A POLARCARDIOGRAPH COMPUTER

by

WILLIAM KEITH RAE PARK

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in the Department

of

Electrical Engineering

We accept this thesis as conforming to the

standard required for candidates for the

degree of MASTER OF APPLIED SCIENCE

Members of the Department of

Electrical Engineering

THE UNIVERSITY OF BRITISH COLUMBIA

November 1954

ABSTRACT

Vectorcardiograms have proved useful in the diagnosis of heart disorders. However, such information as the variation of the magnitude and angle of the vector with time is not directly obtainable from a vectorcardiogram. An electronic device which would present the magnitude and angle of the vector as continuous functions of time, or "polarcardiograph" as it is named, would be useful in electrocardiographic research.

It is shown that such a device, which must compute the polar co-ordinates of points from their respective Cartesian co-ordinates, can be constructed if analogue multipliers, subtractors and adders are available as well as a two-phase sinusoidal voltage source and a device for generating a voltage proportional to the phase difference of two sinusoidal signals.

A search of the literature revealed that a similar device had already been constructed, the major difference between it and the present machine being the manner in which multiplication is achieved.

The principal difficulty involved in the design of the computer was the development of a simple and accurate multiplier using a pentagrid tube. A mathematical analysis of the dependence of the plate current on the two control-grid voltages was made to determine the operating conditions under which such a tube has an output voltage proportional to the product of the two input voltages.

The polarcardiograph was built using the pentagrid-tube multipliers, and when tested proved to have an overall accuracy well within that required for normal electrocardiographic purposes.

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ACKNOWLEDGMENT

The author expresses his indebtedness to members of the Department of Electrical Engineering at the University of British Columbia, especially to Dr. A.D. Moore for his guidance throughout the research project and to Dr. F. Noakes for arranging for assistance whenever it was needed.

The polarcardiograph was developed with the assistance of Dr. G.E. Dower, following his suggestion that such a device might be useful in electrocardiographic research. The major portion of the project was supported by the Medical Board Fund of the Vancouver General Hospital.

The author's post-graduate studies were made possible through the British Columbia Electric Railway Company Limited Graduate Scholarship, which he was awarded in 1953.

A POLARCARDIOGRAPH COMPUTER

I INTRODUCTION

The device to be described is an electronic analogue computer which converts the Cartesian coordinates (x,y) represented by the input voltages to the equivalent polar coordinates (r,θ) represented by the output voltages. Since the device is specially designed for electrocardiographic work, a short description of electrocardiography is in order.

Probably little can be said about the electrical voltages which can be measured on the surface of and inside a human body (or animal body for that matter) during a heartbeat without raising some controversy. Certainly one can state that these voltages and their associated current fields are generated in the heart, if only because their magnitudes rise as measurements are made closer to that organ. Since the only known form of current in living tissue is ionic movement, it can also be stated that the voltages which produce an electrocardiogram are due to ionic currents originating in the heart muscle cells.

Potential measurements made on living cells indicate that in the resting state the interior of a cell is negative with respect to its external environment, and that the potential difference across the cell membrane is about 80 millivolts. This figure is of the same order of magnitude for nearly all living cells. When the cell becomes activated the potential across the cell membrane reverses, with the interior of the cell becoming about 40 millivolts positive with respect to the external environment.

A study of the ion concentrations inside and outside of a resting cell yields the following information:

Ion Concentration Outside	Ion Concentration Inside
K^+ ... 2.5 mEq	K^+ ... 120 mEq
Na^+ ... 120 "	Na^+ ... 20 "
Cl^- ... 120 "	Cl^- ... 10 "
	and organic anions 130 milliequivalents/litre.

It has been found that potassium and chloride ions are free to cross the cell membrane of the resting cell whereas sodium and the organic anions are not. Due to their concentration difference, potassium ions will leave, making the cell interior negative. When the negative potential of the interior of the cell is sufficient to hold the potassium ions against the pressure created by their concentration difference, equilibrium results. The calculated value of this equilibrium potential for the known concentration differences agrees fairly well with the experimental value of -80 millivolts.

Activation of the cell is thought to involve a change in permeability of the cell membrane which allows sodium ions to cross it at about two hundred times the speed with which potassium ions may cross it. Sodium ions then rush in, causing the interior of the cell to become positive. This positive condition is maintained for a short period during which the membrane permeability to both ions is relatively low; then the permeability to potassium ions rises and potassium ions leave the cell thus re-establishing the negative potential difference across the cell membrane. In muscle cells the changes in the cell potential seem to be directly related to changes in the tendency of the contractile elements (actomyesin) to contract. In order that the cell may be returned to the exact state that it was in before activation, the sodium ions that entered during the activation period must be removed. The removal of these sodium ions against the concentration pressure difference is accomplished by some metabolic process known as a "sodium pump".[1] The sodium pump thus provides the energy of activation, although not necessarily the energy of contraction.

The ions flowing into and out of a cell constitute an electric current. This current, if allowed to flow through the remainder of the body, would produce voltage drops across its various parts. Although the genesis of these electrocardiographic voltages is controversial, it is agreed that they must ultimately arise from these ionic currents in the heart muscle cells.

The normal electrocardiogram, as shown in Figure 1, consists of a slow small-amplitude P wave, a more rapid QRS complex, and a slow T wave. The P wave arises from auricular activity and the QRS from ventricular activity. The T wave is associated with the recovery process of the ventricles. An electrocardiogram, such as the one illustrated, is obtained by amplifying the voltage obtained between two electrodes attached to the patient and recording the voltage variation as a function of time. Electrode connections are normally made to each arm and to one leg (usually the left leg) as well as to the front of the left chest and occasionally to the back of the right chest.

Electrocardiograms obtained from these leads are used to diagnose disorders of the heart. The most accurate information present in an electrocardiogram is the rate and rhythm of the heartbeat. Any other information obtained must come from comparison of the wave shape under consideration with the shape of waveforms whose corresponding heart disorder has already been correlated with clinical and post-mortem studies. Such correlation has been carried out over a period of more than thirty years and the results are useful in determining such information about the heart as the origin of the initiating pulse or "pacemaker", the speed and direction of propagation of the activation potentials to the ventricles and the state of the ventricular muscle. Disorders of the ventricular muscle diagnosed by such means include enlargements of either of the ventricles, death or injury of portions of the muscle and abnormalities of the muscle physiology such as deficient oxygen supply, which might result from impairment of the circulation to the heart muscle, or electrolyte disturbances, which might be produced by kidney disease.[2]

Since improved correlation can be obtained by placing more leads on the patient, the best results might be expected from the greatest number of leads. A vectorcardiogram, which may be thought of as being equivalent to an infinite number of electrocardiograms, does in fact yield information not present in an electrocardiogram.

A vectorcardiogram is generated by suitably choosing two patient leads and considering one lead voltage to be the

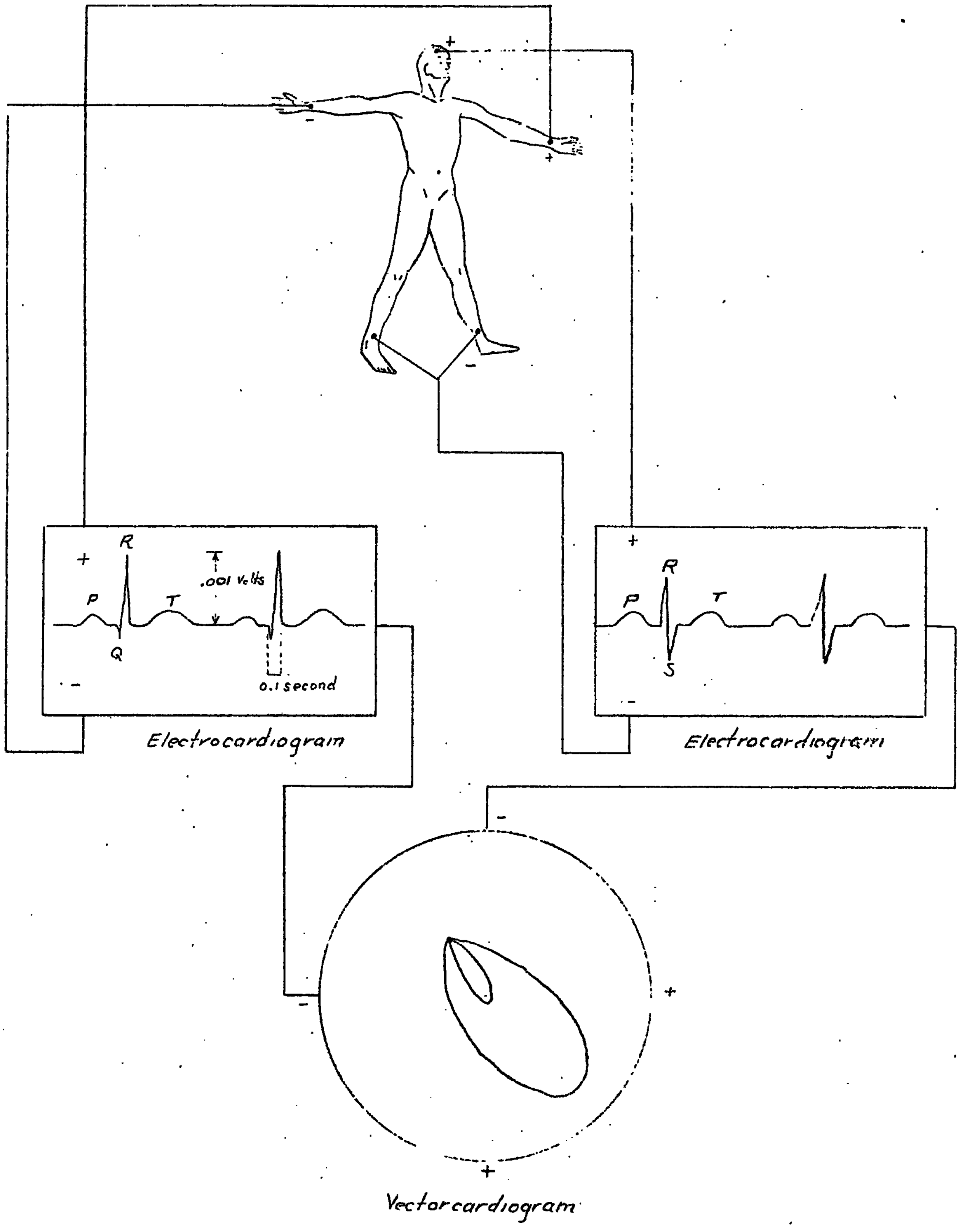


Figure 1
Schematic Electrocardiograph Arrangements

horizontal and the other the vertical component of a vector drawn from the origin in Cartesian coordinates, as shown in Figure 1. The locus of the tip of this vector is called a vectorcardiogram.[3,4] The logic for such a procedure is supported by the heart-vector theory, which considers the possibility of replacing the heart's electrical activity by a current dipole of variable strength and orientation. The dipole, of course, may be replaced by a vector. The conduction of the current of this dipole throughout the remainder of the body is then considered to give rise to the voltage drops which produce an electrocardiogram.

A vectorcardiogram is obtained at the present time by driving the horizontal and vertical deflection plates of a cathode-ray tube with the horizontal and vertical components of the vector respectively. The resulting vector locus displayed on the face of the tube is photographed during one heartbeat.

One disadvantage of the vectorcardiogram obtained in this way immediately becomes obvious when it is considered that there is no time separation of the events of the tracing. Much overlapping occurs, especially near the origin, so that valuable information about the recovery phase of the heart cycle, which is characterized by lower frequency components and lower voltages, is lost. Another disadvantage is that only one cycle is obtained, giving no opportunity to study changes that might occur from heartbeat to heartbeat.

A device whose outputs are the polar coordinates of the vector locus as functions of time might possibly have the advantages of vectorcardiograph without its disadvantages. There is also the possibility that the clinical and post-mortem correlations might be improved by displaying the information obtained in a new way and by displaying new information such as the angular velocity of the vector. Whether or not such a "polarcardiograph" will be of value must be determined by experiment. The development of the polarcardiograph is the subject of the present thesis.

Some idea of the need for a device which will give new information about electrocardiography may be obtained by considering that there are about 8000 electrocardiograms taken annually at the Vancouver General Hospital. Of the abnormal tracings obtained, about 60% are interpreted as "nonspecific". It is hoped that the use of the polarcardiograph will reduce the size of this nonspecific group.

II PRINCIPLE OF THE POLARCARDIOGRAPH

For the purpose of analysis, assume that there exists a heart-vector voltage whose instantaneous magnitude is H (when measured at the surface of the body) at some instantaneous angle α. Further, assume that electrodes positioned correctly on the surface of the body will carry the horizontal and vertical component voltages C and V respectively, which are given by:

$$C = H \cos \alpha$$
$$V = H \sin \alpha \; .$$

It is desired to construct an electronic device which, with C and V as the input voltages, will compute output voltages proportional to:

$$H = \sqrt{C^2 + V^2}$$
$$\text{and } \alpha = \tan^{-1} \frac{V}{C} \; .$$

The problem is solved by first generating two sinusoidal signals which are equal and constant in amplitude but 90° out of phase. Denote these signals by:

$$e_1 = E \sin(\omega t+\theta)$$
$$e_2 = E \sin(\omega t+\theta+90^\circ) = E \cos(\omega t+\theta) \; .$$

Multiply e_1 and e_2 by C and V respectively and add the products. The resulting sum S is given by:

$$
\begin{aligned}
S &= Ce_1 + Ve_2 \\
&= E\sin(\omega t+\theta)\cdot H\cos\alpha + E\cos(\omega t+\theta)\cdot H\sin\alpha \\
&= HE\sin(\omega t+\theta+\alpha)\,.
\end{aligned}
$$

This is the desired result since the magnitude of S is proportional to the magnitude of H and the phase of S is different from that of e_1 by the heart-vector angle α. The form of the computor is shown in the block diagram of Figure 2, where for convenience θ has been chosen as 45°.

A search of the literature revealed that a similar device had already been constructed by R. McFee.[5] The difference in the two devices lies in the manner in which multiplication is achieved. Multiplication in McFee's computer is performed by tubes with a single control-grid in a transformer-coupled circuit normally known as a balanced modulator, whereas the present device uses tubes with two control grids and hence does not require isolation transformers. It is hoped that the use of this form of multiplier will result in a device which, while being more compact and less costly to build, will at the same time be more accurate.

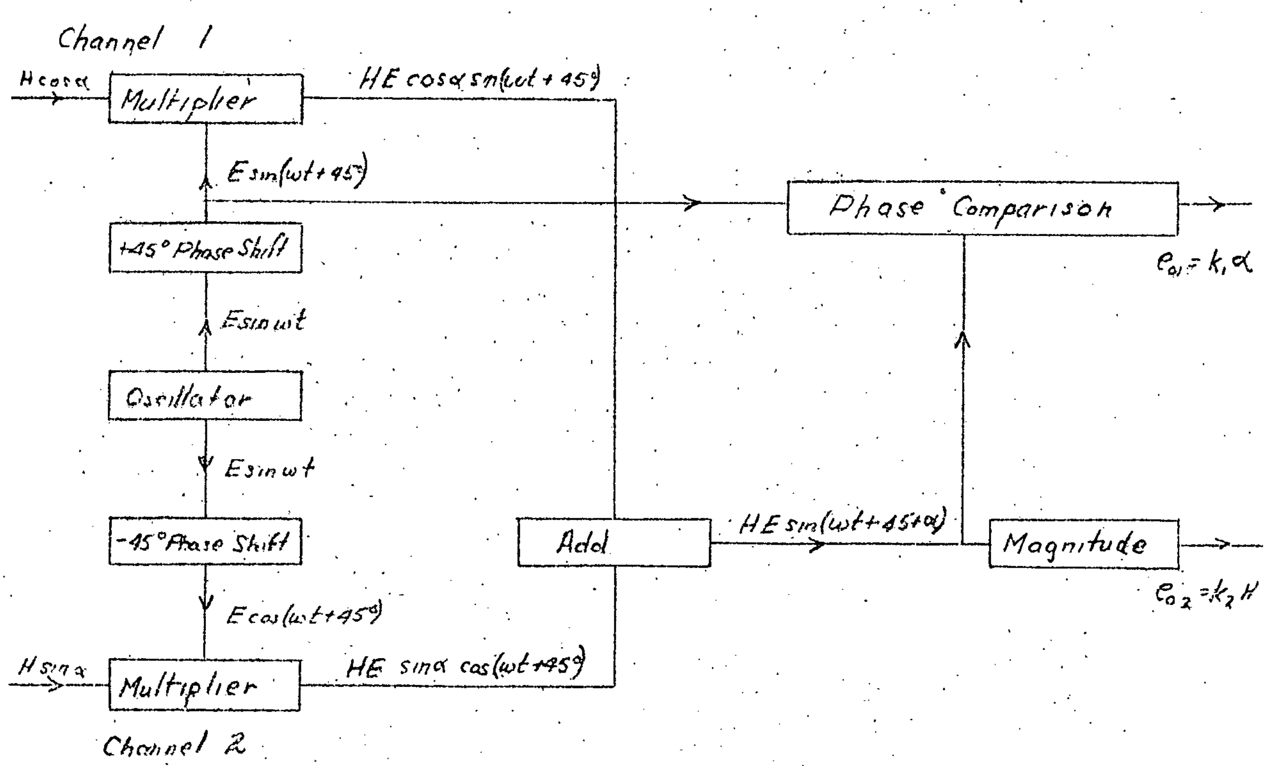


Figure 2
Schematic Computer

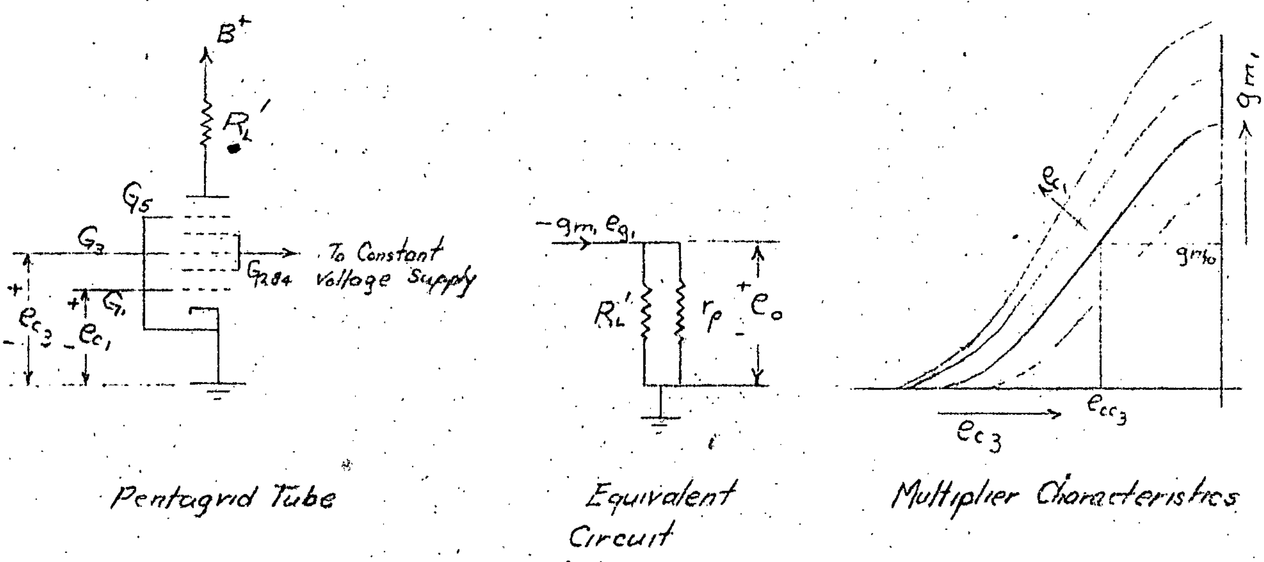


Figure 3
Pentagrid Tube Circuit

III THE MULTIPLIER

Pentagrid tubes of the type normally used for frequency conversion in radio receivers have effectively two control grids, called G_1 and G_3 in conventional notation as shown in Figure 3(a). If an operating point exists where the transconductance of the first grid is linearly dependent upon the signal applied to the third grid, e_{c3}, as illustrated in Figure 3(c), then the use of the tube as an analogue multiplier is possible.

The equivalent circuit of the pentagrid tube with a resistive load R_L' is shown in Figure 3(b). The output voltage, e_o, for constant e_{c3} ($=E_{cc3}$) and for constant screen-grid (grids 2 and 4) supply voltage, in response to a signal voltage e_{g1}, is given by:

$$e_o = -g_{m1}\, e_{g1}\, R_L$$

$$\text{where } R_L = \frac{R_L' \, r_p}{R_L' + r_p} \;.$$

If g_{m1} follows the linear law illustrated in Figure 3(c), then g_{m1} may be expressed by the relation:

$$g_{m1} = g_{m1o} + K(e_{c3} - E_{cc3})$$

$$= g_{m1o} + Ke_{g3} \;.$$

Inserting this expression for g_{m1} in the expression for e_o yields:

$$e_o = -(g_{m1o} + Ke_{g3})e_{g1}\, R_L$$

$$= -g_{m1o}\, e_{g1}\, R_L - Ke_{g1}\, e_{g3}\, R_L \;.$$

The second term in the above expression, $-\, Ke_{g1}\, e_{g3}\, R_L$, is the desired product output. When there is no signal applied to the third grid an output voltage $-g_{m1o}\, e_{g1}\, R_L$ remains. This zero signal output voltage must be cancelled by a second signal. If a second similar tube is operated with the same signal applied to G_1, and if the outputs of the two tubes are then subtracted, only the product term will remain. The signal applied to G_3 of the second tube could obviously be zero for satisfactory performance, but a more detailed analysis shows that making it equal and opposite to that applied to G_3 of the first tube gives optimum performance.

A mathematical analysis is required to determine what effect the dependence of g_{m1} on e_{c1} will have on the output and what shape the g_{m1} surface must have if accurate multiplication is to be obtained. With a plate resistance r_p such that $r_p >> R_L'$, and with the screen-grid voltage a constant, the plate current i_b is a function of e_{c1} and e_{c3} only. That is,

$$i_b = i_b(e_{c1}, e_{c3}).$$

Define E_{cc1}, E_{cc3}, as the operating-point grid voltages and express the functional relationship above in the form:

$$i_b = i_b(E_{cc1} + e_{g1},\; E_{cc3} + e_{g3})$$

where e_{g1} and e_{g3} are the signal voltages applied to G_1 and G_3 respectively. Expanding the above expression as a Taylor series expansion of a function of two variables yields:

$$i_b = i_b(E_{cc1}, E_{cc3}) + e_{g1}\frac{\partial i_b}{\partial e_{c1}} + e_{g3}\frac{\partial i_b}{\partial e_{c3}}$$

$$+ \tfrac{1}{2}e_{g1}\frac{\partial^2 i_b}{\partial e_{c1}^2} + e_{g1}\ e_{g3}\frac{\partial^2 i_b}{\partial e_{c1}\ e_{c3}}$$

$$- \tfrac{1}{2}e_{g3}^2\frac{\partial^2 i_b}{\partial e_{c3}^2} + \ldots\ldots\ldots$$

The quantities $\frac{\partial i_b}{\partial e_{c1}}$ and $\frac{\partial i_b}{\partial e_{c3}}$ are the transconductances g_{m1} and g_{m3} for grids G_1 and G_3 respectively. The incremental component of the plate current i_p is the only part that is of interest. Inserting the definitions for transconductance in the expression for i_p gives:

$$i_p = e_{g1}\ g_{m1} + e_{g3}\ g_{m3}$$

$$+ \tfrac{1}{2}e_{g1}^2\frac{\partial}{\partial e_{c1}}g_{m1} + e_{g1}\ e_{g3}\frac{\partial}{\partial e_{c3}}g_{m1}$$

$$+ \tfrac{1}{2}e_{g3}\frac{\partial}{\partial e_{c3}}g_{m3} + \frac{1}{6}\ e_{g1}^3\frac{\partial^2}{\partial e_{c1}^2}g_{m1}$$

$$+ \tfrac{1}{2}e_{g1}^2\ e_{g3}\frac{\partial^2}{\partial e_{c1}e_{c3}}g_{m1} + \tfrac{1}{2}e_{g1}\ e_{g3}^2\frac{\partial^2}{\partial e_{c3}^2}g_{m1}$$

$$+ \frac{1}{6}e_{g3}^3\frac{\partial^2}{\partial e_{c3}^2}g_{m3} + \ldots\ldots\ldots$$

The voltage output from the tube is given by:

$$e = -i_p\ R_L = -i_{p1}\ R_L \quad .$$

A second similar tube is required to cancel the zero signal output voltage $-g_{m1}\, e_{g1}\, R_L$ obtained when $e_{g3} = 0$. The signal applied to G_1 of this second tube must be e_{g1} if cancellation of the output from the first tube is to be achieved. The signal applied to G_3 of the second tube is yet to be determined but it will be related to that applied to G_3 of the first tube and hence will be denoted by ke_{g3}. The plate current of this second tube will be:

$$
\begin{aligned}
i_{p2} = {} & e_{g1}\, g_{m1} + ke_{g3}\, g_{m3} \\
& + \tfrac{1}{2}e_{g1}^2 \frac{\partial}{\partial e_{c1}} g_{m1} + ke_{g1}\, e_{g3} \frac{\partial}{\partial e_{c3}} g_{m1} \\
& + \tfrac{1}{2}k^2\, e_{g3}^2 \frac{\partial}{\partial e_{c3}} g_{m3} + \cdots\cdots
\end{aligned}
$$

If the two output voltages are subtracted the resultant voltage is expressed as:

$$
\begin{aligned}
e_{o1} - e_{o2} &= - R_L(i_{p1} - i_{p2}) \\
&= - R_L\Big\{e_{g3}\, g_{m3}(1-k) + e_{g1}\, e_{g3}(1-k)\frac{\partial}{\partial e_{c3}} g_{m1} \\
&\quad + \tfrac{1}{2}(1-k^2)\, e_{g3}^2 \frac{\partial g_{m3}}{\partial e_{c3}} + \tfrac{1}{2}(1-k)e_{g1}^2\, e_{g3} \frac{\partial^2 g_{m1}}{\partial e_{c1} e_{c3}} \\
&\quad + \tfrac{1}{2}(1-k^2)\, e_{g1}\, e_{g3}^2 \frac{\partial^2}{\partial e_{c3}^2} g_{m1} \\
&\quad + \tfrac{1}{6}e_{g3}^3\, (1-k^3) \frac{\partial^2}{\partial e_{c3}^2} g_{m3} + \cdots\cdots\Big\}.
\end{aligned}
$$

Obviously, the output voltage will be a product of the input voltages only if the term $e_{g1}\, e_{g3}\, (1-k) \frac{\partial}{\partial e_{c3}} g_{m1}$ exists and if all other terms in the above expression vanish. The requirement that this term exist means that the transconductance g_{m1} must be dependent upon e_{c3} at the operating point. If all of the remaining terms are to vanish independently of the form of e_{g1} and e_{g3}, severe restrictions will be placed on the required tube characteristics at the operating points. In particular, it is necessary that $g_{m3} = 0$, which is never true for a pentagrid tube. Therefore, some restrictions must be placed on the allowable frequency components contained in e_{g1} and e_{g3}. For the purpose of analysis consider:

$$e_{g1} = E_1 \sin \omega t$$

$$e_{g3} = E_3 \sin \theta t.$$

The difference voltage becomes:

$$\begin{aligned} e_{o1} - e_{o2} = - R_L \Big\{ & E_3 \sin \theta t\, g_{m3}(1-k) \\ & + E_1 \sin \omega t\, E_3 \sin \theta t\, (1-k) \frac{\partial}{\partial e_{c3}} g_{m1} \\ & + \tfrac{1}{2}(1-k^2)\, E_3^2 \sin^2 \theta t \frac{\partial g_{m3}}{\partial e_{c3}} \\ & + \tfrac{1}{2}(1-k)\, E_1^2 \sin^2 \omega t\, E_3 \sin \theta t \frac{\partial^2}{\partial e_{c1} e_{c3}} g_{m1} \\ & + \tfrac{1}{2}(1-k^2)\, E_1 \sin \omega t\, E_3^2 \sin^2 \theta t \frac{\partial^2}{\partial e_{c3}^2} g_{m1} \\ & + \frac{1}{6}(1-k^3)\, E_3^3 \sin^3 \theta t \frac{\partial^2}{\partial e_{c3}^2} g_{m3} + \cdots\cdots \Big\} . \end{aligned}$$

After simplification, the output voltage is:

$$e_{o1} - e_{o2} = - R_L\Big\{(1-k)\ g_{m3}\ E_3 \sin \theta t$$

$$+ \tfrac{1}{2}(1-k)E_1E_3\left[\cos(\omega t+\theta t)-\cos(\omega t+\theta t)\right] \frac{\partial}{\partial e_{c3}}\, g_{m1}$$

$$+ \tfrac{1}{4}(1-k^2)\ E_3^2\ \left[1-\cos 2\theta t\right] \frac{\partial g_{m3}}{\partial e_{c3}}$$

$$+ \tfrac{1}{4}(1-k)E_1^2E_3\left[1-\cos 2\omega t\right] \sin \theta t\ \frac{\partial^2}{\partial e_{c1} e_{c3}}\, g_{m1}$$

$$+ \tfrac{1}{4}(1-k^2)E_1E_3^2\ \sin\omega t\left[1-\cos 2\theta t\right] \frac{\partial^2}{\partial e_{c3}^2}\, g_{m1}$$

$$+\tfrac{1}{24}(1-k^3)E_3^3\left[3 \sin \theta t-\sin 3\theta t\right] \frac{\partial^2}{\partial e_{c3}^2}\, g_{m3}$$

$$+ \ldots\ldots\ldots \Big\} .$$

Assume that $\omega >> \theta$ and that a bandpass filter with band centre at ω can be built to attenuate voltages at frequencies represented by 0, θ, 2θ, 3θ, and 2ω, 3ω, and so forth, to negligible values. The voltage output from this filter is given by:

$$e = - R_L\Big\{\tfrac{1}{2}(1-k)\ E_1E_3\left[\cos(\omega t-\theta t) - \cos(\omega t+\theta t)\right] \frac{\partial}{\partial e_{c3}}\, g_{m1}$$

$$+ \tfrac{1}{4}(1-k^2)\ E_1E_3^2\ \left[\sin\omega t -\cos 2\theta t \sin \omega t\right] \frac{\partial^2}{\partial e_{c3}^2}\, g_{m1}$$

$$+ \tfrac{1}{8}E_1^3E_3\ \sin \omega t\ \sin \theta t(1-k)\ \frac{\partial^3}{\partial e_{c1}^2 e_{c3}}\, g_{m1}$$

$$+ \frac{1}{24} E_1 E_3^3 (1-k^3) \sin \omega t \left[3 \sin \theta t - \sin 3\theta t\right] \frac{\partial^3}{\partial e_{c3}^3} g_{m1}$$

$$+ \ldots\ldots\ldots \Big\} .$$

The signal applied to G_3 of the second tube is given as ke_{g3}. If no signal were applied to this grid, k in the above expression for the output voltage would be zero. This form of operation might be called unbalanced operation as opposed to the balanced or double-ended operation obtained when the signal applied to G_3 of the second tube is equal and opposite to that applied to G_3 of the first tube (i.e., $k = -1$).

For unbalanced operation, it is required that

$$\frac{\partial^2}{\partial e_{c3}^2} g_{m1} = \frac{\partial^3}{\partial e_{c3}^3} g_{m1} = \frac{\partial^n}{\partial e_{c3}^n} g_{m1} = 0$$

where n is an integer greater than unity. This condition will be satisified if the curves of g_{m1} plotted against e_{c3} for constant values of e_{c1} are straight lines. These lines need not be parallel nor equally spaced with respect to e_{c1}, but, if they are not parallel, some additional requirements are placed upon the shape of the g_{m1} surface or upon the form of the signal voltages by terms such as the third term of the above output voltage expression, $\frac{1}{8}E_1^3 E_3 \sin \omega t \sin \theta t(1-k) \frac{\partial^3}{\partial e_{c1}^2 e_{c3}} g_{m1}$. It is easily shown that this term is one of the general set of terms represented by $\frac{\partial^n}{\partial e_{c1}^n}\left(\frac{\partial g_{m1}}{\partial e_{c3}}\right)$, n an even integer, and that if g_{m1} varies linearly with e_{c3}, it is the only type of term which remains in the filtered output voltage which does not necessarily contribute to the product. Vanishing of this type of term would be most easily obtained by requiring that the curves of g_{m1} versus

e_{c3} for constant values of e_{c1} be parallel, although this is not necessary. All that is required is that the slope of the g_{m1} curves be linearly dependent upon e_{c1}. In practice such an operating region would be difficult to locate.

However, for the special case in which e_{g1} contains only one frequency component of constant amplitude, the set of terms represented by $\frac{\partial^n}{\partial e_{c1}^n}\left(\frac{\partial g_{m1}}{\partial e_{c3}}\right)$ contribute to the product because E_1^3, E_1^5, and so forth are just constants. If e_{g1} is not of constant amplitude the error introduced by these higher order derivatives will be small if the g_{m1} surface is reasonably smooth over the operating range, since the product of the value of the derivative and its Taylor series expansion coefficient rapidly becomes negligible with respect to $\frac{\partial g_{m1}}{\partial e_{c3}}$ as more terms are considered.

For balanced operation ($k = -1$) all terms containing $(1-k^2)$, $(1-k^4)$ and so forth, vanish. Under this condition, in order to obtain an output proportional to the product of the two inputs it is required only that E_1 be constant and that

$$\frac{\partial^n}{\partial e_{c3}^n} g_{m1} = 0$$

for n an odd integer greater than unity. That is, for balanced operation, the g_{m1} curves plotted against e_{c3} need not be straight lines, a simple square law curvature, for example, being satisfactory. Certainly balanced operation will give greater accuracy than will unbalanced operation, as might be concluded from comparison of the multiplier pair with a push-pull audio amplifier.

Maximum accuracy will be obtained from the tubes by operating them balanced in the most linear portion of their characteristics.

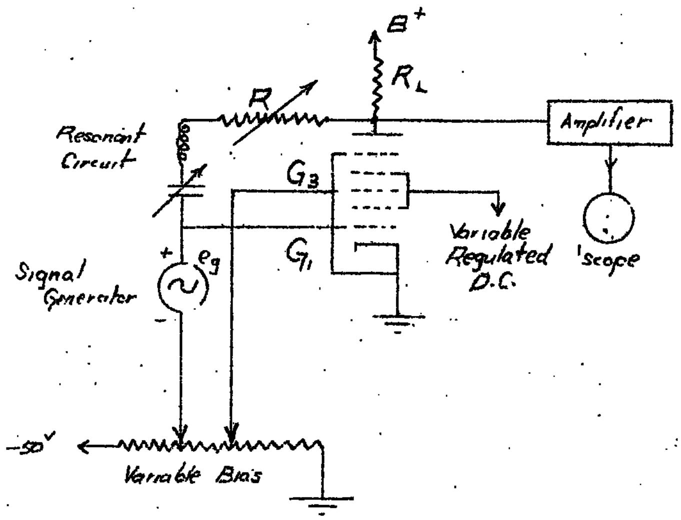
A bridge for measuring the transconductance of a tube was constructed following the circuit of Figure 4. At balance (no signal output) the relation

$$\frac{e_g}{R} = g_m e_g \qquad \text{holds, or}$$

$$g_m = \frac{1}{R} .$$

Plate voltage had little effect on the measured value of the transconductance over a swing of more than 100 volts, so an operating value of approximately 250 volts was chosen.

A large number of measurements covering the useful working range of a type 6SA7 tube were made and the useful portion of the results are displayed on the graphs of Figure 6. A suitable operating point is seen in the region $e_{c1} = -3$ to -6 volts and $e_{c3} = -1.5$ to -7.5 volts for a screen-grid voltage between 90 and 100 volts. The linearity and allowable signal levels were considered sufficient to warrant the construction of a complete multiplier consisting of two multiplier tubes and a subtractor. This unit when constructed and tested proved to be accurate enough for the polarcardiograph application.



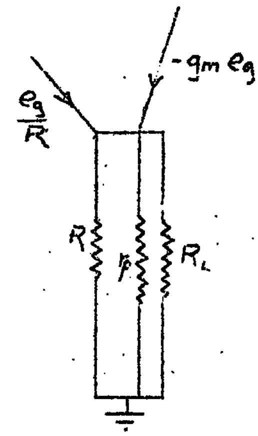
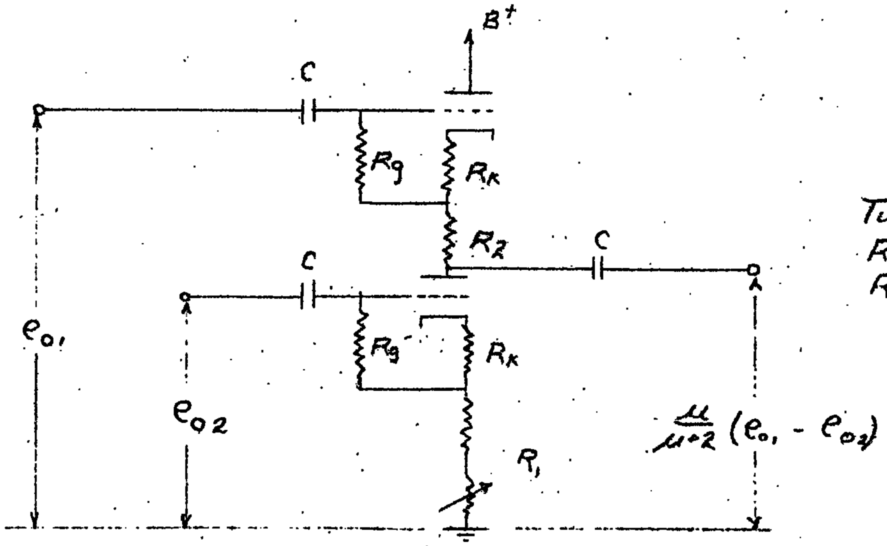
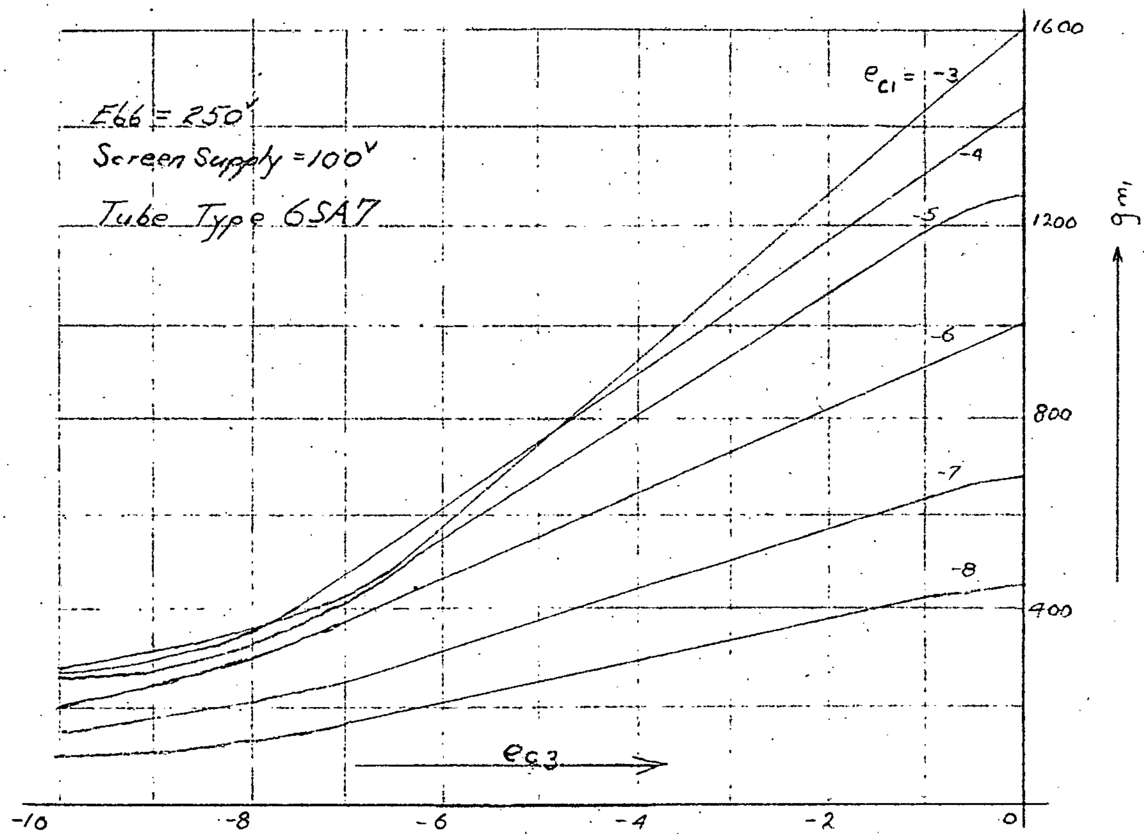


Figure 4
Transconductance Bridge



Tube: 6SN7
$R_g = 0.33^M$
$R_K = 1^K$ $R_1 = R_2 = 30^K$

Figure 5
Subtraction Circuit



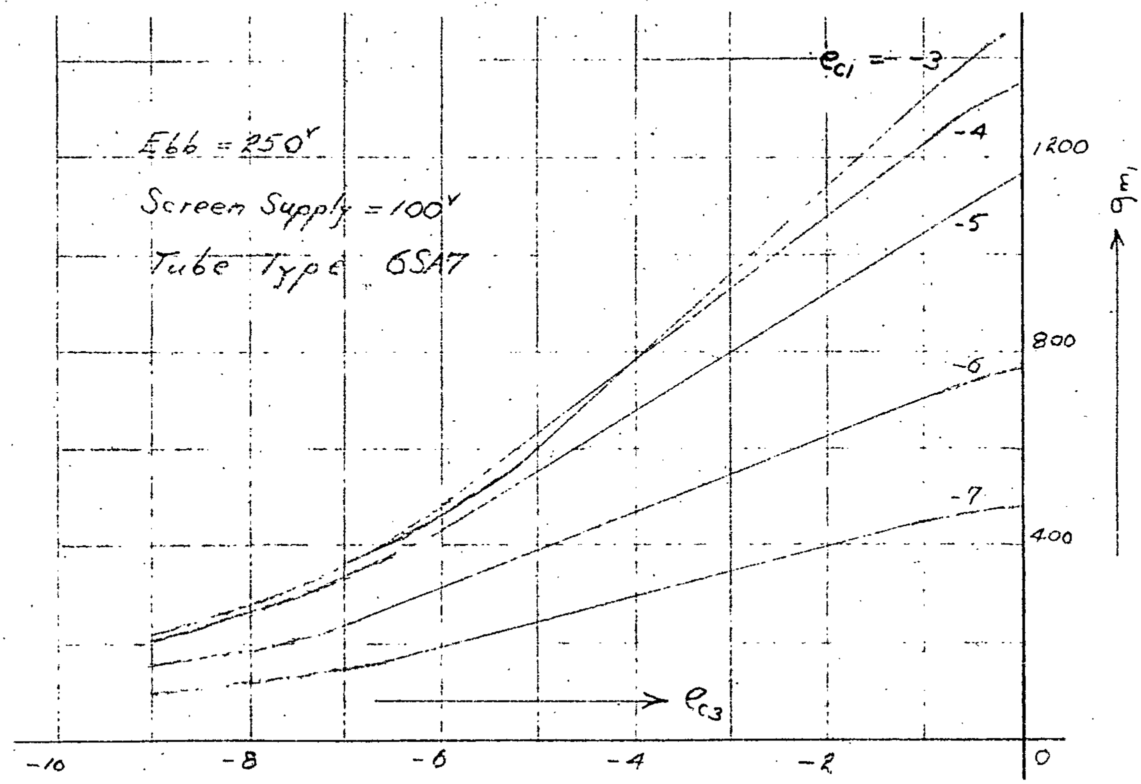


Figure 6
6SA7 Characteristics

IV THE COMPLETE COMPUTER CIRCUIT

Subtraction of the outputs from the two multiplier tubes is accomplished by the cathode-follower type of subtraction circuit shown in Figure 5. If $R_1 = R_2$, $\mu_1 = \mu_2$, $r_{p1} = r_{p2}$ and if the capacitor impedances are negligible, the output voltage is:

$$\frac{\mu}{\mu+2}(e_{o1}-e_{o2}).$$

If $r_{p1} \neq r_{p2}$, $\mu_1 \neq \mu_2$, $R_1 \neq R_2$, the output voltage is given by:

$$e_o' = \frac{-\mu_2 e_{o2}\left[(R_1+R_K)(1+\mu_1)+r_{p1}\right]+\mu_1 e_{o1}\left[(R_2+R_K)(1+\mu_2)+r_{p2}\right]}{r_{p1}+r_{p2}+(\mu_1+1)(R_1+R_K)+(\mu_2+1)(R_2+R_K)}.$$

For subtraction to be achieved the coefficients of e_{o1} and e_{o2} must be equal. This condition is satisfied when

$$\mu_2 \ (R_1+R_K)(1+\mu_1)+r_{p1} = \mu_1 \ (R_2+R_K)(1+\mu_2)+r_{p2}.$$

Rewritten in terms of R_1 this expression requires that

$$R_1 = \frac{\mu_1(1+\mu_2)}{\mu_2(1-\mu_1)} R_2 + \frac{\mu_1-\mu_2}{\mu_2(1+\mu_1)} R_K + \frac{\mu_1 r_{p2}-\mu_2 r_{p1}}{\mu_2(1-\mu_1)}$$

If R_1 is made adjustable the above condition can be satisfied. In practice this adjustment is made when $e_{o1} = e_{o2}$, that is, when the two subtractor inputs are equal. The adjustment of R_1 to give zero output voltage is referred to as "balancing the subtractor".

Because the above circuit is of the cathode-follower type, it is inherently stable against circuit parameter variations. Calculations indicate that, for the circuit values used, an unbalance of 10% between the μ's or R's produces less than 1% error in the output. Moreover, less than 10% of the input signal at any grid appears as grid-to-cathode swing, and therefore the assumption of tube linearity used in the above analysis is justified even when the signal approaches ten times the operating bias.

The complete computer would require two subtractors and one adder following the multipliers. However, the identical process can be carried out by using two adders and one subtractor as illustrated in Figure 7. The advantage is that addition can be accomplished without the use of vacuum tubes as shown in Figure 8. The two voltages to be added are represented by e_{11} and e_{22} in series with their associated internal impedences R . The two sources are coupled through isolation resistors R_i to a common resistor R_g. The voltage across R_g is given by:

$$e' = (e_{22}+e_{11}) \frac{R_g}{R+R_i+2R_g}$$

which is the required addition.

The resistor R_i is made approximately 100 times larger than the internal resistance R so that the voltage sources are independent of each other. The common resistor R_g must be made large to prevent excessive attenuation of the signal. In the present case, the input impedance of a cathode-follower isolation amplifier serves as the resistor R_g. This amplifier was also required in order to present a low-impedance source to the subtractor. When a high-impedance source is presented to

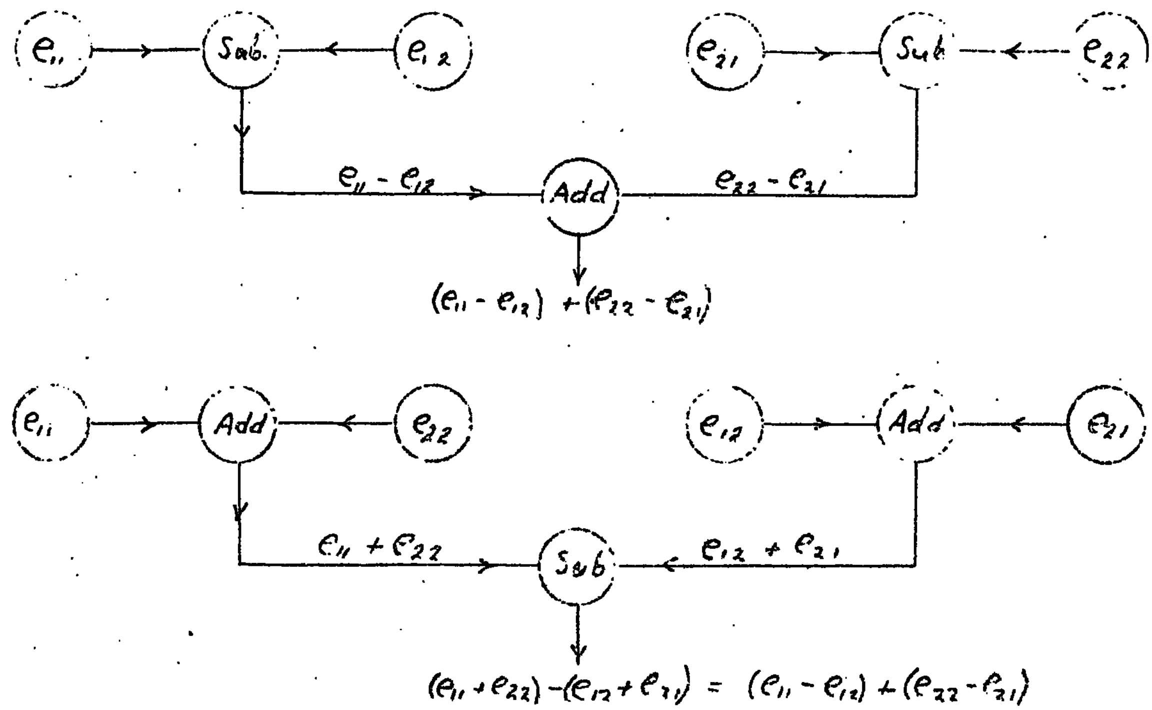


Figure 7
Two Possible Computer Arrangements

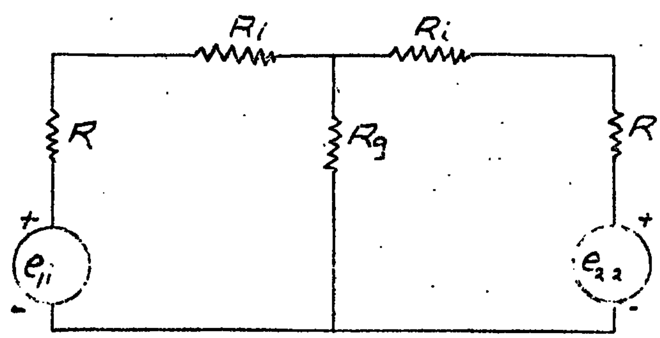


Figure 8
Resistive Addition

the subtractor it cannot be balanced solely by a variable resistor, as assumed previously, due to the effects of stray capacitance between the subtractor grids and ground. This capacitive unbalance can be compensated for by a trimmer condenser on one of the subtractor grids, but the setting of this trimmer condenser is dependent upon the setting of the variable resistor R_1 in the subtractor circuit, as well as upon the setting of trimmer condensers required in the multiplier plate circuits. Little capacitive balancing is required on the subtractor when it is operating with a low-impedance input.

In the computer circuit of Figure 9, it is seen that one plate load resistor of each multiplier pair is variable. When no signal is applied to G_3 of either tube in a pair, the output voltages from both tubes should be equal so that cancellation can take place in the subtractor. The variable plate load resistor on one tube allows the outputs to be adjusted to equality. This adjustment is referred to as multiplier balancing.

Due to phase shift caused by stray capacitance, exact multiplier balancing cannot be achieved solely by a variable resistor. A small trimmer condenser in one plate circuit of each multiplier pair is required to achieve an accurate balance. The settings of these trimmer condensers are somewhat dependent upon the settings of the variable resistors, but should normally require changing only when a multiplier tube is changed.

The mathematical analysis predicted the necessity of a band-pass filter with a pass band extending from $\omega - \theta$ to $\omega + \theta$ radians per second, where ω represents the frequency of the oscillator and θ represents the maximum frequency contained in

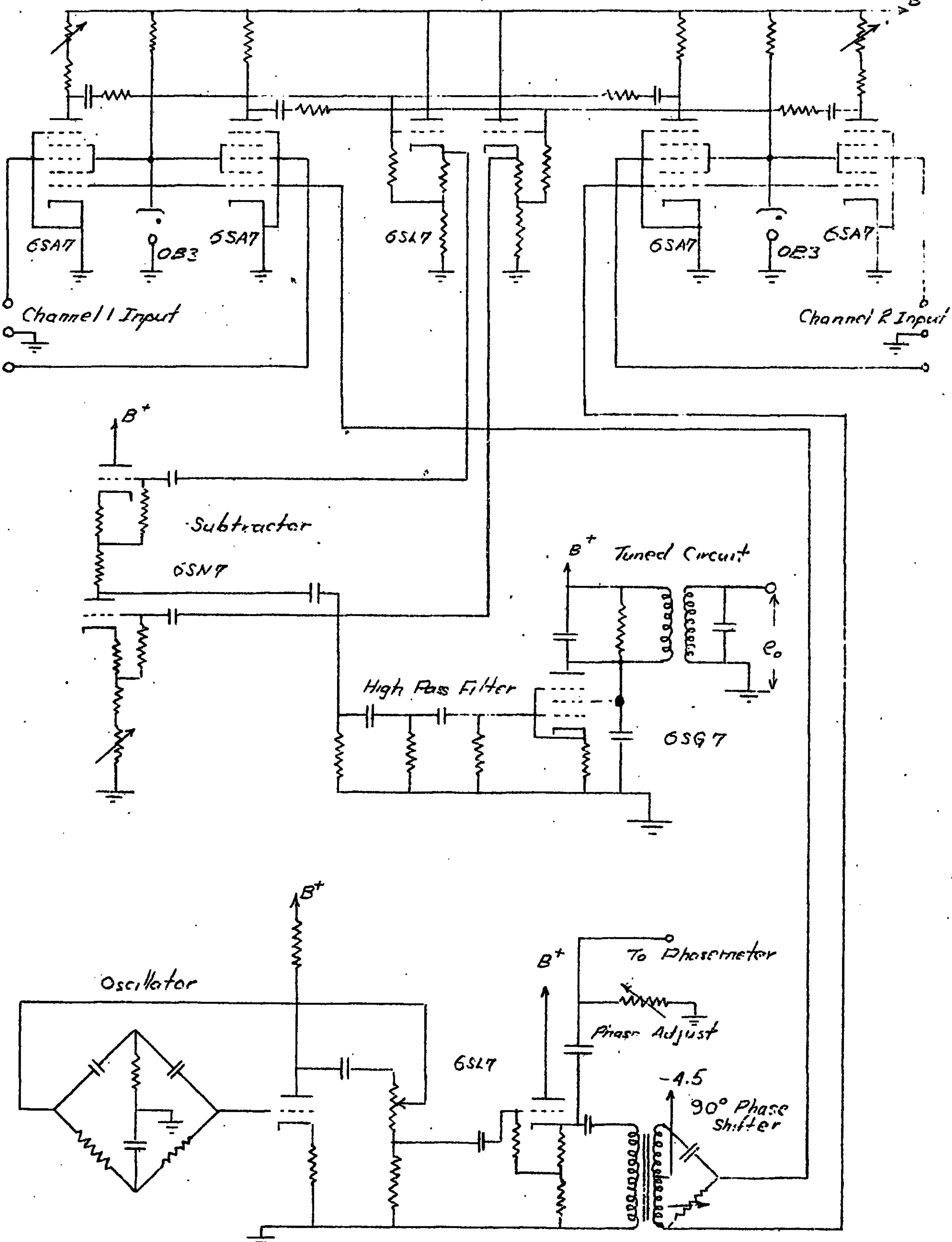


Figure 9
Computer Circuit

the patient voltages used for the horizontal and vertical inputs to the computer. The electrocardiographic voltages contain frequencies from about one cycle per second up to about 300 cycles per second, although a frequency response up to 100 cycles per second seems to be adequate for recording equipment. To allow 100 cycles per second for the electrocardiograph frequencies and $\pm$ 100 cycles per second for oscillator frequency drift requires a pass-band width of 400 cycles per second.

The filter, as shown in Figure 9, consists of a pentode-driven, double-tuned circuit whose design centre frequency is that of the oscillator. Because the phase-measuring equipment becomes increasingly difficult to build as the frequency of the input signals is raised, the oscillator frequency should be kept as low as possible. Four kilocycles per second was chosen as the lowest frequency for which a double-tuned filter circuit could be built with a reasonably flat response ($\pm 2\frac{1}{2}\%$) over the 400 cycles per second pass band.

From the mathematical analysis, the difference voltage from one multiplier pair is given by:

$$e_{o1} - e_{o2} = - R_L \Big\{ E_3 \sin \theta t \; g_{m3}(1-k) + E_1 \sin \omega t \; E_3 \sin \theta t (1-k) \frac{\partial}{\partial e_{c3}} g_{m1} + \dots\dots \Big\} .$$

The first term of the above expression is the amplified electrocardiograph input voltage. Because the transconductance g_{m3} is considerably greater than the quantity $\frac{\partial g_{m1}}{\partial e_{c3}}$, this amplified electrocardiograph voltage is greater than the desired product voltage. To prevent the large low-frequency voltage from overdriving the filter amplifier a high-pass R-C filter was placed at the subtractor output as shown in Figure 9.

Since the low-frequency components of the electrocardiograph input signals makes ac bypassing of resistors impractical, bias is supplied to the multiplier tubes from a battery, and screen voltage is supplied from voltage-regulator tubes. One regulator tube was used for each multiplier pair to prevent coupling through the screens, which results in high-frequency oscillation.

V THE OSCILLATOR

The oscillator, a resistive-capacitive twin-T type, features good frequency stability and low harmonic content. The output from the isolation amplifier is adjusted to approximately 20 volts rms by means of the potentiometer in the feedback path.

The phase shifter, as shown in Figure 9, uses a centre-tapped audio-frequency transformer with a suitable transformation ratio. Ninety-degree phase shift is obtained when the voltages across the resistor and condenser are equal. The oscillator output (see Figure 9) for the phasemeter is obtained directly from the cathode follower through an R-C phase shifter. Phase shift is required at this point to compensate for phase shift in the R-C high-pass filter in the subtractor output.

VI MAGNITUDE AND PHASE OUTPUTS

The voltage appearing at the output of the tuned circuit, which is given by HE $\sin(\omega t + 45^\circ + \alpha)$, is a sinusoidal signal of radian frequency ω whose rms value is directly proportional to the magnitude of H. If this signal is rectified and averaged, the result is directly proportional to H. Half-wave rectification is performed by a vacuum diode and averaging by the electromechanical recorder, since it can respond only to the quasi-dc component of the rectified sine wave.

One type of phasemeter which is not ambiguous about 180° is that which utilizes an Eccles-Jordan trigger circuit, or flip-flop, in its output.[6,7,8] As shown in the schematic diagram of Figure 10, the two signals whose phase is to be compared are first amplified and clipped until they are "square waves". Differentiation then produces negative- and positive-going pulses marking the sides of the square waves. The positive-going pulses are removed by a vacuum diode and the negative pulses are applied to the flip-flop. The first negative pulse will cause one tube of the flip-flop to conduct and the next negative pulse will cause it to cease conduction; hence the average conduction time of either tube will be proportional to the phase difference of the two input waves. In practice, connections are made to the plates of both tubes for balanced output since the recording instrument is equipped for balanced input. Again, no filtering is required

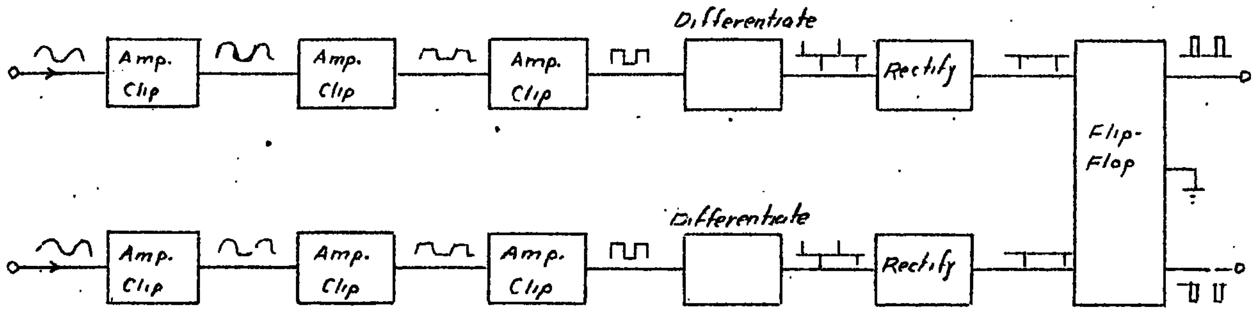


Figure 10
Schematic Phasemeter

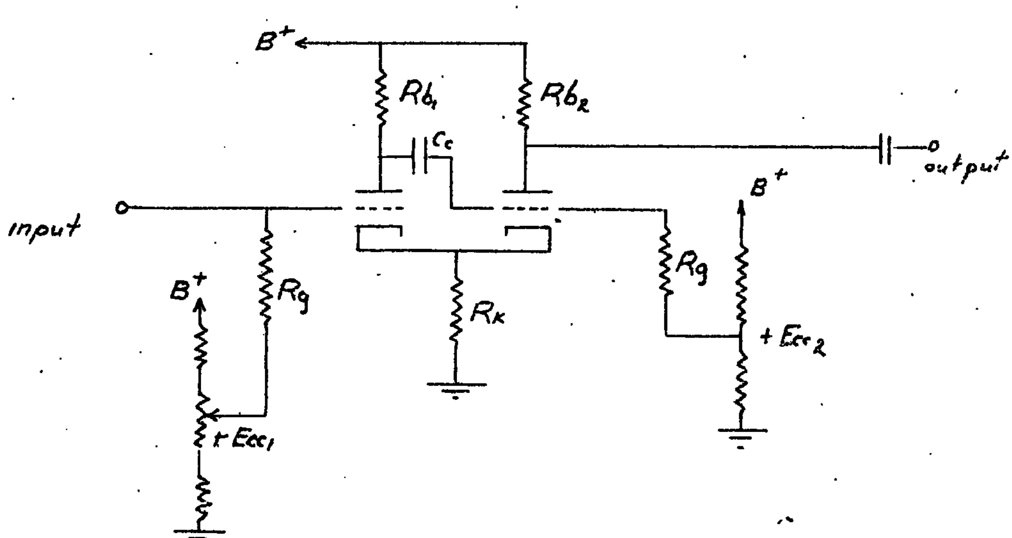


Figure 11
Cathode-Coupled Clipper Circuit

since the recorder itself is effectively an electromechanical low-pass filter.

Four kilocycles per second corresponds to a phase change of 360° in 250 micro-seconds. That is, one degree of phase change corresponds to roughly 0.7 micro-seconds. If the phase is to be measured within $\pm 2^{\circ}$ of 180°, the flip-flop must be capable of distinguishing between two pulses less than two micro-seconds apart and the pulses themselves should be less than a micro-second wide. Further, if the phase reading is to be accurate, the negative trigger pulses must mark the exact point where the sinusoidal waves cross the zero axis, going negative. This last condition can be achieved by insuring that the clipping action is balanced, that is, that both halves of the square wave are equal in width.

The cathode-coupled clipper[9], shown in Figure 11, is easily adjusted to give symmetrical clipping by changing the positive bias E_{cc1}. The regenerative feedback resistor R_{b1} increases the amplification of the circuit, thus causing clipping to take place at lower input levels. Since it is desired to have no output signal when there is no input signal, the linear and non-linear gains of the clipper must be less than unity. Three stages are cascaded in order to provide a reasonably square output wave with about 0.5 volts rms input. Decoupling of the first two stages of each channel is necessary to prevent zero-signal oscillation of the system. Differentiation is accomplished by using a series R-C differentiator with a time constant of 1×10^{-7} seconds. A high-gain pentode amplifier with a low plate load resistance raises the pulse level to approximately 50 volts. The resulting pulse width, as viewed on an oscilloscope, appears to be less than one micro-second.

The positive pulses from the differentiator are removed by a diode, the cathode of which is biased positive with respect to the plate so that the pulse generator is effectively disconnected from the trigger circuit except when a negative pulse is being applied, as shown in the complete circuit diagram, Figure 13. This arrangement prevents the low impedance of the pulse generator from loading the flip-flop.

The trigger circuit is designed for dc stability[10], the values of the resistors and cross-coupling condensers being chosen as low as possible so that minimum switching time can be obtained.

VII INPUT CIRCUITS

If the phase output is to be correct, the angle must be measured from the correct origin. Since the first stages of the electrocardiograph amplifiers are ac coupled and the average of the heart vector components is not zero, the origin must be re-established at the input to the computer. For this purpose, the computer has been equipped with a cathode-ray tube, the horizontal and vertical deflection plates of which are driven by the horizontal and vertical components of the heart vector, respectively.

With no signal applied to the computer input, the spot on the cathode-ray tube is centred. This rest point is marked on the face of the tube and the computer is balanced to give zero output. The multiplier inputs are taken from tap points on a resistor connected between the deflection plates of the cathode-ray tube. When the electrocardiograph amplifiers and a patient are connected to the computer, the cathode-ray tube displays a vectorcardiogram, the origin of which is clearly visible as a bright spot since the beam is stationary during the resting period between heart beats. The centring controls on the input amplifiers are then used to bring the origin back to the marked point on the face of the cathode-ray tube.

If a noise signal is present, it may be partially masked by deflecting the origin of the vectorcardiogram slightly to the right of the marked point on the cathode-ray tube. This procedure results in a large error in the angles associated with a small signal and correspondingly smaller error in the angles associated with a larger signal. For this reason a magnitude recording should be taken simultaneously with a phase recording so that the probable validity of the phase readings may be estimated.

VIII THE COMPLETE INSTRUMENT

Controls for balancing the subtractor, the multipliers, the input amplifiers and for equalizing the overall gains of the two channels are available to the operator. Balance is checked visually by observing an electron-ray tube which is connected to the tuned-circuit output. A rotary switch makes suitable connections for balancing purposes. The balancing procedure is carried out with no signal applied at the input, that is, the spot on the face of the cathode-ray tube must be centred.

In position "1", the rotary switch ties the two subtractor grids together. The variable resistor in the subtractor is then adjusted until the electron-ray tube indicates zero output.

In position "2", the rotary switch connects the oscillator grids, that is G_1 of both tubes of channel one, to bias. Channel two is then balanced by adjusting the variable plate load resistor of one of its tubes until there is zero output indicated by the electron-ray tube. In position "3", the rotary switch interchanges the connections to allow for balancing of channel one by a similar procedure.

In position "4" of the rotary switch, the oscillator grids of both channels are tied to one side of the phase shifter, and at the same time, equal but opposite standardized dc signals are introduced to the input amplifiers. If the input amplifiers have equal gains the spot on the face of the cathode-ray tube will move away from the origin at 45° to the horizontal. The input amplifiers are adjusted to equal gain by ganged variable resistors in the plate circuits of the input amplifiers. A line at 45° to the horizontal has been drawn on the face of the cathode-ray tube for this purpose. When the input amplifiers have been balanced (i.e., the spot on the cathode-ray tube face lies at some point on the 45° line), the overall gains of both computer channels are equalized by means of ganged variable resistors in the grid circuits of the multipliers. At balance the electron-ray tube indicates zero output.

Position "5" of the rotary switch returns all connections to normal and the machine is then ready for operation.

The preceding balancing procedures are essentially independent of each other and need be carried out only during the warm-up period. Balancing checks should be made before every recording during the first half-hour of operation.

Aside from focus and intensity controls on the cathode-ray tube, no other controls were available to the operator on the original device. However, a few trial runs showed that the machine was quite difficult to use and could not be expected to give accurate results when operated by an average technician.

When originally designed, the input amplifiers were direct-coupled to the electrocardiograph amplifiers. Drift in the electrocardiograph machines was thus amplified, and centring of the vector origin became almost impossible when drift was worse than usual. Capacitive coupling from the amplifiers, with variable time constants, was substituted for the direct coupling. When there is little drift a time constant of 16 seconds can be selected. When there is a comparatively large amount of drift, a time constant as low as ½ second can be selected enabling some results to be obtained at the expense of the low-frequency components in the electrocardiograph waveform. The operator will be required to record the time constant used.

In order to check that the operator has centred the vector origin, a push switch which temporarily short-circuits the magnitude output has been installed. The operator will be required to hold this switch down for one or two heartbeats when making a recording. The zero value of the magnitude will then show on the recording, and if the baseline of the magnitude tracing is discernible, it will be possible to tell if the machine has been zeroed correctly by inspecting the recording. The error in zeroing will also be obtainable and can be used to determine the worth of the corresponding angle recording.

The angle recording is marked at $\pm 180^\circ$, $\pm 90^\circ$, 0° by means of the "angle calibrate" switch which applies a dc signal to each grid of the input amplifiers in the appropriate order. Usually either $+180^\circ$ or -180° will be calibrated. However, instability of the flip-flop in this region may occasionally prevent either point from being marked on the recording. The angle is calibrated by holding the switch briefly in each of four positions. This can only be done after the computer has been balanced and only if no signal is being applied from the patient. The recorder, of course, must be running. The "angle calibrate" switch must be returned to the "off" position when a recording is taken.

Any angle from 0° to 360° can be subtracted from the angle output by means of a quadrant selector and a calibrated phase shifter. The quadrant selector changes the output angle in steps of 90° by interchanging the horizontal and vertical inputs and their polarities, the rotation of the vectorcardiogram being visible on the cathode-ray tube. The calibrated phase shifter varies the angle from 0° to -90° by advancing the phase of the comparison signal. This rotation is not evident on the vectorcardiogram. Angle shifting is helpful in avoiding the flip-flop switchover point at $\pm 180^{\circ}$. The operator can record the amount of phase shift used, although it is possible that the waveform of the angle output is more important than the numerical value of the angle.

IX ACCURACY TESTS

The overall accuracy of the machine was checked by applying sinusoidal signals from a two-phase source to the input terminals. Since the vector locus for such voltages is a circle whose centre is at the origin, the angle output should be a "sawtooth" waveform as shown in Figure 12(a).

Actual machine outputs for circular inputs are shown in Figures 12(b) and 12(c), representing the maximum and minimum allowable signal levels respectively. The maximum signal level of 4.5 volts rms is determined by non-linearity of the magnitude output, although the angle output waveform remains substantially the same up to 6 volts rms input. Satisfactory performance is also obtained with signal levels as low as 0.1 volts rms, indicating a dynamic range better than 45:1.

As the lowest test frequency available was 20 cycles per second, the filter required to remove the 4 kilocycles per second carrier frequency reduced the transient response of the output sufficiently to prevent the triangular wave from extending over the full range of -180° to $+180^{\circ}$. It is felt that at the normal fundamental input frequency of one cycle per second the transient response is adequate.

As a further check on the transient response, voltages representing a small circle with centre displaced from the origin on the $\pm 180^{\circ}$ line were applied to the input terminals. The resulting angle output waveform is shown in Figure 12(d). The time required to switch from -180° to within 5% of $+180^{\circ}$ is of the order of 0.01 seconds, or about 1% of the duration of the heart cycle. The flip-flop switchover will therefore appear as a nearly vertical line on a polarcardiograph tracing.

The magnitude output versus input is plotted in Figure 12(e) and shows reasonable linearity from 0 to 4.5 volts rms input. The non-linear portion at low signal levels may be due to a small amount of unbalance. In operation, this non-linearity will be small with respect to error produced by noise and incorrect centring.

These tests indicate that the overall accuracy of the machine is sufficient for normal electrocardiographic purposes, provided it is used with a recorder having a frequency response of at least 300 cycles per second.

X CONCLUSION

The polarcardiograph is an electronic analogue computing device which computes voltages proportional to the magnitude and angle of the heart vector from voltages proportional to its components in Cartesian co-ordinates.

The computer was designed using analogue multipliers, adders, a subtractor, a local oscillator producing a two-phase sinusoidal source and a phasemeter capable of generating a voltage proportional to the phase difference of two sinusoidal signals. The device was designed to use the output voltages of standard electrocardiograph machines for input voltages and its output can be recorded by any strip recorder of sufficient bandwidth.

The device is different from a similar one cited in the literature in the manner in which multiplication is achieved. Multiplication in the present machine is performed by pentagrid type vacuum tubes. The operating conditions under which a pentagrid tube will perform multiplication were derived mathematically. Such operating conditions were achieved and were incorporated in the design of the computer.

The complete instrument when tested proved to have an accuracy well within that originally required.

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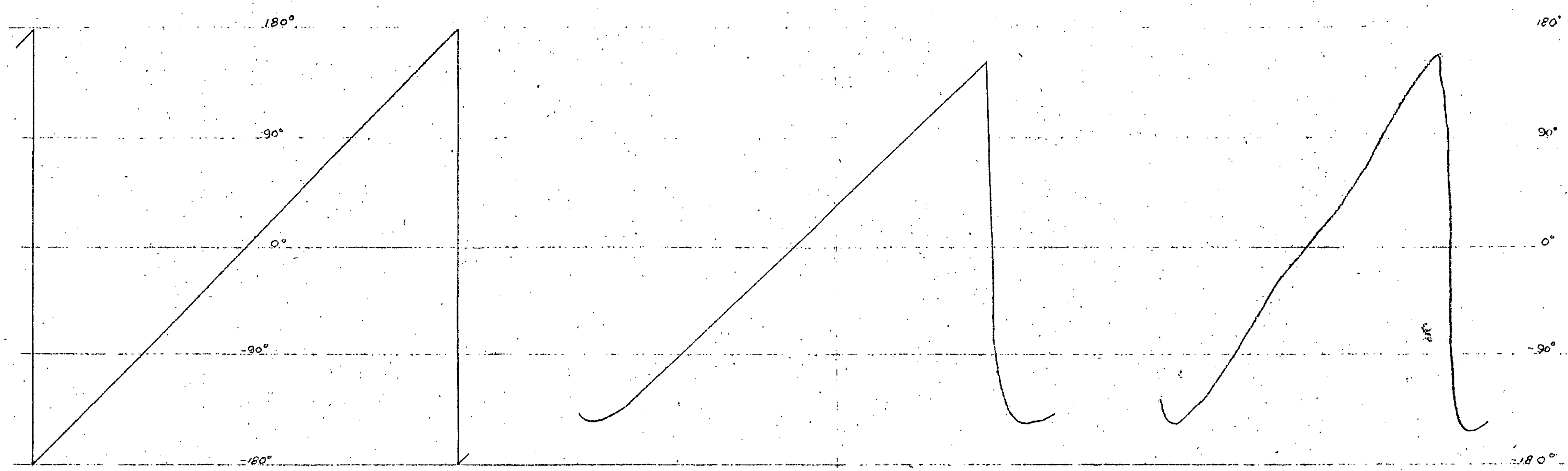
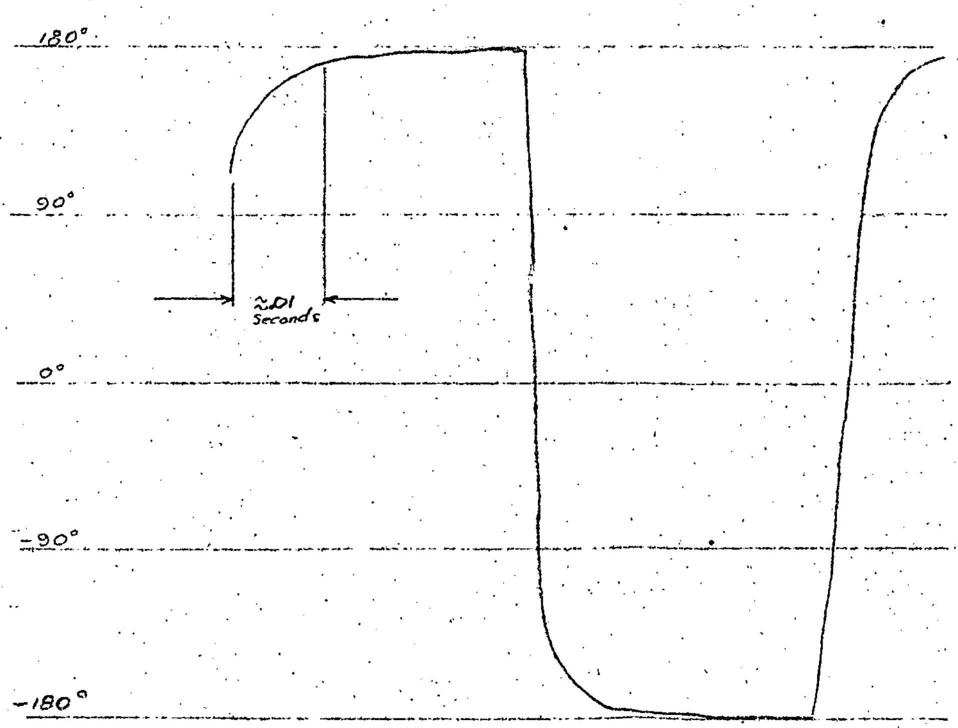


12(a) Theoretical Phase Output

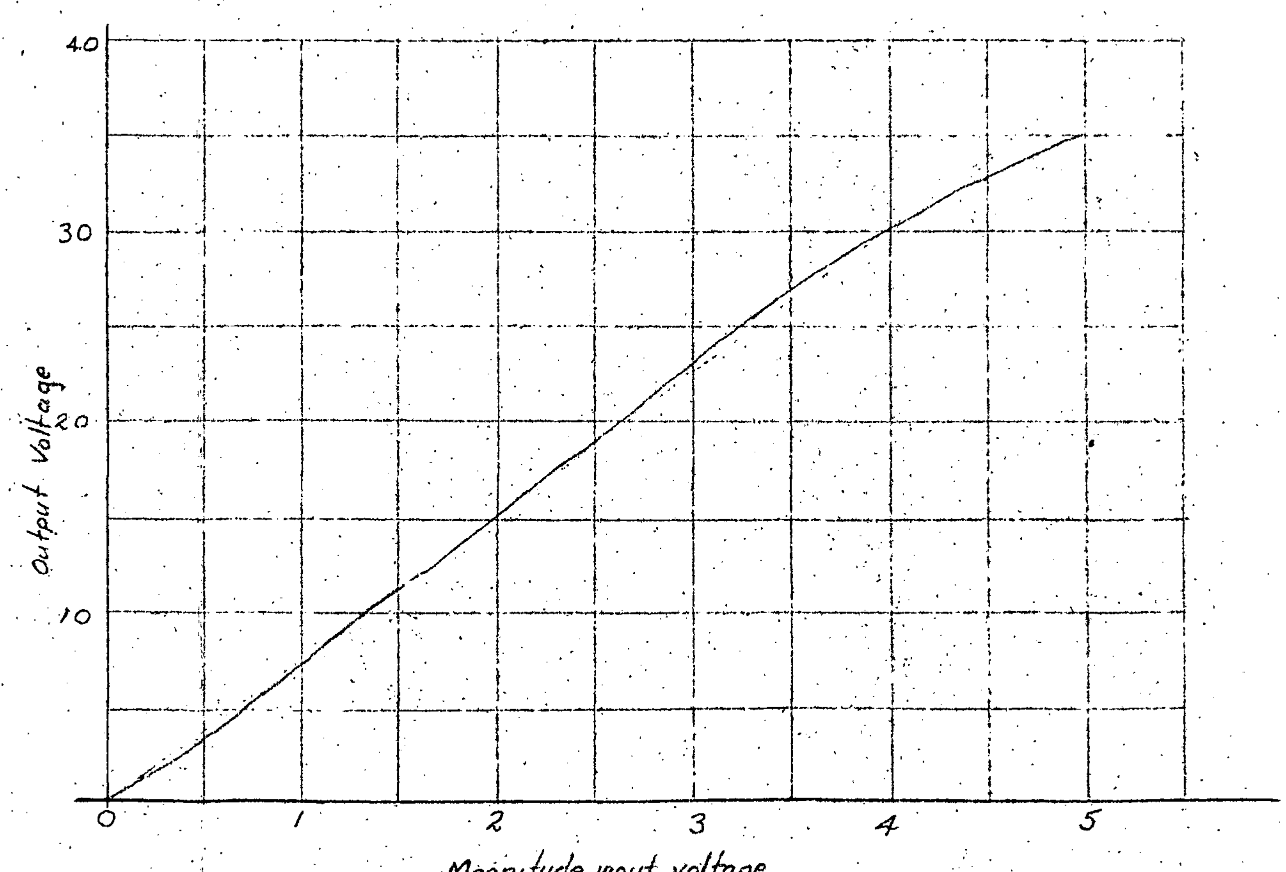
12(b) Actual Phase Output 4.5 V rms input

12(c) Actual Phase Output 0.1 V rms input

Figures 12(b) and 12(c) traced from an oscilloscope. Different "X" Amplitudes.



12(d) Phase Output - Displaced Circle Input



12(e) Magnitude Linearity

Figure 12 Computer Outputs

All resistance values are megohms unless otherwise indicated. 1K = 1000 ohms 1Ω = 1 ohm

Polarcardiograph Computer

Circuit Diagram as Built Nov. 1954
at the
University of British Columbia
Department of Electrical Engineering

All capacitance values are micro-micro farads unless otherwise indicated 1μ = 1 microfarad