A SURVEY OF LOW-NOISE NUCLEONIC AMPLIFIERS

by

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B.A.Sc., University of British Columbia, 1961

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF APPLIED SCIENCE

In the Department of
Electrical Engineering

We accept this thesis as conforming to the
standards required from candidates for the
degree of Master of Applied Science

Members of the Department
of Electrical Engineering

The University of British Columbia
AUGUST 1963
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ABSTRACT

In nucleonic energy determinations, pulses from capacitive sources must be amplified with the addition of a minimum amount of noise. The problems encountered in this were considered both theoretically and experimentally.

The "classical" theory of Gillespie for systems using ion chambers and tube amplifiers is generalized to include systems using the modern solid-state detectors and amplifiers. Expressions are found relating the "equivalent input noise charge" to the characteristics of the detector, the preamplifier, and the pulse-shaping network. A relationship is derived between the equivalent noise charge and the conventional noise figure of an amplifier. It shows that amplifiers suitable for nucleonic work have noise figures much lower than 1 db.

The theoretical study shows that conventional vacuum tubes, Nuvistors, and field effect transistors are the best active devices for this application. Junction transistors, tunnel diodes, parametric amplifiers and Masers are shown to be unsuitable.

Experimental measurements made on preamplifiers built with tubes, Nuvistors, and field effect transistors confirm the theoretical predictions with good accuracy. When the detector capacitance is 20pf, the preamplifiers exhibited noise charges of 310, 360 and 670 electronic charges respectively. The E810F tube is superior to other tubes currently used, while the field effect transistor is, at present, the best solid-state device.
ACKNOWLEDGEMENT

The project was carried out under a National Research Council grant to the Electrical Engineering Department, and an Atomic Energy of Canada Limited grant to the Physics Department. In addition to these general grants, the author was a recipient of an NRC studentship for the 1962–63 university term and a UBC Graduate Student Scholarship for the 1961–62 term. This financial assistance is gratefully acknowledged.

The author is also indebted to many member of the faculty, staff and graduate student body of both the Physics and Electrical Engineering departments. Particularly helpful were Professor F. K. Bowers of the Electrical Engineering Department and Dr. B. L. White of the Physics Department whose patience in seeing the project through to completion was apparently unbounded.
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1. INTRODUCTION

Nuclear reactions are generally studied by measuring the energy distribution of their products.\(^{1,1}\) For charged particles, the most common technique utilizes ionization chambers or solid-state detectors to convert the energy into charge. When the conversion is linear, the energy spectrum becomes a pulse height spectrum and can be measured electronically with a pulse height analyzer or "kicksorter".

Many experiments have energy spectra with closely spaced peaks. Often the peaks are not resolved by the kicksorter because they are broadened by fluctuations in the detector yield and by noise in the electronic equipment. At present, either of these factors may be the dominant cause of broadening in a given experiment.\(^{1,2}\) However, they are additive, and a reduction in either one results in an improvement in resolution. Therefore constant efforts are being made to improve both detectors and amplifiers.

The general theory of low-noise nucleonic amplification with tubes was presented in 1953 by Gillespie.\(^{1,3}\) Since then, many new electronic devices have been developed that appear promising for nucleonic work. Studies of Nuvistors\(^{1,4}\), and junction transistors\(^{1,5}\) have been made, but as yet no comprehensive survey has been reported that generalized the work of Gillespie to include all devices. For this reason, a study was carried out by the author as an electrical engineering project in cooperation with the nuclear physics group of the UBC
Physics department. The study was intended to survey all possible amplifiers, to compare their nucleonic resolution, and to suggest ways of improving the resolution now obtainable.

Two methods were derived to compare devices. Firstly the nucleonic resolution was related to the noise figure commonly reported in manufacturers' specifications. Secondly, each device deemed suitable from reported noise figures was represented by a noise model similar to that used for tubes. The theory of Gillespie can then be applied with slight modification and provides a second, more accurate comparison.

From these results it was possible to predict that none of the new devices surveyed will give as good resolution as a conventional tube. However, in situations where the size of tubes or their power requirements make them unsuitable, theory showed that Nuvistors and field effect transistors (f.e.t.'s) are the best devices available.

Amplifiers were built with the "best" available tube, Nuvistor, and f.e.t. respectively. Their noise levels were as predicted by the theory. The 7586 Nuvistors studied have been used for some time in nucleonic amplifiers, but the E810F (7788) tubes and 2N2386 f.e.t.'s were studied in detail for the first time.

It was found that the E810F gives the lowest noise levels of any device currently available. A note reporting this result was prepared by the author and Dr. B.L. White of the Physics Department and has been accepted for publication in the Review of Scientific Instruments.
The theory, experimental methods and results are reported in more detail than is usual in a thesis because of the lack of a modern text on low-noise nucleonic techniques. It is hoped that the thesis will help to fill this void for nuclear physics students.

1.1 Thesis Outline

The investigation of noise in nucleonic systems is reported in the following sequence. For the remainder of Chapter 1, a "typical" counting system is discussed. This section is intended primarily as an introduction to the techniques of proportional counting, but it also serves to introduce some important preliminary results needed in the noise analysis. For instance, it is shown that the use of matching networks is impractical for this type of system, and that the noise analysis can be carried out with any feedback loops removed.

Chapter 2 is devoted to theoretical considerations of the "general" nucleonic system. The general noise model of the system is introduced and the effect of the frequency response on both signal and noise is found. The basic criteria for judging nucleonic amplifiers, the "equivalent noise charge", is then defined and expressions for it found from the results of the signal and noise derivations. To provide a link with communications work, the noise figure is discussed and its relationship to the noise charge found. From the relationship found it is possible to narrow the field of possible amplifiers
before detailed discussions of individual devices are given.

In Chapter 3, tube, Nuvistor, f.e.t. and junction transistor noise models, conforming to the general model, are found theoretically. The f.e.t. is considered in more detail than the others because it is relatively unfamiliar. A brief discussion of noise in tunnel diodes and parametric amplifiers is also given showing that they are not suitable for this application. At the end of the chapter, the results of the theoretical discussion are summarized and the relative merit of the devices predicted.

The experimental methods for finding the noise models of the E810F, 7586, and 2N2386 are described in Chapter 4 and the measured noise parameters given. With the aid of these parameters, the choice of bias point is discussed and a more detailed comparison of the devices given.

Chapter 5 gives a description of the four preamplifiers built to check the theory and measurements of the preceding chapters. The noise charge of the preamplifiers was determined by the methods described in section 5.2, with the results reported in section 5.3.

In Chapter 6, the final chapter, the conclusions drawn from the theoretical and experimental work are summarized briefly.

1.2 "Typical" Nucleonic Counting System

The problems encountered in measuring energy spectra can be explained most easily by considering the "typical" counting
The detector produces a signal, proportional to the energy of each incident particle, which is amplified by the wide-band preamplifier and fed to the post-amplifier. The frequency response of the post-amplifier (represented in Fig. 1.1 by the "pulse shaping network") is chosen to maximize the output signal-to-noise ratio. From the post-amplifier, the shaped pulses enter the pulse height analyzer where they are sorted into a number of channels (typically 100 or 256) according to their amplitude. In each channel the pulses are counted electronically and the total stored until the experiment is finished. The energy spectrum found in this way is then "read out" with an electric typewriter, an X-Y recorder, or some other output device.

1.2.1 The Signal Produced by Particle Detectors

Until recently, the most commonly used particle detector
was the ionization chamber (1.6) in which the particle loses energy creating ion pairs in gas at the average rate of 30 electron-volt per ion pair. Now, however, the ion chamber has been largely replaced by the various types of solid-state detectors. (1.7) In solid-state detectors, the signal is produced by the creation of hole-electron pairs. Since this requires an average energy of only 3.5 ev, the solid-state detectors give a larger charge and therefore superior resolution.

With both types of detectors, the charges created by the passage of the particle require a finite "collection time" to traverse the detector and be collected in the external circuit. During this time, a current \( i(t) \) flows from the detector, so that on a microscopic time scale the detector is a current source as shown in Fig. 1.2(a).

For ionization chambers of large dimensions, \( T_c \) may be several microseconds, while for semiconductor detectors it is generally less than 10 ns. (1.8) Since the output pulses usually have rise times on the order of microseconds for minimum noise, the charge in the solid-state detector can be assumed to arrive instantaneously. In the case of the ionization chamber, the charge does not arrive instantaneously, but noise calculations can be carried out as if it does, and a correction made later to account for the finite rise time, (see Gillespie (1.9) for a calculation of correction factors). For this thesis then, the detectors can be represented as a source of charge \( Q \) with an internal capacitance \( C_d \) as shown in Fig. 1.2(b). The internal resistance of the source can be neglected because it will be
much larger than the impedance of $C_d$ at all frequencies of interest.

![Circuit Diagram](image)

(a) as a current source  
(b) as a charge source

Fig. 1.2 The Particle Detector

The voltage generated by the signal charge is a step of amplitude $Q/(C_d + C_i)$ where $C_i$ is the input capacitance of the preamplifier. It will be shown later that the resistive portion of the input impedance must be large for minimum noise (see section 2.2.1) so that the voltage step decays slowly. When several particles are detected in succession, the input voltage is as shown in Fig. 1.3. The steps are differentiated into pulses by the limited-frequency pass band of the post-amplifier. This reduces their amplitude by a factor $S$, where $S$ is the peak response of the pulse shaping network to a unit step. At the output then each particle is represented by a pulse of amplitude

$$
\left(\frac{Q}{C_d + C_i}\right) \times A_1 A_2 S.
$$

($A_1$, $A_2$ are the amplifier voltage gains).
If it is assumed that the experiment being conducted with the "typical" system results in an energy spectrum with two peaks, the output from the kicksorter would look like Fig. 1.4(a) in the absence of noise. The peaks are well separated since they are broadened only by "straggling" in the energy of the incident particles. When noise is introduced at any point in the system, it modulates the pulse amplitude randomly. Therefore pulses from monoenergetic particles acquire a Gaussian amplitude distribution with a standard deviation equal to the rms noise $n_0$ (1.10). The effect of noise on the "typical" experiment is shown in Fig. 1.4(b). The peaks now overlap slightly and must be separated by fitting two Gaussian curves to the shape of the total spectrum. Clearly the accuracy with which this can be done diminishes as the noise increases and the peaks broaden. With the best detectors and amplifiers...
it is currently possible to obtain spectral line widths equivalent to about 5 kev of particle energy.\(^{(1,2)}\)

Compared to the typical particle energies of hundreds of kev, this is small, but it still restricts the accuracy of many experiments severely.

From this discussion, it is evident that in nucleonic work, noise is a problem even when large signal-to-noise ratios are obtained because of the need to resolve pulses of nearly equal amplitude. This contrasts with binary communications systems such as PCM where an adequate signal-to-noise ratio is one that enables pulses to be distinguished from spaces.

The dependence upon the height of the pulses rather than simply their presence also means that the amplifiers must have extremely good gain stability because a shift in gain during
a long experiment broadens spectral lines in much the way noise
does. Stability is generally achieved by applying negative
feedback around all amplifiers. In section 1.4, the two types
of feedback commonly used are compared.

The spectrum of Fig. 1.4(b) is the relative energy
spectrum for the experiment. To find the absolute energy the
output voltage/input energy calibration for the system must be
found. This is done most accurately by introducing particles of
known energy (e.g. Polonium alphas of 5.3 Mev) at the input and
locating their spectral line in the output. An alternative,
and generally simpler calibration is performed with pulses of
known charge from a pulse generator. If the detector yield
is known, the spectral peak from the pulse generator represents
the location of a known energy and thus serves as a calibration
peak. Besides giving the absolute energy of the peaks, the
calibration can be used as a check on the long-term stability
of the system with respect to changes in the detector or
amplifiers.

With this rather brief description of a "typical"
experiment, the basic nature of the problem has been defined.
To summarize: an instantaneous charge from a capacitive source
is to be amplified with the addition of a minimum amount of
noise. The shape of the output pulse is not important, only
its signal-to-noise ratio is. Further, the amplifiers must be
highly stable with respect to gain drifts.

1.3 Methods of Coupling the Detector to the Amplifier

Coupling the detector directly to the amplifier resulted
in a signal $V = i(t) \cdot Z_i \approx Q/(C_d + C_i)$. With a narrow-band signal, $i(t)$, this voltage could, in principle, be increased to $V = i(t) \cdot R$, where $R$ is the parallel combination of $R_b$ and $R_i$ (see Fig. 1.5), by "tuning out" the source and amplifier capacitances. The larger signal would not affect the signal-to-noise ratio with respect to the detector or input circuit noise, but it would decrease the effect of noise arising later in the system.

Can a similar increase be obtained for impulsive, nucleonic signals by using an appropriate coupling technique? It will be shown that some improvement is theoretically possible, but energy conservation prevents the signal from being as large as $i(t)R$.

![Diagram](image)

**Fig. 1.5 Matching Detector to Amplifier**

### 1.3.1 The Passive Coupling Network

Consider Fig. 1.5, which shows the detector coupled to the amplifier by the passive network $N$. The function of $N$ is to "tune" $C_d$ and $C_i$. If $N$ does this, a signal $i(t)$ from the detector results in a voltage larger than $Q/C_d$ across $C_d$.
indicating that $C_d$ now stores a charge larger than $Q$. Since the
detector charge is strictly limited by the particle energy and
detector yield, the extra charge must have been supplied by the
passive network. The signal is of one polarity only so each
pulse requires additional charge from the network. But a passive
network can supply charge to the external circuit only on a
reciprocating basis; otherwise it is acting as an energy source.
Therefore the network can not be built.

An active network could "tune" $C_d$ and $C_{in}$, but it can be
shown that it would increase the noise at least as much as it
increased the signal so that no net improvement would be
achieved.

1.3.2 The "Best" Passive Coupling Network

Even though a passive coupling network cannot give a
drastic improvement in signal level it can cause some improve-
ment by transferring all the energy initially stored on $C_d$
to $C_i$ at some later time. If this is done, the energy on $C_i$
is given by $E_i$.

$$E_i = \frac{1}{2} \frac{Q_i^2}{C_i} = \frac{1}{2} \frac{Q^2}{C_d} \quad \cdots (1.1)$$

Solving for $Q_i$ and the input voltage $V_i$ under this condition gives:

$$Q_i = Q \sqrt{\frac{C_i}{C_d}} \quad V_i = Q \sqrt{\frac{C_i C_d}{C_i C_d}} \quad \cdots (1.2)$$
The improvement in $V_i$ relative to the direct coupled value $Q/(C_d + C_i)$ is plotted in Fig. 1.6 and shows that delay line coupling gives a worthwhile increase in signal level. Unfortunately, a delay line with characteristic impedance of (say) 10 Meg between capacitors of (say) 20pf requires inductors of 2000 henries, and is therefore impractical.

![Diagram of improvement in signal level](image

**Fig. 1.6 Improvement in Signal Level**

### 1.3.3 Coupling with an Ideal Transformer

The curve above shows that the improvement is least when $C_d = C_i$, indicating that when direct coupling is used the "matched" condition is the best. It is, in fact, the condition for maximum instantaneous energy transfer from $C_d$ to $C_i$. By coupling the detector to the amplifier with an ideal transformer, this condition can be achieved for all $C_d$ and $C_i$. If the transformer has a voltage stepdown ratio of $r$, then the detector generates an apparent charge $rQ$ from a capacitance $r^2C_d$. 
The voltage across \( C_i \) is \( V_i \) which has a maximum \( V_{\text{max}} \) at \( r_{\text{max}} \).

\[
V_i = \frac{Q}{C_i + r^2 C_d}; \quad V_{\text{max}} = \frac{Q}{2\sqrt{C_i C_d}}; \quad r_{\text{max}} = \sqrt{\frac{C_i}{C_d}}
\]

The maximum occurs when the reflected capacitances are equal and this yields a pulse half as great as that achieved with the lossless delay line. The transformer coupling scheme is of interest only when \( C_i \) and \( C_d \) are very unequal, since losses in practical transformers would otherwise more than offset the increase in signal level.

From these arguments, it can be seen that in most cases direct coupling of detector to amplifier cannot be improved upon for maximum signal-to-noise ratio.

1.4 Comparison of Charge Sensitive and Voltage Sensitive Configurations

Nucleonic preamplifiers can be divided into the two general classes, "voltage sensitive" and "charge sensitive", depending upon whether the feedback used to obtain stability is resistive or capacitive. Schematics of these two types of amplifiers are shown in Figs. 1.7(a) and (b). In the voltage sensitive configuration, the feedback is applied to the cathode of the first stage rather than directly to the input to ensure that the feedback is frequency-independent. In the charge sensitive or Miller integrator configuration, capacitive feed-
back is taken directly from the output to the input.

\[
\frac{-V_o}{V_{in}} = A_{ol}
\]

**Fig. 1.7(a) Voltage Sensitive Amplifier**

\[
A_{cl} = \frac{A_{ol}(R_{fb} + R')}{R_{fb} + R' + A_{ol}R'}
\]

\ ...(1.4)

**Fig. 1.7(b) Charge Sensitive Amplifier**

1.4.1 Closed-loop Gain

The closed-loop gain of the voltage sensitive amplifier is given by Eqn. 1.4. In this expression \( R' \) is the parallel combination of \( R_c \) and the impedance seen looking into the cathode of the first tube. If a charge is dumped on the input, the output voltage \( V_o \) is given by Eqn. 1.5(a) which reduces to Eqn. 1.5(b) when \( A_{ol} \) is large.
For the charge sensitive amplifier, $Q$ gives rise to a voltage $V_{\text{in}}$

$$V_{\text{in}} = \frac{Q}{C_{t2}} - \frac{V_o C_{fb}}{C_{t2}} \quad \text{where} \quad C_{t2} = (C_d + C_i + C_{fb}) \quad \ldots (1.6)$$

$V_{\text{in}}$ is amplified by $A_{o1}$, so that the output voltage $V_o$ is given exactly by Eqn. 1.7(a) and for large $A_{o1}$ by Eqn. 1.7(b).

(a) $V_o = \frac{QA_{o1}}{(C_{fb} A_{o1} + C_{t2})}$

(b) $V_o = \frac{Q}{C_{fb}} \quad \ldots (1.7)$

Comparison of Eqn. 1.5(b) and 1.7(b), shows that the output of the charge sensitive amplifier depends upon fewer parameters than does the output of the voltage sensitive one. In particular it is independent (for large gain) of the detector and tube capacitances. This characteristic of the charge sensitive configuration makes it easier to calibrate and also gives it greater stability.

1.4.2 System Calibration

When the energy versus pulse-height calibration of the nucleonic system is checked with the pulse generator, an accurately known step voltage $V_c$ is applied to the test capacitor $C_c$. A charge $Q_c$ given by Eqn. 1.8 then passes to
the amplifier as the calibrating signal.

\[ Q_c = \frac{C_i C_c V_c}{(C_i + C_c)} \]  \( \cdots (1.8) \)

For the voltage sensitive amplifier, \( C_i \) is just the capacitance \( C_t \), defined earlier. For the charge sensitive amplifier the Miller capacitance \( C_{fb}(1 + A_{ol}) \) is added to \( C_t \) so that \( C_i \) is now much greater than any reasonable value of \( C_c \). Therefore the test charge is \( C_c V_c \) for this case and the system can be calibrated without measuring the input and detector capacitance. If \( C_c \) is made very small, the same condition prevails for the voltage sensitive amplifier; but it is difficult to obtain very accurate capacitors of much less than \( 1 \text{pf} \) so calibration for this type of amplifier usually requires a measure of the input capacitance.

1.4.3 Stability

Another advantage of capacitive feedback is that it provides closed-loop gain stability with respect to drifts in both the open-loop gain and the detector capacitance, while voltage feedback stabilizes gain drifts only. This can be seen by considering Eqn. 1.9 and the derivatives for the two types of feedback.

\[ \Delta V_o = \frac{\partial V_o}{\partial A_{ol}} \Delta A_{ol} + \frac{\partial V_o}{\partial C_t} \Delta C_t \]  \( \cdots (1.9) \)

It has been assumed that the feedback components are of
high stability and only the open-loop gain or the total capacitance can drift.

For the voltage sensitive amplifier, the derivatives are found from Eqn. 1.5(a) to be

\[
\frac{\partial V_o}{\partial A_{o1}} = \frac{QR^2}{C_t(R + A_{o1}R^')^2} = \frac{V_o R}{(R + A_{o1}R^')A_{o1}}, \quad \text{where } R = R_{fb} + R^'
\]

\[
\frac{\partial V_o}{\partial C_t} = \frac{Q A_{o1}R}{C_t^2(R + A_{o1}R^')} = \frac{-V_o}{C_t}
\]

Substituting these in Eqn. 1.9 and adding absolute values for the worst case, gives Eqn. 1.10 relating a fractional change in either \(A_{o1}\) or \(C_t\) to the resulting fractional change in \(V_o\).

\[
\frac{\Delta V_o}{V_o} = \frac{R}{(R + A_{o1}R^')} \left(\frac{\Delta A_{o1}}{A_{o1}}\right) + \left(\frac{\Delta C_t}{C_t}\right) \quad \cdots (1.10)
\]

For the charge sensitive amplifier the derivatives are found from Eqn. 1.7(a) to be:

\[
\frac{\partial V_o}{\partial A_{o1}} = \frac{V_o C_t^2}{(A_{o1}C_{fb} + C_t^2)A_{o1}}; \quad \frac{\partial V_o}{\partial C_t} = \frac{-V_o}{(A_{o1}C_{fb} + C_t^2)}
\]

Therefore Eqn. 1.11 relates the change in output to the change in gain and capacitance for the charge sensitive amplifier.
Comparison of Eqn. 1.10 and 1.11 shows that the charge sensitive configuration, as claimed, stabilizes both gain and capacitance drifts while the voltage sensitive configuration stabilizes gain drifts only.

1.4.4 Input Drift at High Count Rates

The input voltage shown in Fig. 1.3 builds up at high count rates until it reaches a maximum dc level determined by the count rate and time constant. The maximum shift at an average count rate \( r \), can be shown to be \( V_s = T_{\text{in}} V_i r \), where \( T_{\text{in}} \) is the input time constant and \( V_i \) the amplitude of the input pulses. For the voltage sensitive amplifier,

\[
T_{\text{in}} = C_t R, \text{ where } R = R_i R_b / (R_i + R_b); V_i = Q/C_t
\]

For the charge sensitive case,

\[
T_{\text{in}} = R \left( A_{o1} C_{fb} + C_{t2} \right); V_i = Q/(A_{o1} C_{fb} + C_{t2})
\]

Therefore both configurations have the same input shift, \( V_s = BQr \), at a given count rate. If the input bias can change by a maximum amount \( V_{\text{max}} \), then both amplifiers can handle the maximum count rate \( r_{\text{max}} \) given by Eqn. 1.12.
\[ r_{\text{max}} = \frac{V_{\text{smax}}}{QR}. \] \hspace{1cm} \text{...}(1.12)

For 1mev particles, an input resistance of 100 meg., and a maximum bias shift of (say) 0.1 volts, \( r_{\text{max}} \) is \( 2.2 \times 10^4 \) counts per second. For smaller allowable bias drift, and higher particle energy, \( r_{\text{max}} \) is even smaller and may in fact be too small for some experiments. This difficulty can be overcome with the charge-sensitive, but not the voltage-sensitive configuration, by tying \( R_i \) to the output rather than to the ground. This adds dc feedback to the amplifier and reduces the effective input resistance to \( R_i/A_{ol} \). The maximum count rate is then increased by a factor \( A_{ol} \) to \( r_{\text{max}} = A_{ol}V_{\text{smax}}/QR_i \). This is near the upper cutoff frequency of most preamplifiers and therefore imposes no restriction on high count rate operation.

1.4.5 Noise in Feedback Amplifiers

The "charge sensitive" configuration has been shown to be superior to the "voltage sensitive" configuration for both stability and experimental convenience. To see how their noise performance compares it is necessary to consider the general effect of feedback on the signal-to-noise ratio.

It is often stated that the S/N ratio of an amplifier is not altered by the application of feedback because both the signal and the noise are reduced by the same factor. While this explanation appears to account adequately for the small
effect that feedback generally has on noise performance, it is not strictly true and must be applied with caution, particularly to wide band systems such as nucleonic preamplifiers.

A more accurate statement of the effect of feedback is given by the following theorem (1.11): "the instantaneous S/N current ratio in a lead short-circuiting the output of an amplifier is equal to the instantaneous S/N voltage ratio in the normal load impedance."

As a consequence of this theorem, any feedback loop can be removed from the output and grounded without affecting the ratio of the instantaneous signal to the instantaneous noise. The resolution is determined not by the instantaneous S/N ratio but by the ratio of the peak signal to the rms noise. To find this quantity, the spectral density of the instantaneous noise must be integrated over the frequency pass band. Because feedback generally changes the frequency response of the amplifier, the required S/N ratio differs for the open and closed-loop preamplifiers. However, the noise performance of the preamplifier can be determined correctly from the open-loop system if the closed-loop frequency response is used.

In nucleonic systems, the primary frequency response limitations are introduced later by the post-amplifier and the preamplifier response causes only a second-order effect. Therefore, the original statement that feedback does not affect noise performance is very nearly correct.

It will be shown later that the addition of $C_{fb}$ to the input capacitance reduces the S/N ratio of the charge sensitive
amplifier, while the noise introduced to the input cathode by
the unbypassed feedback resistor: does the same for the
voltage sensitive amplifier.

However, both these effects are small and the two con-
figurations do give roughly the same S/N ratio. Because the
"charge sensitive" configuration is superior in the other aspects
it is most often used.
2. SIGNAL, NOISE AND THE FREQUENCY RESPONSE

In Chapter 1, the nucleonic system was defined and the effect of noise on pulse height resolution was mentioned. The effect of feedback was discussed to show that the closed-loop system can be replaced by an open-loop system for signal and noise analysis. In this chapter, the effect of the frequency response upon the signal and noise of a general nucleonic system will be found, and the criteria for judging the noise performance of the system will be discussed.

2.1 The Effect of Frequency Response on Nucleonic Signals

The input voltage signal of Fig. 1.3 can be shaped into pulses suitable for height analysis by limiting the frequency response in a variety of ways - the most common of which are double delay-line clipping\(^{(2.1, 2.2)}\) and RC clipping\(^{(2.3)}\). Delay-line clipping generates narrower pulses than RC clipping for a given signal-to-noise ratio; hence it is used primarily in high count-rate experiments. RC clipping on the other hand, lends itself more readily to experimental noise measurements because of the ease with which the frequency response can be altered.

The simplest RC pulse shaping network uses isolated elementary integrating and differentiating networks with time constants \(T_1\) and \(T_2\) respectively. Gillespie has carried out a detailed analysis of this network and has shown that the
signal-to-noise ratio is a maximum for \( T_1 = T_2 = \tau \). This simple result has been used by almost all authors in discussing the noise of particular amplifiers(2.5,2.6,2.7) and was also used for the experimental portion of this study.

However, for theoretical calculations of noise it was found necessary to add a second "parasitic" integration time constant, \( T_3 \) (less than \( \tau \)), to the basic system to account for unavoidable high frequency effects that limit the pass band further. In the overall system, \( T_3 \) corresponds to the high frequency cut-off of the preamplifier, while in the noise measurements of individual devices it is the input time constant of the test circuit (see Section 4.2). With the three time constants, the frequency response is given by the function \( P(s) \) of Eqn. 2.1(a). Upon putting \( T_1 = T_2 = \tau \) and \( T_3 = x\tau \), \( P(s) \) takes the form of Eqn. 2.1(b) which is more convenient for finding the response of the system to a signal.

\[
\begin{align*}
(a) \quad P(s) &= \frac{sT_1}{(1 + sT_1)(1 + sT_2)(1 + sT_3)} \quad \ldots (2.1) \\
(b) \quad P(s) &= \frac{1}{x\tau^2} \frac{s}{(s + 1/\tau)^2(s + 1/x\tau)} \quad \ldots (2.1)
\end{align*}
\]

The input voltage is a step, \( Q/C_\tau \), and in the frequency domain it is represented by \( S_1(s) = Q/sC_\tau \). The signal, normalized to unit input voltage, in the frequency domain therefore is just the denominator of Eqn. 2.1(b). Taking the inverse Laplace transform with the aid of Heaviside's expansion results in
Eqn. 2.2(a) for \( s(t) \), the signal in the time domain. In Fig. 2.1, \( s(t') \) is plotted for several different values of \( x \).

(a) \[
\begin{align*}
\frac{1}{(1-x)} \left[ \frac{1}{(1-x)} e^{-t'/x} + \left( t' - \frac{x}{(1-x)} \right) e^{-t'} \right]
\end{align*}
\]

where \( t' = \frac{t}{T} \) \( \ldots (2.2) \)

(b) \[
\begin{align*}
\frac{e^{-t'}}{(1-x)} \left( t' - \frac{x}{(1-x)} \right) \quad (\text{for small } x) \quad \ldots (2.2)
\end{align*}
\]

Eqn. 2.2(a) is not readily maximized with respect to time. However if it is assumed that \( x \) is much less than 1 (i.e. the parasitic time constant is much smaller than \( \tau \)), the first exponential term can be ignored and the simpler, approximate expression 2.2(b) written.

Differentiation of Eqn. 2.2(b) shows that at \( t' = \left( 1 - \frac{x}{(1-x)} \right) \) the signal reaches the peak value \( S \) given by Eqn. 2.3.

\[
S(x) = \frac{1}{(1-x)} e^{-1/(1-x)} \quad (\text{for } x < 0.3) \quad \ldots (2.3)
\]

When \( x = 0 \), the maximum is \( S(0) = e^{-1} \) occurring at \( t' = 1 \), the well known result for a pass band of equal integration and differentiation time constants. Eqn. 2.3 holds up to \( x = 0.3 \) with less than 3% error but for larger \( x \) the error increases rapidly, being over 7% when \( x = 0.4 \). \( S(x) \) is plotted in Fig. 2.2. The values for small \( x \) were found from Eqn. 2.3, and for large \( x \) from Fig. 2.1.
Fig. 2.1 Step Response of Pulse Shaping Network

Fig. 2.2 Peak Response of Pulse Shaping Network
2.2 Noise in the Nucleonic System

2.2.1 General Noise Model for the Nucleonic System

Noise is generated throughout the system by a variety of physical processes but its effect at the output is completely independent of its origin. For this reason it was assumed that all noise results from a noise voltage, \( n \), at the input of a noise-free system as illustrated in Fig. 2.3(a).

\[ \begin{align*}
\bar{e}_n^2 &= w_e \Delta f \\
I_n^2 &= w_p \Delta f
\end{align*} \]

(a) Single Generator Model \hspace{1cm} (b) Two Generator Model

Fig. 2.3 General Noise Models

\( n \) generally consists of two components, one dependent and one not dependent on the input admittance \( Y_t \). These are represented in Fig. 2.3(b) by a shunt current generator \( i_n \) and a series voltage generator \( e_n \) respectively. In the most general case, \( e_n \) and \( i_n \) are at least partially correlated but for the frequencies of interest in nucleonic applications, they can be taken to be completely uncorrelated with little or no
error (2.8). With this assumption, noise voltages resulting from \( i_n \) and \( e_n \) add as mean squares and the total equivalent mean square input noise voltage \( \bar{v}^2 \) is given by \( \bar{v}^2 = \bar{e}^2 + \left| \frac{i_n}{n} \right|^2 \).

In Fig. 2.4 the noise sources are decomposed further, into amplifier noise generators \( e_n \) and \( i_a \), and other generators representing detector and input circuit noise.

\[
\begin{align*}
    w_d &= 2Q_e I_d \\
    w_a &= 2Q_e I_L \\
    w_b &= \frac{4kT}{R_b} \\
    w_e &= w_w + w_f = 4kT R_i + K_f \\
    w_a &= 2Q_e I_L \\
    w_b &= \frac{4kT}{R_b} \\
    w_e &= w_w + w_f = 4kT R_i + K_f
\end{align*}
\]

Fig. 2.4 Detailed Noise Model of Nucleonic System

As a general rule, the amplifier noise arises predominantly in the input stage with small contributions from the second and subsequent stages. This is because the noise of the first stage is amplified before adding to the noise of the subsequent stages. The amplifier noise generators \( e_n \) and \( i_a \) can therefore be taken to be the noise generators of the active device used for the first stage. In most active devices \( i_a \) results from shot noise in the input leakage current, so that it is convenient to take the spectral density of \( i_a \) to be \( w_L = 2Q_e I_L \), where \( Q_e \) is the electronic charge. \( I_L \) is defined by this expression to be the equivalent amplifier
leakage current. It is not necessarily the measurable dc leakage current and may even be frequency dependent.

The voltage generator for all devices is found to contain two distinct components; one frequency-independent or "white", $e_w$ with spectral density $w_w$; and a second "flicker" or "excess" component, $e_f$, with a spectral density $w_f$ that is inversely dependent upon frequency. $e_w$ is commonly taken to result from thermal noise in a fictitious "equivalent noise resistor" $R_n$, located at the input as shown. Therefore $w_w = 4kT R_n$, where $k$ is Boltzmann's constant and $T = 290^\circ K$. The flicker noise is sometimes lumped with the white noise in one frequency-dependent equivalent noise resistor but evaluation of the overall system noise eventually requires the separation of the two for purposes of integration. Therefore, they will be treated separately from the start, and the flicker noise characterized by its own parameter $K$, which relates $w_f$ to the frequency as $w_f = (K/f)$. The three parameters, $I_L$, $R_n$ and $K$, then represent the noise of all the active devices and therefore of all amplifiers considered in this thesis.

In addition to the noise of the amplifier, the bias resistors contribute thermal noise currents $i_i$ and $i_b$ with spectral densities of $w_i = 4kT/R_i$ and $w_b = 4kT/R_b$ respectively, and the detector contributes a shot noise current $i_d$ with spectral density $w_d = 2Q_e I_d$. $I_d$ is the detector leakage current. If the detector is connected directly to the amplifier as shown, all current generators can be added together to give the spectral density $w_p$ of the original parallel current generator $i_n$. 
\[ w_p = 2Q_e I_e \text{ where } I_e = I_L + \frac{4kT}{2Q_e R_i} + \frac{4kT}{2Q_e R_b} + I_d \] 

This noise current causes a series input voltage \( i_n / Y_t \) with spectral density \( w_S \) given by Eqn. 2.5:

\[ w_S = \frac{2Q_e I_e R^2}{(1 + \omega^2 R^2 C^2)} \text{, where } R = \frac{R_i R_b}{R_i + R_b} \]

From Eqn. 2.4, it can be seen that the thermal noise of the bias resistors behaves like shot noise in a leakage current \( 4kT/2Q_e R \). This has a magnitude of 1 nA for a 49.5 meg value of \( R \). Since the leakage currents encountered in tubes and ion chambers are of this order, very large bias resistors must be used.

Eqn. 2.5 suggests that both the thermal noise and the leakage current noise can be reduced simply by reducing \( R_i \) or \( R_b \) until \( w_S = 4kT R \). The entire shunt noise contribution then appears as an addition to \( R_n \). Since \( R_n \) is typically between 100 and 500 ohms, \( R \) must also be of this order so that its noise is not excessive. If it were this small, however, the signal would be attenuated to such an extent by the parallel RC input admittance that the net effect would be a decrease in the signal-to-noise ratio. Therefore, the first method discussed, reducing input thermal noise through the use of very large bias resistors, is to be preferred.
2.2.2 Effect of Frequency Response on Noise

So far the amplifier and detector noise has been discussed in terms of spectral densities only. The effect of the frequency pass band upon the total noise of the system will now be found. To do this, the spectral components are multiplied by the squared frequency response of the system, and integrated over the entire frequency range. This results in Eqn. 2.6 for \( n^2 \), the equivalent input noise. Eqn. 2.7 is then found by substituting the expressions for \( w_w, w_p, \) and \( w_f \) from Fig. 2.4.

\[
\overline{n^2} = \int_{0}^{\infty} \left[ w_w |P(\omega)|^2 + w_p \left| \frac{P(\omega)}{Y_t} \right|^2 + w_f |P(\omega)|^2 \right] \frac{d\omega}{2\pi} \quad \ldots (2.6)
\]

(a) \[
\overline{n^2} = \int_{0}^{\infty} \left[ 4kT \overline{r_n} |P(\omega)|^2 + \frac{2QeI_e |P(\omega)|^2}{C_t^2 \omega^2} + 2\pi K \frac{|P(\omega)|^2}{\omega^2} \right] \frac{d\omega}{2\pi} \quad \ldots (2.7)
\]

(b) \[
\overline{n^2} = 4kT \overline{r_n} I_1 + \frac{2QeI_e}{C_t^2} I_2 + 2\pi K I_3 \quad \ldots (2.7)
\]

The integrands \( I_1, I_2, I_3 \) are found in terms of the time constants by substituting Eqn. 2.1(a) for \( P(s) \) in Eqn. 2.7. The resulting integrals all have the form of Eqn. 2.8 with \( m \) differing for each integral as noted. The integrals are evaluated in Appendix I.

\[
I = \frac{T_1^2}{2\pi} \int_{0}^{\infty} \frac{\omega^m d\omega}{(1 + \omega^2 T_1^2)(1 + \omega^2 T_2^2)(1 + \omega^2 T_3^2)} \quad m = 2 \text{ for } I_1
\]
\[
\quad m = 0 \text{ for } I_2
\]
\[
\quad m = 1 \text{ for } I_3
\]

\ldots (2.8)
When $T_1 = T_2 = \tau$ and $T_3 = x\tau$ the results are:

(a) $I_1 = \frac{1}{8\tau(1 + x)^2}$

(b) $I_2 = \frac{\tau(1 + 2x)}{8(1 + x)^2}$

(c) $I_3 = \frac{2x^2 \ln(x) + (1-x^2)}{4\pi(1-x^2)^2}$

(d) $I_3 \approx \frac{1}{4\pi(1 + x)}$

The effect of the "parasitic" time constant $T_3$ upon each type of noise is shown in Fig. 2.5 by plotting $\sqrt{I_k(x) / I_k(0)}$ for each integral. The approximate form of $I_3$ given by Eqn. 2.9(d) can be seen to be within a few percent of $I_3$ for all $x$ of interest so it will be used in all subsequent equations. In particular, it is substituted with $I_1$ and $I_2$ in Eqn. 2.7 to give Eqn. 2.10 for the total rms equivalent input noise voltage

$$\bar{n}(x) = \frac{1}{(1 + x)} \left[ \frac{kT n}{2\tau} + \frac{Q_e I_e \tau(1 + 2x)}{4C_t^2} + \frac{K}{2} (1 + x) \right]^{\frac{1}{2}}$$

\[\text{Fig. 2.5 Effect of } T_3 \text{ on Noise Voltage Components}\]
The overall effect of x depends upon the relative size of $I_L$, $R_n$ and K, and cannot therefore, be found exactly at this time. However, it can be found approximately if it is assumed that x is large enough to cause a significant change in any term only when $\tau$ is small (since $T_m < \tau$ for all $\tau$). Applying some foresight to Eqn. 2.10, it is apparent that the condition of small $\tau$ results in the white noise term predominating over the others because of its inverse dependence upon $\tau$. The effect of x on $\bar{n}$ can thus be taken to be just the effect of x on the white noise term and Eqn. 2.10 can be simplified to Eqn. 2.11. This approximation was found to be adequate for determining the noise of the amplifier alone (see section 5.2); but when a large leakage current is added from the detector, the leakage current noise is not insignificant before the effect of x is felt. For this case the exact expression for $\bar{n}$ must be used.

$$\bar{n}(x) \approx \frac{1}{(1 + x)} \left[ \frac{kTR_n}{2\tau} + \frac{Q_e I_e \tau}{4C_t^2} + \frac{K}{2} \right]^{1/2} = \frac{\bar{n}(0)}{(1 + x)}$$

$\ldots$(2.11)

2.3 The "Equivalent Input Noise Charge"

2.3.1 Definition of the Equivalent Input Noise Charge

In previous sections expressions were derived for the signal and noise in terms of the system time constants and the parameters of the general noise model. The signal-to-noise
ratio is the basic measurement of the accuracy with which an experiment can be performed; but since it is a function of the input signal level, it is not a suitable criterion for comparing the noise performance of individual amplifiers. They must be judged and compared on the basis of the normalized signal-to-noise ratio. This is independent of the signal level, yet is easily converted into the resolution given by the amplifier in any experiment.

The signal-to-noise ratio is \( \frac{S}{C/n} \) where \( S \) is the peak output given in Fig. 2.2 and \( n \) is the rms noise given by Eqn. 2.10. Normalizing the signal-to-noise ratio with respect to the input charge \( Q \) gives \( S/Ctn \) in units of \((\text{charge})^{-1}\).

This is seldom used directly but is inverted and defined to be the "equivalent input noise charge", \( Q_n \). \( Q_n \) is then used as the basic criterion for judging nucleonic amplifier noise. In order that two amplifiers be compared under identical conditions, it is necessary to specify not only the noise charge but also the frequency response and detector capacitance used when the measurement or calculation was made.

While \( Q_n \) has been defined here as the inverse of the normalized signal-to-noise ratio, it is often defined directly as "that quantity of charge which applied instantaneously to the amplifier input produces an output pulse of amplitude equal to the rms noise output voltage" \((2.5)\). This equivalent definition is more useful for experimental work than the one given above because it defines the noise charge in the manner in which it is measured. It does however, have the disadvantage
that the basic relationship between noise charge and the signal-to-noise ratio is implied rather than stated. This can lead to the error of setting $Q_n = C_t \bar{n}$, ignoring the effect of the frequency response on the (implied) unit signal\(^{(2.5,2.9)}\).

For ease in converting the noise charge to percent resolution, $Q_n$ is generally expressed in either electronic charges or "equivalent KeV particle energy". The first conversion simply requires the dividing of $Q_n$ by $Q_e$, the electronic charge. The resulting quantity $N = \frac{Q_n}{Q_e}$ can then be converted into particle energy units by dividing $N$ by the "detector yield" of 1 ion pair per 30 electron-volts for a typical ion chamber, or 1 hole-electron pair per 3.5 electron-volts for a solid state detector. This is then the rms noise expressed in energy units. To give a direct measurement of the resolution, the rms quantities are generally converted into the Full noise line Width at one Half Maximum height. This requires multiplication by 2.35, the ratio of the FWHM to the standard deviation of a Gaussian distribution. These conversions are summarized below in Eqn. 2.12.

\[
\begin{align*}
(a) \quad Q_n &= \frac{C_t \bar{n}}{S} \quad \text{(in coulombs)} \\
(b) \quad N &= \frac{Q_n}{Q_e} = \frac{C_t \bar{n}}{SQ_e} \quad \text{(in electronic charges)} \\
(c) \quad \text{FWHM} &= 47 \times 10^{-3} N \quad \text{(in KeV for ion chambers)} \\
(d) \quad \text{FWHM} &= 8.23 \times 10^{-3} N \quad \text{(in KeV for solid-state detectors)}
\end{align*}
\]

\(...(2.12)\)
An alternative method of specifying the normalized signal-to-noise ratio is to state separately the equivalent input noise voltage $\bar{n}$ and the total input capacitance. This requires a knowledge of the amplifier input capacitance, a parameter not required for the noise charge specification. Because this capacitance is difficult to measure, and because of the unnecessary complication of specifying two quantities rather than one, the use of noise voltages as a criterion of amplifier noise performance is not recommended and not often used.

2.3.2 Equations for Noise Charge

Eqn. 2.12(b) defines the equivalent noise charge $N$ in terms of the peak signal discussed in section 2.1 and the equivalent input noise found in section 2.2. The effect of the "parasitic" time constant was to decrease both these quantities, the signal as shown in Fig. 2.2, and $\bar{n}$ as shown by the white noise curve of Fig. 2.5. Since the noise charge depends directly upon $\bar{n}$ and inversely upon $S$, the effect of $x$ upon $N$ is just the ratio of the changes caused in $\bar{n}$ and $S$. This can be represented by a "correction factor" $C(x)$ found in Eqn. 2.13. Using this factor, $N(x)$ is then found in Eqn. 2.14.

$$C(x) = \frac{\bar{n}(x)}{\bar{n}(0)} \frac{S(0)}{S(x)} \approx \frac{(1-x)}{(1+x)} e^{x/(1-x)} \quad \cdots (2.13)$$

(for $x < 0.3$)
\[ N(x) = C(x)N(0) = C(x)C_t \frac{\bar{n}(0)}{Q_e S(0)} \]

\[ = C(x) \frac{C_t e}{Q_e} \left[ \frac{4kT n}{2 \tau} + \frac{Q_e I_e \tau}{4C_t^2} + \frac{k}{2} \right]^{\frac{1}{2}} \] ...

(2.14)

\(C(x)\) is plotted in Fig. 2.6. It was evaluated directly for small \(x\), but where the approximate form of \(S\) no longer holds, it was found graphically from the curves of Figs. 2.2 and 2.5.

It can be easily shown that \(N(0)\) is a minimum when the white noise and leakage current noise terms are equal. When they are equal, \(\tau\) is so long that \(C(x)\) has negligible effect; the minimum of \(N(x)\) and \(N(0)\) then occur at the same time constant \(\tau_{\text{min}}\).

\(\tau_{\text{min}}\) is given by Eqn. 2.15 and the minimum noise charge \(N_{\text{min}}\) by Eqn. 2.16.
For convenience in later calculations, numerical values for the various constants have been substituted in Eqns. 2.14, 2.15, and 2.16 to obtain the expressions below. In them, $C_t$ is in pf, $R_n$ in ohms, $\tau$ in $\mu$sec, $I_e$ in nanoamps, $K$ in volts$^2$/cycle, and the resulting $N$'s are in electronic charges.

$$N(x) = C(x) \cdot 1.71 \cdot C_t \left[ \frac{0.198 R_n}{\tau} + \frac{4 \times 10^3 I_e \tau}{C_t^2} + \frac{K \times 10^{14}}{2} \right]$$

...(2.14')

$$\tau_{\text{min}} = 0.0224 \cdot C_t \sqrt{\frac{R_n}{10 I_e}}$$

...(2.15')

$$N_{\text{min}} = 1.71 \cdot C_t \left[ 0.178 \sqrt{10 R_n I_e} + \frac{K}{2} \cdot 10^{14} \right]^{1/2}$$

...(2.16')

These three equations relate the noise charge of the system to the various noise parameters and the time constants. They were derived by assuming that the white noise term predominates whenever $x$ is large enough to affect the noise appreciably; they apply therefore, only to this case. Prediction of noise charge when this assumption does not apply can be made
by using \( \bar{n}(x) \) and \( S(x) \) directly without the simplification of factoring \( C(x) \). However, it was found that for the amplifiers built during this study, the approximate expressions were completely adequate; hence they were used exclusively.

### 2.3.3 Calculation of Noise Charge of a Feedback Amplifier

In Chapter 1 it was shown by appealing to the basic theorem on noise in feedback amplifiers that the signal and noise can be found for an open-loop system if the closed-loop frequency response is used. At that time an example was not given because the effect of the frequency response had not yet been discussed. Now that it has, \( N \) will be found for a system with two feedback loops to clarify the effect that feedback has on the signal and noise. The two-loop system shown in Fig. 2.7 represents a typical nucleonic preamplifier. \( C_{fb1} \) is the planned charge feedback loop discussed previously, and \( C_{fb2} \) the parasitic Miller feedback of the first stage. The high frequency cutoff of the preamplifier is accounted for by \( x \) in \( P(s) \), leaving \( A_1 \) and \( A_2 \) frequency-independent.

![Fig. 2.7 Feedback Loops of Typical Preamplifier](image-url)
If $C_{fbl} = C_{fb2} = 0$, the signal and noise are given by Eqns. 2.17 and 2.18 (note that $v_o$ and $\bar{n}$ have been referred to the input by dividing by $A_1A_2$).

$$v_{o1} = \frac{Q}{C_{t1}} S(x_1) \quad \text{where} \quad C_{t1} = C_i + C_d \quad \ldots(2.17)$$

$$\bar{n}_1 = \bar{n}(x_1) = \left[ 4kT R_n I_1(x_1) + \frac{2Q_e I_e I_2(x_1)}{C_{t1}^2} + 2\pi K I_3(x_1) \right]^{\frac{1}{2}} \quad \ldots(2.18)$$

The closed-loop system is most easily analyzed by applying superposition to all instantaneous voltages fed to the input. For the signal this results in the following expression:

$$v_o = \left[ \frac{Q}{C_{t2}} - \frac{v_o C_{fbl}}{A_2 C_{t2}} - \frac{v_o C_{fb2}}{C_{t2}} \right] A_1A_2$$

Solving for $v_o$ results in Eqn. 2.19, in which $C_{t2} = C_i + C_d + C_{fbl} + C_{fb2}$ and $x_2$ is the $x$ for the closed-loop system. Generally $x_2 < x_1$. $A_1A_2$ has again been removed to refer $v_o$ to the input.

$$v_{o2} = \frac{QS(x_2)}{(C_{fb2} + C_{fbl}A_1 + C_{fb2}A_2)} = \frac{QS(x_2)}{C'} \quad \ldots(2.19)$$
For the series noise sources the instantaneous total is:

\[ n_s = A_1 A_2 \left( e_n - \frac{C_{fb1} n_s}{A_2 C_{t2}} - \frac{C_{fb2} n_s}{C_{t2}} \right) \]

while for shunt noise source it is:

\[ n_p = A_1 A_2 \left( \frac{i_n}{Y_{t2}} - \frac{C_{fb1} n_p}{A_2 C_{t2}} - \frac{C_{fb2} n_p}{C_{t2}} \right) \]

lumping these together and integrating over all frequencies gives Eqn. 2.20 for the total rms noise.

\[ \overline{n_2} = \frac{C_{t2}}{C_t} \left[ 4kTB_1 I_1(x_2) + \frac{2Q_e I_2(x_2)}{C_{t2}} + 2\pi K I_3(x_2) \right] \]

\[ = \frac{C_{t2}}{C_t} \overline{n}(x_2) \quad \ldots (2.20) \]

From Eqns. 2.17 and 2.18, the open-loop noise charge can be found to be

\[ N_1 = \frac{C_{t1} \overline{n}(x_1)}{S(x_1)} \quad \ldots (2.21) \]

and from Eqns. 2.19 and 2.20 the closed-loop noise charge is

\[ N_2 = \frac{C_{t2} \overline{n}(x_2)}{S(x_2)} \quad \ldots (2.22) \]

These expressions differ only in the values of \( C_t \) and \( x \).
The difference in $C_t$ results from the addition of the two feedback capacitors while the change in $x$ results from the change in the high frequency response when feedback is applied. The calculation therefore verifies, for this case, the verbal proof given earlier that the closed-loop system can be analyzed by opening the feedback loops and placing the feedback components in shunt with the input.

An important result of this analysis is the demonstration that the noise charge depends upon the true parasitic capacitance at the input of the amplifier and not the apparent capacitance observed when the amplifier is in operation. The cascode configuration (see Chapter 5) is therefore chosen for the input stage because of its inherent low noise properties and not, as is often stated, because of its low Miller capacitance.

2.4 The Noise Figure

2.4.1 The Noise Figure of a Nucleonic System

The noise charge criterion for judging system noise is used only in nucleonic work. In other applications such as communications, the Noise Figure $F$, or the Noise Temperature $T_e = T_o (F-1)$ is used. Why then is the noise figure not used for this application as well? The reason is that $F$ gives an erroneous indication of the conditions for optimum noise performance for this type of amplifier. This can be seen by applying the usual definition of $F$ below to the nucleonic system.
F = total output noise power with the input termination in place

Output noise power resulting from the noise-free amplification of the source noise

From this, the spot noise-figure F is given by Eqn. 2.23, while the broad-band, or integrated noise-figure \( F_{\text{int}} \) is given by Eqn. 2.24.

\[
F = \frac{-n^2}{I_d/\omega C_t^2} = 1 + \frac{(I_e - I_d)}{I_d} + \frac{4kTR_n C_t^2}{2Q_e I_d} + \frac{2\pi K \omega C_t^2}{2Q_e I_d}
\]  \( \cdots (2.23) \)

\[
F_{\text{int}} = 1 + \frac{(I_e - I_d)}{I_d} + \frac{4kTR_n C_t^2}{2Q_e I_d \tau^2} + \frac{4K C_t^2}{2Q_e I_d \tau}
\]  \( \cdots (2.24) \)

Both these expressions tend to a minimum at low frequencies, or when the detector noise is large. This does not correspond to the condition for minimum noise charge found earlier but simply indicates that the noise figure is low when the source noise predominates. Literal application of the definition of F therefore results in an erroneous determination of the optimum operating conditions for the nucleonic system.

If in the definition of F the denominator is taken to be the noise inherent in the signal due to statistical fluctuations in the particle energies, F is given by \( F = 1 + \frac{n^2}{s^2} \), where \( s^2 \) is the noise associated with the input signal. This F is a minimum when the noise charge is a minimum, and represents properly the degradation in the input signal-to-noise ratio. By defining F in this way, a noise figure for the
system has been found, but it gives no information not already expressed in the noise charge.

2.4.2 The Relationship Between N and F

Even though the noise figure is not suitable for specifying the noise performance of nucleonic amplifiers, a relationship does exist between the noise figure of a resistively terminated amplifier and the noise charge for the same amplifier when it is driven from a capacitive source. This relationship can be derived by considering the spot noise figure \( F \) for the amplifier represented by the noise model of Fig. 2.8.

\[
\begin{align*}
\bar{I}_g^2 &= \frac{4kT}{R_g} \Delta f; & \bar{I}_n^2 &= \sigma_p \Delta f; & \bar{e}_n^2 &= \nu_e \Delta f
\end{align*}
\]

\( F \) is given by Eqn. 2.25 as the ratio of the total mean square input noise to the mean square input noise voltage resulting from the source noise current \( i_g \).
\[
F = \left( \frac{-2}{4kT} \frac{n}{RgZ_{\text{in}}} \right)^2 = 1 + \frac{w_p Rg}{4kT} + \frac{w_e}{Rg} \frac{2}{4kTR} + \left| 1 + \frac{Rg}{Z_{\text{in}}} \right|^2 \quad \ldots (2.25)
\]

\(Z_{\text{in}}\) is in general complex but since we are concerned with the spot noise figure, it can be taken to be constant and equal to \(|Z_{\text{in}}|\). Eqn. 2.25 can therefore be rewritten as Eqn. 2.26. \(F\) is a minimum when the two terms in \(Rg\) are equal. Thus the optimum source resistance \(R_{go}\) is given by Eqn. 2.27 and the minimum noise figure by Eqn. 2.28(a) and (b).

\[
F = 1 + \frac{w_e}{2kT |Z_{\text{in}}|} + \frac{Rg}{4kT} \left( w_p + \frac{w_e}{|Z_{\text{in}}|^2} \right) + \frac{w_e}{4kTRg} \quad \ldots (2.26)
\]

\[
R_{go}^2 = \frac{\frac{w_e |Z_{\text{in}}|^2}{w_e + |Z_{\text{in}}|^2} \frac{2}{w_p}} \quad \text{(note that } R_{go} \leq |Z_{\text{in}}|) \quad \ldots (2.27)
\]

(a) \(F_{\min} = 1 + \frac{w_e}{2kT |Z_{\text{in}}|} + \frac{2R_{go}}{4kT} \left( w_p + \frac{w_e}{|Z_{\text{in}}|^2} \right) \quad \ldots (2.28)\)

(b) \(F_{\min} = 1 + \frac{w_e}{2kT |Z_{\text{in}}|} + \frac{2w_e}{4kTR_{go}}\)

Solving for \(w_p\) and \(w_e\) from these equations results in Eqn. 2.29 and 2.30 relating the noise generators of the general noise model to: (i) the minimum spot noise figure \(F_{\min}\) of an amplifier, (ii) the optimum source resistance \(R_{go}\), and (iii) the input impedance \(Z_{\text{in}}\) at the frequency for which \(F_{\min}\) is given.
To simplify the remaining calculations it is assumed that the flicker component can be ignored, so that \( w_e = w_w \). Substituting \( w_e \) and \( w_p \) in Eqn. 2.6 for \( \bar{n}^2 \), integrating over a frequency pass band defined by \( T_1 = T_2 = \tau \) and \( T_3 = 0 \), and substituting the result in Eqn. 2.12(b), results in Eqn. 2.31 relating \( N \) to \( F min \). Eqn. 2.31 can be minimized by choosing \( \tau_{min} \) to make the two components equal, with the result shown in Eqn. 2.32. (Here \( T_i = R \_go \_C_t \))

\[
N = \frac{C_e \sqrt{2kT(F_{min}-1)R_{go}}}{Q_e} \left[ \frac{\tau}{8T_i^2} \left( \frac{|Z_{in}| - R_{go}}{|Z_{in}|} \right) + \frac{|Z_{in}|}{8\tau (|Z_{in}| + R_{go})} \right]^{\frac{1}{2}} \]

...(2.31)

\[
N_{min} = \frac{e^{\sqrt{2kT(F_{min}-1)C_t}}}{2Q_e} \left[ \frac{1 - \frac{R_{go}}{|Z_{in}|}}{1 + \frac{R_{go}}{|Z_{in}|}} \right]^{\frac{1}{4}} \]

...(2.32)

Eqn. 2.31 and 2.32 give the desired relationship between the noise charge of an amplifier driven from a capacitative source and the noise-figure of the same amplifier driven from a resistive source. It should be stressed that even though the single-frequency noise figure is used to find the noise charge, the resulting equation gives the general broad-band noise charge with the flicker term ignored. Through the use of this
equation, amplifiers reported for communications applications can be rapidly evaluated for possible use in nucleonic work. This is important because in recent years large numbers of papers have appeared in communications literature reporting the noise figures of amplifiers built with the many new solid state devices. In addition, the manufacturers' specifications for transistors usually give only $F$ as the standard noise criterion for the device (2.11,2.12). Thus a preliminary choice of the best transistor or other device for a nucleonic amplifier can be made by using Eqn. 2.32 to determine the expected noise charge.

As an example of the use of Eqn. 2.32, $N_{\text{min}}$ can be calculated for the 2N2386 field effect transistor from the following data given in the specifications. $F_{\text{min}} = .25\text{db} = 1.03$ at $R = 1\text{ Meg, } f = 1\text{kc}$. The f.e.t. has an input capacitance of 20pf so $|Z_{\text{in}}| = 8\text{ meg}$. If the limiting case of zero detector capacitance is taken, $C_\text{t} = 20\text{pf}$. Substitution of these quantities in Eqn. 2.32 gives $N_{\text{min}} = 560$, or FWHM = 4.6Kev for a solid state detector. The 2N2386 was found to exhibit approximately this noise charge in practice. (See section 5.3, Fig. 5.9). The calculation indicates that the noise figure of an amplifier must be considerably less than 1 db if it is to be suitable for nucleonic work.
3. NOISE SOURCES IN ACTIVE DEVICES

Now that the general nucleonic system has been analyzed, the amplifier noise will be considered separately. In the general noise model, amplifier noise was taken to result entirely from the noise of the input stage and was characterized by the parameters $L$, $R_n$ and $K$ of the two-generator noise model. A consideration of amplifier noise therefore entails finding $L$, $R_n$ and $K$ for each active device that might be used in a nucleonic amplifier.

This is done theoretically in the following sections for tubes, Nuvistors, field effect transistors and junction transistors. These devices all exhibit noise figures less than 1 db at high source resistance and should therefore give low noise charge as well (see section 2.3). Brief discussions of the more "exotic" tunnel diode, parametric, and Maser amplifiers are then given to show why they are not suitable for nucleonic applications.

3.1 Noise in Tubes and Nuvistors

Noise in the conventional tube and its modern miniaturized cousin, the Nuvistor, is produced by the same processes. At frequencies less than, say 20 Mc., the most important of these are the shot effect in plate and grid currents and flicker (excess) noise in the plate current. These are, for all practical purposes, completely uncorrelated.
3.1.1 Shot Noise in Tubes and Nuvistors

In a tube or Nuvistor operating under temperature-limited conditions, the emission of an electron from the cathode is a random event depending in no way upon the emission of other electrons. The rate of charge emission, therefore, fluctuates statistically about the mean value giving rise to the plate current fluctuation known as "shot noise". It can be shown that the shot noise can be represented by a current generator $i_s$ in the plate circuit with a power spectral density $w_s = 2Q_e I_p$, where $Q_e$ is the electronic charge (see Fig. 3.1).

$$w_f = kT; \quad w_w = 4kT R_n; \quad w_L = 2Q_e (|I_g^+| + |I_g^-|); \quad w_s = 2Q_e I_p$$

![Fig. 3.1 Triode Noise Model](image)

This result does not hold, however, when the plate current is limited by a potential minimum between plate and cathode (so-called space-charge limited operation) because now the emission of one electron affects subsequent emission. This can be explained simply as follows. The depth of the potential minimum depends strongly upon the space charge density in the vicinity of the cathode. A fluctuation in the cathode emission therefore causes an opposing fluctuation in the potential minimum depth. This in turn
induces cathode emission that compensates for the initial current fluctuation and reduces the spectral density by a factor $\Gamma^2$ to $\omega_s = 2q_e I_p \Gamma^2$, where $\Gamma^2 < 1$. $\Gamma$ has been found experimentally and theoretically to be proportional to $g_m/I_p$ where $g_m$ is the transconductance of the tube, so that $\omega_s \alpha 2q_e g_m$ for space-charge limited operation. To represent this type of noise in the general noise model, $i_s$ is assumed to result from the noise voltage $e_w = i_s/g_m$ at the input. The shot noise is then represented by the equivalent noise resistor $R_n$ where $R_n = c/g_m$. The constant $c$ varies between 2.5 and 4 depending upon tube geometry and cathode temperature and is most easily found experimentally.

3.1.2 Grid Leakage Noise

The second important source of noise in tubes and Nuvistors is shot noise in the grid leakage current $I_g$. The observable $I_g$ is the algebraic sum of two components $I_g^+$ and $I_g^-$. $I_g^+$, a positive current flowing from the grid, results from the capture by the grid of positive ions left in the tube after evacuation, and from emission of electrons from the grid. Grid emission is a result of impurities on the grid or of the photo-electric effect (3.4, 3.5). $I_g^-$ on the other hand, results from the grid capturing electrons from the main plate current. The approximate dependence of these two components upon grid bias is shown in Fig. 3.2. It can be seen that $I_g^-$ reduces very quickly to zero for negative grid bias while $I_g^+$ varies more slowly. The bias at which the two components are equal is the potential at which the
grid floats on open circuit.

Since these two currents arise independently and are not smoothed by the space charge they both exhibit full, uncorrelated shot noise. This noise occurs at the input so it can be represented by the current generator \( i_a \) with spectral density \( w_L = 2Q_e (|I_g^+| + |I_g^-|) \), in the position shown in Fig. 3.1.

3.1.3 Flicker Noise

The remaining source of noise in the triode is flicker or excess noise in the plate current. This type of noise is not well understood but it is thought to result from fluctuations in the emissivity of the cathode material \( 3.6 \). Such fluctuations are generally slow, hence the flicker noise spectrum contains
predominately low frequency components. It has been found experimentally that it can be adequately represented over the frequency range 1 c to 50 Kc by the generator $i_f$ with an inverse frequency dependent spectrum. For the general noise model, $i_f$ is referred to the grid, as was the shot noise, giving $e_f$ with $w_f = K/f$. $K$ must be determined experimentally, but for most tubes $K = 10^{-13}$ within 50%\(^{(3.7)}\).

### 3.1.4 Partition Noise in Multigrid Tubes

In pentodes and other multigrid tubes, part of the cathode current is diverted from the anode to a positive grid. This random partitioning of the cathode current results in additional noise which may be regarded as a decrease in the space-charge smoothing factor $\Gamma$. It can be shown that $\Gamma^2$ for multigrid operation is given by $\Gamma_m^2 = I_s/I_c + I_p \Gamma_c^2/I_c$ where $I_s$, $I_c$, and $I_p$ are the screen grid, cathode and anode currents respectively and $\Gamma^2$ is the triode smoothing factor\(^{(3.8)}\). This additional source of noise is avoided simply by tying the screen grid to the plate and using the tube as a triode.

### 3.2 Equivalent Circuit and Noise Model for the Field Effect Transistor

The field effect transistor (f.e.t.) was one of the earliest semiconducting devices discussed in detail in the literature\(^{(3.9)}\); but in spite of this early and apparently promising start it has never achieved widespread use. The
principal reasons for this have been the relatively high cost of f.e.t.'s caused by fabrication difficulties, and the rapid development of cheap, versatile junction transistors capable of outperforming them in almost any circuit. Recently however, fabrication techniques have improved, resulting in commercially available f.e.t.'s and renewed interest in their use for applications where they are superior to junction transistors. One such application is the low-noise amplification of signals from high impedance sources.

A quantitative analysis of the operation of the f.e.t. was given first by Shockley then in more detail by Dacy and Ross. For this thesis a simple qualitative description of its operation based on the work of the above authors will have to suffice because of space limitations. It should be noted at the outset that the f.e.t. operates by a process completely different from any junction transistor and is not simply a special type of junction transistor.

This process is one of voltage control rather than current control so the f.e.t. is most conveniently thought of as a solid state analogue of the vacuum tube. Another difference between the f.e.t. and the junction transistor is that current in the f.e.t. is carried exclusively by majority carriers whereas current in the junction transistor is carried by both majority and minority carriers. For this reason, the f.e.t. is sometimes called a "unipolar" transistor to distinguish it from the "bipolar" junction transistor.

Both n-channel and p-channel f.e.t.'s are made but only
the p-channel type is described here. The description can be applied to the n-channel F.E.T. by changing all voltage and current polarities.

3.2.1 Theory of F.E.T. Operation

Fig. 3.3 shows a schematic picture of the p-channel F.E.T. It consists simply of a conducting "channel" of heavily doped p-type silicon or germanium sandwiched between two "gates" of n-type material. The interfaces between channel and gate form p-n junctions. Associated with each of these is a carrier-depleted region that extends into the channel to a depth determined by the potential difference between channel and gate at any given point. Current flows through the channel between physically identical terminals to which the names "source" and "drain" are assigned according to the direction of current flow. In the description below it is assumed that current flows in the positive x-direction.

(a) non pinch-off operation
First consider the operation when the gate is short circuited to the source ($V_{gs} = 0$). A negative potential $V_{ds}$ applied to the drain causes a current $I_d$ to flow in the channel. At any point $x$, the reverse bias on the channel-gate junction is approximately $V(x) = I_d R_c x$, where $R_c$ is the average channel resistance per unit length. (see Fig. 3.3(a)). The increasing reverse bias with increasing $x$ results in a deepening penetration of the carrier-depleted regions into the channel. If $V_{ds}$ is high enough, sufficient current will flow to cause the depletion layers to meet at $P$, leaving the channel from $P$ to the drain free of carriers (see Fig. 3.3(b)). The current required to cause this so-called "pinch-off" is determined by the average resistance between the source and $P$ and also by the reverse bias needed to cause the depletion layers to penetrate the whole channel. Since both of these quantities depend only upon device dimensions and materials, little change in current will be
effected by a further increase in $V_{ds}$. Any such increase is simply absorbed across the high impedance of the carrier depleted region of the channel.

If now a reverse bias is applied to the gate, the carrier depleted regions extend part way across the channel, even in the absence of channel current. The effect of channel current upon the shape of the depletion layers is the same as before, but because of the initial penetration, pinch-off occurs at a lower current and lower drain voltage than with zero gate bias. Again the current is relatively independent of the drain voltage as long as the voltage is large enough to cause pinch-off. From this elementary argument, it is evident that when the f.e.t. is operated with the channel pinched off, it will exhibit $I_d$ vs $V_{ds}$ characteristics (Fig. 3.4) like those of a pentode.

![Graph](image)

Fig. 3.4 Typical Static Characteristics of F.e.t.

In the region of operation where the depletion layers do not completely penetrate the channel, the shape of the curves can
be calculated from a consideration of the potential function in the channel and its effect on the carrier's present there. This calculation is done completely in Shockley's paper (3.9) and results in expressions for the output conductance $g'_o$, the transconductance $g'_m$, and the I/V curves in terms of the device dimensions and applied gate potential. The portion of the I/V characteristic thus calculated is shown in Fig. 3.4 to be again similar to that of a pentode.

3.2.2 F.e.t. Equivalent Circuit

From the physical considerations of the f.e.t. operation, the equivalent circuit shown in Fig. 3.5(a) can be drawn. In it $g'_o$ and $g'_m$ are the quantities calculated by Shockley for the ideal f.e.t., $C'_{gs}$, $C'_{gd}$, and $C'_{sd}$ represent the effect of the distributed junction capacitance between the various terminals, and the generators $i_c$ and $i_g$ represent the noise calculated by the theory of van der Ziel (see sections 3.2.4 and 3.2.5). These theoretically predicted quantities together characterize the "ideal" f.e.t. The "ideal" f.e.t. cannot be fabricated, however, because it is impossible to extend the gate-channel junction for the full length of the channel material and small bulk resistances are left in series with both the source and drain. These are represented in Fig. 3.5(a) by $r_s$ and $r_d$ respectively.

$r_s$ and $r_d$ supply feedback to the ideal f.e.t. so that the parameters observed for the complete device differ from the theoretical ones. The observed parameters are those of the
simplified equivalent circuit shown in Fig. 3.5(b). The F.e.t. will normally be operated in the pinch-off region for linearity. In this case $g'_m \gg g'_o$ and the "simplified" parameters are related to the "intrinsic" parameters as shown in Eqn. 3.1. The
"simplified" and "intrinsc" parameters may differ as much as 20% at high $g'_m$.

\[(a) \quad g_m = \frac{g'_m}{1 + g'_m r_s} \quad \quad (b) \quad g_o = \frac{g'_o}{1 + g'_m r_s} \quad \quad \cdots (3.1)\]

\[(c) \quad C_{gs} = C'_{gs} (1 - g_m r_s) \quad (d) \quad C_{gd} = C'_{gd} (1 + g_m r_d)\]

It was shown in section 2.4 that the noise charge of the amplifier depends upon the unfedback capacitance of the first stage. Apparently then $C_{gs}$ and $C_{gd}$ must be found from $C'_{gs}$ and $C'_{gd}$ for use in calculations. This is not necessary, however, for the noise parameters of the simplified circuit have also been altered by feedback (see section 3.2.4). Therefore the parameters of each circuit are self-consistent.

3.2.3 Noise Model of F.e.t.

Noise in the f.e.t. is represented in Fig. 3.5(a) by the generators $i_g$ at the gate, $i_c$ in the channel and the thermal voltage generators $e_s$ and $e_d$ associated with the parasitic resistors $r_s$ and $r_d$. The location of these generators is roughly the same as those in the tube (see Fig. 3.1) but because the f.e.t. is a semiconducting device, the noise is produced differently. The complete theory of channel and gate noise has been presented in two recent papers by A. van der Ziel (3.11, 3.12). There have been only a few published reports of experimental
verification of the theory and they indicate that the theory does not give as accurate a prediction of the noise performance as does the theory for tubes discussed earlier. The reason for this is the difficulty in measuring the parasitic resistors $r_s$ and $r_d$ so that their contributions of thermal noise and feedback can be calculated. Brunke (3.13) showed however, that if $r_s$ and $r_d$ are calculated from the maximum $g_m$, the zero drain voltage $g_o$, and the observed noise at pinch-off, then the experimental and theoretical results can be made to agree throughout the non-pinchoff region. An outline of van der Ziel's theory of channel and gate noise in which noise sources are found in terms of the other parameters of the f.e.t. follows.

3.2.4 Channel Noise in F.e.t.

Channel noise represented by $i_c$ is the main source of white noise in the f.e.t. It results from modulation of the channel current by thermal fluctuations in the channel width, and is therefore nearly equal to the thermal noise of the channel conductance. The expression derived by van der Ziel for the spectral density of $i_c$ in the non-pinchoff region of operation is

$$w_c = 4kT g_{\text{max}} Q$$

where $g_{\text{max}}$ is the transconductance at pinch-off for the same source-gate bias, and $Q$ is a complicated but slowly varying function of the source and drain potentials. Although the theory was derived for non-pinchoff operation, van der Ziel claims that it holds approximately in the pinch-off region as long as the drain voltage is not too large.
When the gate is short circuited, the thermal noise of $r_s$ and $r_d$ adds to the channel noise to produce a total noise current $i_o$ in the external circuit. The spectral density of $i_o$ is given by Eqn. 3.2. In this expression $w_s$, $w_d$, and $w_c$ are the spectral densities of the external noise currents due to $r_s$, $r_d$, and the channel noise respectively.

\[ w_o = w_s + w_d + w_c \]  \hspace{1cm} (3.2)

If the capacitors are ignored and pinch-off operation assumed, these can be written by inspection of the equivalent circuit to be;

(a) $w_s = \frac{4kT r_s g_m^2}{(1 + g_m' r_s)^2}$

(b) $w_d = \frac{4kT r_d g_o^2}{(1 + g_m' r_s)^2}$

(c) $w_c = \frac{4kT g_m' Q}{(1 + g_m' r_s)^2}$ (note that $g_{\text{max}} = g_m'$ at pinch-off)

Substitution of these expressions in Eqn. 3.2 results in Eqn. 3.4 for the spectral density of the total noise current.

\[ w_o = \frac{4kT g_m^2}{(1 + g_m' r_s)^2} \left[ r_s + \frac{Q g_m'}{g_m^2} + \frac{r_d g_o^2}{g_m^2} \right] \] \hspace{1cm} (3.4)

With the output noise expressed in this way, it can be seen that the noise resistor $R_n$ is just the term in the brackets.
Because the output conductance $g'_o$ is much smaller than the transconductance $g'_m$, the contribution from $r_s$ is much smaller than the others and can be ignored. $R_n$ is then given by Eqn. 3.5(a).

\[
\begin{align*}
(a) \quad R_n & \ll r_s + \frac{Q}{g_m} \\
(b) \quad R_n & \ll r_s (1 - Q) + \frac{Q}{g_m} = \frac{Q}{g_m}
\end{align*}
\]

Substitution of $g'_m = g_m/(1 - g_m r_s)$ in Eqn. 3.5(a) results in (b), relating $R_n$ to the measurable transconductance. $Q$ is expected to be near unity, so the first term of 3.5(b) is negligible. Thus the noise resistor of the f.e.t. is inversely related to $g_m$ as was that of the tube. The experimental results verified this type of behaviour (see section 4.3) but the value of $Q$ was 30% higher than theoretically predicted. This discrepancy is not surprising in view of the many approximations made in accounting for the noise of the parasitic resistors.

### 3.2.5 Gate Noise in F.e.t.

In addition to the channel noise, there will be shot noise in any gate leakage current, and excess noise with characteristic $1/f$ frequency dependence in the channel current. As with grid leakage in tubes, the gate leakage current is made up of two independent components $I^+_g$ and $I^-_g$ each of which exhibit full shot noise. This noise is represented as $i_{gw}$, the white component
of gate noise for which \( w_g = 2Q_e \left( |I_g^+| + |I_g^-| \right) \). The excess noise is accounted for by the frequency-dependent part of the noise voltage generator \( e_n \) in the same way as it was with tubes, the constant \( K \) being determined experimentally.

The noise sources discussed until now have all had direct analogues in the tube and Nuvistor noise circuits. There is however, one type of noise exhibited by the f.e.t. which has no low frequency analogue in the tube. This is the correlated gate noise that results from capacitive feedthrough of the channel noise to the gate. It resembles the induced grid noise that occurs in tubes at high frequencies. The power spectrum of this noise when represented by a current generator at the gate, is \( w_g = H(z)4kTR_n \omega^2 C_{gs}^2 \) where \( H(z) \) is near \( 0.4^{(3.12)} \). \( \omega \) is partially correlated with the channel noise but it can be assumed to be uncorrelated with little error \( (3.12) \). With this assumption, the noise model of the f.e.t. conforms to the general model in which two uncorrelated noise sources represent all noise.

For the f.e.t. then; \( I_L = |I_g^+| + |I_g^-| + H(z)4kTR_n \omega^2 C_{gs}^2 /2Q_e \),

and \( R_n = r_s (1-Q) + Q/g_m \).

The \( \omega^2 \) dependence of the final term in \( I_L \) makes this source of noise important at high frequencies and high input impedance. In nucleonic amplifiers, however, \( I_L \) is shunted by the net input capacitance, \( C_t \); so this term contributes a noise voltage with a white spectral density of \( H(z)4kTR_n (C_{gs}/C_t)^2 \) to the input. The effect of the frequency-dependent gate noise is therefore to cause an apparent increase in \( R_n \) to
\[ R'_n = R_n \left( 1 + \frac{H(z)C_{gs}^2}{C_t^2} \right) \]. The increase is generally small because \( H(z) = 0.4 \), and \( C_{gs} \ll C_t \).

3.3 Noise Model for Junction Transistors

For junction transistors, \( I_L \) and \( R_n \) can be found by referring the white noise sources of the basic T noise model (see Fig. 3.6(a)) to the input of the common emitter equivalent circuit. In the basic noise model, \( i_e \) represents shot noise in the emitter current, \( e_b \) thermal noise in the base spreading resistor, and \( i_p \) partition noise in the collector current. The transistor also exhibits 1/f noise in the emitter current and the collector leakage current. This can be represented at the input by the flicker constant \( K \) as with the other devices. For transistors, \( K \) varies widely and must be determined experimentally; hence it will not be discussed further.

(a) Basic Junction Transistor Noise Model
For the calculation of equivalent input noise, the basic noise model is modified as shown in Fig. 3.6(b). All current sources have been changed to their equivalent voltage sources, the base has been taken as the input, and $C_e$ has been ignored on the assumption that $f \ll 1/2\pi r_e C_e$. From this circuit, the collector current resulting from the signal and from each noise source can be found as follows:

Let $e_1 = e_{x1} + e_{n1}$ and $e_2 = e_{x2} + e_{n2}$; where $e_{x1}$ and $e_{x2}$ are external voltage sources in the first and second loops respectively, and $e_{n1}$ and $e_{n2}$ are the respective internal noise sources. The loop equations for the circuit are then:

$$
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix} =
\begin{bmatrix}
(Z_s + r_b' + r_e) & (r_e) \\
(r_e - \alpha Z_c) & (r_e + Z_c(1-\alpha))
\end{bmatrix}
\begin{bmatrix}
i_b \\
i_c
\end{bmatrix} \ldots (3.6)
$$
Solving for $i_c$ by Cramer's rule gives,

$$I_c = \frac{e_2 (Z_s + r'_b + r_e) - e_1 (r_e - \alpha Z_c)}{(Z_s + r'_b) [r_e + Z_c (1-\alpha)] + r_e Z_c} \ldots (3.7)$$

If this current is assumed to result from a voltage $e_{x1}$ at the input, then $i_c$ can be expressed as,

(a) $i_c = Y'_f e_{x1}$ where $Y'_f = \frac{i_c}{e_{x1}}$ \hspace{1cm} (when $e_{x2} = e_{n1} = e_{n2} = 0$)

(b) $i_c = Y'_f \left[ e_1 - \frac{(Z_s + r'_b + r_e)}{(r_e - \alpha Z_c)} e_2 \right] \ldots (3.8)$

(Note that $Y'_f$ differs from the true forward transconductance $Y_f$ because $e_{x1}$ is in series with the source impedance). $Y'_f$ can be found from Eqn. 3.7 to be;

(a) $Y'_f = \frac{-(r_e - \alpha Z_c)}{(Z_s + r_e) [r_e + Z_c (1-\alpha)] + r_e Z_c}$

(b) $Y'_f \approx \frac{\alpha}{r_e (1 + Z_s/Z_c) + Z_s/\beta} \quad \text{(when } r_e \ll Z_c \text{)} \ldots (3.9)$

(c) $Y'_f \approx \frac{\alpha}{r_e} \quad \text{(when } \frac{Z_s}{\beta} \ll r_e \ll Z_c \text{ and } Z_s \ll Z_c \text{)}$

Taking now the signal and each noise source separately, the Eqns. 3.10 for the collector current can be found.
(a) signal \[ e_1 = e_{in}; e_2 = 0 \quad i_{cs} = Y_f' e_{in} \]

(b) emitter noise \[ e_1 = e_e; e_2 = e_e \quad i_{ce} = Y_f' e_e (1 + Z_s / Z_c) \]

(c) base noise \[ e_1 = e_b; e_2 = 0 \quad i_{cb} = Y_f' e_b \]

(d) partition noise \[ e_1 = 0; e_2 = -\frac{i}{p_c} Z_i \quad i_{ce} = -Y_f' e_b (Z_s + r_e + r'_b) / \]

\[ ... (3.10) \]

(In equations (b), (c) and (d), it was assumed that \( r_e, r_b \ll Z_c \).

From these expressions it is evident that the three sources of noise can be represented at the input of the amplifier by two generators as required. If one collects all terms that depend on \( Z_s \) for the shunt current generator and all terms that do not depend on \( Z_s \), for the series voltage generator, the respective spectral densities are:

\[ (a) \quad \nu_L = \frac{2Q_e I_c}{\alpha^2} \left[ 1 - \frac{|\alpha|^2}{\alpha_o} \right] + \frac{2kT r_e}{Z_c^2} \]

\[ ... (3.11) \]

\[ (b) \quad \nu_w = 4kT r_b' + 2kT r_e + \frac{2Q_e I_e}{\alpha^2} \left[ 1 - \frac{|\alpha|^2}{\alpha_o} \right] (r_e + r_b')^2 \]

Considerable simplification of these expressions can be effected if a particular transistor is considered. One of the best low-noise transistors at present is the 2N930 which exhibits a noise figure of less than 1db \((3.16)\). The collector impedance of the 2N930 is 8 pf shunted by approximately 20 megohm;
so the second term of 3.11(a) can be ignored. The transistor will be operated well below the cut-off frequency to limit the partition noise; hence \(|\alpha| \approx \alpha_0\). Incorporating these modifications in the expressions for \(w_L\) and \(w_e\) gives:

\[
\begin{align*}
(a) \quad w_L & = \frac{2Q_e I_e}{\beta} \\
(b) \quad w_w & = 4kT(r_b' + 2r_e) + \frac{2Q_e I_e}{\beta} (r_e + r_b')^2
\end{align*}
\]

...(3.12)

Since \(r_e \approx kT/Q_e I_e\), the second term of (b) can be expressed as \(2kT r_e/\beta + \text{terms in } r_b' \text{ of the same order of magnitude. This term is therefore much smaller than the first and can be ignored.}

For the junction transistor the constants \(I_L\) and \(R_n\) are then, \(I_L \approx I_e/\beta\) and \(R_n \approx r_b' + 2r_e = r_b' + 2kT/Q_e I_e\).

It must be stressed that these estimated parameters for the transistor are optimistic because all the terms that were ignored during the course of the calculation would have added to either the noise resistor or the apparent leakage current. In addition, the 1/f noise was ignored. It will be seen in section 3.6 that even with these helpful approximations, transistors theoretically give much poorer noise performance than any of the devices discussed so far.

3.4 Modern "Exotic" Low-Noise Amplifiers

3.4.1 The Tunnel Diode

The tunnel or Esaki diode has often been mentioned in the
literature as a useful low-noise amplifying device. In support of these claims, noise figures of 4 to 5db for high frequency communications amplifiers are cited\(3.17,3.18\). At high frequencies, the noise figures obtainable from tubes and transistors are at least this high so that in comparison the tunnel diode is a "low-noise" device.

However, it was shown in section 2.3 that for nucleonic amplifiers the minimum noise charge is obtained with the device that gives the lowest noise figure regardless of frequency. Since tubes\(3.19\), Nuvistors\(3.1\), f.e.t.'s\(3.20\), and junction transistors\(3.16\) all give noise figures below 1 db at low frequencies, they are vastly superior to tunnel diodes for nucleonic applications. Apart from this argument, the tunnel diode can be dismissed as being unsuitable for an even more basic reason. It is a negative resistance device and therefore provides power gain by "cancelling" the source and load resistance. With capacitive nucleonic sources the resistive portion of the terminations has no effect upon the system gain. The negative resistance of the tunnel diode cannot therefore, be used to amplify the signal.

3.4.2 Parametric Amplifiers and Masers

Two other types of low-noise amplifiers that have caused considerable interest of late are the parametric amplifier\(3.21\) and the Maser\(3.22\). Like the tunnel diode, both of these amplifiers are useful mainly at frequencies too high for the proper operation of tubes or transistors. If they are to be
useful for amplifying pulses, then, a modulator must be devised to convert the low frequency pulse signal to a modulated rf signal. The modulator must add less noise than a conventional amplifier would if the low noise capabilities of the parametric amplifier or Maser are to be realized.

One modulator that shows promise for such low-noise work combines modulation with parametric amplification through the use of two variable capacitors (varactors) in a bridge circuit. The bridge is pumped at rf and gives an output of constant amplitude until a signal unbalances the bridge and modulates the output. The rf output impedance is considerably lower than the input impedance seen by the low-frequency signal; so power gain as well as modulation is achieved. The results of noise measurements made on this circuit have recently been published. The measurements were made at frequencies below 100 cps and indicate that the circuit is relatively free from 1/f type noise. However, it has an equivalent noise resistor of at least 1K which compares unfavourably with the $R_n$'s found for the other devices.

It can be concluded then, that while the bridge modulator-amplifier shows great promise for low-noise amplification at low frequencies, it cannot compete yet with more conventional amplifiers at frequencies of interest for nucleonic amplifiers. The fact that modulation cannot be achieved without adding more noise than a conventional amplifier rules out the use of negative-resistance parametric amplifiers and Masers for this application too.
The possibility of using the non-linear capacitance of the p-n particle detector as the variable reactance in a parametric amplifier was considered early in this study. This idea had to be abandoned however, because the bias conditions for detection and amplification are not easily compatible. Linear energy resolution of detected particles requires a deep depletion layer in the detector hence a large reverse bias. On the other hand, parametric amplification requires a high slope of the capacitance versus voltage curve hence a small reverse bias.

3.5 Summary of Theoretical Results

With the brief arguments of the previous section, the modern "exotic" amplifiers have been shown to be unsuitable for nucleonic applications. The choice of an input device therefore reduces to one of the four "conventional" active devices discussed in section 3.1 through 3.3. The noise sources for the four have been expressed in the notation of the general noise model so that a comparison of their expected noise can now be made.

3.5.1 Choice of Tube, Nuvistor and F.e.t. Types

The parameters $I_L$ and $R_n$ for the four devices are summarized in Table 3.1.
Table 3.1 Theoretical Noise Parameters

<table>
<thead>
<tr>
<th>Tubes</th>
<th>$I_L$</th>
<th>$R_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuvistors</td>
<td>$</td>
<td>I^+</td>
</tr>
<tr>
<td>f.e.t.</td>
<td>$</td>
<td>I^+</td>
</tr>
<tr>
<td>Junc. Trans.</td>
<td>$\approx I_e/\beta$</td>
<td>$\approx r'_b + 2r_e$ or $r'_b + \frac{2kT}{Q_eI_e}$</td>
</tr>
</tbody>
</table>

The equations of Table 3.1 give another criterion (apart from reported noise figure) for choosing suitable active devices for low-noise nucleonic amplifiers. If it is assumed that the grid leakage currents are roughly the same for all high quality tubes, then the best tube is the one with the highest transconductance, and the lowest parasitic capacitance. The latter affects the noise mainly at low detector capacitance when $C_t \approx C_i$; so the most important single parameter is the transconductance. The tubes currently available that exhibit the highest transconductance are the WE 416B microwave triode with a $g_m$ of 50ma/v, and the Phillips E810F (7788) pentode with a $g_m$ of 60ma/v. Besides having slightly higher $g_m$, the E810F is also considerably cheaper than the 416B so it was chosen for this study. Other commonly used tubes such as the 417A triode, the E83F pentode and the 6AK5 pentode were not considered because their noise performance has already
been studied extensively and cannot be expected to compare with that of the newer E810F.

The criterion of highest $g_m$ also applies to the Nuvistors and f.e.t.'s. Hence the choice of the 7586 and the 2N2386 which gave the highest $g_m$ at the time the study was started. F.e.t.'s with higher transconductance are now available (e.g. 2N2498). The theoretical and experimental consideration given for the 2N2386 apply to these newer types also.

The 2N930 was chosen mainly on the strength of its low noise figure. However, the fact that it exhibits high current gain at very low current, a feature not often found in transistors, could have been used as an alternative criterion.

3.5.2 Comparison of Noise Performance

If in Table 3.1 the extreme values for $c$ and $Q$ are taken; $r_s$, $r_b'$, and the frequency-dependent gate term are ignored (assume $C_t > C_{gs}$); and typical values for the other parameters are taken from manufacturers' specifications, Table 3.2 results:

<table>
<thead>
<tr>
<th>Tubes (E810F)</th>
<th>$I_L$</th>
<th>$R_n$ (range)</th>
<th>$C_i$</th>
<th>Bias Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuvistor (7586)</td>
<td>10na</td>
<td>45 to 73Ω</td>
<td>25pf</td>
<td>$I_p = 30$ ma $g_m = 55$ ma/v</td>
</tr>
<tr>
<td>f.e.t. (2N2386)</td>
<td>1na</td>
<td>225 to 400Ω</td>
<td>8pf</td>
<td>$I_s = 8$ ma $g_m = 10$ ma/v</td>
</tr>
<tr>
<td>Junc. Trans. (2N930)</td>
<td>55na</td>
<td>5000 Ω</td>
<td>20pf</td>
<td>$I_e = 0.01$ ma $\beta = 180$</td>
</tr>
<tr>
<td></td>
<td>1µa</td>
<td>250 Ω</td>
<td>6pf</td>
<td>$I_e = 0.2$ ma $\beta = 200$</td>
</tr>
</tbody>
</table>

Table 3.2 Theoretical Comparison of Noise Parameters
This table gives an idea of the magnitude of noise parameters that can be expected. In spite of the uncertainty in $c$ and $Q$, the devices can be ranked in the order shown for expected minimum noise levels. For most detector capacitances, the E810F is best because it has the lowest $R_n$. However, as $C_d$ tends to zero, the Nuvistor may be superior because of its low parasitic capacitance.

The f.e.t. might be expected to out-perform the Nuvistor because its product, $I_L R_n$ is lower (see Eqn. 2.16 for $N_{\text{min}}$). However, in practical applications $I_L$ has bias resistor and detector noise added (c.f. section 2.2) so the f.e.t. leakage current is always swamped and the net shunt current generator is roughly as large as for the Nuvistor. The two are therefore comparable theoretically except for the lower capacitance of the Nuvistor.

The two sets of parameters for the 2N930, show that junction transistors are vastly inferior to any of the other devices except when the detector leakage current is large. By equating the product $R_n (I_L + I_d)$ for the 2N930 and the 2N2386, it can be shown that the two devices give the same noise charge when $I_d = 0.05/\beta R_n \leq 1\mu A$ ($R_n$ is for the f.e.t.). Since most detectors have leakage currents much smaller than $1\mu A$, the junction transistor was rejected as a suitable device for low-noise nucleonic amplifiers, and excluded from further study.

A more detailed comparison of the devices is given at the end of Chapter 4. By that time the experimentally determined $I_L$'s and $R_n$'s have been discussed so the uncertainty in the various parameters is reduced.
4. MEASUREMENT OF NOISE PARAMETERS

From theoretical considerations and a knowledge of typical \( g_m \)'s, \( \beta \)'s, and \( I_g \)'s, it was possible to estimate the relative noise performance of tubes, Nuvistors, f.e.t.'s and junction transistors. However, the parameters found in this way were not accurate enough for use in designing amplifiers, and had to be supplemented by the measurements described in this chapter.

The measuring techniques were not original, but they are described in detail because the descriptions given in the literature do not explicitly mention all the precautions that must be taken. The measurements are described in detail for tubes only, but the same methods were used for Nuvistors and f.e.t.'s with the modifications noted.

4.1 Tube and Nuvistor Parameters

4.1.1 Grid Current Measurement Techniques

\( I^+ \) and \( I^- \) were estimated for the E810F and 7586 from the net d.c. grid leakage current \( I_g \) measured with the circuit shown in Fig. 4.1.

The measuring sequence starts with the key in the grid circuit open and the grid at a potential \( V_g = I_g R_g \). Under this condition, the potentiometer \( P_2 \) is adjusted until no current flows in the galvanometer. The key is then closed and, since the resistance of \( P_1 \) is much less than \( R_g \), the grid potential...
Fig. 4.1 Grid Current Test Circuit

changes to $V_t$. This change in grid potential is amplified by the tube and detected by the galvanometer. Adjustment of $P_1$ until the galvanometer reads zero again makes $V_t$ equal to $V_g$. The grid voltage can then be measured across the relatively low impedance of $P_1$ and the grid leakage current calculated to be $I_g = V_g/R_g$.

4.1.2 Grid Current Measurement Results

Values of $I_g$ were obtained in this way throughout the region of useful plate conditions by varying $V_{bb}$ and $R_k$, then plotted against $V_g$. Typical curves for the E810F are shown in Fig. 4.2. From these curves $I_g^+$ and $I_g^-$ can be estimated because of the known exponential behaviour of $I_g$ at low current (4.1).
For convenience in choosing bias points, the grid current data was then plotted as curves of constant $I_g$ and constant $I_L$ in the plane of plate characteristics. (Note: $I_g = I_g^+ + I_g^-$; $I_L = |I_g^+| + |I_g^-|$.)

Fig. 4.3 gives the curves for the E810F and Fig. 4.4 for the 7586 Nuvistor.

4.1.3 $R_n$ and K Measurement Technique

The parameters $R_n$ and $K$ representing shot and flicker noise were found by the resistive substitution method using the system shown in Fig. 4.5. This system consists of the tube being tested, which acts as a preamplifier, a high-gain post-amplifier which defines the pass band and a Hewlett-Packard
Fig. 4.3 Grid Leakage Current for the B810F Tube

\[ I_D = |I_g^+ + I_g^-| \]

\[ I_g = I_g^+ + I_g^- \]

Fig. 4.4 Grid Leakage Current for the 7586 Nuvistor

\[ I_D = |I_g^+ + I_g^-| \]

\[ I_g = I_g^+ + I_g^- \]
400D VTVM which serves as a detector. The noise passing to the VTVM is, for all practical purposes, entirely due to the shot and flicker noise of the tube plus the thermal noise of the test resistor $R_t$. This is ensured by making the grid resistor so small that the grid current noise is negligible, making $R_L$ so large that the thermal noise current in the load is negligible, and by using a low-noise post-amplifier.

![Resistive Substitution Test System](image)

Fig. 4.5 Resistive Substitution Test System

The experimental procedure consists of first short-circuiting the input grid and observing the rms noise voltage $\overline{e_1}$ on the VTVM, then inserting the test resistor $R_t$ and observing the new noise voltage $\overline{e_2}$. $R_t$ generates a noise current with spectral density $4kT/R_t$ in parallel with the input impedance of the test circuit; so the additional noise added by $R_t$ has the spectral density $4kT|Z_{in}|^2/R_t$. In many resistive substitution measurements, it is assumed that $R_t$ delivers full thermal noise to the input of the tube. However, it was found
in these measurements that the shunting effect of \( Z_{in} \) was appreciable even at moderate frequencies, because of the large Miller capacitance of the triodes being tested. When this is taken into account, the noise voltages \( e_1^2 \) and \( e_2^2 \) measured are given by Eqns. 4.1 and 4.2.

\[
\frac{e_1^2}{e_0^2} = \int_0^\infty (4kTR_n + \frac{K}{f}) |A(f)|^2 \, df \quad \text{(where } A(f) \text{ is the gain function)} \tag{4.1}
\]

\[
\frac{e_2^2}{e_0^2} = \int_0^\infty (4kTR_n + \frac{4kTR_t}{(1+\omega^2T_{in}^2)} + \frac{K}{f}) |A(f)|^2 \, df
\]

(\text{where } T_{in} = R_{t}C_{in}) \tag{4.2}

The measurements were all made with single, equal integration and differentiation time constants, \( \tau \). The integrals for this case have been evaluated previously (see Chapter 2.2) so that \( e_1^2 \) and \( e_2^2 \) can be written as;

\[
\frac{e_1^2}{e_0^2} = \left[ \frac{kTR_n}{2\tau} + \frac{K}{2} \right] A \quad \text{(where } A \text{ is the nominal mid-band gain)} \tag{4.3}
\]

\[
\frac{e_2^2}{e_0^2} = \left[ \frac{kTR_n}{2\tau} + \frac{kTR_t}{2\tau(1+x)^2} + \frac{K}{2} \right] A \text{(where } x = T_{in}/\tau) \tag{4.4}
\]

By taking the ratio of these equations \( R_n \) can be found to be:
(a) \[ R_n = \frac{R_t y}{(1-y)(1+x)^2} - \frac{K\tau}{kT} \quad \text{(where } y = \frac{e_1}{e_2}) \]

(b) \[ R_n = \frac{R'_n}{(1+x)^2} - \frac{K\tau}{kT} \]

(c) \[ R_n = R'_n - \frac{K\tau}{kT} \quad \cdots (4.5) \]

In Eqn. 4.5(b) \( R'_n \) is the apparent noise resistance without correction for flicker or input time constant, while in Eqn. 4.5(c) \( R''_n \) is the combined shot and flicker noise resistor. Taking the difference between two values of \( R''_n \) measured at different frequencies gives Eqn. 4.6 for \( K \).

\[ K = \frac{\Delta R''_n kT}{\Delta \tau} \quad \cdots (4.6) \]

With the results expressed in this way, the calculation of \( R_n \) and \( K \) proceeds as follows. From the voltage ratios, \( R'_n \) is found and corrected for the input time constant. Values of \( R''_n \) measured at different time constants are then used in Eqn. 4.6 to find \( K \). Finally \( R''_n \) is corrected for the flicker contribution to give \( R_n \).

The measurement is independent of the gain and absolute calibration of the system, making the resistive substitution method superior to any direct measurement method.

4.1.4 \( R_n \) and \( K \) Measurement Results

The measurements were made throughout the useful region of operation for two E810F tubes and for two 7586 Nuvistors as well as at selected biases for 8 other E810F's. Two test resistors were used as
a check, and the measurements were made at three values of
\( \tau \): 0.08, 0.8, and 8 \( \mu \)sec. From these measurements, \( R_n \) and \( K \)
were computed as in the following calculation for one of the E810F
tubes.

Table 4.1 below shows the measured data. The first
column gives the plate supply voltage and plate current which
locates the data in the plate characteristics. \( \bar{e}_1 \) and \( \bar{e}_2 \) are
the rms noise voltages in arbitrary units measured with the VTVM,
the second value of \( \bar{e}_2 \) being from the second resistor substituted.
In this case, the two values used were 100 and 200 ohms.
\( \bar{e}_0 \) is the noise contributed by the post amplifier with the test
circuit replaced by its approximate output impedance. In
most of the measurements, this contribution to the noise was
small, but when the plate current in the test circuit was low,
resulting in a low first-stage gain, a correction had to be
made for this factor too.

<table>
<thead>
<tr>
<th>( V_{bb} )</th>
<th>( I_b )</th>
<th>( R_t )</th>
<th>( \bar{e}_1 )</th>
<th>( \bar{e}_2 )</th>
<th>( \bar{e}_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>200v</td>
<td>20ma</td>
<td>100</td>
<td>1.50</td>
<td>2.18</td>
<td>.30</td>
</tr>
<tr>
<td>200v</td>
<td>20ma</td>
<td>200</td>
<td>1.50</td>
<td>2.56</td>
<td>.30</td>
</tr>
</tbody>
</table>

Table 4.1 Data for E810F \( R_n \) Calculation

\( \tau = .08 \mu \)sec \( \quad R_L = 4k \)

In Table 4.2, the calculations are performed in sequence.
First the post-amplifier noise is subtracted to give the true values of $e_1^2$ and $e_2^2$ which are then used to determine the two values of $R'_n$. The effect of doubling the input time constant is evidenced by the larger value of $R'_n$ obtained with the 200 ohm test resistor.

The difference in the two values of $R'_n$ can be used to estimate the input time constant correction since $R'_n$ must be the same in both cases. That is,

$$\frac{R'_{n1}}{(1+x_1)^2} = \frac{R'_{n2}}{(1+x_2)^2} \quad \ldots(4.7)$$

Since $x_2 = 2x_1$, $x_1$ can be found thus;

$$x_1 = \frac{1}{2} \frac{\sqrt{r} - 1}{\sqrt{r}} \quad \text{where } r = \frac{R'_{n1}}{R'_{n2}} \quad \ldots(4.8)$$

$x$ can also be determined from a direct measurement of the input capacitance of the test circuit and a calculation of $T_{in}$ and $x$ for the particular test resistor and time constant being used. In most cases both methods of determining $x$ were used with consistent results. In the final column of Table 4.2, $R''_n$, the combined shot and flicker noise resistor is given. The correction factors $1/(1+x)^2$ used were obtained from many measurements of $R'_n$. Hence they do not make the two values of $R''_n$ exactly equal as they would if they were obtained from this sample alone.
After $R''_n$ was calculated in this way for all operating conditions, $K$ was estimated from the frequency dependence of the results using Eqn. 4.6. The results of this calculation were roughly the same for both tube types and for the Nuvistors, giving $K \leq 10^{-13}$, as long as the plate current and power dissipation were well below the rated maximum.

This is in accord with the theory. A more detailed analysis of the dependence of flicker noise upon plate current and voltage was not attempted because in the cases of interest to this study the flicker noise formed only a small part of the total noise and could be adequately represented by the constant $10^{-13}$.

Using this value of $K$, $R''_n$ was adjusted to give the true equivalent noise resistor $R'_n$. The correction was only 1.5 ohms at the time constant of 0.08 sec., so that the uncertainty in $K$ does not affect appreciably the accuracy of $R'_n$.

In Figs. 4.6, and 4.7, the values of $R_n$ for the best E810F, and 7586 Nuvistor respectively are plotted as curves of constant $R'_n$ in the plate characteristic plane, as were the grid leakage currents. The variation in $R_n$ was found to be less than 20\% for the E810F but almost 75\% for the two Nuvistors tested.
Fig. 4.6 Equivalent Noise Resistor for the E810F Tube

Fig. 4.7 Equivalent Noise Resistor for the 7586 Nuvistor
This could indicate that the spread in parameters is greater for Nuvistors than for tubes but with only two samples to study, such a conclusion is rather premature. To see if the shot noise measured agreed with the predicted behaviour, the constant relating $R_n$ to $g_m$ was determined for the E810F. It was $3.5 \pm 20\%$ for plate currents between 5ma and 20ma, which is within the range predicted by theory.

4.2 Measurement of F.e.t. Parameters

Extensive measurements of the f.e.t. noise and admittance parameters were made in order to become acquainted with this relatively new device, and to obtain information for use in the design of f.e.t. circuits. The admittance parameter measurements are of only secondary importance to the study of f.e.t. noise performance, so they are reported in Appendix II.

4.2.1 F.e.t. Static Characteristics

The static characteristics of the two f.e.t.'s available were measured and are shown in Fig. 4.8. To check that the f.e.t. is symmetric, as claimed in the theory, the nominal source and drain were interchanged with no change in the static characteristics. Most of the other parameters were spot checked with the f.e.t. inverted with the same result, indicating that the source and drain are indistinguishable for this type of f.e.t. The large difference in the zero bias current probably results from a difference in channel width and indicates that good control of f.e.t. parameters has not yet been achieved. For low-noise
Fig. 4.8(a) Static Characteristics of F.e.t. #1

Fig. 4.8(b) Static Characteristics of F.e.t. #2
amplifier circuits this is not crucial because the input device, be it a tube or an f.e.t., is usually selected for minimum noise, but for any circuit where selection of components is not desirable, this limits the design considerably.

4.2.2 F.e.t. Gate Leakage Measurements

To measure the gate leakage current, the f.e.t. was connected as shown in Fig. 4.9. This circuit operates like the grid leakage current test circuit Fig. 4.1.

The leakage current \( I_g \) was measured throughout the useful region of operation with the results shown in Figs. 4.10(a) and (b). For the bias conditions shown, \( I_g \) is equal to the reverse leakage because, unlike tubes, p-n junctions exhibit no significant forward current until a slight forward bias is applied. For this reason the separate components of gate current need not be found unless operation with the gate forward biased is
Fig. 4.10(a) Gate Leakage Current for F.e.t. #1

Fig. 4.10(b) Gate Leakage Current for F.e.t. #2
anticipated. In any case, the leakage current is so small that its noise will generally be much smaller than the detector leakage noise and further calculations of the positive and negative components result in no useful information.

4.2.3 F.e.t. Noise Resistor Measurements

The noise resistor and flicker constant for the f.e.t. were measured in the same way for the f.e.t. as for tubes. It is, however, necessary now to modify the theory to include the frequency-dependent gate noise. This noise does not affect the measurement of gate leakage current and need only be included in the resistive substitution measurement of the channel and flicker noise.

The short circuit output $e_1$ remains the same as for the tube, but to $e_2$, the output with the test resistor in place, a contribution $e_g$ is added. $e_g$ is given by:

$$
e^2_g = \frac{4k TH(z) R_n C_{gs}^2 R_t^2}{2\pi} \int_0^\infty \frac{\tau^2 \omega^4 d\omega}{(1 + \omega^2 \tau^2)^2 (1 + \omega^2 x^2 \tau^2)}$$

(4.9)

where $x = T_{in}/\tau$ and the other variables have their usual meaning.

The integral is evaluated in Appendix I as I4. Taking this result and substituting it in Eqn. 4.9 gives Eqn. 4.10 for the noise due to the frequency-dependent gate noise.

$$
e^2_g = \frac{k TH(z) R_n B_t^2 C_{gs}^2 (2+x)}{2x \tau^2 (1+x)^2}$$

(4.10)

To compare this with the contribution for channel noise,
the ratio $e_n^2/e_g^2$ is taken. The contribution from channel noise is $kTR_n/2\tau(1+x)^2$, so that the ratio is:

$$\frac{e_n^2}{e_g^2} = \frac{2x\tau^2}{(2+x)H(z)R_t^2C_s^2}$$

...(4.11)

$$\approx \frac{T_{in}\tau}{H(z)R_t^2C_s^2} \quad \text{if } x \ll 2$$

Substituting typical values of $H(z) = 0.4$, $R_t = 2K$, $C_s = 8\text{pf}$, $T_{in} = C_{in}R_t = 2K \times 40\text{pf}$, result in $e_n^2/e_g^2 \approx 10^3\tau$, where $\tau$ is in $\mu\text{secs}$.

As long as the measurements are made at time constants longer than (say) $0.1 \mu\text{sec.}$, then the frequency-dependent gate noise will not affect the determination of the channel and flicker noise.

The calculation in section 3.2.5 showed that the frequency-dependent gate noise is not an important component of the noise in the amplifiers built. Therefore, no attempt was made to confirm the theoretical expression for it by extending the resistive substitution measurements to shorter time constants where $e_g^2$ would be large enough to detect.

The circuit of Fig. 4.11, which is equivalent to the test circuit used for tubes (see Fig. 4.5) was used for the resistive substitution measurements on the f.e.t. The data obtained at time constants of 0.32, 1.6 and 8.0 $\mu\text{secs}$ with the two test resistors, 1K and 2K, was reduced to $R_n$ and $K$ by calculations like those described in Section 4.1 for tubes. In Figs. 4.12(a) and (b)
Fig. 4.11 F.e.t. Resistive Substitution Test Circuit

curves of constant \( R_n \) are plotted in the drain characteristic plane.

<table>
<thead>
<tr>
<th>f.e.t. #1 ( V_{ds} = 3v )</th>
<th>f.e.t. #2 ( V_{ds} = 2v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_d (\text{ma}) )</td>
<td>( R_n (\text{K}) )</td>
</tr>
<tr>
<td>.2</td>
<td>3.3</td>
</tr>
<tr>
<td>.4</td>
<td>1.45</td>
</tr>
<tr>
<td>.6</td>
<td>1.15</td>
</tr>
<tr>
<td>.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 4.3 F.e.t. Noise Resistors

In Table 4.3, the product \( R_n g_m \) is formed for different drain currents in f.e.t. #1 and #2. \( Q \) is roughly 50% and 30% higher than predicted for f.e.t. #1 and #2 respectively. As was mentioned in the theory, this is probably a result of the approximations made in accounting for the effect of the parasitic channel resistors.
Fig. 4.12(a) Equivalent Noise Resistor for F.e.t. #1

Fig. 4.12(b) Equivalent Noise Resistor for F.e.t. #2
The 25% variation in $Q$ with current is not excessive because $Q$ is not expected to be a constant but is rather a slowly varying function of the bias voltages.

The flicker noise was found to be roughly 10 times that observed for tubes so that $K$ could be determined more accurately. In f.e.t. #1, $K = 8 \times 10^{-13} \pm 10\%$, for all drain current above 0.2 ma; in f.e.t. #2, $K = 6 \times 10^{-13} \pm 10\%$ for all drain currents above 0.4 ma. Below these currents, $R_n$ becomes large enough to make the determination of $K$ difficult, but apparently $K$ increases at very low currents.

4.3 Summary of Experimental Noise Parameters

At the end of chapter 3, a brief comparison of the expected noise performance of tubes, Nuvistors and f.e.t.'s was made using the theoretically determined noise parameters. Because of the uncertainty in these parameters, the comparison was brief and indicated only that the E810F was expected to be the best device. Now that the noise parameters for the devices are known for all possible operating conditions, a more detailed prediction of the noise performance will be made.

4.3.1 Choice of Bias Points

The minimum noise charge for a nucleonic system occurs when the product $P = R_n I_e = R_n (I_L + I_{ex})$ is a minimum. $I_{ex}$ is the sum of external leakage currents not affected by changes in bias point. The effect of changing the bias point can be seen from a consideration of Fig. 4.13.
In this diagram, some $R_n$ and $I_L$ curves for the E810F are superimposed, and $I_{ex}$ is included by increasing the value of the $I_L$ curves as shown. At the points A and B where the $R_n = 80$ curve intersects the $I_L = 2$ curve, $P$ is equal to $80(I_{ex} + 2)$. If the bias point is moved to the right of B or to the left of A along the $R_n = 80$ curve, the leakage current increases; hence $P$ increases. Between A and B however, the leakage current and the product decrease until a minimum is reached at the point $X_1$, where the two curves are tangent. A complementary argument following the constant leakage current curves gives the point $X''$ as another point of minimum $P$. The locus of all points of equal slope therefore form the curve of optimum bias points for any $I_{ex}$. As $I_{ex}$ increases, $I_L$ becomes less important in the product so the bias point moves up the curve to lower $R_n$. 

Fig. 4.13 Bias Point For Minimum Noise
Locating the optimum bias point exactly from grid current measurements is difficult because of the uncertainty in separating $I_g$ into its two components. Therefore, the amplifiers built had adjustable bias circuits (see section 5.2.1). The argument above does show however, that the bias condition for minimum noise at fixed $I_e$ is neither that where the grid is biased sufficiently negative to reduce $I_g$ to its positive component, \((4.2)\) nor that where the grid is floating \((4.3)\). By allowing the grid to float, grid bias resistor noise is eliminated and $I_{ex}$ is reduced. This could conceivably result in a decrease in $P$, even though the tube is not biased on the curve of optimum bias for fixed $I_{ex}$. However, two factors combine to make this unlikely. Firstly, the detector leakage current is generally much larger than that attributed to the bias resistors, so that the fractional improvement in $I_{ex}$ is small. Secondly, in moving from point $X$ to $A$, $I_L$ increases rapidly because of the exponential increase in $I_g$ and opposes the improvement in $I_{ex}$.

In Table 4.4, $R_n$ and $I_L$ values for different points along the optimum bias line are collected and the product $P = R_n I_e$ is evaluated for external leakage currents of 0, 1, and 10na.

<table>
<thead>
<tr>
<th>$I_b$ (ma)</th>
<th>$R_n$ (K)</th>
<th>$I_L$ (na)</th>
<th>$P_0$</th>
<th>$P_1$</th>
<th>$P_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>.22</td>
<td>.4</td>
<td>.088</td>
<td>.306</td>
<td>2.28</td>
</tr>
<tr>
<td>10</td>
<td>.125</td>
<td>1.2</td>
<td>.15</td>
<td>.275</td>
<td>1.39</td>
</tr>
<tr>
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<tr>
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<td>.070</td>
<td>3.0</td>
<td>.21</td>
<td>.280</td>
<td>.91</td>
</tr>
</tbody>
</table>

Table 4.4 Collected Noise Data for Optimum Bias Points for the E810F Tube
It can be seen that the addition of an external leakage current of only 10na moves the optimum bias point to a plate current of 20 ma. Since most solid state detectors have at least this much leakage, the tube will generally be biased at as high a current as possible for minimum system noise. At plate currents above 20 ma, the transconductance of the E810F increases slowly, so $R_n$ does not continue to decrease. Also the flicker noise increases as the plate dissipation increases. The net result is that the best bias point for the E810F is at a plate current of 20 ma and plate voltage of roughly 110 volts.

Similar considerations for finding minimum $P$ apply to the Nuvistor because of the similarity in the $R_n$ and $I_L$ curves. Collected bias point data for the Nuvistor are shown in Table 4.4. It shows roughly the same behaviour as that for the E810F and indicates that the optimum bias point for most amplifiers is at the maximum current of 10 ma.

<table>
<thead>
<tr>
<th>$I_b$(ma)</th>
<th>$R_n$</th>
<th>$I_L$</th>
<th>$P_0$</th>
<th>$P_1$</th>
<th>$P_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.000</td>
<td>.8</td>
<td>.8</td>
<td>1.8</td>
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</tr>
<tr>
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<td>.370</td>
<td>6.0</td>
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<td>2.59</td>
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<tr>
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<td>9.0</td>
<td>2.88</td>
<td>3.20</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 4.5 Collected Noise Data for Optimum Bias Points for the 7586 Nuvistor

The f.e.t. has leakage currents of fractions of 1 na
so that the noise of $I_L$ will always be swamped by detector noise. Therefore, the f.e.t. is biased always at maximum current for minimum $R_n$. For f.e.t. #2, then, a bias current of 2.3 ma at a drain voltage of 6 volts was chosen, giving $R_n = 600$ ohms.

4.3.2 Calculated Curves of Minimum Noise Charge

The data in Tables 4.3 and 4.4 and the parasitic capacitances obtained from the manufacturers' specifications were used in Eqn. 2.16 to find $N_{min}$, and in Eqn. 2.15 to find $\tau_{min}$. Curves of $N_{min}$ vs. $\tau_{min}$ are shown in Fig. 4.14. The curves were extended to short time-constants by using the data for the highest current in Eqn. 2.14 for $N$. The extended curves show the noise charge that results when the time constant is made shorter than $\tau_{min}$ (for high count rates or other reasons). The f.e.t. curves were calculated entirely from Eqn. 2.14, because the bias point is assumed fixed. Data for the 2N930 curves was taken from the manufacturers' specifications and the calculation made ignoring $r'_b$, the flicker noise, and high frequency effects in $i_p$. The $N$'s predicted for the 2N930 are therefore optimistic, particularly at short time constants.

In addition to the curves shown for $C_d = 20pf$, similar curves were calculated for $C_d = 0pf$, 60pf, and 100pf. From each curve the absolute minimum $N$ was found and the curves of $N_{min}$ vs $C_d$ plotted (see Fig. 4.15).

The information displayed in Figures 4.14 and 4.15 is self-explanatory and shows that the tentative conclusions of section 3.5 were substantially correct. The E810F and Nuvistor exhibited parameters within the predicted range. As expected,
Fig. 4.14(a) $N_{\text{min}}$ vs. $\tau_{\text{min}}$ for 1 nA Detector Leakage Current

Fig. 4.14(b) $N_{\text{min}}$ vs. $\tau_{\text{min}}$ for 10 nA Detector Leakage Current
Fig. 4.15(a) $N_{\text{min}}$ vs. $C_d$ for 1 na Detector Leakage Current

Fig. 4.15(b) $N_{\text{min}}$ vs. $C_d$ for 10 na Detector Leakage Current
the noise of the E810F was less at all but the lowest detector capacitance. The f.e.t. on the other hand, exhibited considerably more flicker and channel noise than expected, and as a result is inferior to the Nuvistor even with no external noise. It is, however, still much superior to the junction transistor. Therefore, for applications where the ruggedness and low power consumption of solid-state devices are required the f.e.t. emerges as the best device.
5. PREAMPLIFIER DESIGN AND PERFORMANCE

In Chapter 2 it was assumed that amplifier noise can be reduced to the noise of the input stage. On the basis of this assumption, the equivalent noise charge of nucleonic amplifiers was then discussed in Chapters 3 and 4 in terms of predicted and measured noise parameters of various active devices. To check the validity of the noise charges predicted in this way, four preamplifiers were built and their noise charge measured. Their design and performance is described in this chapter.

5.1 Design and Construction of Preamplifiers

Of the four preamplifiers built, three were of the "charge sensitive" type using respectively E810F (7788) tubes, 7586 Nuvistors, and 2N2386 field effect transistors for the input stages. The fourth, also using E810F's, was built in the "voltage sensitive" configuration for comparison with the charge sensitive E810F stage. Because of the superior noise performance of the E810F, time was taken to make the tube circuits into practical nucleonic preamplifiers with large open-loop gains. The Nuvistor and f.e.t. stages, on the other hand, were built primarily to test the noise theory of the thesis and do not have sufficient gain for use in nucleonic systems. The good performance of the modified tube amplifiers indicated that the other amplifiers could be similarly modified for use as practical nucleonic preamplifiers. The modifications were not carried out because of the time and component limitations, but they are discussed in conjunction with the circuit descriptions.
The preamplifiers all use "cascode", or grounded-cathode-grounded-grid input stages. This circuit is commonly used for low-noise amplifiers because it gives the lowest noise of any two stage configuration (5.1). All the input stages were biased at high current levels in accordance with the discussion of section 4.3.1. The noise charge was measured in the absence of detector noise, and in this case lower bias currents would have improved the performance. However, any improvement obtained in this way is illusory because the low current amplifiers give higher system noise than the high current one when the inevitable detector noise is added. The preamplifiers are of conventional design; so they are discussed only briefly and expressions for their gain are derived in Appendix III.

Note: all resistors in kOhms, all capacitors in \( \mu \)F, unless noted.

Fig. 5.1 E810F Charge-Sensitive Preamplifier
5.1.1 E810F Charge Sensitive Preamplifier

Fig. 5.1 shows the E810F "charge sensitive" preamplifier circuit. The input cascode of two triode-connected E810F's (T₁ and T₂) has its load "bootstrapped" by the E810F pentode stage (T₃) to increase the ac load seen by the cascode without increasing the dc load. The circuit gain is therefore increased without the need for excessively high plate supply voltages and load resistors. Isolation of the dynamic cascode load and a low output impedance are provided by the output cathode follower (T₄). Feedback through $C_{fb}$ renders the circuit "charge sensitive", while feedback through the input grid resistor $R_{fb}$ reduces the tendency of the input to drift at high count rates. The potentiometers $P₁$ and $P₂$ in the bias circuit are used to adjust the dc conditions of the input tube to the optimum value. $P₁$ has sufficient range for all possible plate voltages, but $P₂$ gives only a fine adjustment of the plate current defined by the choice of $R₁$, $R₂$, and $Rₖ$. With the values of these resistors shown, the nominal cascode current is 20mA.

The cascode stage without "bootstrapping" has a voltage gain $A_c$ as given by Eqn. 5.1 derived in Appendix III. In this expression $μ₁$ and $r_{p₁}$ refer to $T₁$, $μ₂$ and $r_{p₂}$ to $T₂$.

$$A_c = \frac{μ₁(μ₂ + 1)R_L}{R_L + r_{p₂} + r_{p₁}(μ₂ + 1)} \leq g_mR_L \text{ for large } μ₂ \quad ...(5.1)$$

With a cascode current of 20mA; $r_{p₁} = r_{p₂} = 1.3K$ and $μ₁ = μ₂ = 50$, so that $A_c = 208$. When the load is divided and bootstrapped, the voltage gain is given by Eqn. 5.2 for which the maximum occurs when $R₁' = R₂$ (see Appendix III). Choosing $R₁ = R₂$.
$A_{cb} = \frac{\mu_1 (\mu_2 + 1) R_1' (1 + g_{m3} R_2)}{R_1 + R_2 + r_{p2} + r_{p1} (1 + \mu_2) + R_1' g_{m3} R_2}$

where $R_1' = \frac{R_1 R_3}{R_1 + R_3}$

...(5.2)

approximates this condition fairly well, since $R_3$ is 20K and reduces $R_1$ by only a small amount. Substituting the values for the various tube parameters and resistors gives $A_{cb} = 1750$. The measured values for $A_c$ and $A_{cb}$ were in good agreement with the predicted values, being 200 and 1600 respectively. The dynamic cascode load also increases the gain of the first tube above the value of unity it normally has in a cascode circuit so that the Miller capacitance of $T_1$ is added to the input capacitance. The analysis of the amplifier with two feedback loops in chapter 2 showed, however, that this does not affect the noise performance.

The charge sensitivity and gain stability of the circuit can be calculated from Eqns. 1.7 and 1.11 of section 1.4. Eqn. 1.7 shows that the output pulse amplitude for fixed input charge differs by only 5% for detector capacitances between 0pf and 150pf, indicating good charge sensitivity. From Eqn. 1.11, the closed-loop gain stability can be found to be 100 times better than the open-loop gain stability when $C_t$ is 30pf and 10 times better when $C_t$ is 300pf. The effect of input capacitance drifts is reduced by the same amount. These figures indicate that the stability of the amplifier is adequate for long-term nucleonic experiments. The stability can be increased further, if necessary, by increasing $C_{fb}$. However, this decreases the output signal and might necessitate the use of a lower-noise post amplifier than was required with $C_{fb} = 2pf$.

With no special precautions taken to improve the high-frequency
response of the circuit, rise times of 42 ns and 10 ns for 2 pf and 5 pf feedback capacitors respectively are measured. Attempts to reduce the rise time further by increasing the amount of feedback resulted in large overshoot and eventual oscillation, so that for the basic circuit, the minimum rise time can be taken to be 10 ns. For all but the very fastest coincidence work then, the circuit gives an adequate rise time.

5.1.2 E810F Voltage Sensitive Preamplifier

For a low-noise input stage, the voltage sensitive E810F circuit shown in Fig. 5.2 employs a modified version of the input cascode used in the charge sensitive preamplifier. This is followed by a grounded-cathode pentode stage to boost the open-loop gain, and an output cathode follower to provide a low output impedance. The cascode has a gain of 80, and the pentode stage a gain of 50 so that the overall open-loop voltage gain is 4000.

Feedback is applied from the output through $R_{fb}$ and the phase lead capacitor to the input cathode, thus forming the conventional voltage sensitive preamplifier (5.2). The closed-loop voltage gain, and rise time are determined by the amount of feedback, hence by the size of $R_{fb}$. For $R_{fb} = 1K$ the closed-loop gain was 130 and the rise time 40 ns. (Note that the effective cathode resistor is 7.5 ohms formed from 10 ohms shunted by $1/g_{m1}$.) Decreasing $R_{fb}$ to 220 ohms reduced the gain to 30 and the rise time to 12 ns, but a further decrease resulted in the circuit oscillating at high frequencies. For timing purposes then, the voltage sensitive and charge sensitive amplifiers are equivalent. However, the long-term gain stability of the voltage sensitive amplifier cannot be expected to be as good since th
Fig. 5.2 E810F Voltage Sensitive Preamplifier

The feedback compensates only for drifts in open loop gain and not for changes in capacitance at the input. The amount of improvement in gain stability is 30 times and 75 times for 1k and 200 ohm feedback resistors respectively.

5.1.3 Nuvistor Test Amplifier

The Nuvistor test amplifier consisted simply of a cascode of two Nuvistors and an output cathode follower (see Fig. 5.3). As it
stands the Nuvistor cascode has a voltage gain of 90, so that the 2pf feedback capacitor does not make the circuit truly charge sensitive except for very low source capacitance. However, the circuit was intended for use only as a noise test circuit, and for this purpose the charge sensitivity is adequate.

![Circuit Diagram]

**Fig. 5.3 Nuvistor Test Preamplifier**

To convert the Nuvistor test amplifier into a practical nucleonic preamplifier, the gain must be increased considerably and the input grid resistor must be tied to the output. Because of the similarity between Nuvistors and tubes, a Nuvistor circuit could be designed directly from the E810P circuit simply by scaling the currents and voltages appropriately. Another possibility would be to use a Nuvistor input with junction transistors elsewhere in the circuit. This mixed circuit would require a somewhat different design than the tube circuits so that the transistors can be isolated from the relatively high Nuvistor supply voltages.
5.1.4 F.e.t. Preamplifier

The manufacturing tolerances for f.e.t.'s are relatively poor, and the maximum current of the two available for this experiment differed considerably (as it presumably would for any other two). Since the noise resistor is a minimum when the current is a maximum (see Fig. 2.21), the "best" low-noise f.e.t. is the one with the highest maximum current. Use of the "best" low-noise f.e.t. as the common-source stage of a cascode of two f.e.t.'s poses a problem because the common-gate f.e.t. cannot be biased at a current high enough for optimum performance of the input f.e.t. Therefore, either the bias current must be lowered, or ac coupling must be used between stages. The first solution results in an increase in noise, while the second results in an increase in the low frequency cut-off unless a very large capacitor is used to couple to the low impedance common-gate stage. In addition to these design problems the expense of f.e.t.'s makes it desirable to avoid the use of two when an adequate substitute can be found.

All these problems can be overcome through the use of a mixed f.e.t.-junction transistor cascode circuit of the type shown in Fig. 5.4. Bootstrapping of the cascode load has been used in this circuit in an attempt to increase the gain to a suitable level for practical use, but it resulted in a maximum gain of only 230. An analysis of the circuit in Appendix III shows that the gain is given roughly by Eqn. 5.3.

\[ A \approx \frac{g_m \beta_3 R_1 R_2}{R_1 + r_e \beta_3} \quad (\text{if } r_c \text{ is very large}) \]

\[ A \approx \frac{g_m R_1 R_2}{r_e} \quad (\text{if } \beta_3 \to \infty) \]

\[ \ldots (5.3) \]
Note: all resistors in kohms; all capacitors in \( \mu \)F; unless noted.

Fig. 5.4 F.e.t. Preamplifier

Thus \( A \) can be increased by choosing \( T_2 \) to have large collector resistance, \( T_3 \) to have high \( \beta \), and by increasing both \( R_1 \) and \( R_2 \). With good transistors, this circuit does, therefore, offer the possibility of large enough gain for practical use.

An amplifier was built using the f.e.t. followed by two common emitter stages, but it was found that the phase shift through this circuit was too large for the feedback to be effective. Phase-lead interstage coupling and transistors with higher \( f_t \), could overcome this drawback but the circuit showed less promise than the bootstrapped cascode circuit.

The rise time of the f.e.t. preamplifier depended upon the type of transistors used as well as the amount of feedback. With 2N1304's and 2N1305's the rise time was 0.5 and 0.7 \( \mu \)sec. for 2 pf and 5 pf feedback capacitors respectively, while the use of 2N705's and 2N706's reduced the rise time to 0.3 and 0.1 \( \mu \)secs. The rise
times are still considerably longer than those obtained with the tube and Nuvistor stages, indicating that the f.e.t. circuit is suited mainly to "slow" experiments.

5.1.5 Construction Techniques for Low-Noise Preamplifiers

The choice of input device, bias point, and circuit configuration for low-noise nucleonic preamplifiers has been simplified by this and other studies of amplifier noise. However, to achieve the predicted noise levels in practice, the conditions of complete input isolation and lack of excess component noise which was assumed implicitly in the calculations, must also be reproduced in the amplifiers. The construction techniques used to do this are discussed below with reference to the E810F preamplifiers only, but they apply to the other stages as well.

To isolate the E810F preamplifier from stray signals, the tubes were shielded with conventional tube shields, and the remainder of the circuit was built in a closed copper chassis. Input signals and dc power were fed into the chassis in shielded cables, all dc lines were decoupled within the chassis, and the tube filaments were supplied from a dc filament supply (see Appendix IV). In addition to these precautions, the base of the input tube T1 was shielded separately to isolate the input from the rest of the circuit (see Fig. 5.5(a)). The signal and test inputs enter this inner shield through the rf connectors shown. When one was not in use it was capped to prevent pickup of noise on the input.
Since the purpose of the inner shield is to isolate the high impedance input from noise sources within the circuit, the shield shown in Fig. 5.5(b) might be superior to the one used. It excludes the cathode components, filament leads and the plate lead which could all introduce noise to the input and has only the feedback lead passing through it. The shield of Fig. 5.5(a) was found to be adequate for the circuit built, but the more selective shield is recommended for use in other circuits of this type.

The problem of excess noise in components was overcome by using axial-lead wire wound resistors for $R_1$, $R_2$, $R_3$ and $R_k$, which are the only resistors whose noise could form an appreciable portion of the output noise. Of the others, $R_{fb}$ carries essentially no current and hence generates no excess noise, while the remainder are all either ac bypassed or occur in a location where their noise is swamped by the amplified input noise.
5.2 Measurement of Preamplifier Noise Performance

The noise performance of the four preamplifiers was evaluated for various detector capacitances and system time constants by standard procedures \((5.2.3)\). A test input charge was applied to the input and the output signal-to-noise ratio \(S/n\) was observed. The noise charge was then found from \(Q_n = Q_c n/S\). During these measurements, the detector capacitances were simulated by placing a capacitor \(C_d\) across the preamplifier input (see Fig. 5.6), and the time constants were set by the Dynatron pulse amplifier. A low-noise broad-band post amplifier described in Appendix IV was used to couple the preamplifier to the Dynatron amplifier so that the noise of the latter (\(\approx\)1 v rms at the output) did not affect the overall signal-to-noise ratio.

\[
Q_c = V_c C_c \left[ \frac{C_{in} + C_d}{C_d + C_{in} + C_c} \right],
\]

where \(C_{in}\) is the total amplifier input capacitance. When \((C_{in} + C_d) > 200\) pf, (as it
is for all the charge sensitive preamplifiers), the input charge can be taken to be $V_c C_c$ with at most 1% error. However, with a voltage sensitive preamplifier, the input capacitance must be determined and the complete expression for $Q_c$ used.

The basic input charge was determined by measuring the mercury-relay supply voltage with a dc VTVM and $C_c$ with a Boonton Q-meter. Since $C_c$ was only 2pf, the accuracy of this measurement was estimated to be about 5%. Later, the input pulses were compared to the accurately known charge pulses generated in a solid-state detector by Polonium alpha particles. This checks the charge measured with the meters and also gives a direct energy-unit calibration of the system. Again the accuracy was estimated to be 5%.

At the output of the system, the signal-to-noise ratio can be measured directly with an oscilloscope and a VTVM, or as a pulse height spectrum with a kicksorter. Both methods were used, but because of the time required to accumulate an accurate pulse height spectrum at 60pps, the kicksorter method was used only to check some of the direct measurements.

When noise is measured with the HP 4000D VTVM, the voltage indicated by the meter must be multiplied by 1.13 to obtain the true rms noise output. This is because the meter is calibrated to read the rms of sinusoidal, not random signals. An additional correction is necessary at short time constants to account for the high frequency losses in the meter. The frequency response of the VTVM was measured and found to be limited at high frequencies by (approximately) a double time constant of 0.02 μsec. Thus the observed noise must be increased further by the factor $(1 + x')^2$.
where \( x^* = 0.02/T \). The net "correction factors" that were applied to the raw noise voltages are given in Table 5.1 below. The use of a broader band VTVM for the noise measurement would have avoided any error that might result from approximating the high frequency response of the meter with two time constants. However none was available for the experiment so the HP 400D had to be used.

<table>
<thead>
<tr>
<th>( \tau ) in ( \mu )secs.</th>
<th>0.08</th>
<th>0.16</th>
<th>0.32</th>
<th>0.8</th>
<th>1.6</th>
<th>3.2</th>
<th>8.0</th>
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<td>correction factor</td>
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<td>1.43</td>
<td>1.28</td>
<td>1.18</td>
<td>1.15</td>
<td>1.14</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Table 5.1 Meter High Frequency Corrections

5.3 Results of Noise Charge Measurements

Using the direct measurement method described in the previous section, curves of noise charge versus \( \tau \) (with detector capacitance as a parameter) were obtained for the four preamplifiers. Figs. 5.7 and 5.8 show the curves for the E810F charge-sensitive and voltage-sensitive preamplifiers, respectively, Fig. 5.9 for the Nuvistor preamplifier, and Fig. 5.10 for the f.e.t. preamplifier. In Figs. 5.7(b), 5.8(b), 5.9(b) and 5.10(b), the noise charge is plotted with detector capacitance as the variable to show the effect of a change in detector capacitance more clearly.

For each preamplifier, the theoretical curves, shown as broken lines, were calculated from the noise parameters given in Chapter 4. The calculations were similar for all the circuits and can be explained briefly with the aid of the following sample calculations for the E810F preamplifier.
Fig. 5.7(a) $N$ vs. $\tau$ for E810F Charge Sensitive Preamplifier

Fig. 5.7(b) $N$ vs. $C_d$ for E810F Charge Sensitive Preamplifier
Fig. 5.8(a) $N$ vs. $\tau$ for E810F Voltage Sensitive Preamplifier

Fig. 5.8(b) $N$ vs. $C_d$ for E810F Voltage Sensitive Preamplifier
Fig. 5.9(a) N vs. $\tau$ for Nuvistor Preamplifier

Fig. 5.9(b) N vs. $C_d$ for Nuvistor Preamplifier
Fig. 5.10(a) $N$ vs. $\tau$ for F.e.t. Preamplifier

Fig. 5.10(b) $N$ vs. $C_d$ for F.e.t. Preamplifier

$C_d = 68\text{pf}$

$C_d = 47\text{pf}$

$C_d = 30\text{pf}$

$C_d = 10\text{pf}$

$C_d = 0\text{pf}$

Measured with Meters

Theoretical Curves

$\tau = 1\mu\text{sec}$

$\tau = 8\mu\text{sec}$
For a cascode current of 20 ma the noise parameters in the E810F are found from Figs. 4.8 and 4.3 to be $R_n = 70$ ohms, $I_L = 3\text{na}$ and $K = 10^{-13}$. The grid resistor contributes an apparent leakage current of $I_i = 0.5\text{na}$ and the net input capacitance of the tube is 25 pf. Substitution of these quantities in Eqn. 2.14 results in Eqn. 5.4 for the noise charge of the E810F circuit.

$$N = C(x)1.71(27 + C_d) \left[ \frac{13.8}{\tau} + \frac{13.5 \times 10^3 \tau}{(27 + C_d)^2} + 5 \right]^{1/2} \ldots (5.4)$$

If it is assumed that the high frequency response of the preamplifier can be approximated for noise calculations by the single time constant $T_3$, then $T_3 = \frac{\text{rise}}{2.2} = 0.18 \mu\text{sec}$. With these values of $T_3$, the "correction factor" $C(x)$ can be found from Fig. 2.6 to be .86, .91, and .95 at $\tau = 0.1$, 0.2 and 0.4 $\mu$secs respectively and negligible at all longer $\tau$'s. Using Eqn. 5.4 and these values of $C(x)$ the calculation of $N$ for $C_d = 10\text{pf}$ is carried out in Table 5.2.

<table>
<thead>
<tr>
<th>$\tau$</th>
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<th>Shot</th>
<th>Grid</th>
<th>Sum</th>
<th>$N(o)$</th>
<th>Corr. Fac.</th>
<th>$N(x)$</th>
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<td>.96</td>
<td>144</td>
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<td>650</td>
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<td>5</td>
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<td>502</td>
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<td>19.2</td>
<td>31.1</td>
<td>352</td>
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</tr>
</tbody>
</table>

Table 5.2 Calculation of $N$ for E810F Preamplifier ($C_d = 10\text{pf}$)
The validity of the assumption that white noise predominates for x large enough to affect the noise charge is demonstrated by the calculation. As the detector capacitance increases, the assumption becomes even more accurate because of the dependence of the leakage current noise upon the inverse square of $C_t$. Theoretical curves were obtained for the other conditions and the other devices by similar calculations using the data in Table 5.3.

<table>
<thead>
<tr>
<th>Device</th>
<th>Tube (E810F)</th>
<th>Tube (E810F)</th>
<th>Nuvistor (7586)</th>
<th>f.e.t. (2N2386)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_h$ (ma)</td>
<td>20</td>
<td>20</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>$R_n$ (ohms)</td>
<td>70</td>
<td>80</td>
<td>370</td>
<td>$600 \left[ 1 + 0.4 \left( \frac{10}{C_t} \right)^2 \right]$</td>
</tr>
<tr>
<td>$I_L$ (na)</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>$I_i$ (na)</td>
<td>0.5</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$K(v^2/c)$</td>
<td>$10^{-13}$</td>
<td>$10^{-13}$</td>
<td>$10^{-13}$</td>
<td>$6 \times 10^{-13}$</td>
</tr>
<tr>
<td>$t_r$ ($\mu$sec)</td>
<td>0.04</td>
<td>0.04</td>
<td>0.06</td>
<td>0.3</td>
</tr>
<tr>
<td>$T_3$ ($\mu$sec)</td>
<td>0.018</td>
<td>0.018</td>
<td>0.027</td>
<td>0.136</td>
</tr>
<tr>
<td>$C_i$ (pf)</td>
<td>25</td>
<td>25</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>$C_{fb}$ (pf)</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.3 Data for Calculation of N for Preamplifiers

It can be seen from all the curves that the theoretical noise charge for any of the three devices can be very nearly achieved. The divergence between the experimental and theoretical curves is in
most cases within the expected experimental accuracy. However, there are some systematic differences which can probably be interpreted as follows. Firstly, the minimum noise is never achieved, indicating that some second-stage, post-amplifier, or stray input noise is added to that calculated. Secondly, noise charges lower than the theoretical values were measured at short time constants indicating that the corrections applied to the experimental values for meter losses, or to the theoretical values for the effect of $T_3$ were perhaps inadequate. Undoubtedly a more detailed approximation to the high frequency response of the meter and preamplifiers would show that observed noise can be predicted accurately at all frequencies.

The results of the noise charge measurements verify the predictions of section 4.3.2. That is, the E810F is the best low noise device except at very low detector capacitance, where the Nuvistor is superior. Of the two configurations used for the E810F, the charge-sensitive one exhibited lower noise, particularly at high frequencies, because in the voltage-sensitive amplifiers the additional noise generated by the unbypassed feedback resistor increases $R_n$ from 70 to 80 ohms. At low frequencies the voltage sensitive preamplifier was inferior mainly because the grid resistor used was 22 meg rather than 100 meg as was used in the charge sensitive circuit. Decreasing the feedback resistor and increasing the grid resistor should then make the voltage-sensitive amplifier more nearly competitive with the charge-sensitive one.

All the curves show the noise exhibited by the amplifiers alone, and the minimum noise levels in these curves do not indicate the optimum time-constants for the amplifiers when they are used with
detectors. The differences were discussed in Section 4.3.2 and at that time it was pointed out that the addition of the detector noise, the optimum time constants are considerably shorter than those given here.
6. CONCLUSIONS

The study reported in this thesis showed that in spite of the many recent advances in solid-state electronics, conventional tube amplifiers still give the best nucleonic resolution. A triode-connected E810F was found to be the best tube for all detector capacitances above 10pf. Below that, some older tube types and the 7586 Nuvistor are superior.

Of the "modern" solid-state devices, the field effect transistor was found to be the most suitable for use in nucleonic amplifiers. The 2N2386 noise levels were 80% higher than those obtained with the E810F, but they were still roughly half those expected with junction transistors. Therefore, in applications where minimum size and power dissipation are more important than minimum noise, the f.e.t. should be used in preference to the junction transistors now commonly used. The more "exotic" tunnel diode and parametric amplifiers were shown to be unsuitable for low-noise nucleonic work.

Apart from these general conclusions regarding the relative merit of various amplifiers, the study produced many subsidiary results. Preliminary discussions of the "typical" counting system showed that matching networks between source and amplifier will not generally improve the noise performance. Also the effect of feedback in reducing the input capacitance of (say) a cathode follower cannot be used to reduce the noise charge.

Of the alternative feedback configurations, the "charge sensitive" one has greater stability and provides greater experimental convenience. It has been stated in other reports
that the charge sensitive configuration also has poorer resolution because $C_{fb}$ is added to the total capacitance. However, it was found for the E810F that the feedback resistor noise added in the voltage sensitive configuration causes an even larger change in noise charge. Therefore the charge sensitive configuration is superior in this aspect too.

The noise of each device can be represented by two noise generators, one in series and one in shunt with the input. By writing the spectral densities of the generators, in the same form as those of the usual tube noise generators, the "classical" theory of Gillespie can be applied to all amplifiers. To account for the high-frequency cutoff of the preamplifier, a "correction factor" has been applied to the noise charge equations derived for a frequency response defined by single, equal integrating and differentiating time constant. The "correction factor" was small, but it resulted in better agreement between theory and experiment than is usually obtained.

The noise charge is a minimum when the "white" components of the two generators are equal. Because of this, the minimum noise charge for any device is determined by its product $R_n I_L$, where $R_n$ is the equivalent noise resistor and $I_L$ is the equivalent input leakage current. In most applications, $I_L$ is swamped by the detector leakage current so $R_n$ is the most important parameter in determining the minimum noise. To be competitive with the E810F, a device must have an equivalent noise resistor less than 80 ohms.

$R_n$ and $I_L$ are not usually given in device specifications. Instead, the noise figure, $F$ is quoted. Using Eqn. 2.32, the
noise charge can be found from $F$, giving an alternative criterion for judging whether an amplifier is suitable for this application. To compete with f.e.t.'s a device must exhibit a noise figure less than 0.2 db, and to compete with tubes, less than 0.1 db.

It is possible to predict $R_n$ and $I_L$ for tubes, Nuvistors, f.e.t.'s and junction transistors with enough accuracy to rank their noise performance in the order given. However, to design amplifiers and predict their exact noise performance, $R_n$ and $I_L$ and the flicker noise constant $K$ must be found experimentally. $R_n$ and $K$ can be found by the resistive substitution method, and $I_L$ from dc measurements of leakage current. By plotting the measured parameters in the plate characteristics (source characteristics for the f.e.t.), the optimum bias points for different detector leakage currents can be found. It was shown for tubes and Nuvistors that these points are near the potential at which the grid floats on open circuit. As the detector leakage increases, the optimum bias point moves to higher current. The input stage of a general purpose amplifier should therefore be biased at as high a current as possible. Similar arguments apply for the f.e.t.

To achieve the predicted noise levels, amplifiers must effectively eliminate second stage noise. It was shown that the cascode configuration does this. However, the voltage gain of the basis cascode is not large enough to make charge feedback fully effective, so its load is commonly "bootstrapped". The E810F preamplifier demonstrated that bootstrapping increases the gain as required, without affecting the noise performance. Thus it can also be used to increase the gain of the Nuvistor circuit. Increasing the gain of the f.e.t. circuit requires the use of
junction transistors of higher $\beta$ than those used.

With the completion of this study, the "classical" work on low-noise nucleonic amplifiers has been updated to include the latest tubes and solid-state devices. The potential of the E810F and the 2N2386 for low-noise nucleonic amplifications has been assessed for the first time. The assessment showed that the E810F is superior to all other devices when minimum noise is required, while the field effect transistor is superior when both low noise and solid-state circuitry are required.
Evaluation of Integrals Used in Theory

The four integrals used in the theory \((I_1, I_2, I_3, I_4\) in Chapt. 2; \(I_4\) in Chapt. 4) all have the general form of Eqn. 1.1, so they can be expanded in partial fraction and integrated as shown to give Eqn. 1.2.

\[
I = \frac{T^2}{2\pi} \int_{0}^{\infty} \frac{\omega^m d\omega}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)(1+\omega^2 T_3^2)}
\]

\(m = 2\) for \(I_1\), \(m = 0\) for \(I_2\), \(m = 1\) for \(I_3\) ... I.1

\[
I = \frac{T^2}{2\pi} \int_{0}^{\infty} \left[ \frac{A}{1+\omega^2 T_1^2} + \frac{B}{1+\omega^2 T_2^2} + \frac{C}{1+\omega^2 T_3^2} \right] d\omega
\]

\[
I = \frac{T^2}{2\pi} \left\{ \frac{\text{Atan}^{-1}(\omega T_1)}{T_1} + \frac{\text{Btan}^{-1}(\omega T_2)}{T_2} \right\} \bigg|_{\omega=0}^{\infty} + \frac{\text{Ctan}^{-1}(\omega T_3)}{T_3} \bigg|_{\omega=0}^{\infty}
\]

\[
I = \frac{T^2}{4\pi} \left[ \frac{A}{T_1} + \frac{B}{T_2} + \frac{C}{T_3} \right]
\]

... I.2

The constants \(A, B,\) and \(C\) can be found for \(I_1, I_2\) and \(I_4\) by substituting successively \(\omega^2 = -T_1^2, \omega^2 = -T_2^2, \omega^2 = T_3^2\) Eqn. I.3. I.3 relates the numerator of the expanded integrand to the original expression. For \(I_3\), when \(m\) is odd, the constants are imaginary so this one integral must be treated separately (see Eqn. I.5 and following).

\[
\omega^n = A(1+\omega^2 T_2^2)(1+\omega^2 T_3^2) + B(1+\omega^2 T_1^2)(1+\omega^2 T_3^2) + C(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)
\]

... I.3
For $I_1$, $I_2$ and $I_4$ the denominators of the constants are the same and are given by Eqn. I.4. The numerators for each integrand and each constant are given in Table I.1.

(a) Den $[A] = (T_1^2 - T_2^2)(T_1^2 - T_3^2)$
(b) Den $[B] = (T_2^2 - T_3^2)(T_1^2 - T_2^2)$
(c) Den $[C] = (T_1^2 - T_3^2)(T_2^2 - T_2^2)$

...(1.4)

<table>
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<th></th>
<th></th>
<th>Num.(A)</th>
<th>Num.(B)</th>
<th>Num.(C)</th>
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<tbody>
<tr>
<td>$I_1$</td>
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<td>$T_2^4$</td>
<td>$-T_2^4$</td>
</tr>
<tr>
<td>$I_2$</td>
<td>0</td>
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<td>$-T_2^4$</td>
<td>$T_3^4$</td>
</tr>
<tr>
<td>$I_3$</td>
<td>4</td>
<td>1</td>
<td>$-1$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table I.1 Partial Fraction Constants' Numerators

Upon substituting these constants in Eqn. I.2 and simplifying, Eqn. I.5 for $I_1$, $I_2$ and $I_3$ result.

(a) $I_1 = \frac{T_1^2}{4(T_1 + T_2)(T_2 + T_3)(T_1 + T_3)}$
(b) $I_2 = \frac{T_1^2(T_1T_2 + T_1T_3 + T_2T_3)}{4(T_1 + T_2)(T_2 + T_3)(T_1 + T_3)}$

(c) $I_3 = \frac{T_1^2(T_1 + T_2 + T_3)}{4T_1T_2T_3(T_1 + T_2)(T_2 + T_3)(T_1 + T_3)}$

...(1.5)

To evaluate $I_3$, the original integral is expanded as shown in Eqn. I.6, so that A, B and C will be real. The expanded integral can be evaluated by inspection to be Eqn. I.7. This expression vanishes
at the lower limit. At the upper limit it can be evaluated by letting $\omega \to \infty$ after substituting $A$, $B$ and $C$. The constants were found as before by equating the numerators of the expanded and unexpanded integrand. For $I_3$ and this new expansion, $A$, $B$ and $C$ are the same as for $I_2$ and result in Eqn. I.8 for $I_3$,

$$I_3 = \frac{T_1^2}{2\pi} \int_0^\infty \left[ \frac{A}{(1+\omega^2T_1^2)} + \frac{B}{(1+\omega^2T_2^2)} + \frac{C}{(1+\omega^2T_3^2)} \right] d\omega \quad \ldots I.6$$

$$I_3 = \frac{T_1^2}{2\pi} \left\{ \frac{A}{2T_1^2} \ln(1+\omega^2T_1^2) \right\}_0^\infty + \frac{B}{2T_2^2} \ln(1+\omega^2T_2^2) \right\}_0^\infty + \frac{C}{2T_3^2} \ln(1+\omega^2T_3^2) \right\}_0^\infty$$

$$I_3 = \frac{T_1^2}{2\pi} \left\{ \frac{T_1^2T_2^2\ln(T_1/T_2)}{(T_1^2 - T_2^2)(T_1^2 - T_3^2)(T_2^2 - T_3^2)} + \frac{T_1^2T_3^2\ln(T_3/T_1)}{T_1^2} \right\} \ldots I.7$$

$$I_3 = \frac{T_1^2}{2\pi} \frac{T_1^2T_2^2\ln(T_1/T_2)}{(T_1^2 - T_2^2)(T_1^2 - T_3^2)(T_2^2 - T_3^2)} + \frac{T_1^2T_3^2\ln(T_3/T_1)}{T_1^2} \right\} \ldots I.8$$

Eqns. I.5, and I.8 are the expressions of the four integrals for arbitrary $T_1$, $T_2$ and $T_3$. In the theory the special case when $T_1 = T_2 = T$, and $T_3 = xT < T$ is used exclusively. For this case, the following simplified expressions for the integrals can be found.

(a) $I_1 = \frac{1}{8 \ T(1+x)^2}$

(b) $I_2 = \frac{T(1+2x)}{8(1+x)^2}$

(b) $I_3 = \frac{2x^2 \ln x + (1-x)^2}{4\pi(1-x^2)^2}$

(c) $I_4 = \frac{(2+x)}{8xT^3(1+x)^2}$
APPENDIX II

F.e.t. Admittance Parameters

The admittance parameters were measured in the manner recommended by the manufacturer. In the case of the input and output admittances $Y_i$ and $Y_o$, a second direct measurement was also made on a Heathkit Q-meter. The measurements were all relatively straightforward so the description here will be limited to a brief outline of the experimental procedure, diagrams of the test jigs, and graphs of the results obtained.

a) Forward Transadmittance

The forward transadmittance, $Y_f$, was measured with the circuit shown in Fig. II.1. In this circuit, the drain current resulting from the 10 mv gate signal $V_1$, causes a voltage $V_2 = V_1 Y_f \cdot 100$ in the 100 ohm load resistor. Hence $|Y_f|$ (in Millimhos) = $V_2$ (in mv). The load is small enough that the output can be considered to be short circuited as is required for a measure of $Y_f$. Curves of $Y_f$ vs. drain current for the two f.e.t.'s are shown in Fig. II.5. Measurements were made at frequencies of 10 kc, 100kc, and 1 mc. At 1mc the value of $Y_f$ has started to fall slightly. This behaviour can be predicted if the parasitic capacitances are included in the calculation of $g_m$ from $g_m^i$ but that calculation is not particularly interesting or instructive and will be omitted.

b) Reverse Transadmittance $Y_r$

The jig of Fig. II.2 gives $Y_r$ (which is $sC_{gd}$ in the equivalent circuit) only if the input can be considered to be short circuited.
To see that the 10K gate resistor effectively short circuits the input, consider the expression for the gate current in terms of the admittance parameters. It is, $I_g = Y_1 V_1 + Y_r V_2$, but $V_1 = I_g 10^4$, so that $I_g (1 - 10^4 Y_1) = Y_r V_2$. Now $10^4 Y_1$ is much less than unity and the 10K gate resistor acts as an effective short circuit, as is required. The 1 volt signal $V_2$ causes a current $Y_r$ to flow in the gate resistor giving rise to a voltage $V_2 = 10^4 Y_r$ so that $Y_r$ (in umhos) = $1V_1$ (in mv). It was assumed that $Y_r$ can be approximated by the single capacitor $C_{gd}$ which was calculated and plotted in the static characteristic plane (see Fig. II.6). From these curves, it can be seen that $C_{gd}$ is relatively independent of the drain current in the pinch-off region.

c) Input Admittance $Y_1$

Because of the importance of $Y_1 = s(C_g + C_{ds})$ in the prediction of noise, it was measured in two separate ways. First directly on a Heathkit Q-meter using the test circuit of Fig. II.3(a), then

---

Fig. II.1 Jig to Measure $Y_f$ of F.e.t.
indirectly using the jig of Fig. II.3(b). The direct method is obvious while the indirect method can be understood by considering the equation $I_g$ above. With the source short circuited to the drain, $V_2$ is now zero and $I_g = Y_1 V_1$ where $V_1 = V'_1 - 100\text{mv}$. Substituting this in the equation and solving for $Y_1$ gives $Y_1 = 10 \frac{V_2}{(100-V_2)}$, ($V_2$ in mv, $Y_1$ in $\mu$hmhos) or, approximately $Y_1 = 0.1V_2$. Both methods gave substantially the same results, and these are displayed in Fig. II.7 as curves of constant capacitance in the drain characteristic plane.
d) Output Admittance $Y_o$

The imaginary and real parts of the output admittance, $Y_o$, were measured separately using the two circuits shown in Figs. II.4(a) and (b). The operation of the direct method is again self-evident, while the indirect method follows from the expression $I_d = Y_f V_{gs} - Y_o V_{gd}$. In this case $V_{gs} = 0$, so the drain current is entirely due to the affect of $V_{gs}$ and $Y_o$. Now $V_2 = 100 I_d$ and $V_{gd} = V - V_2$ which upon substitution in the equation for $I_d$ gives $Y_o$ (in $\mu$mhos) $= 10V_2/(1000 - V_2) = 10V_2$, where $V_2$ is in mv. The results of the direct measurements showed that the capacitive component of $Y_o$ is indistinguishable from $C_{gd}$ indicating that $C_{ds}$ can be ignored. The output conductance $g_o$ obtained from the other measurement was checked by comparing it with the slope of the static characteristics then plotted in Figs. II.8(a) and (b) against the drain current.
Fig. II.4(a) Jig to Measure $I_o$ of F.e.t. Directly

Fig. II.4(b) Jig to Measure $I_o$ of F.e.t. Indirectly
Fig. II.5(a) $y_f$ of F.e.t. #1

Fig. II.5(b) $y_f$ of F.e.t. #2
Fig. II.6(a) $C_{dg}$ of F.e.t. #1

Fig. II.6(b) $C_{dg}$ of F.e.t. #2
Fig. 11.7(a) $C_i = C_{gs} + C_{gd}$ of F.e.t. #1

Fig. 11.7(b) $C_i = C_{gs} + C_{gd}$ of F.e.t. #2
Fig. II.8(a) $V_o$ of F.e.t. #1

Fig. II.8(b) $V_o$ of F.e.t. #2
Derivation of Preamplifier Gain Expressions

a) Gain of Tube Cascode with "Bootstrapped" Load

The "bootstrapped" cascode of Fig. III.1(a) can be represented by the equivalent circuit of Fig. III.1(b).

The various grid voltages can be seen to be: 

\[ V_{g1} = V_{in}, \]
\[ V_{g2} = \mu_1 V_{g1} - I_p r_{pl}, \]
\[ V_{g3} = -I_p R_2. \]

When these values are substituted in the equivalent circuit, it can be simplified to the form shown in Fig. III.2 below.

\[
\begin{align*}
(\mu_1\mu_2 V_{in} - \mu_2 I_p r_{pl}) & - r_{p2} + R_2 \\
\mu_1 V_{in} & - r_{pl} \\
\end{align*}
\]
The current $I_p$ around the loop is:

$$I_p = \frac{V_{in} \mu_1 (\mu_2 + 1)}{R_1' + R_2 + r_{p2} + r_{pl}(1 + \mu_2) + R_1'g_m R_2}$$  \hspace{1cm} \ldots \text{III.1}$$

The output voltage is:

$$V_o = -I_p R_1'(1 + g_m R_2)$$  \hspace{1cm} \ldots \text{III.2}$$

Therefore the gain is:

$$A_{cb} = \frac{\mu_1 (\mu_2 + 1) R_1'(1 + g_m R_2)}{R_1' + R_2 + r_{p2} + r_{pl}(1 + \mu_2) + R_1'g_m R_2}$$  \hspace{1cm} \ldots \text{III.3}$$

To find the gain of the unbootstrapped cascode, let $R_2 = 0$ and $R_1' = R_L$, where $R_L$ is the load resistance. The cascode gain is then:

$$A_c = \frac{\mu_1 (\mu_2 + 1) R_L}{R_L + r_{p2} + r_{pl}(1 + \mu_2)}$$  \hspace{1cm} \ldots \text{III.4}$$

The optimum values of $R_1$ and $R_2$, can be found by maximizing the approximate expression III.5 with respect to $R_1$. In III.5 it has been assumed that $\mu \gg 1$, and the sum of $R_1$ and $R_2$ has been taken to be $R$.

$$A_{cb} \approx \frac{\mu_1 \mu_2 R_1 (R_1 - R_1') g_m}{R + r_{p2} + \mu_2 r_{pl} + R_1 (R - R_1') g_m}$$  \hspace{1cm} \ldots \text{III.5}$$

Differentiation with respect to $R_1$ results in:

$$\frac{dA_{cb}}{dR_1} = \frac{\mu_1 \mu_2 g_m (R - 2R_1)}{(\text{Denominator})^2}$$  \hspace{1cm} \ldots \text{III.6}$$
The derivative is zero only when $R_1 = R/2$, therefore the maximum gain is achieved when the two load resistors are equal.

b) **Gain of F.e.t.-Junction Transistor Combination Circuit**

To determine the gain of the F.e.t.-junction transistor circuit, consider Fig. III.3 below. In III.3(b) the cascode has been replaced by a current generator $g_m V_{in}$ with internal resistance $r_o$ in accordance with Norton's theorem. The voltage gain of the circuit is then:

$$A = \frac{g_m R_L}{R_L + r_o},$$

where $R_L$ is the apparent load resistance.

![Diagram](image)

(a) F.e.t.-junction Transistor Circuit  (b) Simplified Circuit

Fig. III.3

The apparent load resistance can be found as follows. The current $I$ divides between $R_1$ and $T_3$ so that the voltage drop across $R_1$ equals the voltage drop across the (approximate) base-emitter resistance $r_e/(1-\alpha)$. Thus,
The voltage change caused by the current is,

\[ V_o = I_1 \beta_3 R_2 \quad \text{(assume } \beta_3 \gg 1; \ R_2 \ll R_3) \]

Therefore the apparent load resistance \( R_L \) is;

\[ R_L = \frac{V_o}{I} = \frac{\beta_3 R_1 R_2}{R_1 + \frac{r_e}{(1 - \alpha)}} \quad \text{...III.9} \]

Upon substitution of III.9 in III.7, it is seen that

\[ A = \frac{g_m r_o \beta_3 R_1 R_2}{\beta_3 R_1 R_2 + r_o R_1 + \frac{r_e}{(1 - \alpha)}} \quad \text{...III.10} \]

The output resistance of the cascode can be seen by inspection of Eqn. III.4 to be approximately \( r_{pl} (1 + \mu_2) \) for the tube circuit. In the transistor case \( r_{pl} \) becomes \( 1/g_o \) of the f.e.t., and \( \mu_2 \) becomes \( r_{e2}/r_{e2} \), the voltage transfer ratio for the junction transistor \( T_3 \). This will in general be larger than \( R_L \); so the approximate gain expression Eqn. III.11 can be used.

\[ A \approx \frac{g_m \beta_3 R_1 R_2}{R_1 + \frac{r_e}{(1 - \alpha)}} \approx \frac{g_m \beta_3 R_1 R_2}{R_1 + r_e \beta_3} \quad \text{III.11} \]
Subsidiary Electronic Equipment

A) Post-Amplifiers

Two low-noise post-amplifiers were built to couple the pre-amplifiers to the Dynatron amplifier during the noise charge measurements. The tube circuit shown in Fig. IV.1, is a conventional ring-of-three circuit, with two grounded-cathode stages followed by an output cathode follower. The input tube is triode-connected for minimum noise. Feedback from the output to the cathode of the input tube can be used to vary the gain from the open loop value of 5000 to a minimum of 10.

The transistor post-amplifier shown in Fig. IV.2 is of similar design, the voltage gain being provided by two grounded emitter stages and the low output impedance by an emitter follower. In this case feedback is taken from the second stage rather than the output, and the gain is adjusted by varying the feedback resistor. The open loop gain is roughly 2000 while the closed loop gain is adjustable from 20 to 220.

B) DC Filament Supply

The dc filament supply used with the E810F and Nuvistor pre-amplifiers is shown in Fig. IV.3. Cascaded series regulating circuits reduce the ripple without the use of large chokes or capacitors. The first stage reduces the voltage from 15 to 9 volts and the ripple from 0.5 volts to 50mv (at 2 amps out). The output voltage is set with the second stage which reduces the ripple further to 2mv (at
2 amps). The power transistors are operated well below their maximum ratings, but they are bypassed by the resistors to provide a further margin of reliability. At output currents below 1 amp, the current drawn by the resistors is large enough to affect adversely the regulation circuits. They can therefore be "switched out" of the circuit when only small currents are required. The effect of changes in the output current upon the ripple and the output voltage are shown in Fig. IV.4.
Note: all resistances in Kohms, all capacitances in μF, unless noted.

Fig. IV.1 Tube Post-amplifier

Note: all resistances in Kohms; all capacitances in μF, unless noted.

Fig. IV.2 Transistor Post-amplifier
Note: all resistors in Kohms; all capacitors in μF unless noted.

Fig. IV.3 D-c Filament Supply

Fig. IV.4 Ripple and Output Voltage of D-c Filament Supply
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