THE THEORY OF GEOPHYSICAL SURVEYING BY THE HIGH FREQUENCY ELECTROMAGNETIC METHOD

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THE THEORY OF GEOPHYSICAL SURVEYING BY THE HIGH FREQUENCY ELECTROMAGNETIC METHOD.

Part 1.

General Principles of Electromagnetic Prospecting.

In the high frequency electromagnetic method of prospecting, an alternating magnetic field is applied to an area by means of an oscillator and vertical coil antenna. The field from the antenna induces a current in any orebody, which may exist underneath the area being prospected, by reason of the greater conductivity of the ore as compared with the country rock. This induced current, in turn, produces a secondary field at the surface which may be investigated by means of suitable instruments.

Suppose, for instance, we consider the coil antenna shown in fig. 1, which carries a current,

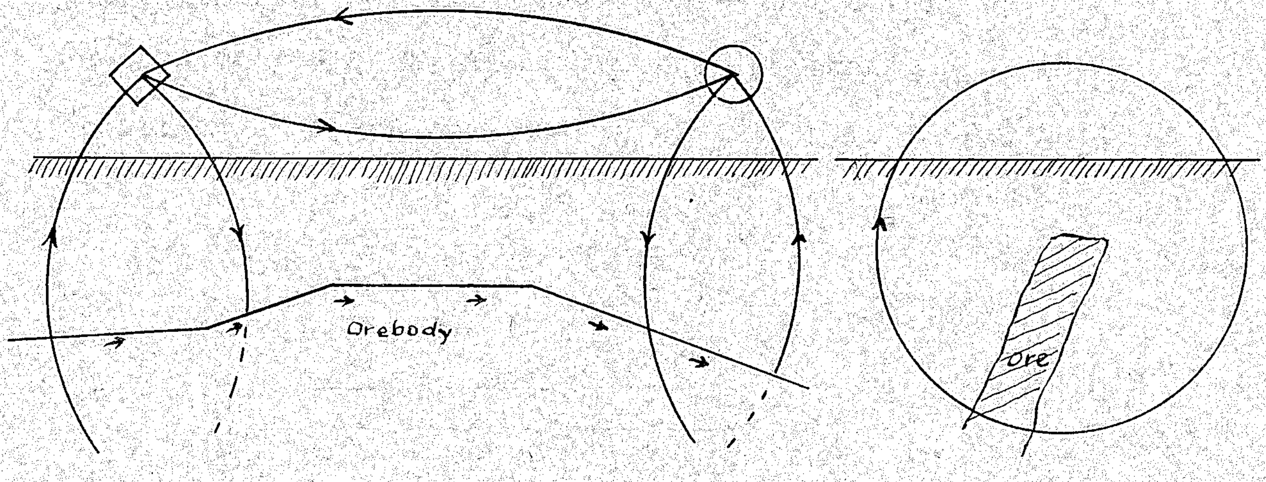


Fig 1.

$i_1 = I_1 \sin \omega t$, where i_1 is the instantaneous current.

Then if the mutual inductance between the antenna and the circuit in the vein is M,

e_2 = instantaneous secondary e.m.f.

sign $= \omega M I_1 \cos \omega t = \omega M I_1 \sin(\omega t - \frac{\pi}{2})$

and

i_2 = instantaneous secondary current

$$= \frac{\omega M I_1}{\sqrt{R_2^2 + X_2^2}} \sin(\omega t - \frac{\pi}{2} - \phi)$$

where R_2 = resistance of path in the orebody

X_2 = reactance of the orebody

and $\tan \phi = \frac{X_2}{R_2}$

The secondary field will be in phase with the current in the orebody and, therefore, at the point P, we have two separate fields; the primary field from the oscillator, $\phi_1 = \Phi_1 \sin \omega t$, and the secondary field, $\phi_2 = \Phi_2 \sin(\omega t - \frac{\pi}{2} - \phi)$ This is shown vectorially in fig 2 for a point P_1, situated to the left of P (see fig 1b). Assuming the instantaneous direction of ϕ_1 to be horizontal, the secondary field, ϕ_2, is at an angle depending upon the position of P_1 with respect to P. If P_1 is to the left

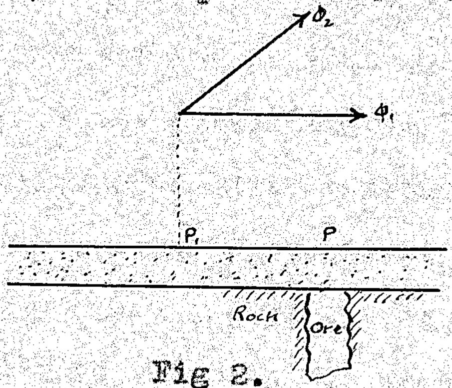


Fig 2.

of P (as in the diagram), the field ϕ_2 will point diagonally upwards to the right, while if P_1 is to the right of P, will point downward to the left. Fig 3

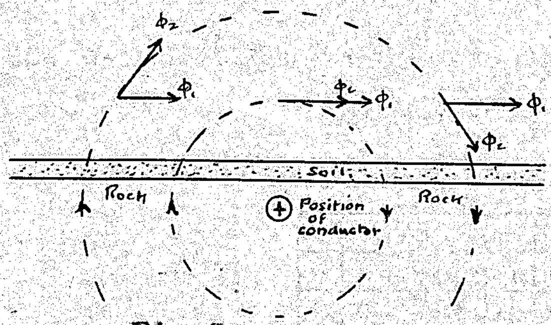


Fig 3.

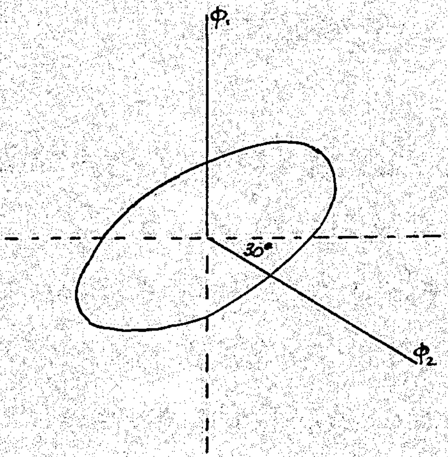
shows the vectors in varying positions in a traverse of the conductor.

If the plane of the coil antenna is kept directed towards the points P, P_1, P_2 etc, the horizontal field ϕ_1 will remain approximately constant for one traverse line, providing PP_1, P_2P_1, P_2P, remain small compared with the distanse away from the transmitting loop Thus a comparison of the two components of the field is possible. This is effected by means of one or more search coils and detecting instruments.

<u>Elliptical Polarization of Resultant Field.</u>

Since there are two vectors ϕ_1 and ϕ_2, which differ both in space and in time phase, we cannot compound them by the parallelogram law. They can however, be compounded instantaneously, i.e. for instantaneous values at a particular time of the cycle, when it is found that the resultant vector is variable in length and revolves with the angular velocity ω of the primary and secondary fields, and that the path of its extremity traces out an ellipse.

As an example of this, refer to fig 4. Here we have two vectors $\phi_1 = 1 \sin\theta$ and $\phi_2 = 1 \sin(\theta - 30°)$. They differ in space by an angle $\frac{\pi}{2} + 30°$. Values of ϕ_1 and ϕ_2



θ	30°	60°	90°	120°	150°	180°	210°	240	270°	300°	330°
ϕ_1	+0.5	+0.86	+1.0	+0.86	+0.5	0	−0.5	−0.86	−1.0	−0.86	−0.5
ϕ_2	0	+0.5	+0.86	+1.0	+0.86	+0.5	0	−0.5	−0.86	−1.0	−0.86

Fig 4.

are given for various values of θ. Plotting these instantaneous values of ϕ_1, ϕ_2, and compounding corresponding values by the parallelogram law, we find the instantaneous values of the resultant flux vector, which traces out an ellipse.

Furthermore, it will be shown in Part 11 that all out of phase fields of the same frequency can be compounded into one resultant field; so that there is a single plane ellipse, and one only, at any point in the field, due to any system of conductors. A field of this nature is said to be elliptically polarized.

<u>The Determination of the Ellipse</u>.

For the purposes of geophysical prospecting, it is unnecessary completely to determine the ellipse; we need only obtain the direction of its minor axis and an approximate idea of the eccentricity. The latter can most easily be found by determining the ratio of lengths of the axes. The most successful method in use at present simply finds the direction of the minor axis.

The Single Coil Method of Prospecting.

It is evident that, in the event of there being no orebody in the vicinity, the primary field is the only field at the point under consideration; in this case a single search coil, rotated about an axis horizontal and in the same plane as the antenna, will be in a horizontal plane when it is not threaded by any flux. When an ore deposit is present, the coil will also be horizontal directly above the deposit, as at point A in fig 5., where ϕ_1 and ϕ_2 add directly to

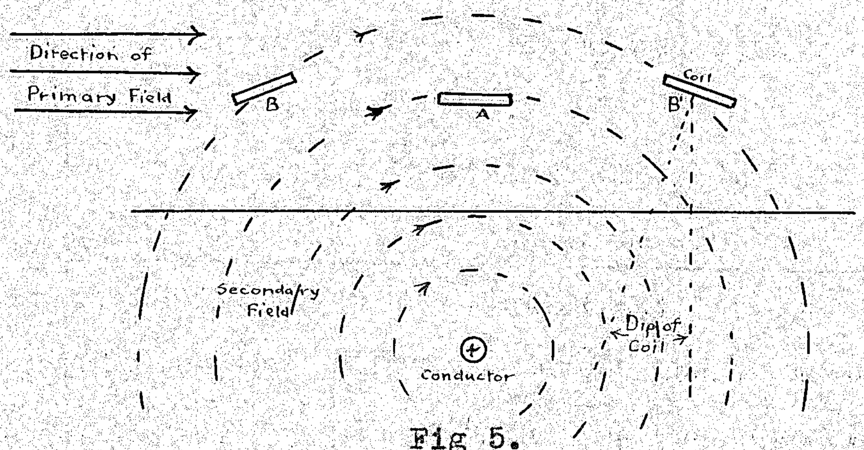


Fig 5.

form one unidirectional resultant. At points B and B_1, the position of minimum pick up will be at an angle as indicated. If the field is elliptically polarized, as is usually the case, the minimum intensity# in the detecting unit will be difficult to determine. This disadvantage is inherent in the single coil method. Sometimes it is possible to bring the secondary field into phase by altering the frequency; this is objectionable since it gives non-uniform conditions for interpreting a set of readings, if they are all taken at different frequencies.

Thus it will be seen that the normal to the plane of the coil points towards the buried conductor. Fig 6 shows the orientation of the field ellipses in the traverse across a buried conductor below

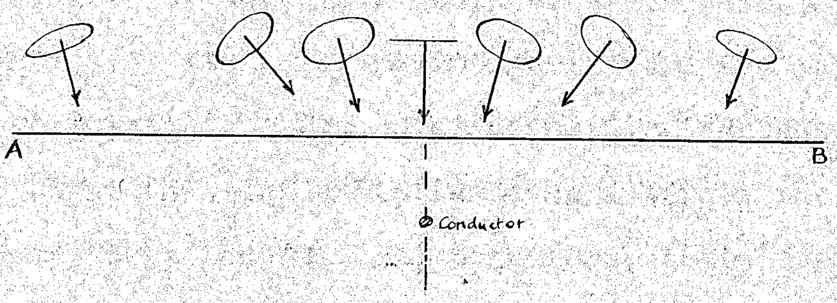


Fig 6.

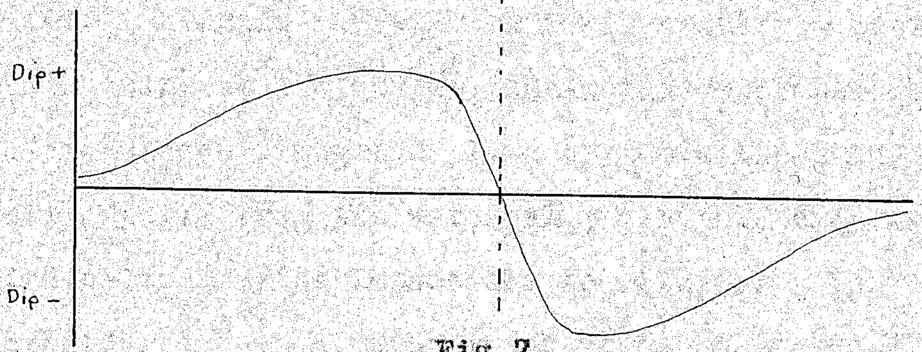


Fig 7.

the point P. The dotted lines represent the normals to the plane of the search coil, and the angles made by these with the vertical is termed the dip of the coil In fig 7., the dips are shown plotted for a traverse from A to B. Note that the dip increases at first and then rapidly decreases to zero over the orebody: after crossing the orebody, the dip rapidly increases in the opposite direction, rises to a maximum and finally decreases as the secondary field grows weaker with distance. Outside the points A and B , the field from the orebody is too weak to have much effect compared with the primary field, and the dip becomes zero again.

The single coil apparatus has been used very successfully and is the basis of the methods used by the Radiore Co and other geophysical concerns now operating. Its chief advantage is ease and rapidity of manipulation. The outstanding disadvantage would seem to be that the elliptical character of the field is completely ignored, and therefore, that an excellent source of information about the orebody is neglected. Again in rough or hilly country , especially with veins at low angles, the method operates at a disadvantage since the plane of the ellipse may depart greatly from the vertical. In this case a zero indication may be impossible to obtain.. For example consider a vein dipping at 45° as shown in the block diagram fig 8a. The hillside slopes at 45° and the oscillator is situated downhill in the direction AB. In fig 8b, the oz axis is horizontal and parallel with thelower edge MN of the block diagram. ab, a'b' and a"b" are projections of the vector ϕ_2 in the xy, xz and yz planes respectively, while ac, a'c' and a"c" are the corresponding projection of the vector ϕ_1. It is

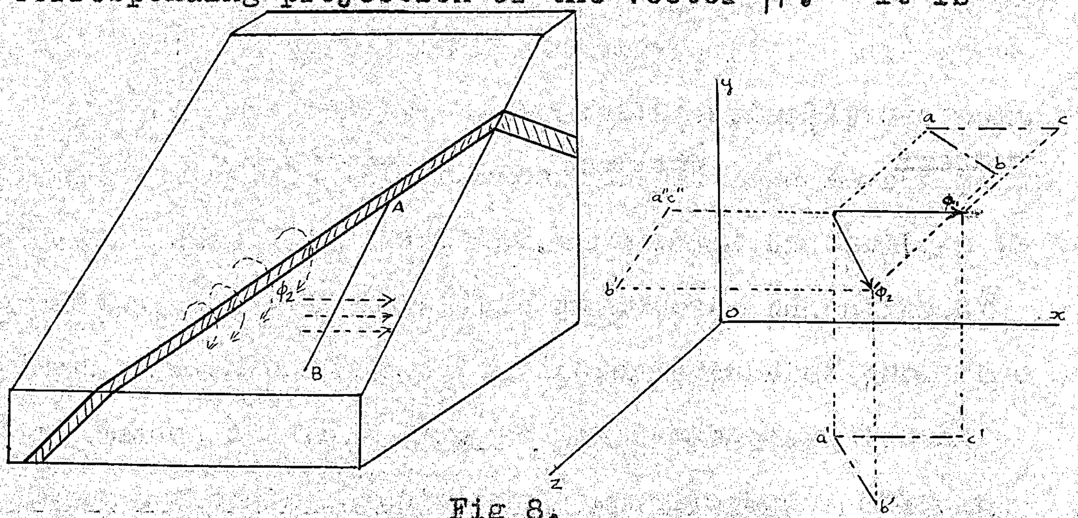


Fig 8.

difficult to draw the ellipse clearly without complicating the diagram, but it can be seen that the ellipse is flat lying with the major axis, depending on the phase difference, in the approximate direction R, pointing diagonally downward to the right. In this case, it would be impossible to place a single search coil with its plane at right-angles to the plane of the ellipse, parallel to ϕ_1, and also with it's fixed axis sighted towards, and lying in, the plane of the antenna, all at the same time. This means that a zero indication would not be obtained in any position of the coil.

The Triple Coil Apparatus.

It is evident from the above discussion that a single search coil can give the general direction of the axes of the ellipse, but that, in order to have more definite information about the field, some other form of search coil is necessary. The triple coil apparatus is an attempt to deal with this matter; it has been experimented with in the laboratory but is still in the experimental stage.

In this method, three coils, mutually at right angles, are used. Since the ellipse lies in one plane only, a coil placed in this plane will be threaded by no flux, and will therefore give zero indication in the detector. This is the sole purpose of one of the coils, which is used to place the other two at right angles to the plane of the ellipse. This is the first adjustment and may be understood by referring to fig 9. Here the ellipse is assumed to be in

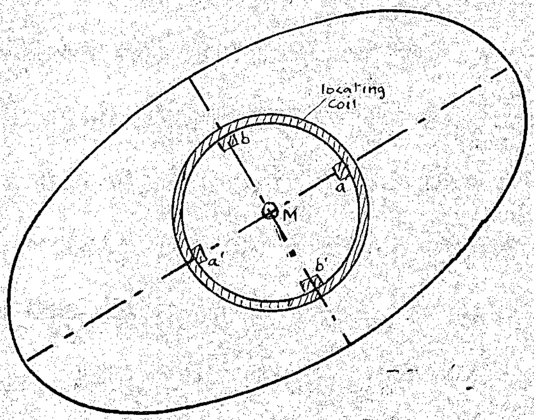


Fig 9.

the plane of the paper. The "locating coil" is shown in the plane of the ellipse, in which position it is threaded by no flux.

The other two coils, a and b, are shown in section and are called the "direction finding" coils. They are mutually at right angles to the locating coil and have an equal number of turns. One of them, the coil b, is variable so that any percentage of the number of turns in coil a can be cut in.

Suppose ,now, that the whole coil assembly be rotated about the axis M, which is perpendicular to the plane of the ellipse, to the position bb' , aa'; in this position, the variable coil will be perpendicular to the major axis, the fixed coil will be perpendicular to the minor axis and the locating coil wil still be in the plane of the ellipse.

Again, in this position, the voltages induced in each direction finding coil will be proportional to the coil area, the number of turnsin the coil and the major or minor axis of the ellipse. Thus, by adjusting the number of turns in the variable coil , the voltages may be made equal and may be made to balance each other out, when connected in a suitable way to a detector. In this way ,the ratio of major to minor axis can be found, using the number of turns and the area of the coils in the calculation.

The scheme of connections is shown diagrammatically and in simplified form in fig 10. Since the

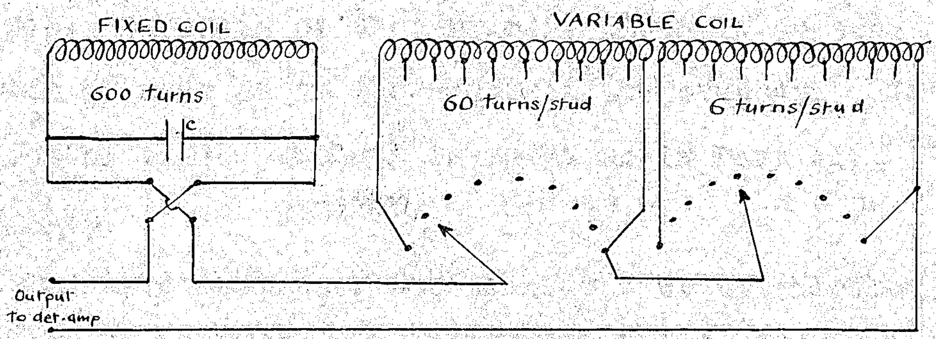


Fig 10.

phase of the voltage in the fixed coil differs from that in the variable coil by $\frac{\pi}{2}$, a condenser is connected across the former and tuned to resonance. The voltage across the condenser C is balanced against the voltage induced in the turns in circuit in the variable coil. A detector is connected to the points AB, where at balance, the voltage should be zero. The locating coil has been omitted fron the diagram for simplicity: it is cut in across the points AB when required by means of a two-way switch.

It is evident from the bare outline given here that dip and strike of the plane of the ellipse is obtainable from readings taken with the locating coil; also, that the dips and directions of major and minor axes in the plane of the ellipse, and the ratio of the axes, can be obtained from the setting of the direction finding coils.

For a complete determination of the ellipse, the actual, not relative, values of the axes are required. Fortunately, this would not seem to be necessary: it can, however, be obtained indirectly from the field strength of the primary field if an assumption is made as to the shape of the orebody. The subject of field strength is taken up in Part 11.

Apparatus.

A bare outlineof the principle of the electromagnetic methods of prospecting has been given above. A detailed discussion will follow in part 11. Meanwhile, let us consider the apparatus necessary.

A. The Transmitter.

The transmitting apparatus may be divided into three units: theoscillator, the coil antanna and the power supply.

(1) <u>The Oscillator</u>. This varies according to the need for portability. In level country, where transportation is not a difficulty, a powerful outfit is possible. Usually a single tube having 15 to 25 watts anode dissipation is employed, coupled directly to the antenna. A UX210 or, better, aW.E.212D, working on reduced anode voltage, is sufficient. Some companies use two or more 46's as a class B amplifier of output from a small oscillator. A pure wave form is a great advantage in obtaining a balance at the receiving set, so that anything like maximum output from the transmitter is not possible. Two tubes, working as a push-pull buffer amplifier, would eliminate even harmonics and would appear to be the best. In addition to purity of wave form, it is necessary for the three coil method , to have constant frequency. The oscillator is mounted in a stout wooden case: it can be constructed so thatvthe weight is about 20 pounds.

(2) <u>The Coil Antenna.</u> A number of patterns of antennas have been tried. The Imperial Geophysical Experimental Survey in Australia used a loop consisting of ten turns ow wire, 8 feet square, mounted on a pole about 15 feet long: the whole antenna was collapsible. The Radiore Co uses a small circular

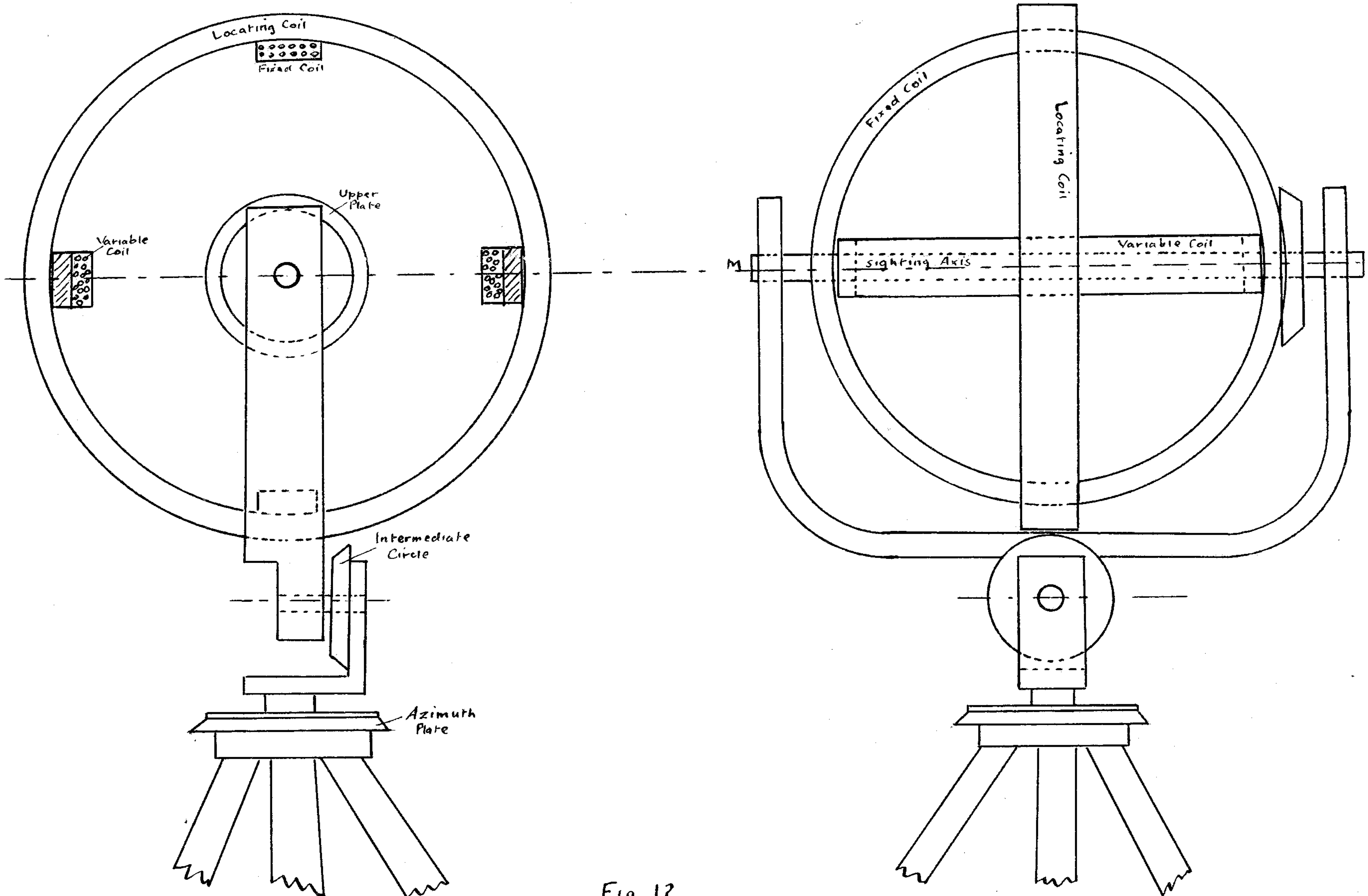
coil, "doughnut" shape, having many turns, mounted on atripod withturntable. Other companies use large triangular or irregular loops, supported on poles. A requirement for a large primary field is that the coil which is used as the tank inductance of the oscillator should have as large an inductance and as low an effective resistance as possible. A compromise is necessary since it is an advantage to have a compact coil which is rotatable on the mounting.

(3) <u>The Power Supply</u>. This is the great difficulty when transportation is a problem. A voltage of 500-1000is used for the oscillator plate supply. Hand-cranked generators have been successfully employedfor the single coil method, but the requirement of constant field strength and frequency would seem to demand a steadier source of current for the three coil method. A light air-cooled gasoline motor, driving a small 110 volt a.c. generator with tube rectifier, could be built to weigh less than 100 lbs. This is not unduly heavy, considering that one set up of the transmitter will cover a radius of about 2500 feet, or four fullsized claims. The I.G.E.S. used a storage battery and dynomotor, which would be satisfactory were available. if facilities were available for charging accumulators. The whole transmitting outfit would weigh in the neighbourhood of 200 lbs and would require two pack horses to transport.

B. <u>The Receiving Apparatus</u>.

(1) <u>The Single Coil Method</u>. The single coil is mounted on a tripod, having horizontal and vertical graduated circles, similar to a transit. The coil itself has 50-100 turns of fine wire on a fibre hoop, up to 2 feet in diameter. Sights are set along the diametral axis about which the coil is pivoted so that it may be alignedproperly with the transmitting antenna. The vertical plate records degrees and minutes of angle around this axis: this is used to indicate dip of coil. The horizontal plate records azimuth angles of the coil and is operated in the same way as a transit. These graduated circles do not need the same degree of accuracy as those of the transit: a r reading to ten minutes is more accurate than the setting of the coil.

The detector-amplifier unit operates into a pair of headphones. Unmodulated transmission was employed by the I.G.E.S., with autodyne reception: in this case the received signal in the phones is proportional to the first power of the signal voltage. However, it is very doubtful whether it is advisable to introduce any oscillations into the receiving circuits Some American prospecting companies modulate the transmitter: in this case, the received signal is proportional to the square of the signal voltage, so that an extra stage of amplification is needed. The



Locating Coil
Fixed Coil
Upper Plate
Variable Coil
Intermediate Circle
Azimuth Plate
Fixed Coil
Locating Coil
Variable Coil
Sighting Axis
M
M'

Fig 12

detector unit is carried on the back of the operator or attached to the tripod. The whole set is compact and light. Fig 11 shows a typical circuit, as used

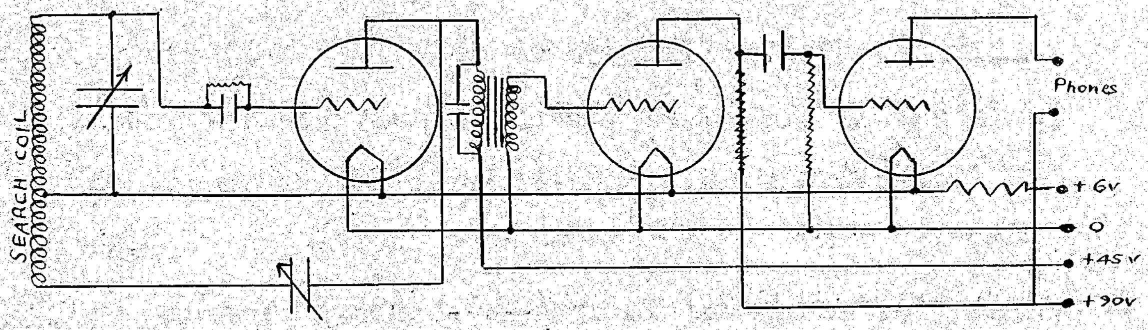


Fig 11.

by the I.G.E.S. The tapping is taken 1/3 of the way along the search coil to produce oscillation. Tuned transformer coupling in the first stage of amplification reduces harmonics, and is followed by a stage of resistance amplification. The total step-up ratio is about 200.

(2) <u>The Three Coil Method</u>. The three search coils are fixed rigidly at right angles to each other. The axis of rotation, MM', of the coils is perpendicular to the locating coil (see fig 12): this is also the sighting axis. Sighting is done through the hollow spindle MM', to which is fixed the graduated circle for finding the dip of the axes in the plane of the ellipse. A U-shaped arm carries the ends of this spindle: to the bottom of the U is attached a hinge with graduated circle. The hinge allows the U to place the locating coil in any plane normal to the U itself. Finally, the wholecoil assembly is rotatable

about a horizontal plate with graduated circle. Thus There are three circles to be read in any region near an orebody. The whole system seems very complicated, but study of the diagrams will help. Actually, when the elliptical structure of the field is thoroughly grasped, the adjustments should take very little time, as the approximate orientation of the ellipse is known.

Another point in favour of the three coil method is that the fixed direction finding coil can be used as a single search coil for regions where there are no indications of an ore body. It would normally be so used, and the other coils would only come into play where the elliptical character of the field was marked. Thus the three coil method includes the single coil method.

The selector switch panel can be made very compact and will probably be mounted inside the coil assembly. The panel contains two selector switches to alter the number of tumns in the variable coil, a reversing switch for the fixed coil, and a switch to connect either the locating coil or the fixed direction finding coil to the detector.

The detector, which will be mounted on one of the legs of the tripod, will be either a detector-amplifier head-phone set or a vacuum tube voltmeter. The latter would be preferable , if sufficient sensitivity can be combined with a rugged construction.

Field Procedure.

The standard method of electromagnetic prospecting, using a single search coil has been described, together with a second method using three coils. Since my object has been to attempt to develop the latter form of apparatus, with a view to the more complete determination of the field ellipse, the field procedure will be described from the standpoint of the three coil apparatus. The procedure for a single coil is included, as the more complex apparatus is used in this way for preliminary investigation.

It is assumed that in indication of a possible orebody has been obtained by geological or mining work, and that the region is to be examined geophysically to determine the location and course of the ore. A base line is first established along the strike of the probable vein system and traverse lines are run at suitable interval of say 100 feet. The coil antenna is then set up on the base line inaa convenient position to examine the area.

To examine the field with the fixed direction finding coil , usedas a single coil, the receiving set is set up, levelled, and a sight taken through the hollow spindle back at the oscillator. The oscillator is then set in operation and the coil antenna is rotated by an assistant so that the axis of the coil assemblycoincides with the vertical plane containing the coil antenna. The switch on the selector panel is set so that the fixed direction finding coil is connected to the detector. Upon revolving the coil assembly round the sighting axis, a position of minimum indication is given in the headphones. If no orebody is present, the fixed direction finding coil should be horizontal when this adjustment has been made.

If there is an ore deposit present, the direction finding coil will not be horizontal, and, in all probability , the minimum indicationin the detector will be very broad. In this case, the dip reading of the fixed direction finding coil is entered in the note book and the apparatus is used to measure the elliptical field.

To measure the elliptical field, the switch is first set to cut in the locating coil. The whole assembly is then rocked about the intermediate axis toward or away from the direction of the oscillator (in the plane of the coil antenna), until the zero position is found. It is possible that the lower plate will have to be unclamped and the coils rotated slightly about the vertical axis, in conjunction with the rocking motion, to bring the locating coil accurately into the plane of the ellipse. When the plane of zero indication has been found, the two lower circles are clamped. The direction finding coils are now at right angles to the plane of the ellipse.

The next operation is to determine the dips and relative lengths of the axes of the ellipse. T The switch is again set so that the fixed direction finding coil only is in circuit, and the coils are rotated around the sighting axis until a position of minimum intensity is found. The minor axis of the ellipse will then be normal to the fixed coil. The second switch is then set so that the fixed and variable coils are balanced against each other, and finally, the selector switches and reversing switch are manipulated until a balance is obtained.

The result of this operation is that three angles and one ratio are obtained. The reading of the lower plate gives the azimuth angle of the strike of the plane of the ellipse; the reading of the intermediate graduated circle gives its dip: while the upper circle gives the dips of the major and minor axes in the plane of the ellipse. Usually, the plane of the ellipse will be practically vertical, and these latter readings may be used uncorrected; in the case of a flat lying ellipse, the actual azimuth and inclination of the axes may be easily computed.

The Choice of Frequency.

The voltage induced in the orebody, and that induced in the coils of the receiver, are proportional to the frequency; the first is proportional to the first power, and the second to the square. Therefore, a high frequency would appear to be of advantage in inducing a large current in the orebody, and hence, in producing a large secondary field. Unfortunately, this is only partially the case, since the higher the frequency, the greater the absorption by eddy currents in the overburden and overlying rocks. A point is reached where an increase in frequency produces a reduction in inductive effect, owing to the low penetration of the magnetic field. This absorption effect will be discussed in Part II. From experimental results in caves and tunnels, it was found that frequency of 20-30 kilocycles gave the best results for penetration.[1] It would therefore appear that some frequency in this band would be the best choice. However, the design of the oscillator will enter into the problem, and it will probably be found that there is difficulty in obtaining enough inductance in the antenna circuit for efficient operation, and that a higher frequency will give better results.

1. Eve & Keys. Nature, vol. 124, page 178, 1929.

Electrical Properties of Rocks and Minerals.

It will be evident from the foregoing discussion, that it is the difference in relative conductivity, between the ore minerals and the rocks and overburden surrounding them, which makes the geo-electrical methods possible. We shall now examine the resistivity of these materials.

The following table is taken from the report of the Imperial Geophysical Experimental Survey, and is the result of measurements made on rock in place. The values of resistivities are given in ohms per cm. cube.

	Material.	Resistivity. ohms/cm. cube
A.	Crystalline Rocks. Igneous rocks,	$2\times10^4 - 10^5$
B.	Consolidated Sedimentary Rocks. Shales, Slates, Limestones, etc.	$10^3 - 5\times10^4$
C.	Unconsolidated Formations. Clays, Sands, Glacial Deposits, etc.	$50 - 10^4$
D.	Ore Minerals. (Selected Samples).	
	Sphalerite, Hematite, Stibnite, etc.	$10^3 - 10^6$
	Chalcopyrite, Bornite, Chalcosite, Pyrite, Galena, Pyrrhotite, etc.	$10^{-4} - 1$
E.	Underground Water.	
	Normal Water. (potable).	$10^3 - 10^5$
	Saline Water. 1% NaCl.	75
	5% " .	15
	10% " .	8.25
	20% " .	5.1

The importance of resistivity measurements being made on rock in place, lies in the fact that the presence of water decreases the values considerably. Measurements made in the laboratory on dry specimens, usually show resistivity values many hundreds of times those given in the table. This shows that conduction in rocks is mainly of an electrolytic character, whereas in minerals, the reverse is the case conduction being metallic.

The presence of soluble salts in the water permeating rocks and overburden is likely to have very deleterious effects upon the success of this method of prospecting. First, it diminishes the resistivity ratio between rocks and ore: secondly, it produces a screening awtion, which causes absorp- of the electromagnetic waves. In some districts, as in parts of Australia, wherethe underground waters are highly saline, prospecting by geoelectrical methods may be impossible. In British Columbia, little trouble should be experienced since the waters are, for the most part, fresh and of high resistivity. Again, the presence of soluble salts may give rise to large out of phase fields owing to the relatively low resistivity: these, however, can usually be distinguished at once from orebodies, owing to their uniform quality.

Upon starting to operate in a new district, the average resistivity of country rocks and overburden should be determined by measurements upon material in place. Any standard earth resistance system may be employed, as explained in books on geophysical surveying. The average resistivity of the orebodies cannot usually be obtained on ore in place and must be made on selected samples.

In determining the phase of the secondary field, the resistance and inductance of the ore material must be taken into account. On this matter, there is a pronounced difference of opinion between leading geophysicists. Jakosky[1] considers that the reactance of an orebody is predominantly capacitative, especially in the case of disseminated, faulted and broken ores. In this event, the impedance would decrease with frequency increase. He states that in his experience, many ores, which were practically non-conductors at low frequencies or with direct current, show a very low impedance to high frequency currents. This was found to be the case in desert regions, where no moisture occurs in the orebodies.

On the other hand, Sundberg[2] believes that

1. J.J.Jakosky "Geophysical Prospecting" A.I.M.E. 1929
2. Sundberg " " "

the opposite is the case, and that the reactance, if any, is predominantly inductive. If this is so, the impedance will increase with frequency: it is interesting to note that Sundberg uses low frequency (500-1000 cycles) methods, principally, so that he may be said to have the courage of his convictions.

The theory will be advanced in Part II that the distribution of high frequency currents (skin effect) alone determines the reactance of the orebody, which is nearly constant and equal to the effective resistence. If this is true, the phase angle of the secondary field due to the currents in the ore, is constant and equal to 45° lagging. It would be interesting to test this theory out by measurements on an actual orebody at varying frequencies.

Interpretation of Results.

Upon completion of the field work in an area, the results are taken to the office and a set of "index curves" for each traverse are constructed. The index curve is a graphical way of finding the approximate depth of a conductor below the surface from a consideration of the minor axis of the ellipse. It is more or less empirical, the assumption being made that the primary vector is horizontal and the secondary vector vertical. The construction is as follows, fig 13,

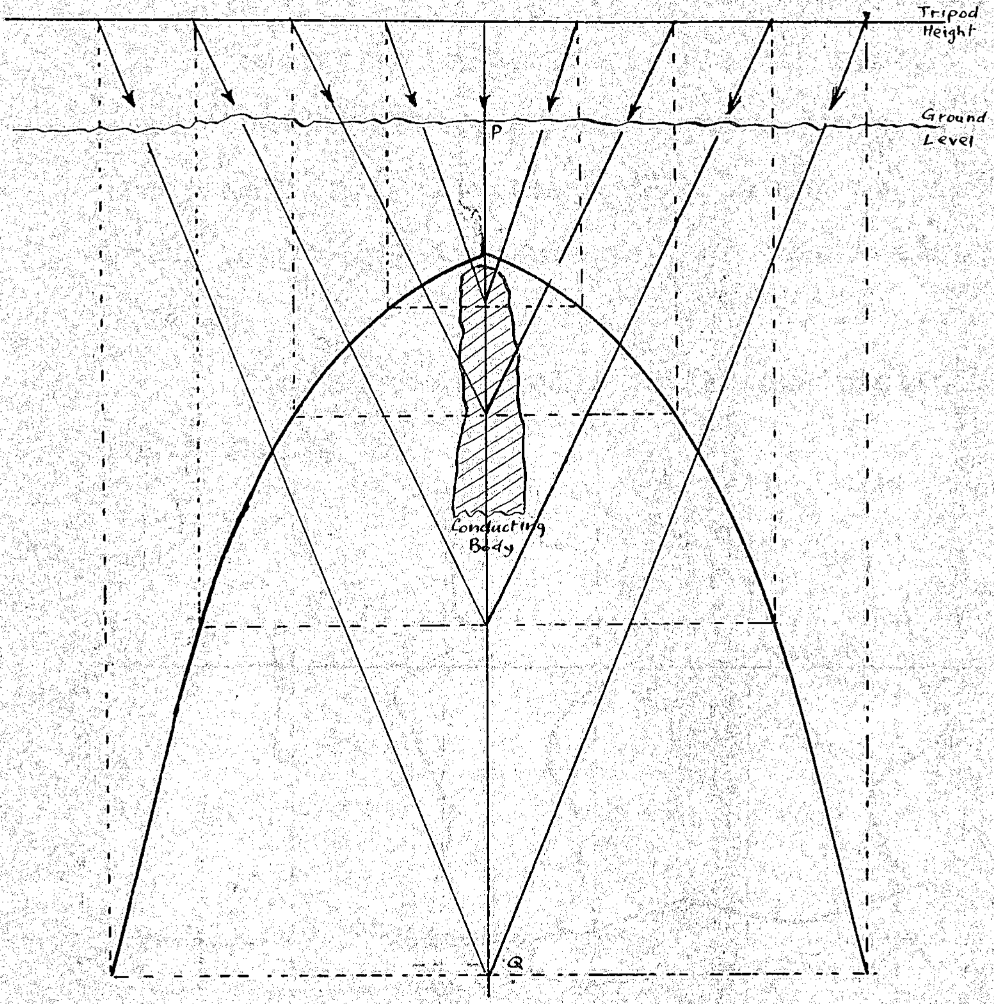


Fig 13.

The conducting body is assumed to be below the point P, and, since the dip angles are the same on each side of P, it is vertical. Continue the dip vectors until they meet the line PQ: then draw the horizontal to meet the vertical line through the point on the traverse where the dip was taken. This procedure gives one point on the curve. Other points are located by a similar construction. The curve in this case, is approximately a parabola, with axis vertical. The depth of the orebody does not necessarily coincide with the apex of the curve owing to the refraction of the electromagnetic waves at the surface: actually, it will be somewhat lower. This construction is of great value since, however shaky it's mathematical foundation may be, it has been found accurately to represent the actual state of affairs.

Traverses and index curves are plotted as shown in fig 14,

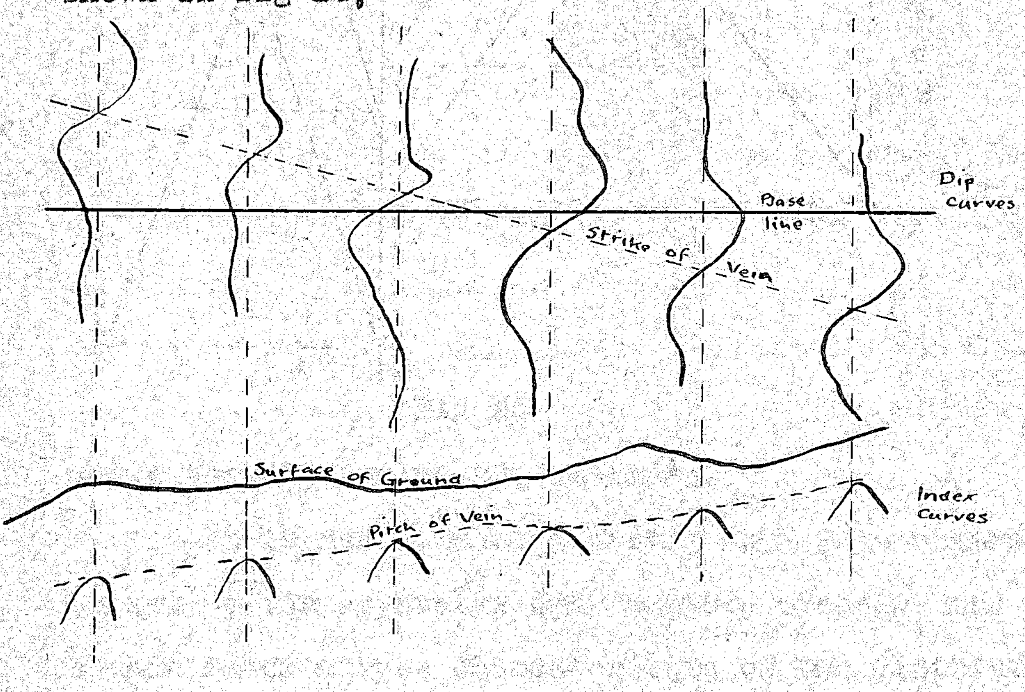


Fig 14.

By making these drawings for each part of the area surveyed, the strike and pitch of underground ore-bodies may be calculated.

A dipping orebody is characterized by unequal dips on either side of the apex. The index curve has the shape shown in fig 15. It is evident that diamond drilling should be started on the side

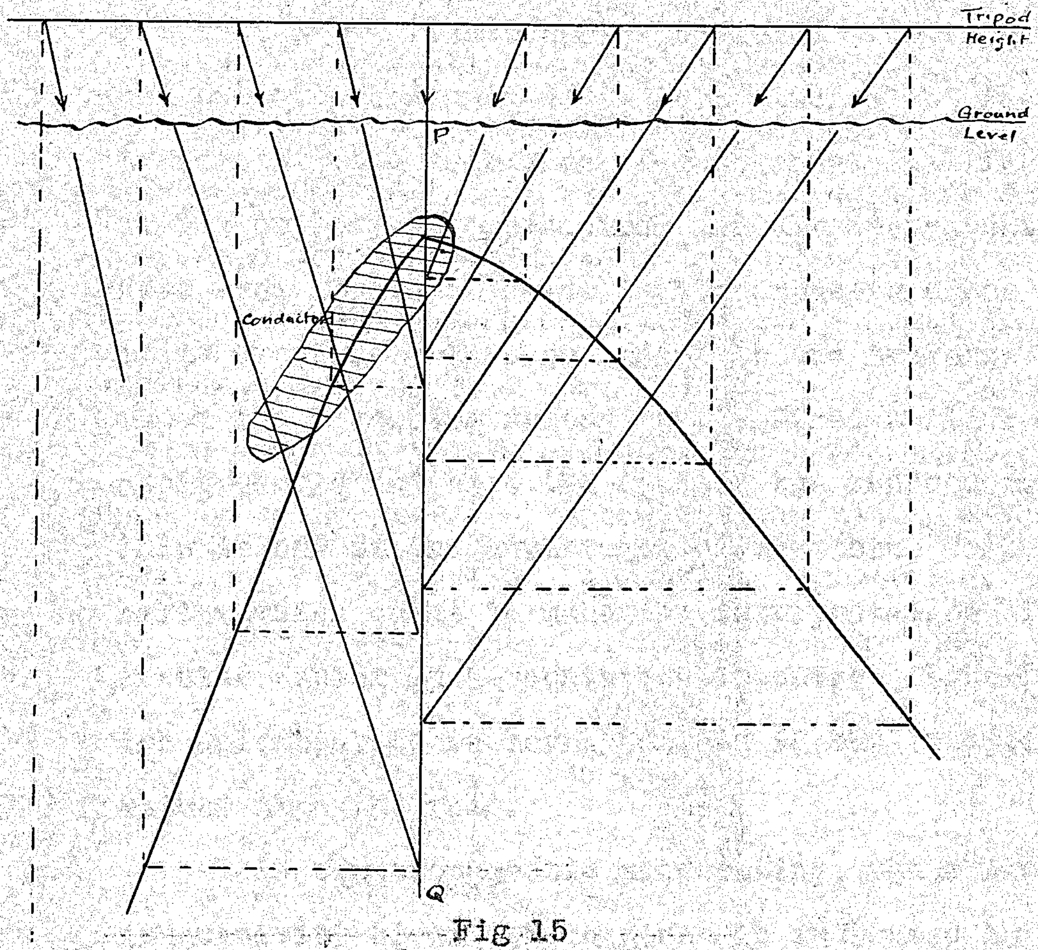


Fig 15

where the dip angles are smallest.

It is usual, when starting work in a district to set up the apparatus over a known orebody, and obtain index curves characteristic of the district. These curves may then be compared with those from an unknown area.

In rugged topography, the form of the index curve will be different in each cas ,depending on local irregularities. The direction finding coils should show up to best advantage in this kind of work, where the observation stations are not necessarily at the same elevation. The proximity of the orebody can then be estimated entirely by the degree of elliptical polarization of the field. Vertical out of phase components may be plotted along the traverse: the magnitude of the vectors willincrease as the orebody is approached, rapidly decrease to zero over it, and increase in the opposite sense as it is crossed. Thus the point on the traverse , at which the reversing switch to the fixed coil is operated, should indicate the apex of the conductor. This is one of the advantages of the three coil apparatus which ought to make the interpretation of results easier in a mountainous country , like British Columbia, where level terrain is the exception rather than the rule.

In electromagnetic prospecting, the ratio of conductivity of different zones is indicated by the instrument. Hence allindications obtained do not necessarily indicate orebodies. Wet shear zones or schistose graphitic rocks will often give indications similar to ore. Geological evidence and practical experience in the district must be relied on fordistinguishing these features. Further study is necessary, along this line, in the field.

The accurate interpretation of geophysical data can only be made in conjunction with existing geological and mining data. Undergground structure is usually very complex; ore shoots swell and pinch, veins have irregular dips, and geological features are complicated by numerous faults. The geophysicist must also be a geologist and should be assisted by the best local knowledge available. These methods are no longer in their infancy, but they must be intelligently applied to give satisfactory results. The common belief that the existance of mineral is indicated by a mysterious electrical appliance is erroneous. Each indication of mineral must be examinedon it's own merits and correlated with the geology of the terrain. One application of geophysical work is the elimination of barren areas before the expense of drilling or underground work is incurred.

Costs of Geophysical Exploration.

Approximate costs are not easy to estimate. They vary with the nature of the ground, thickness of vegetation, accessibility for transportation, etc. An average party of 3 to 5 men should cover from 8 to 25 acres per day, exclusive of preliminary surveying, laying out of base lines and traverses, The I.G.E.S. gives $3.25 per acre as the average cost under Australian conditions. In British Columbia, The cost should be much the same. A few thousand dollars laid out in this way may save many thousands in later development. These methods are for intensive examination of areas suspected of mineralisation. They are, in general, not suitable for reconnaissance prospecting,as there are cheaper methods for this work.

Part II.

Mathematical Consideration of the Electromagnetic Fields in the Region of an Orebody.

General Case of Two Fields differing in Direction and Time Phase. Consider the case shown in fig 16. The vector $H_2 \sin(\omega t - \alpha)$ makes an angle $\frac{\pi}{2} + \theta$ with the vector $H_1 \sin \omega t$.

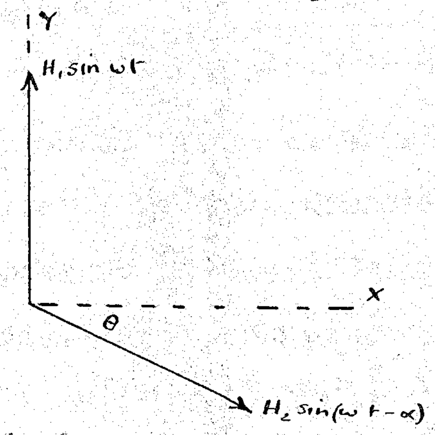


Fig 16.

Resolving these vectors along X and Y axes, we have

$$X = H_2 \sin(\omega t - \alpha) \cos\theta$$
$$Y = H_1 \sin \omega t - H_2 \sin(\omega t - \alpha) \sin\theta$$

Next eliminate ωt, $\qquad H_2 = \dfrac{X}{\sin(\omega t - \alpha)\cos\theta}$

and $\qquad \omega t = \sin^{-1}\dfrac{X}{H_2 \cos\theta} + \alpha$

Substituting,

$$Y = H_1 \sin\left\{\sin^{-1}\frac{X}{H_2\cos\theta} + \alpha\right\} - X\frac{\sin\theta}{\cos\theta}$$

$$= H_1\left\{\sin\left(\sin^{-1}\frac{X}{H_2\cos\theta}\right)\cos\alpha + \cos\left(\sin^{-1}\frac{X}{H_2\cos\theta}\right)\sin\alpha\right\} - X\frac{\sin\theta}{\cos\theta}$$

$$= H_1\left\{\frac{X}{H_2\cos\theta}\cos\alpha + \frac{\sqrt{H_2^2\cos^2\theta - X^2}}{H_2\cos\theta}\sin\alpha\right\} - X\frac{\sin\theta}{\cos\theta}$$

and squaring, and rearranging,

$$\frac{H_2^2\cos^2\theta - X^2}{H_2^2\cos^2\theta}\sin^2\alpha = \frac{Y^2}{H_1^2} - \frac{2XY}{H_1}\left\{\frac{\cos\alpha}{H_2\cos\theta} - \frac{\sin\theta}{H_1\cos\theta}\right\}$$
$$+ X^2\left\{\frac{\cos^2\alpha}{H_2^2\cos^2\theta} - \frac{2\cos\alpha\sin\theta}{H_1 H_2\cos^2\theta} + \frac{\sin^2\theta}{H_1^2\cos^2\theta}\right\}$$

This equation is of the general type,

$$Ax^2 + 2Bxy + Cy^2 = 1$$

which is the equation of an ellipse. The semi-axes are given by the relation

$$\frac{1}{m^2} = \frac{1}{2}(A+C) \pm \frac{1}{2}\sqrt{(A-C)^2 + 4B^2}$$

The above discussion can be extended to cover any number of fields in three dimensions by adding the axial components of the different fields. Thus the resultant axial components are:

$$X = H_x^1 \sin(\omega t + \alpha) + H_x^2 \sin(\omega t + \alpha') + H_x^3 \sin(\omega t + \alpha'')$$

$$Y = H_y^1 \sin(\omega t + \alpha) + H_y^2 \sin(\omega t + \alpha') + H_y^3 \sin(\omega t + \alpha'')$$

$$Z = H_z^1 \sin(\omega t + \alpha) + H_z^2 \sin(\omega t + \alpha') + H_z^3 \sin(\omega t + \alpha'')$$

where H_x^1 & H_y^1, H_z^1, H_x^2 H_y^2 H_z^2 &, H_x^3 H_y^3 H_z^3, are components of resultant fields H_1 H_1 H_3 respectively, and $\alpha\,\alpha'\,\alpha''$ are the respective phase angles. Then

$$X = H_x^1\{\sin\omega t\cos\alpha + \cos\omega t\sin\alpha\} + H_x^2\{\sin\omega t\cos\alpha' + \cos\omega t\sin\alpha'\} + H_x^3\{\sin\omega t\cos\alpha'' + \cos\omega t\sin\alpha''\}$$

$$Y = H_y^1\{\quad do \quad\} + H_y^2\{\quad do \quad\} + H_y^3\{\quad do \quad\}$$

$$Z = H_z^1\{\quad do \quad\} + H_z^2\{\quad do \quad\} + H_z^3\{\quad do \quad\}$$

and simplifying,

$$X = \{H_x^1\cos\alpha + H_x^2\cos\alpha' + H_x^3\cos\alpha''\}\sin\omega t + \{H_x^1\sin\alpha + H_x^2\sin\alpha' + H_x^3\sin\alpha''\}\cos\omega t$$

$$Y = \{H_y^1\cos\alpha + H_y^2\cos\alpha' + H_y^3\cos\alpha''\}\sin\omega t + \{H_y^1\sin\alpha + H_y^2\sin\alpha' + H_y^3\sin\alpha''\}\cos\omega t$$

$$Z = \{H_z^1\cos\alpha + H_z^2\cos\alpha' + H_z^3\cos\alpha''\}\sin\omega t + \{H_z^1\sin\alpha + H_z^2\sin\alpha' + H_z^3\sin\alpha''\}\cos\omega t$$

The coefficients of $\sin\omega t$ and $\cos\omega t$ are constants, and therefore, we obtain two further equations.

$$R_1 = C_1 \sin\omega t$$

$$R_2 = C_2 \cos\omega t$$

where R_1, R_2, are the resultant vectors from the compounding of the sine terms and the cosine terms respectively. The vectors R_1 and R_2 are 90° out of phase, so we obtain a single resultant vector, which rotates in one plane and is elliptically polarized.

It is therefore evident that, when many fields of like frequency exist, all differing in space and in time phase, there is always one plane through any point, which is parallel to the resultant field, and, furthermore, that the resultant field is elliptically polarized in that plane. In the three coil method, the locating coil finds this plane, while the direction finding coils determine the ratio of the axes of the ellipse and their direction.

Criteria for Complete Determination of the Ellipse.

For a complete determination of the ellipse, we need the following information.

(1) The direction of one axis. This is usually the minor axis and is approximately given by the single coil method. It is exactly defined by three angles.

(2) The direction of the other axis:

since the axes are mutually at right angles, this may be calculated.

(3) The magnitude of the axes. The actual magnitudes can only be obtained by assuming an approximate shape for the orebody, calculating the equation of the corresponding ellipse and using the measurements obtained at the point. The ratio of the axes may be obtained as explained above, and is sufficient.

(4) The phase of one of the axes with respect to the primary field. This cannot be measured. The problem requires further thought as the information might prove very useful if it could be easily obtained.

__Calculation of The Field for a Simple Case.__

Let us now turn to the calculation of a very simple case, i.e. that of a long straight conductor M, of small radius, at depth d. The origin of coordinates is at O. The return conductor is considered to be at a depth great enough that the field due to it is negligible. Consider the field at point P.

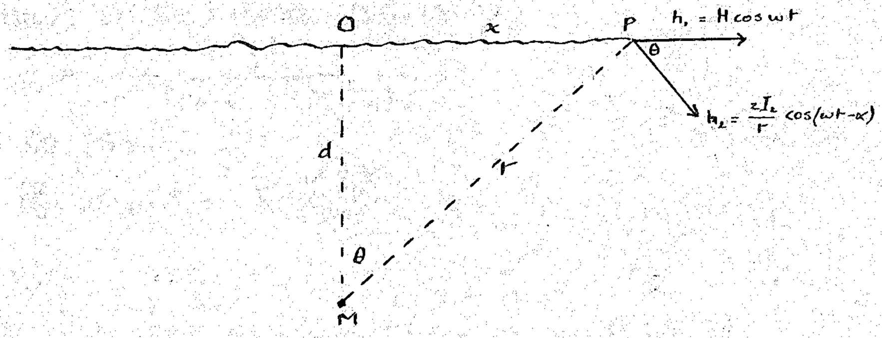


Fig 17.

Let $h_1 = H_1 \cos\omega t$ and $h_2 = \frac{2I_2}{r}\cos(\omega t-\alpha)$ be the primary and secondary fields at the point P, distant x from O. Fig 18 shows the vectors at the point P.

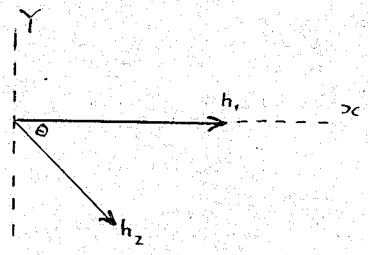


Fig 18.

$$h_2 = \frac{2I_2}{\sqrt{x^2+d^2}}\cos(\omega t-\alpha)$$

Resolving along axes,

$$Y = -h_2 \sin\theta$$

$$X = h_1 + h_2\cos\theta$$

or

$$\left.\begin{aligned} Y &= -\frac{2I_2}{\sqrt{x^2+d^2}}\sin\theta\cos(\omega t-\alpha) \\ X &= H_1\cos\omega t + \frac{2I_2}{\sqrt{x^2+d^2}}\cos\theta\cos(\omega t-\alpha) \end{aligned}\right\} \quad (1)$$

Let $\frac{2I_2}{\sqrt{x^2+d^2}} = A$ and $H_1 = B$

Then we have two equations,

$$Y = -A\sin\theta\cos(\omega t-\alpha) \qquad (2)$$

$$X = B\cos\omega t + A\cos\theta\cos(\omega t-\alpha) \qquad (3)$$

Next eliminate ωt : from (2)

$$A = -\frac{Y}{\sin\theta\cos(\omega t-\alpha)}$$

Substitute in (3)

$$X = B\cos\omega t - Y\frac{\cos\theta}{\sin\theta} \qquad (4)$$

Again

$$\omega t = \cos^{-1}\left(-\frac{Y}{A\sin\theta}\right) + \alpha$$

$$\therefore \cos\omega t = -\frac{Y}{A\sin\theta}\cos\alpha + \frac{\sqrt{A^2\sin^2\theta - Y^2}}{A\sin\theta}\sin\alpha$$

Substitute in (4)

$$X = -\frac{BY}{A\sin\theta}\cos\alpha + \frac{B\sin\alpha\sqrt{A^2\sin^2\theta - Y^2}}{A\sin\theta} - Y\frac{\cos\theta}{\sin\theta}$$

$$\therefore X\sin\theta + \frac{BY}{A}\cos\alpha + Y\cos\theta = \frac{B}{A}\sin\alpha\sqrt{A^2\sin^2\theta - Y^2}$$

Squaring and dividing by $\sin^2\theta$

$$X^2 + \frac{2XY}{\sin^2\theta}\left\{\frac{B}{A}\sin\theta\cos\alpha + \sin\theta\cos\theta\right\} + \frac{Y^2}{\sin^2\theta}\left\{\frac{B^2}{A^2} + 2\frac{B}{A}\cos\alpha\cos\theta + \cos^2\theta\right\} = B^2\sin^2\alpha \qquad (5)$$

Now substitute in the values,

$$\sin\theta = \frac{x}{\sqrt{x^2+d^2}} \qquad \cos\theta = \frac{d}{\sqrt{x^2+d^2}} \qquad B = H_1 \qquad A = \frac{2I_2}{\sqrt{x^2+d^2}}$$

and we get,

$$X^2 + 2XY\left[\frac{H_1^2(x^2+d^2)}{2I_2 x}\cos\alpha + \frac{d}{x}\right] + Y^2\left[\frac{H_1^2(x^2+d^2)^2}{4I_2^2x^2} + \frac{H_1(x^2+d^2)d}{I_2x^2}\cos\alpha + \frac{d^2}{x^2}\right] = H_1^2\sin^2\alpha \qquad (6)$$

This equation is of the type.

$$AX^2 + 2BXY + CX^2 = 1$$

and the lengths of the semi-axes are given by:

$$\frac{1}{m^2} = \frac{1}{2}(A+C) \pm \frac{1}{2}\sqrt{(A-C)^2 + 4B^2} \qquad (7)$$

and the angle θ between the major axis and the vertical is defined by:

$$\tan 2\theta = \frac{2B}{A-C} \qquad (8)$$

When the appropriate values of A, B, and C, obtained from eqn. (6), are substituted in eqns. (7) and (8), our information about the ellipse is complete.

Before we can use these equations, some relationship between H_1 and I_2 must be found. An accurate computation of I_2 is not generally possible, owing to the uncertainty regarding the electrical properties of the circuit and absorption in the overburden. However, we will make the simplifying assumption that the primary and secondary fields are equal at the point O: then,

$$H_1 = \frac{2 I_2}{d}$$

and substituting in equation (6), we get,

$$X^2 + 2XY\left[\frac{x^2+d^2}{dx}\cos\alpha + \frac{d}{x}\right] + Y^2\left[\frac{(x^2+d^2)^2}{d^2x^2} + 2\,\frac{x^2+d^2}{x^2}\cos\alpha + \frac{d^2}{x^2}\right] = H_1^2\sin^2\alpha \qquad (9)$$

We will further assume that the reactance of the orebody is negligible. It was shown on page 1 that the secondary field lags the primary field by an angle $\frac{\pi}{2} + \phi$, where $\phi = \frac{X}{R}$; in this case, $\phi = 0$ and $\alpha = \frac{\pi}{2}$. The expression, then, is further simplified to

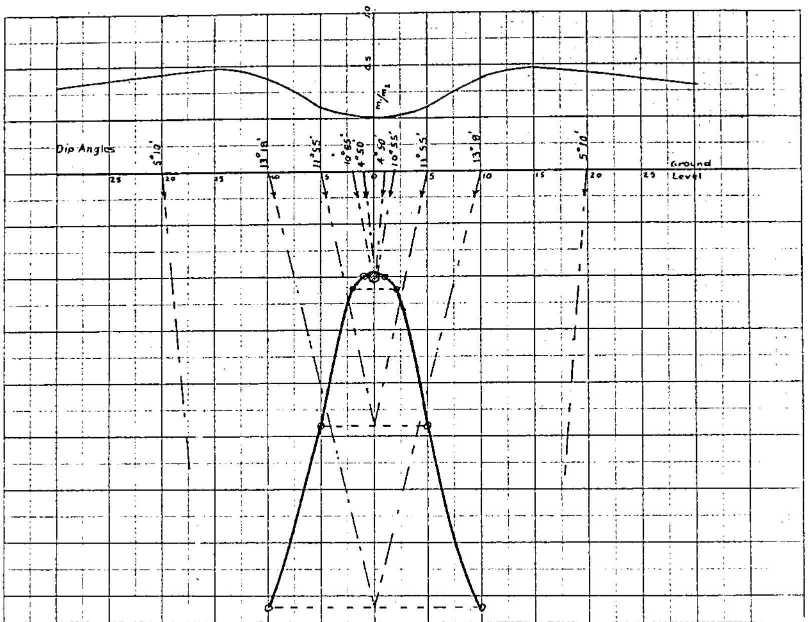
$$X^2 + 2XY\frac{d}{x} + Y^2\left[\left(\frac{x}{d} + \frac{d}{x}\right)^2 + \frac{d^2}{x^2}\right] = H_1^2 \qquad (10)$$

Applying eqn. (7), we find that the ratio of the axes is

$$\frac{m_1}{m_2} = \sqrt{\frac{1 + \left(\frac{d}{x} + \frac{x}{d}\right)^2 + \frac{d^2}{x^2} - \sqrt{\left(1 - \left(\frac{x}{d} + \frac{d}{x}\right)^2 - \frac{d^2}{x^2}\right)^2 - 4\frac{d^2}{x^2}}}{1 + \left(\frac{x}{d} + \frac{d}{x}\right)^2 + \frac{d^2}{x^2} + \sqrt{\left(1 - \left(\frac{x}{d} + \frac{d}{x}\right)^2 - \frac{d^2}{x^2}\right)^2 - 4\frac{d^2}{x^2}}}} \qquad (11)$$

Again, applying eqn. (8), the angle θ is given by

$$\tan 2\theta = \frac{2\frac{d}{x}}{1 - \left(\frac{x}{d} + \frac{d}{x}\right)^2 - \frac{d^2}{x^2}} \qquad (12)$$



CALCULATED INDEX CURVE

Fig 19

In fig 19 is shown a calculated traverse of the conductor, with ellipses drawn to scale; the ratio of the axes is given as a curve on a horizontal base of feet. The conductor is presumed to be buried at a depth of 10 feet below the point O. The index curve is plotted by extending the minor axis to meet the vertical through O, and then taking the corresponding ordinate on the vertical line through the observation station. An examination of the diagram shows that the ratio of the axes is zero above the conductor, increases to a maximum at about ~~ten~~ 20 feet to right or left and decreases again at points farther removed. Again, the dip of the minor axis is zero at O, increases to a maximum at about ten feet to right or left and decreases again at points further removed. The index curve is seen to be of complex form and to come to an apex, just above the conductor. This is the type of curve given by the three coil apparatus.

The single coil apparatus gives a different shape of curve. By taking eqn. (9) and letting $\alpha = 0$, we get the following expression;

$$X^2 + 2XY\left[\frac{x^2+d^2}{dx} + \frac{d}{x}\right] + Y^2\left[\frac{(x^2+d^2)^2}{d^2x^2} + 2\,\frac{x^2+d^2}{x^2} + \frac{d^2}{x^2}\right] = 0$$

or, $$\left[X + Y\left\{\frac{x^2+d^2}{dx} + \frac{d}{x}\right\}\right]^2 = 0 \qquad (13)$$

Thus by ignoring the phase angle α, the elliptical polarization dissappears. The angle of dip is given by,

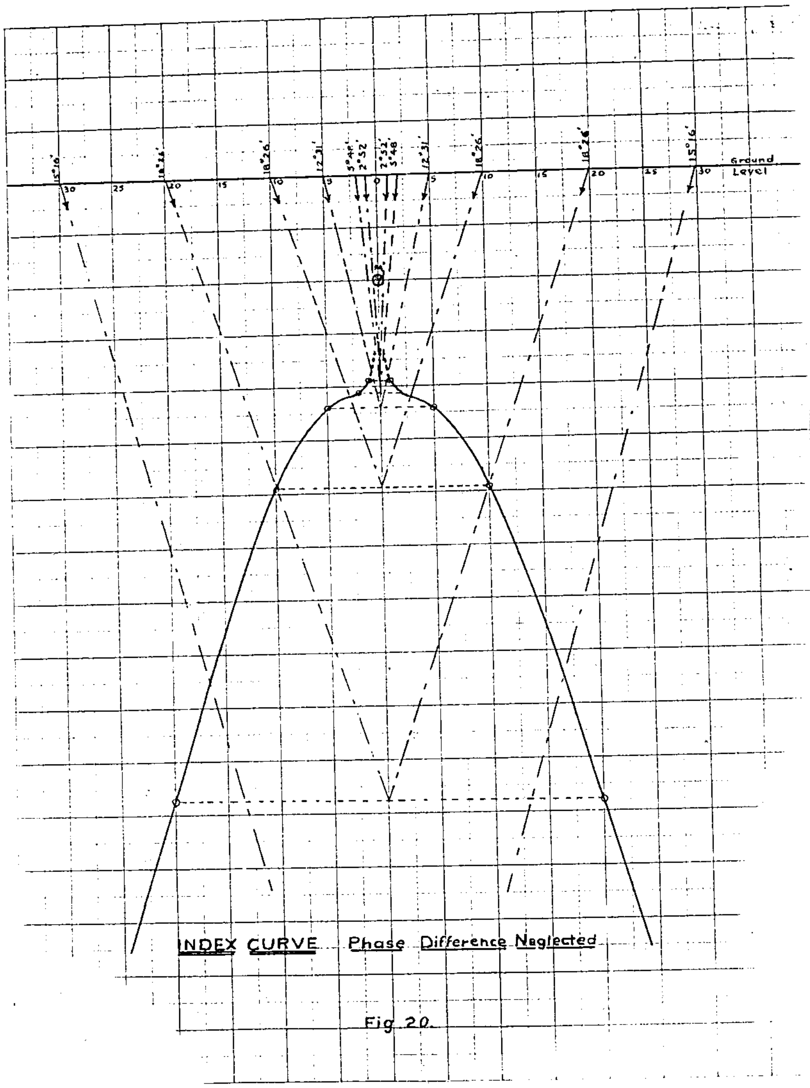


Fig 20.

$$\tan\theta = \frac{Y}{X} = -\frac{dx}{x^2+2d^2} \qquad (14)$$

The index curve is shown, plotted on this basis in fig 20. The curve comes to a much sharper apex; there is difficulty in determining the exact position of the conductor.

More complicated cases such as dipping veins and flat lying deposits , can be treated in the same manner and will also give theoretical solutions. Allowance may also be made for the return conductor. The object of calculations of this kind is to get an idea of the type of observation to expect in the field. Each different shape of conductor gives it's own type of curve; therefore, calculations of all the cases should be made before going into the field. Much work has been done on models, particularly in the case of the sphere.[1] This is an easy case to calculate and the results have confirmed the theoretical work by experiment. Work has also been done on drainpipes,[2] with good results.

The need for this work lies in the fact that, while it is easy to obtain thesolutions for the field given the shape of the conductor, the converse is not easy and is better obtained empirically.

(1) Mason, Max. "Geophys. Prospecting" A.I.M.E.1929
(2) Report I.G.E.S. p.286 Camb. Univ Press. 1931

The Nature of the field from the Oscillator.

It is now necessary to consider the nature of the magnetic field which originates at the oscillator. The argument follows Maxwell's fundamental principles, and will be followed throughout in detail, in order to obtain convenient expressions for practical use later.[1]

Equations of the Electromagnetic Field. In considering the nature of the electromagnetic field, we have two quantities to examine. First, there is the magnetic circuit, which may be symbolized by,

$$\mathbf{B} = \mu \mathbf{H} = \mathbf{H} + 4\pi \mathbf{I} \qquad (15)$$

where $\mathbf{B}$ = flux density, $\mathbf{H}$ = magnetic field intensity and $\mathbf{I}$ = intensity of magnetization in the region, due to the magnetizing force $\mathbf{H}$: μ = permeability of the medium. These quantities, except μ, are vectors, though they normally operate in the same direction in an isotropic medium. Also we have the relationship,

$$\nabla \cdot \mathbf{B} = 0 \qquad (16)$$

This is a statement of Gauss' Law, and postulates that as much magnetism leaves any region as enters it.

Secondly, we have the electric circuit,

$$\mathbf{D} = k \mathbf{E} = \mathbf{E} + 4\pi \mathbf{P} \qquad (17)$$

(1) The material for this discussion was taken from the following sources:
J.H.Jeans "Electricity and Magnetism" C.U.P.
Page. "Intro. to Theoretical Physics"
Gibb and Wilson "Vector Analysis"

where $\mathbf{D}$ = electric displacement or induction in the region, $\mathbf{E}$ = electric intensity which produces the polarization $\mathbf{P}$ in the dielectric: k is the dielectric constant of the medium. The latter is ~~only~~ $\frac{1}{4\pi}$ times the dielectric constant as usually measured, but is used here in this form to retain the symmetry between equations 15 and 17. These quantities, also, are vectors though normally in the same line, i.e. in vacuum or isotropic medium. We have also, Poissons equation,

$$\nabla . \mathbf{D} = \nabla . (\mathbf{E} + 4\pi \mathbf{P}) = -4\pi\rho \qquad (18)$$

where ρ density of free electricity in the region.

Ampere's rule states that the line integral of the magnetic force $\mathbf{H}$, round any region, is equal to 4π times the surface integral of the current density $\mathbf{J}$, taken over the same region. Therefore,

$$\int_o \mathbf{H} . d\mathbf{r} = 4\pi \int_\sigma \mathbf{J} . \mathbf{n}\, d\sigma \quad \#$$

where $\mathbf{n}$ is unit vector in direction of vector $d\sigma$.

This can be transformed into the following, by Stokes Theorem,

$$\int_o \mathbf{H} . d\mathbf{r} = 4\pi \int_\sigma \mathbf{J} . \mathbf{n}\, d\sigma = \int_\sigma \nabla \times \mathbf{H} . \mathbf{n}\, d\sigma$$

or, in other words,

$$\mathrm{Curl}\ \mathbf{H} = 4\pi \mathbf{J} \qquad (19)$$

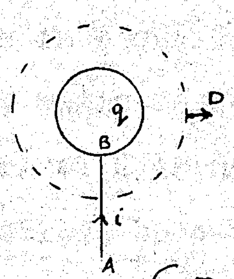
\# The following integral notation is used.
$\int_o$ denotes line integral
$\int_\sigma$ denotes surface integral
$\int_v$ denotes volume integral.

There are points to notice about this result; first, that it applies only for steady direct current: in magnetostatics,

$$\text{Curl}\,\mathbf{H} = -\nabla \times \nabla \psi \equiv 0$$

where ψ is the magnetic potential. Secondly, $\nabla . \mathbf{J} = 0$ for steady direct current. This latter statement says that current does not collect at any point, or that as much leaves a region as enters it.

Let us now passon to a region where the current density is changing. Consider the case of a body which is being charged by means of a current i flowing through a wire AB (see fig). Let charge



on body at any instant, be q. Describe a surface around the body, and call the displacement thereon .

Then, $\int_\sigma \mathbf{D} . \mathbf{n}\, d\sigma = 4\pi q$

and , differentiating,

$$\int_\sigma \frac{\partial \mathbf{D}}{\partial t} . \mathbf{n}\, d\sigma = 4\pi \frac{\partial q}{\partial t}$$

But i = $\mathbf{J}$,integrated over the area of the wire = $\frac{\partial q}{\partial t}$

$$\therefore \int_\sigma \frac{\partial \mathbf{D}}{\partial t} . \mathbf{n}\, d\sigma = -4\pi \int_\sigma \mathbf{J} . \mathbf{n}\, d\sigma$$

The negative sign before the second integral is on account of the inward flow of current, whereas $\mathbf{n}$, the unit vector normal tomthe surface element $d\sigma$, is drawn with the positive direction outward, From this we get,

$$\int_\sigma \left(\frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J}\right).\mathbf{n}\, d\sigma = 0$$

and, by Gauss' vector theorem,

$$\int_\sigma \left(\frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J}\right).\mathbf{n}\, d\sigma = \int_v \nabla.\left(\frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J}\right) dv = 0$$

and, therefore,

$$\nabla.\left(\frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J}\right) = 0 \qquad (20)$$

$\frac{\partial \mathbf{D}}{\partial t}$ is called the "displacement current"; it produces all the effects of a true current in regions where the electric displacement is changing. We may therefore write the total current, actual and displacement, into eqn. 19. We then get, for regions where the current density is changing,

$$\text{Curl } \mathbf{H} = \frac{4\pi}{c}\left(\mathbf{J} + \frac{1}{4\pi}\frac{\partial \mathbf{D}}{\partial t}\right) \qquad (21)$$

Here $\mathbf{H}$ is in electromagnetic units, while $\mathbf{J}$ and $\mathbf{D}$ are in electrostatics units: the constant c is the ratio between the units, which is 3×10^{10}. The curl of H is not necessarily zero as in magneto-statics, because, while the current density J may zero, as in the region outside a wire carrying current, the displacement current may have a value, if the current is changing.

We have now got the equations of the electric and magnetic circuits themselves, and also a cross relation between the magnetic intensity H and the displacement D. We now proceed to develop another cross relationship between the electric intensity E and the magnetic flux density B.

Faradays law states that the line integral of the electric intensity, round any closed circuit, is numerically equal to the time rate of decrease of magnetic flux through it: in other words,

E.M.F. = e = $\frac{dN}{dt}$ where N is the number of line linkages. Therefore

$$e = \int_0 \mathbf{E}.d\mathbf{r} = -\int_\sigma \frac{\partial \mathbf{B}}{\partial t}.\mathbf{n}\, d\sigma$$

and by Stokes' Theorem

$$\int_0 \mathbf{E}.d\mathbf{r} = \int_\sigma \nabla\times\mathbf{E}.\mathbf{n}\, d\sigma = -\frac{1}{c}\int_\sigma \frac{\partial \mathbf{B}}{\partial t}.\mathbf{n}\, d\sigma$$

and, therefore,

$$\mathrm{Curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad (22)$$

<u>Wave Propagation of Electrical Effects.</u>

In eqns. 21 and 22, we have obtained the necessary information to proceed. Since we are primarily interested in the region outside the wire carrying the current, we may put $\mathbf{J} = 0$. Then

$$\mathrm{Curl}\,\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{K}{c}\frac{\partial \mathbf{E}}{\partial t} \qquad (21a)$$

$$\mathrm{Curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\frac{\mu}{c}\frac{\partial \mathbf{H}}{\partial t} \qquad (22a)$$

Substituting eqn. 22a in 21a,

$$\nabla\times\nabla\times\mathbf{H} = \frac{K}{c}\frac{\partial}{\partial t}\left(\nabla\times\mathbf{E}\right) = -\frac{\mu K}{c^2}\frac{\partial_2 \mathbf{H}}{\partial t^2}$$

but $\nabla\times\nabla\times\mathbf{H} = \nabla\nabla.\mathbf{H} - \nabla.\nabla\mathbf{H}$

$$= -\nabla.\nabla\mathbf{H} \quad \text{since} \quad \nabla.\mathbf{H} \propto \nabla.\mathbf{B} = 0$$

$$\therefore \nabla.\nabla\mathbf{H} = \frac{K\mu}{c^2}\frac{\partial_2 \mathbf{H}}{\partial t^2} \qquad (23)$$

Similarly

$$\nabla\times\nabla\times\mathbf{E} = -\frac{\mu K}{c^2}\frac{\partial_2 \mathbf{E}}{\partial t^2}$$

$$= \nabla\nabla.\mathbf{E} - \nabla.\nabla\mathbf{E} = -\nabla.\nabla\mathbf{E}$$

since $\nabla.\mathbf{E} = 0$ where no free charge

$$\therefore \nabla.\nabla\mathbf{E} = \frac{\mu K}{c^2}\frac{\partial_2 \mathbf{E}}{\partial t^2} \qquad (24)$$

These are extensions of Laplace's equation to cover regions in the vicinity of moving electric charges. They do not hold for regions containing any free charges. In a gas free charges are negligible.

These equations, by the following transformations ca be put in another form. Let E_x E_y E_z be scalar magnitudes of the vector E along the axes of a right handed coordinate system: also let $\mathbf{i}$ $\mathbf{j}$ $\mathbf{k}$ be unit vectors along the axes. Then

$$\nabla.\nabla \mathbf{E} = \nabla.\nabla\left[\mathbf{i}E_x + \mathbf{j}E_y + \mathbf{k}E_z\right]$$
$$= \frac{\mu K}{c^2}\frac{\partial_2}{\partial t^2}\left[\mathbf{i}E_x + \mathbf{j}E_y + \mathbf{k}E_z\right]$$

therefore we have for the x component

$$\nabla.\nabla E_x = \frac{\mu K}{c^2}\frac{\partial_2 E_x}{\partial t^2} = \frac{\partial_2 E_x}{\partial x^2} + \frac{\partial_2 E_x}{\partial y^2} + \frac{\partial_2 E_x}{\partial z^2}$$

Similar equations hold for y and z components: also for H_x H_y H_z . We have then, six equations as follows

$$\left.\begin{aligned}\frac{\partial_2 E_x}{\partial t^2} &= \frac{c^2}{\mu K}\left[\frac{\partial_2 E_x}{\partial x^2} + \frac{\partial_2 E_x}{\partial y^2} + \frac{\partial_2 E_x}{\partial z^2}\right]\\ \frac{\partial_2 E_y}{\partial t^2} &= \frac{c^2}{\mu K}\left[\frac{\partial_2 E_y}{\partial x^2} + \frac{\partial_2 E_y}{\partial y^2} + \frac{\partial_2 E_y}{\partial z^2}\right]\\ \frac{\partial_2 E_z}{\partial t^2} &= \frac{c^2}{\mu K}\left[\frac{\partial_2 E_z}{\partial x^2} + \frac{\partial_2 E_z}{\partial y^2} + \frac{\partial_2 E_z}{\partial z^2}\right]\end{aligned}\right\} \quad (25)$$

$$\left.\begin{aligned}\frac{\partial_2 H_x}{\partial t^2} &= \frac{c^2}{\mu K}\left[\frac{\partial_2 H_x}{\partial x^2} + \frac{\partial_2 H_x}{\partial y^2} + \frac{\partial_2 H_x}{\partial z^2}\right]\\ \frac{\partial_2 H_y}{\partial t^2} &= \frac{c^2}{\mu K}\left[\frac{\partial_2 H_y}{\partial x^2} + \frac{\partial_2 H_y}{\partial y^2} + \frac{\partial_2 H_y}{\partial z^2}\right]\\ \frac{\partial_2 H_z}{\partial t^2} &= \frac{c^2}{\mu K}\left[\frac{\partial_2 H_z}{\partial x^2} + \frac{\partial_2 H_z}{\partial y^2} + \frac{\partial_2 H_z}{\partial z^2}\right]\end{aligned}\right\} \quad (26)$$

These are the general differential equations of an effect similar to a wave motion, propagated through the medium with a velocity $\frac{c}{\sqrt{\mu K}}$. In air or vacuum, the values of μ and K are both unity: in this case, the wave travels with velocity c, which is the same as the velocity of light.

Plane Electromagnetic Waves.

Consider the foregoing equations applied to a plane wave advancing along the x axis: take the electric intensity vector, then

$$\frac{\partial_2 E}{\partial t^2} = \frac{c^2}{\mu K}\frac{\partial_2 E}{\partial x^2} \tag{27}$$

and the solution of this equation is

$$e = E \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \tag{28}$$

This equation states that the instantaneous value of the electric force describes sinusoidal variations both with respect to time and with respect to space: that, in the periodic time T, the wave has travelled a distance equal to th e wave length λ and therefore ~~also~~ that the velocity of propagation of the wave is $\frac{\lambda}{T}$.

Now in the case of a plane wave moving in any direction, equation 28 can be written

$$e = E \sin 2\pi\left(\frac{s}{\lambda} - \frac{t}{T}\right) = E\epsilon^{-2\pi j\left[\frac{s}{\lambda} - \frac{t}{T}\right]} \tag{29}$$

where s is the distance measured along the line of propagation of the wave. Let $\mathbf{n}$ be unit vector along this same line, then

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{K}\frac{\partial}{\partial z} = \mathbf{n}\frac{\partial}{\partial s}$$

and by eqn. 21a,

$$\text{Curl } \mathbf{H} = \nabla \times \mathbf{H} = \mathbf{n}\frac{\partial}{\partial s} \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

and therefore, $$\mathbf{n} \times \frac{\partial \mathbf{H}}{\partial s} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \tag{30}$$

Similarly, by eqn. 22a,

$$\mathbf{n} \times \frac{\partial \mathbf{E}}{\partial t} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{31}$$

These are the wave equations, and can be solved by substituting for the operators $\frac{\partial}{\partial s}$ and $\frac{\partial}{\partial t}$. By eqn. 29

differentiating,

$$\frac{\partial e}{\partial s} = \frac{\partial}{\partial s}(e) = -\frac{2\pi j}{\lambda} E \epsilon^{-2\pi j\left(\frac{s}{\lambda} - \frac{t}{T}\right)} = -\frac{2\pi j}{\lambda} e$$

$$\therefore \frac{\partial}{\partial s} = -\frac{2\pi j}{\lambda}$$

and, similarly, $\frac{\partial}{\partial t} = \frac{2\pi j}{T}$

Substituting these results in eqns. 30 and 31,

$$-j\frac{2\pi}{\lambda} \mathbf{n} \times \mathbf{H} = \frac{1}{c} j \frac{2\pi}{T} D$$

and letting v velocity of wave ,

$$\mathbf{H} \times \mathbf{n} = \frac{v}{c} D \qquad (32)$$

The negative sign is eliminated by changing the order of the cross product. Similarly,

$$\mathbf{n} \times \mathbf{E} = \frac{v}{c} \mathbf{B} \qquad (33)$$

It will be easily seen from these equations that the vectors D,B and n are all at right angles. For instance, dotting both sides with n in eqn. 32,

$$H \times n . n = \frac{v}{c} D.n$$

$$= H. n \times n$$

$$= 0 \quad \text{since } n \times n = 0 \text{ always.}$$

Similarly $\mathbf{B.n}$ is always zero. This shows that the vectors D and B are at right angles to n. Again.

$$\frac{v^2}{c^2} D.B = H \times n . n \times E = H \times n \times n . E = 0$$

showing that D and B are at right angles to each other.

The diagram, fig 21, shows how the electric and magnetic vectors are related. D and B form a right handed system with n, and are in the wave front. Note that E is not coincident with D unless the medium is isotropic.

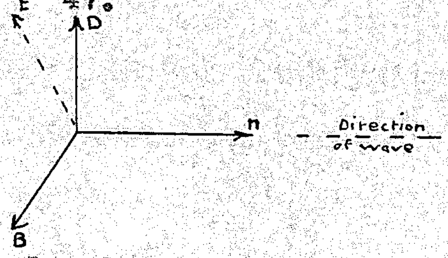


Fig 21.

Energy of the Wave. Poyntings Theorem.

The energy passing any surface may be shown to be the vector product of E and H as follows:

$$\nabla \times H = \frac{K}{c}\frac{\partial E}{\partial t}$$

$$\nabla \times E = -\frac{\mu}{c}\frac{\partial H}{\partial t}$$

also
$$\nabla \times H.E = \frac{K}{c} E.\frac{\partial E}{\partial t}$$

$$\nabla \times E.H = -\frac{\mu}{c} H.\frac{\partial H}{\partial t}$$

subtracting

$$\frac{K}{c}E.\frac{\partial E}{\partial t} + \frac{\mu}{c}H.\frac{\partial H}{\partial t} = \nabla \times H.E - \nabla \times E.H$$

Again $\nabla.(E \times H) = \nabla.(E \times H)_{E\,const.} + \nabla.(E \times H)_{H\,const.}$

$$= -E.\nabla \times H + H.\nabla \times E$$

Therefore $\frac{1}{2}\frac{K}{c}\frac{\partial (E^2)}{\partial t} + \frac{1}{2}\frac{\mu}{c}\frac{\partial (H^2)}{\partial t} = -\nabla.(E \times H)$

Integrating for the whole of the volume under consideration, and dividing by 4π, we get

$$\int_v \frac{1}{c}\left[\frac{\partial}{\partial t}\left(\frac{KE^2}{8\pi} + \frac{\mu H^2}{8\pi}\right)\right] dv + \frac{c}{4\pi}\int_v \nabla.(E \times H)\, dv = 0$$

and changing by Gauss' Theorem to surface integral form
$$\int_v \frac{1}{c}\left[\frac{\partial}{\partial t}\left(\frac{KE^2}{8\pi} + \frac{\mu H^2}{8\pi}\right)\right] dv + \frac{c}{4\pi}\int_\sigma E \times H.n\, d\sigma = 0$$

This theorem is really a statement of the law of conser vation of energy, and states that the rate of change of energy , integrated over the whole regional volume, is equal to the energy passing the surface. In an anisotropic medium, the Poynting energy vector $\frac{c}{4\pi}(E \times H)$ is not necessarily in the direction perpendicular to the wave front, owing

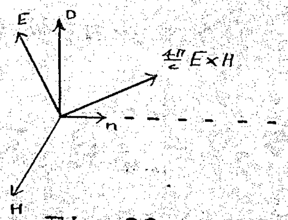


Fig 22.

to E not being coincident with D (fig 22.). The wave then sidesteps through the medium (fig 23),

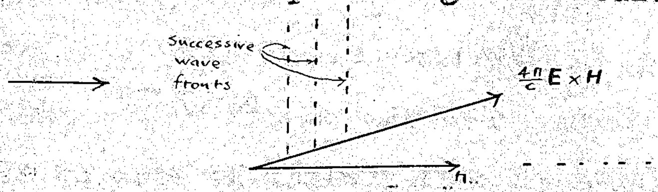


Fig 23

being always perpendicular to n.

The energy per unit volume is $\frac{1}{8\pi}(\kappa E^2 + \mu H^2)$.

By taking the scalar product of H with both sides of eqn. 33, and letting μ be unity,

$$\frac{v}{c} H.H = n \times E.H = \frac{v}{c} D.E$$

Hence $H^2 = \kappa E^2$

Therefore energy per unit volume

$$= \frac{1}{8\pi}(\kappa E^2 + H^2) = \frac{\kappa E^2}{4\pi} = \frac{H^2}{4\pi}$$

This shows that magnetic and electric energy per cm^3 are equal.

The Vector Potential.

We have seen by eqn. 16, $\nabla.B = 0$. Now let A be a vector such that

$$\text{Curl } A = B = \nabla \times A$$

Also $\nabla.B = \nabla.\nabla \times A$ which is identically zero.

Again, $\nabla\times\mathbf{B} = \nabla\times\nabla\times\mathbf{A}$

$$= \nabla\nabla.\mathbf{A} - \nabla.\nabla\mathbf{A}$$

Now $\nabla.\mathbf{A}=0$, since A is a vector depending on J and Div J is zero.

$$\therefore \quad \nabla\times\mathbf{B} = -\nabla.\nabla\mathbf{A}$$

Also $\nabla\times\mathbf{H} = 4\pi\mathbf{J}$, and in free space H = B,

Therefore, $\nabla\times\mathbf{B} = -\nabla.\nabla\mathbf{A} = 4\pi\mathbf{J}$

$$= -\nabla.\nabla\left[\mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z\right]$$

$$= 4\pi\left[\mathbf{i}J_x + \mathbf{j}J_y + \mathbf{k}J_z\right]$$

Hence $\nabla.\nabla A_x = -4\pi J_x$

Next we draw an analogy from Laplace's equation, where $\nabla.\nabla\phi = -4\pi\rho$, where ρ = volume density of electricity and ϕ = potential $= \int_v \frac{\rho\,dv}{r}$. In this case, ρ corresponds to J, a vector, and ϕ corresponds to A, another vector. Hencewe can write

$$A_x = \int\frac{J_x\,dv}{r} \qquad A_y = \int\frac{J_y\,dv}{r} \qquad A_z = \int\frac{J_z\,dv}{r}$$

and multiplying by $\mathbf{i}$ $\mathbf{j}$ $\mathbf{k}$ and adding

$$\mathbf{A} = \int\frac{\mathbf{J}\,dv}{r} \qquad (36)$$

where A is called the "vector potential" of the current J. The vector potential at any point in a current carrying region is found from the current density J by exactly the same process that the electric scalar potential, ϕ, is found from the volume density, ρ, except that the integration in the former case, is a vector integration, Whereas, in the latter case it is a ~~vector~~ scalar integration. The magnetic field vector, then, is found from the vector potential by the relation, $\mathbf{B} = \text{Curl}\,\mathbf{A}$.

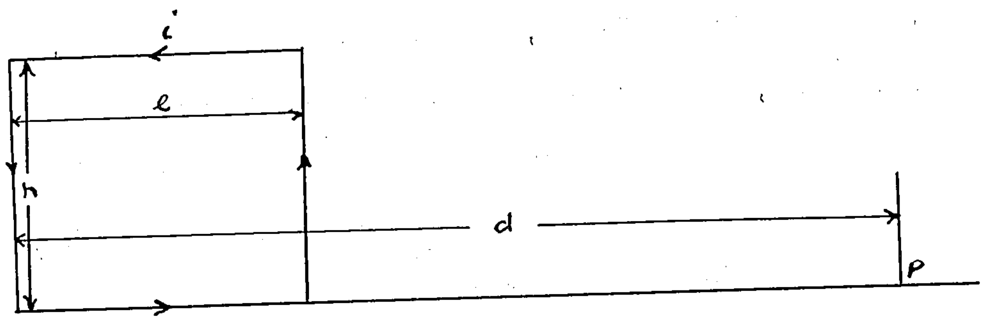


Fig 24.

The Magnetic Field from the Oscillator.

We will now try to find an expression for the magnitude of the field from the oscillator. The following treatment is adapted from Dellinger[1].

The radiation from an antenna is usually calculated by taking the electric and magnetic field equations for a Herzian doublet, modified according to the type of antenna under consideration. In this case, we are primarily interested in the magnetic induction field: we are only interested in the radiation effect in so far as we wish to avoid the so-called "aerial effect" in our search coils. We will therefore, calculate the magnetic field directly from the vector potential.

The instantaneous value of the vector potential, due to the current density J in the vertical conductor, is

$$\mathbf{A} = \int \frac{J\,dv}{r} = \int \frac{\frac{i}{a}\,a\,ds}{r} = \int \frac{i\,ds}{r}$$

since $J = \frac{i}{a}$, and a ds = dv, where a = area of wire, ds = element of length. Therefore at the point P (see fig 24), $\mathbf{A} = \frac{ih}{d}$

since h is small compared with d.

Since we are dealing with a rapidly alternating current, we cannot assume that the electromagnetic effects are propagated instantaneously: it requires a time $\frac{d}{c}$ for the effect to be transmitted to the point P. Now the current in the coil is assumed to be everywhere the same and equal to

(1) J.H. Dellinger, U.S. Bur. Stds, Vol 15, 1919. "Radiation from an antenna"

$$i = I_m \sin \omega t$$

then in our expression for vector potential, we must take the current as it was an instant of time $\frac{d}{c}$ beforeand therefore,

$$A_1 = \frac{h I_m}{d} \sin \omega (t - \frac{d}{c})$$

where A_1 is the vector potential due to the left hand wire, and

$$A_2 = - \frac{h I_m}{d-l} \sin \omega (t - \frac{d-l}{c})$$

due to the right hand wire. Therefore the total vector potential at the point P is

$$A = \frac{h I_m}{d} \sin \omega (t - \frac{d}{c}) - \frac{h I_m}{d-l} \sin \omega (t - \frac{d-l}{c})$$

Now by eqn. 25, $H = \text{curl } A$

$$= \frac{1}{10} \frac{\partial A}{\partial d}$$

since we are considering a straight conductor, having no vector potential in any other plane than that containing the coil, and perpendicular to the direction of d. The factor 1/10 was introduced to convert to amperes.

Now differentiating A , partially with respect tod

$$H = - \frac{h \omega I_m}{10 c d} \cos \omega (t - \frac{d}{c}) - \frac{h I_m}{10 d^2} \sin \omega (t - \frac{d}{c})$$
$$+ \frac{h \omega I_m}{10 c (d-l)} \cos \omega (t - \frac{d-l}{c}) + \frac{h I_m}{10 (d-l)^2} \sin \omega (t - \frac{d-l}{c})$$

and writing d for d-l. since l is very small compared with d

$$H = - \frac{h \omega I_m}{10 c d} \left\{ \cos \omega (t - \frac{d}{c}) - \cos \omega (t - \frac{d-l}{c}) \right\} \mp \frac{h I_m}{10 d^2} \left\{ \sin \omega (t - \frac{d}{c}) - \sin \omega (t - \frac{d-l}{c}) \right\}$$

$$= - \frac{2}{10} \frac{h \omega I_m}{c d} \sin \omega (t - \frac{d - \frac{l}{2}}{c}) \sin \frac{\omega l}{2c} + \frac{2 h I_m}{10 d^2} \cos \omega (t - \frac{d - \frac{l}{2}}{c}) \sin \frac{\omega l}{2c}$$

If the coil contains N turns, H will be increased N times: also substituting effective values for I,

$$H = -\frac{2}{10}\,\frac{Nh\omega I}{cd}\sin\frac{\omega l}{2c} + \frac{2}{10}\,\frac{NhI}{d^2}\sin\frac{\omega l}{2c}$$

but $\frac{\omega}{c} = \frac{2\pi}{\lambda}$ and $\sin\frac{\omega l}{2c} = \frac{\omega l}{2c}$ if l is small compared with the wave length. The complete expression is

$$H = -\frac{4\pi^2}{10}\,\frac{NhlI}{\lambda^2 d} + \frac{2\pi}{10}\,\frac{NhlI}{\lambda d^2} \qquad (37)$$

The first term represents the radiation field and the second, the induction field. The former var- inversely with the first power of d, while the latter varies inversely as d squared. Therefore the induction field falls off rapidly as the distance from the oscillator is increased. If we equate the two fields, we find that, at a distance $\frac{\lambda}{2\pi}$, the two fields are equal. Theoretically, this is the limiting distance from the oscillator at which work may be done. In practice, the search coils act as an aerial before this distance is reached, and produce a signal loud enough to drown the response from an orebody. At 50 K.C., $\lambda = 6000$ meters, $d = \frac{\lambda}{2\pi} = \frac{6000}{2\pi}$ or approximately 3000 ft. We should limit our observations to a dis tance of 2500 ft. from the aerial, and less if possible. Four full sized claims can thus be covered from one set up in this way, if the oscillator is located at a central point.

For an oscillator having an anode dissipation of 25 watts with 500 volts on the plate, the cirulating current in the coil antenna would be about 40 m.amps. If the arêal had h=6 ft, l = 6ft and N = 10 turns, then substituting these values in eqn. 37, we find that the induction field at 50 K.C. would be given by $H = \frac{1.4}{d^2} \times 10^{-2}$ gauss

$= 1.52 \times 10^{-9}$ gauss at 100 ft.

$= 0.38 \times 10^{-9}$ " " 200 "

$= 1.52 \times 10^{-11}$ " " 1000 "

This gives an idea of the small field strength to be measured. This is the direct field, the secondary field would, in general, be much smaller. Since the field strength is directly proportional to the current in the antenna, it is important to make this current as lagge as possible: for this reason, a large tank inductance is desirable as forming the antenna coil. The coil must be air spaced to keep the effective resistance and charging current low. The Radiore "doughnut" coil should be very good it this respect. Limitations are placed on the number of turns in the coil by the factors just mentioned, so they cannot be increased indefinitely.

Penetration of Electromagnetic Waves.

We will now turn to the question of the penetration of waves through rock and overburden. Jeans[1] has indicated a way in which the problem of wave propagation through a medium with conduction may be tackled. The conduction current must be included in Maxwells equation for the curl of H.

(1) J.H.Jeans "Electricity And Magnetism" Ch 18

In the vector notation,

$$\nabla \times \mathbf{H} = \frac{1}{c}\left(\frac{\partial D}{\partial t} + s\,E\right)$$

The medium has conduction currents and displacement currents. (s = conductivity)

$$\frac{\partial D}{\partial t} = k\frac{\partial E}{\partial t} = \frac{\partial E}{\partial t} + 4\pi\frac{\partial P}{\partial t}$$

Now let $\quad E = E_0\,\epsilon^{j\omega\left(t - \frac{lx+my+nz}{c}\right)}$

$$\frac{\partial E}{\partial t} = E_0\,j\omega\,\epsilon^{j\omega\left(t - \frac{lx+my+nz}{c}\right)} = j\omega E$$

and

$$E = \frac{1}{j\omega}\frac{\partial E}{\partial t}$$

Substitute in above eqn,

$$\nabla \times \mathbf{H} = \frac{1}{c}\left[k\frac{\partial E}{\partial t} + \frac{s}{j\omega}\frac{\partial E}{\partial t}\right]$$

$$= \frac{1}{c}\frac{\partial E}{\partial t}\left[k - \frac{js}{\omega}\right]$$

This is the same equation as for space, where the dielectric constant k, is replaced by a new constant $k_1 = k - \frac{js}{\omega}$.

The wave equations may be worked out in this manner, and complex expressions obtained for wave slowness. In this way, the attenuation and phase change can be calculated theoretically. The following treatment is due to Zenneck[1], but has been altered to conform to the right handed axes of modern convention. It affords a useful picture of what happens to a wave travelling over a conducting medium.

In fig 25, let the direction of propagation be the x axis, which is taken in the surface of the earth. The z axis is positive in the upward direction to conform with the right handed convention.

(1) J. Zenneck "Über die Fortpflanzung ebener Elektromagnetischer Wellen längs eine Leitefläche und ihre Beziehung zur drahtlosen Telegraphie" Annalen der Physic vol 23 p 846 1907 Translated by J.A Fleming ~~"Electrician"~~ "Engineering" June 1909

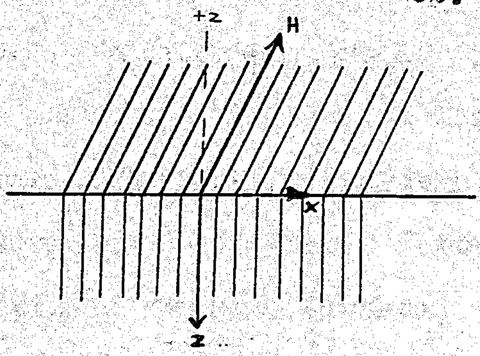


Fig 25

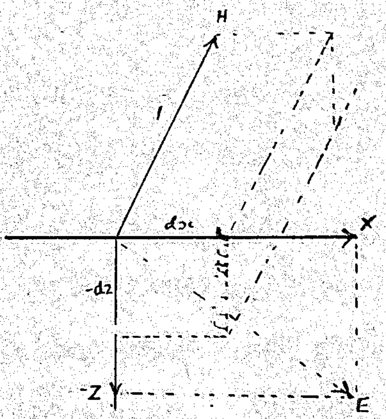


Fig 26

Let k = dielectric constant in e.s.u.

μ = magnetic permeability in e.m.u.

ρ = resistivity in ohms/cm. cube.

s = specific conductivity = $\frac{9\times10^{11}}{\rho}$ e.s.u.

f = frequency = $\frac{\omega}{2\pi}$

c = 3×10^{10} = wave velocity

λ = wave length

$\frac{2\pi}{\lambda}$ = q = wave slowness.

Again let the axial components of electric intensity be X Y Z. Referring to fig 25, it is evident that, if the magnetic vector H be considered as along the Y axis, it will have no component along the X and Z axes. Similarly, the electric vector will have no component along the Y axis, see fig 26.

Now, referring to the discussion which has gone before, we have the following relations: these are repeated here for convenience.

$$\mathbf{D} = k\mathbf{E} \qquad (17)$$

$$\mathbf{B} = \mu\mathbf{H} \qquad (15)$$

$$\int_0 \mathbf{H}.d\mathbf{r} = 4\pi\int_\sigma \mathbf{J}.\mathbf{n}\, d\sigma = \int_\sigma \nabla\times\mathbf{H}.\mathbf{n}\, d\sigma \qquad (19)$$

$$\int_0 \mathbf{E}.d\mathbf{r} = \int_\sigma \nabla\times\mathbf{E}.\mathbf{n}\, d\sigma = -\frac{1}{c}\int_\sigma \frac{\partial \mathbf{B}}{\partial t}.\mathbf{n}\, d\sigma \qquad (22)$$

Applying these to the element of volume $1 \times dx \times dz$ in fig 26, we find that there is an induced current -Zdx, through the ~~same~~ area $1 \times dx$, owing to the magnetic force H: and, also a displacement current -jωkZdx, through the same area due to component -Z of the electric intensity. Total current is therefore $-\left[s + j\omega k\right] Z$.

Taking the line integral of H round this area

$$\int_0 H.dr = H - \left(H + \frac{\partial H}{\partial x}dx\right)$$

$$= -\frac{\partial H}{\partial x}dx$$

and by eqn. 19, this is egual to 4π times the current enclosed by this area. Therefore

$$\frac{4\pi}{c}\left[s + j\omega k\right] Z = \frac{\partial H}{\partial x} \qquad (38)$$

Now let H vary sinusoidally with space and time

$$H = A\epsilon^{j(\omega t + qx)}$$

$$\frac{\partial H}{\partial x} = jqH$$

and substituting in eqn. 38.

$$\frac{4\pi}{c}\left[s + j\omega k\right] Z = jqH \qquad (39)$$

Similarly taking the area $1 \times dz$

$$\frac{4\pi}{c}\left[s + j\omega k\right] X = -\frac{\partial H}{\partial z} \qquad (40)$$

Next take the area $dx \times dz$, By eqn. 22, we have

$$\text{Curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

In this case the magnitudes of the scalar components are X, Y and Z. Therefore

$$\text{curl } \mathbf{E} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ X & Y & -Z \end{vmatrix} = \mathbf{i}\left[-\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}\right] + \mathbf{j}\left[\frac{\partial X}{\partial z} + \frac{\partial Z}{\partial x}\right] + \mathbf{k}\left[\frac{\partial X}{\partial Y} - \frac{\partial Y}{\partial x}\right]$$

and , since there is no component along the Y axis

$$\text{Curl}\, \mathbf{E} = \mathbf{j}\left[\frac{\partial X}{\partial z} + \frac{\partial Z}{\partial x}\right] = -\frac{1}{c}\frac{\partial B}{\partial t}$$

The same result can be reached another way

$$\int_0 \mathbf{E}.d\mathbf{r} = X dx + \left[-Z + \frac{\partial Z}{\partial x}dx\right]dz - \left[X - \frac{\partial X}{\partial z}dz\right]dx - \left[-Z dz\right]$$

$$= \left[\frac{\partial Z}{\partial x} + \frac{\partial X}{\partial z}\right]\partial x\, dz$$

Since E and H vary sinusoidally,

$$\frac{\partial X}{\partial z} + jqZ = j\frac{\omega\mu H}{c} \qquad (41)$$

We have now three equations , numbers 39, 40 and41, between X, Z and H. Eliminating X and Z from these

$$\frac{\partial_2 H}{\partial z_2} + \left[q^2 + j\frac{4\pi\omega\mu}{c^2}(s + j\omega k)\right]H = 0 \qquad (42)$$

$H = \epsilon^{jBz}$ is a solution of this equation, where

$$B^2 = q^2 + j\frac{4\pi\omega\mu}{c^2}\left[s + j\omega k\right] \qquad (43)$$

Asva complete solution of the three equations,

$$H = A\epsilon^{jBz}\epsilon^{j[\omega t + qx]} \qquad (44)$$

$$X = \frac{jBcA}{4\pi[s + j\omega k]}\epsilon^{jBz}\epsilon^{j[\omega t + qx]} \qquad (45)$$

$$Y = \frac{jqcA}{4\pi[s + j\omega k]}\epsilon^{jBz}\epsilon^{j[\omega t + qx]} \qquad (46)$$

Next apply these equations to a second parallelopiped $1 \times dx \times dz$, in the air above the interface

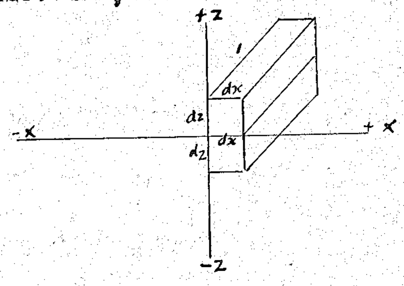


Fig 27.

The same equations hold on either side of the interface, except that s and k are different: also, the constants B and A are different. Let values in air be s k A B and in dielectric be s′ k′ A′ B′

Then by eqn 43,

$$\left.\begin{aligned} B^2 - q^2 &= j\,\frac{4\pi\omega\mu}{c^2}\left[s + j\omega K\right] \\ B_1^2 + q_x^2 &= j\,\frac{4\pi\omega\mu}{c^2}\left[s_1 + j\omega K_1\right] \end{aligned}\right\} \quad (47)$$

We next insert the boundary conditions at the interface, where x and z are zero and t is zero. Substituting in eqn. 44, we find that $A = A'$. Again, when $X = X_1$ at the interface

$$\frac{B_1}{s_1 + j\omega K_1} = \frac{B}{s + j\omega K} \quad (48)$$

Writing T and T_1 for the denominators in eqn. 48, and P for $\frac{4\pi\omega\mu}{c^2}$,

$$\left.\begin{aligned} B^2 - q^2 &= j\,PT \\ B_1^2 - q^2 &= j\,PT_1 \\ \frac{B}{T} &= \frac{B_1}{T_1} \end{aligned}\right\} \quad (49)$$

Hence

$$\left.\begin{aligned} B^2 - jPT &= B_1^2 - jPT_1 \\ B^2 &= jP\frac{T^2}{T_1 + T} \\ B_1^2 &= jP\frac{T_1^2}{T_1 + T} \\ q^2 &= -jP\frac{T_1 T}{T_1 + T} \end{aligned}\right\} \quad (50)$$

Again from eqns. 45 and 46

$$\frac{X}{Z} = -\frac{B}{q} = -\sqrt{\frac{T}{T_1}} = -\sqrt{\frac{s + j\omega K}{s_1 + j\omega K_1}} \quad (51)$$

In the case of air s is zero

$$\frac{X}{Z} = -\sqrt{\frac{j\omega K}{s_1 + j\omega K_1}} = -\sqrt{\frac{j\frac{\omega K}{s_1}}{1 + j\frac{\omega K_1}{s_1}}}$$

Now let $\tan 2\phi = \frac{1}{j\frac{\omega K_1}{s_1}}$ then

$$\epsilon^{j2\phi} = \cos 2\phi + j\sin 2\phi$$

$$\therefore \epsilon^{j2\phi} = \frac{\frac{\omega K_1}{S_1}}{\sqrt{1+\frac{\omega^2 K_1^2}{S_1^2}}} + j\frac{1}{\sqrt{1+\frac{\omega^2 K_1^2}{S_1^2}}}$$

$$= \sqrt{1+\frac{\omega^2 K_1^2}{S_1^2}}\;\frac{j}{1+j\frac{\omega K_1}{S_1}}$$

but $$\frac{X}{Z} = -\sqrt{\frac{j\frac{\omega K}{S_1}}{1+j\frac{\omega K_1}{S_1}}} = -\sqrt{\frac{\frac{\omega K}{S_1}\,\epsilon^{j2\phi}}{\sqrt{1+\frac{\omega^2 K_1^2}{S_1^2}}}}$$

$$= -\sqrt{\frac{\frac{\omega K}{S_1}}{\sqrt{1+\frac{\omega^2 K_1^2}{S_1^2}}}}\;\epsilon^{j\phi}$$

Thus X and Z differ in phase by an angle ϕ, where $\tan 2\phi = \frac{S_1}{\omega K_1}$. The electrbc intensity E is the resultant of these two components,and traces out an ellipse. The major axis is inclined forward. The magnetic vector H, being parallel to the interface, is still plane polarized, although if the inter face wese inclined, as in hilly country it would not be so any longer.

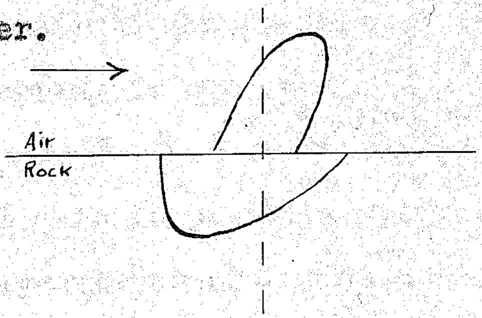


Fig 28.

We have seen that the wave front of the primary field is distorted, owing to the electrical properties of the earth over which the wave is travelling. The distortion is shown diagrammatically in fig 28. There is a discontinuity at the earth's surface. The expression,eqn. 52 differs from that obtained by Zenneck in that the sign of the exponent is changed: this is because the Z axis was taken upwards to conform to convention.

The principle object of Zenneck's work on this subject was to find the loss in amplitude of a radio wave, travelling over a conducting medium. He found that, where the wave travels over a good conductor, such as sea water, there is little loss in energy due to eddy currents below the interface. If , however, the medium conducts but poorly, as in rock and soil, there is considerable penetration and consequent loss in energy. This phase of the argument is not of interest in geophysical surveying owing to the short distances between the oscillator and the search coil. We may, however, proceed to some interesting conclusions regarding the penetration of waves into the earth.

Referring to eqn. 44

$$H = A \epsilon^{jBz} \epsilon^{j(\omega t + qx)}$$

Since the Z axis is positive upwards, it is evident that negative values of Z give decreasing amplitude of the wave as the distance from the surface increases At the surface

$$H = A_1 \epsilon^{j(\omega t + qx)}$$

Let B be a complex quantity

$$B = -(a + jb)$$

We wish to find the depth at which the amplitude is $\frac{1}{e}$ of that at the surface. Accordingly, let $z = -\frac{1}{b}$

$$H = H_0 \epsilon^{-j(a+jb)z} \qquad (53)$$
$$= H_0 \epsilon^{j\frac{a}{b}} \epsilon^{-1}$$

Hence $\frac{1}{b}$ is the depth from the surface at which the wave amplitude has decreased to $\frac{1}{e}$. Notice that the phase of the wave is also altered by the factor .

We also know that from eqn. 51,

$$B = \sqrt{\frac{T}{T_1}}\, q$$

We have next to find a complex expression for the wave slowness q in the wave front. By eqn. 50

$$q^2 = -j P \frac{T_1 T}{T_1 + T}$$

$$q^2 = -j \frac{4\pi\omega\mu}{c^2} \frac{(s + j\omega k)(s_1 + j\omega k_1)}{s + s_1 + j\omega(k + k_1)}$$

if $s = 0$

$$= \frac{4\pi\omega\mu k}{c^2} \frac{s_1 + j\omega k_1}{s_1 + j\omega(k_1 + k)} \qquad (54)$$

Now let $q = m + jm_1$ and $s_1 + j\omega k_1 = \epsilon^{j\phi}\sqrt{s_1^2 + \omega^2 k_1^2}$ where $\tan\phi = \frac{\omega k_1}{s_1}$

Therefore substituting in eqn. 53

$$q = \sqrt{\frac{4\pi\omega^2 k\mu}{c^2}} \sqrt{\frac{\sqrt{s_1^2 + \cancel{j}\,\omega^2 k_1^2}}{\sqrt{s_1^2 + \omega^2 (k_1 + k)^2}}}\; \epsilon^{j\left(\frac{\phi_1}{2} - \frac{\phi_2}{2}\right)}$$

where $\tan\phi_1 = \frac{\omega k_1}{s_1}$ and $\tan\phi_2 = \frac{\omega(k_1 + k)}{s_1}$

$$\therefore q = \frac{\omega}{c} \sqrt{\frac{\sqrt{s_1^2 + \omega^2 k_1^2}}{\sqrt{s_1^2 + \omega^2 (k_1 + k^2)}}}\; \epsilon^{j\left(\frac{\phi_1}{2} - \frac{\phi_2}{2}\right)}$$

Again, since $4\pi k = 1$ for air and $\mu = 1$ for air and rock, also since

$$B = \sqrt{\frac{s_1 + j\omega k_1}{s + j\omega k}}\, q = \sqrt{\frac{\sqrt{s_1^2 + \omega^2 k_1^2}}{\sqrt{s^2 + \omega^2 k^2}}}\, q\, \epsilon^{j\left(\frac{\phi_1}{2} - \frac{\phi}{2}\right)}$$

where $\tan\phi = \frac{\omega k}{s}$

$$\therefore B = \frac{\omega}{c} \frac{\sqrt{s_1^2 + \omega^2 k_1^2}}{\sqrt{\sqrt{s^2 + \omega^2 k^2}\sqrt{s_1^2 + \omega^2 (k_1 + k)^2}}}\; \epsilon^{j\left(\phi_1 - \frac{\phi_2 + \phi}{2}\right)}$$

$$= a + jb.$$

Expanding the exponential term

$$a+j\mathcal{b} = \frac{\omega}{c}\frac{\sqrt{s_1^2+\omega^2 K_1}}{\sqrt{\sqrt{\omega^2 K^2}\sqrt{s_1^2+\omega^2(K_1+K)^2}}}\left\{\cos\left(\phi_1 - \frac{\phi_2+\phi}{2}\right) + j\sin\left(\phi_1 - \frac{\phi_2+\phi}{2}\right)\right\} \quad (55)$$

Equating real and imaginarybparts,

$$\mathcal{b} = \frac{\sqrt{\omega}}{c}\frac{\sqrt{s_1^2+\omega^2 K_1^2}}{\sqrt{K\sqrt{s_1^2+\omega^2(K+K_1)^2}}}\sin\left(\phi_1 - \frac{\phi_2+\phi}{2}\right) \quad (56)$$

Let us now turn to a practical example of a wave penetrating rock of fairly high resistivity (e.g quartz diorite). Let $\rho = 10^5$ ohms per cm. cube, $K_1 = \frac{3}{4\pi}$ for rock and $K = \frac{1}{4\pi}$ for air. Also let the frequency be 30,000# cycles. We then calculate the following quantities

$$s_1 = \frac{9\times10^{11}}{10^5} = 9\times10^6$$

$$\tan\phi = \frac{\omega K}{s} = \frac{2\pi(30000)}{0} = \infty \qquad \therefore \phi = 90^\circ$$

$$\tan\phi_1 = \frac{\omega K_1}{s_1} = \frac{2\pi(30000)3}{4\pi(9\times10^6)} = 0.005 \qquad \phi_1 = 0^\circ 17'$$

$$\tan\phi_2 = \frac{\omega(K_1+K)}{s_1} = 0.0066 \qquad \phi_2 = 0^\circ 23'$$

$$\therefore \sin\left(\phi_1 - \frac{\phi_2+\phi}{2}\right) \simeq \frac{1}{\sqrt{2}}$$

$$\mathcal{b} = \frac{\sqrt{2\pi(30000)}}{3\times10^{10}}\sqrt{\frac{81\times10^{12}+20.25\times10^8}{\sqrt{81\times10^{12}+36\times10^8}}}\sqrt{4\pi}\,\frac{}{\sqrt{2}} = \frac{1}{9.2\times10^3}$$

Therefore the depth $\frac{1}{e}$ at which the amplitude has decreased to $\frac{1}{e}$ of that at the surface is 9.2×10^3 cm or 92 meters. This is a considerable depth due to the comparatively low frequency of the waves and the comparatively high resist#vity. Actually orebodies have been located to a depth of 400 ft. However the usual maximum depth is 200 feet, so that the above result may be said to agree with practise. The we-weakness of all theoretical methodsof treatment of this problem lies in the fact that the earth is taken as being a homogeneous dielectric, having conductivity, which is farvfrom being the case. Eve and Keys[1] give the formula

$$H = H_0 \epsilon^{-2\pi d\sqrt{\frac{\mu f}{\rho}}}$$

for field strength at a depth d. This formula results from following an argument similar to that of Steinmetz in his treatment of the distribution of alternating flux in conductors[2]. It does not appear to be correct in this instance as it ignores the dielectric peoperties of the medium. If this formula is applied to the above example, the depth at which $H = \frac{1}{\epsilon} H_0$ is found to be 492 meters, a considerably different result. The above formula was meant to apply to metallic conduction and not to wave propagation through rock.

Again, it is found in practice that it is not possible to obtain any results in a region of high ground conductivity, due to the screening action of the overburden. The I.G.E.S. in the course of it's work in the Moonta district of South Australia[3], met with highly conducting overburden, saturated with saline water, which rendered attempts at electrical prospecting abortive. The resistivity of

(1) Eve and Keys "Applied Geophysics" Camb.Univ Pr
(2) Steinmetz "Transient Phenomena and Oscillations"
(3) I.G.E.S.Report p.112.

this overburden was 271 ohms/cm.cube. Using the previous method of calculation, it is found that the field will be reduced to $\frac{1}{e}$ of it's value at the surface by passing through 1.54 meters of this formation. Furthermore, the eddycurrents induced in this screening layer of overburden, were found to give a uniform indication, similar to ore, over the whole territory.

The change in phase due to the overburden.

We will now proceed to apply Zenneck's reasoning to examine the change in phase of the wave, due to the conductivity and capacitance of the overburden. In eqn. 53 we saw that

$$H = H_0 \epsilon^{-jaz} \epsilon^{bz}$$

and from eqn. 55, we found the value of $a+jb$. Therefore equating reals, we find that

$$a = \frac{\sqrt{\omega}}{c} \frac{\sqrt{S_1^2 + \omega^2 K_1^2}}{\sqrt{K_1 \sqrt{S_1^2 + \omega^2 (K+K^2)}}} \cos\left(\phi_1 - \frac{\phi_2 + \phi_3}{2}\right) \quad (57)$$

Applying eqn. 57 to the example of overburden, with resistivity of 10^5 ohms/cm.cube and dielectric constant of 3 . Then $a = \frac{1}{9200}$. Suppose we wish to find the change in phase at a depth of 50 meters: Inserting a value of z = 5000 cms

$$\epsilon^{-j\phi} = \epsilon^{-j\frac{5000}{9200}}$$

and $\phi = 31^\circ 10'$. Therefore the phase difference of H at a depth of 50 meters is $31^\circ 10'$ behind that of H at the surface. This would be repeated when the return wave comes back to the surface.

Distribution of Current in the Orebody.

It was mentioned above that the alternating flux produced by the oscillator is rapidly reduced as it penetrates into the ore deposit, which may be treated as a solid conductor. Steinmetz gives[1]

$$H = H_0 \epsilon^{-2\pi d\sqrt{\frac{\mu f}{\rho}}}$$

The result is that the depth of penetration of flux into such material as pyrite, is only about 1 cm at 30 K.C. Therefore the larger the surface area of the deposit, the greater will be the indication at the receiver.

Again, the current distribution in the deposit is dependent on the frequency due to "skin effect". The theory is well known and will not be given here.[2] The effective impedance of a conductor is

$$Z_e = r l \sqrt{0.4\pi\lambda\mu f}\,(1 + j) \times 10^{-8} \qquad (58)$$

Where r = d.c. resistance

λ = conductivity

μ = permeability

f = frequency

l = distance from centre of conductor to circumference

So that there is no current at the centre: it is confined to a small skin on the outside of the conductor. For conductors of large dimensions such as orebodies, and for high frequencies, the current only penetrates a fraction of a centimeter. In this case

(1) Steinmetz "Transient Phenomena and Oscillations" p 362 et seq

(2) Steinmetz loc. cit. p 376 et seq.

the effective resistance is proportional to the diameter of the conductor: therefore narrow veins provide better indications than massive bodies of greater total volume.

Again, from eqn. 58, there is an effective reactance equal to the effective resistance: it would seem, therefore, that the phase angle due to the passage of current in the conductor, is always 45° and furthermore, that it is impossible for an orebody to show a capacitative reactance on this account. Experimental work in the field should verify or disprove this deduction. If this is correct, it would seem that all the discussions on the subject of reactance of deposits, in the published articles, are fruitless.

Summary of Phase Relationships.

If $H_1 \sin \omega t$ is the primary field, the secondary field will be given by

$$H_2 \sin\left(\omega t - \frac{\pi}{2} - \phi_1 - \phi_2\right)$$

where ϕ_1 is the phase angle due to the effective impedance of the deposit, while ϕ_2 is due to the transmission of the waves through the overburden. For the same orebody and setup of apparatus,

$$\tan \phi_1 = \frac{x_e}{r_e} = 1 \quad \text{for high frequencies,}$$

therefore $\phi_1 = \frac{\pi}{4} = 45°$. Again,

$$\phi_2 \propto d\sqrt{f}$$

so that the resultant field can be brought into phase by increasing the frequency, since by this means $\frac{\pi}{2} + \phi_1 + \phi_2$ may be made equal to 180° or 360°. This accords with practical knowledge.

A further deduction of the approximate depth of an orebody might be made from the amount by which it is necessary to raise the frequency at the oscillator to bring the primary and secondary fields into phase. Again this deduction must be checked in the field.

The whole question of the phase of the secondary field is very complicated and depends on many unknown factors. There is nothing in the published articles on geophysical prospecting which indicates that an attempt has been made to solve it, possibly because those, who have practical experience, consider that it cannot be solved. The analysis has only been attempted here in order to show the nature of the problem and to get a better understanding of the mechanism of this geophysical method.

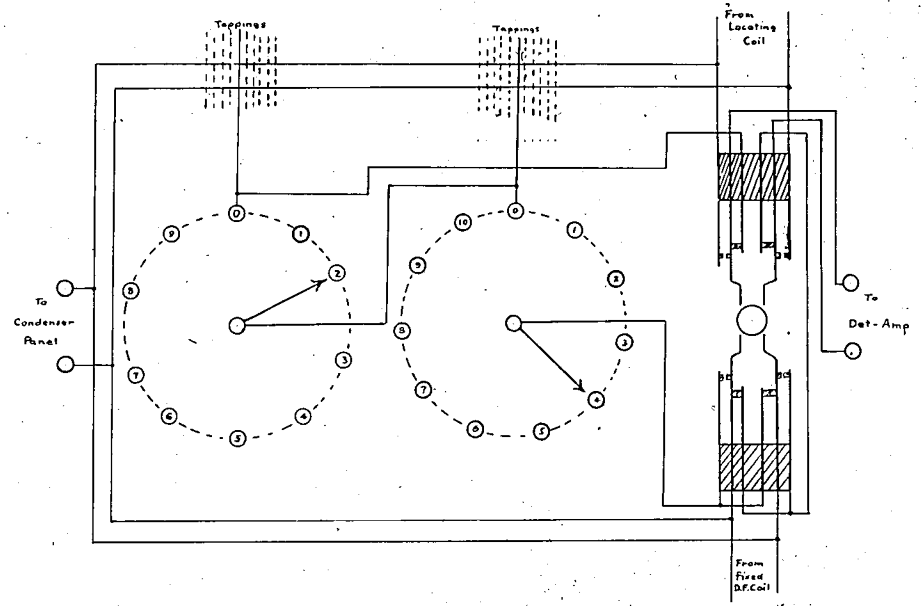
APPENDIX.

Laboratory Test of Three Coil Apparatus.

In order to get data and experience for the construction and design of apparatus to be used in the field, it was decided to constrict a model of the apparatus, and to attempt to measure an artificial magnetic field with elliptical polarization, in the laboratory.

The Search Coils. It was originally intended to take the measurements through the whole frequency band from 500 cycles to 50 K.C. However, in the design of the model search coils, no allowance was made for the distributed capacitance in the type of winding adopted, since the effect of this factor could only be ascertained through experience: in consequence, various disturbing factors were introduced into the experiment, which rendered the measurements at the higher frequencies unsuccessful.

The coils, as constructed, were three in number and had diameters, 11¼", 12", 12¾"; they were wound on wooden frames and mounted at right angles to each other by means of pieces of ebonite, cut to the correct shape. The inner coil was arranged as the locating coil and had 600 turns. The 12" coil also had 600 turns, with 9 taps at 60 turns and 10 taps at 6 turns. The outer coil acted as the fixed direction finding coil and also had 600 turns. The



SELECTOR PANEL

inductances, calculated by Nagaoka formula[1], were approximately 200 to 240 millihenries. The size of the wire was #34, enamelled. The coils were placed in a wooden cradle , shaped so that they could be tu turned through any angle.

The Selector Panel. The wiring diagram is shown in the figure. The panel itself was ebonite, with brass selector switches and contacts. The reversing and locating coil switch was conveniently made from a 4-way telephone switch, taken from an old telephone switchboard. It was found to be more convenient to mount the panel separately, rather than inside the coils, although this arrangement necessitated the use of long leads from the tappings to the panel, a factor which was later found to introduce errors.

The Test Field. The elliptically polarized test field was produced by two large loops, 9 feet square, having 10 turns each. These were wound on a rough lumber framework , erected in the laboratory. The current in the loops was supplied by various oscillators. For audio frequencies, a Cambridge Instrument Co source, calibrated on a mutual inductance frequency bridge , was the source of current. For higher frequencies, up to 20 K.C.,a dynatron oscillator, with Campbell variable standard inductanse and Tinling standard air condenser in the oscillatory arm of the circuit, was used. Above 20 K.C., a a Hartley type oscillator, from the electrostatic sweep of a General Electric cathode ray oscillograph was employed: this instrument was calibrated at the factory.

The current in the two test field coils was varied in phase by a condenser in the horizontal coil circuit; the circuit is shown in fig 29. When first operated, the electrostatic coupling between the test field coils and the search coils, was found to be so great that no results could be obtained at all. This difficulty was partially overcome by wrapping the coils with aluminium foil and connecting the latter to earth. The shielding effect given in this way, was not complete owing to the difficulty of shielding the leads to the various instruments and the coils of the oscillators. This difficulty persisted throughout the experiment.

The inductance of the large coils, as measured and calculated gave concordant results. The values of inductance and resistance were

Horizontal coil 1.340 mH and 15.81 ohms

Vertical coil 1.365 mH and 17.12 ohms.

<u>The Detector-Amplifier Unit</u>.

This unit was built up from salvaged radio parts from several discarded sets. At first, it took the form of a vacuum tube voltmeter-amplifier, resistance coupled, using the General Radio circuit.[1]

(1) Radio Engineer's Handbook. p 161

The step up ratio of voltage was 200. This apparatus was not successful owing to the difficulty of obtaining a sufficiently sensitive voltmeter, for the output stage, from apparatus at hand in the laboratory.

The unit was then rearranged to be a 4-tube detector amplifier. One tube gave amplification at radio frequency, and was coupled to the autodyne detector by transformer coupling. The detector was followed by two stages of transformer coupled audio amplification. This arrangement was also abandoned, owing to difficulty with the radio frequency transformer at the comparatively low radio frequency of 20-50 K.C.

Finally the unit was rebuilt to be a three tube detector amplifier unit with headphones. Transformer audio coupling was used. The tubes were W.E.101F telephone repeater tubes, with filament voltage of 4 volts, and 6 volts negative grid bias. It worked well at audio frequencies and would probably have been satisfactory at higher frequencies, if measurements had been possible at these frequencies. The general scheme of connections is shown in fig 29.

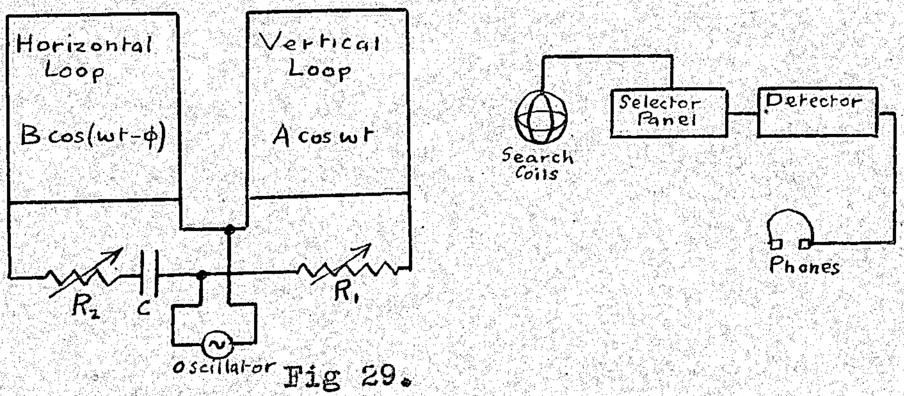


Fig 29.

The Test Magnetic Field.

Let $A\cos\omega t$ be field due to vertical loop

$B\cos(\omega t-\phi)$ " " " " horizontal loop.

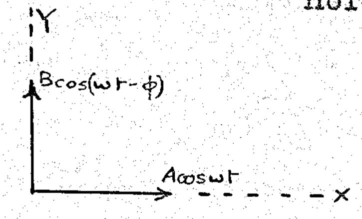


Fig 30

Resolving $\quad X = A\cos\omega t$

$$Y = B\cos(\omega t-\phi)$$

$$= B\cos\omega t\cos\phi + B\sin\omega t\sin\phi$$

Eliminate ωt $\quad \cos\omega t = \dfrac{X}{A}$

$$\sin\omega t = \frac{\sqrt{A^2-X^2}}{A}$$

$$Y = \frac{BX}{A}\cos\phi + B\frac{\sqrt{A^2-X^2}}{A}\sin\phi$$

Squaring and rearranging

$$\frac{Y^2}{B^2} - 2\frac{XY}{AB}\cos\phi + \frac{X^2}{A^2} = \sin^2\phi$$

This is the equation of the ellipse. The ratio of the axes is given by

$$\frac{m_1}{m_2} = \frac{\left(\frac{1}{B^2}+\frac{1}{A^2}\right) - \sqrt{\left(\frac{1}{A^2}-\frac{1}{B^2}\right)^2 - \frac{16\cos^2\phi}{A^2B^2}}}{\left(\frac{1}{A^2}+\frac{1}{B^2}\right) + \sqrt{\left(\frac{1}{A^2}-\frac{1}{B^2}\right)^2 - \frac{16\cos^2\phi}{A^2B^2}}}$$

Now A and B are proportional to the current in the loops. Therefore

$$A = c\frac{E}{\sqrt{R^2+\omega^2L_1^2}} \qquad B = c\frac{E}{\sqrt{R'^2+\left(\omega L_2-\frac{1}{\omega C}\right)^2}}$$

where c is constant of proportionality

E is common voltage applied to both loops.

R, R' are total resistances

L_1, L_2 are inductances

C is capacity added to horizontal loop.

Settings.										Calculated Values		Observed Values	
f	R_1	R_2	$C \mu F$	X_c	X_1	X_2	R	R'	ϕ	θ	$\frac{m_1}{m_2}$	θ	$\frac{m_1}{m_2}$
500	9000	0	0.02	15920	4.25	4.25	9017	15	90°	0	0.32	6°	0.28
1000	9000	0	0.01	15920	8.48	8.48	9017	15	90°	0	0.32	3°	0.36
2000	9000	0	0.01	7960	12.75	12.75	9017	15	90°	90°	0.782	95°	0.68
3000	9000	0	0.01	5300	17.00	17.00	9017	15	90°	90°	0.347	85°	0.39
4000	9000	0	0.01	3980	21.25	21.25	9017	15	90°	90°	0.196	88°	0.22
5000	9000	0	0.01	3300	25.50	25.50	9017	15	90°	90°	0.135	89°	0.17
6000	9000	0	0.01	2650	29.75	29.75	9017	15	90°	90°	0.0868	85°	0.10
7000	5000	0	0.01	2280	34.00	34.00	5017	15	90°	90°	0.208	85°	0.17

By making the capacitative reactance in horizontal loop circuit large compared with ωL_1 and ωL_2, and also making R' as small as possible, the current, and hence the magnetic field of the horizontal loop, may be made to have a phase difference of practically 90°. The terms $R'^2 \omega^2 L_1^2$ and $\omega^2 L_2^2$ may be neglected. Also the term $\frac{16\cos^2\phi}{A^2B^2}$ is zero if $\phi = 90°$. The equation of the ellipse is then

$$\frac{X^2}{A^2} + \frac{Y^2}{B^2} = 1$$

and the major axis is either horizontal or vertical, according as A or B is greatest. If these factors are neglected the expression reduces to

$$\frac{m_1}{m_2} = \frac{R^2 + \frac{1}{\omega^2C^2} - \sqrt{\left(R^2 - \frac{1}{\omega^2C^2}\right)^2}}{R^2 + \frac{1}{\omega^2C^2} + \sqrt{\left(R^2 - \frac{1}{\omega^2C^2}\right)^2}} = \frac{1}{\omega^2C^2R^2}$$

Measurements at Audio Frequencies.

The table on the opposite page shows the various resistances and reactances for frequencies from 500-7000 cycles, together with calculated and observed values of $\frac{m_1}{m_2}$ and θ. The smallness of the terms $\omega^2L_1^2$, $\omega^2L_2^2$ and R'^2 shows the justification for neglecting them.

The angle θ was measured with an Abney Hand Level, placed on the coils. The results show a discrepancy with calculated values, at all frequencies. The balance setting was very broad at the lower frequencies and the results hadcto be averaged in these cases. This somewhat dissappointing result can be directly traced to the following causes.

(1) Braadness of Balance Setting.

The note in the headphones could never really be made to dissappear. This effect may be considered under the following subdivisions.

(a) The capacity coupling between the test field coils and the search coils caused a continuous signal to be heard in the phones. The test field were shielded with aluminium foil as described above; it was , however , not possible to shield all leads, nor to shield the output transformer of the oscillator or buffer amplifier. The effect increases with frequency, and makes this method of testing the apparatus impossible at radio frequencies.

(b) The leads to the selector panel are situated in the magnetic field, and so had an emf induced in them. These were made much longer than necessary, since this effect was not antiwipated when the apparatus was designed.

(c) There was considerable difficulty in tuning the fixed coil accurately. The coils were all wound with too many turns of wire; the signal was extremely loud in the phones, and the number of turns could havee been reduced by about 80% with advantage, considering the amplification employed. Coupling also probably occurred between turns of the search coils owing to their large number.

(d) Capacitative coupling between the loops and the wiring of the detector itself caused errors. It was discovered at the end of the experiment, that the detector would give a small note in the headphones, even if unconnected with the coils.

(2) <u>Errors in the Angle of Inclination of the Axes.</u>

In all probability, this was caused by distortion of the field in the loop, due to lagge masses of iron, in the form of electrical machinery, in the laboratory. This machinery was all situated to the north of the apparatus, since the latter was placed next to the south wall of the building. The discrepancy was about the same at all frequencies.

In addition to the above disturbing factors, the presence of a slight 60 cyclehum may be mentioned. This was considered to be due to the transformers in the transformer vault, and possibly , to a slight leakage between the a.c. and d.c. circuits of the laboratory , since the latter was used for B battery voltage for amplifier and oscillator.

<u>Work at High Frequency.</u>

An attempt was made to raise the frequency to 20- 30 K.C., and to make measurements in this band. The oscillator from the cathode ray oscillograph was used for this purpose; it was connected to the loops through a line from the meter room at the other end of the laboratory.

Various systems were tried to bbtain an audible note. First, autodyne detection, with tuned plate and grid coils in the detector stage, coupled to the radio frequency transformer in the preceding stage, was tried. This was abandoned owing to the difficulty of constructing a suitable radio frequency transformer for such low frequencies. Second, the vacuum tube voltmeter was used, and later, abandoned for reasons given above. Finally, buzzer modulation in the grid circuit of the oscillator gave an audible note in the phones.

Measurements in this frequency band failed for two reasons: first, owing to capacity coupling between the coils , as outlined above. This was especially bad because of the length of line (about) 20 yards), from the oscillator and the test loops, and the high frequency. Second, it was found impossible to tune the fixed direction finding coil on account of the large natural wavelength of the coil

Thus, while measurements were not successful in this band, the reasons are not far to seek: therefore, we cannot assume that the method will not be successful in the field. While the fields to be measured in the field will tend to be smaller than in the laboratory, given correct design of the coils, the disturbing factors will be absent

Summary of Conclusions.

(1) The fact that results were obtained on the low frequencies, suggests that the principle of the apparatus is sound, but that careful design is essential.

(2) This method of testing the apparatus is unsuitable for high frequencies. A better method would be to select a suitable underground conductor, such as an iron pipe, and make actual measurements in the field.

(3) A considerable amount of information has been gained about the design of the search coils. A comparatively few turns of wire, say 200, will be used on the next outfit. This will reduce the capacity of the winding, since they will be space wound, and the coil will have a small natural wave length. The difficulty of support for th3 winding will be increased: this, however, should not be insurmountable. Again, the leads from the tappings and coils to the selector panel will be as short as possible. The latter will be mounted inside the coil assembly, as also will be the tuning condenser.

(4) Radio frequency amplification, in the detector set, is not suitable for the comparatively low frequencies employed. Autodyne detection, therefore, cannot be used, owing to instability when tuning the search coils. For this reason , it will be necessary to modulate the transmitter. Simple chopper

or buzzer modulation in the grid circuit will be satis factory. An extra stage of amplification can be added if necessary.

(5) Very careful control of the frequency is necessaryas the fixed direction finding coil must be tuned accurately. The oscillator should be calibrated with great care against a standard wavemeter. There should be four settings to give say 20,30,40, and 50 K.C. The search coil should have condensers corresponding to these frequencies, which can be cut in by a switch on the selectar panel, according to which frequency is being used. This tuning should be done permantly in the laboratory, since it cannot be done as accurately in the field.

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