A STUDY OF FRICTION INDUCED VIBRATION

by

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Department of Mechanical Engineering,
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Date: May, 1962.
Frictional vibrations, which occur when two solid bodies are rubbed together, are analyzed mathematically and observed experimentally. In the mathematical analysis, the non-linear differential equation of motion during the slip period is derived making use of the experimental friction-velocity curve. A qualitative graphical solution of this differential equation of motion is presented to illustrate the general form and behavior of the motion. The experimental friction-velocity curve is then linearized allowing the differential equation of motion to undergo standard analytical solution. The experimental investigations were carried out using unlubricated steel surfaces and six different supporting systems. The experiments were confined to sliding in the negative slope region of the friction curve for the particular surfaces used. The effects of load, stiffness and velocity of the translating surface are considered and the results suggest that the decay of the vibrations, as the speed of the moving surface is increased, corresponds in form to the friction-velocity curve for the surfaces used. Using the original analytical relationship describing the shape of the negative slope region of the friction curve, the theoretical results are altered accordingly. Good correlation is obtained between the analytical results and the experimental observations.
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\( F_{K} \)  force of kinetic friction at sliding speeds greater than \( \mathcal{U}_{M} \), (lbs.)

\( F_{M} \)  minimum force of friction, (lbs.).

\( F_{S} \)  force of static friction, (lbs.).

\( \Delta F \)  difference between \( F_{S} \) and \( F_{M} \), (lbs.).

\( K \)  stiffness of the elastic system, (lbs./in.).

\( m \)  mass of the vibrating parts, (lbs. sec.\(^2\)/in.).

\( r \)  structural damping coefficient of the elastic system, (lbs. sec./in.)

\( R \)  resultant effective viscous damping coefficient, (lbs. sec./in.).

\( s_{N} \)  slope of the negative slope portion of the friction-velocity curve (secs./in.).

\( s_{P} \)  slope of the positive slope portion of the friction-velocity curve, (secs./in.).

\( t \)  time, (secs.).

\( t_{K}, t_{S} \)  time of the slip and stick periods respectively, (secs.).

\( T \)  period of the motion, (secs.).

\( \mathcal{U} \)  relative velocity of sliding, (in./sec.).

\( \mathcal{U}_{M} \)  that value of relative velocity of sliding that produces a minimum coefficient of friction, (in./sec.).

\( V \)  table velocity, (in./sec.).

\( V_{C} \)  critical table velocity, (in./sec.).

\( W \)  total normal load between the surfaces, (lbs.).

\( x, \dot{x}, \ddot{x} \)  displacement, velocity and acceleration of the slider respectively.

\( x_{B}, x_{S} \)  displacement of the slider due to the breakaway and static coefficients of friction respectively, (in.).

\( x_{M} \)  displacement of the slider due to the minimum coefficient of friction, (in.).
LIST OF SYMBOLS (cont'd)

\( x_{\text{MIN}} \): the minimum displacement of the slider, (in.).

\( \alpha \): amplitude of vibration, (in.).

\( \psi \): the dimensionless parameter: \( \frac{\Delta F}{m \sqrt{\omega_d}} \)

\( \phi \): the dimensionless damping parameter: \( \frac{\nu}{\sqrt{1 - \nu^2}} \)

\( \mu, \mu_s \): coefficient of friction, and coefficient of static friction respectively.

\( \mu_b, \mu_k, \mu_m \): breakaway, kinetic and minimum coefficients of friction respectively.

\( \mu_k \): is the coefficient of friction at sliding speeds greater than \( \mu_m \).

\( \nu \): combined damping ratio.

\( \nu_1 \): damping ratio for the elastic system.

\( \nu_2 \): damping ratio for the surfaces.

\( \omega \): natural circular frequency of the supporting system, (rad./sec.)

\( \omega_d \): damped natural circular frequency of the supporting system, (rad./sec.).
ACKNOWLEDGEMENT

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CHAPTER ONE

1. INTRODUCTION

2. FRICTIONAL VIBRATIONS - A LITERATURE SYNOPSIS

3. SUMMARY
1. INTRODUCTION

If one of two surfaces in frictional contact is driven slowly forward while the other is elastically suspended to a fixed position, it is found that sliding is not continuous, but rather proceeds with a series of "sticks" and "slips".

In position-control servomechanisms operating at creep speeds, the presence of these self-excited frictional vibrations, apart from causing increased wear, destroys the accuracy and sensitivity of any final positioning movement.

The purpose, therefore, of this investigation is to study not only the form of the vibration but the influence of various parameters defining the supporting system, on the stability of the frictional vibrations. It is hoped that the results, presented in the form of functional relationships, will be of general value in design.

The investigation was carried out using clean, unlubricated flat steel surfaces, the preparation of which is carefully defined. The driven surface was moved with pure translation thus eliminating any rotational effects.

 Provision was made in the experimental apparatus for variation in load, stiffness, and velocity of the driven surface.
2. FRICTIONAL VIBRATIONS - A LITERATURE SYNOPSIS

Early investigations on the sliding of bodies at low velocities under boundary friction revealed that the motion may not be a continuous process. Lord Rayleigh (1)* discusses briefly the motion of the violin string under the action of the bow, but offers no detailed development of the motion since "some of the details were still obscure". In 1929, Wells (2), in an effort to measure kinetic boundary friction at low speeds, discovered an unstable region where "alternate sticking and slipping took place under certain circumstances". No very serious attempt was made at that time to explain the behavior in this region.

Thomas (3) studied frictional vibrations employing analytical and graphical techniques. The differential equation of motion of the vibrating slider was solved and various cases were studied. On the phase plane, Thomas shows the trajectories representing the solution of the equation of motion for the simple harmonic case with no damping. Under lubricated conditions a viscous damping term is added to the differential equation and the phase plane representation is then altered accordingly.

Kaidanovsky and Haikin (4) studied frictional vibrations in sliding systems having friction varying with velocity, and they observed that a necessary condition for such vibrations is the existence of a region in which the friction decreases as the velocity increases. In such circumstances equilibrium is unstable. As a matter of interest they point out that the relation between friction and speed for hydrodynamic lubrication possesses a

* Numbers in brackets designate references which are listed in the Bibliography. A Supplementary Bibliography is included for completeness.
narrow region where friction decreases with increasing velocity. In the narrow region representing boundary lubrication the slope of the curve is negative while in the hydrodynamic region the slope is positive. They suggest that equilibrium is unstable in the boundary or negative slope region while sliding in the hydrodynamic region is inherently stable.

Blok (5) confirmed that frictional relaxation oscillations depend upon the particular shape of the friction-velocity curve. His analytical treatment is based on the linearized friction-velocity curve and he extends his analysis to show that the oscillations depend on the amount of damping in the supporting system. He develops a dimensionless parameter,

\[ \psi = \frac{\sqrt{UK_m}}{F_s} \]

and shows that frictional vibrations will not occur in a sliding system if the damping ratio, \( \nu \), is larger than some critical value depending on the magnitude of \( \psi \). If the damping in the system is equal to the critical value, then he shows that the system hovers between vibration and smooth sliding.

Bowden and Leben (6) carried out a series of experiments on the friction between sliding metals in the absence of a lubricating film. They show that the frictional force does not remain constant during sliding and that the process may not be continuous. Sliding may proceed with a series of "sticks" and "slips". They suggest that friction between metals can be attributed in part to localized cold welding processes. Since isolated asperities carry the load, the result is that excessive local pressures are established thus giving rise to cold welding or adhesion (20). During their experiments, simultaneous measurements were taken of the surface temperature. This revealed large fluctuations in temperature, and, at the instant of slip
there was a sudden temperature "flash". Thus in addition to the cold welding process they suggest that hot welding processes also exist which contribute to the surface damage.

Further quantitative investigations as to the peak temperatures achieved during slip were carried out by Morgan, Muskat and Reed (7).

Dudley and Swift (8) employed Liénard's method of graphical construction of integral curves in their analysis of frictional vibrations. The advantage in this graphical approach is that direct use is made of the experimental friction-velocity curve. It is not necessary to express this curve analytically or to linearize it. The graphical construction is applied to various shapes of experimental friction curves and the results show that under certain conditions oscillations build up or persist while in other cases they decay. The method serves to illustrate the solution of the governing differential equation of motion previously introduced by Blok (5).

In 1948, Bristow (9) confirmed that the existence of the negative slope region was a necessary condition for the excitation of frictional vibrations. His observations indicated that the motion is dependent on the relative magnitude of the damping coefficient of the supporting system. During the "stick" period, slight relative movement between the two surfaces was noted which resulted in a decrease in the amplitude of vibration as the velocity of the moving surface increased. However, no experimental relationship was presented to show the form of this amplitude decrease.

Rabinowitz (10) found that two surfaces undergo slight displacement before the static friction coefficient falls to some kinetic value. From his experiments, which were non-relaxation tests, it was noted that the
static friction persisted for a displacement of the order of 40 microinches. Wide variation in this figure could exist depending upon the general surface conditions. As previously suggested by Bowden, when metal surfaces are in contact, plastic flow occurs and metallic junctions are formed. Rabinowitz suggested that preliminary displacement is the yield of the junctions prior to their shearing. A junction of somewhat greater strength is formed after a longer period of rest, the process being analogous to creep. This results in increased crosssectional area of the junctions which then require larger shearing forces. Thus it appears that the coefficient of static friction becomes a function of the "time of rest".

Accordingly in 1953, Burwell and Rabinowitz (11) studied the influence of this "time of rest" on the strength of junctions. It was concluded from the experiments that a finite time is required for a junction to reach its full strength. There are however, some principle features of this strength increase that are still obscure.

The effects of frictional vibrations in servomechanisms is discussed by H. Lauer (12). He describes the difficulties in the operation of a simple positioning servo arising from non-linear friction effects. The non-linear differential equation describing these effects is studied by means of graphical phase plane displacements.

Derjaguin, Push and Tolstoi (13) have presented a very detailed theoretical analysis based on Blok's earlier work. Linearization of the experimental friction-velocity curve was necessary in order to solve the differential equation of motion during slip. However, they fail to adequately define the linearized friction-velocity curve upon which their analysis is based. Boundary conditions, representing the region of systems for which
frictional vibrations are impossible, are applied to the equations for velocity and acceleration. This results in a parameter $\phi$, which is similar in form to that developed by Blok.

\[
\phi = \frac{\Delta F}{\sqrt{\mu K_m}}
\]

where $\Delta F$ represents the difference between the static and kinetic forces of friction. In applying the boundary conditions they do not define whether or not the table velocity is in the positive or negative slope region of the friction-velocity curve.

In high precision machines operating under servo-control, it is necessary to minimize the value of critical velocity in order to provide sensitive response. Following the theoretical analysis of Derjaguin, Push and Tolstoi (13), Singh (14), (18) and (19) presents an experimental study of critical velocity. Effort is made to present the necessary conditions that must exist in the supporting system which will produce a minimum critical velocity. Concentrating on structural and surface damping ratios, results are presented in graphical form which serve to predict the magnitude of critical velocities for a wide range of damping. Singh concludes that to suppress or eliminate frictional vibrations it is necessary to:

1. Reduce the difference between the static and kinetic forces of friction.
2. Increase the stiffness-inertia ratio.
3. Increase the damping in the system.
3. **SUMMARY**

To make full use of the existing information on the mechanics of the phenomenon and to serve as a starting point for the present investigation, a concise summary will be included at this point.

1. The negative slope region of the friction-velocity curve is a necessary condition for the excitation of frictional vibrations in sliding systems.

2. The vibration depends on the magnitude of the damping in the system.

3. Slight relative adjusting movement between the surfaces during the "stick" period has been observed.

4. A preliminary displacement immediately prior to slip has been noticed by previous investigators and it has been suggested that this movement represents the yield of the junctions in shear.

5. To decrease the critical velocity for any given system it is necessary to:
   a) decrease the difference between the static and kinetic coefficients of friction.
   b) raise the stiffness of the supporting system.
   c) increase the damping in the system.
CHAPTER TWO

1. THEORETICAL ANALYSIS OF FRICIONAL VIBRATIONS
1. THEORETICAL ANALYSIS OF FRICTIONAL VIBRATIONS

Consider the general case of a slider of mass "m" restrained by an elastic system of elasticity "K" and in contact with a plane surface which is driven with a constant velocity "V". Such a system is shown diagrammatically in Fig. 1, in which "r" represents the coefficient of viscous structural damping of the supporting system.

![Diagram of sliding system](image)

**Fig. 1 Diagrammatic representation of the sliding system.**

The displacement "x" of the slider is measured relative to the position of the unstrained spring. If no relative motion exists between the slider and the lower surface, then the equation describing the motion of the slider may be written as:

\[ r \dot{V} + K \dot{V} = \mu_s W \]  

During the "stick" period equation 1 governs, and the displacement of the slider is represented by the portion of the curve labelled AB in Fig. 2. Up to point B, the force of static friction is capable of withstanding the sum of the spring and damping forces. Since the force of friction cannot exceed the static value, displacement of the elastic system beyond point B results in relative motion between the slider and the moving surface.
The form of the "stick slip" oscillation.

The slip period then occurs during which the slider moves rapidly from B to C. For many surfaces, the friction-velocity relationship takes the general form illustrated in Fig. 3. If relative motion exists between the surfaces the equation describing the motion of the slider can be written as:

\[ m \ddot{x} + r \dot{x} + Kx = \mu W \]

where \( \mu \) is the coefficient of friction between the surfaces, and varies as illustrated in Fig. 3.

For the case of light damping, comparatively large slip velocities are encountered as suggested by the slope of the curve BC in Fig. 2. The reason for these high velocities in the underdamped case is apparent from
Fig. 3 since the coefficient of friction achieves a low kinetic value very rapidly. This allows the term "Kx" in equation 2 to govern, returning the slider to point C in Fig. 2. At point C, the velocity of the slider becomes equal to the velocity of the moving surface and a "stick" period commences. The cycle ABC is repeated continuously, the motion being referred to as "stick-slip" sliding.

It should be mentioned that the damping is less than aperiodic, otherwise the displaced slider will drift asymptotically from point A of Fig. 2 back to a displacement corresponding to the velocity of the moving surface. Under these conditions further relaxation oscillations are impossible unless the velocity of the moving surface drops to zero momentarily. This would result in another transient followed by smooth sliding.

Since the differential equation of motion of the slider during slip (equation 2) is a second order non-linear equation, the motion can be represented graphically on a phase plane diagram which consists of a plot of velocity versus position. The general zero slope isocline method will be used to establish the phase plane plot (15). Scale restrictions do not permit the application of the method to accurate analysis of normal relaxation-type oscillations but qualitative use of the phase plane diagram does serve to illustrate the general behavior.

Rewriting the differential equation 2 in the form:

\[ \ddot{x} + \frac{K}{m} \dot{x} + \omega^2 x = \frac{\mu W}{m} \]

and letting \( \dot{x} = y \) we have:

\[ \dot{y} + \frac{K}{m} y + \omega^2 y = \frac{\mu W}{m} \]
For the zero slope isocline we set
\[ \frac{dy}{dt} = 0 \]
giving:
\[ \mu = \frac{\mu W}{K} - \frac{r \cdot y}{K} \]  \hspace{1cm} 3

Selecting various values of \( \dot{x} = y \) and obtaining the corresponding values of \( \mu \) from the experimental friction curve similar to that shown in Fig. 3, and substituting these values into equation 3 results in the zero slope isocline curve plotted on the phase plane in Fig. 4. Using Liénard's graphical construction the slopes of the trajectories of the differential equation can be drawn on the phase plane. The complete slip trajectory for the initial condition represented by point B can be drawn as shown by following, in a clockwise direction, the short slope lines in the integral field.

Fig. 4 is drawn for a table velocity in the negative slope region of the friction curve and this results in a well defined "stick-slip" oscillation of approximate amplitude AB. The effects of static friction becoming operative at point A virtually cut off the phase plane diagram and abruptly stops the spiralling trajectory. Fig. 5 shows a system operating in the positive slope region of the friction curve where the possibility exists for the spiralling trajectory to "miss" the vertical \( \dot{x} = V \) axis at some point D. Trajectories that do not encounter the axis \( \dot{x} = V \) are not influenced by the effects of static friction consequently further sticking is impossible and these trajectories spiral in to the stable point E. The value of \( \dot{x} \) at point E is zero so that smooth sliding results (13). The effects of structural damping in the system is described by the term \( -\frac{r \cdot y}{K} \).
in equation 3. Increasing the value of "r" produces a family of straight lines having increasing slope through the origin of the phase plane. Adding this component to the first term of equation 3 results in a zero slope isocline possessing a steeper positive slope on the phase plane. This in turn results in a more rapid spiralling in of the whole integral field to the stable point E.

Returning to Fig. 3, the friction-velocity curve consists of two portions; an exponentially-shaped portion at low $\mathcal{U}$ values, and a linear
Fig. 5 Graphical solution for the motion in the supercritical zone portion at higher $\mathcal{U}$ values. The friction-velocity curve can be expressed quite accurately by the relationship*:

$$\mu = (\mu_s - \mu_m) e^{-\frac{2 \mu_m}{u_m} \ln \left( \frac{\mu_s - \mu_m}{\partial \mu_m - \partial u_m} \right)} + s_p \mathcal{U} + \mu_m$$

Since

$$\mathcal{U} = \mathcal{V} - \mathcal{X}$$

* Refer to Appendix 1 for the derivation of this equation.
Substitution yields:

\[ \mu = (\mu_s - \mu_m) e^{-\frac{\mu_s - \mu_m}{\mu_s - \mu_m}} + A_p (V - \dot{V}) + \mu_m \quad (6) \]

Substitution of equation 6 into equation 2 produces a non-linear differential equation by virtue of the exponential term. It is now apparent that the slider is under large negative damping for a short period immediately after slip. Thus each oscillation is initially self-excited by the negative slope of the friction-velocity curve.

In order to solve the non-homogeneous second order differential equation 2 by analytical methods the coefficients in equation 2 must be constant. This condition is fulfilled only when \( \mu \) is a single linear function of \( U \). Correspondingly Fig. 6 illustrates the linearized friction-velocity curve which will be used in the following analytical solution. Essentially, relationship 5 becomes:

\[ \mu = \mu_m + A_p (V - \dot{V}) \quad (7) \]

The effect of static friction, \( \mu_s \), will be added as a boundary condition in the solution. This approach is used by Derjaguin, Push and Tolstoi (13).
Re-writing equation 2 and substituting \( \gamma \), gives:

\[
m \ddot{x} + (\alpha + \alpha_\rho W) \dot{x} + K \dot{x} = W (\mu_m + \alpha_\rho V)
\]

Expressing the coefficient of \( \dot{x} \) as \( R \) and the term on the right as \( F_K \), we have the general form:

\[
m \ddot{x} + R \dot{x} + K x = F_K
\]

The solution to this differential equation can be written as:

\[
x = \frac{F_K}{K} + e^{-\phi \omega_d t} \left[ A \cos \omega_d t + B \sin \omega_d t \right]
\]

Differentiation with respect to time yields:

\[
\dot{x} = \omega_d e^{-\phi \omega_d t} \left[ (B - \Phi A) \cos \omega_d t - (A + \Phi B) \sin \omega_d t \right]
\]

and

\[
\ddot{x} = -\omega_d^2 e^{-\phi \omega_d t} \left[ \left\{ A(1 - \phi^2) + 2 \Phi B \right\} \cos \omega_d t + \left\{ B(1 - \phi^2) - 2 \Phi A \right\} \sin \omega_d t \right]
\]

where

\[
F_K = W (\mu_m + \alpha_\rho V)
\]

\[
\nu = \frac{R}{2 \sqrt{Km}}
\]

\[
\nu_1 = \frac{\alpha_\rho W}{2 \sqrt{Km}}
\]

\[
\nu_2 = \frac{\alpha_\rho W}{2 \sqrt{Km}}
\]
\[
\omega^2 = \frac{K}{m}
\]

17

\[
\omega_d = \omega \sqrt{1 - \nu^2}
\]

18

\[
\phi = \frac{\nu}{\sqrt{1 - \nu^2}}
\]

19

In order to evaluate the constants of integration, A and B, the following boundary conditions apply:

When \( t = 0 \), the slider is on the verge of slip, represented by point B in Fig. 2, point B in Fig. 4 and point F in Fig. 5. Therefore, from equation 1:

\[
\rho = \frac{F_s - \tau V}{K}
\]

20

also

\[
\rho' = V
\]

21

Equations 10, 11 and 12 become:

\[
\rho = \frac{F_k}{K} + \frac{V(1 - \nu^2)}{\omega_d} e^{-\phi \omega_d t} \left[ (\psi - 2\phi) \cos \omega_d t + (1 + \phi[\psi - \phi]) \sin \omega_d t \right]
\]

22

\[
\rho' = V e^{-\phi \omega_d t} \left[ \cos \omega_d t - (\psi - \phi) \sin \omega_d t \right]
\]

23

\[
\rho'' = -\nu \omega_d e^{-\phi \omega_d t} \left[ \psi \cos \omega_d t + \phi (\frac{\phi}{\omega_d^2} - \psi) \sin \omega_d t \right]
\]

24

where

\[
\psi = \frac{\Delta F}{m \nu \omega_d} = \frac{W (\mu_s - \mu_m)}{m \nu \omega_d}
\]

25
Since the following experimental investigations are limited to very small values of \( R \) and \( V \), powers of \( \nu' \) and terms in \( \nu \) in equations 22, 23 and 24 can be neglected, giving:

\[
\nu = \frac{F_m}{K} + \left( \frac{F_s - F_m}{K} \right) e^{-\nu'\omega t} \left[ \cos \omega t + \nu' \sin \omega t \right]
\]

\[
\ddot{\nu} = -\frac{\omega}{K} \left( F_s - F_m \right) e^{-\nu'\omega t} \left[ \sin \omega t \right]
\]

\[
\dddot{\nu} = -\frac{\omega}{K} \left( F_s - F_m \right) e^{-\nu'\omega t} \left[ \cos \omega t - \nu' \sin \omega t \right]
\]

Referring to Fig. 2 the minimum displacement can be written as:

\[
\nu_{\text{MIN}} = \frac{W\mu_m}{K} - \frac{W(\mu_s - \mu_m)}{K} e^{-\nu'\pi}
\]

Thus the amplitude of vibration becomes:

\[
\lambda = \frac{W(\mu_s - \mu_m)}{K} \left( 1 + e^{-\nu'\pi} \right)
\]

From equation 8 it will be noticed that the positive slope, \( s_p \), of the friction-velocity curve contributes to the viscous damping in the system. The structural viscous damping coefficient \( r \) is effectively increased by an amount \( s_p W \) which can be referred to as "surface damping". The corresponding viscous surface damping ratio \( \nu'_s \) is given by equation 16.

The magnitude of the amplitude of vibration is seen to depend on the ratio \( \frac{W}{K} \) and the difference between the coefficients of static and kinetic friction.
CHAPTER THREE

1. DESCRIPTION OF EXPERIMENTAL APPARATUS
   a) THE VIBRATION APPARATUS
   b) THE APPARATUS FOR FRICTION MEASUREMENT

2. INSTRUMENTATION

3. PREPARATION OF THE SLIDING SURFACES
1.a) THE VIBRATION APPARATUS

The work of previous investigators shows that frictional vibrations depend largely on the characteristics of the supporting system as well as the shape of the friction-velocity relationship for the surfaces involved. It has been shown that the following parameters of the supporting system are important:

1) Stiffness of the elastic system
2) Normal load on the surfaces
3) Structural damping in the supporting system
4) Velocity of the traversing surface

Accordingly, the experimental apparatus, constructed by the author, was designed to allow convenient variation in some of the above parameters. The apparatus, in its present form, however, has no provision for variation in structural damping. With respect to stiffness, load, and traversing velocity, the influence of these parameters on the resulting form of the vibration will be shown.

For still greater versatility, it was desired to keep the mass of the vibrating member independent of the normal load on the surfaces. To accomplish this, it is necessary that the elastic system transmit the normal load, consequently the suspension was designed in the form of a cantilever beam, shown in Fig. 7, which possesses a high natural frequency in the normal direction. The stiffness of the cantilever beam in the transverse direction is easily altered by a simple change in length which can be accomplished by adjusting the clamping block shown in Fig. 7. The normal load W, is applied by means of the loading system which pivots the cantilever beam assembly about the axis A-A. This axis is supported on an adjustable
base (not shown) by means of two ball bearing blocks.

To eliminate any curvature effects caused by rotating turntables it was decided to employ pure translation of the lower surface. This was accomplished by the use of a 2\(\frac{3}{4}\) inch diameter power screw on which a large threaded nut is guided transversely by sliding ways. The power screw is driven by a thyratron tube-controlled variable speed motor coupled to the power screw by two worm drive reduction gear boxes giving a combined ratio of 700 : 1. The resulting maximum translational speed of the lower surface is 0.030 inch per second.

In friction measurements it is essential that the sliding surfaces under examination are maintained in constant uniform contact at all times throughout the sliding process. To accomplish this condition, with a minimum amount of precision machine work, the self-aligning joint illustrated in Fig. 8 was designed. It will be observed from Fig. 8 that the upper specimen or slider is free to rotate in two planes about axes that are perpendicular to each other. Consequently this self-aligning joint allows for slight adjusting movement of the upper specimen to occur during the sliding process. This is an important feature of the apparatus.

The mass, \(m\), of the vibrating parts is made up of the total weight of this joint plus a proportion of the beam weight that participates in the motion.

The structural damping coefficient, \(r\), of the cantilever beam, determined by free vibration tests, was found to be viscous in form but quite small in magnitude. A brief description of the free vibration tests, as well as the general calibration procedure for the apparatus, will be found in Appendix 2.
FIG. 7 DIAGRAMMATIC SKETCH OF THE APPARATUS

FIG. 8 THE SELF-ALIGNING JOINT
FIG. 9 PHOTOGRAPHS OF THE VIBRATION APPARATUS.
1. b) THE APPARATUS FOR FRICTION MEASUREMENT

As previously mentioned, the investigations were carried out using unlubricated steel surfaces. To obtain the experimental friction-velocity relationship for the steel surfaces, illustrated qualitatively in Fig. 3, a subsidiary friction apparatus was used. This was necessary since slip velocities in the neighborhood of 3 inches per second were recorded on several trial tests. This speed is far beyond the range of the vibration apparatus. To produce such comparatively large speeds, the subsidiary friction apparatus was designed using a horizontally mounted 16 inch diameter revolving steel plate made of the same material as the steel surfaces under study. The rotational speed of the steel plate can be controlled by means of a variable speed electric motor. Curvature effects, which were considered to have negligible effect in the friction-velocity measurements, were however minimized by conducting the experiments at the largest possible radius on the steel plate. Since the cantilever beam suspension system is easily removed from the vibration apparatus, this unit was used to support the upper friction surface.
2. INSTRUMENTATION

Deflection of the slider is measured by means of a Brush "Type One" linear displacement electromechanical transducer coupled to a Brush Model BL 202 direct writing magnetic oscillograph so that a permanent recording of slider movement is obtained. The moving core of the transducer is connected to the slider by means of a small tension spring. The oscillograph is electrically coupled to a Brush Model BL 320 amplifier which produces a minimum sensitivity of 1.2 scale divisions on the oscillograph chart per .001 inch slider displacement. Consequently the maximum amplitude that can be recorded is .0040 inch corresponding to full chart width.

To produce a convenient means of centering the trace of the vibration on the oscillograph chart, the complete transducer is mounted on an adjustable base or crossfeed. By means of a handwheel, graduated to read .0005 inch displacement, the transducer can be positioned to "center" the trace of the vibration. From the oscillograph charts of the vibration, using the sensitivity as listed above, slider deflections of the order of .0005 inch can be accurately determined. Thus the accuracy of the crossfeed and the oscillograph are matched.

Table 1 indicates the range of the variables that can be controlled by adjustments in the apparatus.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Range</th>
<th>Convenient Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>W</td>
<td>0 to 30 pounds</td>
<td>2 pounds</td>
</tr>
<tr>
<td>Stiffness</td>
<td>K</td>
<td>12 to 180 pounds per inch</td>
<td>13 pounds per inch</td>
</tr>
<tr>
<td>Table Velocity</td>
<td>V</td>
<td>0 to .03 inch per second</td>
<td>.00001 inch per second</td>
</tr>
</tbody>
</table>

TABLE 1. Range of the controlled variables.
3. PREPARATION OF THE SLIDING SURFACES

Considerable care and attention was devoted to the preparation of the sliding surfaces before any friction or vibration tests were made. The best criterion, of course, is reproducibility of results. Previous experimenters have used many "recommended" finishing and cleaning techniques but there seems to be general agreement in the literature that the best procedure is to expose a fresh surface by cutting or abrasion. In addition, the experiments must be performed as soon as possible after the finishing and cleaning process. In this investigation no provision was made in either apparatus for removing freshly accumulated contaminant material from directly in front of the slider, however the experiments were performed directly after the surface treatment process with a time interval of only a few minutes.

The surfaces were first prepared from C 1020 (cold rolled) steel, then de-greased using tri-chloroethylene. Using grade number 80-C, Behr-Manning "Tufbak" durite waterproof emery paper, the surfaces were polished in the direction of sliding. The emery paper was used dry, and after polishing, the surfaces were thoroughly washed again with tri-chloroethylene. After one experimental traverse of the lower surface, both surfaces were refinished and cleaned.

The surface roughness of the specimens produced by this finishing process was measured by means of a Brush "Surfindicator" Model BL-110, equipped with a power traverse. The range of the Surfindicator is 1 to 1000 microinches and the instrument is calibrated to measure the RMS (root mean square) roughness height in microinches. Measurements were made at close intervals along each friction surface in directions both parallel and
perpendicular to the direction of sliding. RMS roughness readings are listed in Table 2 using the following symbols:

\[ Y_{//} = \text{average RMS surface roughness value in microinches parallel to the direction of sliding.} \]

\[ Y_{\perp} = \text{average RMS surface roughness value in microinches perpendicular to the direction of sliding.} \]
CHAPTER FOUR

1. EXPERIMENTAL RESULTS
   a) FRICTION-VELOCITY CURVES
   b) FRICTIONAL VIBRATION RESULTS
1. a) FRICTION VELOCITY CURVES

The friction-velocity curves for the steel specimens are shown in Fig. 10. The coefficient of static friction was found to be quite consistent with the value .49. The surfaces were allowed to remain at rest for a period of about 10 minutes before a static test was taken. The coefficient of kinetic friction was found to be load dependent as indicated by the curves of Fig. 10. The value of the slope, for the positive slope region of the friction curves, is small, with the friction rising more rapidly with the lighter load. The experimental friction curves consist of an average of four tests. The mean deviation in coefficient of friction for the four tests was ± .04 giving a relative error with respect to \( \mu_M \) of ± 20%. From the average curves of Fig. 10, the mean deviation is ± .007 giving a relative error with respect to \( \mu_M \) of ± 3%. Corresponding loads of six pounds and ten pounds were used in the vibration experiments.

Considerable care was taken to accurately determine the value of \( U_M \), and indications from a reasonable number of tests predicted this value to be of the order of .06 inches per second. This is an average value and the mean deviation from this average was found to be as much as ± 50%. Thus the value of \( U_M \) is quite variable and in general it can be stated that the coefficient of friction achieves a minimum value very rapidly, resulting in a steep negative slope.

The linear equations shown in Fig. 10 serve to describe the experimental curves with reasonable accuracy. The equations describing the positive slope region compare in form with equation 7.
SURFACES: C 1020 COLD ROLLED STEEL
ROUGHNESS: \( y_f = 8 \mu \text{ in.} \quad y_d = 15 \mu \text{ in.} \)
80-C EMERY PAPER

**RELATIVE VELOCITY \( \mu \) (INS./SEC.)**

\[ \mu = -4.4 \mu + .49 \]
\[ \mu = .0045 \mu + .233 \]
\[ \mu = .00050 \mu + .215 \]

**LOAD**
- \( W = 6 \text{ LBS.} \)
- \( W = 10 \text{ LBS.} \)

**Fig. 10 - Graph of Coefficient of Friction versus Relative Velocity.**
1. b) FRICTIONAL VIBRATION RESULTS

Table 2 gives the values of the parameters defining the six systems which were used in the experimental investigation. Subsidiary measurements used in the determination of $K$, $W$, $\omega$, and $r$ are described in Appendix 2.

Corresponding to these six systems, the oscillograph records of the vibrations are shown in Figs. 11 to 16. The vibrations proceed in a direction from left to right across each figure thus making the slip process proceed in a direction from the bottom to the top of each figure.

Four tests were conducted for each system at the table velocities noted. From the resulting 64 oscillograph traces, reasonable consistency in the features of the vibrations was observed. For example, the mean deviation in maximum displacement for system 1 at $V = .0015$ inch per second is 8% with respect to equilibrium position while for system 6 at $V = .030$ inch per second the mean deviation in maximum displacement is 13% with respect to equilibrium position. Deviations for the intermediate systems lie within this range.

To obtain an estimate of the maximum slip velocity, the maximum oscillograph chart speed of 5 inches per second was used. From these high speed traces which are mounted on the right hand side of Figs. 11 to 16, larger scale plots were made of the slip period thus obtaining the velocity variation. The maximum slip velocity achieved during each slip period is listed in Table 2. The low chart speed on the oscillograph was used to obtain the general form of the vibration and these low speed results are mounted on the left hand side of Figs. 11 to 16. Each displacement scale
FIG. 11 TYPICAL TRACES OF THE OSCILLATION FOR SYSTEM I.
SCALE: 1 DIVISION = .0012 INCHES SLIDER DISPLACEMENT.
FIG. 12  TYPICAL TRACES OF THE OSCILLATION FOR SYSTEM 2.
SCALE: 1 DIVISION = .0012 INCHES SLIDER DISPLACEMENT.
FIG. 13 TYPICAL TRACES OF THE OSCILLATION FOR SYSTEM 3.
SCALE: 1 DIVISION = .0012 INCHES SLIDER DISPLACEMENT.
FIG. 14 TYPICAL TRACES OF THE OSCILLATION FOR SYSTEM 4.

SCALE: 1 DIVISION = .0012 INCHES SLIDER DISPLACEMENT.
FIG. 15 TYPICAL TRACES OF THE OSCILLATION FOR SYSTEM 5.
SCALE: 1 DIVISION = 0.0012 INCHES SLIDER DISPLACEMENT.
FIG. 16 TYPICAL TRACES OF THE OSCILLATION FOR SYSTEM 6.
SCALE: 1 DIVISION = .0012 INCHES SLIDER DISPLACEMENT.
<table>
<thead>
<tr>
<th>System Number</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$K$</th>
<th>$W$</th>
<th>$\omega$</th>
<th>$r$</th>
<th>$m$</th>
<th>$\nu_1$</th>
<th>$V$</th>
<th>$\dot{x}_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Micro-Inches</td>
<td>Micro-Inches</td>
<td>lbs/in.</td>
<td>lbs</td>
<td>Radians/sec.</td>
<td>lbs-sec./in.</td>
<td>lbs-sec.²/in.</td>
<td>in./sec.</td>
<td>in./sec.</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>12.8</td>
<td>7.4</td>
<td>155</td>
<td>10</td>
<td>300</td>
<td>0.046</td>
<td>0.0017</td>
<td>0.045</td>
<td>0.0015</td>
<td>-2.2</td>
</tr>
<tr>
<td>2</td>
<td>15.2</td>
<td>8.1</td>
<td>119</td>
<td>10</td>
<td>240</td>
<td>0.036</td>
<td>0.0021</td>
<td>0.036</td>
<td>0.0015</td>
<td>-2.7</td>
</tr>
<tr>
<td>3</td>
<td>18.6</td>
<td>9.6</td>
<td>85</td>
<td>10</td>
<td>205</td>
<td>0.026</td>
<td>0.0020</td>
<td>0.032</td>
<td>0.0015</td>
<td>-3.1</td>
</tr>
<tr>
<td>4</td>
<td>12.1</td>
<td>7.2</td>
<td>62.6</td>
<td>6</td>
<td>180</td>
<td>0.027</td>
<td>0.0019</td>
<td>0.039</td>
<td>0.0030</td>
<td>-3.8</td>
</tr>
<tr>
<td>5</td>
<td>12.9</td>
<td>7.5</td>
<td>41.4</td>
<td>6</td>
<td>150</td>
<td>0.026</td>
<td>0.0018</td>
<td>0.048</td>
<td>0.015</td>
<td>-2.01</td>
</tr>
<tr>
<td>6</td>
<td>18.3</td>
<td>8.3</td>
<td>38.4</td>
<td>6</td>
<td>135</td>
<td>0.025</td>
<td>0.0021</td>
<td>0.054</td>
<td>0.015</td>
<td>-2.5</td>
</tr>
</tbody>
</table>

**TABLE 2** Numerical Values of the Parameters for the Six Experimental Systems.
division on the oscillograph traces corresponds to a slider displacement of 1.2 thousandths of an inch. This sensitivity is noted on Figs. 11 to 16.

It will be observed that the amplitude of vibration depends on the velocity of the lower surface for each system. Increasing this table velocity for system 1 say, greatly reduces the amplitude of vibration and in general, for the remaining systems, the amplitude decreases with an increase in table velocity. In addition, the amplitude decreases as the ratio \( \frac{W}{k} \) decreases which is readily predicted by the theory (equation 30). From the data provided by the oscillograph charts, Figs. 17 to 22 illustrate graphically the variation in amplitude with table velocity for each system. Figs. 24 to 29 illustrate the rate of decay of amplitude of vibration with increasing table velocity for the six systems. Under frictional vibration no true static condition exists during the stick period since a process of continual adjustment between surface asperities takes place, consequently the experimental points giving the maximum displacement, in Figs. 17 to 22, are not proportional to the static coefficient of friction. Hence this apparent "static" coefficient of friction under vibration conditions will be referred to henceforth as the "breakaway coefficient of friction", \( \mu_B \). This term seems appropriate since the slider actually breaks away from the lower traversing surface in such a manner as to produce a relationship with table velocity which is approximately exponential in shape. With this in mind, the experimental points were extrapolated beyond the point of maximum table velocity in an effort to determine where the amplitude of vibration seems to die out. With the aid of Figs. 24 to 29, it appears that the amplitude dies out at a table velocity corresponding to the value of \( U_M \) from the friction-velocity curve. Thus, from these results, the maximum slider displacement seems to correspond to the value
of kinetic friction at the particular table velocity. Accordingly, using
the appropriate ratio of \( \frac{W}{K} \), the upper points in Figs. 17 to 22 were con-
verted to produce Fig. 23, a plot of \( \mu_B \) versus table velocity. Since
the experimental results suggest that the breakaway coefficient of friction
corresponds to the particular table velocity, equation 4A, with \( U \) replaced
by \( V \), takes the form:

\[
\mu_B = (\mu_s - \mu_M) e^{-\frac{V}{V_m} \ln\left(\frac{\mu_s - \mu_m}{\alpha \mu_M}\right)} + \mu_M
\]

Using the following experimental values:
\( \mu_s = .49 \), \( \mu_M = .22 \) and \( U_m = .06 \) inch per second, equation 31
becomes:

\[
\mu_B = .27 e^{-80V} + .22
\]

This equation is shown by the solid curve in Fig. 23. Thus the breakaway
coefficient of friction varies in the same manner as the friction-velocity
curve for the surfaces under examination. Using this experimentally observed
fact, equations 26, 27, 28, 29, and 30 can be easily altered in accordance
with the relevant friction-velocity curve, to predict with reasonable
accuracy the dynamic performance of the slider for table velocities in the
negative slope region. Replacing the constant \( \mu_s \) with the breakaway
coefficient, the above mentioned equations become:

\[
\nu = \frac{W}{K} \left[ \mu_m + (\mu_B - \mu_M) e^{-2\omega M t} \left( \cos \omega M t - 2 \sin \omega M t \right) \right]
\]

\[
\nu_{\text{MAX.}} = \frac{\mu_B W}{K}
\]

\[
\nu_{\text{MIN.}} = \frac{W}{K} \left[ \mu_m - (\mu_B - \mu_M) e^{-2\omega M t} \right]
\]
\[ \dot{\mu} = \frac{-W}{m \omega} (\mu_b - \mu_m) (\sin \omega t) e^{-\gamma \omega t} \] 36

\[ \dot{\mu}_{\text{MAX}} = \frac{-W}{m \omega} (\mu_b - \mu_m) \sin (\tan^{-1} \frac{\nu}{\omega}) \left[ e^{-(\tan^{-1} \frac{\nu}{\omega})} \right] \] 37

\[ \ddot{\mu} = \frac{-W}{m} (\mu_b - \mu_m) e^{-\gamma \omega t} \left[ \cos \omega t - \nu \sin \omega t \right] \] 38

\[ \alpha = \frac{W}{K} (\mu_b - \mu_m) (1 + e^{-\gamma \mu}) \] 39

The upper solid curves in Figs. 17 to 22 represent equation 34 while the lower curves represent equation 35. The solid curves in Figs. 24 to 29 represent equation 39 and the curves in Figs. 30 to 35 represent equation 37.
Fig. 17 Graph of Slider Displacement versus Table Velocity - System I.

TABLE VELOCITY (INCHES/SECOND)

SLIDER DISPLACEMENT ("x" (INCHES))

Experimental

Theoretical

Speed Limit of the Apparatus
Fig. 18
Graph of Slider Displacement versus Table Velocity - System 2.

Slider Displacement "\( \nu \)" (inches).

Table Velocity "\( v \)" (inches/second).

Theoretical — Experimental.
Graph of Slider Displacement versus Table Velocity - System 3.
Fig. 21: Graph of Slider Displacement versus Table Velocity - System 5.

Velocity (inches/second)

Experimental

Theoretical
Fig. 22
Graph of Slider Displacement versus Table Velocity — System 6.

SLIDER DISPLACEMENT (INCHES).

TABLE VELOCITY "v" (INCHES/SECOND).

THEORETICAL
EXPERIMENTAL

- 111 -
**Fig. 23** Graph Showing the Relationship between the Breakaway Coefficient of Friction and the Table Velocity for the Six Systems.
Fig. 24
Graph of Amplitude of Vibration versus Table Velocity - System 1.
Fig. 25

Graph of Amplitude of Vibration versus Table Velocity — System 2.

Amplitude of Vibration \( \lambda \) (inches).

Table Velocity \( v \) (inches/sec).

Theoretical — Experimental.
Fig. 26 Graph of Amplitude of Vibration versus Table Velocity — System 3.

Amplitude of Vibration "\( \alpha \)" (Inches).

Table Velocity "v" (Inches/Second).
Figure 21: Graph of Amplitude of Vibration vs. Table Velocity - System 4.

Table Velocity \( \times ( \text{inches/second}) \)

Amplitude of Vibration \( \alpha \) (inches)
AMPLITUDE OF VIBRATION "α" (INCHES).

Table velocity "v" (inches/second).

-Theoretical
-Experimental
Fig. 2.9 Graph of Amplitude of Vibration versus Table Velocity - System 6.

Table Velocity v. (Inches/Second)

Amplitude of Vibration "\( \chi \)" (Inches)

Experimental

Theoretical
Fig. 30
Graph of Maximum Slip Velocity Versus Table Velocity - System 1.

Maximum Slip Velocity $\dot{n}_{\text{max}}$ (Inches/Sec.).
Fig. 3.1 Graph of Maximum Slip Velocity Versus Table Velocity System 2.

Maximum Slip Velocity $\dot{\lambda}_{\text{max}}$ (Inches/Second).

Table Velocity $\dot{\nu}$ (Inches/Second).

Experimental ○ Theoretical —
Fig. 32
GRAPH OF MAXIMUM SLIP VELOCITY VERSUS TABLE VELOCITY – SYSTEM 3.
Fig. 33  Graph of Maximum Slip Velocity versus Table Velocity — System 4.

Maximum Slip Velocity $\dot{w}_{\text{max}}$ (Inches/Second).

- Table Velocity ($v$)
- Theoretical
- Experimental

- Table of Data
MAXIMUM SLIP VELOCITY $\dot{\eta}_{\text{MAX}}$ (INCHES/SECOND).

Fig. 34 Graph of Maximum Slip Velocity versus Table Velocity — System 5.
Fig. 35
Graph of Maximum Slip Velocity versus Table Velocity—System 6.

Maximum Slip Velocity $u_{\text{max}}$ (inches/second).

Table Velocity $v$ (in. / sec.)

Theoretical
Experimental ○
CHAPTER FIVE

1. DISCUSSION OF RESULTS
2. CONCLUSIONS
3. RECOMMENDATIONS
1. DISCUSSION

The experimental results show the relationship between the amplitude of vibration, $\alpha$, and the velocity of the lower traversing surface, $V$. Frictional vibrations were produced using six supporting systems having various ratios of $\frac{W}{K}$. The supporting systems were characterized by relatively constant viscous or structural damping. The magnitude of the damping coefficients was small consequently the influence of structural damping on the resulting vibration was considered negligible. From the results of the friction-velocity measurements, it is apparent that the investigations of frictional vibrations were performed for table velocities in the negative slope region of the friction-velocity curve. The shape of this negative slope region was approximated by an exponential relationship having certain defined end conditions. The vibration experiments show that the decay of maximum slider displacement is also exponential in shape with similar conditions of velocity and coefficient of friction. In addition, the vibration seemed to vanish at a point corresponding to the "knee" of the friction-velocity curve. The graphical approach suggested that for table velocities in the positive slope portion of the friction-velocity curve, trajectories could "miss" the vertical axis $\dot{x} = V$ and proceed to completion. This represents the case of smooth sliding. Table velocities in the negative slope region were shown to produce stable limit cycles.

It was therefore concluded that the maximum slider displacement seemed to correspond to the value of the kinetic friction at a particular table velocity. The analytical solution was then altered, in view of the experimental indications, making use of the general exponential expression for the shape of the negative slope region of the friction-velocity curve.
Theoretical and experimental results were then compared and reasonable correlation was found.

Figs. 17 to 22 are essentially the envelope of the vibration as a function of table velocity and in general there is strong agreement between theory and experiment in the velocity range \(0 < V \leq 0.002\) inch per second. For this velocity interval, Figs. 24 to 29 predict a mean deviation between theory and experiment of about \(-0.006\) inch for system 1 and \(+0.002\) inch for system 6, giving relative errors, with respect to equilibrium position, of \(-10\%\) and \(+2\%\) respectively. Intermediate systems are seen to lie within this range. With this accuracy equation 39 may be of value in predicting the positioning error that can be expected in final positioning movement under servo control. The accuracy of equation 39 is poor at velocities in the range \(0.02 < V < 0.03\) inch per second since corresponding relative errors are \(-4\%\) and \(120\%\) approximately.

Figs. 30 to 35 compare the experimental and theoretical results for the maximum slip velocity for the six systems. While the experimental results show considerable scatter, it will be observed that the theory serves to predict general trends in the variation of maximum slip velocity with table velocity. Enlarging the experimental records to obtain the velocity variation can introduce error which could account for some of the scatter in the values of maximum slip velocity.

It was shown that frictional vibrations are self excited with each oscillation being driven for a very short period by heavy negative damping. This is apparent when the relationship describing the negative slope region:

\[ \mu = \mu_s + \alpha_n U \]
is substituted into equation 2. Using the experimental value of $s_N$ from Fig. 10 we find $v_a \cong -40$ for system 1. The existence of the negative slope region serves to excite the vibrations. For the positive slope region we find that $v \cong v_1$ since the numerical value of $s_p$ from Fig. 10 is very small.

The term "effective surface damping" was introduced to describe the component that is added to the value of $r$ in equation 8 by virtue of the slope of the friction curve.

The oscillograph records of the vibrations shown in Figs. 11 to 16, illustrate several interesting features of the phenomenon. It will be observed that at low table velocities, breakaway of the slider is very sudden while at increasing table velocities breakaway becomes less sudden accompanied by decreased accelerations and slip velocities. The junction forming process at the end of each slip period is marked by comparatively low accelerations as illustrated by the high speed slip traces. In some cases it will be seen (Fig. 12 (a)) that the junction forming process consists of slight irregularities suggesting the presence of minute frictional vibrations.

At low table velocities, the "stick" period is quite uniform and with the sensitivity used shows no visible relative motion. At higher table velocities the "stick" period shows slight "sub-relaxation oscillations" of which Fig. 13 (c) is an example. Possibly due to increased activity and adjustment between the surfaces at higher table velocities the "stick" period becomes less stable. No doubt some degree of adjustment does occur between the surfaces at low table velocities and this slight movement becomes noticeable at higher table velocities.
Fig. 13 (a) is a good example of "dwell" that was sometimes noticed prior to the slip period. The relative motion between the surfaces usually produced no movement of the slider, the slider remaining motionless for periods up to about two seconds or sliding distances of the order of .003 inch. A slip period usually followed the dwell.
2. CONCLUSIONS

Frictional vibrations between unlubricated sliding steel surfaces have been studied in some detail. Traversing velocities were confined to the negative slope portion of the friction-velocity curve. The conclusions to be derived from the work are:

1. The vibrations are basically a non-linear phenomenon.
2. Each oscillation is self-excited. For a very short period the system is driven by large negative damping by virtue of the steep negative slope portion of the friction-velocity curve for the surfaces in question.
3. The investigations suggested that the form of the decay of vibration amplitude and the shape of the friction-velocity curve in the negative slope region were similar.
4. The amplitude of vibration seemed to die out at a point corresponding to the "knee" of the friction-velocity curve.
5. In order to suppress or eliminate frictional vibrations, the linearized theory predicts that the necessary conditions are:
   a) Reduce the ratio $\frac{W}{K}$ by either decreasing the load between the surfaces or increasing the stiffness of the supporting system.
   b) Decrease the difference between the static and kinetic ($\mu_m$) coefficients of friction.
   c) Operate the system at a traversing velocity in excess of $\frac{U_m}{U}$ for the surfaces used.

It is considered that the investigation has served to point out some of the important features of the phenomenon.
3. RECOMMENDATIONS

Future experimental investigations should be carried out using a modified apparatus capable of producing higher traversing velocities. The present apparatus is limited to speeds of the order of .030 inches per second. Table velocities in excess of this value prevent adequate observation of the vibrations. If velocities in the range of .1 to .15 inches per second could be achieved then extrapolation of the results would be unnecessary.

The limitations on maximum amplitude that can be recorded by the present instrumentation methods suggest that the stiffness of the cantilever beam should be increased slightly to a range of about 200 to 400 pounds per inch. Since there is sufficient sensitivity adjustment remaining in the equipment, deflections of about 25 microinches could still be recorded satisfactorily.

To observe the effect of damping, the present apparatus could be easily modified by installing an electromagnetic viscous-type damper on the end of the slider. This would provide a convenient and flexible variable damping system.
APPENDICES

1. DERIVATION OF THE ANALYTICAL RELATIONSHIP DESCRIBING THE SHAPE OF THE FRICTION-VELOCITY CURVE.

2. CALIBRATION OF THE APPARATUS AND SUBSIDIARY TECHNIQUES.
APPENDIX 1. DERIVATION OF THE ANALYTICAL RELATIONSHIP DESCRIBING THE SHAPE OF THE FRICTION VELOCITY CURVE.

The friction velocity curve shown qualitatively in Fig. 3 can be closely approximated by an analytical relationship of the form:

\[ \mu = C_1 e^{-C_2 U} + \sigma_p U + \mu_m \]  

where the terms " \( \sigma_p U + \mu_m \) " describe the positive slope region and the exponential term describes the negative slope region. Referring to Fig. 3, it is clear that \( C_1 = \mu_s - \mu_m \). At the point \( (U_M, \mu_M) \), the numerical value of \( C_1 \) must be reduced to some negligible quantity to allow the remaining linear terms to describe the positive slope portion. This point is the effective cutoff point for the exponential term. Introducing a 1% relative error with respect to \( \mu_m \), the boundary condition for evaluation of the constant \( C_2 \) becomes:

\[ U = U_M, \quad \mu = \mu_M + 0.01 \mu_M \]  

giving:

\[ \mu = (\mu_s - \mu_M) e^{-\frac{U}{U_M} \ln \left( \frac{\mu_s - \mu_M}{0.01 \mu_M} \right)} + \mu_M \]  

for the negative slope region. For the complete friction velocity curve, equation 3A becomes:

\[ \mu = (\mu_s - \mu_M) e^{-\frac{U}{U_M} \ln \left( \frac{\mu_s - \mu_M}{0.01 \mu_M - \sigma_p U_M} \right)} + \sigma_p U + \mu_M \]  

which compares with equation 4.
APPENDIX 2. CALIBRATION OF THE APPARATUS AND SUBSIDIARY TECHNIQUES

a) Calibration of the Loading System

The normal load $W$, between the sliding surfaces is adjusted by applying suitable weights to the loading pan illustrated in Fig. 7 and Fig. 9. For various weights on the loading pan, the normal load produced at the slider was measured by means of a strain ring. The strain ring was loaded directly by the slider so that known loads $W$, were tabulated as functions of beam length and total weight on the loading pan.

b) Calibration of the Transducer

To determine the relationship between pen deflection and transducer displacement for a given sensitivity setting on the amplifier, the moving core of the transducer was placed in contact with the end of the moving table. On the opposite end of the moving core a dial gage reading to .0001 inch was used to record deflections of the core produced by slight movements in the table.

c) Calibration of the Stiffness of the Cantilever Beam

To determine the stiffness of the cantilever beam for various lengths, the strain ring was mounted horizontally on the driven surface by means of suitable brackets. The axis of the strain ring was placed coaxial with the center-line of the slider and the deflecting load was transmitted from the end of the slider to the strain ring by means of a small steel ball. The dial gage was placed on the opposite end of the slider so that deflections were readily obtained. It was not possible to use the transducer to record deflections in this instance since the amplifier and oscillograph were used to record loads on the strain ring.
d) Experimental Determination of the Coefficient of Viscous (Structural) Damping "$r$"

The evaluation of this coefficient was carried out using free vibration tests in which the sliding surfaces were separated allowing the cantilever beam to vibrate freely after being given an initial displacement. The equation governing the motion is similar to equation 2 except $W = 0$ yielding a homogeneous differential equation. The oscillograph trace of the free damped vibration was drawn to a larger scale and then plotted on semi log paper. The envelope was drawn over the peaks of both positive and negative amplitudes thus giving the value of $r$. From this, the natural frequency of the system could also be calculated. Four free vibration tests were carried out for each of the systems of Table 2.

e) Velocity of the Driven Surface

To determine the table velocity $V$, for various settings on the thyratron speed control console, the transducer was placed directly against the end of the moving table. Knowing the speed of the oscillograph chart, the slope of the resulting trace produced the corresponding value of table velocity for the particular console setting. A graph was then drawn giving the relationship between velocity $V$ and console setting $N$. 
BIBLIOGRAPHY


SUPPLEMENTARY BIBLIOGRAPHY


