DYNAMIC LOADS ON SPUR GEAR TEETH

by

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ABSTRACT

The load on gear teeth while in operation has long been an open question. So many variable and uncertain factors are involved that it is not surprising that many different formulas and rules have been proposed from time to time. Few, if any, systematic experiments were made to obtain data on this subject until about 1923, when the ASME Special Research Committee on the Strength of Gear Teeth was organized and the Lewis gear-testing machine was built. The results of this Committee's work were published in an ASME Research Publication, 1931, entitled "Dynamic Loads on Gear Teeth".

For the experiment in this thesis, a gear testing machine was designed and built in order to investigate the dynamic loads on gear teeth. Strain gages were applied to one of the test gear teeth and the main shaft, and were calibrated to give an indication of tooth load and shaft torque respectively. A static tooth load calibration on a strain indicator was made by means of a load arm, a cable and dead weights. The static calibration helped to set the indicator so that the reading shown on it during the dynamic load test could be converted to the true load applied. During the dynamic load test of each test series, photographs of the oscillograph traces were taken at different machine speeds and at several different values of nominal torque.

The principal object of these tests was to determine loads between gear teeth under operating conditions by using strain gages and electronic recordings. At the present time, the experimental work is limited to involute spur gears only.

It is suggested by most gear experts, that the dynamic tooth load may be considered as the sum of the static load and the increment load. The experiments show clearly that the precise gears have no increment loads on their teeth, and that the measured maximum tooth loads are almost the same at different test speeds.

The formula for evaluating the dynamic load on spur gear teeth, developed by Earle Buckingham, gives slightly higher values than the measured loads at low speed (approximate pitch line velocity 90 fpm), but diverges widely from the present experimental results at high speed (approximate pitch line velocity 2800 fpm).

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CONTENTS

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CHAPTER I

BRIEF HISTORY[1]

The dynamic load is the maximum momentary load imposed on the gear teeth by the conditions of service, including the influence of errors in the gear teeth themselves. The nature and extent of these dynamic loads on gear teeth have been studied by many investigators and it seems that it still remains an open question.

One of the earliest efforts to account for the increase in dynamic load due to velocity was the use of a velocity factor. Several gears were tested to destruction under static load, and the results compared those from other gears of the same size, strength and material tested to destruction at various speeds. The ratio of the dynamic failure load to the static failure load is called the velocity factor.

Oscar Lasche[2] first considered the effects of tooth errors in 1899 and came to the conclusion that the tooth elasticity affected the permissible error. In other words, a greater error is permissible with elastic teeth than with rigid teeth, because they can absorb the instantaneous forces without disturbing the rotating masses.

In 1908, Ralph E. Flanders[3] discussed the nature of dynamic loads and stated "After some reflection, the writer has come to the conclusion that a variation in the strength of perfectly formed gearing, due to a variation in the velocity, can be due to but one thing -- impact caused by the imperfect meshing of otherwise perfectly shaped teeth, deformed by the load they are transmitting".

Marx[4] ran a number of tests at various pitch line velocities at Stanford University between 1911 and 1915. The results of these tests, (1) emphasized the importance of contact ratio (2) questioned the commonly used velocity factors, and (3) indicated that the cutting accuracy had a pronounced effect upon the strength at high speed.

Between the years 1900 and 1931 the idea that the dynamic load could be considered as the sum of the transmitted load and an incremental load gradually gained support. This equation may be expressed as

$$W_d = W + W_i$$

where W_i is the incremental load and is due to tooth-form and spacing errors, unbalance, fluctuating loads and the deformation of the teeth under load. W is the transmitted load which is the useful compoment of force which is transferred from one gear to another during action.

Research under the auspices of the ASME Special Research Committee on the Strength of Gear Teeth was organized in 1923. The research was conducted at the Massachusetts Institute of Technology by Earle Buckingham[5]. Buckingham's report gave the following equation for dynamic loads.

$$W_d = W + \sqrt{F_a\,(2F_2 - F_a)}$$

where F_a is the acceleration load which is practically independent of the applied load and F_2 is the force required to deform the teeth the amount of the effective tooth error.

This Buckingham equation generally gives values of the dynamic load which are several times the nominal load. This led Professor W. O. Richmond to doubt the validity of Buckingham's method and to decide on an experimental program for the purpose of measuring actual tooth loads.

Buckingham wrote a mathematical analysis of the mechanics of gearing which was published in his book "Analytical Mechanics of Gears"[1] in 1949. Darle W. Dudley commented in his book "Modern Gear Design"[6] -- "The Buckingham method gives values of the dynamic load which are slightly high but it is the best method available and will show clearly the effect of masses, shaft stiffnesses, and tooth errors in producing dynamic overloads".

Professor W. A. Tuplin [7] has derived a formula for computing the dynamic increment of a pair of high-speed gears from known design factors. His works are based on the wedge analogy for the insertion or withdrawal of error between meshing gear teeth. The equation for dynamic increment force , F, is

$$F = k\, e_e$$

where k is the stiffness of a two-gear system and e_e is the effective error as determined from the design curve, which is used to find the ratio of effective error to actual error from the ratio of time consumed by introduction of error to natural period of vibration.

In 1955, Reswick[10] developed an expression for dynamic tooth loads based on a study of a simple physical mechanism which was used to simulate gears in action and gave physical insight into the dynamic behaviour of gears. Reswick's prediction for dynamic tooth loads are in general, in some cases, agreement with those predicted by Buckingham's equation.

Professor G. Niemann and H. Rettig[11] have made an attempt to measure the actual dynamic tooth forces on a pair of loaded gear with the help of some indicators (which are similar to the function of strain gages). In their experiments the following conclusions were made and the report was published in January 1957, (1) Dynamic incremental tooth forces occur even in the accurate cut gears, (2) with a small applied load the influence of the dynamic incremental forces is significantly large, (3) increasing the applied load by, for example, 100% the tooth root stress and tangential tooth pressure increase less than 100%, since the dynamic incremental forces do not increase to the same extent, (4) by increasing the speed, the tooth root stress increases less than the tangential tooth pressure.

In order to measure dynamic loads on gear teeth, a dynamical gear testing machine, and a strain gage apparatus, which are very similar in principal with the one devised in this research, were devised by M. Utagawa[12] in 1956. The testing machine is a power-feed back type and has two pairs of gears loaded against each other on two parallel shafts. One pair is test gears and the other is speed up gears. The applied toque is measured by strain gages attached to the torsion shaft. And the strain gage for measuring tooth stress is attached to the tooth flank near the fillet curve at the back face of a gear tooth. The stress-cycles or strain-cycles of the tooth are observed through a cathode-ray oscilloscope. In the experiments, Utagawa found that the measured stresses at the spot where a strain gage is attached were nearly equal to the calculated results. Loads and strains are proportional at the same acting point. The frequency of the dynamic load variation almost coincided with the natural frequency of the vibration system formed by the gear and pinion and the mating teeth. Increment load becomes larger as the velocity increases, and the contact ratio has influence on the dynamic load.

CHAPTER II

DESCRIPTION OF THE GEAR TESTING MACHINE

In order to obtain data on the dynamic loads acting on gear teeth under operating conditions, a testing machine was constructed in 1956 under the guidance of Professor W. O. Richmond, Department of Mechanical Engineering, The University of British Columbia. The general arrangement of the testing machine is shown in Fig. 1.

In the machine, the pair of test gears, 'E' and 'F', is loaded against a pair of master gears 'D'. The gears have 24 teeth of 4 diametral pitch. The master gears are 2 inches in face width and have a 20-degree pressure angle. The machine is designed for test gears of 1 inch face width.

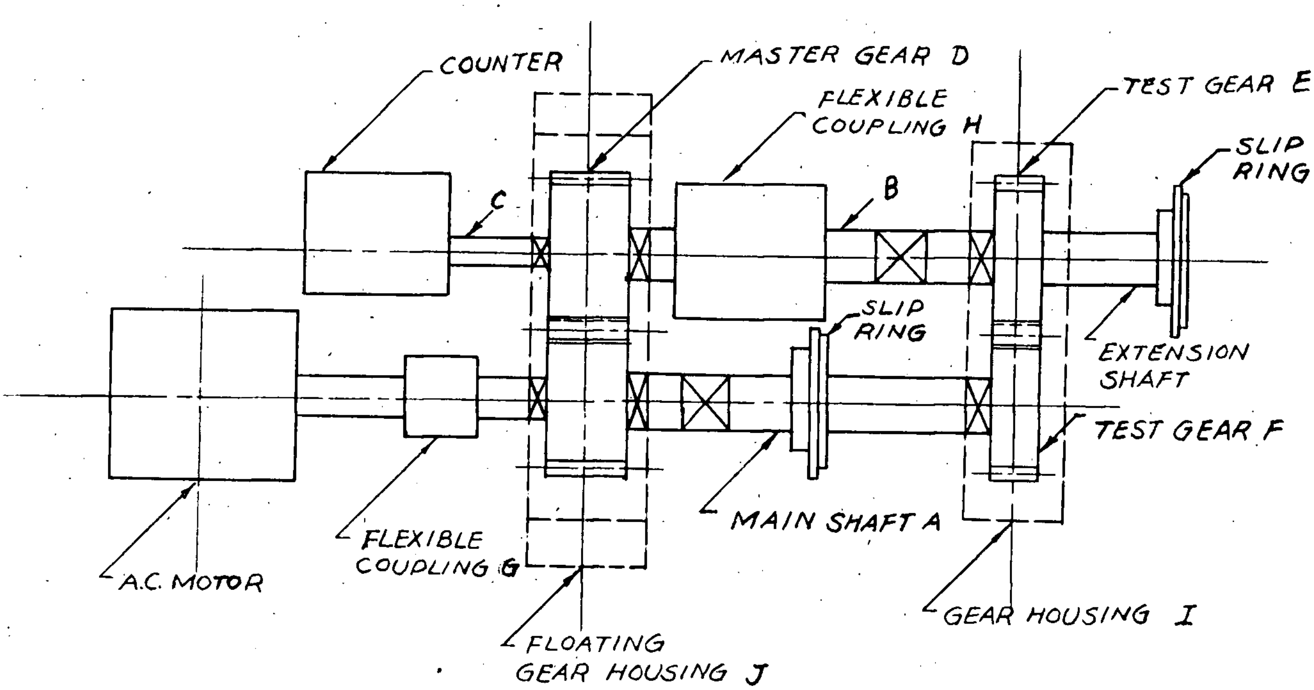


Fig. 1 A SKETCH OF GEAR TESTING MACHINE

The pair of master gears is enclosed in a floating housing 'J' which can pivot about the main shaft 'A' as shown in Fig. 2. Rotation of the housing is prevented by a set screw which contacts a load ring located beneath. Tightening of the screw which bears on the load measuring ring tilts the housing thus producing torque in the shaft and tangential load on the teeth of the master gears 'D'. Equal and opposite torques are produced in the shafts 'A' and 'B'.

The main shaft 'A' with the test gear on one end and the master gear on the other end is supported by ball bearings and is connected to a driving motor by a small coupling 'G'. The machine is driven by a 3 hp induction motor operating at 1780 rpm for high speed tests. For low speed tests, a small ¼ hp, 1725 rpm, driving through a gear reducer with a reduction ratio of 30 to 1 is used, so that the test gears revolve at about 60 rpm. The driving motor supplies friction and bearing losses.

Two secondary shafts, 'B' and 'C', parallel to the main shaft 'A' are connected by an American D-11 coupling 'H' of the Oldham type. The larger secondary shaft 'B' carries the test gear 'E' which is loaded against the other test gear 'F' carried by the main shaft in the gear housing 'I'. At the end of this shaft an attached extension shaft with slip rings on it projects through the plexiglass cover plate. The conductors which transmit the signal of the gear tooth strain gages pass through the core of the hollowed extension shaft and connect to the slip rings. The smaller secondary shaft 'C' carries a master gear in the pivotted housing 'J'.

The torque in the shaft is measured by means of four SR-4 strain gages cemented to the shaft. The gages are of type C-1, with a resistance of 502 $\pm$ 2 ohms, the gage factor is 3.35 $\pm$ 1%. The gages are cemented at a 45° angle with respect to the shaft axis to measure the principal tensile and compression

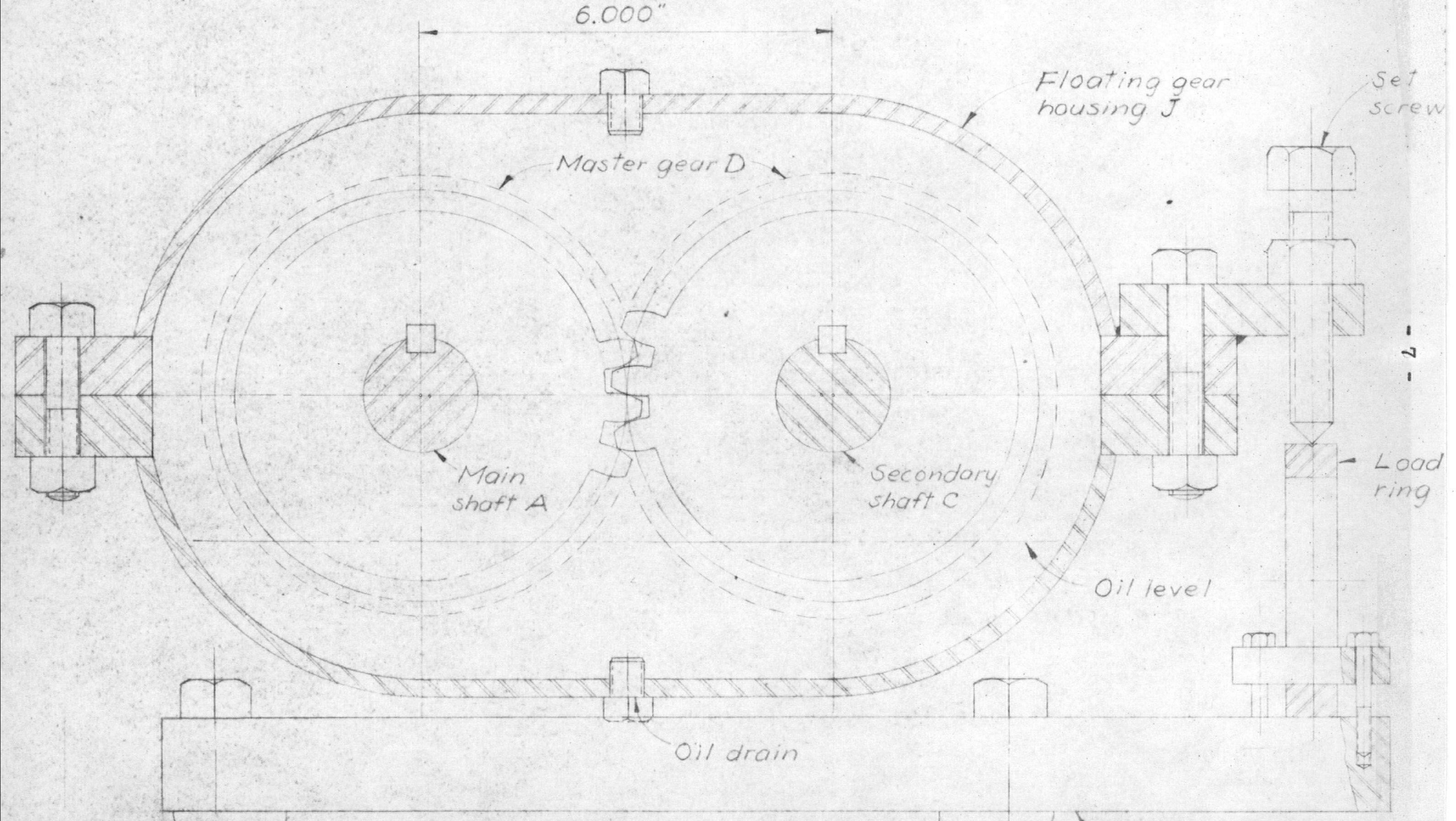


Fig. 2

Section on ℄ Floating Gear Housing 'J'

strains induced by the torque. The gages are connected in a Wheatstone-bridge circuit, as shown in Fig. 3 to slip rings which are mounted on the main shaft. Conducting brushes attached to the stationary frame complete the electrical connections between the gages and the external electronic instrumentation. With torsional moments as indicated, gages R_1 and R_3 are strained in tension and gages R_2 and R_4 in compression. This conventional arrangement of strain gages gives four active bridge legs on the rotating shaft.

For measuring gear tooth loads, four SR-4 strain gages of type C-7, with a resistance of 505 $\pm$ 3 ohms, gage factor 3.27 $\pm$ 2% are placed at the fillets of the middle tooth as shown in Fig. 4. The sensitive wires inside the gages are parallel with the line drawn through the corner of the tooth and tangent to the fillet, and the center line of the gage is at the fillet. These gages are connected in a Wheatstone-bridge circuit as shown in Fig. 4; the leads being brought out to the electronic instruments via the slip ring assembly shown in Fig. 5a.

The two strain gage bridges are connected through a balancing and calibrating switch to an Ellis BAM-1 strain indicator. The output from the indicator is displayed on a cathode ray oscilloscope. The switch enables either the shaft torque or the gear tooth load to be selected for display on the screen of the cathode ray tube.

The sweep of the oscilloscope is initiated, or triggered, by a contact point on the slip ring disc for the tooth strain gage bridge (Fig. 5a). The location of this contact point is adjusted so that the sweep starts slightly before the tooth with strain gages makes contact with another tooth. The speed of the sweep is adjusted so that the tooth load record occupies a major portion of the screen.

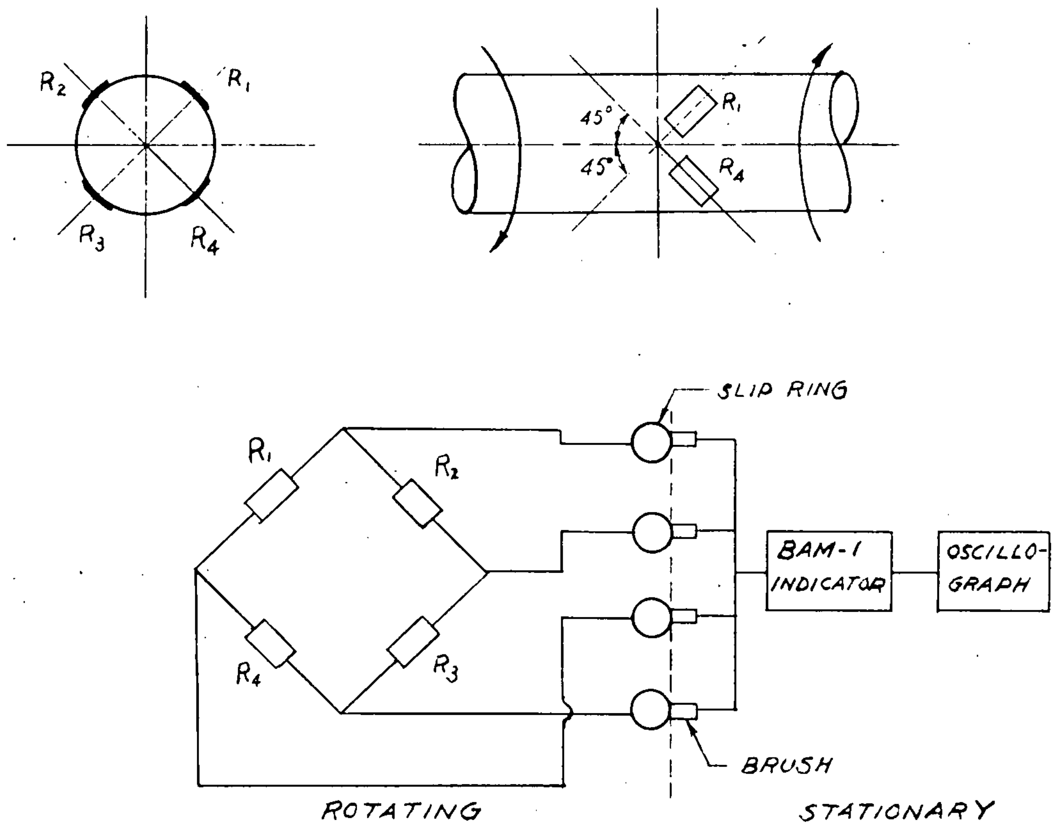

R2
R1
R3
R4
45°
45°
R1
R4
SLIP RING
R1
R2
R4
R3
BAM-1 INDICATOR
OSCILLO-GRAPH
BRUSH
ROTATING
STATIONARY

Fig. 3

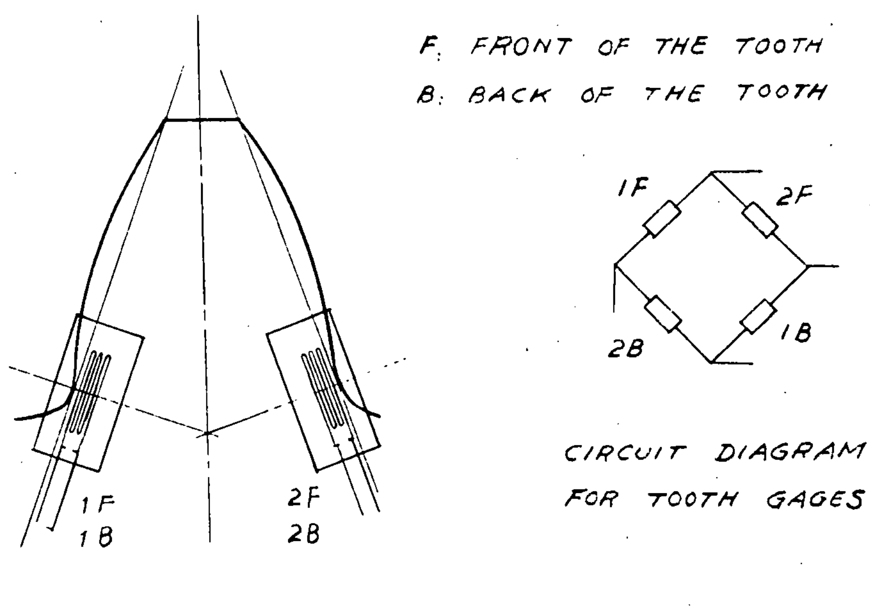

F: FRONT OF THE TOOTH
B: BACK OF THE TOOTH
1F
2F
2B
1B
1F
1B
2F
2B
CIRCUIT DIAGRAM
FOR TOOTH GAGES

Fig. 4

To relate the speed of the cathode ray trace to the pitch line velocity of the gears, the device shown in Fig. 5b is used. A voltage pulse is generated in the coil when a tooth passes the magnitized core and this voltage variation is recorded to determine the tooth space interval.

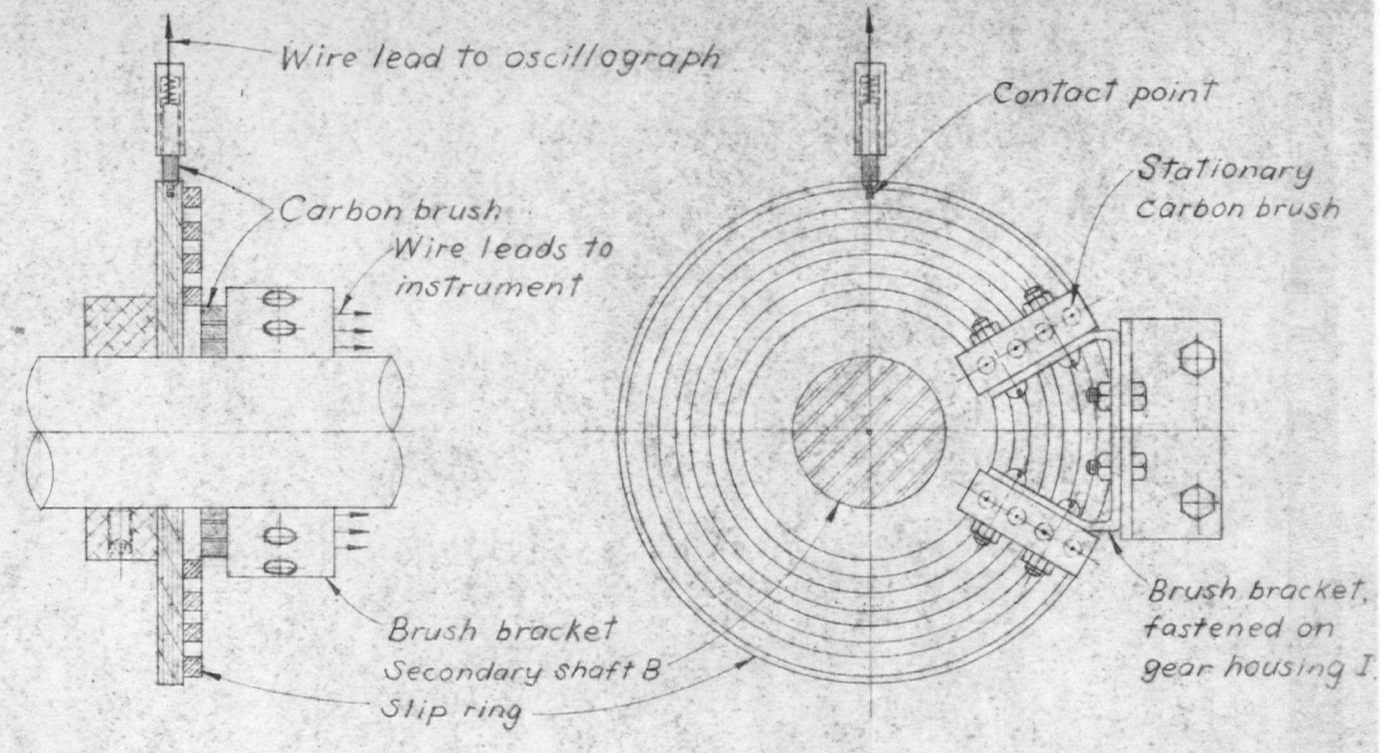


Fig. 5a Slip Ring and Carbon Brushes

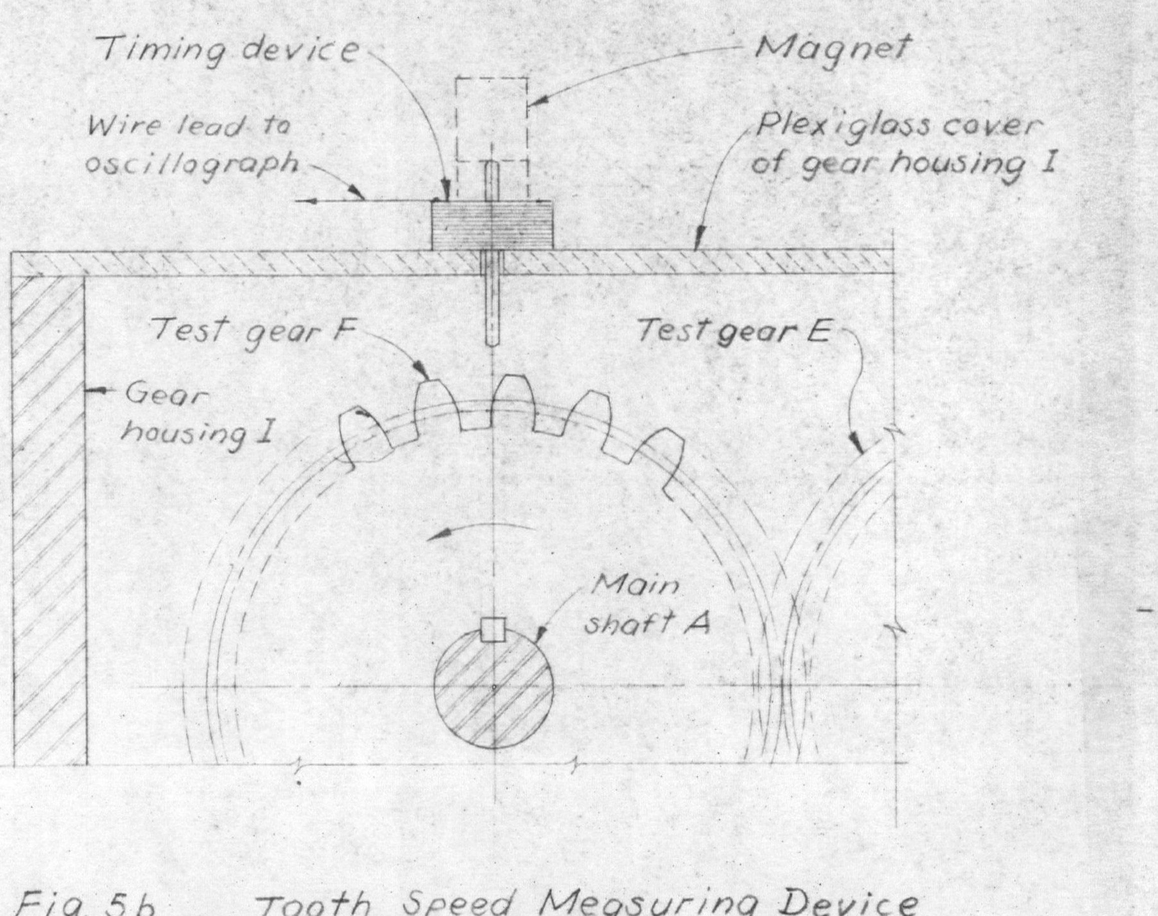


Fig. 5b Tooth Speed Measuring Device

CHAPTER III

CALIBRATION AND TEST PROCEDURE

A. Calibration of Torque Measuring Bridge on the Main Shaft.

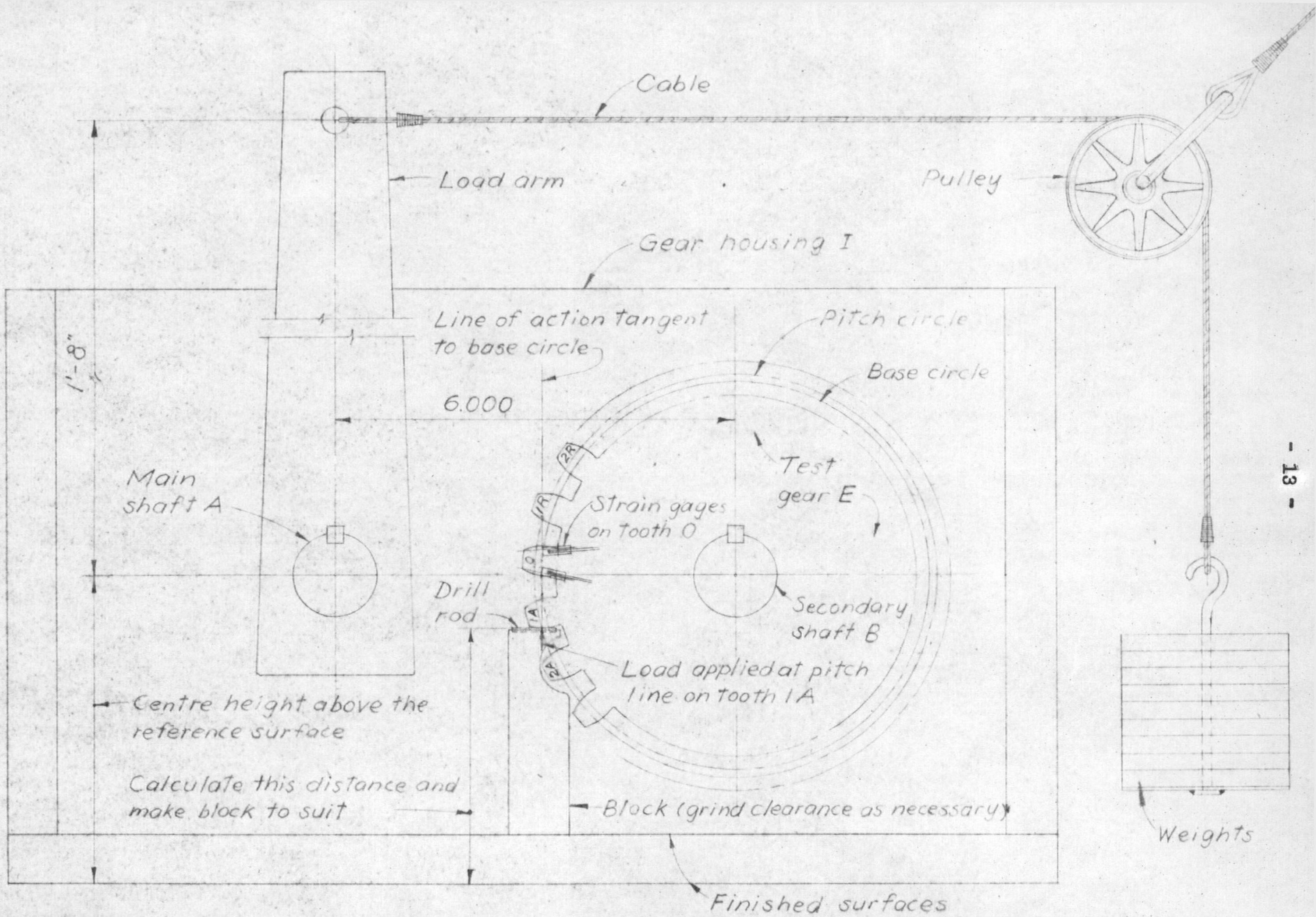
The strain-gage bridge on the main shaft for measuring the torque transmitted by the test gears was calibrated by applying weights at the end of a 20 inch lever. The arrangement of lever and weights is shown in Fig. 6.

It was hoped that the torque could also be obtained by measuring the reaction of the pivotted housing containing the master gears by a steel ring with strain gages attached. However, the reading was adversely affected by the adjacent flexible coupling, so that reliable results were not obtained.

B. Calibration of Bridge Measuring Tooth Load.

The output of the strain gage bridge attached to the root of the gear tooth was calibrated by the arrangement shown in Fig. 6. The output of this bridge is proportional to the bending stress or bending moment at the root of the tooth. This is not directly proportional to load because of the changing moment arm as the contact point travels down the face of the tooth. Thus the calibration had to take into account the position of the force on the tooth as well as the load magnitude. Hence the tooth load conversion charts (Fig. 7) are needed to obtain the load acting on the tooth.

For the purposes of reference, gear teeth with strain gages attached may be labelled as shown in Fig. 6, namely, teeth 2A, 1A, O, 1R, and 2R; where A means teeth ahead of middle tooth O, and R means teeth



Static Calibration of Bridge Measuring Tooth Load

Fig. 6

behind the middle tooth.

Strain gages on the tooth O were connected in a bridge circuit (see Fig. 3). This connection gives almost no reading for the loads applied to teeth 2A, 1A, 1R, and 2R, but a maximum reading for the loads applied to tooth O.

Loads tangent to the base circle of the gear and normal to the tooth surface were applied at the tip of the tooth, on the pitch line, and at the point on the tooth profile at the end of the tooth engagement. The loads were applied by putting a special designed steel block together with an eighth inch diameter steel drill rod underneath the tooth as shown in Fig. 6. The total height of the block and the rod was accurately calculated for each of the three positions previously described.

During static calibration adjustment had to be made in the coupling to allow the gear to take up the desired position and yet maintain the loading arm at right angles to the cable supporting the weights.

The tooth load conversion charts for different pairs of gears were plotted on a tooth contact cycle basis. The vertical scale of the oscillograph display will have to be corrected by this chart to give the true tooth loads.

A sample of the tooth load conversion chart is shown in Fig. 7.

C. Test Procedure When Measuring Dynamic Load.

The following procedure was followed in the dynamic load tests. After the machine was started the desired load was applied utilizing shaft torque measurement as a guide. The readings for each run included the speed, the average torque and oscillograph records of the shaft torque variation, the dynamic load record from the tooth, and a record indicating the tooth speed. Tests were run at two speeds, slow operation at about 60 rpm (92.4 fpm pitch line velocity) and fast operation at nearly 1800 rpm (2825 fpm pitch line velocity). Records were taken for torque values of 500, 1000, 1500, and 2000 in-lbs.

Tests were carried out on three pairs of gears, two pairs were 20° full depth involute cut by the National Research Council, and one pair was 14½° full depth cut by the form cutter method at the Canadian Sumner Iron Works. One pair of NRC gears had bushings installed in the bores, which made the tooth pitch circle eccentric to the bore of the gears.

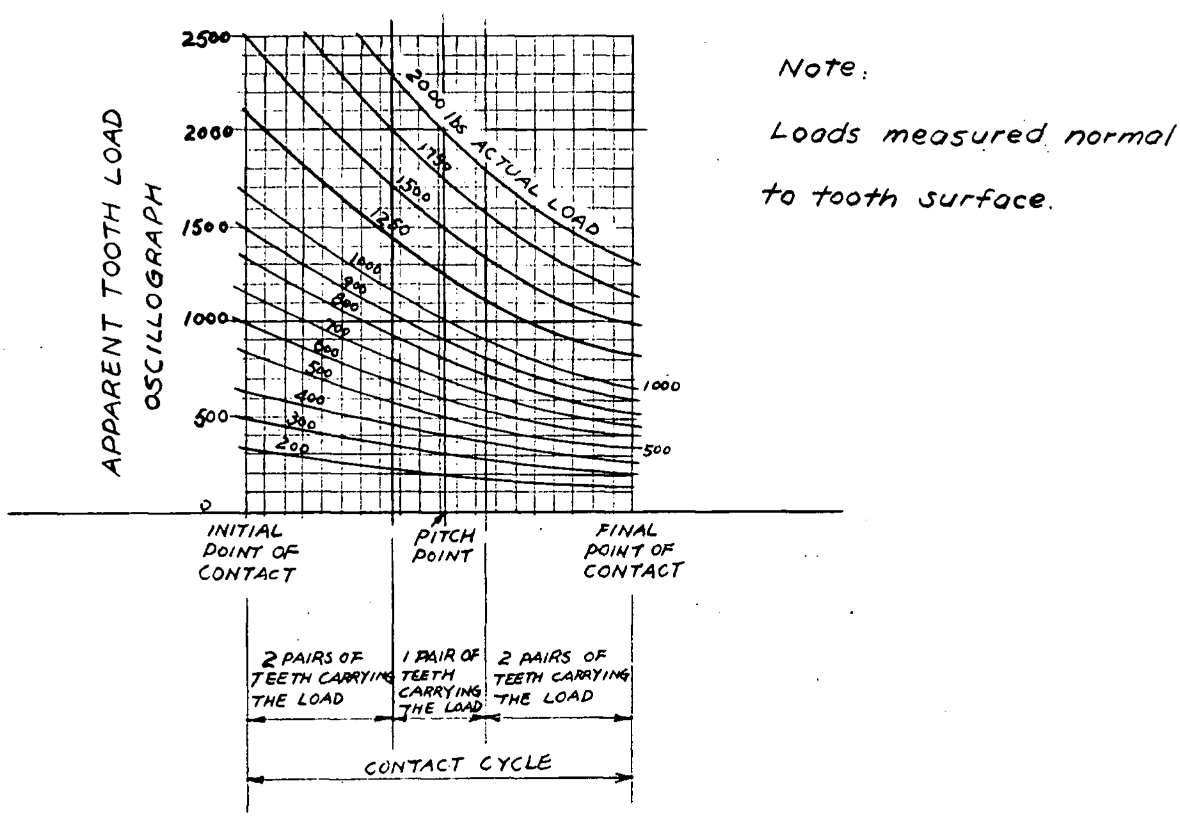


Fig 7 TOOTH LOAD CONVERSION CHART OF THE FIRST PAIR OF NRC MADE 20-DEG. INVOLUTE SPUR GEARS

CHAPTER IV

EXPERIMENTAL RESULTS

A. First Test Series - NRC Involute Spur Gears (20-degree pressure angle)

This test series was run in two parts, first with the gears set so that the eccentricity of the bores were in opposite directions, and second with the eccentricity in the same direction.

a) Machine at low speed

The typical records for the first test series are shown in Fig. 8. Fig. 8a shows a typical shaft torque variation record obtained from the strain gages on the main shaft. The straight line at bottom of the diagram represents the zero reading of the instrument. The upper curve is the torque variation under a measured torque of 1950 in-lb., when the gears were rotating at 56 rpm (88 fpm pitch line velocity). The nature of the torque variation for a full revolution is shown in Fig. 8b. The tooth load variation obtained at 88 fpm pitch line velocity under a measured torque of 1950 in-lb. is shown in Fig. 8c. By using the tooth load conversion chart (Fig. 7) the corresponding acting tooth load diagram was plotted in Fig. 8d. In the diagram, it was found that the load on tooth 0 where the strain gages were attached increased gradually from zero value at the initial point of contact to a maximum value which was achieved when tooth 0 carried the full load. The load decreased gradually as the pair of following teeth came into contact, and the load became zero at the final point of contact. The similar tooth load diagrams (Fig. 10a and 10b) were obtained for the second test series at a measured torque of 2020 in-lb.

As the torque values were reduced, the shaft torque variation maintained the same pattern as shown in Fig. 8a and 8b. However, the tooth load curve shows an increase in the duration of maximum load as illustrated in Figs. 8c to 8j. At higher load, the maximum tooth force only appeared during the period when one tooth carried the whole load. As the torque was reduced to a lower value, the tooth load reached a maximum value earlier and this condition persisted for nearly half the contact cycle.

b) <u>Machine at high speed.</u>

All the records obtained at high speed (Fig. 9 and Fig. 11) have the same pattern as at low speed, but vibration effects were present.

The upper curve in Fig. 9a shows a typical record of shaft torque variation obtained at 1775 rpm (2786 fpm pitch line velocity) under a measured torque of 1600 in-lb. The lower curve in the same diagram represents the zero reading of the instrument. The shaft torque pattern of second test series at a measured torque of 2320 in-lb. is shown in Fig. 11a.

The irregularity of the zero curve may be explained as follows:

Firstly, the slip rings on the main shaft are composed of two halves with the result that a gap and an irregularity exists at the junction of the two halves. The discontinuity of the rings produces vibration of the brushes and a variation of the brush pressure. At 60 rpm this change in pressure is very small. Since the change of pressure is proportional to the square of the speed, the change in pressure at 1800 rpm is 900 times as great as at 60 rpm. This pressure vibration causes the contact resistance to vary. This is one of the cause of ripples in the zero curve. Also the ripple is much effected by dust on slip rings -- clean rings give a smoother signal.

Secondly, generator effect may have had some influence. The four strain gages which compose the Wheatstone-bridge circuit, form a closed circuit. If the closed circuit cuts a magnetic field (the earth's field or a field due to the current in nearby conductors) then voltage will be set up between the terminals of the circuit. The magnitude of this voltage was not determined.

Fig. 9b shows the shaft torque variation for a full revolution under a measured torque of 1600 in-lb.

The tooth load variation at high speeds is illustrated in Figs. 9c to 9h. Figs. 9c and 9d represent the tooth load variation under a measured torque of 2200 in-lb.

Reducing the nominal torque value to 1000 in-lb., more ripples appeared in the tooth load pattern and a longer period of maximum load is evident as shown in Fig. 9h. The ripple represents the natural frequency of elastic system which is 2770 cps by measuring from the record.

The typical records of second test series at high speed are shown in Figs. 11a, 11b, and 11c. For these records the machine was run at 1795 rpm (2820 fpm pitch line velocity) with a measured torque of 2320 in-lb.

As mentioned earlier, the oscillograph records were analysed by plotting maximum measured tooth loads against the torque applied on the shaft to the same time scale, as shown in Fig. 12.

With the eccentricity of the gears in opposite direction curve 2 was obtained. Curve 2 represents the test series run at low speed, and curve 3 is the high speed result. Both curves are fairly close to the theoretical prediction (curve 1) in which the forces on the tooth are equal to the torque applied divided by the base circle radius of the gear. For the same torque value, the maximum tooth loads at low speed are higher than the theoretical curve while the maximum tooth loads at high speed are slightly lower than the theoretical curve.

With the eccentricity of the gears in the same direction curve 5 for low speed, and curve 4 for high speed were obtained. The maximum tooth load at low speed (curve 5) was slightly greater than at high speed (curve 4) the same as in the previous test. Both curves 4 and 5 are farther from the theoretical curve and this may be due to interference between teeth.

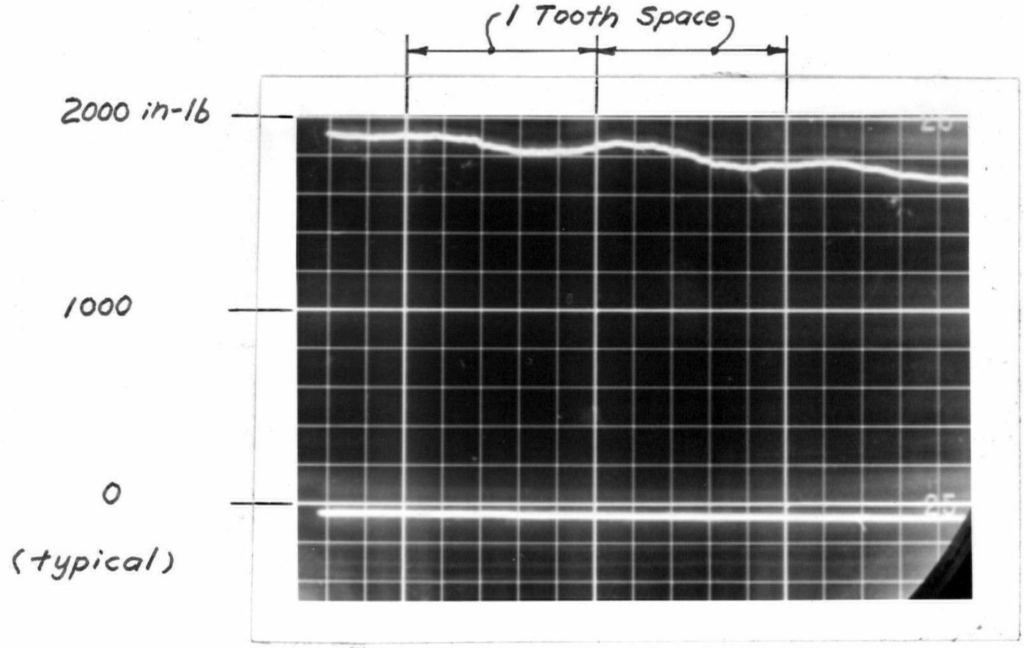


Fig.8a Shaft torque variation under a measured torque of 1950 in-lb at low speed. (Test No. 5)

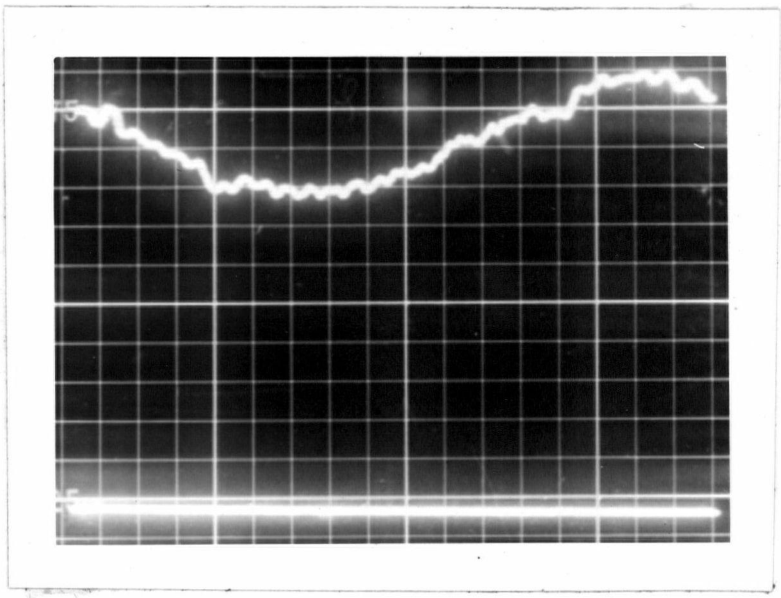


Fig. 8b Shaft torque variation for full revolution at 1950 in-lb at low speed. (Test No. 5)

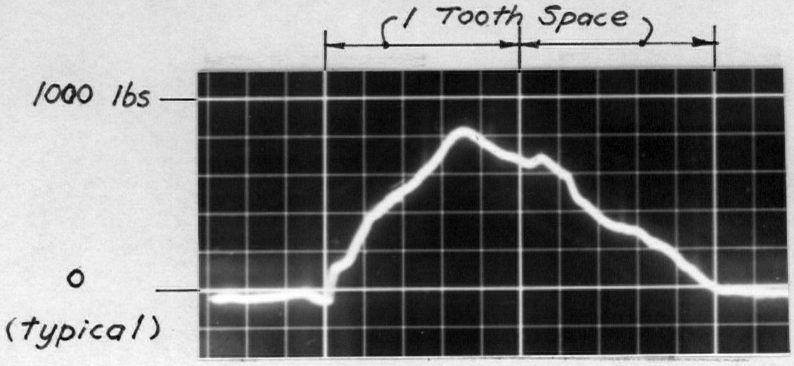


Fig.8c Tooth load variation at a measure torque of 1950 in-lb at low speed. (Test No. 5)

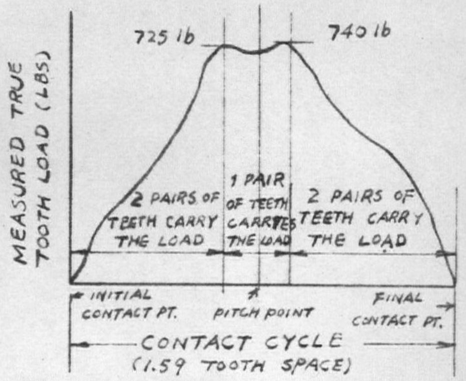


Fig.8d Force diagram of Fig. 8c.

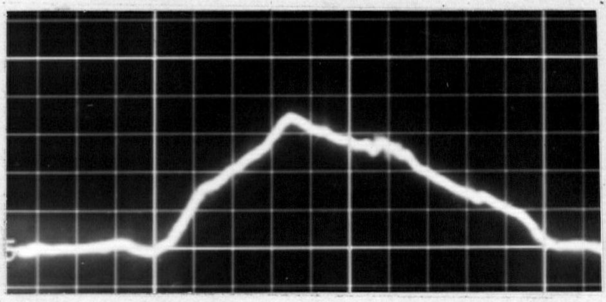


Fig.8e Tooth load variation at a measured torque of 1500 in-lb at low speed (Test No. 5)

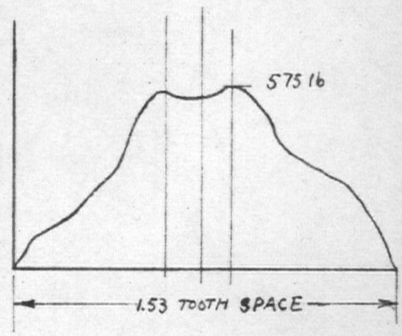


Fig. 8f Force diagram of Fig.8e.

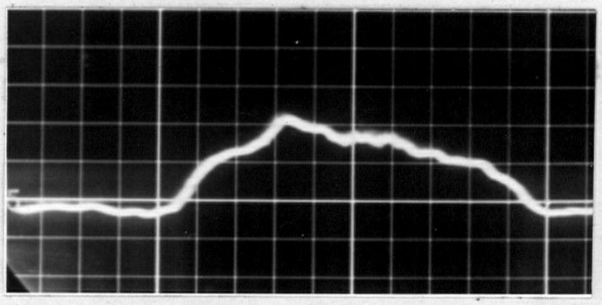


Fig.8g Tooth load variation at a measured torque of < 1100 in-lb at low speed. (Test No. 5)

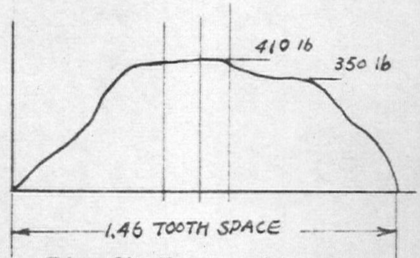


Fig.8h Force diagram of Fig.8g.

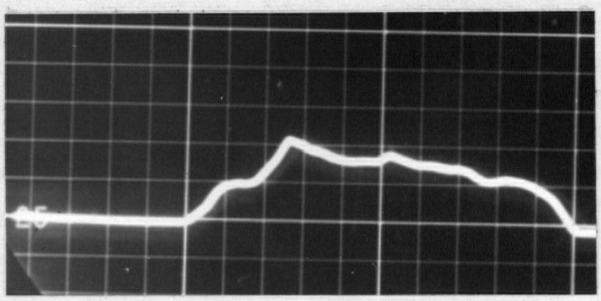


Fig. 8i Tooth load variation at a measured torque of 410 in-lb at low speed. (Test No. 5.)
(Double Scale)

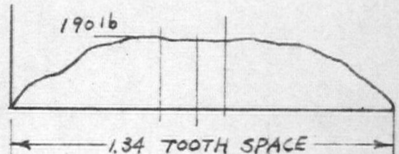


Fig. 8j Force diagram of Fig. 8i

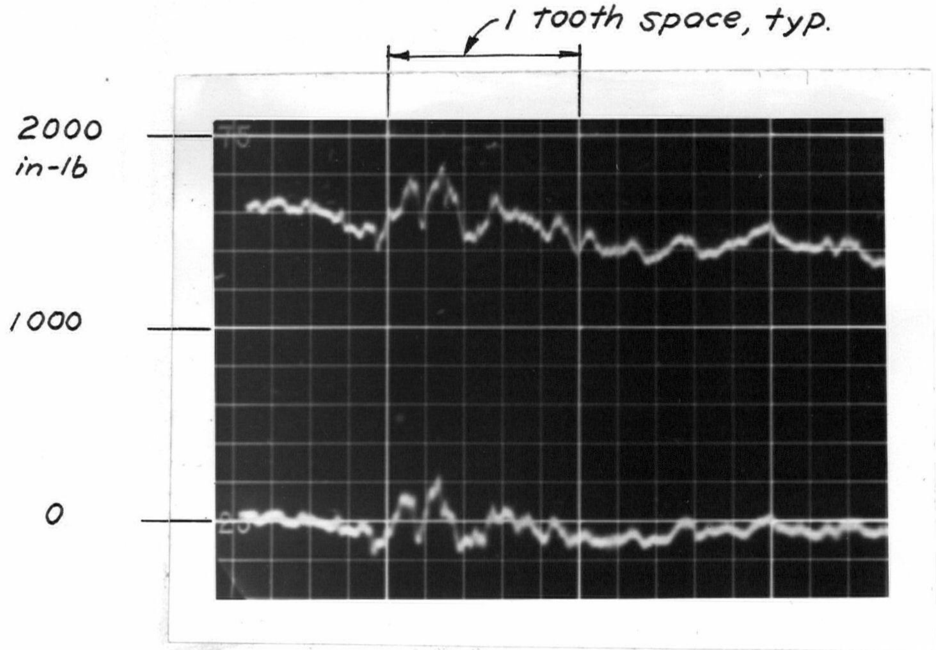


Fig. 9a Shaft torque variation at a measured torque of 1600 in-lb at high speed (Test No. 6)

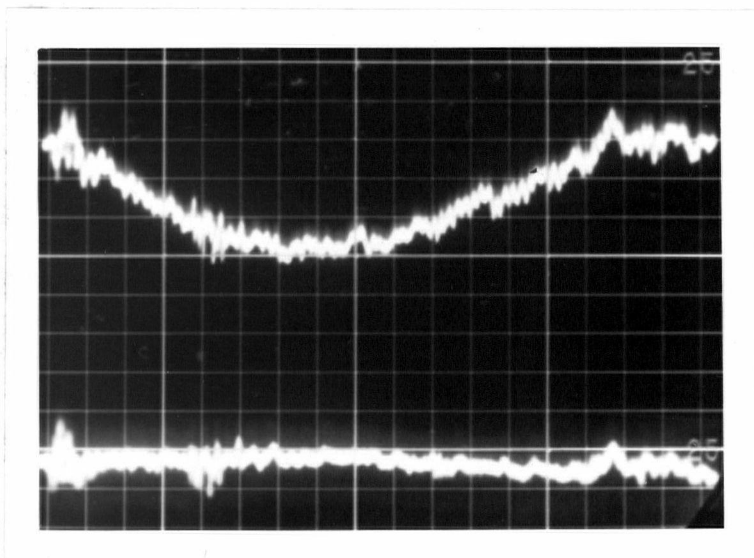


Fig. 9b Shaft torque variation for full revolution at 1600 in-lb at high speed. (Test No. 6)

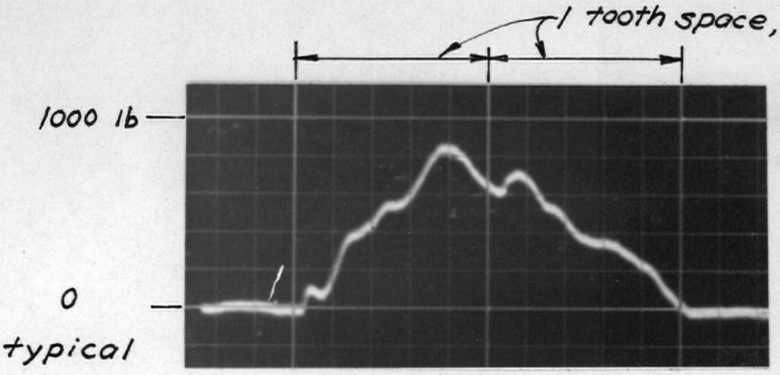


Fig.9c Tooth load pattern at a measured torque of 2200 in-lb at high speed. (Test No. 6)

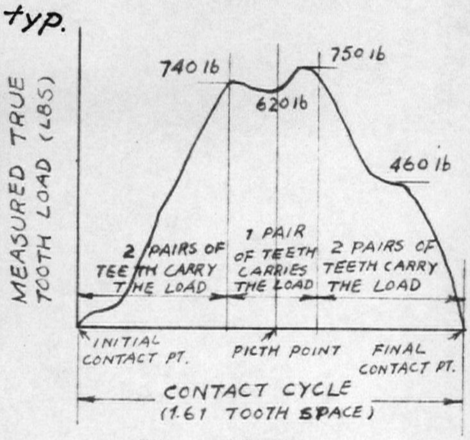


Fig. 9d Force diagram of Fig. 9c.

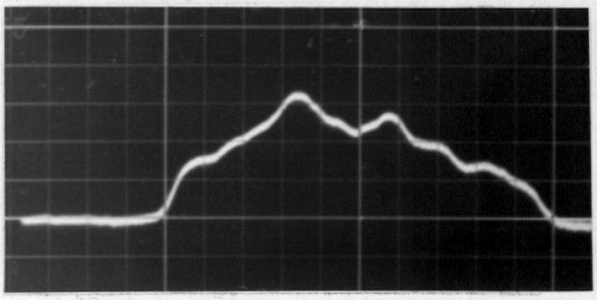


Fig. 9e Tooth load pattern at a measured torque of 1600 in-lb at high speed (Test No. 6)

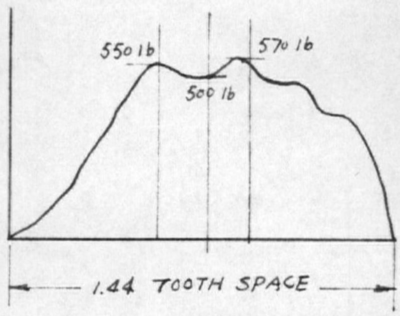


Fig. 9f Force diagram of Fig. 9e.

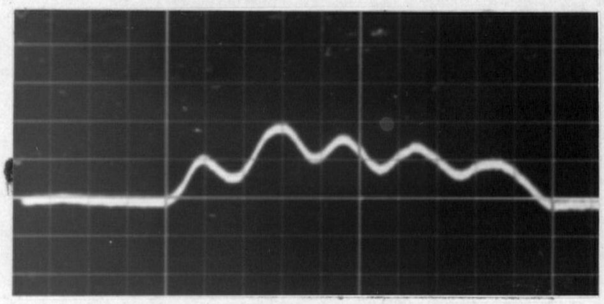


Fig. 9g Tooth load pattern at a measured torque of 950 in-lb at high speed. (Test No. 6)

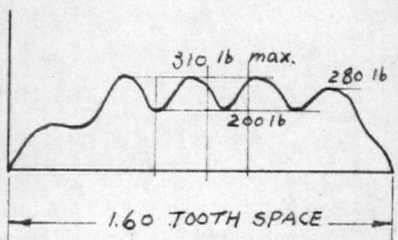


Fig. 9h Force diagram of Fig.9g (frequency 2770 cps)

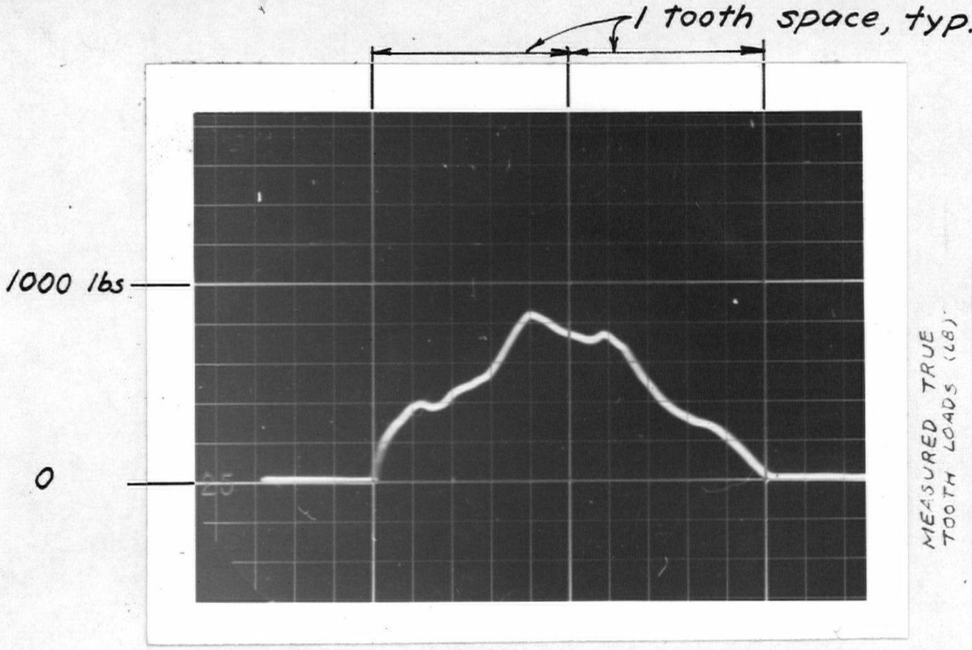


Fig. 10a Tooth load pattern at a measured torque of 2020 in-lb at low speed. (Test No. 8)

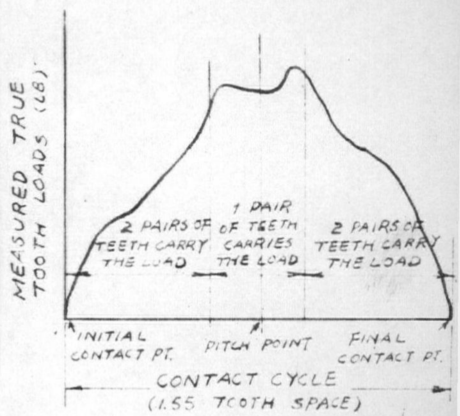


Fig. 10b Force diagram of Fig. 10a.

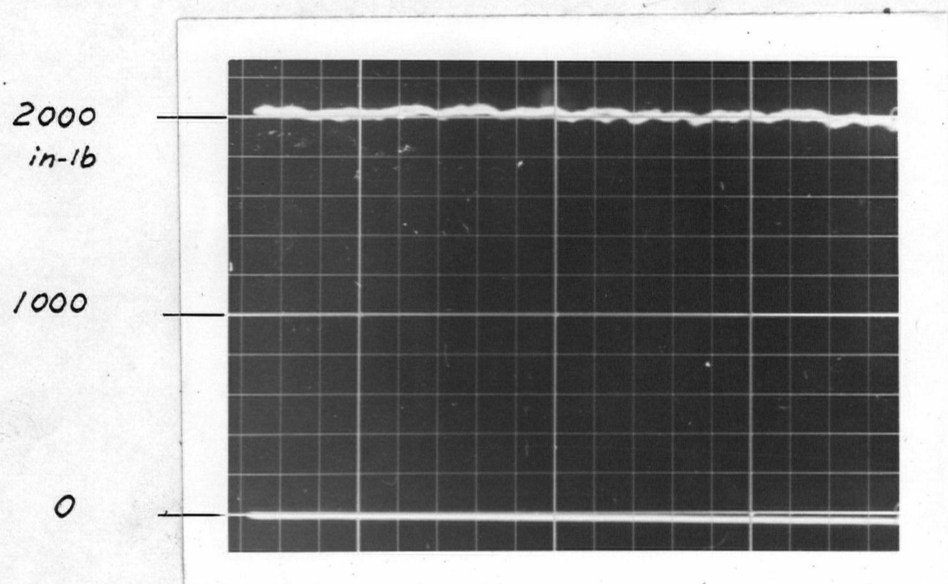


Fig. 10c Shaft torque pattern at a measured torque of 2020 in-lb at low speed. (Test No.8)

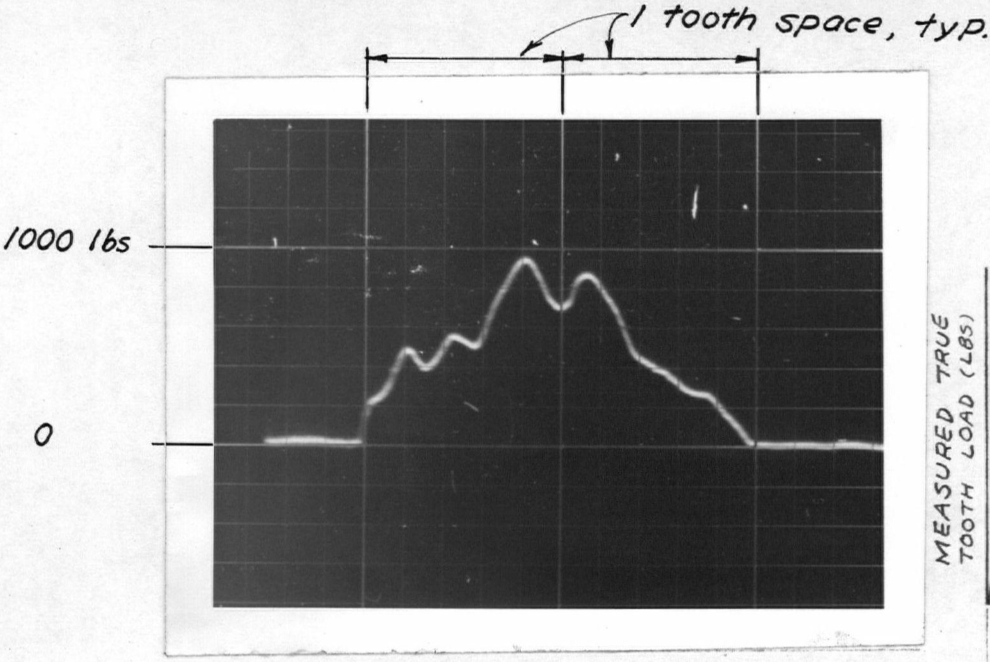


Fig.11a Tooth load pattern at a measured torque of 2320 in-lb at high speed (Test No. 7)

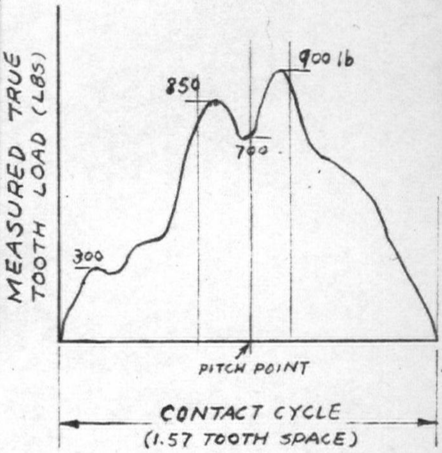


Fig. 11b Force diagram of Fig. 11a.

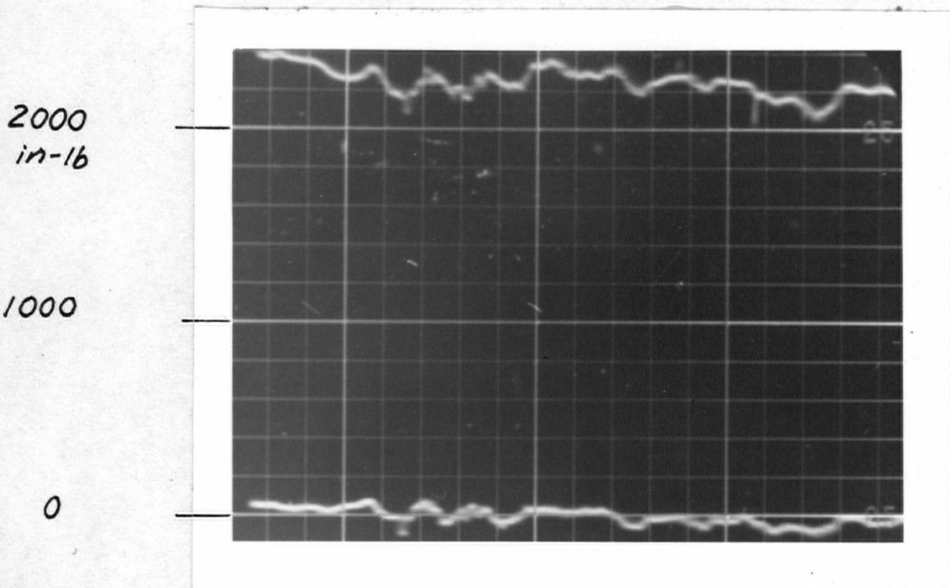


Fig. 11c Shaft torque pattern at a measured torque of 2320 in-lb at high speed (Test No. 7)

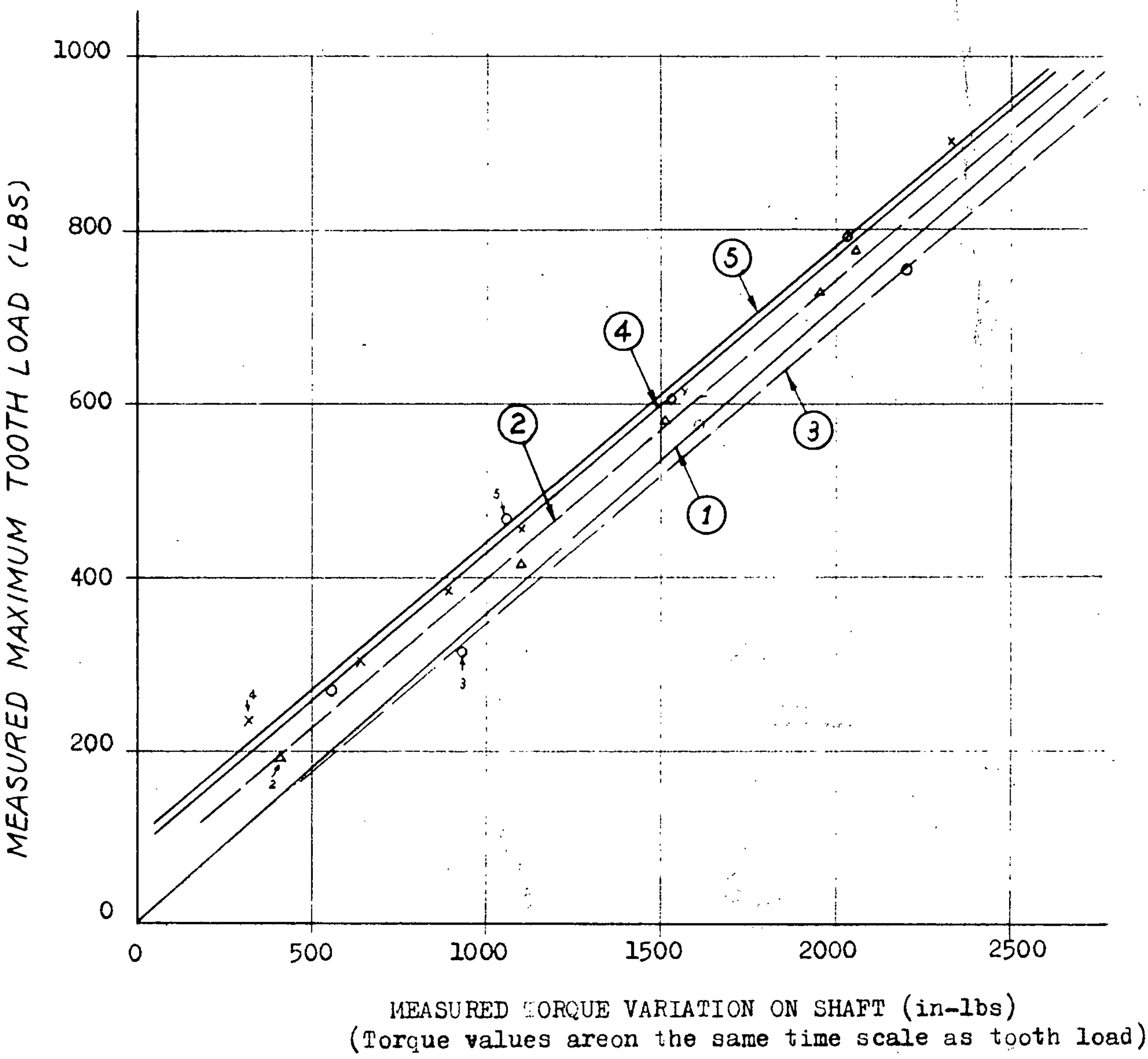


Fig. 12 Analysis of oscillograph records with the first pair of NRC gears (20-deg. pressure angle)

Curve 1 Theorteical tooth load curve
Curve 2 Maximum eccentricity vs minimum eccentricity at low speed.
Curve 3 Maximum eccentricity vs minimum eccentricity at high speed.
Curve 4 Both maximum eccentricities meshed at high speed.
Curve 5 Both maximum eccentricities meshed at low speed.

B. Second Test Series - Canadian Sumner Form Cut Gears (14½ degree pressure angle)

The second test series were conducted according to the same general procedure as detailed for the first test series, with the exception that after each test, one of the gears was rotated one third of a turn so that a different tooth on one gear was meshed with the tooth on the other gear to which the strain gages had been attached. This was done in order to compare the characteristic of the load pattern for different pairs of mating teeth.

The results of the three series of tests are shown in figures 13 to 18 for the different meshing conditions.

From the characteristic loading patterns, one can see that one tooth only carries the load, there is no sharing of load as in more accurately cut gears. The vibration is pronounced in the high speed tests. Essentially similar load patterns were obtained for all three meshing conditions.

The records were analysed by plotting curves with the torque variation on the shaft as the abscissa, and the maximum true tooth loads as ordinates.

In Fig. 19, curve 1 represents the theoretical tooth load which equals the applied torque divided by the base circle radius of the gear. In running the test, the pair of teeth in contact were marked 'A' and curve 2 was obtained at low speed, then one gear was shifted by one third of a revolution so that the tooth marked 'B' on the driving gear was in contact with tooth 'A' on the driven gear, and curve 3 was obtained at low speed. Similarly, curve 4 was obtained when a pair of teeth marked 'C' and 'A' were meshed together.

Curves 2, 3, and 4 which are plotted from the experimental data, diverge appreciably from the theoretical prediction (curve 1). This is very likely due to calibration errors.

At high speed, severe vibration is evident in the records thus making analysis difficult.

From a comparison of measured maximum tooth loads in the experimental results, it can be seen that the Canadian Sumner Gears have far greater tooth loads than the NRC gears.

When analysing the tooth load records as a first approximation, the tooth can be considered to be an undamped linear single-degree of freedom system subjected to a load which is gradually applied and then maintained indefinitely. The detailed analyses are given in Chapter V.

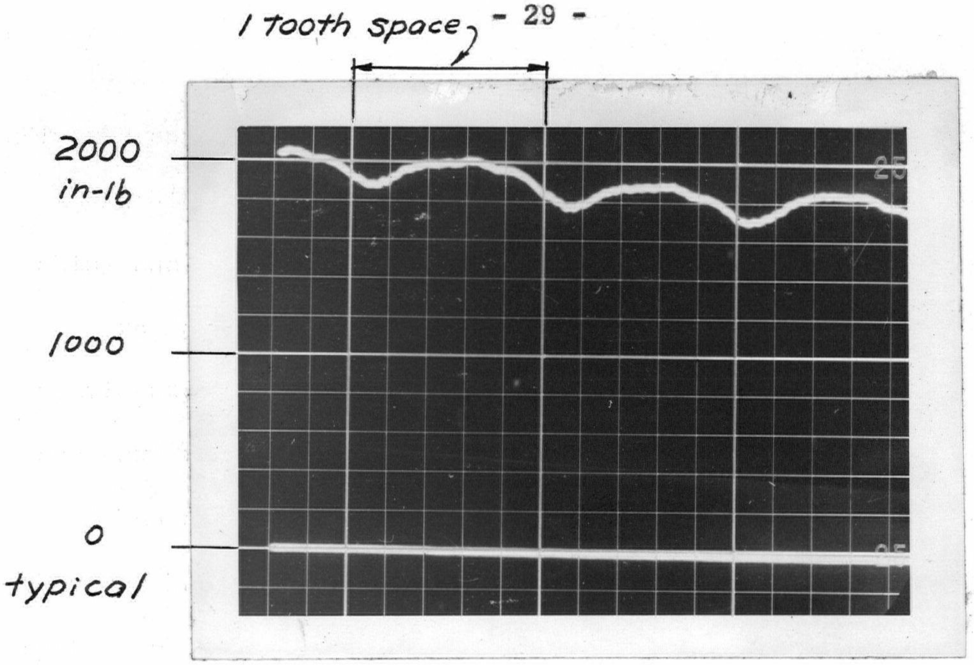


Fig 13a Shaft torque variation at a measured torque of 2000 in-lb at low speed (Test No. 9)

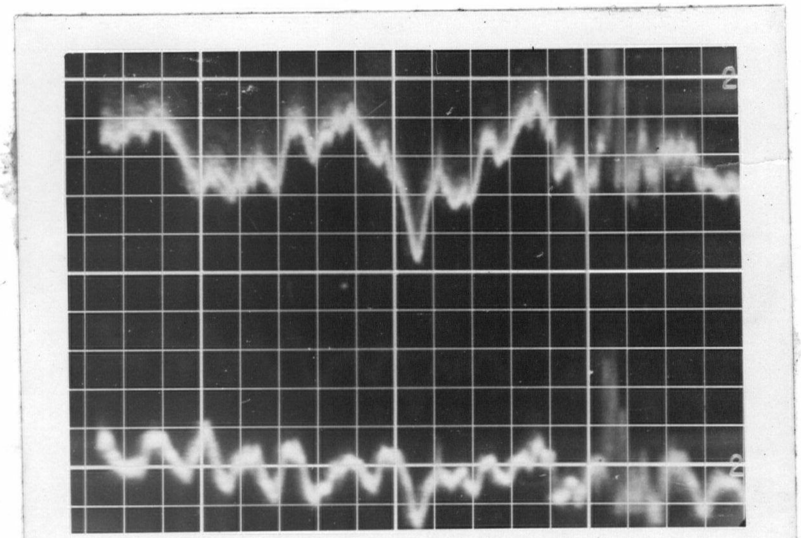


Fig. 14a Shaft torque variation at a measured torque of 1620 in-lb at high speed (Test No. 10)

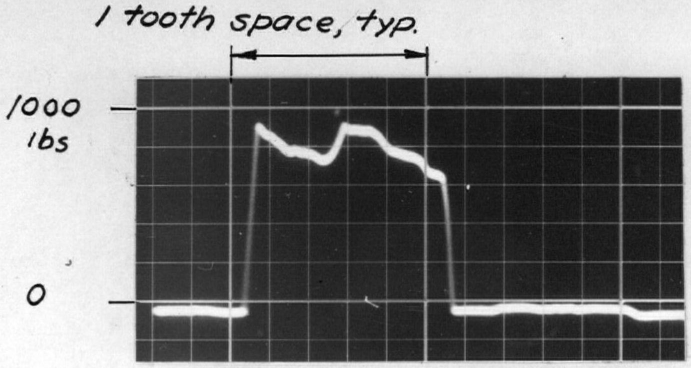


Fig. 13b Tooth load pattern at a measured torque of 2000 in-lb at low speed. (Test No. 9)

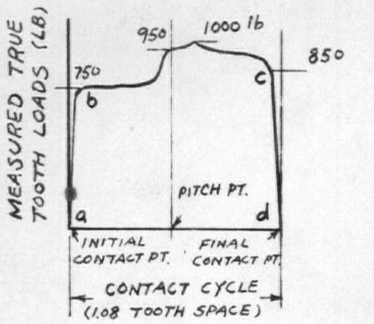


Fig.13c Force diagram of Fig. 13b

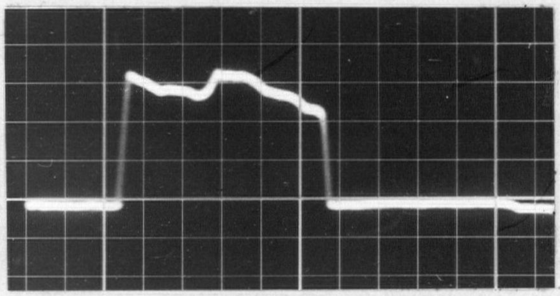


Fig. 13d Tooth load pattern at a measured torque of 1450 in-lb at low speed (Test No. 9)

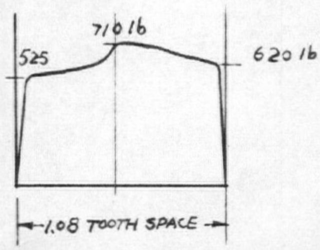


Fig. 13e Force diagram of Fig. 13d

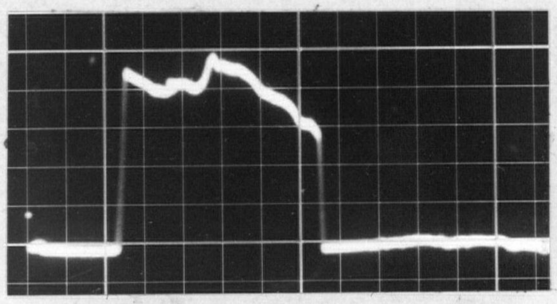


Fig. 13f Tooth load pattern at a measured torque of 1050 in-lb at low speed (double scale)

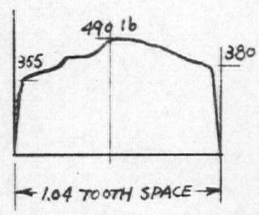


Fig. 13g Force diagram of Fig. 13f

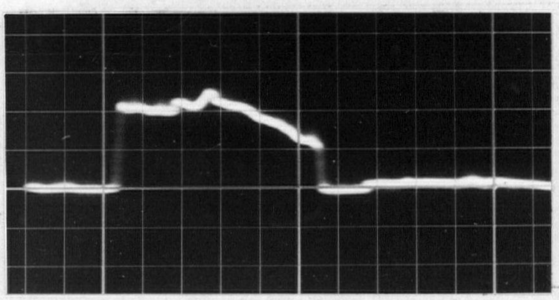


Fig. 13h Tooth load pattern at a measured torque of 560 in-lb at low speed (double scale)

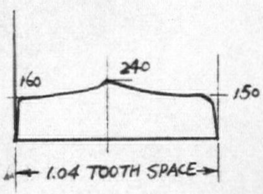


Fig. 13i Force diagram of Fig. 13h

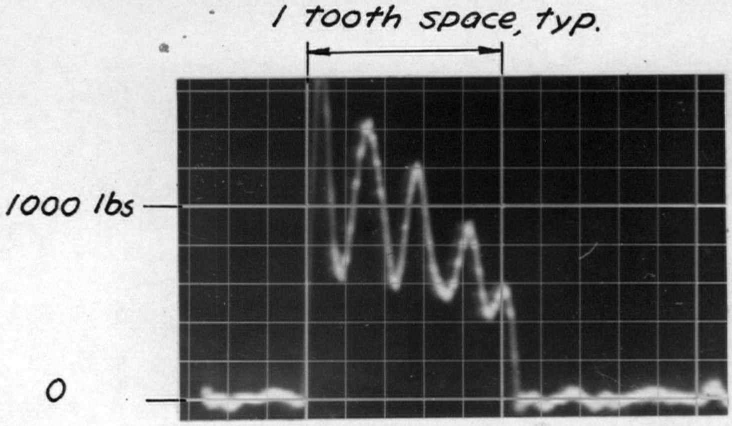


Fig.14b Tooth load pattern at a measured torque of 1620 in-lb at high speed (Test No. 10)

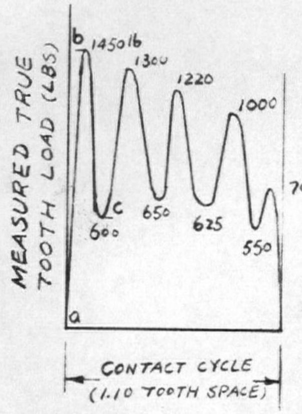


Fig. 14c force diagram of Fig. 14b. (frequency = 2860 cps)

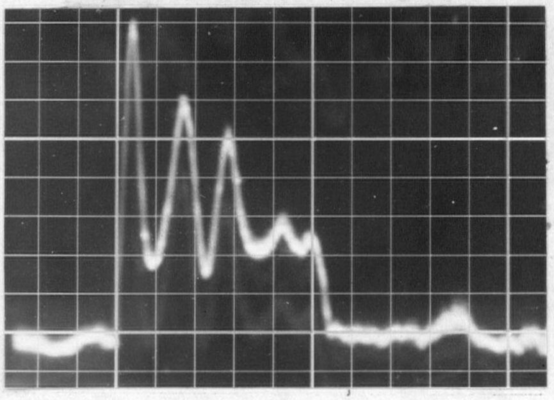


Fig. 14d Tooth load pattern at a measured torque of 1500 in-lb at high speed (Test No. 10)

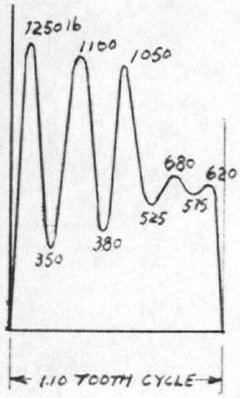


Fig. 14e Force diagram of Fig. 14d.

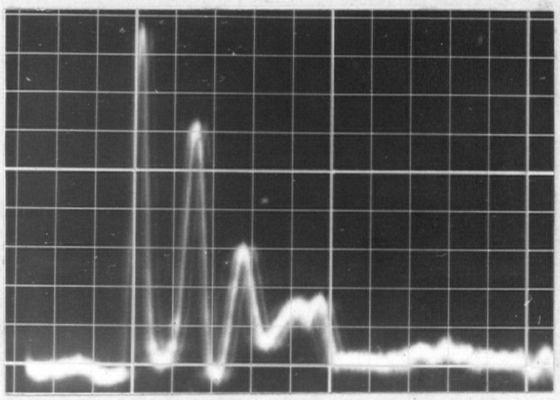


Fig.14f Tooth load pattern at a measured torque of 1000 in-lb at high speed (Test No. 10)

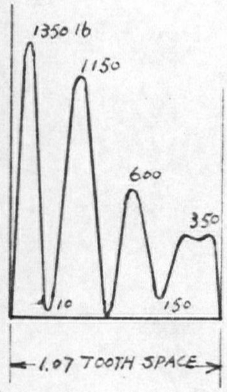


Fig. 14g Force diagram of Fig. 14f.

2. The tooth marked A on the driven gear meshed with the tooth B on the driving gear.

The records in this case were of the pattern as in the preceding case, so only some typical diagrams are shown below.

a) Machine at low speed.

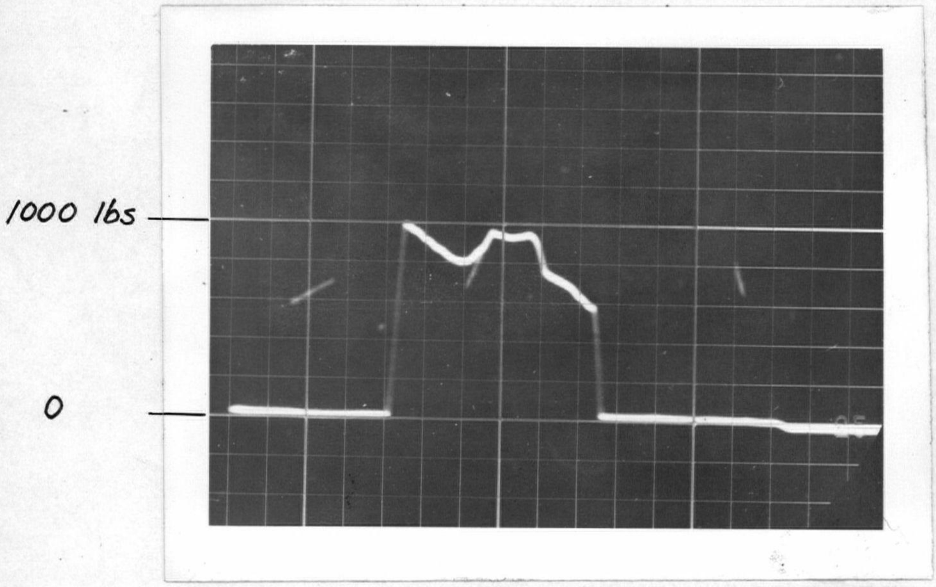


Fig.15a Tooth load pattern at a measured torque of 2000 in-lb at low speed. (Test No. 12)

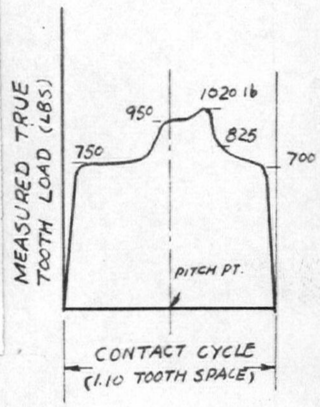


Fig. 15b Force diagram of Fig. 15a.

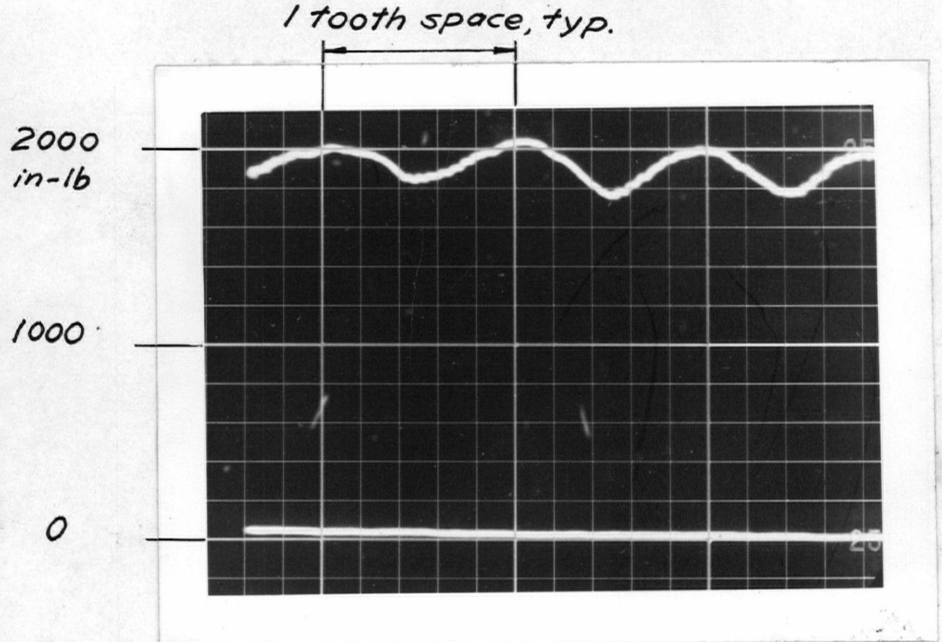


Fig. 15c Shaft torque variation at a measured torque of 2000 in-lb at low speed (Test No. 12)

b) Machine at high speed.

Some typical records are shown in the following diagrams.

The measured frequency of the tooth load pattern is 3000 cps.

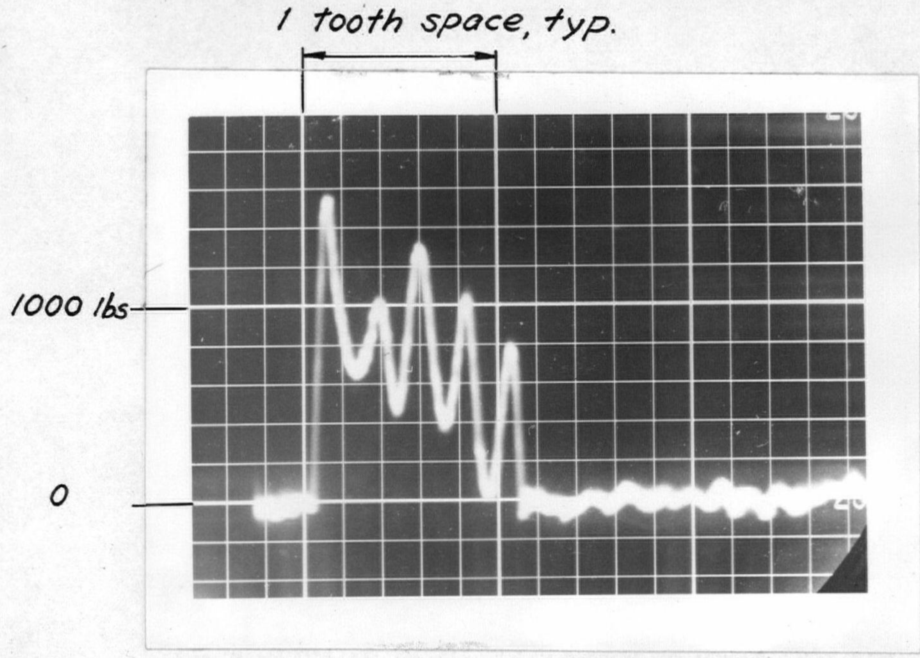


Fig. 16a Tooth load pattern at a measured torque of 2000 in-lb at high speed. (Test No. 11)

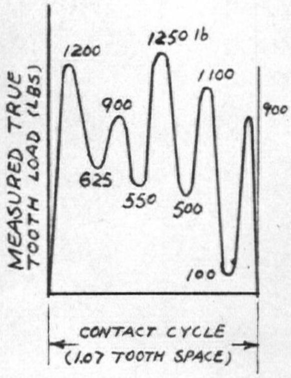


Fig. 16b Force diagram of Fig. 16a.

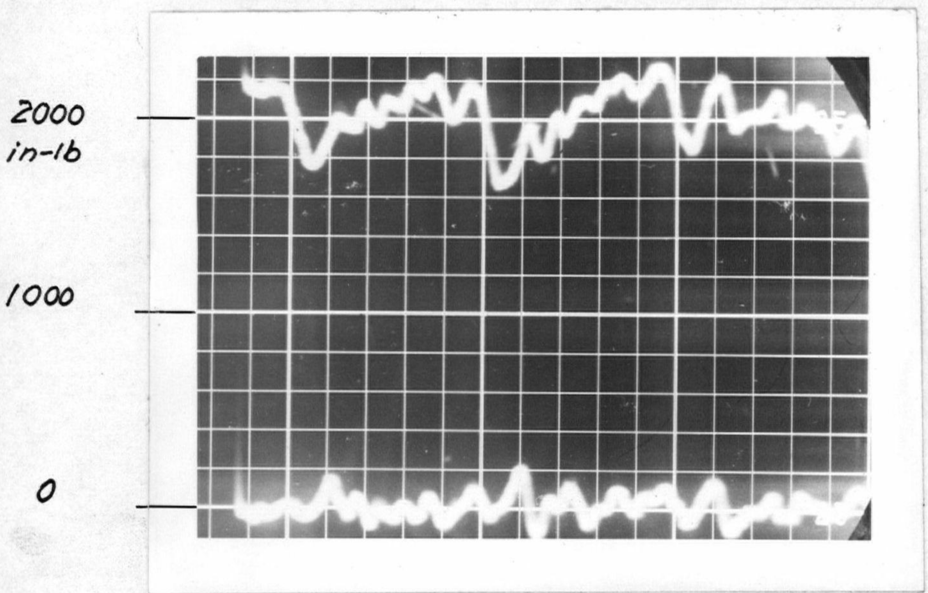


Fig. 16c Shaft torque variation at a measured torque of 2000 in-lb at high speed. (Test No. 11)

3. The tooth marked A on the driven gear meshed with the tooth C on the driving gear.

Some typical diagrams are shown below.

a) Machine at low speed.

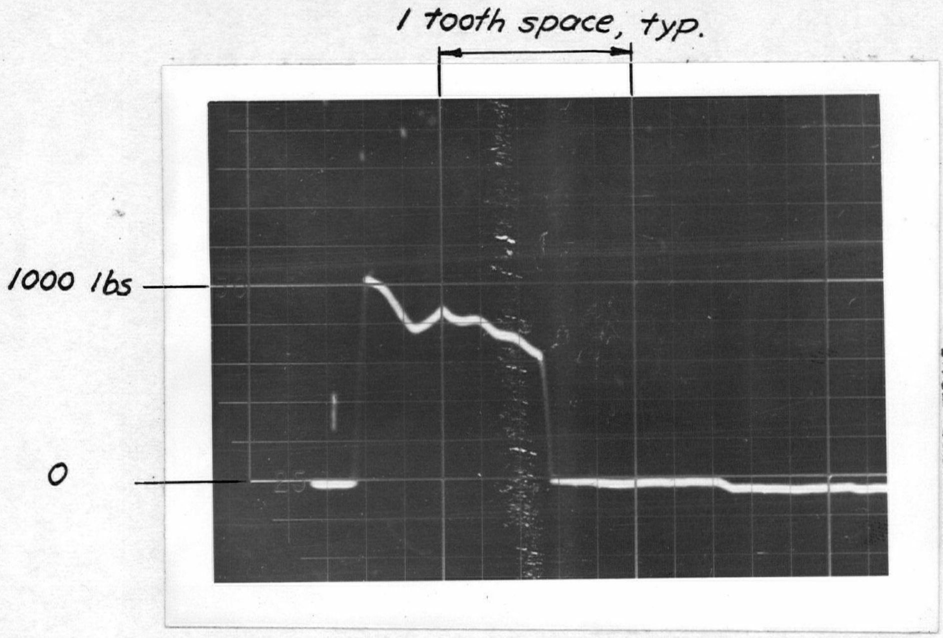


Fig. 17a Tooth load pattern at a measured torque of 1920 in-lb at low speed. (Test No. 13)

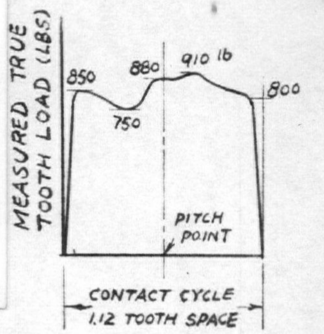


Fig. 17b Force diagram of Fig.17a.

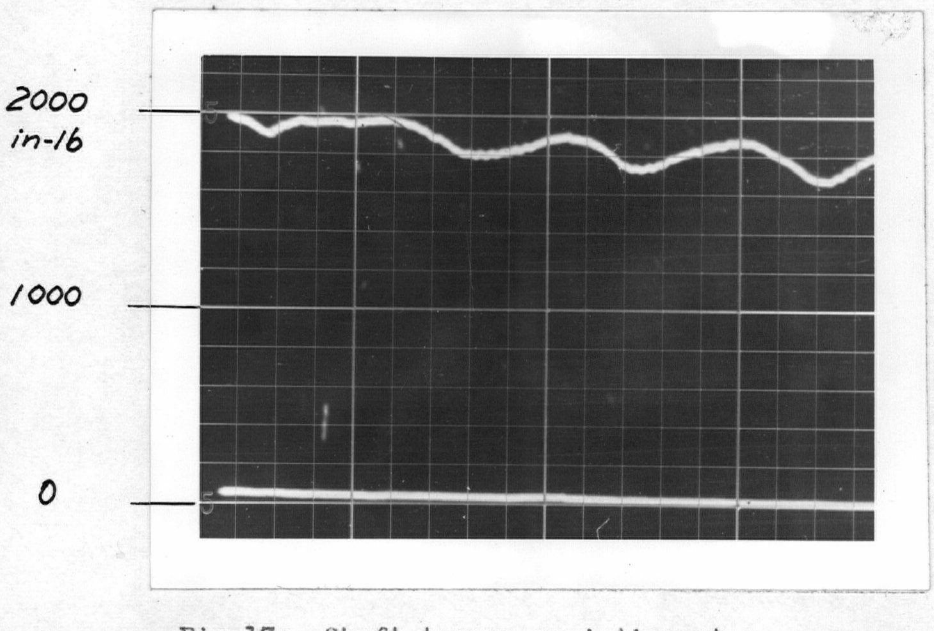


Fig.17c Shaft torque variation at a measured torque of 1920 in-lb at low speed. (Test No. 13)

b) Machine at high speed.

The measured frequency of the tooth load pattern and the shaft torque pattern were 3000 and 2890 cycles/sec respectively.

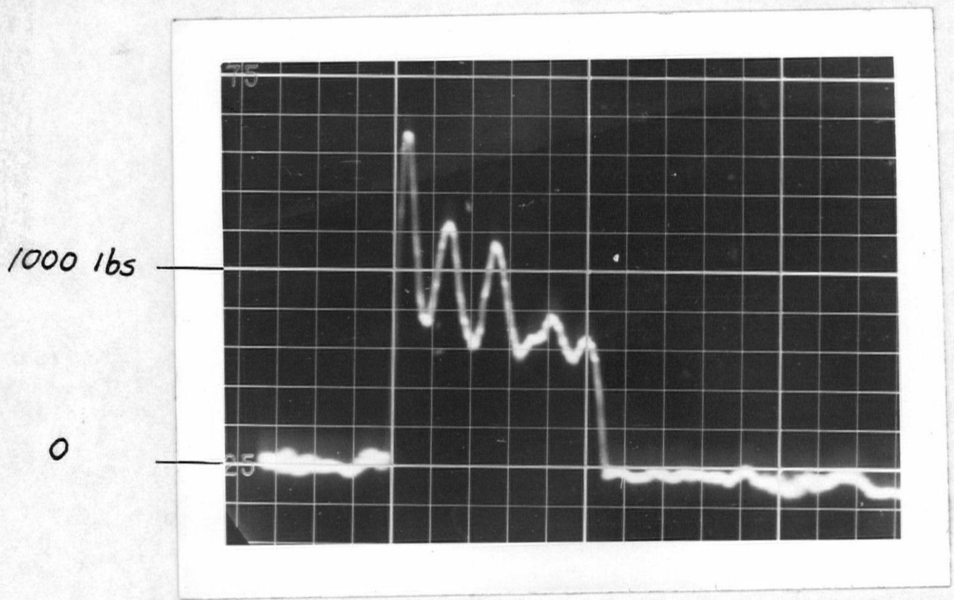


Fig. 18a Tooth load pattern at a measured torque of 2000 in-lb at high speed. (Test No. 14)

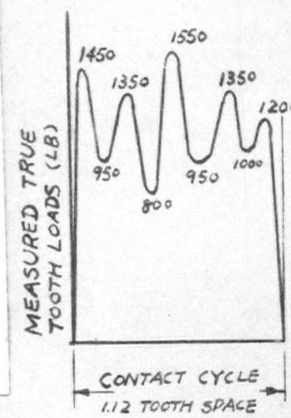


Fig. 18b Force diagram of Fig. 18a.

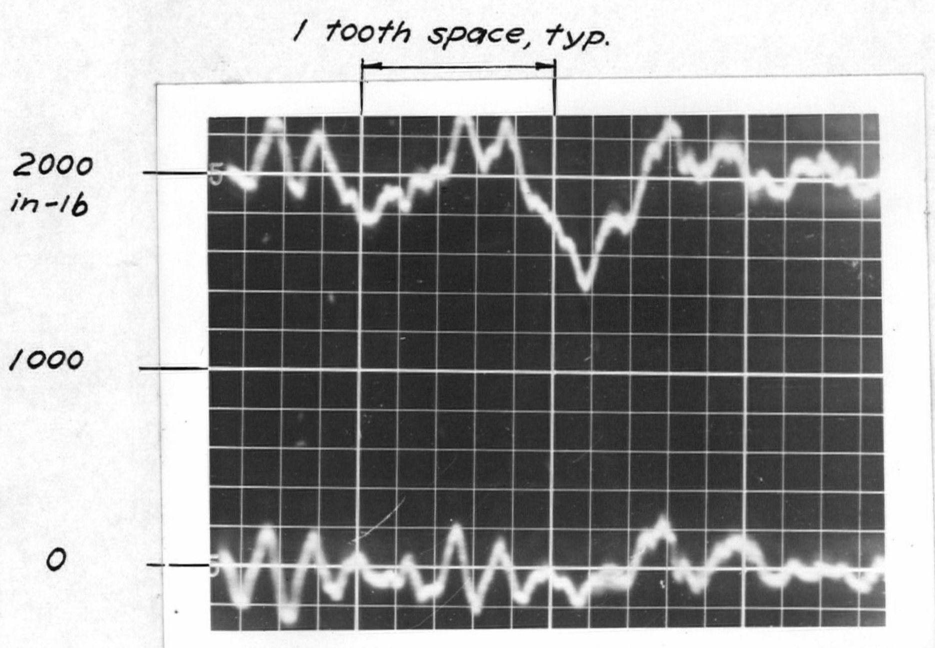


Fig. 18c Shaft torque variation at a measured torque of 2000 in-lb at high speed. (Test No. 14).

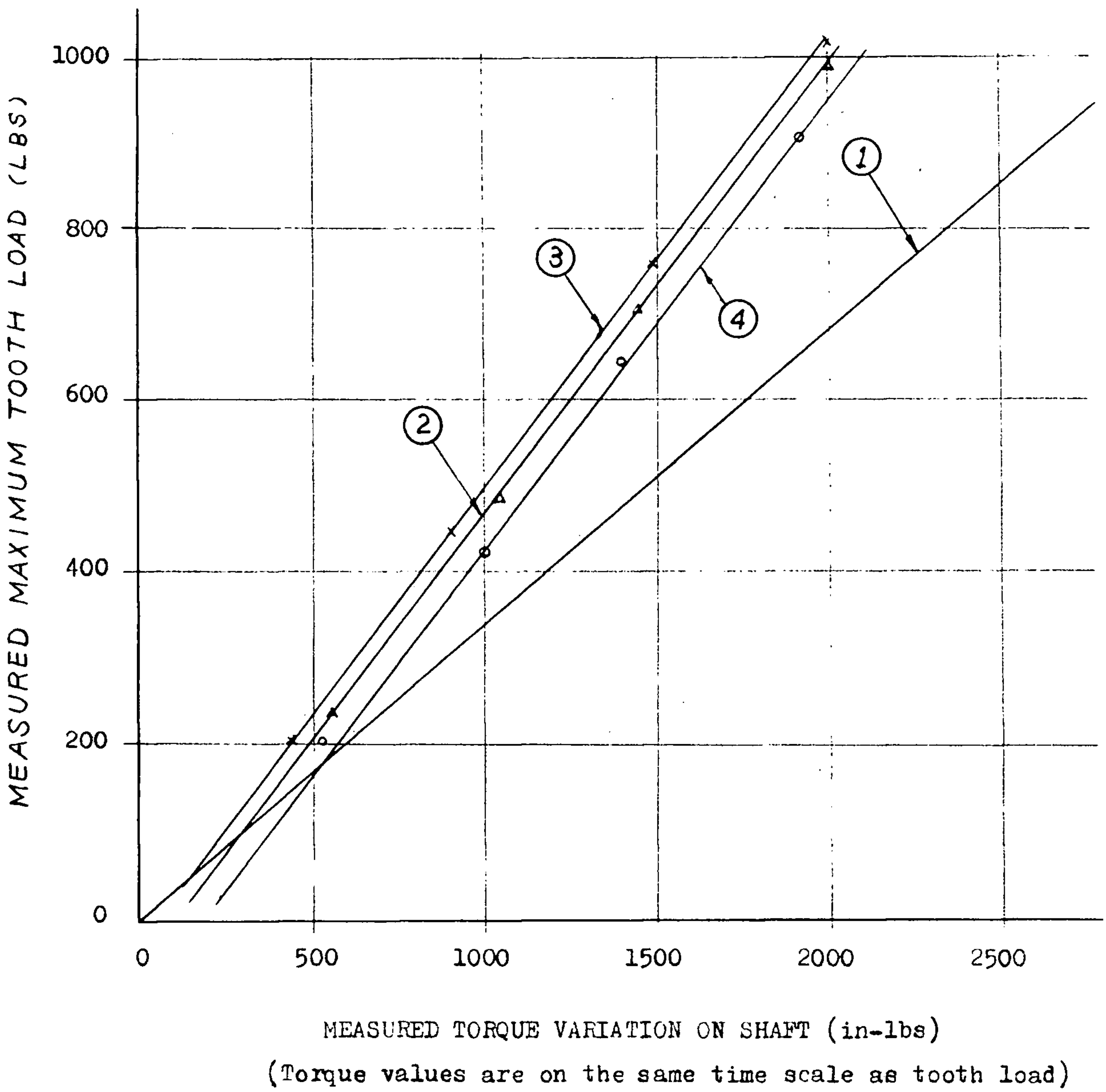


Fig. 19 Analysis of oscillograph records with the Canadian Sumner Gears (14 1/2 deg. pressure angle)

Curve 1 Theoretical tooth load curve.
Curve 2 Tooth 'A' meshed with tooth 'A' at low speed.
Curve 3 Tooth 'A' meshed with tooth 'B' at low speed
Curve 4 Tooth 'A' meshed with tooth 'C' at low speed.

C. <u>Third Test Series - Second Pair of NRC Involute Spur Gears (20-degree pressure angle)</u>

The third test series were conducted according to the procedure detailed for the first test series with the exception that after each test one of the gears was rotated one third of a turn so that a different tooth on one gear was meshed with the tooth on the other gear to which the strain gages had been attached.

The results of the three series of tests are shown in figures 20 to 25 for the different meshing conditions.

The torque variation obtained in these tests fluctuates less than the first pairs of NRC gears and closes to a straight line. The reduced torque fluctuation tended to suggest that the second pair of NRC gears had fewer tooth errors than the first pair.

From the tooth load recordings one can see clearly the load is shared by the teeth during the beginning and end of the contact cycle.

At high speed the oscillograph recordings have the same general pattern as at low speed, except that there were some vibration effects.

Referring to Fig. 21d, hi represents the average force on a pair of teeth which are approaching the end of contact. The sudden reduction in force represented by the line gh indicates that a new pair of teeth have come into contact at 'a' and have assumed a portion of the load. Results of experiments indicate that a new pair of teeth must suddenly come into contact causing a very rapid increase in force (curve ab, Fig. 21d). The line cd shows that the load was shared by two pairs of teeth until the load jumped to its maximum value when one tooth started to carry the whole load. When the action passes point e, the vibrating motion appears to decay and settle down to an average maximum value represented by fg.

The oscillograph records obtained from the tests of the second pair of NRC gears were analysed and their results are shown in Fig. 26. In the diagram, curve 1 represents the theoretical tooth load which is equal to the applied torque divided by the base circle radius of the gear. Curve 2 resulted with both 'A' teeth of the pair of gears meshed together at low speed while curve 3 is the result at high speed. Curves 4 and 5 were obtained at low speed and at high speed respectively with one gear shifted one third of a revolution. Similarly, curves 6 and 7 were obtained after one gear was again shifted by one third of a revolution.

In these tests the majority of the measured tooth loads were lower than the theoretical load for the same applied torque. This is either due to the calibration errors, or could be explained by the Reswick's[10] study in the following paragraph. Also, the larger the torque value, the greater the range between the theoretical value and the measured load. For the same pair of meshed teeth the measured tooth loads at low speed are sometimes higher than the loads at high speed; whereas in the tests of the first pair of NRC gears, the tooth loads at low speed always had a slightly higher value than at high speed.

J. B. Reswick[10] developed an expression for dynamic tooth loads based on a study of a simple physical mechanism. "From first principles", stated Reswick, "equations have been developed which predict values for dynamic tooth loads that are in general agreement with those predicted by Buckingham's equation for 'lightly loaded gears'+ but differ somewhat in the case of 'heavily loaded gears'*". Results developed from experiments of the model indicated that in many heavily loaded gears the total dynamic load may be actually less than the static load determined from the transmitted power, and the dynamic loads may be ignored, but the dynamic load is important in lightly loaded gears.

\+ The term "lightly loaded gears" applies to those gears in which the deflection due to the steady power load is very much smaller than the manufacturing error. In this case, only the pair of teeth involved in the engagement is deflected.

* The term "heavily loaded gears" is applied to those gears in which the deflection due to the steady power load is equal to or greater than the manufacturing tooth error. Under these conditions the load is carried by two pairs of teeth during tooth engagement.

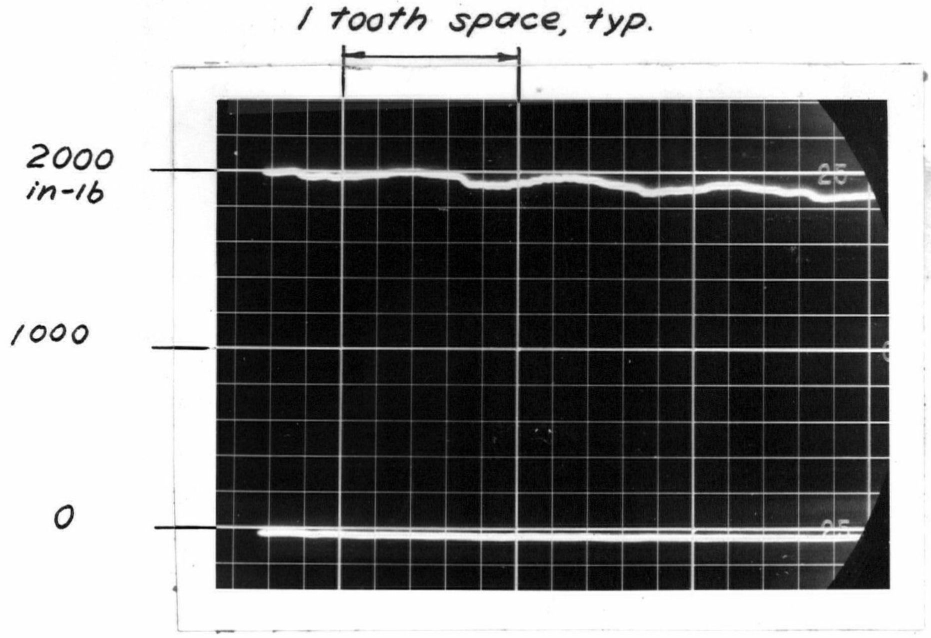


Fig. 20a Shaft torque variation at a measured torque of 1980 in-lb at low speed. (Test No. 16)

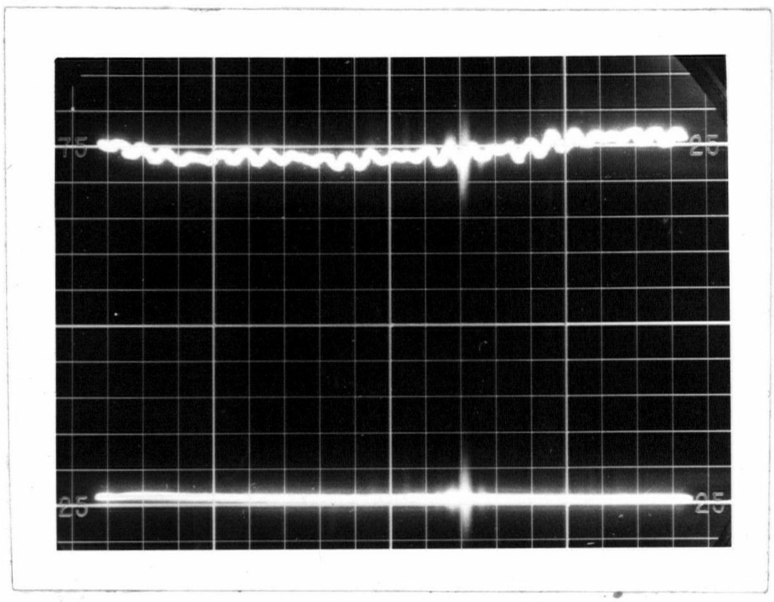


Fig. 20b Shaft torque variation for full revolution at 1980 in-lb at low speed. (Test No. 16).

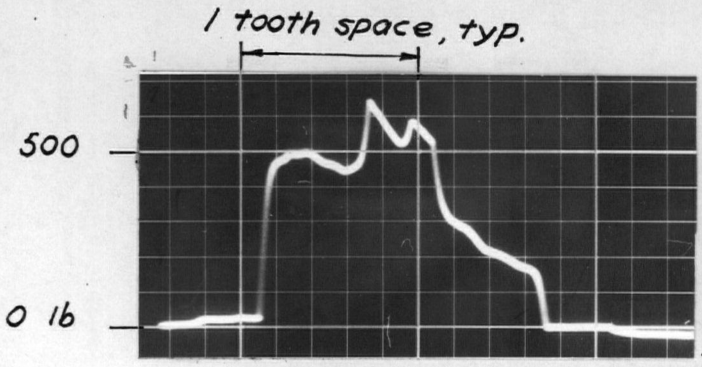


Fig. 20c Tooth load variation at a measured torque of 1980 in-lb at low speed-double scale (Test No.16)

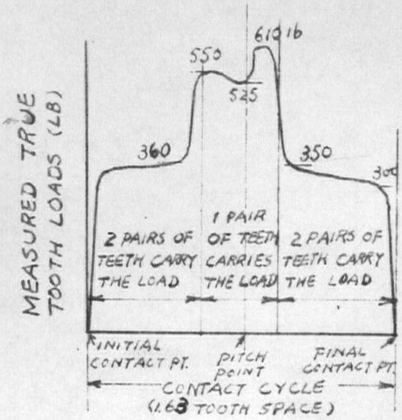


Fig.20d Force diagram of Fig. 20c.

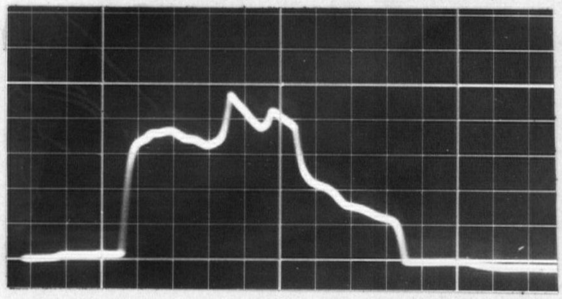


Fig.20e Tooth load variation at a measured torque of 1450 in-lb at low speed-double scale(Test No.16)

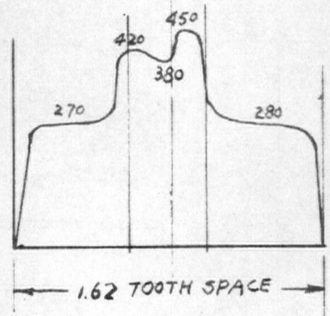


Fig. 20f Force diagram of Fig.20e.

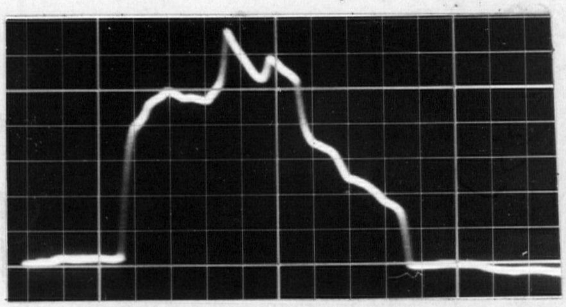


Fig.20g Tooth load variation at a measured torque of 960 in-lb at low speed-quadruple scale.(Test No.16)

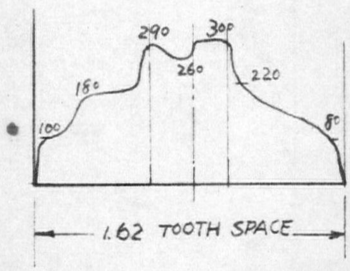


Fig.20h Force diagram of Fig. 20g.

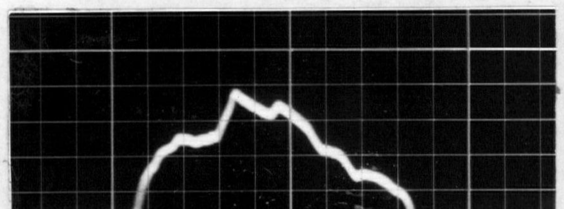


Fig. 20i Tooth load variation at a measured torque of 500 in-lb at low speed-quadruple scale.(Test No.16)

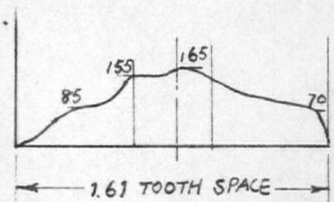


Fig.20j Force diagram of Fig. 20i.

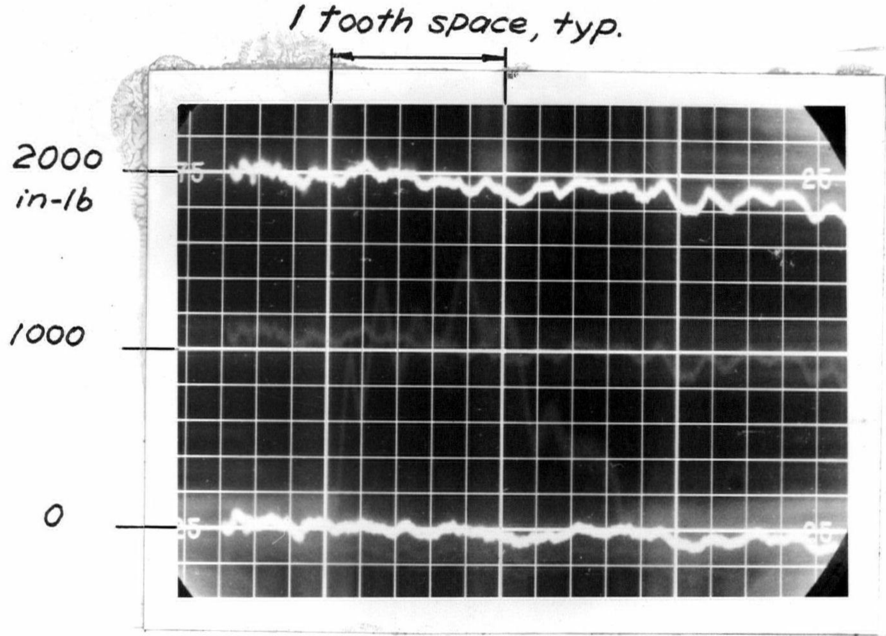


Fig. 21a Shaft torque variation at a measured torque of 2000 in-lb at high speed. (Test No. 17)

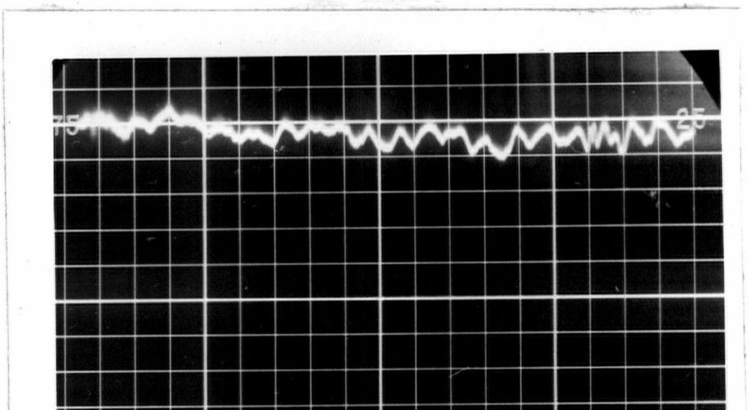


Fig. 21b Shaft torque variation for full revolution at 2000 in-lb at high speed. (Test No. 17)

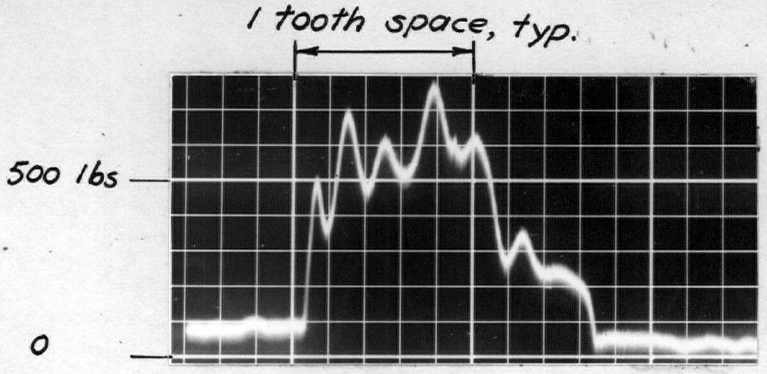


Fig. 21c Tooth load pattern at a measured torque of 2000 in-lb at high speed-double scale. (Test No. 17)

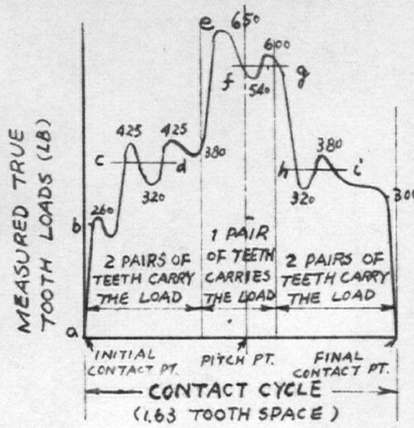


Fig. 21d Force diagram of Fig. 21c

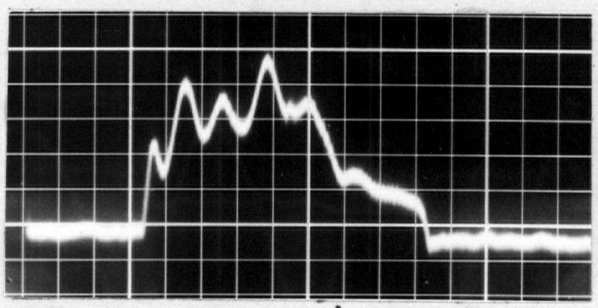


Fig. 21e Tooth load pattern at a measured torque of 1450 in-lb at high speed-double scale (Test No. 17)

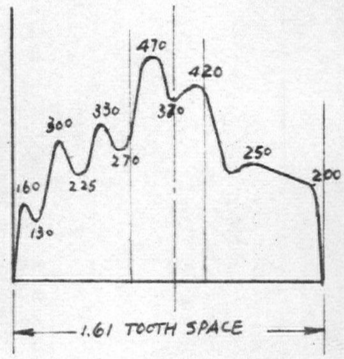


Fig. 21f Force diagram of Fig. 21e.

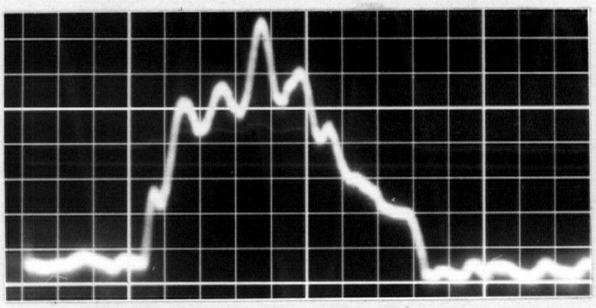


Fig. 21g Tooth load pattern at a measured torque of 1050 in-lb at high speed-quadruple scale. (Test No. 17)

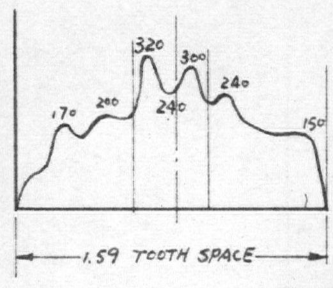


Fig. 21h Force diagram of Fig. 21g

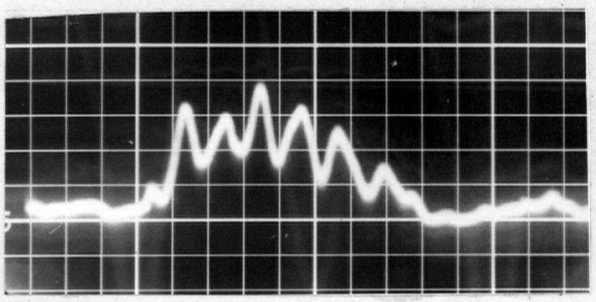


Fig. 21i Tooth load pattern at a measured torque of 570 in-lb at high speed-quadruple scale. (Test No. 17)

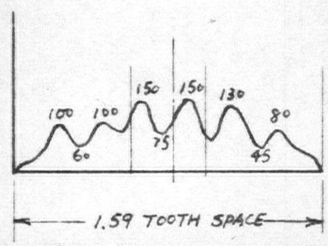


Fig. 21j Force diagram of Fig. 21i

2. The tooth marked A on the driven gear meshed with the tooth B on the driving gear.

   a) Machine at low speed

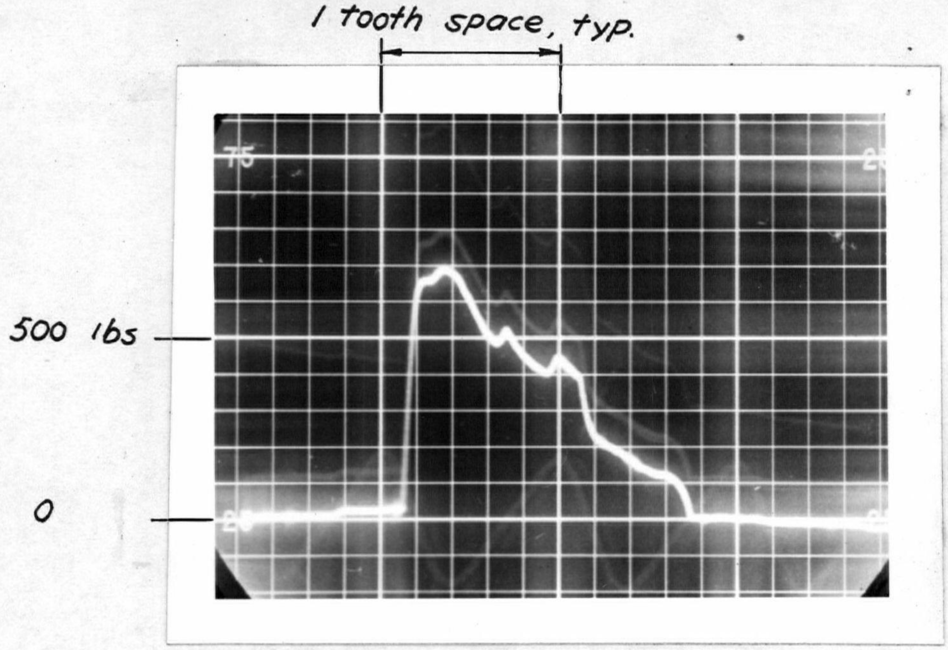


Fig.22a Tooth load variation at a measured torque of 1450 in-lb at low speed-double scale (Test No.19)

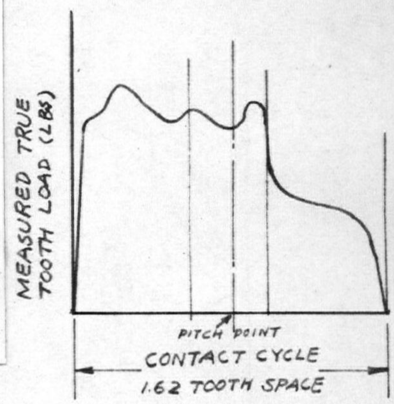


Fig.22b Force diagram of Fig. 22a

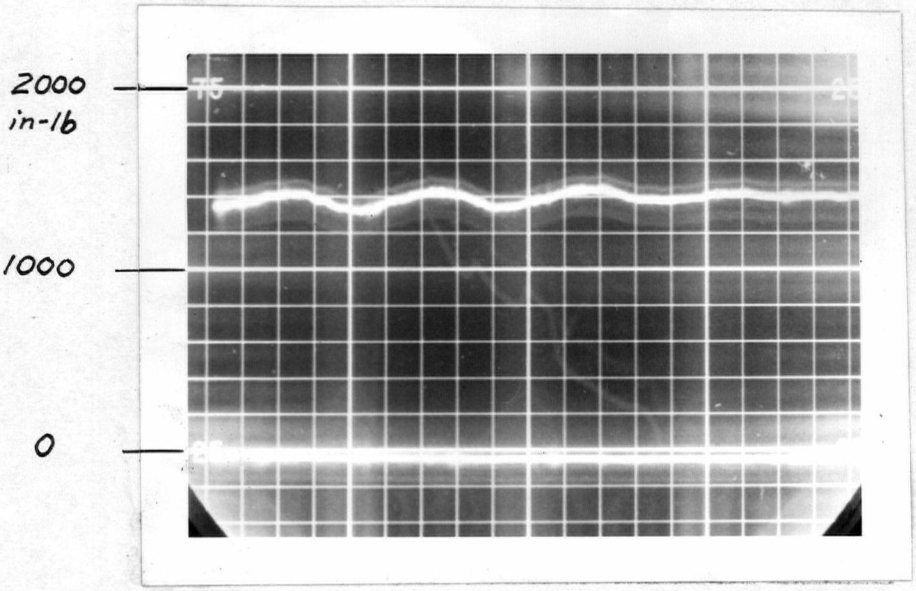
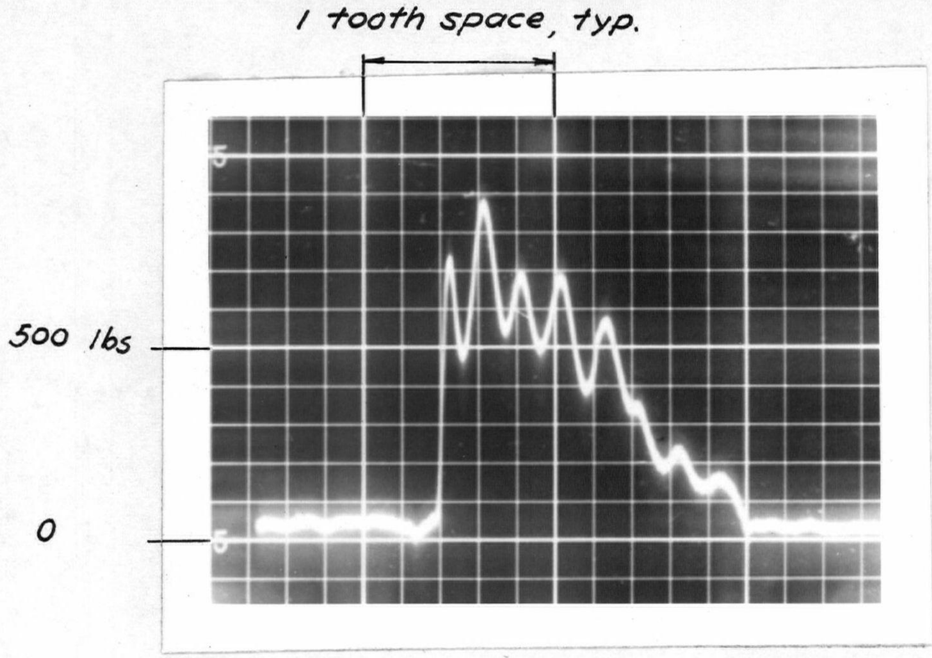


Fig.22c Shaft torque variation at a measured torque of 1450 in-lb at low speed (Test No. 19)

Fig 23
P. 67

b) Machine at high speed.



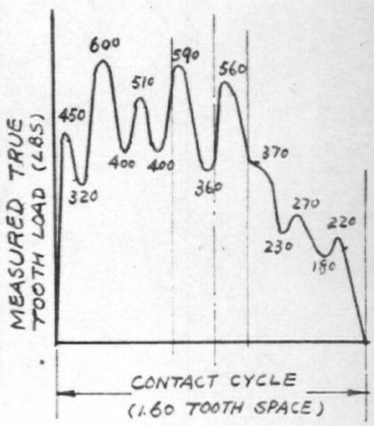


Fig. 23a Tooth load pattern at a measured torque of 1550 in-lb at high speed-double scale. (Test No. 18)

Fig. 23b Force diagram of Fig. 23a

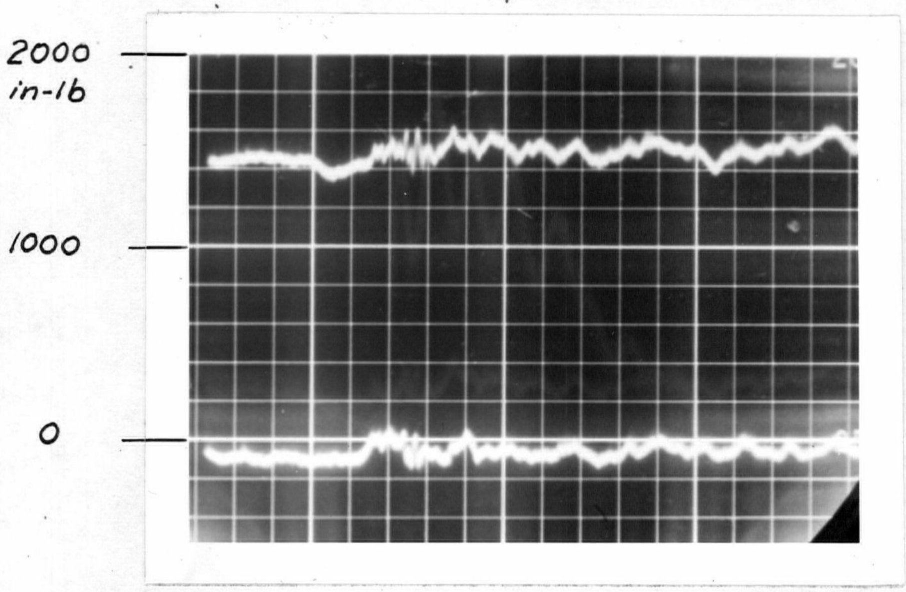


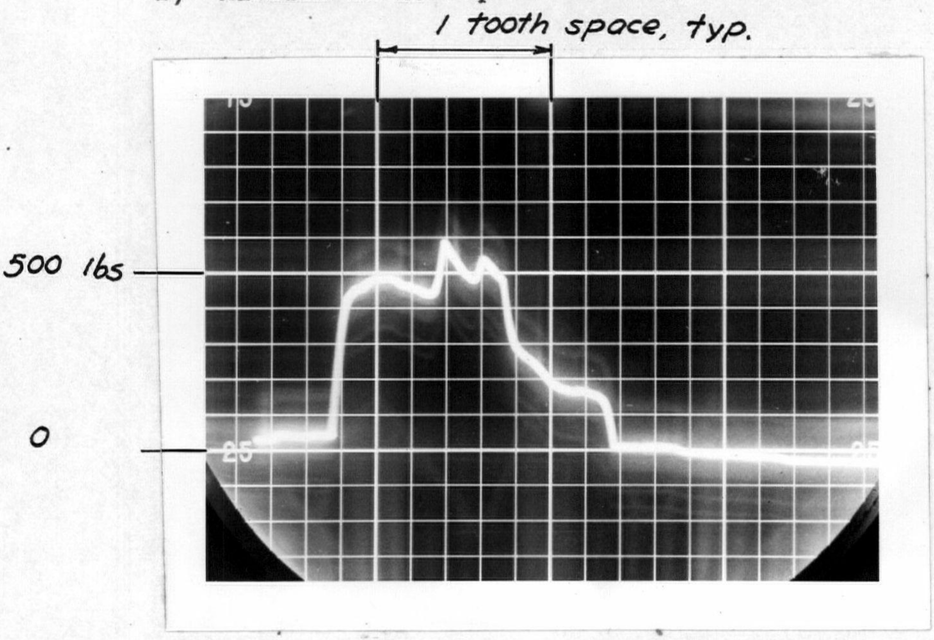
Fig. 23c Shaft torque variation at a measured torque of 1550 in-lb at high speed. (Test No.18)

24
25
9

3. The tooth marked A on the driven gear meshed with the tooth C on the driving gear.

The results of this test were very similar to the first test series in which both "A" teeth were meshed together. One typical recording for each run is shown in the following diagrams.

a) Machine at low speed



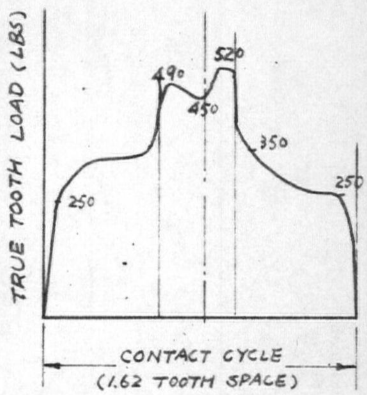
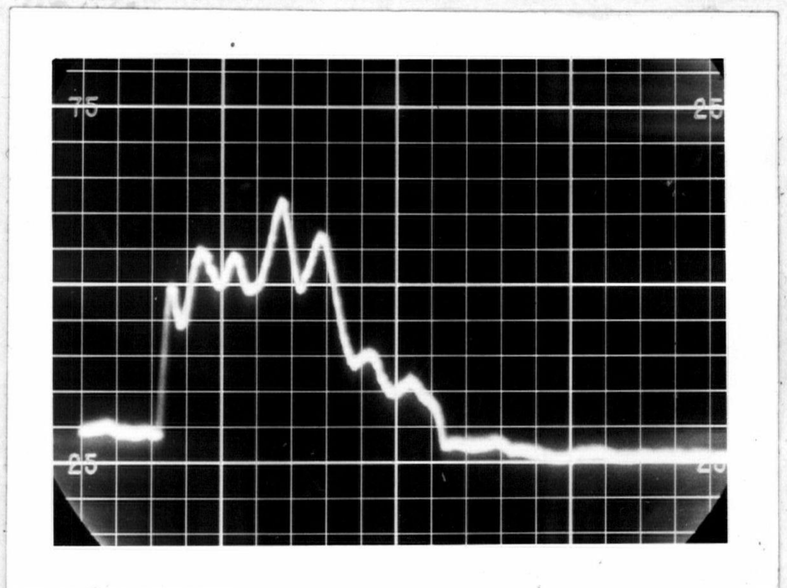


Fig.24a Tooth load pattern at a measured torque of 1800 in-lb at low speed-double scale. (Test No. 20)

Fig.24b Force diagram of Fig. 24a

b) Machine at high speed.



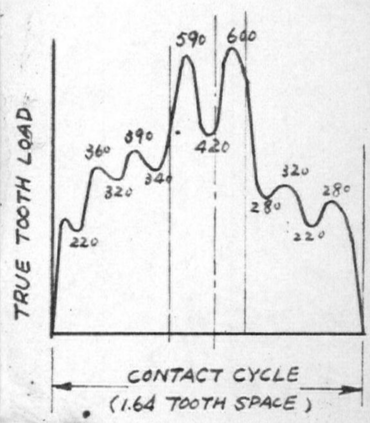


Fig. 25a Tooth load pattern at a measured torque of 2220 in-lb at high speed-double scale (Test No. 21

Fig. 25b Force diagram of Fig. 25a. (frequency 5100 cps)

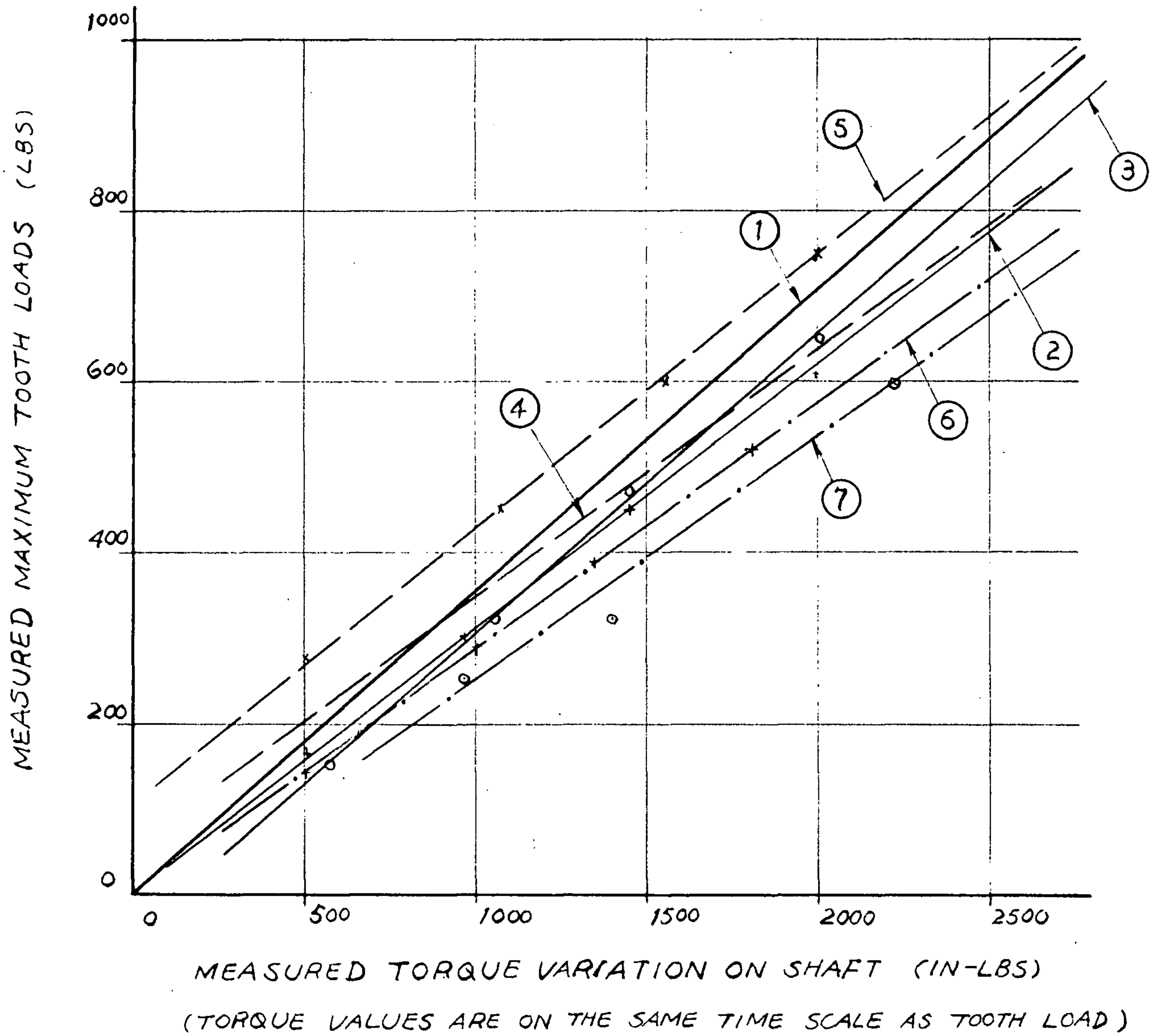


Fig 26 ANALYSIS OF OSCILLOGRAPH RECORDS WITH THE SECOND PAIR OF NRC GEARS (20-DEG. PRESSURE ANGLE)

Curve 1	Theoretical tooth load curve.
Curve 2	Tooth 'A' meshed with tooth 'A' at low speed.
Curve 3	Tooth 'A' meshed with tooth 'A' at high speed.
Curve 4	Tooth 'A' meshed with tooth 'B' at low speed.
Curve 5	Tooth 'A' meshed with tooth 'B' at high speed.
Curve 6	Tooth 'A' meshed with tooth 'C' at low speed.
Curve 7	Tooth 'A' meshed with tooth 'C' at high speed.

CHAPTER V

COMPARISON OF CALCULATED DYNAMIC LOADS AND ACTUAL MEASURED RESULTS

A. Buckingham's Formula[8]

Earle Buckingham developed a general formula for calculating the dynamic load on the spur gear tooth. This formula covers average conditions rather than specific ones, particularly where the values of the many factors are unknown or indeterminate.

Variable	Values for Test Gears
m_p = mass effect of pinion at R_1	
m_g = mass effect of gear at R_2	
R_1 = pitch radius of pinion	= 3 in.
R_2 = pitch radius of gear	= 3 in.
N_1 = number of teeth in pinion	= 24
N_2 = number of teeth in gear	= 24
P = circular pitch	$= \frac{\pi}{P_d} = \frac{\pi}{4}$
F = face width of gears	= 1 in.
V = pitch line velocity ft/min (See Table I)	
C_m = mass factor, which depends on the pitch-line velocity (See Table I)	

$$m_p = \frac{P^2 N_1 F}{64} \left[\frac{.002\ R_1^2 + 1}{.001\ R_1^2 + 1} \right] = 0.233$$

$$m_g = \frac{P^2 N_2 F}{64} \times \left[\frac{.002\ R_2^2 + 1}{.001\ R_2^2 + 1} \right] = 0.233$$

Therefore, the effective mass at the pitch line of the gear is:

$$m = C_m \left[\frac{m_p\ m_g}{m_p + m_g} \right] = 0.1166\ C_m$$

TABLE I Mass Factors - Values of C_m (See Reference 8)

Gear Speed		C_m
RPM	Pitch-line velocity ft/min	
55	86.5	8.448
56	88.0	8.420
57	89.5	8.393
58	91.1	8.364
59	92.7	8.336
60	92.4	8.307
1775	2786	1.327
1780	2796	1.325
1785	2804	1.323
1790	2811	1.321
1795	2820	1.320
1800	2825	1.319

Let W = total applied load, lbs.

$$= \frac{33{,}000\ hp}{V} = \frac{\text{applied torque}}{\text{pitch circle radius}}$$

C = deformation factor, depending upon tooth error and element of gears (See Table II)

TABLE II Values of Deformation on Factor C (See Reference 8)

Material	Tooth Form	Error in Action (in)					
		.0005	.001	.002	.003	.004	.005
Steel & Steel	14 ½ deg.	800	1600	3200	4800	6400	8000
	20° Full Depth	830	1660	3320	4980	6640	8300

Then

$$f_1 = .0025 \left[\frac{R_1 + R_2}{R_1 R_2}\right]^2 \cdot mV^2 = .000129\ C_m V^2$$

$$f_2 = FC + W$$

$$f_a = \frac{f_1\ f_2}{f_1 + f_2}$$

The total dynamic load is

$$W_d = W + \sqrt{f_a(2f_2 - f_a)}$$

1. First pair NRC Spur Gears:

Assuming the gears were "precision gears," the tooth error is .0006 inch, therefore from Table II, C = 996.

TABLE III Data for Evaluating dynamic loads.

a) Machine at low speed (Test No. 5)

Meas. Torque in-lb	Meas. rpm	V ft/min	C_m	m	f_1	w lb	f_2	f_a	W_d lb	Meas. max. tooth loads lb
1950	56	88	8.420	.982	8.44	650	1646	8.40	816	725
1500	56	88	8.420	.982	8.44	500	1496	8.39	658	575
1100	57	89.5	8.393	.978	8.70	367	1363	8.65	520	410
410	58	91.1	8.364	.975	9.00	137	1133	8.92	280	190

b) Machine at high speed (Test No. 6)

2200	1775	2786	1.327	.1547	1334	733	1730	753	2160	750
1600	1780	2796	1.325	.1545	1338	533	1530	712	1840	570
930	1780	2796	1.325	.1545	1338	310	1305	660	1445	310

Table III shows that the measured maximum tooth loads at high speed are very similar to those at low speed, and in both speeds the measured tooth loads are slightly higher than the static load, w, determined from the transmitted torque for the same applied torque. The measured tooth load in the tests at low speed is nearly 11% greater than the static load. At high speed, the measured tooth loads are very close to the static load. Therefore, in these tests the dynamic loads may actually be neglected. The value of the dynamic loads calculated by Buckingham's formula at low speed showed a 12.5% larger value than the measured load at a torque of 1950 in-lb and were 47% greater at 410 in-lb. At high speed, W_d, is 190% larger than the measured load at a torque of 2200 in-lb and increased to 360% at 930 in-lb.

TABLE IV The experimental results of the first pair of NRC gears.

Maximum eccentricity of one gear ~~is~~ meshed with the minimum eccentricity of the other.

Machine at low speed (Test No. 5)

Meas. Torque in-lb	Meas. Max. tooth load lbs	Length of contact cycle in tooth space	Torque at nominal value	Equivalent Max. tooth load.
1950	725	1.59	2000	744
1500	575	1.53	1500	575
1100	410	1.46	1000	373
410	190	1.34	500	232

Machine at high speed (Test No. 6)

2200	750	1.61	2000	682
1600	570	1.44	1500	535
930	310	1.60	1000	334
--	--	--	500	--

Maximum eccentricities of both gears meshed together.

Machine at low speed (Test No. 8)

2020	790	1.55	2000	782
1520	600	1.55	1500	593
1050	460	1.49	1000	438
550	260	1.45	500	236

Machine at high speed (Test No. 7)

2320	900	1.57	2000	775
1590	610	1.56	1500	575
1100	450	1.57	1000	410
640	300	1.40	500	234

2. Second pair of NRC Gears:

This pair of NRC gears were "precisely ground gears" with an estimated tooth error .0006 inches, Table II gives C = 996.

TABLE V Data for evaluating dynamic loads.

a) Machine at low speed (Test No. 16)

Meas. Torque in-lb	Meas rpm	V ft/min.	C_m	m	f_1	W	f_2	f_a	W_d lb	Meas. Max.Tooth Loads lb.
1980	56	88.0	8.420	.982	8.44	660	1656	8.39	826	610
1450	57	89.5	8.393	.978	8.70	483	1480	8.65	643	450
960	57.5	90.3	8.379	.977	8.85	320	1316	8.80	472	300
500	58	91.1	8.364	.975	9.00	167	1163	8.94	310	165

b) Machine at high speed (Test No. 17)

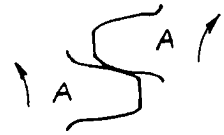
2000	1780	2796	1.325	.1545	1338	667	1663	742	2050	650
1450	1785	2804	1.323	.1543	1342	483	1480	703	1740	470
1050	1790	2811	1.321	.1541	1347	350	1346	673	1517	320
570	1790	2811	1.321	.1541	1347	190	1186	630	1200	150

The test results indicate the interesting fact that not only are dynamic loads non-existent, but also that the measured maximum tooth loads are slightly lower than the static loads. Reswick's[10] experiments demonstrated that in many heavily loaded gears the total dynamic load may be actually less than the static load determined from the transmitted power. Therefore, it would appear that there is existing evidence in support of the present results.

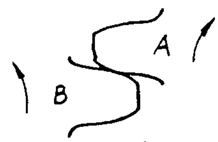
The calculated dynamic loads, W_d, by Buckingham's formula, are larger than the measured loads at low speed. The percentage of larger value from a torque of 1980 in-lb to 500 in-lb are 35%, 43%, 57% and 88% respectively. For the machine at high speed, the deviations of the experimental results from the Buckingham predictions are even greater. The theoretical values of W_d are greater than the measured loads by the following percentages 215%, 270%, 375% and 750% progressing from the highest to the lowest torque value respectively.

TABLE VI The Experimental Results of the second pair of the NRC Gears.

Machine at low speed (Test No. 16)

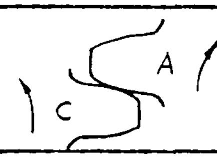


Meas. Torque in-lb	Meas.max. tooth load lbs	Length of contact cycle in tooth space.	Torque at nominal value	Equivalent max. tooth load. lbs.
1980	610	1.63	2000	616
1450	450	1.615	1500	465
960	300	1.62	1000	313
500	165	1.61	500	165
Machine at high speed (Test No. 17)				
2000	650	1.63	2000	650
1450	470	1.61	1500	485
1050	320	1.59	1000	305
570	150	1.59	500	132



Low Speed (Test No. 19)

2000	600	1.63	2000	600
1450	480	1.62	1500	496
1000	350	1.61	1000	350
510	200	1.60	500	196
High Speed (Test No. 18)				
2000	750	1.64	2000	750
1550	600	1.60	1500	580
1070	450	1.60	1000	420
500	275	1.58	500	275



Low Speed (Test No. 20)

1800	520	1.62	2000	578
1300	390	1.61	1500	450
1000	290	1.62	1000	290
500	140	1.58	500	140
High speed (Test No. 21)				
2220	600	1.64	2000	540
1400	320	1.61	1500	343
960	250	1.60	1000	260
480	—	--	500	--

3. Canadian Summer Iron Works Ltd.

Involute Spur Gears (14 ½ deg. Pressure Angle)

Assuming these gears were "First class commercial gears," the error in action is 0.0026 inch, and from Table II, C = 4160.

TABLE VII Data for Evaluating Dynamic Loads.

a) Machine at low speed (Test No. 9)

Meas. Torque in-lb	Meas. rpm	V ft/min	C_m	m	f_1	W lb	f_2	f_a	W_d lb	Meas. max. tooth loads lbs.
2000	56	88.0	8.420	.982	8.44	667	4825	8.43	950	1000
1450	57	89.5	8.393	.978	8.70	483	4643	8.69	765	710
1050	57.5	90.3	8.379	.977	8.85	350	4510	8.84	630	490
560	58	91.1	8.364	.975	9.00	187	4347	8.89	465	240

b) Machine at high speed (Test No. 10)

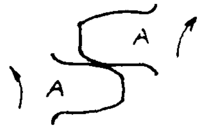
1620	1795	2820	1.320	.154	1360	540	4700	1050	3500	1450
1500	1795	2820	1.320	.154	1360	500	4660	1050	3445	1250
1000	1800	2825	1.319	.1538	1364	333	4500	1045	3220	1350

In these tests the dynamic load was 50% greater than the static load at low speed and nearly 160% of the static load at high speed. Thus the dynamic loads can no longer be neglected. The Buckingham formula gives rather close results at low speed, but gives a predicted value 250% larger than the measured load at high speed.

A further analysis of the Canadian Summer gears will be made in the following pages.

TABLE VIII The Experimental Results of the Canadian Sumner Gears.

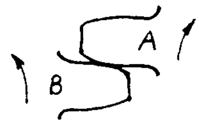
Machine at low speed (Test No. 9)



Meas. Torque in-lb	Meas.Max. tooth load lbs.	Length of contact cycle in tooth space	Torque at nominal value in-lb	Equivalent max. tooth load. lbs.
2000	1000	1.08	2000	1000
1450	710	1.08	1500	735
1050	490	1.04	1000	470
560	240	1.04	500	214

High speed (Test No. 10)

1620	1450	1.10	2000	1790
1500	1250	1.10	1500	1250
1000	1350	1.07	1000	1350
—	—	—	500	—

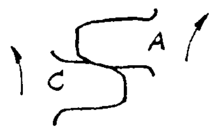


Machine at low speed (Test No. 12)

2000	1020	1.10	2000	1020
1500	765	1.09	1500	765
900	450	1.08	1000	500
450	210	1.07	500	234

High speed (Test No. 11)

2050	1200	1.07	2000	1170
1500	1300	1.07	1500	1300
1000	1500	1.07	1000	1500
—	—	—	500	—



Machine at low speed (Test No. 13)

1920	910	1.12	2000	950
1400	650	1.10	1500	700
1000	430	1.09	1000	430
450	230	1.08	500	255

High speed (Test No. 14)

2150	1300	1.12	2000	1200
1300	1200	1.12	1500	1380
1000	900	1.08	1000	900
—	—	—	500	—

Analysis of Load Cycle of Canadian Sumner Gears.

For purposes of calculating the response of the system to a force impulse function, consider an undamped linear single-degree of freedom system subjected to a load which is gradually applied and then maintained indefinitely[9], (Fig. 27b).

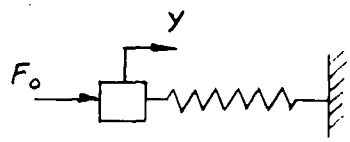


Fig. 27a

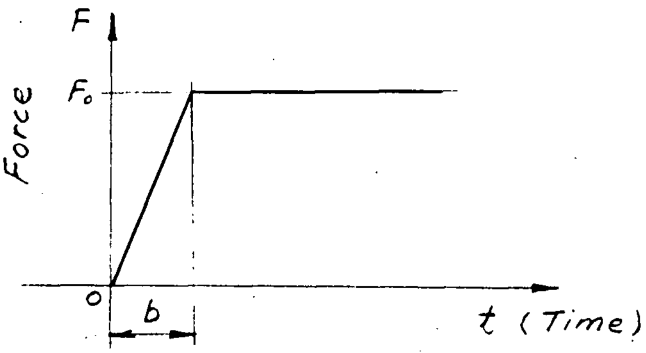


Fig. 27 b

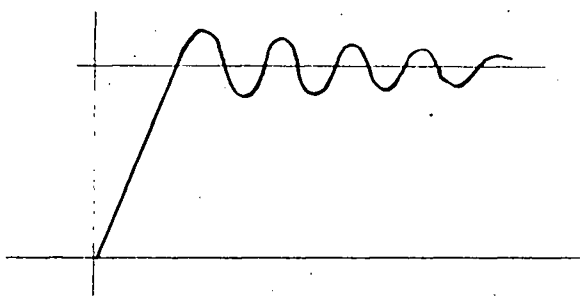


Fig. 27 c

If the applied force is $f(t)$, the equation of motion becomes

$$m \frac{d^2y}{dt^2} + K y = f(t)$$

where $f(t) = \frac{Fo\ t}{b}$ for $0 < t < b$

$= 0$ for $t < 0$

If $\omega_n = \sqrt{\frac{K}{m}}$

where ω_n = undamped angular frequency,

K = spring constant

m = mass of the system having a forced vibration.

$$\text{then } \frac{d^2y}{dt^2} + \omega_n^2 y = \frac{f(t)}{m} \tag{1}$$

If at time t = 0, the initial displacement $f(0)$, and the initial velocity $f'(0)$ are zero, then by method of Laplace Transformation Eq. (1) becomes

$$y(t) = \frac{F_o}{mb\,\omega_n^3} \left\{ \omega_n t - \sin \omega_n t \right\}$$

$$= \frac{F_o}{K} \left\{ \frac{t}{b} - \frac{1}{\omega_n b} \sin \omega_n t \right\} \quad \text{when } 0 \leq t \leq b \tag{2}$$

$$y(t) = \frac{F_o}{mb\,\omega_n^3} \left\{ (\omega_n t - \sin \omega_n t) - \left[\omega_n (t-b) - \sin \omega_n (t-b) \right] \right\}$$

$$= \frac{F_o}{mb\,\omega_n^3} \left\{ \omega_n b + \sin \omega_n (t-b) - \sin \omega_n t \right\}$$

$$= \frac{F_o}{K} \left\{ 1 + \frac{1}{\omega_n b} \left[\sin \omega_n (t-b) - \sin \omega_n t \right] \right\} \qquad (3)$$

when $t \geqslant b$

$\frac{F_o}{K}$ represents the static deflection of the system under the action of a steady force F_o. Letting $\frac{F_o}{K} = Y_{st}$, Equations (2) and (3) can then be written to express the results in terms of a magnification or response factor $y(t)/y_{st}$.

$$\frac{y(t)}{y_{st}} = \frac{t}{b} - \frac{1}{\omega_n b} \sin \omega_n t \qquad \text{for } 0 \leqslant t \leqslant b \qquad (4)$$

$$\frac{y(t)}{y_{st}} = 1 + \frac{2}{\omega_n b} \operatorname{Cos} \omega_n \left(t - \frac{b}{2}\right) \operatorname{Sin} \frac{\omega_n b}{2} \quad \text{for } t \geqslant b \qquad (5)$$

The maximum value of the response ratio is defined as the dynamic load factor.

Then Dynamic Load factor $= 1 + \frac{2}{\omega_n b} \sin \frac{\omega_n b}{2}$ (6)

Fig. 14b is the high speed tooth load pattern of the Canadian Sumner gears at a measured torque of 1620 in-lb

ad - length of tooth load contact cycle

$= \frac{1.13''}{1''} = 1.13$ tooth space.

bc - length of one tooth carrying the load

$= 0.95$ tooth space

ab - the length for the load transferring to the tooth, or the

time interval $= \frac{60 \times .11}{1780 \times 24} = .000275$ sec.

cd - the length for the load drop or the time interval

$$= \frac{60 \times .07''}{1780 \times 24} = 0.000175 \text{ sec.}$$

The calculated natural frequency of the vibration system formed by the gear and pinion and the mating teeth is about 3300 c/sec. The formula derived by W.A. Tuplin[13] is as follows:

$$f = \frac{1}{2\pi} \sqrt{k\left(\frac{r_1^2}{I_1} + \frac{r_2^2}{I_2}\right)}$$

and

$$\frac{r_1^2}{I_1} = \frac{1}{M_1} \qquad \frac{r_2^2}{I_2} = \frac{1}{M_2}$$

where

f	=	Natural frequency	c/sec.
k	=	Equivalent teeth stiffness	lb/in.
r_1, r_2	=	Pitch radius	in.
I_1, I_2	=	Moment of inertia of rotating masses about their axes of rotation	lb - in - sec^2.
M_1, M_2	=	Mass of gear	lb - sec^2/in.

In the formula all rotating masses are ignored except the gears themselves, and each gear is regarded as a ring with metal thickness under the teeth equal to the pitch of the teeth and radius of gyration as equal to the pitch radius.

The measured frequency of the tooth load pattern in Fig. 14b is 2860 cycles/sec, which agrees qualitatively with the calculated frequency.

Therefore:

$$\omega_n = 2\pi f = 2\pi \times 2860 \text{ rad/sec.}$$

$$b = 0.000275 \text{ sec} \quad \text{(length ab in Fig. 14b).}$$

$$\frac{\omega_n b}{2} = 1/2 \times 2\pi \times 2860 \times 0.000275 = 2.47 \text{ rad.}$$

$$\text{Dynamic load factor} = 1 + \frac{2}{\omega_n b} \times \sin\left(\frac{\omega_n b}{2}\right)$$

$$= 1.252$$

The steady force Fo from consideration of the torque is:

$$\frac{1620}{3 \cos 14\tfrac{1}{2}^\circ} = 560 \text{ lbs.}$$

then

$$F \max. = Fo \left\{ 1 + \frac{2}{\omega_n b} \times \sin\left(\frac{\omega_n b}{2}\right) \right\}$$

$$= 560 \times 1.252 = 700 \text{ lbs.}$$

Comparing the value of $F_{max.}$ found above with the measured maximum tooth load (1450 lbs), there is no satisfactory agreement.

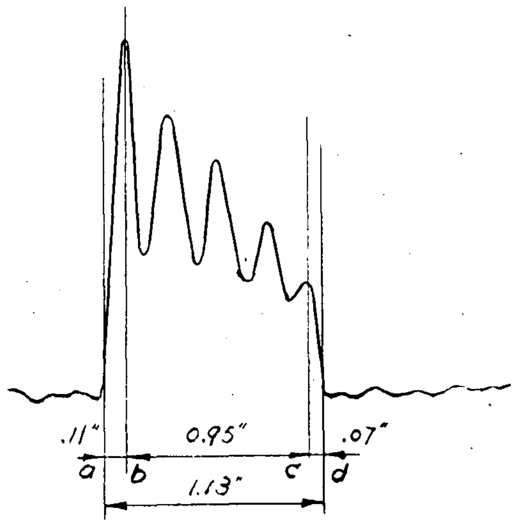


Fig. 14b Tooth load pattern at a measured torque of 1620 in-lb at high speed (Test No. 10).

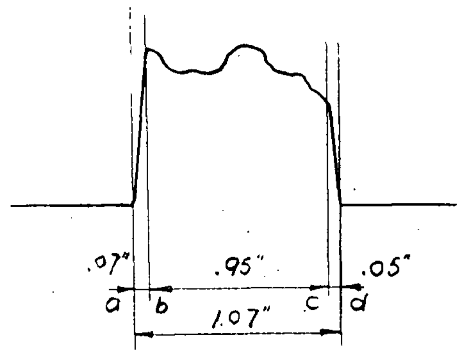


Fig. 13b Tooth load pattern at a measured torque of 2000 in-lb at low speed (Test No. 9).

For analyzing the tooth load pattern of the Canadian Sumner Gears at low speed, a particular pattern at a measured torque of 2000 in-lb is shown in Fig. 13b.

ad - length of tooth load contact cycle

= 1.07"/1" = 1.07 tooth space.

bc - one tooth carrying the load

= 0.95 tooth space

ab - length for the load transferring

to the tooth, or the time interval

$= \frac{60 \times .07}{57 \times 24} = .00307$ sec.

cd - the length for the load drop or the time interval $= \frac{60 \times .05}{57 \times 24}$

= .0022 sec.

Taking the natural frequency as 2860 rad/sec

then

$$\omega_n = 2\pi f = 2\pi \times 2860 \text{ rad/sec} \qquad b = .00307 \text{ sec.}$$

$$\frac{\omega_n b}{2} = 1/2 \times 2\pi \times 2860 \times .00307 = 27.6 \text{ rad.}$$

The steady force $F_o = \dfrac{2000}{3 \cos 14\frac{1}{2}^o} = 690$ lbs.

The maximum force is therefore:

$$F_m = F_o \left\{ 1 + \frac{2}{\omega_n b} \sin \frac{\omega_n b}{2} \right\} = 865 \text{ lbs.}$$

The value of the maximum force found above (865 lbs) agrees fairly well with the corresponding measured maximum tooth load (1000 lbs).

CHAPTER VI

CONCLUSIONS

A. Accurately Cut Gears

The test results indicate that in accurately cut gears the maximum load between teeth is not appreciably increased by dynamic effects. The load is shared by the teeth during the beginning and end of the contact cycle, and the period of load engagement of a given pair of teeth is quite close to the theoretical value.

At high speeds, vibration is set up when the load is transferred to a single tooth, the frequency of load vibration being that of the natural frequency of the tooth system. This vibration is the cause of the dynamic load effect. The test results show that the increase in tooth load due to this effect is not very large.

It appears that the Buckingham dynamic load equation greatly overestimates the additional load due to dynamic effects.

In analyzing the dynamic tooth load, Buckingham assumed that a single pair of teeth are carrying the load at the critical phase of the load transfer. Hence, the Buckingham formula always gives larger values than actual measurements. In this connection D. W. Dudley[6] commented: "Even though the Buckingham method may not always give an answer that will agree with test data, it is still the best method available at present".

B. Form Cut Gears

In form cut gears the tips and flanks of the teeth depart from the involute profile, and only about the middle third of the teeth is actually of involute profile. As a result of this, the teeth do not share the load, but only one pair of teeth carry the load at a time. There is thus a sudden transfer of load from one tooth pair to the other. At high speed this sudden transfer results in a severe tooth load vibration. The added load due to dynamic effects is thus much larger than with accurately cut gears.

Even though the dynamic load effects in these gears are larger than with accurately cut gears, the Buckingham dynamic load equation still overestimates the value of the dynamic load.

C. <u>General Conclusions</u>

The tooth profile shape and the accuracy in manufacture, i.e. the contact ratio, have profound influence in the dynamic incremental forces.

Buckingham's equation always overestimates the value of the dynamic tooth force especially in the higher speed cases.

The frequency of the dynamic load variation is close to the natural frequency of the vibration system formed by the gear and the pinion as masses and the mating teeth as a spring.

The dynamic tooth load are linearly proportion to the applied torque. Increasing the applied torque by 100%, the measured tooth loads also increase about 100%.

With a small applied torque, the length of the tooth contact cycle decreases slightly, more vibration effect occurs and the maximum tooth load is maintained for a longer period.

In accurately cut gears, the speed does not effect the dynamic tooth forces. The maximum tooth loads at high speed tests (approximately 2800 fpm pitch line velocity) are very close to those at low speed tests (approximately 90 fpm pitch line velocity). The dynamic tooth loads are very small and may be ignored.

The question of dynamic loads on gear teeth has produced a large number of isolated observations and analyses. Much needs to be done in this field both analytically and experimentally, before a clear and exact picture as well as reliable predictions can be made of the complex dynamic load phenomena.

APPENDIX

INSTRUMENTATION

I. BAM-1 BRIDGE AMPLIFIER AND METER

The BAM-1 Bridge Amplifier Meter is an instrument which can be utilized for both dynamic and static work with SR-4 strain gages, Statham pickups and similar resistance type transducers. The DC stability of the meter is within 100 microvolts per hour. Frequency response is flat within 5% from DC up to 25,000 cps and slowly drops off at higher frequencies. The DC transistor pre-amplifier has a gain of 150.

For static measurements the meter on the BAM-1 reads tension to the right and compression to the left of zero in the middle. The scales are arbitrary with a simple system of calibration to make them read directly in terms of units being measured. The bridge circuit is initially balanced with the meter at zero. Thermoelectric effects are balanced out. For dynamic measurements the BAM-1 Bridge Amplifier meter was coupled to a DuMont 350 DC Cathode-ray Oscilloscope.

II. DU MONT TYPE 350 CATHODE-RAY OSCILLOGRAPH

The Du Mont Type 350 Cathode-ray Oscillograph is an instrument that can be used for many general applications. The high gain of this instrument assures that it can be used directly with many types of transducers, while DC amplification provides that the lowest frequency portions of signals will be faithfully reproduced.

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