AN ANALYSIS OF THE MULTIVIBRATOR CIRCUIT

by

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Introduction

The multivibrator was first demonstrated by Abraham and Bloch[1] in 1919. Since that time many attempts have been made to work out a satisfactory mathematical analysis of its operation. To quote E. B. Moullin[2] "the multivibrator system does not lend itself to exact analysis or to an investigation by Fourier series". For example, van der Pol[3] has obtained an expression for the conditions for maintenance of oscillations and he finds that the steady state is made possible by assuming residual induction in the system. Van der Pol sets up a second degree differential equation for which no exact mathematical solution is known (at present) to exist. Hund[4] also sets up differential equations with which to explain the action of a multivibrator system but he makes no attempt to solve these equations. Perhaps one of the best papers on the multivibrator system is that of Watanabe[5], wherein he presents a theoretical discussion and graphical analysis of its operation. Later we shall compare our results with Watanabe's.

1. Abraham and Bloch, Ann. der Phys., vol. 12, p.237, 1919.
2. Moullin, E.B., "Radio Frequency Measurements", 1931.
3. Van der Pol, B., I.R.E., vol.22, p.1051, 1934.
4. Hund, A., "High Frequency Measurements". p.54, 1933.
5. Watanabe, Y., I.R.E., vol.18, p.327, 1930.

It is the purpose of this paper to try to give a workable analysis which will predict with some degree of accuracy the frequency and waveforms of the multivibrator circuit. The analysis presented will be verified, as far as possible, by actual experimental results.

The Multivibrator Circuit.

The multivibrator is essentially a two-stage resistance-coupled amplifier with the output of the second stage fed back into the grid of the first stage as shown in Fig. 1.

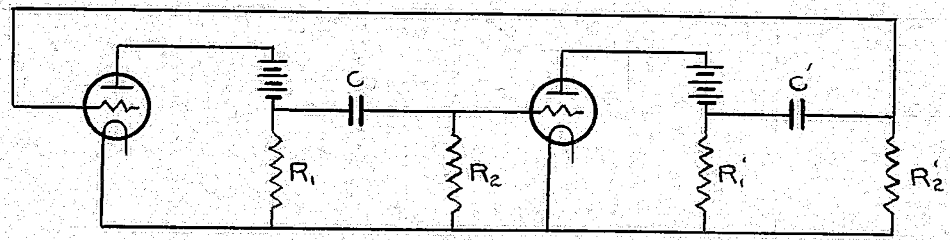


Fig. 1

Oscillations of the multivibrator type can occur in any audio-frequency amplifier employing a grid leak and a grid condenser, provided only that there is some connection between output and input by which the amplifier can supply its own input in the proper phase and in sufficient magnitude. Oscillations of this type occur in many audio-frequency amplifiers as a result of energy fed back through a common plate impedance. Such oscillations are frequently called "motor-boating" because their frequency is very low and is comparable to the sound of a motor-boat engine.

Multivibrators are chiefly used as a source of harmonics in the measurement of frequency.

Harmonics[1] as high as the 300th. can be detected in the output. The fundamental can be made an integral part of an injected control frequency, and thus a high degree of accuracy can be obtained in frequency measurements. The wave-form of the multivibrator is greatly distorted which restricts its use as a low frequency generator. It is, however, finding some use as a time base in oscilloscope circuits.

The Operation of a Multivibrator.

Since there is no resonant circuit (see Fig.1), the generated frequency of a multivibrator depends upon the time constants of the circuit. Such an arrangement will oscillate because each tube produces a phase shift of 180°. It may be noted here that van der Pol[2] treats multivibrators as a special case of "relaxation oscillators", that is oscillators which do not depend upon tuned circuits but only upon the time constants or "time of relaxation" of the circuits employed.

The operation is started by a slightly positive voltage irregularity at the grid of tube No.1. This small voltage is amplified by the two tubes and then reapplied to the grid of tube No.1 in a very much enlarged form, to be again amplified and so. This action takes place almost instantly. The result of repeated amplification is to cause the grid of tube No.1 to

1. Dye, D.W., Phil. Trans. of Ro. Soc., Vol.224A, p.259,1923-4.
2. Van der Pol, B., Phil. Mag., Vol.2, p.978, 1926.

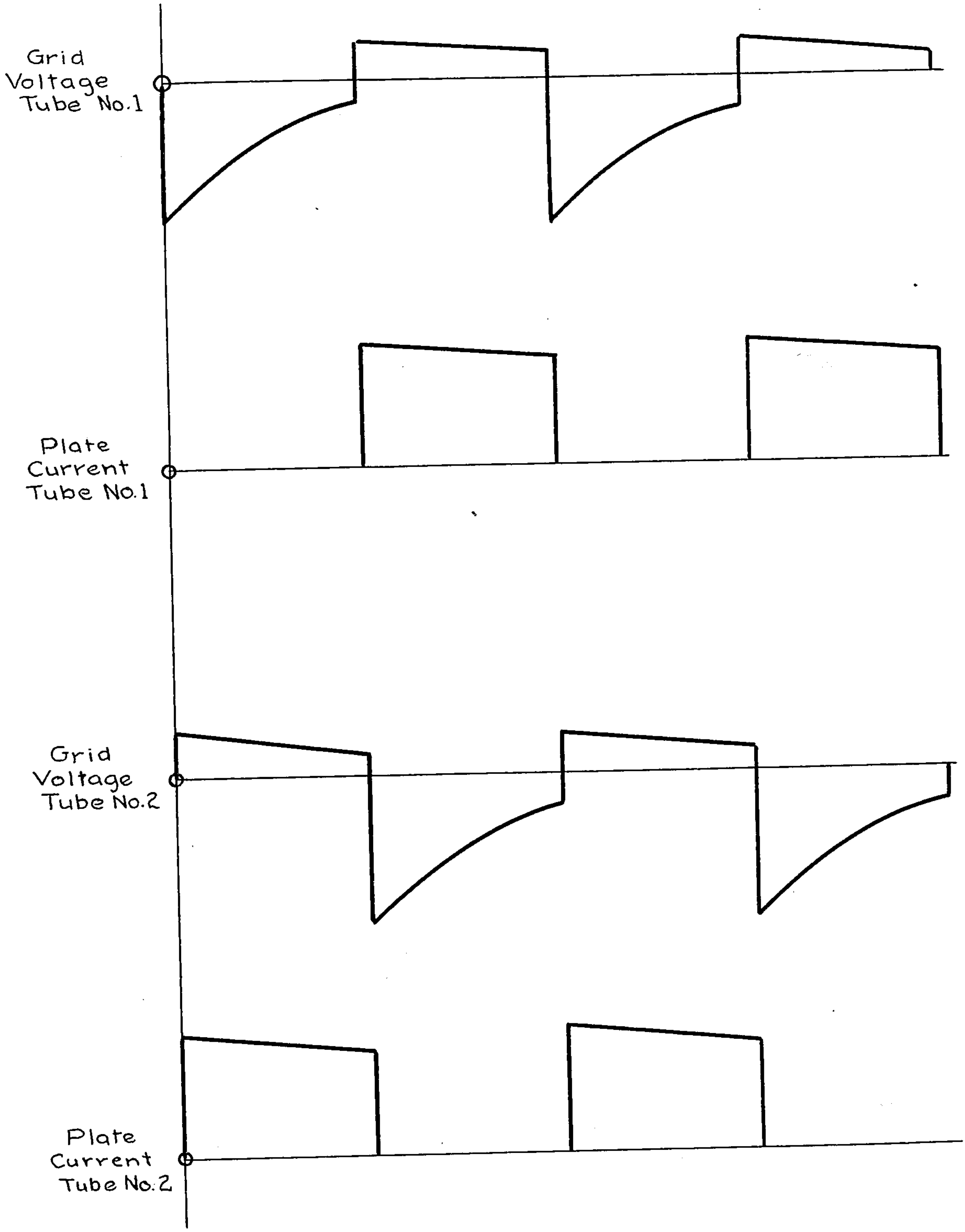


PLATE I

(To face page 4.)

increased its potential suddenly and to make the grid potential of tube No.2 to decrease just as suddenly (i.e. become more negative). These considerations[1] are shown ideally in Plate I. This rapid change is brought to a stop when the plate current of one of the tubes, in this case tube No.2, is cut off; for then all amplification ceases and for the moment one tube is drawing a heavy plate current while the other tube takes little or no plate current. This condition is unstable, because the leakage through the grid-leak resistances discharging the grid-leak condensers gradually brings the grid potentials back to normal. As the grid potential of tube No. 2 becomes less negative as a result of this grid-leak action, a point (cut-off) is reached where plate current in tube No. 2 will flow. Amplification is now possible and any minute irregularity that increases the grid voltage of tube No.2 will be amplified. This will build up a very sudden positive voltage on the grid of tube No. 2 while causing the grid of the first tube to go so negative that amplification is rendered impossible. From the above it can be seen that the tubes operate alternately, when one tube suddenly acquires a large positive grid potential, the other acquires a large negative grid potential.

In general, the frequency of the oscillations produced by the above action depends upon the rate at which the grid vol-

1. Hull & Clapp, I.R.E., Vol. 17, p.252, 1929.

tages decay through the grid-leak resistances and condensers associated with each tube. Hence, the time to complete one cycle is made up of two parts, the time constants of each of the two grid-leak resistances and condensers. This frequency is given approximately by the formula

$$f = \frac{1}{R_2C + R_2'C'} \quad \text{c.p.s} \tag{1}$$

where (see Fig.1.)

f =fundemental frequency

R_2=Grid-leak resistance tube No. 2

R_2'= " " " " No. 1

C =Grid Condenser tube No. 2

C' = " " " No. 1

The above expression does not take into account the plate re-resistances, the dynamic plate resistances or the amplification constants of the tubes.

There is little doubt that the frequency is influenced to some extent by the resistances in the plate circuit and by the characteristics of the tubes themselves. Dye[1] and others have observed errors as high as 25% between the actual observed frequency and the frequency as given by equation (1). Terman[2] states that "the frequency is determined primarily by the grid-leak resistance and grid condenser capacity, but it is also influenced by the remaining circuit constants, tube characteristics and electrode voltages". Later it will be shown in part the effect of some of these circuit constants upon the frequency of the multivibrator. The practical frequency range is from

1. Dye, D.W., "loc. cit."
2. Terman, F.E., "Measurements in Radio Engineering", p.131, 1935.

about 1 cycle per minute to 100,000 per second which is much the same as any audio-amplifier circuit.

Analysis of the Multivibrator Circuit.

In order to facilitate a practical solution, the following assumptions were made and are believed justifiable.

1. The circuit is a two-stage resistance--coupled amplifier with output coupled back to input.
2. Each tube produces a phase shift of 180°, causing the output of the second tube to supply an input to the first tube that is exactly the right phase to sustain oscillations.
3. The time constant of the oscillations generated is made up of the time constant of the first tube and its connecting network plus the time constant of the second tube and its associated network. These are not necessarily equal unless each circuit has the same parameters.
4. Straight line characteristics are assumed for triodes operated at normal voltages. This is equivalent to assuming "μ" and dynamic plate resistance constant.
5. Grid current can be neglected.
6. Any triode for practical purposes can be reduced to an equivalent network.[1]

1. Colebrook, F.M., J.I.E.E., vol.67, p.157, 1929.

If only alternating current components are considered, for audio-frequencies the multivibrator circuit can be represented as in Fig. 2.

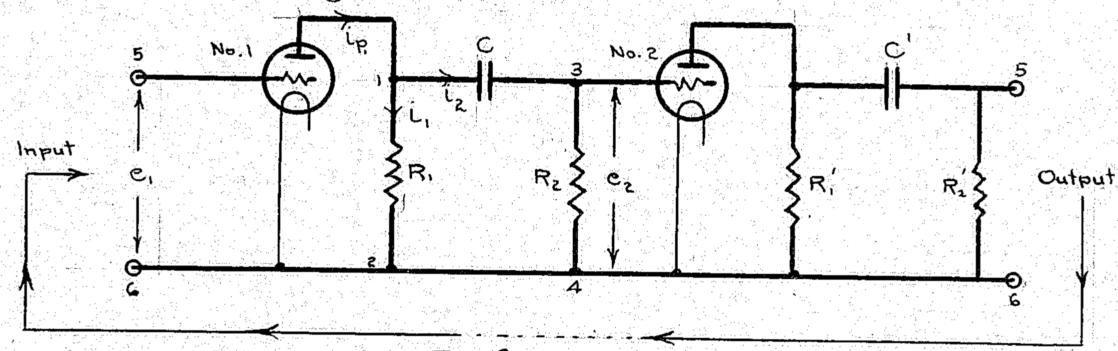


Fig. 2

Where

R_2 and R_2' are grid leak resistances.

C and C' are grid condensers.

R_1 and R_1' are plate resistances.

For mathematical convenience assume that the circuit is symmetrical, hence $R_1 = R_1'$, $C = C'$, $R_2 = R_2'$ and tube No.1 = tube No.2.

Let

e_1 = any instantaneous voltage applied to tube No.1

e_2 = instantaneous voltage appearing across points 3 and 4

i_p = the a.c. plate current of tube No.1

r_p = the dynamic plate resistance.

μ = the amplification factor.

p = the Heaviside operator

Since it was assumed that any triode operated at normal voltages has essentially a straight line characteristic we have when tube No.1 is in operation

$$i_{p_1} = \frac{-\mu e_1}{r_p + Z_{12}} \quad (1) \qquad (2)$$

where Z_{12} is the total impedance between points 1 and 2.

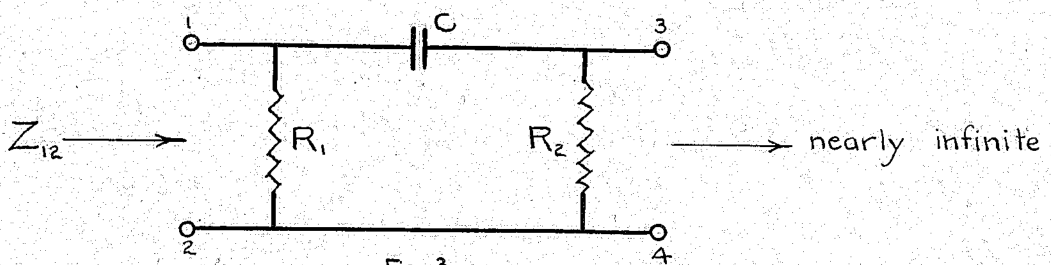


Fig. 3

Hence Z_{12} is given by

$$Z_{12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{pC}}} \qquad (3)$$

Simplifying

$$Z_{12} = \frac{(pCR_2 + 1)R_1}{pC(R_2 + R_1) + 1} \qquad (4)$$

Tube No.1

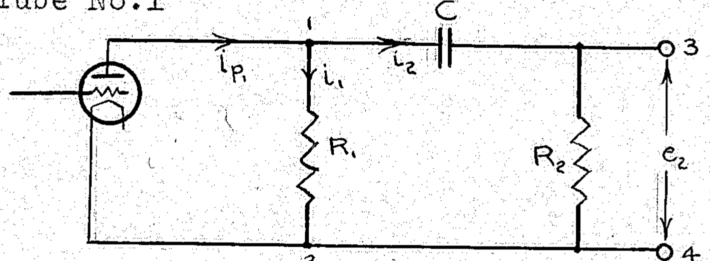


Fig. 4

The plate current of tube No.1 divides into two branches as shown Fig.4, the "C"-branch carries current i_2 while the "R" -branch carries current i_1. These currents can be expressed by

$$i_1 = \frac{R_2 + \frac{1}{pC}}{R_1 + R_2 + \frac{1}{pC}} \cdot i_{p_1} \qquad (5)$$

and $$i_2 = \frac{R_2}{R_1 + R_2 + \frac{1}{pC}} \cdot i_{p_1} \qquad (6)$$

Substituting equation (2) in equation (6)

$$i_2 = \frac{pCR_1}{pC(R_1+R_2)+1} \cdot \frac{-\mu e_1}{r_p + Z_{12}} \qquad (7)$$

1

The voltage across points 3 and 4 is given by e_2 or $i_2 R_2$

$$\therefore \quad e_2 = i_2 R_2$$

$$\text{or} \quad e_2 = \frac{pCR_1R_2}{pC(R_1+R_2)+1} \cdot \frac{-\mu e_1}{\left\{ r_p + \frac{(pCR_2+1)R_1}{pC(R_1+R_2)+1} \right\}}$$

$$= \frac{-CR_1R_2\mu \cdot pe_1}{pC(R_1r_p + R_2r_p + R_1R_2) + r_p + R_1}$$

$$= \frac{-CR_1R_2\mu \cdot pe_1}{C(R_1r_p + R_2r_p + R_1R_2)\left\{ p + \frac{r_p + R_1}{C(R_1r_p + R_2r_p + R_1R_2)} \right\}} \qquad (8)$$

Let

$$K = \frac{\mu}{1 + \frac{r_p}{R_2} + \frac{r_p}{R_1}} \qquad (9)$$

$$\text{and} \quad b = \frac{r_p + R_1}{C(R_1R_2 + R_1r_p + R_2r_p)}$$

$$= \frac{1}{CR_2\left\{ 1 + \frac{R_1 r_p}{R_2(r_p + R_1)} \right\}} \qquad (10)$$

Substituting equations (9) and (10) in equation (8) we have

$$e_2 = -K \cdot \frac{p}{p+b} \cdot e_1 \qquad (11)$$

This equation represents the operational solution of the transient through tube No.1 and its associated network when any voltage e_1 is applied to its grid. The voltage e_2 is determined by the solution of equation (11). A similar equation is obtained for the transient through tube No.2 and its associated network when it becomes operative.

Determining the Fundamental Frequency.

From previous considerations the voltage e_1 applied to the grid at the beginning of each cycle reached its maximum value instantaneously through successive amplifications. In all respects e_1 has the form of a steady e.m.f. suddenly applied to a network at time zero (beginning of the cycle). These conditions are equivalent to Heavisides "unit Function". For convenience let the maximum value of e_1 be E_1. Hence equation (11) becomes

$$e_2 = -K \frac{p}{p+b} E_1 \, 1 \qquad (12)$$

The solution of which is

$$e_2 = -K E_1 \varepsilon^{-bt} \qquad (13)$$

The time constant for tube No.1 and its associated network is determined by equation (13) which is an exponential function. Let T_1 be the time constant for this first portion of the cycle.

Hence

$$T_1 = \frac{1}{b}$$

where

$$b = \frac{1}{CR_2\left\{1 + \frac{R_1 r_p}{R_2(R_1+r_p)}\right\}}$$

$$T_1 = CR_2\left\{1 + \frac{R_1 r_p}{R_2(R_1+r_p)}\right\} \tag{14}$$

A similar expression can be obtained for T_2, the time constant which determines the second portion of the cycle. Since each tube functions alternately, T_2 must be evaluated by solving separately for the transient through tube No.2 and its associated network. Since it was assumed that the multivibrator has symmetrical circuits which is the usual case, the time constants for each portion of the cycle will be equal; i.e., T_1 equals T_2.

Therefore the time constant for the complete cycle for symmetrical circuits is given by T where

$$T = T_1 + T_2 \tag{15}$$

$$= 2CR_2\left\{1 + \frac{R_1 r_p}{R_2(R_1+r_p)}\right\}$$

The frequency will, of course, be given by

$$f = \frac{1}{T}. \tag{16}$$

where

f = fundamental frequency in c.p.s.
T = time constant of a complete oscillation.

It might be well to compare equation (15) with the results from other sources. For example the equation usually given for a symmetrical multivibrator circuit is

$$T = 2 \cdot CR_2. \tag{17}$$

This equations differs from (15) by the factor $\left\{1 + \frac{R_1 r_p}{R_2(R_1+r_p)}\right\}$ Later we shall examine this correction factor more closely with the aid of graphs. Again Watanabe[1] gives the following formula for a symmetrical circuit.

$$T = 2\, CR_2 \log_\varepsilon \frac{E_1 - e_c}{e_c} \tag{18}$$

where

E_1 = the maximum value of e_1 appearing on the grid.

e_c = the residual voltage retained by the condensers after discharge.

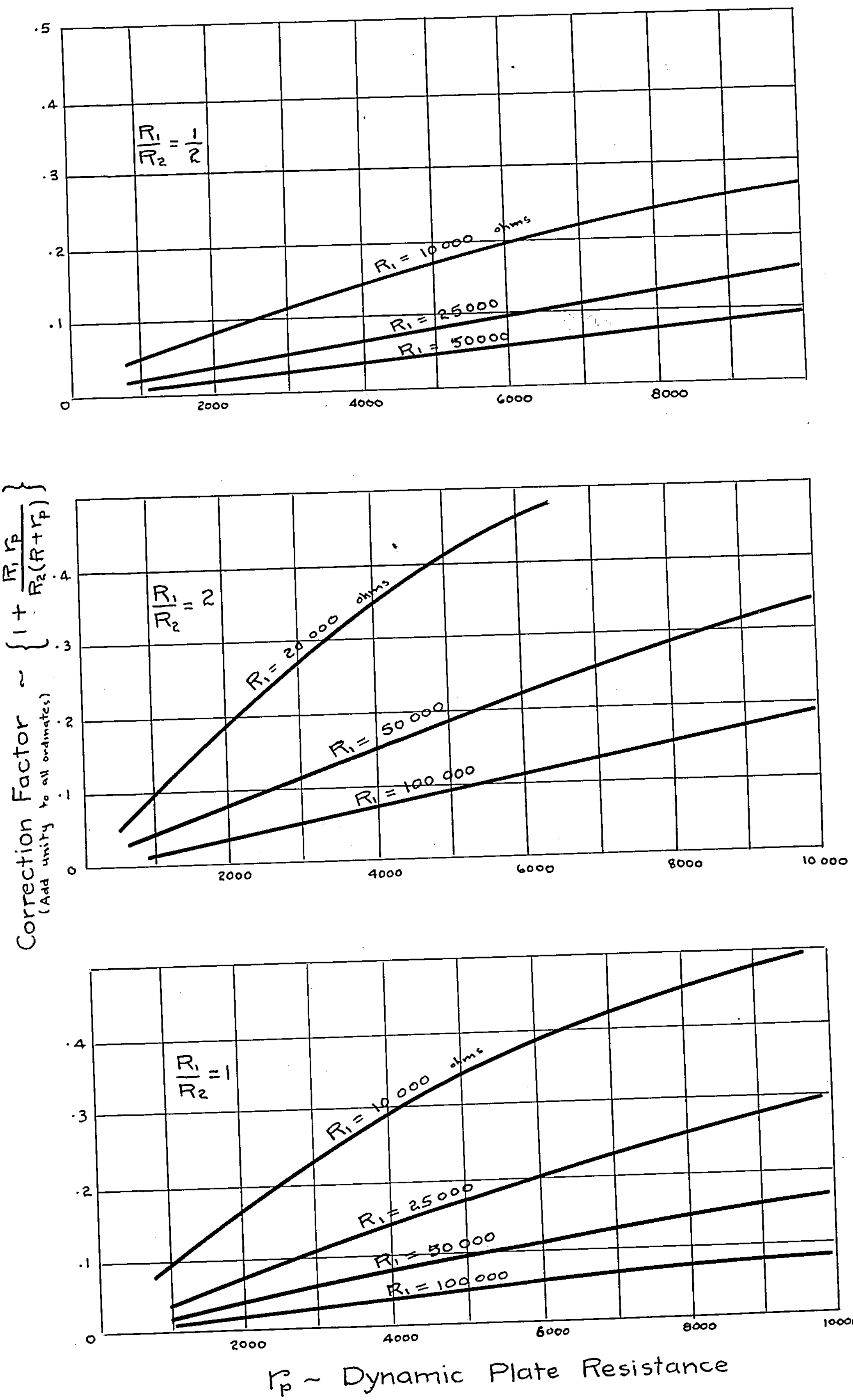
Unfortunately he does not state how this expression has been derived. The values of E_1 and e_c or their ratios can be obtained from oscillograms. The expression given by Watanabe differs from (15) by the "correction" factor

$$\log_\varepsilon \frac{E_1 - e_c}{e_c} \tag{19}$$

In general E_1 is several times larger than e_c, thus this correction factor is comparable to unity.

An examination of many papers on the frequencies of multivibrators bears out the fact that the actual observed time constant is always greater than the time constant as given by $T = 2 \cdot CR_2$. For example Dye[2] calculated the time constant of a symmetrical circuit using equation (17) and obtained a time constant of 930×10^{-6} seconds, but the observed time constant was actually 1000×10^{-6} seconds, where the circuit constants were $C = .0062\,\mu f$, $R_1 = R_2 = 75\ 000$ and r_p = (estimated) 8000 ohms. Hence the calculated result would have

1. Watanabe, Y., "loc. cit."
2. Dye, D.W., "loc. cit."



R₁/R₂ = 1/2
R₁ = 10 000 ohms
R₁ = 25000
R₁ = 50000
R₁/R₂ = 2
R₁ = 20 000 ohms
R₁ = 50 000
R₁ = 100 000
R₁/R₂ = 1
R₁ = 10 000 ohms
R₁ = 25000
R₁ = 50 000
R₁ = 100 000
Correction Factor ~ {1 + R₁ rₚ / R₂(R + rₚ)}
(Add unity to all ordinates)
rₚ ~ Dynamic Plate Resistance
0
2000
4000
6000
8000
10 000
.1
.2
.3
.4
.5

PLATE II

to be multiplied by a correction factor equal to 1.07 (approx.) to obtain the observed results. Using the correction factor as obtained from equation (15) we have for the above values

$$\text{Correction factor} = 1 + \frac{R_1 r_p}{R_2(R_1 + r_p)}$$

$$= 1 + \frac{75000 \times 8000}{75000 \times 83000}$$

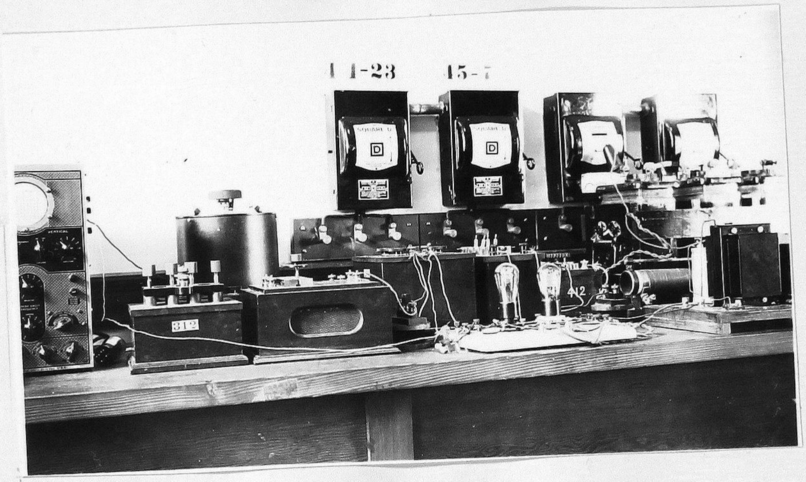
$$= 1.09$$

which compares favourably with the experimental result of 1.07. This would indicate that the correction factor as obtained from equation (15) in our analysis is of the right order and magnitude.

To get a clear idea of the magnitude of this correction factor as given by equation (15) namely,

$$\left\{ 1 + \frac{R_1 r_p}{R_2(R_1 + r_p)} \right\}$$

values were calculated for various circuit parameters and dynamic plate resistances. The results are shown by the graphs in Plate II. It can be seen that theoretically the error in the time constant can be as high as 50 per cent. In practice, however, these extreme values are never used. The graphs also show that for good results R_1 should be less than R_2. This bears out Watanabe's assumption as stated in his analysis. It may also be noted that best operation, in so far as frequency prediction is concerned, should be obtained with tubes having low values of dynamic plate resistance, and R_1 and R_2 should at all times be much larger than the dynamic plate resistance.



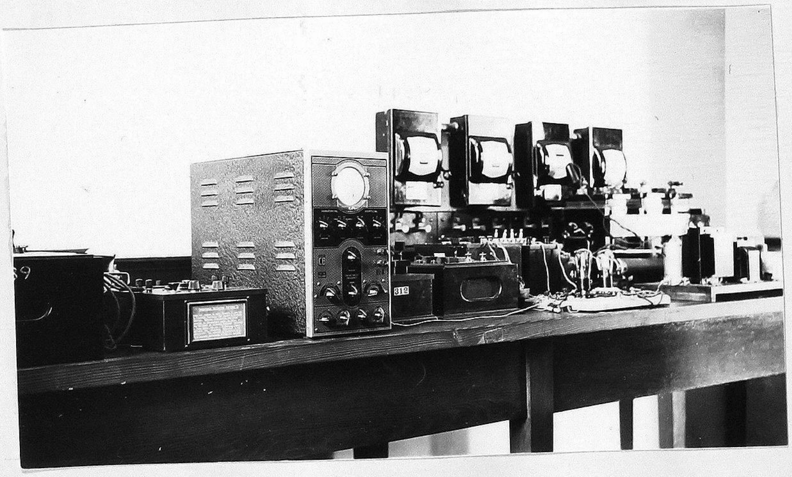


PLATE III

Experimental Check on Frequency Determination.

An experimental symmetrical multivibrator was constructed using Leeds and Northrup precision resistances and condensers. The general layout is shown in Plate III. Many different types of triodes were tried such as 71-A's, 01-A's, 45's, 30's, 31's, 26's and 112-A's. Oscillograms were obtained by means of a cathode-ray oscilloscope[1].

The frequency of the fundamental was determined by means of a modified Campbell bridge[2]. This bridge gave sharp resonance points when used with a Moullin vacuum tube voltmeter. However the range was limited from about 400 c.p.s. to 6000 c.p.s.. To check the accuracy of the bridge an electrically driven tuning fork rated at 1000 c.p.s. was used. The results after several trials gave the mean frequency of the tuning fork as 994 c.p.s.

In the experiment tubes were operated at or near normal voltages. No attempt was made to measure dynamic plate resistances as it was felt that the manufacturers' tables would give values that were accurate enough for all practical purposes. Leads to the various pieces of apparatus were short and straight. The frequencies generated appeared to be stable with no indication of "drift". In fact once the oscilloscope was synchronized with the multivibrator no other adjustments were necessary.

1. Clough--Brengle Co.
2. See Appendix "A".

TRIODE: 26

r_p = 7600 ohms at 135V : $R_1 = R_2$ = 50000 ohms

C = C' µf	Calc'd Time Const. $2CR_2$	Obser'd Freq. c.p.s	Obser'd Time Constant	Obser'd Correction Factor	Calc'd Correct. Factor
.025	2.5 $\times 10^{-3}$	340	3.1 $\times 10^{-3}$	1.17	1.13
.02	2.0	370	2.7	1.14	↓
.01	1.0	920	1.09	1.09	
.008	.8	1100	.91	1.14	
.005	.5	1850	.52	1.07	
.004	.4	2180	.46	1.15	
.003	.3	3050	.33	1.09	
.002	.2	4800	.21	1.05	
			Mean	1.12	1.13

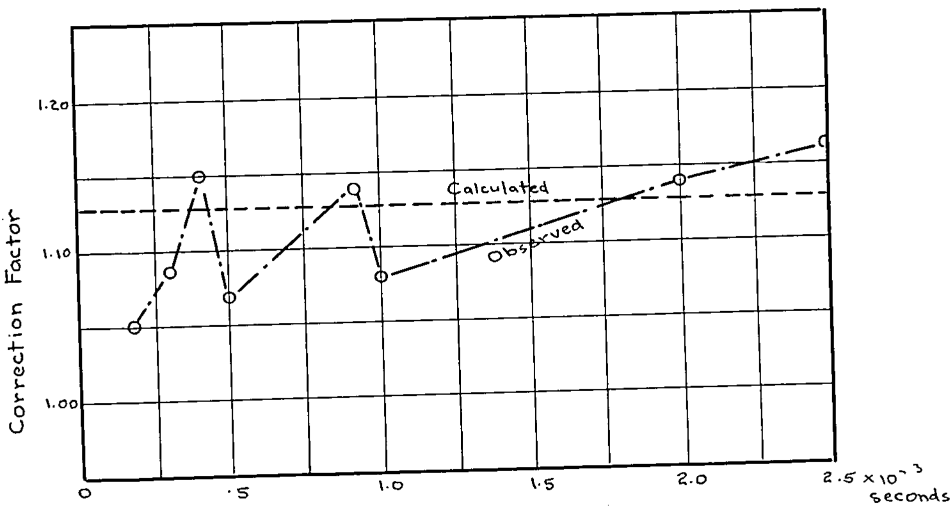
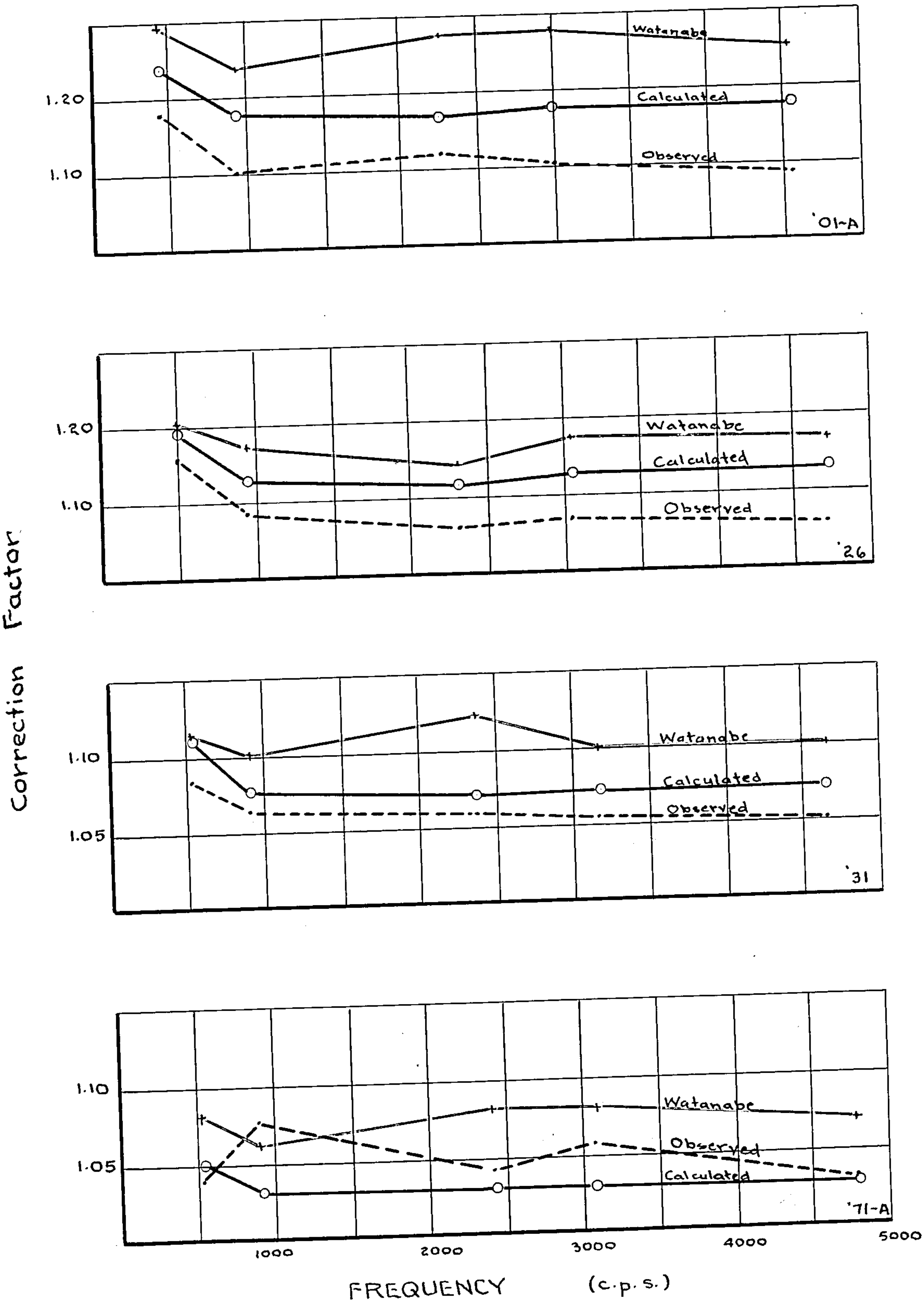


Fig. 4

TABLE I.



Graphs to accompany Table II.

Tube*	R_1 ohms	R_2 ohms	C μf.	$2CR_2 \times 10^{-3}$	Obs. Freq. c.p.s.	Obs. Time Constant $\times 10^{-3}$	Correction Factors		
							Observ'd	Calc'd $1+\frac{R_1 r_p}{R_2(R_1+r_p)}$	Watanabe $\lg \frac{E_1-E_c}{E_c}$
'01-A $r_p = 10000$ 135 V.	50000	50000	.01	1.0	910	1.1	1.10	1.17	1.24
	50000	35000	.025	1.75	485	2.0	1.18	1.24	1.29
	35000	50000	.004	.40	2320	.43	1.13	1.16	1.28
	50000	50000	.003	.30	3020	.34	1.11	1.17	1.28
	50000	50000	.002	.20	4600	.218	1.09	1.17	1.25
26 $r_p = 7600$ ~ 135 V.	50000	50000	.01	1.0	920	1.09	1.09	1.13	1.17
	50000	35000	.025	1.75	491	2.1	1.16	1.19	1.20
	35000	50000	.004	.4	2350	.43	1.06	1.12	1.15
	50000	50000	.003	.3	3050	.33	1.09	1.13	1.18
	50000	50000	.002	.2	4750	.21	1.05	1.13	1.17
31 $r_p = 4100$ ~ 135 V.	50000	50000	.01	1.0	940	1.06	1.07	1.08	1.10
	50000	35000	.025	1.75	555	1.8	1.08	1.11	1.11
	35000	50000	.004	.4	2400	.42	1.06	1.07	1.12
	50000	50000	.003	.3	3150	.32	1.06	1.08	1.10
	50000	50000	.002	.2	4710	.21	1.05	1.08	1.10
'71-A $r_p = 1820$ ~ 135 V.	50000	50000	.01	1.0	940	1.06	1.07	1.03	1.06
	50000	35000	.025	1.75	540	1.85	1.04	1.05	1.08
	35000	50000	.004	.4	2400	.42	1.04	1.03	1.08
	50000	50000	.003	.3	3100	.32	1.06	1.03	1.08
	50000	50000	.002	.2	4800	.21	1.03	1.03	1.07

* "Technical Manual", Hygrade Sylvania Corp., 1937.

TABLE II

The results of a typical run for a symmetrical circuit using '26 tubes are shown in Fig.4 (Table I). In this case, R_1 and R_2 were constant while the condensers (C) were varied together so as to cover the indicated frequency range.

A further type of check was decided upon in the form of a comparison between the correction factors as obtained from the following

1. $\dfrac{\text{Actual observed time constant}}{2\ CR_2}$.

2. $1 + \dfrac{R_1 r_p}{R_2(R_1+r_p)}$ calculated from circuit values.

3. Watanabe's factor $\log_\varepsilon \dfrac{E_1 - e_c}{e_c}$ which was obtained from oscillograms.

The results of the above using various tubes and circuit values in symmetrical circuit are shown in Table II. Some difficulty was experienced in evaluating Watanabe's correction factor from the oscillograms since e_c is small compared to E_1. A slight error in measuring e_c results a much larger error in the correction factor. On the whole the agreement is quite close.

Waveform Analysis.

In the following analysis only symmetrical multivibrator circuits are considered. Although the same reasoning can be applied to unsymmetrical circuits. It should also be noted that only the alternating components are treated, as the direct current componencts can easily be superimposed upon the others. The cathode-ray oscilloscope tracings show alternating components only. These tracings were copied directly from the oscilloscope screen.

The Waveform of the Voltage Across the Plate Resitance R_1.

Consider the portion of the cycle during which tube No.1 is in operation and plate current ip. is flowing,

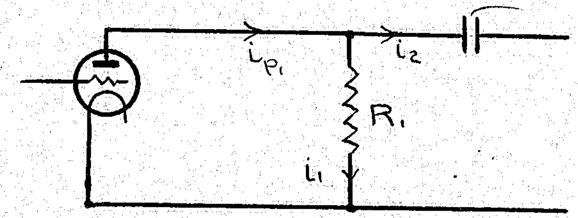


Fig. 5

from equation (2) we have

$$i_{p_1} = \frac{-\mu e_1}{r_p + Z_{12}}$$

and from equation (6)

$$i_1 = \frac{R_2 + \frac{1}{pC}}{R_1 + R_2 + \frac{1}{pC}} \cdot i_p$$

Substituting and simplifying in equation (6) we have

$$i_1 = \frac{(pR_2C + 1)}{[pC(R_1+R_2)+1]} \frac{(-\mu e_1)}{(r_p + Z_{12})}$$

$$= \frac{(pCR_2+1) \cdot -\mu e_1}{r_p(pCR_1 + pCR_2+1) + pCR_2R_1 + R_1}$$

$$= \frac{(pCR_2+1)(-\mu e_1)}{pC(R_1r_p + R_2r_p + R_1R_2) + r_p + R_1} . \qquad (20)$$

Let the instantaneous voltage across R_1 be given by e_{R_1}

$$\therefore e_{R_1} = i_1 R_1$$

$$= \frac{R_1(pCR_2+1)(-\mu e_1)}{C(R_1r_p + R_2r_p + R_1R_2)\left\{p + \frac{r_p + R_1}{C(R_1r_p + R_2r_p + R_1R_2)}\right\}} \qquad (21)$$

Since

$$b = \frac{r_p + R_1}{C(R_1 r_p + R_2 r_p + R_1 R_2)}$$

and $$K = \frac{\mu}{\left(1 + \frac{r_p}{R_1} + \frac{r_p}{R_2}\right)}$$

$$\therefore\ e_{R_1} = \frac{(pCR_2 + 1)(-\mu e_1)}{CR_2\left(1 + \frac{r_p}{R_1} + \frac{r_p}{R_2}\right)(p+b)} \tag{22}$$

$$= \frac{-K\left(p + \frac{1}{CR_2}\right)}{(p+b)}\cdot e_1$$

Let $$\frac{1}{CR_2} = a$$

$$\therefore\ e_{R_1} = \frac{-K(p+a)}{p+b}\cdot e_1 \tag{23}$$

The solution of equation (23) for the first portion of the cycle during which e_1 takes the form of steady e.m.f. suddenly applied with a maximum value of E_1 is

$$e_{R_1} = -E_1 K \frac{p+a}{p+b}\cdot 1$$

$$= -E_1 K\left[\frac{a}{b} + \varepsilon^{-bt}\left(1 - \frac{a}{b}\right)\right] \tag{24}$$

From previous considerations

$$\frac{a}{b} = \frac{CR_2}{CR_2\left\{1 + \frac{R_1 r_p}{R_2(R_1 + r_p)}\right\}}$$

$$= \frac{1}{1 + \frac{R_1 r_p}{R_2(R_1 + r_p)}} < 1$$

An examination of equation (24) is shown in Fig. 6 for various ratios of $\frac{a}{b}$.

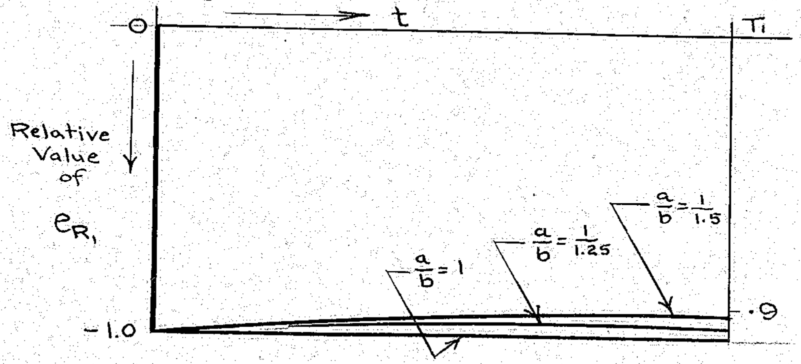


Fig. 6

During the second portion of the cycle no plate current flows in Tube No.1, hence the voltage appearing across R_1 is directly dependent upon the discharge of the condenser in the circuit of tube No. 1.

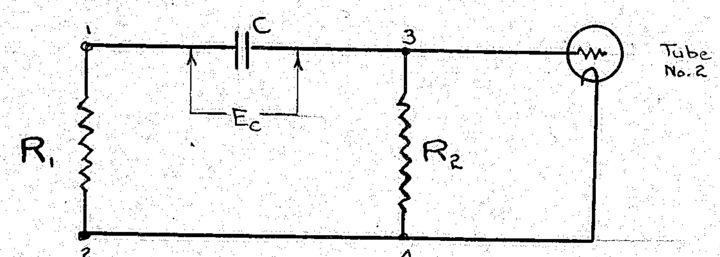


Fig. 7

This circuit is represented in Fig. 7.

Let

e'_{R_1} = instantaneous voltage across R_1 during this portion of the cycle.

E_c = maximum voltage on the C.

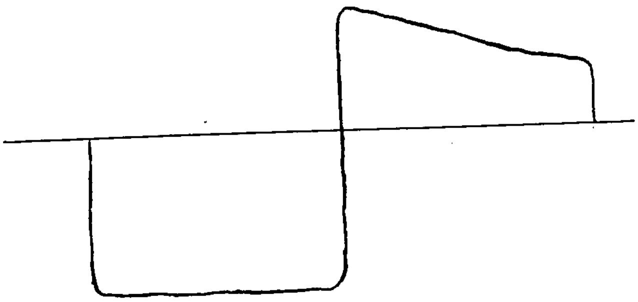
ic = instantaneous current due to E_c.

Therefore

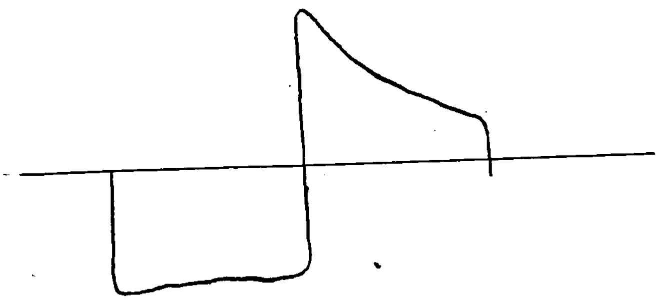
$$e'_R = i_c R_1$$

$$= \frac{E_c}{R_1 + R_2 + \frac{1}{pC}} \cdot R_1 \quad 1.$$

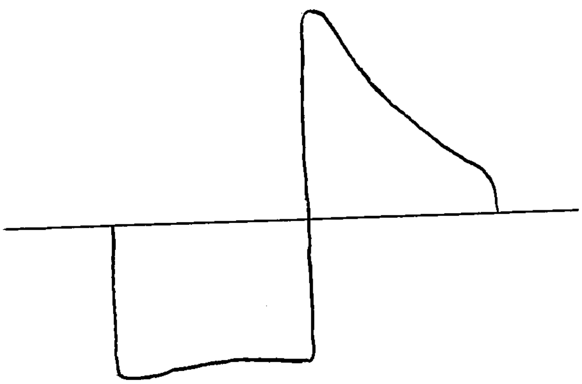
$$= E_c \frac{R_1}{R_1 + R_2} \cdot \frac{p}{\left\{p + \frac{1}{CR_1 + CR_2}\right\}} 1. \qquad (25)$$



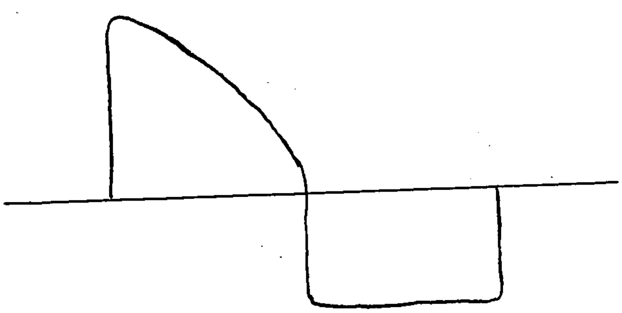
Tube '31
$R_1 = 50\,000$



Tube '26
$R_1 = 50\,000$



Tube '30
$R_1 = 35\,000$



Tube '71
$R_1 = 50\,000$

PLATE IV

Waveforms of Voltages across R_1

which has as a solution

$$e'_{R_1} = E_c \frac{R_1}{R_1 + R_2} \cdot \varepsilon^{-\frac{t}{CR_1 + CR_2}} \qquad (26)$$

This equation is an exponential. Combining this equation with equation (24), the waveform of the a.c voltages appearing across R_1 can be ideally represented as in Fig.8.

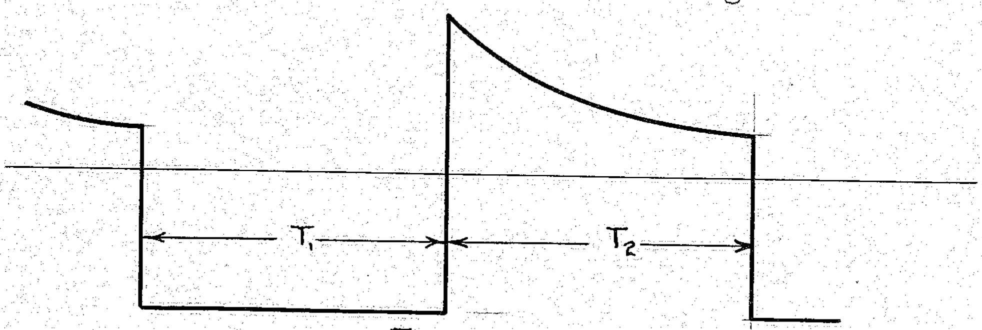


Fig. 8

From this we should expect a rectangular waveform during T_1 and an exponential during T_2. The oscillograms shown in Plate IV correspond closely with the theoretical waveforms.

The Waveform of the Voltage across Condenser C.

During the first portion of the cycle, plate current is flowing in tube No.1, therefore the voltage across condenser C will be given by

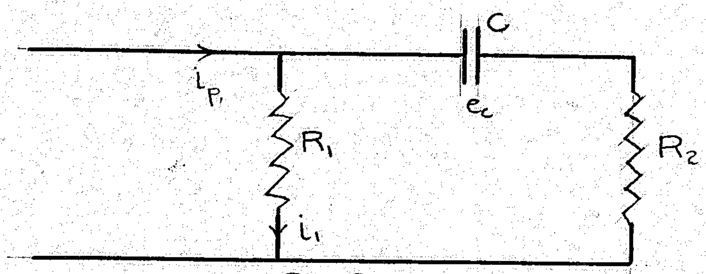


Fig. 9

$$e_c = \frac{1}{pC} \cdot \frac{R_1}{R_1 + R_2 + \frac{1}{pC}} \cdot i_{P_1} \quad 1. \qquad (27)$$

but

$$ip = \frac{-\mu e_1}{r_p + Z_{12}} \quad (1)$$

Substituting and simplifying equation (27)

$$\therefore e_c = -\frac{R_1}{pC(R_1+R_2)+1} \cdot \frac{\mu e_1}{r_p + Z_{12}}$$

$$= -\frac{\mu R_1}{pC(R_1R_2 + R_1 r_p + R_2 r_p) + r_p + R_1} \cdot e_1$$

$$= \frac{-k}{CR_2(p+b)} \cdot e_1 \, 1 \qquad (28)$$

Where k and b have the same meaning as given previously in equations (9) and (10) namely

$$k = \frac{\mu}{1 + \frac{r_p}{R_1} + \frac{r_p}{R_2}}$$

$$\text{and } b = \frac{1}{CR_2\left\{1 + \frac{R_1 r_p}{R_2(r_p+R_1)}\right\}}$$

Equation (28) has for a solution during the first portion of the cycle

$$e_c = -\frac{E_1 K}{CR_2 b}(1 - \varepsilon^{-bt}) \qquad (29)$$

where E_1 as before is maximum value of e_1.

If $CR_2 = \frac{1}{a}$ then equation (29) becomes

$$e_c = -E_1 K \frac{a}{b}(1 - \varepsilon^{-bt}) \qquad (30)$$

The form of this equation for various ratios of $\frac{a}{b}$ is shown in Fig. 10.

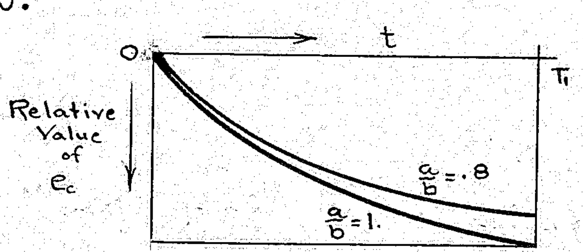


Fig. 10

Typical Waveforms of Voltages across Condensers "C".

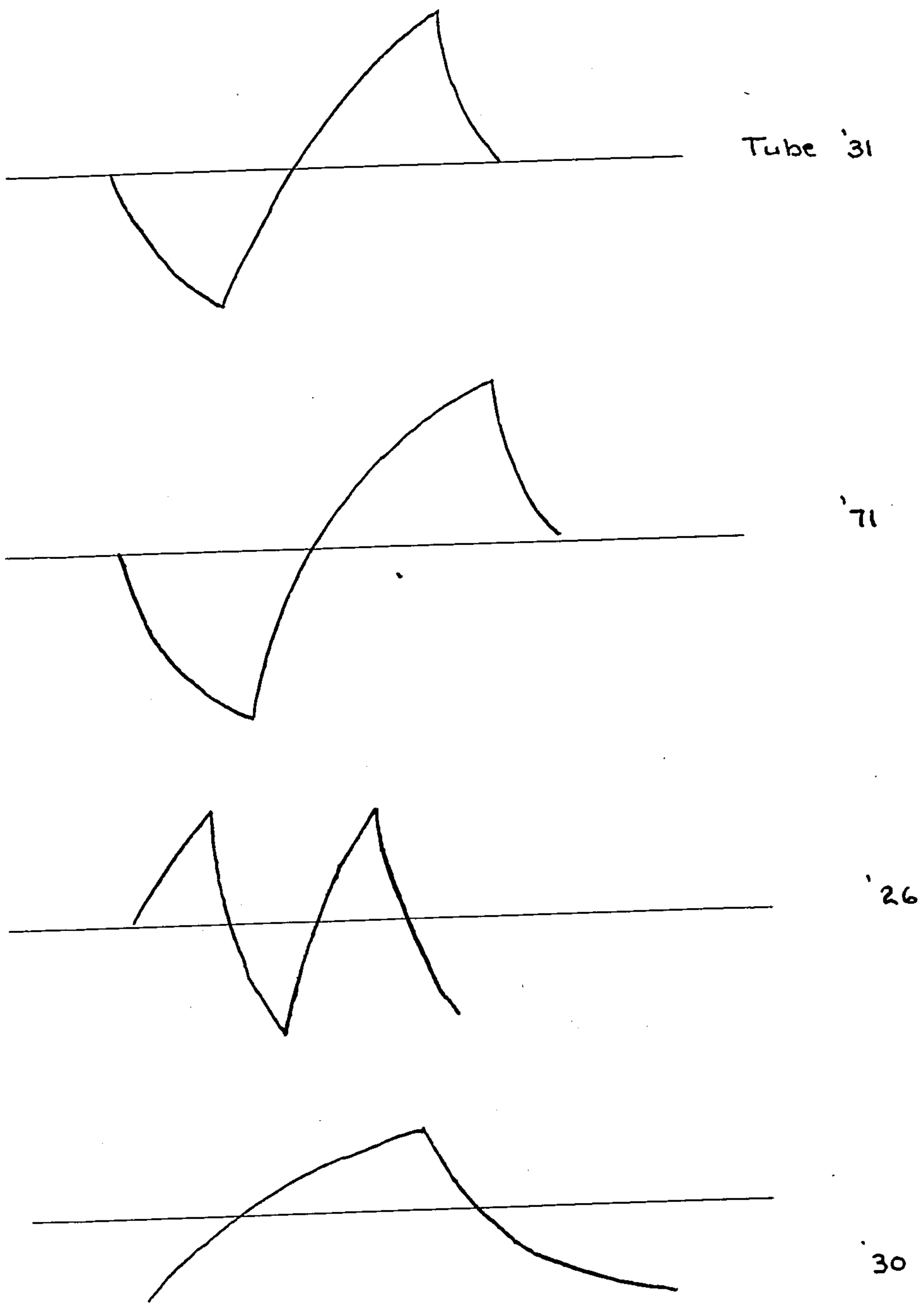


PLATE V.

During the second portion of the cycle, no plate current flows in tube No.1, thus the condenser C discharges.

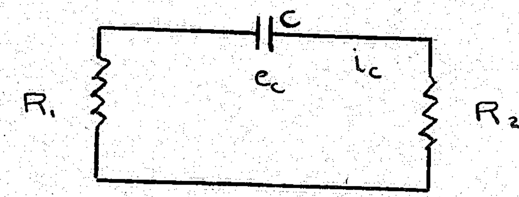


Fig. 11

Let

$e_c' =$ the instantaneous value of the e.m.f. across C .

$E_c =$ Maximum value of e_c' at the beginning of this portion of the cycle .

$i_c' =$ the instantaneous current due to e_c' .

Hence

$$e_c' = -\frac{i_c}{pC}$$

but

$$i_c' = \frac{-E_c}{R_1 + R_2 + \frac{1}{pC}}$$

$$\therefore\ e_c' = \frac{-E_c}{pC(R_1 + R_2) + 1} \qquad (31)$$

which has for a solution

$$e_c' = -E_c \cdot \varepsilon^{-\frac{t}{CR_1 + CR_2}} . \qquad (32)$$

The graph of this equation (32) is an exponential which is to be expected. The combined waveform for the whole cycle is shown ideally in Fig. 12.

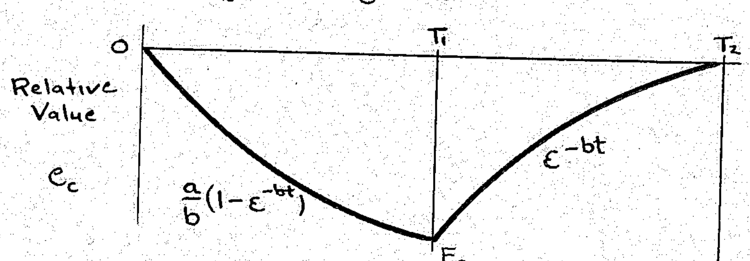
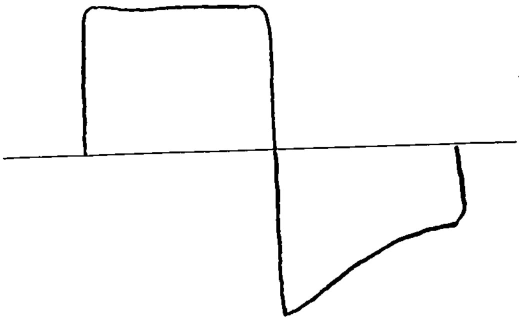
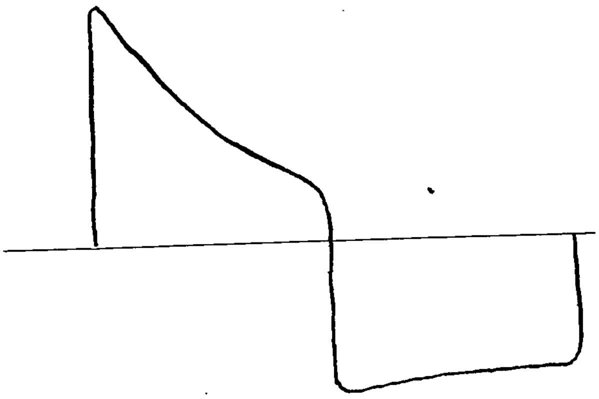


Fig. 12.

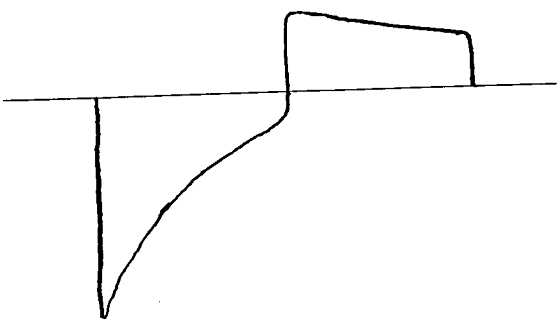
Typical Waveforms of Voltages across Resistance "R_2"



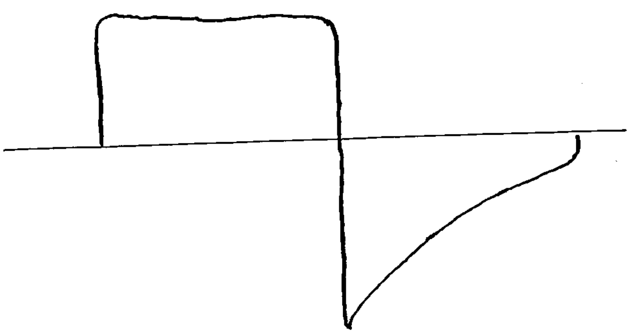
'71
$R_2 = 50000$



'26
$R_2 = 50000$



'31
$R_2 = 35000$



'01-A
$R_2 = 50000$

PLATE VI.

Again the oscillograms shown in Plate V can be compared with Fig. 12.

The Voltage Waveforms across the Grid Resistance R_2.

The final waveform to be discussed is that appearing across the grid-leak resistance (R_2). From previous discussions it was shown that this voltage has essentially a rectangular form. However the voltage during the second portion of the cycle for the circuit of tube No. 1 will be determined by the condenser discharge and will be given by

$$e_{R_2} = E_c \frac{R_2}{R_1 + R_2} \varepsilon^{-\frac{t}{CR_2 + CR_1}} \tag{32}$$

which is easily obtained from equation (26). The complete waveform is shown ideally in Fig. 13.

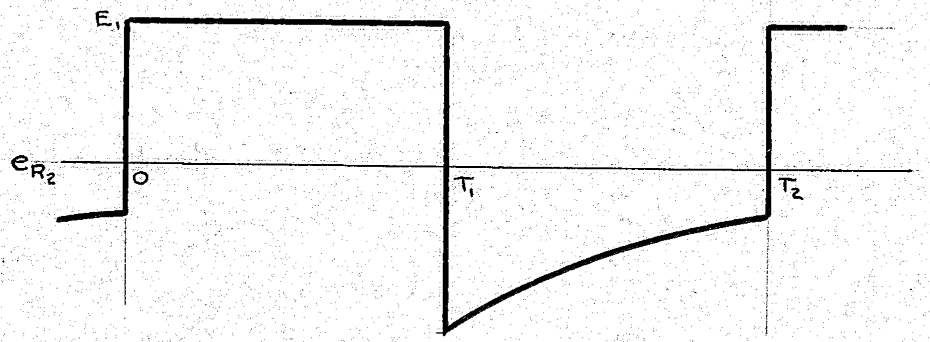


Fig. 13

The oscilloscope tracings are shown in Plate VI. It will be noted that the actual waveform is closely rectangular during T_1, while during T_2 the waveform did not appear to follow the exponential form quite as closely for low values of R_2.

Conclusions.

Unfortunately, the limitations of available apparatus prevented a full investigation of the multivibrator through wide ranges of frequency and circuit parameters. The results obtained from this study may be summarized as follows:

1. The frequency can be predicted with a fair degree of accuracy from the circuit parameters.
2. The waveforms as observed are accounted for theoretically and show close agreement with the predicted forms.
3. The effect of varying the plate voltage was not studied quantitatively but in general the frequency decrease in plate voltage.
4. There is field for further study in unsymmetrical circuits and also "multivibrator" types of oscillations appearing in single tetrode circuits.[1]

Acknowledgements

The writer wishes to thank Dr. H.J. MacLeod for his many helpful suggestions and for the priviledge of using the Electrical laboratory. Thanks are also due to Prof. W.B. Coulthard for examining many of the mathematical equations and G.I. Goumeniouk for the loan of much equipment from the Physics Department.

1. See Appendix "B".

Appendix A

Campbell Frequency Bridge[1]

The circuit arrangement for this bridge is shown in Fig.1

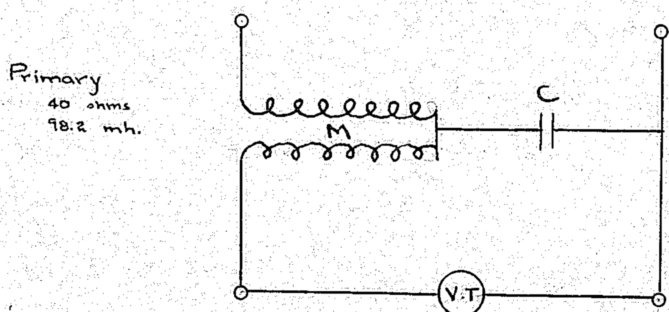


Fig. 1

where M is a Campbell variable mutual inductance standard which has a range from —30 to +110000 microhenries and C is a standard air condenser.

The method is quick and results obtained are accurate. The resonance point is sharp and easily found. At balance the frequency is given by the following

$$\omega^2 = \frac{1}{MC}$$

$$f = \frac{1}{2\pi\sqrt{MC}} = \frac{159200}{\sqrt{M_\mu \cdot C_\mu}} \quad \text{c.p.s.}$$

A further modification of the above is shown in Fig.2.

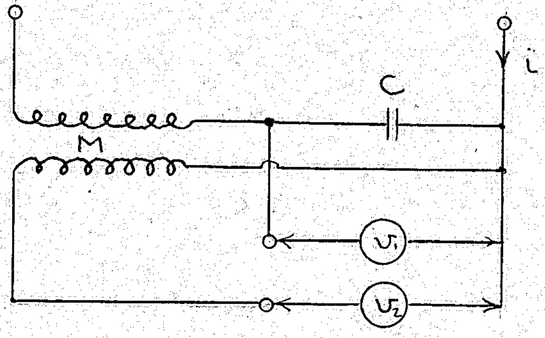


Fig. 2

1. Moullin, "loc. cit."

The frequency is obtained as follows

$$v_1 = \frac{i}{\omega C}$$

$$v_2 = \omega M i$$

$$\omega^2 = \frac{v_2}{v_1 MC}$$

$$\text{or} \qquad f = \frac{1}{2\pi}\sqrt{\frac{v_2}{v_1 MC}} \qquad \text{c.p.s.}$$

The condenser C in Fig.2 may be replaced by a pure resistance R and hence the frequency will be given by

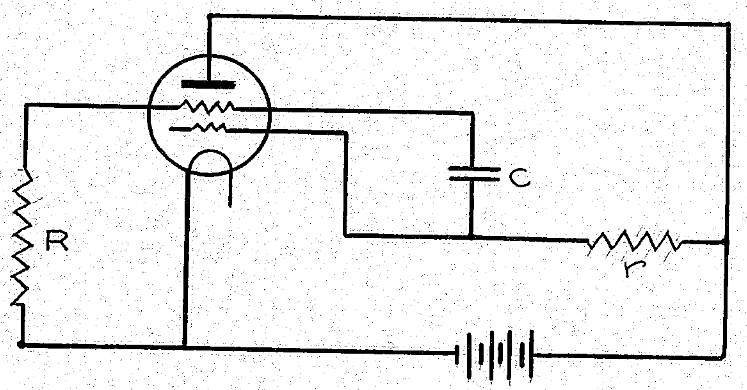
$$f = \frac{v_2 R}{2\pi v_1 M} \qquad \text{c.p.s.}$$

The impedance introduced into the multivibrator circuit by any of the above bridge circuits is small at the freqencies measured. Errors caused by harmonics can be neglected as given in an analysis by Moullin and others[1].

1. Hund, A., "High Frequency Measurements", p.209, McG.-Hill, 1933.

Appendix B

Oscillations similar to those of a multivibrator can be produced by a single tetrode in a resistance-capacitance circuit, such as illustrated below



The time constant of the oscillations produced is given by

$$T = CR. \text{ (approx.)}$$

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