AN ANALYSIS OF RESOURCE ALLOCATION IN THE PRODUCTION OF MARKET EGGS

by

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B.S.A., University of British Columbia, 1948

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE OF AGRICULTURE

in the Department

of

Agricultural Economics

We accept this thesis as conforming to the standard required from candidates for the degree of MASTER OF SCIENCE IN AGRICULTURE

Members of the Department of

Agricultural Economics

THE UNIVERSITY OF BRITISH COLUMBIA

October, 1960

ABSTRACT

Farmers are confronted continually with the necessity of revising and reorganizing their production plans in order to maximize the net returns from the available resources. This need arises from the dynamics of price and yield fluctuations, which can only be estimated within a range. If future changes in prices and yields could be predicted with absolute certainty, a single plan could be formulated which would specify the resource combination at each point in time, for each change in techniques, and for each price situation.

The present study involves an investigation of the resources employed by commercial market egg producers in the Lower Fraser Valley and Vancouver Island areas during the period of 1949 to 1951 to determine (1) the deviation of the actual resource combinations employed during each year from the theoretically optimum combination required for maximum net returns and (2) the effectiveness of alterations to the resource combination made by producers in attempting to adjust their operations to variations in the input-output price relationships.

The production function method of analysis was used because it recognized the basic functional relationships in the production process and provided a quantitative analytical technique founded on general economic principles. The analysis

was based on input- output data compiled from detailed records of 66, 57 and 45 commercial market egg enterprises for the respective years 1949, 1950 and 1951 ending on September 30. For the purpose of this analysis, the numerous individual resources employed in the production of market eggs were aggregated into the categories of (1) land, buildings and equipment, (2) laying flock, (3) labor, (4) feed, and (5) other cash expenses. A Cobb-Douglas production function was derived for each year by the least-squares method of fitting a linear multiple regression equation. The marginal value products of the resource categories, with all inputs fixed at their geometric means, were estimated by partial differentiation of the production function with respect to each input variable.

As indicated by the coefficient of multiple determination, about 95 per cent of the variance in total output (gross income) from the market egg enterprises during each year was explained by the five input categories. According to the t-test, coefficients of the following input categories were statistically different from zero at the five per cent significance level: laying flock and feed in 1949; feed and other cash expenses in 1950; and laying flock and feed in 1951. All coefficients had a value less than 1.0, indicating diminishing marginal returns to all input categories. Returns to scale, as measured by the sum of the coefficients, were constant for each year.

In each year, the marginal value products were larger for the input categories of laying flock and feed than for the other inputs. In view of this persistent inequality, it was apparent (1) that the theoretically optimum combination of resources for these egg enterprises was not attained in any of the three years and (2) that the adjustments in resource inputs from year to year did not constitute a major improvement in the resource combination.

The marginal value products of the various resources revealed the (average) results of the decisions taken by producers in using the resources at their disposal for the production of market eggs. The failure to achieve the most profitable combination of these resources was attributed to restrictions imposed by (1) production techniques and practices that prevented quick and precise adjustments to the input of certain resources, (2) inflexibility and indivisibility of resources, and (3) imperfect knowledge of output and prices in the future. Some of these restrictions may preclude the possibility of effective improvement in resource use in the short run.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES -- Continued

LIST OF FIGURES

ACKNOWLEDGEMENT

The author wishes to express sincere appreciation to Dr. Walton J. Anderson, Chairman of the Department of Agricultural Economics, for his assistance and suggestions in the development and writing of this thesis. He also wishes to acknowledge the guidance and criticism received from the Graduate Committee members, Dean Blythe A. Eagles, Professor Jacob Biely, Dr. Nora E. Neilson, Professor T. I. Matuszewski and Dr. Joseph J. Richter.

The author is also indebted to Dr. J. F. Booth, Director of the Economics Division, Canada Department of Agriculture, for permission to use the data on market egg enterprises which was collected by the British Columbia Regional Office of the Economics Division.

AN ANALYSIS OF RESOURCE ALLOCATION IN THE PRODUCTION OF MARKET EGGS

INTRODUCTION

As managers, farmers are concerned with combining the limited resources at their disposal so as to maximize net returns or profits. Although some farmers may forego this objective in favor of more leisure or for reasons of other ideals or beliefs, the question of optimum allocation of resources must be related to the economic end of maximizing profits. If a greater value of product can be obtained from the same resources, or fewer resources can be used for the same value of product, then possibilities exist for an increased net return through adjustments in the combination of resources. The optimum use of given resources is attained only when the net product is at a maximum, and no re-arrangement of the resources will result in an output which yields a greater net value.

There are several main obstacles to achieving an optimum resource combination. Inefficient use of resources may occur as a result (1) of imperfect knowledge of the physical input-output relationships or (2) of uncertainty as to the course of future prices of resources and products. Also, a reluctance to abandon familiar methods of production, inflexibility in the quantity and use of some resources, and limited capital all contribute to resource combinations which are not optimum.

Statement of the Problem

The main categories of resources used for market egg production include land, buildings, equipment, laying birds, feed, labor and a group of miscellaneous items. In the aggregate, certain changes occurred from year to year in both the quantity of resources employed and the quantity of eggs produced during the period from 1949 to 1951; the input of feed and layers declined slightly from 1949 to 1950, and then increased in 1951 to a level above that of 1949; labor input decreased substantially from 1949 to 1950, and increased by a small amount in 1951. At the same time, changes in feed and egg prices were such that the feed-egg price ratio was least favorable in 1950 and most favorable in 1951. Therefore, it would seem that producers responded to the narrowed margin between feed and egg prices in 1950 with a small reduction in the input of some resources, particularly those which could be easily and quickly adjusted. On the other hand, encouraged by the improved feed-egg price relationship of 1951, they attempted to take advantage of the situation by substantially increasing the inputs of most of these same resources. As a consequence of the adjustments in resource combination, total production of eggs fell slightly from 1949 to 1950, but rose in 1951 well above the 1949 output.[1]

Two closely related aspects of the problem of resource combination in market egg production were investigated in the following analysis. Essentially, this required estimation of the marginal value productivities of certain groups of resources employed in market egg enterprises during each of the three years. These estimates of value products made it possible to observe any departure of the actual resource combinations employed during each year from the theoretically optimum combination required for maximum profit. This could be achieved by comparing the marginal value products with the respective marginal costs (or prices) of resources. Secondly, the effectiveness of alterations to the resource combination made by producers in attempting to adjust their operations to variations in the feed-egg price relationship can be appraised through the changes in the marginal value product/marginal cost ratios which occurred from year to year.[2]

Method of Analysis

Estimates of the marginal value productivity of the resource categories were derived by the production function

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[1]See Appendix I and II for a description of the physical and financial organization of the market egg enterprises, and for details of other data used in the following analysis.

[2]See Appendix III for a summary of the theoretical concepts relevant to the problem of economic efficiency of resource use in the short run for a single enterprise.

method of analyzing input-output data for farms. This technique recognizes the basic functional relationships that underlie the production process. It incorporates these functional relationships based on general economic principles into a quantitative method of analysis. For this reason, the production function method offers certain advantages over the more conventional method of tabular analysis as a technique for measuring resource productivity.[1]

The tabular method of analysis (also called the method of direct comparison) ordinarily depends on the grouping and cross-classification of data and the calculation of averages to reveal relationships between various "efficiency factors" and a "measure of returns" such as management return, labor income, or other "residual" profit figure.[2] Management return or a similar measure is often used to indicate the effectiveness of resource allocation on farms. The method of calculating this residual return implies that the current market price of each resource is equal to its marginal value productivity. This equality could be expected to occur under long-run stable

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[1]For a comparison of these methods of analysis, see Earl O. Heady, Glen L. Johnson, and Lowell S. Hardin (eds.), Resource Productivity, Returns to Scale, and Farm Size, (Ames: Iowa State College Press, 1956), pp. 151-159.

[2]For a discussion and illustration of this method of analysis, see G. W. Forster, Farm Organization and Management, (rev. ed.; New York: Prentice-Hall, Inc., 1946), pp. 97-112. For definition and calculation of many of the efficiency factors and measures of returns, see John D. Black et al., Farm Management, (New York: The Macmillan Co., 1949), pp. 489-503; G. W. Forster, Farm Organization and Management, (rev. ed.; New York: Prentice-Hall, Inc., 1946), pp. 167-193; and John A Hopkins and Earl O. Heady, Farm Records and Accounting, (4th ed.; Ames: Iowa State College Press, 1955), pp. 163-203.

competitive conditions, but not necessarily in the short-run. When resources are under-priced relative to their actual productivity, the return to the residual factor represents an amount which is not due to management alone, but also includes a portion that is due to the productivity of other resources whose productivity is thereby underestimated. Thus, if management is set up as the residual claimant on income, resources may appear to be used more efficiently on large farms than on small farms when, in fact, a larger "residual" return to management could be caused entirely by the greater quantity of resources, and not by any difference in production practices or the kind of resources employed. If resources are assigned prices greater than their true productivity, equally misleading conclusions on resource efficiency could be drawn from the under-estimated return to management. Thus, the productivity of any one factor, when calculated by the residual method, is incorrectly estimated to the extent that the market price assigned to other resources deviates from the true marginal value productivity of these resources.

The imputation of returns to factors of production by the residual method implies that (1) the market price of each resource is equal to its marginal value product; (2) the total physical or value product can be divided into shares so that (a) each factor receives a reward equal to its marginal productivity, and (b) the rewards so computed exactly exhaust the total physical or value product. The implicit assumption in the residual procedure is that the exact return to each factor can be imputed, and that neither an excess nor a deficit of the total product will remain after imputation of returns to all factors. However, the returns to all factors computed in this manner amount to the total product under only one condition: the elasticity of production must equal 1.0. With respect to a single-factor production function, this condition exists when the production function is linear and homogeneous throughout all ranges, that is, a straight line input-output curve. It is also attained on a production function characterized by diminishing marginal resource productivity at the point where the marginal product equals the average product of the resource. Thus, if the elasticity of production is greater than 1.0, total returns to all factors (when imputed according to the marginal product of each factor) exceed the total product. On the other hand, if the elasticity is less than 1.0, the shares imputed to resources amount to less than the total product. Consequently, imputation of factor returns by the residual procedure does not exactly distribute a total product among the resources with which it was produced, except when the elasticity of production for each factor is equal to 1.0.

The residual method gives the actual marginal value productivity of only one factor of production when market prices coincide with the marginal value products of other factors, and when the elasticity of production is 1.0 for all factors. If the production function is not linear throughout a relevant range of resource use, or when farmers have not attained a resource combination denoted by the linear portion of a production function, then the elasticity of production will differ from 1.0 and factor returns imputed on the basis of the marginal product of each factor will not add exactly to the total product. However, there is no reason for allocating the residual surplus or deficit of the total product to land or any other single resource.[1]

The assumption, in the tabular method of analysis, of a linear relationship between input of resources and output of product also implies a constant return to all units of input. This means that using an additional unit of a resource always increases the total product by some constant amount regardless of the previous level of input. Such an assumption is implicit in recommendations that advocate the maximum physical output per unit of input. Similarly, recommendations that larger profits are possible through continuous increases in output per unit of input presume a constant marginal product, in which case there would be no point beyond which increased physical production might lead to decreased profits. A linear production function, however, cannot be accepted as an input-output model except in special situations. It may provide a sufficient explanation for greater profits when output is increased in the short run. It seldom applies, however, to the input-output relationship for a fixed technical unit such as an acre or an animal.

Many production processes for the farm as a divisible

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[1]Earl O. Heady, Economics of Agricultural Production and Resource Use, (New York: Prentice-Hall, Inc., 1952), pp. 402-414.

producing unit involve a linear input-output relationship. This relationship, of course, does not apply to a single animal or acre as a technical unit which is indivisible. Yet the total animals and acres that form part of a farm of fixed size, or of a farm with a fixed set of buildings and equipment, can be considered as divisible inputs. In this sense, a linear relationship normally exists between the input of animals or land and the output of product. When technical units (animals and acres) as well as other resources that enter directly into the production process (feed, seed, fertilizer, etc.) are varied in a fixed proportion, the input-output ratio for the farm is likely to remain fairly constant until fixed resources (operator and family labor, buildings and equipment) are fully utilized. For example, each additional acre of land used for a crop requires an equal input of seed, fertilizer, machinery and labor, and can be considered to add an equal increment to output. Similarly, for a farm with a fixed acreage and fixed services in the form of labor, buildings and equipment, the input-output relationship is linear over a certain range as both feed and livestock numbers are varied from zero to the capacity of the limiting factor. However, as soon as the stock of services represented by the fixed resources becomes fully engaged in the production process, any further increase in the composite input of variable factors may lead to a decline in the additional output of product.[1]

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[1]For a discussion of the short-run farm production function, see Earl O. Heady, Economics of Agricultural Production and Resource Use, (New York: Prentice-Hall, Inc., 1952), pp. 78-83.

The production function method of analysis is based on concepts that allow a relaxation of these restrictions imposed by the tabular method. Thus, it is capable of providing estimates of resource productivity that are more realistic for an investigation of economic efficiency in resource use. However, this does not imply that the tabular method should be rejected completely as a technique in farm management analysis. On the contrary, it is particularly useful for summarizing factual data in studies that are primarily historical and descriptive in nature rather than analytical or predictive. This method can also be applied effectively when emphasis is given to determining and examining the characteristics of different sizes and types of farms, the variables associated with farm profits, and average input-output ratios such as crop yield per acre, milk production per cow and feed requirements per animal. In addition, preliminary and supplementary analyses of farm input-output data by the tabular method are often essential to the application of other techniques such as activity analysis.

THE ANALYTICAL MODEL

The term "production function" is used to describe the input-output relationship since any observed relationship between variables corresponds to a functional relationship between the variables. For example, the output of product is determined by the quantity of input such as seed, labor, land and other resource services. As illustrated in Appendix III, a single-variable production function can be shown as a two-column table with the factor inputs and the resultant total output of product listed in separate columns. Also, a graph may be used as a geometrical presentation of a production function with the factor input measured on the horizontal x-axis and the product output measured on the vertical y-axis. In addition, a production function may be expressed as an algebraic equation of the form

$$Y = f(X),$$

which indicates that a functional relationship is assumed to exist between a single product Y and a single variable factor X. In the usual situation where production of a commodity requires several resources, a production function is expressed more accurately in the form

$$Y = f(X_1, X_2, X_3).$$

Here, Y refers to a single product and X_1, X_2, and X_3 refer to specific factors of production.

A production function in this general form indicates all factors that contribute to the output of product. This symbolic representation does not specify any quantitative relationships between variables. These are expressed only by writing the production function in an algebraic form such as

$$Y = a + bX_1, \text{ or}$$

$$Y = aX_1^{\,b}X_2^{\,c}.$$

The letters a, b and c denote constants. The values of these constants specify the quantitative relationship between the output of product and the inputs of resources, or the amount by which the product Y changes as the inputs of factor X_1 or X_2 are varied.

Input-output relationships or production functions can be expressed by many different types of equations. However, a particular relationship is not described with equal accuracy by all equations and, conversely, any one equation is not adequate to express all relationships. Each type of equation is capable of accurately representing only certain types of relationships. Several different equations might be fitted to the same set of data, but considerable variation would occur in the accuracy with which each equation described the true relationship. The equation of a straight line fitted to data involving a linear relationship will accurately express the true relationship shown by the data. Any attempt to represent this relationship by any other equation would only result in a distorted and often misleading expression of the true relationship. The true nature of a relationship is revealed only within the limitations of the particular equation that is used. Consequently, the choice of an equation to represent the relationship between two or more variables must depend on a logical analysis of the relationship as well as the ability of any given equation to express the empirical relationship.

Homogeneity of quality in the factors of production and in the product is necessary for an input-output relationship to have any significance. Similarly, the input-output relationship cannot be defined if variation in the quality of product occurs with different levels of input. The production function also relates to a specific time period and to a certain technique of production since the input-output relationship changes with the method of production.

Several types of mathematical functions comprised of various combinations of squared, cubed, cross-product, square root, exponential and reciprocal terms have been used to estimate the relationship between total output of product and two or more inputs. Each function imposes definite restrictions on the relationships that can be expressed because the mathematical characteristics of each function contain certain implicit assumptions about the nature of the relationships. Consequently, the particular function should be selected for its ability to represent adequately the theoretical model which describes the general form of the input-output relationship. The selection of a function to satisfy this requirement becomes increasingly difficult with the complex relationships introduced by several resource categories and, as a result, some degree of compromise is often a practical necessity.

The function selected to represent the input-output relationships in the production of market eggs was of the general form

$$Y = aX_1^{b1}X_2^{b2}....X_n^{bn}.$$

Commonly known as the Cobb-Douglas production function, it was first used by Cobb and Douglas to estimate the proportions of total product attributable to labor and capital in manufacturing industries.[1] It was also employed by Douglas and his associates in later studies that involved a similar analysis for various regions and time periods.[2] A function of this general form was among those used by Nicholls in an analysis of labor productivity in a meat packing plant,[3] and has been applied extensively in the analysis of resource productivity on farms when the main objective was estimation of marginal value productivities of inputs for a sample of farms.

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[1]See Charles W. Cobb and Paul H. Douglas, "A Theory of Production", The American Economic Review, XVIII (March, 1928), pp. 139-165; and Paul H. Douglas, The Theory of Wages, (New York: The Macmillan Company, 1934).

[2]For example, see Marjorie L. Handsaker and Paul H. Douglas, "The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria," The Quarterly Journal of Economics, LII (1937-1938), pp. 1-36 and 215-254; M. Bronfenbrenner and Paul H. Douglas, "Cross-sectional Studies in the Cobb-Douglas Function," The Journal of Political Economy, XLVII (December, 1939), pp. 761-785; and Grace T. Gunn and Paul H. Douglas, "The Production Function for American Manufacturing in 1914," The Journal of Political Economy, L (August, 1942), pp. 595-602.

[3]W. H. Nicholls, Labor Productivity Functions in Meat Packing, (Chicago: University of Chicago Press, 1948).

Characteristics of the Cobb-Douglas Production Function

The Cobb-Douglas function possesses several desirable properties for the analysis of the productivity of resources. Compared with other functions, estimation of fewer constants or regression coefficients is required for the same number of variables. It also permits either increasing or decreasing marginal productivity of each input variable with fewer terms (and regression coefficients) than are required by other types of functions. This reduces the loss in degrees of freedom and saves time in computation. Also, a statistical test of significance can be readily applied to each regression coefficient. Since the Cobb-Douglas function is linear in the logarithms, it can be fitted by the least squares method of linear multiple regression. After the parameters of the function have been derived, it is relatively easy to compute marginal productivities of the resource inputs from the partial derivative of the function in respect to each resource.

The regression coefficients directly provide the elasticity of production of the respective factors which is the percentage change in output that would result from a one per cent increase in input of that factor with all other factors held constant. If an elasticity of production (regression coefficient) is less than one and greater than zero, the level of factor input falls within the rational area of production. Under this condition, additional inputs of the factor result in a decreasing marginal product.

The sum of the elasticities (regression coefficients) indicates the returns to scale associated with the resource combination represented by the production function. A sum of elasticities equal to one indicates constant returns to scale since a one per cent increase in all factor inputs would add the same percentage increase to the output of product; a sum equal to less than one indicates decreasing returns to scale, and a sum greater than one shows increasing returns.

There are other mathematical properties inherent in the Cobb-Douglas function which may present certain limitations to an adequate expression of factor-product and factor-factor relationships. This function imposes a constant elasticity of production for each factor over all ranges of input. In other words, it implies that the proportional change in output of product remains constant for any given proportional change in the input of a factor, regardless of the level of factor input.

The Cobb-Douglas function also imposes constant elasticities of substitution for all levels and combinations of factor inputs. This assumes that the slope of successive iso-product curves is the same at the points of intersection with a given scale line, or that the rate of factor substitution remains constant at all levels of output for a fixed proportion of factor inputs. Under this condition, the isoclines and scale lines are straight lines, and the proportion of different resources in the least cost combination does not vary with level of output.

The Cobb-Douglas function expresses either constant, increasing, or decreasing marginal productivity, but not a combination of these conditions. Consequently it would not be an appropriate function for expressing a factor-product relationship covering the range of increasing and decreasing marginal returns.

Iso-product contours under the Cobb-Douglas function become asymptotic to the input axes. The contours never intersect the axes, suggesting that the product can never be produces with one resource alone. This implies that complementarity of resources, or very nearly so, occurs in the extreme ranges of factor combination, but that substitution of resources is possible throughout the central portion of the iso-product contour.

A high degree of correlation between input variables may distort any estimates of resource productivity and, in some cases, may prevent the determination of true productivities. For practical purposes, it may be necessary to assume that joint factors, or perfect complementarity of factors, are present and that independent productivities do no exist. Under this assumption, the complementary factors would be treated as a single composite input.

Some Applications of the Cobb-Douglas Production Function in Analysis of Resource Productivity on Farms

Production functions, as a means of expressing the functional relationship between resource inputs and product output, have been used for two main purposes. First, they have been used to compute physical input-output ratios for technical units of production such as quantity of fertilizer applied per acre of crop, quantity of feed fed per animal, and combinations of feeds in a ration. Second, they have been used as a diagnostic tool to estimate the productivity of general categories of resources to provide an indication of the efficiency with which resources are employed on farms.

Use of the Cobb-Douglas function has been somewhat limited in studies of the physical input-output relationships in the application of variable resources to a fixed technical unit. In many cases, some other equation was more appropriate, both logically and statistically, for defining these physical relationships, particularly when only one or two variable resources were being examined. However, the Cobb-Douglas function has not been entirely disregarded in analyses of this aspect of resource utilization. In an exploratory study based on data from an experiment designed for a different purpose, Heady used a Cobb-Douglas function in deriving a total product function for milk with forage and grain as variable inputs. An isoquant (iso-product) equation also was obtained, from which marginal rates of substitution of forage for grain were estimated.[1] Another publication contains the results of a feeding experiment purposely designed for investigation of the input-output relationship in pork production with corn and a protein supplement as variable feed inputs. A Cobb-Douglas function was one of two

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[1]Earl O. Heady, "A Production Function and Marginal Rates of Substitution in the Utilization of Feed Resources by Dairy Cows," Journal of Farm Economics, XXXIII (November, 1951), pp. 485-498.

types of equations fitted to several groups of data. Functions were derived also for estimating marginal physical products, corn-protein combinations that produce a given gain in weight (iso-product curve), and marginal rates of corn/protein substitution.[1]

Many studies of resource productivity relationships for the farm as a unit, involving at least four or five categories of resources, have relied on the Cobb-Douglas production function. Tintner and Brownlee were among the first to apply this method of analysis. Using business records of 468 Iowa farms for 1939, they estimated the marginal value productivities and production elasticities for six classes of resources on five farm types (dairy, hog, beef feeder, crop and general). Total product was measured by gross income in dollars. The categories of resource inputs included land (in acres), labor (in months), farm improvements (in dollars), liquid assets (in dollars), working assets (in dollars) and cash operating expenses (in dollars).[2]

In a subsequent study by Heady, production functions were derived from a random sample of Iowa farms for the year 1939. The original data was collected by the survey method. The total value of products resulting from the year's operations

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[1]Earl O. Heady, Roger C. Woodworth, Damon Catron, and Gordon C. Ashton, "An Experiment to Derive Productivity and Substitution Coefficients in Pork Output," Journal of Farm Economics, XXXV (August, 1953), pp. 341-354.

[2]Gerhard Tintner and O. H. Brownlee, "Production Functions Derived from Farm Records," Journal of Farm Economics, XXVI (August, 1944), pp. 566-571.

was used as the measure of total output. Inputs were classified as real estate (inventory value of land and buildings), labor (in months), machinery and equipment (value of machinery and cash expenses for repairs, fuel and oil), livestock (value of livestock, livestock expenses and value of feed fed to livestock) and cash operating expenses (fertilizer, twine, custom work, etc.). Five groups of farms were defined according to location within the state in order to examine the returns to specific resources used on land in various productivity ratings, and to compare the returns for areas with different combinations of land, labor and other resources. A grouping was made also according to type of farm (crop, hog, dual purpose and dairy, general, and special). Finally, the farms in the sample were classified as large or small on the basis of total capital. This grouping was made to test a hypothesis of a range of increasing as well as decreasing returns to scale.[1]

The main objectives of a study by Heady and Shaw were to measure the marginal value productivity of resources used in different farming regions, and to predict the effect of different quantities of resources on the value of product produced. Random samples of farms were selected in 1951 for the Piedmont area of Alabama, North Central Iowa, Southern Iowa, and the dry-land wheat area of Montana. These areas differed considerably in the kinds and quantities of resources employed on farms and in the total amount of output per farm. Separate

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[1]Earl O. Heady, "Production Functions from a Random Sample of Farms," Journal of Farm Economics, XXVIII (November, 1946), pp. 989-1004.

production functions for livestock and crops were computed for each region to enable a comparison of resource productivity in primary (crop) production and secondary (livestock) production. The resource inputs for crops were classified as cropland (in acres), labor on crops (in months) and all capital services used in crop production (seed, fertilizer, tractor fuel, repairs, depreciation on machinery, etc.). The resource categories for livestock consisted of labor used on livestock (in months) and all capital inputs for livestock (value of grain, hay, pasture and other feeds, purchase value of feeder stock, depreciation and repairs for livestock buildings and equipment, etc.).[1]

An approach to the problem of deriving an independent production function for each enterprise on diversified farms was reported by Beringer. A single function derived from data for farms with several enterprises was considered unsatisfactory because such a function is unlikely to be a true expression of the input-output relationships for any one enterprise. It was reasoned that enterprise functions for multiple enterprise farms are independent of each other, and usually differ from the corresponding functions derived for specialized single enterprise farms. Data was obtained from detailed cost accounts for 27 dairy-hog farms in north-western Illinois. Separate production functions for the dairy, hog and crop enterprises, and a composite production function for the farm were derived from

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[1]Earl O. Heady and Russell Shaw, Resource Returns and Productivity Coefficients in Selected Farming Areas," <u>Journal of Farm Economics</u>, XXXVI (May, 1954), pp. 243-257.

the data. Input categories for both livestock enterprise functions included labor (in hours), feed (in dollars), cash expenses (in dollars), machinery investment (in dollars), livestock investment (in dollars), and housing (in animal units). Only four resource classifications were used for the crop enterprise function: labor (in hours), land (in acres), cash expenses (in dollars) and machinery investment (in dollars). Feed was excluded as an input variable in the composite function for the farm. Marginal value productivities were estimated for the various input categories used in the four production functions. The possibility of obtaining more meaningful productivity estimates from the enterprise functions was examined by comparing the enterprise functions with the composite farm function.[1]

Changes in the combination of resources employed on 146 commercial farms in north-central Illinois between the periods of 1936-39 and 1950-53 were studied by Swanson. A production function was derived for each period from data contained in account books kept on these farms. Resource inputs were classified as land investment, buildings and soil improvements, livestock investment, labor, power and machinery, and purchased feed. It was assumed that a reasonably efficient combination of resources had been attained on these farms during the years 1936 to 1939. The production elasticity necessary in the 1950-53 period to provide the marginal value productivity that prevailed

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[1] Christoph Beringer, "Estimating Enterprise Production Functions from Input-Output Data on Multiple Enterprise Farms", Journal of Farm Economics, XXXVIII (November, 1956), pp. 923-930.

in 1936-39 was calculated for each class of resources. These adjusted production elasticities were compared with the actual elasticities for 1950-53 to specify any significant differences in the resource combinations employed during the two periods.[1]

Reference could be made to many other studies of resource productivity for the farm unit. Those cited above were selected as illustrations of the problems to which the Cobb-Douglas production function has been applied as a method of analysis.

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[1]Earl R. Swanson, "Resource Adjustments on 146 Commercial Corn-belt Farms, 1936-1953", Journal of Farm Economics, XXXIX (May, 1957), pp. 502-505.

LIMITATIONS OF PRODUCTION FUNCTION ANALYSIS

The production function method of analyzing farm input-output data is subject to certain limitations which originate mainly in the aggregation of numerous resources into a few input categories. Basic characteristics of the data prevent the selection of a small number of independent input categories so that each retains a completely separate, distinct and additive relationship to the dependent output variable. If the input variables are not independent, the derived coefficients will not be an accurate expression of the functional relationships between resource input and product output. This difficulty may be partly overcome by combining the individual inputs into categories that are neither good substitutes not good complements for each other.

The input variables are not completely substitutable and divisible. Many resources used in agricultural production can be replaced by others to only a limited degree, or are available only in discontinuous and indivisible units. Consequently, something short of perfect substitutability and divisibility is attained by grouping resources into a small number of input categories.

Management is not included as a factor affecting output because a quantitative measurement of this input is most difficult. Thus, its omission from the analysis must introduce a bias into estimates of the effects of other inputs that are measured and taken into account.

The production function derived from a group of farms does not indicate the relationships that exist within an individual farm. The aggregation of resource inputs necessary to make the function manageable tends to obscure the diversity in quality of resources and in techniques of production that is normally found on a group of farms. Unless the farms in the sample are homogeneous with respect to methods of production, quality of resources used and kind of products produced, the estimated relationships will not be descriptive of any single farm. They are more likely to represent a "hybrid" of several different functions. Differences in resource qualities may be reconciled to some extent by expressing inputs in terms of their dollar values, but differences in production techniques are not necessarily reflected by input values. Some additional homogeneity is introduced by grouping and stratification of farms according to one or more factors or characteristics. However, the results of a production function analysis are not widely applicable to predicting the outcome of operations on individual farms.

Selection of the input categories and allocation of individual items to the appropriate category are among the more specific limitations to application of the production function method of analysis. Although many individual items must be aggregated in some manner, only a small number of input categories can be used in order to minimize the amount of mathematical computation and the loss of degrees of freedom. All of the important economic factors that contribute to the output of product, and that are also subject to quantitative measurement, are grouped into categories which, ideally, are functionally independent of each other. These categories then become the input variables in the equation chosen to describe the input-output relationships. If the input variables are not functionally independent, the result will be a large proportion of unexplained variability in the dependent variable, in this case, the total output of product. The value of the regression coefficients may be biased upwards if joint relationships exist with other factors that were not considered. Some form of preliminary analysis to determine the relationships between proposed input categories would serve as a guide to selecting categories that conform as near as possible to the requirements for independence.

In addition to the problems of identification and separation of the input categories, there are a number of important problems of measurement associated with a production function analysis. The most satisfactory method of measuring production function variables is in terms of physical units that express a standard quantity and quality of each factor. However, this is a practical impossibility for categories such as equipment and real estate due to the difficulty of reducing resource inputs of inherently different sizes and qualities to a common physical unit. In many cases, the only alternative is to measure these inputs in dollar values, which restricts application of the results of the analysis to a specific cost-price situation. With resource inputs and product output measured in terms of current prices, projection of the results is limited to situations that provide similar factor-product price relationships. On the other hand, if inputs and output are valued at long-time normal prices, the input-output relationships revealed by the analysis may be extended over a wider range of conditions. This, in turn, involves the selection of an appropriate period on which the long-time prices may be based.

Production functions can be derived from data that are expressed in either physical or value units. All quantities of input and output in the production function for a group of farms could be measured in dollar value. Ordinarily, however, inputs such as land and labor are measured in physical units, and output and other input categories are measured in dollars. Even though the product output and all resource inputs are measured in dollars, the technical relationships are the same as if all data were expressed in physical units; the value production function is only a translation of the physical production function.

These limitations indicate the necessity for caution in using the production function method of analysis and in interpreting the results obtained. A production function analysis of farm business data is most useful as a diagnostic device for defining problems that require further investigation with more specific techniques. The Cobb-Douglas function provides measurements of resource efficiency based on the performance of a group of farms. It is only a guide to specific resource use on the individual farm.

ANALYSIS OF THE EMPIRICAL DATA

Source of the Data

Empirical data used in this thesis were obtained from records of poultry enterprises[1] which tend to fall into one of the following types: (1) specialized full-time poultry farms on which income from other farm products and non-farm income were small; (2) part-time farms usually operated by persons in semi-retirement or who had a full-time occupation other than farming, with most of the farm income supplied by a small to medium size poultry flock, but with non-farm income often exceeding farm income; (3) full-time farms with poultry as a major or minor enterprise combined with one or more other enterprises such as small fruits or dairy. The data were obtained almost entirely from the specialized poultry farms. A few of the part-time farms were included and all enterprises of less than 200 laying birds were excluded.

Detailed information on the poultry enterprise was collected by the account book method for a twelve-month period starting at October 1. Enumerators completed the beginning inventory record and instructed the farm operator on the entries required in the account book. Supplementary forms were supplied

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[1]These poultry enterprise records were obtained in the Lower Fraser Valley and Vancouver Island areas of British Columbia by the Economics Division, Canada Department of Agriculture, and cover the three-year period of October 1, 1948 to September 30, 1951.

as an aid to keeping a daily record of such items as egg production, culling and mortality, and producers were asked to retain egg sales statements, feed bills and other similar records of receipts and expenses for the poultry enterprise. Most farms were visited twice during the year to check the account books or to assist in making the entries. At the end of the year a final call was made to complete the ending inventory and to collect the completed account book.

The inventories included values for each item under land, buildings, equipment, flock, feed and supplies, along with the description and quantity of these items. Current accounts included:

(1) Records of daily egg production, mortality and culling for hens and pullets;

(2) Amounts sold and cash receipts from market eggs, hatching eggs, fowl, fryers and broilers;

(3) The quantity and cost of each kind of feed fed to layers and young stock;

(4) Other expenses entirely chargeable to the poultry enterprise such as litter, chicks and other poultry purchased, hired labor, brooder fuel, disinfectant, repairs to poultry buildings and equipment;

(5) Amounts charged to the poultry enterprise for electricity, insurance, real estate taxes, and operating costs of car, truck and tractor, as estimated from the total farm expense for each item;

(6) Labor time in hours, as estimated by each producer,

divided between caring for the laying flock and raising young birds.

A total of 299 completed account books, each covering a period of 12 months, was obtained during this study. Some producers co-operated throughout the three-year period of the study, others participated for only one or two years. Consequently, the information collected in each year pertained to a somewhat different group of poultry enterprises.

Income from the sale of market eggs was the major source of revenue for more than one half of the enterprises. For the remainder, market eggs were in most cases the main product, supplemented by the sale of hatching eggs and/or poultry meat (fryers and broilers) on a seasonal basis. Cull layers sold as fowl were a source of income common to all enterprises. Thus, it was possible to define four distinct types of poultry enterprise based on the combination of products. These were designated as:

(1) Specialized market egg;

(2) Combination market and hatching egg;

(3) Combination market egg and poultry meat;

(4) Combination market egg, hatching egg and poultry meat.

All data used in this section were taken from records for the specialized market egg enterprises. This group was selected with the objective of obtaining some degree of homogeneity in the general characteristics of enterprise organization, production methods and kind of product. It includes a total of 168 records comprised of 66, 57 and 45 records for the respective years 1949, 1950 and 1951 ending on September 30.

Method of Analysis

Total output and the various input categories selected for analysis of the data by the production function were designated as follows:[1]

X_1, total output measured in dollars;

X_2, annual input of real estate and equipment measured in dollars;

X_3, laying flock input measured in layer years;

X_4, labor input measured in hours;

X_5, feed input measured in dollars;

X_6, other cash inputs measured in dollars.

Output is a function of the five input categories which include the total resources used for the market enterprise. Accordingly, the production function can be expressed in the general form of

$$X_1 = f(X_2, X_3, X_4, X_5, X_6).$$

In the logarithmic form of the Cobb-Douglas function, the input-output relationship is specified as:

$$\log X_1 = \log a + b_2 \log X_2 + b_3 \log X_3 + b_4 \log X_4 + b_5 \log X_5 + b_6 \log X_6.$$

Values for total output and for each of the input categories, calculated for each enterprise from data contained in the enterprise records, were converted to logarithms to allow estimation

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[1]See Appendix IV for details of the items aggregated in total output and in the input categories.

of the function in this form. The Cobb-Douglas production function for each year was derived from the respective sets of logarithmic values by the least-squares method of fitting a linear multiple regression equation.[1]

The Results and Their Statistical Reliability

The following production functions were derived:

Year 1949

$$\log X_1 = \log 0.4250 - 0.0106\log X_2 + 0.2600\log X_3 - 0.0829\log X_4 + 0.7919\log X_5 + 0.0442\log X_6.$$

Year 1950

$$\log X_1 = \log 0.4792 - 0.0587\log X_2 + 0.1514\log X_3 + 0.0130\log X_4 + 0.6514\log X_5 + 0.2323\log X_6.$$

Year 1951

$$\log X_1 = \log 0.4847 - 0.0206\log X_2 + 0.2464\log X_3 + 0.0102\log X_4 + 0.7375\log X_5 + 0.0282\log X_6.$$

Statistical measures relating to the reliability of the production functions are given in Table 1.

The standard error of estimate indicates the discrepancy between estimates of total output based on the production function and the actual total outputs contained in the sample. It is a measure of the range of error that could be expected in estimates of total output. The statistical meaning of this measure is that the total output estimated from the production

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[1]Computation of the regression coefficients, and the tests for reliability of the regression coefficients and the production function are outlined in Appendix V.

function would probably deviate from the actual total output by less than one standard error of estimate in 68 per cent of the cases and by less than two standard errors in 95 per cent of the cases.

TABLE 1

MEASURES OF RELIABILITY OF THE PRODUCTION FUNCTION

	1949	1950	1951
Adjusted standard error of estimate (log value)	0.0550	0.0754	0.0608
Range of two standard errors for an estimated total output of $1,000: From	776	707	756
To	1,288	1,415	1,323
Range of two standard errors for an estimated total output of $10,000: From	7,762	7,066	7,558
To	12,882	14,151	13,231
Adjusted coefficient of multiple determination	0.9526	0.9381	0.9463
Adjusted coefficient of multiple correlation	0.9760	0.9686	0.9728
Standard error of coefficient of multiple correlation	0.0057	0.0079	0.0076
Significance of coefficient of multiple correlation (t-value)	36.201[a]	29.227[a]	27.937[a]

[a]Significant at one per cent level of probability ($P = 0.01$).

Since the standard error of estimate is expressed as a logarithm, it must be applied to the logarithmic value of an estimated total output. The range of error when converted from logarithms to natural numbers does not remain constant and the confidence intervals are not symmetric, but varies directly and proportionally with the size of the estimated output. For this reason, the range of two standard errors of estimate for total outputs of \$1,000 and \$10,000 are given in Table 1. Since the output of nearly all enterprises was between \$1,000 and \$10,000, these ranges indicate the approximate limits within which both the estimated and the actual outputs could be expected to fall. Although the range of error in estimates appears large, it would be expected that 95 per cent of the actual outputs are within these limits.

As indicated by the coefficients of multiple determination, approximately 95 per cent of the variance in total output (gross income) of the market egg enterprises is explained by the input categories of real estate and equipment, laying flock, labor, feed, and other cash expenses. Inputs other than these apparently had little effect on total output.

The coefficients of multiple correlation show that a high degree of association exists between total output and the collective input categories. (Values as high as 0.97 are somewhat exceptional, since a value of 1.0 for the correlation coefficient indicates perfect correlation). All of the multiple correlation coefficients are statistically significant at the one per cent level of probability.

The regression coefficients of the production functions derived for each year, along with the standard errors and t-values, are listed in Table 2. A regression coefficient in the Cobb-Douglas function is the production elasticity of the respective resource category. It indicates the percentage change in total output that is associated with a one per cent increase in the input of a resource, with the input of all other re-

TABLE 2

REGRESSION COEFFICIENTS, STANDARD ERRORS, VALUES OF t, AND PROBABILITIES

	1949	1950	1951
Regression coefficients (production elasticities):			
Real estate and equipment (X_2)	-0.0106	-0.0587	-0.0206
Laying flock (X_3)	0.2600	0.1514	0.2464
Labor (X_4)	-0.0829	0.0130	0.0102
Feed (X_5)	0.7919	0.6514	0.7375
Other cash expenses (X_6)	0.0442	0.2323	0.0282
Sum of elasticities	1.0026	0.9894	1.0017
Standard error of regression coefficients:			
Real estate and equipment	0.0367	0.0600	0.0498
Laying flock	0.1134	0.1773	0.1469
Labor	0.0573	0.0708	0.0564
Feed	0.1461	0.2330	0.1891
Other cash expenses	0.0386	0.0570	0.0606
Values of t:			
Real estate and equipment	0.289	0.978	0.413
Laying flock	2.293	0.854	1.677
Labor	1.446	0.184	0.181
Feed	5.412	2.796	3.901
Other cash expenses	1.145	4.075	0.465
Probabilities:[a]			
Real estate and equipment	0.387	0.168	0.341
Laying flock	0.014	0.199	0.051
Labor	0.081	0.427	0.429
Feed	b	0.004	b
Other cash expenses	0.130	b	0.323

[a]Probability that an equal or greater value of the observed regression coefficient could have occurred because of sampling variation.

[b]Less than 0.0005.

sources held constant. For example, the production elasticity (regression coefficient) of the laying flock input in 1949 is 0.2600. This means that a one per cent increase in the input of laying flock alone produced an increase of 0.26 per cent in total output.

The magnitude of the production elasticity also reveals the nature of marginal returns to the production factor. Diminishing marginal returns exist for the resource categories with an elasticity of production that is positive and less than unity. Total output increases, but at a decreasing rate, as additional inputs of any one of these resource categories are combined with fixed quantities of all other resources. Negative marginal returns are indicated for the resource categories with a negative elasticity of production. Precisely interpreted, this means that further increases in the input of any one of these resource categories causes a decline in total output. Such conditions, however, appear inconsistent with the levels of resource use and production that are usually found on farms. Normally, the input of any one resource would not be extended to the point where additional applications caused a reduction in the total product. Since none of the negative elasticities differ significantly from zero (at the five per cent level of probability), it is more realistic to assume that the marginal return is zero, and that the total output is unaffected by increasing the input of these resource categories.

Returns to scale, as indicated by the sum of elasticities, are constant for all practical purposes. Constant returns to scale exist when a proportionally equal increase in all resources is accompanied by the same proportional increase in product. For example, a ten per cent increase in the input of all resource categories would enlarge the total output of product also by ten per cent.

The standard error of a regression coefficient indicates the reliability of the regression coefficient as a measure of the functional relationship between an input category and the output of product. The regression coefficients are only the best estimates of the input-output relationships in market egg production as derived from a sample of enterprises and, therefore, are likely to deviate from the true parameters due to chance variation in the sample. The standard error of a regression coefficient provides a measure of the probable range of the difference between the estimated and the true input-output relationship. A regression coefficient and its standard error are interpreted as follows: The chances are 68 out of 100 (i.e., P = 0.68) that the true regression coefficient falls within the range of one standard error on either side of the estimated regression coefficient. For example, the probability is 0.68 that the true regression coefficient of the laying flock input in 1949 falls between 0.26 plus and minus 0.11, or between 0.37 and 0.15. Similarly, the probability is 0.95 that the true regression coefficient falls between 0.26 plus and minus two standard errors, or between 0.48 and 0.04. Stated in terms of the input-output relationship for 1949, an addition of one per cent to the laying flock input would increase the total output by 0.15 to 0.37 per cent in two out of three cases, and by 0.04 and 0.48 per cent in 19 out of 20 cases.

A regression coefficient was appraised for significance according to the probability that it differed from zero due to sampling variation, as determined by the t-test. The probabilities (in Table 2) indicate that the regression coefficients for laying flock and feed in 1949, for feed and other cash expenses in 1950, and for laying flock and feed in 1951 differ significantly from zero at the five per cent level of significance. Except for the labor input in 1949, the coefficients for all other input categories are non-significant at the ten per cent level of significance.

INTERPRETATION OF THE RESULTS

The average levels of input for the various resource categories, as measured by the geometric mean, are shown in Table 3. The estimated marginal value products (in the same table) relate specifically to these input levels because a marginal product derived from the Cobb-Douglas function varies with the quantity (input level) of resources. The marginal value products are the returns which, on the average, could be expected from the addition of one more unit of the respective resource categories. For example, increasing the laying flock input by one layer year in 1949 would have added $2.25 in output of product, with other inputs held constant at the level of the geometric means. Similarly, an additional dollar spent on feed in 1949 would have returned $1.33 in output of product.

The marginal value products in the second part of Table 3 are expressed as dollars per unit of input used in deriving the production function. The laying flock and labor inputs are measured in layer years and hours, respectively, while the other inputs are measured in dollars. In the third section of the table, all marginal productivities are expressed as dollars per dollar of input. In this form, a marginal value product indicates the addition to total output of product obtained from increasing the input of a particular resource category by one dollar, when all inputs are at the level of the geometric means.

Furthermore, it is equivalent to the ratio of the marginal value product to the factor price.

TABLE 3

GEOMETRIC MEANS AND MARGINAL VALUE PRODUCTS OF INPUT CATEGORIES

	1949	1950	1951
Geometric means:			
Total output ($)	4,022	3,274	5,431
Real estate and equipment ($)	316	278	321
Laying flock (layer years)	464	418	491
Labor (hours)	1,909	1,486	1,542
Feed ($)	2,396	2,231	2,740
Other cash expenses ($)	285	257	326
Marginal value products per unit of input:[a]			
Real estate and equipment ($)	-0.13	-0.69	-0.35
Laying flock ($)	2.25	1.19	2.73
Labor ($)	-0.17	0.03	0.04
Feed ($)	1.33	0.96	1.46
Other cash expenses ($)	0.62	2.96	0.47
Marginal value products per dollar of input:			
Real estate and equipment ($)	-0.13	-0.69	-0.35
Laying flock ($)[b]	1.09	0.55	1.18
Labor ($)[c]	-0.33	0.06	0.07
Feed ($)	1.33	0.96	1.46
Other cash expenses ($)	0.62	2.96	0.47

[a]Estimated at geometric means of total output and input categories.

[b]Based on the average value of a pullet layer: $2.07, $2.18 and $2.32 for the respective years.

[c]Based on the average hourly wage paid to hired labor: $0.52, $0.49 and $0.59 for the respective years.

In view of the inequality of the marginal value products per dollar of input, the theoretically optimum combination of resources was not attained for these market egg enterprises in any of the three years. In general, considerably higher returns are indicated for inputs of laying flock and feed than for the other resource categories. Subject to any practical limitations imposed by technical conditions of production and physical characteristics of the resources employed, additional inputs of laying flock and feed would have been most effective in increasing profits (or decreasing losses) in 1949 and 1951. An adjustment of this nature would have been inappropriate in 1950 for two reasons; the marginal value products of the laying flock and feed inputs were exceeded (1) by their marginal costs and (2) by the marginal value product of other cash expenses.

It is equally apparent that adjustments in the resource inputs from year to year, made in response to variations in the factor-product price relationships, did not constitute a major improvement in resource allocation. If these changes in the quantity of inputs had resulted in a closer approximation to the optimum resource combination, the inequality of the marginal value products would have been reduced in successive years. However, there is little evidence to support this proposition. For example, the marginal value products of the real estate and equipment input and the laying flock input differ by $1.22 in 1949, by $1.24 in 1950, and by $1.53 in 1951. Likewise, the difference in the marginal value products of laying flock and labor is $1.42 in 1949, $0.49 in 1950, and $1.25 in 1951. Comparing other pairs of marginal value products in this manner reveals a similar erratic fluctuation in their differences.

At this point, it is possible to demonstrate that estimates of resource productivity derived by the production function and the residual methods of analyzing farm input-output data may lead to quite different conclusions about the relative efficiency of resource combinations. In applying the residual method of analysis, labor return or some similar measure computed as a residual quantity serves as an indicator of efficiency in resource use. The average labor return for the market egg enterprises was $1,191 in 1949, $617 in 1950, and $2,296 in 1951 (Appendix I, Table 8). Accordingly, it would be concluded that the economic efficiency of the resource combinations employed during these three years was lowest in 1950 and highest in 1951. This distinct fluctuation in efficiency of resource use implied by the variation in labor return is contrary to the generally inefficient resource combinations employed each year that is indicated by the continued inequality in the marginal value products per dollar of input.

A further deficiency of the measures of residual returns is that they provide only a general index of profits or resource efficiency and do not point out how the resource combination could be altered to gain a larger net income. Removal of this limitation has been attempted by using the average returns per unit of resource input, such as the return per hour of labor, for comparison with a pre-determined standard of performance. These measures, however, retained all the inherent defects of the residual method, and, as a result, could be misleading as indicators of the adjustments in inputs needed to increase net income. For example, the average return per hour of labor computed by the residual method was $0.55 in 1949,

$0.26 in 1950, and $1.26 in 1951 (Appendix I, Table 8). Following the procedure of the residual method, any departure from the most profitable input of labor could be determined by comparing this return with the average cost of hired labor of $0.52, $0.49 and $0.59 in the respective years. On this basis, the labor input would appear to be near the optimum in 1949, excessive in 1950, and restricted in 1951. On the other hand, the marginal value product of labor (Table 3) indicates that an excessive amount of labor, relative to other inputs, was employed in all years. Assuming that the cost of unpaid labor was the same as the wage paid to hired labor, a smaller input of labor would have increased the net income during each year.

Within the limitations of the production function method of analysis, the marginal value products of the various resource categories reveal the (average) results of the decisions made by producers in combining the resources at their disposal for the production of market eggs. There are a number of restrictions associated with production techniques and practices, rigidity and indivisibility of resources, and imperfect knowledge of production and prices in the future that partially explain the failure of producers to achieve the optimum or most profitable combination of resources. Some of these restrictions also may reduce and even prevent the possibility of effectively improving the allocation of resources within a short period of time.

The physical indivisibility of the buildings required for the poultry enterprise precluded a prompt adjustment in the input of housing services. A laying house, once it was built, became a fixed quantity which could not be expanded and contracted to exactly meet the fluctuations in housing requirements arising from variations in the size of laying flock within the production period. The maximum capacity was established within a narrow range by the floor area of the building. Overcrowding for any extended period of time was almost certain to create problems in sanitation, control of disease and other factors related to maintainence of the physiological condition of the layers. Consequently, the construction of new buildings was the only effective means of obtaining additional housing. A current need, however, could not be met in this way due to the time required for planning and erection of a building. Furthermore, the stock of housing services supplied by a building was not depleted during a single production period, but was extended into the future over the life span of the building. For this reason, a new laying house would not likely be constructed unless the capital outlay could be recovered from the anticipated increase in income from additional housing services in the following years. Income from the market egg enterprise, particularly in 1949 and 1950, was not conducive to optimistic expectations for the future, and undoubtedly deterred most producers from extending the facilities for housing layers. Aside from the depressed prospects for future income, many producers probably had little or no inclination toward any physical expansion of the enterprise. For some producers, the production of market eggs was undertaken on a small scale to supplement their income from other sources, and was not necessarily essential to their livelihood. In other cases, producers in the older age groups were often physically unable to do the work required in the operation of a larger enterprise.

The lack of flexibility in the laying house capacity and the current practices in laying flock replacement contributed to under-utilization of the laying houses for most of the year. The number of layers reached a maximum usually in September and October when pullets raised for flock replacement had been placed in the laying houses. Since additions to the laying flock were seldom made during the production period, culling and mortality caused a continual decline in the number of layers. As a result, the number of layers was inadequate for complete utilization of the existing housing facilities except during the first two to three months of the production period. In effect, the practice of adding replacements to the laying flock only once a year created the need for housing facilities in excess of that actually required. Evidence of this condition is provided in Table 4 (Appendix I) by the ratio of the number of layer years (as a measure of the actual size of laying flock for the year) to the flock inventory at October 1 (as an approximation of the laying house capacity). On this basis, laying houses were occupied at 77, 76 and 83 per cent of their capacity for the respective years.

Most of the equipment required for the laying flock was subject to the same conditions that were responsible for under-utilization of the existing housing services. Installations such as feeder, water system and water troughs formed an integral part of a laying house. Although this equipment was less durable than the building, it did not require annual replacement but, with occasional minor repairs, remained serviceable over a period of several years.

The generally accepted practice among producers of rearing pullets for replacement of the laying flock necessitated specialized buildings and equipment that were unemployed for several months during the year. For the typical producer who purchased all his chicks in a single lot, this was a seasonal activity which extended over a six month period beginning in late February or early March. Under normal weather conditions, the brooder houses and brooding equipment were used for about two months, and seldom for more than three months. These buildings and equipment then remained idle for the rest of the year, with the occasional exception of a brooder house that served also as a shelter for young pullets on range. Shelters, feeders, water troughs and other range equipment were essential for three to four months while the growing pullets were kept on range, but were not used for any other purpose during the year.

Apart from the small area occupied by the laying houses, land was needed only as range for young pullets. Although this land was quite necessary under the prevailing practices in raising pullets, it also was generally unused for as much as nine months of the year. In addition, the amount of range land was frequently excessive in relation to the number of pullets raised. Many producers were not compelled to limit the range area to actual requirements because alternative and competitive uses for the land did not exist on these single enterprise farms.

This applied particularly to producers with small flocks and to those owning several acres of land suitable for poultry range.

These conditions, undoubtedly, were largely responsible for the low marginal value product of the real estate and equipment input in each of the years. For all practical considerations, the land, buildings and equipment associated with the market egg enterprise constituted a fixed quantity of input that could not be quickly modified. At the same time, the current production practices were such that the services supplied by these resources were either under-employed or unused for several months during the year. Also, most of the annual costs incurred in providing these resources (i.e., depreciation, interest on investment, taxes and insurance) were unaffected by using the services less intensively. As a result, the amounts of land, buildings and equipment necessary to meet a maximum requirement during part of the year were in excess of that actually needed at other times. In effect, the under-utilized and unused real estate and equipment represents an excessive application of these resources relative to the input of other resources. However, the inherent inflexibility of these resources combined with the usual production practices offered little opportunity to avoid this condition of waste and inefficiency in resource use.

In contrast to the rigidity of input of buildings and equipment, the inputs of all other resources were much more flexible. The greater flexibility of these inputs originated primarily in one or both of the following conditions: (1)

divisibility of the resource into small units which enabled more refined adjustment in the level of input at any time, and (2) complete consumption of the resource in the production process within a single production period. As a result, the inputs of these resources could be controlled more effectively by the market egg producer, and adjustments could be readily made in the level of input which, in his judgement, were appropriate to changes in the input-output price relationship.

For all but a very few producers who purchased layers during the year, the maximum input of laying flock was established by the number of layers housed at the beginning of the production period, that is, the pullets raised for flock replacement plus any layers retained in the flock for their second year of production. However, changes in the initial size of laying flock could be made from year to year, within the capacity of the laying houses, by either increasing or decreasing the number of pullets and/or hens. The initial flock inventory[1] indicates that producers accomplished this adjustment mainly by varying the number of hens.

In effect, the number of pullets had been determined in advance by the number of chicks that were purchased five to six months previously. The depressed prices for eggs during February and March,[2] when the decision was made regarding chick purchases, did not encourage producers to increase their laying

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[1]See Table 4, Appendix I.

[2]See Table 9, Appendix II.

flock for the coming year. At the same time, expectations of higher prices for eggs during the year ahead may have deterred any plans for reduction of the laying flock. Confronted with these alternatives, producers apparently chose to compromise by neither increasing nor decreasing the number of pullets in the laying flock at October 1 in both 1949 and 1950.

The higher annual production of eggs by a pullet, as compared with a hen, influenced the majority of producers to follow the practice of completely replacing the laying flock each year. Despite this advantage of the all-pullet flock, some producers preferred to keep some layers through at least part of a second period of production. These layers were not usually selected until the onset of the moulting period in August and September. Apparently, the current price of eggs was not a major consideration of producers in deciding on the number of layers to be kept for a second year. Although egg prices had advanced sharply in August and September of 1949, the number of hens in the laying flocks at October 1 had been reduced from the previous year. In contrast to 1949, egg prices during August and September of 1950 had remained relatively stable at a lower level, but laying flocks contained a larger number of hens. Presumably, producers relied to a greater extent on their expectations of egg prices for the year ahead, which proved to be correct for both years, and also on current and anticipated production costs, particularly as affected by the price of feed.

Within the limit imposed by the initial size of laying flock, producers possessed a large measure of control over the laying flock input for the year as a whole. Except for the inevitable decrease caused by mortality, the size of laying flock could be reduced easily and quickly at any time by culling any number of layers that still remained in the flock. The annual mortality of layers relative to the initial number of layers was approximately the same for each year, varying only from 11 to 12 per cent for hens and from 14 to 16 per cent for pullets. Therefore, assuming a random distribution of mortality within each year, fluctuations from year to year in the net size (input) of laying flock, measured in layer years,[1] can be attributed almost entirely to the effects of culling.

Producers reduced the laying flock input slightly from 1949 to 1950, but expanded it substantially in 1951. Although this adjustment was partly the result of fluctuation in the initial number of layers, it was achieved also by varying the length of time that layers were retained in the flock. For the flock as a whole, a layer was kept for an average of 282 days in 1949, 276 days in 1950 and 302 days in 1951. However, an appreciable difference existed between hens and pullets in this respect. A hen was kept for an average of 190, 193 and 255 days in the respective years, or approximately nine weeks longer in 1951 than in either of the two previous years. In comparison, a pullet was retained for a longer and more nearly equal period; 308 days in 1949, 298 days in 1950 and 318 days in 1951. Thus, producers reduced the laying flock input for 1950 mainly by

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[1]See Table 14, Appendix I.

culling pullets at an earlier date than in 1949. On the other hand, they expanded the flock input for 1951 by delaying the culling of both hens and pullets to a later date. Most of this increase over the 1950 input was gained from the longer period that hens were retained in the flock, which amounted to 65 days as compared with 20 days for pullets. Furthermore, the initial flock inventory for 1951 contained more hens but the same number of pullets.

Appraised on the basis of the marginal value product per dollar of input, the laying flock input was close to the optimum in 1949 although a larger input would have produced a small increase in profits. Producers, in their anxiety over the unfavorable feed-egg price relationship that existed from December through July,[1] may have been premature in their decisions on culling, and so reduced the flock input more than was necessary. They decreased the flock input for 1950 in response to a further decline in the feed-egg price ratio, but the marginal value product indicates that this reduction should have been much larger. As previously stated, the smaller input of laying flock for 1950 resulted mainly from increased culling of pullets. Considering that a hen produced 30 per cent fewer eggs than a pullet,[2] increased culling of hens would have been more appropriate in adjusting the flock input. Moreover, it is doubtful that any significant economic benefit was derived from retaining hens in the laying flock during a year when egg prices

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[1]See Table 10, Appendix II.

[2]See Table 4, Appendix I.

were so depressed. Producers expanded the flock input for 1951 to take advantage of the vastly improved egg prices that prevailed from January through September, but still fell short of attaining the most profitable input. Extension of the average productive period to 255 days for hens and 318 days for pullets indicates that producers delayed culling as long as possible in order to obtain the maximum use of the available layers. Consequently, limitations imposed by the initial size of laying flock prevented any further addition to the flock input for 1951. This restriction would not have existed except for the inability of producers to predict the sharp advance in egg prices during 1951. Otherwise, it is probable that they would have raised a larger number of pullets during the previous year.

Total feed input for the market egg enterprise was divided between the laying flock and the young pullets raised for flock replacement, with the larger proportion (at least 80 per cent) going to the laying flock. Young pullets consumed approximately the same quantity of feed each year because the number of pullets raised, as shown by the laying flock inventory at October 1, did not vary. Consequently, changes in the feed requirements of the laying flock were mainly responsible for variations in the quantity of feed used by the enterprise. Feed for the laying flock amounted to 626, 619 and 660 cwt., and cost $2,270, $2,317 and $2,542 in 1949, 1950 and 1951 respectively. Since the cost of this feed accounted for 65 to 70 per cent of all cash costs for the enterprise, there was considerable incentive for producers to exercise all possible control over the feed input.

Total feed input for the laying flock could be completely controlled through the number of layers kept during the year and the amount of feed fed per layer. However, the possibility of adverse effects on egg production deterred producers from varying the level of feeding except within relatively narrow limits. Some evidence of this restriction is provided by the small fluctuations in the annual consumption of feed by layers which, expressed in pounds of feed per layer year, increased from 110 pounds in 1949 to 113 pounds in 1950 and decreased to 106 pounds in 1951. As a result, producers were forced to rely almost entirely on adjustments in the size of laying flock for regulation of the feed input. Thus, the conditions that influenced the decisions of producers in regard to the flock input (that is, the initial number of layers and the culling of layers during the year) were equally pertinent to the feed input. In fact, culling was done primarily to maintain a minimum input of feed relative to the output of eggs, and only incidentally to reduce the number of layers. In attaining this objective, the removal of unproductive layers from the flock was most essential because they consumed nearly as much feed as layers in full production.

Producers were under little or no restraint in making adjustments to the labor input. Labor for the market egg enterprise was provided almost entirely by the producers and their families, so that an adequate supply was readily available at all times. In addition, there were inherent inflexibilities in the labor input which impeded a precise alteration in the amount of labor used in conformity to variations in the labor requirement of the enterprise. Although producers possessed virtually unrestricted control over the labor input, its persistently low marginal value product can be attributed mainly to the use of an excessive amount of this resource relative to other resources employed in the market egg enterprise.

Several circumstances contributed to this seemingly extravagant use of labor for the poultry enterprise. Foremost among these was the absence of any strong inducement for most producers to make a deliberate effort toward minimizing the labor input. Since labor was supplied mainly by the producers and their families, the small cash outlay for labor had no moderating influence on the total amount of labor that was used. In addition, there was generally no other enterprise on these farms to provide competition for the use of this unpaid labor. As a result, many producers were able to accomplish the necessary work at a leisurely pace. Most small and medium size enterprises were only a part-time activity in which labor efficiency received no particular emphasis. Furthermore, many of these enterprises were operated by persons in older age groups whose physical capacity for work was less than that of younger and more active persons.

Only an incomplete and somewhat superficial explanation can be given for variations in the marginal value product of other cash expenses because of the several heterogeneous and unrelated items that comprise this input category. However, at least 50 per cent of these cash expenses was incurred in each year as the cost of purchased chicks. Also, the marginal value product of these cash expenses fluctuated in the opposite direction to changes in the level of egg prices. Although this relationship could be entirely coincidental, it does suggest the possibility that the practice of raising pullets for layer replacements provided producers with the largest return in 1950 when egg prices were low, and conversely, with the smallest return in 1951 when egg prices were high.

APPENDIX I

PHYSICAL AND FINANCIAL ORGANIZATION OF THE MARKET EGG ENTERPRISE

The following is a description of some of the characteristics of the land, buildings, equipment, laying flock and production practices associated with the market egg enterprises.

Except for building sites, the land requirements of the poultry enterprise were largely limited to range for young pullets during the summer months. Almost without exception, layers were confined to the laying house throughout the year. Generally, the land used for poultry had been cleared and seeded at some time with a mixture of grass and clover. However, native wild grasses and weeds often predominated as a result of inadequate maintenance of the original seeding. The amount of land used for poultry range was determined more by the total acreage in the farm unit than by the number of young birds raised. On small parcels of less than three acres, young pullets usually had access to all land that was not required for the dwelling and other residential purposes, regardless of the number of young birds raised. On larger farm units, the amount of land available for poultry was less restricted which, in some cases, induced an excessive use of land for this purpose. Only the land actually used for poultry during the year was recorded in the inventory. This excluded any additional land that would be used in following years if rotation of the poultry range was practised to control contamination of the soil by diseases and parasites. Land values varied from \$50 to \$500 per acre. In general, the larger enterprises were located on the more expensive land. However, value of the land appeared to be related mainly to the size of parcel, the presence or possibility of other agricultural uses, and whether the land was used primarily for agricultural or residential purposes and, as a consequence, provided little indication of the suitability of the land for poultry production.

Poultry buildings included laying houses, brooder houses and any part of other buildings that was used for egg and feed storage. Occasionally, a vacant pen in the laying house was used as a brooder house. Brooder houses frequently served also as shelters for young pullets on range.

Most laying houses were one-storey, wooden frame buildings on a foundation of concrete blocks, cedar blocks or poured concrete. Wooden flooring was most common, although concrete floors were used in a few cases. Open screened windows for lighting and ventilation of the building extended along one side of the laying house. A room for feed storage was usually incorporated in the laying house. Storage space for eggs was generally provided in the basement or other cool area in the operator's dwelling, and seldom in a separate building specially constructed for this purpose. In most laying houses, manure under the perches was collected on a board platform raised above the floor level, and was removed at least twice a week and often daily. A less common arrangement for manure collection consisted of a pit under the perches which was cleaned only once or twice during the year. Individual laying nests were provided in many laying houses, although the larger community nest accommodating several layers at the same time was favored by some producers. Lights were installed in nearly all laying houses and, in a few instances, were controlled by means of an automatic time-switch.

Typically, a laying house contained from two to four laying pens, although a few houses contained six or more pens. The average laying pen had a floor area of 512 square feet, which is approximately equal to dimensions of 20 x 25 feet. However, there was some variation in both the floor area and the dimensions of laying pens. The floor area was between 350 and 449 square feet in most cases, and was seldom larger than 750 square feet. Many laying pens were 20 x 20 feet in size, but the dimensions ranged from 16 to 24 feet wide and from 20 to 40 feet long.

Based on the operators' estimates of the normal capacity of their laying pens, the average laying pen with 512 square feet of area would accommodate 141 layers, providing 3.6 square feet of floor per layer. However, the number of layers that could be placed in a laying pen depended largely on the size of the pen. Consequently, the estimated normal capacity ranged from 100 to 149 layers for most laying pens and seldom exceeded 300 layers.

The laying houses and the feed storage room for the average enterprise contained 3,255 square feet of floor area with a normal capacity, again based on the operators' estimates, of 857 layers. The cost of replacing the laying houses was estimated at $3,060, or $0.94 per square foot of floor area. This was the cost, as estimated by the operators, of replacing the existing buildings at the prices of materials and labor that prevailed during the period of the study. It included the cost of nest, perches, dropping boards or pits, and electrical wiring and fixtures, but excluded water and feed equipment.

In most cases, brooder houses were single-wall frame construction with no insulation except double-flooring, and were permanently located on some kind of foundation. Ventilation usually was provided by adjustable windows in the front of the building and ventilators in the roof, although louvered air outlets in the gables or under the eaves were also used instead of roof ventilators. The dimensions varied considerably, but two-thirds of the brooder houses were 10 to 14 feet wide and 10 to 18 feet long. A fairly standard size of brooder house, measuring 12 x 14 feet, provided floor space for approximately 400 chicks.

Brooder houses for the average enterprise contained 439 square feet of floor area. The estimated replacement cost was $527, or $1.20 per square foot of floor area.

Other building requirements of the poultry enterprise included a place for cleaning, packing and storing eggs which was usually located in part of the basement or a porch in the farm house. Occasionally, part or all of a shed was used for storage of tools, equipment, litter and other supplies. The replacement cost of these buildings, or the portions used for poultry, amounted to $160.

The main items of equipment in the laying house were the water system and mash feeders. Water under pressure was piped to float-controlled water troughs in each laying pen or to a tap inside the laying house on two-thirds of the farms. On the remainder, water was carried in pails to each laying pen from a source outside the laying house. The mash portion of the layer ration usually was fed from shallow troughs, mounted on low stands, which were filled at least once daily. Self-feeders equipped with hoppers to contain mash for several days feeding were used on nearly 20 per cent of the farms. Other laying house equipment included a wheelbarrow, feed pails or a feed cart, egg baskets, and hand tools for cleaning the laying pens. The hover type of brooder heated by oil or electricity, with a rated capacity of 500 chicks, was most popular. Water fountains and chick feeders completed the brooder house equipment. A standard set of equipment on the range consisted of shelters, feeders, water fountains, and fencing. Range shelters for young stock were typically low, open-side, movable structures with mesh wire under the perches. Many of these shelters were 8 x 10 feet or 10 x 12 feet in size, providing room for 120 and 210 young pullets, respectively. Egg cleaning was done by hand with an abrasive buffer in most cases, although a circular cleaner powered by an electric motor was used on a few farms. Egg cases were supplied to the producers by the wholesale buyers.

With the exception of the larger enterprises, most laying flocks were composed entirely of one breed. Leghorn, New Hampshire and the Leghorn-New Hampshire cross were by far the most prevalent breeds, accounting for nearly 90 per cent of all layers. Leghorns predominated in both the small and large size flocks, while New Hampshires out-numbered all other breeds in the medium size flocks. The Leghorn-New Hampshire cross was most prominent in the smaller flocks.

Laying birds were divided into two classes according to age. Layers in their first year of production were classed as pullets, and those that had completed one year of production were classed as hens.[1] For all farms, about 75 per cent of the laying flock was replaced annually, as indicated by the percentage of total layers at September 30 that were classified as pullets. Complete replacement of the laying flock was a common practice among the smaller enterprises. However, many of the larger flocks were only partially replaced each year, and so contained hens as well as pullets. Some depletion of the laying flock resulted throughout the year from routine culling and losses due to disease and other causes. The main disposal of layers normally occurred toward the end of the production year, usually in August and September. At that time, young pullets starting to lay were taken from the range and placed in the laying houses.

Nearly all pullets were raised from day-old chicks purchased from a commercial hatchery during the period of

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[1]In this study, layers over 18 months of age were classed as hens.

February 15 to April 15. An average of 81 per cent of the chicks reached the laying age of approximately five and one-half months. All brooding and range losses, including mortality, losses due to predatory animals, and unhealthy or poorly developed birds that were destroyed, amounted to 16 per cent. The remaining three per cent included cockerels missed in sexing and pullets culled on the range. In a few cases, part of the laying flock replacements was obtained by purchasing ready-to-lay or laying pullets in the late summer and early fall, or by purchasing immature pullets and raising them to laying age.

All feed, with the exception of small quantities of oats grown on a few of the larger farms, and most supplies used by the poultry enterprise were purchased from feed and farm supplies companies located in the larger towns. Most poultry producers had feed delivered as it was required, usually once a week, and normally had only a small quantity on hand.

Wheat and oats comprised most of the whole grain that was fed in the layer ration, although barley and corn were included occasionally. Poultry producers purchased the various grains separately and then combined them in the ration to meet the requirements of the laying flock. Poultry mashes, on the other hand, were mixed by the feed manufacturers according to a standard formula. Although most feed manufacturers would prepare mash according to a specified formula at some extra charge, nearly all producers fed the standard laying mash containing 19 per cent protein. A few producers substituted a breeders mash, containing slightly less protein (18 per cent) and a higher concentration of vitamins, for part or all of the standard laying mash. In some cases, a vitamin supplement was added to the ration during the winter months. The general practice, however, was to feed the standard laying mash as it was prepared by the feed manufacturer.

Several kinds of feed, particularly mashes, were used in raising young pullets in order to meet the changes in nutritional requirements at various stages of growth. Although several methods and feed combinations were used, the most common practice can be described as follows:

(1) Chick starter mash was fed for the first five or six weeks, with chick grain introduced by mixing it with the mash or placing it in separate feeders;

(2) At five to six weeks of age, the mash ration was gradually changed to a growing or developing mash of lower protein content;

(3) At the same age or before, the feeding of oats and wheat was started. Usually, this grain was medium size or coarsely ground until the pullets could utilize whole grain.

(4) As the pullets neared laying age, the growing mash was replaced with laying mash.

Labor for the laying flock was required largely for the routine daily chores such as feeding and watering, cleaning dropping boards, and collecting, cleaning and packing eggs. The seasonal jobs included removal of manure from dropping pits, complete cleaning and disinfecting of laying pens, and moving young pullets from the range to laying houses. Most of the labor in raising young pullets was required for daily feeding and watering, and other less frequent jobs such as relocating feeders, water troughs and shelters on the range. Other jobs done only once or twice a year included preparation of brooders and brooder houses for chicks, and thorough cleaning and disinfecting of brooder houses, shelters and range equipment.

The operator and his family supplied nearly all of the labor for the poultry enterprise. Hired labor was employed mainly on the larger enterprises, but usually for only one or two weeks to assist with seasonal work.

The data presented in Table 4 provide additional information on the market egg enterprises. It indicates some of the changes that occurred from year to year during the period covered by the study.

The average number of layers at October 1 ranged between 723 and 753 over the three year period. This small variation was due entirely to the number of hens in the laying flock, with the number of pullets remaining unchanged from year to year.

The poultry producers were asked to value their laying birds according to the prices prevailing at the beginning of each survey year. Thus, these values, especially for hens, reflect changes in the market price for fowl. The consistent increase in the value of pullets is influenced partly by a continuous rise in the major costs, particularly feed costs, of raising pullets.

TABLE 4

SOME AVERAGES FOR MARKET EGG ENTERPRISES, LOWER FRASER VALLEY AND VANCOUVER ISLAND, 1949-51

	1949	1950	1951
Flock inventory (no. of layers):[a]			
Hens	161	151	182
Pullets	572	572	571
Total	733	723	753
Value of layers ($ per bird):			
Hens	1.34	1.28	1.49
Pullets	2.07	2.18	2.32
Mortality (no. of layers):			
Hens	18	18	21
Pullets	91	82	84
Total	109	100	105
Culling (no. of layers):			
Hens	82	77	146
Pullets	362	396	301
Total	444	473	447
Price received for culls ($ per bird):	1.24	1.19	1.63
Layer-years (no.):			
Hens	84	80	127
Pullets	483	467	497
Total	567	547	624
Eggs laid per layer-year (no.):			
Hens	140	147	148
Pullets	209	207	211
Flock	199	198	193
Price received for eggs (¢ per doz.):	45.5	41.9	54.8
Feed per layer-year (lbs.):			
Grain	51.6	52.8	48.4
Mash	58.7	60.4	57.4
Total	110.3	113.2	105.8
Feed per dozen eggs (lbs.):			
Grain	3.13	3.26	3.00
Mash	3.56	3.72	3.55
Total	6.69	6.98	6.55
Price of feed for layers ($ per cwt.):			
Grain	3.27	3.29	3.39
Mash	3.94	4.14	4.24
Total ration	3.63	3.74	3.85
Feed-egg price ratio:	12.5	11.2	14.2
Labor per layer-year (hrs.):	3.28	2.77	2.48

[a]At October 1, 1948, 1949 and 1950, respectively.

Relative to the initial size of flock, layer mortality was nearly the same for each year--15, 14 and 14 per cent in 1949, 1950 and 1951, respectively. The annual mortality for hens was slightly less than for pullets, ranging from 11 to 12 per cent for hens and from 14 to 16 per cent for pullets.

More extensive culling of the laying flock was practiced in 1950 than in the other two year; 65 per cent of the total flock was culled in 1950 and about 60 per cent in 1949 and 1951. Culling of hens increased from 51 per cent in the first two years to 80 per cent in 1951. Culling of pullets, on the other hand, was higher in the first two years, rising from 63 per cent in 1949 to 69 per cent in 1950 and then falling to 53 per cent in 1951. A considerably higher price was received for cull layers, when sold as fowl, in 1951 than in either of the two preceding years.

Culling and mortality removed 70 to 80 per cent of the layers from the flock at some time during each year. Consequently the flock inventory at the beginning of the survey year was inadequate for measurihg the size of flock for the year as a whole. This difficulty was overcome by using the layer-year as the standard unit of measurement. In so doing, layers that remained in the flock for less than the year were converted to fractions of layer-years.[1] The total layer-years then provide

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[1]A layer-year is the equivalent of one layer in the flock for one year. For example, two layers in the flock for 150 and 215 days, respectively, are equal to one layer in the flock for 365 days, or one layer-year. Thus, the number of layer-years were calculated by aggregating the days that individual layers were in the flock, and dividing the total by 365.

a measure of the net size of laying flock for the year.

The net size of laying flock, measured in layer-years, decreased slightly from 1949 to 1950, but increased substantially in 1951. Variations in the initial flock inventory, as well as in the average length of time each layer was retained in the flock, contributed to this fluctuation in flock size. A hen was kept in the laying flock for an average of 190, 193 and 255 days in 1949, 1950 and 1951, respectively, or approximately nine weeks longer in 1951 than in either of the previous years. There was less difference in the length of time a pullet was retained; 308 days in 1949, 298 days in 1950, and 318 days in 1951. For the flock as a whole, a layer was kept in the flock for 282, 276 and 302 days in the respective years. Apparently, variations in the initial number of hens and in the average length of time a hen was retained in the laying flock were the main causes of the changes in the net size, particularly the large increase in 1951.

The rate of production, as measured by the number of eggs per layer-year, remained reasonably constant for both hens and pullets. A hen produced approximately 30 per cent less eggs than a pullet in all years. Consequently, the explanation of the slight decline in flock rate of production lies in the larger number of hens that were retained in the flock for a longer period.

The average price received for eggs sold during the year declined moderately from 1949 to 1950, and then rose quite sharply in 1951.

Feed input for the laying flock increased by nearly three pounds per layer-year from 1949 to 1950, and then decreased by more than seven pounds in 1951. The feed inputs per layer-year shown in Table 4 are averages for the flock, including both hens and pullets. Since the 1951 flock contained a larger proportion of hens, adjustments in the level of feeding consistent with the rate of egg production could have caused feed input for the flock to decline from the previous years. However, the quantity of feed per dozen eggs indicates that the level of feeding was not in constant proportion to the rate of egg production. In view of this, it is concluded that the level of feeding was slightly higher in 1950 than in 1949, but lower in 1951 than either of the previous years.

The proportion of grain and mash in the layer ration was practically constant at 46 per cent grain and 54 per cent mash. However, there was a small but consistent substitution of mash for grain in successive years, amounting to one pound per 100 pounds of ration during the three year period.

The average price of grain in the layer ration did not rise to any extent until 1951. The price of mash for layers, on the other hand, increased by 20 cents in 1950 and 10 cents in 1951. As a result, increases in the price of feed for the laying flock averaged 11 cents per 100 pounds in both 1950 and 1951.

Total feed cost per layer-year amounted to \$4.00, \$4.24 and \$4.07 in 1949, 1950 and 1951, respectively. This, of course, reflects both the price of feed and the quantity of feed used. The larger quantity of feed purchased at a higher price caused an increase in feed costs per layer-year during 1950. However, the still higher price in 1951 was more than compensated by a reduction in the quantity of feed used, thereby causing a decrease in feed costs per layer-year.

Of greater importance is the cost of feed required to produce a dozen eggs, since it includes the effects of the level of egg production. Feed inputs per dozen eggs cost 24.3, 26.1 and 25.2 cents during the respective years. With essentially no change in the flock rate of production from 1949 to 1950, the larger quantity of more expensive feed used in 1950 carries through completely as higher feed costs per dozen eggs. Although a dozen eggs was produced at a lower feed cost in 1951, part of the gain realized from the reduction in feed input was lost through the lower rate of egg production.

The feed-egg price ratio indicates the pounds of layer ration that are equal in value to a dozen eggs.[1] Variations in this price ratio from year to year show the relative changes that occurred in feed and egg prices. With an increase in feed prices and a decrease in egg prices from 1949 to 1950, the feed-egg price ratio declined, indicating the increase in feed prices relative to egg prices and also the smaller quantity of feed that could be purchased with a dozen eggs. The feed-egg price ratio in 1951 was considerably higher than in the two previous

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[1]The feed-egg price ratio was calculated by dividing the average price received for a dozen eggs by the average price paid for a pound of the composite grain and mash ration for layers.

years. Although feed and egg prices advanced during this year, the increase was proportionately greater for eggs. As a result, a dozen eggs in 1951 would purchase 26 per cent more feed than in 1950, and 14 per cent more than in 1949.

The substantial reduction in labor requirements of the laying flock, as indicated by the hours of labor per layer-year, cannot be attributed to the widespread adoption of any new labor-saving equipment and production techniques. Severe weather conditions during the winters of 1948-49 and 1949-50 probably added to the daily chore time required for care of the laying flock and maintenance of the laying pens. Seasonal work such as cleaning the laying houses would not vary to any extent from year to year and, consequently, some small decline in the hours of labor per layer-year might be expected in 1951 as a result of the larger laying flock. However, it is quite probable that weather conditions and flock size provide only a partial explanation of the consistent decline in labor input for the laying flock. Due to the difficulty of obtaining accurate estimates of labor time, there is also the possibility that the downward trend in labor requirement could be the result of bias in the original data.

Changes in the average investment in the poultry enterprises during the three year period occurred mainly in the value of buildings and flock (Table 5). In general, the larger inventory value in 1951 was caused primarily by rising prices, since any physical expansion of either buildings or poultry flock was only incidental.

TABLE 5

AVERAGE INVENTORY VALUES FOR MARKET EGG ENTERPRISES, LOWER FRASER VALLEY AND VANCOUVER ISLAND, 1949-51

	1949	1950	1951
	$	$	$
Land	743	752	755
Buildings	2,479	2,546	2,814
Equipment	383	374	369
Feed	51	45	34
Supplies	14	14	8
Poultry flock	1,406	1,392	1,735
Total	5,076	5,123	5,715

The complete specialization of these enterprises in market egg production is illustrated by the summary of receipts presented in Table 6. Eggs and cull layers sold as fowl, which are joint product of the market egg enterprise, provided at least 96 per cent of the cash receipts each year. Egg sales alone accounted for approximately 85 per cent of the cash receipts in each year. Other cash receipts and the value of poultry products consumed on the farm amounted to less than $250.

Receipts from sale of market eggs, then, determined to a large extent the amount of income from the poultry enterprise. Egg receipts, of course, depended on the quantity of eggs sold and the market price. The quantity sold decreased from 9,029 dozen in 1949 to 8,835 dozen in 1950. Thus, the decline in egg receipts and also in total cash receipts during 1950 was caused more by lower prices than by curtailment of production. Egg output expanded moderately in 1951 under the influence of more favorable prices, with sales of eggs increasing to 9,770 dozen. However, the higher price for eggs, rather than the increase in production, was responsible for most of the gain in receipts during 1951.

TABLE 6

AVERAGE RECEIPTS FOR MARKET EGG ENTERPRISES, LOWER FRASER VALLEY AND VANCOUVER ISLAND, 1949-51

	1949	1950	1951
	$	$	$
Cash receipts:			
Eggs	4,108	3,702	5,354
Fowl	542	544	709
Chicken	28	14	21
Chicks and breeders	53	44	15
Feed sack refunds	79	50	44
Manure	10	8	12
Co-op dividends	11	15	5
Total	4,831	4,377	6,160
Poultry products consumed on farm:			
Eggs	46	38	65
Meat	19	23	26
Total	65	61	91
Change in value of poultry flock inventory:	-112	-196	233

The year-to-year fluctuations in expenses for the poultry enterprise (Table 7) were much smaller than in receipts. After remaining practically unchanged from 1949 to 1950, expenses moved upward in 1951. Cash expenses, like cash receipts, were dominated by a single item. Feed purchases accounted for about 86 per cent of the annual cash expenses. All other items, except for purchase of chicks, involved only small cash outlays. Purchases of feed and chicks required approximately 92 per cent of the cash expenditures during each year. The non-cash charges for depreciation and interest amounted to just over $300 in 1951 which was slightly higher than in previous years.

TABLE 7

AVERAGE EXPENSES FOR MARKET EGG ENTERPRISES, LOWER FRASER VALLEY AND VANCOUVER ISLAND, 1949-51

	1949	1950	1951
	$	$	$
Cash expenses:			
Purchased feed	2,861	2,868	3,343
Farm grown feed	12	9	5
Purchased litter	56	38	40
Farm grown litter	4	3	1
Chicks and other stock	202	219	258
Hired labor	27	59	45
Electricity	20	21	23
Brooder fuel	14	18	16
Medicine and disinfectant	27	28	35
Taxes and insurance	34	31	33
Operation of car, truck and tractor	33	29	42
Repairs to equipment	4	3	5
Repairs to buildings	15	11	17
Small tools purchased	5	3	6
Other	12	21	28
Total	3,326	3,361	3,897
Depreciation:			
Buildings	84	84	98
Equipment	59	58	52
Total	143	142	150
Interest on investment:			
Land	30	30	30
Buildings	99	102	113
Equipment	15	15	15
Total	144	147	158

Totals of 626, 619 and 660 cwt. of feed for layers were purchased at costs of $2,270, $2,317 and $2,542 in the respective years. Despite a small reduction in quantity purchased, feed costs for layers rose slightly from 1949 to 1950 as a result of the higher prices. The larger expenditures for layer feed during 1951 was caused by a moderate increase in quantity purchased as well as the continued advance in prices.

The cost of feed used to raise pullets for replacement of the laying flock declined from $591 in 1949 to $551 in 1950. Since this decrease was nearly equal to the increase in feed costs for layers, total feed costs for the enterprise remained almost unchanged from 1949 to 1950. With feed costs for young stock rising to $801 in 1951, the additional cost of feed for the enterprise was shared almost equally by layers and young stock.

TABLE 8

AVERAGE RETURNS FOR MARKET EGG ENTERPRISES,[a]
LOWER FRASER VALLEY AND VANCOUVER ISLAND, 1949-51

	1949	1950	1951
Net cash income ($)	1,505	1,016	2,263
Family labor earnings ($)	1,164	558	2,251
Labor return ($)	1,191	617	2,296
Return on investment ($)	296	-55	1,454
Return per hour of labor ($)	0.55	0.26	1.26
Rate of return on investment (%)	5.8	-1.1	25.4

[a]Calculated by the residual method.

Some of the standard measures of net returns as derived in analyzing farm business data by the residual method are presented in Table 8. All of these measures indicate that net income from market eggs dropped in 1950 well below the 1949 level, but increased in 1951 to exceed the two preceding years by a considerable margin.

The measures of net returns in Table 8 were calculated as follows:

Net cash income--total cash receipts minus total cash expenses.

Family labor earnings--net cash income minus net decrease in total inventory value, or plus net increase in total inventory value, plus value of poultry products consumed on farm, minus interest on capital investment. The net change in total inventory value included depreciation charges. The return imputed to capital investment was calculated at four per cent of the total inventory value.

Labor return--family labor earnings plus wages paid to hired labor.

Return on investment--net cash income minus net decrease in total inventory value, or plus net increase in total inventory value, plus value of poultry products consumed on farm, minus value of operator and unpaid family labor. The net change in total inventory value included depreciation charges. The value of operator and unpaid family labor was based on the average hourly wage paid to hired labor, that is 52, 49 and 59 cents per hour in 1949, 1950 and 1951 respectively. Operator and family labor amounted to 2,118, 1,670 and 1,739 hours in the respective years.

Return per hour of labor--labor return divided by total hours of labor, including operator, family and hired labor.

Rate of return on investment--return on investment expressed in percentage of the total inventory value.

APPENDIX II

THE FEED-EGG PRICE RELATIONSHIP

The summaries of receipts and expenses (Tables 6 and 7) demonstrate that the net income from market egg production was determined mainly by egg receipts and feed costs. In turn, egg receipts depended on the price of eggs and the quantity of eggs sold, and feed costs depended on the price of feed and the quantity of feed purchased. The producer had some control over the quantities of eggs sold and feed purchased but, in both cases, had to accept prices as they occurred.

The quantity of feed required to produce a dozen eggs varied only slightly from year to year (Table 4). Consequently, most of the differences in net income, as determined by egg receipts and feed costs, can be traced to variations in egg prices and feed prices. In order to show the nature of these variations, the market prices for Grade "A" Medium eggs and for three kinds of feed at the mid-point of each month throughout the period of the study are listed in Table 9. The average price received by producers for all grades and sizes of eggs was close to the market price for Grade "A" Medium during this period and, as stated previously, the layer ration consisted mainly of wheat, oats and laying mash.

Consistently low egg prices prevailed throughout the second year, particularly from July to September when the

TABLE 9

EGG AND FEED PRICES AT VANCOUVER, OCTOBER, 1948 TO SEPTEMBER, 1951

	Oct., 1948 to Sept.,1949	Oct., 1949 to Sept.,1950	Oct., 1950 to Sept.,1951
Grade "A" Medium eggs (¢ per doz.):[a]			
October 15	51	51	48
November 15	53	51	48
December 15	41.5	38	57
January 15	41.5	31	40
February 14	40	36	40
March 15	40	36	47
April 15	38	36	51
May 15	40	37	56
June 15	41	39	57
July 15	47	46	65
August 15	54	48	59
September 15	55	48	53
No. 5 wheat ($ per cwt.):[b]			
October 15	3.45	3.45	3.30
November 15	3.45	3.50	3.15
December 15	3.55	3.45	3.05
January 15	3.55	3.45	3.15
February 14	3.55	3.45	3.25
March 15	3.55	3.45	3.35
April 15	3.55	3.45	3.35
May 15	3.55	3.45	3.40
June 15	3.55	3.45	3.45
July 15	3.55	3.45	3.65
August 15	3.45	3.30	3.65
September 15	3.45	3.30	3.65
Feed oats ($ per cwt.):[b]			
October 15	2.80	2.60	2.95
November 15	2.70	2.80	2.95
December 15	2.85	2.75	3.20
January 15	2.65	2.75	3.40
February 14	2.65	2.75	3.45
March 15	2.55	2.90	3.55
April 15	2.70	3.20	3.55
May 15	2.70	3.45	3.45
June 15	2.70	3.55	3.30
July 15	2.70	3.40	3.05
August 15	2.75	3.05	2.95
September 15	2.60	2.95	2.95

TABLE 9 -- Continued

	Oct., 1948 to Sept., 1949	Oct., 1949 to Sept., 1950	Oct., 1950 to Sept., 1951
Laying mash ($ per cwt.):[b]			
October 15	3.70	4.10	4.25
November 15	3.70	4.25	4.15
December 15	3.90	4.10	4.15
January 15	3.90	3.95	4.15
February 14	3.90	3.95	4.15
March 15	3.90	3.95	4.15
April 15	3.95	4.05	4.25
May 15	3.95	4.15	4.25
June 15	3.95	4.25	4.25
July 15	3.95	4.25	4.25
August 15	4.05	4.25	4.10
September 15	4.10	4.25	4.10

[a]Price to producer, from Egg and Poultry Market Report; 1948 to 1950 (Ottawa: Marketing Service, Canada Department of Agriculture) and Poultry Products Market Report; 1951 (Ottawa: Marketing Service, Canada Department of Agriculture).

[b]Price picked up at Vancouver, from Markets Bulletin; (Victoria: Markets Branch, British Columbia Department of Agriculture).

season peak normally occurred. During the third year, however, egg prices advanced to a much higher level than in the two preceding years, especially in December and from March to August. The change in feed prices, on the other hand, was more gradual and regular. The price of wheat dropped slightly in the second year but, after a further decline early in the third year, it increased rapidly above the first year's level. Although the oats price moved erratically, its trend was generally upward during the second and third years. The price of laying mach increased slowly but persistently throughout the first year, fluctuated within a narrow range at a moderately higher level in the second year, and remained generally steady but still somewhat higher for most of the third year.

A more important aspect of the changes in these prices is the variation in feed prices relative to egg prices. With the quantity of feed required to produce a dozen eggs remaining nearly constant, net returns would increase as feed prices became lower in relation to egg prices, and vice versa. This price relationship is indicated by the feed-egg price ratio, which shows the pounds of feed equal in value to a dozen eggs. Thus, as the feed price increases in relation to the egg price, the feed-egg price ratio declines because less feed can be purchased with the receipts from a dozen eggs. Conversely, when the feed price decreases relative to the egg price, the feed-egg price ratio increases.

The feed-egg price ratios for wheat, oats and laying mash (Table 10) show the price relationship between eggs and an individual feed. Since the layer ration was composed mainly of these three feeds, a feed-egg price ratio was calculated for a ration consisting of 31 per cent wheat, 15 per cent oats and 54 per cent laying mash. As indicated by the ratio of prices for the composite layer ration and eggs, the feed-egg price relationship which confronted producers of market eggs was least favorable during the second year, and most favorable during the third year of the study. Compared month by month with the first year, this price ratio was lower throughout all of the second year, and was higher during the third year except in October, November, January, February and

September. However, it was only in October and November of the third year that the price ratio dropped appreciable below the first year. The maximum and minimum values of the price ratio (15.3 and 10.4 in the first year, 13.9 and 8.6 in the second year, and 16.8 and 10.6 in the third year) further emphasize the conditions with regard to feed and egg prices.

TABLE 10

FEED-EGG PRICE RATIOS,[a] BASED ON VANCOUVER PRICES, OCTOBER, 1948 TO SEPTEMBER, 1951

	Oct., 1948 to Sept.,1949	Oct., 1949 to Sept.,1950	Oct., 1950 to Sept.,1951
No. 5 wheat:			
October 15	14.8	14.8	14.5
November 15	15.4	14.6	15.2
December 15	11.7	11.0	18.7
January 15	11.7	9.0	12.7
February 14	11.3	10.4	12.3
March 15	11.3	10.4	14.0
April 15	10.7	10.4	15.2
May 15	11.3	10.7	16.5
June 15	11.5	11.3	16.5
July 15	13.2	13.3	17.8
August 15	15.7	14.5	16.2
September 15	15.9	14.5	14.5
Feed oats:			
October 15	18.2	19.6	16.3
November 15	19.6	18.2	16.3
December 15	14.6	13.8	17.8
January 15	15.7	11.3	11.8
February 14	15.1	13.1	11.6
March 15	15.7	12.4	13.2
April 15	14.1	11.2	14.4
May 15	14.8	10.7	16.2
June 15	15.2	11.0	17.3
July 15	17.4	13.5	21.3
August 15	19.6	15.7	20.0
September 15	21.2	16.3	18.0

TABLE 10 -- Continued

	Oct., 1948 to Sept.,1949	Oct., 1949 to Sept.,1950	Oct., 1950 to Sept.,1951
Laying mash:			
October 15	13.8	12.4	11.3
November 15	14.3	12.0	11.6
December 15	10.6	9.3	13.7
January 15	10.6	7.8	9.6
February 14	10.3	9.1	9.6
March 15	10.3	9.1	11.3
April 15	9.6	8.9	12.0
May 15	10.1	8.9	13.2
June 15	10.4	9.2	13.4
July 15	11.9	10.8	15.3
August 15	13.3	11.3	14.4
September 15	13.4	11.3	12.9
Total layer ration:[b]			
October 15	14.6	13.9	12.8
November 15	15.3	13.4	13.1
December 15	11.4	10.3	15.5
January 15	11.5	8.6	10.7
February 14	11.1	9.9	10.6
March 15	11.1	9.9	12.3
April 15	10.4	9.6	13.2
May 15	11.0	9.7	14.5
June 15	11.3	10.0	14.8
July 15	12.9	11.9	16.8
August 15	14.7	12.7	15.6
September 15	15.0	12.8	14.0

[a]Pounds of feed equal in value to one dozen eggs, calculated from prices in Table 9.

[b]Based on a ration containing 31 per cent wheat, 15 per cent oats, and 54 per cent laying mash.

APPENDIX III

THEORETICAL CONCEPTS[1]

Two sets of relationships are relevant to the formulation of a theoretical solution to the problem of economic efficiency of resource use in the short run[2] for a single enterprise. The first set is the input-output relationship including the factor-factor or resource substitution relationship. These define the response of output in physical terms to changes in factor combinations. The second set is the price ratios between factors and products which provide a criterion for specifying the combination of factors and the product output level that will maximize profits or minimize costs.

The Input-Output Relationship

The input-output relationship is outlined here in terms of the transformation of factors into product when one factor varies and all other factors remain constant. The problem is

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[1]For a detailed and fully illustrated presentation of the concepts outlined in this section, see Earl O. Heady, Economics of Agricultural Production and Resource Use, (New York: Prentice-Hall, Inc., 1952), pp. 21-199.

[2]A short-run condition of production exists when the quantities of some resources are fixed at any level, regardless of the number of fixed resources and the level at which each is fixed. In the long-run condition, variations in the input of all resources is possible.

one of intensity of factor use, that is, what amount of the variable factor will give the optimum (most profitable) output. Usually, one factor alone cannot be varied. However, the principle applies equally when more than one factor is varied in proportion to the fixed factors.

The input-output relationship between a single variable input, with the quantity of other resources held constant, and the output of a single product can take one of three general forms, namely, constant, decreasing or increasing returns. Constant returns occur when each additional unit of the variable factor applied to the fixed factor results in equal additions to total output of product; the ratio of total output to total input remains constant. Decreasing returns to the variable factor occur when each additional unit of input adds less to total output than the previous unit. Under this condition, the output/input ratio declines. Increasing returns to a single factor exist when each successive unit of the variable resource adds more to the total product than the previous unit of input. When increasing productivity of a variable factor occurs, the output/input ratio increases.

Many input-output relationships include two of the conditions stated above. Most common is one that combines both increasing returns and decreasing returns to the variable factor. The theoretical model based on this combination of relationships is as follows: If the quantity of one resource is increased by equal increments with the quantities of other resources held constant, the increments to total product may increase at first but will decrease after a certain point. This statement is illustrated in Table 11 and Figure 1. Increasing returns to the variable factor X exist for all increments of input up to and including the sixth unit. Up to this level of input, the total product increases at an increasing rate, and the total product curve (Yp in Figure 1) is convex to the x-axis. Decreasing returns occur after the sixth unit of input. The total product continues to increase but by successively smaller amounts, causing the total product curve to bend toward the x-axis of the graph.

TABLE 11

RELATIONSHIP OF RESOURCE INPUT TO TOTAL, AVERAGE AND MARGINAL PRODUCTS (HYPOTHETICAL DATA)

Input of Variable Factor X	Output of Product Y	Output/Input Ratio or Average Product Y/X	Additional or Marginal Product $\Delta Y/\Delta X$
0	0		
2	2	1.00	1
4	8	2.00	3
6	18	3.00	5
8	26	3.25	4
10	32	3.20	3
12	36	3.00	2
14	38	2.71	1
16	36	2.25	-1
18	30	1.67	-3

The average and marginal products are basic quantitative measurements derived from this general input-output model. The average product, or average productivity, is the amount of product per unit of input of the variable factor. It can be expressed by the ratio Y/X, where Y is the total product and X is the total input of the variable factor. When the input-output relationship is linear, the average product remains constant. If the input-output relationship represents increasing returns, the average product increases as more of the variable factor is employed. Conversely, under the conditions of decreasing returns, the average product of the variable factor decreases as more units are applied to the fixed factor. The marginal product is the quantity added to the total product by an additional unit of the variable factor. It is the ratio of the increment in total output to the increment in input of the variable factor. This ratio can be expressed as $\Delta Y/\Delta X$, when ΔY is the change in product output and ΔX is the change in factor input.[1]

The relationship between marginal and total products is as follows: As long as the marginal product is increasing, the total product increases at an increasing rate. Beyond the point where the marginal product is at a maximum, the total product continues to increase but at a decreasing rate, and

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[1]Only an approximation of the marginal product is obtained by dividing the increment in total product by the increment in input of the variable factor. For example, an increase from six to eight units in factor input results in an increase from 18 to 26 units in product output, as shown in Table 11. The marginal product of four units does not relate specifically to the eighth unit of input, but rather it is the average marginal product of all fractional inputs between six and eight units of the variable factor. This is true because the change in input of factor X is not infinitely small. However, the exact marginal product at a given level of factor input can be calculated by differentiation of the mathematical equation that expresses the functional relationship between factor input and product output. The marginal product as a derivative relates to a change in factor input that is infinitely small (approaches the limit zero).

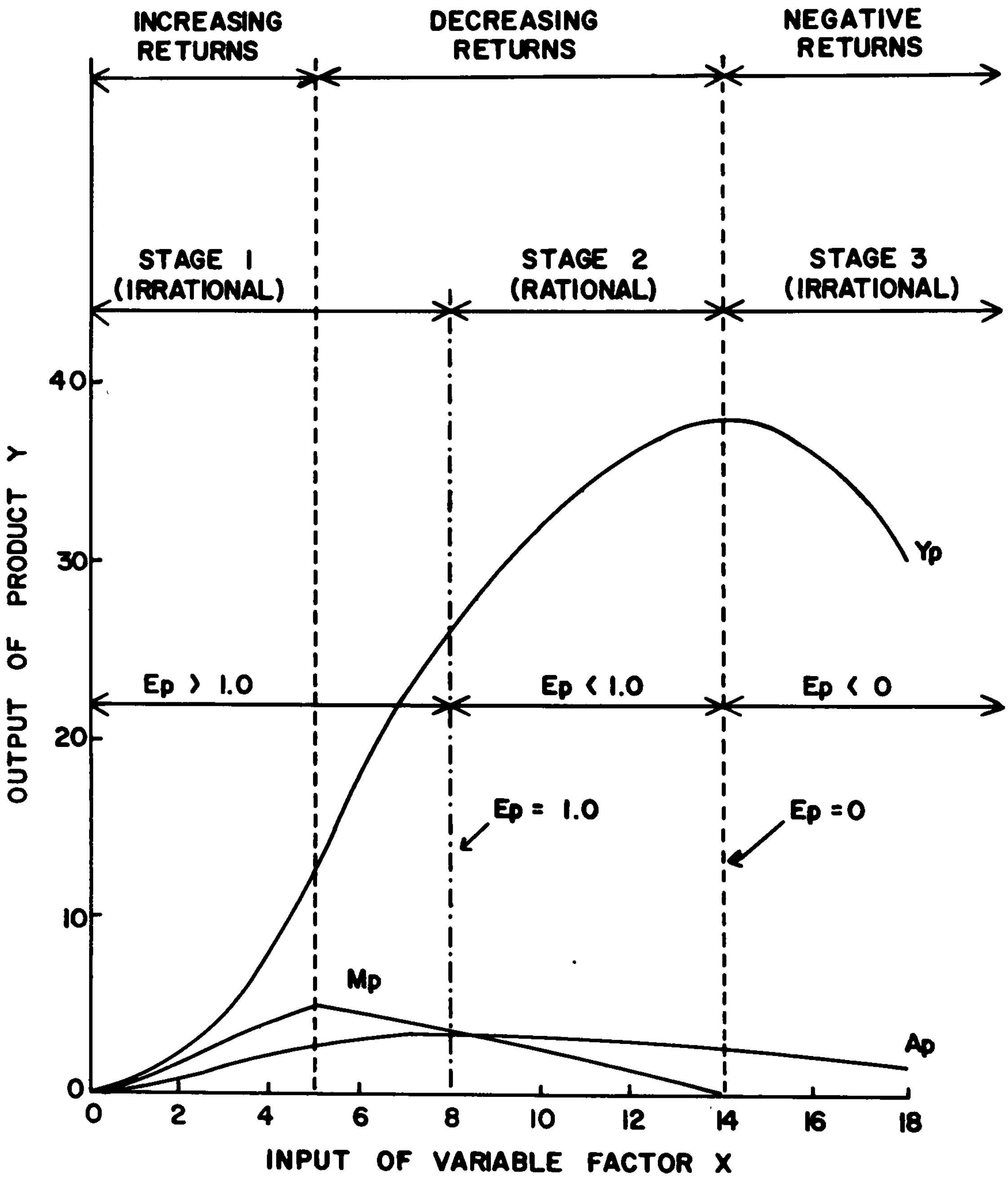

FIGURE I. — Relationship of input of a single variable resource to total, average and marginal products.

reaches a maximum when the marginal product is zero. Then, as the total product declines, the marginal product becomes negative.

The ranges of increasing, decreasing and negative returns to a single variable factor can be defined in terms of the marginal product. Returns to an additional unit of input increase until the maximum marginal product is reached, are constant at that point, and decrease thereafter. Negative returns are indicated by a marginal product less than zero.

Certain relationships also exist between the average and marginal products. The average product of a variable factor increases as long as it is exceeded by the marginal product, even though the marginal product may be declining. The average and marginal products are equal at the maximum average product. Finally, the average productivity of a resource decreases if the marginal product is less than the average product.

The elasticity of production is defined as the percentage change in product output as compared with the percentage change in factor input. It can be expressed in equation form as

$$Ep = \frac{\Delta Y/Y}{\Delta X/X}.$$

This equation can also be written as

$$Ep = \frac{\Delta Y/\ X}{\Delta Y/X},$$

which shows that the elasticity of production equals the ratio of the marginal product to the average product.

The value of the elasticity of production depends on the nature of the input-output relationship. Production elasticity is equal to 1.0 when returns to the variable factor are constant. An elasticity of less than 1.0 is always associated with decreasing returns to the variable factor. However, when the elasticity of production is greater than 1.0, either increasing or decreasing returns may prevail. Although increasing returns can only occur when the elasticity exceeds 1.0, decreasing returns also are possible under this condition of elasticity. The range of decreasing returns associated with a production elasticity greater than 1.0 extends from the maximum marginal product to the maximum average product (see Figure 1).

Certain relationships between the elasticity of production and the total, average and marginal products can also be defined. The elasticity of production is equal to 1.0 at the maximum average product, at which point the average and marginal products are equal. It is greater than 1.0 up to the maximum average product, and less than 1.0 between the maximum average product and the maximum total product. Production elasticity equals zero at the maximum total product and becomes less than zero (or negative) as the total product declines.

Principles of Resource Allocation

As outlined in the following, the principles of resource allocation relate to the transformation of a single variable factor into a single product when all other resources are held constant, with the objective of maximizing profits.

The general input-output relationship which includes increasing, decreasing and negative marginal returns can be divided into segments denoted as the three stages of production. As illustrated in Figure 1, stage 1 extends to the input of the variable factor that results in the maximum average product. Stage 2 covers the range of inputs between the maximum average product and the maximum total product. Stage 3 includes all inputs that have a negative marginal product and extends over the entire range of declining total product. An input-output relationship with increasing marginal returns throughout would fall entirely in stage 1. However, when decreasing marginal returns occur at all levels of input, the input-output relationship might include all three stages of production.

A level of resource use that falls in stage 1 is uneconomic because greater returns can always be obtained by using a larger quantity of the variable resource. The application of additional amounts of the variable resource in stage 1 increases the average productivity of all previous inputs. Also, a larger product is obtained from the fixed factors as well as from each additional unit of the variable factor. Thus, the product of neither the variable factor nor the fixed factors can be maximized in stage 1. Instead of restricting the application of a variable resource to the fixed factors before the limit of stage 1 is reached, the combination of fixed and variable resources can always be rearranged within stage 1 to secure a larger product. A greater product from given resources can be gained by leaving idle some of the factor that was otherwise considered as "fixed". This is possible even when the available amount of the variable factor is limited. For example, suppose that column 1 of Table 11 refers to pounds of fertilizer applied to an acre of grain land and column 2 is the yield per acre. Also assume that only 800 pounds of fertilizer are available for use on 200 acres of land. If the fertilizer is applied to all of the land at the rate of four pounds per acre, the total production is 1600 bushels. However, if half the land is left idle and the fertilizer is applied to only 100 acres at the rate of eight pounds per acre, total production increases to 2600 bushels. Thus, more product is obtained from the same quantity of fertilizer and less land. Economic returns also must increase, except when the product has no value attached to it.

Stage 3 also is an area of uneconomic and irrational production where the total product can be increased by using a smaller quantity of resources. The only difference, compared with stage 1, is that some of the variable factor is withdrawn from use. Again assuming that Table 11 is an input-output relationship of fertilizer applications on grain land, 3600 pounds of fertilizer applied to 200 acres at 18 pounds per acre would result in a total product of 6000 bushels. However, if only 2800 pounds of fertilizer are used at 14 pounds per acre, the total production increases to 7600 bushels. In this case, a larger product is gained by adjusting the combination of fixed and variable resources through reduction of the variable resource (fertilizer). As long as the product has a value, economic returns also increase when resources are recombined in this manner within stage 3.

Consequently, irrational and technically inefficient production exists if resources can be rearranged to give either (1) a larger product from the same resources, or (2) the same product from a smaller aggregate of fixed and variable resources. This condition exists for any resource combination falling in stage 1 and stage 3. In cases of irrational production, the adjustment in resource combination required to increase economic returns can be specified without knowledge of the resource and factor prices. Profits are always increased when any rearrangement of resources results in a larger product from the same resources or the same product from less resources. Irrational production indicates that a greater value of product can always be produced with the same or a smaller aggregate of resources.

Because stage 1 and stage 3 are irrational ranges of production, the problem of resource allocation is often considered only within stage 2, or in terms of an input-output relationship with a production elasticity of less than 1.0 but more than zero. This, however, does not suggest that farm resources are not combined in an irrational manner. Irrational combinations of resources can and do exist for several reasons, including capital limitations, indivisibility of resources, uncertainty, ignorance, and even an apathetic attitude of the farm operator.

Even without knowledge of resource and product prices, it is evident that profits can be maximized only when the variable factor is applied to the fixed factor at a rate that falls within stage 2. However, it is impossible to determine the exact combination of variable and fixed resources required within stage 2 to maximize profits without reference to the prices of the variable factor and the product.

Economic efficiency requires a resource combination that will maximize profits. Attainment of this condition involves a decision to use one of the several resource combinations that are possible within stage 2, the rational area of production. The problem is to specify the amount of a variable factor to be combined with fixed resources in order to maximize profits. Selection of the most efficient combination of resources can be made only in terms of the appropriate price ratio. In the transformation of a single variable factor into a single product, the relevant price ratio is the factor-product price ratio. It is expressed as Px/Py, when Px is the price of the variable factor X, and Py is the price of the product Y.

The level at which a variable factor should be applied to fixed factors for profit maximization is determined by the following condition: The factor/product price ratio must equal the marginal physical productivity of the variable factor. This condition can be expressed in equation form as

$$Px/Py = \Delta Y/\Delta X.$$

Another way of writing this equation is

$$(Px)(\Delta X) = (Py)(\Delta Y),$$

which indicates that profits are maximized when the change in the variable input and the change in the product output are equal in value.

The necessary conditions for maximum profits are illusstrated in Table 12. As presented there, profits are maximized when four units of the variable factor are used. At this level of variable input, the factor/product price ratio equals the marginal physical product, or

$$Px/Py = \Delta Y/\Delta X = 50.$$

Also, the changes in value of the variable factor input and the product output are equal, or

$$(Px)(\Delta X) = (Py)(\Delta Y) = 50.$$

In other words, the increase in cost of the variable factor is equalled by the increase in value of the product.

TABLE 12

OPTIMUM LEVEL OF APPLYING A VARIABLE FACTOR TO FIXED FACTORS (HYPOTHETICAL DATA)

Input of Variable Factor X	Output of Product Y	Marginal Physical Product $\Delta Y/\Delta X$	Value of Added Factor[a] $(Px)(\Delta X)$	Value of Added Product[b] $(Py)(\Delta Y)$
0	142			
1	214	72	$50	$72
2	284	70	50	70
3	347	63	50	63
4	397	50	50	50
5	440	43	50	43
6	475	35	50	35

[a]Price of the variable factor, Px = $50 per unit.

[b]Price of the product, Py = $1 per unit.

Inequality of the factor/product price ratio and the marginal physical product indicates a variable input that is inconsistent with a maximum profit. When the factor/product price ratio is greater than the marginal physical product, or

$$Px/Py > \Delta Y/\Delta X,$$

the cost of an additional unit of the variable factor exceeds the value of the additional output of product. In this case, profits can be increased by using less of the variable factor. If the factor/product price ratio is less than the marginal physical product, or

$$Px/Py < \Delta Y/\Delta X,$$

the cost of adding a unit of input is less than the value of the additional product. Consequently, using a larger amount of the variable leads to increased profits.

The conditions defining the most profitable quantity of a variable resource to be combined with fixed resources can also be presented graphically. As a first step, it is necessary to explain the nature of factor/product price ratios. Lines A and B in Figure 2 indicate the quantities of a factor and a product that are equal in value under price ratios of 25/1 and 50/1, respectively. As the factor price increases relative to the product price, Px/Py becomes larger and the price ratio line assumes a steeper slope. Conversely, a decrease in the factor price relative to the product price causes Px/Py to become smaller and reduces the slope of the price ratio line. However, variations can occur in the prices of both factor and product, either simultaneously or independently, and in the same or opposite direction. Consequently, the factor/product price ratio and the slope of the price ratio line change with every disproportionate variation in factor and product prices.

The input of a variable factor required for maximum

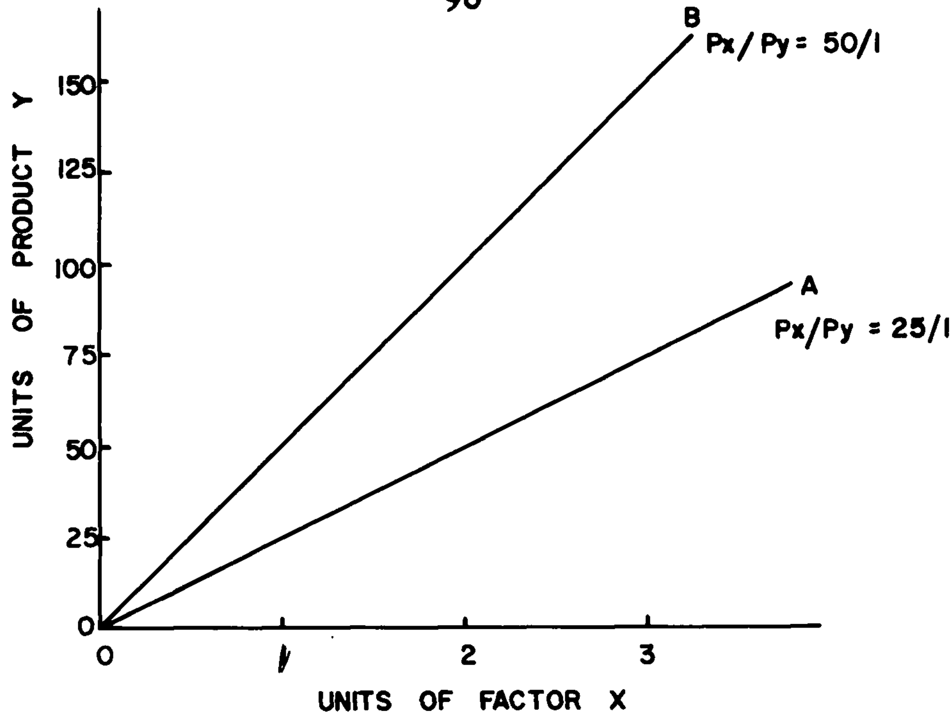

FIGURE 2.— Factor/product price ratios.

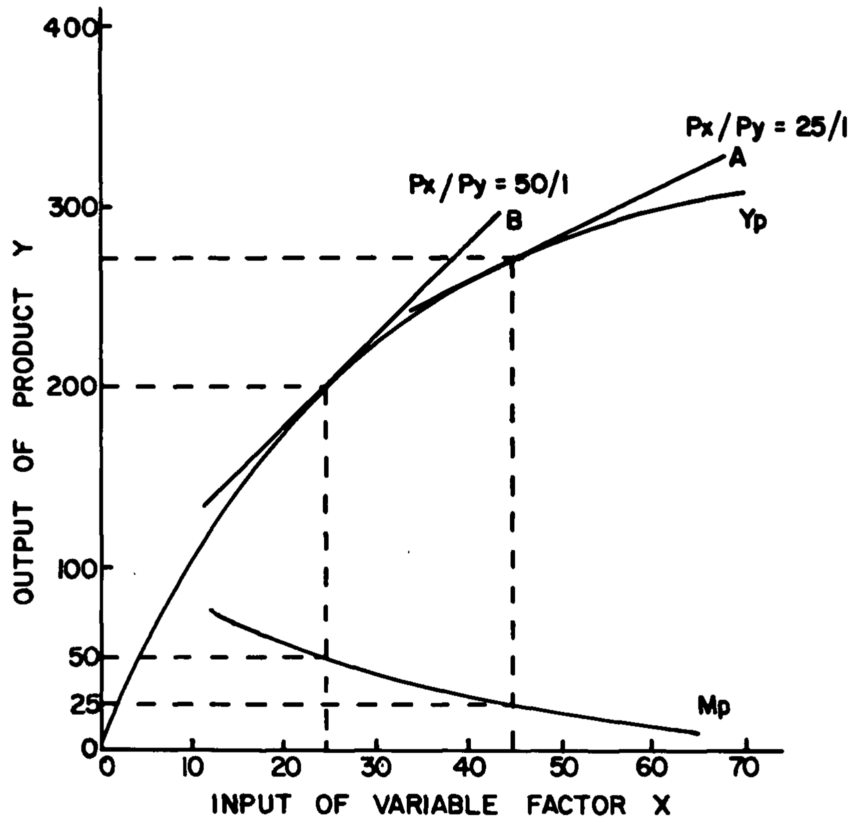

FIGURE 3.— Equation of the factor/product price ratio and the marginal physical product for maximum profits.

profits is denoted by tangency of the total product curve and the price ratio line. The slope of the input-output curve denotes the marginal product of the variable factor, and the price ratio line indicates the factor/product price ratio. Since two lines have the same slope at a point of tangency, the factor/product price ratio and the marginal product are equal at the point of tangency of a price ratio line and the physical input-output curve.

This condition is presented in Figure 3, where A and B are price ratio lines and the curve Yp represents an input-output relationship. When Px/Py is 25/1, tangency of the price ratio line A with the total product curve Yp indicates that profits are maximized with an input of 45 units of the variable factor X. The marginal physical product (curve Mp) is 25 units at this input, so that Px/Py equals $\Delta Y/\Delta X$.

An increase in the factor price relative to the product price is represented by a shift in the price ratio line from A to B (Figure 3). As a result, input of the variable factor must be reduced to regain the condition of maximum profits. On the other hand, if the factor price decreases relative to the product price, profits can be maximized only by increasing the input of the variable factor.

The more or less continual fluctuation of most, if not all, factor/product price ratios has certain implications in resource allocation. First, these changes necessitate frequent adjustments in the proportion of variable and fixed resources in order to maximize profits. In addition, variations in factor and product prices create uncertainty about the future. Since producers, as users of resources, can only anticipate future prices, their ability to maximize profits depends on the accuracy of their estimates of price changes.

Value productivity relationships differ from physical input-output relationships only in the unit of measurement; the output of product is measured in dollars instead of a physical unit such as pounds or bushels. Consequently, the total value product (Yv) is equal to the total physical product multiplied by the product price, and

$$Yv = (Yp)(Py).$$

The average value product (Av) is derived by multiplying the average physical product by the product price, or by dividing the total value product by the input of variable factor, and

$$Av = (Ap)(Py) = Yv/X.$$

The marginal value product is derived by multiplying the marginal physical product by the product price, or by calculating the change in total value product associated with an additional unit of variable input, and,

$$Mv = (Mp)(Py) = \Delta Yv/\Delta X.$$

The condition for maximum profits, stated in terms of value productivity, requires equality of the marginal cost and the marginal value product of a resource. Likewise, profits are maximized when the marginal cost and marginal revenue of a unit of product are equal. However, the optimum input of a variable resource is not indicated directly by an input-output relationship that relates the physical input of a variable resource to the value of output. It can be specified only by reference to marginal cost of the resource and the marginal value product.

The marginal cost of a resource is the cost of the last unit added to the total input. Since any quantity of a resource can generally be purchased at the current price by farm operators, the marginal cost of a resource is equal to the price of the resource regardless of the quantity used. Consequently, variations in marginal cost of a resource will occur only through changes in the prevailing price of a resource.

Profit maximization in terms of value productivity can be illustrated by the data in Table 13. With the price (and marginal cost) of the variable factor at $50, an input of four units will maximize profits when marginal value productivity is based on a product price of $1.00 per unit. If the factor price rises to $70 and the product price remains unchanged at $1.00, the optimum factor input is reduced to two units. However, an increase in product price to $2.00 per unit would require a variable input of six units for maximum profits. Thus, any change in either the factor price or the product price compels an adjustment in the variable input that will equalize the marginal cost of the factor and the marginal value product; otherwise, the maximum level of profits cannot be maintained.

Up to this point, an unlimited supply of the variable resource has been assumed in determining the input required for maximum profits. However, the quantity of any specific resource, such as fertilizer, land or labor, available to a farmer is frequently limited by the funds at his disposal. This available stock of a resource may be insufficient to provide the level of input necessary for maximum profits. The problem then is the allocation of a limited stock of a resource to obtain the largest possible profit. Since any quantity of product can be sold at the same price under the competitive conditions of a farm, any increase in the total physical product from given resources will also increase net income, because total cost of the resources remains constant. Consequently, organization of the resources to obtain the largest physical product will result in the largest profit, although not necessarily the maximum profit.

TABLE 13

MARGINAL COST OF A FACTOR AND MARGINAL VALUE PRODUCT (HYPOTHETICAL DATA)

Input of Variable Factor	Output of Product	Marginal Physical Product	Marginal Cost of Variable Factor when Price is		Marginal Value Product when Price is	
			$50	$70	$1.00	$2.00
0	142					
1	214	72	$50	$70	$72	$142
2	284	70	50	70	70	140
3	347	63	50	70	63	126
4	397	50	50	70	50	100
5	440	43	50	70	43	86
6	475	35	50	70	35	70

Suppose that only 12 units (cwt.) of fertilizer are available for use on six acres of land that is identical in quality. The most efficient use of this limited supply of fertilizer is attained when the marginal physical productivity of fertilizer is the same on each acre of land. This principle can be illustrated with the marginal physical products shown in Table 13. Each additional unit of input results in a smaller marginal physical product, or successively smaller increments to the total product. Consequently, the fertilizer must be applied at the same rate per acre in order to equalize its marginal physical product on each acre. By applying two units of fertilizer to each of the six units of land, the total product is 1704 units. Any other allocation of the fertilizer results in a smaller total product because the marginal product of part of the fertilizer would not be as large as possible.

The Factor-Factor Relationship

The factor-factor or resource substitution relationship refers to the transformation of two or more variable resources into a product. The problem now centres on determination of the optimum combination of a number of variable resources in producing some constant amount of product. In the interests of simplifying the following outline, only two factors are considered to be variable, while all others are held at some fixed level, in the production of a single product. Although more than two variable resources are ordinarily involved in a production process, the two-factor relationships are equally applicable to any number of variable resources.

The relationship between the input of two variable factors, X_1 and X_2, and the output of a single product Y can be shown as a two-way table such as Table 14. This table actually consists of a series of single factor input-output relationships with either X_1 or X_2 fixed at different levels and input of the other allowed to vary. It shows that input of both factors can be increased, either proportionately or otherwise, to gain a larger output of product. It also indicates that several different combinations of X_1 and X_2 can be used to attain the same level of output. Consequently, three distinct types of adjustment in the factor inputs are possible: (1) Input of one factor can be increased or decreased while the other is held constant; (2) Input of both factors can be either increased or decreased simultaneously; (3) One factor can be increased and the other decreased in quantity to produce the same amount of product. The last of these adjustments involves the factor-factor or resource substitution relationship in which the replacement of one factor with the other is possible within a certain range in the production of a constant output of product.

A two-factor input-output relationship can also be presented graphically as a series of isoquants or iso-product curves (Figure 4). Each iso-product curve indicates all of the possible combinations of X_1 and X_2 that yield a specified quantity of product Y. Figure 4 also shows that X_1 and X_2 are substitute resources because a range of input combinations, wherein an increase in one resource replaces a decrease in the other, exists at each level of input.

The rate of factor substitution may be either constant or decreasing in the production of a fixed output of product. Under a constant rate of substitution, one factor replaces the other at the same ratio throughout all factor combinations at a fixed level of output. A continuous and linear isoquant is characteristic of resources that substitute at a constant rate. Factor substitution at a decreasing rate exists when successive increments in the input of one factor replace a decreasing quantity of the other factor. In this case, the isoquant is a curved line convex to the axes of the graph. As indicated in Figure 4, it assumes a greater slope as factor X_1 is increased, with each additional unit of X_1 replacing a smaller amount of X_2. Conversely, the curve declines in slope as X_2 is substituted for X_1, but only at a decreasing rate.

TABLE 14

RELATIONSHIP BETWEEN INPUT OF TWO VARIABLE FACTORS AND OUTPUT OF PRODUCT (HYPOTHETICAL DATA)

Input of Factor X_1	Input of Factor X_2									
	0	1	2	3	4	5	6	7	8	9
	-Output of Product Y-									
0	0	0	0	0	0	0	0	0	0	0
1	0	6	7	8	8	7	6.5	6	5.5	5
2	0	10	12	13	14	15	16	16.5	16	15.5
3	0	12	16	18	19	20	21	22	23	24
4	0	12	20	22	24	25	26	27	28	29
5	0	11	22	26	28	30	31	32	33	34
6	0	10	24	30	32	35	36	37	38	39
7	0	9	24	32	36	39	41	42	43	44
8	0	8	24	34	40	42	44	46	48	49
9	0	7	23	36	41	45	48	50	52	54

The marginal rate of substitution refers to the amount by which one resource (X_2) is decreased as input of the other resource (X_1) is decreased by one unit. It can be expressed as the ratio $\Delta X_2/\Delta X_1$, when ΔX_2 is the change (decrease) in X_2 and ΔX_1 is the change (increase) in X_1. Calculated by this method, the marginal rate of substitution is an average between two distinct combinations of resources.[1] When two factors substitute for each other at a constant rate, the marginal rate of substitution does not vary. However, if the rate of factor substitution diminishes, the marginal rate of substitution becomes progressively smaller as one factor replaces the other in the production of a constant product (Table 15).

TABLE 15

DIMINISHING RATE OF FACTOR SUBSTITUTION
WITH OUTPUT FIXED AT 100 UNITS
(HYPOTHETICAL DATA)

Input of Factor X_1	Input of Factor X_2	Change in Factor X_1 ΔX_1	Change in Factor X_2 ΔX_2	Marginal Rate of Substitution $\Delta X_2/\Delta X_1$
50	62			
55	49	5	-13	-2.6
60	40	5	- 9	-1.8
65	34	5	- 6	-1.2
70	29	5	- 5	-1.0
75	25	5	- 4	-0.8
80	22	5	- 3	-0.6

The elasticity of substitution is defined as the relative change in the quantities of two resources which combine in

[1]The exact marginal rate of substitution refers to a single point on the iso-product curve. It must be computed as a derivative of the isoquant equation, expressed as dX_2/dX_1 or dX_1/dX_2, where the change in X_2 or X_1 becomes infinitely small.

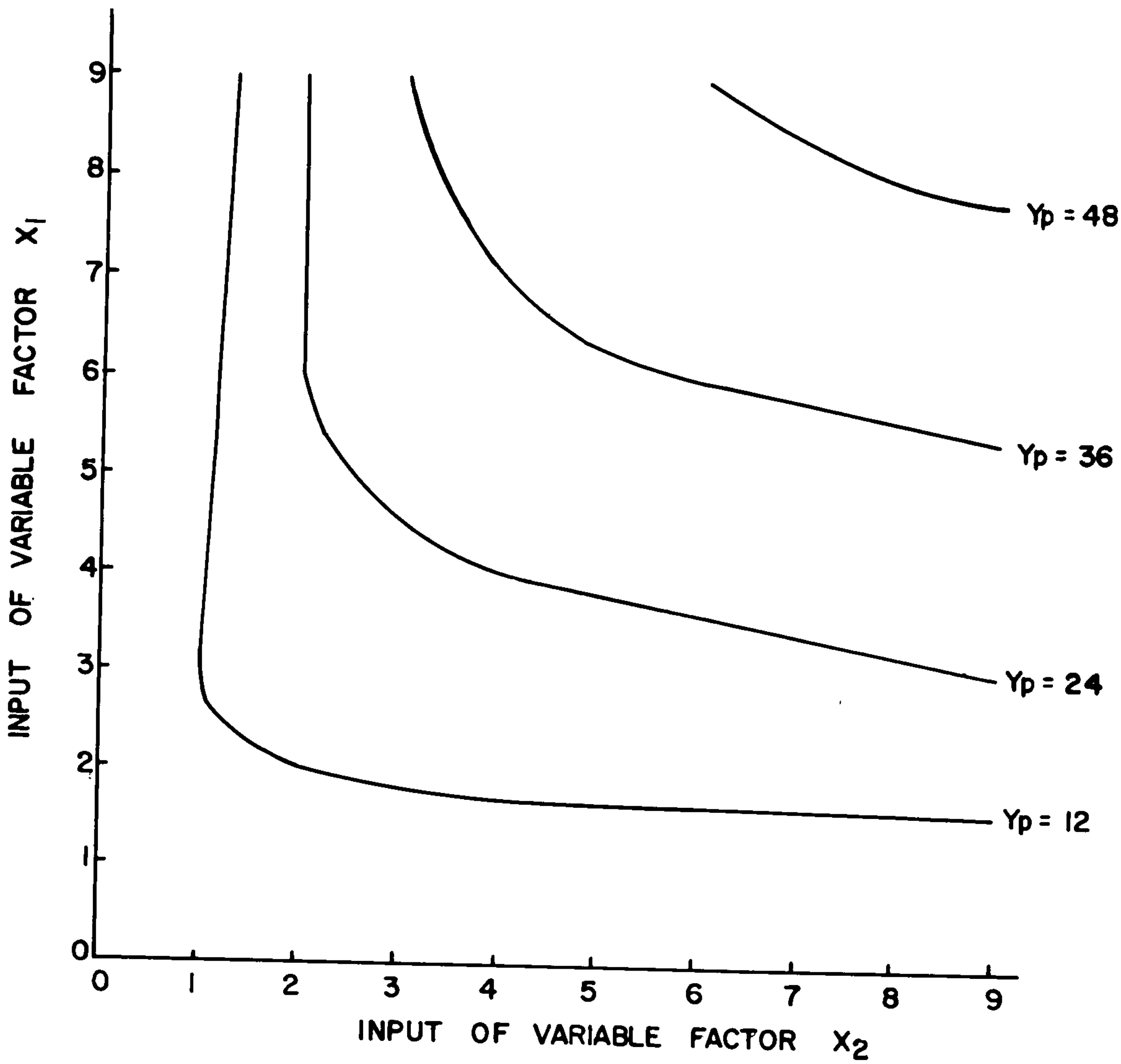

FIGURE 4.— Isoquants or iso-product curves.

producing a fixed amount of product. In substituting factor X_1 for X_2, the elasticity of substitution can be calculated as

$$E_s = \frac{\Delta X_2}{X_2} \Big/ \frac{\Delta X_1}{X_1},$$

where X_1 and X_2 are the original quantities of the two factors, ΔX_1 is the change in factor X_1 and ΔX_2 is the change in factor X_2.[1] The elasticity of substitution is always negative for substitute resources, and indicates how fast the slope of the iso-product curve changes or how rapidly the marginal rate of substitution declines.

Resources can be either technical substitutes or technical complements. The extreme condition of technical complementarity involves resources that combine only in a fixed proportion. In this case, there is only one combination of resources for producing each quantity of product. It is impossible to maintain a given level of output by factor substitution. Also, the total product is unaffected by adjustments to the input of one factor alone. A larger output is obtained only by simultaneous increases in the input of both factors.

There are other cases of technical complements where further reduction in input of one factor cannot be replaced by an increase in another factor. Many resources employed in agriculture are of this nature, serving both as substitutes and technical complements over different ranges of input combi-

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[1]This method of calculation gives the average elasticity for a range of factor combinations, or for a portion of the iso-product curve. The elasticity at a specific point on the product curve must be computed by calculus and in reference to an infinitely small change in resource inputs.

nations. Iso-product curves for complementary or limitational resources of this nature are shown in Figure 5. If output is to be maintained at 10 units, factor X_1 can be substituted for X_2 only up to the point where the iso-product curve becomes vertical. Conversely, replacement of X_1 with X_2 ends where the iso-product curve becomes horizontal. Thus, factor substitution is possible over a range of input combinations but, beyond a certain point, some minimum input of one factor is required to maintain the level of output.

The two resources are substitutes within the range of input combinations delineated by the ridge lines OA and OB (Figure 5), and they are complements for all combinations falling outisde of the ridge lines. Factor X_2 is complementary with X_1 for the vertical portions of the iso-product curves above OA because (1) no further substitution of X_1 for X_2 is possible without a decrease in total product, and (2) X_2 must be increased along with X_1 to gain an increase in total product. These conditions also apply in respect to the relationship of X_1 with X_2 for the portions of the iso-product curves falling below OB.

The difference between technical substitutes and technical complements in resource combination can now be stated more specifically. Resources are technical substitutes when their marginal rate of substitution is negative or less than zero. In Figure 5, the sign of $\Delta X_2/\Delta X_1$ is negative for all factor combinations that are indicated by the portion of each iso-product curve falling within the ridge lines OA and OB. Within

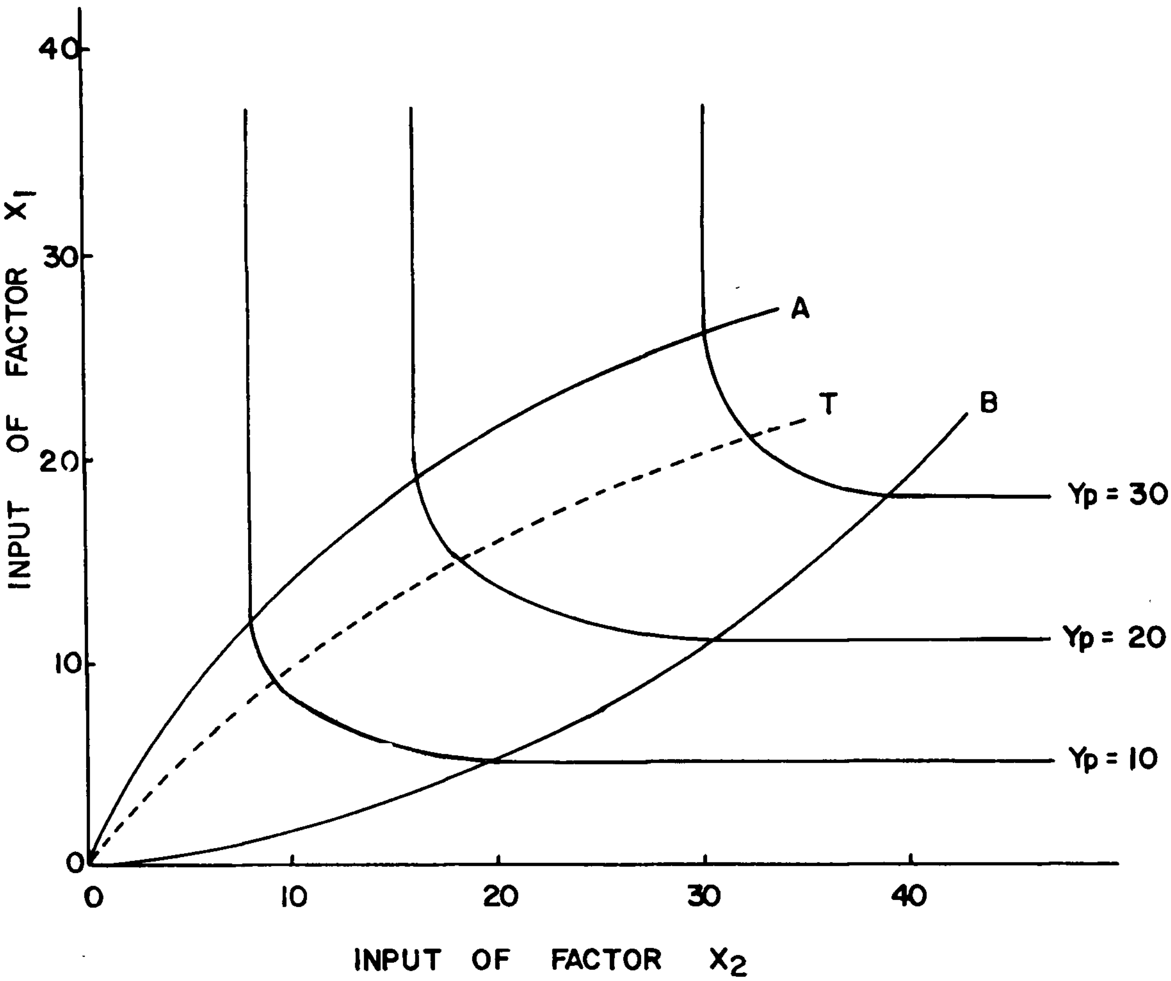

FIGURE 5.— Ridge lines and isocline indicating equal marginal rates of substitution at different levels of output.

these limits, an increase or positive change in X_1 is always associated with a decrease or negative change in X_2. Resources are technical complements when the marginal rate of substitution is zero or greater. The substitution ratio is zero along the vertical portion of each iso-product curve (above OB in Figure 5) because none of the minimum amount of X_2 required to maintain a level of output can be replaced by an increase in the input of X_1. In some extreme cases, it is possible for the input of one factor to be carried to such a high level that the other factor must also be increased to maintain a constant output of product. Under this condition, the ratio of change in factor inputs is positive.

An irrational combination of resources is indicated by a marginal rate of substitution that is equal to, or greater than zero. Under conditions of limited substitution, the area of irrational resource use begins at the point on an iso-product curve where the two factors become complementary. Any factor combination for a constant product that falls outside of the ridge lines (OA and OB in Figure 5) is irrational because increased input of one factor either allows no reduction in input of the other factor or requires that input of the other factor also be increased. Conversely, the same quantity of product can be obtained by using less of one or both resources. The limits of rational resource combination are marked by the ridge lines, which define the points of zero factor substitution on a family of iso-product curves. Factor combinations falling between the ridge lines are rational because a larger input of one factor permits a reduced input of the other factor at a given level of output.

As already noted, the marginal rate of substitution usually diminishes as more of one factor and less of another factor is used to produce the same quantity of product. However, as output is raised to higher levels, factor substitution may be at a greater or lesser rate. The slope of iso-product curves for successively larger products may become steeper or flatter, depending on whether the marginal rate of substitution declines more rapidly or more slowly than at the preceding output. Also, a change often occurs in the range of factor combinations within which substitution is possible. Nevertheless, the marginal rate of substitution can be equal at different levels of output.

These conditions are illustrated by the iso-product contours in Figure 5. The line OT defining the points of equal marginal rate of substitution is called an isocline. The ratio $\Delta X_1/\Delta X_2$ is equal at all points where the isocline intersects an iso-product curve. There is a continuous isocline for each marginal rate of substitution that is common to all constant product curves. The ridge lines also are isoclines in the sense that they denote a substitution rate of zero. Equal substitution rates, however, do not necessarily occur on all iso-product curves; rates of substitution may be found at lower levels of production which do not exist at a higher level.

Resource Combination and Cost Minimization

Resource substitution presents the problem of combining factors in a way to minimize the cost of producing a given amount of product. Alternatively, the maximum economic product, measured in profits at the farm, is obtained from given resources only when each unit of output is produced with the minimum cost or outlay of resources. Resources that substitute continuously in the production of a given output constitute a major area of resource substitution relationships. Confronted with the many factor combinations that are possible under conditions of continuous substitution, some criterion is required to indicate which of the several alternatives is most desirable. The relevant indicator for profit maximization is the <u>factor price ratio</u>.

Knowledge of the factor price relationship is unnecessary for making adjustments in factor combinations when the marginal rate of factor substitution is equal to or greater than zero. Such irrational resource combinations can be rejected as uneconomic because the aggregate input of resources could be reduced without lowering the level of output. As long as the same physical output can be produced with less of one or more factors, net profit is not at a maximum. Rational combinations of resources are associated with the portion of the iso-product curve characterized by negative and diminishing marginal rates of factor substitution. Since a smaller input of one factor must be compensated by a larger input of another factor within the range of rational resource combinations, the optimum combination cannot be selected without reference to the factor price relationship.

The principle of cost minimization can be stated as follows: If two or more factors are employed in the production of a single product, cost is at a minimum when the ratio of factor prices is inversely equal to the marginal rate of substitution of the factors. This condition is expressed by the equation

$$\Delta X_2/\Delta X_1 = Px_1/Px_2,$$

where $\Delta X_2/\Delta X_1$ is the marginal rate of substitution of factor X_1 for factor X_2, Px_1 is the price per unit of X_1, and Px_2 is the price per unit of X_2.

This principle of cost minimization is best illustrated when two factors substitute at a diminishing marginal rate. The average marginal rate of substitution of X_1 for X_2 under an assumed factor-factor relationship is shown in Table 16. When factor prices are \$1.80 per unit of X_1 and \$1.00 per unit of X_2, 100 units of output are produced at a minimum cost with an input combination of 55 to 60 units of X_1 and 40 to 40 units of X_2. The average marginal rate of factor substitution ($\Delta X_2/\Delta X_1$) of 1.8 within this range of inputs equals the factor price ratio (Px_1/Px_2) of 1.8. A single least-cost factor combination is not indicated because the substitution rate is an average for the range of inputs rather than the exact substitution rate at a specific input combination.

Adjustments in the factor inputs are required to retain a least-cost combination when a variation in one or both factor prices causes a change in the price ratio. With factor prices of \$1.60 for X_1 and \$2.00 for X_2 (Table 16) the price ratio (Px_1/Px_2) becomes 0.8. Equating this price ratio with the marginal rate of substitution indicates that costs are minimized with an input combination of 70 to 75 units of X_1 and 25 to 29 units of X_2. All other combinations of factor inputs involve a larger total cost under these factor prices.

TABLE 16

MINIMIZATION OF COSTS UNDER A DIMINISHING MARGINAL RATE OF FACTOR SUBSTITUTION (HYPOTHETICAL DATA)

Input of Resources Required to Produce 100 Units of Output		Marginal Rate of Substitution of X_1 for X_2	Cost of Producing 100 Units with Factor Prices of:	
			X_1 = \$1.80	X_1 = \$1.60
X_1	X_2	$\Delta X_2/\Delta X_1$	X_2 = \$1.00	X_2 = \$2.00
50	62		\$152	\$204
55	49	2.6	148	186
60	40	1.8	148	176
65	34	1.2	151	172
70	29	1.0	155	170
75	25	0.8	160	170
80	22	0.6	166	172

An <u>iso-cost line</u> represents graphically all the possible combinations of two factors that can be purchased at a given total cost. If two factors X_1 and X_2 cost \$3 and \$1 per unit, respectively, the iso-cost line AB in Figure 6 shows the different quantities of X_1 and X_2 that are available for a constant expenditure of \$30. Iso-cost lines are always linear for factors that can be purchased in any quantity without affecting the market price. The slope of the iso-cost line denotes the factor price ratio. The slope of line AB (or Px_2/Px_1) is 1/3, indicating that one unit of X_1 can be purchased at the same cost as three units of X_2.

An iso-cost line can be constructed for any constant total outlay for the two factors. Line CD in Figure 6 represents the iso-cost line for an expenditure of \$60 for X_1 and X_2 with prices remaining at \$3 and \$1, respectively. It has the same slope as AB because the factor price ratio is unchanged. However, since CD represents a larger total outlay, it falls at a higher level than AB.

Any disproportionate change in factor prices alters the factor price ratio and, consequently, causes a change in the slope of the iso-cost line. This effect is shown in Figure 7 by the iso-cost lines AB and CD, both of which represent a constant expenditure of \$60 for X_1 and X_2. Line AB indicates a price ratio (Px_2/Px_1) of 1.0/1.5 at prices of \$6 for X_1 and \$4 for X_2. If the price of X_1 decreases to \$2 and the price of X_2 increases to \$10, the price ratio is changed to 5.0/1.0. The iso-cost line, as represented by CD, then assumes a steeper slope and rotates toward the axis of the factor that has become relatively cheaper. Line AB indicates that only 0.67 unit of X_1 can be purchased for the cost of 1.0 unit of X_2. However, with factor prices as indicated by CD, 5.0 units of X_1 are equal in cost to 1.0 unit of X_2.

The least-cost combination of factor inputs for a given

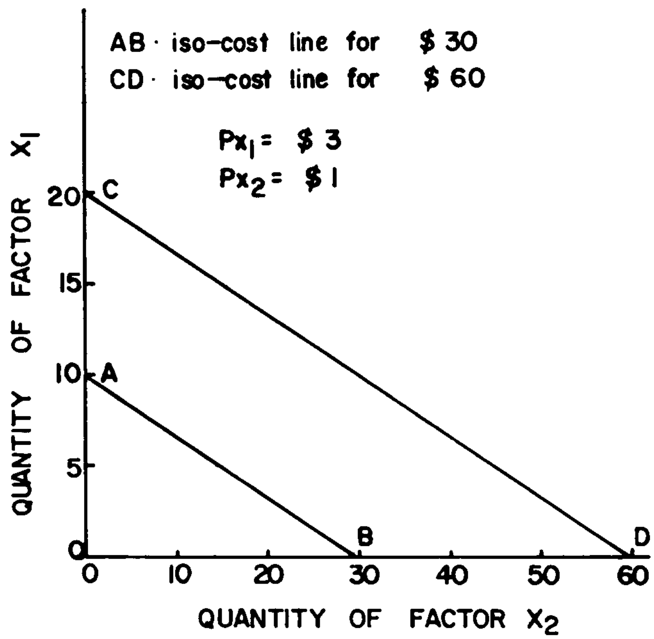

FIGURE 6.—Iso-cost lines for constant factor prices.

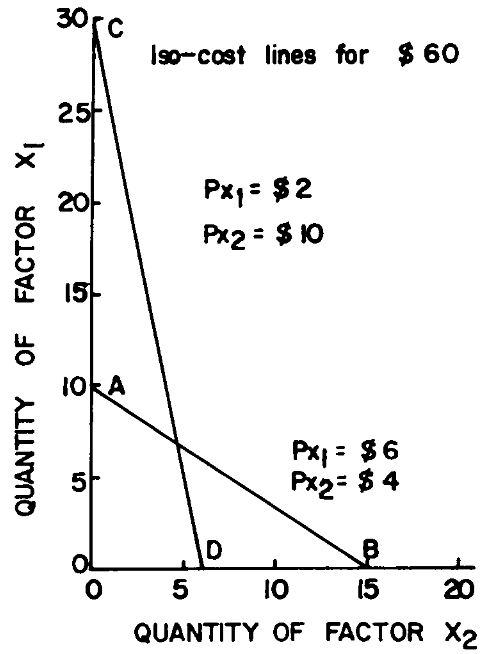

FIGURE 7.—Iso-cost for different factor prices.

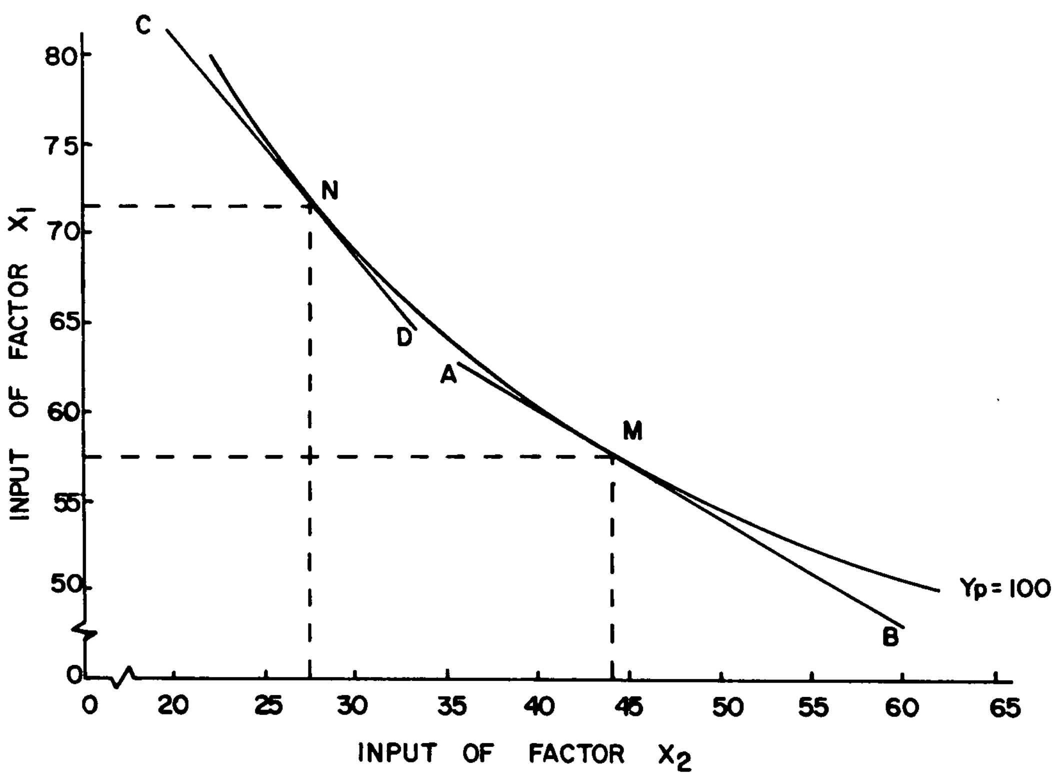

FIGURE 8.—Least-cost combination of factor inputs as indicated by tangency of iso-cost lines and iso-product curves.

output of product is denoted by the point of tangency of an iso-cost line with an iso-product curve (Figure 8). At this point, the factor price ratio and the marginal rate of factor substitution are equal. Line AB is an iso-cost line with a slope (Px_2/Px_1) of 1.0/1.8, when the factor prices are \$1.80 for X_1 and \$1.00 for X_2. It is tangent to the iso-product curve for 100 units of output at point M, indicating inputs of 57.5 units of X_1 and 44 units of X_2 for minimum total cost. If the factor prices change to \$1.60 for X_1 and \$2.00 for X_2, the iso-cost line shifts to CD with a slope of 1.0/0.8. Tangency with the iso-product curve at point N indicates that 71.5 units of X_1 and 27.5 units of X_2 minimize the total cost of factor inputs. As would be expected, the relatively cheaper factor X_1 is substituted for the more expensive factor X_2 as a result of the variation in factor prices. Input of X_1 is increased from 57.5 to 71.5 units, and input of X_2 is decreased from 44 to 27.5 units.

The fact that more than two variable factors are required in most agricultural production processes does not invalidate the conditions for minimizing the cost of factor inputs. The principle can be extended to any number of factors. If three substitute resources, X_1, X_2, and X_3, are used in a production process, the total cost of factor inputs for a given level of output is minimized when

$$\Delta X_1/\Delta X_2 = Px_2/Px_1,$$
$$\Delta X_1/\Delta X_3 = Px_3/Px_1, \text{ and}$$
$$\Delta X_2/\Delta X_3 = Px_3/Px_2.$$

Conditions of Optimum Resource Combination in the Short Run

The principles of factor substitution have been presented up to this point under the assumption of a limited supply of resources for the producing unit. However, if resources are available in unlimited quantities, then two problems in resource combination must be considered. Since an input-output relationship involving two or more variable factors includes both the factor-product and the factor-factor relationships, the optimum level of output and the optimum combination of variable resources must be selected. This requires a decision on (1) how to combine resources with a fixed technical unit as output is expanded from zero to the most profitable level (the factor-product relationship), and (2) how to combine the variable resources for minimum costs at each level of output (the factor-factor relationship).

The attainment of minimum costs as output is expanded to the most profitable level poses the question of whether resource inputs should be increased in a fixed or in a variable proportion. The answer depends on the factor price ratio and on changes in the marginal rate of factor substitution when the level of output is raised, as illustrated in Figure 9. The lines marked IP are iso-product curves for different levels of production. The factor price relationship is indicated by the slope of the iso-cost lines denoted as EC. Tangency of the iso-cost line and the iso-product curve specifies the least-cost combination of inputs for each output. The expansion line B[1] drawn through these minimum cost points shows that a relatively greater proportion of factor X_1 should be employed as output is increased, when costs are minimized for each particular output. This results from the higher rate of substitution of X_1 for X_2 that is associated with increased output.

Limitations on capital for the acquisition of additional resources, among other conditions, prevent most farmers from extending production to the optimum level. Typically, they must attempt to gain the largest possible profit from a stock of various resources that is restricted to rather narrow limits. Under these conditions, the farmer is confronted with the task of (1) allocating a given quantity of resources between technical units producing a single product in a manner to maximize the physical product and its value, and (2) allocating a given quantity of resources between alternative commodities and enterprises in a manner to maximize the total value of production.

The analysis of Figure 9 indicates that, for a farm producing a single product with factors X_1 and X_2, profits can be maximized by (1) equating the factor price ratio Px_2/Px_1 with the marginal rate of factor substitution $\Delta X_1/\Delta X_2$ at each level of output, and (2) extending output until the marginal cost of resources equals the marginal value of product. When this condition is attained, the combination of resources cannot be

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[1]The expansion line is actually an isocline; both mark a point on each iso-product curve where the marginal rate of factor substitution is the same.

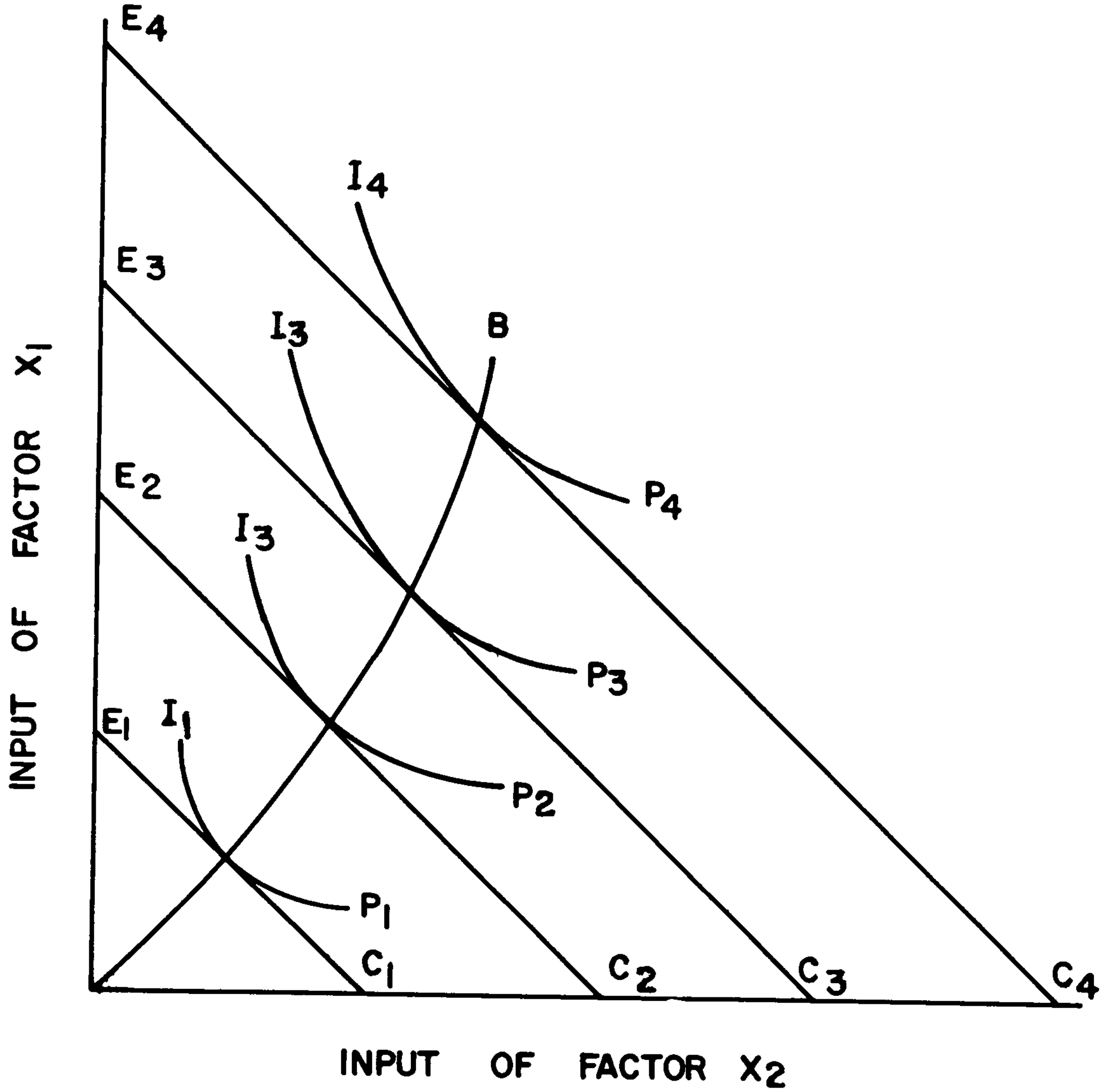

FIGURE 9.— Profit maximization and the expansion path.

rearranged to increase net profit, and the marginal <u>value</u> productivities of all resources are equal. If the quantity of resources necessary to achieve this ultimate position are not available and cannot be obtained as the result of limited capital or other conditions, then output will be restricted to some lower level and the full maximization of profit will be impossible.

At the points of tangency of the iso-product curves and the iso-cost lines shown in Figure 9, the marginal physical products of factors X_1 and X_2 are equal, or

$$\Delta Y/\Delta X_1 = \Delta Y/\Delta X_2.$$

Denoting the marginal physical products as MPx_1 and MPx_2, respectively, this equation can be written as

$$MPx_1 = MPx_2.$$

Since profit is maximized at each level of output by employing the resource combinations indicated by the point of tangency, the marginal physical product of factor X_1 equals the factor/product price ratio, or

$MPx_1 = Px_1/Py$ and similarly

$MPx_2 = Px_2/Py$. Then

$$\frac{MPx_1}{MPx_2} = \frac{Px_1/Py}{Px_2/Py} = \frac{Px_1}{Px_2}, \text{ and}$$

$$\frac{MPx_2}{MPx_1} = \frac{Px_2}{Px_1}.$$

If three factors X_1, X_2 and X_3 are used to produce a single product Y, profits are at a maximum for each level of output when

$$\frac{MPx_1}{MPx_2} = \frac{Px_1}{Px_2},$$

$$\frac{MPx_1}{MPx_3} = \frac{Px_1}{Px_3}, \text{ and}$$

$$\frac{MPx_2}{MPx_3} = \frac{Px_2}{Px_3}.$$

This condition of profit maximization can also be expressed as

$$\frac{MPx_1}{Px_1} = \frac{MPx_2}{Px_2} = \frac{MPx_3}{Px_3},$$

which indicates that, for each level of output, the ratio of the marginal physical product to the factor price is equal for all factors.

The marginal value product of a factor is obtained by multiplying the marginal physical product by the product price. For example, the marginal value product of X_1 is $(Py)(MPx_1)$. Consequently, the condition for maximum profits at each level of output can also be stated as

$$\frac{(Py)(MPx_1)}{Px_1} = \frac{(Py)(MPx_2)}{Px_2} = \frac{(Py)(MPx_3)}{Px_3} = k.$$

This means that profits are maximized when the ratio of the marginal value product to the factor price is the same for all factors, and equals the constant "k". For a farm with unlimited capital, factor inputs would be increased in the combinations indicated by the expansion line until the level of output reached the point where the marginal value productivity of each factor equals the factor price, and the value of "k" is 1.0. In other words, $1 invested in an additional input of any factor results in an increase of $1 in value of the product.

However, if a farm has limited capital for the acquisition of resources, output will be restricted to some lower level. The conditions for maximum profit are still denoted by the above equation, except that the constant "k" has a value greater than 1.0. The ratio of marginal value product to the factor price is the same for all factors, but is greater than 1.0. In this situation, the marginal value productivity of each resource exceeds the price (marginal cost) of the resource. Consequently, more profit could be gained by expanding output to a higher level but, with a limited amount of capital, the resources needed for this expansion of output cannot be acquired. Thus, with insufficient resources or other conditions serving to restrict the level of output, the resource combination required for maximum profits is attained when the marginal value product/factor price ratio is equal, but greater than 1.0, for all factors.

APPENDIX IV

AGGREGATION OF TOTAL OUTPUT AND THE INPUT CATEGORIES IN THE PRODUCTION FUNCTION

Total output and the various input categories in the production functions derived for the market egg enterprise were measured and classified in the following manner:

X_1, total output measured in dollars:

egg sales,

<u>plus</u> bird sales (fowl, chicken, chicks and breeders),

<u>plus</u> manure sales,

<u>plus</u> patronage dividends from co-operatives,

<u>plus</u> market value of eggs and poultry meat used in the farm home,

<u>plus</u> any increase in flock inventory value at end of the record year as compared with start of the record year,

<u>minus</u> any decrease in flock inventory value at end of the record year as compared with start of the record year.

X_2, real estate and equipment input measured in dollars:

building depreciation (2.5 per cent of estimated replacement cost),

<u>plus</u> interest on investment in buildings (4.0 per cent of current depreciated value),

<u>plus</u> building repairs,

<u>plus</u> insurance,

<u>plus</u> interest on investment in land (4.0 per cent of current value),

<u>plus</u> taxes,

<u>plus</u> equipment depreciation (15.0 per cent of current depreciated value),

<u>plus</u> interest on investment in equipment (4.0 per cent of current depreciated value),

<u>plus</u> equipment repairs,

<u>plus</u> small tools purchased,

<u>plus</u> operating costs for car, truck and tractor.

X_3, laying flock input measured in layer years:

pullet layer years,

<u>plus</u> hen layer years adjusted to equivalent of pullet layer years (hen layer years <u>multiplied by</u> the ratio of the average value of a hen to the average value of a pullet; these ratios are 0.647 for 1949, 0.588 for 1950, and 0.640 for 1951, as calculated from the initial laying flock inventory).

X_4, labor input measured in hours:

operator labor,

<u>plus</u> family labor,

<u>plus</u> hired labor.

X_5, feed input measured in dollars:

purchased grain,

<u>plus</u> purchased mash,

<u>minus</u> refund on feed sacks returned,

<u>plus</u> feed supplements,

<u>plus</u> value of farm grown feed.

X_6, other cash inputs measured in dollars:

purchased litter,

<u>plus</u> value of farm grown litter,

<u>plus</u> brooder fuel,

<u>plus</u> electricity,

<u>plus</u> medicine and disinfectant,

<u>plus</u> shell,

<u>plus</u> grit,

<u>plus</u> purchased chicks and other stock,

<u>plus</u> other cash expenses.

APPENDIX V

COMPUTATION OF REGRESSION COEFFICIENTS AND TEST OF STATISTICAL SIGNIFICANCE

Solution of the appropriate set of normal equations[1] yields the regression coefficients, or value of the "b's" for the production function. The constant "a" in the production function is calculated by substituting the means of the variables (in logarithms) and the "b" values in the following equation:

$$a = \bar{X}_1 - b_2\bar{X}_2 - b_3\bar{X}_3 - b_4\bar{X}_4 - b_5\bar{X}_5 - b_6\bar{X}_6.$$

The standard error of estimate[2] is a measure of the reliability of the production function for estimating total output. It indicates the accuracy with which estimates of total output may be expected to approximate the actual output values contained in the sample. The standard error of estimate adjusted for the size of sample ($\bar{S}$) is obtained by taking the square root of $\bar{S}^2$. The value of $\bar{S}^2$ is given by the formula:

$$\bar{S}^2 = \frac{\Sigma(x_1^2) - \left[b_2(\Sigma x_1x_2) + b_3(\Sigma x_1x_3) + b_4(\Sigma x_1x_4) + b_5(\Sigma x_1x_5) + b_6(\Sigma x_1x_6)\right]}{n - m}$$

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[1]For the method of deriving and solving the normal equations, see Mordecai Ezekiel, Methods of Correlation Analysis (2d ed.; New York: John Wiley and Sons, 1950), pp. 198-205 and 459-469.

[2]Ibid., pp. 128-135 and 208-210.

where x_1, x_2, etc. are deviations of the variables X_1 and X_2 (in logarithms) from their respective means (in logarithms);

n equals the number of sets of observations or records in the sample;

m equals the number of constants in the production function, including "a" and the "b's".

The <u>coefficient of multiple determination</u>[1] measures the proportion of total variance in output that is explained by the several input categories. It was calculated from the formula:

$$R^2 = \frac{b_2(\Sigma x_1x_2) + b_3(\Sigma x_1x_3) + b_4(\Sigma x_1x_4) + b_5(\Sigma x_1x_5) + b_6(\Sigma x_1x_6)}{\Sigma(x_1^2)}.$$

Adjusting for the size of sample was obtained by calculating $\bar{R}^2$ as

$$\bar{R}^2 = \frac{1 - (1 - R^2)(n - 1)}{n - m}$$

The adjusted <u>coefficient of multiple correlation</u> ($\bar{R}$) is a measure of the degree of correlation between output and the collective inputs. It was calculated by taking the square root of the adjusted coefficient of multiple determination, or

$$\bar{R} = \sqrt{\bar{R}^2}$$

The <u>standard error of the coefficient of multiple correlation</u> was computed from the formula

$$\sigma_R = \frac{1 - R^2}{\sqrt{n - m}}.$$

The t-test was used to test the statistical significance of the multiple correlation coefficient. The value of "t" was

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[1]Ezekiel, <u>op.cit</u>., pp. 136-143 and 210-213.

computed from the expression

$$t = \frac{R\sqrt{n - m}}{\sqrt{1 - R^2}}.$$

By referring to a table of t-values, it is possible to establish the probability that the multiple correlation coefficient differs from zero due to chance alone.[1] The multiple correlation coefficient is accepted as significantly greater than zero if the computed t-value exceeds the t-value stated in the table for an arbitrarily selected level of probability (usually 0.05), with the degrees of freedom equal to n - (m - 1).

The <u>standard errors of the regression coefficients</u> were obtained from the following formulas:[2]

$$\sigma b_2 = \bar{S}\sqrt{c_{22}}$$
$$\sigma b_3 = \bar{S}\sqrt{c_{33}}$$
$$\sigma b_4 = \bar{S}\sqrt{c_{44}}$$
$$\sigma b_5 = \bar{S}\sqrt{c_{55}}$$
$$\sigma b_6 = \bar{S}\sqrt{c_{66}}$$

Statiscal significance of a regression coefficient was determined by applying the t-test in the same manner as for the multiple correlation coefficient. The t-value was obtained by calculating the ratio of the regression coefficient to the unbiased estimate of its sampling standard deviation, that is

$$t = \frac{b}{\sigma b}.$$

By comparing this ratio with the values in a t-table, it was

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[1]Ezekiel, <u>op. cit.</u>, pp. 322-325.

[2]Ezekiel, <u>op. cit.</u>, pp. 469-472.

possible to ascertain the probability that the regression coefficient differed from zero due to chance.

The marginal value productivity of each resource category was estimated with the level of all inputs fixed at their geometric mean. The marginal value products were derived by partial differentiation of the production function with respect to each input variable. For the Cobb-Douglas function, the derivative with respect to X_2 (the marginal value product of input X_2) is given by the equation

$$\frac{dX_1}{dX_2} = \frac{b_2 X_1}{X_2},$$

where b_2 is the regression coefficient of input X_2;

X_1 is the geometric mean of total output X_1;

X_2 is the geometric mean of input X_2.

The geometric mean of a variable was computed as the antilog of the arithmetic mean of the values of that variable expressed in logarithms. For example,

$$\text{geometric mean of } X_1 \text{ (total output)} = \text{antilog } \frac{\Sigma \log X_1}{n},$$

where $\Sigma \log X_1$ is the sum of the logarithmic values of X_1;

n is the number of records (X_1 values) in the sample.

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