discrimination value, the test reliability, and its curricular validity, it may be concluded that the decimal fraction computation test was a suitable testing instrument by which to measure the second independent variable.
IV. DESCRIPTION OF THE OTIS SELF-ADMINISTERING TEST OF MENTAL ABILITY

The Otis Self-Administering Test of Mental Ability, Intermediate Examination, Form A, shown in Appendix D, is the testing instrument used to measure the third independent variable.

This well-established and widely known test requires only a brief description concerning three matters: the purpose of the test, the criteria used by the author to judge the validity of each item contained in it, and the reported reliability.

The purpose of the test, according to the author, is to predict the rate at which a student can progress through school. The Otis Intelligence Quotient is, therefore, a relative numerical indication of brightness. In the Manual of Directions there is no statement of the extent to which the Intermediate Examination does, in fact, serve its avowed purpose. There is a meagre report concerning the correlation between scores in the Higher Examination and "scholarship". This report is that of the Principal of a High School in Maine who found a correlation of approximately .58 between scores in the Higher Examination and the "scholarship" of about 400 students in Grades 11 and 12.

The author states: "The method of standardization is perhaps the best assurance as to the validity of the tests".10

In this standardization procedure the criterion used to judge validity was the ability of each item to discriminate between two
groups - a so-called "good group" and "poor group".

The only distinction between the two groups was that the median age of the good group was over two years less than that of the poor group. They had reached the same average educational status, therefore, but at different rates. Only those items were included in the test which distinguished between the students who progressed slowly and the ones who progressed rapidly. The entire standardization group was composed of about 2000 high school students in three cities located in California, Illinois, and Minnesota.

Finally, the last matter to be described about the test is the reliability. The reliability was determined by means of correlation between different forms of the same test. For the Intermediate Examination an average correlation of .948 was found between Forms A and B when these two forms were administered to two groups composed altogether of 427 cases. In one group Form A was administered first, while in the other group Form B was administered first. The probable error of a score in the Intermediate Examination is reported to be slightly over $2\frac{1}{2}$ points in half the cases. The author states that "this means also that the probable error of an I.Q. is about $2\frac{1}{2}$ points.

The Otis Self-Administering Tests of Mental Ability, Intermediate Examination, is designed for Grades 4 to 9. It is, therefore, extremely appropriate for Grade 7. Before deciding on the Otis, the experimenter inquired into the number of pupils who had previously written Form A, or any of the other forms, of the Intermediate Examination. Since the
elementary schools in Surrey at the time of the experiment included Grade 8, the practice is to administer the Otis test immediately prior to the entrance of the pupils into the High Schools. It was discovered that a negligible number of pupils participating in the experiment had written any form of the Intermediate Examination.

V. DESCRIPTION OF THE STANFORD ACHIEVEMENT TEST
(ADVANCED READING TEST: FORM E)

The Stanford Advanced Reading Test: Form E, shown in Appendix D, is the testing instrument used to measure the fourth independent variable. Composed of two sub-tests: paragraph meaning and word meaning, it forms part of the Stanford Advanced Battery of Achievement Tests for Grades 7, 8, and 9. Form E is one of five alternate forms available in the 1940 edition which has been superceded by the 1953 revision.

Provision is contained in the test for converting the raw score into an equated score which makes possible many interpretations of the test results. However, to avoid implications involving the normative group this conversion was not made. Instead, the final raw score for each pupil in this variable was obtained simply by finding the average of the original raw score in each of the two sub-tests, and disregarding the fraction where it occurred. In all respects, except in the matter of converting the raw scores into equated scores, the publisher's directions were strictly adhered to.
Because the conversion table was not used, the standardization data supplied by the authors is not presented as a basis for determining the suitability of this test for the present study, except for reporting its reliability. In place of this standardization data the following data, which pertains to the administration of the test at the beginning of the experiment, is presented for this purpose in Tables XXIV and XXV. It may be concluded from a study of these tables that the test distributed the scores in a satisfactory manner.

The split-half reliability coefficients of the test, corrected by the usual Spearman-Brown formula based on random samples of pupils from 34 school systems in the standardization population, is reported to be .841 for the Paragraph Meaning and .907 for the Word Meaning. The average of these coefficients, .874, is accepted as the reliability of the Stanford Advanced Reading Test when used in the present situation.

VI. CONCLUSION

Chapter IV contains a description and evaluation of each of the four tests used to measure the five variables involved in this study. Two administrations of Farquhar's test, at the beginning and at the end of the experiment, provide respective measures of one of the independent variables and of the criterion variable. The other tests were administered at the beginning of the experiment to measure the remaining three independent variables.
### TABLE XXIV

**FREQUENCY OF SCORES IN STANFORD READING TEST ADMINISTERED AT THE BEGINNING OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS**

(Maximum: 47)

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Score</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>1</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>37</td>
<td>2</td>
<td>19</td>
<td>17</td>
</tr>
<tr>
<td>36</td>
<td>0</td>
<td>18</td>
<td>5</td>
</tr>
<tr>
<td>35</td>
<td>3</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>34</td>
<td>4</td>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>33</td>
<td>3</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>32</td>
<td>2</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td>31</td>
<td>4</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>29</td>
<td>4</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>1</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>27</td>
<td>5</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>3</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>2</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>23</td>
<td>9</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>21</td>
<td>11</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

### TABLE XXV

**SUMMARY OF STATISTICAL DETAIL IN STANFORD READING TEST ADMINISTERED AT THE BEGINNING OF THE EXPERIMENT TO THE 147 PARTICIPATING SUBJECTS**

<table>
<thead>
<tr>
<th>Median</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.24</td>
<td>20.37</td>
<td>7.277</td>
</tr>
</tbody>
</table>
These descriptions and evaluations afford assurance that the tests provided efficient measurements of the variables for which they were used.

The initial differences between the treatment groups in the four independent variables are held constant in the analysis of covariance technique. The numerical extent to which these variables would otherwise have been responsible for the pupils' achievement on the criterion variable is stated in the multiple regression analysis which follows. No additional variables are considered to have exercised an appreciable influence on this achievement.

This statistical control of all the important concomitant influences, together with the careful imposition of actual controls in the plan and administration of the experiment, enables any differences between the groups in the criterion variable to be attributed to the treatments involved. Except for the materials of instruction used, the treatments are intended to be identical. The materials of instruction differ only in the characteristic of manipulability.

The method of imposing the actual controls in the plan and administration of the experiment is described in Chapter III; the method of imposing the statistical control over the independent variables is described in Chapter V.
FOOTNOTES


4 Ibid; p. 119.

5 Ibid, p. 440

6 Ibid, p. 125

7 Farquhar, op. cit., p. 7

8 Supra, p. 82


10 See the Manual of Directions accompanying the Otis Self-Administering Tests of Mental Ability, p. 12.
CHAPTER V

THE STATISTICAL ANALYSIS

I. INTRODUCTION

The problem to be analyzed statistically in this study involves two teaching treatment groups. The experimental group, composed of two classes with a net number of 59 subjects, was taught with the use of manipulative aids. The control group, composed of three classes with a net number of 88 subjects, was taught with the use of static representations of the aids used by the experimental group. Referred to as "visualization" materials, these aids are intended to possess characteristics identical to those of the manipulative materials in all details except the capacity to be manipulated.

The classes selected were matched on the basis of size. By a method described elsewhere, each class was then assigned at random to its treatment group.\(^1\)

Before the commencement of the experiment tests were administered to all subjects to provide measures of the four control variables. At the conclusion of the experiment one of these tests was readministered to provide a measure of the criterion variable.

The original data is contained in Appendix F.

Broadly stated, the hypothesis to be tested is that there is no significant difference between the achievement of the two treatment groups
on the criterion when the initial differences, measured by the four control variables, have been removed or held constant. In other words, according to this hypothesis, any difference in the mean scores of the two groups on the criterion, after allowances have been made for chance differences in the mean level of achievement in the control variables, may be accounted for entirely by chance fluctuations in random sampling.

The allowances for initial differences are to be made in terms of the multiple regression of the criterion measure \( Y \) on the control measures \( X_1; X_2; X_3; X_4 \).

Analysis of Covariance is the statistical procedure used to test this hypothesis.

Commenting on the use of the analysis of covariance in research, Edwards states:

In particular, it is applicable to those situations where the matching of groups is not feasible prior to the assignment of the subjects to the experimental conditions, but where some measure of initial performance may be obtained after the assignment. In experiments of this sort, the analysis of covariance may be effectively used to reduce the error mean square in the test of significance.\(^2\)

Lindquist states:

... through a purely statistical control we can secure the same precision in the evaluation of the treatment effect as if we had experimentally controlled the X-factor by actually matching the groups with reference to X ...\(^3\)

Garrett states:

Covariance analysis is especially useful to experimental psychologists when for various reasons it is impossible or quite difficult to equate control and experimental groups at the start: ... Through covariance one is able to effect
adjustments in final or terminal scores which will allow for differences in some initial variable.\(^4\)

These statements attest to the suitability of analysis of covariance to the present problem in which no attempt was made to match the groups with reference to any of the control variables.

A further explanation of the applicability of covariance analysis may be made with reference to Table XXVI, which shows the means and standard deviations obtained by each of the treatment groups in the criterion variable and the four control variables.

It will be observed that the means of the two groups in each variable differ very slightly. Likewise, except in the case of the variability on the Otis Test (\(X_3\)), the standard deviations of the two groups in each variable differ very slightly.

Even though the difference between the group means of the control variables does not seem large enough to influence greatly the difference in the means of the criterion variable, the analysis of covariance, which represents an extension of analysis of variance to allow for the correlation between criterion and control scores, is worthwhile. The correlation of means between groups, and the correlation of variables within groups, will increase the precision of the test of significance through altering the mean square (both within groups and between groups) used as the error term by the regression of \(Y\) on each of the \(X\) variables.

Certain degrees of the correlations referred to could change an insignificant \(F\) value obtained in an analysis of variance of the
## TABLE XXVI

Means and Standard Deviations Obtained by Each Treatment Group in the Criterion Variable and the Four Control Variables

|                | Criterion Variable (Y) | Independent Variables
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(X₁)</td>
<td>(X₂)</td>
</tr>
<tr>
<td><strong>Means</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>14.492</td>
<td>7.797</td>
</tr>
<tr>
<td><strong>Standard Deviations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>5.315</td>
<td>3.718</td>
</tr>
</tbody>
</table>

---
criterion into a significant F value, after the application of the analysis of covariance. Needless to say, the adjustment from the analysis of covariance may also give results just the opposite of this.

By means of an analysis of these correlations a preliminary examination, as given in Subsection 5 of this chapter, may be undertaken to determine whether the analysis of covariance will prove efficient in detecting differences between the means of the two groups on the criterion Y.

In the present problem, where there is little difference between the means of the two treatment groups in each variable, the primary purpose of the analysis of covariance is to increase the precision of the test of significance. In both the analysis of variance and the analysis of covariance the F value used in the test of significance of the means is obtained by dividing the mean square between groups by the error mean square within groups. Both the correlation of the means between groups (the tendency for the group with the higher mean on each of the X variables to have the higher mean on the Y variable) and the correlation of individual scores within groups (the tendency for subjects within each group who achieve high scores in each of the X variables to achieve high scores also on the Y variable) determine the nature and extent of the adjustment in the numerator and denominator of the F ratio that will result from the application of the analysis of covariance.
Using the covariance analysis in the present problem for the purpose of making allowances for differences between the means of the experimental and control groups in the X variables follows unquestionably as a secondary objective.

II. AN ANALYSIS OF VARIANCE OF EACH OF THE FOUR INDEPENDENT VARIABLES \( X_1, X_2, X_3, X_4 \)

The first step in the application of the covariance technique to the present problem is to analyze the data for each of the independent variables in the usual manner of an analysis of variance.

The purpose of this step is to test the hypothesis that the scores of the two treatment groups in each of the independent variables are in reality random samples drawn from the same normally distributed population and, further, that the means between the groups in each variable differ only through the fluctuations of sampling.

The rationale of the analysis of variance, by which this hypothesis is tested, is stated comprehensively by Lindquist:

The basic proposition (of the analysis of variance) is that from any set of \( r \) groups of \( n \) cases each, we may, on the hypothesis that all groups are random samples from the same population, derive two independent estimates of the population variance, one of which is based on the variance of group means, the other on the average variance within groups. The test of this hypothesis then consists of determining whether or not the ratio \( F \) between these estimates lies below the value in the table for \( F \) that corresponds to the selected level of significance.\(^5\)
The variance of the group means, the first independent estimate of the population variance, is represented by the sum of the square of the deviations of the mean of each group from the general mean. Each of these squared deviations is weighted or multiplied by the number of subjects in each group in order to put them on a per individual measure basis. The greater the difference in the group means, the larger the sum of squares between the groups. The sum of squares based upon variation of group means for two treatment groups is equal to

\[ k_e (M_e - M)^2 + k_c (M_c - M)^2 \]

where \( k \) designates the number of subjects, \( M \) designates the grand mean for the total sample, and the subscripts designate the treatment group. Mentioning the methods of finding the sum of squares between groups (the deviations of the mean of each group from the general mean), Wert, Neidt, and Ahmann say: "the first (shown above) is relatively easy to understand but usually time consuming to compute; the second is mathematically identical and is more generally used". This second formula is

\[ \Sigma x^2 \text{ between groups} = \frac{(\Sigma x_e)^2}{k_e} + \frac{(\Sigma x_c)^2}{k_c} - \frac{(\Sigma x)^2}{N} \]

where \( N \) refers to the number of subjects in the total sample.

The variance within groups, the second independent estimate of the population variance, is derived from the sum of squares of the
deviations of each score from the mean of its own treatment group. Unlike the variance for the total sample, it is free from any influence of the difference in the means between the treatment groups. The sum of squares within groups is

\[(S_1 - M_g)^2 + (S_2 - M_g)^2 + \ldots + (S_n - M_g)^2\]

where \(S_1, S_2, \ldots\) etc. designate the scores of individual subjects in the variable concerned, and the subscript "g" refers to the treatment group to which the subjects belong. Explaining the two methods of finding the sum of squares within groups, Wert, Neidt, and Ahmann write: "Here again, the first (shown above) is self-explanatory and the second saves time." The second method suggested by these authors is one in which the within sum of squares is not directly computed. It is found by subtracting the sum of squares between groups from the total sum of squares. This sum of squares for the total sample may be found directly from the original measures without first subtracting the mean. The formula used in this case is

\[\sum x^2_{\text{total}} = \sum x^2 - \frac{\left(\sum x\right)^2}{N}\]

Thus, the sum of squares within groups is the difference between the sum of squares for the total and the sum of squares between groups.

This is shown as follows:

\[
\left\{\sum x^2 - \frac{\left(\sum x\right)^2}{N}\right\} - \left\{\frac{\left(\sum x_e\right)^2}{k_e} + \frac{\left(\sum x_c\right)^2}{k_c} - \frac{\left(\sum x\right)^2}{N}\right\}
\]
Therefore,
\[ x^2_{\text{within groups}} = \sum x^2 - \left\{ \frac{(x_e)^2}{k_e} + \frac{(x_c)^2}{k_c} \right\} \]

The sum of squares between groups and the sum of squares within groups are each divided by the number of degrees of freedom involved. These calculations yield, respectively, independent estimates of the population variance between groups and within groups.

On the assumption that the groups making up a total series of measurements are random samples from a single normally distributed population, the two foregoing independent estimates of the population variance may be expected to differ only within the limits of the chance fluctuations that occur from random sampling.

To test this null hypothesis the ratio of the variance between groups to the variance within groups is expressed as a quotient, called an F value. This F value is then compared with the .05 and .01 points of the variance ratios tabled by Snedecor.\(^3\)

The value at .05, given in the table for a particular number of degrees of freedom, is the value which would be exceeded only 5% of the time as a result of sampling variation if the null hypothesis were true. Therefore, an F value which equals or exceeds the tabled value at the .05 level has a probability equal to or less than 5%. This means that there is only 1 chance in 20, or less than 1 chance in 20, that an F value as large as this could be obtained by sampling variation. Consequently, a result that happens as seldom as this by
chance would be indicative of systematic differences between treatment effects, and so the null hypothesis would be rejected at the 5% level of significance.

Further, if the F value equals or exceeds the tabled value at the .01 level, it means that there is only 1 chance in 100, or less than one chance in 100, that a value as large as this could be obtained by sampling variation. Such a result, said to be significant at the 1% level, would be even more convincing evidence on which to reject the null hypothesis.

On the other hand, if the F value falls short of the tabled value at the .01 level, it has a probability greater than 1%. This means that there is more than 1 chance in 100 that an F value as large as this could be obtained by sampling variation. Consequently, a result that happens as frequently as this by chance would not be indicative, at the 1% level of significance, of systematic differences between treatment effects, and so the null hypothesis would be considered tenable at this level.

Further, if the F value falls short of the tabled value even at the .05 level, the probability is greater than 5%. This means that there is more than 1 chance in 20 that a value as large as this could be obtained by sampling variation. A result that happens as frequently as this by chance would be even less indicative of systematic differences between treatment effects, and so the null hypothesis would be considered tenable at the 5% level of significance.
In accordance with the procedures presented in the preceding pages of this subsection, an analysis of variance was computed for each variable in the whole battery of test controls used in this experiment, namely: \(X_1, X_2, X_3,\) and \(X_4\). In addition, an analysis of variance was computed for the variable used as the criterion measure, namely: \(Y\).

In each case the formulae used are those shown on pages 114 and 115. The data required for substitution in these formulae are contained in Table XXVII. It may be noted in passing that the sums of scores and the sums of squares of scores shown in this table, as well as the sums of cross products shown in Table XXXIV (page 130) and used in a subsequent step, may all be secured in a single operation on an automatic Monroe computing machine. The accumulating sums are carried in the machine, and only the totals recorded.

As an example of the procedures employed in applying these data to the formulae referred to, the calculations of the sums of the squares of the variable \(X_1\), for the total sample and within the subgroups, are shown below.

The calculation of the sum of squares for the total sample:

\[
\Sigma X_1^2 = \Sigma X_1^2 - \frac{(\Sigma X_1)^2}{N}
\]

\[
= 12142 - \frac{(1234)^2}{147}
\]

\[
= 12142 - \frac{1522756}{147}
\]

\[
= 12142 - 10358.88435
\]

\[
= 1783.11565
\]
### TABLE XXVII

**SUMS OF SCORES IN THE FIVE VARIABLES, AND SUMS OF SQUARES OF SCORES, ARRANGED BY CLASSES, FOR EACH TREATMENT GROUP AND FOR THE TOTAL SAMPLE**

<table>
<thead>
<tr>
<th></th>
<th>General Prince</th>
<th>Prince Charles</th>
<th>Total for Experimental Group</th>
<th>Fleetwood</th>
<th>Hjorth</th>
<th>Road</th>
<th>Simon Cunningham</th>
<th>Total for Control Group</th>
<th>Total for Both Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma y$</td>
<td>318</td>
<td>537</td>
<td>855</td>
<td>576</td>
<td>333</td>
<td>343</td>
<td>1,252</td>
<td>2,107</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_1$</td>
<td>176</td>
<td>284</td>
<td>460</td>
<td>359</td>
<td>191</td>
<td>224</td>
<td>774</td>
<td>1,234</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_2$</td>
<td>357</td>
<td>679</td>
<td>1,036</td>
<td>578</td>
<td>432</td>
<td>450</td>
<td>1,460</td>
<td>2,496</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_3$</td>
<td>2,498</td>
<td>3,892</td>
<td>6,390</td>
<td>4,143</td>
<td>2,656</td>
<td>2,914</td>
<td>9,713</td>
<td>16,103</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_4$</td>
<td>434</td>
<td>767</td>
<td>1,201</td>
<td>776</td>
<td>480</td>
<td>538</td>
<td>1,794</td>
<td>2,995</td>
<td></td>
</tr>
<tr>
<td>$\Sigma y^2$</td>
<td>4,778</td>
<td>8,827</td>
<td>13,605</td>
<td>9,912</td>
<td>5,075</td>
<td>5,311</td>
<td>20,298</td>
<td>33,903</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_1^2$</td>
<td>1,518</td>
<td>2,600</td>
<td>4,118</td>
<td>3,911</td>
<td>1,829</td>
<td>2,284</td>
<td>8,024</td>
<td>12,142</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_2^2$</td>
<td>5,949</td>
<td>13,197</td>
<td>19,146</td>
<td>10,268</td>
<td>8,040</td>
<td>8,388</td>
<td>26,696</td>
<td>45,842</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_3^2$</td>
<td>273,822</td>
<td>425,174</td>
<td>698,996</td>
<td>471,773</td>
<td>291,380</td>
<td>330,904</td>
<td>1,094,057</td>
<td>1,793,053</td>
<td></td>
</tr>
<tr>
<td>$\Sigma x_4^2$</td>
<td>9,042</td>
<td>18,253</td>
<td>27,295</td>
<td>18,206</td>
<td>10,846</td>
<td>12,458</td>
<td>41,510</td>
<td>68,805</td>
<td></td>
</tr>
</tbody>
</table>


The calculation of the sum of squares within the subgroups:

\[
\Sigma x_1^2 = \Sigma x_1^2 - \left\{ \left( \frac{\Sigma x_{1E}}{k_E} \right)^2 + \left( \frac{\Sigma x_{1C}}{k_C} \right)^2 \right\} \\
= 12142 - \left\{ \left( \frac{460}{59} \right)^2 + \left( \frac{774}{88} \right)^2 \right\} \\
= 12142 - \left\{ \frac{211600}{59} + \frac{599076}{88} \right\} \\
= 12142 - 10394.1250 \\
= 1747.8750
\]

The sum of squares between groups was not directly computed. It was found by subtracting the sum of squares within groups from the sum of squares for total.

Table XXVIII shows these three sums of squares for each of the five variables involved in the study.

**TABLE XXVIII**

**SUMS OF SQUARES OF SCORES IN THE FIVE VARIABLES, IN DEVIATION FORM, FOR THE TOTAL SAMPLE, AND FOR WITHIN, AND BETWEEN, THE TREATMENT GROUPS**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Total</th>
<th>Within Groups</th>
<th>Between Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma y^2 )</td>
<td>3702.66667</td>
<td>3700.20031</td>
<td>2.46636</td>
</tr>
<tr>
<td>( \Sigma x_1^2 )</td>
<td>1783.11565</td>
<td>1747.87750</td>
<td>35.23815</td>
</tr>
<tr>
<td>( \Sigma x_2^2 )</td>
<td>3460.93878</td>
<td>3427.81510</td>
<td>33.12368</td>
</tr>
<tr>
<td>( \Sigma x_3^2 )</td>
<td>29062.46259</td>
<td>28911.13347</td>
<td>151.32912</td>
</tr>
<tr>
<td>( \Sigma x_4^2 )</td>
<td>7784.42177</td>
<td>7784.38906</td>
<td>0.03271</td>
</tr>
</tbody>
</table>
### TABLE XXIX

**ANALYSIS OF VARIANCE OF PERFORMANCE OF THE TV/O TREATMENT GROUPS ON THE INDEPENDENT VARIABLE \( X_1 \)**

*(Farquhar Test of Understanding - First Administration)*

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>146</td>
<td>1783.11565</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td>145</td>
<td>1747.87750</td>
<td>12.05433</td>
<td></td>
</tr>
<tr>
<td>Between groups</td>
<td>1</td>
<td>35.23815</td>
<td>35.23815</td>
<td>2.923</td>
</tr>
</tbody>
</table>

\[
F,_{145} = \frac{35.23815}{12.05433} = 2.923
\]

*From Table F, df 1/145*

- \( F \) at .05 level = 3.91
- \( F \) at .01 level = 6.81

### TABLE XXX

**ANALYSIS OF VARIANCE OF PERFORMANCE OF THE TWO TREATMENT GROUPS ON THE INDEPENDENT VARIABLE \( X_2 \)**

*(Decimal Fraction Computation Test)*

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>146</td>
<td>3460.93878</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td>145</td>
<td>3427.81510</td>
<td>23.64010</td>
<td></td>
</tr>
<tr>
<td>Between groups</td>
<td>1</td>
<td>33.12368</td>
<td>33.12368</td>
<td>1.401</td>
</tr>
</tbody>
</table>

\[
F,_{145} = \frac{33.12368}{23.64010} = 1.401
\]

*From Table F, df 1/145*

- \( F \) at .05 level = 3.91
- \( F \) at .01 level = 6.81
TABLE XXXI
ANALYSIS OF VARIANCE OF PERFORMANCE OF THE TWO TREATMENT GROUPS ON THE INDEPENDENT VARIABLE $X_3$
(Otis S.A. Test of Mental Ability, Intermediate, Form A)

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>146</td>
<td>29062.46259</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td>145</td>
<td>28911.1347</td>
<td>199.38713</td>
<td></td>
</tr>
<tr>
<td>Between groups</td>
<td>1</td>
<td>151.32912</td>
<td>151.32912</td>
<td>0.759</td>
</tr>
</tbody>
</table>

\[
F_{1, 145} = \frac{151.32912}{199.38713} = 0.759
\]

From Table F
\[
\text{df } 1/145
\]
\[
F \text{ at .05 level } = 3.91 \\
F \text{ at .01 level } = 6.81
\]

TABLE XXXII
ANALYSIS OF VARIANCE OF PERFORMANCE OF THE TWO TREATMENT GROUPS ON THE INDEPENDENT VARIABLE $X_4$
(Stanford Advanced Reading Test: Form E)

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>146</td>
<td>7784.42177</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td>145</td>
<td>7784.38906</td>
<td>53.68544</td>
<td></td>
</tr>
<tr>
<td>Between groups</td>
<td>1</td>
<td>.03271</td>
<td>.03271</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

\[
F_{1, 145} = \frac{0.03271}{53.68544} = 0.0006
\]

From Table F
\[
\text{df } 1/145
\]
\[
F \text{ at .05 level } = 3.91 \\
F \text{ at .01 level } = 6.81
\]
The summary of the analyses of variance for the independent variables $X_1, X_2, X_3,$ and $X_4$ is recorded in Tables XXIX to XXXII, inclusive.

An examination of these tables reveals the partition of the total sum of squares into the two independent estimates of the population variance referred to in Lindquist's quotation on page 112. One of these estimates is based on the variance within groups; the other on the variance of the group means (between groups).

Along with this partition of the total sum of squares into two parts there is a corresponding partition of the total number of degrees of freedom. This partition may be shown as follows:

<table>
<thead>
<tr>
<th>Sum of Squares</th>
<th>General Number of Degrees of Freedom</th>
<th>Specific Number of Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within groups</td>
<td>$N - r$</td>
<td>145</td>
</tr>
<tr>
<td>Between groups</td>
<td>$r - 1$</td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$N - 1$</td>
<td>146</td>
</tr>
</tbody>
</table>

where $N$ is the total number of subjects and $r$ is the number of treatment groups.

In Snedecor's table the $F$ value at the .05 level of significance for 1 and 150 (the tabled value nearest to 145) degrees of freedom is 3.91, while at the .01 level it is 6.81.

The $F$ values in each of these four analyses of variance fall considerably short of the value required for significance at the .05 level.

Thus, the null hypothesis is tenable. The difference between
the means of the experimental and control groups in each of the independent variables is less than may be expected through the fluctuations of sampling. It may be concluded that the scores of both groups in all four independent variables are in reality random samples drawn from the same normally distributed and homogeneous population.

III. AN ANALYSIS OF VARIANCE OF THE CRITERION VARIABLE Y

The second step in the application of the covariance technique to the present problem is to analyze the data for the dependent or criterion variable in the usual manner of an analysis of variance. It will be remembered that the re-administration of Farquhar's test at the close of the experiment provided the measure of the criterion variable Y. The first administration of this test at the beginning of the experiment supplied the measure of one of the independent variables, namely: $X_1$.

In accordance with the procedures outlined in the preceding subsection, the analysis of variance was computed and is summarized in Table XXXIII.

In this preliminary analysis of the Y-means no allowance has been made for the initial differences between the groups. It is seen that the resulting F value (.097) falls far short of significance at the .05 level; it is, in fact, considerably less significant than the F value (2.92) obtained in the analysis of the results of the first administration of Farquhar's test.
### TABLE XXXIII

ANALYSIS OF VARIANCE OF PERFORMANCE OF THE TWO TREATMENT GROUPS ON THE CRITERION VARIABLE Y

*Farquhar Test of Understanding - Final Administration*

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>146</td>
<td>3702.66667</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td>145</td>
<td>3700.20031</td>
<td>25.51862</td>
<td></td>
</tr>
<tr>
<td>Between groups</td>
<td>1</td>
<td>2.46636</td>
<td>2.46636</td>
<td>0.097</td>
</tr>
</tbody>
</table>

\[
F_{1, 145} = \frac{2.46636}{25.51862} = 0.097
\]

From Table F: \( \frac{1}{145} \)

- F at .05 level = 3.91.
- F at .01 level = 6.81

The purpose of the remaining computations in this statistical treatment is to make allowance in the analysis of the criterion scores (Y) for individual differences in the control scores \( (X_1, X_2, X_3, X_4) \) obtained at the beginning of the experiment.

The general procedure by which this purpose is accomplished involves the prediction of the criterion variable from known values of the control variables. If the deviation of the control scores of any pupil from the general means of these scores is known, the amount by which the pupil's criterion score would be expected to deviate from the criterion mean may be computed. This expectation, which is based entirely on initial performance in the control tests without regard for
the methods of teaching the two groups, constitutes the prediction of Y by $X_1$, $X_2$, $X_3$, and $X_4$. It is otherwise referred to as the regression of Y on these control variables though, as Garrett points out, the "original meaning of 'stepping back' to some stationary average is not necessarily implied".\(^9\)

Certain basic assumptions inherent in this prediction or regression procedure are discussed later.\(^10\)

The difference between the predicted sum of squares of the criterion and the actual sum of squares of the criterion is known as the residual sum of squares or the sum of squares of errors of estimate. The relationship may be shown in this way:

\[
\begin{align*}
\left\{ \text{Original sum of squares of criterion} \right\} & - \left\{ \text{Sum of squares of criterion predicted on control variable scores} \right\} = \left\{ \text{Adjusted sum of squares of criterion (Sum of squares of residuals or sum of squares of errors of estimate)} \right\} \\
\left\{ \Sigma Y^2 \right\} & - \left\{ \text{Sum of squares due to regression} \right\}
\end{align*}
\]

The residuals or errors of estimate are the sums of squares based upon the variation remaining in Y after that portion which can be attributed to the regression of Y on the X variables has been taken into account. In other words, the original sums of squares of the criterion, as shown in Table XXXIII, are adjusted so that the variability contributed to these sums of squares by the control scores $X_1$, $X_2$, $X_3$, and $X_4$ is removed or held constant. This adjustment, of course, concerns the sum of squares for total, the sum of squares within groups, and the sum of squares between groups.
When these adjusted sums of squares are calculated a further analysis, similar to that presented in Table XXXIII, is made of the criterion means to ascertain whether these means between the two treatment groups have become significantly different as a result of taking into account the individual differences in the control variables. This analysis is located in Subsection VIII of this chapter.

IV. COMPUTATION OF THE SUMS OF CROSS PRODUCTS IN DEVIATION FORM FOR EACH PAIR OF VARIABLES

The third step in the application of the covariance technique to the present problem is to compute the sums of cross products in deviation form for each pair of variables. Four prediction variables and one criterion variables involve ten pairs of cross products.

The analysis of covariance represents an extension of the analysis of variance in that it takes into account the regression of $Y$ on the $X$ variables. The dependence of regression upon the relationship between the $Y$ scores and each of the $X_1, X_2, X_3, X_4$ scores may be expressed in Edwards' words: "It is the presence of correlation or association between the two that makes prediction possible, and the efficiency or accuracy of such predictions is a function of the degree or strength of the relationship that exists".\textsuperscript{11}

The formulae used to compute the sums of squares due to regression, or in other words the predicted sums of squares of the criterion, are derived from the correlation formula:
Lindquist traces the derivation which results in
\[ \sum xy = \sum(x - \bar{x})(y - \bar{y}) + \sum xy \]
It is understood that this summation is for the total sample.

Then he writes:

"Thus we see that the total sum of the PRODUCTS (of deviations) may be analyzed into two components, just as the total sum of SQUARES (of deviations) may be analyzed for either variable considered alone. The components of the total sum of products (of deviations from the general mean) are the sum of the products of deviations from the group means and \( n \) times the sum of the products of the group means (each mean expressed as a deviation from the general mean).

The COVARIANCE of two variables for a sample is the mean of the PRODUCTS of their deviations from their means, just as the VARIANCE of a single variable is the mean of the SQUARES of the deviations."

Stated in other words, it may be said that the total sum of crossproducts may be analyzed into two components, just as in the analysis of variance it was possible to analyze the total sum of squares into two components.

The first component is the sum of cross products within groups. It is based upon the deviations of the individual scores from the means of the treatment group of which they are a part.

The second component is the sum of cross products between groups. It is based upon the deviations of the means of each treatment group from the general mean of the total sample.

These two components correspond, respectively, to the two independent estimates of the population variance, namely: the sum of

\[ r_{xy} = \frac{\sum xy}{N \sigma_x \sigma_y} \]
squares within groups and the sum of squares between groups.

Referring to his own derivation of the above formula for the sum of cross products for the total, Edwards says that it does not represent the most convenient method of calculating the sum of cross products. Instead, it is easier to take the values of \( X \) and \( Y \) from zero origin and to apply a correction term to the products of the original values.

The resulting formulae are:

\[
\sigma_{x_1y}^{\text{total}} = \sigma_{x_1y} - \frac{\sigma_{x_1y}^{\text{between groups}}}{N}
\]

\[
\sigma_{x_1y}^{\text{between groups}} = \frac{(\sigma_{x_1y})_e}{k_e} + \frac{(\sigma_{x_1y})_c}{k_c} - \frac{\sigma_{x_1y}}{N}
\]

The sum of cross products within groups is the difference between the sum of cross products for the total and the sum of cross products between groups. This is shown as follows:

\[
\sigma_{x_1y} - \frac{\sigma_{x_1y}^{\text{between groups}}}{N} \bigg\{ \frac{(\sigma_{x_1y})_e}{k_e} + \frac{(\sigma_{x_1y})_c}{k_c} - \frac{\sigma_{x_1y}}{N} \bigg\}
\]

Therefore,

\[
\sigma_{x_1y}^{\text{within groups}} = \sigma_{x_1y} - \bigg\{ \frac{(\sigma_{x_1y})_e}{k_e} + \frac{(\sigma_{x_1y})_c}{k_c} \bigg\}
\]

In all cases the subscripts designate the treatment groups, \( k \) designates the number of subjects in the treatment group referred to, and \( N \) designates the number of subjects in the total sample.
The data required for substitution in these formulae, are contained in Tables XXVII (page 118) and XXXIV (page 130).

As an example of the procedures employed in applying these data to the formulae, the calculations of the sums of cross products of the combination of variables \( X_1 Y \), for the total sample and within the subgroups, is shown below. This exemplifying parallels that on pages 117 and 119 for the sums of squares.

The calculation of the sum of cross products \( X_1 Y \) for the total sample:

\[
\sum_{x_1y_{\text{total}}} = \sum_{x_1y} - \frac{\sum_{x_1\sum y}}{N}
\]

\[
= 19268 - \frac{(1234)(2107)}{147}
\]

\[
= 19268 - \frac{2600038}{147}
\]

\[
= 19268 - 17687.33333
\]

\[
= 1580.66667
\]

The calculation of the sum of cross products \( X_1 Y \) within the subgroups:

\[
\sum_{x_1y_{\text{within groups}}} = \sum_{x_1y} - \left\{ \frac{(\sum_{x_1\sum y})_e}{k_e} + \frac{(\sum_{x_1\sum y})_c}{k_c} \right\}
\]

\[
= 19268 - \frac{(855)(460)}{59} + \frac{(1252)(774)}{88}
\]

\[
= 19268 - \frac{393300}{59} + \frac{969048}{88}
\]

\[
= 19268 - 6666.10169 + 11011.90909
\]

\[
= 19268 - 17678.01078
\]

\[
= 1589.98922
\]

As in the case of the sum of squares, the sum of cross products between groups was not directly computed. It was found by subtracting
<table>
<thead>
<tr>
<th></th>
<th>General Montgomery</th>
<th>Prince Charles</th>
<th>Total for Experimental Group</th>
<th>Fleetwood</th>
<th>Hjorth</th>
<th>Road Simon</th>
<th>Total for Control Group</th>
<th>Total for Both Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma x_1 y$</td>
<td>2,559</td>
<td>4,518</td>
<td>7,077</td>
<td>6,062</td>
<td>2,854</td>
<td>3,275</td>
<td>12,191</td>
<td>19,268</td>
</tr>
<tr>
<td>$\Sigma x_2 y$</td>
<td>5,127</td>
<td>10,386</td>
<td>15,513</td>
<td>9,708</td>
<td>6,119</td>
<td>6,370</td>
<td>22,197</td>
<td>37,710</td>
</tr>
<tr>
<td>$\Sigma x_3 y$</td>
<td>35,163</td>
<td>59,352</td>
<td>94,515</td>
<td>65,778</td>
<td>36,894</td>
<td>39,448</td>
<td>142,120</td>
<td>236,635</td>
</tr>
<tr>
<td>$\Sigma x_4 y$</td>
<td>6,119</td>
<td>12,189</td>
<td>18,308</td>
<td>12,794</td>
<td>6,780</td>
<td>7,654</td>
<td>27,228</td>
<td>45,536</td>
</tr>
<tr>
<td>$\Sigma x_1 x_2$</td>
<td>2,803</td>
<td>5,439</td>
<td>8,242</td>
<td>6,030</td>
<td>3,486</td>
<td>4,046</td>
<td>13,562</td>
<td>21,804</td>
</tr>
<tr>
<td>$\Sigma x_1 x_3$</td>
<td>19,388</td>
<td>31,327</td>
<td>50,715</td>
<td>40,962</td>
<td>20,779</td>
<td>25,866</td>
<td>87,627</td>
<td>138,342</td>
</tr>
<tr>
<td>$\Sigma x_1 x_4$</td>
<td>3,363</td>
<td>6,512</td>
<td>9,875</td>
<td>8,051</td>
<td>3,960</td>
<td>5,098</td>
<td>17,109</td>
<td>26,984</td>
</tr>
<tr>
<td>$\Sigma x_2 x_3$</td>
<td>39,310</td>
<td>73,811</td>
<td>113,121</td>
<td>66,700</td>
<td>47,245</td>
<td>51,395</td>
<td>165,340</td>
<td>278,461</td>
</tr>
<tr>
<td>$\Sigma x_2 x_4$</td>
<td>6,812</td>
<td>14,466</td>
<td>21,274</td>
<td>13,038</td>
<td>8,631</td>
<td>9,619</td>
<td>31,288</td>
<td>52,762</td>
</tr>
<tr>
<td>$\Sigma x_3 x_4$</td>
<td>47,962</td>
<td>85,253</td>
<td>133,215</td>
<td>89,376</td>
<td>53,782</td>
<td>61,900</td>
<td>205,058</td>
<td>338,273</td>
</tr>
</tbody>
</table>
the sum of cross products within groups from the sum of cross products for total.

Table XXXV shows these three sums of cross products for each of the ten pairs of variables.

An explanation of the reason that deviation scores rather than raw scores are used in both the sums of squares and the sums of cross products is given on page 141.

Tables XXVIII (page 119) and XXXV (page 132) contain the essential data to be used in the calculation of the regression coefficients. This calculation is found in Subsection VI. These data, however, are first used in Subsection V, where a preliminary examination is made of the conditions under which analysis of covariance is worthwhile.

V. AN EXAMINATION OF THE CONDITIONS UNDER WHICH AN ANALYSIS OF COVARIANCE WILL INCREASE THE PRECISION OF THE TEST OF SIGNIFICANCE

The fourth step in the application of the covariance technique to the present problem is to compute the correlation coefficients of the means between treatment groups and of the individual scores within groups. Though not an actual part of the covariance procedure, a description of these two correlation coefficients should give a good indication of the conditions under which an analysis of covariance will prove efficient in detecting differences between the means of the groups on the Y variable. In addition, this description should be helpful in interpreting the final test of significance.
TABLE XXXV

SUMS OF CROSS PRODUCTS OF SCORES IN THE FIVE VARIABLES, IN
DEVIATION FORM, FOR THE TOTAL SAMPLE, AND FOR WITHIN, AND
BETWEEN, THE TREATMENT GROUPS

<table>
<thead>
<tr>
<th>Cross Products</th>
<th>Total</th>
<th>Within Groups</th>
<th>Between Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma x_1 y$</td>
<td>1580.66667</td>
<td>1589.98922</td>
<td>-9.32255</td>
</tr>
<tr>
<td>$\Sigma x_2 y$</td>
<td>1934.00000</td>
<td>1924.96148</td>
<td>9.03852</td>
</tr>
<tr>
<td>$\Sigma x_3 y$</td>
<td>5825.33333</td>
<td>5844.65254</td>
<td>-19.31921</td>
</tr>
<tr>
<td>$\Sigma x_4 y$</td>
<td>2607.66667</td>
<td>2607.95070</td>
<td>-0.28403</td>
</tr>
<tr>
<td>$\Sigma x_1 x_2$</td>
<td>851.18367</td>
<td>885.34822</td>
<td>-34.16455</td>
</tr>
<tr>
<td>$\Sigma x_1 x_3$</td>
<td>3164.43537</td>
<td>3091.41102</td>
<td>73.02435</td>
</tr>
<tr>
<td>$\Sigma x_1 x_4$</td>
<td>1842.29932</td>
<td>1841.22574</td>
<td>1.07358</td>
</tr>
<tr>
<td>$\Sigma x_2 x_3$</td>
<td>5038.63265</td>
<td>5109.43220</td>
<td>-70.79955</td>
</tr>
<tr>
<td>$\Sigma x_2 x_4$</td>
<td>1908.12245</td>
<td>1909.16333</td>
<td>-1.04088</td>
</tr>
<tr>
<td>$\Sigma x_3 x_4$</td>
<td>10188.06803</td>
<td>10185.84322</td>
<td>2.22481</td>
</tr>
</tbody>
</table>
Nature of correlation of means between treatment groups

The first part of this subsection deals with the correlation of the means between the treatment groups. It will be remembered that this correlation refers to the tendency for the group with the higher mean on each of the X variables to have the higher mean on the Y variable.

Where only two groups are involved the correlation must, of course, be either +1 or -1. This correlation may be computed by the Pearson product-moment method. The formula used when deviations are taken from the means of the two distributions is

\[ r_{x'y'} \text{ (between)} = \frac{\sum x'y'}{\sqrt{\sum x'^2 \times \sum y'^2}} \]

Substituting the appropriate values from the between groups source of variation in Tables XXVIII (page 119) and XXXV (page 132) results in the following:

\[ r_{x'y'} \text{ (between)} = \frac{-9.32255}{\sqrt{35.23815 \times 2.46636}} = \frac{-9.32255}{\sqrt{86.9099656340}} = \frac{-9.32255}{9.32255} = -1 \]

In Appendix G similar calculations are shown for each of the remaining three correlations \( x_2'y' \), \( x_3'y' \) and \( x_4'y' \).

An examination of the means in Table XXVI (page 110) reveals that in three of these four pairs of variables there is a negative
correlation. In other words, in these cases the group with the higher mean in the one variable has the lower mean in the other. In Table XXXV (page 132) it is seen that in these three pairs of variables the sums of cross products are negative. In analysis of variance, to produce significant differences in the means of the treatment groups it is desirable, where there are only two groups, that the sums of cross products between groups be negative, or where there are more than two groups, that the sums of cross products be as near zero as possible.

In view of the fact that three of the sums of cross products between groups in this experiment are negative, it would seem likely that the sum of squares due to regression within groups will exceed the sum of squares due to regression for total. This anticipation is justified by subsequent calculations.\(^\text{15}\)

Such a condition inevitably results in the sum of squares of residuals between groups becoming larger than the original sum of squares between groups.

This sum of squares of residuals between groups, when divided by the number of degrees of freedom for that source of variation, becomes the mean square between groups which forms the numerator of the F ratio.

The nature of the correlation of the means between the treatment groups is such that it indicates that the precision of the test of significance will be increased in this problem by the application of the analysis of covariance.
Nature of correlation of individual scores within each treatment group

The second part of this subsection deals with the correlation of individual scores within groups. It will be remembered that this correlation refers to the tendency for subjects within each group who achieve high scores in each of the X variables to achieve high scores also on the Y variable.

The higher the correlations within groups between the criterion variable and each of the independent variables, the larger will be the sum of squares due to regression, and, consequently, the smaller will be the sum of squares of residuals. The error mean square, which is the variance obtained by dividing the sum of squares of residuals by the number of degrees of freedom within groups, will likewise become smaller. Since this forms the denominator of the F ratio, it will be seen that the strengths of the correlations referred to affect directly the precision of the final test of significance of the means between the two treatment groups.

A formula exists by which the extent of the reduction in the adjusted error variance may be estimated on the basis of the strength of the correlations. These correlations, corrected for attenuation, will now be calculated.

The Pearson product-moment formula, shown on page 133, is used. Four correlations must be calculated, namely: $x_1y$, $x_2y$, $x_3y$ and $x_4y$. The necessary data are found in the within groups source of variation in Tables XXVIII (page 118) and XXXV (page 132). As an example of the
procedure employed in applying these data to the formula, the calculation of the within group correlation between $x_1$ and $y$ is shown below.
In Appendix G similar calculations are shown of the within group correlations for each of the remaining pairs of variables $x_2y$, $x_3y$, $x_4y$.

$$r_{x_1y} \text{ (within)} = \frac{1589.98922}{\sqrt{5700.20031 \times 1747.87750}}$$

$$= \frac{1589.98922}{\sqrt{5467496.8673420250}}$$

$$= \frac{1589.98922}{2543.1273}$$

$$= .63$$

Each of the correlations computed in this way has been corrected for attenuation to give an intrinsic correlation between two series of measures with postulated perfect reliability. The formula used to obtain this correction for attenuation is

$$r_{xy} = \frac{r_{x_1y}}{\sqrt{r_{x_1x_1} \times r_{yy}}}$$

where $r_{x_1x_1}$ and $r_{yy}$ refer to the reliabilities of the tests involved.

For convenience, a summary of these reliabilities previously reported in the various tables of Chapter IV, is reproduced in Table XXXVI.

As an example of the procedure employed in applying the data to the formula, the correction for attenuation of the correlation within groups of $x_1y$ is shown on page 137. In Appendix G similar calculations of corrections for attenuation are shown for each of the remaining correlations $x_2y$, $x_3y$, and $x_4y$. 
\[ r = \frac{.63}{\sqrt{.549 \times .541}} \]
\[ = \frac{.63}{\sqrt{.297009}} \]
\[ = \frac{.63}{.545} \]
\[ = \text{greater than unity} \]

Table XXXVII summarizes the correlation data obtained from the calculations exemplified above.

**TABLE XXXVI**

**SUMMARY OF RELIABILITIES OF TESTS EMPLOYED TO PROVIDE MEASURES OF THE CRITERION VARIABLE AND THE FOUR INDEPENDENT VARIABLES**

<table>
<thead>
<tr>
<th>Test</th>
<th>Variable</th>
<th>Reliability</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farquhar's Test (final admin.)</td>
<td>Y</td>
<td>.541</td>
<td>Hoyt modification of K.R. Formula</td>
</tr>
<tr>
<td>Farquhar's Test (trial admin.)</td>
<td>X₁</td>
<td>.549</td>
<td>same</td>
</tr>
<tr>
<td>Decimal Computation Test</td>
<td>X₂</td>
<td>.821</td>
<td>same</td>
</tr>
<tr>
<td>Otis Test</td>
<td>X₃</td>
<td>.948</td>
<td>comparable-forms</td>
</tr>
<tr>
<td>Stanford Test</td>
<td>X₄</td>
<td>.874</td>
<td>Split-half</td>
</tr>
</tbody>
</table>

**TABLE XXXVII**

**PEARSON PRODUCT MOMENT COEFFICIENT OF CORRELATION, AND INTRINSIC CORRELATION, WITHIN GROUPS OF THE CRITERION VARIABLE WITH EACH OF THE INDEPENDENT VARIABLES**

<table>
<thead>
<tr>
<th>Independent and Criterion Variables</th>
<th>Product-Moment Coefficient of Correlation</th>
<th>Intrinsic Correlation (after correction for attenuation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁y</td>
<td>.63</td>
<td>greater than unity</td>
</tr>
<tr>
<td>x₂y</td>
<td>.54</td>
<td>.81</td>
</tr>
<tr>
<td>x₃y</td>
<td>.57</td>
<td>.80</td>
</tr>
<tr>
<td>x₄y</td>
<td>.49</td>
<td>.71</td>
</tr>
</tbody>
</table>
The average intrinsic correlation may be assumed to be approximately .73.  

This coefficient may now be substituted in the following formula, which was first referred to on page 135.

\[
\text{Sum of squares within groups} = \frac{r}{\sum (n - 1) - 1} \frac{(1 - r^2_{xy \text{ (within)})}}{1}
\]

Edwards refers to this as a variation of the formula used to calculate the standard error of estimate.

\[
(1 - .73^2) \\
(1 - .5329) \\
.467
\]

In his treatment of this particular point, Lindquist states: "The ratio between the adjusted error variance and the unadjusted error variance is very nearly equal to \((1 - r^2_{\text{w}})\)." Shown in the form of a proportion, this becomes:

\[
\frac{\text{Adjusted error variance}}{\text{Unadjusted error variance}} = .467
\]

Substituting the mean square variance within groups reported in Table XXXIII (page 124):

\[
\frac{\text{Adjusted error variance}}{25.51862} = .467
\]

Adjusted error variance = approximately 12
This reveals that the error mean square used in the final test of significance, after allowances have been made for the regression of the Y variable on each of the X variables, will be approximately 12. This compares to 25.51862, the original mean square used in the first test of significance before any allowances were made for regression. Since this reduces the denominator of the F ratio by over one-half its original value, it is apparent that the precision of the experiment will be more than doubled by reason of the within groups correlation alone.

**Summary of correlation conditions**

In summary, it may be said that the foregoing examination of the correlation between groups and the correlation within groups indicates that the F value used in the test of significance will be substantially increased through the application of an analysis of covariance.

The first indication is that the correlation of the means between groups has been found to be such that the numerator of the original variance ratio (2.46636, as shown in Table XXXIII - page 124) will be increased by taking into account the regression of Y on the X variables.

The second indication is that the correlation of individual scores within groups is sufficiently high (.73) to permit a considerable reduction in the denominator of the same variance ratio (25.51862).

Only an extremely insignificant difference in achievement
between the two groups was detected by the analysis of variance of
the Y scores alone (F = .097). Because of the nature of the correlations
described in this subsection, this difference will inevitably be more
pronounced and may indeed be significant when multiple regression is
taken into account, and the results tested against the mean square
for error in the analysis of covariance.

The prospect of this outcome warrants the continuation of the
present statistical treatment.

VI. CALCULATION OF THE COEFFICIENTS OF REGRESSION

The fifth step in the application of the covariance technique
to the present problem is to calculate the coefficients of regression.
This step is necessary in a multiple regression analysis; that is, where
more than one X variable is involved in the prediction of the criterion
variable.

In a four variable regression problem the general regression
equation in deviation form is

\[ y = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 \]

When the expression \[ \Sigma (y - a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4)^2 \]
is differentiated with respect to \( a_1, a_2, a_3, \) and \( a_4 \), respectively,
and each of the derivatives is set equal to zero the resulting normal
equations are

\[
\begin{align*}
\sum x_1 y &= a_1 \sum x_1^2 + a_2 \sum x_1 x_2 + a_3 \sum x_1 x_3 + a_4 \sum x_1 x_4 \\
\sum x_2 y &= a_1 \sum x_1 x_2 + a_2 \sum x_2^2 + a_3 \sum x_2 x_3 + a_4 \sum x_2 x_4 \\
\sum x_3 y &= a_1 \sum x_1 x_3 + a_2 \sum x_2 x_3 + a_3 \sum x_3^2 + a_4 \sum x_3 x_4 \\
\sum x_4 y &= a_1 \sum x_1 x_4 + a_2 \sum x_2 x_4 + a_3 \sum x_3 x_4 + a_4 \sum x_4^2
\end{align*}
\]

An explanation should now be made of the reason that the deviation form of the general regression equation is used in preference to the raw score form.

Wert, Neidt, and Ahmann offer this explanation:

As in the case of single variable regression, the deviation score method can be used to calculate the prediction equation for multiple regression. The general equation in deviation form differs from the equation in raw score form in that the C term has again disappeared. Thus the number of normal equations necessary has been reduced by one. Whereas the raw score computations require one more normal equation than the number of prediction variables present, the deviation score method requires the same number of normal equations as prediction variables used. This labor-saving aspect is the principal advantage of the deviation score method over the raw score method, particularly as the number of prediction variables increases.

Only the coefficients of the total regression equations and of the within groups regression equations need be calculated. These coefficients provide the necessary data to obtain the sums of squares of residuals for the total sample and for within the treatment groups. The sum of squares of residuals between the treatment groups is the difference between these two sums. The coefficients of the between groups regression equations, therefore, do not need to be calculated.
Calculation of the coefficients of the total regression equation

To obtain these coefficients it is necessary to substitute in the four normal equations the appropriate deviation values of the sums of squares and of cross products for the total sample. Substitution in the normal equations of these values, which are contained in Tables XXVIII (page 119) and XXXV (page 132) yields:

\[1580.66667 = 1783.11565a_1 + 851.18367a_2 + 3164.43537a_3 + 1842.29932a_4\]
\[1934.00000 = 851.18367a_1 + 3460.93878a_2 + 5038.63265a_3 + 1908.12245a_4\]
\[5825.33333 = 3164.43537a_1 + 5038.63265a_2 + 29062.46259a_3 + 10188.06803a_4\]
\[2607.66667 = 1842.29932a_1 + 1908.12245a_2 + 10188.06803a_3 + 7784.42177a_4\]

Table XXXVIII indicates the values of the regression coefficients \(a_1, a_2, a_3,\) and \(a_4\) (for total) which result from the solving of these four simultaneous equations. The values were checked by substituting them in the original equations and obtaining identities.

Calculation of the coefficients of the within groups regression equation

To obtain these coefficients the appropriate deviation values, also contained in Tables XXVIII (page 119) and XXXV (page 132), of the sums of squares and of cross products for within groups are substituted in the normal equations. The equations become:

\[1589.98922 = 1747.87750a_1 + 885.34822a_2 + 3091.41102a_3 + 1841.22574a_4\]
\[1924.96148 = 885.34822a_1 + 3427.81510a_2 + 5109.43220a_3 + 1909.1633a_4\]
\[5844.65254 = 3091.41102a_1 + 5109.43220a_2 + 28911.13347a_3 + 10185.84322a_4\]
\[2607.95070 = 1841.22574a_1 + 1909.1633a_2 + 10185.84322a_3 + 7784.38906a_4\]
### TABLE XXXVIII

REGRESSION COEFFICIENTS OF THE TOTAL REGRESSION EQUATION AND THE WITHIN GROUPS REGRESSION EQUATION

<table>
<thead>
<tr>
<th>Regression Coefficient</th>
<th>Total Regression Equation</th>
<th>Within Groups Regression Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>.585892389</td>
<td>.608249961</td>
</tr>
<tr>
<td>$a_2$</td>
<td>.287384266</td>
<td>.271002746</td>
</tr>
<tr>
<td>$a_3$</td>
<td>.07888538</td>
<td>.084037579</td>
</tr>
<tr>
<td>$a_4$</td>
<td>.022633764</td>
<td>.014727310</td>
</tr>
</tbody>
</table>

Table XXXVIII indicates also the values of the regression coefficients $a_1$, $a_2$, $a_3$, and $a_4$ (for within groups) which result from the solving of these four simultaneous equations. The values were checked as in the manner of the previous solutions.
VII. CALCULATION OF THE SUMS OF SQUARES OF RESIDUALS

The sixth step in the application of the covariance technique to the present problem is to calculate the sums of squares of residuals, otherwise known as the errors of estimate.

As previously shown, the sum of squares of residuals is obtained by subtracting the sum of squares due to regression from the original sum of squares of the criterion. This relationship is represented by the formula:

\[
\text{Sum of squares of residuals} = \sum y^2 - (a_1 \sum x_1 y + a_2 \sum x_2 y + a_3 \sum x_3 y + a_4 \sum x_4 y)
\]

The sum of squares of residuals is based upon the variation remaining in Y after due allowance has been made for the regression of Y on each of the X variables. Through covariance, therefore, a statistical control has been maintained over those unmatched pupil abilities which purport to be measured by the tests selected.

Exercising this statistical control over these variables, which are considered to be the most important ones in influencing the pupils' performance on the criterion, permits a precise evaluation of the treatment effects.

Substituting in the above equation the appropriate values obtained from Tables XXVIII (page 119), XXXV (page 132), and XXXVIII (page 143) results in the following:
Sum of squares of residuals for total = 3702.66667 - (.585892389)(1580.66667)
= 3702.66667 - (.287384266)(1934.00000)
= 3702.66667 - (.078888583)(5825.33333)
= 3702.66667 - (.022633764)(2607.66667)
= 3702.66667 - 926.10057149897463
= 3702.66667 - 555.801170.44400000
= 3702.66667 - 459.55229190637139
= 3702.66667 - 59.02131199944588
= 3702.66667 - 2000.47535
= 1702.19132

Sum of squares of residuals for within = 3700.20031 - (.608249961)(1589.98922)
= 3700.20031 - (.271002746)(1924.96148)
= 3700.20031 - (.084037579)(5844.65254)
= 3700.20031 - (.014727310)(2607.95070)
= 3700.20031 - 967.1108810542042
= 3700.20031 - 521.66984702422408
= 3700.20031 - 491.17044955780066
= 3700.20031 - 38.40809842361700
= 3700.20031 - 2018.35928
= 1681.84103

Since the coefficients of regression between groups were not calculated, the sum of squares of residuals between groups are obtained by finding the difference between the two sums of squares already calculated, thus:

\[
\begin{align*}
\text{Sum of squares of residuals for total} & \quad \text{Sum of squares of residuals for within} \\
\text{Sum of squares of residuals between groups} &= \text{Sum of squares of residuals between groups}
\end{align*}
\]

Substituting the appropriate values produces the following:

\[
1702.19132 - 1681.84103 = 20.35029
\]

Table XXXIX contains a summary of the computation of the sums of squares of residuals.
### TABLE XXXIX

**SUMMARY OF SUMS OF SQUARES OF RESIDUALS**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares of Criterion</th>
<th>Sum of Squares due to Regression</th>
<th>Sum of Squares of Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3702.66667</td>
<td>2000.47535</td>
<td>1702.19132</td>
</tr>
<tr>
<td>Within</td>
<td>3700.20031</td>
<td>2018.35928</td>
<td>1681.84103</td>
</tr>
<tr>
<td></td>
<td>2.46636</td>
<td>20.35029</td>
<td></td>
</tr>
</tbody>
</table>

An examination of this table confirms the general accuracy of the preliminary analysis contained in Subsection V of this chapter.

The nature of the correlation between groups has made the sum of squares of residuals between groups considerably larger than the original sum of squares of the criterion.

In addition, the degree of the correlation within groups (.75) has made the sum of squares of residuals within groups less than one-half as large as the original sum of squares of the criterion.

The effect upon the F value of these adjustments in the original sums of squares of the criterion is seen in the summary of the analysis of covariance contained in Subsection VIII.
VIII. CALCULATION OF F VALUE AND APPLICATION OF TEST OF SIGNIFICANCE

The seventh step in the application of the covariance technique to the present problem is to calculate the F value and apply the test of significance to the adjusted group means.

The analysis of covariance of the performance of the two treatment groups on the criterion variable Y is summarized in Table XL.

This analysis may be compared with the preliminary analysis of the Y means contained in Table XXXIII (page 124), where no allowance was made for initial differences between the groups in the control variables.

In presenting a comparison of the two tables, an explanation should be made concerning the change in the degrees of freedom. In the analysis of covariance an additional degree of freedom was lost for each of the four prediction variables through the reduction in variability imposed by the calculation of the regression coefficients for total and within. This reduces the degrees of freedom for each of these sources of variation to 142 and 141 respectively. Since a new regression coefficient was not calculated in obtaining the adjusted sum of squares between groups, no additional degree of freedom is lost.

It was stated previously that in the present problem, where there is little difference between the means of the two treatment groups
### TABLE XL

ANALYSIS OF COVARIANCE OF PERFORMANCE OF THE TWO TREATMENT GROUPS ON THE CRITERION VARIABLE Y

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Sum of squares of residuals</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>142</td>
<td>1702.19132</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within groups</td>
<td>141</td>
<td>1681.84103</td>
<td>11.9280</td>
<td></td>
</tr>
<tr>
<td>Adjusted Means</td>
<td>1</td>
<td>20.35029</td>
<td>20.3503</td>
<td>1.706</td>
</tr>
</tbody>
</table>

\[
F_{1, 141} = \frac{20.3503}{11.9280} = 1.706
\]

From Table F

\[
df = 1/141
\]

F at .05 level = 3.91
F at .01 level = 6.81

In each variable, the primary purpose of the analysis of covariance is to increase the precision of the test of significance. An examination of Table XL reveals the extent to which the analysis fulfilled this purpose both through increasing the mean square between groups, and through decreasing the error mean square within groups.

It will be noted that the mean square between groups (the numerator of the variance ratio) has been increased from 2.4664 in the original analysis of variance to 20.3503 in the final analysis of covariance. At the same time, the error mean square within groups (the denominator of the variance ratio) has been decreased from 25.5186 to 11.9280.
In the latter case, where a formula exists by which this reduction may be estimated on the basis of known correlations, the extent of the adjustment in the error variance was precisely anticipated. While the resulting F value (1.706) still falls short of significance at .05, it represents an increase in the precision of the experiment of over 17 times the F value obtained in the original analysis of the criterion means.

However, since the difference in the adjusted criterion means remains insignificant despite the statistical control of the X variables, it can be concluded with reasonable certainty that the difference which does exist is due to sampling fluctuations rather than to a real treatment effect.

Therefore, the statistical analysis contained in this chapter confirms the null hypothesis. This hypothesis states that pupils who are taught with the use of certain specified manipulative materials in the manner prescribed by this experiment achieve an understanding of decimal fractions that is not significantly different from the achievement of pupils who are taught with the use of visualization materials which bear characteristics similar to those of manipulative aids in all details except the capacity to be manipulated.

IX. ASSUMPTIONS UNDERLYING THE DERIVATION OF ANALYSIS OF COVARIANCE

Several assumptions underlie the application of the analysis of covariance to the present problem. To be able to draw valid conclusions
respecting the effect upon the criterion of the teaching treatments, it is necessary that the assumed conditions actually exist in the design and conduct of the experiment. Wert, Neidt, and Ahmann emphasize this necessity: "The more the data in an investigation depart from the strict fulfillment of the assumptions the more likely is the investigator to reach erroneous conclusions". 23

Lindquist lists these assumptions as follows:

(1) The subjects in each treatment group were originally drawn either (a) at random from the same parent population, or (b) selected from the same parent population on the basis of their X-measures only - the selection being random with reference to all other factors for any given value of X.

(2) The X-measures are unaffected by the treatments.

(3) The criterion measures for each treatment group are a random sample from those for a corresponding treatment population.

(4) The regression of Y on X is the same for all treatment populations.

(5) This regression is linear.

(6) The distribution of adjusted scores for each treatment population is normal.

(7) These distributions have the same variance.

(8) The mean of the adjusted scores is the same for all treatment populations. 24

Referring to these conditions which establish the validity of the procedure, Lindquist writes:

Judging by past applications of the method of analysis of covariance in educational and psychological research, the assumptions underlying the test of the hypothesis of equal treatment effects are, in general, in greater need of critical
attention than is true with most, if not all, of the designs previously considered. Generally the method has been employed with little regard to the conditions under which the test is valid, and instances are numerous in which one or more of the conditions have clearly not been satisfied.\textsuperscript{25}

The same author then deals specifically with each assumption.

Dealing with Assumption 1, he states:

The first condition, concerning the manner of selection of the treatment groups, has perhaps most often been violated with serious consequences.\textsuperscript{26}

Lindquist describes one misconception which contributes to this violation:

\ldots they (experimenters) seem to have assumed that the method eliminates the effects of any systematic differences that may have existed originally among the treatment groups, even though some of these differences may be quite independent of the X variable employed.\textsuperscript{27}

He then presents two examples to illustrate unwise reliance upon analysis of covariance to remove systematic differences. These examples involve the use of analysis of covariance in an experimental comparison of three ways of teaching fourth grade arithmetic.

The first example is one in which

\ldots throughout the first semester the classes had had different arithmetic teachers, who had not only differed in personal effectiveness but also had used somewhat different methods of teaching arithmetic. Suppose the teacher of the class that was later to use experimental Method A used a method much like Method A, so that when the experiment began the pupils were able at once to use the experimental method with near maximum effectiveness. Suppose, however, that the teacher of the class that was later to use Method B had used a method which conflicted with Method B, so that considerable time was required early in the experiment before the pupils were able to use this method effectively. In this case, no "adjustments" based on initial intelligence test scores, or even on initial arithmetic achievement test scores, could possibly account for the effects of these differences upon the final adjusted means of the treatment groups.\textsuperscript{28}
In the present study careful precautions were observed in the class selection to avoid the inadvertent introduction into the experiment of the inadequately controlled systematic differences described by Lindquist.

The three main criteria for the selection of the five classes were based primarily on the qualifications of the teachers involved. Among these criteria due importance was given to selecting teachers who had been following reasonably conventional methods in their everyday teaching practices and who, though enthusiastic, were nevertheless disinterested in the manipulative and visualization methods of teaching decimal understanding.

The experimenter is unaware of any characteristics in the performance of the five teachers selected which could possibly work to the advantage or disadvantage of any class participating in the experiment.

No need existed, therefore, to introduce any of the control variables for the planned or incidental purpose of imposing invalid controls over any of the systematic differences noted by Lindquist in the foregoing quotations.

The second example is one in which

... the classes were originally selected not at random but so as to differ markedly with reference to some trait or characteristic related to the criterion variable in the experiment. Suppose, for example, that the classes had been selected according to ability and interest, that the abler and more industrious students had been assigned to one class and the least able to another, and that appropriate modifications in instruction had been used with these classes during the first semester. Suppose then that an initial achievement provided the X-measures used in the analysis of covariance. In this case, not only
would Assumption 1 would be invalid, but differences in regression (Assumption 4) and in variability of adjusted scores (Assumption 7) or even differences in the nature of the regression (Assumption 5) might well be expected. Nevertheless, many applications of this type also may be found reported in the research literature.  

In the present study Subsection II of this chapter contains the report of the analysis of variance which was applied to each of the independent variables $X_1$, $X_2$, $X_3$, and $X_4$. The F values obtained in these analyses, and shown in Tables XXIX to XXXII inclusive, (pages 120 and 121), are 2.293, 1.401, 0.759, 0.0006, respectively. The F value required for significance with the given number of degrees of freedom is 3.91 at the .05 level. These data support the hypothesis that the scores of both treatment groups in all four variables are in reality random samples drawn from the same normally distributed and homogeneous population, and that the means between the groups in each variable differ only through the fluctuations of sampling.

Statistical evidence is thus available to assure the validity of Assumption 1.

Dealing next with Assumption 2, Lindquist writes:

If the X-measures are taken at the beginning of the experiment or before, they could obviously not be affected by the treatments no matter what X may represent.  

In the present study each of the four X-measures was obtained before the administration of the teaching methods. On this account the X-measures are assuredly unaffected by the treatments.  

Lindquist then concludes his discussion of the importance of the assumptions underlying the test of significance of the treatment effect:
Of the remaining assumptions, perhaps the most critical in practice is the assumption (Condition 4) that the regression of Y on X is the same for all treatment populations. Decisions concerning the validity of the other assumptions—linearity of regression, normality of distribution, and homogeneity of variance—must generally represent judgments based on \textit{a priori} considerations like those discussed in earlier chapters, since available statistical tests of the validity of these assumptions are both low in power and difficult to apply. A statistical test of homogeneity of regression, however, is readily available...

This statistical test of homogeneity of regression, performed to satisfy Assumption 4, is presented in the following subsection.

\textbf{X. STATISTICAL TEST OF HOMOGENEITY OF REGRESSION}

Regression refers to a correlation relationship between the criterion variable and each of the independent variables. From such correlations it is possible to determine the relative weight with which each independent variable "enters in" or contributes to the criterion, independently of the other factors.

To be homogeneous this weight which each independent variable contributes to the criterion must be the same within the limits of sampling error for all treatment groups.

To satisfy Assumption 4 in the present study, homogeneity in this respect between the two treatment groups must be proven for four pairs of variables, namely: $X_1Y$, $X_2Y$, $X_3Y$, and $X_4Y$. However, to establish this proof it is considered sufficient to test the homogeneity of regression between the two treatment groups of only two of these pairs, namely: $X_1Y$ and $X_4Y$. 
These pairs were selected because, as shown in Table XLI, they contribute, respectively, the most and the least to the proportion of the entire variance accounted for by the use of the complete battery of four variables.

The data in Table XLI is obtained from the calculations of the sum of squares of residuals for within which were reported on page 145. It may be noted that the within group regression variance offers the most nearly unbiased estimate of the regression of Y on the X variables because it is free from any influence of systematic differences in the means of the two treatment groups.

**TABLE XLI**

WITHIN GROUP VARIANCE ACCOUNTED FOR BY THE USE OF EACH OF FOUR INDEPENDENT VARIABLES: X₁, X₂, X₃, X₄.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name of Variable</th>
<th>Variance</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁</td>
<td>Initial Understanding of Concepts of Decimal Fractions</td>
<td>967.111</td>
<td>.479</td>
</tr>
<tr>
<td>X₂</td>
<td>Computational Ability in Decimal Fractions</td>
<td>521.670</td>
<td>.259</td>
</tr>
<tr>
<td>X₃</td>
<td>Mental Ability (Otis Test)</td>
<td>491.170</td>
<td>.243</td>
</tr>
<tr>
<td>X₄</td>
<td>Reading Ability (Stanford Test)</td>
<td>38.408</td>
<td>.019</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>2018.359</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Specifically, the problem involved in this subsection is to determine whether the regression coefficients of the two pairs of variables $X_1^Y$ and $X_4^Y$ differ significantly in the two treatment groups. This involves the testing of two hypotheses which, stated in terms of beta coefficients, are:

$$\beta_{E_{1E}X_1} = \beta_{C_{1C}X_1} \quad \text{and} \quad \beta_{E_{4E}X_4} = \beta_{C_{4C}X_4}$$

If these null hypotheses remain tenable in the test of significance, it may be said that the values of $b_{y_{E_{1E}X_1}}$ and $b_{y_{C_{1C}X_1}}$ do not differ significantly, and neither do the values of $b_{y_{E_{4E}X_4}}$ and $b_{y_{C_{4C}X_4}}$. It will follow, then, that the regressions between the two treatment groups are homogeneous, and that Assumption 4 has been fulfilled.

**Test of the Null Hypotheses that $\beta_{E_{1E}X_1} = \beta_{C_{1C}X_1}$**

A statement of the purpose of each calculation involved in the testing of this hypothesis is included in the sequence of enumerated steps which follow:

1. Sum of squares of errors of estimate for the variables $X_1$ and $Y$ in the two treatment groups.

For the Experimental and Control groups independent calculations are made of the sum of squares based upon the variation remaining in $Y$ after due allowance has been made for the regression of $Y$ on $X_1$. The sum of these two calculations divided by the number of degrees of freedom produces the sum of squares of errors of estimate for
the variables $X_1$ and $Y$ in the two treatment groups. These calculations are presented by the following formula, in which the sums of squares and of cross products are expressed in deviation form:

$$s_{yx_1^2} = \frac{(\sum y_E^2 - \frac{(\sum x_{1E}y_E)^2}{\sum x_{1E}^2}) + (\sum y_C^2 - \frac{(\sum x_{1C}y_C)^2}{\sum x_{1C}^2})}{k_E + k_C - 4}$$

It may be noted that the sums of squares and cross products in this formula are the basic quantities referred to, respectively, on pages 114 and 128. They are obtained from raw scores in the following manner:

$$\sum x_1^2 = \sum x^2 - \frac{(\sum x_1)^2}{N}$$

$$\sum x_1y = \sum x_1y - \frac{\sum x_1\sum y}{N}$$

When these equivalents are substituted in the deviation formula shown above, the raw score formula shown on the first line of page 159 is obtained.

The necessary raw score data is contained in Tables XXVII (page 118) and XXXIV (page 130).

The denominator in the formula refers to the degrees of freedom available for the sum of squares being calculated. Two degrees
of freedom are lost due to the restrictions imposed by $y_E^2$ and $y_C^2$. Two additional degrees are lost through the calculations of the regression coefficients $b_{y_E x_{1E}}$ and $b_{y_C x_{1C}}$.

The complete calculations, shown on the next page result in the following:

$$s_{yx_1}^2 = 15.661343$$

2. Standard error of estimate.

This is a measure of the average errors of estimate or prediction. It represents the scatter of the $Y$ values around the regression line, and is found by taking the square root of the variance obtained in the foregoing step.

$$s_{yx_1} = \sqrt{15.661343} = 3.957$$

3. Regression Coefficients of $y$ on $x_1$ for the Experimental and Control Groups.

The regression coefficient of $y$ on $x_1$ may be written

$$b_{yx_1} = \frac{\sum x_1 y}{\sum x_1^2}$$

At this point certain data obtained in the calculations of Step 1 are summarised for convenience in Table XLII. These data are used in the present step and the one immediately following.
Step 1 in the test of the null hypothesis that $\beta_{Y_{1E}} = \beta_{Y_{1C}}$

**Calculation of Sum of Squares of Errors of Estimate for the Variables $X_1$ and $Y$ in the Two Treatment Groups**

\[
S_{YX_1}^2 = \left\{ \left( \Sigma Y_E^2 - \frac{(\Sigma Y_E)^2}{k_E} \right) - \left( \frac{\Sigma (X_{1E}Y_E - \Sigma X_{1E}Y_E)}{k_E} \right)^2 \right\} + \left\{ \left( \Sigma Y_C^2 - \frac{(\Sigma Y_C)^2}{k_C} \right) - \left( \frac{\Sigma (X_{1C}Y_C - \Sigma X_{1C}Y_C)}{k_C} \right)^2 \right\}
\]

\[
S_{YX_1}^2 = \left\{ \begin{array}{l} 
13605 - \frac{(855)^2}{59} - \left( \frac{7077 - (460)(855)}{59} \right)^2 \end{array} \right\} + \left\{ \begin{array}{l} 
20298 - \frac{(1252)^2}{88} - \left( \frac{12191 - (774)(1252)}{88} \right)^2 \end{array} \right\}
\]

\[
S_{YX_1}^2 = \left\{ \begin{array}{l} 
1214.746 - \frac{(410.898)^2}{531.559} \end{array} \right\} + \left\{ \begin{array}{l} 
2485.455 - \frac{(1179.091)^2}{1216.318} \end{array} \right\}
\]

\[
S_{YX_1}^2 = \left\{ \begin{array}{l} 
897.120 \end{array} \right\} + \left\{ \begin{array}{l} 
1342.452 \end{array} \right\}
\]

\[
S_{YX_1}^2 = 15.661343
\]
TABLE XLII
SUMS OF SQUARES AND CROSS PRODUCTS OF VARIABLES $x_1$ AND $y$, CALCULATED INDEPENDENTLY FOR EXPERIMENTAL AND CONTROL GROUPS

<table>
<thead>
<tr>
<th>Sum of Squares</th>
<th>Sum of Cross Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum x_{1E}^2$</td>
<td>$\sum x_{1E}y_E$</td>
</tr>
<tr>
<td>531.559</td>
<td>410.898</td>
</tr>
<tr>
<td>$\sum x_{1C}^2$</td>
<td>$\sum x_{1C}y_C$</td>
</tr>
<tr>
<td>1216.318</td>
<td>1179.091</td>
</tr>
</tbody>
</table>

Using the appropriate data in the regression coefficient formula results in

\[
b_{yE}x_{1E} = \frac{410.898}{531.559} = .773
\]

\[
b_{yC}x_{1C} = \frac{1179.091}{1216.318} = .969
\]

4. Standard Error of the Regression Coefficients $b_{yE}x_{1E}$ and $b_{yC}x_{1C}$

This is represented by the formula:

\[
s_{b_{yE}x_{1E}} = \frac{s_{y1}}{\sqrt{\sum x_{1}^2}}
\]
Using the appropriate data in this formula results in

\[
 s_{b_{y_{E1E}}} = \frac{3.957}{\sqrt{531.559}} = .172 \\
 s_{b_{y_{C1C}}} = \frac{3.957}{\sqrt{1216.318}} = .113
\]

5. Standard error of the difference between the regression coefficients \( b_{y_{E1E}} \) and \( b_{y_{C1C}} \).

This is represented by the formula:

\[
 s_{b_{1-b_{2}}} = \sqrt{s_{b_{y_{E1E}}}^2 + s_{b_{y_{C1C}}}^2} = \sqrt{(.172)^2 + (.113)^2} = .206
\]

6. Test of significance of the difference between the regression coefficients \( b_{y_{E1E}} (.773) \) and \( b_{y_{C1C}} (.969) \).

The t value is obtained by dividing the difference between the regression coefficients by the standard error of the difference between the regression coefficients.
This is represented by the formula:

\[ t = \frac{b_{y_E^X_{1E}} - b_{y_C^X_{1C}}}{s_{b_1} - s_{b_2}} \]

\[ = \frac{.969 - .773}{.206} \]

\[ = .951 \]

This t value may be evaluated by entering the t table with\( k_E + k_C - 4 = 143 \) degrees of freedom. For the two-tailed test of significance of the null hypothesis that\( \beta_{y_E^X_{1E}} = \beta_{y_C^X_{1C}} \), a t of 1.976 would be required at the .05 level of significance, while 2.609 would be required at the .01 level. Since the observed value of t is only .951, the null hypothesis remains tenable. It may be said, therefore, that the regression coefficients \( b_{y_E^X_{1E}} \) and \( b_{y_C^X_{1C}} \) do not differ significantly.

Test of the Null Hypothesis that\( \beta_{Y^E_4} = \beta_{Y^C_4} \)

In the testing of this hypothesis the procedure is identical to that followed in the treatment of the foregoing hypothesis.

1. Sum of squares of errors of estimate for the variables \( X_4 \) and \( Y \) in the two treatment groups.

The necessary raw score data is contained in Tables XXVII (page 118) and XXXIV (page 130).
The complete calculations, shown on the next page, result in the following:

\[ s_{yx_4}^2 = 19.755776 \]

2. Standard error of estimate.

\[ s_{yx_4} = \sqrt{19.755776} \]

= 4.445

3. Regression Coefficients of \( y \) on \( x_4 \) for the Experimental and Control Groups.

At this point certain data obtained in the calculations of Step 1 are summarized for convenience in Table XLIII. These data are used in the present step and the one immediately following.

**TABLE XLIII**

SUMS OF SQUARES AND CROSS PRODUCTS OF VARIABLES \( x_4 \) AND \( y \), CALCULATED INDEPENDENTLY FOR EXPERIMENTAL AND CONTROL GROUPS

<table>
<thead>
<tr>
<th>Sum of Squares:</th>
<th>Sum of Cross Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum x_{4E}^2 ) = 2847.525</td>
<td>( \sum x_{4E}y_E ) = 903.678</td>
</tr>
<tr>
<td>( \sum x_{4C}^2 ) = 4936.864</td>
<td>( \sum x_{4C}y_C ) = 1704.273</td>
</tr>
</tbody>
</table>
Step 1 in the test of the null hypothesis that $\beta_{Y_E X_4E} = \beta_{Y_C X_4C}$

CALCULATION OF SUM OF SQUARES OF ERRORS OF ESTIMATE FOR THE VARIABLES $X_4$ AND $Y$ IN THE TWO TREATMENT GROUPS

$$S_{YX_4}^2 = \frac{\left\{ \sum Y_E^2 - \left( \sum Y_E \right)^2 \right\} \left( \sum X_{4E}^2 - \left( \sum X_{4E} \right)^2 \right) \sum X_{4E}^2 - \left( \sum X_{4E} \right)^2 \sum X_{4E} \sum Y_E}{k_E + k_C - 4}$$

$$S_{YX_4}^2 = \frac{\left\{ 13605 - \frac{(855)^2}{59} - \frac{18308 \cdot (1201)(855)^2}{59} \right\} \left( 27295 - \frac{(1201)^2}{59} \right) \sum X_{4E}^2 - \left( \sum X_{4E} \right)^2 \sum X_{4E} \sum Y_E}{k_E + k_C - 4}$$

$$S_{YX_4}^2 = \frac{\left\{ 1214.746 - \frac{(903.678)^2}{2847.525} \right\} \left\{ 2485.455 - \frac{(1704.273)^2}{4936.864} \right\}}{143}$$

$$S_{YX_4}^2 = \frac{\left\{ 927.959 \right\} \left\{ 1897.117 \right\}}{143}$$

$$S_{YX_4}^2 = 19.755776$$
Using the appropriate data in the regression coefficient formula results in

\[ b_{yE^4E} = \frac{903.678}{2847.525} = .317 \]

\[ b_{yC^4C} = \frac{1704.273}{4936.864} = .345 \]

4. Standard Error of the Regression Coefficients \( b_{yE^4E} \) and \( b_{yC^4C} \).

Using the appropriate data in the formula results in

\[ s_{b_{yE^4E}} = \frac{4.445}{2847.525} = .083 \]

\[ s_{b_{yC^4C}} = \frac{4.445}{4936.864} = .063 \]

5. Standard error of the difference between the regression coefficients \( b_{yE^4E} \) and \( b_{yC^4C} \).

Using the appropriate data in the formula results in

\[ s_{b_1-b_2} = \sqrt{(.083)^2 + (.063)^2} = .104 \]
6. Test of significance of the difference between the regression coefficients $b_{y_{4E}^x}$ (.317) and $b_{y_{4C}^x}$ (.345).

Using the appropriate data in the formula results in

$$t = \frac{.345 - .317}{.104}$$

$$= .269$$

This $t$ value may be evaluated by entering the $t$ table with $k_E + k_C - 4 = 143$ degrees of freedom. For the two-tailed test of significance of the null hypothesis that $\beta_{y_{4E}^x} = \beta_{y_{4C}^x}$, a $t$ of 1.976 would be required at the .05 level of significance, while 2.609 would be required at the .01 level. Since the observed value of $t$ is only .269, the null hypothesis remains tenable. It may be said, therefore, that the regression coefficients $b_{y_{4E}^x}$ and $b_{y_{4C}^x}$ do not differ significantly.

A non-significant difference between treatment groups has thus been proven for the regression coefficients of $X_1Y$ and $X_4Y$, two of the four pairs of variables involved in this experiment.

It will be recalled that the independent variables $X_1$ and $X_4$ made, respectively, the greatest and the least contribution to the prediction of the criterion. Accordingly, the $t$ value involving each of these variables with the criterion are .951 ($X_1Y$) and .269 ($X_4Y$).
The t values of the other two pairs of variables, \(X_2Y\) and \(X_3Y\), will lie between these two extreme t values. Consequently, the difference between the treatment groups of the regression coefficients of \(X_2Y\) and \(X_3Y\), taken separately, are not significant.

It may be said, therefore, that the weight which each independent variable contributes to the criterion variable is the same within the limits of sampling error for both of the treatment groups in this experiment. This completes the test of homogeneity of regression.

The fulfillment of all the assumptions noted by Lindquist makes it possible to draw valid conclusions respecting the effect upon the criterion of using certain manipulative materials in group instruction. These conclusions are contained in Chapter VI.

XI. THE USE OF A BINARY COMPUTER (ALWAC III-E) IN PERFORMING AUTOMATIC COVARIANCE COMPUTATIONS

The computations involved in the covariance analysis reported in the preceding subsections of this chapter were performed with the assistance of an automatic Monroe computing machine. The raw data, in addition to being treated in this way, were processed through the electronic binary computer, Alwac III-E.

The Alwac III-E operates on the binary counting system. In this system each digit position assumes only two discrete values, 0 and 1. Numbers of higher value are indicated by increasing the next most
significant digit and repeating the sequence.

The most time-consuming part of the computational procedure is the initial preparation of a programme, containing the technical details by which the computer executes the required processes. Once prepared, however, the programme provides for the treatment of any data.

The programme used in the treatment of the present data was written by Dr. T. Hull of the Department of Mathematics, University of British Columbia. It was designed to accommodate a maximum of eight variables in the performance of the various covariance calculations.

As the first step in dealing with the present data, a Flexowriter was used to punch on tapes the pupils' scores in the five variables. On the tapes these scores appear in the binary form. The next step was to process the punched tapes through another Flexowriter which operates in conjunction with the computer.

The resulting computations, typed by this Flexowriter and completed within approximately five minutes, produced automatically the following results: the means and standard deviations of each of the five variables, a five by five correlation matrix and covariance matrix, and the coefficients for the regression of the first on the remaining four variables.

Except for minor discrepancies which are attributable to differences in the formulae employed, the results obtained in the Alwac computations agree with those that have already been reported in this chapter.

The only function performed by the author was to operate the first mentioned Flexowriter to record the raw scores.
FOOTNOTES

1 Supra, p. 50


7 Ibid

8 Most statistic textbooks reproduce this table which is taken from Snedecor: Statistical Methods, Iowa State College Press, Ames, Iowa.

9 Garrett, op. cit., p. 154

10 Infra, pp. 149-154


12 E. F. Lindquist, Statistical Analysis in Educational Research, pp. 183-184


14 While the correlation of each of the ten combinations of variables is involved in the calculation of the regression coefficients, it is the correlation of only the criterion variable with each of the independent variables (four pairs in all) which is involved in the calculation of the sum of squares due to regression. It is the estimation of this sum which is the concern of this subsection.
Since the intrinsic correlation of $x_1y$ is greater than unity, the uncorrected coefficient of .63 was used to arrive at this fraction.


Lindquist, *Design and Analysis of Experiments in Psychology and Education*, p. 327.

This derivation procedure, in greater detail, is contained in Wert, Neidt, and Ahmann, *op. cit.*, p. 241.

Ibid

An explanation, from which the derivation of this formula may be deduced, is contained in Wert, Neidt, and Ahmann, *op. cit.*, pp. 235 et seq.

Supra, p. 111

Wert, Neidt, and Ahmann, *op. cit.*, p. 183

Lindquist, *Design and Analysis of Experiments in Psychology and Education*, p. 323.

Ibid, p. 328

Ibid,

Ibid.

Lindquist, *Design and Analysis of Experiments in Psychology and Education*.

Supra, pp. 48 et seq.

Lindquist, *Design and Analysis of Experiments in Psychology and Education*, pp. 329 et seq.

Ibid, p. 330

Since the two-tailed test of significance of this $t$ value is based upon both tails (positive and negative) of the distribution of $t$, there is the probability of obtaining a positive or negative $t$. In the formula the two terms of the numerator may be arranged in either order. To obtain a positive numerator in the present case, the $b \sqrt{C \times 10}$ term has been made the minuend.

The abridged $t$ table contained in most statistics textbooks is from Table IV of Fisher: *Statistical Methods for Research Workers*, (Edinburgh: Oliver & Boyd, Ltd.) Additional entries (over 30 df) are taken from Snedecor: *Statistical Methods*, (Ames, Iowa: Iowa State College Press).
CHAPTER VI
SUMMARY AND CONCLUSIONS

I. SUMMARY

**Purpose of the experiment.** This experiment was undertaken to secure data upon which to determine the effectiveness of the group instruction use of certain manipulative aids in contributing to an understanding of particular decimal concepts.

Since manipulability of a concept is the most essential characteristic of manipulative aids, this study seeks to determine the effectiveness of these particular aids by isolating this characteristic as the experimental variable.

**Background and justification.** The movement toward meaningful arithmetic learning emphasizes the need to find teaching materials which make effective contributions to the pupils' understanding of concepts.

This study may justifiably be included in the quest for these materials because, though subjective opinions are common, objective studies involving manipulative aids are not only few in number but are not designed specifically to determine their effectiveness.

**Problems proposed by the investigation.**

1. Do pupils who are taught with the use of certain manipulative aids in the manner prescribed by this experiment achieve an
understanding of decimal fractions that is significantly different from the achievement of pupils who are taught with the use of visualization materials similar to the manipulative aids in all details except manipulability?

2. What is the relative weight with which each of the four independent variables, initial understanding of the processes involved in decimal fractions, computational ability in decimal fractions, mental ability, and reading ability, "enters into", or contributes to, the criterion variable independently of the treatment groups?

3. When the concomitant influences represented by the four independent variables referred to in Problem 2 are held constant by means of analysis of covariance, do pupils taught with the use of manipulative aids achieve an understanding significantly different from the understanding achieved by pupils taught with the use of visualization materials?

4. For which treatment group — experimental or control — is there the higher correlation between achievement on the criterion variable and achievement on each of the independent variables?

Procedure. The effectiveness of the manipulative materials was determined by comparing the achievement on a criterion measure of an experimental group of 59 subjects and a control group of 88 subjects. These groups were composed, respectively, of two and three classes, which were first of all selected in accordance with certain criteria, then matched on the basis of size, and finally, assigned at random to each treatment group.
Teaching treatments, prescribed by a series of 11 lessons (including 3 review lessons) for each group, were identical except with respect to the materials of instruction. These materials were, in turn, intended to possess similar characteristics except with respect to that of manipulability. This characteristic emerged, therefore, as the experimental variable.

The criterion measure is Farquhar's Test of Understanding of the Processes Involved in Decimal Fractions.

The hypothesis tested is that no significant difference exists between the achievement of the two treatment groups on the criterion variable.

By means of a battery of four tests (Farquhar's Test, a Decimal Fraction Computation Test, the Otis Test of Mental Ability, and Stanford Reading Test), measures were obtained of pupil abilities in areas which were considered to influence achievement on the criterion. The efficiency of each test for its purpose was fully investigated. An analysis of variance was made of the results of each of these tests. In all four cases the differences between the treatment groups were found not to exceed those which could be attributed to fluctuations of sampling. These differences were then controlled statistically by the analysis of covariance, which allows for the correlation between criterion and independent variable scores.

The resulting F value, though substantially larger than the F value obtained in the original analysis of variance of the criterion
variable, remained insignificant at the .05 level, and the null hypothesis was sustained.

II. CONCLUSIONS

Summary of results. Results of the experiment, stated as direct answers to the problems proposed by the investigation, are as follows:

1. There is no significant difference between the achievement on the criterion test of the pupils taught with the use of the manipulative materials and those taught with the use of the visualization materials. (The F value obtained in the analysis of variance is 0.097, while an F value of 3.91 is required at the .05 level of significance.)

2. Of the total influence which the four independent variables exerted upon the achievement on the criterion test, the percentage contributed by each variable, independently of treatments, is as follows: (1) initial understanding of the processes involved in decimal fractions - 48%; (2) computational ability in decimal fractions - 26%; (3) mental ability - 24%; (4) reading ability - 2%.

3. When the concomitant influences represented by the four independent variables are held constant by the statistical procedure of analysis of covariance, there is still no significant difference between the achievement on the criterion test of the pupils taught
with the use of manipulative materials and those taught with the use of visualization materials. As a result of holding constant these concomitant influences, however, the F value obtained in the analysis of covariance became 1.706.

4. Table XLIV shows for each treatment group the correlation between achievement on the criterion variable and achievement on each of the independent variables.

### TABLE XLIV

**PEARSON PRODUCT MOMENT COEFFICIENTS OF CORRELATION BETWEEN ACHIEVEMENT ON THE CRITERION TEST AND ACHIEVEMENT ON EACH OF THE INDEPENDENT VARIABLES, ARRANGED ACCORDING TO TREATMENT GROUPS**

<table>
<thead>
<tr>
<th>Group</th>
<th>$X_1 Y$</th>
<th>$X_2 Y$</th>
<th>$X_3 Y$</th>
<th>$X_4 Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>.51137</td>
<td>.46417</td>
<td>.65998</td>
<td>.48592</td>
</tr>
<tr>
<td>Control</td>
<td>.67816</td>
<td>.57482</td>
<td>.53172</td>
<td>.48654</td>
</tr>
</tbody>
</table>

It will be noted that the largest difference in correlations between the treatment groups is between $X_1$ and $Y$ (achievement on the initial test of understanding and achievement on the final test of understanding). Even this difference, when tested by transforming the r's into Fisher's z-function, was found to be non-significant.
(The procedures for calculating the Pearson Coefficients of Correlation and for determining the significance of the difference between correlations are shown in Appendix G.)

Neither group, therefore, has a significantly higher correlation between achievement on the criterion variable and achievement on each independent variable.

**Interpretation of results.** The first interpretation deals with the fact that the F value obtained in the analysis of covariance of the criterion variable (1.706) was larger than the F value obtained in the analysis of variance of the same variable. (0.097).

The control group achieved the higher mean in three of the four independent variables (Table XXVI - page 110), namely: initial understanding of the processes involved in decimal fractions (F value is 2.923), mental ability (F value is 0.759), and reading ability (F value is 0.0006).

The experimental group achieved the higher mean in the remaining independent variable, namely: computational ability in decimal fractions (F value is 1.401).

Despite this initial advantage of the control group (though is no case was the difference between group means significant), the experimental group at the end of the instructional period achieved the higher mean in the criterion test (F value is 0.097).
The initial understanding of decimal fractions, in which the largest difference between group means existed, was also the variable which was most predictive of achievement on the criterion, independently of the treatment groups. Therefore, it was the statistical control of this variable in particular which increased the final F value obtained in the analysis of covariance to 1.706.

The second interpretation deals with the correlations for both treatment groups between achievement on the criterion variable and achievement on each of the independent variables.

The lack of any significant difference between the various correlations of the two treatment groups shows that the level of the pupils' ability in respect to the four areas considered in no way determined the effectiveness of the manipulative aids in contributing to an understanding of decimal concepts.

Summary of conclusions. The data obtained from the investigation, leads to the following inferences and conclusions.

1. There is no advantage, or disadvantage, for an unselected group of Grade VII pupils in being taught the rationalization of specific decimal fraction concepts by group demonstration through the media of certain instructional materials which are concrete and movable as opposed to certain other materials which are static representations of these materials, and which are thereby intended to possess similar characteristics in all details except that of manipulability.

2. The manipulative materials used in this investigation are neither more nor less effective than the static representations as
media for conveying an understanding of specific decimal fraction concepts to Grade VII pupils of any particular capacity in the following areas: initial understanding of decimal fraction processes, computational ability in decimal fractions, mental ability, and reading ability.

3. It must not be inferred that any generalization concerning the effectiveness of these specific materials of instruction, which were used in this investigation exclusively by the teacher for group demonstration purposes, would be applicable also to similar materials if they were used in a teaching procedure in which the pupils themselves participated individually in the manipulative activity.

4. It must not be inferred that any generalization concerning the effectiveness of the specific manipulative aids used in this investigation, in a brief teaching assignment devoted exclusively to the rationalization of processes, would be applicable also to the same materials if they were used in a teaching assignment of longer duration, and/or a teaching assignment in which the emphasis on the WHY of the processes was taught concurrently with, or preceded, the emphasis on the HOW of the processes.

5. Independently of treatment groups, the achievement on the initial test of understanding of the processes involved in decimal fractions was the variable most predictive of achievement on the final test of understanding. Computational ability in decimal fractions and mental ability each shared approximately one-half the predictive
capacity of the initial test of understanding. Reading ability was a negligible predictor of achievement on the final test of understanding.

Implications of these conclusions and suggestions for further study. While these inferences and conclusions were warranted by the data, the complex nature of the teaching and learning assignments in the investigation necessitates that certain reservations be made with respect to these inferences and conclusions.

Teacher comments show that the presentation of certain concepts via manipulative devices caused difficulties in pupil learning. This indicates a need for further investigation in actual learning situations of better manipulative ways to represent arithmetical ideas simply and clearly.

Indirectly, teacher comments indicate that the manipulative materials may be more effective when teachers are trained specifically in the concomitant philosophy and instructional procedures relevant to this medium of conveying meanings.

Conclusions 1 and 2 should be accepted with the reservation that if the teachers and pupils had been more accustomed to this method of presenting the concepts the results may have been different. How much more effective these materials may be when the instructional period is longer and when teachers are trained in their use is a problem for further investigation.

Conclusions 3 and 4, submitted in the form of precautions against unwise inferences, may profitably be made the subjects for
future experimentation. One appropriate investigation would be to
determine the effectiveness of manipulative aids used in a teaching
procedure in which the pupils participated in the manipulative
activity. Another appropriate investigation would be to determine
the effectiveness of manipulative aids used in a teaching procedure
in which the HOW-WHY sequence was varied from that followed in the
present study.

The implications relating to these two conclusions were
stated in Subsection VIII of Chapter III.
BIBLIOGRAPHY

A. BOOKS


### B. DISSERTATION ABSTRACTS


### C. PERIODICALS


D. UNPUBLISHED THESIS


E. YEARBOOKS


### APPENDIX A

**COMMUNICATION OF ADMINISTRATIVE ARRANGEMENTS**

**TO THE TEACHERS PARTICIPATING IN THE STUDY**

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial letter to the teachers conveying information concerning the experiment</td>
</tr>
<tr>
<td>Orientation notes</td>
</tr>
<tr>
<td>Evaluation form</td>
</tr>
<tr>
<td>Summary of lesson objectives</td>
</tr>
<tr>
<td>Schedule of lessons</td>
</tr>
</tbody>
</table>
School Board Office,  
Box 66,  
Cloverdale, B.C.,  
February 11, 1957.

Dear

At last I am able to write to you in connection with the experiment in arithmetic about which Mr. Niedzielski spoke to you early in January. Meanwhile I have been continuing the preparations for it. Before writing to you I wanted to see the preparatory work reach a stage of completion where I could be reasonably sure that the arrangements would proceed in accordance with a plan. I believe that stage has now been reached.

First, let me say that the experiment is scheduled for the period extending from Thursday, March 21st to Wednesday, April 10th, instead of the latter two weeks of February as was originally planned.

While it is desirable that there should be no extensive treatment of decimals before that time, there is no reason why classes involved in the experiment should not proceed with some of the ordinary computation processes in decimals.

It is realized that the work in percentage depends in part upon facility in work with some decimal processes. Participation in the experiment should not impede normal progress in this phase of the Grade VII arithmetic.

Second, you will no doubt be interested in the way in which participating in the experiment would effect you personally.

1. I think you will enjoy it. Prior to the experiment the lessons and procedure will have been approved by the local authorities and by the U.B.C. College of Education. You can be assured that the project will be the result of a good deal of thought and advice from many people.

2. The experiment is a matter of current interest in educational research. It is concerned with the method of teaching decimal fractions meaningfully. The necessity of teaching for meaning has now been established by research, but the method of doing so is as yet quite unsupported. This experiment is an attempt to contribute some scientific conclusion, however, infinitesimal, to the existing body of knowledge in this field.

(Page two follows)
3. You will probably be interested in the labor involved from your point of view. The experiment consists of approximately 12-15 daily lessons to be taught to classes comprising two sections: one an experimental group and the other a control group. The experimental group will be composed of two Grade VII classes, and the control group will be composed of two, or possibly three, Grade VII classes.

The lessons for each group are complete in mimeograph form with all the teaching materials and work sheets and other tests supplied. No advance preparation is needed except a thorough reading of the content of each day's lesson as it is supplied on the form. To facilitate this preparation it will be necessary to hold two orientation meetings with each group by itself. Times which are mutually convenient for all will be arranged.

The lessons are one hour in length. Approximately half of this time will be spent in teaching or in exercises of a group work nature. Prepared assignments for individual seat work occupy the remaining half of the lesson. These assignments are designed so that they may be marked in class by the pupils themselves.

Prior to the experiment a group intelligence test, an achievement test, and another test relevant to the area of the experiment will be administered to each class. The marking of these tests will be done quite independently of the participants in the experiment. At the conclusion of the experiment another test will be administered which will also be marked and recorded independently of the participants.

In the interests of the experiment I feel that this is all the detail I can supply at this time. However, if you have any questions or comments concerning your own personal involvement in its administration, please do not hesitate to let me know and I shall be very glad to reply to them.

May I thank you for the expression given to Mr. Niedzielski of your willingness to participate in the experiment. I look forward with anticipation to your cooperation, and, I am quite certain that you will derive some benefit and satisfaction in the project.

Very sincerely yours,

George J. Greenaway
ORIENTATION NOTES

The purpose of this experiment in which you are engaged is to determine the advantage (or disadvantage) for an unselected group of Grade VII pupils of being taught in group situations with the use of instructional materials which are concrete and moveable as opposed to those which are merely static representations of these manipulative devices.

The one experimental variable, therefore, is the type of instructional materials used. Otherwise, the lesson procedures are identical. The various concomitant variables: the pupils' intelligence, reading ability, initial computational skill and initial understanding of decimal fractions, these will be controlled in the statistical analysis of the data.

In order to make identical for both groups the teaching that results from the prescribed lesson procedures, it is imperative that the instructions and time limits contained in the lesson plans be observed closely.

For the purpose of establishing uniform familiarity with these instructions two pre-instructional meetings for each group will be held. Finally, for the purpose of evaluating the experiment there will be a joint meeting of both groups immediately following the final testing.

In preparation for this concluding meeting it is necessary to make two requests:

First, a diary should be kept of your experiences in the teaching of these lessons. Whether reported upon verbally or in
writing at the final meeting, this will provide an instructive account of your own personal experiences in the teaching of these lessons. This account will make an invaluable contribution to the worth of this study.

Second, an evaluation form, which will be distributed near the end of the experiment, should be completed. This form represents a structured interrogation into some aspects of your experience with the teaching of these lessons. Its purpose is to ensure a report by each participant upon common areas of interest and concern in the performance of the experiment. The completion of this form will contribute further to the value of this study.
The following evaluation form is designed to obtain opinions upon certain fundamental issues involved in this experiment.

The practical experience you have had in the actual teaching of these lessons will make your opinions valuable in the final assessment of this problem.

You are invited, therefore, to be frank in your answers and comments. Apart from containing an indication of the group to which you belong, this form need reveal no further identification.

This form is intended to supplement, not to replace, whatever notes and observations you have accumulated in the diary of your experiences in the teaching of these lessons.

In Parts One and Two of this form the GREEN letters in the column at the left identify the lesson objectives bearing the same letter on the accompanying Summary of Lesson Objectives.

**PART ONE**

For the purpose of attaining each objective in this series of lessons, how effective were the teaching materials placed at your disposal?

<table>
<thead>
<tr>
<th>Lesson Objective</th>
<th>Very Effective</th>
<th>Reasonably Effective</th>
<th>Not Effective</th>
<th>Remarks</th>
</tr>
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<tbody>
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<tr>
<td>Lesson Objective</td>
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</tbody>
</table>
From the standpoint of the pupils' maturity and accumulation of experience with arithmetical concepts, is the end of Grade VII a suitable time to teach each of the objectives in this series of lessons?

<table>
<thead>
<tr>
<th>Lesson Objective</th>
<th>Very Suitable</th>
<th>Reasonably Suitable</th>
<th>Not Suitable</th>
<th>Remarks</th>
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</table>
This series of lessons contains a somewhat concentrated effort to teach the meaning of certain decimal concepts. If this effort were spread uniformly throughout the whole period of instruction on decimal fractions in Grades VI and VII, how do you think the final outcome would be effected?

<table>
<thead>
<tr>
<th>Lesson Objective</th>
<th>Very Suitable</th>
<th>Reasonably Suitable</th>
<th>Not Suitable</th>
<th>Remarks</th>
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</thead>
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**PART THREE**
SUMMARY OF LESSON OBJECTIVES

LESSON I

INTRODUCTION: THE HINDU-ARABIC DECIMAL SYSTEM OF NOTATION

Lesson Objectives of Part One

1. To present a brief history of the art of reckoning.

2. To convey an appreciation of the simplicity and convenience of our presently used Hindu-Arabic decimal system of notation as compared to earlier methods, especially the Roman system.

Lesson Objective of Part Two

3. To show visually the structure of the decimal number system.

LESSON II

IDENTIFICATION AND MEANING OF PLACE NAMES IN MIXED DECIMAL FRACTIONS

Lesson Objectives of Part One

1. To show that the decimal system of whole number notation may be extended to the right of the ONES' place.

2. To emphasize that the ONES' place is the centre of this extended system of notation, and that the other place names are symmetrical around it.

3. To provide a familiarization with the decimal fraction place names.

Lesson Objective of Part Two

4. To compare the relationship in size of the various positional values.

LESSON III

REDUCTION OF DECIMALS TO COMMON FRACTIONS

Lesson Objective of Part One

1. To consider decimals as a special form of common fractions having denominators of 10, 100, 1000 etc., that is, any power of 10.

Lesson Objective of Part Two

2. To show how decimal fractions indicate the numerator and denominator of equivalent common fractions.
Lesson Objective of Part Three
3. To provide practice in the reading and writing of decimal fractions.

LESSON IV
THE USE OF ZERO AS A PLACE HOLDER

Lesson Objective of Part One
1. To demonstrate the use of zero as a place holder.

Lesson Objective of Part Two
2. To demonstrate the use of zero as a terminal cipher.

LESSON V
CHANGING THE LOCATION OF THE DECIMAL POINT: ITS EFFECT ON THE VALUE OF THE EXPRESSION

Lesson Objective of Part One
1. To demonstrate the effect upon the value of a decimal fraction of moving the decimal point.

Lesson Objective of Part Two
2. To demonstrate the effect upon the location of the decimal point of multiplying or dividing a decimal fraction by a power of 10.

LESSON VI
ROUNDING DECIMAL FRACTIONS

Lesson Objective of Part One
1. To illustrate the significance of rounding decimal fractions.

Lesson Objective of Part Two
2. To demonstrate various applications of the rounding of decimal fractions.

Lesson Objective of Part Three
3. To indicate why UNLIKE decimal fractions must be changed to LIKE decimal fractions (that is, with the same understood denominator) in order that they may be added or subtracted.
LESSON VII
DIVISION INVOLVING DECIMAL FRACTIONS

Lesson Objective of Part One
1. To explain the significance of performing division involving a decimal fraction.

Lesson Objective of Part Two
2. To demonstrate the significance of moving the decimal point in performing divisions involving decimal fractions.

LESSON VIII
MISCELLANEOUS CONCEPTS INVOLVING DECIMAL FRACTIONS

Lesson Objective of Part One
1. To convey the significance of changing a common fraction to a decimal fraction.

Lesson Objective of Part Two
2. To illustrate the reason for the placement of the decimal point in the product obtained by the multiplication of decimal fractions.

Lesson Objective of Part Three
3. To develop an understanding of the importance in the addition of decimal fractions of aligning columns according to place value.

Note: The GREEN letters at the left indicate the consecutive enumeration of the various Lesson Objectives involved in this series of lessons.

In Parts One and Two of the accompanying evaluation form the GREEN letters in the column at the left identify the lesson objective bearing the same letter on this summary.
## SCHEDULE OF LESSONS

<table>
<thead>
<tr>
<th>DAY</th>
<th>DATE</th>
<th>LESSON NUMBER</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>13 May</td>
<td>Lesson I</td>
<td>Introductory Lesson: &quot;The Hindu-Arabic System of Notation&quot;.</td>
</tr>
<tr>
<td>Tuesday</td>
<td>14 May</td>
<td>Lesson II</td>
<td>&quot;Identification and Meaning of Place Names in Mixed Decimal Fractions&quot;.</td>
</tr>
<tr>
<td>Wednesday</td>
<td>15 May</td>
<td>Lesson III</td>
<td>&quot;Reduction of Decimals to Common Fractions&quot;.</td>
</tr>
<tr>
<td>Friday</td>
<td>17 May</td>
<td>Lesson IV</td>
<td>&quot;The Use of Zero as a Place Holder&quot;.</td>
</tr>
<tr>
<td>Monday</td>
<td>20 May</td>
<td>HOLIDAY</td>
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</tr>
<tr>
<td>Tuesday</td>
<td>21 May</td>
<td>Lesson V</td>
<td>&quot;Changing the Location of the Decimal Point; Its effect on the value of the Expression&quot;.</td>
</tr>
<tr>
<td>Wednesday</td>
<td>22 May</td>
<td>Lesson VI</td>
<td>&quot;Rounding Off Decimal Fractions&quot;.</td>
</tr>
<tr>
<td>Thursday</td>
<td>23 May</td>
<td>REVIEW LESSON</td>
<td>Cumulative Review of concepts previously taught in Lessons I to VI, plus recapitulation of Lesson Exercises.</td>
</tr>
<tr>
<td>Friday</td>
<td>24 May</td>
<td>Lesson VII</td>
<td>&quot;Division Involving Decimals&quot;.</td>
</tr>
<tr>
<td>Monday</td>
<td>27 May</td>
<td>Lesson VIII</td>
<td>&quot;Reduction of Common Fractions to Decimal Fractions&quot;; also &quot;Some Major Concepts Involved in the Multiplication and Addition of Decimal Fractions&quot;.</td>
</tr>
<tr>
<td>Tuesday</td>
<td>28 May</td>
<td>FINAL REVIEW</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** If for any reason you wish to communicate with me in regard to your participation in this Experiment, please phone the Surrey School Board Office (CLOVERDALE 2-1551 or 2-1561) during the day or my home (LAKEVIEW 2-2073) during the evening.
APPENDIX B

THE LESSONS

Background material for the teaching of Lessons I, II, and III (WHITE SHEETS - distributed to teachers of both treatment groups) . . . . . . 200

Lessons I to VIII, inclusive (PINK SHEETS - distributed to teachers of the experimental group) 215

Lessons I to VIII, inclusive (YELLOW SHEETS - distributed to teachers of the control group) . . 250
LESSON I

INTRODUCTION: THE HINDU-ARABIC SYSTEM OF NOTATION

1. Lesson Objectives:

1. To present a brief history of the art of reckoning.

2. To convey an appreciation of the simplicity and convenience of our presently used Hindu-Arabic decimal system of notation as compared to earlier methods, especially the Roman system.

3. To show visually the structure of the decimal number system.

2. Lesson Preparation:

The procedure of teaching to attain the above three objectives is divided into two parts. The first part deals with the achievement of the first two objectives; the second deals with the achievement of the third objective.

The former involves the use of no special teaching materials. Sections (a), (b), and (c) of the background material contain the necessary information for the verbal presentation of this part of the lesson. These sections should be carefully studied in advance for that purpose.

The latter does involve the use of special teaching materials. These materials are listed at the beginning of the second part of the lesson procedure. Section (d) of the background material contains the theoretical information necessary for the meaningful presentation of the second part of the lesson.

Finally, an evaluative exercise in the form of a worksheet entitled "The Decimal System of Notation" should be available for distribution at the end of the teaching presentation.

3. Background:

(A) DISADVANTAGES OF THE ANCIENT SYSTEMS OF NOTATION

Reckoning is one of the oldest arts practised by man. The number systems used by the early civilizations had some very great disadvantages. For example, the cuneiform or wedge-shaped characters which formed the notation of the Babylonians were complicated in design and difficult to reproduce. The seven numerals used by the Romans, though convenient to write, operated in a very cumbersome system.

The consequences of these disadvantages were that the ancients did not use their numerals for counting and calculating purposes. They used them only for recording that which had been previously counted or calculated by other means. To illustrate, a sheep-herder, desiring to calculate
the number of sheep he may have lost during the day, would place a pebble in a pile as each sheep left the pasture. When the sheep returned at night he would remove a pebble from the pile. The number of pebbles left over would represent the number of sheep still left. If necessary to record his losses, he would resort at this stage to the use of numerals. It is true that more refined devices, such as the abacus, were used for calculating. But always the number system worked on principles different from those of the calculating device. The arts of calculating and recording were distinctly separate. The numerals were simply not devised to aid in number thinking.

(B) HISTORICAL DEVELOPMENT OF THE HINDU-ARABIC NOTATION SYSTEM

Many centuries passed with no progress in the art of number. Eventually, the first step was reached in the development of a more sensible and more easily managed symbolism. In about the fifth century of the Christian era the Hindus developed the nine numerals of the present system. Without the zero, however, the nine symbols were an unsystematic disarray of numerals, possessing the same disadvantages of the earlier systems. Nevertheless, the use of the Hindu numerals spread to Arabia.

About the tenth century the zero was invented to complete the Hindu-Arabic system of notation. Despite the amazing transformation which the zero made in the simplicity and utility of the system, the Hindu-Arabic notation was slow to replace the ponderous Roman system. It was about two centuries later that it was brought into Western Europe, and it was not until the sixteenth century that it came into general use.

(G) ADVANTAGES OF THE HINDU-ARABIC NOTATION SYSTEM

The importance of the invention of the zero in this development should not be overlooked. It has been well said that "zero is the catalyst that brings together static numeral signs into a dynamic system of number thinking". By enabling the existing symbols, 1 to 9 inclusive, to possess a place value as well as a face value, the zero gave the Hindu-Arabic system a function no previous system ever possessed.

The new system extended its use beyond the recording function to which the earlier systems were confined; it could also be used for counting and calculating. Unlike Roman numeration, the Hindu-Arabic system appropriated to itself the principle of positional value which facilitated in a practical way the ancient process of calculating on the abacus. The long-separated arts of calculating and recording were united at last into a single whole system.

Furthermore, if the structure of the number system, including the principle of place value, is understood, the numerals are an aid in the number thinking that accompanies the
performance of the calculation. If the structure of the number system is not understood, the performance of the calculation, even though possible, will be on a completely mechanical level in accordance with some prescribed rule.

(D) THE STRUCTURE OF THE NUMBER SYSTEM

(i) The number system is based on a grouping by tens

It is presumed that the base of our number system is ten because primitive man used his fingers in counting. The number 28, for example, came to mean that all the fingers had been used two times (2 groups of 10 fingers), and that 8 of them had been used a third time.

The ten-ness of the system is why it is called a decimal system. The word "decimal" is derived from the Latin "decimus" which means tenth and "decem" which means ten. Nine is as far as we go in our number system without regrouping and starting over again with one. Ten in any group join to make one in the position of next higher value.

(ii) The number system has place value

Each position has a value ten times as much as the position immediately to the right, or one-tenth as much as the position immediately to the left. A comparison of our number system with the Roman system, which does not have place value, illustrates the significance of this principle.

<table>
<thead>
<tr>
<th>Roman Numerals</th>
<th>Hindu-Arabic Numerals</th>
</tr>
</thead>
<tbody>
<tr>
<td>I has a value of one. It is called one.</td>
<td>1 has a value of one. It is called one.</td>
</tr>
<tr>
<td>II has a value of one and one. It is called two.</td>
<td>II has a value of one ten and one one. It is called eleven.</td>
</tr>
<tr>
<td>III has a value of one, one, and one. It is called three.</td>
<td>III has a value of one hundred, one ten, and one one. It is called one hundred, eleven.</td>
</tr>
</tbody>
</table>

(This system is based on an additive or subtractive principle.) (This system is based on a place value principle.)

The above comparison shows that in Roman numerals the "I" always has a value of one, regardless of its position. In Hindu-Arabic numbers the "I", or any other symbol, has a value which depends on its location.

(iii) The number system uses zero, or cipher, as a place holder

Let us look at the number two thousand eight. This means
two thousand, no hundreds, no tens, eight ones. When the number is written in this way, the needlessness of indicating that there are no hundreds and no tens is apparent.

However, when the same number is expressed in symbols the denomination of each numeral (that is: the fact that 2 refers to thousands and 8 refers to ones) is indicated only by the position occupied by the 2 and 8. These positions are:

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

The zero, by indicating that a place is empty, serves to keep the numerals in the proper place. It has been called the place holder because it fills an empty place in a number written in symbols in order to protect the values of the other numerals which lie in the other places to the left in a whole number, or other places to the right in a fractional number. This protective function of zero explains why it is not necessary, in expressing twenty in symbols, to use the zeros in the following way:

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The first two zeros are unnecessary because, unlike the third zero in the ones column, they have no numeral to protect.

(iv) The number system may be extended to the right of the Ones' place to provide a notation of decimal fractions.

A significant feature of the Hindu-Arabic number system is that fractional parts may be expressed simply by extending the numeral places to the right of the ones' place, thus:

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>
The three principles underlying the structure of the whole number system; (1) ten-ness (2) place value and (3) place holding, apply also to the decimal fraction notation.
1. **Lesson Objectives:**

1. To show that the decimal system of whole number notation may be extended to the right of the ONES' place.

2. To emphasize that the ONES' place is the centre of this extended system of notation, and that the other place names are symmetrical around it.

3. To provide a familiarization with the decimal fraction place names.

4. To compare the relationship in size of the various positional values.

2. **Lesson Preparation:**

The procedure of teaching to attain the above four objectives is divided into two parts. The first part deals with the achievement of the first three objectives; the second part deals with the achievement of the fourth objective. While both parts will contribute in some measure to all objectives, it is desirable that the primary intent of each part be kept clearly in mind as the lesson proceeds.

Both parts involve the use of special teaching materials. These materials are listed at the beginning of each part of the lesson procedure.

Section (A) of the background material recapitulates briefly that portion of Lesson I which dealt with the four basic principles in the structure of the Hindu-Arabic number system. Section (B) contains the background orientation relevant to Objective 1; section (C) is similarly relevant to Objectives 2 and 3, while section (D) is relevant to Objective 4.

Finally, an evaluative exercise is provided for distribution at the end of the teaching presentation.
3. Background:

(A) REVIEW OF THE PRINCIPLES UNDERLYING WHOLE NUMBERS

The background of Lesson I dealt with the four essential principles underlying the structure of the Hindu-Arabic number system:

1. it is based on a grouping by tens
2. it uses position to determine the value of a symbol in a number
3. it uses zero as a place-holder to keep symbols in their appropriate positions
4. it may be extended to the right of ONES' place to provide a notation of decimal fractions.

Showing in a meaningful way the application of the first three of these principles in whole numbers was one of the objectives of Lesson I.

(B) DECIMAL FRACTIONS ARE AN INTEGRAL PART OF THE WHOLE NUMBER SYSTEM WITH THE SAME COMMON PRINCIPLES

The fourth principle serves as the means of introducing decimal fraction notation in Lesson II. This method of introducing decimal fractions as an integral part of our decimal system of number is one of two ways suggested by Brueckner and Grossnickle in "How to Make Arithmetic Meaningful". The other way suggested by them is to consider decimals as a special form of common fractions having denominators of ten or some power of ten. This latter approach will be used in a subsequent lesson to reinforce the pupils' development of an understanding of decimal fraction concepts.

If pupils regard decimals from the outset as a logical extension of the whole number system, they will readily recognize the application of the first three principles is decimal fractions as well as in whole numbers. Teaching pupils to regard decimals in this way constitutes the first objective of Lesson II.
(C) Significant Features in the Relationship between the Integral and Fractional Parts of a Mixed Decimal Expression

Treatng decimal fractions as an integral part of the decimal system establishes the need to show clearly the relationship between the whole and fractional parts of a mixed decimal expression. This necessitates emphasizing the following:

1. **The ONES' place is the centre of our system of notation.**

   The prominence of the decimal point should not be allowed to detract from the importance of the ONES' place. The primary function of the decimal point is to designate the location of the ONES' digit. The point occupies no column or place in the number system.

   As a matter of fact it might be more logical to place the decimal point, or some other identifying mark such as a bar, either above the ONES' digit or below it. This would remove from the point the feature which Buckingham in "Elementary Arithmetic: Its Meaning and Practice" describes as "the purely incidental mark of distinction between ONES and TENTHS".

   It is interesting to note that some cultures use identifying marks other than the decimal point as we know it. Taylor and Hills state in "Arithmetic for Teacher-training Classes" that: "The number which we write as 16,357 has been written in these forms: 16,3'5'7''; 16°,3'5''7''; 16,(o)3(1)5(2)7; 16)357." In France and Germany, they point out, it would be written 16,357. However, the purpose, if not the form, of the identifying mark is common to all cultures. It is to indicate the position of the ONES' column. All other symbols have their values determined in relation to this column.

2. **The other positional values are symmetrical around the ONES' place**.
This is illustrated in the following schematic representation:

<table>
<thead>
<tr>
<th>THOUSANDS</th>
<th>HUNDREDS</th>
<th>TENS</th>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
</table>

It should be noted that the importance of emphasizing the symmetry of positional values around the ONES' place is to overcome the disadvantage that derives from the incidental distinction given to the decimal point by being placed between the ONES and TENTHS. Emphasizing the position of the decimal rather than the position of the ONES' place leads to such apparent discrepancies as the following: four places are needed to represent THOUSANDS whereas only three places are needed to represent THOUSANDTHS. If positional values are looked upon as being symmetrical around the ONES' place then the more logical generalization can be made that three places on EITHER side of the ONES' place represents THOUSANDS and THOUSANDTHS.

Teaching pupils to observe the centrality of ONES in our number system, and to note the correspondence between the TENS and TENTHS, HUNDREDS and HUNDREDTHS, etc. constitutes the second objective of Lesson II.

In the teaching procedure that follows, the same technique of instruction is used for the attainment of Lesson Objectives 1 and 2. During the process of this instruction the pupils will have ample opportunity to become familiar with the decimal fraction place names, thus providing for the attainment of Lesson Objective 3.

(D) THE VISUAL RELATIONSHIP IN SIZE OF DIGIT POSITIONS

The teaching procedure designed for the attainment of the first three objectives is intended to give pupils a general understanding of the positional values extending from THOUSANDS...
to THOUSANDTHS. Within the limitations of a reasonable amount of instructional materials it is difficult to make an adequate visual presentation of the actual relationship in size of such a wide range of positional values.

The attainment of Lesson Objective 4 is intended to give pupils a visual experience with a limited range of positional values extending from the ONES' place to the THOUSANDTHS' place. The conceptual learning obtained therefrom will be readily transmitted to the wider range.
LESSON III

REDUCTION OF DECIMALS TO COMMON FRACTIONS

1. Lesson Objectives:

1. To consider decimals as a special form of common fractions having denominators of 10, 100, 1000 etc., that is, any power of 10.

2. To show how decimal fractions indicate the numerator and denominator of equivalent common fractions.

3. To provide practice in the reading and writing of decimal fractions.

2. Lesson Preparation:

The procedure of teaching to attain the above three objectives is divided into three parts: one part for each objective.

The first part does not involve the use of special teaching materials; the materials to be used in the second and third parts are listed at the beginning of each part of the lesson procedure.

Sections (A), (B) and (C) of the Background material are relevant, respectively, to the three lesson objectives.

Finally, an evaluative exercise is provided for distribution at the end of the teaching presentation.

3. BACKGROUND:

(A) DECIMAL FRACTIONS ARE A SPECIAL FORM OF COMMON FRACTIONS

In the background material of Lesson II reference was made to two suggestions for the introduction of decimal fractions given by Brueckner and Grossnickle in "How to Make Arithmetic Meaningful". One way is to consider decimals as an integral part of our decimal system of number. This approach was used in Lesson II.

The other way is to consider decimal fractions as a special form of common fractions having denominators of any power of 10. This
approach is used in Lesson III. Buswell and Brownell reinforce this opinion when they state in their manual to teaching "Arithmetic We Need": "Once pupils understand that decimals are fractions, the denominators of which are not visible and are always 10 or a multiple of 10, they will have developed a real understanding of the meaning of decimals."

Decimal fractions may, therefore, be regarded as a selected part of all common fractions, namely: those whose denominators are a power of 10. Any fraction which has a denominator of 10, 100, 1000, etc. is a decimal fraction. As Spitzer says in "The Teaching of Arithmetic": "... it is the fact that the denominators of decimals are all powers of tens which makes decimals unique, and not the use of the decimal point". Although general usage has established decimal fractions as those fractions in which a decimal point is used, it should not obscure the fact that they are simply common fractions with unwritten, but understood, denominators of some power of 10. Consideration of decimal fractions in this way is helpful for developing meaningful insights into areas of decimal work such as:

1. the reading and writing of decimal fractions
2. the reduction of common fractions to decimals
3. the changing of measurement or terminating decimals to higher terms (for example, changing 5.1 to 510 hundredths)
4. the rounding of decimal fractions to lower terms (for example, changing 0.942 to 9 TENTHS).

These points will be illustrated at the appropriate places in this, and subsequent, lessons.

(B) INTERPRETATIONS OF THE NUMERATOR AND DENOMINATOR OF A DECIMAL FRACTION

In Section (A) of this background material we have seen that decimal fractions are simply common fractions having denominators of 10, 100, 1000, etc.

The numerator of a decimal point is indicated directly by the
number to the right of the decimal point. It is read as a whole number. For example, in .425 the numerator is 425 and is read four hundred twenty five; in .000425 it is also 425 and is read the same way. The position of the decimal point does not change the value of the numerator.

The denominator of a decimal fraction, being unwritten, must be interpreted from the name of the last-used decimal place to the right. Lesson II (Objective No. 3) provided familiarization with the decimal fraction place names which will enable pupils to make this interpretation. Thus, 1.2 is one and two tenths as a mixed fraction while it is twelve tenths as an improper fraction. Similarly, .12 is twelve HUNDREDTHS, .012 is twelve THOUSANDTHS, etc. In each case it is the name of the last-used place to the right of the decimal point which indicates the "invisible" denominator.

Lesson III (Objective No. 2) is intended to show the reason why the denominator of a decimal fraction may be interpreted from the name of the last-used decimal place to the right. In addition, this objective is intended to show why the position of the last digit after the decimal point actually determines the value of a decimal fraction. For example, in .12 it is the fact that the digit 2 is in the HUNDREDTHS' position which determines the value of the fraction. This may be explained in this way: since the 1 is in the first position to the right of ONES' place it represents 1 TENTH which is equivalent to 10 HUNDREDTHS. Together with the 2 already in the HUNDREDTHS' position this makes 12 HUNDREDTHS.
Certain generalizations will result from interpreting the numerator and denominator of a few decimal fractions. One such generalization is: if the last-used decimal place is one place to the right of the ONES' place the fraction represents TENTHS; if it is two places to the right of the ONES' place it represents HUNDREDTHS, etc. A somewhat more mechanical form of the same generalization is: the denominator of a common fraction will have one zero for every figure to the right of the decimal point in the equivalent decimal fraction.

(C) THE READING AND WRITING OF DECIMAL FRACTIONS

The first objective in this lesson is to lead the pupils to understand that decimal fractions are merely common fractions with unwritten, but understood, denominators. Furthermore, they are a selective type of common fractions because the denominators are always 10, 100, 1000, etc., that is, some power of 10. The second objective is to lead pupils to understand the significance of the last-used decimal place to the right of the ONES' place in determining the numerator and denominator of the common fraction equivalent to the decimal fraction. These two objectives should give pupils an understanding that decimals are fractions, the denominators of which, though unwritten, are a power of 10. According to Buswell, Brownell and Sauble in the Manual to TEACHING ARITHMETIC WE NEED, this is the basis for developing "a real understanding of the meaning of decimals."

If these two objectives are attained, this third objective concerning the reading and writing of decimal fractions will have been achieved as well. This procedure follows Spitzer's recommendation of writing the common fraction of decimals as a means of reading the decimal written with the decimal point. In the decimal fraction .036, for example, if the pupil understands that the unwritten denominator must be a power of 10; that the power is determined by the place value
of the 6; and, further, that the numerator is determined by converting the 3 TENTHS into THOUSANDBTHS and adding it to the existing 6 THOUSANDBTHS to give 36/1000, then he will have a meaningful insight into the reading of .036.
4. Procedure of Teaching:

1. **Achievement of Lesson Objectives 1 and 2 (Time: 12 minutes)**

The contents of Sections (a), (b), and (c) of the background material may be discussed within approximately the allotted time. In order to achieve the above objectives it is suggested that the material provided should be referred to in general terms. The emphasis should be on the spontaneity of the presentation rather than on too rigid an adherence to minute detail.

If the objectives of the lesson are reached successfully it should motivate the pupils to explore and experiment with decimals in subsequent lessons.

2. **Achievement of Lesson Objective 2 (Time: 18 minutes)**

**Materials:**

Three place value charts, 210 tickets of which 200 should be in groups of 10 held together with an elastic band, and 10 loose tickets.

**To illustrate:**

(i) the ten-ness (or decimal nature) of our number system.
(ii) positional value, or the idea that the numeral at the right of a whole number has an ONES' value, the next numeral on the left has a TENS' value, and the next on the left has a HUNDREDS' value.

**Steps:**

1. Set the three place value charts on the bottom ledge of the blackboard.

   **Note:** in making the two illustrations noted above, Steps 7 to 9 inclusive show the situations which require a regrouping from the ONES' position to the TENS' position. Step 9 emphasizes the relationship in actual value between various digits in these two given positions.

2. Place the single tickets on the ONES' chart while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

3. Explain that we must regroup when we reach 10. Provide a small square immediately above the charts as shown in the illustration at the top of page 216. These squares may be drawn on the blackboard.

4. Explain that in these squares we customarily write only one figure to indicate the number of tickets on each place value chart. This illustrates the need to regroup when 10 has been reached in any one position.

5. Remove the 10 single tickets and substitute one bundle of 10 tickets by placing it on the TENS' board. Write the
symbols in the small squares.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Symbol</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HUNDREDS</th>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>XXXXXXX</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(regroup to form 10)</td>
</tr>
</tbody>
</table>

6. Emphasize the fact that 1 bundle on the TENS' board is composed of 10 times as many tickets as 1 ticket on the ONES' board.

7. Continue to replace single tickets on the ONES' chart while counting 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

8. As in step 5, regroup from ONES' place to TENS' place. Repeat the details of this regrouping as often as is considered necessary.

9. As in step 6, draw attention frequently to such facts as:
   (a) 4 on the ONES' chart represents one-fifth as many tickets as 2 on the TENS' chart.
   (b) 8 on the TENS' chart represents forty times as many tickets as 2 on the ONES' chart, etc.

Note: Step 10 shows a situation which requires a regrouping from both the ONES' position to the TENS' position and the TENS' position to the HUNDREDS' position. In other words it illustrates situations which require two successive regroupings. Step 11 emphasizes the relationship in actual value between various digits in these three given positions.

10. On the place value charts show 135 as follows: 12 bundles of 10 tickets on the TENS' chart and 15 single tickets on the ONES' chart. Emphasize the fact that in the squares above each chart we write only one figure to indicate the number of tickets on that particular chart, and emphasize also that it takes 10 in one position to equal 1 in the adjacent position on the left. Following the emphasis on these details proceed to perform the regrouping to obtain 1 ticket on the HUNDREDS' chart, 3 on the TENS' chart, and 5 on the ONES' chart.

11. As in steps 6 and 9, draw attention to such facts as:
   (a) 1 on the HUNDREDS' chart represents 20 times as many tickets as 5 on the ONES' chart.
   (b) 5 on the ONES' chart represents 1/6 as many tickets as 3 on the TENS' chart.

12. Draw from pupils, out of the experience they have had with the foregoing relationships, generalizations framed around the following:
   (a) The number system is based on a grouping by tens.
(b) The number system has place value. This means that each numeral in a number possesses a value assigned by the "place" it occupies in the number. Each "place" has a value ten times as much as the "place" immediately to the right, or one-tenth as much as the position immediately to the left.

13. At this point a very brief comparative description may be made of the principles underlying the Roman numeral system of notation (see heading (d) (ii) of the background material).

To illustrate:

(iii) the use of zero, or cipher, as a place holder.

Steps:

1. Show 9 bundles on the TENS' chart and 9 tickets on the ONES' chart. Then add one ticket to the ONES' chart. Regroup.

2. Explain that a figure must be written to show each place in the three place number, even though the charts in two of the places are empty. This is the PLACE HOLDING FUNCTION OF ZERO in our number system. Show that zero has a protecting role to keep 1 in the third space from the right, or on the HUNDREDS' chart.

3. Add some single tickets, say 7, to the ONES' chart.

4. While referring to the place value charts deal with the number shown under three headings:

(a) How it reads - one hundred seven. (Note: this is a convenient point at which to explain that the use of "And" in the reading of a number is reserved exclusively to indicate the connection between the ONES' place and the TENTHS' place. It is never used either in a whole number or in a fractional number.)

(b) What it means - one hundred, no tens, seven ones.

(c) How it is written -
Achievement of Lesson Objectives 1, 2, and 3 (Time: 15 minutes)

Materials:

Seven place value charts, Chart indicating the decimal point. Twenty-five cards in each of seven different colors. (Note: this substitution for the bundles of cards used with the place value charts in Lesson 1 is necessitated by the impracticability of using cards smaller than the ONES' cards to represent units less than ONE.

Note: Steps 1 and 2 which follow are intended to meet Lesson Objective 1, while Step 3 is intended to meet Objective 2. All three steps should contribute to the attainment of Objective 3.

Steps:

1. Set the place value chart to represent ONES on the bottom ledge of the blackboard.

   Place the decimal point to the right of the ONES' chart.

   Explain that the purpose of the decimal point is to identify the ONES' digit. It may be of interest to draw attention to the method explained on the second page of this lesson by which people elsewhere identify the ONES' digit. Explain that when the ONES' digit has been located all other digits obtain their values from the position they occupy in relation to the ONES' place. A number, like 34573, is quite meaningless unless we know which digit stands for unity.

2. Place additional charts to secure the arrangement shown below:

   ![Diagram]

   Represent 25 THOUSANDTHS on the charts, and then, by performing the necessary regrouping point out that the following three principles, which were shown in Lesson 1 to form the structure of whole number system, apply also to decimal fractions: (1) ten-ness; (2) place value; (3) place holding function of zero.

   The same treatment may be applied to other representations, such as: (a) 25 HUNDREDTHS; (b) 25 TENTHS

3. Remove the charts used in Step 2, and then assign successive pairs of pupils to come forward to place charts in positions which are symmetrical around the ONES' chart, as shown in the diagrams.
The lowest row of charts shows the arrangement when all have been placed on the bottom ledge of the blackboard.

Draw lines on the blackboard above the charts, as shown in the illustration above, to emphasize the symmetry around the ONES' place.

While the charts are in this position, discussion should be held which points out the following:

(a) the central position occupied by the ONES' place.

(b) The symmetry of the other place values around the ONES' place.

(c) the various value relationships whereby each place represents a value ten times as large as the place next to it on the right, one hundred times as large as the second place to it on the right, etc.

Illustrate these relationships with specific examples shown on the charts, e.g.:

(i) a 4 in the ONES' place is ten times as large as 4 in the TENTHS' place.
(ii) a 7 in the TENS' place is one thousand times as large 7 in the HUNDREDTHS' place.
(iii) a 2 in the TENTHS' place is one-fiftieth as much as 1 in the TENS' place (that is, the 1 in the TENS' place actually represents 100 TENTHS, which is fifty times larger than 2 TENTHS).

4. To conclude this portion of the lesson, three generalizations should be drawn from pupils at this stage:

(a) The following principles which underlie the whole number system apply also to decimal fractions (Objective 1):

(i) **Place value** - each position assigns to a digit a particular value.
(ii) Ten-pose - the value assigned to a digit in one position is ten times larger than the value assigned to it in the position next to it on the right, etc.

(iii) Place-holding function of zero - in order to "protect" the value of numerals by keeping them in the required positions, zeros are needed to record whatever empty positions exist BETWEEN the decimal and numerals in the most extreme positions to the left or right of the decimal point.

Note: it may be mentioned in passing that zeros fill another function quite apart from a place-holding function. This function, as well as the place-holding function, will be dealt with more fully in Lesson IV.

(b) The arrangement of positions around the ONES' place is symmetrical (Objective 2):

(i) the position which is third from the ONES' place (fourth from the decimal) on the left, and third from the ONES' place on the right are THOUSANDS and THOUSANDTHS respectively.

(ii) the position which is second from the ONES' (third from the decimal) on the left, and second from the ONES' place on the right are HUNDREDS and HUNDREDTHS respectively.

(iii) the position which is next to the ONES' place (second from the decimal) on the left, and next to the ONES' place on the right are TENS and TENTHS respectively.

Achievement of Lesson Objective 4 (Time: 15 minutes)

Materials:

Decimal place value cards. Four cards to represent the following place values: one whole, one tenth, one hundredth, one thousandth. A fifth card bears the decimal point.

Note: The achievement of this objective should enable pupil to formulate a meaningful generalisation respecting the comparison of decimal fractions, e.g.: which is larger - .379 or .38?

Pupils who have become accustomed to making comparisons of whole numbers only may find the comparison of decimal fractions less obvious than it first appears to them. The Winston textbook "Thinking with Numbers" contains a drawing, shown at the left, which may be presented on the blackboard to pupils to emphasise that one must learn to check conclusions in arithmetic. In comparing decimal fractions, as in comparing the lengths of these 30 inch lines, "You cannot always be sure".
1. Choose pupils each to carry a place value card and take their positions facing the class while holding the card in full view.

Start with the ONE card, followed by the decimal point, then follow:

(a) with the TENTH card - explain that if the card on the left were shown it would be represented by 1 bundle of 10 cards. This would be in the TENS' position.

(b) with the HUNDREDTH card - explain that if the card located in the corresponding position to the left of the ONE were shown it would be represented by 10 bundles of 10 cards.

(c) with the THOUSANDTH card - explain that if the card located in the corresponding position to the left of the ONE were shown it would be represented by 10 bundles of 100 cards.

This arrangement may be represented on the blackboard, thus:

While pupils are in this position discuss the manner in which we would arrange the following in order of size, beginning with the largest:

(a) 1.1  (b) .011  (c) 11  (d) .11  (e) 1.11

2. Let the pupils holding the ONE card and the THOUSANDTH card be seated.

Proceed to compare two decimal fractions, e.g., .25 and .3 in this way:

Assign pupils to take up their positions behind the card bearers as shown in the diagram. The x's represent .25 and the o's represent .3. Pupils representing .25 may be referred to as Team (a), while those representing .3 may be referred to as Team (b).

Discuss why Team (b) represents a larger fraction than Team (a).

3. The same procedure may be followed in showing the reasoning involved in arranging the following according to size:

(a) .5  (b) .05  (c) 5.5  (d) .055  (e) .55
4. To conclude this portion of the lesson, the following generalisation should be drawn from pupils after those standing have resumed their seats.

"Decimal fractions can be ranked in order of size by comparing the absolute value of the digits in the corresponding places, thus:

(a) the largest of several decimal fractions will be the one with the largest figure in the TENTHS' place.

(b) if the figures in the TENTHS' place are equal, then the largest fraction will be the one with the largest figure in the HUNDREDTHS' place.

(c) if the figures in the HUNDREDTHS' place are equal, then the largest fraction will be the one with the largest figure in the THOUSANDTHS' place.
**Procedure of Teaching:**

**PART ONE**

**Achievement of Lesson Objective 1 (Time: 8 minutes)**

To consider decimals as a special form of common fractions having denominators of 10, 100, 1000 etc., that is, any power of 10.

**Materials:**

No special materials required.

**Steps:**

1. Write the following series of common fractions on the blackboard:
   
   (a) \( \frac{1}{2} \)  
   (b) \( \frac{9}{100} \)  
   (c) \( \frac{3}{4} \)  
   (d) \( \frac{11}{20} \)  
   (e) \( \frac{7}{10} \)  
   (f) \( \frac{1}{8} \)  
   (g) \( \frac{3}{1000} \)  
   (h) \( \frac{2}{5} \)  
   
   (i) \( \frac{1}{16} \)  
   (j) \( \frac{3}{25} \)  
   (k) \( \frac{17}{10000} \)  
   (l) \( \frac{7}{30} \)  
   (m) \( \frac{3}{8} \)  
   (n) \( \frac{1}{60} \)  
   (o) \( \frac{19}{50} \)  
   (p) \( \frac{1}{7} \)  

2. **Verbal Explanations:**

   (a) Explain what is meant by "a power of 10". Obviously, it is beyond the scope of the pupils' comprehension at this stage to explain that it means "the index of 10". Consequently, it will suffice to explain that in effect it means 10 multiplied by itself any number of times, or 10 by itself, thus: 10, 100, 1000, etc.

   The meaning of "a power of 10" should be made distinct from the meaning of "a multiple of 10" which means 10 multiplied, not by itself any number of times, but by any number, for example: 5, 8, 12, 20, 30, etc., to give these respective multiples of 10: 50, 80, 120, 200, 300 etc.

   (b) Explain that while all the fractions written on the board are common fractions, those with a denominator of a power of 10 may also be regarded as decimal fractions, even though it is customary practice in writing decimal fractions to omit writing the denominator and to indicate it indirectly by the use of a decimal point.

3. Form two columns on the blackboard, and at the top of each write headings as follows:

<table>
<thead>
<tr>
<th>Fractions which may be considered only as common fractions</th>
<th>Fractions which may be considered as Decimal fractions</th>
</tr>
</thead>
</table>

Under the appropriate heading enter each of the fractions already written on the blackboard.
Achievement of Lesson Objective 2 (Time: 12 minutes)

To show how decimal fractions indicate the numerator and denominator of equivalent common fractions.

Materials:
Four place value charts, namely: ONES', TENTHS', HUNDREDS', THOUSANDS'. 25 yellow tickets, 15 each of blue and green tickets, and 5 red tickets.

Steps:

Note: The two points stated below should be clearly emphasized after each of the following three representations contained in Part Two of this Lesson.

1. The position of the last digit after the decimal point determines the value of a decimal fraction. That is, each of the digits in the decimal positions preceding the last place may in turn be converted to the place value of the last position after the decimal point.

The number so obtained determines the NUMERATOR of the equivalent common fraction.

At the same time the particular place value of the last occupied position indicates the DENOMINATOR of the equivalent common fraction.

2. When a decimal fraction is changed to a common fraction, the denominator has ONE ZERO for every figure to the right of the decimal point.

1. Perform representations of the following three fractions as indicated:

(a) Represent .025 on the place value charts, writing the number above the charts as shown:

\[
\begin{array}{cccc}
\text{ONES} & \text{TENTHS} & \text{HUNDREDS} & \text{THOUSANDS} \\
0 & 0 & 2 & 5 \\
\end{array}
\]

Remove the two green tickets from the Hundredths' Board and substitute 20 yellow tickets on the THOUSANDS' board, thus:

\[
\begin{array}{cccc}
\text{ONES} & \text{TENTHS} & \text{HUNDREDS} & \text{THOUSANDS} \\
0 & 0 & 2 & 5 \\
\end{array}
\]

Emphasize clearly the two points stated above in green.
(b) Represent .12 on the place value charts, writing the number above the charts as shown:

![Place Value Charts]

Remove the blue ticket from the TENTHS' board and substitute 10 green tickets on the HUNDREDTHS' board, thus:

![Updated Place Value Charts]

Emphasize clearly the two points stated above in green.

(c) Represent 2.3 on the place value charts, repeating the procedure as in (a) and (b) above.

Note: step 2 below is merely an extension of (c) above and shows that the two points noted above may be used to explain the conversion of an integral number into an improper fraction. In this case, of course, it is the position of the terminating zero which determines the value of the improper fraction.

2. Perform representations of the following as indicated:

(a) Represent 2.0 on the place value charts. Remove the two red tickets, and substitute 20 blue tickets on the TENTHS' chart, thus:

![Updated Place Value Charts]

(b) Though the manipulation involved in the following need not be undertaken, proceed to explain, nevertheless, that

2.00 would be shown as 200 green tickets on HUNDREDTHS' chart;

2,000 would be shown as 2000 yellow tickets on the THOUSANDTHS' chart.

Emphasize, as in Step 1, the significance of the last-used position to the right of the decimal point in determining the numerator and the denominator of the improper fraction. For example, 2.00 is 200 HUNDREDTHS; 2,000 is 2000 THOUSANDTHS.
Achievement of Lesson Objective 3 (Time: 10 minutes)

To provide practice in the reading and writing of decimal fractions.

Materials:

No special materials required.

Steps:

Note: The achievement of Lesson Objective 2 will enable pupils to visualize the common fraction equivalent of a decimal fraction. It is this ability to visualize the common fraction form which, according to Spitzer, provides a good procedure for the reading of decimals. Therefore, the first step below presents, at a more abstract level, the same method used in the achievement of Lesson Objective 2.

1. Write the decimal fraction 0.256 on the blackboard. Then explain the meanings for this decimal that are shown below:

   0.256 means
   0.0200 (200 THOUSANDTHS)
   0.0050 (50 THOUSANDTHS)
   0.0006 (6 THOUSANDTHS)

   0.256 is read "two hundred fifty-six thousandths".

2. Explain that in reading a mixed decimal like 115.231 we connect the whole number and the fraction by "AND". In the reading of decimals the word "AND" is reserved for this purpose and is never used, with one exception, in either the integral or fractional portion of the mixed decimal.

   Thus,

   115.231 is read "one hundred fifteen AND two hundred thirty-one THOUSANDTHS".

   847 is read "eight hundred forty-seven thousandths".

   800.047 is read "eight hundred AND forty-seven THOUSANDTHS".

   The exception is in the reading of a decimal fraction containing a common fraction, for example:

   4.12½ is read "four AND twelve and one-half HUNDREDTHS".

   0.07 is read "one seventh of a TENTH".

3. Explain that in reading a NON-TERMINATING or INFINITE decimal fraction like 3.1416 it is common usage to read this as a telephone number; thus:

   3.1416 may be read "three DECIMAL (or POINT) One-four-one-six!"

4. Explain that in reading a TERMINATING or FINITE decimal fraction such as might be obtained as a measurement by the use of a micrometer, for example .0500, would be read "five hundred TEN THOUSANDTHS". In such cases as No. 3 and 4 it is custom, rather than rule, which determines the most acceptable method of reading.
PART ONE

Achievement of Lesson Objective 1. (Time: 20 minutes)

To demonstrate the use of zero as a place holder.

Materials:

Decimal place value charts. Four charts to represent the following place values: one whole, one tenth, one hundredth, one thousandth. A fifth chart bears the decimal point, and a sixth chart bears the zero symbol. Also, decimal place value cards as shown on next page.

Steps:

Note: Steps 1, 2, and 3 demonstrate visually the use of zero as a place holder.

1. Set the five cards shown below on the bottom ledge of the blackboard.

   The illustration indicating the position of the decimal point to the right of the ONE card is for diagrammatic convenience only. In actually setting out these cards it will present the visual symmetry of the different place values more effectively if the decimal point is placed in front of the red board toward the right edge, instead of being placed entirely to the right as is done in the diagram.

   ![Cards illustration]

2. Remove the TENTH’S card.

   Explain the necessity to fill the empty place, otherwise the HUNDREDTH’S and THOUSANDTH’S cards will be located one and two places respectively to the right of the decimal point. According to the generalization learned in 4 (b) of Lesson II these cards must now be considered to represent TENTHS and HUNDREDTH respectively.

   Therefore, if it is intended merely to remove the TENTH’S card and leave the HUNDREDTH’S and THOUSANDTH’S cards with their original value, then a zero must be used to fill the empty place "to protect" the values of these cards.

   Accordingly, insert the card bearing the zero in the empty place.

3. Restore the cards to their original positions and this time remove both the TENTH’S AND HUNDREDTH’S cards and follow the procedure as in Step 2.
Note: Steps 4, 5, and 6 demonstrate visually the effect upon the value of a mixed decimal fraction of inserting a zero immediately after the Decimal point.

4. Set the four cards shown below on the bottom ledge of the blackboard. (Note: follow the instruction contained in Step 1 above in regard to the placement of the board containing the decimal point)

   (a)  (b)  (c)

Figure 1

5. Then insert the ZERO immediately after the decimal point, thus:

   (a)  (b)  (c)  0

Figure 2

6. Since, however, the second and third cards from the ONES' place must be HUNDREDTHS and THOUSANDTHS respectively, these two cards must be replaced to give the following arrangement:

   (d)  0  (e)  (f)  (g)

Figure 3

By comparing the arrangement shown in Figure 1 with that shown in Figure 3 it should be pointed out that we have, in effect, taken \(\frac{1}{10}\) of card (b) to give us card (f), and we have taken \(\frac{1}{10}\) of card (c) to give us card (g).

Since we have not, of course, in any way altered the ONE'S card, it cannot be said that we have taken one-tenth of the original mixed decimal expression.

All that can be said is that inserting the zero immediately after the decimal point has the effect of reducing the value of the mixed decimal expression.

Note: Steps 7 and 8 demonstrate visually the effect upon the value of a simple decimal fraction of inserting a zero immediately after the decimal point.

7. Set the three cards shown below on the bottom ledge of the blackboard.
6. Then insert the ZERO immediately after the decimal point, thus:

![Fig. 55](image)

However, as in Step 6, Cards (A) and (b) must be changed to give:

![Fig. 6](image)

Unlike the previous example, this illustration shows that inserting the zero immediately after the decimal point in a simple fraction has the effect of making the value of the new fraction EXACTLY ONE-TENTH of the value of the original fraction.

9. To conclude this portion of the lesson, two generalisations should be drawn from pupils at this stage:

(a) If a zero is inserted after the decimal point in a mixed decimal expression it has the effect of reducing the value of the expression.

(b) If a zero is inserted after the decimal point in a simple decimal expression it makes the value ONE-TENTH as much as it was originally.

PART TWO

Achievement of Lesson Objective 2 (Time: 10 minutes)

To demonstrate the use of zero as a terminal cipher.

Materials:

Decimal place value cards. Two cards to represent the following place values: one whole, one tenth. A third card bears the decimal point, and a fourth card bears the zero symbol.

Steps:

1. Set the three cards shown below on the bottom ledge of the blackboard.

2. Then annex the ZERO immediately to the right of the TENTH’S card, thus:

3. Draw attention of pupils to the following two points:
(a) a Terminal Zero, unlike a place holding zero, is annexed to the end of a decimal fraction.

(b) a Terminal Zero does not change the actual value of a decimal fraction, but it does change the SIGNIFICANCE of it.

This change in SIGNIFICANCE or MEANING which results from adding a Terminal Zero will be discussed in Lesson VI.

At this point it will be sufficient to point out that adding the zero in the above example enables the fraction to be read "ONE and TEN HUNDREDTHS" instead of "ONE and ONE TENTH".

This indicates that the decimal fraction is accurate to the nearest HUNDREDTH. Without the terminal ZERO it is accurate only to the nearest TENTH.

4. To conclude this portion of the lesson, the following generalization should be drawn from pupils at this stage:

The addition of a terminal zero to a decimal fraction does not change the value of the fraction but it does change the significance of the fraction.
PART ONE

Achievement of Lesson Objective 1 (Time: 15 minutes)

To demonstrate the effect upon the value of a decimal fraction of moving the decimal point.

Materials:

Three place value charts: ONES', TENS', HUNDREDS'. Chart indicating the decimal point. Tickets: 1 single, 1 packet of 10; 1 packet of 100.

Three place value cards: TENTH, HUNDREDTH, THOUSANDTH.

Steps:

Note: Steps 1 and 2 demonstrate visually the effect upon the value of the decimal fraction of moving the decimal point to the left.

Steps 3 and 4 demonstrate visually the effect upon the value of the decimal fraction of moving the decimal point to the right.

Step 5 is the final step in the induction, and contains a generalization which should be drawn from pupils as a result of their experience with the first four steps.

1. Set the three charts shown below on the bottom ledge of the blackboard.

   Place a single card or ticket on the ONES' chart, a packet of 10 on the TENS' chart, and a packet of 100 on the HUNDREDS' chart.

   Move the decimal point one place to the left as indicated by the red arrow at the top of the diagram.

   Explain: Since the place immediately to the left of the decimal point must always be the ONES' place, this makes it necessary to consider that the packet of 10 at present shown on the TENS' chart has, in effect, been reduced to 1 single ticket.

   Likewise, the tickets shown on the adjacent charts must be reduced to one-tenth the original amount in order to maintain the principle of TEN-NESS.

2. Move the decimal point two places to the left of the original location as indicated by the green arrow at the bottom of the diagram.

   Repeat the appropriate explanation given in step 1.
3. Set the three cards shown below on the bottom ledge of the blackboard.

![Diagram showing three cards with arrow indicating movement of decimal point]

Move the decimal point **one place** to the right as indicated by the red arrow at the top of the diagram.

**Explain:** Since the place immediately to the left of the decimal point must always be the ONES' place, this makes it necessary to consider that the representation of ONE-TENTH (immediately to the left of the new location of the decimal point) has, in effect, been increased to ONE.

Likewise, the representations shown on adjacent cards must be increased to ten times the original size in order to maintain the principle of TENNESS.

4. Move the decimal point **two places** to the right of the original location as indicated by the green arrow at the bottom of the diagram.

Repeat the appropriate explanation given in step 3.

5. To conclude this portion of the lesson, the following generalization should be drawn from pupils at this stage:

**(a)** For every place that a decimal point is moved to the right in a number, it has the effect of multiplying the number by TEN. That is, if the decimal point is moved one place to the right, the number becomes **10** times larger; if it is moved **two places** to the right, the number becomes **100** times larger, etc.

**(b)** For every place that a decimal point is moved to the left in a number, it has the effect of dividing the number by TEN. That is, if the decimal point is moved one place to the left, the number is reduced to **ONE-TENTH** its original value; if it is moved **two places** to the left, the number is reduced to **ONE-HUNDREDTH** its original value, etc.

---

**PART TWO**

**Achievement of Lesson Objective 2 (Time: 15 minutes)**

To demonstrate the effect upon the location of the decimal point of multiplying or dividing a decimal fraction by a power of 10.

**Materials:**

Same as for Part One.

**Steps:**
Note: Part Two of this Lesson is the converse to Part ONE. The steps in this part, therefore, are parallel to those contained in the first part.

Steps 1 and 2 demonstrate visually the effect upon the location of the decimal point of dividing a number by a power of 10.

Steps 3 and 4 demonstrate visually the effect upon the location of the decimal point of multiplying a number by a power of 10.

Step 5 is the final step in the induction, and contains a generalisation which should be drawn from pupils as a result of their experience with the first four steps.

1. Set the three charts shown below on the bottom ledge of the blackboard.

Place a single card or ticket on the ONES chart, a packet of 10 on the TENS chart, and a packet of 100 on the HUNDREDS chart.

Divide this number shown, that is 111, by 10. This means dividing each place by 10, and so we get:

Since the one ticket or 1 unit must be identified by the decimal point it is, consequently, necessary to adjust the location of the decimal point by moving it one place to the left, as shown by the red arrow.

2. Replace the tickets in order to indicate 111. This time divide each each place by 100, and so we get:

Explain the necessity to make the adjustment in the location of the decimal point as shown by the green arrow.
3. Set the three cards shown below on the bottom ledge of the blackboard:

(Multiply by 10)

This represents .111. Let us now multiply this decimal fraction by 10, thus:

It is now necessary to adjust the location of the decimal point in order to put it beside the card that stands for ONE. That is, when the number is multiplied by 10 it is necessary to move the decimal point one place to the right. See red arrow.

4. Repeat the illustration given in step 3; applying it this time to demonstrate the need to move the decimal point two places to the right when the number is multiplied by 100.

5. To conclude this portion of the lesson, the following generalisation should be drawn from pupils at this stage:

(a) When a decimal fraction is multiplied by 10, 100, 1000, etc., (that is, some power of 10) the decimal point is moved one place to the right for every zero in the multiplier.

(b) When a decimal fraction is divided by 10, 100, 1000 etc., that is, some power of 10) the decimal point is moved one place to the left for every zero in the divisor.
PART ONE

Achievement of Lesson Objective 1 (Time: 15 minutes)

To illustrate the significance of rounding decimal fractions.

Materials:
Wall rule with movable indicator.

Steps:

Note: The significance of rounding decimal fractions is shown by comparing the variation in a measurement rounded only to UNITS to the variations in measurements rounded successively to TENTHS and HUNDREDTHS.

1. The scale indicated below represents a longer section of the wall rule used in this lesson. Draw this representation on the blackboard.

   ![Scale Representation]

   (a) Explain that when we say that a line is 2 inches long we signify by this indication merely that the length is CLOSER TO 2 INCHES THAN IT IS TO 1 INCH or 3 INCHES. The rather considerable amount of variation in length permitted is indicated by the area marked in RED.

   It should be evident that in order to round a measurement number to the NEAREST unit it is necessary to know at least the number of TENTHS involved in the measurement.

   (b) Explain that when we say that a line is 2.0 inches long we signify by this indication that the length this time is CLOSER TO 2.0 INCHES THAN IT IS TO 1.9 INCHES or 2.1 INCHES. The more restricted amount of variation in length permitted by this designation is indicated by the area marked in PURPLE.

   It should be evident in this case that in order to
round a measurement number to the NEAREST TENTH it is necessary to know at least the number of HUNDREDTHS involved in the measurement.

Finally, explain that when we say that a line is 2.00 inches long we signify by this indication that the length this time is CLOSER TO 2.00 INCHES THAN IT IS TO 1.99 INCHES or 2.01 INCHES. The even more restricted amount of variation in length permitted by this designation is indicated by the very small area marked in GREEN.

It should be evident in this case that in order to round a measurement number to the NEAREST HUNDREDTH it is necessary to know at least the number of THOUSANDTHS involved in the measurement.

2. Refer to the wall rule.

Point out how this represents only a portion of the blackboard illustration shown in Step 1.

Let us say that the length of a line is 1.67 units. This means that this measurement is rounded to the nearest HUNDREDTH, and that in order to be able to effect this degree of rounding it is necessary to know the length of the line in THOUSANDTHS, or, in other words, to know that the length lies somewhere between 1.665 and 1.674.

In the chart above, the small area shaded in GREEN indicates the area of this variation; and, in the chart below, an enlargement of this same area of variation is reproduced.

Now the indicator on the wall rule to show the very small variation in length that could be permitted when the length of a line is described as 1.67 units.

Point out on the wall rule that as we successively reduce the accuracy of rounding we increase the variation in the length of the line represented by the measurement. That is to say, point out that if this line were rounded to the nearest TENTH it would be
Lesson VI (Page 2)

1.7, and show on the wall rule that this description of its length would entitle it to be placed between 1.65 and 1.74.

This variation in enlarged form is shown on the chart below:

```
  6  5
  10ths 10ths
  
  3
  100ths
```

Finally, point out that if this line were rounded to the nearest UNIT it would be 2, and show on the wall rule that this description of the length would entitle it to be placed between 1.5 and 2.4.

In all these cases of rounding, if the fraction is equal to, or greater than, one-half of the fractional interval, the fraction will be raised to the next highest interval.

3. Repeat Step 2 with other illustrations on the wall rule.

Assume, for example, that the length of a line is 1.32.

Show the variation in length permitted when this line is rounded successively to: (a) HUNDREDTHS (b) TENTHS (c) UNITS.

Show that in rounding a number to HUNDREDTHS it is necessary to know the number of THOUSANDTHS; in rounding to TENTHS it is necessary to know the number of TENTHS; and in rounding to UNITS it is necessary to know the number of TENTHS.

4. To conclude this portion of the lesson, three generalisations should be drawn from pupils at this stage:

(a) In rounding a mixed decimal fraction to the nearest whole number, if the number of TENTHS is 5 or greater, add 1 to the whole number.

In rounding a decimal fraction to the nearest TENTH, if the number of HUNDREDTHS is 5 or greater, add 1 to the number of TENTHS, etc.

(b) In rounding a mixed decimal fraction to the nearest whole number it is necessary to know the number of TENTHS. In rounding a number to the nearest TENTH it is necessary to know the number of HUNDREDTHS.

(c) After rounding has been completed, the place occupied by the last digit or zero indicates the accuracy of the measurement. For example, 2.060 is accurate to the nearest THOUSANDTH.
Lesson VI (Page 4)

PART TWO

Achievement of Lesson Objective 2 (Time: 8 minutes) 238

To demonstrate various applications of the rounding of decimal fractions.

Materials:

Wall rule with movable indicator.

Steps:

Note: Decimal fractions are frequently expressed to a degree of accuracy beyond that required for a particular purpose. The following steps show visually how approximations of such decimal fractions may be made by various applications of rounding.

1. Assume the length of a line to be 1.837. Indicate on the Wall rule the very small variation in length that would be permitted by this very accurate description of length.

2. For convenience we may round this mixed decimal expression to HUNDREDTHS if the purpose for which the measurement was being used warranted it, and report it as 1.84 or 184 HUNDREDTHS.

Remind pupils of the point that was emphasized in Part Two of Lesson III concerning the importance of the last-used position after the decimal point. Thus, in 1.84, when we convert everything to the position occupied by the 4 we get 184 HUNDREDTHS.

Point out that this measurement, 1.84 or 184 HUNDREDTHS is accurate to the nearest HUNDREDTH, and that IN ORDER TO OBTAIN THIS DEGREE OF ACCURACY WE MUST FIRST, BEFORE ROUNDING, KNOW ALSO THE NUMBER OF THOUSANDTHS.

3. For even greater convenience, 1.837 may be rounded to TENTHS.

4. Demonstrate on the blackboard how 1.596 could be expressed as:

(a) 1.60 (read "one and sixty hundredths")

or 160 HUNDREDTHS.

(b) 1.6 or 16 TENTHS.
Lesson VI (Page 5)

PART THREE

Achievement of Lesson Objective 3 (Time: 7 minutes)

To indicate why UNLIKE decimal fractions must be changed to LIKE decimal fractions (that is, with the same understood denominator) in order that they may be added or subtracted.

Materials:

No special materials required.

Steps:

Note: Step 1 refers to non-measurement numbers which may be counted as discrete, non-continuous entities.

Step 2 refers to measurement numbers, that is, those specifically indicated in a problem or situation to represent inches, pounds, or some other unit of measurement which can never be "entirely" exact.

1. When the numbers do NOT mean inches, or some other measurement, fill the empty spaces with zeros, for example:

\[
\begin{align*}
0.8 & \quad \text{change to} \quad 0.800 \\
0.65 & \quad \text{change to} \quad 0.650 \\
0.239 & \quad \text{change to} \quad 0.239 \\
\end{align*}
\]

2. When the numbers represent measurements, as in the examples below, it is necessary to find the number with the fewest decimal places and round all the other numbers to that number of places, for example:

\[
\begin{align*}
0.8 & \quad \text{change to} \quad 0.8 \quad \text{Note: it is understood} \\
0.65 & \quad \text{change to} \quad 0.7 \quad \text{that these numbers refer} \\
0.239 & \quad \text{change to} \quad 0.2 \quad \text{to inches, pounds, etc.} \\
\end{align*}
\]

3. To conclude this portion of the lesson, the following generalization should be drawn from pupils at this stage:

"The sum or difference of measurement numbers will be accurate only to the fractional unit of the number that has the fewest decimal places."
DIVISION INVOLVING DECIMAL FRACTIONS

PART ONE

Achievement of Lesson Objective 1 (Time: 6 minutes)

To explain the significance of performing division involving decimal fractions.

Materials:

No special materials required.

Steps:

Note: In division involving decimal fractions frequently the placement of the decimal point is governed only by meaningless rule. The purpose of Part One of this lesson is to interpret the reason for the placement of the decimal point in a quotient.

The division of common fractions is used as a means of developing this interpretation.

The time limit devoted to Part One imposes very great restrictions on the thoroughness with which this topic may be discussed. For this reason it is necessary to restrict the examples shown, and deal only with ones such as the following divisions. Such curiosity may be aroused by this incomplete presentation as will make profitable a more complete presentation outside the area of this experiment.

1. Present the following examples on the blackboard:

(a) $8.6 \div 16.34$  
(b) $14 \div 5.14$  
(c) $9.8 \div 7.052$

The above examples have been selected because none of them requires the addition of zeros to the dividend. It may be explained, however, if the need arises, that the same principle holds in the case of non-terminating or infinite quotients, where

The above examples may be worked out by different pupils on the blackboard.

2. When the quotients have been obtained demonstrate by means of divisions involving common fractions that in the case of:

Example (a) HUNDREDTHS divided by TENTHS is Tenths.
(b) THOUSANDTHS divided by HUNDREDTHS is Tenths.
(c) THOUSANDTHS divided by TENTHS is Hundreths.
Lesson VII (Page 2)

PART TWO

Achievement of Lesson Objective 2 (Time: 22 minutes)

To demonstrate the significance of moving the decimal point in performing divisions involving decimal fractions.

Materials:

Flannel Board with supply of paper prepared with the appropriate design.

Steps:

Note: Steps 1 to 4, inclusive, refer to examples where a whole number is divided by a decimal.

1. Write on the blackboard the following division question:

   \[ .5) 3 \]

   Point out that when a whole number is divided by a simple fraction the answer is LARGER than the dividend.

   This may revolutionise somewhat the concept children may have gained in previous grades in which it was believed that if a number were divided it would automatically mean that the quotient would be smaller than the dividend.

   To illustrate this, apply Sheet 1 of the accompanying materials to the flannel board.

   \[
   \begin{array}{c}
   \text{DIVIDEND} \\
   \hline
   \end{array}
   \quad \begin{array}{c}
   \text{DIVISOR} \\
   \hline
   \end{array}
   \]

   \[ \text{The quotient is larger than the integral dividend when the divisor is less than one.} \]

   Sheet 1

   Supply the answer to the division question already written on the blackboard.

2. Write on the blackboard the second division question:

   \[ .125) 1.5 \]

   With the participation of the pupils, identify .125 as the decimal equivalent of \( \frac{1}{8} \).

   When this has been done apply Sheet 2 to the flannel board.

   \[
   \begin{array}{c}
   \text{DIVIDEND} \\
   \hline
   \end{array}
   \quad \begin{array}{c}
   \text{DIVISOR} \\
   \hline
   \end{array}
   \]

   Sheet 2
Supply the answer to the division question already written on the blackboard.

Out of the above two steps pupils should have gained an understanding of the fact that when a whole number is divided by a divisor less than ONE, the quotient will be greater than the dividend.

3. Apply Sheet 3 to the flannel board. This is shown below:

<table>
<thead>
<tr>
<th>DIVIDEND</th>
<th>DIVISOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram of rectangles]</td>
<td>[Diagram of 2/3]</td>
</tr>
</tbody>
</table>

To illustrate that \(2\div\frac{2}{3}\) is equal to \(6\div\frac{2}{3}\).

Note: Circles rather than rectangles are supplied.

The difficulty in this division is obvious.

Let us multiply the DIVISOR by 3 in order to make it a whole number. This gives us a new DIVISOR of 2.

By means of various SIDE-EXAMPLES on the blackboard show that the quotient remains unchanged when the DIVISOR and the DIVIDEND are each multiplied by the same number.

Accordingly, let us multiply the DIVIDEND by 3 also. THIS gives us a new dividend of 6.

From Sheet 4 take representations of whole units and apply these to the flannel board to the right of the original problem.

4. Write the division \(4\div2\) on the blackboard, and discuss with pupils the need to multiply the DIVISOR by 10 in order to remove the decimal fraction, and also to multiply the DIVIDEND by 10 in order to compensate for the change in the DIVISOR.

Out of Steps 3 and 4, pupils should have gained an understanding of the following two facts:

(a) A division is made easier when the DIVISOR is multiplied by a quantity which will make it a whole number.

(b) The quotient remains unchanged when the DIVIDEND is multiplied by the same amount as the DIVISOR.

Note: Steps 5 to 8, inclusive, refer to examples where a decimal fraction is divided by a decimal fraction.

5. As in Step 3, use the flannel board to explain that when the divisor is a fraction, the division is more easily performed if the divisor is made a whole number.

Apply the representations shown below, and contained on Sheet 5,
Alongside, and to the right of Sheet 5, apply to the flannel board the contents attached to Sheet 6.

This is intended to illustrate that in the division:

\[ \frac{3.2}{4} \]

when the DIVISOR is multiplied by 10 to make it 4, and the DIVIDEND multiplied by 10 also, the division process becomes much easier to perform.

It illustrates, too, that when the DIVIDEND, as well as the DIVISOR, is multiplied by the same amount, the quotient remains unchanged.

Stated in another way, it may be said that if the decimal point is moved the same number of places, AND IN THE SAME DIRECTION, in the DIVIDEND and the DIVISOR, the answer remains unchanged.

6. Discuss in what way the answer would be altered if, instead of moving the decimal point the same way in both the DIVIDEND and the DIVISOR, the point were moved ONE PLACE TO THE LEFT IN THE DIVIDEND and ONE PLACE TO THE RIGHT IN THE DIVISOR.

Illustrate the division \[ \frac{1.2}{10} \] on the blackboard, and point out that if the decimal point were moved to the RIGHT in the DIVIDEND and to the LEFT in the DIVISOR, the answer would be 100 times larger than it should be.

7. Similarly, discuss in what way the answer would be altered if, instead of moving the decimal point the SAME way in both the DIVIDEND and the DIVISOR, the point were moved ONE PLACE TO THE LEFT in the DIVIDEND and to the RIGHT in the DIVISOR.

Using Sheet 7, illustrate the division \[ \frac{1.2}{100} \] on the flannel board to show that the answer would be only 1/100 of what it should be.
6. To conclude this lesson, the following three generalisations should be drawn from pupils:

(a) When a whole number is divided by a simple fraction the quotient (answer) will be larger than the dividend.

(b) In dividing with decimals, the divisor may be made a whole number by multiplying it by a given amount, provided the dividend also is multiplied by the same amount.

(c) In dividing with decimals, if the decimal point is moved ONE PLACE IN OPPOSITE DIRECTIONS in the DIVIDEND and the DIVISOR, the answer will be EITHER 100 times greater than, or 1/100 as great as, it should be.
LESSON VIII

MISCELLANEOUS CONCEPTS INVOLVING DECIMAL FRACTIONS

PART ONE

Reduction of Common Fractions to Decimal Fractions

Achievement of Lesson Objective 1 (Time: 12 minutes)

To convey the significance of changing a common fraction to a decimal fraction.

Materials:

Flannel Board with supply of paper prepared with the appropriate designs.

Steps:

1. Set the Flannel Board on the bottom ledge of the blackboard.

   Apply the symbols attached to Sheet 7 to the left side of the Flannel Board, as shown below:

   Numerator is the DIVIDEND
   Denominator is the DIVISOR

   \[ \frac{1}{4} \]

   Note: Materials are also supplied for illustrating \( \frac{3}{8} \)

   Explain that a common fraction merely indicates an unperformed division, and that the changing of this common fraction into a decimal fraction INVOLVES THE PERFORMANCE OF THIS DIVISION.

   In this division the numerator of the fraction becomes the DIVIDEND and the denominator becomes the DIVISOR.

   Since 1 is not evenly divisible by 4, it is necessary to convert the 1 to TENTHS. Apply the appropriate paper to the Flannel Board to represent this conversion.

   Since 10 is not evenly divisible by 4, it is necessary to convert the 10 to HUNDREDTHS. Apply the appropriate paper to the Flannel Board to represent this conversion.

   Since this numerator is now divisible by 4, perform the division by writing 25 HUNDREDTHS on the blackboard to the right of the Flannel Board, and then express this as a decimal fraction, .25.

2. Copy on the blackboard other illustrations, such as the following:
3. To conclude this portion of the lesson, two generalisations should be drawn from pupils at this stage:

(a) Converting a common fraction to a decimal fraction involves a division in which the numerator of the fraction becomes the dividend, and the denominator becomes the divisor.

(b) Before performing this division it is necessary to add zeros to the numerator. Adding these zeros really amounts to converting the NUMERATOR from ONES to TENTHS, HUNDREDTHS, THOUSANDTHS, or whatever smaller unit is required to obtain a suitable decimal fraction equivalent.

PART TWO

Multiplication Involving Decimal Fractions

Achievement of Lesson Objective 2 (Time: 12 minutes)

To illustrate the reason for the placement of the decimal point in the product obtained by the multiplication of decimal fractions.

Materials:

Wall rule with movable indicator.

Steps:

1. Hang the Wall Rule from the moulding at the top of the blackboard.

   Regard the distance between 1 and 2 on the rule as 1 whole unit.

   Show on the rule by means of moving the indicator:

   (a) 1/10 of (which means times) 1 whole unit is 1 TENTH,
   or, in other words, .1 times 1 equals .1

   That is, TENTHS TIMES UNITS EQUALS TENTHS.
(b) \( \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} \)

or, in other words, \( .1 \times .1 = .01 \)

That is, TENTHS TIMES TENTHS EQUALS HUNDREDTHS.

2. In the same way explain that in the question:

\[ 19.8 \times 7.6 = 150.48 \]

the decimal point is located in this place in the answer because TENTHS TIMES TENTHS IS HUNDREDTHS.

3. Point out how the value of this product would be altered if the decimal in the first number were changed two places to the left, for example, and changed one place to the right in the second number.

Thus, instead of \( 19.8 \times 7.6 \) we would now have

\[ .198 \times 76 \]

This product would have to be expressed in THOUSANDTHS, because THOUSANDTHS times ONES (76 ones) equals THOUSANDTHS.

The original was expressed in HUNDREDTHS. Therefore, the act of changing the decimal points as we did had the effect of making the value of the fraction exactly \( 1/10 \) of what it was at first.

4. If time permits, repeat this procedure contained in Steps 2 and 3 with the following example:

The product of \( 4.86 \) and \( 6.9 \) is \( 33.534 \) (HUNDREDTHS times TENTHS is THOUSANDTHS).

Point out in what way the value of this product would be affected if the decimals were moved into the following positions:

(a) \( 48.6 \times .69 \) (Answer remains unchanged)
(b) \( .486 \times 6.9 \) (Answer is \( 1/10 \) of what it was).

**PART THREE**

Addition involving decimal fractions

Achievement of Lesson Objective 3 (Time: 6 minutes)

To develop an understanding of the importance in the addition of decimal fractions of aligning columns according to place value.
Materials:

No special materials required.

Steps:

Note: Pupils often fail to line up decimal points when they write decimals in addition problems. Errors resulting from this may not be detected because of the failure to recognize what the decimal in the sum must mean. Writing the sum correctly should be rationalized in terms of place value.

1. Copy the following chart on the blackboard:

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

This visualization should be used to impress upon pupils the fact that the necessity to align the decimals under one another is merely to ensure that numbers with similar place values will be added.

In an addition involving decimal fractions it is no more correct to add a 1 in the TENTHS place to a 1 in the HUNDREDTHS place than it is to add 1/10 and 1/100 without changing them to a common denominator.

2. It should be pointed out that where there is an addition involving decimals derived from measurements, as in the case at the right, these quantities should not, from a practical point of view at least, be added as they stand. The number 12.3 does not necessarily mean 12.30. It may mean anything from 12.25 to 12.34, inclusive. If such measurements have been obtained, and they are to be added, the only sensible thing to do is to ROUND ALL TO TENTHS, that is, round so that all the measurements are expressed to the same number of places.

Emphasize the fact that in an example such as the one shown in Step 2, the answer will be accurate ONLY to the nearest TENTH.
3. To conclude this portion of the lesson, two generalizations should be drawn from pupils at this stage:

(a) In the case of the addition of measurement numbers involving decimal fractions, the sum will be accurate only as far as the last-used place value of the number containing the fewest number of decimal places.

(b) In the addition of decimal fractions all figures with the same place value should be placed in the same column.
Procedure of Teaching:

1. **Achievement of Lesson Objectives 1 and 2 (Time: 12 minutes)**

   The contents of Sections (a), (b), and (c) of the background material may be discussed within approximately the time. In order to achieve the above objectives it is suggested that the material provided should be referred to in general terms. The emphasis should be on the spontaneity of the presentation rather than on too rigid an adherence to minute detail.

   If the objectives of the lesson are reached successfully it should motivate the pupils to explore and experiment with decimals in subsequent lessons.

2. **Achievement of Lesson Objective 3 (Time: 18 minutes)**

   **Materials:**

   Six visualization cards numbered 1, 2, 3, 4, 5, 6. (Numbers are indicated on the reverse side.)

   **To illustrate:**

   (i) the ten-ness (or decimal nature) of our number system.

   (ii) positional value, or the idea that the numeral at the right of a whole number has a ONES' value, the next numeral on the left has a TENS' value, and the next on the left has a HUNDREDS' value.

   **Steps:**

   1. Hang Cards 1 and 2 from the moulding at the top of the blackboard and in the positions shown in the diagrams below.

      Note: in making the two illustrations noted above, Steps 2 to 6 inclusive show the situations which require a regrouping from the ONES' position to the TENS' position. Step 9 emphasises the relationship in actual value between various digits in those two given positions.

      2. Point to each of the block in the first row of the ONES' column while counting 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
3. Explain that we must regroup when we reach 10. Provide a small square immediately below the Cards as shown in the illustration at the top of the next page. These squares may be drawn on the blackboard.

4. Explain that in these squares we customarily write only one figure to indicate the number of blocks in any one column position. This illustrates the need to regroup when 10 has been reached.

---

<table>
<thead>
<tr>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="chart1.png" alt="Diagram of Blocks" /></td>
</tr>
</tbody>
</table>

Chart No. 1

---

<table>
<thead>
<tr>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="chart2.png" alt="Diagram of Blocks" /></td>
<td></td>
</tr>
</tbody>
</table>

Chart No. 2

---

5. Draw a line on the blackboard, as shown, connecting this first row of 10 blocks with the equivalent representation in the TENS column in Chart No. 2.

6. Emphasize the fact that 1 block in the TENS column represents 10 times as many blocks as 1 block in the ONES column.

7. Continue to count to 20, pointing this time to each of the blocks in the second row in the ONES column.

8. As in Step 3, explain that we must regroup when we reach another group of 10 blocks. Once again, draw a line on the blackboard connecting this second row of 10 blocks with the equivalent representation in the TENS column in Chart No. 2. Write symbols in squares.
9. As in Step 6, draw attention frequently to such facts as:

(a) 4 on the ONES' chart represents one-fifth as many blocks as 2 on the TENS’ chart.

(b) 8 on the TENS’ chart represents forty times as many blocks as 2 on the ONES’ chart, etc.

Note: Step 10 shows a situation which requires a regrouping from BOTH the ONES' position to the TENS' position and the TENS' position to the HUNDREDS' position. In other words it illustrates situations which require two successive regroupings. Step 11 emphasizes the relationship in actual value between various digits in these three positions.

10. Hang Cards 3 and 4 in the same position formerly occupied by Cards 1 and 2. Card 3 has 12 blocks in the TENS' column and 15 blocks in the ONES' column. Emphasize the fact that in the squares below each card we write only one figure to indicate the number of blocks on that particular card, and emphasize also that it takes 10 in one position to equal 1 in the adjacent position on the left. Following the emphasis on these details proceed to perform the regrouping to obtain the result shown on Card 4: 1 block on the HUNDREDS' card, 3 blocks on the TENS' card and 5 blocks on the ONES' card.

11. As in Steps 6 and 9, draw attention to such facts as:

(a) 1 in the HUNDREDS' position (on Card No. 4) represents 20 times as many blocks as 5 in the ONES' position.

(b) 5 in the ONES' position (on Card No. 4) represents 1/6 as many blocks as 3 on the TENS' chart.

12. Draw from pupils, out of the experience they have had with the foregoing relationships, generalizations framed around the following:

(a) The number system is based on a grouping by tens.

(b) The number system has place value. This means that each numeral in a number possesses a value assigned by the "place" it occupies in the number. Each "place" has a value ten times as much as the "place" immediately to the right, or one-tenth as much as the position immediately to the left.

13. At this point a very brief comparative description may be made of the principles underlying the Roman numeral system of notation (see heading (d) (ii) of the background material).
To illustrate:

(iii) the use of zero, or cipher, as a place holder.

Steps:

1. Hang Cards 5 and 6 in the same positions formerly occupied by Cards 3 and 4. Card No. 5 has 9 blocks in the TENS' column and 10 blocks in the ONES' column. This represents 100 and involves a regrouping as shown on Card No. 6.

   ![Diagram of place value columns]

   In the space on the blackboard below each column position write the appropriate symbol.

2. Explain that a figure must be written to show each place in the three place number, even though the columns in two of the places are empty. This is the PLACE HOLDING FUNCTION OF ZERO in our number system. Show that zero has a protecting role to keep 1 in the third space from the right, or the HUNDREDS' column.

3. Draw on the blackboard a representation of Card No. 6 and show on this representation one block in the HUNDREDS' column, and seven blocks in the ONES' column.

4. While referring to the blackboard drawing described in Paragraph 3 above deal with the number shown under three headings:

   (a) How it reads - one hundred seven. (Note: this is a convenient point at which to explain that the use of "and" in the reading of a number is reserved exclusively to indicate the connection between the ONES' place and the TENS' place. It is never used either in a whole number or in a fractional number.

   (b) What it means - one hundred, no tens, seven ones.

   (c) How it is written -

   ![Number representation]
Achievement of Lesson Objectives 1, 2, and 3 (Time: 15 minutes)

Materials:
Three visualization cards numbered 7, 8, 9. (Numbers are indicated on the reverse side)

Note: Steps 1 and 2 which follow are intended to meet Lesson Objective 1, while Step 3 is intended to meet Objective 2. All three steps should contribute to the attainment of Objective 3.

Steps:

1. Hang Cards No. 7 and 8 from the moulding at the top of the blackboard and in the positions shown in the diagrams below.

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDS</th>
<th>THOUSANDS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Card No. 7

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDS</th>
<th>THOUSANDS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Card No. 8

Point to the decimal point.

Explain that the purpose of the decimal point is to identify the ONES' digit. It may be of interest to draw attention to the methods explained on the third page of this lesson by which people elsewhere identify the ONES' digit. Explain that when the ONES' digit has been located all other digits obtain their values from the position they occupy in relation to the ONES' place. A number, like 34873, is quite meaningless unless we know which digits stand for unity.

2. In pointing to, and explaining, the regrouping shown on Visualization Cards 7 and 8, emphasize the fact that the following three principles, which were shown in Lesson I to form the structure of the whole number system, apply also to decimal fractions: (1) ten-ness; (2) place value; (3) place holding function of zero.
Make an outline of Cards 7 and 8 on the blackboard and represent on Card 7 such other fractions as (a) 25 HUNDREDTHS (b) 25 TENTHS.

Perform on the representation of Card 8 on the blackboard the necessary regrouping in order to emphasize further the three principles noted immediately above.

3. Remove Cards 7 and 8 and replace with Card 9, shown below:

```
THOUSANDS  HUNDREDS  TENS  ONES  TENTHS  HUNDREDTHS  THOUSANDTHS
2       2       2       2       2       2       2
```

Draw lines on the blackboard below the charts, as shown in the illustration above, to emphasize the symmetry around the ONES' place.

While the charts are in this position, discussion should be held which points out the following:

(a) the central position occupied by the ONES' place.

(b) the symmetry of the other place values around the ONES' place.

(c) the various value relationships whereby each place represents a value ten times as large as the place next to it on the right, one hundred times as large as the second place to it on the right, etc.

Illustrate these relationships with specific examples written on the blackboard in the appropriate place under the card, e.g.:

(i) in the number 4.4 - a 4 in the ONES' place is ten times as large as 4 in the TENTHS' place.

(ii) in the number 77.77 - a 7 in the TENS' place is one thousand times as large as 7 in the HUNDREDTHS' place.

(iii) in the number 212.2 - a 2 in the TENTHS' place is one-fiftieth as much as a 1 in the TENS' place (that is, the 1 in the TENS' place actually represents 100 TENTHS', which is fifty times larger than 2 TENTHS).

4. To conclude this portion of the lesson, two generalizations should be drawn from pupils at this stage:

(a) The following principles which underlie the whole number system apply also to decimal fractions (Objective 1):

(i) Place value - each position assigns to a digit a particular value.

(ii) Ten-ness - the value assigned to a digit in one position is ten times larger than the value assigned to it in the position next to it on the right, etc.

(iii) Place-holding function of zero - in order to "protect" the value of numerals by keeping them in the required positions, zeros are needed to record whatever empty positions exist.
BETWEEN the decimal and numerals in the most extreme positions to the left or right of the decimal point.

Note: it may be mentioned in passing that zeros fill another function quite apart from place-holding function. This function, as well as the place-holding function, will be dealt with more fully in Lesson IV.

(b) The arrangement of positions around the ONES' place is symmetrical (Objective 2):

(i) the position which is third from the ONES' place (fourth from the decimal) on the left, and third from the ONES' place on the right are THOUSANDS and THOUSANDTHS respectively.

(ii) the position which is second from the ONES' (third from the decimal) on the left, and second from the ONES' place on the right are HUNDREDS and HUNDREDTHS respectively.

(iii) the position which is next to the ONES' place (second from the decimal) on the left, and next to the ONES' place on the right are TENS and TENTHS respectively.

Achievement of Lesson Objective 4 (Time: 15 minutes)

Materials:

Visualization card No. 10. (Number is indicated on the reverse side)

Note: The achievement of this objective should enable pupils to formulate a meaningful generalization respecting the comparison of decimal fractions, e.g.: which is larger - .379 or .38? Pupils who have become accustomed to making comparisons on whole numbers only may find the comparison of decimal fractions less obvious than it first appears to them. The Winston textbook "Thinking with Numbers" contains a drawing, shown at the left, which may be presented on the blackboard to pupils to emphasize that one must learn to check conclusions in arithmetic. In comparing decimal fractions, as in comparing the lengths of these 30 inch lines, "You cannot always be sure".

Steps:

1. Hang Card NO. 10 from the moulding at the top of the blackboard

Point to the ONE card, followed by the decimal point. Then continue to draw attention to the other places to the right of the decimal as follows:
(a) the TENTH chart - explain that if the chart located in the corresponding position to the left of the ONE were shown it would cover an area 10 times larger than the area of the ONE chart. This would be in the TENS' position.

(b) the HUNDREDTH chart - explain that if the chart located in the corresponding position to the left of the ONE were shown it would cover an area 100 times larger than the area of the ONE chart. This would be in the HUNDREDS' position.

(c) the THOUSANDTH chart - explain that if the chart located in the corresponding position to the left of the ONE were shown it would cover an area 1000 times larger than the area of the ONE chart. This would be in the THOUSANDS' position.

A representation of Card NO. 10 should be put on the blackboard, together with the extensions to the left of the ONES' place, as shown on the diagram at the bottom of the previous page.

With the assistance of the Card and diagram on the board, discuss the manner in which we would arrange the following in order of size, beginning with the largest:

(a) 1.1 (b) .011 (c) 11 (d) .11 (e) 1.11

2. Proceed to compare two decimal fractions, e.g., .25 and .3 in this way:

```
  a b
  x x
  o x
  x o x
```

Under the TENTHS' and HUNDREDTHS' columns of Card No. 10, as in the example above, use x's to represent .25 and o's to represent .3. By referring to the visualization explain why the .3 is larger than the .25.

3. The same procedure may be followed in showing the reasoning involved in arranging the following according to size:

(a) .5 (b) .05 (c) 5.5 (d) .055 (e) .55

4. To conclude this portion of the lesson, the following generalization should be drawn from pupils after the completion of the above.

"Decimal fractions can be ranked in order of size by comparing the absolute value of the digits in the corresponding places, thus:
(a) the largest of several decimal fractions will be the one with the largest figure in the TENTHS' place.

(b) If the figures in the TENTHS' place are equal, then the largest fraction will be the one with the largest figure in the HUNDREDTHS' place.

(c) If the figures in the HUNDREDTHS' place are equal, then the largest fraction will be the one with the largest figure in the THOUSANDTHS' place.
PART ONE

Achievement of Lesson Objective 1 (Time: 8 minutes)

To consider decimals as a special form of common fractions having denominators of 10, 100, 1000 etc., that is, any power of 10.

Materials:

No special materials required.

Steps:

1. Write the following series of common fractions on the blackboard:
   
   \[
   \begin{aligned}
   (a) & \frac{1}{2} & (b) \frac{9}{100} & (c) \frac{1}{4} & (d) \frac{11}{20} & (e) \frac{7}{10} & (f) \frac{1}{8} & (g) \frac{3}{1000} & (h) \frac{3}{40} \\
   (i) & \frac{1}{16} & (j) \frac{3}{25} & (k) \frac{172}{10,000} & (l) \frac{7}{30} & (m) \frac{3}{8} & (n) \frac{1}{60} & (o) \frac{19}{50} & (p) \frac{1}{7}
   \end{aligned}
   \]

2. Verbal Explanations:

   (a) Explain what is meant by "a power of 10". Obviously, it is beyond the scope of the pupils' comprehension at this stage to explain that it means "the index of 10". Consequently, it will suffice to explain that in effect it means

   10 multiplied by itself any number of times, or 10 by itself, thus: 10, 100, 1000, etc.

   The meaning of "a power of 10" should be made distinct from the meaning of "a multiple of 10" which means 10 multiplied, not by itself any number of times, but by any number, for example: 5, 8, 12, 20, 30, etc., to give these respective multiples of 10: 50, 80, 120, 200, 300 etc.

   (b) Explain that while all the fractions written on the board are Common fractions, those with a denominator of a power of 10 may also be regarded as decimal fractions, even though it is customary practice in writing decimal fractions to omit writing the denominator and to indicate it indirectly by the use of a decimal point.

3. Form two columns on the blackboard, and at the top of each write headings as follows:

   | Fractions which may be considered only as common fractions | Fractions which may be considered as Decimal fractions |

   Under the appropriate heading enter each of the fractions already written on the blackboard.
Achievement of Lesson Objective 2 (Time: 12 minutes)

To show how decimal fractions indicate the numerator and denominator of equivalent common fractions.

Materials:

Three visualization cards numbered 7, 8, 9. (Numbers are indicated on the reverse side)

Steps:

Note: The two points stated below should be clearly emphasized after each of the following three representations contained in Step 1 of this Lesson procedure.

1. The position of the last digit after the decimal point determines the value of a decimal fraction. That is, each of the digits in the decimal positions preceding the last place may in turn be converted to the place value of the last position after the decimal point.

   The number so obtained determines the **numerator** of the equivalent common fraction.

   At the same time the particular place value of the last occupied position indicates the **denominator** of the equivalent common fraction.

2. When a decimal fraction is changed to a common fraction, the denominator has ONE ZERO for every figure to the right of the decimal point.

   Provide representations of the following three fractions as indicated:

   (a) Hang Cards 7 and 8 from the moulding at the top of the blackboard. Point to the representation on Card 8 (lower diagram) and explain how 2 **HUNDREDTHS** and 5 **THOUSANDTHS** may be converted to the representation shown on Card 7 (upper diagram).

   In other words, when the 2 **HUNDREDTHS** have been converted to **THOUSANDTHS**, and added to the 5 **THOUSANDTHS** already there, it shows the importance of the position of the last digit after the decimal point in determining the value of a decimal fraction.

   Emphasize clearly the two points stated above in green.

   (b) Draw on the blackboard representations of Cards 7 and 8 and then illustrate .12 on these representations as shown below:

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>
In other words, when the 1 TENTH has been converted to HUNDREDTHS, and added to the 2 HUNDREDTHS already there, it shows the importance of the position of the last digit after the decimal point in determining the value of the decimal fraction.

**Emphasize clearly the two points stated above in green.**

(c) Illustrate 2.3 on the blackboard representation and follow the procedure outlined in (b).

**Note:** Step 2 below is merely an extension of (c) above and shows that the two points noted above may be used to explain the conversion of an integral number into an improper fraction. In this case, of course, it is the position of the terminating zero which determines the value of the improper fraction.

2. Hang Card 9 from the moulding at the top of the blackboard.

(a) Refer to the section of this chart shown below:

```
<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
```

Imagine the numbers on this section to be as represented above.

Explain that if the two ONES were converted to TENTHS there would be 20 TENTHS.

(b) Refer then to the section of this chart shown below:

```
<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Imagine the numbers on this section to be represented above.

Explain that if the two ONES were converted to HUNDREDTHS there would be 200 HUNDREDTHS; or, if converted to THOUSANDTHS there would be 2000 THOUSANDTHS.

In each case the two points noted in green on the previous page should be emphasized.
PART THREE

Achievement of Lesson Objective 3 (Time: 10 minutes)

To provide practice in the reading and writing of decimal fractions.

Materials:
No special materials required.

Steps:

Note: The achievement of Lesson Objective 2 will enable pupils to visualize the common fraction equivalent of a decimal fraction. It is this ability to visualize the common fraction form which, according to Spitzer, provides a good procedure for the reading of decimals. Therefore, the first step below presents, at a more abstract level, the same method used in the achievement of Lesson Objective 2.

1. Write the decimal fraction 0.256 on the blackboard. Then explain the meanings for this decimal that are shown below:

<table>
<thead>
<tr>
<th>Decimal Fraction</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.200 (200 THOUSANDTHS)</td>
<td>200/1000</td>
</tr>
<tr>
<td>0.050 (50 THOUSANDTHS)</td>
<td>50/1000</td>
</tr>
<tr>
<td>0.006 (6 THOUSANDTHS)</td>
<td>6/1000</td>
</tr>
<tr>
<td>0.256 (256 THOUSANDTHS)</td>
<td>256/1000</td>
</tr>
</tbody>
</table>

0.256 is read "two hundred fifty-six thousandths".

2. Explain that in reading a mixed decimal like 115.231 we connect the whole number and the fraction by "AND". In the reading of decimals the word "AND" is reserved for this purpose and is never used, with one exception, in either the integral or fractional portion of the mixed decimal.

Thus,

115.231 is read "one hundred fifteen AND two hundred thirty one THOUSANDTHS".

.347 is read "eight hundred forty-seven thousandths".

800.047 is read "eight hundred AND forty-seven THOUSANDTHS".

The exception is in the reading of a decimal fraction containing a common fraction, for example:

4.12½ is read "four AND twelve and one-half HUNDREDTHS".

0.07 is read "one seventh of a TENTH".

3. Explain that in reading a NON-TERMINATING or INFINITE decimal fraction like 3.1416 it is common usage to read this as a telephone number, thus:

3.1416 may be read "three DECIMAL (or POINT) One-four-one-six!"

4. Explain that in reading a TERMINATING or FINITE decimal fraction such as might be obtained as a measurement by the use of a micrometer, for example .0500, would be read "five hundred TEN-THOUSANDTHS". In such cases as No's 3 and 4 it is customary rather than a rule, which determines the most acceptable method of reading.
PART ONE

Achievement of Lesson Objective 1 (Time: 20 minutes)

To demonstrate the use of zero as a place holder.

Materials:

Cards 9 and 10. Also two pieces of blank paper to be used in covering up certain spaces on Card 9, and a piece of paper bearing a zero symbol to be used with Card 10.

Steps:

Note: Steps 1, 2, and 3 demonstrate visually the use of zero as a place holder.

1. Hang Card 9 from the moulding at the top of the blackboard. Cover up the three sections at the left, thus leaving exposed the part shown below:

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Cover the 2 in the TENTHS' place.

   Explain the necessity to fill the empty space, otherwise the 2's in the HUNDREDTHS' and THOUSANDTHS' places will be located one and two places respectively to the right of the decimal point. According to the generalization learned in 4 (b) of Lesson II these 2's must now be considered to represent TENTHS' and HUNDREDTHS' respectively.

   Therefore, if it is intended merely to remove the 2 Tenths and leave the 2 Hundredths and 2 Thousandths in their original places, then a zero must be used to fill the empty space "to protect" the original place value of the 2 Hundredths and 2 Thousandths.

   Accordingly, hang up in the appropriate place the sheet bearing the zero.

3. Cover the 2 in the HUNDREDTHS' position as well, thus leaving only the 2 in the THOUSANDTHS' position exposed.

   Explain, as in Step 2, the necessity to insert zeros as place holders "to protect" the value of the 2 in the THOUSANDTHS' Place.
Lesson IV (Page 2)

Note: Steps 4, 5, and 6 demonstrate visually the effect upon the value of a mixed decimal fraction of inserting a zero immediately after the decimal point.

4. Hang Card No. 10 from the moulding at the top of the blackboard. Cover up the THOUSANDTH representation, leaving this arrangement, as shown in Figure 1.

5. Immediately under Card No. 10 (shown in Figure 1), draw on the blackboard the representation shown in Figure 2.

This shows that a zero has been inserted between the decimal point and the TENTH.

The insertion of this ZERO causes a displacement of the TENTH and the HUNDREDTH, as shown in Figure 2.

6. Since, however, the second and third positions from the ONE'S place must be HUNDREDTHS AND THOUSANDBDHs respectively, it is necessary to make the appropriate alteration, shown in Figure 3, which should also be drawn on the blackboard immediately under Figure 2.

By comparing the arrangement shown in Figure 1 with that shown in Figure 3 it should be pointed out that we have, in effect, taken 1/10 of position (b) to give us position (f), and we have taken 1/10 of position (c) to give us position (g). See arrows indicating this.

Since we have not, of course, in any way altered the ONE'S position, (position (a) still remains as position (d) ), it cannot be said that we have taken one-tenth of the original mixed decimal expression.

All that can be said is that inserting the zero immediately after the decimal point has the effect of reducing the value of the mixed decimal expression.
Note: Steps 7 and 8 demonstrate visually the effect upon the value of a simple decimal fraction of inserting a zero immediately after the decimal point.

7. Continue to use Card No. 10. Cover up the ONE'S place and the THOUSANDTH'S place, leaving the arrangement as shown in Figure 4.

8. Then insert the ZERO immediately after the decimal point. Show this by drawing on the blackboard immediately under Card 10 the representation shown in Figure 5.

This figure shows that the insertion of the ZERO causes a displacement of the TENTH and HUNDREDTH.

As in step 6, since the second and third positions from the ONE'S place must be HUNDREDTHS and THOUSANDTHS respectively, it is necessary to make the appropriate alteration, shown in Figure 6, which should also be drawn on the blackboard immediately under Figure 5.

Unlike the previous example (described in Steps 4, 5, and 6), this illustration shows that inserting the zero immediately after the decimal point in a simple fraction has the effect of making the value of the new fraction EXACTLY ONE-TENTH of the value of the original fraction.

As shown by the arrows, inserting a zero immediately after the decimal point in a simple fraction causes a displacement which reduces each place to 1/10 its original value.

9. To conclude this portion of the lesson, two generalizations should be drawn from pupils at this stage:

(a) If a zero is inserted after the decimal point in a mixed decimal expression it has the effect of reducing the value of the expression.

(b) If a zero is inserted after the decimal point in a simple decimal expression it makes the value ONE-TENTH as much as it was originally.
Achievement of Lesson Objective 2 (Time: 10 minutes)

To demonstrate the use of zero as a terminal cipher.

Materials:
Card 10, and a piece of paper bearing a zero symbol.

Steps:
1. Hang Card 10 from the moulding at the top of the blackboard. Cover up the HUNDREDTH's and THOUSANDTH's representations, leaving the arrangement shown in Figure 1.

2. Immediately under this portion of Card No. 10, draw on the blackboard the arrangement shown in Figure 2, which shows that ZERO has been annexed immediately to the right of the TENTH's place.

3. Draw attention of pupils to the following points:
   (a) a Terminal Zero, unlike a place holding zero, is annexed to the end of a decimal fraction.

   (b) a Terminal Zero does not change the actual value of a decimal fraction, but it does change the significance of it.

   This change in SIGNIFICANCE or MEANING which results from adding a Terminal Zero will be discussed in Lesson VI.

   At this point it will be sufficient to point out that adding the zero in the above example enables the fraction to be read "ONE and TEN HUNDREDTHS" instead of "ONE and ONE TENTH".

   This indicates that the decimal fraction is accurate to the nearest HUNDREDTH. Without the terminal zero it is accurate only to the nearest TENTH.
4. To conclude this portion of the lesson, the following generalization should be drawn from pupils at this stage:

The addition of a terminal zero to a decimal fraction does not change the value of the fraction but it does change the significance of the fraction.
Achievement of Lesson Objective 1 (Time: 15 minutes)

To demonstrate the effect upon the value of a decimal fraction of moving the decimal point.

Materials:

Visualization Cards 9 and 11. Pieces of paper sufficient to cover the portions of Card 9 that are not being used.

Steps:

Note: Steps 1 and 2 demonstrate visually the effect upon the value of the decimal fraction of moving the decimal point to the left.

Steps 3 and 4 demonstrate visually the effect upon the value of the decimal fraction of moving the decimal point to the right.

Step 5 is the final step in the induction, and contains a generalization which should be drawn from pupils as a result of their experience with the first four steps.

1. Hang Visualization Card 9 from the moulding at the top of the blackboard. Cover up the following 2's: Thousands, Tenths, Hundredths, Thousandths, thus leaving the portion of the Card shown below.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Emphasise the point that the number represented is composed of 2 Hundreds, 2 Tens, and 2 Ones.

As indicated above, draw an arrow (red, in this illustration) to indicate the movement of the decimal point one place to the left.

Explain: Since the place immediately to the left of the decimal point must always be the Ones' place, this makes it necessary to consider that the original 2 Tens have now, in effect, been reduced to 2 Ones.

Likewise, the other 2's shown in adjacent positions must be reduced to one-tenth the original place value in order to maintain the principle of Ten-ness.

2. As indicated by the green arrow in the illustration above, draw an arrow on the blackboard to indicate the movement of the decimal point two places to the left of the original location. Repeat the appropriate explanation given in step 1.
3. Hang Visualization Card 10 from the moulding at the top of the blackboard. Cover up the ONE. This leaves:

As indicated by the red arrow in the illustration above, draw an arrow on the blackboard to indicate the movement of the decimal point one place to the right.

Explain: Since the place immediately to the left of the decimal point must always be the ONES' place, this makes it necessary to consider that the representation of ONE-TENTH (immediately to the left of the new location of the decimal point) has, in effect, been increased to ONE.

Likewise, the representations shown on adjacent places (that is, the TENTH and HUNDREDTH places) must be increased to ten times the original size in order to maintain the principle of TEN-NESS.

4. As indicated by the green arrow in the illustration above, draw an arrow on the blackboard to indicate the movement of the decimal point two places to the right.

Repeat the appropriate explanation given in step 3.

5. To conclude this portion of the lesson, the following generalisation should be drawn from pupils at this stage:

(a) For every place that a decimal point is moved to the right in a number, it has the effect of multiplying the number by TEN. That is, if the decimal point is moved one place to the right, the number becomes 10 times larger; if it is moved two places to the right, the number becomes 100 times larger, etc.

(b) For every place that a decimal point is moved to the left in a number, it has the effect of dividing the number by 10. That is, if the decimal point is moved one place to the left, the number is reduced to ONE-TENTH its original value; if it is moved two places to the left, the number is reduced to ONE-HUNDREDTH its original value, etc.

PART TWO

Achievement of Lesson Objective 2 (Time: 15 minutes)

To demonstrate the effect upon the location of the decimal point of multiplying or dividing a decimal fraction by a power of 10.
Materials:

Same as for Part One.

Steps:

Note: Part Two of this Lesson is the converse to Part ONE. The steps in this part, therefore, are parallel to those contained in the first part.

Steps 1 and 2 demonstrate visually the effect upon the location of the decimal point of dividing a number by a power of 10.

Steps 3 and 4 demonstrate visually the effect upon the location of the decimal point of multiplying a number by a power of 10.

Step 5 is the final step in the induction, and contains a generalization which should be drawn from pupils as a result of their experience with the first four steps.

1. Hang Visualization Card 9 from the moulding at the top of the blackboard. Cover up the same portion of the Card as in Part ONE, leaving the following:

<table>
<thead>
<tr>
<th>HUNDREDS</th>
<th>TENS</th>
<th>ONES</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

Emphasize the point that the number 222 is composed of 2 HUNDREDS, (or 200); 2 TENS (or 20); and 2 ONES (or 2). These may be written in the appropriate places on the blackboard as shown above.

Divide each of these by 10. This, too, may be written on the blackboard under Line (a), as shown below:

<table>
<thead>
<tr>
<th>20</th>
<th>2</th>
<th>2/10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2 TENS)</td>
<td>(2 ONES)</td>
<td>(2 TENTHS)</td>
</tr>
</tbody>
</table>

Since the ONES must be identified by the decimal point, it is, consequently, necessary to adjust the location of the decimal point from its original position (Figure 1) to one place to the left, as shown by the red arrow in Line (b).

2. Erase Line (b) from the blackboard, and proceed to develop from Line (a), this time to show what happens to the position of the decimal point when the number is divided by 100.

Line (b), therefore, becomes:
Since the ONES must be identified by the decimal point, it is, consequently, necessary to adjust the location of the decimal point by moving it from its original position to two places to the left, as shown by the green arrow in Line (b).

3. Hang Visualization Card 10 from the moulding at the top of the blackboard. Cover up the ONE. This leaves:

![Visualization Card 10](image)

(Multiply by 10)

This represents .111. Let us now multiply this decimal fraction by 10, thus:

![Figure 1](image)

The representation shown in Figure 1 should be drawn on the board directly underneath Visualization Card 10.

It is now necessary to adjust the location of the decimal point in order to put it beside the card that stands for ONE. That is, when the number is multiplied by 10 it is necessary to move the decimal point one place to the right. See red arrow, which should also be drawn on the blackboard in the appropriate place.

4. Repeat the illustration given in Step 3; applying it this time to demonstrate the need to move the decimal point two places to the right when the number is multiplied by 100.

5. To conclude this portion of the lesson, the following generalisation should be drawn from pupils at this stage:

(a) When a decimal fraction is multiplied by 10, 100, 1000, etc., (that is, some power of 10) the decimal point is moved one place to the left for every zero in the divisor.

(b) When a decimal fraction is divided by 10, 100, 1000 etc., (that is, some power of 10) the decimal point is moved one place to the left for every zero in the divisor.
ACHIEVEMENT OF LESSON OBJECTIVE 1 (Time: 15 minutes)

To illustrate the significance of rounding decimal fractions.

Materials:

Visualization Card No. 11.

Steps:

Note: The significance of rounding decimal fractions is shown by comparing the variation in a measurement rounded only to UNITS to the variations in measurements rounded successively to TENTHS and HUNDREDTHS.

1. Draw the following scale on the blackboard:

![Scale Diagram]

(a) Explain that when we say that a line is 2 inches long we signify by this indication merely that the length is closer to 2 inches than it is to 1 inch or 3 inches. The rather considerable amount of variation in length permitted is indicated by the RED area.

It should be evident that in order to round a measurement number to the nearest UNIT it is necessary to know at least the number of UNITS involved in the measurement.

(b) Explain that when we say that a line is 2.0 inches long we signify by this indication that the length this time is closer to 2.0 inches than it is to 1.9 inches or to 2.1 inches. The more restricted amount of variation in length permitted by this designation is indicated by the PURPLE area.

It should be evident in this case that in order to round a measurement number to the nearest TENTH it is necessary to know at least the number of TENTHS involved in the measurement.

(c) Finally, explain that when we say that a line is 2.00 inches long we signify by this indication that the length this time is closer to 2.00 than it is to 1.99 or to 2.01 inches. The even more restricted amount of variation in length permitted by this designation is indicated by the GREEN area.

It should be evident in this case that in order to round a measurement number to the nearest HUNDREDTH it is...
necessary to know at least the number of THOUSANDTHS involved in the measurement.

2. Hang Visualization Card No. 11 from the moulding at the top of the blackboard. Show diagrammatically how this represents only a portion of the blackboard illustration shown in Step 1.

Let us say that the length of a line is 1.67 units. This means that this measurement is rounded to the nearest HUNDREDTH, and that in order to be able to effect this degree of rounding it is necessary to know the length of the line in THOUSANDTHS; or, in other words, to know that the length lies somewhere between 1.765 and 1.774.

Point out on this chart that as we successively reduce the accuracy of rounding we increase the variation in the length of the line represented by the measurement. That is to say, point out that if this line were rounded to the nearest TENTH it would be 1.7 and show that this variation would entitle it to be placed between 1.75 and 1.84. And further, point out that if this line were rounded to the nearest UNIT it would be 2 and show that this variation would entitle it to be placed between 1.5 and 2.4.

In all these cases of rounding, if the fraction is equal to or greater than one-half of the fractional interval, the fraction will be raised to the next highest interval.

3. Repeat with other illustrations. Assume, for example, that the length of a line is 1.68, or again, 1.32. Repeat the same procedure as in Step 2.

4. To conclude this portion of the lesson, three generalizations should be drawn from pupils at this stage:

(a) In rounding a mixed decimal fraction to the nearest whole number, if the number of TENTHS is 5 or greater, add 1 to the whole number.

(b) In rounding a mixed decimal fraction to the nearest whole number it is necessary to know the number of TENTHS. In rounding a number to the nearest TENTH it is necessary to know the number of HUNDREDTHS.

(c) After rounding has been completed, the place occupied by the last DIGIT or ZERO indicates the accuracy of the measurement. For example, 2,060 is accurate to the nearest THOUSANDTH.
Achievement of Lesson Objective 2 (Time: 8 minutes)

To demonstrate various applications of the rounding of decimal fractions.

Materials:

Visualisation Card No. 11.

Steps:

Decimal fractions are frequently expressed to a degree of accuracy beyond that required for a particular purpose. The following steps show visually how approximations of such decimal fractions may be made by various applications of rounding.

1. Assume the length of a line to be 1.837. Indicate on the Visualisation Card the very small variation in length that would be permitted by this very accurate description.

2. For convenience we may round this mixed decimal expression to HUNDREDTHS, and report it as 1.84 OR 184 HUNDREDTHS.

Remind pupils of the point that was emphasised in Part Two of Lesson III concerning the importance of the last-used position after the decimal point. Thus, in 1.84, when we convert everything to the position occupied by the 4 we get 184 HUNDREDTHS.

Point out that this measurement, 1.84 or 184 HUNDREDTHS, is accurate to the nearest HUNDREDTH, and that IN ORDER TO OBTAIN THIS DEGREE OF ACCURACY WE MUST FIRST, BEFORE ROUNDING, KNOW ALSO THE NUMBER OF THOUSANDTHS.

3. For even greater convenience, 1.837 may be rounded to TENTHS. As shown in the diagram above, point out on the Visualisation Card that this may be rounded to 1.8 OR 18 TENTHS.

Repeat
Repeat the various points made in Step 2 above.

4. Demonstrate on the blackboard how 1.596 could be expressed as:

(a) 1.60 (read "one and sixty hundredths")

or 160 HUNDREDTHS.

(b) 1.6 or 16 TENTHS.
Achievement of Lesson Objective 3 (Time: 7 minutes)

To indicate why UNLIKE decimal fractions must be changed to LIKE decimal fractions (that is, with the same understood denominator) in order that they may be added or subtracted.

Materials:
No special materials required.

Steps:

Note: Step 1 refers to non-measurement numbers which may be counted as discrete, non-continuous entities.

Step 2 refers to measurement numbers.

1. When the numbers do NOT mean inches, or some other measurement, fill the empty spaces with zeros, for example:

\[
\begin{align*}
0.8 & \quad \text{change to} \quad 0.800 \\
0.65 & \quad \text{change to} \quad 0.650 \\
0.239 & \quad \text{change to} \quad 0.239 \\
\end{align*}
\]

2. When the numbers represent measurements, as in the example below, it is necessary to find the number with the fewest decimal places and round all the other numbers to that number of places, for example:

\[
\begin{align*}
0.8 & \quad \text{change to} \quad 0.8 \\
0.65 & \quad \text{change to} \quad 0.7 \\
0.239 & \quad \text{change to} \quad 0.2 \\
\end{align*}
\]

Note: it is understood that these numbers refer to inches, pounds, etc.

3. To conclude this portion of the lesson, the following generalization should be drawn from pupils at this stage:

"The sum or difference of measurement numbers will be accurate only to the fractional unit of the number that has the fewest decimal places."
DIVISION INVOLVING DECIMAL FRACTIONS

PART ONE

Achievement of Lesson Objective 1 (Time: 8 minutes)

To explain the significance of performing division involving decimal fractions.

Materials:

No special materials required.

Steps:

Note: In division involving decimal fractions frequently the placement of the decimal point is governed only by meaningless rule. The purpose of Part One of this lesson is to interpret the reason for the placement of the decimal point in a quotient.

The division of common fractions is used as a means of developing this interpretation.

The time limit devoted to Part One imposes very great restrictions on the thoroughness with which this topic may be discussed. For this reason it is necessary to restrict the examples shown, and deal only with ones such as the following divisions. Such curiosity may be aroused by this incomplete presentation as will make profitable a more complete presentation OUTSIDE THE AREA OF THIS EXPERIMENT.

1. Present the following examples on the blackboard:

(a) 8.6)16.34 
(b) .14) .584 
(c) 9.8)7.056

The above examples have been selected because none of them requires the addition of zeros to the dividend. It may be explained, however, if the need arises, that the same principle holds in the case of NON-TERMINATING or INFINITE quotients where

The above examples may be worked out by different pupils on the blackboard.

2. When the quotients have been obtained demonstrate by means of divisions involving common fractions that in the case of:

Example (a) HUNDREDS divided by TENTHS is Tenths.
(b) THOUSANDS divided by HUNDREDS is Tenths.
(c) THOUSANDS divided by TENTHS is Hundredths.
Achievement of Lesson Objective 2 (Time: 22 minutes)

To demonstrate the significance of moving the decimal point in performing divisions involving decimal fractions.

Materials:

Visualization Card No. 12. For use in this lesson, each section should be regarded as 1/10 of the dividend and of the divisor.

Steps:

Note: Steps 1 to 4 inclusive refer to examples where a whole number is divided by a decimal.

1. Write on the blackboard the division $1 \div 1$ and illustrate the answer on Visualization Card No. 12, shown below:

```
   .1 .1 .1 .1 .1 .1 .1 .1 .1 .1
```

Point out that when a whole number is divided by a simple fraction the answer is larger than the dividend.

This may revolutionize somewhat the concept children may have gained in previous grades in which it was believed that if a number were divided it would automatically mean that the quotient would be smaller than the dividend.

2. Though it is not easy to illustrate visually, explain that when the divisor (lower section of the Visualization Card) is a fraction, the division is more easily performed if the divisor is made a whole number.

Illustrate this with such an example as the following:

6 divided by 3/10 is not as easy to divide as

60 divided by 3.

3. Refer on the Visualization Card to the division:

```
   .1 \div 1
```

Show that if the divisor is multiplied by 10 to give 1; and if the dividend is also multiplied by 10 to give 10, the quotient will be the same.
4. At this point two generalizations should be drawn from pupils:

(a) When a whole number is divided by a simple fraction the quotient (answer) will be larger than the dividend.

(b) When both the dividend and the divisor are multiplied by the same number the quotient remains the same.

Note:
Steps 5 to 8 inclusive refer to examples where a decimal fraction is divided by a decimal fraction.

5. As in Step 2, use Visualization Card 12 to explain that when the divisor is a fraction, the division is more easily performed if the divisor is made a whole number.

Illustrate this with such an example as the following:

\.4)\.32 is more easily divided when changed to 4)32

The visualization shown below of this example should be drawn on the blackboard and used to supplement the visualization medium contained on Card 12.

As in Step 3, illustrate on the Visualization Card that if the DIVIDEND and the DIVISOR are each multiplied by the same number, the quotient remains unchanged.

Stated in another way, it may be said that if the decimal point is moved the same number of places, AND IN THE SAME DIRECTION, in the DIVIDEND AND the DIVISOR the answer remains unchanged.

6. Discuss in what way the answer would be altered if, instead of moving the decimal point the same way in both the DIVIDEND and the DIVISOR, the point were moved ONE PLACE TO THE LEFT IN THE DIVISOR and ONE PLACE TO THE RIGHT IN THE DIVIDEND.

Illustrate the division \( .1 \div .2 \) on the visualization card to show that the answer would be 100 times larger than it should be.
Illustrate the division \( \frac{1}{2} \) on the Visualization Card to show that the answer would be 100 times larger than it should be.

7. Similarly, discuss in what way the answer would be altered if, instead of moving the decimal point the same way in both the DIVIDEND and the DIVISOR, the point were moved ONE PLACE TO THE RIGHT IN THE DIVISOR and ONE PLACE TO THE LEFT IN THE DIVIDEND.

Illustrate the division \( \frac{1}{2} \) on the Visualization Card to show that the answer would be only 1/100 of what it should be.

8. At this point two further generalizations should be drawn from pupils:

(a) In dividing with decimals, the divisor may be made a whole number by multiplying it by a given amount, provided the dividend also is multiplied by the same amount.

(b) In dividing with decimals, if the decimal point is moved ONE PLACE IN OPPOSITE DIRECTIONS in the DIVIDEND and the DIVISOR, the answer will be EITHER 100 times greater than, or 1/100 as great as, it should be.
LESSON VIII

MISCELLANEOUS CONCEPTS INVOLVING DECIMAL FRACTIONS

PART ONE

Reduction of Common Fractions to Decimal Fractions

Achievement of Lesson Objective 1 (Time: 12 minutes)

To convey the significance of changing a common fraction to a decimal fraction.

Materials:

Visualisation Card 12. (Note: For use in this lesson each space in the upper section (dividend) and in the lower section of this card should be regarded as one whole unit instead of 1/10 of a unit, as was the case when the card was used in Lesson VII)

Steps:

1. Hang Visualisation Card 12 from the moulding at the top of the blackboard.

While referring to this visualization, explain that a common fraction merely indicates an unperformed division, and that the changing of this common fraction into a decimal fraction involves the performance of this division. In this division the numerator of the fraction becomes the dividend and the denominator becomes the divisor.

Demonstrate the conversion of the fraction \( \frac{1}{4} \) to a decimal fraction.

Point out one section on the upper part of the illustration. Let this represent the numerator of 1. Since 1 is NOT evenly divisible by 4 it is necessary to convert the 1 whole, (as shown in Lesson III, Part Two, Step 2) into a smaller denomination which will be divisible by 4.

Illustrate that changing the 1 into 10 TENTHS does not permit it to be divided by 4. Consequently it is necessary to change it into 100 HUNDREDTHS.

2. Copy other illustrations, such as the following, on the board:

<table>
<thead>
<tr>
<th>Common Fraction</th>
<th>Change to Tenths</th>
<th>Change to Hundredths</th>
<th>Change to Thousandths</th>
<th>Decimal Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{5}{10} )</td>
<td>( \frac{50}{100} )</td>
<td>( \frac{500}{1000} )</td>
<td>.5</td>
</tr>
<tr>
<td>( \frac{3}{8} )</td>
<td>( \frac{30}{8} )</td>
<td>( \frac{300}{8} )</td>
<td>( \frac{3000}{8} )</td>
<td>.375</td>
</tr>
</tbody>
</table>
3. TO CONCLUDE THIS portion of the lesson, two generalisations should be drawn from pupils at this stage:

(a) Converting a common fraction to a decimal fraction involves a division in which the numerator of the fraction becomes the dividend, and the denominator becomes the divisor.

(b) Before performing this division it is necessary to add zeros to the numerator. Adding these zeros really amounts to converting the NUMERATOR from ONES to TENTHS, HUNDREDTHS, THOUSANDTHS, or whatever smaller unit is required to obtain a suitable decimal fraction equivalent.

PART TWO

Multiplication Involving Decimal Fractions

Achievement of Lesson Objective 2 (Time: 12 minutes)

To illustrate the reason for the placement of the decimal point in the product obtained by the multiplication of decimal fractions.

Materials:

Visualization Card II (As used in Lesson VI)

Steps:

1. Hang Visualization Card II from the moulding at the top of the blackboard.

Regard the distance between 1 and 2 on the card as 1 whole unit.

Show on the card that:

(a) 1/10 of (which means times) 1 whole unit is 1 TENTH.

or, in other words, .1 times 1 equals .1

TENTHS TIMES UNITS EQUALS TENTHS

(b) 1/10 times 1/10 equals 1/100

or, in other words, .1 times .1 equals .01

TENTHS TIMES TENTHS EQUALS HUNDREDTHS.

2. In the same way explain that in the question 19.8 times 7.6 the decimal point is located in this place in the answer, 150.48 because

TENTHS TIMES TENTHS IS HUNDREDTHS.

3. Point out how the value of this product would be altered if
the decimal in the first number were changed two places to the left, for example, and changed one place to the right in the second number.

Thus, instead of $19.8 \times 7.6$, we would now have $0.198 \times 76$.

This product would have to be expressed in THOUSANDTHS, because THOUSANDTHS times ONES (76 ones) equals THOUSANDTHS.

The original was expressed in HUNDREDTHS. Therefore, the act of changing the decimal points as we did had the effect of making the value of the fraction exactly $1/10$ of what it was at first.

4. If time permits, repeat this procedure contained in Steps 2 and 3 with the following example:

The product of $4.86$ and $6.9$ is $33.534$ (HUNDREDTHS times TENTHS is THOUSANDTHS).

Point out in what way the value of this product would be affected if the decimals were moved into the following positions:

(a) $48.6 \times 6.9$ (Answer remains unchanged)
(b) $0.486 \times 6.9$ (Answer is $1/10$ of what it was).

PART THREE

Addition involving decimal fractions

Achievement of Lesson Objective 3 (Time: 6 minutes)

To develop an understanding of the importance in the addition of decimal fractions of aligning columns according to place value.

Materials:

Visualization Card 13.

Steps:

Note: Pupils often fail to line up decimal points when they write decimals in addition problems. Errors resulting from this may not be detected because of the failure to recognize what the decimal in the sum must mean. Writing the sum correctly should be rationalized in terms of place value.
Lesson VIII (Page 4)

1. Hang Visualisation Card 13 from the moulding at the top of the blackboard.

<table>
<thead>
<tr>
<th>ONES</th>
<th>TENTHS</th>
<th>HUNDREDTHS</th>
<th>THOUSANDTHS</th>
</tr>
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<tr>
<td>1</td>
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</table>

This visualization should be used to impress upon pupils the fact that the necessity to align the decimals under one another is merely to ensure that numbers with similar place values will be added.

In an addition involving decimal fractions it is no more correct to add a 1 in the TENTHS place to a 1 in the HUNDREDTHS place than it is to add 1/10 and 1/100 without changing them to a common denominator.

2. It should be pointed out that where there is a division involving decimals derived from measurements, as in the case at the right, these quantities should not, FROM A PRACTICAL POINT OF VIEW AT LEAST, be added as they stand. The number 12.3 does not necessarily mean 12.30. It may mean anything from 12.25 to 12.34, inclusive. If such measurements have been obtained, and they are to be added, the only sensible thing to do is TO ROUND ALL TO TENTHS, that is, to round so that all the measurements are expressed to the same number of places.  

| 12.3 inches | 8.65 " | 14.059 " | 6.4 " |

(In practical work round this to TENTHS)  

3. Emphasize the fact that in an example such as the one shown in Step 2, the answer will be accurate ONLY to the nearest TENTH.

3. To conclude this portion of the lesson, two generalisations should be drawn from pupils at this stage:

(a) In the case of the addition of measurement numbers involving decimal fractions, the sum will be accurate only as far as the last-used place value of the number containing the fewest number of decimal places.

(b) In the addition of decimal fractions all figures with the same place value should be placed in the same column.
APPENDIX C

THE PUPILS' WORKSHEETS

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</table>
THE DECIMAL SYSTEM OF NOTATION

Write the letter of the best answer on the answer sheets provided.

1. Which of the following is the largest?
   (A) 1346  (B) 6341  (C) 1000  (D) 5999  (E) 2997

2. Which one of the following is represented by the 7 in 37829?
   (A) seven hundred  (B) seven-tenths
   (C) seven thousand  (D) seventy thousand
   (E) seven

3. If you changed the number 73069 so that the 3 was in the 9's place and the 9 was in the 3's place, how would the new number compare with 73069?
   (A) It would be larger  (B) It would be smaller
   (C) It would be the same size  (D) Can't tell
   (E) It can't be done

4. If you re-arranged the figures in the number 53429, which of the following arrangements would give the largest number?
   (A) 95,324  (B) 95,432  (C) 59,432
   (D) 95,234  (E) 95,243

5. Which of the following numbers is the smallest?
   (A) 11890  (B) 10999  (C) 19000
   (D) 17999  (E) 18999

6. If you re-arranged the figures in the number 43,125, which of the following arrangements would give the smallest number?
   (A) 54,321  (B) 21345  (C) 12,345
   (D) 14,532  (E) 13,245

7. If the figures in 86,473 were re-arranged, which of the following would place the largest figure in the thousand's place?
   (A) 73,648  (B) 38,467  (C) 76,483
   (D) 87,643  (E) 86,734

8. If the figures in 23,469 were re-arranged, which of the following would place the smallest figure in the tens' place?
   (A) 46,932  (B) 96,432  (C) 69,234
   (D) 34,629  (E) 92,346

9. Which of the following has a 3 in the hundreds' place?
   (A) 23,069  (B) 86,231  (C) 49,563
   (D) 39,043  (E) 42,304
10. Which of the following has a 4 in the ten-thousands' place?
   (A) 423,104  (B) 643,142  (C) 438,116
   (D) 374,942  (E) 763,420

11. In the number 3,944 the 4 on the right represents a number how
    many times as large as the 4 on the left?
   (A) 1/10  (B) 1/2  (C) 1  (D) 5  (E) 10

12. Which of the following statements best tells why we write a
    zero in the number 4039 when we want it to say "four thousand
    thirty-nine?"
   (A) Because the number would say "four hundred thirty-nine" if
       we did not write the zero.
   (B) Writing the zero helps us to remember the number correctly.
   (C) Writing zero tells us that there are no hundreds in the
       number 4039.
   (D) Because the number would be wrong if we left the zero out.

13. About how many tens are there in 6452?
   (A) 6.5  (B) 65 1/2  (C) 654
   (D) 6,540  (E) 65,000
Worksheet No. 2

IDENTIFICATION AND MEANING OF PLACE NAMES
IN MIXED DECIMAL FRACTIONS

Write the letter of the best answer on the answer sheets provided.

1. The value of 2 in .024 is how many times the value of the 4?
   (A) 20  (B) 1/2  (C) 10  (D) 5  (E) 50

2. Which of the following methods is best for determining the value of the 7 in 3748?
   (A) Its position in the number
   (B) Its size when compared with other figures in the number
   (C) Its size when compared with the whole number 3748
   (D) Its size among the numerals from 1 to 9
   (E) Its position in the number and its size

3. The value of the 1 in 2.41 is what fractional part of the value of the 2?
   (A) 1/2  (B) 1/100  (C) 1/50  (D) 1/200  (E) .05

4. The value of 3 written two places to the right of ONES' place is:
   (A) .3  (B) .03  (C) 30  (D) .003  (E) 300

5. Which of the following numbers has the figure 4 written in the HUNDREDTHS' place?
   (A) 4486.453  (B) 3682.474  (C) 3271.043  (D) 34444.424

6. The value of 6 in the number 1.683 is how many times the value of the 3?
   (A) 100  (B) 1/200  (C) 2  (D) 200  (E) 1/2

7. Digit (a), as marked in the following number, is how many times digit (b):
   \[ \begin{array}{c}
   (a) \\
   (b) \\
   \end{array} \]
   \[ \begin{array}{c}
   3 \ 2 \ 5 \ . \ 7 \ 2 \\
   100 \\
   1/100 \\
   \end{array} \]
   \[ \begin{array}{c}
   \text{(A) 100} \\
   \text{(B) 1/200} \\
   \text{(C) 10} \\
   \text{(D) 1/1000} \\
   \text{(E) 1000} \\
   \end{array} \]

8. Which of the following numbers is the largest?
   (A) .3248  (B) .4  (C) .3249  (D) .329  (E) .3328

9. In (a)(b) the digit marked (a) is how many times the digit .0 8 4 marked (b)?

10. Which of the following numbers is the greatest?
    (A) .3  (B) .295  (C) .11  (D) .101  (E) .301
11. The largest of several decimal fractions will be the one with the largest digits in
   (A) the TENTHS' place
   (B) the HUNDREDTHS' place
   (C) the THOUSANDTHS' place
   (D) any place

12. The main purpose of the decimal point is to indicate the digit in:
   (A) HUNDREDS' place   (B) HUNDREDTHS' place   (C) ONES' place
   (D) TENTHS' place     (E) TENS' place.

13. The following numbers: .0163; .02; .1; .0897; .0911, when arranged in order of size from largest to smallest would be:
   (A) .0911; .1; .0897; .02; .0163
   (B) .0911; .0897; .0163; .1; .02
   (C) .1; .0911; .0897; .02; .0163
   (D) .1; .02; .0163; .0897; .0911
   (E) .0163; .02; .0897; .0911; .1

14. The largest expression of the following is
   (A) .16  (B) 1.6  (C) .016  (D) .0016  (E) 16.0
THE READING AND WRITING OF DECIMAL FRACTIONS

Worksheet No. 3

Write the letter of the best answer on the answer sheets provided.

1. Out of the following common fractions select those which may also be regarded as decimal fractions:

   (A) \( \frac{2}{5} \)  (B) \( \frac{9}{17} \)  (C) \( \frac{7}{100} \)  (D) \( \frac{8}{50} \)  (E) \( \frac{3}{10} \)

2. In every decimal fraction there is an unwritten denominator which is always:
   (A) 10  (B) 50  (C) a multiple of 10  (D) a power of 10  (E) 100

3. Express each of the following decimal fractions in words:
   (a) 0.362  (b) 0.0375  (c) 200.007  (d) 0.1 \( \frac{1}{9} \)  (e) 0.120
   (f) 5.754989  (g) 0.0560  (h) 0.34\( \frac{3}{9} \)

4. Write these decimals with common fractions, as in example:
   Example: \( \frac{.15}{100} = \frac{15}{100} \)  (a) \( .031 \)  (b) \( 2.02 \)  (c) \( .875 \)

5. Change the following mixed decimals to improper fractions as in example:
   Example: 1.25 is 125 HUNDREDTHS or 125/100
   (a) 4.75  (b) 10.00  (c) 1.05

6. The unwritten denominator of a decimal fraction is understood to possess one zero for:
   (a) every figure to the right of the decimal point
   (b) every zero to the right of the decimal point
   (c) every figure, except the zeros, to the right of the point.

7. The unwritten denominator of a decimal fraction is determined by:
   (a) the number of zeros after the decimal point
   (b) the place value of the last-used decimal place
   (c) the size of the largest digit after the decimal point
   (d) the size of the first digit after the decimal point

8. The numerator of a decimal fraction is determined by:
   (a) the position of the last digit after the decimal point
   (b) the number of digits after the decimal
   (c) the size of the first digit after the decimal point.
THE FUNCTIONS OF ZERO IN DECIMAL FRACTIONS

Write the letter of the best answer on the answer sheets provided.

1. Adding two zeros to the right of a whole number is the same as:
   (A) Adding 10 to the number
   (B) Adding 100 to the number
   (C) Multiplying the number by 10
   (D) Multiplying the number by 100
   (E) Dividing the number by 100

2. Crossing off a zero from the right side of a whole number has the same effect as:
   (A) Subtracting 10 from the number
   (B) Subtracting 100 from the number
   (C) Multiplying the number by 10
   (D) Multiplying the number by 1
   (E) Dividing the number by 10

3. Adding two zeros to the right of a mixed decimal expression like 8.53 has the same effect upon the value of the expression as:
   (A) Adding 10 to the expression
   (B) Adding 100 to the expression
   (C) Leaving the expression unchanged
   (D) Multiplying the expression by 10
   (E) Multiplying the expression by 100

4. Inserting a zero BETWEEN THE DECIMAL POINT AND THE 5 in the mixed decimal expression 8.53 has the effect of:
   (A) Multiplying the expression by 10
   (B) Reducing the value of the expression
   (C) Multiplying the expression by 1/10
   (D) Adding 10 to the expression
   (E) Increasing the value of the expression

5. Inserting a zero BETWEEN THE DECIMAL POINT AND THE 5 in the decimal expression .53 has the effect of:
   (A) Multiplying the expression by 10
   (B) Reducing the value of the expression
   (C) Multiplying the expression by 1/10
   (D) Adding 10 to the expression
   (E) Increasing the value of the expression
6. If the length of a board is measured to the NEAREST FOOT, say 7 feet, it is not correct to write this measurement as 7.00 because:
(A) It multiplies the length of the board by 100
(B) It multiplies the length of the board by 10
(C) It adds 100 to the length of the board
(D) It changes the measurement in some other way
(E) It gives an unwarranted degree of accuracy to the measurement.

7. The function of zero as a "place-holder" in a decimal fraction is to:
(A) "Hold" each numeral in the fraction in the required position when no digit is present to perform this function
(B) Give to the fraction a greater degree of accuracy
(C) Spread the digits out to make reading easier
(D) Indicate the number of zeros in the unwritten denominator of the fraction.
Worksheet No. 5

CHANGING THE LOCATION OF THE DECIMAL POINT:
ITS EFFECT ON THE VALUE OF THE EXPRESSION

Write the letter of the best answer on the answer sheets provided.

1. Which of the following numbers is $\frac{1}{100}$ as large as 32.78?
   (A) .3278  (B) 3.278  (C) 327.8  (D) 3278

2. If the number .0857 is changed to 85.7 it becomes:
   (A) $\frac{1}{100}$ as large  (B) $\frac{1}{10}$ as large  (C) 10 times larger
   (D) 100 times larger  (E) 1000 times larger

3. When a number is divided by 1000 the decimal point is moved:
   (A) 2 places to the left  (B) 3 places to the left
   (C) 2 places to the right  (D) 3 places to the right

4. In writing an answer a boy makes the mistake of putting his decimal point two places too far to the left. As a result, his answer is:
   (A) $\frac{1}{10}$ of what it should be  (B) $\frac{1}{100}$ of what it should be
   (C) 10 times what it should be  (D) 100 times what it should be

5. When a number is multiplied by 100 the decimal point is moved:
   (A) 2 places to the left  (B) 3 places to the left
   (C) 2 places to the right  (D) 3 places to the right

6. Moving a decimal point three places to the right has the effect of:
   (A) multiplying the number by 1000
   (B) dividing the number by 1000
   (C) multiplying the number by 100
   (D) dividing the number by 100

7. If a decimal fraction is divided by 10 the decimal point is moved 1 place to the left because:
   (A) the number is increased by 10
   (B) the number is decreased by 10
   (C) the number becomes 10 times as large
   (D) the number becomes $\frac{1}{10}$ as large
Worksheet No. 6

ROUNDING DECIMAL FRACTIONS

Write the letter of the best answer on the answer sheets provided.

1. Round each of these to nearest whole numbers
   (A) 6.5  
   (B) .68  

2. Round each of these to nearest TENTH:
   (A) .36  
   (B) 4.029  
   (C) 7.931  

3. Round each of these to nearest HUNDREDTH:
   (A) .536  
   (B) 4.175  
   (C) 5.782  

4. Assuming that the following numbers have already been rounded indicate fractional unit to which each one is accurate:
   Example: 2.30 is accurate to the nearest HUNDREDTH.
   (A) .490  
   (B) .70  
   (C) 1.87  

5. In order to express the length of a line accurately to the nearest TENTH of an inch it is necessary to measure it to what fraction of an inch?

6. Round the following numbers to whole numbers, tenths, hundredths:
   1.089  
   2.008  
   6.509  
   .7829  
   13.72
Worksheet No. 7

DIVISION INVOLVING DECIMAL FRACTIONS

Write the letter of the best answer, or the answer itself, on the answer sheets provided.

1. When a whole number is divided by a number larger than 1 the quotient is:
   (A) larger than (B) smaller than (C) the same as, the dividend.

2. When a whole number is divided by a number smaller than 1 the quotient is:
   (A) larger than (B) smaller than (C) the same as, the dividend.

3. Divide:
   \[ \frac{2.8}{4.564} \]

4. In question 3 the answer comes out to HUNDREDTHS because:
   (A) There is one figure before the decimal point in the dividend and divisor
   (B) There are two figures in the divisor
   (C) Thousandths divided by tenths are hundredths
   (D) Tenths times tenths is hundredths.

5. Divide:
   \[ \frac{.42}{.7392} \]

6. In question 5 the decimal point may be moved 2 places to the right in the divisor because:
   (A) it must be placed after the two
   (B) it is moved 2 places to the right in the dividend also
   (C) the answer must come out to HUNDREDTHS
   (D) it makes the division easier.

7. Look back at questions 3 and 5 and without dividing write the answers to these divisions:
   (A) \[ \frac{.28}{.4564} \]    (B) \[ \frac{4.2}{7.392} \]

8. In the division \[ \frac{1.6}{9.28} \] if the decimal points are moved into these positions:
   \[ \frac{.16}{92.8} \] the answer will be:
   (A) the same
   (B) 10 times as large
   (C) one-tenth as large
   (D) 100 times as large
   (E) one-hundredth as large.
Worksheet No. 8

MISCELLANEOUS CONCEPTS INVOLVING DECIMAL FRACTIONS

Write the letter of the best answer, or the answer itself, on the answer sheets provided.

1. Which statement best tells why we arrange numbers in addition the way we do?
   (A) It is an easy way to keep the numbers in straight columns
   (B) It helps us to add correctly
   (C) It helps us to add only those numbers in the same position
   (D) It helps us to carry correctly from one column to another
   (E) It would be harder to add if the numbers were mixed.

2. What is the product of:
   (A) TENTHS and ONES (B) TENTHS and TENTHS
   (C) TENTHS and HUNDREDTHS (D) TENS and HUNDREDTHS
   (E) TENS and TENTHS

3. When a whole number is multiplied by a number larger than 1, the product is:
   (A) larger (B) smaller (C) unchanged

4. When a whole number is multiplied by a number smaller than 1, the product is:
   (A) larger (B) smaller (C) unchanged

5. If 2.398 times 87.2 equals 209.1056 without multiplying find the answer to:
   (A) 239.8 times 8.72 (B) 23.98 times .872
   (C) .2398 times 87.2

6. In the question 2.398 times 87.2, if the decimal point is moved 2 places to the right in the first number and one place to the left in the second number the answer is:
   (A) 1/10 as large (B) 10 times as large
   (C) 100 times as large (D) 1/100 as large
   (E) unchanged

7. To change a fraction like 7/8 to a decimal which comes out evenly, before dividing by 8 we must think of 7 as:
   (A) 70 TENTHS (B) 7000 THOUSANDTHS
   (C) 7000 THOUSANDTHS (D) 700 THOUSANDTHS
APPENDIX D

TESTS USED TO MEASURE THE CRITERION VARIABLE AND THE
FOUR INDEPENDENT VARIABLES

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1 This test was used to measure both the criterion variable, and one of the independent variables.
1. In addition of mixed decimal fractions it is important to arrange the numbers so that:
   A. the last figures of all numbers are in the same column
   B. all figures with the same place value are in the same column
   C. the first figures of all numbers are in the same column
   D. none of these.

2. To change a common fraction to a decimal fraction one must know that a common fraction indicates:
   A. multiplication  B. enumeration  C. addition
   D. division  E. subtraction.

3. Adding a zero to the end of a decimal fraction:
   A. makes the value 10 times as much
   B. makes the value 1/10 as much
   C. makes the value 10 more
   D. does not change the value.

4. The largest of several decimal fractions will be the one with the largest figure in:
   A. tenths place    B. hundredths place
   C. thousandths place    D. any place.

5. The number 6.00 has a value of:
   A. 6 hundreds    B. 600 hundreds
   C. 6 hundredths    D. 600 hundredths.

6. If a decimal point is moved two places to the left the number becomes:
   A. one-tenth as large    B. ten times as large
   C. one-hundredths as large    D. one hundred times as large.

7. The number .0170 should be read:
   A. seventeen hundredths
   B. one hundred seventy ten-thousandths
   C. one hundred seventy thousandths
   D. seventeen thousandths.

8. Changing .645 to .0645
   A. does not change the value
   B. makes value 10 times as much
   C. makes value 1/10 as much
   D. makes value 1/100 as much.
9. If the number 42.56 is changed to 42.056 by inserting a zero after the decimal point, the value becomes:
A. unchanged  B. less  C. greater  D. ten times greater  E. one-tenth as much.

10. The value of a decimal fraction is determined by:
A. the size of the first digit after the decimal point
B. the position of the last digit after the decimal point
C. the position of the largest digit after the decimal point
D. the position of the first digit, not including zeros, after the decimal point.

11. Which of the following numbers has the figure "6" in the thousandths place:
A. 4695.5417  B. 6495.1724
C. 4325.2164  D. 4175.6000

12. In the question \( \frac{.5}{16} \) the answer is larger than the number divided because:
A. 16 is more than .5
B. it is the same as multiplied by \( \frac{1}{2} \)
C. dividing a number always gives an answer larger than the number
D. it is the same as finding how many \( \frac{1}{16} \)'s in 16.

13. If a decimal fraction is divided by 1000, the decimal point is moved three places to the left because:
A. the number becomes 1000 times as large
B. the number is increased by 1000
C. the number becomes \( \frac{1}{1000} \) as large
D. the number is decreased by 1000.

14. In the question \( \frac{1.6}{620.54} \) if the decimal point is moved one place to the right in the divisor, and one place to the left in the dividend, the answer will be:
A. one hundred times as great  B. ten times as great
C. one hundredths as great  D. one tenth as great  E. unchanged

15. When a decimal fraction is changed to a common fraction (not reduced), the denominator will have one zero for:
A. every figure to the right of the decimal point
B. every figure, except zeros, to the right of the point
C. every zero to the right of the point
D. none of these

16. Multiplying a decimal by 1000 moves the decimal point:
A. two places to the right  B. three places to the left
C. two places to the left  D. three places to the right.
17. In division with decimals the divisor may be made a whole number before dividing because:
A. you can't divide by a decimal
B. moving the point does not change the value of a number
C. it is more convenient
D. the point in the quotient must be directly above the point in the dividend
E. the value of a fraction is unchanged when both terms are multiplied by the same quantity.

18. The measurement 1.050 inches is accurate to the nearest:
A. tenth of an inch
B. hundredth of an inch
C. thousandths of an inch
D. ten thousandths of an inch.

19. Moving a decimal point 2 places to the right has the same effect as:
A. multiplying the number by 10
B. multiplying the number by 100
C. dividing the number by 100
D. none of these
(\text{a)(b)}

20. In the number: 5 5 5 . 5 5
A. digit (a) is 100 times digit (b)
B. digit (a) is 10 times digit (b)
C. digit (a) is 1/10 of digit (b)
D. digit (a) is 1/100 of digit (b)

21. The number .6925 has a value of about:
A. .69 hundredths
B. 2 hundredths
C. 9 hundredths
D. 692 hundredths.

22. If a number is to be expressed accurately to the nearest hundredth it must be found to at least:
A. one place after the decimal point
B. two places after the decimal point
C. three places after the decimal point
D. four places after the decimal point.

23. In the question: 1.25) 642.3 if the decimal point were located one place to the right in both numbers the answer would be:
A. ten times as large
B. one-tenth as large
C. one hundred times as large
D. one hundredth as large
E. unchanged.

24. If no zeros are added to the dividend, the answer to the question: 4.2) 69.735 will be a two-place decimal because:
A. thousandths divided by tenths is hundredths
B. there are two figures in the divisor
C. tenths times tenths is hundredths
D. there are two places before the point in the dividend.
25. In the question: $6.42 \times 15.7$ if the decimal point were located one place to the right in the first number and two places to the left in the second number the answer would be:
   A. ten times as large       B. one-tenth as large
   C. one hundred times as large  D. one-hundredth as large.

26. In the question: $6.92 \times 74.3 = 514.156$ the decimal point is located at this place in the answer because:
   A. one and two are three
   B. hundredths times tenths is thousandths
   C. tens times hundreds is thousands
   D. there are three places to the left of the point in the numbers multiplied.

27. A "decimal" is a fraction with an unwritten, but understood, denominator which will always be:
   A. one       B. ten       C. any multiple of ten
   D. any power of ten  E. none of these

28. The number 2.134 has a value of about:
   A. 1 tenth       B. 13 tenths       C. 21 tenths
   D. 213 tenths   E. 2.1 tenths.

29. To change a fraction, such as $\frac{3}{4}$, to a two place decimal we divide the numerator by the denominator and we must think of the numerator as:
   A. 3 hundreds       B. 3 hundredths       C. 300 hundredths
   D. 30 hundredths   E. none of these.

30. The sum of: 16.17, 459.4, 142.167, and 2.130 inches will be accurate to the nearest:
   A. inch       B. tenth inch
   C. hundredth inch  D. thousandth inch.
TEST ON DECIMAL FRACTIONS

1. Find the sum of:
   (a) 1.0687   (b) 387.85   (c) 8.975
   9.9345
   8.9784
   5.8459
   7.7956

2. Subtract:
   (a) 1.0687
   176.062
   89.875

3. From 124.40
   take 87.85

4. From 27.08
   take 15.17

5. From 150.000
   take 72.239

6. From 94.72
   take 19.88

7. Multiply each of the following:
   (a) 4 x .2 = ________
   (b) .203 x .3 = ________
   (c) 100 x 8.5 = ________
   (d) .08 x 25 x 1/2 = ________

8. Multiply:
   7.8
   6.4

9. Multiply:
   78.4
   .961

10. Multiply:
    94.36
    8.7

11. Divide each of the following:
   (a) .3) 3.6
   (b) .11) 1.342
   (c) .12) 6
   (d) .4) 1.2
   (e) .2) 10
   (f) 1.25) 62.5

12. Divide:
    .834) 91.74

13. Divide:
    8.9) 708.44

14. What is 3/8 of 6.4?
    Answer: ________

15. Express 1/8 as a decimal.
    Answer: ________
Read this page. Do what it tells you to do.

Do not open this paper, or turn it over, until you are told to do so. Fill these blanks, giving your name, age, birthday, etc. Write plainly.

Name............................................................ Age last birthday...... years
   First name, initial, and last name

Birthday ...................................................... Teacher............... Date ............ 19...
   Month  Day

Grade................................. School ......................... City

This is a test to see how well you can think. It contains questions of different kinds. Here is a sample question already answered correctly. Notice how the question is answered:

Sample: Which one of the five words below tells what an apple is?
1 flower, 2 tree, 3 vegetable, 4 fruit, 5 animal.................................( 4 )

The right answer, of course, is "fruit"; so the word "fruit" is underlined. And the word "fruit" is No. 4; so a figure 4 is placed in the parentheses at the end of the dotted line. This is the way you are to answer the questions.

Try this sample question yourself. Do not write the answer; just draw a line under it and then put its number in the parentheses:

Sample: Which one of the five things below is round?
1 a book, 2 a brick, 3 a ball, 4 a house, 5 a box.........................(  )

The answer, of course, is "a ball"; so you should have drawn a line under the words "a ball" and put a figure 3 in the parentheses. Try this one:

Sample: A foot is to a man and a paw is to a cat the same as a hoof is to a — what?
1 dog, 2 horse, 3 shoe, 4 blacksmith, 5 saddle...............................(  )

The answer, of course, is "horse"; so you should have drawn a line under the word "horse" and put a figure 2 in the parentheses. Try this one:

Sample: At four cents each, how many cents will 6 pencils cost?
...

The answer, of course, is 24, and there is nothing to underline; so just put the 24 in the parentheses.

If the answer to any question is a number or a letter, put the number or letter in the parentheses without underlining anything. Make all letters like printed capitals.

The test contains 75 questions. You are not expected to be able to answer all of them, but do the best you can. You will be allowed half an hour after the examiner tells you to begin. Try to get as many right as possible. Be careful not to go so fast that you make mistakes. Do not spend too much time on any one question. No questions about the test will be answered by the examiner after the test begins. Lay your pencil down.

Do not turn this page until you are told to begin.
EXAMINATION BEGINS HERE.

1. Which one of the five things below does not belong with the others? (Do not write on these dots, lines.)
   1. potato, 2. turnip, 3. carrot, 4. stone, 5. onion

2. Which one of the five words below tells best what a saw is?
   1. something, 2. tool, 3. furniture, 4. wood, 5. machine

3. Which one of the five words below means the opposite of west?
   1. north, 2. south, 3. east, 4. equator, 5. sunset

4. A hat is to a head and a glove is to a hand the same as a shoe is to what?
   1. leather, 2. a foot, 3. a shoestring, 4. walk, 5. a toe

5. A child who knows he is guilty of doing wrong should feel (?)
   1. bad, 2. sick, 3. better, 4. afraid, 5. ashamed

6. Which one of the five things below is the smallest?
   1. twig, 2. limb, 3. bud, 4. tree, 5. branch

7. Which one of the five things below is most like these three: cup, plate, saucer?
   1. fork, 2. table, 3. eat, 4. bowl, 5. spoon

8. Which of the five words below means the opposite of strong?
   1. man, 2. weak, 3. small, 4. short, 5. thin

9. A finger is to a hand the same as a toe is to what?
   1. foot, 2. toenail, 3. heel, 4. shoe, 5. knee

10. Which word means the opposite of sorrow?
    1. sickness, 2. health, 3. good, 4. joy, 5. pride

11. Which one of the ten numbers below is the smallest? (Tell by letter.)
    A 6084, B 5160, C 4342, D 6521, E 9703, F 4296, G 7475, H 2657, J 8839, K 3918

12. Which word means the opposite of pretty?
    1. good, 2. ugly, 3. bad, 4. crooked, 5. nice

13. Do what this mixed-up sentence tells you to do.
    number Write the the in parentheses

14. If we believe some one has committed a crime, but we are not sure, we have a (?)
    1. fear, 2. suspicion, 3. wonder, 4. confidence, 5. doubtful

15. A book is to an author as a statue is to (?)
    1. sculptor, 2. marble, 3. model, 4. magazine, 5. man

16. Which is the most important reason that words in the dictionary are arranged alphabetically?
    1. That is the easiest way to arrange them. 2. It puts the shortest words first. 3. It enables us to find any word quickly. 4. It is merely a custom. 5. It makes the printing easier

17. Which one of the five things below is most like these three: plum, apricot, apple?
    1. tree, 2. seed, 3. peach, 4. juice, 5. ripe

18. At 4 cents each, how many pencils can be bought for 36 cents?

19. If a person walking in a quiet place suddenly hears a loud sound, he is likely to be (?)
    1. stopped, 2. struck, 3. startled, 4. made deaf, 5. angered

20. A boy is to a man as (?) is to a sheep.
    1. wool, 2. lamb, 3. goat, 4. shepherd, 5. dog

21. One number is wrong in the following series. What should that number be? (Just write the correct number in the parentheses.)
    1. 6, 2. 6, 3. 6, 4. 6, 5. 6, 6. 7, 6

22. Which of the five things below is most like these three: horse, pigeon, cricket?
    1. stall, 2. saddle, 3. eat, 4. goat, 5. chirp

23. If the words below were rearranged to make a good sentence, with what letter would the last word of the sentence begin? (Make the letter like a printed capital.)
    nuts from squirrels trees the gather

24. A man who betrays his country is called a (?)
    1. thief, 2. traitor, 3. enemy, 4. coward, 5. slacker

25. Food is to the body as (?) is to an engine.
    1. wheels, 2. fuel, 3. smoke, 4. motion, 5. fire

26. Which tells best just what a pitcher is?
    1. a vessel from which to pour liquid, 2. something to hold milk, 3. It has a handle, 4. It goes on the table, 5. It is easily broken

Do not stop. Go on with the next page.
27. If George is older than Frank, and Frank is older than James, then George is (?) James.
   1 older than,  2 younger than,  3 just as old as,  4 (cannot say which)

28. Count each 7 below that has a 5 next after it. Tell how many 7's you count.
   7 5 3 0 9 7 3 7 8 5 7 4 2 1 7 5 7 3 2 4 7 0 9 3 7 5 7 2 3 5 7 7 5 4 7

29. If the words below were rearranged to make a good sentence, with what letter would the last word of the sentence begin? (Make the letter like a printed capital)
   leather shoes usually made are of

30. An electric light is to a candle as a motorcycle is to (?)
   1 bicycle,  2 automobile,  3 wheels,  4 speed,  5 police

31. Which one of the words below would come first in the dictionary?
   1 march,  2 ocean,  3 horse,  4 paint,  5 elbow,  6 night,  7 flown

32. The daughter of my mother's brother is my (?)
   1 sister,  2 niece,  3 cousin,  4 aunt,  5 granddaughter

33. One number is wrong in the following series. What should that number be?
   3 4 5 4 3 4 5 4 3 5

34. Which of the five things below is most like these three: boat, horse, train?
   1 sail,  2 row,  3 motorcycle,  4 move,  5 track

35. If Paul is taller than Herbert and Paul is shorter than Robert, then Robert is (?) Herbert.
   1 taller than,  2 shorter than,  3 just as tall as,  4 (cannot say which)

36. What is the most important reason that we use clocks?
   1 to wake us up in the morning,  2 to regulate our daily lives,  3 to help us catch trains,
   4 so that children will get to school on time,  5 They are ornamental.

37. A coin made by an individual and meant to look like one made by the government is called(?)
   1 duplicate,  2 counterfeit,  3 imitation,  4 forgery,  5 libel

38. A wire is to electricity as (?) is to gas.
   1 a flame,  2 a spark,  3 hot,  4 a pipe,  5 a stove

39. If the following words were arranged in order, with what letter would the middle word begin?
   Yard Inch Mile Foot Rod

40. One number is wrong in the following series. What should that number be?
   5 10 15 20 25 30 35 40 45 50

41. Which word means the opposite of truth?
   1 cheat,  2 rob,  3 liar,  4 ignorance,  5 falsehood

42. Order is to confusion as (?) is to war.
   1 guns,  2 peace,  3 powder,  4 thunder,  5 army

43. In a foreign language, good food = Bano Naab
t good water = Heto Naab
   The word that means good begins with what letter?

44. The feeling of a man for his children is usually (?)
   1 affection,  2 contempt,  3 joy,  4 pity,  5 reverence

45. Which of the five things below is most like these three: stocking, flag, sail?
   1 shoe,  2 ship,  3 staff,  4 towel,  5 wash

46. A book is to information as (?) is to money.
   1 paper,  2 dollars,  3 bank,  4 work,  5 gold

47. If Harry is taller than William, and William is just as tall as Charles, then Charles is (?) Harry.
   1 taller than,  2 shorter than,  3 just as tall as,  4 (cannot say which)

48. If the following words were arranged in order, with what letter would the middle word begin?
   Six Ten Two Eight Four

49. If the following words were rearranged to make a good sentence, with what letter would the third word of the sentence begin? (Make the letter like a printed capital)
   men high the a wall built stone

50. If the suffering of another makes us suffer also, we feel (?)
   1 worse,  2 harmony,  3 sympathy,  4 love,  5 repelled

51. In a foreign language, grass = Moki
t green grass = Moki Laap
   The word that means green begins with what letter?...
52. If a man has walked west from his home 9 blocks and then walked east 4 blocks, how many blocks is he from his home? ( )

53. A pitcher is to milk as (?) is to flowers.
   1 stem, 2 leaves, 3 water, 4 vase, 5 roots ( )

54. Do what this mixed-up sentence tells you to do.
   sum three Write two the four and of ( )

55. There is a saying, “Don’t count your chickens before they are hatched.” This means (?)
   1 Don’t hurry. 2 Don’t be too sure of the future. 3 Haste makes waste. 4 Don’t gamble ( )

56. Which statement tells best just what a fork is?
   1 a thing to carry food to the mouth, 2 It goes with a knife, 3 an instrument with prongs at the end, 4 It goes on the table, 5 It is made of silver ( )

57. Wood is to a table as (?) is to a knife.
   1 cutting, 2 chair, 3 fork, 4 steel, 5 handle ( )

58. Do what this mixed-up sentence tells you to do.
   sentence the letter Write last this in...i ( )

59. Which one of the words below would come last in the dictionary?
   1 alike, 2 admit, 3 amount, 4 across, 5 after, 6 amuse, 7 adult, 8 affect ( )

60. There is a saying, “He that scatters thorns, let him go barefoot.” This means (?)
   1 Let him who causes others discomforts bear them himself also. 2 Going barefoot toughens the feet. 3 People should pick up what they scatter. 4 Don’t scatter things around ( )

61. If the following words were arranged in order, with what letter would the middle word begin?
   Plaster Frame Wallpaper Lath Foundation ( )

62. In a foreign language, many boys = Boka Hepo
   many girls = Marti Hepo
   The word that means and begins with what letter? ( )

63. A statement which expresses just the opposite of that which another statement expresses is said to be a (?)
   1 lie, 2 contradiction, 3 falsehood, 4 correction, 5 explanation ( )

64. There is a saying, “Don’t look a gift horse in the mouth.” This means (?)
   1 It is not safe to look into the mouth of a horse. 2 Although you question the value of a gift, accept it graciously. 3 Don’t accept a horse as a gift. 4 You cannot judge the age of a gift horse by his teeth ( )

65. Which one of the words below would come last in the dictionary?
   1 hedge, 2 glory, 3 label, 4 green, 5 linen, 6 knife, 7 honor ( )

66. Which statement tells best just what a watch is?
   1 It ticks, 2 something to tell time, 3 a small, round object with a chain, 4 a vest-pocket-sized time-keeping instrument, 5 something with a face and hands ( )

67. Ice is to water as water is to what?
   1 land, 2 steam, 3 cold, 4 river, 5 thirst ( )

68. Which statement tells best just what a window is?
   1 something to see through, 2 a glass door, 3 a frame with a glass in it, 4 a glass opening in the wall of a house, 5 a piece of glass surrounded by wood ( )

69. Which of the five words below is most like these three: large, red, good?
   1 heavy, 2 size, 3 color, 4 apple, 5 very ( )

70. Write the letter that follows the letter that comes next after M in the alphabet ( )

71. One number is wrong in the following series. What should that number be?
   1 2 4 8 16 24 64 ( )

72. An uncle is to an aunt as a son is to a (?)
   1 brother, 2 daughter, 3 sister, 4 father, 5 girl ( )

73. If I have a large box with 3 small boxes in it and 4 very small boxes in each of the small boxes, how many boxes are there in all?
   1 2 4 5 7 8 10 11 12 14 ( )

74. One number is wrong in the following series. What should that number be?
   1 2 4 5 7 8 10 11 12 14 ( )

75. There is a saying, “Don’t ride a free horse to death.” This means (?)
   1 Don’t be cruel. 2 Don’t abuse a privilege. 3 Don’t accept gifts. 4 Don’t be reckless ( )

If you finish before the time is up, go back and make sure that every answer is right.
Name: ___________________________ Age: _______. Grade: ______

Boy or girl: ______ Name of school: ___________________________

City: __________________ State: __________________ Date: __________

<table>
<thead>
<tr>
<th>Test</th>
<th>Score</th>
<th>Grade Equiv.</th>
<th>Age Equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parag. Mean.</td>
<td></td>
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<tr>
<td>Word Mean.</td>
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<tr>
<td>Average Read.</td>
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</tbody>
</table>

Grade Equiv. 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 8.0 9.0 10.0 11.0

Equated Score 20 25 30 35 40 45 50 55 60 65 70 75 *

Age Equiv. 8° 8½ 9° 9½ 10° 10½ 11° 11½ 12° 12½ 13° 13½ 14° 15° 16°

*Values extrapolated above this point.

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24 Interpretation means — 6 petition 7 explanation 8 humility 9 pressure 10 failure
25 Kindred means — 1 delicate 2 gracious 3 humble 4 curious 5 related
26 To prosper is to — 6 endure 7 grieve 8 entertain 9 forgive 10 flourish
27 Nimble means — 1 practical 2 active 3 costly 4 modest 5 dull
28 Conservation means — 6 selecting 7 removing 8 observing 9 connecting 10 protecting
29 Dubious means — 1 doubtful 2 apparent 3 desolate 4 inferior 5 unusual
30 A pavilion is an open — 6 boat 7 forest 8 building 9 store 10 valley
31 To be punctual is to be — 1 bored 2 prompt 3 ashamed 4 worthy 5 determined
32 A lull is a — 6 kettle 7 jar 8 hush 9 link 10 lining
33 Liberality means — 1 gravity 2 havoc 3 impurity 4 hospitality 5 generosity
34 Obvious means — 6 remote 7 reasonable 8 doubtful 9 apparent 10 suitable
35 Competent means — 1 careless 2 useless 3 sincere 4 capable 5 cunning
36 Enthusiastic means — 6 lusty 7 singular 8 shameful 9 zealous 10 subtle
37 Conclusive means — 1 passive 2 variable 3 critical 4 decisive 5 compulsory
38 To amass is to — 6 allay 7 accumulate 8 verify 9 gamble 10 inscribe
39 Reputable means — 1 cordial 2 solemn 3 honorable 4 fortunate 5 prosperous
40 To indorse is to — 6 adjoin 7 magnify 8 disobey 9 affirm 10 disclose
41 To quail is to — 1 quarrel 2 attack 3 mourn 4 tremble 5 trap
42 To reprove is to — 6 preside 7 rebuke 8 regulate 9 replace 10 export
43 To obstruct is to — 1 advance 2 check 3 occupy 4 owe 5 pity
44 Congenial means — 6 original 7 universal 8 successful 9 agreeable 10 refined
45 To contend is to — 1 stroke 2 fasten 3 pardon 4 exchange 5 struggle
46 Void means — 6 empty 7 cruel 8 exact 9 fierce 10 useful
47 An impediment is an — 1 agreement 2 obstacle 3 idiot 4 outline 5 utterance
48 A mediator brings — 6 agreement 7 clashes 8 desolation 9 inspiration 10 reality
49 Equity means — 1 fashion 2 advantage 3 exchange 4 knowledge 5 justice
50 Morbid means — 6 unwholesome 7 impetuous 8 ruthless 9 magnetic 10 monotonous
**DIRECTIONS.** In each exercise one of the five numbered words will complete the sentence correctly. Note the number of this word. Then mark the answer space at the right which is numbered the same as the word you have selected.

**SAMPLES.**

| A rose is a — 1 box 2 flower 3 home 4 month 5 river | A |
| A roof is found on a — 6 book 7 person 8 rock 9 house 10 word | B |
| Bread is something to — 1 catch 2 drink 3 throw 4 wear 5 eat | C |

1. Injury means — 1 haste 2 charm 3 pride 4 praise 5 harm.
2. To arise is to — 6 answer 7 stand 8 sit 9 rest 10 carry.
3. Unoccupied means — 1 unjust 2 useless 3 vacant 4 haunted 5 ignorant.
4. A peg is usually made of — 6 wood 7 paper 8 rock 9 ice 10 sand.
5. To omit is to — 1 bore 2 neglect 3 concern 4 control 5 recover.
6. To defeat is to — 6 abuse 7 assign 8 betray 9 overcome 10 expose.
7. Envious means — 1 shallow 2 social 3 refined 4 enormous 5 jealous.
8. A scoundrel is a — 6 circus 7 shipment 8 villain 9 chronicle 10 loom.
9. To reject is to — 1 engage 2 refuse 3 hasten 4 forbid 5 mourn.
10. To forewarn is to — 6 caution 7 recoil 8 moisten 9 contemplate 10 lengthen.
11. A literary person is a — 1 painter 2 monarch 3 writer 4 rival 5 coward.
12. To prohibit means to — 6 forbid 7 permit 8 assist 9 boast 10 deserve.
13. Stern means — 1 splendid 2 severe 3 joyful 4 wicked 5 eager.
14. Conduct means — 6 effort 7 safety 8 appearance 9 actions 10 features.
15. Exterior means — 1 outer 2 vague 3 ignoble 4 indoors 5 fickle.
16. To violate is to — 6 abuse 7 appeal 8 reward 9 summon 10 tempt.
17. A chart is a — 1 card 2 flag 3 map 4 bowl 5 debt.
18. An alien is a — 6 captive 7 candidate 8 foreigner 9 fortress 10 novelty.
19. Uneasy means — 1 anxious 2 comfortable 3 ashamed 4 unhappy 5 foolish.
20. Seriousness means — 6 fidelity 7 suffrage 8 refinement 9 solemnity 10 displeasure.
21. A prologue is a kind of — 1 knoll 2 meteor 3 introduction 4 pathway 5 platter.
22. A haven is a — 6 breeze 7 package 8 reward 9 verse 10 refuge.
23. A witty person is — 1 silly 2 timid 3 clever 4 meek 5 sly.

Go right on to the next page.
DIRECTIONS. In the paragraphs below, each number shows where a word has been left out. Read each paragraph carefully, and wherever there is a number decide what word has been left out. Then write the missing word in the answer column at the right, as shown in the sample. Write JUST ONE WORD on each line. Be sure to write each answer on the line that has the same number as the number of the missing word in the paragraph.

SAMPLE. 

A-B Dick and Tom were playing ball in the field. Dick was throwing the —A— and —B— was trying to catch it. 

Answer

A. ball

B. 

1-2 Most hawks live on insects, small rodents such as rats, mice, and squirrels, and other destructive animals. Hawks are not particularly fond of chickens and other birds, but some farmers do not realize this. Whenever they see —1—, they want to shoot them because they do not understand that most of their food consists of animals that are —2— to farm crops.

2. 

3-4 Trolls are dwarfs in Norse mythology. They are portrayed as squatty, misshapen figures with evil powers and malevolent natures. They were inclined to thieving and were fond of carrying off children. Sometimes a troll would substitute one of its own offspring for the —3— of a human mother. It was a most unfortunate person who incurred the ill will of a —4—.

4. 

5-6 Benjamin Franklin was one of the most versatile of our great men. He was a statesman, philosopher, writer, publisher, and scientist. In his role of —5— he not only held public office in the United States but also represented the United States in both England and France. As a —6— he is best known for his identification of lightning with electricity.

6. 

7-8-9 In general, insects may be divided into two classes. The group that lives on solid foods has biting mouth parts. The group that lives on liquid foods has long, hollow, sucking mouth parts. The butterfly visits flowers, drawing up its food with its long sucking tube in —7— form. Grasshoppers do untold damage to grain and other farm crops. Because the grasshopper eats —8— food, its mouth parts are of the —9— type.

8. 

9. 

10-11-12 The principal diamond fields of the world are in Africa, Brazil, and Australia. Few persons know, however, that —10— are also found in Arkansas. It is estimated that more than 10,000 of these stones have been taken from the soil of that state. Experts have pronounced the —11— gems equal to the finest —12— produced in Africa, Brazil, or Australia.

11. 

12. 

Go right on to the next page.
13-14-15 Demosthenes was a Greek orator who lived about 200 B.C. He was determined to be an orator although his lungs were weak and his pronunciation faulty. He persevered until at length he surpassed all other —13—. Turning to political life, he devoted his eloquence to speeches opposing the designs on Greece of Philip of Macedon. These famous —14— against Philip by —15— are known as his “Philippics.”

16-17-18 Gypsies are a peculiar vagabond race, now found in many parts of the world. They live in small caravans and earn a livelihood as fortune tellers, tinkers, makers and sellers of basket ware, etc. The —16— can be distinguished from the —17— among whom they rove by their physical appearance and their language as well as by their —18— of living.

19-20-21 Our term “white elephant” for something superfluous or something we do not know what to do with comes from a Siamese custom. In Siam, the white elephant is considered sacred, and anyone possessing one must keep it in a royal and consequently expensive style. Therefore, in the olden days when the king of Siam wished to destroy the fortunes of one of his courtiers he would have a —19— —20— given to the person, who was then obliged to spend so much on its —21— that he usually ruined himself financially.

22 In no other country is dancing so interwoven with folk music as in Spain. The favored dances are the solea, the tango, and the sequidilla. Many of the most popular airs are sung only when used as an accompaniment to —22—.

23-24 The word “infer” means to surmise or conclude from facts or premises, while “imply” means to express indirectly or to hint. For example, one might say: “Mr. Smith —23— that he was interested in Mr. Green’s scheme”; or, in another case, “The man —24— from her remarks that she was not going to be there.”

25-26-27 Desert plants solve in many ways the problem of scarcity of water. The long roots of certain plants penetrate downward to the permanent water layer. Short-rooted plants like the cactus may have hollow stems for the storage of water. Other plants conserve their meager water supply by leathery leaves that prevent water losses by evaporation. Thus we see three ways that desert plants are adapted to inadequate water supplies; namely, by long —25—, the —26— of water supplies during the brief rainy periods, and the possession of structures reducing —27—.
The Smiths bear the predominant surname in the United States. The Browns and the Williamses are exceeded only by them and the Johnsons. Next in order come Jones, Miller, Davis, Anderson, Wilson, and Moore. The two most common American surnames are — and —.

Gregariousness, or the desire to be with people, and solitariness are two opposite traits of character. Though there are people who are almost wholly gregarious and others who much prefer solitude, most people possess both —. When satiated with the company of others, they wish for —, and on the other hand, after a long period of seclusion they develop — interests.

A nineteenth-century poet has said, "Rags are royal raiment when worn for virtue's sake." In other words, it is more noble to do without luxuries and comforts than to — them at the — of one's ideals and honor.

Bacteria have greater resistance to injurious influences than any other known organisms. However, most bacteria are killed like any other — by a brief exposure to — of °-° centigrade.

Although the driver is recognized as the prime factor in traffic accidents, little has been done to teach correct driving habits and skills. For many drivers a traffic — where ignorant drivers may be taught good driving habits is better than a traffic — where poor drivers are fined or otherwise punished.

"Has not your teacher explained to you that if you do not know your arithmetic in this grade what is the chance for success in the next grade?" The preceding sentence as it stands is incorrect, but it can be made into a correct sentence by substituting "— have —" for "what is the."

Charcoal has several properties that make it useful — among which are its resistance to chemical action, its black color, and its ability to absorb large volumes of gases and colored substances. It has been found that charcoal made from peach pits — more poisonous gas than does charcoal from other sources. For this reason — charcoal is used in making gas — for use in wartime.

One advantage of rural life is the close contact with nature which country people enjoy. The children can roam about over the fields picking flowers and hunting for new and strange scenes. Boys can hunt, fish, and swim. Much of our best literature describes the joy of this — with — which country life provides.

End of Test 1. Look over your work.
APPENDIX E

SAMPLES OF THE PROCEDURES USED TO DETERMINE THE SUITABILITY OF THE TESTS FOR THE STUDY

Recording sheets for the preparation of data used in the item analysis of test results 1 . . . . . . 308
Calculation sheets for the preparation of data used in the item analysis of test results . . . . . . 311
Calculation of mean, median, and standard deviation . 313
Calculation of test reliability by using the Hoyt modification of the Kuder-Richardson Formula . . . 314

1 Each of these samples is based on the results obtained from the trial administration of the Farquhar Test to forty Grade VII pupils in White Rock Elementary School.
### RECORDING SHEETS FOR THE PREPARATION OF DATA USED IN THE ITEM ANALYSIS OF RESULTS OBTAINED FROM THE ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL

Top 27% of 40 pupils = 11 pupils

| Part Number (Ranked) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|----------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1                    |   |   | x |   | x |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 19 | 361|
| 2                    | x |   |   |   |   | x |   |   | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 17 | 289|
| 3                    |   |   | x | x |   |   | x | x |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 16 | 256|
| 4                    | x | x |   | x |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 16 | 256|
| 5                    |   |   | x | x | x |   |   |   |   | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 16 | 256|
| 6                    |   |   |   |   |   |   | x | x |   | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 15 | 225|
| 7                    |   | x | x | x | x | x | x | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 13 | 169|
| 8                    | x | x | x | x | x | x | x | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 13 | 169|
| 9                    |   | x | x | x | x | x | x | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 12 | 144|
| 10                   | x | x | x | x | x | x | x | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 12 | 144|
| 11                   | x | x | x | x | x | x | x | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    | 12 | 144|
Middle 46% of 40 pupils = 18 pupils

| Pupil Number (Ranked) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|-----------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 12                    |   |   | x |   |   |   |    | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 13                    |   |   | x | x |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 14                    |   |   |   |   | x | x |    | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 15                    |   |   |   |   |   |   | x |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 16                    |   |   |   |   |   |   | x |   | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 17                    |   |   | x | x |    | x |    | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 18                    |   |   |   |   |   | x |   | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 19                    |   |   |   |   |   |   | x |   | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 20                    |   |   |   |   |   |   | x |   | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 21                    |   |   |   |   |   |   |   |   |   | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 22                    |   |   |   |   |   |   |   |   |   |   | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 23                    |   |   | x | x | x | x |    | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 24                    |   |   | x | x | x | x | x |   | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 25                    |   |   | x | x | x | x | x | x | x | x | x | x | x | x |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 26                    |   |   | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |    |    |
| 27                    |   |   | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |    |    |
| 28                    |   |   | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |    |    |
| 29                    |   |   | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x | x |    |    |

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Bottom 27% of 40 pupils = 11 pupils
**CALCULATION SHEETS FOR THE PREPARATION OF DATA USED IN THE ITEM ANALYSIS OF RESULTS OBTAINED FROM THE ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL**

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### Calculation Sheets for the Preparation of Data Used in the Item Analysis of Results Obtained from the Administration of Farquhar's Test to Forty Grade VII Pupils in White Rock Elementary School

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CALCULATION OF MEDIAN, MEAN, AND STANDARD DEVIATION, OF RESULTS OBTAINED FROM THE TRIAL ADMINISTRATION OF FARQUHAR'S TEST TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL

Median:

Median (50th Centile Point) = $9.5 + \frac{1}{2}$ (1)

= 9.5 + .2

= 9.7

Mean:

Mean = \frac{407}{40} = 10.175

Standard Deviation:

\[
\sqrt{\frac{X^2}{N} - \frac{X^2}{N}} = \sqrt{\frac{4599}{40} - \frac{407}{40}}
\]

= \sqrt{114.975 - 103.530625}

= \sqrt{11.44375}

= 3.382
Calculation of test reliability by using the Hoyt modification of the Kuder-Richardson Formula:

RELIABILITY OF FARQUHAR TEST, BASED ON THE RESULTS OF TRIAL ADMINISTRATION TO FORTY GRADE VII PUPILS IN WHITE ROCK ELEMENTARY SCHOOL

The formula:

$$r_{tt} = \frac{N}{N-1} \times \frac{kS + S_i - T(T+k)}{\frac{kS}{s} - \frac{T^2}{N}}$$

k represents number of pupils
N represents number of items
$S_s$ represents sum of squares of pupils' scores
$S_i$ represents sum of squares of item scores
T represents total of scores for pupils or items

Substituting the figures derived from the calculation sheet,

$$r_{tt} = \frac{30}{29} \times \frac{40(4599) + 7689 - 407(407 + 40)}{(40 \times 4599) - (407)^2}$$

$$= \frac{30}{29} \times \frac{183,960 + 7689 - 181,929}{183,960 - 165,649}$$

$$= \frac{291,600}{531,019}$$

$$= .549$$
The scores obtained by the 147 participating subjects in the tests used to measure the five variables

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<th>Name of Test</th>
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<tr>
<td>$X_1$</td>
<td>Farquhar's Test of Understanding of Processes with Decimal Fractions (First administration at beginning of experiment)</td>
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**SUMS**

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APPENDIX G

SUPPLEMENTARY STATISTICAL CALCULATIONS

Calculation of the correlation of the means between the treatment groups of the criterion variable with each of the independent variables ....... 322

Calculation of the within groups correlation between the criterion variable and each of the independent variables ......... 323

Calculation of the within groups correlation, corrected for attenuation, between the criterion variable and each of the independent variables .... 324

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Method of determining the significance of the difference between two r's ................. 325

(i) Between groups correlation of the mean of \( x_2 \) with the mean of \( y \).

\[
\begin{align*}
  r_{x_2 y} \text{ (between)} &= \frac{9.03852}{\sqrt{33.12368 \times 2.46636}} \\
  &= \frac{9.03852}{\sqrt{81.6949194048}} \\
  &= \frac{9.03852}{9.03852} \\
  &= 1.0
\end{align*}
\]

(ii) Between groups correlation of the mean of \( x_3 \) with the mean of \( y \).

\[
\begin{align*}
  r_{x_3 y} \text{ (between)} &= \frac{-19.31921}{\sqrt{151.32912 \times 2.46636}} \\
  &= \frac{-19.31921}{\sqrt{373.2320884032}} \\
  &= \frac{-19.31921}{19.31921} \\
  &= -1.0
\end{align*}
\]

(iii) Between groups correlation of the mean of \( x_4 \) with the mean of \( y \).

\[
\begin{align*}
  r_{x_4 y} \text{ (between)} &= \frac{-0.28403}{\sqrt{0.03271 \times 2.46646}} \\
  &= \frac{-0.28403}{\sqrt{0.0806746356}} \\
  &= \frac{-0.28403}{0.28403} \\
  &= -1.0
\end{align*}
\]
CALCULATION OF THE WITHIN GROUPS CORRELATION BETWEEN THE CRITERION VARIABLE AND EACH OF THE INDEPENDENT VARIABLES

(i) Within groups correlation between $x_2$ and $y$.

\[
 r_{x_2y} \text{ (within)} = \frac{1924.96148}{\sqrt{3700.20031 \times 3427.81510}} \\
= \frac{1924.96148}{\sqrt{12683602.4956426810}} \\
= \frac{1924.96148}{3561.4045} \\
= .54
\]

(ii) Within groups correlation between $x_3$ and $y$.

\[
 r_{x_3y} \text{ (within)} = \frac{5844.65254}{\sqrt{3700.20031 \times 28911.13347}} \\
= \frac{5844.65254}{\sqrt{106976986.0281453757}} \\
= .57
\]

(iii) Within groups correlation between $x_4$ and $y$.

\[
 r_{x_4y} \text{ (within)} = \frac{2607.95070}{\sqrt{3700.20031 \times 7784.38906}} \\
= \frac{2607.95070}{\sqrt{28803798.8129726086}} \\
= \frac{2607.95070}{5366.9171} \\
= .49
\]
(i) Within groups correlation, corrected for attenuation, between $x_2$ and $y$.

\[ r = \frac{.54}{\sqrt{.821 \times .541}} \]
\[ = \frac{.54}{\sqrt{.444161}} \]
\[ = \frac{.54}{.666} \]
\[ = .81 \]

(ii) Within groups correlation, corrected for attenuation, between $x_3$ and $y$.

\[ r = \frac{.57}{\sqrt{.948 \times .541}} \]
\[ = \frac{.57}{\sqrt{.512868}} \]
\[ = \frac{.57}{.716} \]
\[ = .80 \]

(iii) Within groups correlation, corrected for attenuation, between $x_4$ and $y$.

\[ r = \frac{.49}{\sqrt{.874 \times .541}} \]
\[ = \frac{.49}{\sqrt{.472834}} \]
\[ = \frac{.49}{.687} \]
\[ = .71 \]
CALCULATION OF THE PEARSON PRODUCT-MOMENT COEFFICIENT OF CORRELATION, FOR THE EXPERIMENTAL GROUP, BETWEEN THE CRITERION VARIABLE (Y) AND ONE OF THE INDEPENDENT VARIABLES (X)

\[ r_{xy} = \frac{\Sigma x \cdot y}{N} - \frac{(\bar{x})(\bar{y})}{(\sigma_x)(\sigma_y)} \]

\[ r_{xy} = \frac{7077}{59} - \frac{(7.7966)(14.4915)}{(3.0016)(4.5375)} \]

\[ r_{xy} = .51137 \]

METHOD OF DETERMINING THE SIGNIFICANCE OF THE DIFFERENCE BETWEEN TWO \( r \)'s

The \( r \) between \( X_1 \) and \( Y \) in the experimental group is .51137; the \( r \) between \( X_1 \) and \( Y \) in the control group is .67816. Is the relationship between \( X_1 \) and \( Y \) significantly higher in the control group than in the experimental group?

<table>
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<tr>
<th>Pearson r</th>
<th>Fisher's z</th>
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<tr>
<td>.51137</td>
<td>.56</td>
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<tr>
<td>.67816</td>
<td>.83</td>
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Standard Error of the difference between 2 coefficients

\[ \sigma_{z_1 - z_2} = \sqrt{\frac{1}{N_1 - 3} + \frac{1}{N - 3}} \]

\[ = \sqrt{\frac{1}{59 - 3} + \frac{1}{88 - 3}} \]

\[ = \sqrt{.01786 + .01176} \]

\[ = \sqrt{.02962} \]

\[ = .172 \]
Critical Ratio \[= \frac{z_1 - z_2}{\sigma} \]
\[= \frac{.83 - .56}{.172} \]
\[= 1.57 \]

This CR of 1.57 is below the .05 level of 1.96 (Table of t for use in determining the reliability of statistics). It may be concluded, therefore, that the relationship between \(X_1\) and \(Y\) is not significantly higher in the control group than in the experimental group.