THREE-PHASE FREQUENCY CONVERSION

by

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ABSTRACT

This thesis discusses a principle of continuous speed regulation of AC motors, which is based on frequency conversion, and describes an experimental three-phase converter built for speed test purposes.

The converter accepts power from a three-phase 60 cps. source and control signals from an auxiliary three-phase square-wave generator whose fundamental output frequency can be varied between 70 and 570 cps. When the two three-phase inputs are of opposite sequence, fundamental output frequency can be varied between 10 and 510 cps. The converter is of the static switching type. It consists of three identical units, one in each phase. Each unit has four power transistors which are operated in the on-off mode. Control signals are fed to the base-emitter circuits through isolating transformers.

A free-running multivibrator in the control unit determines the period of the square-wave signals. The signals are taken from three bistable circuits, triggered in proper sequence through a system of gates.

Output voltages from the converter have a relatively high harmonic content. Higher harmonic currents forming zero-sequence systems can be suppressed entirely. Current harmonics of positive and negative sequences must in general be tolerated. A discussion is given on the performance of AC motors when powered by this type of converter.

No-load speed tests on small induction motors confirm the principle on which the experimental work has been based.
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1. INTRODUCTION

A frequency converter may be defined as a device or combination of devices that receives electrical energy with voltage and current components of a given frequency and delivers it at a different frequency. The voltage and number of phases may or may not be changed during the conversion process.

Several types of frequency converters have been built. They are generally classified as either static or rotating. Rotating frequency changers consist in principle of a motor-generator set. The input electrical energy to the motor is converted to mechanical energy and fed to the generator which again converts it to electrical energy. Most of the rotating converters are so-called constant ratio frequency changers, which means that the output frequency has a fixed relation to the input frequency. However, a variable output frequency can be obtained.

Static converters are usually of the AC-to-DC (or DC to AC) type, involving rectifier components. The successful development of high voltage grid-controlled mercury-arc rectifiers has made possible the design of static converters with very high power ratings and high efficiency. When converting DC to AC, the output frequency is controlled by auxiliary equipment and the reactive power required by the load must be supplied from sources on the AC side.

Other types of static frequency converters employ nonlinear elements such as nonlinear reactors or capacitors,
electron tubes and solid state devices.

Frequency converters are applied quite extensively in power engineering. But there are still large fields where such conversion would be desirable, provided it could be achieved with inexpensive and reliable equipment. One such field concerns speed regulation of AC machines, a problem almost as old as the AC machine itself.

The most widely used AC machine is the induction motor. Its ruggedness, low maintenance and reliability in operation is unsurpassed by any other electric motor. But from the point of view of speed control the induction motor leaves much to be desired. At the present time, smooth speed-torque control is usually achieved by changing the input impedance of the motor. This can be done either by varying the rotor impedance, in which case the machine must be of the wound rotor type and have slip rings, or by connecting saturable core reactors in series with the stator windings. Adding resistance in the rotor circuit leads to increased losses and lower efficiency. On the other hand, additional input inductance results in lower power factor. These methods of speed control, therefore, have serious disadvantages.

The speed of an induction or synchronous motor is always closely related to the applied frequency at the input terminals. Assuming that power is available from conventional 60 cps. sources, the speed control problem may be considered a problem of changing the frequency, say 60 cps., to a new frequency \( f \), so that \( \frac{f}{60} \) becomes a controllable ratio.
The purpose of the project reported in this thesis is to seek a solution to the frequency conversion problem which can result in the design of a static converter. The converter should consist of solid-state components only and should first of all be applicable to speed control of conventional AC motors.

A fair amount of research and development work has already been done or is presently conducted along these lines. The results reported so far indicate some success and show that the general approach is to use transistorized variable frequency oscillators and power amplifiers, and take the energy from DC sources\(^1,2,3\).

A different approach has been adopted here, based on some simple theoretical results derived in Chapter 2.
2. AN APPROACH TO SPEED CONTROL OF THREE-PHASE INDUCTION MACHINES BASED ON FREQUENCY CONVERSION

2.1 Rotating mmf's in induction machines

When balanced three-phase voltages are applied to a symmetrical three-phase stator winding of an induction machine, the resulting air-gap mmf can be represented by a travelling space wave of constant amplitude. To demonstrate this for a simple case, where the phase windings are assumed to be sinusoidally distributed and placed 120 electrical degrees apart, let

\[ \omega = \text{angular frequency of input voltage (elect. radians per sec.)} \]

\[ \beta = \text{space angle in elect. degrees, measured from the axis of phase winding a.} \]

\[ F = \text{magnetomotive force (mmf) or magnetic potential difference across the air-gap.} \]

\[ t = \text{time in seconds.} \]

\[ p = \text{number of poles.} \]

\[ a, b, c, \text{ subscripts denoting phases.} \]

With sinusoidally distributed windings, one can write:

\[ F_a = F_a(\omega t) \cos \beta \]

\[ F_b = F_b(\omega t) \cos(\beta - 120) \]

\[ F_c = F_c(\omega t) \cos(\beta - 240) \]

Where \( F_a(\omega t) \), \( F_b(\omega t) \), and \( F_c(\omega t) \) are the individual phase mmf amplitudes at time t. These amplitudes vary with the phase currents so that one can write:

\[ F_a(\omega t) = F_m \cos \omega t \]
\[
F_b(\omega t) = F_m \cos(\omega t - 120) \quad \ldots \quad 2.2 \\
F_c(\omega t) = F_m \cos(\omega t - 240)
\]

The currents in the three phases are balanced and therefore of equal amplitude. Whence also

\[
F_a(\omega t) = F_b(\omega t) = F_c(\omega t) = F_m \tag{\text{max}}
\]

The resultant mmf at any point \( \beta \) and time \( t \) can now be found by superposition of the component mmf's:

\[
F(\beta, t) = F_a + F_b + F_c = F_m \left[ \cos \beta \cos \omega t + \cos (\beta - 120) \cos (\omega t - 120) + \cos (\beta + 240) \cos (\omega t + 240) \right]
\]

\[
= \frac{F_m}{2} \left[ (\cos(\beta + \omega t) + \cos(\beta - \omega t)) + (\cos(\beta + \omega t - 240) + \cos(\beta + \omega t + 240) + \cos(\beta - \omega t + 120 - 120)) \right]
\]

\[
F(\beta, t) = \frac{3}{2} F_m \cos (\beta - \omega t) \quad \ldots \quad 2.3
\]

Equation 2.3 represents the travelling mmf wave. Its direction of rotation is by definition positive. The angular space velocity of this wave in mechanical radians per sec. depends on \( \omega \) and the number of poles in the machine. It represents the synchronous speed of the machine. The speed in electrical radians per sec. is simply equal to \( \frac{d\beta}{dt} \) for \( F(\beta, t) \) equal a constant. Thus

\[
\beta - \omega t = \text{const.},
\]

\[
\frac{d\beta}{dt} = \omega.
\]

In general,
\[ \omega_{\text{mech.}} = \omega_{\text{el.}} \left( \frac{2}{p} \right) = \omega_{\text{synchronous}} = \omega_{\text{sy}} \]

The synchronous speed is

\[ n_{\text{sy}} = \frac{60\omega_{\text{el}}}{2\pi} \frac{2}{p} = \frac{120f}{p} \text{ r.p.m.} \]

In an actual machine the mmf space distribution is not purely sinusoidal because the windings are confined to slots. But by careful design, harmonic content can be kept low. To keep saturation effects low is also largely a matter of design. Therefore, the assumption made in deriving Equation 2.3 can in general be justified.

2.2 MMF waves for modified applied voltages

In this section the line-to-neutral voltages are modified and represented by

\[ e_a = E \sin \omega_s t \sin (\omega_c t - \alpha_a) \]
\[ e_b = E \sin (\omega_s t - 120) \sin (\omega_c t - \alpha_b) \]
\[ e_c = E \sin (\omega_s t - 240) \sin (\omega_c t - \alpha_c) \]

where \( \omega_s \) = angular frequency of the power source
\( \omega_c \) = angular frequency of an auxiliary three-phase control source
\( \alpha \) = time phase angle in the control system

and \( \alpha_a + \alpha_b + \alpha_c = 0 \) or \( 2\pi \)

For convenience, let \( E = 1.0 \) per unit (p.u.). Then one can expand Equations 2.4 and write:

\[ e_a = e_{a1} - e_{a2} \]
\[ = \frac{1}{2} \left[ \cos(\omega_c t - \omega_s t - \alpha_a) - \cos(\omega_c t - \alpha_a + \omega_s t) \right] \]
\[ e_b = e_{b1} - e_{b2} \]
\[ = \frac{1}{2} \left[ \cos(\omega_c t - \alpha_b - \omega_s t + 120) - \cos(\omega_c t - \alpha_b + \omega_s t - 120) \right] \]
\[ e_c = e_{c1} - e_{c2} \]
\[ = \frac{1}{2} \left[ \cos(\omega_c t - \alpha_c - \omega_s t + 240) - \cos(\omega_c t - \alpha_c + \omega_s t - 240) \right] \]

where

\[ e_{a1} = \frac{1}{2} \cos \left( (\omega_c - \omega_s) t - \alpha_a \right) \]
\[ e_{a2} = \frac{1}{2} \cos \left( (\omega_c + \omega_s) t - \alpha_a \right) \]
\[ e_{b1} = \frac{1}{2} \cos \left( (\omega_c - \omega_s) t - \alpha_b + 120 \right) \]
\[ e_{b2} = \frac{1}{2} \cos \left( (\omega_c + \omega_s) t - \alpha_b - 120 \right) \]
\[ e_{c1} = \frac{1}{2} \cos \left( (\omega_c - \omega_s) t - \alpha_c + 240 \right) \]
\[ e_{c2} = \frac{1}{2} \cos \left( (\omega_c + \omega_s) t - \alpha_c - 240 \right) \]

Assume the angles \( \alpha_a, \alpha_b \) and \( \alpha_c \) can be chosen at will in accordance with Equation 2.5, then there are three possibilities of practical interest:

(i) Zero sequence signal applied, that is
\[ \alpha_a = \alpha_b = \alpha_c = 0. \]

In this case the input voltages are balanced. Let \( F_1(\beta,t) \) denote instantaneous value of the mmf wave produced by the voltages \( e_{a1}, e_{b1} \) and \( e_{c1} \), and let \( F_2(\beta,t) \) be similarly related to \( e_{a2}, e_{b2} \) and \( e_{c2} \). As shown above, one then has:

\[ F_1(\beta,t) = F_{a1}(\beta,t) + F_{b1}(\beta,t) + F_{c1}(\beta,t), \]

and one finds:
\[ F_1(\beta, t) = \cos (\omega_c - \omega_s t) \cos \beta + \cos ((\omega_c - \omega_s) t + 120) \]
\[ \times \cos (\beta - 120) + \cos ((\omega_c - \omega_s) t - 120) \cos (\beta + 120) \]
\[ = \frac{3}{2} \cos ((\omega_c - \omega_s) t + \beta) \]

That is
\[ F_1(\beta, t) = F_1 \max \cos ((\omega_c - \omega_s) t + \beta) \] \[ \ldots \] \[ 2.6 \]

In a similar way one finds
\[ F_2(\beta, t) = F_2 \max \cos ((\omega_c + \omega_s) t - \beta - \pi) \] \[ \ldots \] \[ 2.7 \]

The functions \( F_1(\beta, t) \) and \( F_2(\beta, t) \) represent two mmf waves in the air-gap, travelling in opposite directions with different speeds and amplitudes. The amplitude \( F_2 \max \) is less than \( F_1 \max \) because the voltage components that produce them are of equal amplitude while the input impedance increases with increased frequency. If these voltages are applied to an ideal three-phase induction machine, the machine will rotate at a speed proportional to \((\omega_c - \omega_s)\) in negative direction. Since \( F_1 \max \) is larger than \( F_2 \max \), the machine will have a starting torque, but in other respects its behaviour resembles that of the single-phase induction machine.

(ii) Positive sequence signals applied, that is
\[ \alpha_a = 0, \alpha_b = 120^0, \alpha_c = -120^0 \]

In this case
\[ e_{a1} + e_{b1} + e_{c1} = \frac{3}{2} \cos (\omega_c - \omega_s) t, \]
\[ e_{a2} + e_{b2} + e_{c2} = 0. \]

The input voltages are of equal maximum amplitudes, but not balanced. Thus:
\[ F_1(\beta, t) = \cos(\omega_c - \omega_s)t \cos \beta + \cos((\omega_c - \omega_s)t - 120 + 120) \]
\[ \times \cos(\beta - 120) + \cos((\omega_c - \omega_s)t + 120 + 240) \]
\[ \times \cos(\beta + 120) \]
\[ = \cos(\omega_c - \omega_s)t [\cos \beta + \cos(\beta - 120) + \cos(\beta + 120)] = 0 \quad \ldots \quad 2.8 \]

\[ F_2(b, t) = \cos(\omega_c + \omega_s)t \cos \beta + \cos((\omega_c + \omega_s)t - 120) \]
\[ \times \cos(\beta - 120) + \cos((\omega_c + \omega_s)t + 120 - 120) \cos(\beta + 120) \]
\[ = \frac{3}{2} \cos((\omega_c + \omega_s)t + \beta) \quad \ldots \quad 2.9 \]

Hence

\[ F.(\beta, t) = F_{2\text{max}} \cos((\omega_c + \omega_s)t + \beta) \quad \ldots \quad 2.10 \]

From this it is seen that the air-gap mmf's add up to a single wave, rotating in negative direction at an angular speed of \((\omega_c + \omega_s)\) electrical radians per second. If the stator windings are Y-connected with grounded neutral, a zero-sequence current of angular frequency \(\omega_c - \omega_s\) will flow in the neutral. This, of course, is undesirable. Isolated neutral means that the sum of all three line currents is zero at any time. Equation 2.10 still holds, but now the voltage at the neutral becomes \(\frac{1}{3}(e_a + e_b + e_c) = \frac{1}{2} \cos (\omega_c - \omega_s)t\) p.u. (presuming the phase voltages are symmetrical with respect to ground).

(iii) Negative sequence signals applied, that is

\[ \alpha_a = 0, \quad \alpha_b = -120^\circ, \quad \alpha_c = +120^\circ. \]

For this case, it can be shown that
\[ F_1(\beta, t) = F_{1 \text{ max}} \cos ((\omega_c - \omega_s)t - \beta) \] ... 2.11

\[ F_2(\beta, t) = 0 \] ... 2.12

Hence, the resulting mmf wave travels in positive direction at an angular speed of \((\omega_c - \omega_s)\) el. radians per sec.

If the neutral is grounded, a zero-sequence current of angular frequency \((\omega_c + \omega_s)\) flows, and if it is isolated, its voltage becomes \(\frac{1}{2} \cos (\omega_c + \omega_s)t\) p.u. to ground. It is also easy to see what happens when the stator windings are delta-connected. The line currents must be balanced and likewise the line-to-line voltages:

\[ e_{ab} = e_a - e_b \]
\[ = \frac{1}{2} \left[ \cos (\omega_c - \omega_s)t - \cos (\omega_c + \omega_s)t - \cos ((\omega_c - \omega_s)t - 120) + \cos ((\omega_c + \omega_s)t - 120 + 120) \right] \]
\[ = \sqrt{3} \cos ((\omega_c - \omega_s)t + 30) \]

\[ e_{bc} = e_b - e_c \]
\[ = \frac{1}{2} \left[ \cos ((\omega_c - \omega_s) - 120) - \cos ((\omega_c + \omega_s)t - \cos ((\omega_c - \omega_s)t - 120 + 240) + \cos ((\omega_c + \omega_s)t - 120 - 240) \right] \]
\[ = \sqrt{3} \cos ((\omega_c - \omega_s)t - 90) \]

\[ e_{ca} = e_c - e_a \]
\[ = \frac{1}{2} \left[ \cos ((\omega_c - \omega_s)t - 120 + 240) - \cos (\omega_c + \omega_s)t - \cos (\omega_c - \omega_s)t + \cos (\omega_c - \omega_s)t \right] \]
\[ = \sqrt{3} \cos ((\omega_c - \omega_s)t + 150) \] ... 2.13

Whence the line-to-line voltages constitute a positive
sequence system with angular velocity \((\omega_c - \omega_s)\) el. radians per sec. When applied to the idealized stator windings, these voltages produce a space mmf wave in the air-gap that behaves in accordance with Equation 2.11.

Now suppose the voltages described by Equations 1.4 are available, \(\omega_s\) is the constant angular frequency produced by a three-phase local oscillator, then speed control can be achieved by varying \(\omega_c\) instead of \(\omega_s\). The above analysis also indicates that by choosing the three-phase signal sequence as in case (iii), the synchronous speed of the machine can be varied over a wide range above as well as below the "normal" synchronous speed given by \(\omega_s\). This case, therefore, appears to be most promising among the three considered, and is made the principal subject for further investigations. Also, it represents the particular approach to the problem of speed control of three-phase machines adopted in this thesis. The major technical problem associated with this approach is how to generate or produce three-phase voltage components as described by Equations 2.4 or 2.13, such that the power required by the load is taken almost entirely from the \(\omega_s\) source.

Chapters 3 and 4 present some theoretical and practical considerations on this problem.
3. FREQUENCY CONVERSION NETWORKS CONTAINING NONLINEAR PASSIVE ELEMENTS

3.1 General power relations

The question of whether a single modulator type inverter can be used for our purpose will be examined. A nonlinear reactor modulator will be considered, whose non-linear element is an iron core inductor. The energy is supplied by an AC source of frequency \( f_s \) and the signal frequency is \( f_c \). Two general expressions will be derived which show the power-frequency relationships associated with nonlinear elements. The feasibility of the modulator as a power-frequency converter can then be judged. In reference 6, power relations have been derived for the case of a nonlinear capacitor. The subsequent analysis follows a pattern similar to that in reference 6 and leads to the same results.

The characteristic of the nonlinear inductor is assumed to be single-valued, but otherwise its shape is arbitrary. It is given by specifying the current, \( i \), as a function of flux linkage \( \psi \):

\[
i = f(\psi)
\]  \( \ldots 3.1 \)

In general, an infinite number of frequencies can be present, \( f_{m,n} = mf_c + nf_s \), where \( m \) and \( n \) take on all positive and negative integer values, and zero. The magnetic flux linkage in the inductor can be described by a double Fourier series:

\[
\psi = \sum_{m = -\infty}^{\infty} \sum_{n = -\infty}^{\infty} \psi_{m,n} e^{j(mx + ny)}
\]  \( \ldots 3.2 \)
where

\[ x = 2\pi f_c t, \quad y = 2\pi f_s t \] ... 3.3

The voltage across the nonlinear inductor is obtained by taking the total derivative of \( \psi \) with respect to time:

\[ v = \frac{d\psi}{dt} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} V_{m,n} e^{j(mx + ny)} \] ... 3.4

where

\[ V_{m,n} = j2\pi (mf_c + nf_s) \psi_{m,n} \] ... 3.5

The current, \( i \), also may be represented by a double Fourier series, since

\[ i = f(\psi) = f\left[ \psi(x,y) \right] = F(x,y) \] ... 3.6

where \( F(x,y) \) is single-valued and periodic in \( x \) and \( y \). Thus the current can be written as

\[ i = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_{m,n} e^{j(mx + ny)} \] ... 3.7

The coefficients are given by

\[ I_{m,n} = \frac{1}{4\pi^2} \int_{0}^{2\pi} dy \int_{0}^{2\pi} F(x,y) e^{-j(mx + ny)} dx \] ... 3.8

Since \( I_{m,n}^* = I_{-m,-n} \) = the complex conjugate of \( I_{m,n} \), one has:

\[ I_{m,n}^* = \frac{1}{4\pi^2} \int_{0}^{2\pi} dy \int_{0}^{2\pi} F(x,y) e^{j(mx + ny)} dx \] ... 3.9

Multiplying Equation 3.9 by \( jm \psi_{m,n} \) and summing from \(-\infty\) to \(+\infty\) over \( m \) and \( n \), one obtains
By changing the order of integration and the summation on the right hand side, Equation 3.10 becomes

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} j^m \Psi_{m,n} I_{m,n} = \frac{1}{4\pi^2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} j^m \Psi_{m,n} x \left[ \int_0^{2\pi} \int_0^{2\pi} F(x,y) e^{j(mx + ny)} \, dx \right] \quad \ldots \quad 3.10
\]

Differentiating Equation 3.2 with respect to \( x \) yields:

\[
\frac{\partial \psi}{\partial x} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} j^m \Psi_{m,n} e^{j(mx + ny)} \quad \ldots \quad 3.12
\]

Therefore the double summation under the integral in Equation 3.11 equals \( \frac{\partial \psi}{\partial x} \). Equation 3.5 gives for \( \Psi_{m,n} \):

\[
\Psi_{m,n} = \frac{V_{m,n}}{2\pi} (mf_c + mf_s) \quad \ldots \quad 3.13
\]

Combining Equations 3.12 and 3.13 with 3.11 yields

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mV_{m,n}}{mf_c + mf_s} I_{m,n} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} F(x,y) \frac{\partial \psi}{\partial x} \, dx \quad \ldots \quad 3.14
\]

The next step is to transform the last integral to a more suitable form. With \( y \) held constant, \( \frac{\partial \psi}{\partial x} \, dx = d\psi \). In accordance with Equation 3.6, therefore,

\[
\int_0^{2\pi} F(x,y) \frac{\partial \psi}{\partial x} \, dx \rightarrow \int_0^{2\pi} \frac{\psi(2\pi,y)}{\psi(o,y)} \, d\psi \quad \ldots \quad 3.14
\]

and Equation 3.14 takes on the form
\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mV_{m,n} I_{m,n}}{mf_c + n f_s} = \frac{1}{2\pi} \int_{0}^{2\pi} dy \int_{0}^{2\pi} f(\psi) d\psi
\] ...

3.15

Starting again with Equation 3.9, one can next multiply it by \(jn_{m,n}\) and repeat the above procedure. Thus:

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} jn_{m,n} I_{m,n} = \frac{1}{4\pi^2} \int_{0}^{2\pi} \int_{0}^{2\pi} F(x,y) \, dx
\]

\[
x \left[ \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} jn_{m,n} e^{j(mx + ny)} \right]
\]

3.16

\[
\frac{\partial \psi}{\partial y} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} jn_{m,n} e^{j(mx + ny)}
\]

3.17

\[
n_{m,n} I_{m,n} = \frac{nV_{m,n} I_{m,n}}{j2\pi(mf_c + nf_s)}
\]

3.18

Hence, Equation 3.16 becomes:

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{nV_{m,n} I_{m,n}}{mf_c + n f_s} = \frac{1}{2\pi} \int_{0}^{2\pi} dx \int_{0}^{2\pi} F(x,y) \frac{\partial \psi}{\partial y} dy
\]

3.19

The second integral is transformed by changing the variable to \(\psi\), and the limits indicate that the variation of \(\psi\) corresponds to letting \(y\) vary from 0 to \(2\pi\) while \(x\) is kept constant. Hence one finds:

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{nV_{m,n} I_{m,n}}{mf_c + n f_s} = \frac{1}{2\pi} \int_{0}^{2\pi} dx \int_{0}^{2\pi} f(\psi) \frac{\partial \psi}{\partial y} d\psi
\]

3.20

Since \(f(\psi)\) is single-valued and periodic in \(x\) and \(y\), the right hand side of Equations 3.15 and 3.20 must equal zero because:

\[
\int_{0}^{2\pi} f(\psi) d\psi = \int_{0}^{2\pi} f(\psi) d\psi = 0
\]

3.21
It follows that

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mV_{m,n} I_{m,n}^*}{mf_c + nf_s} = 0 \quad \ldots \quad 3.22
\]

\[
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{nV_{m,n} I_{m,n}^*}{mf_c + nf_s} = 0 \quad \ldots \quad 3.23
\]

In the above equations both positive and negative frequencies appear as a result of the Fourier technique. The voltage or current component of a certain amplitude, associated with a particular frequency \( f > 0 \), is split into two components or phasors of half amplitude and frequencies \( +f \) and \( -f \) respectively.

Let \( S_{m,n}, P_{m,n} \) and \( Q_{m,n} \) denote vector, real and reactive powers respectively, flowing into the nonlinear element at frequencies \( \pm |mf_c + nf_s| \). Then:

\[
S_{m,n} = P_{m,n} + jQ_{m,n} = 2 V_{m,n} I_{m,n}^* \quad \ldots \quad 3.24
\]

\[
S_{m,n}^* = P_{m,n} - jQ_{m,n} = 2 V_{m,n}^* I_{m,n} \quad \ldots \quad 3.25
\]

From Equations 3.24 and 3.25 it follows that:

\[
P_{m,n} = V_{m,n} I_{m,n}^* + V_{m,n}^* I_{m,n} \quad \ldots \quad 3.26
\]

\[
Q_{m,n} = j(V_{m,n} I_{m,n}^* - V_{m,n}^* I_{m,n}) \quad \ldots \quad 3.27
\]

Since \( V_{m,n}^* I_{m,n} = V_{-m,-n}^* I_{-m,-n} \), Equations 3.26 and 3.27 can also be written

\[
P_{m,n} = V_{m,n} I_{m,n}^* + V_{-m,-n} I_{-m,-n}^* \quad \ldots \quad 3.28
\]

\[
Q_{m,n} = j(V_{m,n} I_{m,n}^* - V_{-m,-n} I_{-m,-n}) \quad \ldots \quad 3.29
\]

Thus, combining pairs of terms \((VI^*)\) corresponding to the
positive and negative components of each frequency, one can rewrite Equations 3.22 and 3.23 and introduce the powers associated with the various frequencies. It is seen that only the real powers $P_{m,n}$ will appear. Making use of Equation 3.28, Equations 3.22 and 3.23 become:

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{m_{m,n}}{m^2 c + n^2 s} = 0 \quad \ldots \quad 3.30$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{n_{m,n}}{m^2 c + n^2 s} = 0 \quad \ldots \quad 3.31$$

These are the final results. Equations 3.30 and 3.31 provide two independent relations among the powers flowing into a nonlinear reactive element at various frequencies. They have been derived for the case of a nonlinear inductor, but apply equally well to the case of a nonlinear capacitor. The major assumption behind these results was that the B–H characteristic of the inductor be single-valued. Consequently Equations 3.30 and 3.31 show that the sum of all the powers flowing into the element at various frequencies is zero. The presence of hysteresis implies power dissipation in the nonlinear reactive element. Under certain simple but fairly realistic conditions, this hysteresis loss can be taken into account. If the signal level is low compared to the power oscillator level, one can assume that the actual B–H function is only double-valued, and the hysteresis loop is formed by the power oscillator output of frequency $f_s$. According to the literature on this subject, the analysis leading up to Equations 3.15 and 3.20 is not altered by the new condition
for \( i = f(\psi) \). But the values of the double integrals on the right hand side of these equations must be re-evaluated.

Consider the double integral
\[
\frac{1}{2\pi} \int_0^{2\pi} d\psi \int_0^\infty f(\psi) d\psi
\]

Keeping \( y \) constant in the second integral means that the flux variation, when \( x \) varies from 0 to \( 2\pi \), can be represented by a minor loop in the B-H plane. This loop starts and ends at the same point on one of the principal hysteresis loop branches. But according to the condition that \( f(\psi) \) is only double-valued, this minor loop must be confined entirely to one of the hysteresis loop branches. Therefore, the area of the minor loop equals zero, and the above integral equals 0.

Next consider the double integral
\[
\frac{1}{2\pi} \int_0^{2\pi} dx \int_0^\infty f(\psi) d\psi
\]

With \( x = \text{constant} \) and \( y \) varying from 0 to \( 2\pi \), \( \psi \) will travel entirely around the hysteresis loop. This integral therefore represents the area of the loop, or the energy dissipated in the nonlinear reactor during one cycle of the power oscillator frequency. Let this quantity be denoted \( h \) and the average power loss due to hysteresis be denoted \( H \). Then \( H = f_s h \), or \( h = \frac{H}{f_s} \). Under these conditions, Equations 3.30 and 3.31 can be written:
\[
\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{m^p m, n}{m_c f_n + n f_s} = 0 \quad \ldots \quad 3.32
\]
Equation 3.35 indicates that the hysteresis loss is taken from the local power source. The term $H \frac{P_{0,1}}{f_s}$ can thus be combined with the term $\frac{H}{f_s}$ on the left hand side.

3.2 Possibility of application

As concluded in Chapter 2, the object is to establish a circuit which accepts power from two sources, operating at different frequencies $f_s$ and $f_c$, and delivers power at angular frequency $(\omega_c - \omega_s)$. The input power associated with $\omega_c$ is assumed to represent signal power. Now suppose a nonlinear reactor modulator is to be used for this purpose. The system can be illustrated as shown in Fig. 3.1.

![Fig. 3.1 Symbolic representation of converter](image)

$P_c$, $f_c$  \quad Nonlinear reactor modulator  \quad P_0$, $f_0$

$P_s$, $f_s$  \quad $P_c$, $f_c$  \quad $P_s$, $f_s$

$P_c$ = input signal power, $f = f_c$
$P_s$ = input power from the local source, $f = f_s$
$P_0$ = output power at frequency $f_c$.

The power gain, $G_p$, is defined as:

$$G_p = \frac{P_0}{P_c} = f(f_c, f_s)$$  \quad ... 3.34
In analyzing this type of problem, it is customary to assume that currents at only the most important frequencies are allowed to flow in the nonlinear element, all other frequencies being suppressed by ideal filters. Adopting this method or simplification, Equation 3.34 can be solved by the use of Equation 3.32 once the output frequency has been chosen. Now, if one chooses \( f_o = f_c + f_s \) and takes this as the only output frequency that carries a significant amount of power, Equation 3.32 yields:

\[
\sum_{m=0}^{1} \sum_{n=0}^{1} \frac{mP_{m,n}}{mf_c + nf_s} = \sum_{m=0}^{1} \left[ \frac{mP_{m,0}}{mf_c} + \frac{mP_{m,1}}{mf_c + f_s} \right] \\
= \frac{P_{l,o}}{f_c} + \frac{P_{l,1}}{f_c + f_s} = 0 \quad \ldots \quad 3.35
\]

where

\( P_{l,o} = P_c, \quad P_{l,1} = -P_o. \)

Hence

\[
G_p = \frac{P_o}{P_c} = \frac{f_c + f_s}{f_c} = \frac{f_o}{f_c} \quad \ldots \quad 3.36
\]

Equation 3.33 gives:

\[
\sum_{m=0}^{1} \sum_{n=0}^{1} \frac{nP_{m,n}}{mf_c + nf_s} = \frac{P_{o,1}}{f_s} + \frac{P_{l,1}}{f_c + f_s} = \frac{H}{f_s} \quad \ldots \quad 3.37
\]

where

\( P_{o,1} = P_s. \)

Hence

\[
\frac{P_s}{f_s} = \frac{P_o}{f_c} \quad \ldots \quad 3.38
\]

The powers \( P_s, P_c \) and \( P_o \) are all positive and the system
will normally be stable. If the output frequency \( f_o \) is made less than \( f_s \), e.g. \( f_o = f_s - f_c \), the signal power becomes negative according to Equation 3.32. This means that some of the power delivered by the main source at frequency \( f_s \) is converted to frequency \( f_c \) in the nonlinear element and returned to the signal source. Under such conditions the system is potentially unstable. The same conclusion holds for \( f_o = nf_s - mf_c \) where \( m \) and \( n \) are larger than zero. These cases, therefore, will be excluded from the subsequent discussion.

Equation 3.36 indicates that a substantial power gain can be achieved if the frequencies \( f_s \) and \( f_c \) are widely separated. Earlier it was assumed that \( P_s \) was to be supplied directly from a 60 cps source. This appears to be unrealistic because a low power gain results. The frequency \( f_s \) may, however, represent a harmonic of 60 cps. According to Equation 3.36, a substantial power gain can then be obtained provided \( f_c \) has a comparatively low value.

As mentioned above, the expression for \( G_p \) is based on the assumption that the power transfer takes place at a few selected frequencies only, all other frequencies being suppressed by ideal filters. This procedure is very useful, but it is not always justifiable. Although the powers of the unwanted frequencies may be small, in practice they are not likely to vanish entirely. Of course, the individual powers in the above equation can only be determined by a detailed circuit analysis. For the purpose of variable
frequency conversion, the filters to be used must have variable parameters. The design of such filters is further complicated by the fact that successive sideband frequencies $f_s \pm f_c$, $f_s \pm 2f_c$, etc. may come quite close to one another. Even if the output power available at an upper sideband frequency is substantial, the design of suitable filters appears to be practically impossible. Another conclusion, based on the above analysis, is that frequency regulation will be rather poor. Frequency regulation, $f_{\text{reg}}$, is defined as $\frac{f_0 - f_s}{f_s}$. If the power gain specified is 25, $f_{\text{reg, max}}$ is only 4.16%.

The nonlinear reactor modulator, therefore, can be ruled out as a possible solution to the particular frequency conversion problem dealt with here.
4. FREQUENCY CONVERSION NETWORKS CONTAINING NONLINEAR ACTIVE ELEMENTS WITH TIME-VARYING PARAMETERS

4.1 General considerations

The common method of providing power at any frequency is to use variable frequency oscillators and if necessary, power amplifiers. The power is taken from batteries or from AC – DC converters. The power-handling capability of such arrangements is, however, quite limited, especially if sinusoidal output is required. In this case conversion loss may easily amount to more than 50% of input power. An alternative method is to use a square wave inverter with some form of waveshaping or filtering at the output. This does not necessarily improve the overall efficiency, but it does reduce the relative amount of power dissipated in the inverter itself.

With the development of such solid state devices as the transistor, new interest has been taken in static inverter design. The progress in the solid-state art has been very rapid in the past few years, and it is expected that solid-state devices of much greater power handling capability will become available in the near future. But so far, only a few papers have been published which describe transistorized variable frequency power converters for AC machines.¹,²,⁷

These converters are of the switching or square-wave type and consist principally of a three-phase square-wave oscillator and an output amplifier in each phase. Filters have been ommitted in the case of three-phase induction motor
drive. It is claimed that according to laboratory tests, three-phase induction motors perform as well when powered by the square-wave converter as they do when powered from a sine-wave system, provided the stator windings are Y-connected with isolated neutral. Since the switching type converter has high efficiency or low dissipation requirements in the switching device, it is reasonable to assume that future high-power converters will eventually be of this kind.

A switching type converter is one in which the transfer of power is performed by devices that behave essentially as switches. The two types of solid state components presently available for switching purposes are controlled rectifiers and transistors. The electrical characteristic of the controlled rectifier is similar to that of a grid controlled mercury-pool rectifier. It can be turned on by the action of the grid, but once it is on, grid control is lost until the current flow through the element is made zero by other means. The transistor can be turned on and off at will. While firmly on it has a low forward impedance and when off a very high forward impedance. For conceptual purpose, the transistor is often represented by a switch whose opening and closing time is zero.

Several attempts were made to produce time-varying three-phase voltages as described by Equation 2.4 or 2.13. The only acceptable approach appeared to be conversion based on switching type devices such as the transistor. This way, the output wave forms will only be approximations to those given
by Equation 2.4, but at the same time the internal losses in
the converters will become small. The simple converter cir-
cuit developed will now be discussed.

4.2 A single-phase switching type converter

Consider the single-phase circuit in Fig. 4.1. Let \( e_1 \) and \( e_2 \) represent two sinusoidal voltages such that \( e_1 = e_2 = E \sin \omega t \). \( r_L \) is a load resistance.

\[
\begin{align*}
R_1(t) & \quad v & \quad R_2(t) \\
\hline
i_1 & \quad i_L & \quad r_L & \quad i_2 \\
\hline
+e_1 & \quad - & \quad + & \quad -e_2
\end{align*}
\]

Fig. 4.1 Single-phase switching circuit

The time-varying resistors \( R_1 \) and \( R_2 \) vary with time in the following way:

\[
\begin{align*}
R_1 &= r, \quad R_2 = R \quad \text{for} \quad 2n\tau < t < (2n + 1)\tau \\
R_1 &= R, \quad R_2 = r \quad \text{for} \quad (2n + 1)\tau < t < (2n + 2)\tau \\
n &= 0, 1, 2, \ldots
\end{align*}
\]

Assume \( R \gg r_L \gg r \) \( \quad \ldots \quad 4.2 \)

For the above circuit:

\[
\begin{align*}
e_1 &= i_1 \left( R_1 + r_L \right) - i_2 r_L \\
e_2 &= -i_1 r_L + i_2 \left( R_2 + r_L \right) \quad \ldots \quad 4.3 \\
i_L &= i_1 - i_2 \\
e_1 &= e_2 = e
\end{align*}
\]
The load current $i_L$ becomes

$$i_L = e^{\frac{R_2 - R_1}{r_L(R_1 + R_2) + R_1R_2}}$$

and hence the voltage $v$:

$$v = i_L r_L = e^{\frac{R_2 - R_1}{R_1 + R_2 + \frac{R_1R_2}{r_L}}} = e^g$$

... 4.4

In Equation 4.4., $g$ is a function of $R_1$, $R_2$ and $r_L$, hence a function of time. Using the conditions given by Equations 4.1 and 4.2, one finds:

For $2n\tau < t < (2n + 1)\tau$,

$$g(t) = \frac{1 - r}{1 + r + \frac{r}{r_L}} \approx 1$$

For $(2n + 1)\tau < t < (2n + 2)\tau$ ... 4.5

$$g(t) = \frac{\frac{r}{r_L} - 1}{\frac{r}{r_L} + 1 + \frac{r}{r_L}} \approx -1$$

According to Equations 4.5, $g(t)$ can be represented by a square-wave of unit amplitude and period $2\tau$. Denote the fundamental angular frequency of this wave by $\omega_c = \frac{2\pi}{2\tau}$.

Fourier expansion of $g(\omega_c t)$ yields:

$$g(\omega_c t) = \frac{4}{\pi} (\sin \omega_c t + \frac{\sin 3\omega_c t}{3} + \frac{\sin 5\omega_c t}{5} + \ldots)$$

... 4.6

Hence the output voltage $v(t)$ becomes

$$v(t) = \frac{4}{\pi} E \sin \omega_s t (\sin \omega_c t + \frac{\sin 3\omega_c t}{3} + \frac{\sin 5\omega_c t}{5} + \ldots)$$

... 4.7

When the load is resistive, $i$ has the same form as $v$. 
But the above analysis does not hold for \( i \) if the load is partly reactive.

The voltage \( v(t) \) given by Equation 4.7 contains the same voltage components as Equation 2.4 (phase a) and in addition components of smaller amplitude and higher frequency. It is, therefore, an approximation to the desired voltage-time function.

![Fig. 4.2 Mechanical switching arrangement](image)

The resistors \( R_1(t) \) and \( R_2(t) \) in the circuit Fig. 4.1 can be represented by the switching arrangement shown in Fig. 4.2. \( S_1 \) and \( S_2 \) are ideal switches whose opening and closing times equal zero. An electrical arrangement with essentially the same features is shown in Fig. 4.3, using transistors as on-off switches. To perform the switching operations, these transistors are triggered with square-wave signals as indicated in Fig. 4.3, where

\[
e_{c1} = e_{c3}, \quad e_{c2} = e_{c4} = -e_{c1}
\]
The single-phase switching type converter may be arranged as shown in Fig. 4.4. When e > 0, T₁ and T₂ conduct in successive time intervals of length τ, while T₂ and T₄ block. For e < 0, T₃ and T₄ conduct alternately and T₁, T₂ block. In order to simulate the switching operations illustrated in Fig. 4.2, it is also required that each of the transistor pairs T₁, T₃ and T₂, T₄ be turned on and off simultaneously.

Output voltage and current waveform for resistive load is shown in Fig. 4.5, assuming \( \omega_c > \omega_s = 2\pi 60 \text{ radians per second} \).

If the load has an inductive component, the current through it cannot be switched off instantaneously. An attempt to do so in a single-loop circuit with a mechanical switch, results in arcing and a high voltage spike across the
Fig. 4.4 Single-phase converter circuit

Fig. 4.5 Output waveform
switch gap. The corresponding "arching mode" operation of a switching transistor is not possible, the transistor would break down almost immediately. By inspection of the circuit in Fig. 4.4 it will be seen that under certain conditions, transient voltage peaks will not occur. These conditions must be such that the load circuit is never opened.

Consider the case when $e$ is larger than zero and $T_1$ conducts. Then the voltage $v$ equals $e$ approximately. Now let $T_1$ open and $T_2$ close simultaneously. The voltage $v$ then drops to approximately $-e$. It is prevented from going below $-e$ in the case of an inductive load component, because transistor $T_4$ will be turned on at the same time as $T_2$ and acts as a clamp. Similarly, when $T_2$ is turned off, $v$ is prevented from exceeding $+e$ by the action of $T_2$.

So far it has been assumed that the transistors could be turned on and off instantaneously. In most practical applications, switching times $t_{on}$ and $t_{off}$ are of the order of microseconds. Generally, $t_{on}$ is less than $t_{off}$. With increased base drive, $t_{on}$ decreases while $t_{off}$ tends to increase. A typical transient characteristic for common emitter configuration is shown in Fig. 4.6.3,8 During the short time interval when switching takes place, the active transistor pair performs an operation similar to that of a make-before-break switch. Even though the actual transition time is short in comparison with the load time constant, under normal circumstances the load circuit will not be opened so that voltage spikes can build up.
Fig. 4.6 Transient response of common emitter switching circuit

The expression for output voltage derived above, Equation 4.7, presumes ideal voltage source and split-phase transformer. Transfer of reactive power between source and load is not restricted in that case.

4.3 The three-phase switching type converter

To produce an approximation to the system of voltages given by Equation 2.4, three single-phase converters of the type described in the previous section can be used. The principal circuit is illustrated in Fig. 4.7 (control circuits omitted).

Now consider the output voltages from the converter.
Fig. 4.7 Simplified diagram of three-phase converter

Fig. 4.8 Converter system representation for analysis
The input rms line-to-line voltages will be taken as 1.0 p.u. and the winding ratio is chosen as 1:2 for convenience. Further it will be assumed that these transformers are ideal and fed from an ideal three-phase 60 cps voltage source. Under these simplified conditions the circuit can be represented as in Fig. 4.8, where

\[ e_{sa} = \sqrt{2} \sin \omega_s t \]
\[ e_{sb} = \sqrt{2} \sin (\omega_s t - 120) \]
\[ e_{sc} = \sqrt{2} \sin (\omega_s t + 120) \] ... 4.9

The control signal which applies to phase a is given by Equation 4.8. Its effect on the output voltage, here \( v_{an} \), was examined in the previous section for a finite resistive load (Equation 4.7). Equation 4.7 is only true if current is allowed to flow through the switches when closed. To satisfy this condition, one may introduce a parasitic resistive three-phase load in the circuit in Fig. 4.8. This is indicated in Fig. 4.9.

![Diagram of parasitic load](Fig. 4.9 Parasitic load)
In accordance with condition (iii) section 2.2, the control voltages to the three phases must be displaced 120 electrical degrees apart in time and form a symmetrical set of phasors whose direction of rotation is opposite to that of the 60 cps input voltage phasors. The input voltages given by Equation 4.9 form a positive sequence system, hence the fundamental control voltages must form a negative sequence system. The fundamental angular frequency of the control voltages is $\omega_c$. Output line-to-neutral voltages are:

$$e_{an} = \frac{4\sqrt{2}}{\pi} \sin \omega_s t \left[ \sin \omega_c t + \frac{1}{3} \sin 3\omega_c t + \frac{1}{5} \sin 5\omega_c t + \frac{1}{7} \sin 7\omega_c t + \ldots \right]$$

$$e_{bn} = \frac{4\sqrt{2}}{\pi} \sin (\omega_s t - 120) \left[ \sin(\omega_c t + 120) + \frac{1}{3} \sin 3(\omega_c t + 120) + \frac{1}{5} \sin 5(\omega_c t + 120) + \frac{1}{7} \sin 7(\omega_c t + 120) + \ldots \right]$$

$$e_{cn} = \frac{4\sqrt{2}}{\pi} (\omega_s t + 120) \left[ \sin(\omega_c t - 120) + \frac{1}{3} \sin 3(\omega_c t - 120) + \frac{1}{5} \sin 5(\omega_c t - 120) + \frac{1}{7} \sin 7(\omega_c t - 120) + \ldots \right]$$

By trigonometric transformation these can be written:

$$e_{an} = \frac{2\sqrt{2}}{\pi} \left[ \cos(\omega_c - \omega_s)t - \cos(\omega_c + \omega_s)t + \frac{1}{3} \left[ \cos(3\omega_c - \omega_s)t - \cos(3\omega_c + \omega_s)t \right] + \frac{1}{5} \left[ \cos(5\omega_c - \omega_s)t - \cos(5\omega_c + \omega_s)t \right] + \frac{1}{7} \left[ \cos(7\omega_c - \omega_s)t - \cos(7\omega_c + \omega_s)t \right] + \ldots \right]$$
These expressions for the output phase voltages can now be arranged in terms of frequency or sequence components. The sequence components are of particular interest here and will be presented as follows:

\[ e_+ = \frac{2\sqrt{2}}{\pi} \left[ \cos(\omega_c - \omega_s)t - \frac{1}{3} \cos(3\omega_c + \omega_s)t + \frac{1}{7} \cos(7\omega_c - \omega_s)t - \frac{1}{9} \cos(9\omega_c + \omega_s)t + \ldots \right] \quad \ldots 4.11 \]

\[ e_- = \frac{2\sqrt{2}}{\pi} \left[ \frac{1}{3} \cos(3\omega_c - \omega_s)t - \frac{1}{5} \cos(5\omega_c - \omega_s)t + \frac{1}{9} \cos(9\omega_c - \omega_s)t + \ldots \right] \quad \ldots 4.12 \]

\[ e_0 = \frac{2\sqrt{2}}{\pi} \left[ - \cos(\omega_c + \omega_s)t + \frac{1}{5} \cos(5\omega_c - \omega_s)t - \frac{1}{7} \cos(7\omega_c - \omega_s)t + \ldots \right] \quad \ldots 4.13 \]

This is not the conventional form in which symmetrical
components are presented. It has been adopted here because several frequencies are present in each sequence system.

The positive sequence system has the desired fundamental frequency $f_C - f_s$ and in addition components of higher frequencies such as $3f_C + f_s$, $7f_C - f_s$, $9f_C + f_s$, with reduced amplitudes. In the negative sequence system, the fundamental frequency does not appear; nor does it appear in the zero sequence voltage. But the zero sequence voltage contains a component of frequency $f_C + f_s$ with the same amplitude as the fundamental. The system voltages can best be represented by phasor diagrams. This is done in Fig. 4.10, for the components given in Equations 4.11 to 4.13, using phase a as reference for $t = 0$.

![Phasor Diagram](image)

**Fig. 4.10** Phasor representation of system voltages indicating amplitudes relative to the fundamental, direction of rotation and angular velocity. A) positive sequence systems. B) Negative sequence systems. C) Zero sequence systems (pulsating voltages).
Now suppose the load is star-connected with grounded neutral. All sequence voltages can then drive current through the load. But in the case of three-phase AC machine load, the zero-sequence currents produce no desirable torque and should be avoided. This is achieved by isolating the star-point of the windings. A voltage then occurs at the neutral which equals the zero sequence voltage \( e_0 \), (Equation 4.13). Hence the resultant phase voltages for the load become:

\[
e_{aL} = \frac{2\sqrt{2}}{\pi}\left[ \cos(\omega_c - \omega_s)t - \frac{1}{3} \cos(3\omega_c + \omega_s)t + \frac{1}{7} \cos(7\omega_c - \omega_s)t - \frac{1}{9} \cos(9\omega_c + \omega_s)t + \ldots \right] \\
+ \left[ \frac{1}{3} \cos(3\omega_c - \omega_s)t - \frac{1}{5} \cos(5\omega_c + \omega_s)t + \frac{1}{9} \cos(9\omega_c - \omega_s)t + \ldots \right] \quad \ldots 4.14
\]

\[
e_{bL} = \frac{2\sqrt{2}}{\pi}\left[ \cos((\omega_c - \omega_s)t - 120) - \frac{1}{3} \cos((3\omega_c + \omega_s)t - 120) \\
+ \frac{1}{7} \cos((7\omega_c - \omega_s)t - 120) - \frac{1}{9} \cos((9\omega_c + \omega_s)t - 120) + \ldots \right] \\
+ \left[ \frac{1}{3} \cos((3\omega_c - \omega_s)t + 120) - \frac{1}{5} \cos((5\omega_c + \omega_s)t + 120) + \frac{1}{9} \cos((9\omega_c - \omega_s)t + 120) + \ldots \right] \quad \ldots 4.15
\]

\[
e_{cL} = \frac{2\sqrt{2}}{\pi}\left[ \cos((\omega_c - \omega_s)t + 120) - \frac{1}{3} \cos((3\omega_c + \omega_s)t + 120) \\
+ \frac{1}{7} \cos((7\omega_c - \omega_s)t + 120) - \frac{1}{9} \cos((9\omega_c + \omega_s)t + 120) + \ldots \right]
\]
The voltage components in Equations 4.14 to 4.16 are represented in Fig. 4.10 A) and B), and constitute a positive and a negative sequence system. (Or if the common interpretation is used: a set of positive and negative sequence systems).
5. PERFORMANCE OF AC MOTORS WHEN POWERED FROM THE SWITCHING TYPE CONVERTER

This chapter presents a brief examination of AC motor performance when applied voltage to each phase is of the form described above, section 4.3. To start with it will be assumed that these voltages are independent of current flow in the load, and also that the load currents can lag their respective driving voltages. While the first assumption is reasonable, the latter may not be justified, or may require an auxiliary reactive power source. Two types of three-phase AC motors will be considered, the induction motor and the synchronous motor.

5.1 Three-phase induction motor load

All voltage components will produce torque. Since the component of lowest frequency \( (f_c - f_s) \) also has the largest amplitude, motor speed is determined by this frequency. In general, the positive sequence voltages set up a unidirectional electro-magnetic torque which is termed positive since it has the same direction in the air gap as the direction of travel of the mmf-waves associated with positive sequence voltages. Similarly, the negative sequence voltages set up a torque which is termed negative since it acts in a direction opposite to the positive torque. Denote these two torques as \( T_+ \) and \( T_- \) respectively.

Then:

\[
T_+ = T_{f_c - f_s} + T_{3f_c - f_s} + T_{7f_c - f_s} + T_{9f_c - f_s} + \ldots \quad \ldots 5.1
\]
\[ T_\pm = T_3 f_c - f_s + T_5 f_c + f_s + T_9 f_c - f_s + \ldots \]  \[ \ldots 5.2 \]

The subscripts on the right hand side denote associated frequencies. The resultant torque, \( T \), is

\[ T = T_+ - T_- \]  \[ \ldots 5.3 \]

To find approximate expressions for the individual torque components, the conventional induction motor equivalent circuit will be used.\(^4,5\) This circuit is shown in Fig. 5.1.

\[ \text{Fig. 5.1 Circuit of induction motor} \]

\[ V_1 = \text{rms-value of input voltage of frequency } f. \]

\[ x_1 = \text{stator leakage reactance at frequency } f. \]

\[ x_2 = \text{rotor leakage reactance referred to stator, at frequency } f. \]

\[ x_m = \text{magnetizing reactance at frequency } f. \]

\[ r_1 = \text{stator resistance}. \]

\[ r_2 = \text{rotor resistance referred to stator}. \]

\[ s = \text{slip in p.u. of principal synchronous speed}. \]

The electromagnetic torque at input frequency \( f \) is given by the following equation:

\[ T = \frac{p q}{4 \pi f} E_1 I_2 \cos \delta_2 \]  \[ \ldots 5.4 \]
where \( \cos \delta_2 = \frac{\frac{r_2}{s}}{\sqrt{\frac{r_2}{s}^2 + x_2^2}} \)

\( p = \) number of poles
\( q = \) number of phases

It is convenient to transform the above circuit to the one shown in Fig. 5.2. By Thevenin's theorem:

\[ E_{10} = V_1 \frac{jx_m}{r_1 + j(x_1 + x_m)} \]  
\[ R_1 + jX_1 = \frac{jx_m(r_1 + jx_1)}{r_1 + j(x_1 + x_m)} \]

The torque relation can now be written as

\[ T = \frac{pq}{4 \pi f} E_{10}^2 \frac{\frac{r_2}{s}}{(R_1 + \frac{r_2}{s})^2 + (x_1 + x_2)^2} \]  (Nm) \[ \ldots 5.7 \]

As stated above, the reference synchronous speed is determined by the frequency \( f_c - f_s = f_0 \), and the slip \( s \) will be related to this frequency. When a positive sequence voltage of frequency \( f_+ \) is applied to the stator, the apparent slip \( s_+ \) becomes

\[ s_+ = \frac{f_+ - f_0 (1 - s)}{f_+} \] \[ \ldots 5.8 \]
For an applied negative sequence voltage of frequency $f_-$, the apparent slip $s_-\text{ is}$

$$s_- = \frac{f_- + f_0 (1 - s)}{f_-} \quad \ldots 5.9$$

Each of the torque components in Equation 5.1 and 5.2 can now be expressed in terms of corresponding voltages, frequencies, slip $s$ and motor parameters. The condition for adding them together to find resultant positive and negative torques is that no saturation occurs in the motor. Per unit values of the voltages are given in the expressions for $e_+$ and $e_-$, Equations 4.11 and 4.12. In order to simplify the subsequent expressions, the expressions will be restricted to frequencies which make $x_m \gg r_1$. In general, $x_m \gg x_1$, and $x_m \gg x_2$. Equations 5.5 to 5.7 approximated on this basis become

$$E_{10} = V_1 \quad \ldots 5.10$$

$$R_1 + jX_1 = r_1 + jx_1$$

$$T = \frac{pq}{4\pi f} V_1^2 \frac{r_2}{s} \frac{1}{(r_1 + \frac{r_2}{s})^2 + (x_1 + x_2)^2} \quad \ldots 5.11$$

Let $x_1$ and $x_2$ be reactances at frequency $f_s$, and define a new parameter, $k$:

$$k = \frac{f_c}{f_s}, \quad k > 1 \quad \ldots 5.12$$

Thus one has for the fundamental torque component

$$T_{f_c - f_s} = \frac{pqV^2}{4\pi f_s} \frac{k-1}{k-1} \frac{r_2}{s} \frac{1}{(r_1 + \frac{r_2}{s})^2 + (k - 1)^2(x_1 + x_2)^2} \quad \ldots 5.13$$
where
\[ V = V_{f_e} - f_s = \frac{2}{\pi} p.u. \]

As seen from Equations 4.11 and 4.12
\[ V_{if_e} + f_s = \frac{V_{f_e} - f_s}{i} = \frac{V}{i} p.u., \quad i = 1, 3, 5, \ldots \quad 5.14 \]

Combining Equations 5.11, 5.8 and 5.14 gives for the positive high frequency torque components:

\[ T_3 f_c + f_s = \frac{pqV^2}{4\pi f_s} \left( \frac{1}{3} \right)^2 \frac{1}{3k+1} \frac{r_2 (3k + 1)}{2(k+1) + s(k-1)} \]
\[ \frac{r_2 (3k + 1)}{(r_1 + \frac{r_2 (3k + 1)}{2(k+1)+s(k-1)})^2 + (3k+1)^2(x_1+x_2)^2} \]
\[ \ldots \quad 5.15 \]

\[ T_7 f_c - f_s = \frac{pqV^2}{4\pi f_s} \left( \frac{1}{7} \right)^2 \frac{1}{7k-1} \frac{r_2 (7k-1)}{6k + s(k-1)} \]
\[ \frac{r_2 (7k-1)}{(r_1 + \frac{r_2 (7k-1)}{6k+s(k-1)})^2 + (7k-1)^2(x_1+x_2)^2} \]
\[ \ldots \quad 5.16 \]

\[ T_9 f_c + f_s = \frac{pqV^2}{4\pi f_s} \left( \frac{1}{9} \right)^2 \frac{1}{9k+1} \frac{r_2 (9k+1)}{2(4k+1) + s(k-1)} \]
\[ \frac{r_2 (9k+1)}{(r_1 + \frac{r_2 (9k+1)}{2(4k+1)+s(k-1)})^2 + (9k+1)^2(x_1+x_2)^2} \]
\[ \ldots \quad 5.17 \]

For the first three negative torque components, Equation 5.11 combined with Equations 5.9, 5.12 and 5.14 gives:

\[ T_3 f_c - f_s = \frac{pqV^2}{4\pi f_s} \left( \frac{1}{3} \right)^2 \frac{1}{3k-1} \frac{r_2 (3k - 1)}{2(2k-1) - s(k-1)} \]
\[ \frac{r_2 (3k - 1)}{(r_1 + \frac{r_2 (3k - 1)}{2(2k-1)-s(k-1)})^2 + (3k-1)^2(x_1+x_2)^2} \]
\[ \ldots \quad 5.18 \]
\[ T_{5f_c+f_s} = \frac{p q V^2}{4 \pi f_s} \left( \frac{1}{9} \right)^2 \frac{1}{5 k+1} r_2 \frac{(5k+1)}{6k-s(k-1)} \frac{r_2(5k+1)}{\left(r_1+\frac{r_2(5k+1)}{6k-s(k-1)}\right)^2+(5k+1)^2(x_1+x_2)^2} \]

\[ T_{9f_c-f_s} = \frac{p q V^2}{4 \pi f_s} \left( \frac{1}{9} \right)^2 \frac{1}{9k-1} \frac{r_2(9k-1)}{2(5k-1)-s(k-1)} \frac{r_2(9k-1)}{\left(r_1+\frac{r_2(9k-1)}{2(5k-1)-s(k-1)}\right)^2+(9k-1)^2(x_1+x_2)^2} \]

... 5.19

... 5.20

From these expressions one can conclude:

1) Higher harmonic torque components are very small compared to \( T_{f_c+f_s} \) except when \( s \) approaches zero. Since these torques tend to cancel each other, the motor performance will be characterized almost exclusively by Equation 5.13.

2) The available torque for a given \( s > 0 \) decreases with increased reference frequency, i.e. increased \( k \), but it can be varied over a wide range.

At a given reference frequency the torque-slip characteristic takes the form indicated in Fig. 5.3 (assuming relatively low rotor resistance).

Fig. 5.3 Typical torque-slip curve
Neglecting higher harmonic torque components, the slip $s_m$ for which maximum electromagnetic torque $T_m$ occurs, can be derived from Equation 5.13:

$$\frac{dT}{ds}(T_f - f_s) = 0 \quad \mid s = s_m$$

This gives for $s_m$:

$$s_m = \frac{r_2}{\sqrt{r_1^2 + (k-1)^2(x_1+x_2)^2}} \quad \ldots 5.21$$

The slip $s_m$ decreases with increased frequency or synchronous speed. Maximum torque as a function of $k$, $T_{m,k}$, is given by Equation 5.13 with $s = s_m$. Thus one finds:

$$T_{m,k} = \frac{p q V^2}{8 \pi f_s^2} \frac{1}{k-1} \frac{1}{r_1 + \sqrt{r_1^2 + (k-1)^2(x_1+x_2)^2}} \quad \ldots 5.22$$

Note that this relation does not hold for values of $k$ near 1. But it is expected to be fairly accurate for $k > 1.5$. A substantial torque is therefore available at synchronous speeds ranging from about 40 to 200% of the synchronous speed corresponding to $f_s$.

In order to give an idea of how $T_{m,k}$ varies with $k$, and how this torque compares with maximum torque, $T_{m,s}$, at frequency $f_s$ and 1.0 p.u. voltage input, consider the ratio

$$\frac{T_{m,k}}{T_{m,s}}.$$  

Recall that $V$ in Equation 5.22 equals $\frac{2}{\pi}$ p.u. and that reference frequency $f_s$ corresponds to $k = 2$. Hence

$$\frac{T_{m,k}}{T_{m,s}} = \left(\frac{2}{\pi}\right)^2 \frac{1}{k-1} \frac{r_1 + \sqrt{r_1^2 + (x_1+x_2)^2}}{r_1 + \sqrt{r_1^2 + (k-1)^2(x_1+x_2)^2}} \quad \ldots 5.23$$
\( r_1, x_1 \) and \( x_2 \) are not known. But if numerator and denominator on the right hand side of Equation 5.23 are divided by \( r_1 \), the familiar quantity \( Q = \frac{x_1 + x_2}{r_1} \) appears. For a conventional induction motor, \( Q \) has a value of about 5. Thus one can write Equation 5.23:

\[
\frac{T_{m,k}}{T_{m,s}} = \left( \frac{2}{\pi} \right)^2 \frac{1}{k-1} \frac{1 + \sqrt{1 + Q^2}}{1 + \sqrt{1 + (k-1)^2 Q^2}} = 2.47 \frac{1}{k-1} \frac{1}{1 + \sqrt{1 + 25(k-1)^2}} \quad \ldots 5.24
\]

Equation 5.24 is presented graphically in Fig. 5.4.

![Graph](image)

**Fig. 5.4** Maximum relative torque as a function of \( k = \frac{f_c}{f_s} \)

The only way to maintain constant torque at a given slip \( s \), is to vary the applied voltage. But this of course requires extra equipment on the supply side. Both power factor and efficiency will drop with increased \( k \). The power factor decreases because of a larger inductive component in input impedance. Hysteresis and eddy-current losses increase with increased frequency. But although these
drawbacks may be serious enough, they do not necessarily over-
rule the potential advantages of the speed control available.

Concerning the power supply to the induction motor, an
important condition always exists: Reactive power must be free
to oscillate between the magnetic circuits of the machine and
an external source. Examination of the switching converter
described in the previous chapter shows that operation on an
inductive load leads to difficulties. Leakage inductance in
the split-phase transformers can not be disregarded, neither
can one assume ideal coupling between primary and secondary
windings. With resistive converter load, primary transformer
current flows at 60 cps without interruption, whereas an in-
ductive load component requires instantaneous changes in the
primary current. The charging effect of the transistors will
generally be insufficient as a compensation for lagging load
current, but theoretically it presents a possibility to run
induction motors at nearly no load. Loading without addi-
tional compensation for reactive VA is only possible at the
risk of destroying the transistors.

Capacitive compensation at the motor terminals can be
arranged, but appears to become costly. The capacitors must
be adjusted both according to load and synchronous speed.
Another possibility is to use a synchronous condenser to
feed reactive power into the load system, thus establishing
an approximately real load impedance across the converter
output terminals. This solution is recommendable in case
several induction motors are to be operated at the same speed
and fed from a single converter unit. The problem of proper excitation of the synchronous condenser will be discussed in the next section.

5.2 Synchronous motor load

Since supply of reactive power from the converter is restricted, the type of three-phase motor best fitted is the synchronous motor. This motor operates at unity power factor as well as leading or lagging armature current, depending on the degree of excitation at a given shaft load. Its speed is independent of load under stable conditions, and is directly proportional to input frequency. Assuming the motor can be started as an induction motor (no load), with applied voltages as given by Equations 4.14 to 4.16, its speed will be determined by the lowest output frequency from the converter, $f_c - f_s$. Higher harmonic currents will not produce useful torque since their mmf waves do not have a fixed space relation to the flux wave produced by rotor. Compensation for reactive power to the armature takes place at synchronous frequency only. The higher harmonic currents of both positive and negative sequence should be kept low to avoid extra losses in motor and converter. From the point of view of motor design, this condition indicates a small air-gap and no damper windings on the rotor poles. The input impedance for higher harmonic voltages can be increased further by adding series inductance in the rotor excitation circuit.

Excitation of the synchronous motor so as to obtain unity
power factor at various speeds and loads presents a number of technical problems which will not be dealt with here. Only the basic features will be discussed, under simplified conditions.

The following assumptions are made:

a) The motor has a uniform air-gap.
b) No saturation occurs.
c) The armature resistance is negligible.
d) Armature leakage inductance is negligible compared to air-gap inductance.
e) The applied phase voltage is sinusoidal, of frequency \( f \) and constant amplitude.

Assumption b) can be satisfied quite well by careful design when frequency and load ranges are specified; c) and d) simplify to some extent the presentation of current and voltage relations in the machine but cannot be accepted if a more exact analysis is to be carried out. Assumption e) implies that one only considers the fundamental component of input voltage, and neglects voltage drop in the converter. In view of the fact that the useful torque is produced by the fundamental voltage and current components only, part of this assumption is perfectly justified as far as the subsequent analysis is concerned.
The basic phasor diagram for one phase is shown in Fig. 5.5, where

- $V_t$ = terminal voltage, rms value.
- $E_i$ = internal induced voltage, rms value.
- $I_A$ = armature current, rms value.
- $L_d = L_m$ = magnetizing inductance.
- $\phi$ = flux per pole due to rotor mmf.
- $\phi_o$ = flux per pole at no load.
- $\phi_A$ = flux per pole due to armature reaction.

In accordance with the above assumptions

\[
\phi = k_f i_f 
\]

\[
\phi_A = k_A I_A 
\]

$k_f$ and $k_A$ are constants. $i_f$ = field current (DC). The induced voltage $E_i$ is given by

\[
E_i = k_E \omega \phi, \quad k_E = \text{constant}
\]

Also, according to Fig. 5.5,
\[ E_i^2 = V_t^2 + (\omega L d I_A)^2 \quad \ldots \quad 5.28 \]

Substitution of \( E_i \) from Equation 5.27 into 5.28 yields:

\[ k_E^2 \omega \varphi^2 = V_t^2 + (\omega L d I_A)^2 \quad \ldots \quad 5.29 \]

Combine Equations 5.29 and 5.25 and solve for \( i_f \):

\[ i_f = \sqrt{\frac{V_t^2}{(k_E k_f)^2 \omega^2} + \frac{L_d^2}{(k_E k_f)^2} I_A^2} \quad \ldots \quad 5.30 \]

Equation 5.30 shows how the field current depends on applied frequency, voltage and load current, at unity power factor. Let excitation current at no load \((I_A = 0)\) be denoted \( i_{f0} \). From Equation 5.30 it follows that

\[ i_{f0} = \frac{V_t}{k_E k_f \omega} \quad \ldots \quad 5.31 \]

Since \( V_t \) is assumed constant, the no load excitation current must vary as the inverse of applied frequency. This relation, however, does not hold for high synchronous speeds when a substantial amount of torque is required to overcome air friction.

The relation between \( i_f \), \( \omega \) and input power per phase, \( P_{ph} = V_t I_A \), is:

\[ i_f = \sqrt{\frac{V_t^2}{(k_E k_f)^2 \omega^2} + \frac{L_d^2}{(k_E k_f)^2} \frac{P_{ph}}{V_t^2}} \quad \ldots \quad 5.32 \]

The total electromagnetic torque \( T \) is given by

\[ T = \frac{3p}{2} P_{ph} \quad (q = 3) \quad \ldots \quad 5.33 \]
Thus the relation between $i_f$, $\omega$ and $T$ can be written:

$$i_f = \sqrt{\frac{V_t^2}{(kE k_f)^2}} \frac{\omega^2}{\omega^2 + \frac{2}{V_t^2} \frac{L_d^2}{9p^2 (kE k_f)^2} T^2} \quad \ldots \quad 5.34$$

According to Equation 5.34, $i_f$ as a function of $\omega$ has a minimum when the two terms under the square-root sign are equal in magnitude:

$$\frac{\omega}{V_t} \frac{2}{3p} \frac{L_d}{kE k_f} T = \frac{V_t}{kE k_f} \omega$$

and hence

$$(\omega)^{i_f, \text{min}} = V_t \sqrt{\frac{3p}{2L_d T}} \quad \ldots \quad 5.35$$

The current $i_f$ as a function of $\omega$ with constant torque $T$ as a parameter is indicated in Fig. 5.6, where $T_1 < T_2 < T_3$.

![Fig. 5.6 Excitation current as a function of frequency with torque as parameter](image)

The maximum value of $i_f$ depends on design and specified maximum temperature rise in the field windings. The available
torque at a particular frequency is limited by maximum allowable armature current. The torque $T_{\text{max}}(\omega)$ is approximately proportional to $\frac{1}{\omega}$ (Equation 5.33). Since the effective armature resistance increases with increased frequency, $I_{\text{A max}}$ must be reduced unless better cooling is provided at higher speeds. Additional restrictions may be placed on $I_{\text{A max}}$ in connection with a prescribed stability margin.

Theoretically, control of $i_f$ can be based on any of the three quantities $I_{\text{A}}$, $P_{\text{ph}}$, and $T$, in addition to $\omega$. But the best approach in practice would probably be to consider the angle $\delta$ between phase voltage and line current, and make $\delta = 0$ the criterion for correct excitation. Then if $\delta$ is measured at the output of the converter, load can consist of induction motors in addition to the synchronous motor. The synchronous motor acts as a synchronous condenser in this case, or as a combination of synchronous motor and condenser. Note that since $\delta = 0$ is a necessary and sufficient condition for unity power factor operation, it provides a unique key to the field control problem. Effects of speed and load variations, saturation and additional inductive load components will be accounted for automatically by this single condition. Lagging phase angle ($\delta < 0$) indicates that the field current $i_f$ should be increased, and leading phase angle indicates that it could be decreased.

As mentioned in the previous section, control of the applied voltage would improve the torque-frequency characteristic of the induction motor. If for instance the applied
voltage is made proportional to the frequency $f_c - f_s$ (within practical limits), Equation 5.22 takes on the form

$$T_{m,k} = K \frac{k - 1}{r_1 + \sqrt{r_1^2 + (k - 1)^2(x_1 + x)^2}}, \quad K = \text{const.} \quad \ldots \quad 5.35$$

which indicates that the maximum torque is much less frequency dependent than before. To the synchronous motor, operating at unity power factor, a constant voltage/frequency ratio means less variation in field current when torque and speed varies. At no load, the field current would be approximately constant or independent of speed, as can be seen from Equation 5.31. Voltage regulation would also be desirable from the point of view of machine design, since it leads to nearly constant flux densities.

On the other hand, variable ratio transformers and additional voltage-control equipment is necessary. The converter must perform well regardless of actual variations in input rms voltage. This appears to be possible with the type of converter described in Chapter 4, as long as the break-down voltage of the power transistors is not exceeded.
6. EXPERIMENTAL FREQUENCY CONVERTER AND SQUARE-WAVE GENERATOR

6.1 Introduction

It was decided to build an experimental three-phase frequency converter of the type described in Chapter 4, and demonstrate its basic features as a combined power supply and speed control device for AC motors. The control of synchronous speed is of particular interest. An output voltage of about 110 volts rms would be desirable since this is commonly the rated voltage of small three-phase motors. But the maximum output voltage from the converter is limited by the breakdown voltage ($BV_{CE}$) of the transistors. Ordinary power transistors were used, with $BV_{CE}$ specified as 40 volts. As can be seen from Fig. 4.4, maximum voltage across each transistor is $(2e)_{\text{max}} = 2E$. Hence $E$ must be less than 20 volts. This means that the fundamental line-to-line voltage at the load should have an rms value less than $(20) \frac{2}{\pi} \frac{\sqrt{3}}{\sqrt{2}} = 15.6$ volts.

A small three-phase induction motor rated at 20 volts was available so that speed-tests could eventually be carried out at this low voltage.

High power transistors with values of $BV_{CE}$ around 100 volts are available. But in order to provide e.g. 110 V rms line-to-line at the load, the power transistors must withstand nearly 300 volts. Although this load voltage can be achieved with present transistor ratings, either by step-up transformers or by use of several power transistors in series in each branch (Fig. 4.4), the converter becomes more expensive.
and less reliable. However, higher voltage ratings for power transistors which can be expected in the near future, will make this converter more versatile.

The output fundamental frequency is controlled from the square wave generator which triggers the power transistors. A three-phase variable frequency oscillator could have been used as far as no load speed tests of the motor were concerned. But it was decided to build a transistorized control unit, suitable for the particular purpose mentioned above. A number of difficulties were encountered in connection with the design of this square-wave generator, most of them related to the fact that the author was not too familiar with pulse and digital circuits. Only the final arrangement will be described in this chapter. It is realized that this arrangement can still be improved. Some modifications and additional amplification stages become necessary when increased power output from the converter is required.

6.2 General description

A block diagram of converter and frequency control circuits is shown in Fig. 6.1. The blocks a, b and c represent the single-phase converter units. Each of these units consists of four power transistors, arranged and operated as explained in Chapter 4. The supply unit has three single-phase transformers, 115 V delta-connected primary and 2x 13.5 V secondary. Maximum voltage across each power transistor is therefore 2 x 13.5 x 2 = 38.2 V. As indicated in Fig. 4.4,
Fig. 6.1 Block diagram of experimental converter. Blocks a, b and c represent single-phase switching units.
inductive coupling is used to feed triggering signals to the base-emitter terminals.

The square wave generator consists of one multivibrator (a-stable), two differentiating circuits and two inverters, six three-port gates and three flip-flops. Fig. 6.2 shows in more detail how these circuits are arranged.

The output fundamental frequency can be controlled by changing two equal RC time constants in the multivibrator. In Fig. 6.3, the two output voltages from the multivibrator are illustrated by A) and B). These voltages are used to generate two sequences of negative pulses, displaced $180^\circ$ in time, as shown in Fig. 6.3, C) and D). Each pulse sequence is then fed to the corresponding set of gates, $G_{1-3}$ and $G'_{1-3}$, Fig. 6.2. If one gate is open, an incoming negative pulse will turn on the corresponding flip-flop side. This pulse has no effect if that flip-flop side is already on. The gates transmit negative pulses when the control terminals are at zero voltage level, and block if one or both g-terminals have a voltage $v$. Fig. 6.3, E), F) and G) shows output voltage waveforms at flip-flop terminals 1, 2 and 3. The wave-shaping process will be explained next.

Let the period of oscillation in the multivibrator be denoted $t_m$, and let the first pulse appear at $t = 0$. (Fig. 6.3, C). Assume that the flip-flops are initially set so that 1 and 2 are off, 3 is on. Consequently $1'$ and $2'$ are on, $3'$ off. Gates $G_1$ and $G'_2$ are then open. If the first pulse comes in at D (Fig. 6.2), it can only pass through gate $G'_2$. 
Fig. 6.2 Block diagram of three-phase square-wave generator

- FF - flip-flop
- G - gate
- 1, 2, 3, and 1', 2', 3' - output terminals
- c, d - gate input terminals
- g, g' - gate control terminals
Fig. 6.3 Voltage waveforms in the square-wave generator
A), B): multivibrator outputs
C), D): triggering pulses
E), F), G): square-wave output signals
but it has no effect on the output since $2'$ is already on.
The first pulse is therefore assumed to appear at C, the next at D and so on, and the effects can be summarized as follows:

1) $t = 0$. Pulse No. 1 passes through $G_1$ and triggers $FF_1$.
   $$v_1 \rightarrow v, \quad v_1' \rightarrow 0.$$
   $G_3'$ opens, $G_2'$ closes.

2) $t = \frac{1}{2} t_m$. Pulse No. 2 passes $G_3'$ and triggers $FF_3$.
   $$v_3 \rightarrow v, \quad v_3' \rightarrow 0.$$
   $G_2$ opens, $G_1$ closes.

3) $t = t_m$. Pulse No. 3 passes $G_2$ and triggers $FF_2$.
   $$v_2 \rightarrow v, \quad v_2' \rightarrow 0.$$
   $G_1'$ opens, $G_3'$ closes.

4) $t = \frac{3}{2} t_m$. Pulse No. 4 passes $G_1'$ and triggers $FF_1$.
   $$v_1 \rightarrow v, \quad v_1' \rightarrow 0.$$
   $G_3$ opens, $G_2$ closes.

5) $t = 2 t_m$. Pulse No. 5 passes $G_3$ and triggers $FF_3$.
   $$v_3 \rightarrow v, \quad v_3' \rightarrow 0.$$
   $G_2'$ opens, $G_1'$ closes.

6) $t = \frac{5}{2} t_m$. Pulse No. 6 passes $G_2'$ and triggers $FF_3$.
   $$v_2 \rightarrow v, \quad v_2' \rightarrow 0.$$
   $G_1$ opens, $G_3$ closes.

At $t = 3 t_m, 6 t_m, 9 t_m \ldots \ldots$, the process 1) - 6) starts over again. Note that a triggering pulse does not cause opening or closing of gates on the side where it enters. Therefore, control signal to the gates need not be delayed. With the system of gate control shown in Fig. 6.2, the sequence of output voltages $v_1, v_2$ and $v_3$ is fixed, but in order to start this sequence, one of the three flip-flops must be preset to a different state than the other two.

The output power available from the square-wave generator is of course low. The external load to each flip-flop side
Fig. 6.4 Circuit of converter unit, phase a.

Supply transformer: 115/2x13.5 volts, 130 watts
Driver transformers: 400/100 ohms, 0.5 watts
Power transistors: RCA Type No 2N301A (PNP)
Diodes: RAYTHEON Type No. CK848

\[ R = 1.5k \text{ ohm (parasitic load)} \]
\[ R_c = 680 \text{ ohm} \]

\( FF_1 \) - flip-flop
a - output terminal
consists of a resistance in series with two parallel connected base-emitter circuits in the converter. Fig. 6.4 shows the basic arrangement for one phase (phase a). The arrangement is similar for the other two phases.

The output signals from the square-wave generator must be amplified in order to drive the power-transistors properly at converter load currents larger than 0.2 amp. peak value. The simple motor speed tests did not require booster amplifiers at the flip-flop outputs, but tests requiring more power would have been possible with stronger control signals available.

6.3 A-stable multivibrator and pulse forming circuits

Fig. 6.5 shows the circuitry to be dealt with in this section. All four transistors are of the same type, 2N224. Current gain \( \beta = 60 - 120 \). The transistors operate in the on-off mode, and a value of 50 has been used for \( \beta \).

The period of oscillation in the multivibrator, \( t_m \), is largely determined by the time constant \( T = RC \). Since the voltage drop emitter-to-base is only of order 0.1 volt when the transistor is on, and the emitter-to-collector saturation voltage is equally low, one finds

\[
\frac{t_m}{2V_e \frac{e^{\frac{V}{2T}}}{2T}} = V_s
\]

and hence

\[
t_m = 2T \ln 2 \quad \ldots \quad 6.1
\]

The relation between \( t_m \) and \( f_c \) is indicated in Fig. 6.3.

\[
f_c = \frac{1}{3t_m} \quad \ldots \quad 6.2
\]
Fig. 6.5 A-stable multivibrator and pulse forming circuits

Transistors: PHILCO Type No. 2N224

- $R_L = 10 \text{ k.ohm}$
- $R_1 = 68 \text{ k.ohm}$
- $R_{\text{min}} = 47 \text{ k.ohm}$
- $R_2 = 1.8\text{ k.ohm}$
- $R_{\text{max}} = 500\text{ k.ohm}$
- $C_1 = 500 \text{ pF}$
- $C = 0.01 \mu\text{F}$
Combining Equations 6.1 and 6.2 yields for $f_c$:

$$f_c = \frac{1}{6T \ln 2} \quad \text{... 6.3}$$

Equation 6.3 is not strictly true unless the base input resistance is included in the expression for $T$. The values selected for $C$, $R_{\min}$ and $R_{\max}$ yield $f_{c \min} = 70$ cps and $f_{c \max} = 570$ cps. Maximum load current is given by $\beta$ and $(i_{\text{base}})_{\min}$. Since $R_L \ll R_1$ (Fig. 6.5), $R_L$ should be selected so that

$$\frac{V}{R_{\max}} \beta = \frac{V}{R_L} ; R_L = \frac{R_{\max}}{\beta} \quad \text{... 6.4}$$

If the circuit is completely symmetrical, oscillations do not start when the voltage $V$ is applied. Starting can be arranged in a number of ways. One way is to make the circuit unsymmetrical for a short moment by shorting one of the capacitors. In the experimental set-up, the multivibrator starts by itself due to slight dissymmetry.

Next consider circuit II, Fig. 6.5 (circuits II and III are identical). The operating point of the transistor is normally in the saturated region. Minimum output impedance at $C$ (and $D$) equals 5k.ohm. $R_2 = 1.8$ k.ohm. Therefore $R_1 = 50 \frac{9}{6.8} = 68$ k.ohm. When a step voltage rise occurs at $A$, the voltage at $A'$ rises to $2V$ instantaneously and decays with a time constant $= T_1 = R_1 C_1$. The transistor is turned off for a period $\tau_d = T_1 \ln 2$, thus producing a negative pulse at $C$. A negative voltage step at $A$ has no effect on the output at $C$ since whenever this occurs, the inverter transistor is in the on state. The negative pulses produced by circuit II
and III are used to trigger the flip-flops, and must fulfill two conditions concerning: 1) duration $t_d$ and 2) amplitude. The first condition will be considered now, the second later. According to reference 3, $t_d$ can be estimated from the following relation:

$$t_d = \frac{1}{(1 - \alpha_{fb}) \frac{\pi}{2} f_{ab}} \quad \ldots 6.5$$

$\alpha_{fb}$ for the transistor can be taken as 0.98, $f_{ab} = 510$ kc. With these values, Equation 6.5 yields

$$t_d = \frac{10^{-3}}{0.02 \times 2\pi \times 510} = 15.6 \text{ microsec.}$$

The turn-off time for the inverter transistor is relatively short (about 2 microsec.) due to the strong back-bias. Hence the off-time $\tau_d \leq t_d$. With $C_1 = 600$ pF, $\tau_d$ becomes 22 microsec. and $t_d = 20$ microsec. (measured), which is more than sufficient.

6.4 Gates and flip-flops

The six gates are identical, and also the three flip-flops. Fig. 6.6 shows the circuitry of flip-flop FF_1, gates $G_1$ and $G'_1$. Input and output terminals are marked as in Fig. 6.2. The flip-flops were designed for an external load of about 1 k.ohm in each branch. According to transistor specifications, $I_{CBO} = 10$ microamp at $V_{CB} = -12$ volts. The off transistor is back-biased 0.1 volt by selecting $R_4$ 56 k.ohm. Since the flip-flop circuits are conventional, design details will not be included here, nor should it be necessary to explain the characteristic flip-flop operation.
Fig. 6.6 Gates $G_1$, $G'_1$ and flip-flop FF1

Transistors: PHILCO Type No 2N224
Diodes: HUGHES Type No 1N191

$R_L = 1.0$ k.ohm $R_5 = 68.0$ k.ohm $r_g = 10.0$ k.ohm

$R_3 = 18.0$ k.ohm $r_3 = 5.6$ k.ohm $r_2 = 15.0$ k.ohm

$R_4 = 56.0$ k.ohm $C_5 = .01 \mu F$ $C_2 = .02 \mu F$

$C_3 = 200 \mu F$
Next, the gate and triggering system will be explained, referring to blocks $G_1$ and $K$, Fig. 6.6. Terminals 2 and 3' are connected to flip-flop outputs 2 and 3' respectively, as indicated in Fig. 6.2. Hence $v_2$ and $v_3'$ will alternate between zero and $V$ volts, but not simultaneously. A negative pulse at c should only pass through to the base of TR1 when $v_2 = v_3' = 0$. If $v_2$ or $v_3'$, or both equal V, the pulse should not pass the gate. Let the pulse amplitude at c be denoted $v_{pc}$ and consider the case when $v_2$ or $v_3'$, $= V = 12$ volts. With resistance values as given, $i_d = 1.2$ mA, $v_g = 0$ initially. When $v_{pc}$ is applied, $i_d$ drops instantaneously to a value $i_d'$ given by

$$i_d' = \frac{12}{10} - \frac{v_{pc}}{15}$$

Since $v_{pc} < 12$ volts, Equation 6.6 shows that $i_d' > 0.4$ mA, which is sufficient to keep the diode $D_g$ forward-biased, hence $v_g$ remains constant $= 0$. If $v_2 = v_3' = V$, $i_d'$ becomes larger than 0.8 mA. Therefore, the gate blocks as required.

Now if $v_2 = v_3' = 0$, $i_d = 0$ and diode $D_g$ has no appreciable forward bias. A negative pulse applied at C will drive $v_g$ negative. Therefore, the gate transmits a negative pulse under these conditions. In order to turn on TR1, the pulse amplitude at the base, $v_{pb}$, should be 0.3 - 0.4 volts (negative). Base input resistance is taken as 2 k.ohm. $r_3$ is selected 5.6 k.ohm to give a maximum emitter-to-base voltage of 0.4 volts when TR1 is turned on. Due to the capacitors $C_2$ and $C_5$, the triggering voltage at the base decays. Assuming $r_{ib}$ remains constant during the pulse period, $C_2$ and $C_5$
are chosen large enough to limit the decay of the triggering voltage to less than 0.1 volts. \( r_{ib} \) decreases once the transistor is turned on. But the above assumption is safe as far as external triggering action is concerned. When the transistor reaches the conducting state, no external triggering signal is necessary. The analysis on which selection of \( r_3 \), \( C_2 \) and \( C_5 \) has been based, is given in the Appendix.

The value of \( t_d \) given by Equation 6.5 appears to be too large for turn-on action. Actual turn-on time is only 5-6 microsec (measured).

Diode \( D_b \) prevents the base voltage \( v_b \) from rising to cut-off level when the triggering pulse disappears. The charge on \( C_5 \) can be restored due to the presence of \( R_5 \). \( T_5 = (r_3 + R_5) C_5 \) is chosen approximately equal to \( \frac{1}{2f_c \max} \).

Therefore, \( v_5 \) reaches a value close to \( V' = 13.5 \) volts in advance of triggering action. Also, when transistor TR1 is to be turned off, \( v_b \) can rise to cut-off level and \( D_b \) is still back-biased. This is a necessary condition for the flip-flop to function properly.

Some photographed waveforms are shown in Figures 6.7 to 6.10. In Fig. 6.7, one of the output square-wave signals is shown together with one of the pulse trains used for triggering. Fig. 6.8 shows two of the three square-waves in proper time relation. The output voltage, line to converter neutral, is shown in Fig. 6.9, together with the control signal in that phase. Output line-to-line voltage at the converter is shown in Fig. 6.10.
Fig. 6.7 Trigger pulse train and output square-wave
Scaling: 10 volts/div.
.5 msec./div.

Fig. 6.8 Output voltages at flip-flop terminals 1 and 2
Scaling: 10 volts/div.
.5 msec./div.
Fig. 6.9 Output line-to-neutral voltage at converter, and square-wave signal
Scaling: 10 volts/div.
5 msec./div.

Fig. 6.10 Output line-to-line voltage at converter
Scaling: 20 volts/div.
5 msec./div.
7. NO LOAD SPEED TESTS ON THREE-PHASE AC-MOTORS

The low output voltage and current from the converter limit its application to rather unconventionally small three-phase motors. However, the main purpose at this stage is to verify a principle of speed regulation based on frequency conversion, and the amount of power necessary is equivalent to the input motor power at no load.

A small three-phase motor rated at 20 volts was available. It has a rated synchronous speed of 1800 r.p.m. at 60 cps. No load speed tests on this motor confirmed the theory on which these experiments are based, within reasonable speed limits. Continuous variation was achieved between 350 and 5600 r.p.m. The test results are shown graphically in Fig. 7.1, curve A.

Although no direct measurement of torque has been attempted, it is obvious that the torque falls off at very low frequencies and at high frequencies. At low frequencies, increased per unit voltage drop in the armature resistance reduces the internal voltage and hence also the power available at the air-gap. At high frequencies, electromagnetic torque decreases in accordance with Equation 6.13, and finally it is not sufficient to overcome the increased windage.

No difficulty was encountered in operating the inverter on this load. Switching takes place without voltage spikes across the power transistors. The load current approaches sinusoidal wave-form, of frequency $f_c - f_s$.

Encouraged by the test results obtained for the 20 volt
Fig. 7.1 Experimental speed-frequency characteristics $f_c - f_s$

A) 20 volt motor
B) 50 volt motor
motor, one decided to try a larger induction motor. Unfortunately, the ratings of this particular motor were not available, but its rated voltage is estimated to be about 50 volts and rated power about 50 watts. The motor has four poles, distributed three-phase winding in Y-connection with isolated neutral. The original rotor, which has been designed for reluctance-motor operation, was replaced by a squirrel-cage rotor built in the Electrical Engineering Department. The tests on this motor also verified the basic principle of speed control, but mainly due to low phase voltage, the speed could only be varied in the range from 500 to 3000 r.p.m. (Rated synchronous speed at 60 cps input frequency is 1800 r.p.m.) Fig. 7.1, B) shows the experimental speed-frequency curve for this motor. The slip is relatively high, as must be expected when the motor runs with reduced voltage. At low frequencies the peak load current exceeds the limit for proper switching in the converter. That is, the control signals are not sufficiently strong to drive the power transistors into saturation. Hence output voltage will be reduced and distorted, and power dissipation in the converter increases.

An attempt was also made to run a test on a synchronous motor, using the 50 volt machine with salient pole DC excited rotor. This experiment proved to be less successful than the previous two. Although the motor did run in synchronism when started up by other means, it exhibited very low synchronizing torque. Synchronous speed could be varied between 1200 and 2000 r.p.m. by changing the frequency slowly.
There are two major reasons why the synchronous motor did not behave well. The first one is that the applied phase voltage (fundamental) was too low. As explained above, this voltage is limited by the converter. Secondly, pole shoes and cores were made of solid iron instead of laminated iron, thus simplifying the mechanical work involved. This leads to a relatively low negative sequence inductance in the machine. Consequently the torque pulsations due to higher harmonic input currents can become sufficiently large to exceed the synchronizing torque and thereby pull the motor out of synchronism.
8. CONCLUSIONS

The particular principle of speed regulation of three-phase motors which is introduced here, has been confirmed by experiments.

Practical application of the switching type converter is limited by break-down voltage and maximum collector current for the power transistors. The maximum peak value of output fundamental line-to-line voltage is 55% of the transistor break-down voltage. Thus, application to a 110 volt motor requires power transistors that operate safely at 300 volts.

The idea of connecting several transistors in series in each converter branch, to obtain a higher output voltage, does not appear to be practicable.

The switching type converter has low internal dissipation requirements, but the harmonic content in output voltages is high. Output voltages can be represented by a series of symmetrical component systems where all three sequences appear at different frequencies. The dominating positive sequence system is associated with the lowest, or fundamental frequency $f_c - f_s$, and carries most of the power. All sequence systems other than this one are undesirable from the point of view of motor operation and performance. The zero-sequence currents can be suppressed entirely by isolating the load neutral. But the presence of other higher harmonic currents reduces motor efficiency and give rise to pulsating torques in the case of a salient pole synchronous motor load. A synchronous motor
supplied from this converter should exhibit a high negative sequence inductance.

Compensation for reactive volt-amps at the load is desirable. The best method of compensation is to use a synchronous condenser which supplies reactive power at fundamental frequency. Several technical problems arise in connection with synchronous motor or synchronous condenser excitation control, which should be investigated further. Also, the time rate of change of applied frequency is restricted in this case and must be controlled.

The speed control principle outlined in this thesis and applied in the experiments will generally provide good speed regulation at the cost of motor efficiency. It is felt that by careful design of motors and control equipment, efficiency can be maintained at a reasonable level. Since the possible range of speed variation is large, and the speed-torque characteristics can be modified by voltage control, this system possesses some interesting and desirable features. In any case, however, its realization for practical application depends strongly on the development of solid-state components towards higher voltages and power ratings.
APPENDIX. SELECTION OF $r_3$, $C_2$ AND $C_5$

The quiecent state of the gate and triggering circuits, Fig. 6.6, is characterized by the following voltages:

$v_c = V, \ v_5 = V', \ v_g = 0$. For changes from this state, caused by a negative step voltage at $C$, analysis of the response at the base of $T_{R1}$ can be based on the circuit shown in Fig. A-1. $r_{ib}$ is actually shunted by $R_3, R_4$ and $R_5$, but these resistors can be neglected since they are all $> r_{ib} = 2 \ \text{K ohm}$. The branch containing $2C_2$ and $\frac{r_2}{2}$ represents the two (closed) gates $G_2$ and $G_3$.

$C_2$ and $C_5$ must be sufficiently large to prevent the trigger voltage $v_{pb}$ from decaying to a value below 0.3 volts over the switching period. The initial value $v_{pb}$ is chosen approximately 0.4 volts.

![Simplified circuit for analysis of $v_{pb}$](image)

Since the time interval considered is short compared to actual time constants in the circuit, it becomes unnecessary
to carry out a complete transient analysis of the system. Instead, the change in voltage across the capacitors will be taken into account by terms of the form \( \frac{t}{C} \). A small error is thereby committed, but it is of no practical importance here. Consider the above network. At \( t = 0 \), voltage at point C equals \( V \) volts and the capacitors \( C_2 \) and \( 2C_2 \) are charged to \( V \) volts. The switch is opened at \( t = 0 \) and one can write the following equations for \( 0 \leq t \leq t_d \):

\[ i_1(R_2 + \frac{r_2}{2} + \frac{t}{2C_2}) + i_2R_2 + i_3R_2 = V \quad \ldots \quad A.1 \]

\[ i_1R_2 + i_2(R_2 + \frac{r_2}{2} + r_2 + \frac{t}{C_2}) + i_3(r_2 + R_2 + \frac{t}{C_2}) = V \quad \ldots \quad A.2 \]

\[ i_1R_2 + i_2(R_2 + r_2 + \frac{t}{C_2}) \\
+ i_3(R_2 + r_2 + r_3 + r_{ib} + \frac{t}{C_2} + \frac{t}{C_5}) = V \quad \ldots \quad A.3 \]

\[ i_3r_{ib} = v_{pb} \quad \ldots \quad A.4 \]

Substituting numerical values for \( V, R_2, r_2, r_g \) and \( r_{ib} \) in Equations A.1, A.2, A.3, and solving for \( v_{pb} \) yields

\[ v_{pb} = \frac{180 + \frac{12t}{C_2} 10^{-3}}{232.8 + A(r_3 + \frac{t}{C_5} 10^{-3}) + B} \quad \ldots \quad A.5 \]

where

\[ A = 39.9 + 4.24 \frac{t}{C_2} 10^{-3} + \frac{t^2}{C_2^2} 10^{-5} \quad \ldots \quad A.6 \]

\[ B = 17.7 \frac{t}{C_2} 10^{-3} + 0.5 \frac{t^2}{C_2^2} 10^{-6} \quad \ldots \quad A.7 \]

\( r_3 \) in k.ohm, \( t \) in sec. and \( C \) in Farad.
At \( t = 0 \), \( v_{pb} = 0.4 \) volts. According to Equations A.6 and A.7, \( A = 39.9 \) and \( B = 0 \). Substituting these values into Equation A.5 yields for \( r_3 \):

\[
r_3 = \frac{\frac{180}{0.4} - 232.8}{39.9} = 5.45 \text{ k.ohm}.
\]

Hence select \( r_3 = 5.6 \) k.ohm.

At \( t = t_d = 20 \times 10^{-6} \text{ sec.} \), \( v_{pb} = 0.3 \) volts is prescribed. In order to find one of the capacitor values, from Equation A.5, the other one must be chosen. Choose \( C_2 = 0.02 \mu \text{F} \) which appears to be a reasonable value. With these values for \( t \), \( v_{pb} \) and \( C_2 \), Equations A.6 and A.7 yield:

\[
A = 44.24 \\
B = 18.2.
\]

Equation A.5 can now be solved for \( C_5 \), and the result is

\[
C_5 = 6.25 \times 10^{-9} \mu \text{F}
\]

A larger value of \( C_5 \) will reduce the decay of the triggering voltage. To be on the safe side, select \( C_5 = 0.01 \mu \text{F} \).
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