A HIGH TRANSMISSION THIN LENS
BETA-RAY SPECTROMETER

by

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THE REQUIREMENTS FOR THE DEGREE OF
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We accept this thesis as conforming to the standard
required from candidates for the degree of
MASTER OF APPLIED SCIENCE

THE UNIVERSITY OF BRITISH COLUMBIA
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ABSTRACT

A thin lens beta-ray spectrometer has been modified to give a transmission six times that of the unmodified spectrometer at a given resolution.
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The University of British Columbia, Vancouver 8, Canada.

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INTRODUCTION

A. THE JUSTIFICATION OF NUCLEAR SPECTROSCOPY

It has been agreed for many years that the smallest constituent of matter is the atom, which present theories picture as a planetary structure with a central core, possessing nearly all the atomic mass, surrounded by minute particles travelling in orbits around the core. It is also well established that this core, or nucleus, is composed of two kinds of particles, namely positively charged protons and neutral neutrons and that the orbital particles are negatively charged electrons. Certain combinations of protons and neutrons form nuclei that are stable against the loss of either kind of nuclear particle, or nucleon. That is, because of some sort of inter-particle bonding these nuclei will not break up into smaller nuclei or change their structure under normal conditions. It has been found that other nuclei are not stable against change in structure or composition. These unstable nuclei are termed radioactive or excited nuclei.

It is observed that a radioactive nucleus eventually "decays" to a stable nucleus with the simultaneous emission of energy. To explain this, nuclear theories associate an energy level with each nuclear configuration. For a particular nucleus, the lowest energy level is associated with the stable configuration of that nucleus, and is called the ground state of the nucleus. The energy difference between any excited state of a nucleus and its ground state is termed the excitation energy of
the excited nucleus and is the amount of energy the nucleus emits in decaying from the excited state to the ground state.

It is the purpose of nuclear spectroscopy to study the character of the energy emitted during radioactive decay so that a consistent theory may be constructed capable of predicting all observed nuclear phenomena. The properties of the emitted energy give information concerning the angular momentum or spin (measured in units of \( \hbar \)) of the nuclei involved in the decay, the energy levels associated with these nuclei, and the parity of the nuclear states. Parity is a symmetry property of the mathematical function, or wave function, describing each nuclear state. A nucleus is said to possess even or odd parity if the mathematical function describing the state of the nucleus is an even or an odd function respectively.

The energy emitted by a decaying radioactive nucleus is either in the form of electromagnetic radiation (photons or gamma rays) or particle emission, in which the energy is carried away kinetically. Various kinds of particles can be emitted by an excited nucleus but in the low energy region the predominant particle is the beta-particle of which there are two kinds, the negatively charged negatron and the positively charged positron. The negatron is found to be experimentally indistinguishable from the electron.

A typical decay scheme is shown in Fig. 1 where the parent nucleus, containing \( Z \) protons and \( A \) nucleons, may decay to the daughter nucleus, containing \( Z+1 \) protons and \( A \) nucleons, by either of two routes. The parent nucleus may decay directly to the ground state of the daughter nucleus with the emission of the negatron \( \beta^- \), or it may decay first to an excited state of the daughter nucleus with the emission of the negatron \( \beta^- \) followed by the emission of a gamma ray.
PARENT NUCLEUS  
Z protons, A nucleons

DAUGHTER NUCLEUS  
Z+1 protons, A nucleons

FIG. 1 - TYPICAL DECAY SCHEME

FIG. 2 - TYPICAL BETA-RAY SPECTRUM

FIG. 3 - FERMI PLOT
The decay scheme suggests that the negatrons $\beta^-$ and $\beta^+$ are emitted with discrete energies but such is not the case. A typical beta-ray spectrum, as shown in Fig. 2, has a continuous energy distribution up to some end point energy $E_{\text{max}}$ which is observed to be the energy difference between the two energy levels involved in the decay. This apparent deviation from the law of conservation of energy and a further discrepancy in the law of conservation of angular momentum led Pauli to postulate the existence of a new particle, the neutrino.

Fermi, in his theory of beta decay, assumes that during beta decay a beta-particle of energy $E$ and a neutrino of energy $E_{\text{max}} - E$ are emitted simultaneously. In addition, the neutrino emitted is assumed to have an angular momentum of $\frac{\hbar}{2}$ and to be of such a character (no charge and negligible mass) as to be virtually undetectable. Thus the neutrino hypothesis allows us to retain the laws of conservation of energy and of angular momentum.

It is difficult to measure $E_{\text{max}}$ from the experimentally measured beta spectrum because of the tangential approach of the distribution function to the end point. For an accurate determination of $E_{\text{max}}$ it is necessary to turn to the theory of beta decay.

If we assume that the change in angular momentum, $\Delta I$, during the emission of a particular beta-ray is $\Delta I = \pm 1$ or 0 (in units of $\hbar$), then the momentum distribution function for this beta group, from Fermi's theory, is given by:

$$N(p)dp \propto F(E_{\text{max}} - E)^2 p^2 dp$$

where:

- $N(p)dp$ is the number of beta particles emitted per unit time in the momentum interval $(p, p + dp)$. 

1
4.

$E_{\text{max}}$ is the maximum energy observed in the spectrum.

$E$ is the energy of the beta-particle.

$p$ is the momentum of the beta-particle.

$F$ is a function which corrects for the effect of the nuclear coulomb field on the emitted beta-particle.

$F$ is called the Fermi function and is tabulated in the literature.\(^2\)

From equation (1) we see that $\sqrt{\frac{N(p)}{p^2F}} \propto (E_{\text{max}} - E)$. Hence if we plot $\sqrt{\frac{N(p)}{p^2F}}$ as a function of $E$ (Fermi plot) we will get a straight line intersecting the energy axis at $E_{\text{max}}$. If two independent beta groups are present in a spectrum then the end point energy of the second group can be determined by subtracting the contribution of the first beta group from the composite Fermi plot as in Fig. 3.

The theory predicts that a decay or transition in which $\Delta I = \pm 1$ or 0 is the most probable mode of decay. Such a transition is called an allowed transition. Transitions which involve change in parity and spin changes greater than $\pm 1$ are called forbidden transitions. First forbidden and higher forbidden transitions are characterized by the magnitude of the spin change of the transition and any change in parity involved.

For forbidden transitions equation (1) must be modified to:

$$N(p)dp = N_0 P(p)dp \quad (2)$$

where: $N_0$ is a constant and $P(p)dp$ is the probability that a beta-particle is emitted with a momentum in the interval $(p, p+dp)$
\( P(p)dp \) is usually expressed as:

\[
P(p)dp = CF(E_{\text{max}} - E)^2 p^2 dp
\]  

(3)

where: \( C \) is some function such that the modified Fermi plot of \( \sqrt{\frac{N(p)}{Cp^2 F}} \) vs \( E \) is a straight line. The function \( C \) required to do this has been tabulated\(^3\) and is useful in identifying the degree of forbiddeness of the decay.

Using equation (3) we may write that the probability, \( \lambda \), that the nucleus will decay by emission of any electron in the above beta group is:

\[
\lambda = \int P_{\text{max}} P(p)dp
\]  

(4)

where: \( P_{\text{max}} \) is the momentum of an electron of energy \( E_{\text{max}} \).

Substituting (3) into (4) we see that:

\[
\lambda = af
\]  

(5)

where: \( f = \int P_{\text{max}} CnF(E_{\text{max}} - E)^2 p^2 dp \), \( a = \frac{G^2}{2\pi^3} \) (\( G \) is the Fermi constant)\(^3\), and \( n \) is the degree of forbiddeness.

The total decay probability \( \lambda \) has dimensions disintegrations/time, hence \( \frac{1}{\lambda} \) is the mean life of the excited nucleus. The exponential character of radioactive decay leads to the definition of the half-life of a transition.

It can be shown that the mean life \( \tau = \frac{1}{\lambda} \) and the half-life \( T_{\nu_2} \) are related by:

\[
\tau = \frac{T_{\nu_2}}{\ln 2}
\]  

(6)

From (5) and (6) we see that \( \frac{T_{\nu_2}}{\ln 2} = \frac{1}{\lambda} = \frac{1}{af} \) and:

\[
fT_{\nu_2} = \frac{\ln 2}{a}
\]  

(7)

The quantity \( fT \) is called the **comparative half-life** of the transition and has been shown to be related to the degree of forbiddenness of a transition. The logarithm of \( fT_{\nu_2} \) is found to be more convenient to work with\(^4\).
and is a useful way of determining the degree of forbiddenness of a decay by comparing it to the log \( \log(fT_{\nu}) \) values of transitions whose degree of forbiddenness has been established in other ways.

In Fig. 1 we see that if the parent nucleus decays to an excited state of the daughter nucleus then the daughter nucleus will decay to its ground state with the emission of a photon. It is observed that in addition to photon emission electrons of discrete energies are emitted. They are orbital electrons which interact with the nucleus and are ejected from their respective orbits with an energy equal to the transition energy less the binding energy of the orbit from which the electron is ejected. This alternate process of de-excitation is called internal conversion and the electrons thus emitted are called conversion electrons.

If the conversion electron was originally in the K-orbit, it is called a K-conversion electron and similarly for the L, M and other orbits. Because conversion electrons are emitted with discrete energies they appear as sharp peaks in a beta spectrum as shown in Fig. 2. The K-conversion line is more intense because of the greater interaction probability of the K-orbit with the nucleus.

Theory predicts that the probability that a nucleus will decay by internal conversion rather than by the emission of a gamma ray increases with increasing atomic number, decreasing transition energy and increasing spin change of the transition. Hence the probability that the nucleus will decay by internal conversion is a quantity that gives considerable information about the energy levels of the nucleus.
If $\lambda$ is the total probability of decay and $\lambda_\gamma$ and $\lambda_e$ the
probabilities of decay by gamma emission and internal conversion respectively
then we may write:

$$\lambda = \lambda_\gamma + \lambda_e$$  \hspace{1cm} (8)

The ratio of the number of decays by internal conversion to the number of decays
by gamma emission in a given time is called the conversion coefficient and is
given by:

$$\alpha = \frac{\lambda_e}{\lambda_\gamma}$$  \hspace{1cm} (9)

We may further break down $\lambda_e$ into:

$$\lambda_e = \lambda_{e_K} + \lambda_{e_L} + \lambda_{e_M} + \cdots$$  \hspace{1cm} (10)

where $\lambda_{e_K}$ is the probability of decay by internal conversion of a K-orbit electron
and similarly for $\lambda_{e_L}$, $\lambda_{e_M}$ etc. Hence equation (9) may be written as:

$$\alpha = \frac{\lambda_{e_K}}{\lambda_\gamma} + \frac{\lambda_{e_L}}{\lambda_\gamma} + \frac{\lambda_{e_M}}{\lambda_\gamma} + \cdots = \alpha_K + \alpha_L + \alpha_M + \cdots$$  \hspace{1cm} (11)

where: $\alpha_K$ is the K-conversion coefficient

$\alpha_L$ is the L-conversion coefficient etc.

The first six conversion coefficients have been calculated for
various Z values, transition energies and spin and parity changes and have been
tabulated\textsuperscript{6, 7}. Intensity measurements of the beta radiation are necessary to
experimentally determine conversion coefficients. This is in general difficult
as it assumes an accurate knowledge of the efficiency (or the fraction of beta
rays emitted from the source that enter the detector) of the beta-ray spectrometer
being used to measure these radiations. A reasonable assumption made is that
the efficiency of a spectrometer is the same at all energies and hence the ratio
or the $K$-to-$L$ ratio, should be independent of the efficiency.

The $K$-to-$L$ ratio has been calculated theoretically for various $Z$ values, transition energies and spin and parity changes and has been tabulated$^8$. Gamma rays which are not internally converted cannot of course be analyzed by use of a magnetic spectrometer, but if they are allowed to impinge on a thin foil of high atomic number $Z$, they may undergo a process called external conversion. In this process the gamma ray ejects an electron from an orbit of an atom of the foil and as in internal conversion these electrons, or photo-electrons, are emitted with an energy equal to the energy of the gamma ray less the binding energy of the orbit from which the electron is ejected.

Theory predicts that the lifetime of an excited state, defined to be the inverse of the probability of de-excitation by gamma emission, is dependent upon the energy of the transition and the spin and parity changes involved. In many cases these lifetimes are extremely short and elaborate instrumentation has been developed$^9$ which can measure lifetimes down to $10^{-10}$ seconds. Such measurements, when compared to predicted lifetimes, give valuable information on spin and parity changes associated with the transition.

In decay schemes more complicated than that shown in Fig. 1, there may occur cascade processes in which an excited nucleus decays to one or more intermediate excited states before decaying to the ground state, with the emission of two or more gamma rays in succession. The lifetimes of the intermediate states are usually very short and hence the gamma rays appear to be emitted simultaneously or in coincidence. By appropriate pulse height selection,
or energy determination techniques, coincidence measurements are most useful in determining the sequence of de-excitations in a decay scheme.

Gamma rays emitted in a cascade process are correlated through their emission angles. That is, the probability that the second gamma ray in a cascade will be emitted in a certain direction is a function of the direction of emission of the first gamma ray. Such dependence is called angular correlation and is dependent upon the spin changes involved in the cascade. Brady and Deutsch have made angular correlation measurements to determine spin changes in a cascade. Siegbahn has constructed an instrument to measure the angular correlation of internal conversion electrons.

Measurements of the above mentioned nuclear phenomena are made with many different instruments but the various types of beta-ray spectroscopes are the most widely used. These instruments are extensively used to measure beta spectra, internal conversion spectra and photo-electron spectra. It should be noted here that a beta-ray spectroscope is called either a spectrograph, if a photographic plate is used for detection, or a spectrometer, if a counter is used for detection.

All beta-ray spectroscopes employ either electrostatic or magnetic focusing, the latter being predominant. An analysis of the electron focusing of these two types of spectroscopes shows that the electrostatic spectroscope is energy selective while the magnetic spectroscope is momentum selective. The electron trajectories in a magnetic spectroscope are given by
the well known relation:

\[ B_{	ext{ev}} = \frac{mv^2}{\rho} \]  

(12)

where \( B \) is the magnetic flux, \( e \) is the electronic charge, \( m \) is the electronic mass, \( v \) is the electron velocity and \( \rho \) is the radius of curvature of the electron trajectory. The so-called "magnetic stiffness" of the magnetic field, \( B\rho \), is thus related to the electron momentum by:

\[ B\rho = \frac{mv}{e} \]  

(13)

The magnetic stiffness rather than the energy is normally used as the abscissa in representing an electron spectrum.

It is useful in beta-ray spectroscopy to define certain parameters to be used in optimizing the performance of a spectroscope and by which the performance of one spectroscope may be compared with the performance of another.

1) Resolution: Fig. 4 shows a typical line shape for monoenergetic electrons as measured by an instrument having a particular geometry. The finite width of the line is caused by various factors such as scattering, the finite size of the source and slits defining the trajectories, and the inherent spherical aberration of the focusing field. The resolution, usually defined as a percentage, is defined as:

\[ R = \frac{\Delta(B\rho)}{B\rho} \]  

(14)

where \( B\rho \) is the momentum of the focused electrons in gauss - cm, and \( \Delta(B\rho) \) is the line width measured at half height.

2) Transmission: The transmission, \( T \), of a spectroscope, again usually expressed as a percentage, is defined to be the fraction of beta-rays
FIG. 4 - DEFINITION OF RESOLUTION

$\Delta(B_P)$

FIG. 5 - ELECTROSTATIC FIELD SPECTROSCOPE

S - source
D - detector
emitted by the source that reach the detector when the instrument is adjusted to focus these electrons. In other words, the transmission is the fractional solid angle at the source "seen" by the detector.

A related quantity is the gathering power \( \omega \) defined to be the ratio of the acceptance solid angle \( \Omega \), defined by the entrance baffles, to the total solid angle:

\[
\omega = \frac{\Omega}{4\pi}
\]  
(15)

It is obvious that \( T \leq \omega \)

3) **Dispersion**: Dispersion is a measure of the "ability" of a spectroscope to separate adjacent energies. Thus for an instrument to be useful at all the dispersion or line separation must be greater than the line width, which is caused largely by spherical aberration. Dispersion is defined as:

\[
D = \frac{dx}{d(B\phi)}
\]  
(16)

where \( x \) is some coordinate defining the position of the focus.

The majority of cases arising in nuclear spectroscopy require an instrument with only moderate resolution but a higher resolution is often desired when studying complex decay schemes and when "thick" conversion lines distort the shape of the beta spectrum upon which they are superimposed. Similarly high transmission is useful when studying weak sources and for beta-gamma and conversion electron-gamma coincidence work. The following is a description of the focusing principles of various beta-ray spectrosopes.
B. THE FOCUSING PRINCIPLES OF SOME BETA-RAY SPECTROSCOPES

1. Electrostatic Field Spectroscopes

The theory of electrostatic focusing of beta-rays was first developed by Purcell who also built the first instrument of this type in 1938. Several of these instruments have been constructed and the electrostatic fields employed have been usually either spherically symmetric or cylindrically symmetric. A spherically symmetric electrostatic field spectroscope (spherical condenser) is shown in Fig. 5. This instrument utilizes the space focusing property of the geometrical symmetry and has medium resolution at a transmission comparable with the best magnetic spectroscopes.

The main disadvantage of the electrostatic field spectroscope is the high electric field required to focus electrons of only moderate energy. By comparison, a magnetic field of 1000 gauss produces the same radius of curvature as an electric field of 300,000 volts/cm for electrons of relativistic velocities.

2. Magnetic Focusing Spectroscopes

Magnetic focusing spectroscopes may be divided into two groups, flat spectroscopes and lens-type or helical spectroscopes. In the flat spectroscopes the electron trajectories are perpendicular, or nearly so, to the magnetic field while in the helical spectrocope the electron trajectories are mainly in the same direction as the magnetic field.

a) Flat Spectroscopes

i) The Direct-deflection Spectroscope. The first beta-ray spectrocope, developed by Baeyer and Hahn in 1910, was a direct-deflection...
FIG. 6 - DIRECT-DEFLECTION SPECTROSCOPE

FIG. 7 - SEMI-CIRCULAR FOCUSING SPECTROSCOPE

FIG. 8 - THIRD-ORDER FOCUSING PRINCIPLE
flat spectroscope. As shown in Fig. 6 the electrons are simply deflected by an amount which depends upon their energy. Electrons of various energies are thus separated and detected on a photographic plate.

ii) The Semi-circular Focusing Spectroscope. Related to the direct deflection spectroscope of Baeyer and Hahn is the semi-circular or \( \pi \)-spectroscope first developed by Danysz in 1914. This instrument, shown in Fig. 7, exploits the geometric property of semi-circular focusing. The photographic plate used by Danysz and others\(^{16-18}\) was replaced with a counter by Chadwick\(^{19}\) who then discovered the continuous beta-ray spectrum.

iii) The Third-order Focusing Spectroscope. The third-order focusing spectroscope corrects the spherical aberration present in a semi-circular focusing spectroscope. Fig. 8a shows the spherical aberration characteristic of a homogeneous field semi-circular focusing spectroscope. In 1933 Bock\(^{20}\) pointed out that this spherical aberration could be greatly reduced by shaping the magnetic field. The result is the focusing of the median ray at the focus of the two extreme rays as shown in Fig. 8b. A detailed theoretical treatment of the third-order focusing principle has been made by Beiduk and Konopinski\(^{21}\). Instruments of this type have been constructed by Korsunsky, Kelman and Petrov\(^ {22}\) and by Langer and Cook\(^ {23}\).

iv) The Double Focusing Spectroscope. Svartholm and Siegbahn\(^ {24}\) were the first to point out that by suitably shaping the magnetic field of a flat spectroscope it is possible to focus the electrons in the axial direction as well as the radial direction as shown in Fig. 9. Several authors\(^ {25-27}\) have
FIG. 9 - DOUBLE-FOCUSING PRINCIPLE

FIG. 10 - PRISM AND SECTORFIELD SPECTROSCOPES

FIG. 11 - "ORANGE" SPECTROMETER
worked on various aspects of the theory of double focusing while instruments of this type have been built by Hedgran, Svartholm and Siegbahn, Bartlett and Bainbridge and others.

v) **Prism and Sectorfield Spectroscopes.** The prism and sectorfield spectroscopes differ from the above mentioned flat spectroscopes in that both the source and detector lie outside the magnetic field. The focusing principle of the prism spectroscope is illustrated in Fig. 10a. Siday, Korsunsky and others have investigated the focusing properties of such an arrangement while Siday and Silverston have constructed an instrument of this type having a resolution of 0.07%.

Replacing the homogeneous field of the prism spectroscope by a sector field, as shown in Fig. 10b, introduces extra degrees of freedom which may be used to optimize the various focusing characteristics of this type of spectroscope. The sector field may be either homogeneous or inhomogeneous and by suitably shaping the pole faces the fringing field can effect double focusing. Fig. 11 shows the "orange" spectroscope due to Kofoed-Hansen, Lindhard and Nielsen which is effectively six sectorfield spectroscopes in parallel. This instrument has a transmission of 12% at a resolution 2%.

b) **Lens-type Spectrometers**

The lens-type or helical spectrometer was first suggested by Kapitza in 1924. As shown in Fig. 12, monoenergetic electrons emitted from a source on the axis of an axially symmetric field will follow helical trajectories and return to the axis at point P. Due to spherical aberration the
FIG. 13 - SOLENOIDAL SPECTROMETER

FIG. 14 - INTERMEDIATE IMAGE SPECTROMETER
maximum convergence of the trajectories occurs not at the point P but at the "ring focus" F. These trajectories are typical of a lens-type spectrometer whether the magnetic field is homogeneous as in the case of the solenoidal spectrometer or inhomogeneous as in the case of the thick-lens and thin-lens spectrometers.

i) **The Solenoidal Spectrometer.** The solenoidal spectrometer, as shown in Fig. 13, employs the uniform magnetic field characteristic of a solenoid to focus electrons. The first instrument of this type was built by Tricker in 1924. Witcher improved the resolution of the instrument at a given transmission by placing an annular baffle at the ring focus F. Ring focus baffles and optimum operating conditions have been studied by Persico, Hubert, and others.

ii) **The Thick-lens Spectrometer.** The field "shape" of the thick-lens spectrometer reasonably approximates the theoretical field shape producing no spherical aberration. The focusing properties of this instrument have been studied theoretically by Verster and by Paquien and Grivet. Instruments of this type produce the desired field form by enclosing them in iron with the use of suitable pole pieces or by the use of several iron-free coils.

iii) **The Intermediate Image Spectrometer.** An instrument similar to the thick lens spectrometer was constructed by Slätis and Siegbahn in 1949. Focusing was realized by a series of coils. By varying the ratio of the current in the central coils to the total current an "intermediate image"
FIG. 15  THIN-LENS SPECTROMETER
similar to the ring focus is formed at I as shown in Fig. 14. The focus is formed on the axis of the field at F. This instrument has a higher transmission at a given resolution ($T = 8\%, R = 4\%$) than the thick-lens spectrometer.

iv) The Thin-lens Spectrometer. The focusing properties of the thin-lens spectrometer, shown in Fig. 15, were first studied by Deutsch, Elliott and Evans. As in the case of the solenoidal spectrometer, the introduction of an annular slit at the ring focus increases the resolution at a given transmission. The major disadvantage of the thin-lens spectrometer is its inherently large spherical aberration but it has the advantage of having both the source and detector in almost field-free regions. This feature is important in angular correlation experiments. This instrument also has the advantage of being inexpensive as well as requiring less electric power to operate than most of the other magnetic spectrometers.

A comparison of various spectrometers has been made by Persico and Geoffrion. Part of their findings are listed in Table 1.

The ring focus occurring in the trajectories of electrons in a thin-lens spectrometer has been exploited to a certain extent by some workers to increase the efficiency of the instrument. The introduction of an annular slit at the ring focus, as shown in Fig. 15, increases the transmission by a factor of two at a given resolution. Due to the divergence of the electron trajectories past the ring focus, an axial detector must be fairly large to collect most of the electrons and hence is subject to a large background. Conversely, a smaller axial detector operates at a much lower background but only at the price of
## TABLE 1

### Flat Spectroscopes

<table>
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<tr>
<th>Ref.</th>
<th>Type</th>
<th>Iron</th>
<th>(\omega(%))</th>
<th>(R(%))</th>
<th>(\omega/R \times 100)</th>
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<td>.0067</td>
<td>.63</td>
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<td>.127</td>
<td>.7</td>
<td>18.2</td>
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### Helical Spectrometers

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Type</th>
<th>Iron</th>
<th>(\omega(%))</th>
<th>(R(%))</th>
<th>(\omega/R \times 100)</th>
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<tbody>
<tr>
<td>40</td>
<td>Sol. (r)</td>
<td>no</td>
<td>1.0</td>
<td>6.0</td>
<td>16.7</td>
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<tr>
<td>63</td>
<td>Sol. (r)</td>
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<td>5.0</td>
<td>6.5</td>
<td>76.9</td>
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<tr>
<td>64</td>
<td>L.l. (r)</td>
<td>yes</td>
<td>.2, 1</td>
<td>6.7, 7.7</td>
<td>3.0, 13.0</td>
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<tr>
<td>49</td>
<td>L.l. (r)</td>
<td>no</td>
<td>T(_{\text{max}}) = 1.4</td>
<td>1</td>
<td>140</td>
</tr>
<tr>
<td>50</td>
<td>I.i.</td>
<td>yes</td>
<td>8</td>
<td>4</td>
<td>200</td>
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<tr>
<td>51</td>
<td>T.l.</td>
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<td>.18</td>
<td>1.7</td>
<td>10.6</td>
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<tr>
<td>52</td>
<td>T.l.x(r)</td>
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<td>.5</td>
<td>1.5</td>
<td>30</td>
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</table>

\(\omega\) = gathering power, \(R\) = resolution  
Spg = first-order single focusing spectrograph  
Spm = first-order single focusing spectrometer  
D.f. = double focusing  
3s.f. = third-order focusing  
Sol. = solenoidal; (r) = ring focus baffles used  
L.l. = Thick-lens  
I.i. = Intermediate image  
T.l. = Thin-lens
reduced transmission. A detector has been constructed in this laboratory capable of collecting the electrons at the ring focus. Such a detector has the advantage of fairly small dimensions and is thus less subject to detecting "stray" radiations. In addition, the location of the detector ensures complete collection of all electrons passing through the ring focus baffles and hence a high transmission. The development of the scintillation counter has made the realization of such a detector possible.
FIG. 16 - MODIFIED THIN-LENS SPECTROMETER
II

THE MODIFIED THIN-LENS SPECTROMETER

A. PREVIOUS INVESTIGATION

The investigation of ring focus collection in a thin-lens spectrometer was first made in this laboratory by J. A. L. Thompson (unpublished). He determined experimentally the electron trajectories for various geometries of the instrument using an electron gun as a source and a sheet of lucite coated with zinc sulphide as a detector. An attempt was also made by Thompson to reduce the spherical aberration by introducing an axial dipole as suggested by Deutsch, Elliott and Evans but this proved impractical because of cooling difficulties.

The conclusion of this investigation led to the geometry shown in Fig. 16.

The detector used by Thompson, shown in Fig. 17a, consists of a ring of anthracine scintillation crystals "cemented" with a high-viscosity silicone oil (Dow-Corning 200) into a groove on the open lip of a lucite "light-cone". The light-cone is optically coupled, using the same type oil, to a photomultiplier tube. Milley designed a lucite light-cone, shown in Fig. 17b, which much improves the optical coupling between the crystals and the photomultiplier tube.

The coil of the spectrometer used by Thompson and Milley consists of four co-axial coils separated by cooling coils. The second innermost coil was not useable due to a previous accident. The results of Thompson's
FIG. 17a - LIGHT-CONE DUE TO THOMPSON

FIG. 17b - LIGHT-CONE DUE TO MILLEY
investigation of the effect of the dipole in reducing the spherical aberration were not recorded but Milley found that it had no appreciable effect, with the currents available, when all three remaining coils were used.

A second thin-lens spectrometer in this laboratory with all four coils operative was modified in a similar manner to the first instrument with a view towards improving the optical coupling of the detector.

B. PRESENT INVESTIGATION

1. Design and Construction Details.

Because all four coils of the second spectrometer were useable, electron trajectories in this instrument were different than in the first instrument. It was thus necessary to make a completely new investigation of the electron trajectories. The trajectories were calculated according to the method of Deutsch, Elliott and Evans by the Computation Center of the McLennan Laboratory at the University of Toronto. The equation defining the electron trajectories is:

\[ \frac{r''(k^2-A^2)}{(1+r^2)^2} - r'A \frac{dA}{dz} + A \frac{dA}{dr} = 0 \]  

(17)

where:  
- \( A \) is the vector potential per ampere of magnetizing current  
- \( r \) is the radial displacement  
- \( z \) is the axial displacement  
- \( k \) is the electron momentum in gauss-cm/ampere.

The vector potential \( A \) is found from the equation:

\[ A(r,z) = \frac{r}{2} H_0(z) - \frac{r^3}{16} \frac{d^2}{dz^2} H_0(z) + \frac{r^5}{384} \frac{d^4}{dz^4} H_0(z) - \ldots \]  

(18)

In equation (18) \( H_0(z) \) is the axial field, given by:

\[ H_0(z) = \frac{2n\pi}{10L(a_z-a_i)} \left\{ B(a_z) - B(a_i) \right\} \]  

(19)
where: \( n_0 \) is the number of turns in the coil = 2040
\( a_1 \) is the inner radius of the coil = 11.02 cm.
\( a_2 \) is the outer radius of the coil = 28.40 cm.
\( L \) is the axial length of the coil = 17.61 cm.

The function \( B \) is given by:
\[
B(a) = (z + \frac{b}{2}) \ln \left\{ a + \sqrt{a^2 + (z + \frac{b}{2})^2} \right\} - (z - \frac{b}{2}) \ln \left\{ a + \sqrt{a^2 + (z - \frac{b}{2})^2} \right\}
\]

Fig. 12 shows a typical family of trajectories calculated in this manner.

From the calculated trajectories, graphs were made of radial displacement on the focal plane vs. tangent of the angle of emission, as shown in Fig. 18a, at various focal planes for a particular value of \( k \). These graphs indicated the entrance baffles, defined by \( a \) and \( b \), required to produce a focus of "width" \( d \) and "median" ray given by \( c \). The corresponding trajectories are shown in Fig. 18b.

Analysis of these trajectories indicated the regions in which ring foci would occur for different values of momentum, incident ray and geometry parameters. The radial position and annular width of the interesting foci dictated the dimensions of the detector needed to span these foci in an experimental survey. It was also necessary that the dimensions of the detector should allow for the expected discrepancy between calculated trajectories and experimentally observed trajectories, the principal cause of the expected discrepancy being the magnet cooling coils, the dimensions of which were neglected in calculating the magnetic field for the theoretical trajectories.

It was thought that Milley's light-cone could be improved upon in two respects, the effect of each being a priori quantitatively unpredictable.

The anthracene crystals in his light-cone had to be continually replaced because.
FIG. 19a

FIG. 19b

MAGNETIC SHIELD

PHOTOMULT. & PRE-AMP COMPONENTS

TIE-ROD

EMI 6262
of sublimation. Because the silicone oil "cement" slowly flowed out of the detector groove, there was a continual decrease in optical coupling between the crystals and the light cone. A similar fault was occurring at the face of the photomultiplier tube where the silicone oil, which provided the optical coupling between the light-cone and photomultiplier, was leaking away.

A minor design flaw in the region of the scintillation crystal groove present in Milley's light-cone was corrected and a new plastic scintillator (NE 101) was used in place of anthracene crystals. Although anthracene has double the light output of the plastic scintillator, it was hoped that the improved optical coupling between the light-cone and scintillator (almost 50%) would more than compensate for this. Another factor was the removal of the inconvenience of having to replace the anthracene crystals after they sublime. The light-cone designed upon these considerations is shown in Fig. 19a. The plastic scintillator was glued into the groove using bonding agent R 823. A further improvement was made in the coupling of the EMI 6262 photomultiplier tube to the base of the light-cone. A new magnetic shield arrangement was constructed which assured complete coupling before affixing the magnetic shield itself. This arrangement, shown in Fig. 19b, provided 100% coupling through adjustable tie-rods which held the face of the photomultiplier tube firmly and squarely against the base of the light-cone. This detector, like Milley's, was unaffected by the focusing field. It had a low signal-to-noise ratio but was used for a considerable part of the experimental investigation until a second detector was constructed.

To improve the optical coupling it was decided to eliminate the light-cone altogether and simply glue the plastic scintillator (NE 102 - light
FIG. 20 - MAGNETIC SHIELD AND DETECTOR

FIG. 21 - MAGNETIC SHIELD EFFICIENCY
output is 60% that of anthracene) onto the face of a five inch Dumont 6364 photomultiplier. Because of the proximity of the photomultiplier tube to the magnet, a more elaborate magnetic shielding arrangement was required than for the first detector. The shield, shown in Fig. 20, nearly enclosed the detector completely. It was necessary to use brass stand-offs on which to mount the exit baffles as steel stand-offs produced considerable de-focusing within the photomultiplier. The efficiency of the magnetic shield was measured by placing a gamma ray source near the detector and measuring the number of counts per unit time as a function of the magnet current. The efficiency of the shield used is shown in Fig. 21 and indicates that a thicker shield is desirable if high energy spectra are to be studied but the shield shown would be adequate if a correction for the efficiency is applied. Although the signal-to-noise ratio of this detector is tolerable, it is felt that for experimental runs the use of anthracene crystals would greatly improve this factor.

Because of the difficulty of aligning the source, detector and baffles on the axis of the focusing field it was realized that a means of source-centering was required to insure the coincidence of the ring focus with the exit baffles, particularly when narrow entrance and exit baffles were being used. As shown in Fig. 22, such a source-centering device was incorporated into the final design of the instrument. The source-centering device moved the source in two dimensions using a rack and pinion arrangement. The entrance baffles were mounted so as to move rigidly with the source. This device proved successful and invaluable for optimizing the source-center position.
FIG. 22 - MODIFIED SPECTROMETER WITH NEW DETECTOR
2. Experimental Results and Discussion

The equipment was set up as follows. The magnetic field current was electronically controlled by a current regulator. Output pulses from the photomultiplier were fed via a cathode-follower into a standard amplifier. The spectrometer was lined up with the horizontal component of the earth's field while the vertical component was cancelled by means of compensation coils. A detailed description of the electronic control and measurement equipment is given in Milley's thesis.

Preliminary experimental work was done using the experimental arrangement shown in Fig. 19a and Fig. 19b. Using only the entrance baffle defining the angle \( \arctan \frac{1}{2} \) to keep the low angle rays above the lead gamma ray baffle and exposing the entire width of the scintillator ring, the 115 keV conversion line of Eu\(^{152, 154}\) was used to study the "natural" or unbaffled characteristics of the instrument. A survey was made of the variation in peak "shape" as a function of focal plane and source position, both of which are measured from the center of the coil. Fig. 23a shows how the height of the 115 keV conversion line varied with source position and focal plane. The resolution of the line was relatively constant (\(~7\frac{1}{2}\)% over the source positions from -10 cm to -2 cm but improved for larger source positions due to the increasing effect of the spectrometer tube acting as an outside entrance baffle. Fig. 23b shows how the current required to focus the line varied with source position for a given focal plane. At other focal planes studied the curves were almost coincident with the one shown, the difference in focusing current for the focal planes 57.8 cm and 42.8 cm being only .028 amperes. This slight variation
FIG. 23a - PEAK HT. VS SOURCE POS’N AND FOCAL PLANE

SAME PARAMETERS AS IN FIG. 23a.

FIG. 23b - FOCUSING CURRENT VS SOURCE POS’N

FIG. 24 - EFFECT OF MISMATCH ON PEAK SHAPE
of focusing current with focal plane for a given source position indicated that the trajectories are nearly horizontal at the ring focus.

This coarse survey indicated that the source position should be kept between 14 cm and 20 cm. For source positions larger than 20 cm the transmission suffered and for source positions less than 14 cm the current required to focus the electrons was too high, thus limiting the energy range of the spectrometer. It was decided to make the focal plane as large as the size of the spectrometer tube physically allowed to minimize the influence of the focusing field on the photomultiplier. This was particularly important when the second detector was installed.

A simple calculation of the gathering power defined by the entrance baffles indicates that a given emission angle increment, $\Delta \theta$, defined at a high angle of emission, $\theta$, contributes more transmission-wise than the same size increment at a lower angle of emission. The high angle rays undergo greater dispersion than low angle rays but unfortunately they also suffer greater spherical aberration. Hence the angle of emission chosen must be a compromise between spherical aberration and geometrical limitations on the one hand and transmission and dispersion on the other. Because of the many parameters involved, the experimental determination of such a compromise is a time consuming process.

For a given choice of entrance rays it is desired to "match" the size of the exit slot to the size of the ring focus; otherwise a larger slot than necessary will degrade the resolution while a slot that is too small will lower
the transmission. To do this the following procedure was adopted. For a particular exit slot, a set of entrance baffles were installed that defined a ray bundle known to be within the geometrical limitations of the spectrometer tube. The conversion line used (137 kev conversion line of $^{75}\text{Se}$) was then spanned at various source positions and consequently various focal planes because of the inherent geometrical restriction, source-detector distance = constant. This restriction results from moving the spectrometer tube as a whole. The procedure was then repeated with a slightly different outer or inner entrance baffle. Thus by varying the inner and outer entrance baffles one at a time and then spanning the conversion line at various source positions a compromise was reached between transmission and resolution.

It was found that for a constant slot width, the line shape was unaffected by changing the radius of the slot within the limits imposed by the size of the scintillator ring. Thus apparently the character of the focus does not change significantly as it sweeps across the exit slot.

As can be seen from Fig. 18b, a relatively small change in one of the entrance baffles can move the axial position of the ring focus considerably and possibly outside the range of focal planes available in this spectrometer. From Fig. 18a, it can be seen that a mismatch between ring focus width and exit slot size would result if the entrance rays defined by $a$ and $b^1$ are focused at the focal plane $(F,P)_2$. The rays emitted between the angles given by $b^1$ and $b$ that could contribute to the peak intensity without changing the width of the ring focus are baffled out of the entrance ray bundle. The resulting mismatch causes the line shape shown in Fig. 24, the slowly rising leading edge being caused by the loss of
FIG. 25 - CONVERSION ELECTRON SPECTRUM OF SE$^{75}$

EXIT BAFFLES - 10.6, 9.7 cm
ENT. BAFFLES - ARCTAN .34, .475
BACKGROUND - 6,400 c/m
TIME ON EACH POINT - 2 MIN.
FOCAL PLANE - 46.9 cm
SOURCE POSITION - 15.6 cm.
ENT. BARRIERS -- ARCTAN .34, .475
EXIT BARRIERS -- 10.6, 10.7 cm dia.
S.P. -- 15.6 cm
F.P. -- 46.9 cm

ENT. BARRIERS -- ARCTAN .32, .49
EXIT BARRIERS -- 10.6, 9.03 cm dia.
S.P. -- 15.6 cm
F.P. -- 46.9 cm

Fig. 26 - Comparison of Two Instrument Geometries
high emission angle rays on the inner side of the ring focus.

Some of the results obtained by this method are shown in Fig. 25 and Fig. 26. The spectrum in Fig. 25 was taken with a gathering power of 2.2% resulting in a resolution of \( \sim 2 \frac{1}{2} \% \). The value of \( \omega/R \) for this spectrum is .88 compared with a value for \( T/R \) of .1 for the unmodified spectrometer. Fig. 26 compares the 265 keV and 280 keV conversion peaks for different geometries of the instrument. Fig. 26a shows a spectrum taken with \( \omega = 2.2\% \) and \( R \sim 2 \frac{1}{2}\% \) while Fig. 26b shows a spectrum taken with \( \omega = 2.7\% \) and \( R \sim 3\% \). It should be noted that these "matches" are rough because of the comparatively coarse changes in the entrance baffles used.

In conclusion, it is felt that while the detector described exploits the transmission capabilities of the focusing field almost completely, the resolution could be improved by reducing the size of the source and by a better choice of entrance baffles. Investigations at higher resolutions with this instrument will require lower backgrounds than present in the above detector for ease of working. Although the detector suffers from a fairly large background which must be tolerated when studying low energy spectra, the background can be lowered to about 600 counts per minute at 250 keV, and even lower at higher energies. A better signal-to-noise could be achieved by using anthracene crystals in place of the plastic scintillator or by aluminizing the plastic scintillator to reflect escaping scintillation photons back onto the face of the photomultiplier. The latter technique is currently being investigated and shows an improvement in the signal-to-noise ratio by a factor of at least \( 1 \frac{1}{2} \). Another factor influencing the signal-to-noise ratio of the detector is the choice of photomultiplier. This characteristic has been observed.
to vary considerably from one tube to another, and because only one tube of the type used was available it is felt that a selected photomultiplier would improve the signal-to-noise ratio of the detector.

The present limitation on the energy range of the instrument is the efficiency of the magnetic shield. It is proposed to improve this by moving the detector farther from the magnet. This will necessitate shortening the stand-offs on which the source-centering control is mounted.
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