A STUDY ON THE USE OF DIELECTRIC LOADED SLOW WAVE STRUCTURES IN TRAVELLING WAVE TUBES

by

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ABSTRACT

In the first section of this thesis the simple Pierce Theory for the travelling wave tube is discussed and developed. The result of this analysis is a number of equations which produce information about gain and bandwidth.

The next section of the thesis is devoted to a discussion of periodic structures with particular emphasis on dielectric loaded periodic structures.

Finally the Pierce Theory is applied to the dielectric slow wave structure. Results are presented of a study performed to find a correlation between the physical dimensions of the dielectric structure and the gain-bandwidth properties of a travelling wave tube employing the structure. These results, which are in graphical form, can be used to eliminate the initial exploratory design work for travelling wave tubes of this type. A sample use is made of the design curves and the results are compared to a computational check to show both the usefulness and the limitations of the curves.

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1. INTRODUCTION

The travelling wave tube amplifier has received considerable study in recent years because it is a wide band microwave amplifier with a potentially low noise figure. The device consists of a slow wave structure (waveguide in which the phase velocity is less than the velocity of light) with appropriate coupling mechanisms for the input and output of a signal. An electron beam is made to flow in the structure where it can interact with an electromagnetic wave propagating therein. The wave, introduced at the input coupler, travels along the slow wave structure where interaction with the electron beam results in an energy transfer between the beam and the wave.

The number of signal channels which can be transmitted in a given percentage bandwidth increases as the band center frequency is increased. Thus if the percentage bandwidth of an amplifier can be maintained, it is desirable to increase the operating frequency. However the radius and pitch length of the helix slow wave structure, the circuit which has been most commonly used in medium and low power tubes, diminishes as the operating frequency increases. As the size of the helix decreases, the maximum power flow that it can support also decreases and the helix itself becomes increasingly more difficult to construct. The power flow is limited by the heating of the helix metal due to I^2R losses and to electron bombardment from the beam. At operating frequencies above X-band, the problem of finding a more suitable slow wave structure has become important. One possibility is the dielectric loaded structure which is easier to build than the helix and which holds promise of a satisfactory power capacity. This thesis is concerned with determining the suitability of the dielectric structure for use in travelling wave tubes.

The simple Pierce Theory for the travelling wave tube is used throughout this investigation. Although the theory is not rigorous, it does produce a concise solution. Such a solution is very desirable if it is to be used for numerical design. Although more rigorous theoretical developments can be made, they yield results which are much more complicated. The increased numerical labor detracts from the increased accuracy which the refined solutions produce.

The simplicity of the simple Pierce Theory is achieved by neglecting the effects of distributed loss and space charge as well as by restricting the treatment to small signals in order to avoid non-linear equations. Ideal coupling devices are also assumed. A section of the thesis is devoted to a development of the simple Pierce Theory.

A field analysis of the dielectric loaded structure containing discs with a center hole is possible but the solution is only of formal significance because of its complexity. The structure is shown in Figure 1. The hole in the discs is necessary to allow passage of the electron beam.

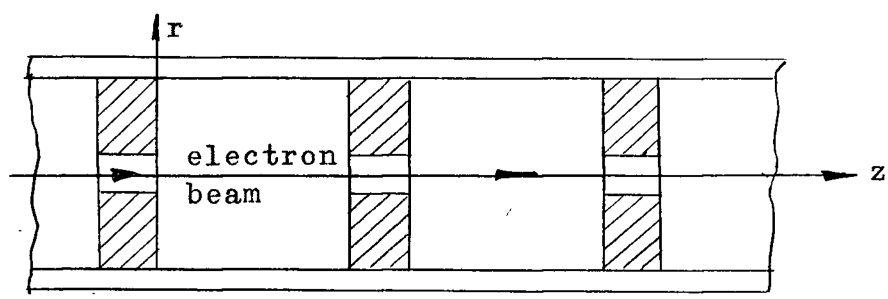

Figure 1. The dielectric structure with a center hole in the discs.

Since much of the work of this study is numerical, a theory which produces a concise result is very desirable. As the field solution for the dielectric structure with solid discs is sufficiently concise, it is used throughout as an approximation to the field solution for the holed disc structure.

The results of this study are presented in a set of design curves relating the gain-bandwidth properties of the dielectric structure when used in a travelling wave tube to the dimensional parameters of the structure. The significance of the errors due to the approximations used are discussed so as to clarify the usefulness of the design curves.

2. THE PIERCE THEORY [1],[2]

2.1 Simplifying Assumptions and Restrictions

In order to reduce the complexity of the problem, a uniformly distributed LC delay line is used to represent the slow wave structure. The shunt susceptance and series reactance of the delay line are chosen so that the phase velocity and the longitudinal E field, $-\frac{\partial V}{\partial z}$, acting on the electrons for unit power flow are the same as those for any given structure. It should be noted that this procedure gives results which compare quite reasonably with those of more complex analyses.

Even with the above simplification, the problem becomes rather complex if, for instance, non-linear electron current flow equations are used. Hence the analysis is restricted to small signals which allows linearization of the electron flow equations. The assumptions that all of the electrons are acted upon by a known field (excluding the field due to local space charge) and that they are displaced only in the longitudinal direction by the ac field are also made to simplify the treatment. The latter assumption can be made sufficiently valid in practice if a strong magnetic focusing field is used. Also, the electrons are assumed to be uniformly distributed in any cross-section which is normal to the direction of beam flow. The force on the electrons due to the magnetic field produced by the displacement current is sufficiently small to neglect for non-relativistic electron velocities to which this work is restricted. Some of the effects deleted by the above assumptions and restrictions can be incorporated into more complex analyses, but the developments in this thesis do not demand these more refined results. As yet a completely satisfactory theory of the travelling wave tube has not been produced; hence, one of the approximate theoretical developments must be resorted to.

With the above assumptions and restrictions, the problem becomes quite manageable. It is easily seen that there are two physical processes occurring in a travelling wave tube: the effect of the circuit voltages on the beam current and the effect of the beam current on the circuit. The latter process can be considered as a perturbation by the electron beam of a wave propagating in the slow wave structure.

2.2 Excitation of Circuit Field by Beam Current

Consider the circuit shown in Figure 2 which is a distributed LC transmission line in close proximity to an electron beam. The circuit will be assumed to be either of infinite length or of finite length with a non-reflective termination.

It will be noted here that three separate currents will be referred to. They are the electron beam current, denoted as the convection current; the circuit current, denoted as the conduction current; and the displacement current.

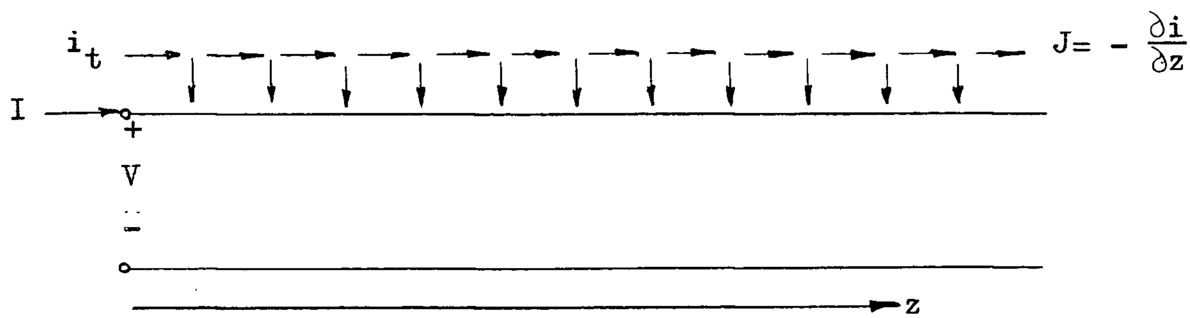

Figure 2 - Equivalent circuit of travelling wave tube

To determine the nature of the coupling of the electron beam to the circuit, consider a small volume of space, such as shown in Figure 3, through which all the convection current flows. The sum of the currents flowing into the volume must be zero to preserve continuity of charge. However, since the convection current varies with distance, there must exist a displacement current to satisfy the above condition. It will be assumed that all the displacement current flows into the circuit, i.e. that unity coupling exists. Since the convection current has been assumed to flow very close to the circuit, the displacement current, J amperes per meter, has only a negligible component in the direction of propagation compared to that which flows directly to the slow wave circuit. Note that the definition of displacement current differs dimensionally from that usually made. The definition used here is more useful since, for a small distance δz, the pertinent current is the total displacement current coupled to the slow wave circuit. The displacement current

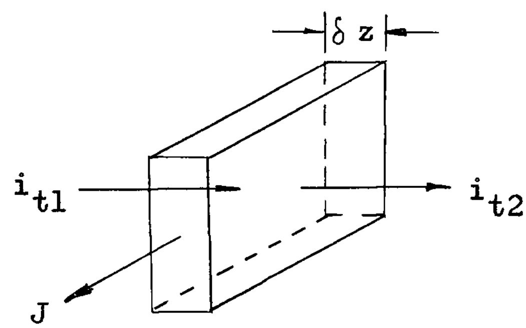

Figure 3 - Volume through which the electron beam passes

is related to the convection current in the following manner:

$$i_{t2} + J\,\delta z - i_{t1} = 0 \tag{1}$$

where

$i_t = -I_o + i$

- total electron convection (beam) current

I_o - average or dc electron convection current

i - ac component of electron convection current

J - displacement current (amperes/meter)

But

$$i_{t2} = i_{t1} + \frac{\partial i}{\partial z}\,\delta z \tag{2}$$

Hence

$$J = -\frac{\partial i}{\partial z} \tag{3}$$

If the slow wave circuit is considered, the ordinary transmission line equations result with the exception that the displacement current is taken into account.

$$\frac{\partial I}{\partial z} = -jBV + J \tag{4}$$

$$\frac{\partial V}{\partial z} = -jXI \tag{5}$$

where

$V = V(z,t)$ - line voltage

$I = I(z,t)$ - line or conduction current

B - shunt susceptance per unit length

X - series reactance per unit length

Differentiation of equation (5) with respect to z and the subsequent substitution of equations (3) and (4) yields the following differential equation:

$$\frac{\partial^2 V}{\partial z^2} + BXV = +jX\frac{\partial i}{\partial z} \tag{6}$$

The time variation of interest for the beam and circuit variables is $e^{j\omega t}$. It is logical to expect V to vary as $e^{-\Gamma z}$ since the effect of the electron beam is to perturb the wave solution for the undisturbed slow wave system. Thus substitution of $e^{j\omega t-\Gamma z}$ into equation (6) yields

$$(\Gamma^2 + BX)V = -jX\Gamma i \tag{7}$$

It is now evident that in the absence of the electron beam the propagation constant reduces to Γ_o, the natural propagation constant of the slow wave structure, where

$$\Gamma_o = j\sqrt{BX} \tag{8}$$

The definition of characteristic impedance in transmission line theory is of use and is given by

$$K = \sqrt{\frac{X}{B}} \tag{9}$$

Equations (8) and (9) can be used to elimiate X and B from equation (7) to produce an expression in terms of more common parameters:

$$V = \frac{-\Gamma_o \Gamma Ki}{(\Gamma^2 - \Gamma_o^2)} \tag{10}$$

2.3 Excitation of Beam Current by the Current Field

Now consider the effect of a circuit wave E field on the electron beam. The two basic equations to be used are the equation of motion of an electron in an E field (11), and the continuity of charge equation (13).

$$F = m\frac{d(u_o + v)}{dt} = -eE$$

$$\frac{dv}{dt} = \eta \frac{\partial V}{\partial z} \tag{11}$$

where

$$E = -\frac{\partial V}{\partial z}$$

- longitudinal electric field intensity

u_o - average or dc velocity of the electrons

v - ac component of electron velocity

m - electron mass

e - charge of the electron $(e > o)$

η - charge to mass ratio of the electron

F - force

The continuity of charge equation for variations in one direction is

$$\frac{\partial}{\partial z}\left(\frac{i_t}{a}\right) = - \frac{\partial}{\partial t}\left(\frac{\rho_o + \rho}{a}\right) \qquad (12)$$

where

ρ_o - average or dc charge per unit length of electron beam

ρ - ac component of linear charge density of electron beam

a - cross-sectional area of electron beam

Simplification of equation (12) yields

$$\frac{\partial i}{\partial z} = - \frac{\partial \rho}{\partial t} \qquad (13)$$

First of all consider the left hand side of equation (11):

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial z}\frac{dz}{dt}$$

$$= \frac{\partial v}{\partial t} + (u_o + v)\frac{\partial v}{\partial z}$$

The small signal restriction allows the above equation to be linearized, i.e. to reduce $(u_o + v)$ to u_o. Since the time variation for the variables is $e^{j\omega t}$, differentiation with respect to t is equivalent to multiplication by $j\omega$. Thus equation (11) becomes

$$\frac{\partial v}{\partial z} + j\,\frac{\omega}{u_o}\,v = \frac{\eta}{u_o}\,\frac{\partial V}{\partial z} \tag{14}$$

The term ω/u_o is of the same form as the phase factor in transmission line or waveguide analyses and in this development may be considered to be the phase factor of some disturbance travelling at the electron velocity. Hence the following definition:

$$\beta_e = \frac{\omega}{u_o} \tag{15}$$

Equation (16) is a basic relation which will be of use in finding an expression relating v and i:

$$(-I_o + i) = (\rho_o + \rho)(u_o + v) \tag{16}$$

Again, the second order terms in the ac variables are deleted to yield

$$(-I_o + i) = \rho_o u_o + \rho u_o + \rho_o v$$

But

$$-I_o = \rho_o u_o$$

Therefore

$$i = \rho u_o + \rho_o v$$

or

$$v = \frac{i - \rho u_o}{\rho_o} \tag{17}$$

The continuity of charge equation can be used to eliminate ρ in equation (17) producing equation (18) which can then be used in equation (14) to yield equation (19):

$$v = \frac{i + \frac{u_o}{j\omega} \frac{\partial i}{\partial z}}{\rho_o} \tag{18}$$

$$\frac{\partial^2 i}{\partial z^2} + 2\, j\beta_e \frac{\partial i}{\partial z} - \beta_e^2\, i = \frac{-j\eta\beta_e I_o}{u_o^2} \frac{\partial V}{\partial z}$$

But

$$u_o = \sqrt{2\eta V_o}$$

where V_o is the voltage through which the electrons are accelerated to give them velocity u_o.

Hence

$$\frac{\partial^2 i}{\partial z^2} + 2j\beta_e \frac{\partial i}{\partial z} - \beta_e^2\, i = -j\, \frac{\beta_e I_o}{2V_o} \frac{\partial V}{\partial z} \tag{19}$$

It can now be seen that if V varies as $e^{-\Gamma z}$ with distance, then the particular solution for i must also. In this case the natural propagation constant , Γ_o, is perturbed slightly to Γ where

$$\Gamma = \Gamma_o - \xi \tag{20}$$

Direct substitution of $e^{-\Gamma z}$ into equation (19) yields

$$i = \frac{j\beta_e I_o \Gamma V}{2V_o (j\beta_e - \Gamma)^2} \tag{21}$$

Hence a voltage which varies as $e^{j\omega t - \Gamma z}$ gives rise to an ac convection current in the beam which varies likewise.

Thus each physical process yields a relation between circuit voltage and ac convection current. These relations (equations (10) and (21)) must hold simultaneously, and forcing them to do so yields four solutions for Γ. It is worth noting that equations (6) and (19) are two simultaneous differential equations for which there are no driving functions. Hence the solution must be given entirely in terms of the four linearly independent functions which result.

2.4 Natural Modes and Their Properties

Equations (10) and (21) can be combined to produce an equation in Γ which defines the allowable values of Γ in terms of the given system parameters:

$$1 = \frac{jKI_o\beta_e \quad \Gamma_o \Gamma^2}{2V_o \; (\Gamma_o^2 - \Gamma^2) \; (j\beta_e - \Gamma)^2} \tag{22}$$

Since equation (22) is of fourth degree in Γ, there are four solutions for Γ and hence, in general, there must be four boundary conditions to completely specify the wave solution. That the problem requires two more boundary conditions than the usual waveguide problem is not surprising since beam conditions must be specified also.

It is to be expected that coupling between the beam and the circuit will occur only if the velocities of the disturbances in each are about equal. If the velocities differ considerably, a segment of the electron beam will alternately see accelerating and decelerating voltages in such rapid succession that the net effect of the voltage wave on it is very small. Thus setting the electron velocity equal to the phase velocity of the slow wave structure in the absence of beam current is quite reasonable. Hence

$$\Gamma_o = j\beta_e \tag{23}$$

The phase velocity of a wave in the slow wave structure is perturbed slightly when the electron beam is introduced. Hence Γ can be expressed in the following form:

$$\Gamma = j\beta_e - \xi \tag{24}$$

Note that this expression for Γ yields only forward waves and since at least one backward wave is to be expected, substitution of equation (24) into equation (22) should produce an equation in ξ of less than the fourth order. In fact, direct substitution with the deletion of small order term yields

$$\xi^3 = -j\beta_e^{\,3} \frac{KI_o}{4V_o} \tag{25}$$

It is now useful to introduce several new parameters. Thus define

$$C^3 = \frac{KI_o}{4V_o} \tag{26}$$

$$\xi = \beta_e C \delta \tag{27}$$

Now only δ remains to be determined to complete the solution of equation (25) where

$$\delta = (-j)^{1/3} \tag{28}$$

which has the solutions

$$\begin{aligned} \delta_1 &= j \\ \delta_2 &= 1 \angle -30^o \\ &= 0.866-j\ 0.5 \\ \delta_3 &= 1 \angle -150^o \\ &= -0.866-j\ 0.5 \end{aligned} \tag{29}$$

Therefore the forward waves have the following z variation:

$$e^{-\Gamma z} = e^{-j\beta_e z}\, e^{\delta C \beta_e z} \tag{30}$$

It can be seen that δ_1 corresponds to a constant amplitude wave which travels slightly faster than the electrons, δ_2 corresponds to an amplified wave whose phase velocity is slightly less than the beam velocity, and δ_3 corresponds to an attenuated wave whose phase velocity is also slightly less than the beam velocity.

The propagation constant for the backward wave follows naturally from the following substitution:

$$\Gamma = -j\beta_e - \xi \tag{31}$$

The final solution for Γ in this case is

$$\begin{aligned}\Gamma_4 &= -j\beta_e + j\,\frac{\beta_e C^3}{4} \\ &= -j\beta_e\left(1 - \frac{C^3}{4}\right)\end{aligned} \tag{32}$$

Since C is a small number, it is obvious that Γ_4 is only very slightly perturbed.

Equations (8) and (9) can be combined to give

$$K = \frac{jX}{\Gamma_o} \tag{33}$$

Since the propagation constant of the slow wave structure has been perturbed, the characteristic impedance is also perturbed. Hence each of the forward waves has an impedance associated with it which is given by

$$\begin{aligned}K_n &= \frac{jX}{\Gamma_n} \\ &= \frac{K}{1 + j\,\delta_n C} \qquad n = 1,2,3 \\ &\cong K(1 - j\,\delta_n C)\end{aligned} \tag{34}$$

The parameter C, as will be shown later, is very important and it is convenient to find a new expression for it which is more useful in applications to slow wave structures. Since the magnitude of the E field, E_o, usually has much more meaning than the voltage V, the variable V will be transformed to E_o. The two are related as follows:

$$E_o = \left| \Gamma V \right| \qquad (35)$$

Also, for the equivalent circuit used, the power flow is given by

$$P = \frac{|V|^2}{2K} \qquad (36)$$

Hence we have

$$2K = \frac{E_o^{\,2}}{\beta_e^{\,2} P} \qquad (37)$$

Thus the new relation for C is given by

$$C^3 = \frac{E_o^{\,2}\, I_o}{8\beta^2 V_o P} \qquad (38)$$

Since the β for each of the forward waves is very nearly β_e, the subscript has been dropped. If V_o/I_o is called the beam impedance, then C^3 is one quarter of the structure impedance divided by the beam impedance. It might be noted that C is often called the gain parameter and K is often called the beam coupling impedance.

2.5 Gain and Bandwidth Expressions

It has now been shown that the linearized differential equations describing the system have the following form of solution:

$$M(z) = e^{-j\beta_e z} \, e^{\delta C \beta_e z}$$

The solutions for δ for the case in which circuit loss and space charge effects are ignored have been found. In general δ will have other values but the solution type remains, to a first approximation, unchanged. Thus if the amplitude of the growing wave is considered at points z_1 and z_2, the following results:

$$\frac{|M(z_2)|}{|M(z_1)|} = e^{xC\beta_e(z_2-z_1)} \tag{39}$$

where

$$\delta = x + jy$$

But (z_2-z_1) can be written as

$$(z_2-z_1) = \frac{2\pi N}{\beta_e} \qquad z_2 > z_1 \tag{40}$$

where N is the number of wavelengths corresponding to $|z_2-z_1|$. Hence

$$\frac{|M(z_2)|}{|M(2_1)|} = e^{2\pi xCN} \tag{41}$$

Thus the logarithmic gain due to this wave can be written as

$$\begin{aligned} G &= 20 \log_{10} e^{2\pi xCN} \\ &= 54.6xCN \end{aligned} \tag{42}$$

If the value of x determined in the previous analysis is substituted into equation (42), it reduces to

$$G = 47.3\ CN \tag{43}$$

Expression (43) does not give the exact gain of the travelling wave tube, however, since there are three waves to be considered (the structure is assumed to be perfectly terminated). To ascertain the initial amplitudes of the three waves, a boundary value problem must be solved. Two of the conditions to be applied here are that the current and velocity modulations are zero at the input:

$$\sum_n v_n = 0 \qquad \sum_n i_n = 0 \qquad n = 1,2,3 \tag{44}$$

Equations (18) and (21) after appropriate manipulation yield

$$v_n = -\frac{\eta \Gamma_o V_n}{u_o \delta_n \beta_e C} \qquad i_n = \frac{j\beta_e I_o \Gamma_o V_n}{2V_o \delta_n^2 \beta_e^2 C^2} \qquad n = 1,2,3 \tag{45}$$

Equations (44) and (45) combine to give

$$\frac{V_1}{\delta_1} + \frac{V_2}{\delta_2} + \frac{V_3}{\delta_3} = 0 \tag{46}$$

$$\frac{V_1}{\delta_1^2} + \frac{V_2}{\delta_2^2} + \frac{V_3}{\delta_3^2} = 0 \tag{46}$$

Obviously the following relation must hold at the input:

$$V_1 + V_2 + V_3 = V \tag{47}$$

Equations (29), (46), and (47) yield

$$V_1 = V_2 = V_3 = \frac{V}{3} \tag{48}$$

where V is the input voltage (z = 0). Hence the voltage at z, denoted by V_z, is seen to be

$$\begin{aligned} V_z &= \frac{V}{3} e^{-j\beta_e z}\left[e^{\frac{-j\beta_e Cz}{2} + \frac{\sqrt{3}}{2}\beta_e Cz} + e^{\frac{-j\beta_e Cz}{2} - \frac{\sqrt{3}}{2}\beta_e Cz} + e^{j\beta_e Cz} \right] \qquad (49) \\ &= \frac{V}{3} e^{-j\beta_e z}\left[1 + 2\cosh\left(\frac{\sqrt{3}\beta_e Cz}{2}\right) e^{\frac{-j3\beta_e Cz}{2}} \right] e^{j\beta_e Cz} \end{aligned}$$

Expression (50) follows directly:

$$\left|\frac{V_z}{V}\right|^2 = \frac{1}{9}\left[1 + 4\cosh^2\left(\frac{\sqrt{3}\,\beta_e Cz}{2}\right) + 4\cos\left(\frac{3\beta_e Cz}{2}\right)\cosh\left(\frac{\sqrt{3}\,\beta_e Cz}{2}\right) \right] \tag{50}$$

To see how $|V_z/V|^2$ varies as a function of z, consider a plot of $10 \log_{10} |V_z/V|^2$ versus CN as shown in Figure 4. Note that the signal does not begin to experience a gain until it has travelled some distance down the tube. This is to be expected since the electron beam requires a finite time to become modulated; and until such time as it does, the beam will not act upon the circuit, i.e. the wave.

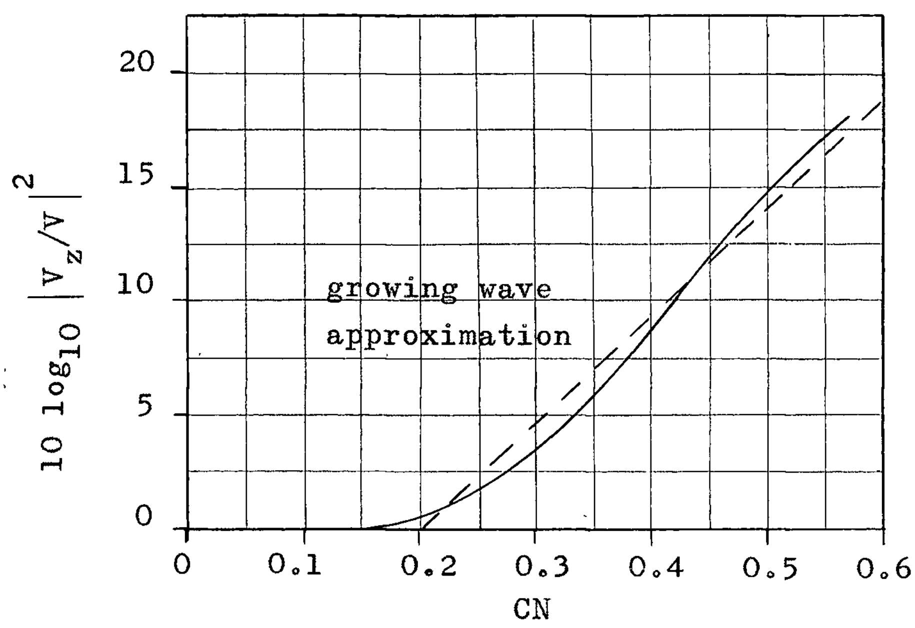

Figure 4. Signal gain as a function of distance

It can be seen from Figure 4 that a straight line approximation to the signal gain function is reasonably accurate for CN > 0.2. The straight line is the gain function for the growing wave and hence, from equation (43), has slope 47.3. It is not surprising that this approximation is reasonably valid since the growing wave will eventually dominate the other two. The exact equation for the straight line approximation is

$$G = A + 47.3\ CN$$

where A is given by the value of V_2 at the input:

$$A = 20 \log_{10} \frac{V_2}{V}$$

$$= -9.54\ \text{db}.$$

Hence

$$G = -9.54 + 47.3\ CN \tag{51}$$

The effect of variations in the mean electron velocity must be considered in order to determine the bandwidth. Using this more general approach, equation (23) must be changed to the following since, in general, the electron speed is to be slightly different from the unperturbed phase velocity.

$$\Gamma_o = j\beta_e\ (1 + Cb) = j\ \frac{\omega}{v_{ph}} \tag{52}$$

It follows through the use of equation (15) that

$$v_{ph}\ (1 + Cb) = u_o \tag{53}$$

In practice with tubes of high overall gain, the gain is reduced by several db. when b is changed by ± 1. For this limited range of b, equation (53) can be written

$$v_{ph} \cong (1 - Cb)\ u_o$$

since $C \ll 1$. Hence the bandwidth is approximately defined by the frequency range for which the phase velocity of the undisturbed wave differs from the electron velocity by less than the fraction $\pm C$. Thus

$$\Delta v_{ph} = \pm C u_o \tag{54}$$

where Δv_{ph} denotes the difference from the phase velocity at the operating point.

3. DIELECTRIC LOADED SLOW WAVE STRUCTURES

3.1 General Theory of Periodic Structures[3]

Before investigating the properties of the dielectric loaded slow wave structure, it is desirable to develop some of the basic theory of periodic structures, i.e. those structures which are physically periodic in space. The basis of this theory is the following theorem by Floquet: for a given mode of the system at a given frequency, the wave function varies from one point in the structure to another point one period away by only a complex constant. Since the exponential function provides a very general way of expressing such a constant, the following will be used:

$$e^{-\gamma z} \tag{55}$$

where γ is a complex number. Note that this function satisfies the requirement of Floquet's theorem at all the points in the structure. However this simple relationship will not allow the boundary conditions in any given structure to be met. This difficulty is overcome by multiplying (55) by factors which are periodic with period L or whole fractions thereof where L is the periodic length of the structure. Thus (55) becomes

$$e^{-\left(\gamma + j\frac{2\pi n}{L}\right) z} \tag{56}$$

where n is an integer.

Since a propagating wave solution is desired, let $\gamma = j\Psi/L$ where Ψ may be real or complex. The obvious procedure from here is to expand the wave function in terms of all the periodic functions above. The component functions in the resulting expansion have the common factor $e^{j\omega t - \gamma z}$. If this common factor is removed from the summation, the resulting series is the well known Fourier series whose component functions form a complete set.[4] Hence any reasonably well behaved field pattern can be approximated to an arbitrary degree of accuracy by a finite portion of the series. Thus

$$U(z,t) = \sum_{n=-\infty}^{\infty} a_n e^{j(\omega t - \phi_n z)} \tag{57}$$

where

$$\phi n = \frac{\Psi + 2\pi n}{L} \tag{58}$$

The function U(z,t) gives the z and t variations for either a fixed point or a fixed curve in the transverse plane. In general the latter applies and the complete wave solution requires the matching of an infinite set of three dimensional wave functions to the boundary conditions. The matching process includes forcing the complete wave solution to become U(z,t) at the aforementioned curve in the transverse plane. Hence the complete wave solution has the form

$$\sum_n N(x,y,n) \quad G(z,t,n)$$

The dielectric slow wave structure to be discussed later only requires a match to U(z,t) at one point in the transverse plane, and it will be seen that the complete wave solution, which is given below, is slightly simpler:

$$N(x,y) \sum_{n} G(z,t,n)$$

Before discussing some of the properties of Ψ, it is useful to determine the phase velocity of a wave in the structure. Examination of equations (57) and (58) shows that there does not exist a unique phase velocity but rather that each component in equation (57) has its own phase velocity. Hence

$$v_{ph} = \frac{\omega}{Re\left(\phi_n\right)} = \frac{\omega}{Re\left(\frac{\Psi}{L}\right) + \frac{2\pi n}{L}} \tag{59}$$

Since the components of equation (57) are distinguished from one another by their phase velocities and not by their frequencies (the frequency being fixed in any given case), they are often called space harmonics. Note, however, that the harmonics exist only as a part of the wave and not as entities unto themselves. Fortunately it is possible to have an interaction between one of the component waves and an electron beam. Consequently equation (59) is of considerable importance.

Ψ is a function of frequency, but not in an arbitrary manner. For the purposes of this discussion consider a lossless structure for which Ψ must be purely real or purely imaginary corresponding to propagating and non-propagating waves respectively. The following heuristic argument can be made to show that the two conditions are possible. The periodic nature of these sytems is such as to cause reflections of the wave. At certain frequencies these reflections will add out of phase and propagation will occur whereas at others they will reinforce to result in reflection of the power. The propagation or pass bands lie between the attenuation or stop bands.

If ω is considered to be a function of Ψ, it can be shown that the function is even and periodic with period 2π provided that the structure is symmetric (A symmetric structure is one for which a reversal in the direction of the central axis produces no physical change). It follows from equation (58) that if Ψ is increased by 2π, the set of ϕ_n's is unchanged. Only the denumeration of the members of the set is altered - each member having its subscript decreased by one. This change in denumeration will have no effect on the outcome of the determination of the constants in the series expansion of the field function with the exception that the original a_n will now be associated with ϕ_{n-1}. In other words, the series solution remains invariant under the stated transformation of the ϕ_n's which proves that the function is periodic.

That ω is an even function of Ψ can be seen by noting the consequences of reversing the flow of power in the structure. If at the same time the definition of the positive z direction is not reversed, then the exponential function in the series has the form

$$e^{j(\omega t + \phi_n z)}$$

Hence it can be seen that the change in direction of power flow can be accounted for in the original solution by changing the sign of the ϕ_n's. Thus, since $\pm\,\phi_n$ are both associated with the same frequency, the function must be even.

As a result of these basic properties of periodic structures, it follows that a typical plot of ω versus Ψ is that of Figure 5, where ω is normalized by $\sqrt{\mu\varepsilon}$. These plots are often referred to as dispersion curves. It should be noted that some periodic structures have dispersion curves which are the inverse of those shown in Figure 5. In other words, the curvature is the reverse of that shown. However, the dispersion curves of the dielectric slow wave structure, which is the periodic structure used in this study, are of the same shape as those shown in Figure 5.

Inspection of this plot shows that L times the slope of the line from the origin to any given point on one of the curves is given by $\omega\sqrt{\mu\varepsilon}\;L/\Psi$ which is the normalized phase velocity, v_{ph}/c. The normalized group velocity is given by L times the slope of the curve since $v_g = d\omega/d\phi$ where $\phi = \Psi/L$.

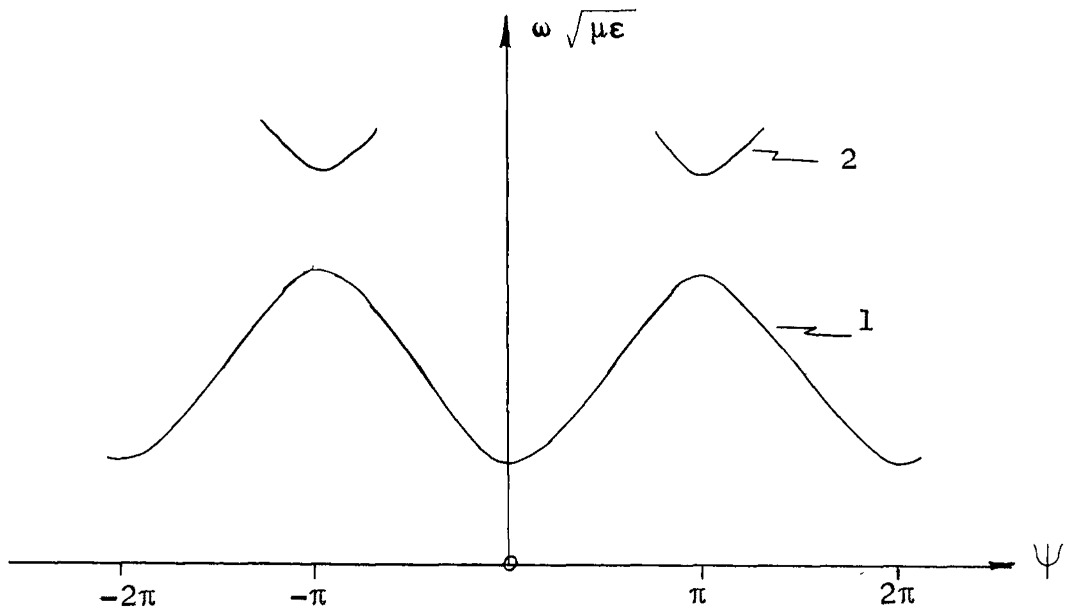

Figure 5. Plot of ω as a function of Ψ

3.2 The Dielectric Structure[5]

For any given structure, the remaining problem is to find the functional relationship between ω and Ψ. This is usually a rather difficult problem resulting in only an approximate answer. On exception, however, is the case of the dielectric loaded slow wave structure for which a closed solution is relatively easy to obtain. This structure consists of a cylindrical metal waveguide loaded periodically with solid dielectric discs as shown in Figure 6. The dielectric discs are denoted by the shaded areas and the intervening areas denote air spaces. It should be noted that the theory to be developed is also valid for the case where the air region is filled with a dielectric different from that of the shaded region.

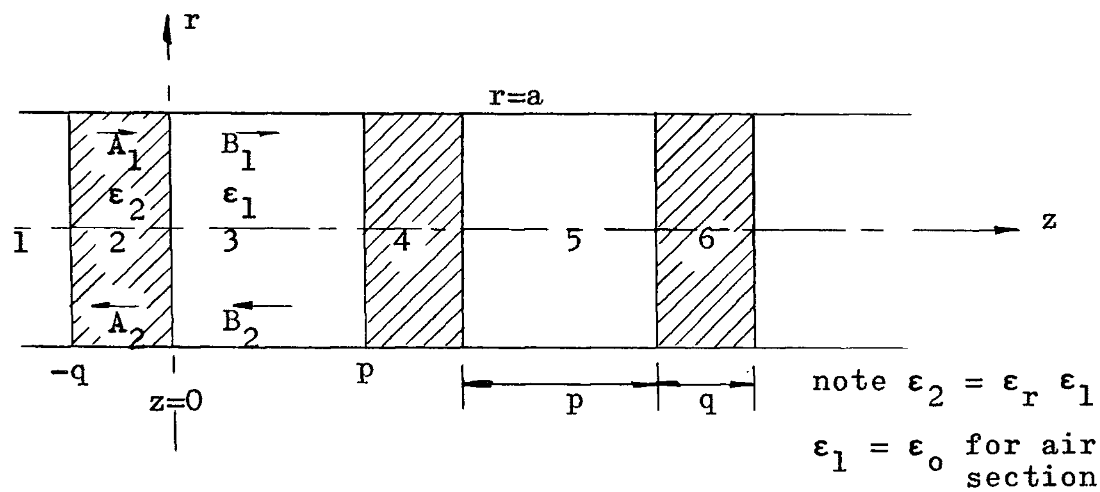

Figure 6. The dielectric structure

The solution of Maxwell's equations for an E_{01} mode in cylindrical waveguide yields the following for the dielectric and air regions respectively:

$$\left.\begin{aligned} E_z &= J_0(kr)\left[A_1 e^{-j\beta_2 z} + A_2 e^{j\beta_2 z}\right] \\ E_r &= \frac{j\omega}{kv_2} J_1(kr)\left[A_1 e^{-j\beta_2 z} - A_2 e^{j\beta_2 z}\right] \\ H_\phi &= \frac{j\omega\varepsilon_2}{k} J_1(kr)\left[A_1 e^{-j\beta_2 z} + A_2 e^{j\beta_2 z}\right] \end{aligned}\right\} \quad (60)$$

and

$$\left.\begin{aligned} E_z &= J_0(kr)\left[B_1 e^{-j\beta_1 z} + B_2 e^{j\beta_1 z}\right] \\ E_r &= \frac{j\omega}{kv_1} J_1(kr)\left[B_1 e^{-j\beta_1 z} - B_2 e^{j\beta_1 z}\right] \\ H\phi &= \frac{j\omega\varepsilon_1}{k} J_1(kr)\left[B_1 e^{-j\beta_1 z} + B_2 e^{j\beta_1 z}\right] \end{aligned}\right\} \quad (61)$$

where

$$\beta^2 = \omega^2 \mu \varepsilon - k^2$$

$$k = S_1/a, \text{ where } S_1 \text{ is the first root of } J_0(x) = 0 \qquad (62)$$

$$v = \frac{\omega}{\beta}, \text{ the phase velocity}$$

and A_1, A_2, B_1, and B_2 are amplitude constants.

Now according to Floquet's theorem, the field solutions for region (4) are given by those for region (2), which are expressed by equations (60), multiplied by $e^{-j\Psi}$. The solutions for region (3) are related to those for region (1) in the same way. To have a valid solution, the sets of equations for regions (2) and (3) must satisfy the required boundary conditions at z = 0, and the sets for regions (3) and (4) must satisfy the same boundary conditions at z = p. This process results in the following set of equations:

$$v_1 A_1 - v_1 A_2 - v_2 B_1 + v_2 B_2 = 0$$

$$\varepsilon_2 A_1 + \varepsilon_2 A_2 - \varepsilon_1 B_1 - \varepsilon_1 B_2 = 0 \qquad (63)$$

$$v_1 A_1 e^{j(2\theta_2 - \Psi)} - v_1 A_2 e^{-j(2\theta_2 + \Psi)} - v_2 B_1 e^{-j2\theta_1} + v_2 B_2 e^{j2\theta_1} = 0$$

$$\varepsilon_2 A_1 e^{j(2\theta_2 - \Psi)} + \varepsilon_2 A_2 e^{-j(2\theta_2 + \Psi)} - \varepsilon_1 B_1 e^{-j2\theta_1} - \varepsilon_1 B_2 e^{j2\theta_1} = 0$$

where

$$2\Theta_1 = \beta_1 p$$

$$2\Theta_2 = \beta_2 q$$

For a unique solution of these equations to exist, it is necessary and sufficient that the following determinant equal zero:

$$\begin{vmatrix} v_1 & -v_1 & -v_2 & v_2 \\ \varepsilon_2 & \varepsilon_2 & -\varepsilon_1 & -\varepsilon_1 \\ v_1 e^{j(2\Theta_2-\Psi)} & -v_1 e^{-j(2\Theta_2+\Psi)} & -v_2 e^{-j2\Theta_1} & v_2 e^{j2\Theta_1} \\ \varepsilon_2 e^{j(2\Theta_2-\Psi)} & \varepsilon_2 e^{-j(2\Theta_2+\Psi)} & -\varepsilon_1 e^{-j2\Theta_1} & -\varepsilon_1 e^{j2\Theta_1} \end{vmatrix} = 0 \quad (64)$$

Expansion and simplification of equation (64) yields

$$\cos \Psi = \cos 2\Theta_1 \cos 2\Theta_2 - 1/2\left[\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right] \sin 2\Theta_1 \sin 2\Theta_2 \quad (65)$$

where

$$Z = \frac{1}{\varepsilon v}$$

This equation is exactly the one sought since it gives a functional relationship between ω and Ψ and can be used in any specific case to yield a dispersion curve. It is often convenient to plot Ψ versus a/λ rather than ω, where λ is the free space wavelength. Since ω and λ are inversely related, the new dispersion curves have the same form as those shown in Figure 5. The convenience of the use of a/λ is due to the fact that it is a dimensionless factor. A dispersion curve with a/λ as the dependent variable becomes frequency dependent only when the radius of the structure is chosen.

It is also useful to have relations between the amplitude coefficients; hence they will now be derived. To find expressions relating A_1, A_2, and B_1 to B_2, the first three equations of the set (63) are rearranged so that B_2 is expressed in terms of the other constants. Division of each equation by B_2 yields

$$\begin{bmatrix} v_1 & -v_1 & -v_2 \\ \varepsilon_2 & \varepsilon_2 & -\varepsilon_1 \\ v_1 e^{j(2\theta_2-\Psi)} & -v_1 e^{-j(2\theta_2+\Psi)} & -v_2 e^{-j2\theta_1} \end{bmatrix} \begin{bmatrix} \frac{A_1}{B_2} \\ \frac{A_2}{B_2} \\ \frac{B_1}{B_2} \end{bmatrix} = \begin{bmatrix} -v_2 \\ \varepsilon_1 \\ -v_2 e^{j2\theta_1} \end{bmatrix}$$

The solution of these equations is (66)

$$\frac{A_1}{B_2} = \frac{\varepsilon_1}{\varepsilon_2} e^{j\Psi} \frac{\cos 2\theta_1 + j\frac{Z_1}{Z_2}\sin 2\theta_1 - e^{-j(2\theta_2+\Psi)}}{e^{j(\Psi-2\theta_1)} - \cos 2\theta_2 - j\frac{Z_2}{Z_1}\sin 2\theta_2} \tag{67}$$

$$\frac{A_2}{B_2} = \frac{\varepsilon_1}{\varepsilon_2} e^{j\Psi} \frac{\cos 2\theta_1 - j\frac{Z_1}{Z_2}\sin 2\theta_1 - e^{j(2\theta_2-\Psi)}}{e^{j(\Psi-2\theta_1)} - \cos 2\theta_2 - j\frac{Z_2}{Z_1}\sin 2\theta_2} \tag{68}$$

$$\frac{B_1}{B_2} = \frac{e^{j(\psi + 2\theta_1)} + j\frac{Z_2}{Z_1}\sin 2\theta_2 - \cos 2\theta_2}{e^{j(\psi - 2\theta_1)} - \cos 2\theta_2 - j\frac{Z_2}{Z_1}\sin 2\theta_2} \tag{69}$$

Other relations such as A_1/A_2, A_1/B_1, etc., could be obtained but those above are sufficient for this thesis.

Another important expression concerning dielectric structures is that relating power flow to field strengths. It might be thought that the total power flow could be computed by adding together the contributions due to each space harmonic, but this procedure is not valid since the space harmonics do not have power flow associated with them. To establish the truth of this statement, assume that the opposite is valid and investigate the consequences of the assumption. To form the Poynting vector the infinite series representing the transverse E and H fields must be multiplied together. For the assumption to be true, the cross terms arising from this multiplication must be of no consequence when the power flow integral is evaluated. However the cross terms do contribute to the power flow. This is not surprising since all the spare harmonics have the same frequency. Thus the assumption is invalid and the original statement is valid. The power calculation using the infinite series leads to a double sum which is laborious to evaluate.

Again the problem is simplified in the dielectric case because the exact field solution is known. As a result of Poynting's theorem, the following integral gives the power flow across any specified cross-sectional area in the structure:

$$P = \int_0^{2\pi}\int_0^{a} \tfrac{1}{2}\mathrm{Re}\left(E_r H_\phi^{*} \right) r \, dr \, d\phi \qquad (70)$$

If the cross-section is in the air region, equations (61) must be used. With the use of a general property of Bessel functions given by (71), the power flow given by (70) reduces to relation (72):

$$\int_0^{a} J_1^{\,2}(kr) r \, dr = \frac{a^2}{2} J_1^{\,2}(ka) \qquad (71)$$

$$P = 2\pi^3 \frac{\varepsilon_o^{\,2} c^2 J_1^{\,2}(ka)}{(ka)^2} a^2 \left(\frac{a}{\lambda}\right)^2 Z_o \sqrt{1-\left(\frac{S_1}{2\pi}\right)^2\left(\frac{\lambda}{a}\right)^2}$$

$$\times \left[\left|B_1\right|^2 - \left|B_2\right|^2 \right] \qquad (72)$$

where

a - radius of structure

λ - free space wavelength

c - velocity of light in vacuo
$= 3(10^8)$ m/sec.

Z_o - free space wave impedance $= \sqrt{\dfrac{\mu_o}{\varepsilon_o}}$

$S_1 = 2.4048$

Before the Pierce theory can be applied to the dielectric structure, the a_n's in equation (57) must be known. In this case the variation of importance is that of the central E_z field. Hence define

$$U(z,t) = E_z(0,0,z,t) \qquad (73)$$

where U(z,t) is given by equation (57). Therefore, equations (73) and (61) yield

$$\sum_{n=-\infty}^{\infty} a_n e^{-j\frac{2\pi n z}{p+q}}$$

$$= e^{j\frac{\Psi z}{p+q}} \left(B_1 e^{-j\beta_1 z} + B_2 e^{j\beta_1 z}\right) \quad \text{for the air region}$$

$$= e^{j\frac{\Psi z}{p+q}} \left(A_1 e^{-j\beta_2 z} + A_2 e^{j\beta_2 z}\right) \quad \text{for the dielectric region} \qquad (74)$$

$$= F(z)$$

where

$$F(z) = E_z(0,0,z,0)e^{j\frac{\Psi z}{p+q}}$$

Note that the functions $e^{-j\frac{2\pi n z}{p+q}}$ form a complete orthogonal set; [4] hence F(z) is expressed exactly by the series in equation (74). Thus E_z (0,0,z,0) must be expressed exactly by

$$E_z\,(0,0,z,0) = F(z)e^{-j\frac{\Psi z}{p+q}}$$

$$= \sum_{n=-\infty}^{\infty} a_n e^{-j\left(\frac{\Psi+2\pi n}{p+q}\right)z}$$

which means that the functions $e^{-j\left(\frac{\Psi+2\pi n}{p+q}\right)z}$ form a complete set. Since the radius of the structure is constant, the functional dependence of the longitudinal E field of the space harmonics on the transverse coordinates is given by one of the Bessel functions - in this case, the zeroth order Bessel function. Hence

$$E_z\ (r,\phi,z,t) = J_o(kr)e^{j\omega t} \sum_{n=-\infty}^{\infty} a_n e^{-j\left(\frac{\Psi+2n\pi}{p+q}\right)z}$$

It follows directly from equation (74) that the a_n's are given by the standard coefficient integral for the Fourier series:

$$a_n = \frac{1}{p+q}\int_0^{p+q} F(z)e^{j\frac{2\pi nz}{p+q}}\,dz \tag{75}$$

The solution of equation (75) can be simplified somewhat since the contribution to the integral from the dielectric region is negligible compared to that from the air region provided $\varepsilon_r >> 1$:

$$a_n \cong \frac{1}{p+q}\int_0^{p} F(z)e^{j\frac{2\pi nz}{p+q}}\,dz \tag{76}$$

4. APPLICATION OF THE PIERCE THEORY TO THE DIELECTRIC LOADED SLOW WAVE STRUCTURE

It can now be seen that given p/a, q/a, a/λ, Ψ, and ε_r as data, equations (69) and (72) yield

$$P = a^2 \left(\frac{a}{\lambda}\right)^2 \sqrt{1-\left(\frac{S_1}{2\pi}\right)^2 \left(\frac{\lambda}{a}\right)^2} \; \left|B_1\right|^2 \; N \left(1-M^2\right) \tag{77}$$

where

$$N = \frac{2\pi^3 \; \varepsilon_o^{\;2} \; c^2 \; J_1^{\;2}(ka) \; Z_o}{(ka)^2}$$

$$= 7.68 \; (10^{-3}) \text{ ohms}^{-1}$$

$$M = \left| \frac{e^{j(\Psi-2\theta_1)} - \cos 2\theta_2 - j\frac{Z_2}{Z_1} \sin 2\theta_2}{e^{j(\Psi+2\theta_1)} + j\frac{Z_2}{Z_1} \sin 2\theta_2 - \cos 2\theta_2} \right|$$

Equations (61), (69), and (76) yield $|a_n|^2$ in terms of $|B_1|^2$:

$$a_n = -jB_1 \left[\frac{e^{j\left[\Psi + 2n\pi - \beta_1(p+q)\right]\frac{p}{p+q}} - 1}{\Psi + 2n\pi - \beta_1 \; (p+q)} \right.$$

$$\left. + M \frac{e^{j\left[\Psi + 2n\pi + \beta_1(p+q)\right]\frac{p}{p+q}} - 1}{\Psi + 2n\pi + \beta_1 \; (p+q)} \right] \tag{78}$$

These results can then be used in conjunction with equation

(37) which reduces to the following in this application:

$$K = \frac{|a_n|^2}{2\phi^2 P} = \frac{|a_n|^2 \left(\frac{p+q}{a}\right)^2 a^2}{2(\Psi + 2n\pi)^2 P} \qquad (79)$$

where the electron beam couples to the n^{th} harmonic. In practice the harmonic of interest is the fundamental forward wave. Since ϕ_o, the phase constant of the fundamental, is related directly to Ψ, the numerical evaluation of K is straight forward. Direct application of equations (26) and (43) furnishes the straight line approximation to the gain. Note that the field solution for the solid disc structure is being used as an approximation to the field solution for the holed disc structure. This approximation is necessary since the field solution for the latter structure is far too complicated for repeated numerical use.(6)

A program to perform the calculation of the beam coupling impedance, K, was written for use with the ALWAC III electronic computer. However, before the program could be used intelligently, a useful correlation between the gain-bandwidth properties and the design parameters had to be found. The design parameters which can be varied are ε_r, p/a, q/a, and λ. The first of these variables to be fixed was ε_r which was done with the aid of the computational results shown in the graphs below. The first curves show how the beam coupling impedance and the required beam velocity vary with ε_r when p/q and $\frac{p+q}{a}$ are fixed. The value of a/λ used in the calculations is that defined on page 44 . The scatter of the points defining the K curve is due to the error in the human judgement required to choose the values of a/λ used.

It can be seen that both K and v_{ph}/c vary so as to produce greater gain at high values of ε_r since the gain parameter varies directly with K and inversely with v_{ph} for a fixed beam current. Figure 8, which gives the variation in the gain per section as a function of ε_r, confirms the above prediction. This plot (Figure 8)

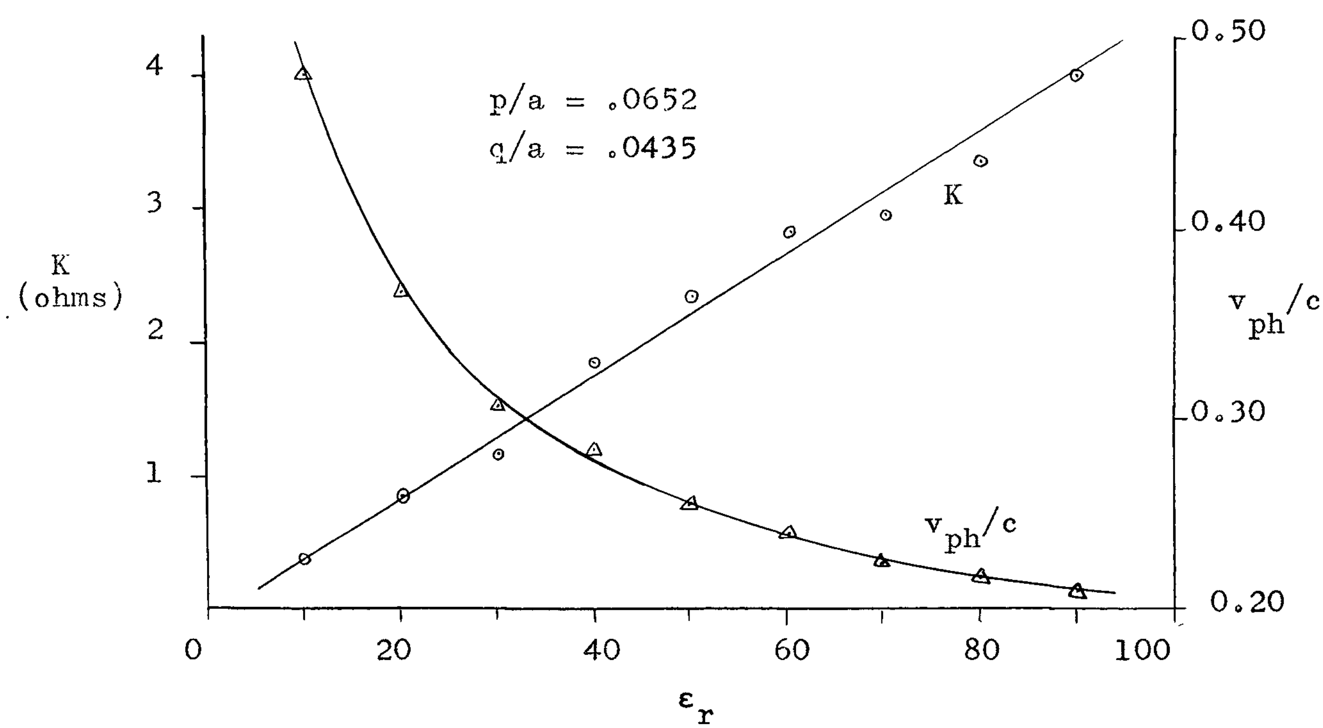

Figure 7. Beam coupling impedance and velocity as a function of ε_r for fixed p/q and $\frac{p+q}{a}$

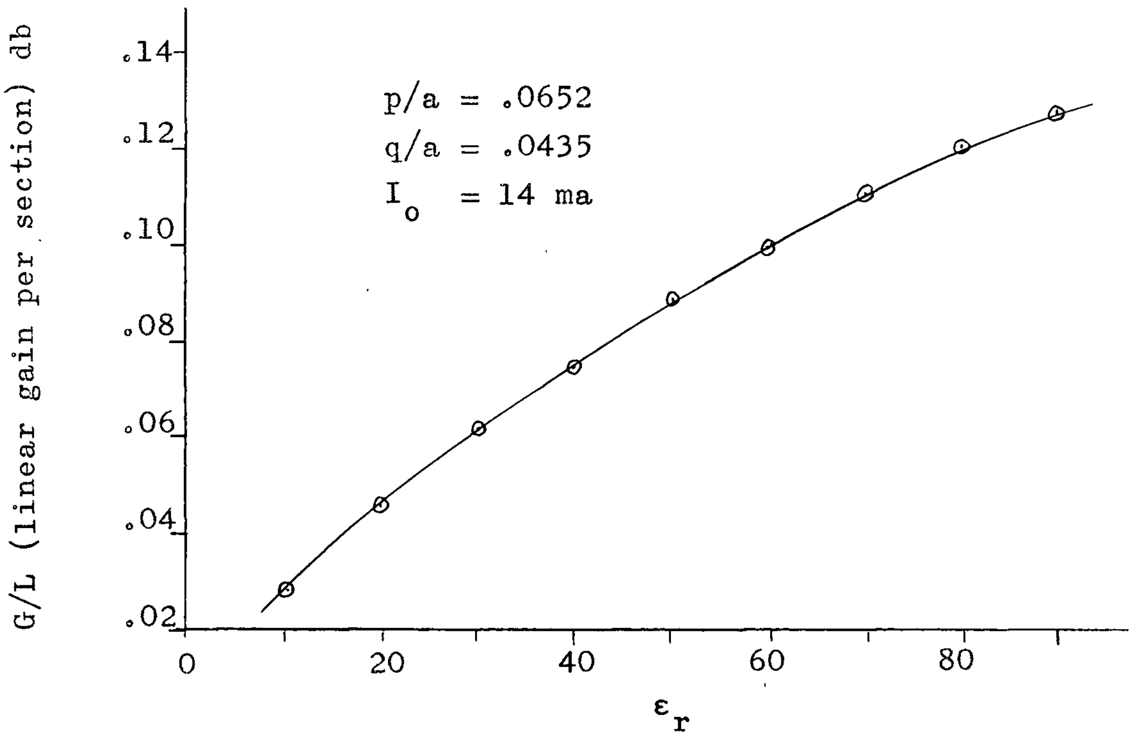

Figure 8. The linear gain per section as a function of ε_r for fixed p/q and $\frac{p+q}{a}$

shows that the number of discs required for any desired overall gain decreases as ε_r for the discs increases. Note that the gain per section referred to here and in the succeeding development is the linear approximation for an assumed beam current of 14 milliamperes. Since the cost of the discs is a significant fraction of the complete manufacturing cost of this type of travelling wave tube, it is obviously desirable that a dielectric with large permittivity be used. Fortunately, a low loss dielectric (titania) with a relative permittivity of approximately 93.5 is available and hence this value is used in the work to follow.

Further correlation between the remaining design

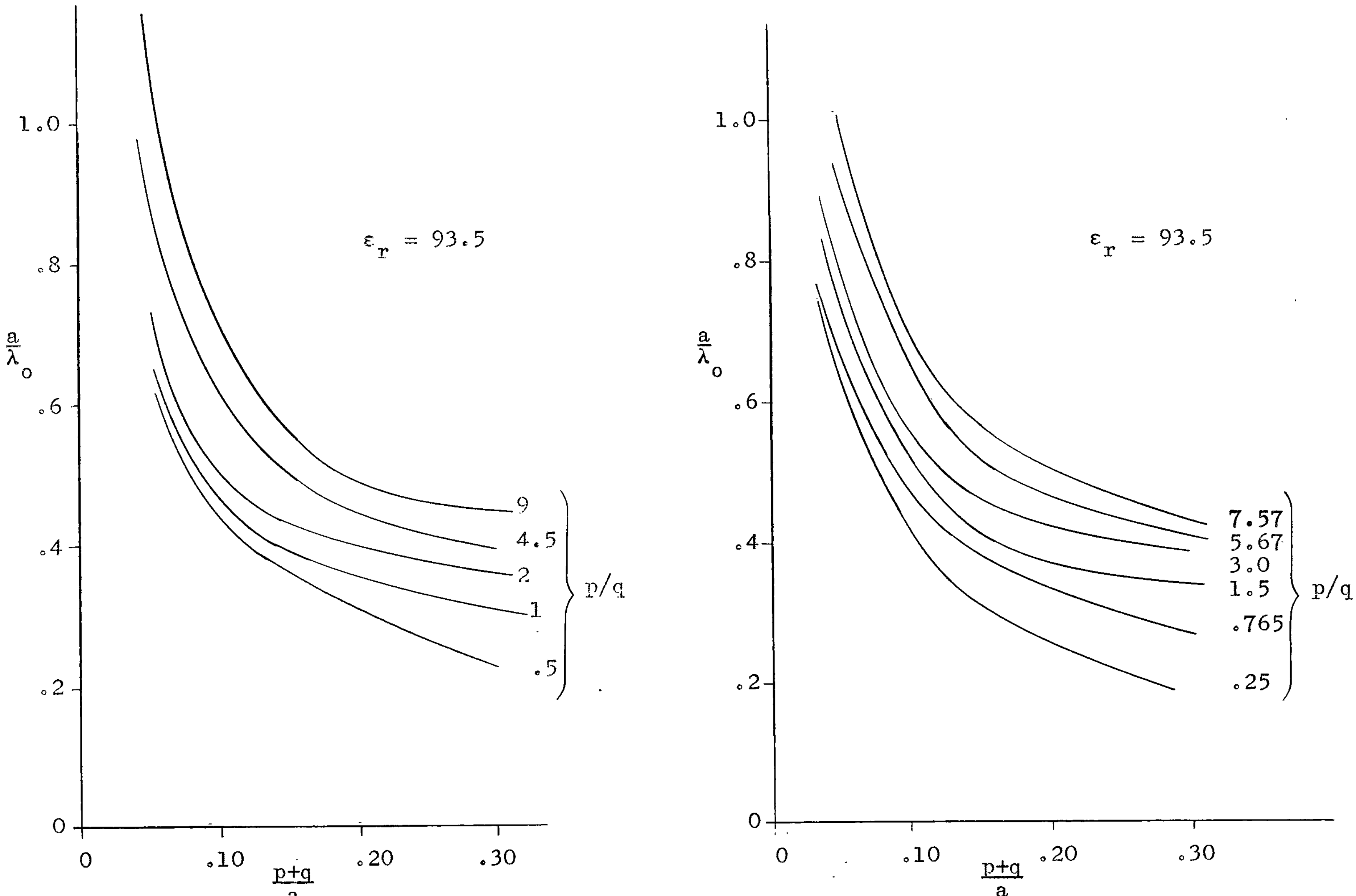

Figure 9. a/λ_o as a function of $\frac{p+q}{a}$ for fixed values of a p/q (the results are presented in two graphs for clarity)

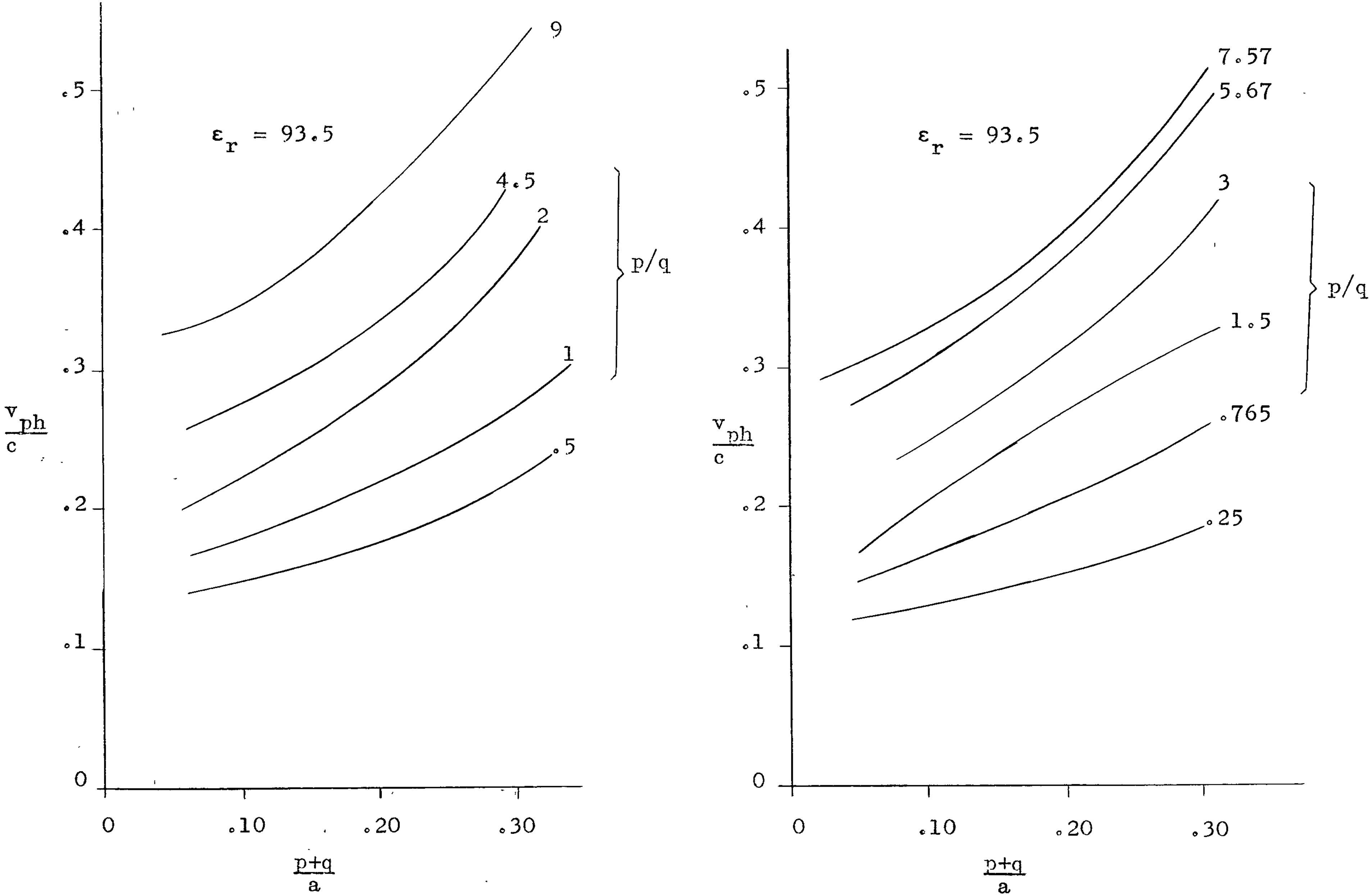

Figure 10. v_{ph}/c as a function of $\frac{p+q}{a}$ for fixed values of p/q

parameters and the travelling wave tube properties arose from a set of curves derived by Mr. Englefield which are shown in Figures 9 and 10. To arrive at these curves it is necessary to place a specific condition on the value of a/λ used, since, for any dispersion curve, an infinite number of points can be chosen.

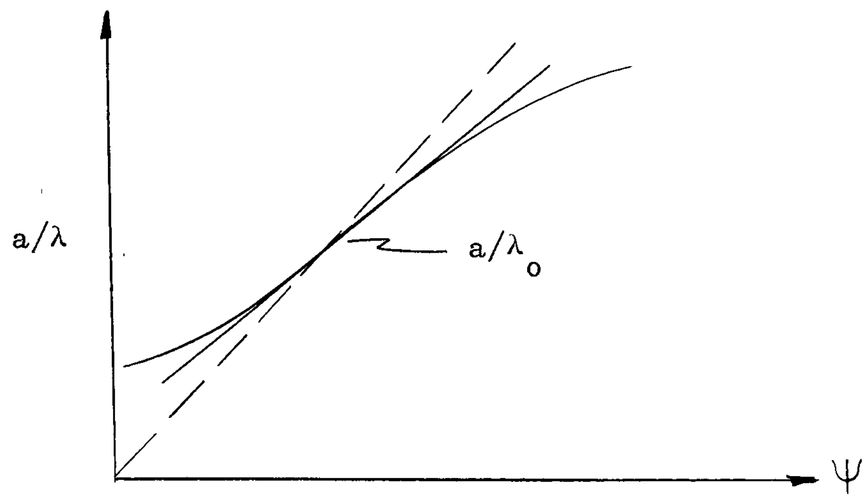

Figure 11. Typical dispersion curve showing the useful value of a/λ

The condition used for this investigation is shown graphically in Figure 11, where a/λ_0, the point in question, denotes the value of a/λ corresponding to the center of the linear section of the dispersion curve for the fundamental wave of the first passband. This is the point of greatest slope of the dispersion curve. Inspection of a set of such curves shows that this is also the point where the difference between the phase and group velocities is a minimum, since

$$v_{ph} = \frac{2\pi cL}{a} \left(\frac{a}{\lambda}\right) \left(\frac{1}{\Psi}\right) \tag{80}$$

$$v_g = \frac{2\pi cL}{a} \frac{d\left(\frac{a}{\lambda}\right)}{d\Psi} \tag{81}$$

It is desirable to pick the point where the above difference is a minimum because, as a result of equation (54), this requirement yields a good bandwidth condition. This fact is most easily seen graphically and is illustrated in Figure 12, where, for a given Δv_{ph}, the bandwidth depends directly upon the extent to which the dispersion curve and the constant phase velocity line are coincident.

Figures 9 and 10 yield, upon specification of a/λ_o, ω, and p/q, sufficient information to fix the dimensions of a structure and to enable, given the beam current, the gain-bandwidth properties to be calculated. To be specific, ε_r is fixed; a/λ_o, ω, and p/q are specified; $\frac{p+q}{a}$ and $\frac{v_{ph}}{c}$ are read from the graphs; and Ψ is calculated using equation (80). This procedure was used to provide data for calculations of the gain parameter and the linear gain per section for a set a/λ_o values. A number of calculations corresponding to specific values of p/q were performed for each choice of a/λ_o. Attempts were then made to find correlation between results obtained and various combinations of design parameters with the result that useful curves were obtained using q and q/a as independent variables. If the variable is q/a, the curves are valid for any frequency; whereas if the variable is q, the curves

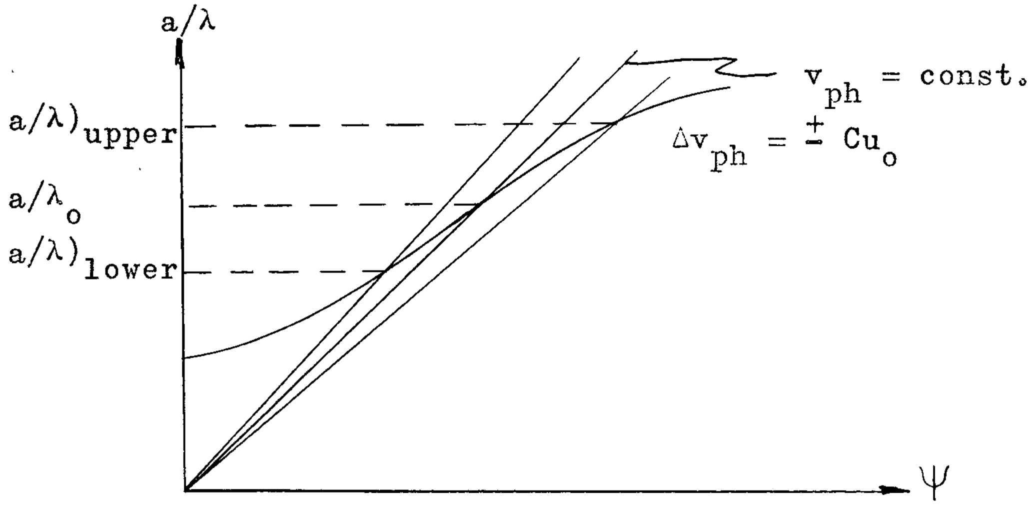

Figure 12. Graph which shows that the bandwidth is increased if, for constant C, $|v_g - v_{ph}|$ is minimized

are valid for the given frequency only. The reason for this frequency dependence is that the choice of an operating frequency only becomes necessary when a is being computed from a/λ_o. Although a^2 appears twice in the calculation previously described, it does in such a way that it cancels out when K is evaluated. Thus to transform the independent variable from q/a to q, an operating frequency must be chosen. The frequency used for this work is 35 Kmc.

The results are shown in Figures 13 to 17. Note that the points do not define a definite set of curves. The error which gives rise to this indeterminancy is due to the difficulty in correctly choosing a/λ_o for any given dispersion curve. In a number of cases, comparative choices made by several people of a/λ_o for a given dispersion curve led to

5-10% discrepancies. Although the curves are somewhat inexact, the information contained in them is sufficiently precise to enable one to rapidly choose a set of parameters that produce results which lie within a preselected range. Thus with all of the initial trial and error design attempts eliminated, one can proceed to more accurate and useful calculations with parameters which are known to give reasonably desirable results.

To illustrate the accuracy which can be expected from the design curves, a specific application will be given. If a structure is to be characterized by q = 0.008 inches and p/q = 1.33, Figures 13,14, and 15 give $a/\lambda_o = 0.44$ and the information listed in Table 1. Since ε_r, λ, and I_o are fixed and p/a, q/a, and Ψ can be calculated from the data available from the curves, a computation can be made to check the information given by the design curves. This has been done and the results are given in Table 1.

Property	Design Curve Results	Computation Check	% Discrepancy
V_o	11.5 kv	11.5 kv	———
K	5 ohms	5.63 ohms	11.2
C	11.8 (10^{-3})	11.98 (10^{-3})	1.5
$\frac{G}{L}$	0.145 $\frac{db}{sec}$	0.15 db	3.3

Table 1. Approximate computation check of a typical design curve result

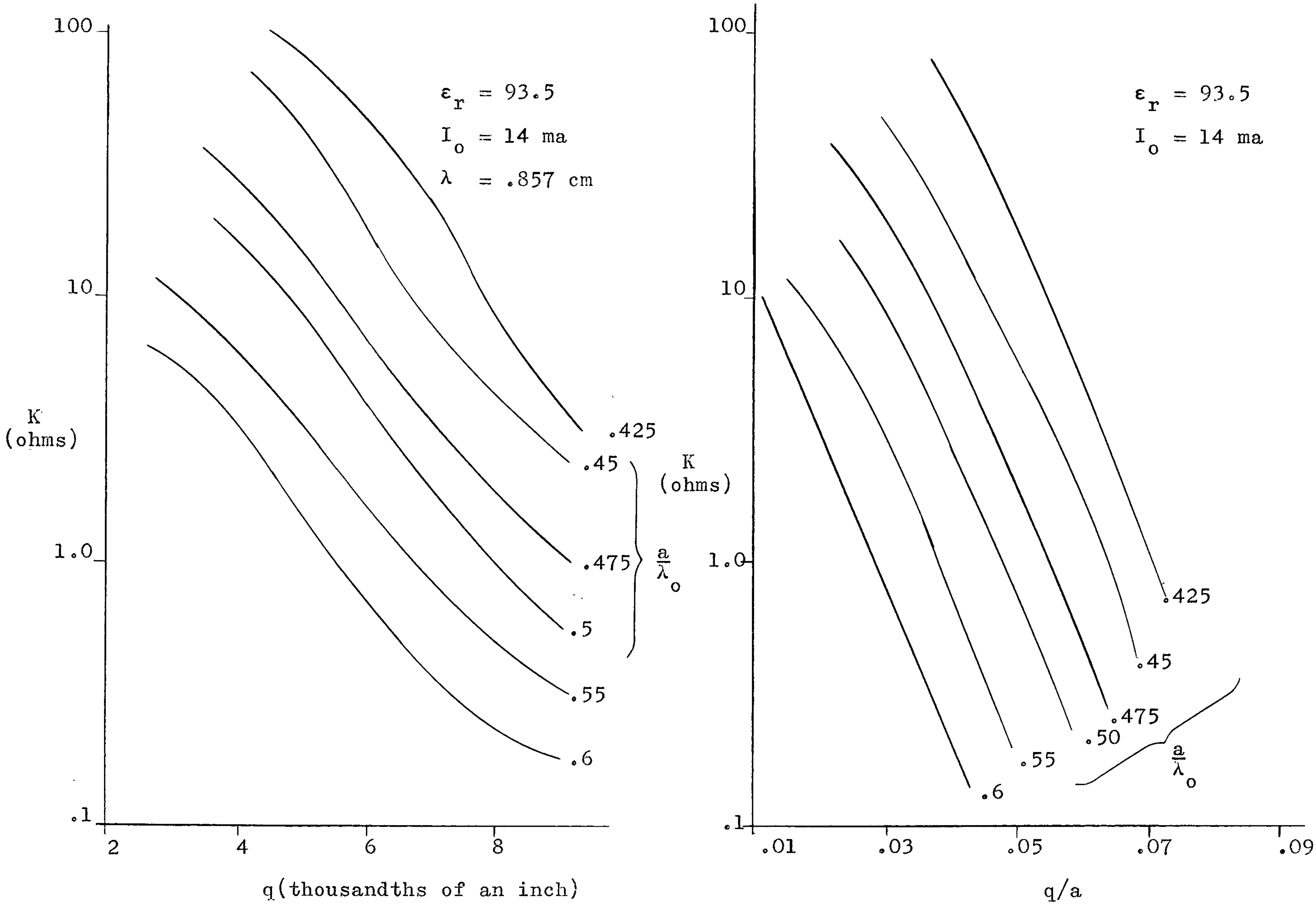

Figure 13. Graphs showing K as a function of q and q/a

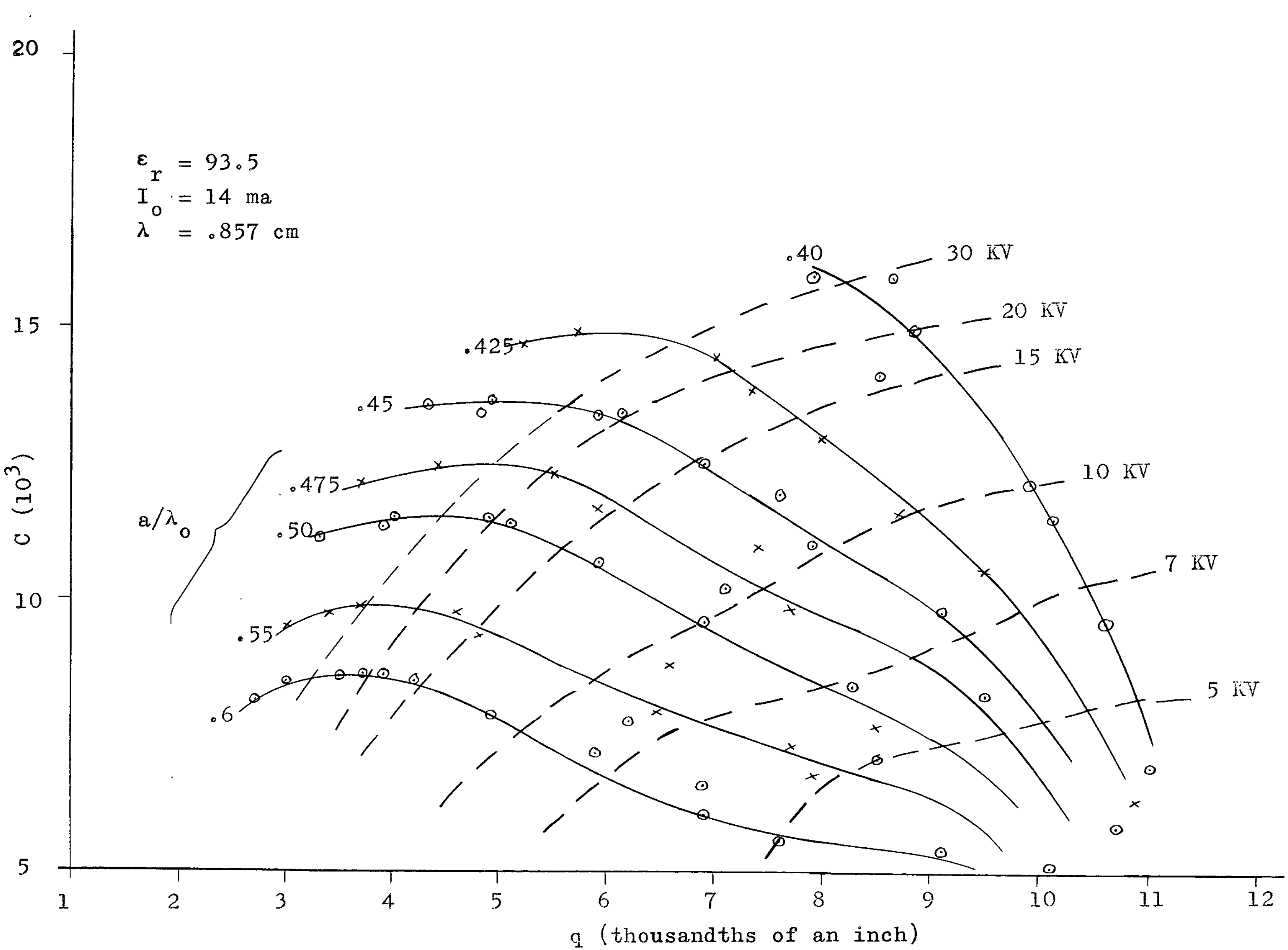

Figure 14. Graph showing C as a function of q for fixed values of a/λ_o and V_o

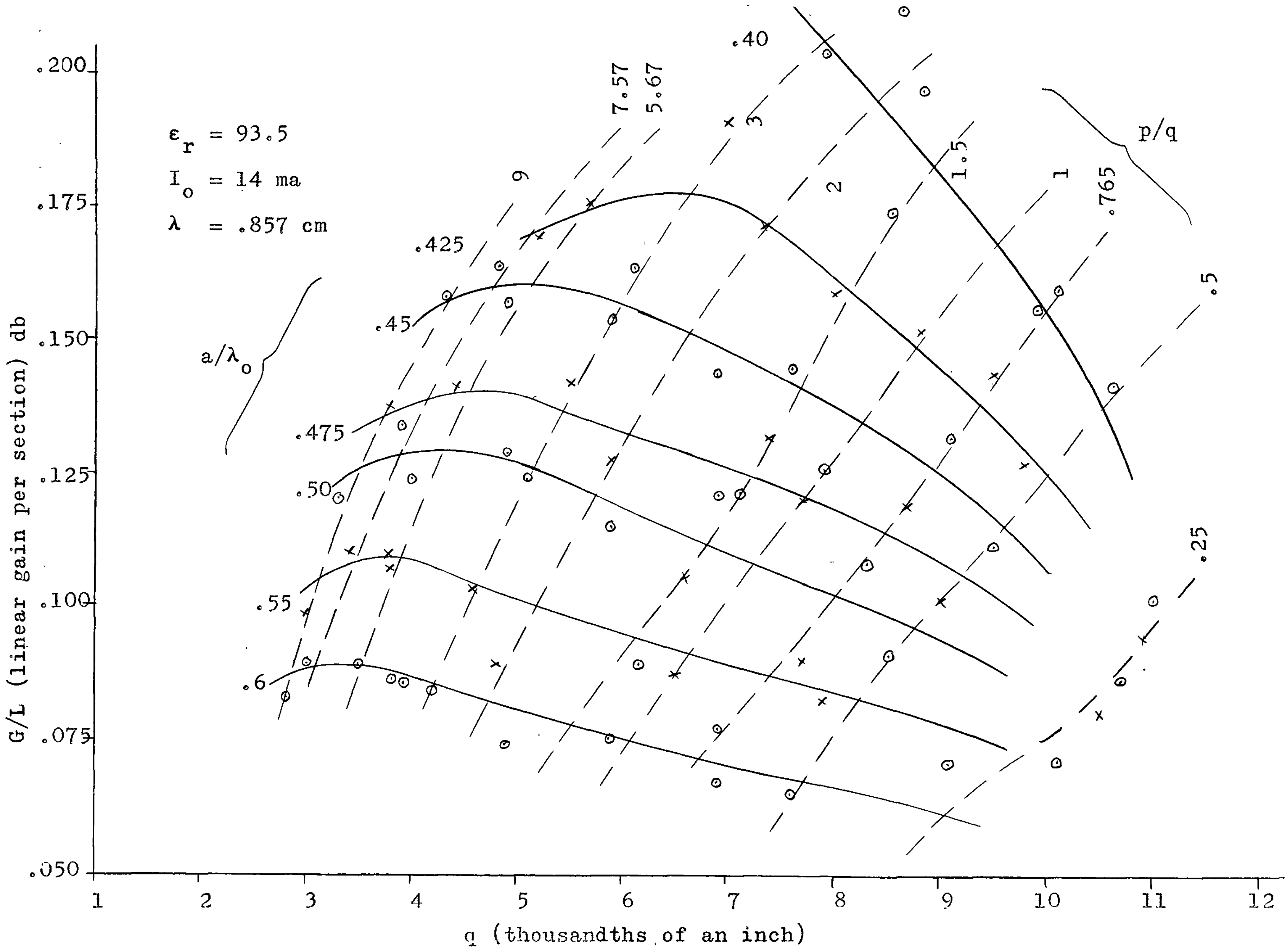

Figure 15. Graph showing G/L as a function of q for fixed values of a/λ_o and p/q

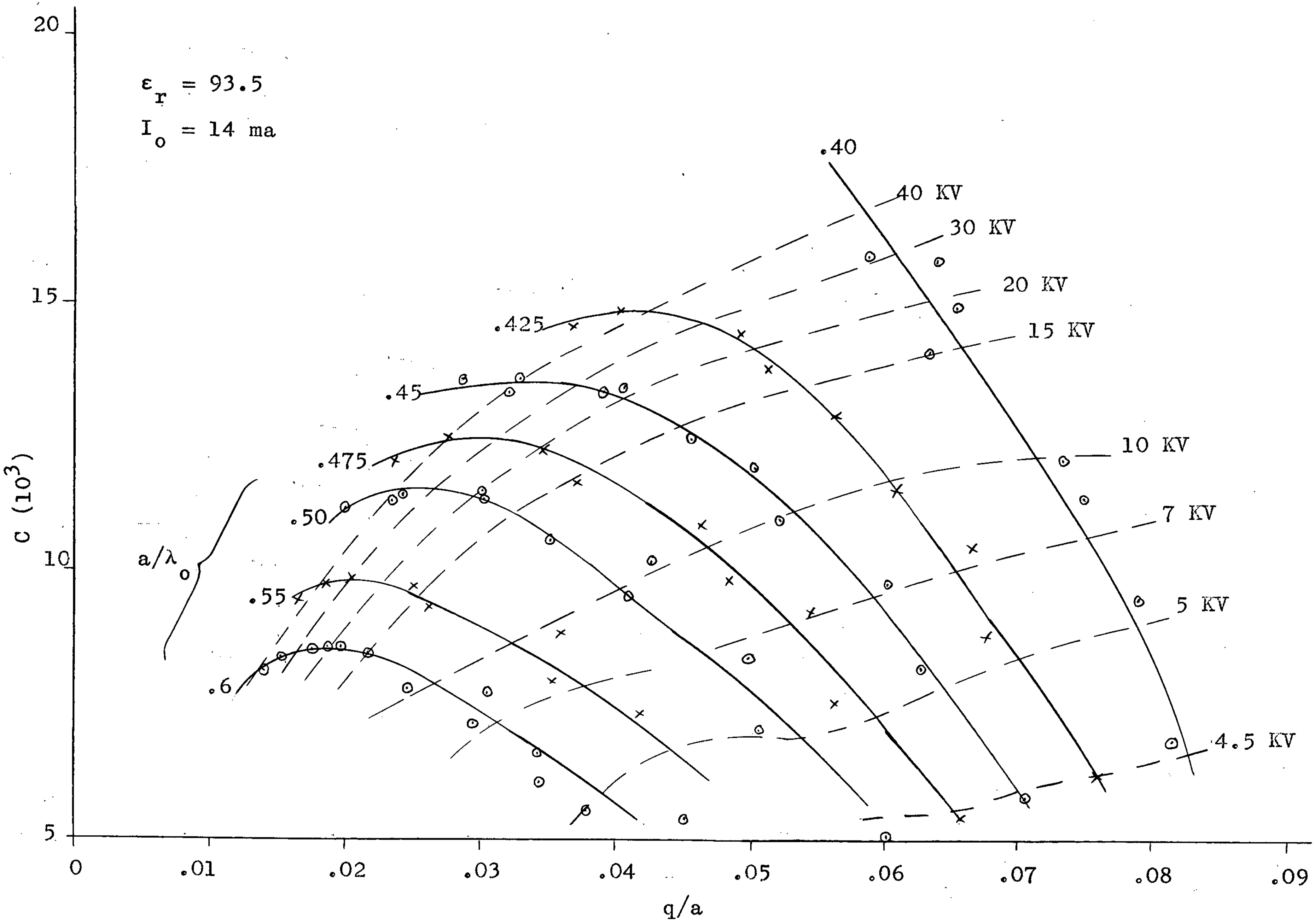

Figure 16. Graph showing C as a function of q/a for fixed values of a/λ_o and V_o

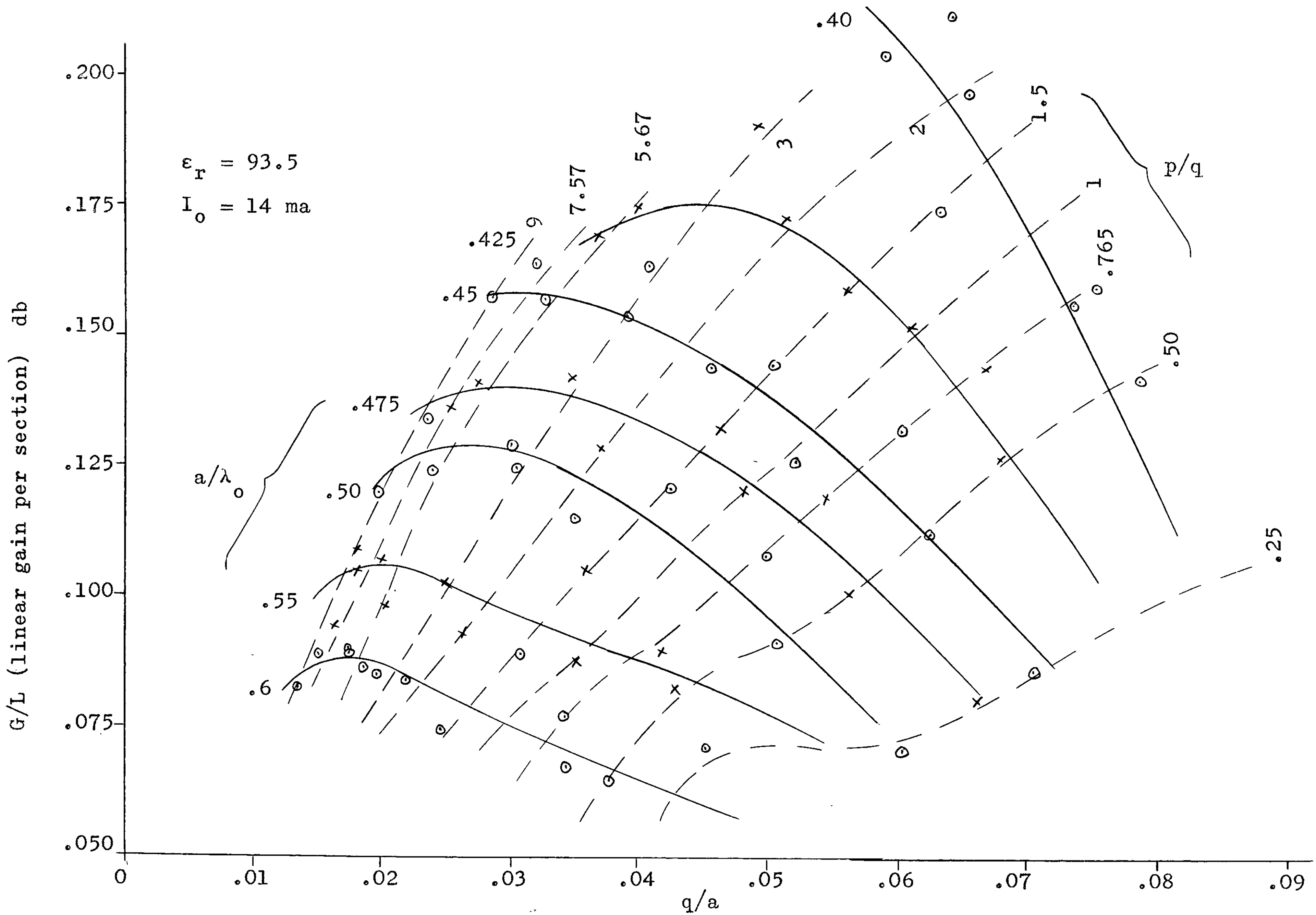

Figure 17. Graph showing G/L as a function q/a for fixed values of a/λ_o and p/q

Objections to the validity of the above comparison may be put forth since the data used for the computational check may not be exactly consistent with reality, i.e. the value of a/λ_o used may not be compatible with the values of

Property	Design Curve Results	Exact Computation Check	% Discrepancy
a/λ_o	0.44	0.44	0
V_o	11.5 kv	11.3 kv	1.8
K	5 ohms	5.552 ohms	9.9
C	11.8 (10^{-3})	11.98 (10^{-3})	1.5
$\frac{G}{L}$	0.145 $\frac{db}{sec}$	0.152 $\frac{db}{sec}$	4.6

Table 2. Exact computation check of a typical design curve result

p/a and q/a used. Such an incompatibility is due to the inherent inaccuracies of the curves and to error in reading data from the curves. A more valid comparison results if sufficient data is chosen from the initial set, given on page 47, to produce a unique dispersion curve. Thus the data for the numerical check is fully consistent with reality. Such a process has been carried out for a dispersion curve fixed by $\frac{p}{q} = 0.072$, $\frac{q}{a} = 0.054$, and $\varepsilon_r = 93.5$. The results are shown in Table 2.

The inaccuracies in the results due to the errors in choosing a/λ_0 are not as disturbing as they might appear. Even if perfect judgement were possible and the points perfectly defined a set of curves, the curves would still be in error due to the approximations used in producing expressions for the coupling impedance, the gain parameter, and the gain. For instance, the circuit loss and space charge effects were ignored. A more serious approximation was the use of the field solution for a solid disc structure.

An approximation solution for the holed disc structure was proposed by R.-Shersby-Harvis et al.[6] Their approach was to replace the actual structure by the rough equivalent shown in Figure 18. The central cylinder $(0 < r < a)$ is filled with air and represents the effect of the holes. The outer cylindrical region $(a < r < b)$ is filled with a solid anisotropic dielectric whose permittivities are chosen so as to approximate the actual structure. Analysis of this structure yields, for phase velocities less than c, an $I_0(kr)$ field variation in the central cylinder where $I_0(kr)$ is the zeroth order Bessel function for an

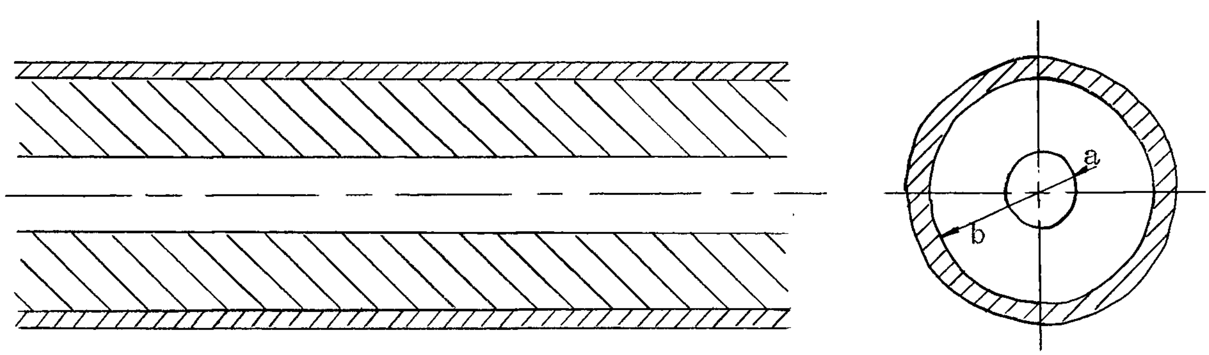

Figure 18. Anisotropic structure of R.-Shersby-Harvie et al

imaginary argument. This type of E_z field dependence upon r is undesirable for solid beam travelling wave tubes since, for maximum gain, the magnitude of the E_z field should be maximum at the center of the structure. This condition is not produced by an $I_0(kr)$ function which increases monotonically with r. These comments again accentuate the main purpose of the results herein: to eliminate the initial exploratory work in the design of travelling wave tubes using dielectric loaded slow wave structures.

5. CONCLUSIONS

An investigation has been carried out to determine the properties of dielectric loaded slow wave structures pertinent to their use in a travelling wave tube. The result of the study was the evolution of a set of design curves which relate to structure dimensions such properties as the coupling impedance, the gain parameter, and the linear gain per section for small signals.

The simple Pierce Theory for the travelling wave tube was used to perform the investigation. This theory is somewhat inaccurate due to the assumptions made in its development. Possibly the most important omission was that of neglecting the effects of the space charge of the beam. It can be shown that the theoretical results produced when space charge effects are included do not differ in form from those obtained in this thesis.[(1),(2)] However the actual numerical results for a given slow wave structure and beam current are different. The effects of circuit loss were also neglected.

The use of the wave solution for the solid disc structure gave rise to another source of error since, in practice, holes must be placed in the discs to permit electron flow. Finally the fallibility of human judgement led to errors in the choice of a/λ_0. However, the aggregate of these errors does not damage the usefulness of the design curves as instruments for initial design.

If further work were to be conducted, it would be useful to find what conditions exist to minimize the difference between the phase and group velocities at the operating point. In this way, one of the conditions for large bandwidths would be satisfied. The effect of the holes in the discs is also worthy of further study. The latter problem which is of great interest is also a difficult one if a concise and manageable solution is desired.[(6),(7)]

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