AN APPLICATION OF LINEAR PROGRAMMING TO LOG ALLOCATION IN THE FOREST INDUSTRY OF BRITISH COLUMBIA

by

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ABSTRACT

This thesis presents an application of linear programming to the question of efficient log allocation in the forest industry of British Columbia. Current procedures for allocating logs among alternative utilization processes are discussed and it is suggested that a more efficient allocation might be obtained through a systematic approach to the problem. The economic necessity of improving net returns to the log supply is emphasized.

A linear programme log-allocation model is presented, based on an integrated-industry in the coastal region of British Columbia. The model encompasses three main categories of log-use, namely sawmilling, plywood production and pulp production, and demonstrates how a given supply of logs may be optimally distributed among these structurally different log-conversion processes.

Emphasis throughout this study is on the structure of the linear programme model, although considerable effort was directed to obtaining realistic data. Solutions of the model, obtained through the services of the Computing Centre at the University of British Columbia, are discussed, and a superficial comparison is made with actual log allocation in the industry. Modifications of the model to suit the log-allocation problem faced by an individual firm in the short-run are discussed and normal comparative-statics applications are considered.

It is pointed out that many of the simplifying assumptions in the model may be relaxed. However, the main limitation to its practical application by industry and government lies in the quality and type of data available. In this respect it is suggested that the linear programme model of this thesis provides a valuable guide to the production data required to improve economic efficiency in the forest industry.

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CHAPTER I

INTRODUCTION

The forest resource of British Columbia is predominately owned by the Crown, but exploited entirely by private enterprise. A question of 'efficiency of resource allocation' arises therefore at two levels: at the forest management level which is in the hands of the Provincial Government, and at the entrepreneurial or private level of firms consuming timber as a raw material. The conditions for optimal resource allocation at each level differ, and give rise to different maximizing criteria.

As owner of most of the forest resource the Provincial Government controls the allocation of timber to the forest industry.[1] To efficiently discharge this function it must be guided by the social aspects of resource exploitation as well as by the demands of the private forest industry, for the ultimate objective of resource management is to maximize the value of the resource to society. The efficient owner is concerned therefore with estimating future benefits and costs associated with the forest resource, and with establishing time rates of preference to be applied in discounting these future benefits to present

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1 The term 'forest industry' as used in this thesis refers collectively to the private firms engaged in extraction and processing of logs. Government administration of the forests is therefore not considered a part of the forest industry.

values. On the strength of these evaluations the owner is able to develop suitable forest-management policies to guide private industry in its exploitation of the resource.

At the industry level efficient resource allocation results from the profit-maximizing behaviour of each private firm. The supply of timber to each firm is one of several input factors which it seeks to allocate in a manner that yields it a maximum over-all net return. Efficient allocation of timber in the forest industry as a whole therefore depends upon the aggregate profit-maximizing behaviour of all firms in the industry, each one optimizing its own production function. The problem is removed from any welfare implications other than those imposed by the government.

This thesis considers the efficiency of log allocation at the second of these two levels, namely, at the level of industrial use. The main objective is to present a systematic approach to the problem of attaining an optimal allocation of logs among various utilization processes, such that the return to a given supply of logs is maximized. It will be useful therefore to elaborate upon current log-allocation procedures in the industry, before considering the log-allocation problem faced by individual firms.

1. LOG ALLOCATION IN THE FOREST INDUSTRY

Two forms of log allocation in the forest industry may be distinguished. First, allocation of logs <u>among</u> competitive firms. Second, allocation of logs <u>within each</u>

firm, between its various alternative utilization processes.

Log Allocation Among Firms.

This problem is solved (theoretically) through the open market for logs, where prices are established by the usual forces of supply and demand and the price system rations the available supply to competitive firms in the market. Not all of the timber in the coastal region goes through the open-log market, but prices established there are used as a basis for completing most "closed" market transactions. In theory the price system should result in maximum economic efficiency in log allocation for the economy as a whole, providing that perfect competition exists throughout (many buyers, many sellers, homogeneous product, free exit and entry of firms, etc.).

In practice it is questionable whether the market is actually "open and competitive".[2] For example, in the case of integrated firms which have their own logging operations it might be argued that "internal" log transactions occurring within the firm are not a part of the open market. In addition there is an element of direct price distortion arising from an interplay between the open-log market and the market for chips and small-wood residues.

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2 "The condition and operation of the open-log market and chip and small-wood marketing in the Vancouver Forest District", Report to the Select Standing Committee of Forestry of the Legislative Assembly of British Columbia, B. C. Forest Service, Victoria, (Jan. 1962) - supplied in confidence to the author.

At the present time the industry accepts that the price paid under contract for wood chips can be related directly to the price established on the Vancouver log market for number three grade hemlock, and vice versa.[3] Any firm in a position to control prices in either of these markets therefore could quite feasibly manipulate prices in the other market.

In the short run, with given utilization capacities and given log supplies for each firm, the aggregate allocation of logs to different utilization processes may not coincide with the optimal allocation plan for the industry as a whole. Such misallocation reflects imperfections in the log-allocation system.

Log Allocation Within Integrated Firms.[4]

The integrated forest products firm is usually confronted with a given or known log supply which it seeks to allocate among its various conversion processes in a manner that maximizes total net return to the enterprise. Profit-maximizing behaviour of the integrated firm therefore involves seeking an optimum allocation of its log supply. The

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3 "The condition and operation of the open-log market and chip and small-wood marketing in the Vancouver Forest District", Report to the Select Standing Committee of Forestry of the Legislative Assembly of British Columbia. B. C. Forest Service, Victoria,(Jan. 1962) - supplied in confidence to the author.

4 In this thesis all references to the firm will assume an integrated forest-products enterprise. Most of the discussion however is equally applicable to smaller firms specializing in a single utilization process, such as sawmilling.

given log supply is usually determined by the firm's own holdings of timber-cutting rights, but it may be augmented by trading or purchasing logs on the open market.

Current Procedures.

The allocation of logs among various conversion processes in an integrated forest products enterprise is presently a haphazard procedure conducted mostly by "rules of thumb". Each integrated firm generally has a centralized co-ordinator responsible for controlling log allocation on a basis of experience, special knowledge, and a talent for assessing the log demand submitted by each mill. In this respect the central log supply control office performs a co-ordination rather than an allocation function, and it is difficult to conceive of any high degree of economic efficiency resulting from the former approach.

It is true that market situations may change rapidly and unpredictably, requiring decisions based on experience and individual judgement. Nevertheless, the lack of a systematic and rigorous analysis of normal log requirements, and the absence of a technique for quickly determining the most efficient (profit-maximizing) allocation of logs, almost certainly results in a high, and unnecessary, degree of inefficiency.

Some important obstacles to developing an effective log-allocation procedure are the lack of sufficient specific data on log yields and production costs, and the complexity of the calculations involved even when the required data are available.

It may be argued that the industry has hitherto lacked an incentive to develop efficient log-allocation procedures, for profits have been high and exploitation characterized by an "extensive" use of a relatively cheap and ubiquitous resource. It will be pointed out below though that these conditions are changing rapidly and that trends in the value of the resource are already such that efficiency in allocating timber among alternate uses demands a more systematic and rigorous approach. The development of such an approach comprises the subject of this study.

II. SCOPE OF THE STUDY

This study examines the problem of efficient allocation of logs in the forest industry. Specifically, the purpose is to develop a solution technique for arriving at the optimal allocation plan for a given supply of logs. The form of the problem therefore is similar to that faced by an integrated forest products enterprise in allocating its logs among alternative utilization plants.

The log-allocation problem lends itself to mathematical solution by linear programming techniques. Thus we shall discuss the elements of "choice", "optimization", and "restraint", and their mathematical analogues in a linear programme model of the allocation problem, before proceeding to a solution and discussion of practical applications.

The emphasis throughout is on the structure of the model. Although considerable effort has been devoted to seeking out the most accurate data available for the purposes of the analysis, no claim is made for a high degree of statistical accuracy. Frequently the data available has been insufficient or in a form poorly suited for the analysis. In these instances (as noted below), extrapolations and assumptions have been made. Nevertheless, it is hoped that the broad lines of the results achieved will provide a useful insight to the efficiency of the log-allocation process.

Thus, while the primary objective is to present a rigorous methodological approach to the problem, more accurate empirical data are required before much reliance can be placed on the numerical results. The study points out the nature of the technical and economic data that are required for the practical application of the solution developed.

Framework of Analysis.

"Linear programming is concerned with the problem of planning a complex of interdependent activities in the best possible (optimal) fashion." [5] The validity of results however depends entirely upon the quality of data supplied to the problem.

Three different approaches were considered to the task of assembling data for the proposed model; these were

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5 A. Charnes, W. W. Cooper and A. Henderson, An Introduction to Linear Programming, Wiley & Sons Inc., N.Y., ch. 1.

to base the model on either,

(i) imaginary data.

(ii) case-study data.

(iii) industry-average data.

The first alternative was discarded on grounds that it would convey little of the practical significance of a linear programme approach to log allocation, and would be of limited interest to those concerned with industry problems. Nor would it specify the shortcomings of existing data. It would be little more than an exercise in linear programming.

The second alternative was discarded for two reasons. Firstly, it was considered unlikely that any firm would be prepared to supply sufficiently detailed data, much of which would probably be regarded as confidential. Secondly, a case study would inevitably reflect special characteristics of the case-study firm.

The third alternative was selected since it would avoid most of the above objections and would be of practical interest to both government and industry. Although the purpose of this study is mainly to demonstrate the application of linear programming to log allocation it was considered important to keep the demonstration as realistic and as general as possible. It will be shown later how both government and industry might modify the "industry-wide" model to analyze their own special problems.

In conjunction with the decision to use industry-average data it was decided to base the model on the concept of an "integrated-industry". The general approach was to treat all the timber-using industries in a given log-market area, drawing on a given supply of logs, as a single completely-integrated entity within which timber could be moved about without cost to its place of highest value-in-use.

Having decided to base the linear programme model on an "integrated-industry" using industry-average data it remained to select an area in which these proposals could be effectively implemented.

The area chosen was that part of the British Columbia forest industry which draws its supply of logs from British Columbia Forest Service Inventory Zone 1 and part of Inventory Zone 2. Inventory Zone 1 covers the lower coastal mainland and Vancouver Island, and the relevant part of Inventory Zone 2 comprises the coastal sector of the Prince Rupert Forest District.[6] This area will henceforth be referred to collectively as the "coastal region."

Guthrie and Armstrong have underlined the geographical "isolation" of the coastal region, bounded on the east by the Coast Range, on the west by the Pacific Ocean and to the north and south by frontiers with the United States.[7]

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6 Continuous Forest Inventory of British Columbia, B.C. Dept. of Lands & Forests, Victoria, B.C., (1958), Tables A-I.

7 J. A. Guthrie, G. R. Armstrong, Western Forest Industry - An Economic Outlook, The John Hopkins Press, Baltimore, (1961), ch.1.

Rankin has pointed out the low cost of water transportation for forest products within the region.[8]

We can summarize the principal reasons for selecting the coastal region as our study area therefore, as follows:

(i) An isolated economic unit. The geographic "isolation" and high mobility of logs within the coastal region make it free from any external complications such as significant log movements into and out of the region.

The major portion of conversion facilities in the region is concentrated in the Gulf of Georgia area, into which most of the coastal log supply flows at low cost. The assumptions of a completely "integrated-industry" are thus not unrealistic.

(ii) A consistent log supply. The commercial timber of the coastal region is relatively uniform with respect to species, size and grades. There are definite biological trends within the region, such as a higher percentage of spruce in the northern sector, but these are substantially offset by cheap transportation

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8 A. G. Rankin, "Cost-Price Relationships in the Forest Industry", The Forestry Chronicle, v. 39, (Mar. 1963), pp. 69-79.

and by the active open market for logs in the Gulf of Georgia.

(iii) <u>A single administrative area</u>. Management of the timber resource in this region is under the sole jurisdiction of the British Columbia Forest Service. This has the desirable result (for our purposes) of uniform and consistent data reporting. In addition this single authority reports to the government of British Columbia only, so that political influences on management of the forests are uniform and consistent.

(iv) <u>A substantial portion of the industry</u>. Latest official figures for British Columbia show that in 1962 approximately 58 percent of the total log-scale for British Columbia was recorded in the coastal region.[9] Thus although the coastal region comprises a relatively small portion of the total area of British Columbia, it accounts for a very significant portion of the current log-scale. This permits the model to be employed with more confidence in demonstrating the implications of a change in forest management policy on the forest industry's profits and consumption pattern.

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9 <u>Annual Report</u>, B. C. Forest Service, Victoria, B.C., (1962).

CHAPTER II

THE CASE FOR IMPROVED LOG ALLOCATION

The main reasons for inefficiency in log allocation have already been referred to; in this chapter the case for improved log allocation in the industry will be examined. It will be shown that improvement in log-allocation offers one of the most promising avenues toward greater economic efficiency in the industry as a whole.

We shall begin by briefly reviewing the forest industry in the coastal region. The trends in industrial behaviour aimed at overcoming some of the increasing difficulties faced by the industry will be indicated, and the most likely future courses of technological development and research will be outlined.

I. THE NATURE OF THE FOREST INDUSTRY

Size.

The forest industry in the coastal region was founded on saw-timber production, which enjoyed a unique advantage derived from the particularly large high-quality trees that grew in abundance along the coast. Technological progress and expanding markets in Canada and abroad led to increasing rates of exploitation of the forests, and to a more intensive use of the logs harvested. At present most of the forest resource in the coastal region, and indeed throughout British Columbia, is under some form of forest management.

The industries which consume logs directly as a raw material are usually divided into three categories; sawmills, plywood mills, and pulp mills. The relative size and growth rate for each of these industries, for selected years from 1954 to 1962, is shown in Table 2.1 on the following page. It is apparent that lumber production is the major industry in terms of total value of production.

The slow per capita growth rate in the lumber industry has been discussed by many writers[11,12,13] who note the low level of research and development expenditure in this industry. In addition, substitute products have undoubtedly influenced the demand for lumber adversely, and reduced its rate of growth. By contrast, the plywood and pulp industries have experienced substantial growth over the past ten years.

Much of the growth in plywood demand can be traced to a more general trend toward the use of components and sheet materials to reduce on-site labour in construction. Zinuvska has stressed the importance of construction labour costs as an external influence on plywood demand.[14]

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11 J. A. Zinuvska, "The Forest Products Mix in a Changing Economy," Proceedings, Soc. of American Foresters, (1960), pp. 59-63.

12 Joseph Zaremba, Economics of American Lumber Industry, Robert Speller Publishers, New York, (1963), ch. 5.

13 J. A. Guthrie and G. R. Armstrong, Western Forest Industry - An Economic Outlook, The John Hopkins Press, Baltimore, (1961), ch. 2 and 3.

14 J. A. Zinuvska, op. cit.

TABLE 2.1

THE VALUE OF FOREST PRODUCTION IN BRITISH COLUMBIA
BY INDUSTRIAL CATEGORIES IN SELECTED YEARS
(millions of dollars)*

	1954	1957	1959	1961	1962
Lumber	270.1	275.4	323.9	359.0	388.3
Plywood	53.0	71.8	75.5	79.8	89.6
Pulp & Paper	156.2	177.4	234.5	258.4	292.0

Source: Annual Report, British Columbia Forest Service, Victoria, B. C., (1962).

* Values include loading and freight charges within the Province.

It should be noted however that this growth in plywood consumption has occurred largely at the expense of lumber consumption in traditional lumber markets.

Growth in the pulp industry has been consistently strong, following the steady increase in paper and paper-products consumption, and augmented by an increasing use of high-grade pulp or cellulose as a raw material for the chemical industries.

<u>Organization of the Industry</u>.

The lumber industry is characterized by a large number of individual enterprises consuming a widely available raw material with relatively low-capital requirements and relatively easy entry. Some writers have suggested that this fragmented structure of the lumber industry has been largely responsible for its lack of technological and market research and thus its sluggish rate of growth. To quote Zinuvska, "...this is an industry structure which might warm the heart of Adam Smith, but it would break the heart of any industrial research director."[15] Nevertheless, while there is a large number of small sawmills on the Pacific Coast there are also several very large lumber producing plants.

In contrast, the number of individual plywood and pulp production plants in the coastal region is relatively small. Latest figures indicate there were sixteen plywood

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15 J. A. Zinuvska, "The Future for Wood in a Competitive Market," paper for presentation at joint meeting Puget Sound and Columbia River sections, Society of American Forester, Longview, Wash., May 4, 1963.

mills[16] and thirteen pulp mills[17] in operation in the coastal region during 1961. Concentration in the plywood and pulp industries is closely associated with large economies of scale, and the high level of technological innovation necessary to expand into world markets.

II. PROBLEMS OF THE INDUSTRY

Cost control in the forest industry seems to vary inversely with the age of the industry. Thus there is less knowledge of unit-costs in the logging and lumber industries than there is in the younger plywood and pulp industries. Inadequate cost control here refers to an insufficient knowledge of unit-costs for each stage of the production process. The problem may exist for two reasons:
(a) insufficient data or (b) risk and uncertainty.

An insufficiency of data may arise merely because the required data is difficult to collect. For example, the logging industry is faced with a heterogeneous supply of raw material (trees) from which it produces an equally heterogeneous product (logs). Great difficulty is experienced therefore in determining the cost of production

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16 Veneer and Plywood Mills, Dominion Bureau of Statistics, Catalogue No. 35-206, Queen's Printers, Ottawa, (July, 1963).

17 Pulp and Paper Mills, Dominion Bureau of Statistics, Catalogue No. 36-204, Queen's Printers, Ottawa, (Oct. 1963).

attributable to <u>each</u> log, particularly since logs of different grade are typically produced from the same felled tree.

A similar problem arises in the lumber industry where a variety of different lumber grades and sizes are frequently produced from the same log. Plywood and pulp mill operations are more amenable to measurement of production data and show a reasonably high degree of cost control.

In addition to the difficulty of collecting unit-cost data there has so far been little incentive to tackle the problem. In the logging and lumber industries of the past the small-owner approach has prevailed, with little or no attention directed to unit-costs as long as the balance sheet showed a profit. Only recently with the emergence of large integrated firms and a growing awareness by the industry of the real limits to its resource base, has there been any attempt to measure unit-costs of production.

(b) Risk and Uncertainty.

Inadequate cost-control in the forest industry due to risk and uncertainty is inevitable. For example, logging costs vary with seasonal weather conditions which influence the fire hazard and the physical difficulties of logging. Some measure of the predictability of these risks is possible but it depends upon data which is lacking in most parts of the province.

The uncertainty of future stumpage values presents a problem throughout the forest industry. Stumpage reflects the true residual of market price less estimated cost of production. Since ownership of the forest resource in most parts of the province remains in the hands of the Crown, and since the government charges stumpage with harvest, these costs to the forest industry are highly unpredictable.[18]

In summary, the logging and log-conversion industries suffer varying degrees of inadequate production-cost control. The lack of unit-costs curtails the measurement of conversion efficiency and productivity which in turn directly affects earnings.

The lack of control over supply costs is reflected in the industry's current haphazard allocation of logs among various conversion processes. Clearly the industry must overcome these cost-control deficiencies if it is to expand and meet the increasing competition for its markets.

Market Problems.

The lumber industry of British Columbia faces strong competition from substitute products, notably plywood, plastics, concrete, aluminium, and steel. Most of this competition has occurred in the traditional markets for lumber where the industry has been forced to either reduce costs, reduce profits, or concede the market. The plywood

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18 A. Milton Moore, Forestry Tenures and Taxes in Canada, Canadian Tax Foundation, Toronto, (1957), ch. 2.

and pulp industries, unlike lumber, have not faced the same degree of competition from substitutes. In fact, they have made inroads into areas previously dominated by lumber, cotton, jute, etc.[19]

The lumber and pulp industries of British Columbia sell a major portion of their output in foreign markets; approximately 77 percent of the lumber output[20] and 89 percent of the pulp output from the coastal region is exported.[21] Since coastal British Columbia accounts for a small fraction of the total world supply of these products this region must, essentially, take world market prices for these products as given.

The coast mills, which are often constructed specifically to serve foreign markets, ship a higher percentage of their output to world markets than does the average mill in the province. However, it is generally agreed that the forest industry of the interior region of British Columbia will account for an increasing share of the total forest industry in the province, as stocks of large-diameter mature timber on the coast are depleted, and improved technology makes possible the harvesting of previously non-commercial stands of small diameter spruce, balsam, pines and fir.[22]

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19 J. A. Guthrie and G. R. Armstrong, op. cit. ch. 6.

20 Production, Lumber Mills, Dominion Bureau of Statistics, Catalogue No. 35-003, Queen's Printers, Ottawa, (accumulated monthly issues 1963).

21 L. Read, Economist & Statistician, Council of Forest Industries, Vancouver, B.C. (1964) - unofficial estimate, since primary source not available.

22 J. A. Guthrie and G. R. Armstrong, op. cit. ch. 5.

As the lumber industry is forced into timber of declining size and quality its position in world markets will become weaker. The industry will be forced to compete in a market for products produced from predominantly second-growth forests, not unlike those found in other parts of the world, and will thereby lose its former natural advantage in large-dimension clear lumber.

Raw Material Supply.

A pervasive problem facing the forest industry is the declining quality of timber. The problem affects the lumber and plywood industries most, for no matter how efficient the logging and log-conversion techniques may become, log quality ultimately governs lumber and plywood quality.

The transition to lower quality raw material is hastened by the fact that lumber and plywood are competitors for the limited stock of high quality logs. The plywood industry in particular prefers large-diameter logs, containing a high percentage of clear wood. The price of these has risen, in the face of limited supply, to the point where log costs now account for up to 60 percent of total plywood costs.[23,24]

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23 W. E. Mayhew, "A New Method of Allocating Costs to Veneer by Grades," For. Prod. Journ., v. 8, (Aug. 1958), pp. 27A-30A.

24 A. G. Rankin, "Cost-Price Relationships in the Forest Industry," The Forestry Chronicle, v. 39, (Mar. 1963), pp. 69-79.

While timber quality is declining the annual harvest is increasing steadily,[25] and the logging industry is being forced to log in more inaccessible areas and at progressively higher elevations. These trends are being accompanied however by technological advances in logging methods, and by more intensive utilization of the raw material on the part of the log-conversion industries.

III. INDUSTRY TRENDS

The forest industry is fully aware of its production and market problems, and has taken steps to overcome them.[26,27] Research and development in technology and marketing have been strongly supported in recent years. The plywood and pulp industry have always been strong in research and development, but only in recent years have the logging and lumber industries made important advances in these fields. Structural changes in the industry have also been significant. These trends are discussed in the following sections.

Research and Development.

(a) Technological Research and Development.

Recent technological advances in all phases of the forest industry have been directed towards lowering unit-production costs, raising over-all utilization of the raw

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25 "Progress to September, 1962 and Future Prospects of the British Columbia Sustained Yield Forestry Program." B.C. F.S., submitted to Tariff Commission of the United States of America, Washington, D.C. October, 1962.

26 J. A. Zinuvska, op. cit.

27 A. G. Rankin, op. cit.

material, and developing new products. Frequently these advances occur concurrently. A great deal of the research effort is carried out by government agencies and the universities, aside from industrial research performed by the larger private enterprises.

(i) Reduction in unit-costs. These gains are made through an improvement in processing efficiency, either by raising output per dollar of operating cost or reducing cost per unit of throughput. Some examples are portable-spar logging, band saws and automated green-chains in sawmills, thinner veneers in plywood production, and continuous digesters in pulping.

(ii) Increased utilization. These are basically physical gains aimed at raising over-all conversion efficiency where higher product yields per unit of raw material lead to higher net returns. Typical examples are chipping of sawmill waste, patching veneer sheets to raise their grade, and recovering chemicals (such as lignosulphonates) from pulp-mill effluents.

(iii) New products. The development of new products may be stimulated by research discoveries or by changing market demands, or some combination of these forces. New products may result from the improvement of existing processes or they may require an entirely new production facility. Some examples are glued - laminated structural beams, mixed specie plywoods, specialty pulps (high alpha cellulose) and wood-extract adhesives.

(b) Market Research and Development.

Both individual firms and industry organizations have made efforts in recent years to improve the market acceptance of wood products, through higher standards of market service and by maintaining competitive prices and quality. In addition, the industry has sought new markets either through product differentiation or through direct promotion of new uses for wood.

(i) Improved market acceptance. Better service to lumber customers has been achieved through bundling of lumber, where each bundle class has a fixed proportion of specified grades of lumber selected to conform with a typical customer's needs. Improved service through the establishment of authorized lumber dealers has also promoted better acceptance.

(ii) New Markets. Industry associations have been particularly active in promoting new uses for wood products through publications and advertisements. For example, drawings and plans for houses, boats, trailers, etc., are offered at subsidized (often zero) prices as an inducement to use wood products. Promotion of overseas markets has been attempted through organized trade missions to foreign markets by prominent British Columbia wood-product manufacturers.

Structural Adjustments.

There is a marked tendency in the forest industry of British Columbia towards integration, both vertical and horizontal. It will be worthwhile to consider here the implications of these two forms of structural adjustment.

(a) Horizontal Integration.

We have noted the competition between the lumber and plywood industries for high-quality logs. In addition, the lumber industry faces competition from the pulp industry for poorer-grade logs. This conflict can be resolved, at least in part, through horizontal integration. Thus the large firm with substantial timber holdings seeks to achieve maximum utilization of its timber by integrating the lumber, plywood and pulp production processes within its own plant. The high-grade peeler logs are used as plywood stock; the bulk of the better grade logs goes to the sawmills; and the remaining logs plus waste from lumber and plywood-mill operations go to the pulp mill. This facilitates the allocation of logs to their most profitable use. However, integration of this sort is practical only within the larger companies.

Some measure of partial integration is secured by the exchange of logs between firms or by the sale of pulp chips. For example, a pulp mill may exchange Douglas fir peeler logs from its own holdings, for hemlock or small logs that a plywood mill wishes to dispose of, or a sawmill may sell its chips to a nearby pulp mill.

(b) Vertical Integration.

The development of the forest industry concurs with Scott's stage <u>II</u> in all aspects, with the possible exception of industrial organization.[28] As Zinuvska has pointed out however, the inducement to invest in the forest industry derives from manufacturing profits in consuming timber (or wood) as a raw material rather than from the returns to growing timber.[29] Thus forest industry firms tend to merge with (invest in) more highly refined wood-manufacturing processes. This is substantiated by the recent trend to vertical integration forwards by many of the larger forest product enterprises.

Vertical integration for large integrated firms provides a degree of stabilization in product prices and sales volumes, through the mutual association of a given productive capacity with a known and captive market. Also, vertical integration frequently results in a broader financial base for the integrated unit, enabling economies of scale through investment in large-capacity plants, which in turn are supported by the guaranteed supplies and markets.

(c) Summary.

Structural adjustments in the forest industry in recent years have resulted in the formation of large integrated forest-product enterprises. Although some social benefits from more complete utilization of the raw material may result, this need not necessarily coincide with greater

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28 A. D. Scott, "The Development of the Extractive Industries," The Can. Jour. Econ. and Pol. Science, v. 28, (Feb. 1962), pp. 70-87.

29 J. A. Zinuvska, op. cit.

economic efficiency. A discussion of the ramifications of government forest policy on structural adjustments within the forest industry, however, is outside the scope of this thesis. Nevertheless it is apparent that the formation of large integrated firms creates a managerial problem for each, in finding the optimal distribution of its available log supply among the various utilization processes. This was discussed in the previous chapter.

IV. CONCLUSIONS

To quote Zinuvska again, "...the challenge of competition in the forest industry today is the challenge of technology and markets." [30] The problem is basically a question of achieving maximum economic efficiency in the use of the forest resource. With strong competition in world markets and in many cases from substitute products, the forest industry as a whole is impelled to lower its unit-costs. In addition, as several writers suggest,[31,32] the forest industry is under considerable pressure to achieve fuller utilization of the raw material, which may not always coincide with attaining maximum economic efficiency. Nevertheless, it is generally agreed that both questions can be solved only through greater expenditures on research and development, in both technology and marketing.

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30. J. A. Zinuvska, op. cit.
31. op. cit.
32. J. A. Guthrie & G. R. Armstrong, op. cit. ch. 6.

As a prerequisite to raising the economic efficiency of log-conversion however, the industry must ensure that it uses its raw material in an optimum manner. Specifically, it is important that each log be allocated to the process where it can earn its highest return. It is clear that this condition is reflected in the question of efficient log allocation discussed in the previous chapter.

In addition to observing the problem of log allocation we have now discussed its relative importance to the future expansion of the forest industry. As a part of the research and development effort called for, improved log allocation offers one of the more promising and fruitful avenues toward greater economic efficiency in the industry as a whole.

CHAPTER III

EXPOSITION OF THE LOG-ALLOCATION PROBLEM

The problem of efficiently allocating logs among alternative utilization processes has been described and its importance emphasized. In this chapter the elements of the problem will be discussed, with a view toward synthesizing a linear programming solution in the following chapter.

Three conditions underly the problem of log allocation in the forest industry, namely:

(1) The objective of profit maximization on the part of the owners of log-utilization plants.

(2) A limited log supply.

(3) Interdependence among the various log-conversion processes.

These conditions which determine the nature of the linear programme model warrant examination in further detail.

Profit Maximization.

It is not unreasonable to assume that each firm in the forest industry attempts to maximize profits and hence seeks to maximize the return to all its imputs, including its log supply.

If the log supply is given, at zero cost, then maximum return, net of manufacturing and conversion costs, constitutes the highest possible sum of entrepreneurial profit on log utilization and economic rent to the log supply. On the other hand, if log costs are deducted from total returns, the maximum return represents maximum entrepreneurial profit (only) that can be earned in log utilization.

Limited Log Supply.

Following the assumptions discussed earlier, we will consider a single integrated-industry with a total log consumption equal to the annual log scale of the coastal region. The log supply is limited in a given year by the actual harvest of timber in the region, plus any carryover of inventory from previous years. In the coastal region this harvest is based almost entirely on natural "old-growth" forests so that the log supply is very heterogeneous with respect to size, quality and specie. Government regulations requiring, among other things, the complete removal of all useable timber from each stand logged, tend to increase the heterogeneity of the harvest by forcing the inclusion of material that would otherwise be left unharvested.

In practice, each firm is free to purchase or trade logs on the open market and thereby attempt to assemble the best supply of logs for its particular operations.

Interdependent Utilization Processes.

It will be expedient to categorize the utilization processes under three headings: lumber production, plywood production, and pulp production. We shall disregard less-important specialty processes which involve the production of such products as shingles, particle-board, etc.

(a) Lumber Production.

In broad terms we may describe this process as the conversion of logs into wood products whose shape and dimension are limited by the size and quality of the log impute.

(b) Plywood Production.

This process may be described as the conversion of logs into wood products whose shape and dimension are not governed,entirely, by the initial log dimension.

(c) Pulp Production.

Pulp production involves decomposing logs into elemental wood fibres, either chemically or mechanically, and the subsequent reconstitution of these fibres into pulp sheets. In the model to follow we shall not consider processing beyond the pulp stage, i.e., into paper products, chemicals, textiles, etc. Pulp can conveniently be regarded as a final product since it actually enters the market in this form.

Process Interdependencies.

There are two types of interdependence to be considered in the log-allocation problem. In the first instance each log-utilization process or activity draws raw material directly from the logging industry and consumes a portion of the limited log supply. Therefore any net increase in consumption in one utilization process in a given year reduces the supply of logs available to the other utilization processes. In an optimizing problem this relationship *per se* is trivial, for logs would be allocated solely to the process which yields the maximum return per log. Nevertheless, this interdependence is important and of practical significance to the industry, for optimal log allocation in this instance depends on the relative prices of the final products.

A second type of interdependence arises through the physical or technological relationships between utilization processes. For example, the pulp industry has become increasingly dependent upon sawmill residues for its raw material (chip) supply.[33] Similarly, the cores remaining after a log has been peeled for veneer are often sawn into lumber, and waste from this operation converted into chips for pulping. Technological interdependencies arise through the production of "composite-products".[34] Typical examples are plywood doors (a combination of plywood and lumber) and laminated beams (a combination of different types of lumber). It should be observed in particular that plywood and pulp are also composite-products, of veneer and chips, respectively. It is clear that a change in the production level of a composite-product will influence the production levels of

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33 J. A. Guthrie & G. R. Armstrong, *Western Forest Industry - An Economic Outlook*, The John Hopkins Press, Baltimore, (1961), ch. 3.

34 We shall define composite-products as products which involve combining two or more wood products produced earlier in the utilization system to form a new product.

its components, and the production levels of other composite-products using any of the same components.

Technological or physical interdependence - that is where the actual level of one process depends on the actual level of some other process(es) - is the key to a linear programme formulation of the log-allocation problem. The objective is to find that optimum combination of processes and process levels which will yield a maximum return to the log supply as a whole.

Summary.

A log-allocation problem arises out of three characteristics of the forest industry: profit maximization, a limited log supply,and technological interdependence among different utilization processes. This is not to suggest that these are the only relevant considerations,for we have overlooked questions of limited process capacity, immobility of factors of production, homogeneity of production functions, etc. Some of these considerations will be discussed later.

It should be noted that each utilization category, lumber, plywood and pulp, presents its own programming problem resulting from a limited supply of raw material with technologically interdependent activities and a profit-maximizing objective.[35,36,37,38] In addition, however, plywood and pulp production involve a recombination of intermediate products (i.e., veneer and chips) to form final products (i.e., plywood and pulp), whereas sawmilling results directly in a final product plus some intermediate products (e.g. chips) for use in other technologically interdependent activities. The contribution of this thesis lies in the construction of a linear programme model which combines these structurally different activities within the confines of a single problem or model.

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35 F. H. Curtis, "Linear Programming the Management of a Forest Property," Journal of Forestry, v. 60, (Sept. 1962), pp. 611-616.

36 N. D. Jackson and G. W. Swinton, "Linear Programming in Lumber Production," For. Prod. Journ. v. XI, (June, 1961), pp. 272-274.

37 E. Koenigsberg, "Linear Programming Applied to the Plywood Industry," For. Prod. Journ. v. XI, (Sept. 1960), pp. 481-486.

38 A. E. Paull, "Linear Programming - A Key to Optimum Newsprint Production," Pulp & Paper Mag. of Can., v. 57, (1956), pp. 145-151.

CHAPTER IV

CONSTRUCTION OF A LOG-ALLOCATION LINEAR PROGRAMME MODEL

The previous chapter dealt with the nature of the log-allocation problem. In this chapter we shall proceed to a more detailed discussion of its main characteristics and consider their mathematical interpretation in a log-allocation linear programme model. A large part of the discussion will relate to the assumptions made concerning the real world and its mathematical analogue. We shall postpone to a later chapter all discussion of the implications of relaxing these assumptions.

I. BASIC OUTLINE OF THE MODEL

Linear programming problems have three basic components:

(1) An objective.

(2) Input restraints.

(3) Alternate interdependent activities.

It has been shown that the log-allocation problem under consideration here contains each of these three elements. Thus the <u>objective</u> is to maximize total net return to an integrated industry which consumes, in a specific period, a given supply of logs in the production of lumber, plywood and pulp. The available supply of logs comprises the <u>input</u> restraints, and the various wood-conversion processes, sawmills, plywood mills, and pulp mills, correspond to

<u>alternate interdependent activities</u>.

Given these three basic components we may describe a linear programme as follows:

Let a_{ij} = the amount of input i required per unit of output from activity j.

p_j = the return per unit-level[39] of activity j.

b_i = the available supply of input i.

X_j = the level of activity j, expressed as a multiple of its unit-level.

Then, in a maximizing problem the objective is to:

Maximize $$\sum_{j=1}^{n} X_j \cdot p_j$$

Subject to: (1) $$\sum_{i=1}^{m} X_j \cdot a_{ij} \leqslant b_i \quad ; \quad j = 1,2, \ldots n$$

(2) $$X_j \geqslant 0$$

Figure I (inside back cover) is a schematic presentation of the log-allocation linear programme model

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39 The unit-level of each activity may be arbitrarily defined to suit the problem. For example, the unit-level of the log-conversion activities in the model is defined as 100 cubic of log consumed. In chapter five we shall systematically define the unit-levels of all activities in the model.

described in this thesis. Each column in the figure consisting of a list of activity coefficients (a_{ij}), represents an activity or process such as sawing a log or manufacturing a plywood; the column headings describe the various activities considered. Each activity is assigned a number from one to 93. The final column on the right-hand side lists the inputs or supply restraints to the model (b_i). The first twelve figures correspond to the log-supply restraints; the remainder refer to intermediate input restraints, which we shall describe later on in this chapter. There are a total of 37 restraints in the model. The bottom two rows of the figure contain the unit-level returns for each activity (p_j), and the corresponding activity levels (X_j).

II. STRUCTURE OF THE MODEL

Allocation Objective.

The objective function, or mathematical expression representing total net return to the enterprise, is a summation of the returns from all activities included in the log-allocation problem. A solution to the problem consists of finding a set of non-negative values for the X_j activity levels which will satisfy the objective function, subject to the supply restraints.

It should be noted that any activity may be defined to suit the problem. The only requirement is that an activity must be interdependent with at least one other activity in the model. In other words, the final level of an activity may not be arbitrarily determined at the start.

Normally we are interested in maximizing total net return, as described in the previous chapter. If the unit-level return of each activity in the model is expressed as a net-return figure, then clearly the summation of activity returns in the objective function will yield a total net return figure. In the model presented in this thesis it will be necessary to consider positive returns and negative returns, corresponding to revenue activities and cost activities, respectively. The summation of these in the objective function however will still yield a total net return figure. This will be demonstrated later, in chapter five.

Activities.

Compilation of the pertinent activities is an important step in the construction of a linear programme model. In this log-allocation problem we have recognized lumber, plywood and pulp-production activities as the basic components of the model, but it is necessary to break these down more thoroughly. For example, logs may be either sawn into lumber, peeled for veneer, or chipped, and the veneer and chips may in turn be sold directly or recombined into plywood or pulp, respectively.

Technological interdependence between activities was noted in the previous chapter as one of the main sources of the allocation "problem". Furthermore, a large measure of this interdependence came from the existence of intermediate products such as veneer and chips, which could be combined into composite products such as plywood and pulp. It is logical therefore that a successful log-allocation model should include separate activities for each method of production and consumption of these intermediate products.

The activities producing intermediate products are referred to as "intermediate-activities", as opposed to "final-activities" which produce a final product consumed outside the model. At an optimum solution the "intermediate-activity" levels would indicate how the given total log supply should be distributed among the various log-conversion processes (sawing, peeling and chipping); while the "final-activity" levels would show how the intermediate products (lumber, veneer and chips) should be disposed of either directly, without further manufacturing, or in combination as part of a final product such as plywood or pulp.

<u>Intermediate Activities</u>.

These activities describe the consumption of logs to produce products such as lumber, veneer and chips. They might therefore be termed cutting activities. In the model these are segregated into three natural categories, sawing, peeling, and chipping. A detailed discussion of the activity coefficients in each activity is given in the next chapter, but it should be noted here that cutting activities are the only activities with coefficients in the Primary Input rows of the model, indicating that they are the only activities which consume logs as input, see Figure I.

Negative signs are given to the activity coefficients and unit-level returns of each intermediate activity, (i.e., sawing, peeling and chipping), see Figure I. This follows from the nature of the intermediate restraints to the problem, discussed in a later section of this chapter.[40] At this point we may note that intermediate activities produce inputs for other activities in the model, (i.e., they produce negative outputs from the model) and incur costs of production which must be deducted from gross revenue to yield a net return figure (i.e., they incur negative returns).

Final Activities.

The final activities include: (a) recombination of intermediate products into final products for sale, and (b) direct selling of intermediate inputs, and primary inputs.

(a) Recombination Activities.

These activities describe the recombination of intermediate inputs into new products. Each activity represents

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40 See Page No. 41.

a unique combination of intermediate products and the activity coefficients describe the relative proportions of intermediate inputs taken in each case. Recombination activities result in "composite-products", described in the previous chapter. Thus, composite products such as plywood and pulp are produced by recombining outputs from cutting activities (veneer peeling, chip chipping).

(b) Selling Activities.

These activities arise from an imbalance occurring between production and consumption of intermediate products. For example, the proportions in which veneers of different grade are required in the manufacture of plywood are unlikely to correspond closely to the proportions in which the veneers are produced from the log, so that there will be some excess veneer of some grades. To account for this surplus we must include activities of selling or disposing of veneer directly, without further manufacture.

A similar situation exists for mixed specie pulps, where we might expect that the technical requirements determining the proportions in which different species of chips must be mixed will differ from the proportions in which the various species are available. Therefore, chip-selling activities will be required to dispose of any chip surpluses.

Finally there is the possibility that the optimum pattern of allocation for the given log supply may not call for 'utilization' of all logs. Hence the model also provides for direct selling of whole logs.

These selling activities, in effect, guarantee that all logs will be "disposed of" in one way or another and that all wood products or uncut logs will be sold. These selling activities correspond to "disposal" activities referred to in the general literature on linear programming,[41] and are essential to generating equality restraints in the model so that a unique mathematical solution is possible.

Restraints.

The restraints to the log-allocation linear programme fall into two categories: primary input restraints and intermediate input restraints. We shall consider these separately.

Primary Input Restraints.

The primary input restraints correspond to the 'available log supply', which is 'given' in this revenue-maximizing problem and consists of a known mixture of different species and different grades of logs. Since the objective function is to maximize net return, these restraints take the form that no more than the given supply of logs may be consumed.

Intermediate Input Restraints.

The intermediate activities produce intermediate

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41 R. Dorfman, P. A. Samuelson and R. M. Solow, Linear Programming and Economic Analysis, McGraw Hill, New York, (1958), ch. 3.

products which may be consumed in both recombination and selling final activities. This feature gives rise to intermediate input restraints in the model. We shall assume that there is to be no accumulation of intermediate products. Subsequently, since intermediate products are both produced and consumed in the model the intermediate input restraints will be zero.

Intermediate inputs with zero-value constraints are the distinguishing feature of this model, and deserve further comment. We have seen that the first stage of any log-conversion process involves a cutting activity which yields a product (lumber, veneer or chips) that becomes an input to either a selling activity or a production activity, or both. Since linear programming theory requires that all activity levels be non-negative there is a danger that "wood" may be counted twice in any given restraint equation (row in Figure I), once in an intermediate activity and again in a final activity.

This difficulty is overcome by assigning negative values to the technical coefficients of each intermediate activity. These negative technical coefficients describe the outturn of intermediate products which are consumed in final activities (recombination or direct selling) where the activity coefficients are positive.

Thus, negative activity coefficients are associated with the production of intermediate products, and positive activity coefficients are associated with the consumption of intermediate products. The condition of "no accumulation" means that each restraint expression must sum to zero. Mathematically, for the model represented in Figure I, this condition may be expressed as follows:

$$\sum_{i=13}^{37} X_j \cdot a_{ij} = 0 \quad ; \quad j = 1,2, \ldots\ldots.93 .$$

Double-counting could occur also in the objective function, but it is avoided in the same way as above. Thus, negative values are assigned to the unit-level returns of each intermediate activity, and positive values to the unit-level returns of each final (recombination or selling) activity. Negative returns in the objective function can be regarded as costs (negative prices) which subtract from total gross revenues to give Total Net Return.

The distinction between primary and intermediate inputs demands emphasis. Primary inputs are 'given' to the model. Total net return is to be maximized with respect to these primary inputs and the primary input restraint conditions take the form of inequalities. On the other hand, intermediate inputs are variables within the optimizing problem. They are the link in the model between primary input and final product but they are in no way a conditional part of the problem.

It may help to realize that the intermediate or cutting activities take primary inputs (logs) as inputs and produce intermediate products (lumber, veneer, chips) as outputs, whereas the final activities, both recombination and selling (excepting log selling), take intermediate products (lumber, veneer, chips) as inputs and produce final products (plywood, pulp, and lumber; veneer and chips) as outputs.

III. ASSUMPTIONS OF THE MODEL

Two types of assumption may be distinguished in the model. Firstly, Fundamental assumptions associated with the mathematical procedures of linear programming. Secondly, Correlation assumptions required to reduce the actual log-allocation problem under consideration to a form amenable to linear programming.

Fundamental Assumptions.

These assumptions are unavoidable as long as we are considering a linear programme model. They have significant interpretations from an economic viewpoint, as we shall discuss below.

(i) The term 'linear programming' refers to the application of a particular mathematical procedure, based on 'linear relationships' and 'linear inequalities', to problems involving choice. Mathematically, the term linear refers to the existence of all variables in the expressions as homogeneous and of degree one. That is there can be no squared or higher-power terms (such as X^2 or X^3) nor any logarithmic or root terms (such as log X or $X^{\frac{1}{2}}$). To the economist this indicates constant returns to scale in each production process or activity.

(ii) The objective function was shown previously to be equal to the summation of the returns from each activity;

viz: $$\sum^{m} X_j \, p_j$$

The X-variable in this function must be homogeneous and of degree one. Therefore, for the objective function to represent a linear relationship, p_j must be a constant; and constant values of p_j in this model imply both infinite elasticities of demand for the final-product activity outputs, and constant costs for the intermediate activity outputs.

(iii) Finally, the linear relationships conditional upon the model mean that, for a given activity, each input is taken in a fixed relative proportion to the other inputs to the activity. In economic terms this means fixed factor proportions. The actual relative proportions taken are given by the technical coefficients of each activity. These coefficients are constants determined at the outset, which together make up the internal structure of the various activities and form the skeleton of the model. The precision of the activity coefficients is critical to the reliability and accuracy of a linear programme solution.

Correlation Assumptions.

These assumptions are mainly to keep the model as simple as possible. We have already discussed the concept of an integrated industry, on which the model in this thesis is based. We shall consider some additional simplifying assumptions below.

(i) We shall assume zero transportation costs between processes so that, for example, chips produced from sawmill waste are identical in value with those produced from veneer waste or from whole logs chipped at the pulpmill. The low-cost water transportation between utilization sites on the coast, and the trend toward integrated utilization plants, (where sawmilling, peeling and pulping facilities are located together at a central site) make this assumption quite plausible.

(ii) We shall assume long-run conditions wherein there is sufficient time for plant capacity to alter to accommodate whatever pattern of log utilization emerges from the optimum allocation plan. Given a fixed log supply, and infinite price elasticity of demand, the linear programme solution would specify the level at which each activity should be operated to achieve maximum total net return.

(iii) We shall assume that the sawing and peeling activities included for each type of log in the model yield the maximum amount of high-grade product possible. This assumption requires some explanation. Within fairly narrow limits the grade of the product produced can be raised by expending more in the production process (e.g., patching veneers to remove knots, or meticulously carving out the highest priced pieces of lumber obtainable from a log). The assumption here does not involve such technical perfection. Rather, it presumes that mills are currently optimizing their economic returns to processing each type of log and that to extract higher-quality material would involve costs (both opportunity and operating costs) in excess of the returns that would be gained. This assumption is required in order to avoid including all technically conceivable methods of cutting each log - a situation that would greatly complicate the model while adding little to its usefulness.

(iv) We shall assume that the integrated industry on which this model is based is comprised of average utilization plants for the coastal region, rather than one single complex. In this manner the industry-average data employed in the model bears some measure of realism to the actual world. The manipulation of this industry-average data and any imputed conclusions from it are of course subject to the assumptions built into the model.

Summary.

These assumptions are necessarily restrictive in the light of constructing a model of such a complex entity as the forest industry. Some of the assumptions are dictated by the nature of linear programming techniques; some are simplifications to make the demonstration as clear as possible. The assumptions discussed above are considered the more important qualifications of the model. Other assumptions such as those implicit in the production functions of the model (i.e., mobility and homogeneity of factor inputs) should not be overlooked in assessing the model's usefulness.

In chapter six we shall consider the implications of relaxing some of the Correlation assumptions by altering the structure of the model, but the Fundamental assumptions will remain.

CHAPTER V

DATA AND SOLUTION OF THE MODEL

The principal objective in this study is to demonstrate an application of linear programming to the log-allocation problem. An integrated industry has been assumed, along with other assumptions, to keep the model reasonably simple. Nevertheless, to render the results as realistic as possible considerable effort has been made to obtain accurate and representative data.

Data used are from published research studies wherever possible. Where usable published data were not available estimates were obtained from experts in the industry. A discussion of the quality of the data follows in chapter six.

The data falls into three categories; activity coefficients, activity unit-level returns, and input (supply) restraints. Tables of data referred to in this chapter are collected together in an appendix at the end of this chapter.

I. ACTIVITY COEFFICIENTS

A discussion of the activity coefficients and their compilation for the log-allocation model falls into two parts. The first deals with the primary-input/log-allocation sector or log-allocation coefficients, the second part covers the intermediate-input/final-product sector or technological coefficients, see Figure I (inside back cover).

1. Log-Allocation Coefficients.

The primary-input log-allocation sector of the model deals only with allocating a given supply of logs among various intermediate activities. There is no consumption or processing of the log involved. All the log-allocation coefficients are therefore unity, and there is only one log-allocation coefficient per intermediate activity.

In this model we have considered twelve different types of log, as shown in Table 5.1. The supply of each type of log may be allocated to any sawing, peeling or chipping (i.e., intermediate) activity or a combination of these.

2. Technological Coefficients.

The technological coefficients describe the technical relationships implicit in each activity. In the intermediate activities they describe the proportions of the several intermediate products produced. In the final recombination activities they describe the proportions in which intermediate products are recombined to form final products. In the selling activities there is no transformation of the item so that all selling activity coefficients are unity.

Technological Coefficients - Intermediate Activities.

The intermediate activities are sawing, peeling and chipping of logs. The unit-level of each is defined as one hundred cubic feet of logs consumed in the activity. An explanation of the data used in compiling these activity coefficients follows.

(a) Log-sawing Activities.

There are twelve log-sawing activities corresponding to the twelve different log categories considered in the problem. While the implication is that any log can be sawn, this may not in fact be true due to technical limitations of size and shape. However, any logs which are technologically unsawable can be bypassed in the selection of log-sawing activities. It was assumed here that any technically sawable log would be directed to an appropriate log-sawing activity, and that economically sawable logs would be determined from these by the linear programming model, in the light of lumber yields, lumber prices and sawing-cost data.

The lumber yield from any log is divided into four categories, based on lumber grades approved by the British Columbia Lumber Manufacturers' Association.[42]

viz.

1. Clear.
2. Select/Merchantable.
3. Construction/Standard.
4. Utility/Economy.

In addition to the lumber yield from each log-sawing activity, account was taken of the production of waste-wood suitable for chipping, and of hog-fuel (including sawdust) suitable only for burning.

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42 Standard Grading and Dressing Rules No. 59. British Columbia Lumber Manufacturers' Association, Vancouver, (Feb. 1961).

The yields of intermediate product (lumber, chips and hog fuel) resulting from each log-sawing activity are shown in Table 5.2, expressed as a percentage of the volume of log consumed (sawn). This data was compiled as averages from numerous different sources, discussed below. The activity coefficients of each log-sawing activity in Figure I are based on the data in Table 5.2, but expressed as decimals. This is so that the coefficients will be readily applicable to any unit-volume of logs consumed; and the unit-volume may be conveniently defined to suit any particular scale of problem and data.

Douglas fir lumber yields were obtained principally from data published by the United States Department of Agriculture, based on studies conducted in United States sawmills west of the Cascade Mountains in Oregon and Washington.[43,44,45,46,47] It was assumed that this Northwestern

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43 E. E. Matson, "Lumber Grade Recovery from Oregon Coast Type Douglas Fir," U.S. Dept. of Agriculture, Pac. N.W. Forest and Range Expt. Sta. Research Paper No. 3, Portland, Oregon, (May, 1952).

44 E. E. Matson, "Lumber Grades from Young-Growth Douglas Fir," U.S. Dept. of Agri., Pac. N.W. Forest & Range Expt. Sta. Research Note No. 79, Portland, Oregon, (Sept. 1952).

45 E. E. Matson, "Lumber Grades from Douglas Fir Peeler Logs," U.S. Dept. of Agric., Pac. N.W. Forest and Range Expt. Sta., Research Note No. 83, Portland, Oregon, (Nov. 1952).

46 E. E. Matson, "Lumber Grades from Old-Growth Douglas Fir Sawmill Logs," U.S. Dept. of Agric., Pac. N.W. Forest and Range Expt. Sta., Research Note No. 125, (Jan. 1956).

47 E. H. Clarke, "Lumber Grade Recovery from Old-Growth Douglas Fir at a Northwestern Oregon Sawmill," U.S. Dept. of Agric., Pac. N.W. Forest and Range Expt. Sta., Research Note No. 191, Portland, Oregon, (Oct. 1960).

United States data was applicable to the British Columbia coastal area, after appropriate adjustments for differences in the grade rules. Some Canadian Douglas fir lumber yield data was obtained from the Forest Products Laboratory, Vancouver, B. C.[48,49]

Hemlock lumber yields are based on Canadian data published by the Forest Products Laboratory in Vancouver[50] and on discussions with mill managers of local forest products companies. Spruce lumber yields are based solely upon estimates supplied by mill managers in the Vancouver area.

The chip yield data for sawmills was interpolated from several sources. In the absence of specific data on chip yields with respect to log grade on the B. C. coast, a correlation between waste-chip production and log diameter was adopted.[51,52] An average diameter for each log

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48 C. F. McBride and J. M. Kinghorn, "Lumber Degrade caused by Ambrosia Beetles," British Columbia Lumberman, v. 44, (July, 1960).

49 J. Dobie, "A Milling Study of 150-year old Douglas Fir," Publication No. 1032, Forest Products Research Branch, Canada, Dept. of Forestry, Ottawa, 1962.

50 C. F. McBride, J. M. Kinghorn, op. cit.

51 B. Dowdle and R. Bain, "Lumber or Chips - A Comparison of Small-log Utilization Alternatives," U.S. Dept. of Agric., Northeastern For. Expt. Sta., (1960).

52 M. E. Hamlin, "Experience Report - Wastewood and Chip Volume Measurement," Amer. Pulpwood Assoc., Northeast Tech. Comm. Minutes, (1956).

grade was determined from the lumber yield data and a calculation made of the chippable waste from sawing each grade of log. The weighted average diameters for each log grade vary with sources of log supply but the estimated chip-yield data is considered representative for the region as a whole.

(b) Log-peeling Activities.

Some grades of log input are not exposed to log-peeling activities due to the limitations of log-peeling technology. In general, the lowest grades of any specie have very poor veneer yields and high peeling and patching costs. The peeling of these logs is therefore unlikely to be included in any profit-maximizing solution to the problem. Accordingly we have excluded peeling activities for the No. 3 grade saw-logs of each specie in the model.

As with the log-sawing activities, figures in each log-peeling activity represent the yields of veneer, chips and hog-fuel, expressed as a percentage of the volume of log consumed (peeled). All veneer yields are on a green-veneer basis; approximately eight percent volume shrinkage occurs during the veneer drying stage but this loss is accounted for later in the plywood production activity coefficients.

The yields of green veneer from Douglas fir logs, shown in Table 5.3 were compiled from published results of an extensive study carried out by the U.S. Department of Agriculture, Pacific Northwest Forest and Range Experimental

Station, Portland, Oregon,[53] and from discussions between the author and technical experts in the plywood industry in Vancouver. The published American study covered a survey of 18 plywood plants in Oregon and Washington, and was based on the veneer yield from more than five million board feet of Douglas Fir logs. However, this study was carried out between 1950 and 1955 and the industry's technology has advanced appreciably in the past ten years, so that considerable weight was given to the current (but estimated and unofficial) data obtained from local experts. Unpublished data by McBride concurs with these unofficial estimates for the coastal region.[54] The hemlock and spruce veneer yields are based solely on estimates by local experts. The data used in the model and shown in Table 5.3 are considered to be a realistic over-all average for coastal B. C.

The production of chippable waste associated with log-peeling activities is based on data quoted by Guthrie and Armstrong for Western Oregon,[55] from data compiled by Guernsey for British Columbia,[56] and from information

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53 E. H. Clarke and A. C. Knauss, "Veneer Recovery from Douglas Fir Logs," U.S. Dept. of Agric. Pac. N.W. Forest and Range Expt. Sta. Research Paper No. 23, Portland, Oregon, (Aug. 1957).

54 C. F. McBride - unpublished results.

55 J. A. Guthrie and G. R. Armstrong, <u>Western Forest Industry - An Economic Outlook</u>, The John Hopkins Press, Baltimore, (1961), p. 138.

56 F. W. Guernsey, "Some Conversion Factors for British Columbia Forest Products," Forest Products Laboratory of Canada publication, V - 1027, Dept. of Northern Affairs and National Resources, Ottawa, (1959).

prepared by the Institute of Forest Products, Washington.[57]

Hog-fuel production is based on data from the same sources. The applicability of all data to conditions in coastal British Columbia was confirmed by local experts.

(c) Log-chipping Activities.

These activities represent the simple process of converting an entire log into pulp chips. Any log can be chipped. Accordingly, there are 12 log-chipping activities to meet the 12 different log categories in the model. It is assumed that there is 100 percent recovery of the log volume, as chips. In practice this represents an over estimate, for fine particles recovered from screening the chips are normally diverted to hog fuel.

Technological Coefficients - Final Activities.

The final activities include both recombination activities and selling activities, as discussed earlier. We shall discuss each of these separately, below.

(a) Plywood Production Activities.

There are nine different veneer grades produced from peeling activities in the model. The number of theoretically possible plywoods which could be manufactured from these veneers depends upon the various veneer thicknesses, panel thickness (number of layers of veneer), and the specific mixtures of veneer for each plywood grade. The number of mathematical possibilities is very large, but many can be readily eliminated in the model as impracticable alternatives.

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57 Anon. Conversion Factors for Pacific Northwest Forest Products, Institute of Forest Products, Seattle, Washington, (June, 1957).

The Douglas-fir plywoods in this model are based upon Canadian Standards Association specifications with regard to the particular veneer grades in each plywood.[58] The actual percentage of each veneer grade in a particular plywood depends upon veneer thickness and panel thickness. For simplicity, a standard panel thickness of 3/8 inch is assumed and the relative proportions of each grade of veneer in the various plywoods is based on data provided by local manufacturers.

The remaining plywood production activities concern mixed-specie and non-Douglas-fir, single-specie plywoods. These are not in fact produced to any large extent on the coast and manufacturing specifications are not readily available. Estimates were obtained however based on Douglas-fir specifications.

The plywood-production activity coefficients are shown in Table 5.4. These are calculated on a dry-veneer basis, accounting for an assumed volume shrinkage of 8 percent for all veneers in the drying stage after peeling. Sanding losses of 14 percent of the green-veneer volume for a 3/8 inch panel are included for all plywoods except Douglas-fir sheathing grade ("sheathing grade" infers unsanded plywood). These adjustments lead to a requirement of approximately 1.28 cubic feet green veneer to produce one cubic foot of sanded plywood, and 1.09 cubic feet

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58 Douglas Fir Plywood - CSA/0121/1961, Canadian Standards Association, Queen's Printers, Ottawa, (1961).

green veneer to produce one cubic foot of unsanded or sheathing plywood. The plywood production coefficients for green veneer are expressed on a basis of the cubic feet green veneer required per cubic foot of plywood produced.

(b) Pulp Production Activities.

Three of the seven pulp production activities in this model are based roughly on the principal types of pulp currently produced in British Columbia. The remainder are hypothetical. The activity coefficients are based on production of one air-dried ton of pulp in each activity. Since the pulp chip inputs are in volume-units and activity outputs in weight units, a conversion factor is built-in to the coefficients to account for this change in units. The calculated activity coefficients are shown in Table 5.5.

(c) Log-selling Activities.

One unit of log input is obviously necessary per unit of output from each log-selling activity (as shown in Figure I). The unit-level of each log-selling activity therefore depends upon the unit-volume selected in the intermediate activities.

(d) Lumber-selling Activities.

The lumber produced from the logs in the model may be considered as an input to the lumber-selling activities. It is assumed that no physical transformation occurs in the selling activities and hence each cubic foot of intermediate input (lumber) results in exactly one cubic foot of lumber sold, so that the lumber-selling activity coefficients each have a value of one. There is a separate and distinct selling activity for each intermediate lumber input, giving a total of twelve.

(e) Veneer-selling Activities.

The veneer selling activities are constructed in a manner similar to the lumber-selling activities. It is assumed that no physical transformation occurs between peeling and selling veneer (the market is for green unsanded veneer). Each cubic foot of intermediate input to a veneer-selling activity corresponds to exactly one cubic foot of veneer sold, so that the veneer-selling activity coefficients have a value of one. There is a separate activity for each different intermediate veneer input giving a total of nine veneer-selling activities.

(f) Chip-selling Activities.

There are only three chip-selling activities, corresponding to the three log species. The total chip supply is produced by the intermediate activities, sawing, peeling and chipping, each of which contributes to one of the intermediate chip-supply inputs. Like the other selling activities, each chip-selling activity has a technical coefficient of one, there being no physical changes between chipping and selling, and one cubic foot of chips sold requires one cubic foot of chips input.

(g) Hog-fuel Selling Activity.

An activity to account for the disposal of hog fuel produced in the sawing and peeling activities is required to make the model complete. Like the other selling activities, one cubic foot of hog fuel sold requires one cubic foot input. The technical coefficient is therefore one.

II. UNIT-LEVEL RETURNS OF THE ACTIVITIES

The unit-level returns of the various activities will be discussed in the same order as the technical coefficients discussed in the preceding section, beginning with the intermediate activities which produce outputs for consumption by the remaining final activities.

The log supply in the model is assumed given, at zero cost. The objective function therefore consists of entrepreneurial profit plus economic rent to the log supply. A discussion of the purely profit-maximizing case will be found in chapter six, dealing with an application of this model to the log-allocation problem of an integrated firm.

Intermediate Activity Unit-level Returns.

(a) Sawing Activity Returns.

When a variety of logs are processed in a sawmill no attempt is made to determine the lumber output per log. Indeed, the collection of such data would be practically impossible in most sawmills without seriously disrupting production. Hence most production data is reported on an average-log basis.

Rather than attempt to assess the sawing cost for each type and grade of log involved in the model it will be assumed that sawing costs per unit volume of log are constant.

Inter-industry comparisons of log-conversion costs by Rankin[59] give an average figure of $26.00 per thousand feet board measure (abbreviated MBM) of lumber produced, for sawing costs in the coastal region of British Columbia. This figure includes selling, administration, depreciation and all manufacturing costs, but is exclusive of log costs.

To convert this figure to a log-volume basis an average lumber yield figure for all logs included in the model was calculated from the data of Table 5.2. This, multiplied by Rankin's average sawing-cost figure gave an average log-sawing cost of $20.00 per hundred cubic feet of log, for each type of log in the model. See Figure I.

(b) Peeling-activity Returns.

Cost data for plywood production are also only available on an average-log basis. The method outlined above to calculate log-sawing costs on a log-volume basis was used to estimate peeling costs per unit of each specie of log allocated to the peeling activities. The calculation gave an average peeling cost of $6.00 per hundred cubic feet of log peeled for all peeling activities in the model. See Figure I.

(c) Chipping-activity Returns.

Chipping costs in the model are based on recent data given by McBride[60] for chipping in the interior region of

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59 A. G. Rankin, "Cost-Price Relationships in the Forest Industry," <u>The Forestry Chronicle</u>, v. 39, (Mar. 1963), pp. 69-77.

60 C. F. McBride, "Barking and Chipping in the Interior of British Columbia," <u>Canada Lumberman</u>, v.83 , (July, 1963), pp.53-55 .

British Columbia, and on the estimates of personnel in the industry. A cost of $2.00 per 100 cubic feet of log chipped was adopted for all logs included in the model, see Figure I.

Recombination Activity Unit-level Returns.

These activities represent plywood production and pulp production,and use the intermediate inputs, veneer and chips. The unit-level returns of these activities are based on recent market prices.

(a) Plywood Production Activity Returns.

Unit-level returns for the pure Douglas-fir plywoods are based on November, 1963 prices quoted by a major Vancouver producer for 3/8 inch panels. The prices were converted to a 100 cubic foot plywood basis to correspond to the units used in the veneer-peeling activities. See Figure I.

Table 5.7 shows the price data used for calculating unit-level returns for the Douglas fir, hemlock, spruce and mixed-specie plywoods in the model. The hemlock, spruce and mixed-specie price data are quite arbitrary.

The unit-level return of each plywood-production activity was calculated by subtracting plywood manufacturing cost from market price. Manufacturing cost is the cost of all stages of plywood production beyond the veneer-selling stage (i.e., after the veneer has been peeled and chipped into standard sizes suitable for shipping or further manufacture). The veneer-peeling activities, as explained earlier, incorporate manufacturing costs up to the green-veneer selling stage.

Plywood-production costs are taken from published data (which unfortunately was several years old) and from the estimates of experts.[61] Costs incurred in the various stages of plywood production are expressed as percentages of sales price in Table 5.7. From these, the unit-level return values in Table 5.6 are calculated by deducting plywood manufacturing costs comprising drying, lay-up and pressing, trimming, patching, sanding (where applicable) and shipping costs from the market price.

(b) Pulp Production Activity Returns.

Pulp production unit-level returns in the model are calculated by deducting pulp production costs, excluding the cost of chips, from recent market prices. The pulp-manufacturing cost adopted in the calculation is $43.00 per air-dry short ton of pulp, for all pulp-production activities in the model. This is an average figure for typical kraft pulp mills in the coastal region, based on estimates by an expert in the field.[62] The market prices for pulp are similarily based on well-informed estimates. Since there is no large open market for this product, these figures are no more than estimates of recent average

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61 W. E. Mayhew, "A New Method of Allocating Costs to Veneer by Grades," For. Prod. Journ., v. 8, (Aug. 1958), pp. 27A-30A.

62 Ian Hudson, Senior Development Engineer, Sandwell & Co. Ltd., Consulting Engineers, Vancouver, B.C.

prices.[63] Most pulps produced in North America are sold in the United States at a uniform delivered-price anywhere in the country. Pulp prices at the mill are therefore greatly influenced by the proximity of the mills' customers.[64]

For the purpose of this analysis an average mill-net price of $115.00 per air-dry short-ton of pulp is adopted. Using this value as a base, market prices are estimated for each of the several pulps in the model. See Table 5.8.

Selling Activity Unit-level Returns.

The selling activity unit-level returns are simply taken from recent market prices, or estimates where market prices are not available. Unlike the unit-level returns of plywood and pulp-production activities discussed in the previous section, actual market data are much more readily available in published form.

(a) Log-selling Activity Returns.

The unit-level returns for log-selling activities are based on current log prices published in the British Columbia Lumberman.[65] Since most log prices in this reference

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63 Most wood pulp in North America is sold under contract at prices announced quarterly by the producers. Most sales are conducted through agents whose standard fee is three percent of selling price, less freight, unless sales are to a captive market in which case commissions are not paid.

64 For example, a pulp mill in the coastal region faces about a ten dollar differential in its mill-net prices between Eastern and Western United States markets; and up to twenty dollars differential between Eastern United States and other world markets where freight costs, paid by the producer, cut more severely into mill-net price.

65 Robert Schultz & Co. Ltd., Log Price Report, British Columbia Lumberman, v.47 (October, 1963), p. 80.

show a $10.00 spread between highest and lowest prices paid, the average or median of the prices shown was selected for the model. See Table 5.9. A conversion factor of six feet-board-measure log scale equals one cubic foot, is used to convert log values to the units used in the model.[66]

(b) Lumber-selling Activity Returns.

Unit-level returns for lumber-selling activities are based on retail prices quoted by a major producer in the coastal region. See Table 5.10.

(c) Veneer-selling Activity Returns.

Market-price data for veneers is difficult to obtain although this product is marketed to a limited extent in the United States. The usefulness of the available data is frequently limited by the grouping of veneer grades under single average prices. Moreover, hemlock and spruce veneers are very rarely sold as such. Most of the data in Table 5.11 are estimates, particularly the values for hemlock and spruce. Neither of these latter species is used for face-veneer at present, so that the A and B grade prices are assumed to be equivalent to Douglas fir C and D grades, which are used exclusively as plywood "core" material.

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66 6 FBM log scale = 1 cubic foot is the generally accepted conversion factor for the coastal region. In the interior of B.C., an official conversion factor of 5.75 FBM log scale = 1 cubic foot is used. See Conversion Factors for Pac. N.W. For. Products, Institute of Forest Products, State of Washington, 303 Andersen Hall, Seattle 5, Washington, 1957.

(d) Chip-selling Activity Returns.

The unit-level returns of chip-selling activities are based on chip prices recently paid in the Vancouver chip market. These prices are available on a "unit" basis,[67] and a conversion factor of 'one unit equals 67 cubic feet solid wood' is used to convert the date to cubic feet, to meet the requirements of the model. See Table 5.12.

III. RESTRAINTS

A distinction has already been drawn between the primary input restraints, which comprise the available supply of logs, and the intermediate input restraints, which stipulate that all intermediate products (lumber, veneer and chips) are to be completely consumed as input to final activities. Since intermediate products are entirely consumed by final activities it follows that the values of the intermediate restraints must be zero. It remains therefore to discuss the sources of data for the primary input restraints.

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67 A "unit" of chips is defined as 200 cubic feet gross, volume of uncompacted chips. The stipulation 'uncompacted' is indefinite, for the several methods of storing and transporting chips lead to various degrees of compaction. A more rigorous quantification that is finding increasing acceptance is the "bone-dry unit," defined as the quantity of pulp chips which will weigh 2400 pounds in an oven-dry condition.

The log supply data are based on the annual log scale of the British Columbia coastal region forest industry. Log scale data for each specie in the model are available in the 1962 Annual Report of the British Columbia Forest Service. In addition, a breakdown of the log scale by log-grades was obtained from industry personnel and statisticians. The log supply data are summarized in Table 5.1.

IV. METHOD OF SOLUTION

Elementary linear programmes involving only a few activities and restraints can be solved by hand using the simplex method.[68] More complicated programmes require machines to handle the very large quantities of data involved. A machine solution of the log-allocation model presented in this thesis was carried out on the IBM 1620 digital computer in the computing centre of the University of British Columbia.

In this chapter we shall briefly describe the format and special features of the machine programme employed in obtaining a solution to the model. Three variations of the model that were solved to demonstrate its practical application will then be outlined. Results of these solutions will be presented and discussed later in chapter six.

IBM 1620 Computer Programme.

The machine programme used is identified as:

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68 W. J. Baumol, Economic Theory and Operations Analysis, Prentice Hall Inc., Calif., (June, 1962), ch. 5.

"1620 Linear Programming Code for Card Input/Output," by Nichols, Nickel and Davis.

Procedural details and mathematical elements of the programme are fully discussed in the above reference, and need not be repeated here. However, some important properties and characteristics of the programme which are of interest to this particular application, follow.

Special Features of the Machine Programme.

The programme is designed for input from cards. All output is on the typewriter although an optional final-matrix punch-out on cards is available. The size of problem which can be handled by this programme is limited by memory-capacity of the computer according to the following relationship.

$$(m + 2) \quad (n + 3) \quad = \quad \frac{\text{memory} - 3920}{10}$$

Where: m is the number of restrictions,
n is the number of 'real' activities, and
'memory' is equal to 40,000 for the particular machine in the University computing centre.

The term 'real' activities refers to those other than the 'disposal' activities. The latter are activities designed to convert the inequality restraint expressions into equalities. For the intermediate input restraints, selling activities were included as the corresponding disposal activities and for computational purposes these selling activities cannot therefore be considered as 'real', in the mathematical sense. For the log-allocation problem shown in Figure I there are m = 37 restraints and n = 57 'real' activities, to give a value of 2340 for the left-hand side of the capacity expression above. The value of the right-hand side of the expression is 3908.

There are three special features of this computer programme which make it quite versatile, and particularly useful to the log-allocation problem under discussion.

(i) Cost or price changes for the various activities, in terms of the original data, can be made without having to reload or re-solve the original matrix. These changes are instituted by inserting a special deck of cards, referred to as the 'Cost Changer' deck, into the over-all programme, usually behind the original-data cards. When entering the programme into the machine a stop is executed after the Cost Changer entry and the required cost change (price change) data can then be entered either by card input or by the typewriter. An unlimited number of cost changes may be made in any single run.

(ii) The restraints of the problem can also be varied without reloading or re-solving the original matrix. These changes are achieved in exactly the same manner as cost changes, by inserting a special deck of cards referred to as the 'RHS Changer', (right-hand side changer) deck. Data for RHS changes may be entered by cards or by typewriter for an unlimited number of changes in any single run. Both cost and RHS Changer decks can be included in the programme. (iii) The over-all programme consists of several relatively independent sub-programmes which can be deleted or altered in many respects without interference with other sub-programmes. Sequential loading of the sub-programmes is automatic. This feature enables a skilled computor programmer to manipulate the various sub-programmes to make adjustments in the actual solution procedure.

Programme Output-Format.

If desired, the results of each iteration in the solution of the problem may be typed out as part of the output. Alternately, the course of the solution may be monitored by setting an appropriate switch on and off periodically. The information monitored for each iteration consists of the iteration number, value of the objective function, the particular variable (activity) removed from the previous solution, and the particular variable inserted into the latest solution.

The value of the objective function increases (in a maximizing problem) with each successive iteration up to the final or optimal value. When the optimal solution has been obtained a Final Basis Output is either punched on cards or typed out, containing the following information:

(i) The final value of the objective function.

(ii) The identification number and input price (cost) of each activity in the Final Basis. (These activities correspond to the ones to be employed to reach an optimum value of the objective function).

(iii) The "level" at which each activity in the Final Basis is to be operated, expressed as a multiple of the activity's "unit-level" defined in the original problem.

(iv) The upper and lower limits to the level of each activity in the Final Basis, beyond which the activity would be dropped from the Final Basis and replaced with a Final Non-Basis activity.

(v) The Final Non-Basis activities which would enter the Final Basis in the event that one of the Final Basis activities exceeded one of its limits.

The remainder of the optimal solution consists of a Final Non-Basis Output containing the following information.

(vi) The identification number and input price (cost) of each activity in the Final Non-Basis.

(vii) The "shadow price" of each non-basis activity. This value represents the penalty to the whole system if a unit of this activity is forced into the final solution. In other words, if a non-basis activity (excluded from the optimal plan) is in fact used or forced into the final plan of activities (contrary to optimal conditions) it will incur a loss to the system equal to its shadow price per unit of the activity used.

(viii) The upper and lower limits to the level at which this activity could be forced into the solution. As more and more of this activity is used the loss incurred goes up, until eventually some other activity in the Final Basis is forced out.

(ix) The Final Basis variables which would leave the solution (be forced out) following the events of (viii) above.

A punch-out on cards of the complete final matrix is optional. When obtained, the matrix, deck punch-out comes complete with control cards, in a format suitable for direct reloading by the normal Data Loader routine.

Transformation of Results.

The output format of the solution programme, outlined above, is difficult to interpret. An auxiliary programme was written therefore to transform the output data into a form more consistent with the log-allocation model and more easily understood by the interested reader.

Using the final solution as input, on cards, this programme adjusts activity levels to a hundred-cubic-foot basis, computes total return for each activity and discards the Final Non-Basis variable-limits. The solution data is collected and rearranged into a more readable form. Maximum value of the objective function and number of iterations required to reach a solution are shown, and each column of figures is given a clearly understood heading including the appropriate units. A sample of the transformation programme output is shown in Appendix I to this thesis.

V. SOLUTION OF LINEAR PROGRAMME

Three variations of the log-allocation linear programme model presented in this thesis were solved, as follows:

APPENDIX

TO

CHAPTER FIVE

follows p. 73

Case I. Using the data presented in chapter five, as shown in Figure I.

Case II. Using the same data as in Case I but with modified activity coefficients for the No. 3 hemlock log-sawing activity.

Case III. Using the same data as in Case II but with a modified log supply.

Details of the modifications in Case II and Case III are shown in Tables 5.13 and 5.14 respectively.

Apart from demonstrating that the Cost Changer sub-programme works effectively, these modifications in Case II and Case III provide a useful insight into practical applications of the model, which we shall discuss in the next chapter.

TABLE 5.1

THE LOG SUPPLY IN THE COASTAL REGION

OF BRITISH COLUMBIA 1962

PERCENTAGE DISTRIBUTION BY GRADES AND AVAILABLE TOTAL SUPPLY

IN MILLIONS OF CUBIC FEET

Log Description	Distribution* (percent)	Available Supply** (million cubic feet)
Douglas Fir #1 peeler	3	6.1
" " #2 "	7	14.2
" " #3 "	14	28.4
" " #4 "	9	18.3
" " #2 sawlog	23	46.7
" " #3 "	44	89.5
Total Douglas Fir:	100	203.2
Hemlock #1 sawlog	5	13.8
" #2 "	18	50.0
" #3	77	213.6
Total Hemlock:	100	277.4
Spruce #1 sawlog	4	1.3
" #2 "	35	11.4
" #3 "	61	19.8
Total Spruce	100	32.5

Sources: *Distribution based on data by industry statisticians. See text pp.

**Available supply on data from: Annual Report, BCFS.

TABLE 5.2

LOG SAWING ACTIVITY TECHNOLOGICAL COEFFICIENTS

PERCENT YIELD OF LUMBER, CHIPS AND HOG FUEL

Log / Output	Douglas Fir						Hemlock			Spruce		
	peelers				sawlogs		sawlogs			sawlogs		
	#1	#2	#3	#4	#2	#3	#1	#2	#3	#1	#2	#3
Lumber:												
Clear	38	32	18	11	11	4	22	11	5	33	16	7
Sel/Merch.	13	14	21	17	17	19	16	16	10	14	17	12
Const/std.	15	15	23	29	29	32	26	27	26	12	26	28
Util/Econ.	6	10	8	9	9	9	6	9	13	8	6	8
Chips:	13	14	16	22	20	22	15	23	32	18	20	25
Hog Fuel:	15	15	14	13	14	14	15	14	14	15	15	14

Source: See Text pp. 51-54.

TABLE 5.3

LOG-PEELING ACTIVITY COEFFICIENTS

PERCENT YIELDS OF GREEN VENEER, CHIPS AND HOG FUEL

Log / Output	Douglas Fir						Hemlock			Spruce		
	peelers				sawlogs		sawlogs			sawlogs		
	# 1	# 2	# 3	# 4	# 2	# 3	# 1	# 2	# 3	# 1	# 2	# 3
A	30	24	17	10	6	-	12	6	-	12	6	-
B	8	8	9	10	12	-	12	12	-	12	12	-
C	23	28	33	38	39	-	29	34	-	29	34	-
Chips	22	21	20	19	18	-	20	20	-	20	20	-
Hog Fuel	17	19	21	23	25	-	27	28	-	27	28	-
Shrinkage	8	8	8	8	8	-	8	8	-	8	8	-

Source: See Text pp. 54-56.

TABLE 5.4

PLYWOOD PRODUCTION ACTIVITY COEFFICIENTS

CUBIC FEET OF GREEN VENEER REQUIRED PER CUBIC FOOT OF FINISHED PLYWOOD *

Plywoods / Veneers	G2S	G/Sol	G1S	S2S	S1S	Sh'g	1	2	1	2	F/H/S	F/H	F/S	H/S	S/H
Douglas Fir - A	.776	.388	.388	-	-	-					.388	-	-	-	-
" B	-	.388	-	.776	.388	-					-	.776	.776	-	-
" C	.504	.504	.892	.504	.892	1.090					-	-	-	-	-
Hemlock A							.388	-			-	-	-	.388	-
" B							.388	.776			.388	-	-	-	-
" C							.504	.504			-	.504	-	-	.892
Spruce A									.388	-	-	-	-	-	.388
" B									.388	.776	.504	-	-	-	-
" C									.504	.504	-	-	.504	.892	-

* Figures on a green veneer basis allowing average 14% sanding loss (where appropriate) and 8% shrinkage loss in drying. See text pp. 56-58.

Source: Douglas fir plywoods - Canadian Standards Association Publication, CSA 0121 - 1961.

TABLE 5.5

PULP PRODUCTION ACTIVITY COEFFICIENTS

CUBIC FEET OF CHIPS (SOLID VOLUME) PER AIR-DRY TON OF PULP

Chip Specie	Yield of Pulp (%)	Density (lb/CF)*	Wood Consumpt'n (CF/ADT)**	Chip Compositions for each Pulp						
				F H 40/60	F H 50/50	F H S 30/50/20	F H S 45/45/10	F 100	H 100	S 100
Douglas Fir	42.3	28.0	152	63.5	79.0	49.0	72.0	152	-	-
Hemlock	41.4	26.5	164	95.5	79.0	81.5	72.0	-	164	-
Spruce	41.4	24.0	181	-	-	32.8	16.0	-	-	181

* Moisture-free weight, green volume. **ADT = Air Dry Ton

<u>Sources</u>:

1. Wood Properties - Canadian Woods - Their Properties and Uses, Appendix Tables, Queen's Printers, Ottawa.

2. Chip Compositions - data supplied by industry personnel. (See text pp. 58).

TABLE 5.6

PLYWOOD MANUFACTURING COSTS AS PERCENT OF SALES PRICE

Production Step	Cost %
1. Logs	47
2. Peeling	10
3. Drying	9
4. Lay-up and Pressing	15
5. Trimming	2
6. Panel Patching	7
7. Panel Sanding	6
8. Warehouse & Shipping	4
	100%

Source: See Text pp. 62-63.

TABLE 5.7

PLYWOOD PRODUCTION ACTIVITY UNIT-LEVEL RETURNS *

CALCULATION OF UNIT-LEVEL RETURNS FROM MARKET

PRICE & MANUFACTURING COST DATA

Plywood Description		Market Price $ per cu.ft.	Manufacturing Costs $/cu.ft. 3/8" plywood	Unit-level Return $/cu.ft. 3/8" plywood
Species	Grade			
Doug.fir	G2S	5.00	2.40	2.60
" "	G/Sol	4.50	2.40	2.10
" "	G1S	4.00	1.80	2.20
" "	S2S	4.40	2.00	2.40
" "	S1S	3.90	1.80	2.10
" "	Sheathing	2.30	1.00	1.30
Hemlock	-	3.00	2.40	0.60
"	-	2.50	2.00	0.50
Spruce	-	3.00	2.40	0.60
"	-	2.50	2.00	0.50
Fir/Hem/Spruce	-	4.00	1.80	2.20
Fir/Hem	-	4.30	2.00	2.30
Fir/Sprc	-	4.25	2.00	2.25
Hem/Sprc	-	2.80	1.80	1.00
Sprc/Hem	-	2.90	1.80	1.10

* See text pp. 62-63.

TABLE 5.8

PULP PRODUCTION ACTIVITY UNIT-LEVEL RETURNS

CALCULATION OF UNIT-LEVEL RETURNS

FROM MARKET PRICES AND MANUFACTURING COST DATA

Pulp	Chip Mix (cu.ft.per ADT pulp*) D.F.	Hem.	Spruce	Assumed Price $/ADT pulp*	Manu-facturing Cost $/ADT pulp*	Activity Unit-level Return $/ADT pulp
a	64	95	-	115.00	43.00	72.00
b	79	79	-	115.00	43.00	72.00
c	49	81	33	125.00	43.00	82.00
d	72	72	16	125.00	43.00	82.00
e	152	-	-	110.00	43.00	67.00
f	-	164	-	110.00	43.00	67.00
g	-	-	181	110.00	43.00	67.00

* ADT = air dry ton

Source: See text pp. 63-64.

TABLE 5.9

LOG SELLING ACTIVITY UNIT-LEVEL RETURNS

Log Description	Market Price Dec. 1963 $/MBM Log Scale	Activity Unit-Level Return $/100 cu. ft.
Douglas Fir #1 peeler	120.00	72.00
" " #2 "	110.00	66.00
" " #3 "	100.00	60.00
" " #4 "	90.00	54.00
" " #2 sawlog	70.00	42.00
" " #3 "	60.00	36.00
Hemlock #1 sawlog	55.00	33.00
" #2 "	50.00	30.00
" #3 "	45.00	27.00
Spruce #1 sawlog	70.00	42.00
" #2 "	55.00	33.00
" #3 "	45.00	27.00

Source: See text pp. 64-65.

TABLE 5.10

LUMBER SELLING ACTIVITY UNIT-LEVEL RETURNS

Specie	Grade	Market Price $/MBM	Activity Unit-Level Return $/CCF lumber
Douglas Fir	Clear	195.00	235.00
	Sel/Mer	110.00	132.00
	Cons/Std	95.00	114.00
	Util/Econ	35.00	42.00
Hemlock	Clear	120.00	144.00
	Sel/Mer	85.00	102.00
	Cons/Std	75.00	90.00
	Util/Econ	35.00	42.00
Spruce	Clear	225.00	270.00
	Sel/Mer	80.00	96.00
	Cons/Std	70.00	84.00
	Util/Econ	35.00	42.00

Source: See Text p. 65.

TABLE 5.11

VENEER SELLING ACTIVITY UNIT-LEVEL RETURNS

Specie	Veneer Grade	Market Price, ($/M/16ths)*	Activity Unit-Level Return $/100 cu.ft.veneer
Douglas Fir	A , B	9.00	173.00
" "	C	5.75	110.00
Hemlock	A , B	5.75	110.00
"	C	4.50	86.50
Spruce	A , B	5.75	110.00
"	C	4.50	86.50

* M/16ths = $1000 \times \frac{1}{16} \times \frac{1}{12}$ cubic feet.

Source: See text pp. 65-66.

TABLE 5.12

PULP CHIP SELLING ACTIVITY UNIT-LEVEL RETURNS

Specie	Market Price $/200 cu.ft."unit"	Activity Unit-Level Return $/100 cu.ft.of chips
Douglas Fir	12.00	18.00
Hemlock	14.00	21.00
Spruce	14.00	21.00

Source: See Text p. 66.

TABLE 5.13

MODIFIED COEFFICIENTS OF ACTIVITY NO. 9; FOR CASE II

Modified sawing activity coefficients for No. 3 hemlock log.

VOLUME OF PRODUCT PER CUBIC FOOT OF LOG CONSUMED

		Original Coefficients* (Case I)	Modified Coefficients** (Case II and Case III)
Lumber:	Clear	0.05	0.02
	Sel/mer	0.10	0.05
	Coast/std	0.26	0.14
	util/econ	0.13	0.12
Chips		0.32	0.40
Hog Fuel		0.14	0.27

* Industry-average data.

**Arbitrarily assumed coefficients.

TABLE 5.14

VOLUME OF LOGS IN MILLION CUBIC FEET,

BY SPECIES AND GRADE

Log Description	Original Log Supply* (Case I and Case II)	Modified Log Supply* (Case III)
Douglas Fir #1 peeler	61	60
" " #2 "	142	140
" " #3 "	284	300
" " #4 "	183	250
" " #2 sawlog	467	550
" " #3 "	895	1000
Hemlock #1 sawlog	138	150
" #2 "	500	750
" #3 "	2136	1350
Spruce #1 sawlog	13	50
#2 "	114	200
#3 "	198	300

* Coastal region data, 1962. (See text pp. 67

**Arbitrarily assumed log supply.

CHAPTER VI

RESULTS AND DISCUSSION

I. RESULTS

The IBM 1620 computer programme outlined in chapter five performed smoothly and no difficulty was encountered in obtaining a solution. Case I required 28 iterations to reach an optimal solution; running time for the main solution programme was approximately twelve minutes with iteration times ranging from ten seconds to 45 seconds per iteration. The solution and final matrix for Case I were punched out on cards. Solutions for Case II and Case III were obtained more easily, since it was quite possible to start with the Case I final matrix, insert the required modifications via the appropriate changer-programme, and reach an optimal solution in two or three iterations.

As described in chapter five the final solution for each case was obtained on cards, rather than typed-out, so that it could be used as input to the solution-transformation programme. A sample of the transformed data is presented in Appendix I to this thesis.

For each case, an optimum solution to the model comprised a maximum value for the objective function and a listing of the final basis and non-basis variables (or activities). The final basis variables specify optimum activities and activity levels, necessary to achieve a maximum value for the objective function. The non-basis variables represent activities excluded from an optimal solution.

Variables numbered from one to 37 inclusive, correspond to intermediate activities in the model, see Figure I. Those specified among the final basis variables comprise the optimum log-allocation plan for the particular case. The remaining variables, numbered from 38 to 94 inclusive, correspond to final activities, and those specified among the final basis variables comprise the optimum mix of final products produced in the particular case. As explained in chapter five, the programme solution includes upper and lower limits to the unit-level return for each final basis variable, and specifies the final non-basis variable that will enter the solution when either limit is exceeded.

Compiled Results

To evaluate the results from each case it will be useful to compile the solution data in tables, shown below, setting forth comparisons between the optimum log allocations, final product mixes, sales volumes, etc.

Table 6.1 shows the total net return for cases I, II and III by each utilization category, namely lumber, plywood and pulp production. These net return figures represent gross returns, less manufacturing costs, excluding the cost of logs. It should be noted that the pulp production figures do not include a deduction for the cost of chips from sawmill and peeling activities. Similarly, the lumber production and plywood production returns do not include returns from the sale of chips to pulp production activities.

The optimum log-allocation plans for each case are shown in Table 6.2, as the percentage of the supply of each type of log that is consumed in each utilization category.

Finally, Tables 6.3, 6.4 and 6.5 show the final output of lumber, plywood and pulp, respectively, for each case. These figures indicate the type and quantity of each final product (lumber, plywood, veneer, pulp, and chips) that would be produced with an optimum allocation of logs, i.e., at a maximum value for the objective function.

Discussion of Results.

The practical significance of these results is limited by the simplifying assumptions applied in the construction of the model. It is possible nevertheless to observe some relevancies to the real world.

For example, according to the model (all cases) no logs should be sold; it is more profitable to convert the total given log supply than sell any part of it. Similarly, no chips should be sold, including those produced from sawmill and plywood mill waste. Since the model is based on the entire log supply of the coastal region we must infer that any sales would be to markets outside the coastal region. Both of these no-sale conditions are substantially met in practice at the present time.[70]

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70 Log exports in 1962, from the whole province, were approximately 1.5% of the coastal region log scale. Chip exports from the province were approximately 0.5% of the coastal region log scale. Annual Report, British Columbia Forest Service, Province of British Columbia, Dept. of Lands and Forests, Victoria, B.C., (1962).

With respect to log allocation, the figures for Case I in Table 6.2 show almost two-thirds of the log supply going to sawmills. In practice approximately 82 percent of the combined fir, hemlock and spruce supply for the coastal region in 1962 was converted to lumber.[71] On the other hand, the percentage of logs allocated to plywood production in Case I of the model is significantly higher than industry practice, (the increase is equal to the difference in lumber production noted above).[72] The percentage of logs allocated to chipping in the model (Case I) is the same as industry practice for 1962.[73] Insofar as the technological information and simplifying assumptions in the model are reasonable, the results of case I indicate that the coastal region forest industry in 1962 misallocated logs to lumber production, which might otherwise have been more profitably utilized in the manufacture of plywood.

In case II, a slight variation in technological coefficients of the No. 3 hemlock log sawing activity results in a significantly modified log-allocation plan. Chipping is now the main activity accounting for almost half of the log input. This pattern of log allocation clearly does not correspond

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71 Annual Report, 1963, British Columbia Lumber Manufacturers Association, Vancouver, (1964).

72 L. Read, Economist and Statistician, Council of Forest Industries, Vancouver, (1964) - personal correspondence.

73 Ibid.

closely with that actually observed in the region. The significant point however is to observe that the log-allocation problem is dominated by a relatively large volume of No. 3 hemlock logs. Thus, if the quality of No. 3 hemlock logs declines as postulated in the modified activity coefficients of case II, the entire allocation of these logs may be shifted from lumber to chips.

In practice of course such a wholesale transfer of logs from one utilization process to another is limited to the long-run, where utilization capacities for each process may be varied. Furthermore, it is possible that a given log supply may be optimally divided between two or more utilization categories. This is evident in case III where the supply of No. 3 spruce is allocated between sawing, 10 percent, and chipping, 90 percent.

The output of Douglas fir lumber specified in Case I, (Table 6.3) is 25 percent below the actual output in 1962 in the coastal region.[74] This reflects the larger portion of logs allocated to plywood production in the case I solution as discussed above. Hemlock and spruce lumber production is moderately higher in case I than actual experience would indicate, but the proportions are reasonably close.[75] Comparisons between case I and case II show that Douglas fir and spruce lumber output is unaffected by the modified activity coefficients

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74 <u>Annual Report 1963</u>, British Columbia Lumber Manufacturers Association, Vancouver (1964).

75 <u>Ibid</u>.

of No. 3 hemlock logs.

In plywood production each case of the model specified two Douglas fir plywoods and one mixed-specie plywood. As discussed earlier, mixed-specie plywoods have only a limited market in British Columbia. In addition, however, the model specified direct sales of each grade of hemlock veneer. As far as can be ascertained from available data there is no market for hemlock veneer, _per se_, in the coastal region. We must conclude therefore that the veneer and plywood production data is inaccurate relative to the lumber and pulp-production data. This is not surprising in view of the divergent sources of data involved in the model. It is of interest however to observe that in each case hemlock veneer is involved in two separate final basis activities; hemlock veneer sales and hemlock - Douglas fir mixed-specie plywood. This is an illustration of the interdependence between these two activities.

The solution data on upper and lower limits to unit-level returns, and the corresponding limiting variables for each final basis variable, are a useful guide to the relative importance of each activity in the optimum solution. For example, activity number six (sawing No. 3, Douglas fir sawlogs) is a final basis variable with a unit-level return (cost) of -$20.00; see Appendix I. The lower limit to this unit-level return figure is -$31.22, beyond which it would be more profitable,from the point of view of total net return to the log supply, to employ activity number 29 (chipping No. 3, Douglas fir sawlogs). There is a "cushion" here of more than $11.00. Conversely, activity number 16 with a unit-level return of -$6.00 has a lower limit of -$6.99, below which it would be replaced by activity number 14. The "cushion" here is $0.99. Clearly, it will take a relatively large (56 percent) decrease in unit-level return (increase in cost) of activity number six before it is replaced, whereas activity number sixteen will be replaced following a similar change of only 16.5 percent.

It should be noted that there is no upper-limiting variable to either of these intermediate activities. In other words, if their unit-level returns are any higher (lower cost) there is no better alternative. Normally, each activity has both upper and lower limiting variables as evident in activity number 38, and other final activities with positive unit-level returns.[76]

Two conclusions follow from these observations on the different cases of the model. First, the validity of any practical conclusions drawn from the solution of the model depends heavily on the quality of the data used. The full value of the model cannot be realized unless accurate data on yields, prices and costs are available.

Second, given confidence in the quality of the data, significant changes in total net return and optimum use of the logs may result from a relatively minor change in yields, prices or costs. In an example, a decrease in the lumber yield from a No. 3 hemlock log resulted in a $3.2 million decrease in total net return (see Table 6.1, cases I and II). It also resulted in a

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76 Some activities, such as No. 11, have an upper-limiting variable but the programme fails to specify a meaningful upper unit-level return figure, due to insufficient space for larger numbers in the machine programme format.

major adjustment in the log-allocation plan (see Table 6.2).

II. APPLICATIONS OF THE MODEL

In this study the acquisition of accurate data and the inclusion of all possible variables in the model was considered of secondary importance to presenting a clear demonstration of the application of linear programming to log-allocation problems. Many simplifying assumptions were included to make this demonstration as simple as possible. In this section we shall consider relaxing some of these simplifying assumptions to make the model more valid.

In chapter four we distinguished between fundamental and correlation assumptions in the model. The fundamental assumptions are basic to linear programming and cannot be relaxed without destroying the linearity of the model. The correlation assumptions however serve merely to simplify the exposition. We may conveniently relax some of the latter assumptions by considering an application of the model to the log-allocation problem faced by an individual firm.

<u>Log-Allocation Linear Programme for an Integrated Firm</u>.

We shall take the approach of modifying the simplified linear programme model presented in this thesis to conform more with the log-allocation problem of an integrated firm. This will involve re-defining the objective function and relaxing some original assumptions.

(a) A Profit Objective-Function.

The first step is to deduct log costs from the total net return objective function. An optimal solution then corresponds to maximum entrepreneurial profit, excluding economic rent to the supply of logs. This correction may be made either by adding log costs to the unit-level return of each intermediate (i.e., log cutting) activity, or by deducting an average log cost from the unit-level return of each final activity. The former procedure is the more accurate although the latter is probably simpler to apply. In either case the unit-level return of log-selling activities would have to be adjusted to zero (or a negative value if expected salvage values for logs were less than purchase price) to avoid double-costing of logs.

(b) Utilization Capacity Restraints.

The next basic modification is to account for the limited utilization capacity of a single firm in the short-run. Previously, the long-run condition of unlimited capacity was assumed. In practice it is the short-run log-allocation problem that is of interest to the firm's current operations, with long-run considerations applicable more to investment and policy decisions, which are discussed later.

Utilization capacity restraints can be readily incorporated into the log-allocation model if no more than the total capacity of each utilization category (sawing, peeling and chipping) need be specified. Each capacity restraint requires an additional row in the matrix of the model, with activity coefficients of unity for each intermediate activity affected by the restraint. Thus, the sawmilling capacity restraint (row) would have a 'one' in each log-sawing intermediate activity column. Total available capacity for each utilization category, expressed in the same units as the primary inputs is entered in the restraint column. The restraint expressions (inequalities) for each limited utilization capacity take the same form as the primary input restraint expressions; i.e., total consumption of each utilization-capacity must be less-than or equal-to the available utilization capacity.

Disposal activities corresponding to the "use" of idle capacity are required for each utilization-capacity restraint. The unit-level returns for these capacity-disposal or idle-capacity activities are given negative values corresponding to the cost of idle capacity, which subtracts from total profit.

A more exact model would include separate capacity restraints for some or all of the intermediate activities in the model, but this would make it considerably more cumbersome. To specify capacity restraints on each original intermediate activity would add as many new restraints, and as many new (disposal) activities, as there were original intermediate activities.

(c) Log Costs.

In the simplified integrated-industry model of this study we assumed that log costs were established on the open market. This is a reasonable assumption for any firm operating within the Vancouver log market. In the event however that a firm does have access to logs (or intermediate products such as chips) of the same grade and quality but from two or more different sources at two or more different prices, then these must be treated as separate inputs to the model. Each one with its own corresponding sawing, peeling, and chipping activities.

Applications of the Model to the Problems of a Firm.

A firm can usefully apply a log-allocation model such as the one outlined above, in two separate ways: (a) in the short-run, (b) in the long-run.

(a) Short-run Applications.

In the short-run the firm seeks to maximize its (short-run) profits subject to given prices, costs, production capacities and level of technology. In this context, the firm, lacking any monopoly or monopsony influence on the market, can take certain data as given, namely:

1. Log Supply: (i) Volume of available logs, by species and grade.
   (ii) Log prices (delivered to the firm).
2. Activities: (iii) Available utilization processes and capacities.
   (iv) Technological coefficients of each utilization process.
   (v) Unit-costs for each utilization process.
3. Products (vi) Demand prices for intermediate and final products.

Given this information the firm can construct and solve a log-allocation programme similar to the one described in this thesis, with the appropriate modifications outlined earlier. This model would be used to determine the maximum profit the firm could earn and the log-allocation plan to be followed in attaining this profit.

These two related pieces of information, maximum profit and optimum log-allocation plan, would serve as useful guides to current operations of the firm, indicating whether, and where, additional profits might be earned through more efficient log allocation.

The effect of a change in composition of the log supply or a change in product prices could be investigated by solving the model a second time with the new data, (as we demonstrated with cases II and III in this study). Changes in maximum profit and optimum log-allocation plan could be noted and evaluated with respect to the original solution.

(b) Long-run Applications.

Long-run changes may be analyzed through the same procedure outlined above, namely, by observing changes in maximum profit and optimum log allocation. In the long run, however, utilization capacities and technological coefficients may vary also, resulting in a change in the basic matrix of the model.

If utilization-capacity restraints are excluded from the model entirely, as in the model presented in this thesis, then the optimum long-run log-allocation plan corresponds to an ideal combination of utilization capacities of the various processes. This solution, <u>ceterus paribus</u>, would provide the firm with a useful guide for investment plans in future expansion.

Alternately, the firm may investigate different utilization capacity combinations resulting from a given amount of new investment in plants, following a benefit-cost analysis procedure for each proposed investment plan.[77]

The model allows a direct comparison between benefit-cost ratios for different investment (expansion) schemes, either on a _ceterus paribus_ basis or including forecast changes in other conditions (such as prices or log supply).

Finally, the firm could employ a linear programme model such as this to evaluate benefits from technological innovation. The industrial research group (or its equivalent) constantly faces a problem of justifying various avenues of research endeavour. An estimate of the effect that future developments in wood conversion techniques, etc., might have on total net return and optimum log allocation would be of great value.

The Log-Allocation Linear Programme Dual.

Reference was made earlier to the "mathematical dual" of a linear programme model and its particular interpretation in the log-allocation problem. To illustrate the nature of the log-allocation dual we shall consider a simple problem in which logs are consumed directly in the production of basic wood products; complications of intermediate products, etc., will be ignored.

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77 J. Davis, D. W. Ross, A. D. Scott and W. R. D. Sewell, _Benefit-Cost Analysis Handbook_, Queen's Printer, Ottawa, 1962.

In the primary problem, similar in form to the model in this study, the objective is to maximize the return to a given supply of logs. Product prices are known and assumed constant; the log supply is fixed. The variables in the problem are the levels of production of various log-conversion processes or activities. A solution to the primary problem comprises selecting the most profitable combination of log-conversion processes, and specifying their corresponding levels of production, subject to the limitations imposed by a restricted, fixed, supply of logs.

Suppose now that it is required to find what proportion of its profits the resource owner (e.g. firm) owes to each type of log in the given supply. To do this the firm will set up accounting prices for these logs which are just high enough to give to these inputs a combined value equal to the total profit of the firm. That is, the firm's profits after paying these imputed prices for logs would be zero. In programming terms, the problem is to find imputed prices for the logs which minimize the total accounting cost of these resources, and yet involve an imputed log-cost per unit of each wood-product no less than the profit per unit of each wood-product.

The distinction may be clarified by considering two linear programme outlines as follows:

The Primary Problem.

Maximize profit:

$$P = p_1 X_1 + p_2 X_2 + p_3 X_3$$

Subject to resource limitations:

$$a_{11} X_1 + a_{12} X_2 + a_{13} X_3 \leqq b_1$$

$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 \leqq b_2$$

and the requirement that no activity levels be negative.

$$X_1 \geqq 0, X_2 \geqq 0, X_3 \geqq 0$$

The Dual Problem.

Minimize Account Costs: (Imputed prices).

$$C = b_1 Y_1 + b_2 Y_2$$

Subject to the requirement that all profits are imputed:

$$a_{11} Y_1 + a_{21} Y_2 \geqq p_1$$

$$a_{12} Y_1 + a_{22} Y_2 \geqq p_2$$

$$a_{13} Y_1 + a_{23} Y_3 \geqq p_3$$

and the requirement that no accounting prices be negative:

$$Y_1 \geqq 0, Y_2 \geqq 0, Y_3 \geqq 0.$$

It should be noted that the same matrix elements apply in the primary and the dual model. The only changes are that rows in the primary become columns in the dual (and conversely); the inequality signs in the restraint expressions are reversed, and the objective function becomes one of minimizing rather than maximizing. There is no difference in the solution technique.

Applications of the Dual.

The dual log-allocation problem described above is probably of more value to a resource owner than to a resource exploiter, since it considers log cost as the optimizing variable rather than product output. In this sense, the term resource owner may encompass three classes of owner: the Provincial Government which owns almost the entire resource in British Columbia;[78] the large integrated firm with its own holdings of timber; the log-buyer for a firm purchasing logs on the open-market.

The log-buyer interpretation is the most rigorous application of the log-allocation dual, for two reasons. Firstly, the log-buyer is concerned only with logs (not trees or timber). Secondly, the log-buyer is predominantly cost-oriented.

The list of imputed log prices generated in the dual solution would indicate to the resource owner the relative value of logs to his particular log-utilization complex.

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78 This is our original interpretation of the term resource owner, used throughout the earlier part of this thesis.

A P P E N D I X

T O

C H A P T E R S I X

follows p 103

TABLE 6.1

TOTAL NET RETURNS, IN LUMBER, PLYWOOD, AND PULP PRODUCTION

Calculated as total gross returns less manufacturing costs, excluding the cost of logs.

FIGURES IN MILLIONS OF DOLLARS

Utilization Category	CASE I	CASE II	CASE III
Lumber Production	120.0	64.7	46.3
Plywood Production and Veneer Sales	134.4	134.4	160.3
Pulp Production and Chip Sales	70.5	125.6	110.5
Hog-Fuel	8.6	5.6	8.2
Total Net Return	333.5	330.3	325.3

TABLE 6.2

OPTIMUM LOG ALLOCATION

Showing percentage of available log supply allocated to various utilization processes, - by species and grade -

	CASE I			CASE II			CASE III		
	Saw'g	Peel'g	Chip'g	Saw'g	Peel'g	Chip'g	Saw'g	Peel'g	Chip'g
Doug. Fir #1 peeler	-	100	-	-	100	-	-	100	-
" " #2 "	-	100	-	-	100	-	-	100	-
" " #3 "	-	100	-	-	100	-	-	100	-
" " #4 "	-	100	-	-	100	-	-	100	-
" " #2 sawl'g	-	100	-	-	100	-	-	100	-
" " #3 "	100	-	-	100	-	-	100	-	-
Hemlock #1 sawl'g	100	-	-	100	-	-	100	-	-
" #2 "	-	100	-	-	100	-	-	100	-
" #3 "	-	-	100	-	-	100	-	-	100
Spruce #1 sawl'g	100	-	-	100	-	-	100	-	-
" #2 "	100	-	-	100	-	-	100	-	-
" #3 "	-	-	100	-	-	100	9.7	-	90.3
Total Log Consumptn.									
million cu.ft.	329.6	163.7	19.8	116.0	163.7	233.4	142.9	205.0	162.1
percentage	64.0	32.0	4.0	22.5	32.0	45.5	28.0	40.0	32.0
ACTUAL CONSUMPTION 1962 - B. C. COAST	82.0	14.0	4.0						

TABLE 6.3

LUMBER SALES BY SPECIES AND GRADE

Volume of lumber sales by four grade categories,* in MILLION cubic feet, and value of lumber sales by specie, in MILLIONS of dollars

	CASE I			CASE II			CASE III		
	FIR	HEM	SPRU	FIR	HEM	SPRU	FIR	HEM	SPRU
Clear	3.580	13.716	2.253	3.580	3.036	2.253	1.945	1.858	1.743
Sel/Mer	17.005	23.568	2.120	17.005	2.208	2.120	14.290	2.138	1.245
Cons/Std	28.640	59.124	3.120	28.640	3.588	3.120	23.620	7.842	1.079
Util/Econ	8.055	28.596	0.788	8.055	0.828	0.788	6.670	8.010	0.205
Totals									
Million cu.ft.	57.280	125.004	8.281	57.280	9.660	8.281	46.525	19.848	4.272
Percent	30	65	5	76	13	11	75	21	4
Million Dollars	66.678	103.541	11.064	66.678	10.171	11.064	52.997	15.093	10.087
ACTUAL 1962 PRODN.	40	57	3						

* See text p. 51 for lumber grades.

TABLE 6.4

PLYWOOD AND VENEER SALES

Volumes of finished plywood and green veneer sold,

in hundreds of cubic feet

Only activities (products) specified in the solution are shown.

Product Description	CASE I	CASE II	CASE III
	(Hundreds of Cubic Feet)		
Plywoods			
Douglas Fir G 1 S	378,814	378,814	413,917
" " S 1 S	72,116	72,116	106,822
Fir/Hem mix	113,607	113,607	119,269
Veneers			
Hemlock A	30,000	30,000	4,950
" B	60,000	60,000	90,000
" C	112,740	112,740	194,888

TABLE 6.5

PULP SALES

Volumes of pulp produced and sold, in air-dry tons

Pulps identified by their chip-mixtures.

Pulp Produced	CASE I	CASE II (air-dry tons)	CASE III
30/Fir/50 Hem/20 Spruce	591,316	591,316	841,857
45 Fir/45 Hem/10 Spruce	175,035	175,035	-
100 Hemlock	121,481	1,007,140	668,107

CHAPTER VII

SUMMARY AND CONCLUSION

The forest industry of British Columbia faces strong competition in its traditional markets from other world producers and from substitute products. To meet this competition it is essential that the industry reduce its unit-costs and raise the economic efficiency of its log-conversion processes. One aspect of this required efficiency is to ensure that each log is consumed in a manner that maximizes total net return to the log supply.

As far as can be ascertained, current log-allocation procedures in the industry result in considerable inefficiency. This arises from two causes; insufficient data on log costs and yields, and the absence of an effective technique for handling the data available.

The problem of determining an optimal distribution of logs among alternate conversion processes is amenable to a linear programme solution. Applications of linear programming to various optimization problems in each sector of the industry have been described in the literature.[79,80,81] The contribution of

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79 F. H. Curtis, "Linear Programming the Management of a Forest Property," Journal of Forestry, v. 60, (Sept. 1962), pp. 611-616.

80 N. D. Jackson & G. W. Swinton, "Linear Programming in Lumber Production," For. Prod. Jour. v. XI, (June, 1961), pp. 272-274.

81 E. Koenigsberg, "Linear Programming Applied to the Plywood Industry," For. Prod. Jour. v. XI, (Sept. 1960), pp. 481-486.

this thesis is a demonstration of the applicability of linear programming to the log-allocation problem faced by the industry as a whole, encompassing all of the principal utilization alternatives.

Fundamental differences in production processes exist between various log-utilization sectors of the industry. Principally, lumber production involves a single breaking-down operation (sawing) to produce a final product, whereas plywood and pulp production require a breaking-down process (peeling and chipping) followed by a recombination process, (glueing and pulping) to produce final products. Reconciling these differences in a linear programme model required introducing the concept of intermediate activities.

Intermediate activities were viewed as processes which produced intermediate products to be consumed as inputs to other activities in the model. They were included in the model with negative activity coefficients and negative activity returns, (i.e., costs). It was assumed that all intermediate products produced would be consumed; i.e., there would be no accumulation of intermediate products.

The scope of the model was limited to the coastal region of British Columbia. Data were assembled from research journals, trade journals and from personal discussions between the author and personnel in the industry with expert knowledge of current operations in the forest industry.

Solutions of the model were obtained through the services of the computing centre at the University of British Columbia. Variations in the model demonstrated its practical value in analyzing the effects of changing log supplies and changing technological coefficients, on total net return and optimum log allocation. It was shown that the model is a powerful tool for investigating short-run and long-run changes in the integrated forest products firm.

It can be concluded that, as a comparative statics economic model, the log-allocation linear programme provides an efficient means of evaluating the consequences of changing economic variables in the industry or firm. Once constructed, the model may be solved an indefinite number of times to investigate various combinations of variables of interest to the analyst.

The many simplifying assumptions on which the model has been constructed can be removed, though to do so would require more data and involve increasing complexity and cost in the solution. Indeed, some of the fundamental assumptions of linear programming may ultimately be removed by constructing a non-linear log-allocation problem, but this degree of complexity is beyond the scope of this thesis.

The principal limitation to the linear programme described in this thesis has been the quality of the data available. The validity of the results, which is governed substantially by the quality of the data employed, determines the usefulness of the model as a practical economic tool of management. Thus, one of the principal contributions of this study has been to emphasize the value of collecting accurate data in the forest products industry. This question has already received considerable attention within the industry and efforts are being directed towards solving the fundamental problem of adequately describing and grading logs and log products (notably lumber).[81,82,83] However, much of the data collected is of the wrong type for this kind of analysis, and several new kinds of data must be collected in order to exploit the full potential of linear-programming analysis. The foundation of allocation models such as the one described in this thesis provide a guide to the production data required.

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81 P. H. Lane, "Evaluating Log and Tree Quality for Wood Products," For. Prod. Jour., v. XIV, (Mar. 1963), pp. 89-93.

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APPENDIX I

follow p. 112

LOG ALLOCATION LINEAR PROGRAMME -RESULTS
NUMBER OF ITERATIONS = 29

MAXIMUM VALUE OF OBJECTIVE FUNCTION = $ 330,359,300 PER YEAR

OPTIMUM LOG ALLOCATION PLAN

VARIABLE	UNIT LEVEL RETURN ($/C.C.F.)	ACTIVITY LEVEL (C.C.F.)	TOTAL RETURN ($X1000)	LOWER LIMIT ($/C.C.F.)	LIMITING VARIABLE	UPPER LIMIT ($/C.C.F.)	LIMITING VARIABLE
6	-20.000	895000.0	-17900.000	-31.2185	29	9999.9000	
7	-20.000	138000.0	-2760.000	-25.1173	20	9999.9000	
10	-20.000	13000.0	-260.000	-63.6560	22	9999.9000	
11	-20.000	114000.0	-2280.000	-35.0928	23	9999.9000	52
15	-6.000	61000.1	-366.001	-13.5323	13	9999.9000	48
16	-6.000	142000.0	-852.000	-6.9923	14	9999.9000	
17	-6.000	284000.0	-1704.000	-33.9281	3	9999.9000	
18	-6.000	183000.0	-1098.000	-38.9924	4	9999.9000	
19	-6.000	467000.4	-2802.002	-30.8572	5	9999.9000	12
21	-6.000	500000.0	-3000.000	-9.3643	8	9999.9000	20
32	-2.000	2136000.0	-4272.000	-13.8536	66	9999.9000	51
35	-2.000	198000.0	-396.000	-5.8068	12	9999.9000	
38	220.000	378814.5	83339.190	213.7109	36	238.5520	89
40	210.000	72116.0	15144.360	199.7134	41	218.7654	36
46	220.000	000.0	.000	201.4480	89	283.3899	23
47	230.000	113606.9	26129.587	225.0001	48	250.5733	41
53	82.000	591316.6	48487.961	80.8777	12	87.4935	34
54	82.000	175034.6	14352.837	78.1454	51	83.6491	12
56	67.000	1007139.5	67478.347	50.7418	51	72.7518	12
70	235.000	35800.0	8413.000	-45.4639	29	309.1249	2
71	130.000	170050.0	22106.500	70.9550	29	262.9911	3
72	115.000	286400.0	32936.000	79.9421	29	200.7145	5
73	40.000	80550.0	3222.000	-84.6506	29	59.8479	14
74	145.000	30360.0	4402.200	121.7395	20	175.5853	8
75	100.000	22080.0	2208.000	68.0168	20	121.0274	8
76	90.000	35880.0	3229.200	70.3181	20	102.4607	8
77	40.000	8280.0	331.200	-45.2885	20	77.3821	8
78	270.000	22530.0	6083.100	175.6697	23	324.3831	12
79	95.000	21200.0	2014.000	6.2185	23	126.7235	12
80	85.000	31200.0	2652.000	26.9506	23	98.5957	12
81	40.000	7879.9	315.196	-211.5476	23	87.5852	12

85	110.000	30000.0	3300.000	60.0207	49	152.6442	20
86	110.000	60000.0	6600.000	81.9635	8	152.6442	20
87	86.000	112742.1	9695.821	76.1048	8	95.9206	48
88	110.000	000.0	.000	84.9073	50	361.5476	23
90	86.000	000.0	.000	76.0794	48	130.3907	23
94	10.000	560880.0	5608.800	-14.0313	8	37.1915	12

NON-BASIS VARIABLES

VARIABLE	UNIT LEVEL RETURN	SHADOW PRICE
1	-20.000	29.4954
2	-20.000	23.7199
3	-20.000	27.9281
4	-20.000	32.9924
5	-20.000	24.8572
8	-20.000	3.3643
9	-20.000	14.5122
12	-20.000	3.8068
13	-6.000	7.5323
14	-6.000	.9923
20	-6.000	5.1173
22	-6.000	43.6560
23	-6.000	15.0928
24	-2.000	86.7777
25	-2.000	70.4006
26	-2.000	60.3058
27	-2.000	50.2109
28	-2.000	45.2788
29	-2.000	11.2185
30	-2.000	22.4743
31	-2.000	15.1570
33	-2.000	50.6392
34	-2.000	19.8760
36	260.000	12.5782
37	210.000	52.5782
39	240.000	12.5782
41	130.000	12.5700
42	60.000	68.7040
43	50.000	78.7040
44	60.000	82.9860
45	50.000	107.2681
48	225.000	4.9999
49	100.000	19.3920
50	110.000	9.7360
51	72.000	5.1137
52	72.000	7.5543
55	67.000	23.9689
57	67.000	40.4097

58	72.000	72.6257
59	66.000	62.2486
60	60.000	58.1538
61	54.000	54.0589
62	42.000	61.1268
63	36.000	33.0665
64	33.000	28.3280
65	30.000	24.0107
66	27.000	11.8536
67	42.000	65.9816
68	33.000	44.2184
69	27.000	30.3424
82	173.000	93.3092
83	173.000	67.5360
84	110.000	20.7982
89	110.000	36.8095
91	18.000	41.8480
92	21.000	19.8536
93	21.000	38.3424

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FIGURE I

SCHEMATIC PRESENTATION OF LOG - ALLOCATION LINEAR PROGRAMME MODEL

INTERMEDIATE ACTIVITIES — LOG SAWING (100 cubic feet of log)

Inputs	Row	1	2	3	4	5	6	7	8	9	10	11	12
PRIMARY INPUTS — LOGS: D.FIR #1 plr.	1	1											
#2	2		1										
#3	3			1									
#4	4				1								
#2 swlg.	5					1							
#3	6						1						
HEMLOCK #1 swlg.	7							1					
#2	8								1				
#3	9									1			
SPRUCE #1 swlg.	10										1		
#2	11											1	
#3	12												1
INTERMEDIATE INPUTS — LUMBER: D.FIR clear.	13	-.38	-.32	-.18	-.08	-.11	-.04	-	-	-	-	-	-
sel/mer.	14	-.13	-.14	-.21	-.15	-.17	-.19	-	-	-	-	-	-
cons/std.	15	-.15	-.15	-.23	-.34	-.29	-.32	-	-	-	-	-	-
util/econ.	16	-.06	-.10	-.08	-.08	-.09	-.09	-	-	-	-	-	-
HEMLOCK clear.	17	-	-	-	-	-	-	-.22	-.11	-.05	-	-	-
sel/mer.	18	-	-	-	-	-	-	-.16	-.16	-.10	-	-	-
cons/std.	19	-	-	-	-	-	-	-.26	-.27	-.26	-	-	-
util/econ.	20	-	-	-	-	-	-	-.06	-.09	-.13	-	-	-
SPRUCE clear.	21	-	-	-	-	-	-	-	-	-	-.33	-.11	-.07
sel/mer.	22	-	-	-	-	-	-	-	-	-	-.14	-.17	-.12
cons/std.	23	-	-	-	-	-	-	-	-	-	-.12	-.26	-.28
util/econ.	24	-	-	-	-	-	-	-	-	-	-.08	-.06	-.08
CHIPS: DOUGLAS FIR	34	-.13	-.14	-.16	-.22	-.20	-.22	-	-	-	-	-	-
HEMLOCK	35	-	-	-	-	-	-	-.15	-.23	-.32	-	-	-
SPRUCE	36	-	-	-	-	-	-	-	-	-	-.18	-.20	-.25
HOG FUEL	37	-.15	-.15	-.15	-.13	-.14	-.14	-.15	-.14	-.14	-.15	-.15	-.14
UNIT-LEVEL RETURN $		-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20
ACTIVITY LEVEL		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}

INTERMEDIATE ACTIVITIES — LOG PEELING (100 cubic feet of log)

Inputs	Row	13	14	15	16	17	18	19	20	21	22	23
D.FIR #1 plr.	1	1		1								
#2	2		1		1							
#3	3					1						
#4	4						1					
#2 swlg.	5							1				
HEMLOCK #1 swlg.	7								1			
#2	8									1		
SPRUCE #1 swlg.	10										1	
#2	11											1
LUMBER: D.FIR util/econ.	16	-.05	-.05	-	-	-	-	-	-	-	-	-
VENEER: DOUGLAS FIR A	25	-.30	-.24	-.30	-.24	-.17	-.10	-.06	-	-	-	-
B	26	-.08	-.08	-.08	-.08	-.09	-.10	-.12	-	-	-	-
C	27	-.23	-.28	-.23	-.28	-.33	-.38	-.39	-	-	-	-
HEMLOCK A	28	-	-	-	-	-	-	-	-.12	-.06	-	-
B	29	-	-	-	-	-	-	-	-.12	-.12	-	-
C	30	-	-	-	-	-	-	-	-.29	-.34	-	-
SPRUCE A	31	-	-	-	-	-	-	-	-	-	-.12	-.06
B	32	-	-	-	-	-	-	-	-	-	-.12	-.12
C	33	-	-	-	-	-	-	-	-	-	-.29	-.34
CHIPS: DOUGLAS FIR	34	-.17	-.16	-.22	-.21	-.20	-.19	-.18	-	-	-	-
HEMLOCK	35	-	-	-	-	-	-	-	-.20	-.20	-	-
SPRUCE	36	-	-	-	-	-	-	-	-	-	-.20	-.20
HOG FUEL	37	-.17	-.19	-.17	-.19	-.21	-.23	-.25	-.27	-.28	-.27	-.28
UNIT-LEVEL RETURN $		-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
ACTIVITY LEVEL		X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}

INTERMEDIATE ACTIVITIES — LOG CHIPPING (100 cubic feet of log)

Inputs	Row	24	25	26	27	28	29	30	31	32	33	34	35
D.FIR #1 plr.	1	1											
#2	2		1										
#3	3			1									
#4	4				1								
#2 swlg.	5					1							
#3	6						1						
HEMLOCK #1 swlg.	7							1					
#2	8								1				
#3	9									1			
SPRUCE #1 swlg.	10										1		
#2	11											1	
#3	12												1
CHIPS: DOUGLAS FIR	34	1	1	1	1	1	1	-	-	-	-	-	-
HEMLOCK	35	-	-	-	-	-	-	1	1	1	-	-	-
SPRUCE	36	-	-	-	-	-	-	-	-	-	1	1	1
HOG FUEL	37	-	-	-	-	-	-	-	-	-	-	-	-
UNIT-LEVEL RETURN $		-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
ACTIVITY LEVEL		X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	X_{30}	X_{31}	X_{32}	X_{33}	X_{34}	X_{35}

FINAL ACTIVITIES — PLYWOOD PRODUCTION (100 cubic feet of plywood)

Inputs	Row	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
VENEER: DOUGLAS FIR A	25	·766	·388	·388	-	-	-	-	-	-	-	·388	-	-	-	-
B	26	-	·388	-	·766	·388	-	-	-	-	-	-	·776	·776	-	-
C	27	·504	·504	·892	·504	·892	1·090	-	-	-	-	-	-	-	-	-
HEMLOCK A	28	-	-	-	-	-	-	·388	-	-	-	-	-	-	·388	-
B	29	-	-	-	-	-	-	·388	·776	-	-	·388	-	-	-	-
C	30	-	-	-	-	-	-	·504	·504	-	-	-	·504	-	-	·896
SPRUCE A	31	-	-	-	-	-	-	-	-	·388	-	-	-	-	-	·388
B	32	-	-	-	-	-	-	-	-	·388	·776	·504	-	-	-	-
C	33	-	-	-	-	-	-	-	-	·504	·504	-	-	·504	·892	-
UNIT-LEVEL RETURN $		260	210	220	240	210	130	60	50	60	50	220	230	225	100	110
ACTIVITY LEVEL		X_{36}	X_{37}	X_{38}	X_{39}	X_{40}	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	X_{46}	X_{47}	X_{48}	X_{49}	X_{50}

FINAL ACTIVITIES — PULP PRODUCTION (Ton, air-dry pulp)

Inputs	Row	51	52	53	54	55	56	57
CHIPS: DOUGLAS FIR	34	·64	·79	·49	·72	1·52	-	-
HEMLOCK	35	·95	·79	·81	·72	-	1·64	-
SPRUCE	36	-	-	·33	·16	-	-	1·81
UNIT-LEVEL RETURN $		72	72	82	82	(67)	67	67
ACTIVITY LEVEL		X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	X_{56}	X_{57}

FINAL ACTIVITIES — LOG SELLING (100 cubic feet of log)

Inputs	Row	58	59	60	61	62	63	64	65	66	67	68	69
D.FIR #1 plr.	1	1											
#2	2		1										
#3	3			1									
#4	4				1								
#2 swlg.	5					1							
#3	6						1						
HEMLOCK #1 swlg.	7							1					
#2	8								1				
#3	9									1			
SPRUCE #1 swlg.	10										1		
#2	11											1	
#3	12												1
UNIT-LEVEL RETURN $		72	66	60	54	42	36	33	30	27	42	33	27
ACTIVITY LEVEL		X_{58}	X_{59}	X_{60}	X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}	X_{68}	X_{69}

FINAL ACTIVITIES — LUMBER SELLING (100 cubic feet of lumber)

Inputs	Row	70	71	72	73	74	75	76	77	78	79	80	81
D.FIR clear.	13	1											
sel/mer.	14		1										
cons/std.	15			1									
util/econ.	16				1								
HEMLOCK clear.	17					1							
sel/mer.	18						1						
cons/std.	19							1					
util/econ.	20								1				
SPRUCE clear.	21									1			
sel/mer.	22										1		
cons/std.	23											1	
util/econ.	24												1
UNIT-LEVEL RETURN $		235	130	115	40	145	100	90	40	270	95	85	40
ACTIVITY LEVEL		X_{70}	X_{71}	X_{72}	X_{73}	X_{74}	X_{75}	X_{76}	X_{77}	X_{78}	X_{79}	X_{80}	X_{81}

FINAL ACTIVITIES — VENEER SELLING (100 cubic feet of veneer)

Inputs	Row	82	83	84	85	86	87	88	89	90
DOUGLAS FIR A	25	1								
B	26		1							
C	27			1						
HEMLOCK A	28				1					
B	29					1				
C	30						1			
SPRUCE A	31							1		
B	32								1	
C	33									1
UNIT-LEVEL RETURN $		173	173	110	110	110	86	110	110	86
ACTIVITY LEVEL		X_{82}	X_{83}	X_{84}	X_{85}	X_{86}	X_{87}	X_{88}	X_{89}	X_{90}

FINAL ACTIVITIES — CHIP SELLING (100 cu. ft.) / HOG-FUEL

Inputs	Row	91	92	93	94
CHIPS: DOUGLAS FIR	34	1			
HEMLOCK	35		1		
SPRUCE	36			1	
HOG FUEL	37				1
UNIT-LEVEL RETURN $		18	21	21	1
ACTIVITY LEVEL		X_{91}	X_{92}	X_{93}	X_{94}

INPUT RESTRAINT (million cu. ft.)

Inputs	Row	Restraint
D.FIR #1 plr.	1	6·1
#2	2	14·2
#3	3	28·4
#4	4	18·3
#2 swlg.	5	46·7
#3	6	89·5
HEMLOCK #1 swlg.	7	13·8
#2	8	50·0
#3	9	213·6
SPRUCE #1 swlg.	10	1·3
#2	11	11·4
#3	12	19·8
LUMBER: D.FIR clear.	13	0
sel/mer.	14	0
cons/std.	15	0
util/econ.	16	0
HEMLOCK clear.	17	0
sel/mer.	18	0
cons/std.	19	0
util/econ.	20	0
SPRUCE clear.	21	0
sel/mer.	22	0
cons/std.	23	0
util/econ.	24	0
VENEER: DOUGLAS FIR A	25	0
B	26	0
C	27	0
HEMLOCK A	28	0
B	29	0
C	30	0
SPRUCE A	31	0
B	32	0
C	33	0
CHIPS: DOUGLAS FIR	34	0
HEMLOCK	35	0
SPRUCE	36	0
HOG FUEL	37	0