DEMODULATOR COMPENSATING NETWORKS

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ABSTRACT

This thesis deals with the study of some demodulator lead-lag networks. Specifically the problem has involved analysis and design, accompanied by experimental verification of a new approach to the realization of phase-lead and phase-lag networks for application in ac servomechanisms.

Analysis has been made of several circuits, different in physical layout but operating on the same basic principle. By computing the parameters which describe the step response of the particular network, an equivalent transfer function is obtained. This transfer function is the describing function for the limiting case of infinite carrier-to-signal-frequency ratio.

Experimental work was done with an electro-mechanical network, capable of generating low-frequency sinusoidal-modulated signals. Phase and amplitude characteristics of an ac lead network, centred at a frequency of 400 cycles per second, were obtained. Since only in the limit $\frac{\omega_c}{\omega_s} \to \infty$ can the network be represented exactly by a describing function, experimental and analytic results for the network were compared to check the limiting describing function as a practical representation.

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Department of Electrical Engineering

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1.0 Introduction

In servomechanism design, compensation is often necessary to meet system specifications. In open-loop systems, problems may arise in achieving a desired transient response. In closed-loop systems, stability margins may require gain-phase compensation. The compensating networks commonly used are lead-lag networks.

In servomechanisms, information may be transmitted by a dc voltage level or with a modulated carrier signal. For dc lead networks, analytic results are well known. Design specification may include, from the point of view of a transient response to a unit-step input, the value of the output for $t = 0$, the value of the output for $t = \infty$, the decay time constant; or, from stability considerations, the maximum amount of available phase shift, and the frequency at which this maximum occurs. Depending on the particular network, one has two or three independent choices of design parameters. Conventionally, compensation in ac systems has been achieved by either of two methods. The first of these is the passive system, with compensation being achieved by the R-L-C elements. The realization of the ac network is obtained by the usual low-pass-to-band-pass transformations. Similar characteristics may be obtained by employing parallel-T and bridge-T networks. These networks are tuned to the carrier frequency and hence these characteristics are quite sensitive to any variation in this carrier frequency. Generally for these networks, the attenuation at the carrier frequency is severe and the phase characteristics are effective over a very limited bandwidth. The second method of achieving ac compensation requires a demodulator, a dc network, and a modulator. Here the envelope is peak detected, the filtered signal is applied to the dc network and then chopped to reintroduce the carrier frequency. The merits of a network of this type are the independence of carrier frequency and the large amount of phase shift available. The main disadvantage of this network is the presence of a filter which limits the amount of available phase lead and the circuit complexity involved.

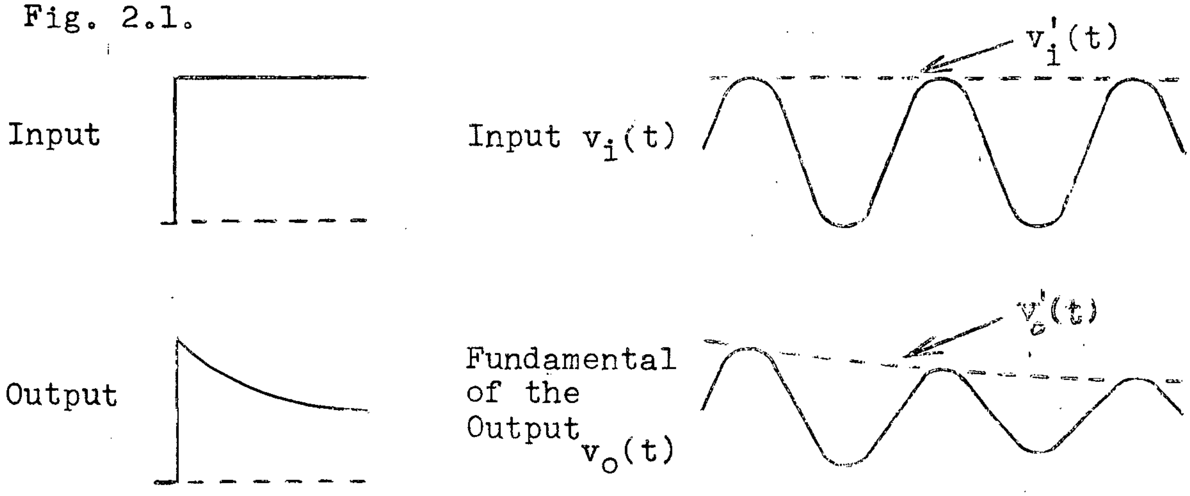
This thesis concerns itself with the detailed study of some demodulator lead-lag networks. These networks have a resemblance to the demodulator-dc-network-modulator-type systems. However they operate on a different principle in that the demodulation operation is not completely carried out and there is no modulator. An analytic technique has been developed based on the study of the transient response to a step input. Parameters describing the transient response have been found and these are used to obtain an equivalent circuit which is valid only for infinite carrier-to-signal-frequency ratio. Previous analytic treatment of various demodulator networks has been superficial because of the difficulty in obtaining equivalent circuits. Using limiting functions, the author has obtained equivalent dc networks. Experimental work was directed towards determining the adequacy of these networks.

2.0 Methods of Analysis

For a non-linear system it is impossible to give a general definition of a transfer function. For certain types of networks, it is convenient to define an equivalent transfer function or describing function of the system. The describing function method uses the fundamental component of the output response to a sinusoidal input signal, to define a transfer function. As the describing function plays the role of the transfer function for the fundamental frequency, a suitable equivalent linear circuit can be obtained.

For demodulator-type networks it is not possible to obtain a describing function which is valid except in the limit as the ratio $\frac{w_c}{w_s} \rightarrow \infty$. The subsequent analysis of these networks relies upon the quasi-stationary nature of the signals. That is, the amplitude of the signals is slowly time-varying but may be assumed constant for a short period of time without appreciable error. In order to estimate the behavior of a particular signal, quasi-stationary Fourier analysis has been applied. Since the theory of these ac lead networks is developed from a transient response viewpoint, consider briefly the transient response of a dc lead network to unit-step input. If a unit-step voltage is applied at the input, the output will suddenly rise to an initial value, followed by a decay to a steady state value as the condenser charges. For an ac lead network, if the input is a suddenly applied unit amplitude sine wave, the envelope of the output will suddenly rise to an initial value followed by decay to steady state value. This is illustrated in

Fig. 2.1.



a. DC Network b. AC Network

Fig. 2.1 Lead Network Response to a Unit-Step Input

Referring to Fig. 2.1, the following parameters are defined which characterize the limiting describing function. These are:

$$a = \frac{v_o^1(0)}{v_o^1(\infty)}, \qquad 2.1$$

$$G = \frac{v_o^1(0)}{v_i^1(0)} \text{ and} \qquad 2.2$$

T, the time constant associated with the response to a step input. $v^1(t)$ represents the envelope of the signal v(t). Then "a" and "T" may be used to define an equivalent dc lead network (see Appendix I). The schematic may be written for the signal fundamental as shown in Fig. 2.2

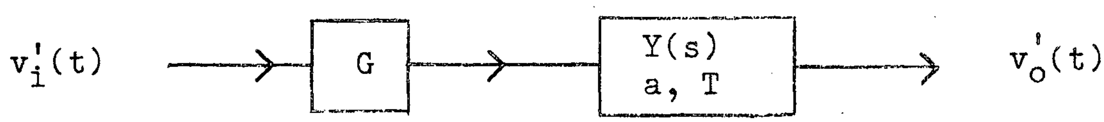


Fig. 2.2 Complete Equivalent Circuit for an AC Lead Network

To account for half or full wave operation and the possible use of transformers, the Transformation Ratio "G" has been introduced.

In order to determine "a" and "T", consider Fig. 2.3, which shows a typical voltage output appearing across a load resistor r_o. The terminals of the condenser are synchronously reversed by a diode bridge so that the current entering C is unidirectional. Both V_o and V_c are taken to be quasi-station-

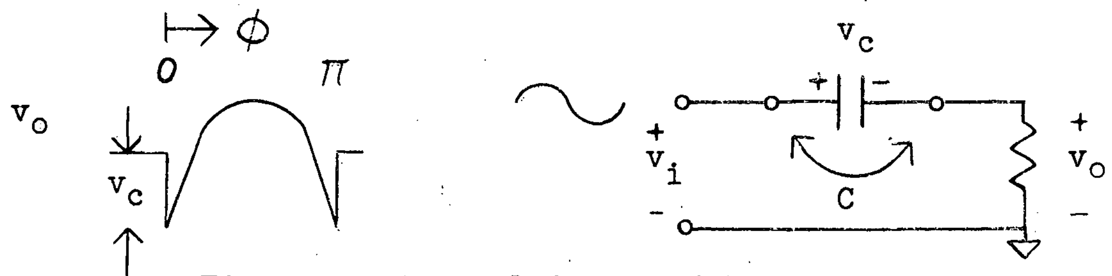


Fig. 2.3 Typical Output Voltage

ary signals. One may write the quasi-stationary average current into the condenser as,

$$\bar{i}(t) = \frac{1}{\pi}\int_0^{\pi} i \, d\phi = \frac{1}{\pi}\int_0^{\pi}\left(-\frac{v_c(t)+\sin\phi}{r_o}\right)d\phi = \frac{1}{\pi}\left(\frac{-\pi v_c}{r_o} + \frac{2}{r_o}\right) \qquad 2.3$$

If the change in v_c per cycle is small, then

$$\Delta v_c = \frac{\bar{i}}{C}\Delta t \qquad : \text{(see Fig. 2.4 below)} \qquad 2.4$$

with $\Delta t = \frac{2\pi}{w_c}$

Rewriting thus: $$\frac{\Delta v_c}{\Delta t} = \frac{\bar{i}}{c} \qquad 2.5$$

It can be seen that

$$\lim_{\frac{w_c}{w_s}\to\infty} \frac{\Delta v_c}{\Delta t} = \frac{dv_c}{dt} \qquad \text{and} \qquad \lim_{\frac{w_c}{w_s}\to\infty} \bar{i} = \frac{-v_c}{r_o} + \frac{2\pi}{r_o} \qquad 2.6$$

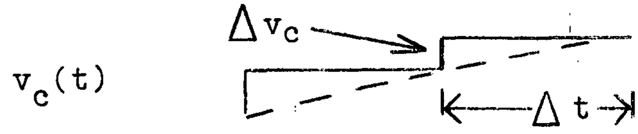


Fig. 2.4 Approximation of Condenser Voltage

The approximate waveform used in the analysis is shown in solid lines, while the actual waveform is shown in dotted lines. The preceeding equations may be solved for v_c giving an exponential function with time constant T. In order to obtain the other two parameters a and G, consider a quasi-stationary Fourier analysis of the output voltage. From Fig. 2.3, it is seen that

$$v_o^1(t) = \frac{1}{\pi}\int_o^{2\pi} v_o \sin\phi \, d\phi = \frac{1}{\pi}\int_o^{2\pi}\left(-v_c+\sin\phi\right)\sin\phi \, d\phi \, .$$

From the definitions of "a" and "G", these parameters are easily obtained. With reference to Fig. 2.2, the complete equivalent circuit has been determined with the aid of the limiting describing function.

For design purposes, it is convenient to have an equivalent dc network resembling as closely as possible the actual circuit. Consider Fig. 2.5. For this circuit one obtains a time constant of $T' = R_eC'$ with $\frac{1}{R_e} = \frac{1}{r_s} + \frac{1}{r_o}$, and the factor $a = \frac{r_s + r_o}{r_o}$

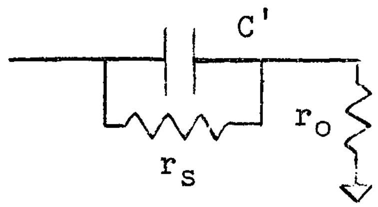


Fig. 2.5 DC Lead Network

The procedure for determining an equivalent circuit is:

(i) Equate load resistors.

(ii) Shunt the condenser with a resistor r_s to account for the attenuation effects.

(iii) With r_s included compute an equivalent resistance R_e through which C' might discharge with time constant T'. To find C', equate T' with the time constant of the ac network.

3.0 Circuit 1.

The first demodulator-lead network to be discussed is the 6-diode gate. A complete description may be found in Appendix II, Reference 1. Briefly, with reference to Fig. 3.2, the control voltage VR blocks the diodes D1 and D2 so that current is passed through the network. The input voltage will then appear at the output. For a better understanding of the operation, the input, output and control waveforms are shown in Fig. 3.1

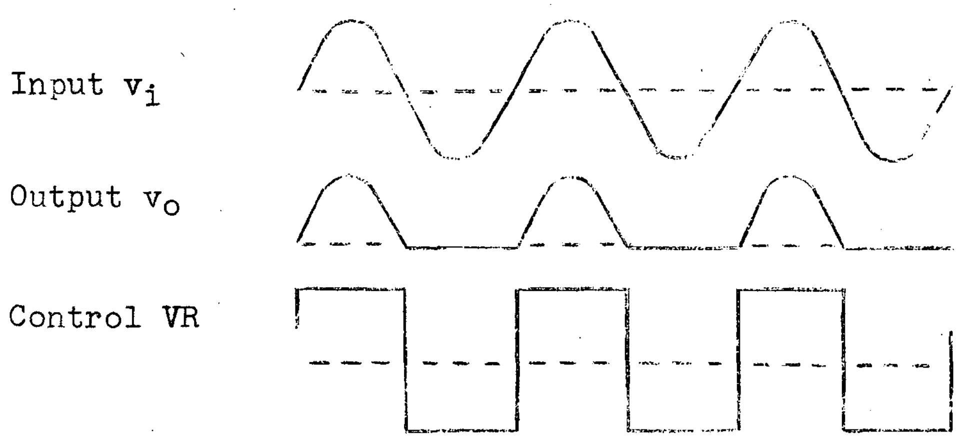
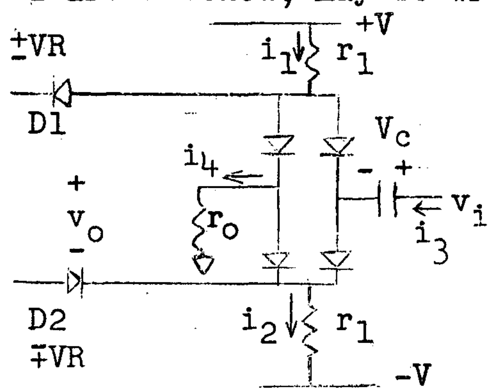


Fig. 3.1 Voltage Waveforms

Consider the circuit shown in Fig. 3.2. The following equations, valid only during the period that D1 and D2 are blocked, may be written for Fig. 3.2:



$$i_1 = \frac{V - v_o}{r_1} \qquad 3.1$$

$$i_2 = \frac{v_o + V}{r_1} \qquad 3.2$$

$$i_2 = i_1 + i_3 \cdot i_4 \qquad 3.3$$

Subtracting 3.1 from 3.2 yields

$$i_2 - i_1 = \frac{2v_o}{r_1} \qquad 3.4$$

Fig. 3.2
Schematic for the Definition of the Quantities to be used in the Analysis.

cont'd.

Substituting 3.4 into 3.3 gives

$$i_3 = i_4 + \frac{2V_o}{r_1} \qquad 3.5$$

Also $$i_4 = \frac{v_o}{r_o} \qquad 3.6$$

Substituting 3.6 into 3.5 yields

$$i_3 = v_o\left(\frac{1}{r_o} + \frac{2}{r_1}\right) = v_o\frac{1}{R} \qquad 3.7$$

with $$\frac{1}{R} = \frac{1}{r_o} + \frac{2}{r_1} \qquad 3.8$$

Then $$v_o = -v_c + v_i \qquad 3.9$$

The problem is to determine v_c as a function of time with the condition $v_c(0) = 0$ and a unit step applied at the input. Consider the circuit shown in Fig. 3.3

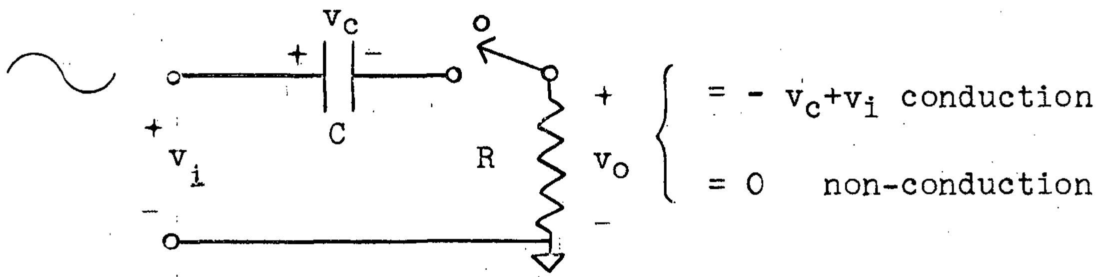


Fig. 3.3 Circuit Operation

The output voltage waveform v_o at any time t is shown in Fig. 3.4

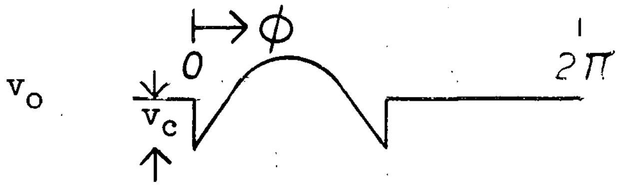


Fig. 3.4 Output Voltage Waveform.

Then according to section 2.0, one may write the quasi-stationary average current into the condenser as

$$i(t) = \frac{1}{2\pi}\int_0^{2\pi} i_3 d\emptyset = \frac{1}{2\pi}\int_0^{2\pi}\frac{v_o}{R}\, d\emptyset = \frac{1}{2\pi}\int^{\pi}\left[\frac{-v_c(t)+\sin\emptyset}{R}\right]d\emptyset = \frac{-v_c(t)}{2R} + \frac{1}{\pi R} \qquad 3.10$$

The differential equation for the condenser voltage is

$$\frac{dv_c}{dt} = \frac{\bar{i}}{C} = \frac{1}{2RC}\left(-v_c(t)+\frac{2}{\pi}\right) \qquad 3.11$$

The solution of 3.11, with the condition $v_c(t=0)=0$, is

$$v_c = \frac{2}{\pi}\left(1-\exp\left(\frac{-t}{2RC}\right)\right) \qquad 3.12$$

In order to obtain the parameters "a" and "G" mentioned in section 2.0, consider the fundamental of the output voltage by applying Fourier analysis to the quasi-stationary signal.

$$v_o'(t) = \frac{1}{\pi}\int_0^{2\pi} v_o \sin\emptyset \, d\emptyset = \frac{1}{\pi}\int_0^{\pi} (-v_c+\sin\emptyset)\sin\emptyset \, d\emptyset = \frac{-2V_c}{\pi} + \frac{1}{2} \qquad 3.13$$

Then one obtains,

$$a = \frac{v_o'(0)}{v_o'(\infty)} = \frac{\frac{1}{2}}{\frac{1}{2}-\frac{4}{\pi^2}} = 5.3 \quad \text{and } G = \frac{v_o'(0)}{v_i'(0)} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

The time constant of the lead network is T = 2RC, which is apparent from 3.12.

For design purposes, it is convenient to have as similar a network configuration as possible to the ac network. From the procedure outlined in section 2.0, the circuit of Fig. 3.5 may be used. For this circuit the time constant is $T' = R_eC'$ with $\frac{1}{R_e} = \frac{1}{R} + \frac{1}{R_s}$ and the parameter a is $\frac{R + R_s}{R}$.

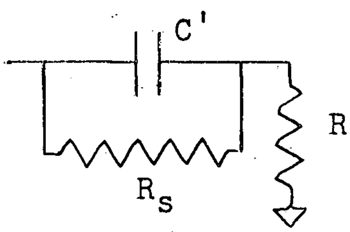


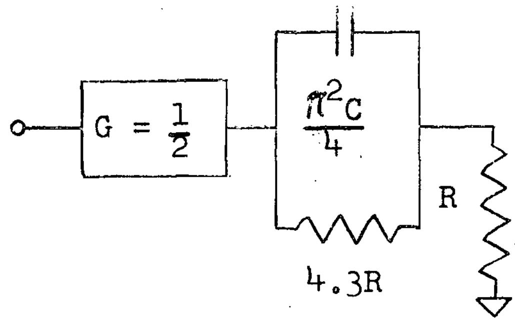
Fig. 3.5 DC Lead Network

Equating parameters, one obtains $\frac{R}{R+R_s} = 1 - \frac{8}{\pi^2}$

or $R_s = 4.3\ R$

Equating time constants it is found that $C' = C\,\frac{\pi^2}{4}$

Then the following equivalent dc lead network may be drawn as shown in Fig. 3.6



$$Y(s) = \frac{1}{a}\left(\frac{1 + aTs}{1 + Ts}\right)$$

$a = 5.3$

$T = 2RC$

Fig. 3.6 Equivalent DC Lead Network

4.0 Circuit 2.

The first published reference to demodulator-lead networks is a brief description given by Diprose (Appendix II, Reference 2). The network is shown below.

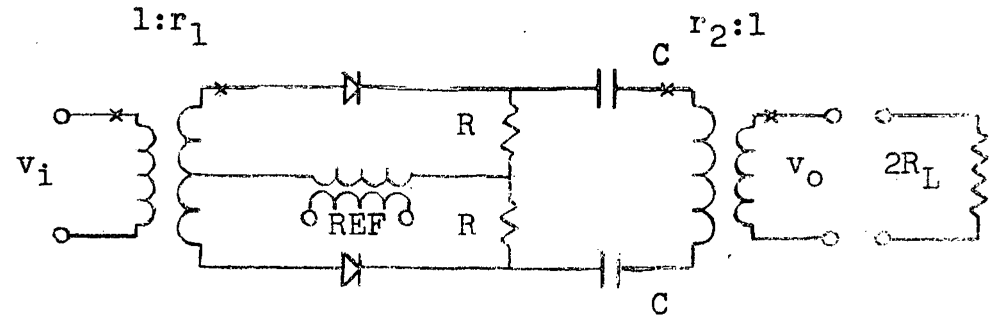


Fig. 4.1 Diprose's Circuit.

The assumptions for network operation are: the voltage level of the reference signal (REF) alone determines whether the diodes conduct or not; and, ideal transformers and ideal diodes are employed. If the turns ratios are conveniently chosen as indicated one may write the following ac equivalent circuit, Fig. 4.2.

with $r_1 = 2$
$r_2 = 1$

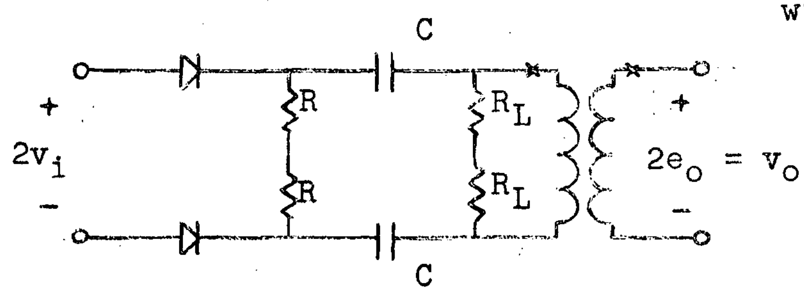


Fig. 4.2 Transformer Equivalent Circuit.

For ease in analysis consider the unbalanced equivalent circuit under various operating conditions. The network has two operating states determined by whether the diodes are conducting or non-conducting. Consider the circuit with the diodes non-conducting and an arbitrary voltage v_c on the condenser as shown in Fig. 4.3.

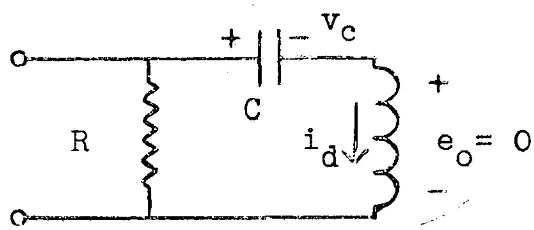


Fig. 4.3 Circuit for Diodes Non-conducting.

As v_c is slowly varying, the output primary terminals are essentially a short circuit and hence the current flowing is given by,

$$i_d = \frac{-v_c}{R} \qquad 4.1$$

The inductance of the primary of the output transformer is assumed to be so large that this current remains unchanged during this conduction cycle of the diodes. The voltage $-v_c$ must now appear across the output.

Fig. 4.4 shows the equivalent circuit when the diodes are conducting.

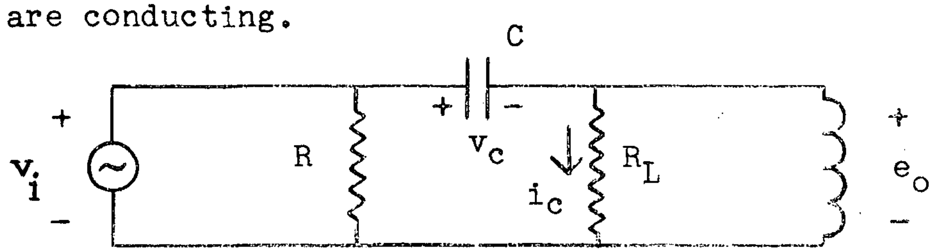


Fig. 4.4 Circuit for Diodes Conducting.

Then clearly,

$$v_i = i_c R_L + v_c \qquad 4.2$$

Hence

$$i_c = \begin{cases} \dfrac{e_i - v_c}{R_L} & \text{Diodes conducting} \\ o & \text{Diodes non-conducting} \end{cases} \qquad 4.3$$

Now $i = i_c + i_d$ 4.4

Then as explained in section 2.0, one may write

$$\frac{dv_c}{dt} = \frac{\bar{i}}{C} \qquad 4.5$$

With the averages taken over the appropriate range, $\bar{i}$ may be found,

$$\bar{i} = \frac{1}{2\pi}\int_0^{2\pi} i \, d\phi = \frac{1}{2\pi}\left[\int_0^{\pi} i_c \, d\phi + \int_0^{2\pi} i_d \, d\phi\right] \qquad 4.6$$

with $\phi = wt$

Substituting and integrating yields,

$$\bar{i} = -v_c\left(\frac{1}{2R_L} + \frac{1}{R}\right) + \frac{1}{\pi R_L} \qquad 4.7$$

Now let $\frac{1}{R_e} = \frac{1}{2R_L} + \frac{1}{R}$ 4.8

and $T = R_e C$ 4.9

The solution of 4.5, with the initial condition that $v_c=0$ for $t=0$, is:

$$v_c = \frac{1}{\pi}\frac{R_e}{R_L}\left(1-\exp\frac{-t}{R_e C}\right) \qquad 4.10$$

Writing the output voltage v_o as a quasi-stationary Fourier series and solving for the amplitude of the fundamental v_o', one obtains, as $v_o = 2e_o$,

$$v_o'(t) = 1-\frac{4v_c}{\pi} \qquad 4.11$$

A lead network may be described by parameters which may be obtained from the response to a unit-step input. These parameters are "a", "T" and "G". Using equation 4.1 and equation 4.2 one obtains:

$$T = R_e C$$

$$a = \frac{v_o'(t=0)}{v_o'(\infty)} = \frac{1}{1-\frac{4R}{\pi^2(2R_L+R)}}$$

$$G = \frac{v_o'(t=0)}{v_i'(0)} = \frac{1}{1} = 1$$

Consider the circuit shown in Fig. 4.5

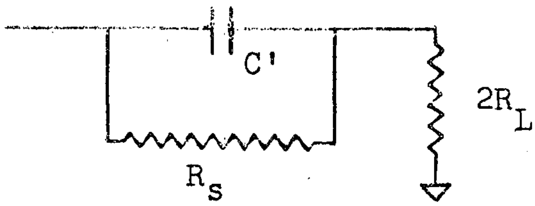


Fig. 4.5 Equivalent Circuit.

Following the procedures outlined at the end of section 2.0, one obtains from (ii),

$$1-\frac{4}{\pi^2}\frac{R}{(2R_L+R)} = \frac{2R_L}{R_s + 2RL}$$

Solving for $\frac{1}{R_s}$ gives: $\frac{1}{R_s} = \frac{\pi^2-4}{8R_L} + \frac{\pi^2}{4R}$

From (iii), it is apparent

$$\frac{1}{R_e'} = \frac{1}{R_s} + \frac{1}{2R_L}$$

Hence equating time constants, yields

$$\frac{C}{R_e'} = \frac{C'}{R_e}$$

$$C' = \frac{C\pi^2}{4}$$

The equivalent circuit is shown in Fig. 4.6

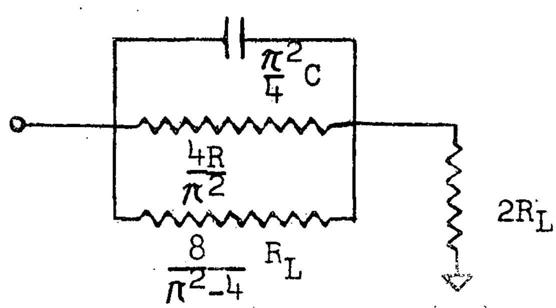


Fig. 4.6 Equivalent Circuit for Diprose's Network.

Then $$a = \frac{1}{1 - \frac{4R}{\pi^2(2R_L+R)}}$$

and $$T = \frac{2R_L R}{2R_L+R}\, C$$

with $$Y(s) = \frac{1}{a}\,\frac{(1 + aTs)}{(1 + Ts)}$$

5.0 Circuit 3.

Continuing the study of ac lead networks one finds in Appendix II, Reference 3 the following circuit:

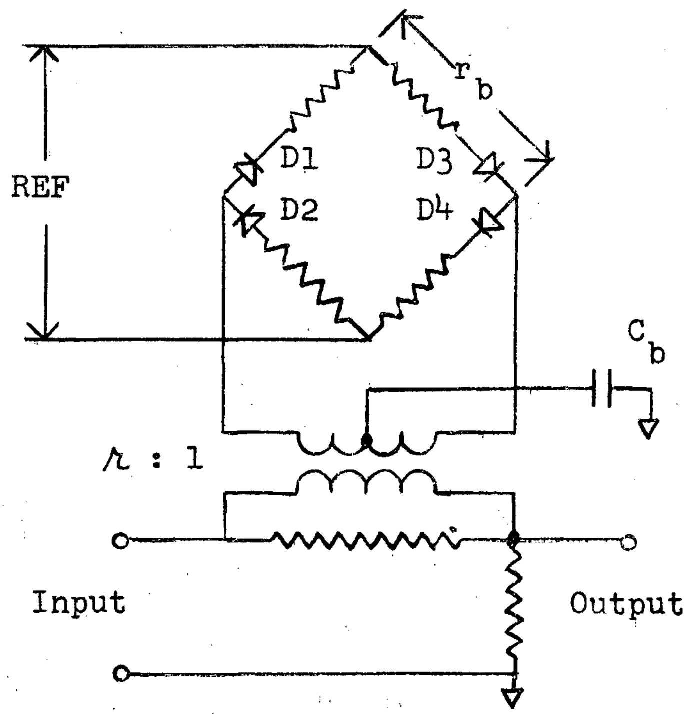


Fig. 5.1 Schematic of Lyons' Network

The sinusoidal input is in synchronism with the reference voltage which controls the diode bridge. It is assumed that the voltage level of the signal is small compared to reference voltage.

In Fig. 5.1, consider the state of network when D1 and D2 are blocked. For an input signal related to the secondary the following circuit applies:

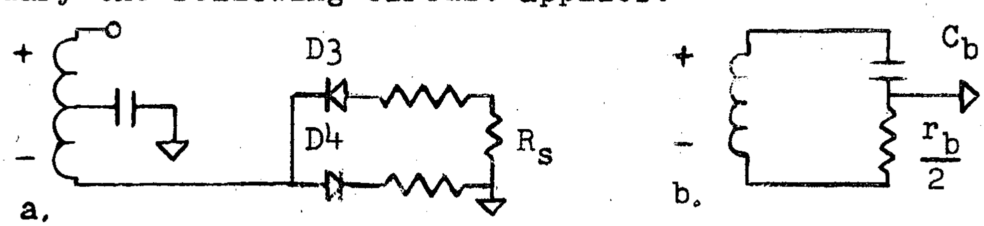


Fig. 5.2 Signal Circuit of the Secondary for State 1.

Neglecting the source impedance, the circuit of Fig. 5.2 (a) may be reduced to Fig. 5.2 (b).

Next consider the state of network when D3 and D4 are non-conducting. Then the input signal related to the secondary sees the following circuit.

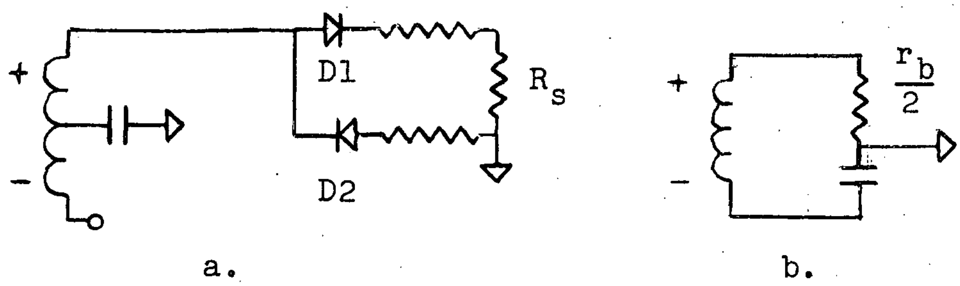


Fig. 5.3 Signal Circuit of the Secondary for State 2.

Using similar assumptions as before this circuit may be reduced as shown. Then for either state of the network the same circuit is applicable for the input signal. Referring circuit components to the primary, one obtains, with the understanding that the condenser must be switched every half cycle, the circuit of Fig. 5.4:

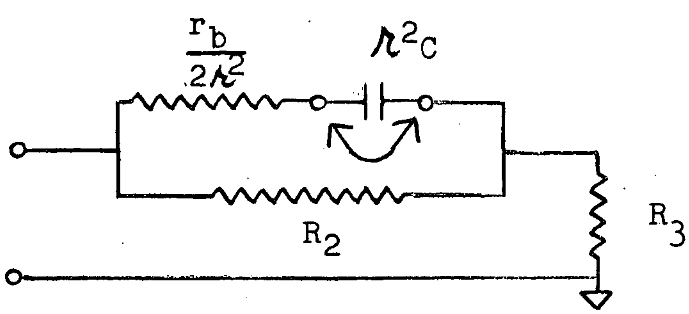


Fig. 5.4 Half-Cycle Equivalent Circuit.

The output response to a unit step input is shown in Fig. 5.5.

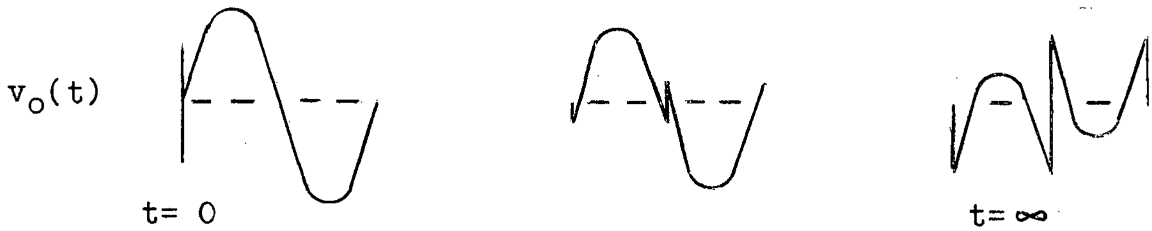


Fig. 5.5 Output Voltage Waveform.

If there is an arbitrary voltage v_c on the condenser, as shown in Fig. 5.6, which is slowly varying, one may write:

$$i_d = +v_c \frac{2}{r_b} \qquad 5.1$$

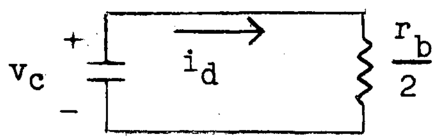


Fig. 5.6 DC Discharge Circuit.

The transformer turns ratio is conveniently chosen so that a ratio of 1:1 exists between the primary and one-half of the secondary. Then the circuit of Fig. 5.7 may be used in the analysis of ac signals.

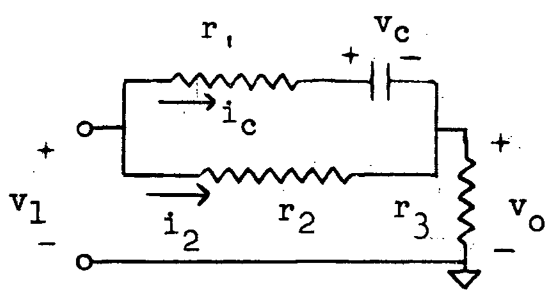


Fig. 5.7 AC Signal Circuit.

With reference to Fig. 5.4

$r_1 = \frac{r_b}{2}$; $r_2 = R_2$

$r_3 = R_3$; $C = C_b$

Let v_1, v_o and v_c refer to input, output and condenser voltages, respectively, as indicated. The following equations can be written:

$$v_1 = r_1 i_c + v_c + (i_c + i_2)\, r_3 \qquad 5.2$$

$$v_1 = r_2 i_2 + (i_c + i_2)\, r_3 \qquad 5.3$$

$$v_o = (i_c + i_2)\, r_3 \qquad 5.4$$

Eliminating i_2 from equations 5.2 and 5.3 gives

$$v_1 - v_c = i_c(r_1 + r_2) + \frac{r_3}{r_3 + r_2}(v_1 - i_c r_3) \qquad 5.5$$

According to section 2.0, it is possible to write the following differential equation,

$$\frac{dv_c}{dt} = \frac{\bar{i}}{C} \qquad 5.6$$

where $\bar{i} = \frac{1}{\pi}\int_0^{\pi} i \, d\phi$ and $i = i_c - i_d$. To compute $\bar{i}$, take a time average of 5.1 and 5.5, noting that $v_1 = \sin \phi$ and v_c is slowly varying. Now define the following equivalent resistances,

$$R_1 = r_1 + \frac{r_2 r_3}{r_2 + r_3} \qquad 5.7$$

and

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{r_1} \qquad 5.8$$

Then

$$\bar{i} = -\frac{v_c}{R} + \frac{2}{\pi}\frac{r_2}{(r_2 + r_3)}\frac{1}{R_1} \qquad 5.9$$

Substituting 5.9 into 5.6 gives,

$$\frac{dv_c}{dt} = \frac{1}{RC}\left[-v_c + \frac{2}{\pi}\frac{r_2}{(r_2 + r_3)}\frac{R}{R_1}\right] \qquad 5.10$$

Solving this differential equation gives, with condition $v_c(0)=0$,

$$v_c(t) = \frac{2}{\pi}\frac{r_2 R}{(r_2 + r_3)\, R_1}\left[1 - \exp\left(\frac{-t}{RC}\right)\right] \qquad 5.11$$

To obtain the network parameters, consider the envelope $v_o'(t)$ of the output $v_o(t)$. With 5.2, 5.3 and 5.4, one may obtain,

$$v_o\left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right] = v_i\left(\frac{1}{r_1} + \frac{1}{r_2}\right) - \frac{v_c}{r_1} \qquad 5.12$$

In order to examine the output fundamental, apply Fourier analysis to 5.12.

$$v_o'(t) = \frac{r_3(r_1 + r_2)}{r_1r_2 + r_1r_3 + r_2r_3} - \frac{4}{\pi}\frac{r_2r_3}{r_1r_2 + r_1r_3 + r_2r_3} v_c$$

Substituting for v_c with 5.11 yields,

$$v_o'(t) = \frac{r_3(r_1 + r_2)}{r_1r_2 + r_1r_3 + r_2r_3} - \frac{8}{\pi^2}\frac{r_2^2r_3}{(r_2 + r_3)^2}\frac{R}{(R_1)^2} \times$$

$$\left(1 - \exp\frac{-t}{RC}\right) \qquad 5.13$$

Then according to the definitions of section 2.0, one has

$$\frac{G}{a} = \frac{r_3(r_1+r_2)}{r_1r_2+r_1r_3+r_2r_3} - \frac{8}{\pi^2}\frac{r_2^2r_3}{(r_2+r_3)^2}\frac{R}{(R_1)^2} \qquad 5.14$$

$$G = \frac{r_3(r_1r_2)}{r_1r_2+r_1r_3+r_2r_3} \qquad 5.15$$

To determine an equivalent circuit by the procedure of section 2.0, consider Fig. 5.8.

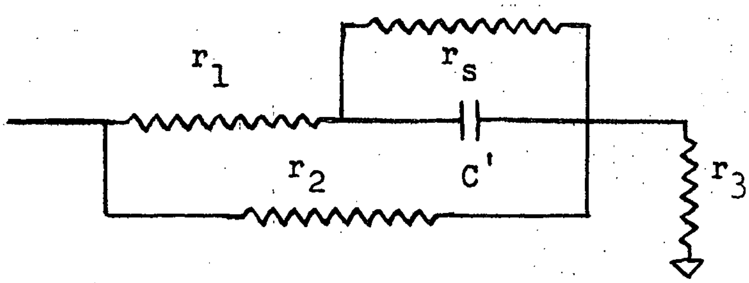


Fig. 5.8 Equivalent Circuit for Lyons' Network.

To account for the attenuation effects, the resistor r_s is introduced into network configuration. To find r_s, the following relations are obtained and solved:

$$\frac{r_3(r_1+r_2)}{r_1r_2+r_1r_3+r_2r_3} - \frac{8}{\pi^2}\frac{r_2^2 r_3 R}{(r_2+r_3)^2 (R_1)^2} = \frac{r_3(r_1+r_2+r_s)}{r_1r_2+r_1r_3+r_2r_3+r_s(r_2+r_3)}$$

The above equation gives $r_s = \frac{8}{\pi^2} r_1 \dfrac{1}{2-\frac{8}{\pi^2} - (1+\frac{8}{\pi^2})\frac{r_2r_3}{r_1r_2+r_1r_3+r_2r_3}}$

To evaluate the equivalent condenser compute the discharge resistance R_e of Fig. 5.8 and then obtain $T_e = R_eC' = RC$. From this procedure one obtains,

$$C' = C(1 + \frac{R}{r_s})$$

For the circuit of Fig. 5.8 one has

$$Y(s) = \frac{G}{a}\frac{1 + aTs}{1 + Ts}$$

with $T = RC$, from 5.13

$\frac{1}{a}$, from 5.14

G, from 5.15

6.0 Circuit 4.

Fig. 6.1 shows the circuit diagram of a demodulator lead network mentioned in Appendix II, Reference 4.

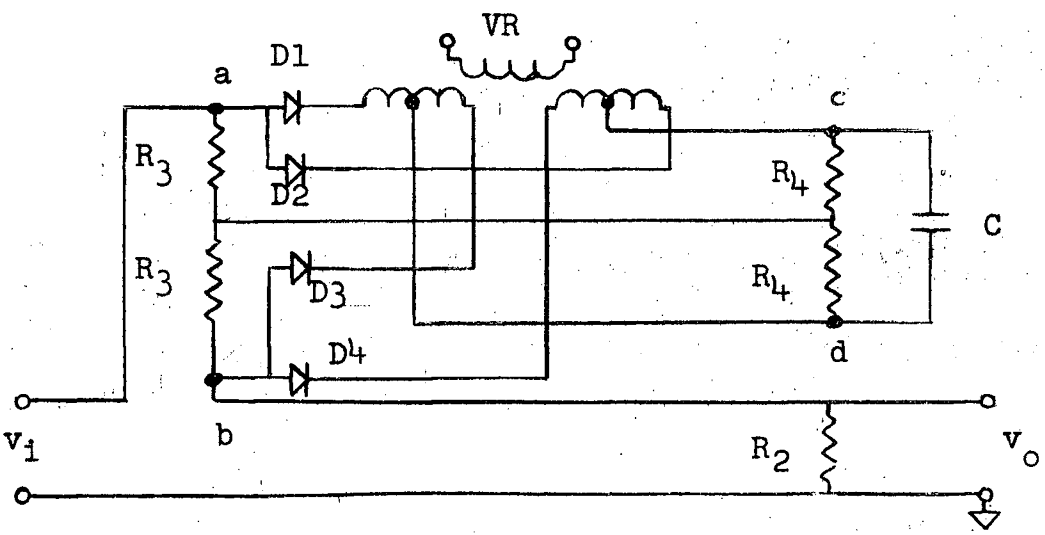


Fig. 6.1 Weiss and Levenstein's Network.

The following assumptions are made in the analysis: (1) ideal diodes are employed, (2) transformers are ideal, (3) the reference voltage controls the diode bridge. This synchronously switches the terminals cd and ab so that the current into the condenser is unidirectional. For a constant amplitude sine input a state of equilibrium is eventually reached whereby the charge added during a cycle is exactly balanced by discharge of the condenser through various resistors. Hence the following waveforms, as shown in Fig. 6.2, occur at the output for this ac step input.

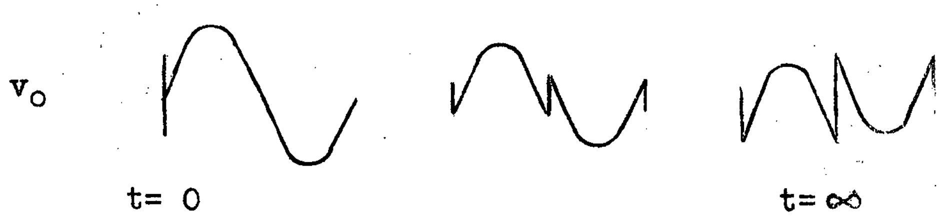


Fig. 6.2 Output Voltage Waveforms.

Then the following circuit in Fig. 6.3 may be considered for analysis with the understanding that the condenser is to be switched every half cycle.

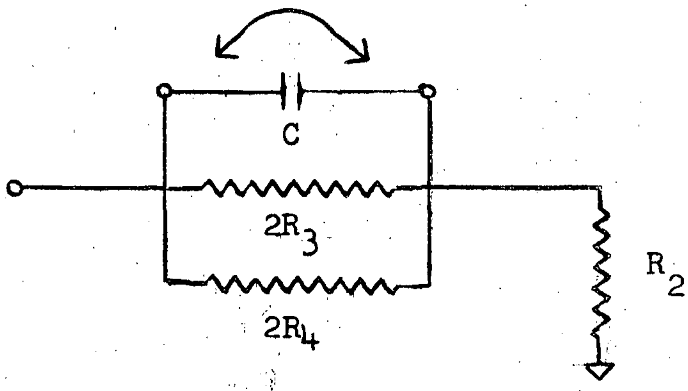
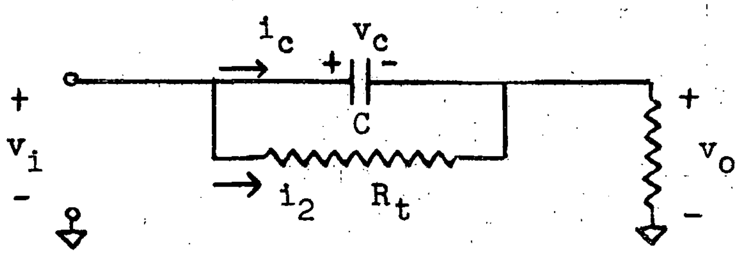


Fig. 6.3 Half-Cycle Equivalent Circuit.

For analysis consider the circuit shown in Fig. 6.4.



$$\frac{1}{R_t} = \frac{1}{2R_3} + \frac{1}{2R_4}$$

Fig. 6.4 AC Charge Circuit.

The following equations may be written:

$$v_i = v_c + (i_c + i_2)\,R_2 \qquad 6.1$$

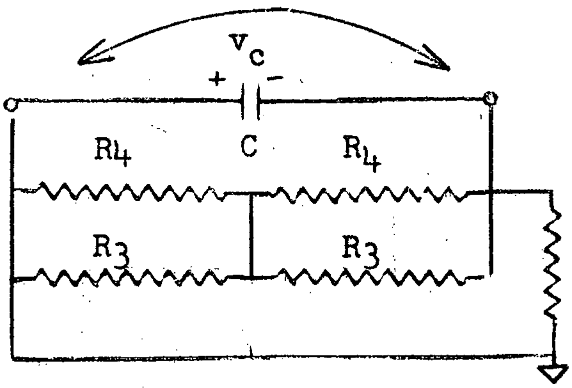
$$v_i = i_2 R_t + (i_c + i_2)\,R_2 \qquad 6.2$$

$$v_o = (i_c + i_2)\,R_2 \qquad 6.3$$

Eliminating i_2 from 6.1 and 6.2 gives,

$$v_i - v_c = i_c R_2 + \frac{R_2(v_i - i_c R_2)}{Rt + R2} \qquad 6.4$$

Consider Fig. 6.5 for the circuit describing the dc discharge. From the description of operation it is apparent that for a voltage v_c on the condenser, discharge will occur through the equivalent resistance R.



$$\frac{1}{R} = \frac{1}{2R_4} + \frac{1}{2R_3} + \frac{1}{R_2}$$

Fig. 6.5 DC Discharge Circuit.

As v_c is slowly varying, one has

$$i_d = \frac{-v_c}{R} \qquad 6.5$$

Now according to section 2.0, it is possible to write the following differential equation,

$$\frac{dv_c}{dt} = \frac{\bar{i}}{c} \qquad 6.6$$

where $\bar{i} = \frac{1}{\pi}\int_0^{2\pi} i.d\phi$, and $i = i_c + i_d$. To compute $\bar{i}$, take the time average of 6.4 and 6.5. By solving these equations for $\bar{i}_c$ and $\bar{i}_d$, one obtains,

$$\bar{i} = \left[-2V_c + \frac{2}{\pi}\frac{Rt}{R_2+R_t}\right]\frac{1}{R} \qquad 6.7$$

Substituting 6.7 into 6.6 yields,

$$\frac{dv_c}{dt} = \frac{1}{RC}\left[-2v_c + \frac{2}{\pi}\frac{R_t}{R_2+R_t}\right] \qquad 6.8$$

Solving this differential equation gives, with the initial condition that $v_c(0) = 0$,

$$v_c(t) = \frac{1}{\pi}\frac{R_t}{R_t+R_2}\left(1-\exp\frac{-2t}{RC}\right) \qquad 6.9$$

If $v_o'(t)$ is the fundamental of the output, then by applying Fourier analysis to the output with the assumption that v_c is slowly varying, one obtains,

$$\tfrac{1}{2}v_o'(t) = \frac{1}{\pi}\int_0^{\pi} v_o \sin\phi \, d\phi = \frac{1}{\pi}\int_0^{\pi}(-v_c+\sin\phi)\sin\phi \, d\phi \qquad 6.10$$

Hence $\quad v_o'(t) = 1 - \dfrac{4v_c}{\pi}$ $\qquad$ 6.11

Using the definitions of a and G as given in section 2.0, one finds,

$$a = \frac{v_o'(0)}{v_o'(\infty)} = \frac{1}{1-\frac{4}{\pi^2}\frac{R_t}{R_t+R_2}} \qquad 6.12$$

$$G = \frac{v_o'(0)}{v_i'(0)} = \frac{1}{1} = 1 \qquad 6.13$$

In section 2.0, a procedure for determining an equivalent circuit has been outlined. Consider the circuit of Fig. 6.6.

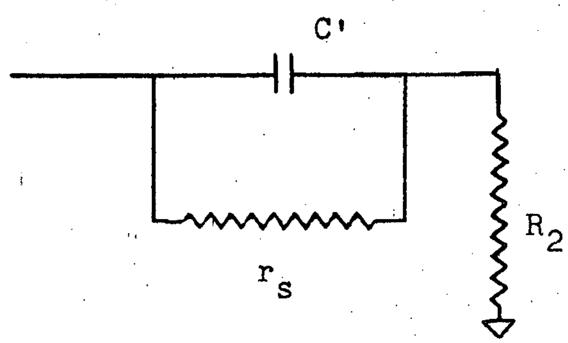


Fig. 6.6 Equivalent Circuit.

To account for attenuation effects and employing equation 6.12, one obtains,

$$1 - \frac{4R_t}{\pi^2(R_t+R_2)} = \frac{R_2}{r_s+R_2} \qquad 6.14$$

Solving for $\frac{1}{r_s}$ gives, $\dfrac{1}{r_s} = \dfrac{\pi^2}{4R_t} + \left(\dfrac{\pi^2}{4} - 1\right)\dfrac{1}{R_2}$ $\qquad$ 6.15

Associate r_e with the time constant T_e of the equivalent circuit, i.e., $T_e = r_e C'$. By inspection one obtains,

$$\frac{1}{r_e} = \frac{1}{r_s} + \frac{1}{R_2} \qquad 6.16$$

Substituting 6.15 into 6.16 gives,

$$\frac{1}{r_e} = \frac{\pi^2}{4}\left(\frac{1}{R_t} + \frac{1}{R_2}\right) \qquad 6.17$$

Equating time constants yields, with the aid of Fig. 6.6 and equation 6.9,

$$\frac{C'}{R} = \frac{C}{2r_e} \qquad 6.18$$

Substituting 6.17 into 6.18, one obtains,

$$C' = C\frac{\pi^2}{8} \qquad 6.19$$

Hence it is apparent that an equivalent circuit is given by Fig. 6.7.

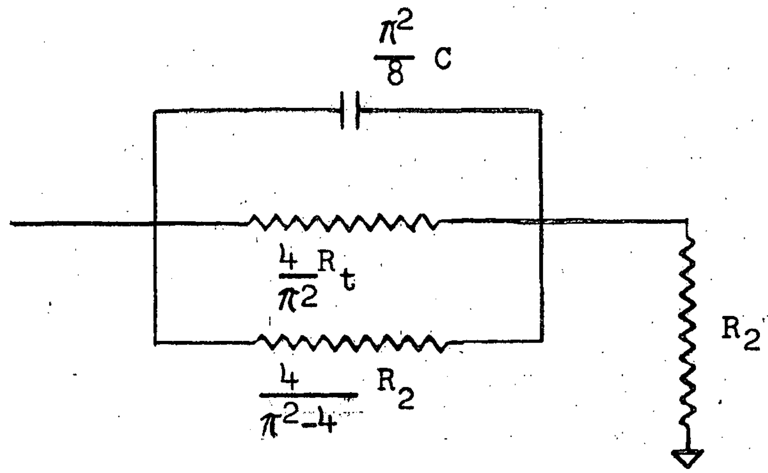


Fig. 6.7 DC Equivalent Lead Network.

Then $\quad Y(s) = \frac{1}{a}\frac{1 + aTs}{1 + Ts}$

with $\quad a = \dfrac{1}{1-\dfrac{4R_t}{\pi^2(R_2+R_t)}}$

$$T = \frac{RC}{2}$$

7.0 Circuit 5.

One finds in Appendix II, Reference 3, the network of Fig. 7.1. It will be shown that this network is an ac lag network.

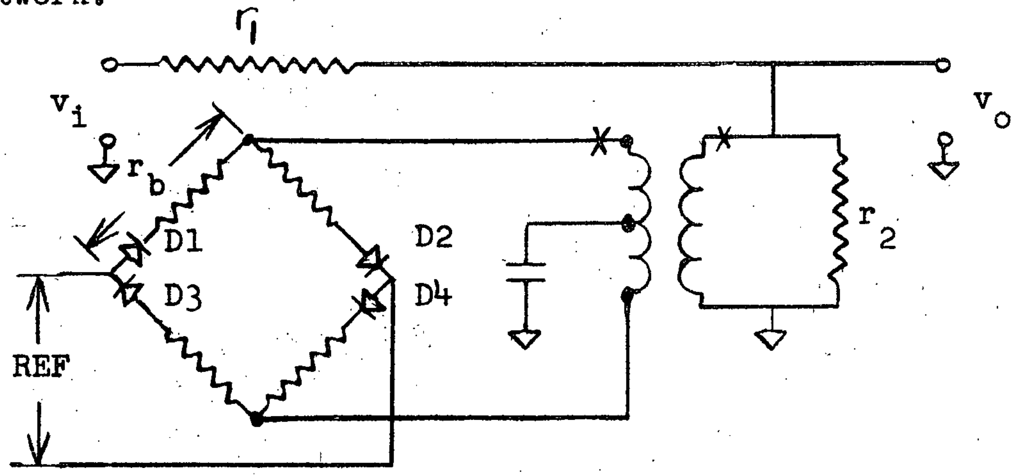


Fig. 7.1 Schematic of Lyons' Lag Network.

The operation of a similar type network has been described in section 5.0.

Consider the state of the network when D1 and D2 are conducting. Then the charge circuit for the condenser is shown in Fig. 7.2.

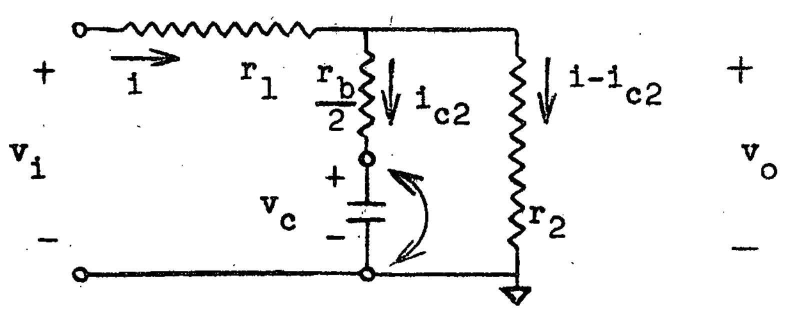


Fig. 7.2 Condenser Charge Circuit.

The diode bridge synchronously reverses the terminals of the condenser so that the current entering is unidirectional.

The following equations may be written:

$$\frac{v_i - v_o}{r_1} = i \qquad 7.1$$

$$v_o - v_c = \frac{i_{c2}\, r_b}{2} \qquad 7.2$$

$$v_o = (i - i_{c2})\, r_2 \qquad 7.3$$

The equations may be solved for i_{c2} in terms of v_i and v_c giving,

$$i_{c2} = Av_i - Bv_c \qquad 7.4$$

with $$A = \frac{2r_2}{2r_1r_2 + (r_1+r_2)r_b}$$

$$B = \frac{2(r_1+r_2)}{2r_1r_2+(r_1+r_2)r_b}$$

Next solving equations 7.1, 7.2, and 7.3 for v_o in terms of v_i and v_c gives,

$$v_o = A_1v_i + B_1v_c \qquad 7.5$$

with $$A_1 = \frac{r_2r_b}{r_1r_b + 2r_1r_2 + r_2r_b}$$

$$B_1 = \frac{2r_1r_2}{r_1r_b + 2r_1r_2 + r_2r_b}$$

The discharge circuit of the condenser is shown in Fig. 7.3 where it is assumed that inductances are negligible for the quasi-stationary type discharge.

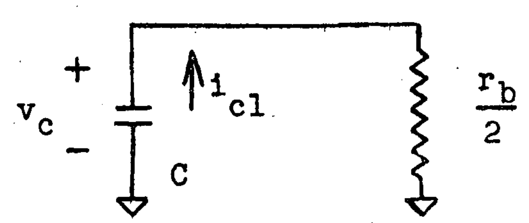


Fig. 7.3 Condenser Discharge Circuit.

Then

$$i_{c1} = \frac{2v_c}{r_b} \qquad 7.6$$

To obtain v_c as a function of time, one may write according to section 2.0,

$$\frac{dv_c}{dt} = \frac{\bar{I}}{C}$$

where $\bar{I}$ is the average net charging current.

Then $$\bar{I} = \bar{I}_{c2} - \bar{I}_{c1} \qquad 7.7$$

Now $\bar{I}_{c2}$ may be computed from 7.4 where v_c is slowly varying and $v_i = \sin\emptyset$, hence

$$\bar{I} = \frac{2A}{\pi} - Bv_c - \frac{2v_c}{r_b}$$

The differential equation becomes

$$\frac{dv_c}{dt} + \frac{1}{T} v_c = \frac{2A}{\pi C} \qquad 7.8$$

with $$\frac{1}{T} = \frac{1}{C}\left(B + \frac{2}{r_b}\right) \qquad 7.9$$

Equation 7.8 may be solved with the condition $v_c(0) = 0$, giving

$$v_c(t) = \frac{2AT}{\pi C}\left(1-\exp\left(\frac{-t}{T}\right)\right) \qquad 7.10$$

To obtain the envelope of the output apply Fourier analysis to the quasi-stationary output signal in 7.5.

$$\frac{1}{2} v_o'(t) = \frac{1}{\pi}\int_0^{\pi} (A_1\sin\emptyset + B_1v_c)\sin\emptyset \, d\emptyset$$

Hence $$v_o'(t) = A_1 + \frac{4B_1}{\pi} v_c(t) \qquad 7.11$$

Then from the definitions of a and G one obtains

$$\frac{1}{a} = \frac{8 \; AB_1T}{\pi^2 \; A_1C} \; ; \text{ and } G = A_1 \; .$$

Following the procedure outlined in section 2.0, the circuit shown in Fig. 7.4 is obtained.

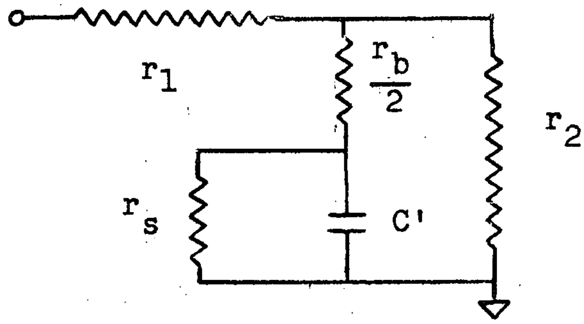


Fig. 7.4 DC Equivalent Circuit.

To account for the steady-state attenuation of a dc step input, one finds, with the aid of 7.11, that

$$r_s = \frac{8K}{\pi^2-8K}\left(R_1+\frac{r_b}{2}\right) \qquad 7.12$$

with $\qquad R_1 = \dfrac{r_1 r_2}{r_1+r_2}$

and $\qquad \dfrac{1}{K} = 2\left(1+\dfrac{R_1}{r_b}\right)$

To obtain C' it is necessary to compute the discharge resistance of Fig. 7.4.

$$\frac{1}{R_e} = \frac{1}{r_s} + \frac{1}{\frac{r_b}{2}+R_1} = \frac{1}{r_s} + B$$

As the time constant is given by 7.9, the following relation holds

$$C' = C\frac{1}{R_e\left(B+\frac{2}{r_b}\right)} = CK\left(1+\frac{r_b}{2r_s} + \frac{R_1}{r_s}\right) \qquad 7.13$$

8.1 Variable Conduction Angle

By varying the conduction angle of the diode bridge it is possible to vary a, t and G. By suitable arrangement of the control voltages of Fig. 3.2, the output voltage will appear as shown in Fig. 8.1, where the conduction angle is $\pi - 2\alpha$.

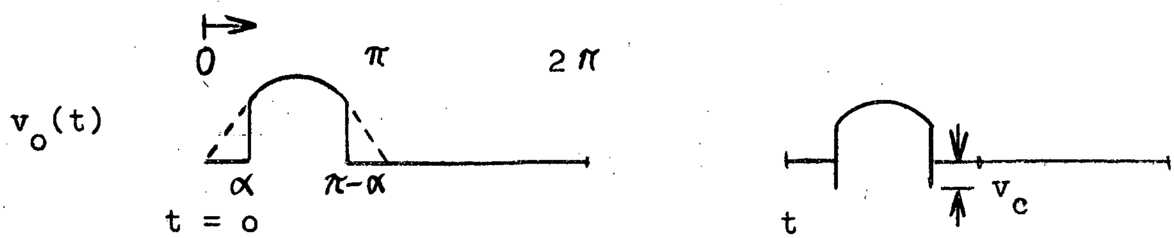


Fig. 8.1 Voltage Output for a Variable Conduction Angle.

During conduction the circuit is as shown in Fig. 8.2.

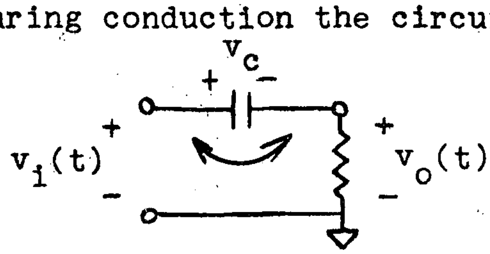


Fig. 8.2 Conduction Circuit.

Then by section 2.0, one may write

$$\frac{dv_c}{dt} = \frac{I}{C}$$

Now it is apparent that

$$I = \frac{1}{2\pi R}\int_{\alpha}^{\pi-\alpha} (-v_c + \sin\emptyset)\, d\emptyset = \frac{1}{2\pi R}\Big[-v_c(\pi-2\alpha) + 2\cos\alpha\Big] \qquad 8.1$$

Substituting into the differential equation and solving yields, with $v_c(0) = 0$,

$$v_c(t) = \frac{2\cos\alpha}{\pi-2\alpha}\left(1-\exp\left[\frac{-(\pi-2\alpha)t}{2RC\,\pi}\right]\right) \qquad 8.2$$

Now applying Fourier analysis to the quasi-stationary output,

$$v_o'(t) = \frac{1}{\pi}\int_0^{2\pi} v_o(t)\ \sin\phi\ d\phi = \frac{1}{\pi}\int_\alpha^{\pi-\alpha} (-v_c+\sin\phi)\sin\phi d\phi$$

$$= \frac{1}{\pi}\left[-2v_c\cos\alpha + \frac{(\pi-2\alpha)}{2} + \frac{\sin 2\alpha}{2}\right] \qquad 8.3$$

The parameters a, T and G may be obtained from definitions in section 2.0. From 8.2, the time constant is

$$T = 2RC\ \frac{\pi}{\pi-2\alpha} \qquad 8.4$$

The factor a is obtained from 8.3, with

$$a = \frac{v_o'(0)}{v_o'(\infty)} = \frac{1}{1-\beta} \qquad 8.5$$

with $\beta = \frac{8\cos^2}{(\pi-2\alpha)(\pi-2\alpha+\sin 2\alpha)}$

The transformation ratio is, then,

$$G = \frac{v_o'(0)}{v_i'(0)} = \frac{1}{2\pi}(\pi-2\alpha+\sin 2\alpha) \qquad 8.6$$

To obtain an equivalen circuit for these parameters, consider Fig. 8.3, and the procedure outlined in section 2.0.

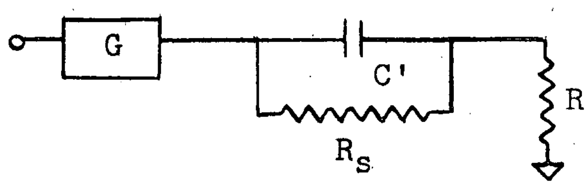


Fig. 8.3 DC Equivalent Lead Network.

Accordingly, one may write,

$$\frac{R + R_s}{R} = a = \frac{1}{1-\beta}$$

Hence $R_s = R\ \frac{\beta}{1-\beta}$

By computing the time constant of the circuit of Fig. 8.3, and equating this with 8.4, one obtains,

$$C' = \frac{2C\ \pi}{\beta(\pi-2\alpha)}$$

This network offers the possibility of varying the maximum available phase shift Øm and the frequency at which it occurs by controlling the conduction angle. These parameters as a function of "a" are given in Appendix I by I.14 and I.16 .

8.2 Servo Mixing Network

In Appendix II, Reference 3, a network is shown capable of employing an ac signal with dc feedback to obtain, as desired, either a lead or lag effect. The schematic is given in Fig. 8.4. The diode bridge operation has been described in section 7.0.

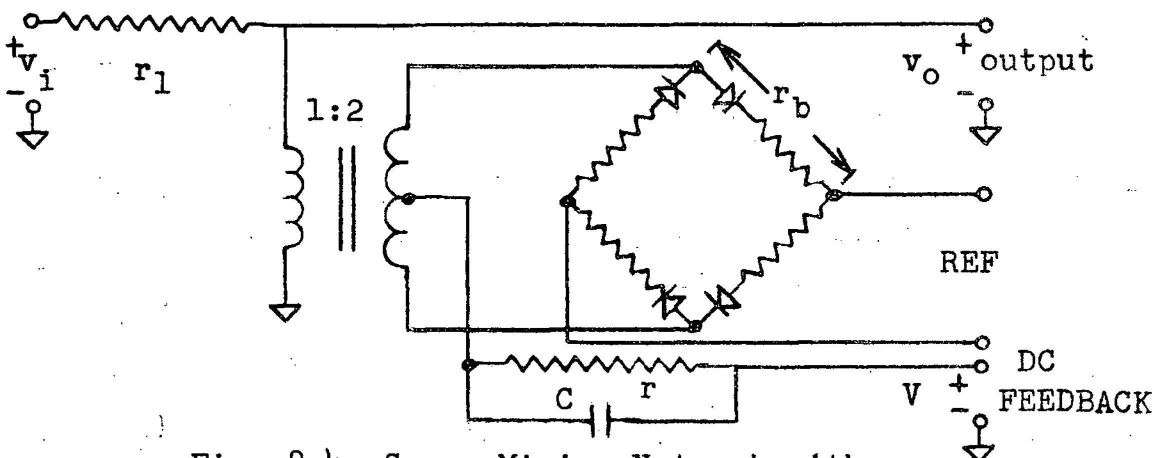


Fig. 8.4 Servo Mixing Network with AC Input and DC Feedback.

Considering only the dc feedback voltage, the circuit in Fig. 8.5 is analysed.

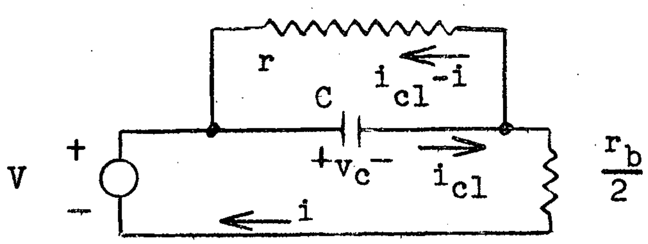


Fig. 8.5 DC Feedback Circuit.

The following equations may be written.

$$V + V_c = i \frac{r_b}{2} \qquad 8.7$$

Hence $$i = \frac{2}{r_b} (V + v_c) \qquad 8.8$$

$$v_c = (i_{c1} - i)r \qquad 8.9$$

Hence $$i_{c1} = i + \frac{v_c}{r} \qquad 8.10$$

Substituting 8.9 into 8.10 gives

$$i_{c1} = \frac{2V}{r_b} + \left(\frac{2}{r_b} + \frac{1}{r}\right) v_c \qquad 8.11$$

Consider only the ac input, the circuit, shown in Fig. 8.6, is analysed.

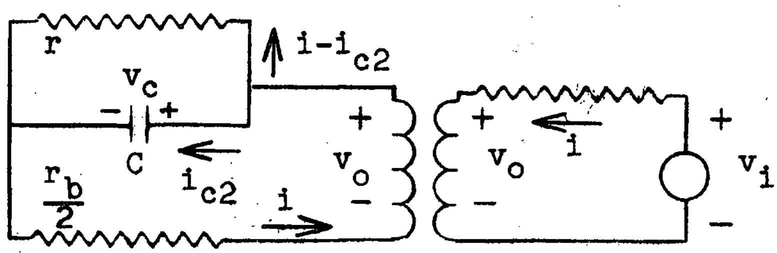


Fig. 8.6 AC Input Circuit.

The following equations may be written.

$$v_c = (i - i_{c2})r \qquad 8.12$$

$$i = \frac{v_i - v_o}{r_1} \qquad 8.13$$

$$v_o - v_c = i \frac{r_b}{2} = \frac{r_b}{2r_1}(v_i - v_o) \qquad 8.14$$

Eliminating the currents and solving for v_o gives,

$$v_o = \frac{v_c}{2r_1 + r_b} 2r_1 + \frac{v_i \, r_b}{2r_1 + r_b} \qquad 8.15$$

Then i_{c2} may be obtained in terms of v_i and v_c from the above equations.

$$i_{c2} = v_i \left(\frac{2}{2r_1 + r_b}\right) - v_c\left(\frac{2}{2r_1 + r_b} + \frac{1}{r}\right) \qquad 8.16$$

In order to obtain the net charging current i_c of the condenser, one has, combining 8.16 and 8.11,

$$i_c = i_{c2} - i_{c1} \qquad 8.17$$

$$= Av_i - Bv_c - \frac{2V}{r_b} \qquad 8.18$$

with

$$A = \frac{2}{2r_1 + r_b} \qquad 8.19$$

$$B = 2\left(\frac{1}{r_b} + \frac{1}{r} + \frac{1}{2r_1 + r_b}\right) \qquad 8.20$$

Now it is possible to write, according to section 2.0,

$$\frac{dv_c}{dt} = \frac{\bar{i}_c}{C}$$

To compute $\bar{i}_c$, consider 8.18. It is assumed that V and v_c are slowly varying and that $v_i = V_i \sin\emptyset$ and V is a dc step voltage. Hence one obtains,

$$\bar{i}_c = A\,\frac{2}{\pi}V_i - Bv_c - \frac{2V}{r_b} \qquad 8.21$$

Substitution into the differential equation gives,

$$\frac{dv_c}{dt} + \frac{1}{T}\,v_c = \frac{K}{T} \qquad 8.22$$

with

$$\frac{1}{T} = \frac{B}{C} \qquad 8.23$$

and

$$K = \frac{1}{B}\left(\frac{2A}{\pi}V_i - \frac{2}{r_b}V\right) \qquad 8.24$$

Solving for v_c with $v_c(0) = 0$ gives

$$v_c(t) = K(1 - \exp -\frac{t}{T}) \qquad 8.25$$

Now applying Fourier analysis to quasi-stationary output gives,

$$\tfrac{1}{2}v_o'(t) = \frac{1}{\pi}\int_0^{\pi} v_o(t)\ \sin\phi\ d\phi$$

$$v_o'(t) = \frac{4}{\pi}\ r_1 A v_c + \frac{Ar_b}{2}\ V_i \qquad 8.26$$

With the aid of 8.24 and 8.26, one may write 8.26 as

$$v_o'(t) = V_i\left(\frac{8\ A^2 r_1}{\pi^2 B}(1-\exp{-\frac{t}{T}}) + \frac{Ar_b}{2}\right) \qquad 8.27$$

$$-V\ \frac{8Ar_1}{\pi B r_b}\ (1-\exp{-\frac{t}{T}}) \qquad 8.27$$

This network offers the possibility of obtaining optimum response with a controlling feedback voltage as the overall effect of the network may be either lead- or lag- compensation depending on the feedback.

From 8.27 one sees that $v_o'(t)$ is composed of two parts: a contribution from the ac signal input and a contribution from the dc signal feedback. This suggests the equivalent circuit of Fig. 8.7 .

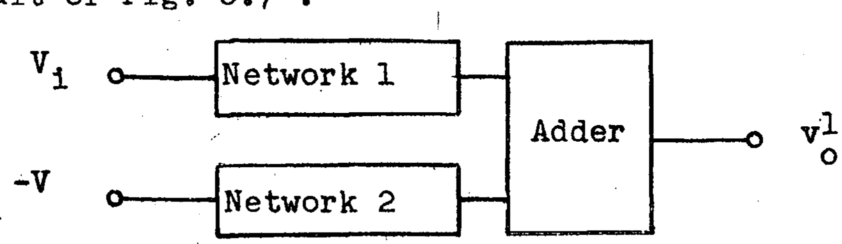


Fig. 8.7 Equivalent Network for the Servo Mixing Network.

To obtain Network 1 consider that part of $v_o'(t)$ which contains V_i. Following the procedure for determining equivalent networks in section 2.0, consider Fig. 8.8.

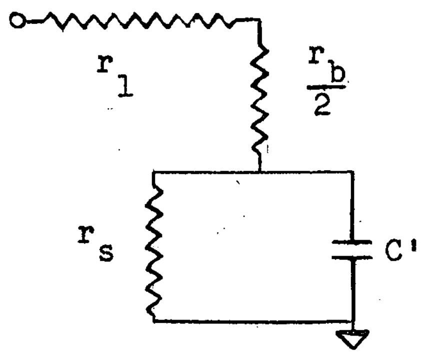


Fig. 8.8 Network 1.

This network must respond to a step input of amplitude V_i as $V_i\left(\frac{8A^2r_1}{\pi^2 B}\ (1-\exp^{-\frac{t}{T}}) + \frac{Ar_b}{2}\right)$.

To account for attenuation at "t" $=\infty$, r_s must satisfy the following equation:

$$\frac{\frac{r_b}{2}+r_s}{r_1+\frac{r_b}{2}+r_s} = A\left(\frac{r_b}{2} + \frac{8}{\pi^2}\ r_1\ \frac{A}{B}\right) \qquad 8.28$$

Solving this equation gives

$$r_s = \frac{8}{\pi^2}\ \frac{1}{B(1-\frac{8}{\pi^2}\frac{A}{B})}$$

Computing the time constant of Network 1 and equating this with 8.23 gives

$$C' = C\ \frac{\pi^2}{8}$$

Next consider the contribution of $v_o'(t)$ associated with the step feedback V. The lag-like response suggest the Fig. 8.9 as an equivalent circuit for Network 2.

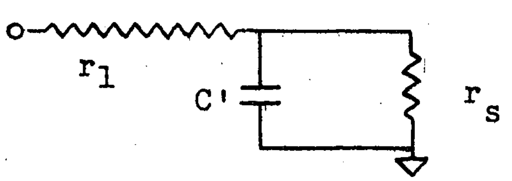


Fig. 8.9 Network 2.

This network must respond to a step input of amplitude V as $V\,\frac{8Ar_1}{\pi Br_b}(1-\exp\frac{-t}{T})$. To account for attenuation at $t = \infty$, the following relation must hold:

$$\frac{r_s}{r_1+r_s} = \frac{8Ar_1}{\pi Br_b}$$

Hence $r_s = r_1 \dfrac{1}{\frac{\pi r_b}{8R}(1+\frac{r_b}{2r_1}) -1}$

Computing the time constant of Network 2 and equating with 8.23 gives,

$$C' = \frac{\pi C}{8}\left[\frac{r_b}{r_1}+\frac{1}{2}\left(\frac{r_b}{r_1}\right)^2\right]$$

9.0 Discussion of the Validity of the Equivalent Circuit.

Consider the network shown in Fig. 9.1. A unit amplitude sine-wave input v_i is applied at t = 0.

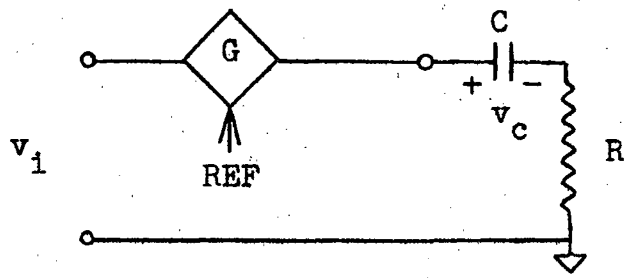


Fig. 9.1 Schematic for Circuit 1.

The operation of the gate G is such that the input to the condenser is a half-wave sine signal. During the non-conduction period of the gate, the condenser input terminal is considered open circuit. During the conduction period of the gate, the signal is assumed to be generated from a zero impedance source.

During the conduction period the following differential equation is valid.

$$v_c + CR\,\frac{dv_c}{dt} = \sin wt \qquad 9.1$$

The solution of 9.1 is

$$v_c = A_n \exp\left(\frac{-t}{T}\right) + P\sin wt + Q\cos wt$$

where $T = RC$, $P = \dfrac{1}{1 + (\omega CR)^2} = \dfrac{1}{1 + (\omega T)^2}$,

$$Q = \frac{-\omega CR}{1 + (\omega CR)^2} = \frac{-\omega T}{1 + (\omega T)^2},$$

and A_n is to be determined for each conduction interval.

As $v_c(0) = 0$, then $A_o = \dfrac{\omega CR}{1 + (\omega CR)^2} = \dfrac{\omega T}{1 + (\omega T)^2}$

For $\dfrac{\omega t}{\pi} = 1$, $v_c(1) = A_o\left[1 + \exp\left(\dfrac{-\pi}{\omega T}\right)\right]$

Let $a = \exp\left(\dfrac{-\pi}{\omega T}\right)$

During the interval $\frac{\omega t}{\pi} = 1$ to $\frac{\omega t}{\pi} = 2$, there is no change in v_c. Then, matching boundary conditions gives

$$A_o(1+a) = A_2 - A_o$$

Hence $\quad A_2 = A_o\ (2+a)$

Now for $\frac{\omega t}{\pi} = 3$, $v_c(3) = A_2 a + A_o$

Similar matching may be done for other times, and the results are, relating to A_o,

$$v_c(1) = A_o\ (1+a)$$

$$v_c(3) = A_o\ (1+2a+a^2)$$

$$v_c(5) = A_o\ (1+2a+2a^2+a^3)$$

In general for $\frac{\omega t}{\pi} = (2n+1)$,

$$v_c(2n+1) = A_o\ \frac{1+a}{1-a}\left(1 - a^{n+1}\right)$$

Let $\quad x = \frac{\pi}{\omega T}$

Then $\quad v_c(2n+1) = \frac{2}{\pi}(1-a^{n+1})\left[1-x^2(0.01799)+ x^4(0.000434)\ldots\right]$

Due to the nature of the operation of the gate G, then

$v_c(2n+1) = v_c\ (2n+2)$

From the analysis of circuit 1 in section 3 one obtains, $v_c(t) = \frac{2}{\pi}\left(1-\exp(\frac{-t}{2RC})\right)$.

To compare the two results set $t = (2n+2)\frac{\pi}{\omega}$. Using the definitions of $a = \exp(\frac{-\pi}{T})$ and $T = RC$, one obtains

$v_c(t) = \frac{2}{\pi}\ (1 - a^{n+1})$

In comparison one has,

$$\frac{v_c(2n+2)}{v_c(t)} = 1-x^2(0.01799)+ x^4(0.000434)\ldots$$

Then it is apparent that

$$\lim_{x \to o} \frac{v_c\ (2n+2)}{v_c(t)} = 1$$

That is, the results from either approach are identical in the limit. Now from the definition of x,

$$\lim_{x \to o} = \lim_{\omega \to \infty} \quad \text{as } x = \frac{\pi}{\omega T} .$$

The limiting case for ω, the carrier frequency, approaching infinity, is equivalent to x approaching zero. Since the error term enters quadratically, the limiting equivalent circuit will be valid for the usual range of operation in servo-systems, i.e., $\frac{\omega_s}{\omega_c} \leqslant \frac{1}{10}$.

10.0 Measurement of Phase and Gain Characteristics of Circuit 1.

The theory of the circuit 1 shown in Fig. 3.2 has been developed using Fourier analysis. Physical results of Fourier analysis may be obtained by employing the time-average output of a multiplier. The system used is shown in Fig. 10.1.

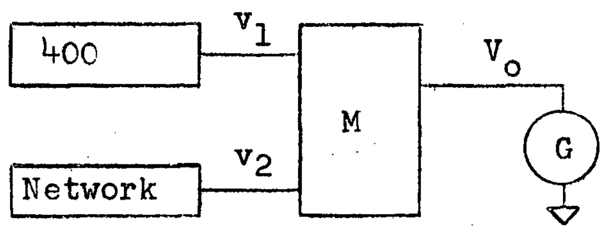


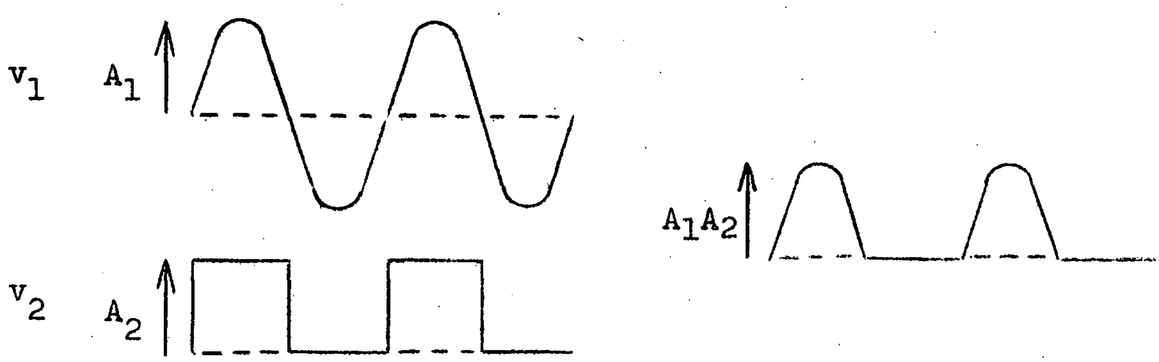
Fig. 10.1 Schematic for Determination of Multiplier Proportionality Constant

For calibration the following waveforms were employed:

v_1 ___ 400 cps voltage of 14.14 peak volts

v_2 ___ 400 cps variable-amplitude square-wave.

The actual waveforms are shown in Fig. 10.2.



a. Input Waveforms

b. Output Waveforms

Fig. 10.2 Multiplier Waveforms.

The multiplier deflection is proportional to the dc level of the output.

$$\overline{\text{Output}} = \frac{1}{2\pi}\int_0^{\pi} A_1 A_2 \sin\phi \, d\phi = \frac{A_1 A_2}{\pi}$$

Now the multiplier's dc output V_o is proportional to the time average of the product of the two inputs. Hence the proportionality constant $\bar{k}$ is defined by

$$V_o = \bar{k}\,\frac{A_1 A_2}{\pi}$$

By suitable measurements it was found that

$$\bar{k} = \left(\frac{2V_o}{A_2}\right)\left(\frac{\pi}{2A_1}\right) = (.579)\,\frac{\pi}{2(14.14)} = 0.06425$$

The measurement of phase-gain characteristics is achieved by the system shown in Fig. 10.3. A modulated carrier is generated by the synchro T1 which is driven by a variable speed motor at the rate ω_s. The modulated signal is applied to a second synchro T2 which is used as a manual phase shifter to vary the phase of the envelope of the signal. This phase-shifted modulated signal is applied to the network. The output of the network is applied to one of the multiplier's inputs. A known comparison signal generated by a resolver is applied to the second multiplier input. The time average of the product of the inputs is recorded by the galvonometer.

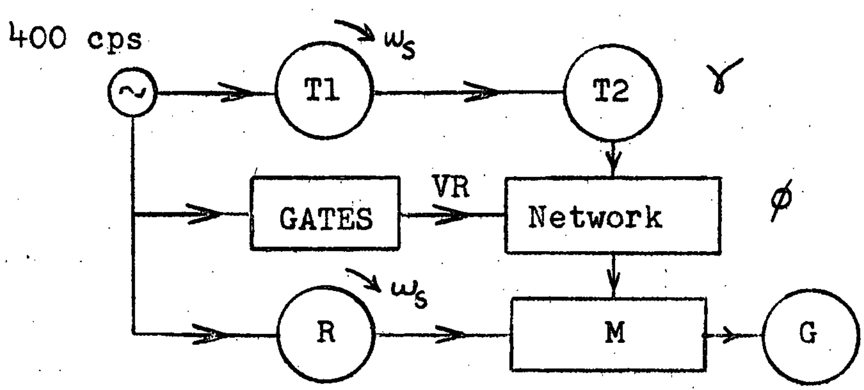
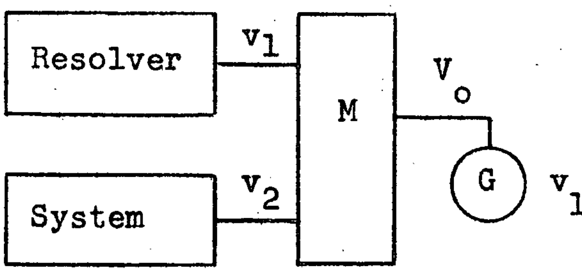


Fig. 10.3 Block Diagram of Measurement System.

For Fig. 10.3, the following terms are defined:

T1	Synchro Transformer	M	Electronic Multiplier
T2	Synchro Transformer	G	Galvonometer
R	Resolver	ω_s	modulation frequency

Consider the following arrangements of multiplier inputs shown in Fig. 10.4.



G Galvonometer

M Electronic Multiplier

$$v_1 \begin{cases} v_{1c} = V_1 \cos \omega_s t \ \sin \omega_c t \\ v_{1s} = V_1 \sin \omega_s t \ \sin \omega_c t \end{cases}$$

$$v_2 = V_2 \sin\ (\omega_s t + \alpha) \sin \omega_c t$$

Fig. 10.4 Multiplier Schematic for Gain-Phase Determinations

In the definition of v_2, α is the combined phase shift introduced by the network and the variable transformer, i.e.,

$$\alpha = \emptyset \text{ (network) } + \gamma \text{(transformer)}$$

Let $$V_{oc} = \bar{k}\ \overline{v_{1c} v_2}$$

$$= \bar{k}\ \overline{V_1 \cos \omega_s t \ \sin \omega_c t \ V_2 \sin(\omega_s t + \alpha) \sin \omega_c t}$$

$$= \bar{k}\ V_1 V_2 \overline{(\sin\alpha \cos^2 \omega_s t + \cos\alpha \cos \omega_s t \ \sin \omega_s t) \sin^2 \omega_c t}$$

With suitable sum and difference angle formulae, one obtains,

$$V_{oc} = \bar{k}\ V_1 V_2 \sin\alpha \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{4} \left| \sin(\omega_c - \omega_s)t + \sin(\omega_c + \omega_s)t \right|^2 d(\omega_s t)$$

$$= \bar{k}\ V_1 V_2 \sin\alpha \frac{1}{2\pi} \cdot \frac{1}{4} . 2 . \pi = \frac{\bar{k}\ V_1 V_2 \sin\alpha}{4}$$

Then V_{oc} is made zero by choosing $\alpha = 0$, i.e., $\emptyset = -\gamma$

Employing v_{1s}, it may be shown similarly

$$V_{os} = \bar{k}\, V_1 V_2 \frac{\cos\alpha}{4}$$

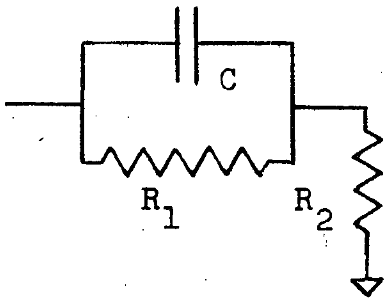
With $\alpha = 0$, one may solve for V_2, obtaining

$$V_2 = \frac{4V_{os}}{\bar{k}\, V_1}$$

For computation of the gain of the network, the peak output amplitude V_2 must be compared with that peak signal V_i which would pass through the system in the absence of any compensating network. If the input signal has a peak V, then as $G = \frac{1}{2}$, one has $\frac{V_i}{V} = \frac{1}{2}$. The gain of the system is

$$Y = \frac{V_2}{V_i} = \frac{2V_2}{V} = \frac{8V_{os}}{\bar{k}\, V_1 V}$$

The experimental studies involved measurement of the gain-phase characteristics as a function of signal frequency. The calculated network parameters a and T provided the gain-phase characteristics of the limiting equivalent circuit. A comparative plot of results is shown in Fig. 10.5. For Circuit 1, a is fixed at 5.3 so that experimental studies involved variation of T. The values of T chosen were 7.25 msecs and 4.21 msecs. These correspond to frequencies of maximum phase shift of 9.6 cps and 16.7 cps respectively.



Y(s)

a

T

$\frac{1}{R}$

Equivalent Circuit

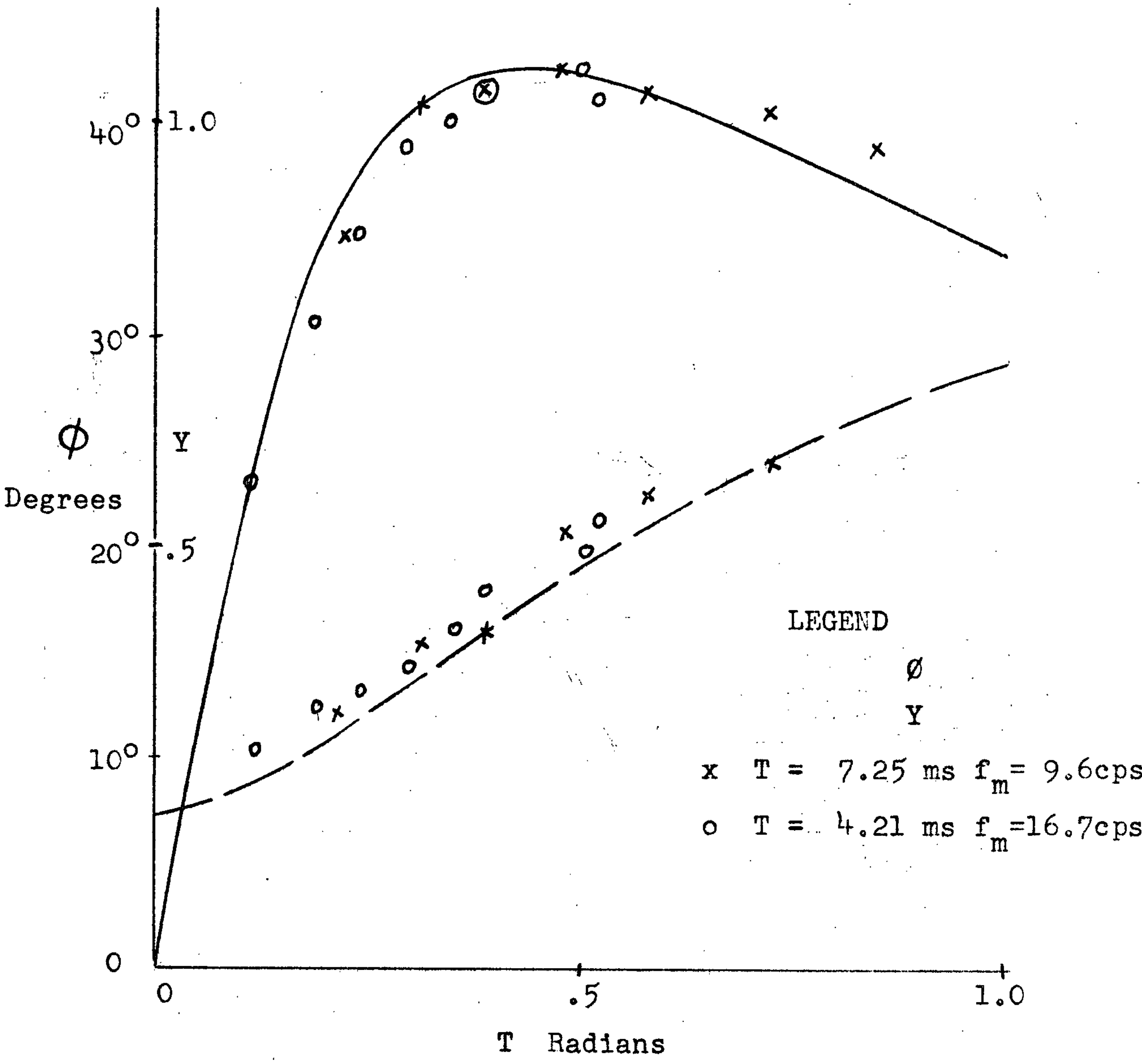


Fig. 10.5

11.0 Conclusions.

By considering the limiting case as $\frac{\omega_c}{\omega_s} \longrightarrow \infty$, a simplified theory was developed to obtain network parameters from the transient response to a step input. With the aid of the parameters, limiting equivalent circuits have been obtained whose qualitative understanding is immediate. Study has been done to determine the range of practical representation of the limiting equivalent circuit. Analysis predicted and experiment confirmed that the limiting circuit was adequate for the usual case in servo-systems where $\frac{\omega_s}{\omega_c} \leqslant \frac{1}{10}$.

It has been shown that basic lead- or lag-compensation may be achieved with several different systems. Further study of compensating networks has revealed that, in special cases, network parameters are controllable by suitable feedback.

With the development of suitable equivalent circuits, carrier-system compensation utilizing demodulator networks can be realized giving wide application. For non-linear systems where adjustable gain-phase characteristics might best achieve specifications, circuits employing feedback-controlled parameters would appear to introduce a new field of interesting possibilities.

Appendix I. DC Lead Network.

Consider the network below.

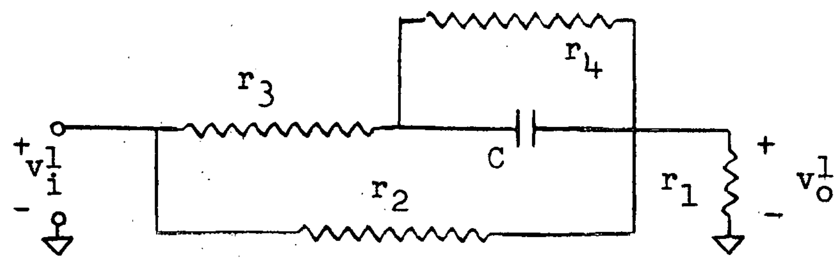


Fig. I.1 DC Lead Network.

The transfer function for this network may be written as

$$Y(s) = G_o \frac{1 + T_1 s}{1 + T_2 s} \qquad \text{I.1}$$

with

$$T_1 = \frac{C(r_2 r_4 + r_3 r_4)}{r_2 + r_3 + r_4} \qquad \text{I.2}$$

$$T_2 = \frac{C(r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4)}{r_1(r_2 + r_3 + r_4) + (r_3 + r_4)r_2} \qquad \text{I.3}$$

$$G_o = \frac{r_1(r_2 + r_3 + r_4)}{r_1(r_2 + r_3 + r_4) + (r_3 + r_4)r_2} \qquad \text{I.4}$$

Then it is apparent that 3 parameters will be sufficient to specify gain and phase characteristics.

Of special interest in the network response to a unit-step input because of the particular development of the theory in the body of the thesis. Then $V_o^1(s) = G_o \frac{(1+T_1 s)}{(1+T_2 s)} \frac{1}{s}$, is the Laplace Transform of the output voltage v_o^1.

In the usual manner the transform may be inverted to yield

$$v_o^1(t) = G_o \left(1-(1-\frac{T_1}{T_2})\exp-\frac{t}{T} \right) \qquad \text{I.5}$$

Then for t = 0 in I.5, one has, $v_o^1(0) = G_o\frac{T_1}{T_2}$ I.6

and for $t = \infty$ in I.5, $v_o^1(\infty) = G_o$ I.7

Combining I.6 and I.7 yields,

$$\frac{v_o^1(0)}{v_o^1(\infty)} = \frac{T_1}{T_2} \qquad \text{I.8}$$

For the network it is often necessary to know the frequency of maximum phase shift and the amount of phase shift at the maximum. From the definition of "a" in section 2.0 one has

$$a = \frac{T_1}{T_2} \qquad \text{I.9}$$

For real frequencies, I.1 may be written

$$Y(j\omega) = |Y| \frac{\exp(j\emptyset_1)}{\exp(j\emptyset_2)} = Y \exp(j\emptyset) \qquad \text{I.10}$$

where $\tan\emptyset_1 = \omega a T_2$ and $\tan\emptyset_2 = \omega T_2$ I.11

and $\emptyset = \emptyset_1 - \emptyset_2$ I.12

The frequency of maximum phase shift is obtained by differentiating I.12.

$$\frac{d\emptyset}{d\omega} = 0 = \frac{d\emptyset_1}{d\omega} - \frac{d\emptyset_2}{d\omega} \qquad \text{I.13}$$

The derivatives of I.13 may be obtained from I.11. These equations yield the following result,

$$aT_2\cos^2\emptyset_1 = T_2\cos^2\emptyset_2$$

The various cosines may be obtained from I.11, so that one has,

$$a \frac{1}{1 + (a\omega T_2)^2} = \frac{1}{1 + (\omega T_2)^2}$$

or, $a(\omega_m T_2)^2 = 1$ I.14

To obtain the amount of phase shift consider $\tan\emptyset = \tan(\emptyset_1 - \emptyset_2)$

Then for the maximum phase shift, it may be shown that

$$\tan\phi_m = \frac{1}{\sqrt{a}}\frac{(a-1)}{2} \qquad \text{I.15}$$

An equivalent form of I.15 is

$$\sin\phi_m = \frac{a-1}{a+1} \qquad \text{I.16}$$

From the transfer function it is apparent that there are three choices to be made in order to specify the phase and amplitude characteristics. These choices may be made in a variety of ways depending upon circumstance. For example they could be chosen on the network's response to a unit-step input. In this case the parameters might be the output amplitude initially, the output amplitude finally, and the time constant for the decay to the steady state. Another basis of choice arises in stability problems where concern might be the frequency of maximum phase shift and the amount of phase shift at that particular frequency.

It should be noted that in Fig. I.1 that r_4 may be eliminated but the circuit would retain its performance characteristics be appropriate modification of r_2 and r_3. Then, essentially, there are four elements to be chosen. The concept of three independent choices has been made previously, while the extra choice or degree of freedom determines the impedance levels.

For some of the circuits discussed in the thesis, Fig. I.1 needs modification. The first case is for $r_4 = \infty$.

Hence $$Y(s) = \frac{r_1\left[1 + Cs\,(r_2+r_3)\right]}{\left[r_1+r_2\right]\left[1 + Cs\left(r_3+\frac{r_1r_2}{r_1+r_2}\right)\right]}$$

As all the important relations have been derived generally, nothing significant arises in this situation.

Of very particular interest is the case of $r_4 = \infty$ and $r_3 = 0$. Then

$$Y(s) = \frac{r_1}{r_1+r_2}\;\frac{1 + Csr_2}{1 + Cs\,\frac{r_1r_2}{r_1+r_2}} = \frac{1}{a}\;\frac{1+aT_2s}{1+T_2s}$$

Now it is apparent that $G_o = \frac{1}{a}$ and $T_1 = aT_2$. Then the parameter a may be checked or determined by measurement. It is apparent for this case that the network requires only two parameters for definition of gain-phase characteristics.

Appendix II. References.

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