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BIPOLAR INDUCTION TORQUE IN A CIRCULAR DISC

by

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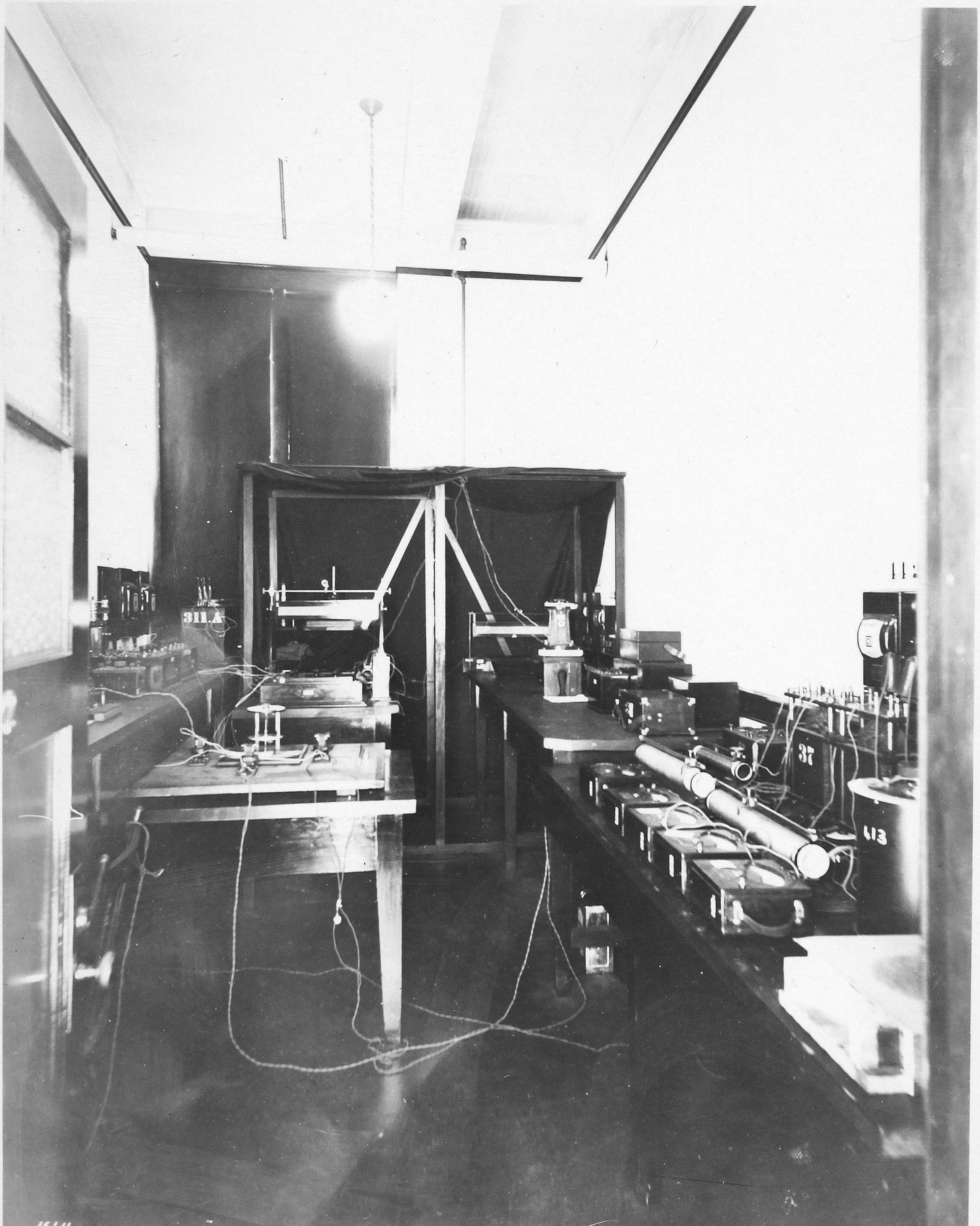
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Bipolar Induction Torque in a Circular Disc.

In order to compute the torque of an induction instrument it is necessary to know the magnitude and direction of the currents induced in the moving element.

In developing the theory of the torque due to bipolar induction in a flat circular disc, it is usually assumed that the eddy currents flow around the inducing magnet in a series of concentric rings.

While this assumption is valid in the case where the centre of the magnet coincides with the centre of the disc, it is not so when the magnet is located eccentrically with respect to the disc axis; and torque formulas based on such an assumption give results considerably in error, the error increasing as the magnets approach the edge of the disc.

Drysdale and Jolley state that the theoretical torque is about 220% of the measured torque, the ratio varying with the distance between the magnets and the disc edge.

Drysdale and Jolley, Electrical Measuring Instruments, Vol. 2, P. 168. Ernest Benn, Ltd., London.

The object of this paper is to investigate the path of the eddy currents induced in a flat circular disc, and to develop a troque formula based on the actual current distribution.

Investigation of Eddy Current Path.

Consider an alternating current field normal to an infinite sheet of conducting material as in Fig. 1.

The resistance of an annular ring of mean radius R and width dR is given by

$$\frac{2\pi R}{dR} \cdot \frac{\sigma}{t}$$

where σ is the specific resistance of the material and t the thickness of the sheet.

The conductance of the ring is given by

$$dG = \frac{t}{2\pi\sigma} \cdot \frac{dR}{R}$$

Since the $e.m.f.$ induced in all rings external to the magnet is the same, the current flowing in a ring of outer radius R_2 and inner radius R_1 is

$$I = E\int_{R_1}^{R_2} dG = \frac{Et}{2\pi\sigma} \log_\epsilon \frac{R_2}{R_1}$$

Next consider a pair of equal alternating current fields of opposite polarity normal to an infinite conducting sheet as shown in Fig. 2.

Let P move in the direction of the resultant electric field, and let F be on the locus of P.

Since PF is the locus of P, the component of the electric field normal to PF must be zero. That is to say there can be no resultant current crossing the locus PF.

The current crossing PF induced by the pole at A

$$I_a = \frac{Et}{2\pi\sigma} \log_\epsilon \frac{R_1}{M}$$

The current crossing PF induced by the pole at B

$$I_b = -\frac{Et}{2\pi\sigma} \log_\epsilon \frac{R_2}{N}$$

The resultant current $I_a + I_b$ must be zero

$$\frac{Et}{2\pi\sigma} \left(\log_\epsilon \frac{R_1}{m} - \log_\epsilon \frac{R_2}{n} \right) = 0$$

$$\frac{R_1}{R_2} = \frac{m}{n}.$$

Next find the equation of the locus of P. Refer to Fig. 3.

Take the origin at O.

$$R_1^2 = (s+x)^2+y^2 = s^2+2sx+x^2+y^2$$

$$R_2^2 = (s-x)^2+y^2 = s^2-2sx+x^2+y^2$$

But $\left[\frac{R_1}{R_2}\right]^2 = \left[\frac{m}{n}\right]^2$

$$m^2(s^2-2sx+x^2+y^2) = n^2(s^2+2sx+x^2+y^2)$$

$$y^2 + \left[x - \frac{m^2+n^2}{2(m-n)}\right]^2 = \frac{m^2n^2}{(m-n)^2} \qquad (1)$$

This is the equation of a family of circles of radius $\frac{mn}{m-n}$ having their centres at

$$x = \frac{m^2+n^2}{2(m-n)}$$

$$y = 0$$

Equation (1) describes the path of the eddy currents induced in a thin infinite sheet by a pair of equal alternating current poles 180° out of phase with each other. A series of these circles together with a sketch of the physical arrangement of the poles is shown in Fig. 4 and 5.

In order to describe the flow circles it is convenient to know the relation between

s = the half distance between the pole centres.

R = the radius of any current path.

e = the distance between the centre of the circle of radius R and the centre of the inducing pole.

This relation is obtained as follows:

$$\begin{aligned} e &= x - s \\ &= \frac{m^2+n^2}{2(m-n)} - \frac{m+n}{2} \\ &= \frac{n^2}{m-n} \end{aligned}$$

$$s = \frac{m+n}{2}$$

$$m = 2s - \overset{n}{\cancel{m}}$$

$$n = R - e$$

$$\therefore e = \frac{(R-e)^2}{2s - R + e - R + e}$$

$$R^2 - 2es - e^2 = 0$$

$$S = \frac{R^2 - e^2}{2e} \qquad (2)$$

$$e = \sqrt{S^2 + R^2} - S \qquad (3)$$

It should be noted from (2) that S is the parameter defining an entire family of current flow circles, and that if the radius of one circle is known together with the distance between its centre and the centre of the pole, then S can be computed; and by substituting this value of S in (3), the value of R corresponding to any value of e can be found, thus defining the entire family of flow lines for that particular case.

A series of flow lines drawn by the above method is shown in Fig. 6.

Consider an alternating current pole located eccentrically with respect to a conducting disc.

Let R' = radius of disc

e' = distance between centre of pole and centre of disc.

Then the boundary of the disc defines one flow line, and by means of equations (2) and (3) the flow lines within the disc can be computed.

In order to obtain some experimental check on the preceding work, a shallow circular trough of mercury was set up in conjunction with an alternating current magnet

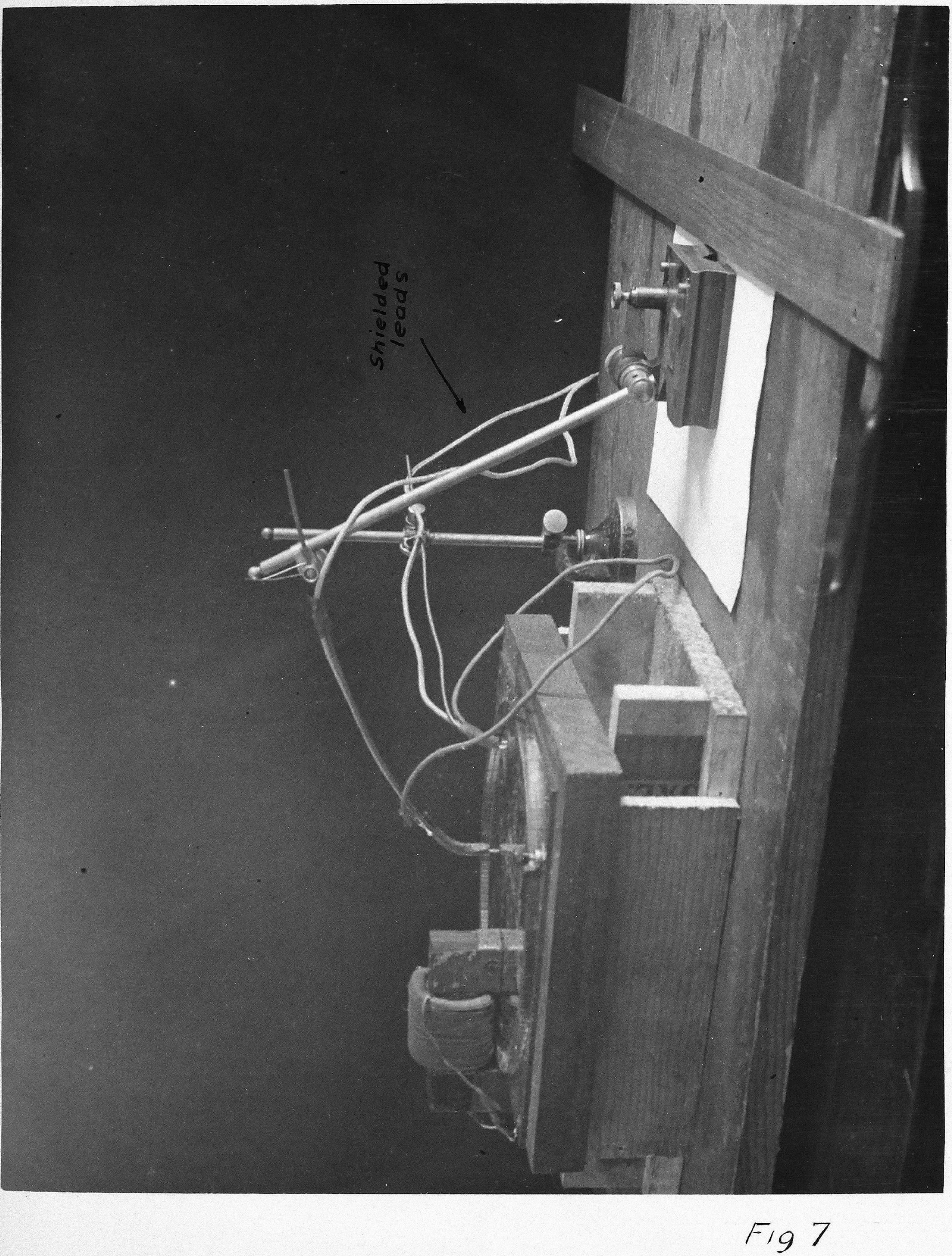


Fig 7

as shown in Fig. 7. Two pointed electrodes made contact with the mercury, and were connected to a vibration galvanometer tuned to the supply frequency, provision being made for moving these contacts and plotting their position relative to the disc.

The equipotential lines in the mercury disc were found and plotted by means of this apparatus. The method gave good results, because a very slight deviation of one electrode from the equipotential line passing through the second electrode caused a large deflection of the galvanometer. A plot of the equipotential lines to full scale is shown in Fig. 6.

The equation of the equipotential loci is the orthogonal trajectory of the original flow line equation (1) and is given by

$$x^2 + (y \mp b)^2 = \sqrt{s^2 + b^2}$$

This defines a family of circles having their centres on the y-axis and passing through the centre of each of the poles.

By means of equations (2) and (3) the y-axis was located for the value of R and ϵ used in the mercury set up, and the series of equipotential and flow circles was drawn.

The equipotential circles described by the above method were found to correspond very well with the experi-

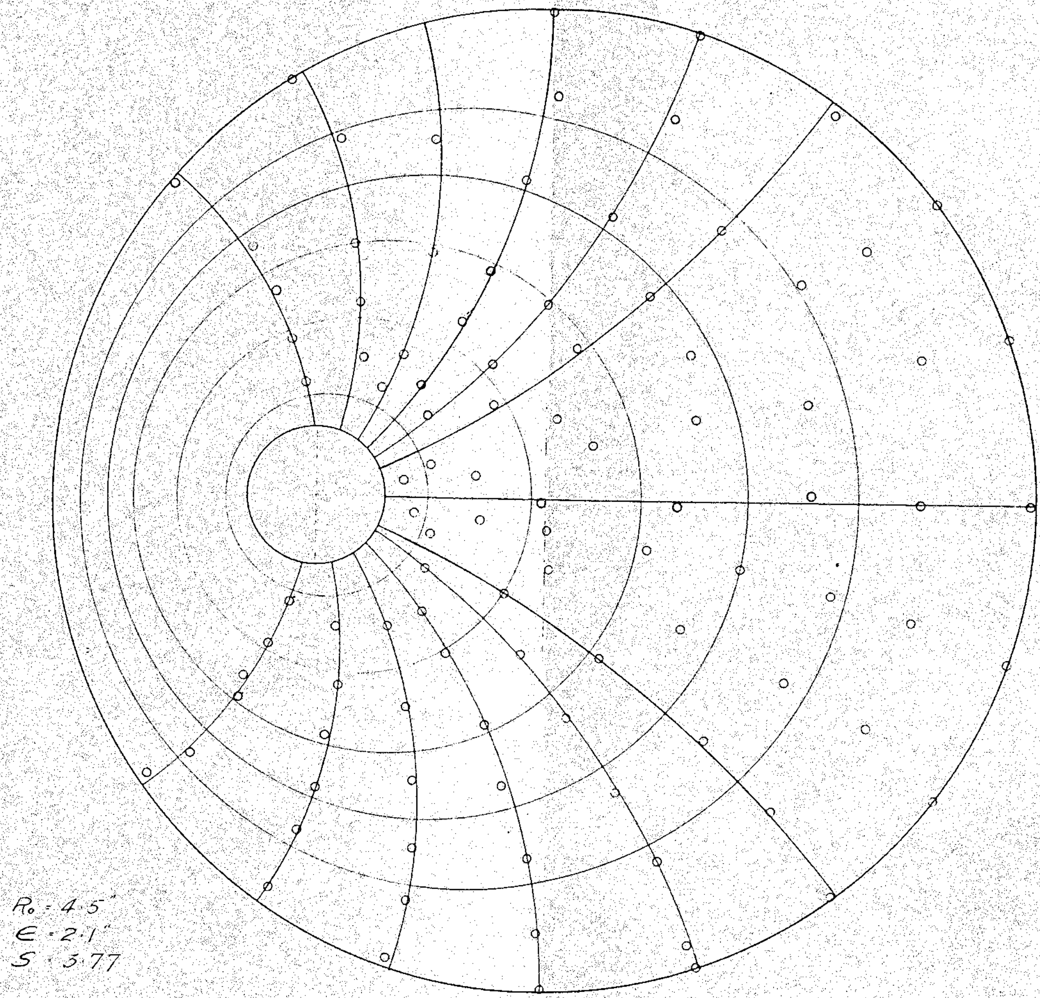


Fig. 6

mental results, and constituted a check on the preceding theory. In Fig. 6, the theoretical equipotential curves are drawn in full, while the experimental points are shown by the small circles.

Calculation of Torque.

The preceding work indicates the path of the currents induced in a disc. The next step is to develop torque relations based on this current distribution.

In order to compute the torque produced by the flow of current induced in the disc by one magnet across the field of the second magnet, it is necessary to know the magnitude and direction of the current.

The fdlowing data is required:

Diameter of disc.
Thickness of disc.
Specific resistance of disc material.
Dimensions of magnets.
Length of air gap.
Location of poles.
Magnitude and phase of flux in each magnet.
Supply frequency.

Refer to Fig. 8.

The first step is to describe the flow line circles through g and h. Then all the current that flows under the pole B due to the emf induced in the disc by the pole A, must flow in the crescent bounded by these two circles.

Construction:

Given e = uv distance between centre of disc and centre of pole A.

R_0 = radius of disc.

Evaluate s by means of equation (2).

Draw the y-axis at a distance s from the centre of pole A.

Bisect uh at K and draw kb normal to uh intersecting the y-axis at b. Join bh and draw hb′ normal to hb and intersecting the x-axis at b′. With centre b′ and radius b′h describe a circle. This circle is the flow line through the point h.

By means of this construction draw flow circles through g and j, the diagonally opposite corner, and centre of the pole, respectively.

Let R_1 = radius of flow circle through g.

R_2 = radius of flow circle through h.

e_1 = distance between centre of R . circle and centre of pole A.

e_2 = distance between centre of R_2 circle and centre of pole A.

T = thickness of disc.

σ = specific resistance microhms/cm^3.

$$e_1 = \sqrt{S^2 + R_1^{\,2}} - S$$

$$e_2 = \sqrt{S^2 + R_2^{\,2}} - S.$$

All dimensions in centimetres.

Knowing the above six factors it is possible to work out the resistance of the ring. The calculation is rather long and is given in the appendix.

The final expression is

$$\text{Resistance} = \frac{2\pi\sigma}{T} \times \frac{10^{-6}}{\log_e \dfrac{e_2 R_1}{e_1 R_2}} \text{ ohms.}$$

The current flowing in the ring can now be computed.

Let ϕ_A = flux in magnet A. R.M.S.

f = supply frequency.

E_A = emf induced in ring by ϕ_A

R_A = resistance of ring surrounding magnet A.

I_A = current flowing in ring due to E_a.

β_B = flux density of magnet B.

θ = electrical phase angle between ϕ_A and ϕ_B.

L = mean length of path of current under magnet B.

Then $E_A = 2\pi f \phi_A 10^{-8}$

$$I_A = \frac{E_A}{R_A} = \frac{E_A \times T \log_e \dfrac{e_2 R_1}{e_1 R_2} \times 10^6}{2\pi\sigma}$$

L is obtained from Fig. 8 as follows:

Let a' be the centre of the flow circle through J (the centre of the pole B). Through J draw Ja intersecting the circles R_1 and R_2 at d and e respectively. Then L may be taken as the effective area of the pole B divided by de. This seems arbitrary!

The force between the current induced by magnet A and the field of magnet B is

$$F = 0.1 \times \beta_B \times I_A \times L \sin\theta .$$

Since the flow lines under the pole B are not parallel the direction of the force varies over the pole face, for example, the force due to the current element at the point h would be along hb' and the force due to the current at g would be along gc'. It is assumed that the resultant force acts along Ja', that is to say, along the normal to the flow line passing under the centre of the pole.

This force can be resolved into two components, one acting through the axis of the disc, and the other tangential to the disc. The torque is due to this second component.

Let ψ = angle $a'Jv$

ρ = radius arm Jv

Then torque = $F\rho \sin\psi$.

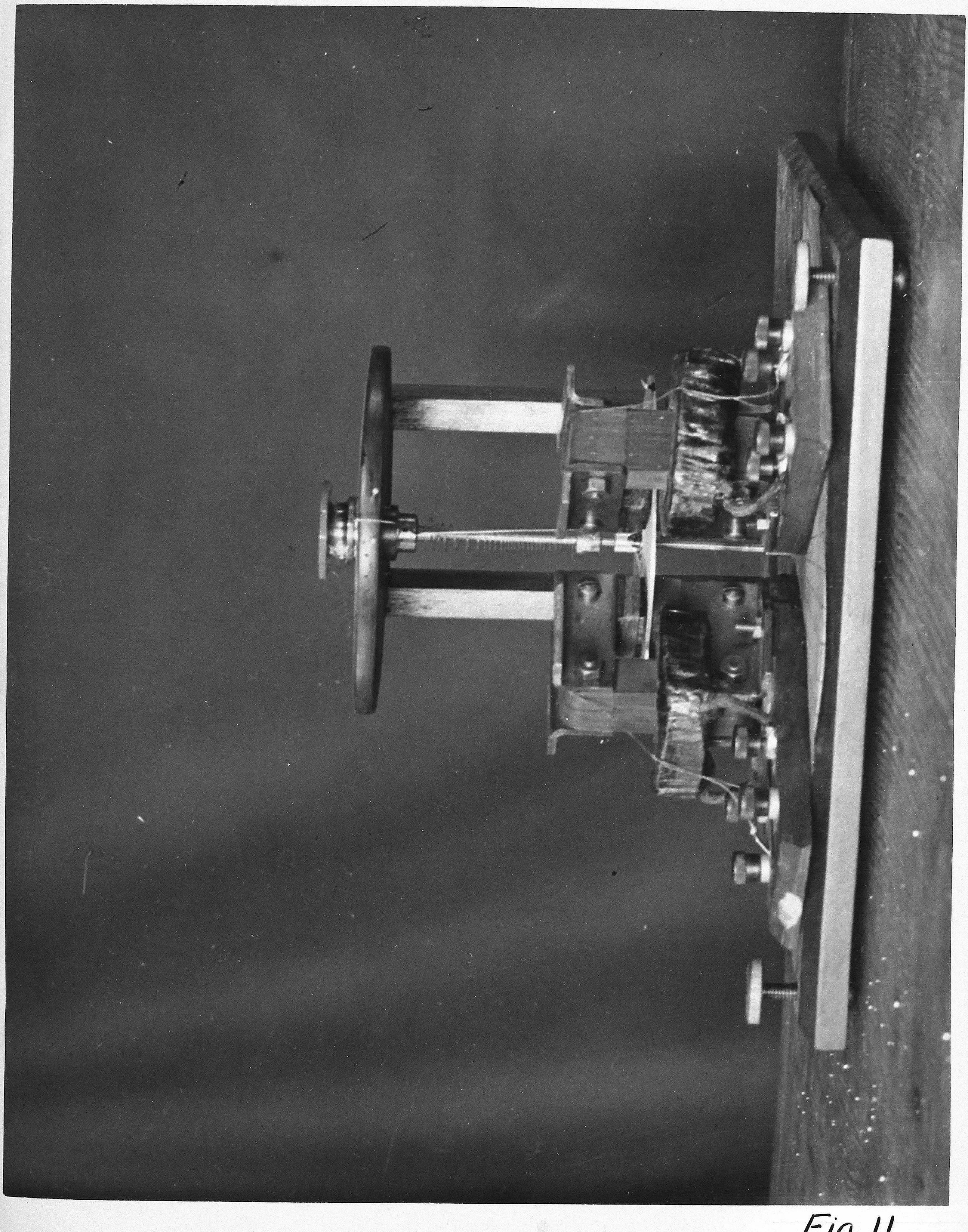


Fig. 11

Since an equal torque is produced by the current I_B flowing under the field ϕ_A. The total torque is twice the above, and can be expressed by

$$T = 0.2 \times \frac{2\pi f \phi_A}{R_A} \times \beta_B \times \angle \varrho \sin\theta \quad \sin\psi \cdot 10^{-8}$$

dyne cm.

In order to check the preceding theory, an aluminum disc mounted on a vertical axis in jewel bearings was fitted with a torsion head as shown in Fig. 11. The instrument was carefully calibrated, and the torque developed by the disc under the influence of two magnets connected to a two phase supply was measured. The flux from each magnet was measured by means of search coils wound on the poles and connected to an AC potentiometer.

The dimensions of the apparatus were as follows:

Diameter of disc	12.7 cm.
Thickness of disc	.1055 cm.
Specific resistance (aluminum)	2.83 microhms per cm. cube.
Dynamometer constant	18.5 dyne cm. per degree.
Pole dimensions	2.2 x 1.9 cm.
Search coil	20 turns.

The results obtained with this apparatus are given in the appendix. When the space angle between the poles was 65°, the calculated torque varied from 63 to 93% of the measured torque, depending on the distance between the magnets and the edge of the disc. When the poles were spaced out to 90° the calculated torque varied from 56.5 to 76.5% of the measured torque.

It should be noted that a number of assumptions and simplifications have been made in developing the preceding theory. It is difficult to find the magnitude and direction of the resultant force due to the current flowing under the magnet, because of the varying current density.

The effects of the disc leakage inductance on the flow contours has not been taken into account. In the case of a low resistance disc, the disc reactance would decrease the magnitude and change the phase angle of the induced currents. The torque would also be modified by the effect of fringing in the driving magnets.

In view of the number of variables and the complexity of the phenomena involved, it is felt that the theoretical results are in about as good agreement with the observed quantities as could be expected.

Apparently very little work has been done on the subject, and it is probable that further research would result in the determination of an accurate expression for the torque.

It would not help your results to take the reactance of the disc ~~[illegible]~~ into consideration, for your calculated torque is from 50 to 90% of the measured value

The penetration of current into the disc is not taken into consideration

Fringing, if taken into account, would reduce the error between observed & calculated values

I believe the error is due to incorrect ~~[illegible]~~ length of path under pole

H.V.

APPENDIX.

Table of results.

Plotted results.

Worked out Example.

Calculation of Resistance.

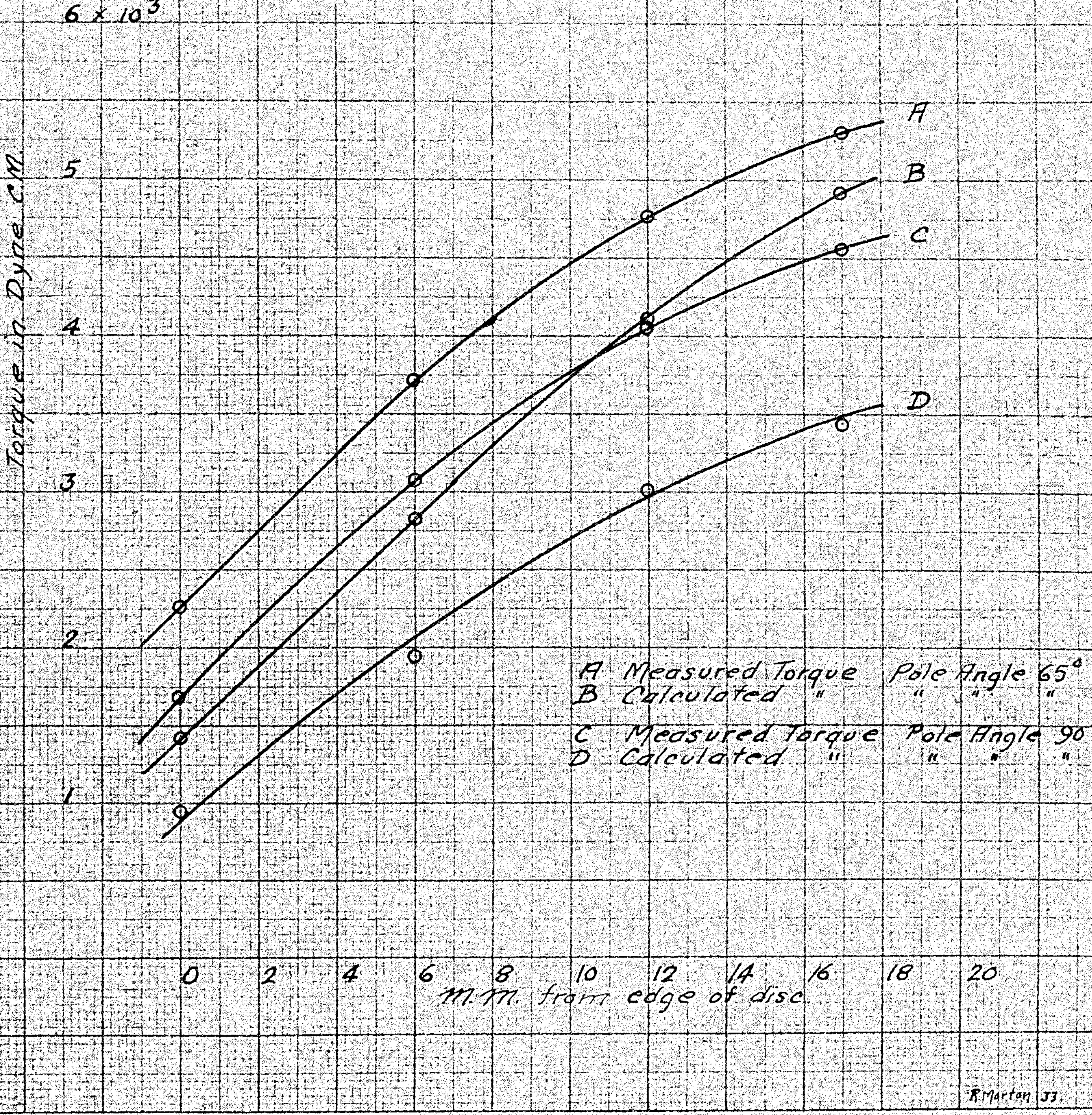
Drawings.

Pole Angle 65°

Distance between edge of pole and Disc Edge. mm.	Measured Torque Dyne Cm.	Computed Torque Dyne Cm.	%
0	2260	1420	63
6	3710	2840	76.5
12	4760	4100	86.0
17	5300	4920	93.0

Pole Angle 90°

0	1680	950	56.5
6	3080	1950	63.
12	4050	3020	74.5
17	4550	3440	76.5



6 × 10³
5
4
3
2
1
Torque in Dyne C.M.
A
B
C
D
A Measured Torque Pole Angle 65°
B Calculated " " " "
C Measured Torque Pole Angle 90°
D Calculated " " " "
0
2
4
6
8
10
12
14
16
18
20
M.M. from edge of disc
R.Marton 33.

A M L^TD

Test No. 13.

Angle between poles 65°.

Distance between edge of pole and edge of disc 6m.m.

E.M.F. induced in 20 turn search coil Pole A 1.220 V
Pole B 1.370 V

Angle between E_a and E_b = = 82.5° $\sin\theta$ = .991.

Dynamometer reading = 201°.

See page 11 for instrument data.

First a plan of the disc was drawn showing the two magnets in their proper location as in Fig. 8.

Then S was computed

$$S = \frac{R_o - e^2}{2e} = \frac{6.35^2 - 4.65^2}{2 \times 4.65} = 2.01.$$

By means of the construction given on page flow circles were described through the points g, h, and j.

Measure R_A = 2.92 cm.

R_B = 5.50 cm.

$$e = \sqrt{S^2 + R^2} - S$$

$$e_A = \sqrt{2.01^2 + 2.92^2} - 2.01 = 1.53$$

$$e_B = \sqrt{2.01^2 + 5.50^2} - 2.01 = 3.87.$$

$$\text{Resistance of ring} = \frac{2\pi\sigma}{T} \times \frac{1}{2.303 \log_{10} \dfrac{e_B R_A}{e_A R_B}}$$

$$= \frac{2\pi \times 2.83}{.105} \times \frac{1}{\log 1.34} = 574 \text{ microhms.}$$

Current flowing in ring

$$= \frac{1.220}{20} \times \frac{10^6}{574} = 10.62 \text{ amps.}$$

Flux density in magnet B

$$\beta_B = \frac{E_B}{2} \times \frac{10^8}{\text{pole area}} \times \frac{1}{20}$$

$$= \frac{1.370}{40 \times 60} \times \frac{10}{4.2} = 432 \text{ lines / cm.}$$

The following quantities are measured from Fig. 8.

$$\angle aJv = \psi = 25.5^\circ \qquad \sin\psi = .430$$

$$de = 2.7 \text{ cm.}$$

$$\text{Mean length of path under pole} = \frac{\text{pole area}}{2.7}$$

$$= \frac{4.2}{2.7} = 1.56 \text{ cm.}$$

$$= \text{Radius arm} = 4.65 \text{ cm.}$$

$$\text{Torque} = 0.2 \times 10.62 \times 432 \times 1.56 \times 4.65 \times .991 \times .430$$

$$= 2850 \text{ dyne cm.}$$

Measured torque = 201 x 18.5 = 3710 dyne cm.

Computed torque as per cent of measured torque = 76.5%.

Calculation of Ring Resistance.

Refer to Fig. 9.

Let R_1 = radius of outer boundary of ring.

R_2 = radius of innter boundary of ring.

e_1 = distance between magnet centre and centre of circle R .

e_2 = distance between magnet centre and centre of circle R_2.

a = $e_1 - e_2$.

σ = specific resistance of disc material.

T = thickness of disc.

Then provided that R_1 is only slightly greater than R_2, the ring resistance is given by

$$W = \frac{2\sigma}{T}\int_0^{\pi} \frac{\frac{R_1 + R_2}{2}\, d\theta}{(a^2+R_1^2+2aR_1\cos\theta)^{1/2} - R_2} .$$

This integral is evaluated as follows:

$$\frac{\sigma(R_1+R_2)}{T}\int_0^{\pi} \frac{\left[\sqrt{A^2+R_1^2+2aR_1\cos\theta} + R_2\right]}{a^2+R_1^2-R_2^2+2aR_1\cos\theta}\, d\theta$$

$$\text{Let } I_1 = \frac{\sigma(R_1+R_2)}{T}\int_0^{\pi} \frac{\sqrt{a^2+R_1^2+2aR_1\cos\theta}}{a^2+R_1^2-R_2^2+2aR_1\cos\theta}\, d\theta$$

$$I_2 = \frac{\sigma(R_1+R_2)R_2}{T}\int_0^{\pi} \frac{d\theta}{a^2+R_1^2-R_2^2+2aR_1\cos\theta}$$

Evaluation of I_1.

Let $\upsilon = \tan\frac{\theta}{2}$

Then $\sin\theta = \frac{2\upsilon}{1+\upsilon^2}$, $\cos\theta = \frac{1-\upsilon^2}{1+\upsilon^2}$ $d\theta = \frac{2\,d\upsilon}{1+\upsilon^2}$

When $\theta = 0$ $\quad \tan\theta = 0$ $\quad \upsilon = 0$

$\theta = \pi$ $\quad \tan\frac{\theta}{2} = \infty$ $\quad \upsilon = \infty$

$$I_1 = \frac{\sigma(R_1+R_2)}{T}\int_0^{\infty} \frac{\sqrt{a^2+R_1^2+2aR_1\left(\frac{1-\upsilon^2}{1+\upsilon^2}\right)}}{a^2+R_1^2+2aR_1\left(\frac{1-\upsilon^2}{1+\upsilon^2}\right)} \times \frac{2d\upsilon}{1+\upsilon^2}$$

$$= \frac{2\sigma}{T}(R_1+R_2)\int_0^{\infty} \frac{\sqrt{(a-R_1)^2\,\upsilon^2+(a+R_1)^2}}{\left\{(a-R_1)^2-R_2^2\right\}^2 + \left\{(a+R_1)^2-R_2^2\right\}} \frac{2d\upsilon}{\sqrt{1+\upsilon^2}}$$

Let $\upsilon = \frac{a+R_1}{a-R_1}\tan\phi$

When $\upsilon = 0$ $\quad \phi = 0$ $\quad du = \frac{a+R_1}{a-R_1}\sec^2\phi\, d\phi$

$\upsilon = \infty$ $\quad \phi = \frac{\pi}{2}$

$$= \frac{2\sigma}{T}(R_1+R_2)\int_0^{\pi/2} \frac{(a+R_1)\sec\phi\,\frac{a+R_1}{a-R_1}\sec^2\phi\, d\phi}{\left[\left((a-R_1)^2-R_2^2\right)\left(\frac{a+R}{a-R}\right)^2\tan^2\phi + \left((a+R_1)^2-R_2^2\right)\right]\sec\phi}$$

$$= m\int_0^{\pi/2}\frac{d(\tan\phi)}{k\tan^2\phi + l}$$

where $k = \left((a-R_1)^2-R_2^2\right)\left(\frac{a+R_1}{a-R_1}\right)^2$

$$\ell = (a+R_1)^2 - R_2^2$$

$$m = \frac{2\sigma}{T} \times \frac{(R_1+R_2)\ (a+R_1)^2}{(a-R_1)}$$

$$I_1 = \frac{m}{k}\int_0^{\pi/2} \frac{d(\tan\phi)}{\tan^2\phi + \ell/k}$$

$$= \frac{m}{k}\sqrt{\frac{k}{\ell}}\left[\tan^{-1}(\tan\phi)\right]_0^{\pi/2}$$

$$= \frac{m}{\sqrt{k\ell}} \cdot \frac{\pi}{2}$$

$$I_1 = \frac{\pi}{2} \times \frac{2\sigma}{T} \frac{(R_1+R_2)\ \dfrac{(a+R_1)^2}{a-R_1}}{\dfrac{a+R_1}{a-R_1}\sqrt{\left((a-R_1{}^2)-R_2^2\right)\left((a+R_1)^2-R_2^2\right)}}$$

$$= \frac{\pi\sigma}{T} \frac{(R_1+R_2)\ (a+R_1)}{\sqrt{\left((a-R_1)^2-R_2^2\right)\left((a+R_1)^2 - R_2^2\right)}}$$

Evaluation of I_2

$$I_2 = \frac{\sigma R_2\ (R_1+R_2)}{T}\int_0^{\pi} \frac{d\theta}{a^2 + R_1^2 - R_2^2 + 2aR_1 \cos\theta}$$

Let $\upsilon = \tan \theta/2$

Then $\cos\theta = \dfrac{1-\upsilon^2}{1+\upsilon^2}$ $\quad d\theta = \dfrac{2d\upsilon}{1+\upsilon^2}$

$$I_2 = \frac{\sigma R_2}{T}(R_1+R_2)\int_0^{\alpha} \frac{\dfrac{2d\upsilon}{1+\upsilon^2}}{a^2+R_1^2-R_2^2+2aR_1\left(\dfrac{1-\upsilon^2}{1+\upsilon^2}\right)}$$

$$= \frac{2\sigma R_2}{T}(R_1+R_2)\int_0^{\alpha}\frac{du}{\left\{(a-R_1)^2-R_2^{\,2}\right\}u^2+\left\{(a+R_1)^2-R_2^{\,2}\right\}}$$

$$= \frac{2\sigma R_2(R_1+R_2)}{T\sqrt{\left((a-R)^2-R_2^{\,2}\right)\left((a+R)^2-R_2^{\,2}\right)}}\tan^{-1}u\sqrt{\frac{(a-R_1)^2-R_2^{\,2}}{(a+R_1)^2-R_a^{\,2}}}\,\Bigg|_0^{\alpha}$$

$$= \frac{\pi\sigma R_2(R_1+R_2)}{T\sqrt{\left((a-R)^2-R_2^{\,2}\right)\left((a+R)^2-R_2^{\,2}\right)}}$$

$$W = I_1 + I_2 = \frac{\pi\sigma(R_1+R_2)(R_1+R_2+a)}{T\sqrt{\left((a-R_1)^2-R_2^{\,2}\right)\left((a+R_1)^2-R_2^{\,2}\right)}}$$

The above expression gives the resistance of a crescent provided that R_1 is only slightly greater than R_2. In order to find the resistance of a thick crescent such as in Fig. 10, it is necessary to change the above expression into a conductance, and integrate the conductances between the outer and inner radii of the thick crescent.

This is done as follows:

$$W = \frac{\pi\sigma}{T}\,\frac{(R_1+R_2)(R_1+R_2+a)}{\sqrt{\left((a-R_1)^2-R_2^{\,2}\right)\left((a+R_1)^2-R_2^{\,2}\right)}}$$

Let c = conductance = $\dfrac{1}{W}$

$$R_2 = R$$

$$R_1 = R + dR$$

$$a = (\rho_1 - \rho_2) = d\rho$$

$$\rho = \sqrt{s^2 + R^2} - s.$$

s in arbitrary.

then $d\rho = \frac{RdR}{\beta}$ where $\beta = \sqrt{s^2 + R^2}$

$$dc = \frac{T}{\sigma\pi}\sqrt{\frac{\left\{\left[\frac{RdR}{\beta} - (R+dR)\right]^2 - R^2\right\}\left\{\left[\frac{RdR}{\beta} + (R+dR)\right]^2 - R^2\right\}}{(2R + dR)\ (2R + dR + \frac{2R^2 dR}{\beta})}}$$

$$= \frac{T}{\sigma\pi}\sqrt{\frac{(2RdR - \frac{2R^2dR}{\beta})\ (2RdR + \frac{2R^2dR}{\beta})}{\left(4R^2 + 4RdR + \frac{2R^2dR}{\beta}\right)}}$$

$$= \frac{T}{\sigma\pi}\ \frac{\sqrt{\frac{4R^2}{\beta^2}(\beta^2 - R^2)(dR)^2}}{4R^2 + \left(4R + \frac{2R^2}{\beta}\right) dR}$$

$$= \frac{T}{\sigma\pi}\ \frac{\frac{2SR}{\beta}\ dR}{4R^2 + \left(4R + \frac{2R^2}{\beta}\right) dR}$$

$$= \frac{2ST}{\sigma\pi} \cdot \frac{RdR}{4R^2\beta + \left(4R\beta + 2R^2\right) dR}$$

Invert denominator

$$= \frac{ST}{\sigma\pi} \times \frac{dR}{2R\beta}$$

$$C = \frac{ST}{2\sigma\pi}\int_{R_b}^{R_a}\frac{dR}{R\sqrt{S^2+R^2}}$$

$$= \frac{ST}{2\sigma\pi} \times \frac{1}{S}\log_e \left.\frac{\sqrt{S^2+R^2}-S}{R}\right|_{R_b}^{R_a}$$

$$= \frac{T}{2\pi\sigma}\log_e \frac{e_b R_a}{e_a R_b}$$

$$\text{Resistance} = \frac{2\pi\sigma}{T} \times \frac{1}{2.303\log_{10}\frac{e_b R_a}{e_a R_b}}.$$

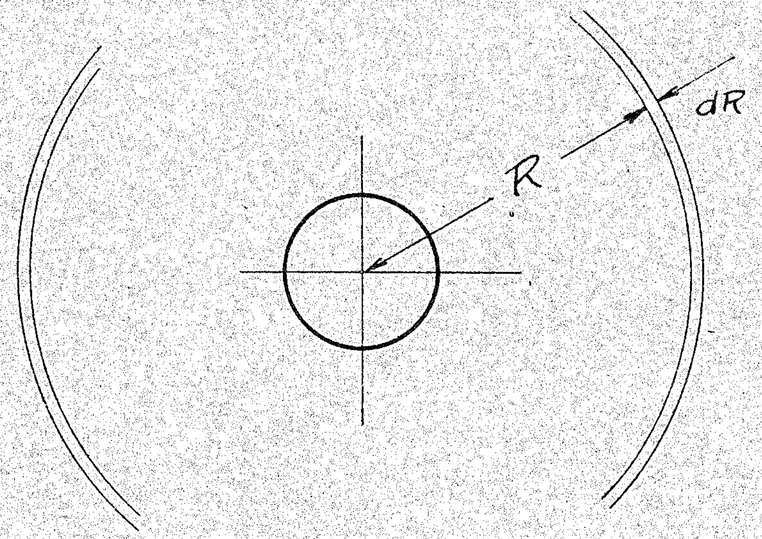


Fig 1.

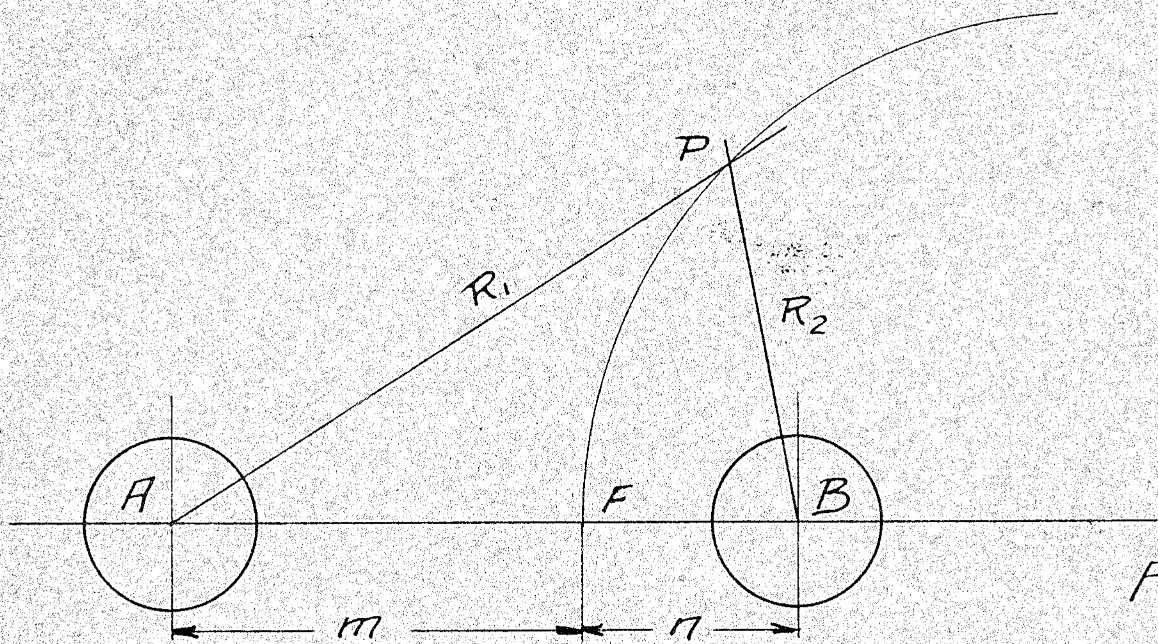


Fig 2.

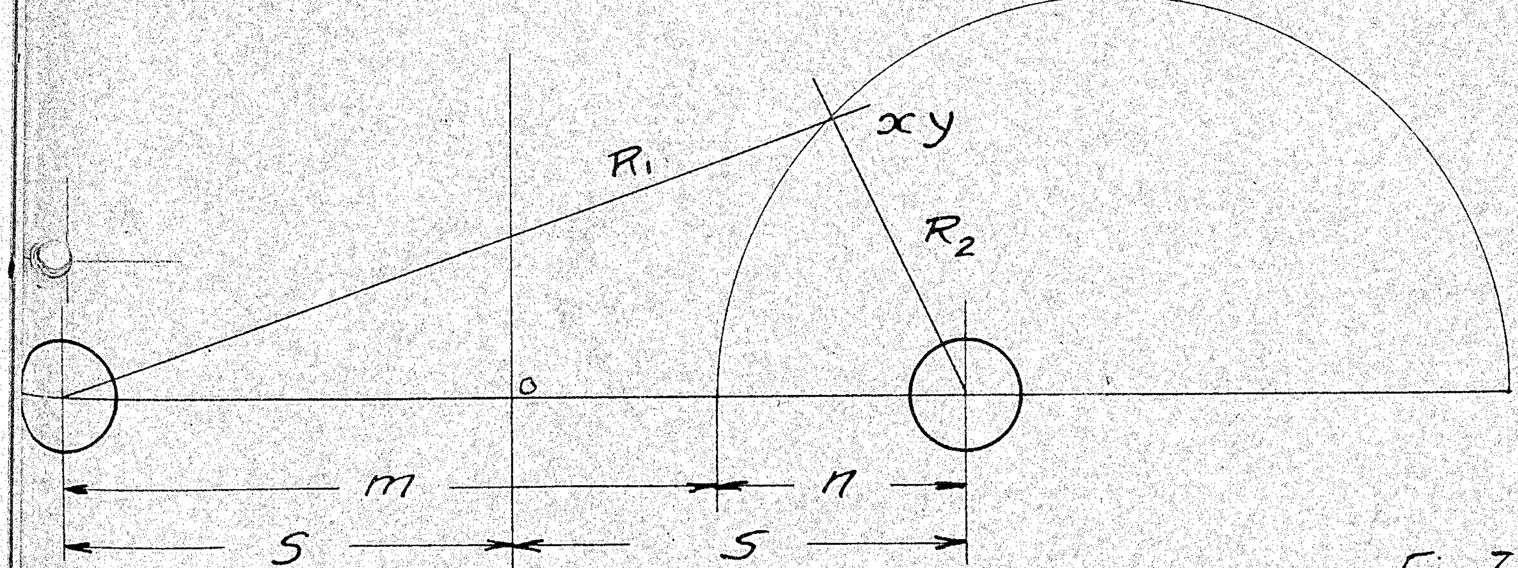


Fig 3.

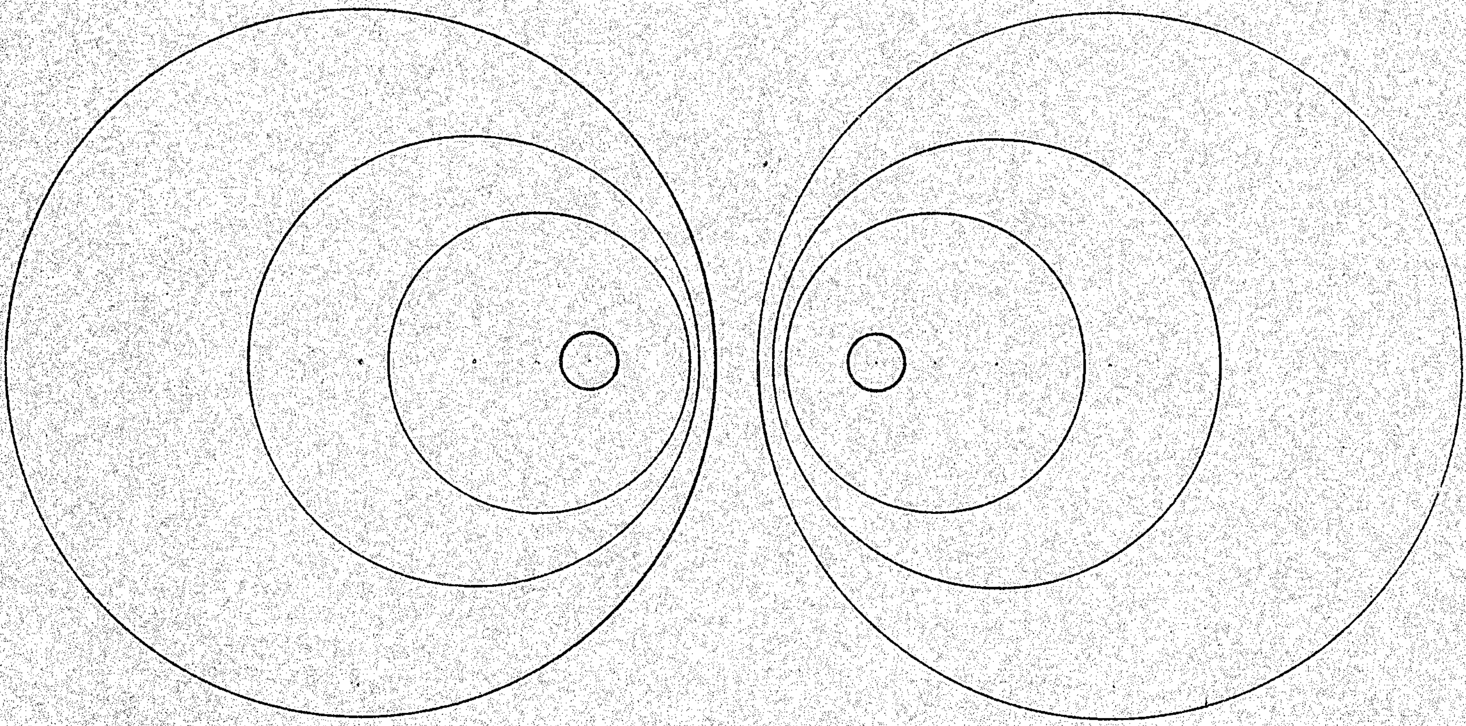


Fig 4.

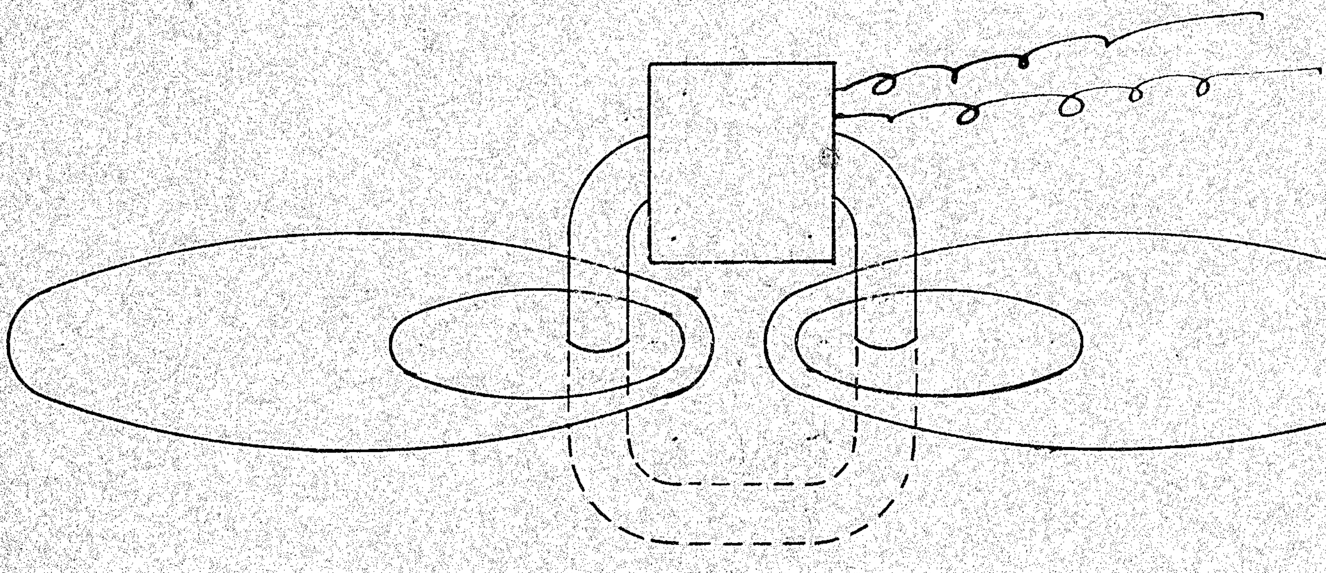


Fig 5.

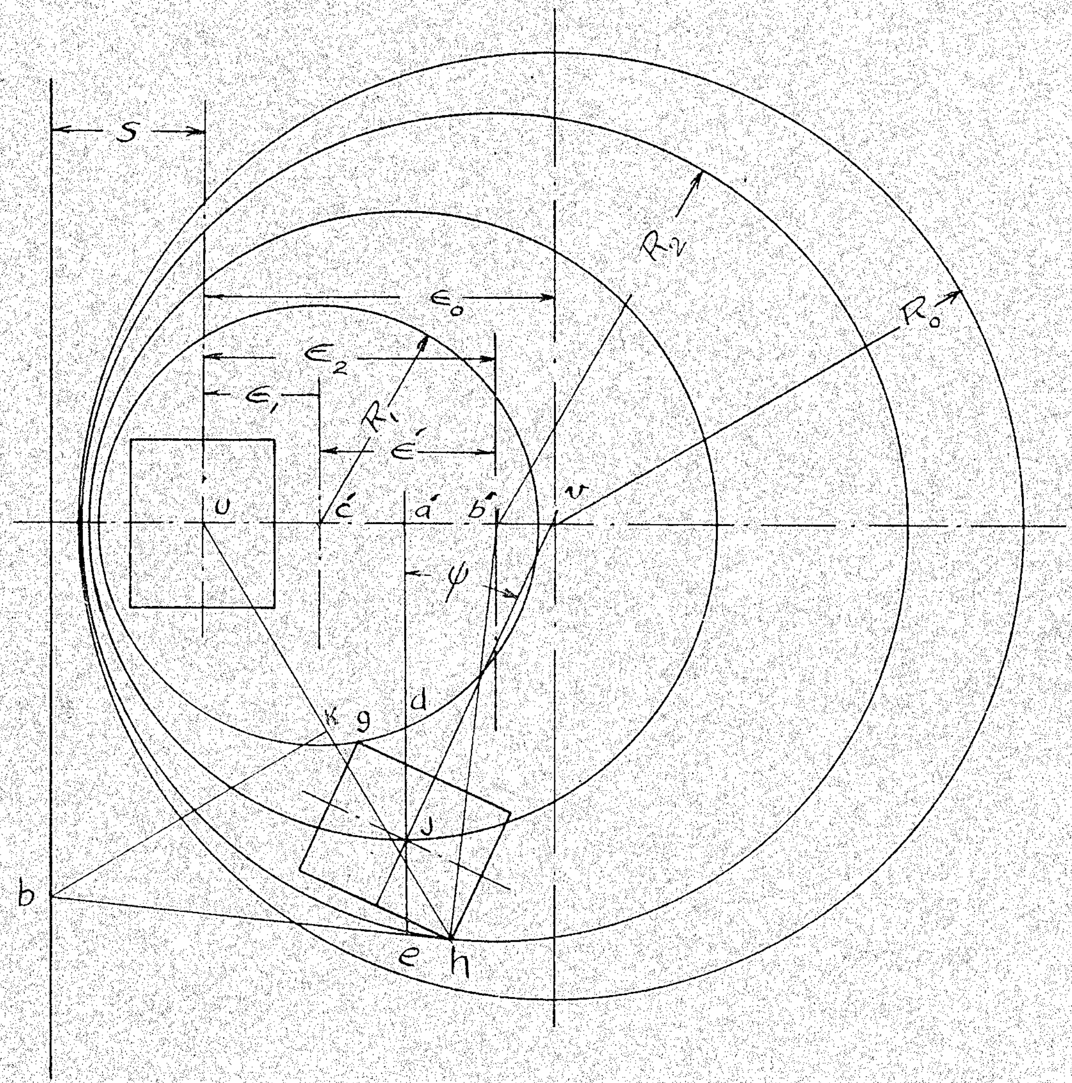

S
ϵ_0
ϵ_2
ϵ_1
R_1
ϵ'
R_2
R_0
u
c'
a'
b'
v
ψ
K
g
d
J
b
e
h

Fig. 8

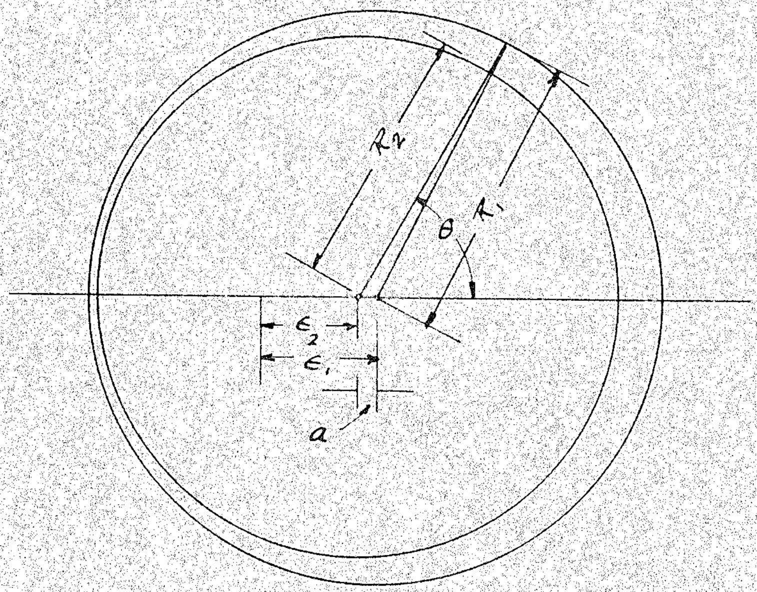


Fig 9

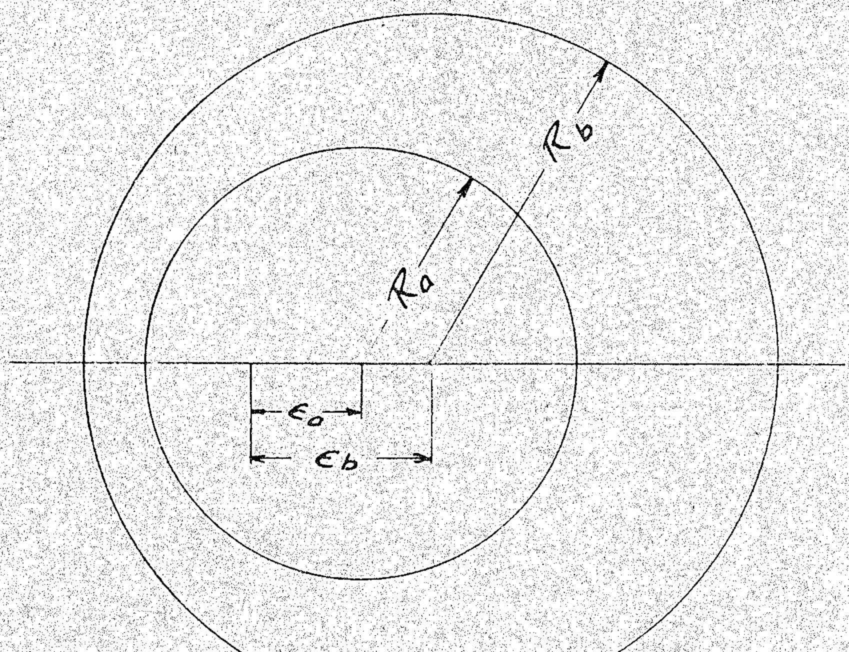


Fig 10