THE DYNAMIC STABILITY OF VOLTAGE-REGULATED AND SPEED-GOVERNED SYNCHRONOUS MACHINES IN POWER SYSTEMS

by

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ABSTRACT

The dynamic stability of a power system has been of constant interest for many years. Among the many contributors are Concordia, Heffron, Phillips, and Messerle.

In most studies, the tie line resistance and the saliency effect were neglected. An attempt has been made in this thesis to study the dynamic stability of the power system in the small including all important effects such as tie line resistance, reactance, saliency, voltage regulator, stabilizer time constants and gains, and governor time constants and gain. Two different methods for studying stability have been applied, namely, the Routh-Hurwitz criterion and the D-partition method.

The important results found in the study are the significant effects of tie line resistance and saliency upon the stability limit from Routh-Hurwitz criterion studies and upon the stability region from studies using the D-partition method. Also, significant are the effects of the machine short circuit ratio, the governor, the exciter and stabilizer time constants and gains, and their coordination.

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1. INTRODUCTION

Since the introduction of the continuously-acting voltage regulator, both theory and practice have shown that a voltage-regulated generator has a greater steady state stability limit in a power system.[1-15]

Normally, the steady state stability limit of a generator in a power system is defined as its maximum steady state load limit without falling out of synchronism.[21] For an unregulated generator, this limit depends on the machine and the system parameters alone.[21,22,23] For a machine with a continuously-acting voltage regulator and governor, negative feedback is introduced so that the power and voltage differences, which may cause machine instability, are constantly monitored and corrected before the machine becomes unstable. In this way, the machine load can be increased beyond the ordinary steady state limit to a new power limit called the dynamic stability limit.[4]

The dynamic stability limit of a generator in a power system primarily depends upon the parameters in the feedback loops introduced by the continuously-acting voltage regulator and governor. However, the machine parameters and the nature of the system to which the machine is connected are also important.

In the development of a modern power system, the ability of the synchronous generator to operate in the dynamic stability region becomes more and more important. This is due to the fact that many synchronous generators have to operate with almost unity power factor or even leading power factors. The steady state stability limit of an under-excited synchronous generator, however, is small unless it is operated in the dynamic stability region with the aid of a voltage regulator. The technical soundness of such an operation has been confirmed in both theory and practice.[3,9,13]

To the theoretical analysis of the dynamic stability of a regulated generator, standard control theory can be applied.[1,4,8] This requires a complete description of the system with differential equations. This is usually considered tedious because high order differential equations with complicated coefficients are involved.

Simplification of the problem is usually made by using only one reactance and one voltage source for the synchronous machine.[17] The results obtained, nevertheless, are not accurate, because many important changes like voltage and current transients, and voltage regulator and governor actions are not included in the equations.[4]

As for synchronous machines, Park's[26] equations can always be used for accurate investigations, since all important machine parameters are included.

Work has been done in this direction since 1944 when Concordia[1] discussed and pointed out the important effects of voltage regulation with respect to stability limit. Park's equations were used and the stability limit of a round-rotor machine operating at unity power factor was studied.

Concordia's work was followed by several others. Among them were Heffron and Phillips[3] who gave a method, also based on Park's equations, for the investigation of a round-rotor generator operating at any power factor. The dynamic stability limits in the leading power factor region were established with the aid of an analogue computer.

Analogue computer methods(3,4,5,6,7) were also developed by Messerle and Bruck,(4-6) and Aldred and Shackshaft(7) in their dynamic stability studies. Messerle and Bruck had gone a step further in taking the prime mover control into consideration. Nyquist criterion was also employed to determine the stability limit of a round-rotor generator. Messerle's work was further extended by Goodwin(8) who, using a network analyser, determined the external impedance and the initial conditions for the stability study of a water wheel generator.

Reports from the C.I.G.R.E.(10,11,12) indicated that similar approaches to the problem of dynamic stability studies have been done in the U.S.S.R. in the past two decades. The two-reaction theory for the synchronous machine has also been employed.

The previous work of Heffron and Phillips, and also that of Messerle have been extended in this thesis. A new method for the study of a salient-pole machine operating at any power factor has been established. The method, includes the round-rotor machine as a special case. The effects of saliency in the round-rotor machine, and the tie-line resistance, which were normally neglected, are taken into consideration. Voltage regulator and governor effects in the under-excited region are investigated. The well known Routh-Hurwitz criterion is used to determine the stability limits. The D-partition method[24] is used for the investigation of the effect of two particular parameters on stability. A digital computer is used for the computations.

2. REVIEW OF GENERAL THEORIES

2.1 Classical Synchronous Machine Theory[22,23,26,27]

When the equations describing the performance of a synchronous machine are written in phase co-ordinates, they become linear equations with time-varying coefficients because the mutual inductances between the stator and rotor circuits are periodic functions of rotor angular position.

In the case of a salient-pole machine the mutual inductances between stator phases are also periodic functions of rotor angular position. This causes difficulties in analysis.

Park[26] introduced the well known d-q transformation with the following assumptions:

1. The stator windings are sinusoidally distributed around the air gap.
2. The effect of the slots on the air gap flux is not appreciable with variation in the rotor angle.
3. Saturation effects are neglected.

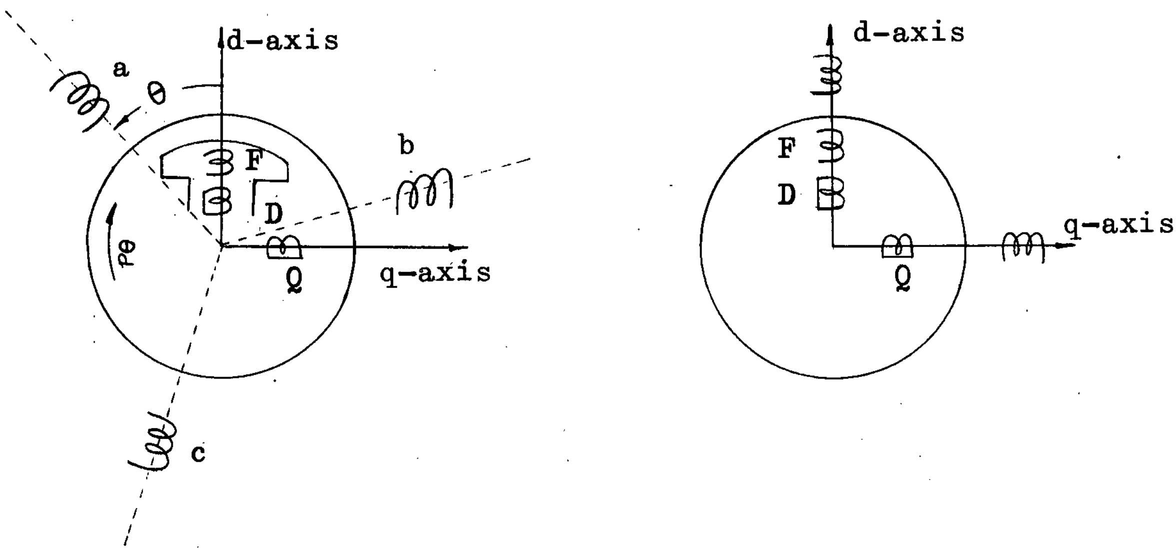
Park's transformation matrix, modified by Yu[27], has the following form:

$$\begin{bmatrix} K_a \\ K_b \\ K_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \cos\Theta & -\sin\Theta \\ \frac{1}{\sqrt{2}} & \cos(\Theta - 120) & -\sin(\Theta - 120) \\ \frac{1}{\sqrt{2}} & \cos(\Theta + 120) & -\sin(\Theta + 120) \end{bmatrix} \begin{bmatrix} K_d \\ K_q \\ K_o \end{bmatrix} \qquad (2\text{-}1)$$

where K_a, K_b, K_c represent variables such as armature currents, flux linkages or voltages in phase co-ordinates and K_d, K_q, K_o the corresponding variables in the direct-, quadrature- and zero-axis coordinates.

When the synchronous machine equations are transformed by the transformation matrix (2-1) from the phase coordinates to the d-q-o coordinates, the time-varying coefficients of the inductances are eliminated.

The transformation, in effect, resolves the three phase windings of the machine into a two-phase model with the direct- and the quadrature-axis windings, and in addition to that a zero axis winding which can be analysed separately.



(a) Synchronous Machine in Phase Coordinates

(b) Synchronous Machine in Park's Coordinates

Fig. 2-1 Synchronous Machine Models

Park's equations in the operator form can be written:

$$
\begin{aligned}
v_d &= p\psi_d - \psi_q p\theta - r_a i_d \\
v_q &= p\psi_q + \psi_d p\theta - r_a i_q \qquad (2\text{-}2) \\
v_o &= p\psi_o - r_a i_o
\end{aligned}
$$

$$
\begin{aligned}
\psi_d &= \frac{G(p)}{\omega} v_{fd} - \frac{x_d(p)}{\omega} i_d \\
\psi_q &= -\frac{x_q(p)}{\omega} i_q \qquad (2\text{-}3) \\
\psi_o &= -\frac{x_o}{\omega} i_o
\end{aligned}
$$

where

$$x_d(p) = \frac{x_d(1 + \tau'_d p)(1 + \tau''_d p)}{(1 + \tau'_{do} p)(1 + \tau''_{do} p)}$$

$$x_q(p) = -\frac{x_q(1 + \tau''_q p)}{(1 + \tau''_{qo} p)} \qquad (2\text{-}4)$$

$$G(p) = \frac{x_{ad}(1 + \tau_{D1} p)}{R_F(1 + \tau'_{do} p)(1 + \tau''_{do} p)}$$

For details of symbols see Appendix I.

Additional equations relating the machine terminal conditions are

$$
\begin{aligned}
v_t^2 &= v_d^2 + v_q^2 \\
T_e &= i_q \psi_d - i_d \psi_q \qquad (2\text{-}5) \\
T_m &= Jp^2\theta + \alpha p\theta + T_e
\end{aligned}
$$

where v_t is the terminal voltage and T_e, the electromechanical energy conversion torque.

2.2 Stability of a Dynamical System

2.2.1 Stability in the small

The system equations describing the performance of a regulating process are usually nonlinear differential equations. However, in the study of a synchronous machine in a power system, a small deviation from an initial equilibrium point is usually considered.

Liapounoff has shown[16,25] that the stability of a non-linear system under small deviations from equilibrium is completely determined by the linearly perturbated system equations which are the first variation of the differential equations. There are also critical cases when the method cannot be applied. Then either all linear perturbation terms vanish or the characteristic roots of the linearized system equations possess a zero root or a pair (or pairs) of purely imaginary roots.

In power system dynamic stability studies, however, it has been assumed that the regulating system is still at the stability boundary when these cases arise.

2.2.2 Stability of a system with linearized differential equations

The stability of a linear differential equation is determined by the homogeneous differential equation describing the free motion of the system. For a system of n^{th}

order the equation of the following form will be obtained:

$$a_o \frac{d^n}{dt} S(t) + a_1 \frac{d^{n-1}}{dt} S(t) + \ldots\ldots\ldots a_n S(t) = 0 \tag{2-5}$$

The solution of which is well known as

$$S(t) = \sum_{i=1}^{n} A_i e^{Z_i t}$$

where Z_i are the roots of the characteristic equation of the form

$$a_o p^n + a_1 p^{n-1} + \ldots \qquad a_n = 0 \tag{2-6}$$

The stability of the system is then determined by the real part of Z_i whose nature can be investigated by a number of criteria, which make it possible to judge the stability without recourse to the computation of Z_i.

2.2.3 The Routh-Hurwitz Stability Criterion

In order to ensure that all roots of the equation (2-6) have negative real parts, it is necessary and sufficient that, with a_o greater than zero, all the diagonal determinants of the array constructed from the coefficients of (2-6),

$$\begin{vmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_o & a_2 & a_4 & & \vdots \\ 0 & a_1 & a_3 & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{vmatrix} \tag{2-7}$$

be greater than zero; i.e.

$$\Delta_1 = a_1 > 0,\ \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_o & a_4 \end{vmatrix} > 0,\ \ldots.$$

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_o & a_2 & a_4 & & \vdots \\ 0 & a_1 & a_3 & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{vmatrix} > 0 \qquad (2\text{-}8)$$

2.2.4 D-partition method for two real parameters[24]

When the effect of two real parameters on the stability of a regulating system is to be considered and if the two parameters can be expressed explicitly in the characteristic equation, the stability boundaries with respect to the two parameters can be determined.

Let the characteristic equation of the system be expressed in the form

$$\gamma P(p) + \tau Q(p) + R(p) = 0 \qquad (2\text{-}9)$$

where γ and τ are the two real parameters under consideration and $Q(p)$, $P(p)$ and $R(p)$ are polynomials with coefficients that are independent of γ and τ.

For a stable system all the zeroes of the characteristic equation must be in the left half z-plane. If the values of γ and τ are allowed to vary continuously so that at least one of the zero locations moves towards and finally reaches the imaginary axis, the system is on the stability limit and becomes oscillatory. The locus of points in the $(\gamma-\tau)$ plane representing such a situation is called the stability boundary. One side of the boundary is the stable region which corresponds to the case of all the zeroes of the characteritic equation lying in the left half z-plane. The other side of the boundary is the unstable region corresponding to the case where at least one of the zeroes of the characteristic equation is in the right half z-plane.

Therefore, if all the possible loci which correspond to the j-axis zeroes of the characteristic equation are found in the $(\gamma-\tau)$ plane, the stability boundary can be determined from the loci. The loci would divide the $(\gamma-\tau)$ plane into stable and unstable regions provided that stable regions exist.

To find the loci of the stability boundary in the $(\gamma-\tau)$ plane, let $p = jw$ in the characteristic equation,

$$\gamma P(j\omega) + \tau Q(j\omega) + R(j\omega) = 0 \qquad (2\text{-}10)$$

Let P, Q and R further be separated into real and imaginary parts:

$$P(j\omega) = P_1(\omega) + jP_2(\omega)$$
$$Q(j\omega) = Q_1(\omega) + jQ_2(\omega)$$
$$R(j\omega) = R_1(\omega) + jR_2(\omega)$$

Then $\gamma P_1(\omega) + \tau Q_1(\omega) + R_1(\omega) + j\left[\gamma P_2(\omega) + \tau Q_2(\omega) + R_2(\omega)\right] = 0$

or

$$\begin{aligned} \gamma P_1(\omega) + \tau Q_1(\omega) + R_1(\omega) &= 0 \\ \gamma P_2(\omega) + \tau Q_2(\omega) + R_2(\omega) &= 0 \end{aligned} \qquad (2\text{-}12)$$

whence

$$\tau = \frac{1}{\Delta}\begin{vmatrix} -R_1(\omega) & P_1(\omega) \\ -R_2(\omega) & P_2(\omega) \end{vmatrix}$$

$$\gamma = \frac{1}{\Delta}\begin{vmatrix} Q_1(\omega) & -R_1(\omega) \\ Q_2(\omega) & -R_2(\omega) \end{vmatrix} \tag{2-13}$$

where

$$\Delta = \begin{vmatrix} Q_1(\omega) & P_1(\omega) \\ Q_2(\omega) & P_2(\omega) \end{vmatrix} \tag{2-14}$$

Nontrivial solutions of γ and τ exist when $\Delta \neq 0$. If $\Delta = 0$ the nontrivial solutions do not exist which means that there is no possible point in the $(\gamma-\tau)$ plane which gives an imaginary-axis root.

By varying ω from $-\infty$ to ∞ all the loci of the j-axis characteristic root can be obtained. In the computation it is sufficient to vary ω from 0 to ∞ since both γ and τ are the ratios of odd polynomials. Note that P_1, Q_1 and R_1 are even, and P_2, Q_2, R_2 are odd polynomials in ω.

Straight line loci are obtained by substituting $\omega = 0$ and $\omega = \infty$ into equation (2-12) because they satisfy it. These lines in the $(\gamma-\tau)$ plane represent the characteristic roots $j0$ and $j\infty$ and can, therefore, be possible stability boundaries.

After the loci are obtained the stable and unstable regions can be identified by some other stability criterion.

3. DERIVATION OF SYSTEM EQUATIONS FOR THE DYNAMIC STABILITY STUDIES

3.1 The System Under Study

A power system normally consists of a number of synchronous machines supplying power to the system at various points. For the dynamic stability study of one particular synchronous machine of the system, the remaining parts of the system are normally considered as an infinite bus to which the machine is connected through an external impedance.

The synchronous machine of the system under study is controlled by a voltage regulator and a speed governor, both of the continuously-acting type.

A schematic diagram of the system is shown in Fig. 3-1.

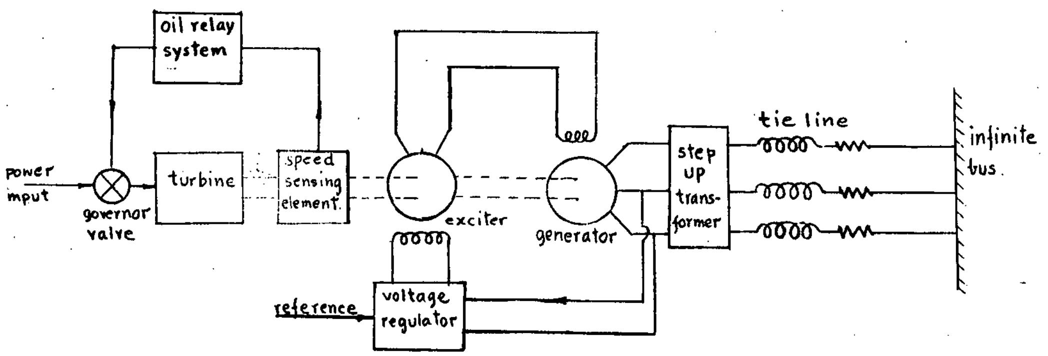


Fig. 3-1 Schematic of System Under Study

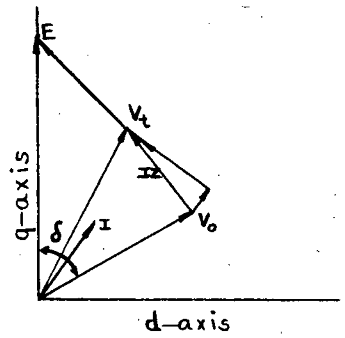


Fig. 3-2 Voltage Phasor Diagram of the Infinite Bus and a Synchronous Generator

3.2 System Equations

3.2.1 The synchronous machine

Park's equations will be applied to the dynamic stability study. In order to find the major effects of the synchronous machine on stability of the system, the following additional assumptions will be made:

1. The induced voltages due to the change of flux linkages $p\psi_d$ and $p\psi_q$ in the d and q axes can be neglected because they are small compared to the speed voltages $\theta\psi_q$ and $\theta\psi_d$ due to the cross excitations.

2. The armature resistance voltage drop can be neglected because it is small.

3. The time constants τ_d', τ_{do}'', τ_q'', τ_{qo}'' and τ_{D1}, being relatively small, will also be neglected.

With the above assumptions,

$$
\begin{aligned}
v_d &= -\psi_q p\theta \\
v_q &= \psi_d p\theta \\
\psi_d &= \frac{G(p)}{\omega} v_{fd} - \frac{x_d(p)}{\omega} i_d \\
\psi_q &= -\frac{x_q(p)}{\omega} i_q \\
T_e &= i_q\psi_d - i_d\psi_q \qquad (3\text{-}1) \\
x_d(p) &= x_d \frac{(1 + \tau_d' p)}{(1 + \tau_{do}' p)} \\
x_q(p) &= x_q \\
G(p) &= \frac{x_{ad}}{R_F(1 + \tau_{do}' p)}
\end{aligned}
$$

The other relevant equations are

$$T_m = Jp\theta + \alpha p\theta + T_e$$
$$v_t^2 = v_d^2 + v_q^2 \tag{3-2}$$

3.2.2 The tie line

An assumption will be made for the external impedance, namely that its resistance and reactance elements are constant.

Two more equations are obtained from the steady state phasor diagram, Fig. 3-2,

$$v_d = v_o \sin\delta + ri_d - xi_q$$
$$v_q = v_o \cos\delta + xi_d + ri_q \tag{3-3}$$

3.2.3 The voltage regulator and the governor

Since a small change in machine terminal voltage causes a small change in the field excitation, the transfer function of a voltage regulator can be written generally as

$$g(p) = \frac{\Delta v_{fd}}{\Delta v_t} \tag{3-4}$$

A similar transfer function can be written for a speed governor,

$$f'(p) = \frac{\Delta T_m}{\Delta p\theta} \tag{3-5}$$

A small change in speed or frequency causes a small change in prime mover input.

The details of the transfer functions are given in

Appendix II.

3.3 Linearization of the System Equations

For the dynamic stability study only small changes of various variables around an initial operating point will be taken into consideration. Hence stability can be determined from a set of linearized equations of the original system equations.

The frequency of the infinite bus is considered fixed. The instantaneous angular displacement, velocity and acceleration of the machine, respectively, can be written as

$$\theta = \theta_o + \delta$$

$$p\theta = \omega + p\delta \quad , \text{ where } p\theta_o = \omega$$

and

$$p^2\theta = p^2\delta$$

Then the dynamic stability of the system can be defined by whether or not δ goes to infinity with time after a small load disturbance.

From equations (3-1) through (3-5), the following linearized system equations are obtained:

$$\Delta v_d = -\Psi_{qo} p\Delta\delta + x_q \Delta i_q$$

$$\Delta v_q = \Psi_{do}\Delta\delta + x_d(p)\Delta i_d + G(p)\Delta v_{fd}$$

$$\Delta v_{fd} = g(p)\Delta v_t$$

$$\Delta T_m = f'(p)p\Delta\delta$$

$$\Delta v_d = v_o \cos \delta_o \Delta\delta + r\Delta i_d - x\Delta i_q$$

$$\Delta v_q = -v_o \sin \delta_o \Delta\delta + x\Delta i_d + r\Delta i_q$$

$$\omega\Delta T_e = i_{do}\Delta v_d + i_{qo}\Delta v_q + v_{do}\Delta i_d + v_{qo}\Delta i_q - T_{eo}\, p\Delta\delta$$

$$\Delta T_m = Jp^2\Delta\delta + \alpha p\Delta\delta + \Delta T_e$$

$$\Delta v_t = \frac{v_{do}}{v_{to}}\Delta v_d + \frac{v_{qo}}{v_{to}}\Delta v_q \triangleq v'_{do}\Delta v_d + v'_{qo}\Delta v_q$$

By defining

$$h(p) \triangleq g(p)\,G(p) \tag{3-8}$$

$$J(p) \triangleq J^2 p + (\alpha - \frac{T_{eo}}{\omega})p - f(p),$$

where $f(p) = pf'(p)$,

and eliminating ΔT_m, ΔT_e, Δv_t from the system equations, the following five homogeneous equations are obtained:

$$\begin{bmatrix} 1 & 0 & -r & x & -v_o \cos\delta_o \\ 0 & 1 & -x & -r & v_o \sin\delta_o \\ 1 & 0 & 0 & -x_q & \Psi_{qo} p \\ -h(p)v'_{do} & -h(p)v'_{qo}+1 & x_d(p) & 0 & -\Psi_{do} p \\ i_{do} & i_{qo} & v_{do} & v_{qo} & \omega J(p) \end{bmatrix} \begin{bmatrix} \Delta v_d \\ \Delta v_q \\ \Delta i_d \\ \Delta i_q \\ \Delta\delta \end{bmatrix} = 0 \tag{3-9}$$

3.4 The Characteristic Equation of the System

This is obtainable from the characteristic determinant of the above matrix equation. The characteristic determinant

$$\Delta = J(p)\left[h(p)A_1 + x_d(p)A_2 + A_3\right] + h(p)\left[A_4p + A_5\right] + x_d(p)\left[A_6p + A_7\right] + \left[A_8p + A_9\right] \qquad (3\text{-}10)$$

where

$$A_1 = -\omega\left[v'_{do}rx_q - v'_{qo}(r^2 + x^2 + x\,x_q)\right]$$

$$A_2 = \omega\left[x + x_q\right]$$

$$A_3 = \omega\left[r^2 + x^2 + x\,x_q\right]$$

$$A_4 = \Psi_{qo}\left[-v'_{do}\left\{v_{do}x + v_{qo}r + i_{qo}(r^2+x^2)\right\} + v'_{qo}\left\{v_{do}r - v_{qo}x + i_{do}(r^2 + x^2)\right\}\right]$$

$$A_5 = v_o \sin\delta_o\left[v'_{do}i_{qo}rx_q - v'_{qo}\left\{v_{do}(x+x_q) + r(v_{qo} + i_{do}x_q)\right\}\right] + v_o\cos\delta_o\left[v'_{do}x_q(v_{do} + i_{qo}x) - v'_{qo}\left\{-v_{do}r + x(v_{qo}+i_{do})\right\}\right]$$

$$A_6 = \Psi_{qo}(v_{qo} - i_{do}x + i_{qo}r)$$

$$A_7 = -v_o\sin\delta_o i_{qo}(x+x_q) + v_o\cos\delta_o(v_{qo} - i_{do}x_q + i_{qo}r)$$

$$A_8 = \Psi_{do}\left[v_{do}(x+x_q) + v_{qo}r + i_{qo}\left\{(r^2+x^2) + xx_q\right\} + i_{do}r\,x_q\right] + \Psi_{qo}\left[-v_{do}r + v_{qo}x - i_{do}(r^2 + x^2)\right]$$

$$A_9 = v_o\sin\delta_o\left[v_{do}(x+x_q) + v_{qo}r + i_{do}r\,x_q\right] + v_o\cos\delta_o\left[-v_{do}r + v_{qo}x + i_{do}xx_q\right]$$

Since the transfer function of the voltage regulator

$$g(p) = \frac{-\mu_e(1+\tau_s p)}{\left\{1+(\tau_e + \tau_s\mu'_s)p + \tau_e\tau_s p2\right\}} \qquad (3\text{-}11)$$

and the transfer function of the speed governor

$$f'(p) = \frac{-\mu_m}{(1+\tau_1 p)(1+\tau_2 p)}$$

or (3-12)

$$f(p) = \frac{-\mu_m p}{(1+\tau_1 p)(1+\tau_2 p)}$$

J(p) and h(p) functions become

$$J(p) = Jp^2 + (\alpha - \frac{T_{eo}}{\omega})p + \frac{\mu_m p}{(1+\tau_1 p)(1+\tau_2 p)}$$

$$h(p) = -\frac{x_{ad}}{R_F}\ \frac{\mu_e (1+\tau_s p)}{(1+\tau'_{do} p)\left\{1 + (\ e+\tau_s \mu'_s)p + \tau_e \tau_s p^2\right\}}$$

$$\triangleq - \frac{\mu'_e (1+\tau_s p)}{(1+\tau'_{do} p)\left\{1+(\tau_e+\tau_s \mu'_s)p + \tau_e \tau_s p^2\right\}}$$

where $\mu'_e \triangleq \mu_e \dfrac{x_{ad}}{R_F}$.

Therefore, the characteristic determinant

$$\Delta = \left[Jp^2+(\alpha - \frac{T_{eo}}{\omega})p + \frac{\mu_m p}{(1+\tau_1 p)(1+\tau_2 p)}\right]\left[\frac{-\mu'_e (1+\tau_s p)A_1}{(1+\tau'_{do} p)\left\{1+(\tau_e+\tau_s \mu'_s)p+\tau_e \tau_s p^2\right\}}\right.$$

$$\left. + \frac{x_d (1+\tau'_d p)A_2}{(1+\tau'_{do} p)} + A_3\right] + \left[\frac{-\mu'_e (1+\tau_s p)}{(1+\tau'_{do} p)\left\{1+(\tau_e+\tau_s \mu'_s)p+\tau_e \tau_s p^2\right\}}\right]\left[A_4 p + A_5\right]$$

$$+ \left[x_d \frac{(1+\tau'_d p)}{(1+\tau'_{do} p)}\right]\left[A_6 p + A_7\right] + \left[A_8 p + A_9\right] \tag{3-14}$$

From equation (3-14), the characteristic equation is obtained:

$$a_o p^7 + a_1 p^6 + a_2 p^5 + a_3 p^4 + a_4 p^3 + a_5 p^2 + a_6 p + a_7 = 0 \qquad (3\text{-}15)$$

where

$$a_o = T_a J\, B_1$$

$$a_1 = T_a B_4 + T_d J\, B_1$$

$$a_2 = T_a B_5 + T_c J\, B_1 + T_d B_4 - \mu'_e\, A_1 J\, \tau_1 \tau_2 \tau_s$$

$$a_3 = T_a B_3 + T_b J\, B_1 + T_c B_4 + T_d B_5 + \mu_m B_1 \tau_e \tau_s -$$

$$-\mu'_e (J A_1 T_E + B_6 \tau_1 \tau_2 \tau_s)$$

$$a_4 = T_b B_4 + T_c B_5 + T_d B_3 + \mu_m \left\{ B_1 (\tau_e + \tau_s \mu'_s) + B_2 \tau_e \tau_s \right\}$$

$$-\mu'_e (J\, A_1 T_F + A_5 \tau_1 \tau_2 \tau_s + B_6 T_E) + J B_1$$

$$a_5 = T_b B_5 + T_c B_3 + \mu_m \left\{ B_1 + B_2 (\tau_e + \tau_s \mu'_s) - \mu'_e\, A_1 \tau_s \right\}$$

$$-\mu'_e \left\{ J A_1 + A_5 T_E + B_6 T_F \right\} + B_4$$

$$a_6 = T_b B_3 + \mu_m (B_2 - \mu'_e A_1) - \mu'_e (A_5 T_F + B_6) + B_5$$

$$a_7 = B_3 - \mu'_e A_5$$

For details of the derivation, see Appendix III.

3.5 The Routh-Hurwitz Determinants of the Characteristic Equation

Routh-Hurwitz Determinants can be written as

$$\begin{vmatrix} a_1 & a_3 & a_5 & a_7 & 0 & 0 & 0 \\ a_0 & a_2 & a_4 & a_6 & 0 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & a_7 & 0 & 0 \\ 0 & a_0 & a_2 & a_4 & a_6 & 0 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 & a_7 & 0 \\ 0 & 0 & a_0 & a_2 & a_4 & a_6 & 0 \\ 0 & 0 & 0 & a_1 & a_3 & a_5 & a_7 \end{vmatrix} \qquad (3\text{-}16)$$

Separately written, they are

$$\Delta_1 = a_1$$

$$\Delta_2 = a_1a_2 - a_0a_3$$

$$\Delta_3 = a_3a_2 - a_1\Delta_0, \text{ where } \Delta_0 = a_1a_4 - a_0a_5$$

$$\Delta_4 = a_4\Delta_3 + \Delta_2(a_1a_6 - a_0a_7 - a_2a_5) + a_0a_5\Delta_0$$

$$\Delta_5 = a_5\Delta_4 - a_3a_6\Delta_3 + \Delta_2(a_2a_3a_7 + a_1a_6a_5 - a_1a_4a_7) - a_0a_3a_7\Delta_0 - a_1(a_1a_6 - a_0a_7)^2$$

$$\Delta_6 = a_6\Delta_5 - a_4a_7\Delta_4 + a_2a_6a_7\Delta_3 + \Delta_2(a_2a_4a_7 - a^2{}_2a_7 - a_0a_6a_5)a_7 + a_2a_1a^2{}_7\Delta_0 + a_0a_7(a_1a_6 - a_0a_7)^2$$

$$\Delta_7 = a_7\Delta_6 \tag{3-17}$$

3.6 The D-partition Equations

To find the effect of relative values of two real parameters on stability, the Routh-Hurwitz Criterion can also be used. The procedure, however, is rather tedious. When the characteristic equation of the system can be expressed explicitly as the function of two real parameters, the D-partition method can be applied.

The characteristic equation of the system under study by the D-partition method can be first written as equation (2-9) and then separated into real and imaginary parts as equation (2-12).

In terms of the μ'_e and μ'_s, which are of particular interest, equation (2-12) becomes

$$\begin{aligned} \mu'_s P_1(\omega) - \mu'_e Q_1(\omega) + R_1(\omega) &= 0 \\ \mu'_s P_2(\omega) - \mu'_e Q_2(\omega) + R_2(\omega) &= 0 \end{aligned} \tag{3-18}$$

The stability boundaries can be determined by solving this equation and by varying ω from zero to infinity.

Detailed derivation of equation (3-18) is given in Appendix III.

4. SYSTEM PARAMETERS AND INITIAL CONDITIONS OF SYNCHRONOUS MACHINES

4.1 System Parameters

The system under study has parameters with the following values.

Synchronous Machines. Synchronous machines of various ratings will behave in exactly the same way as long as the per unit values of the machine parameters are the same. In the analysis the direct- and quadrature-axis reactances of the synchronous machine and the tie line impedance will be varied except for the following parameters which will be kept constant:

$$J = 120\,\pi H = 4000 \text{ p.u.}$$
$$\tau'_{d0} = 2000 \quad \text{p.u.}$$
$$\tau'_{d} = 600 \quad \text{p.u.}$$
$$\alpha = 3.0 \quad \text{p.u.}$$

Voltage Regulator Settings. The voltage regulator settings are assumed, unless otherwise specified, to be normally

$$\tau_e = 800 \quad \text{p.u.}$$
$$\tau_s = 800 \quad \text{p.u.}$$
$$\mu'_e = 30$$
$$\mu'_s = 5$$

Governor Settings. Two different governor transfer functions are considered, unless otherwise specified.

For a governor with one time constant, the gain and the time constant respectively are

$$\mu_m = 25$$
$$\tau_1 = 377 \quad \text{p.u.}$$
$$\tau_2 = 0 \quad \text{p.u.}$$

For a governor with two time constants $\mu_m = 5$ and $\tau_1 = \tau_2 = 300$ p.u. are assumed.

4.2 Initial Conditions of Salient-pole Synchronous Machine

A new method for finding initial conditions, i_{do}, i_{qo}, v_{do}, v_{qo}, Ψ_{do}, Ψ_{qo}, v_{fdo} and δ_o for a salient-pole synchronous machine will be developed.

Consider the steady state. From Park's equations (3-1) to (3-3) we have

$$v_{do} = x_q \, i_{qo} \tag{4-1}$$

$$v_{qo} = \frac{x_{ad}}{R_F} v_{fdo} - x_d \, i_{do} \tag{4-2}$$

$$P = v_{do} \, i_{do} + v_{qo} \, i_{qo} \tag{4-3}$$

$$Q = v_{qo} i_{do} - v_{do} \, i_{qo} \tag{4-4}$$

$$v_{do} = v_o \sin \delta_o + r \, i_{do} - x \, i_{qo} \tag{4-5}$$

$$v_{qo} = v_o \cos \delta_o + x \, i_{do} + r \, i_{qo} \tag{4-6}$$

$$v^2_{to} = v^2_{do} + v^2_{qo} \tag{4-7}$$

In these equations v_{do}, v_{qo}, i_{do}, i_{qo}, v_o, δ_o, v_{fdo} are unknown while v_{to}, P and Q are given. There are 7 equations with 7 unknowns. From (4-3) and (4-1)

$$P = i_{do} i_{qo} x_q + v_{qo} \, i_{qo} \tag{4-8}$$

From (4-4) and (4-1)

$$Q = v_{qo} i_{do} - x_q \, i^2_{qo} \tag{4-9}$$

From (4-7) and (4-1)

$$v_{qo} = \sqrt{v^2_{to} - x^2_q \, i^2_{qo}} \tag{4-10}$$

From (4-9)

$$i_{do} = \frac{Q + x_q \, i^2_{qo}}{v_{qo}} \tag{4-11}$$

From (4-11) and (4-8)

$$P = \frac{(Q + x_q \, i^2_{qo})}{v_{qo}} \, x_q i_{qo} + v_{qo} i_{qo} \tag{4-12}$$

From (4-12) and (4-10)

$$P = \frac{(Q + x_q \, i^2_{qo})}{\sqrt{v^2_{to} - x^2_q i^2_{qo}}} \, x_q i_{qo} + \sqrt{v^2_{to} - x^2_q i^2_{qo}} \; i_{qo} \tag{4-13}$$

Hence

$$i_{qo} = \frac{P \, v_{to}}{\sqrt{(P \, x_q)^2 + (v^2_{to} + x_q Q)^2}} \tag{4-14}$$

$$v_{do} = x_q i_{qo} \tag{4-15}$$

$$v_{qo} = \sqrt{v^2_{to} - v^2_{do}} \tag{4-16}$$

$$i_{do} = \frac{Q + x_q i^2_{qo}}{v_{qo}} \tag{4-17}$$

$$v_{fdo} = (v_{qo} - x_d i_{do}) \, \frac{R_F}{x_{ad}} \tag{4-18}$$

consequently

$$\Psi_{do} = -v_{do} \tag{4-19}$$

$$\Psi_{qo} = v_{qo} \tag{4-20}$$

Next from (4-5) and (4-6)

$$v_o = \sqrt{(v_{do} - r\, i_{do} + x\, i_{qo})^2 + (v_{qo} - x\, i_{do} - r\, i_{qo})^2} \tag{4-21}$$

$$\delta_o = \text{arc tan}\left(\frac{v_{do} - r\, i_{do} + x\, i_{qo}}{v_{qo} - x\, i_{do} - r\, i_{qo}}\right) \tag{4-22}$$

All the initial conditions have thus been found. The method is general and applicable to a salient-pole machine as well as a round-rotor machine, the latter being a special case of the former. The external tie line reactance and resistance have been included.

5. DYNAMIC STABILITY STUDIES USING ROUTH-HURWITZ CRITERION

5.1 Effect of Saliency on the Dynamic Stability

The synchronous saliency in a round-rotor machine has usually been assumed to be zero, but tests on actual machines[14-15] show that some saliency is present. Thus the assumption of non-saliency for round-rotor machines is not valid. In the following, three cases will be studied. The first case, $x_d = x_q$, corresponds to an ideal round-rotor machine; the second case, $x_d = 0.85\ x_q$, a round-rotor machine with saliency effect considered; and the third case, $x_d = 0.6\ x_q$, corresponds to a typical salient-pole machine.

Fig. 5-1 shows the effect of rotor saliency on the dynamic stability of the system under unity and leading power factor operation. The action of the speed governor is discounted by setting $\mu_m = 0$, while the voltage regulator has the normal settings.

Curve 1 indicates the dynamic stability limit of the ideal round-rotor machine with $x_d = x_q = 1.0$ p.u., curve 2 the stability limit of the round-rotor machine with a saliency effect considered ($x_d = 1$, $x_q = 0.85$), and curve 3 the stability limit of the salient-pole machine ($x_d = 1$, $x_q = 0.6$). It can be seen that the rotor saliency increases the dynamic stability limit, especially under leading power factor operation. The family of curves shown in dotted lines corresponds to the system without tie line resistance ($r=0$), and those shown in solid lines corresponds to the system with the tie line resistance included.

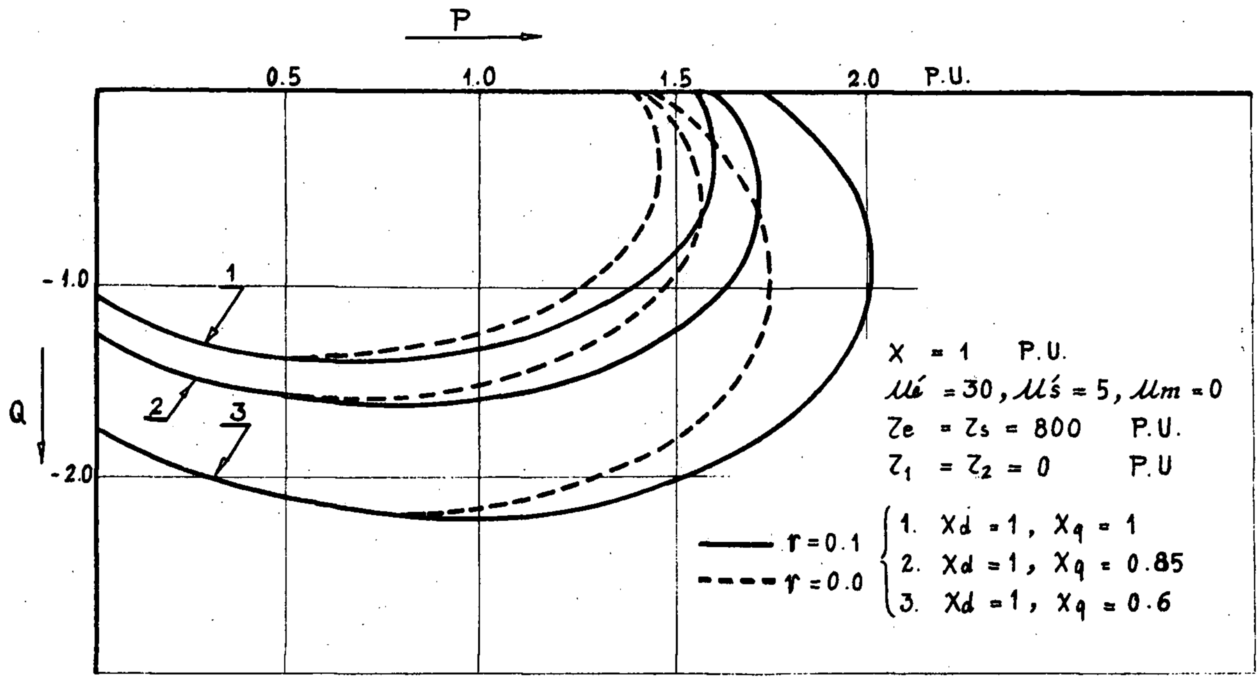


Fig. 5-1 Saliency Effect on Power Limits without Governor

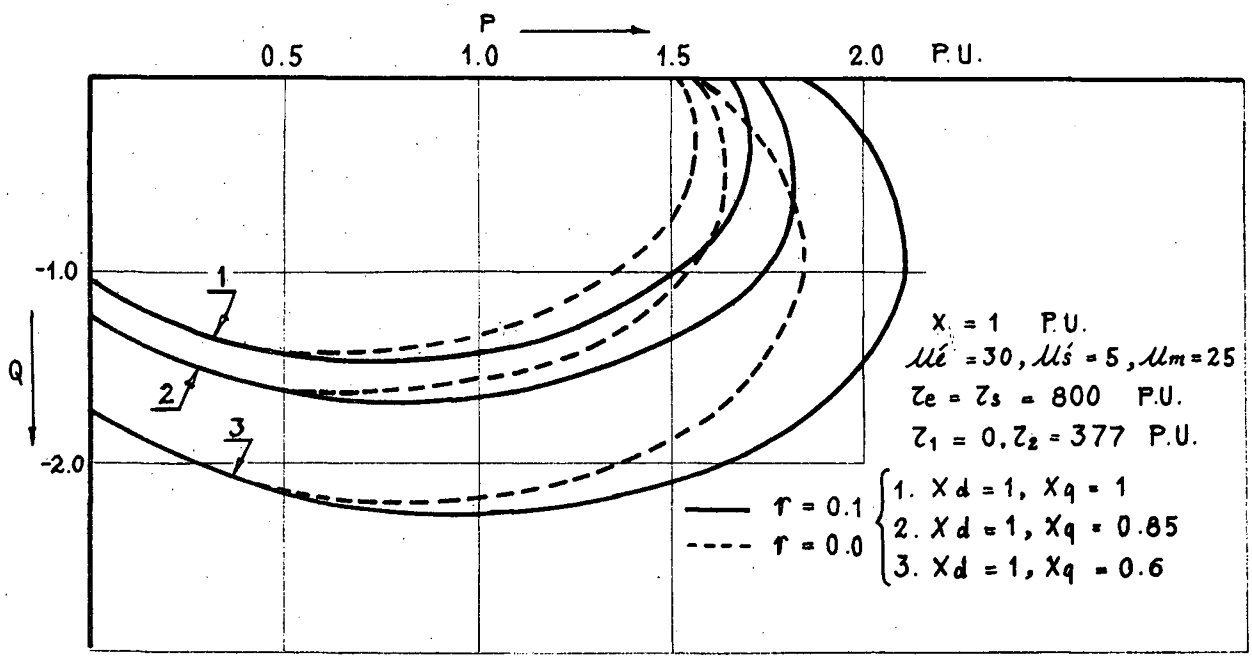


Fig. 5-2 Saliency Effect on Power Limits with a one-time-constant Governor

Similar computations are repeated with governor effect included, and the results are summarized in Fig. 5-2 and 5-3.

In Fig. 5-2 the speed governor of the system has one time constant, while in Fig. 5-3 it has two time constants. The same conclusion can be drawn that the saliency effect of a synchronous machine increases the dynamic stability limit.

5.2 Effect of the Short Circuit Ratio of a Synchronous Machine

The short circuit ratio of a synchronous machine of modern design is approximately equal to $\frac{1}{x_d}$. A high short circuit ratio machine costs more than one with a low short circuit ratio.

Machines designed with low short circuit ratios have limited stability limit if it is operated without a modern voltage regulator, but the stability limit will be increased considerably by using a continuously-acting voltage regulator. Fig. 5-4 shows the effect of the short circuit ratio on the stability limit for an under-excited machine. The dotted lines indicate the power limit of the machine with voltage regulator set at $\mu_e' = 0.01$ and $\tau_e = 3000$ to approximate a system without a continuously-acting regulator, and the solid lines are those for a system with a continuously-acting voltage regulator.

For example, when the short circuit ratio is equal to $\frac{1}{1.5}$ the machine without a voltage regulator can hardly be loaded beyond 0.9 p.u. power at unity p.f. When the continuously-acting voltage regulator is used the limit can be increased to as high as 1.35 p.u. at the same p.f. in the dynamic stability

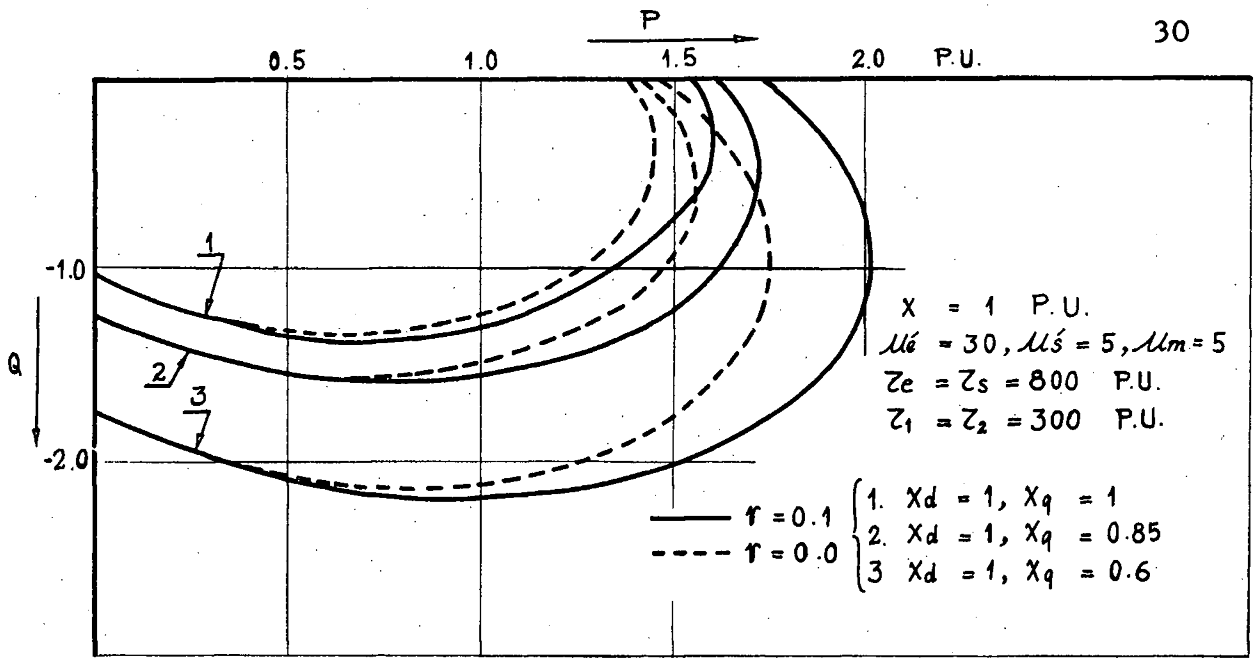


Fig. 5-3 Saliency Effect on Power Limits with a Two-time-constant Governor.

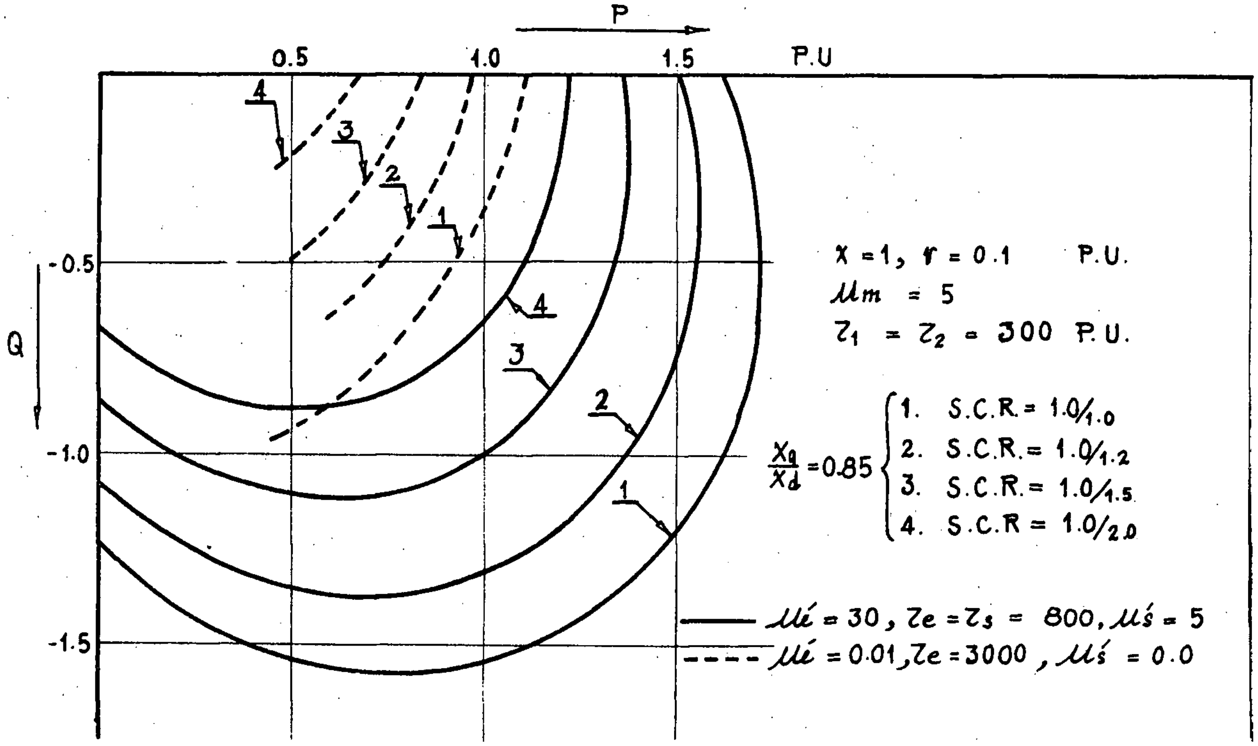


Fig. 5-4 Short Circuit Ratio Effect on Power Limits

region.

5.3 Effect of the Tie Line Impedance

5.3.1 Effect of the line resistance

It is observed from Fig. 5-1, 5-2 and 5-3 that the tie line resistance has a distinct effect on the dynamic stability limit. Larger stability limits are observed when the tie line resistance is included in the computations.

The studies are extended by increasing the resistance value from 0 to 0.3 p.u. while the reactance is kept constant at 1.0 p.u. In all cases the voltage regulator and governor are at normal settings. The results are shown in Fig. 5-5 and 5-6.

Closer examination of Fig. 5-6 further reveals the effect of the line resistance on stability limit at unity power factor and with different degrees of saliency.

5.3.2 The effect of the line reactance

Fig. 5-7 and 5-8 show the results of stability studies by varying the value of tie line reactance. The difference between the stability limit with line resistance included and that without is insignificant, when the tie line reactance is comparatively small. But it becomes significant when the reactance is large. Fig. 5-8 especially shows the results of studies for unity power factor. It also reveals that the smaller the tie line reactance, the larger is the stability limit.

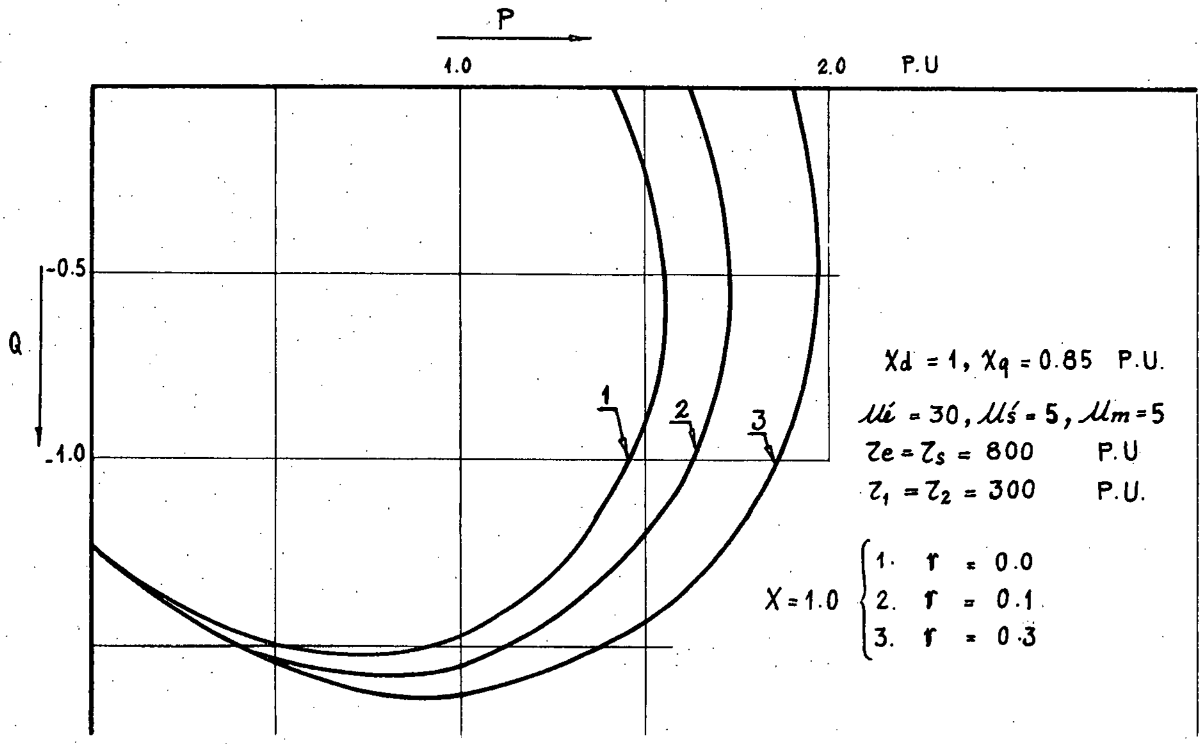


Fig. 5-5 Tie Line Resistance Effect on Power Limits.

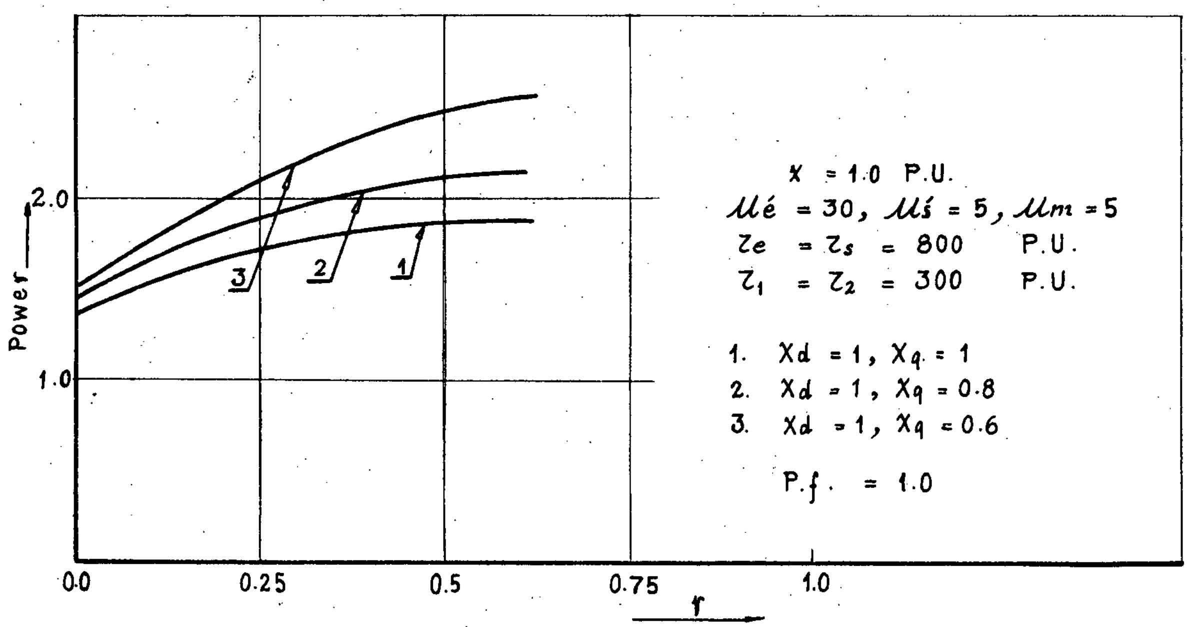


Fig. 5-6 Tie Line Resistance Effect on Power Limits at Unity Power Factor.

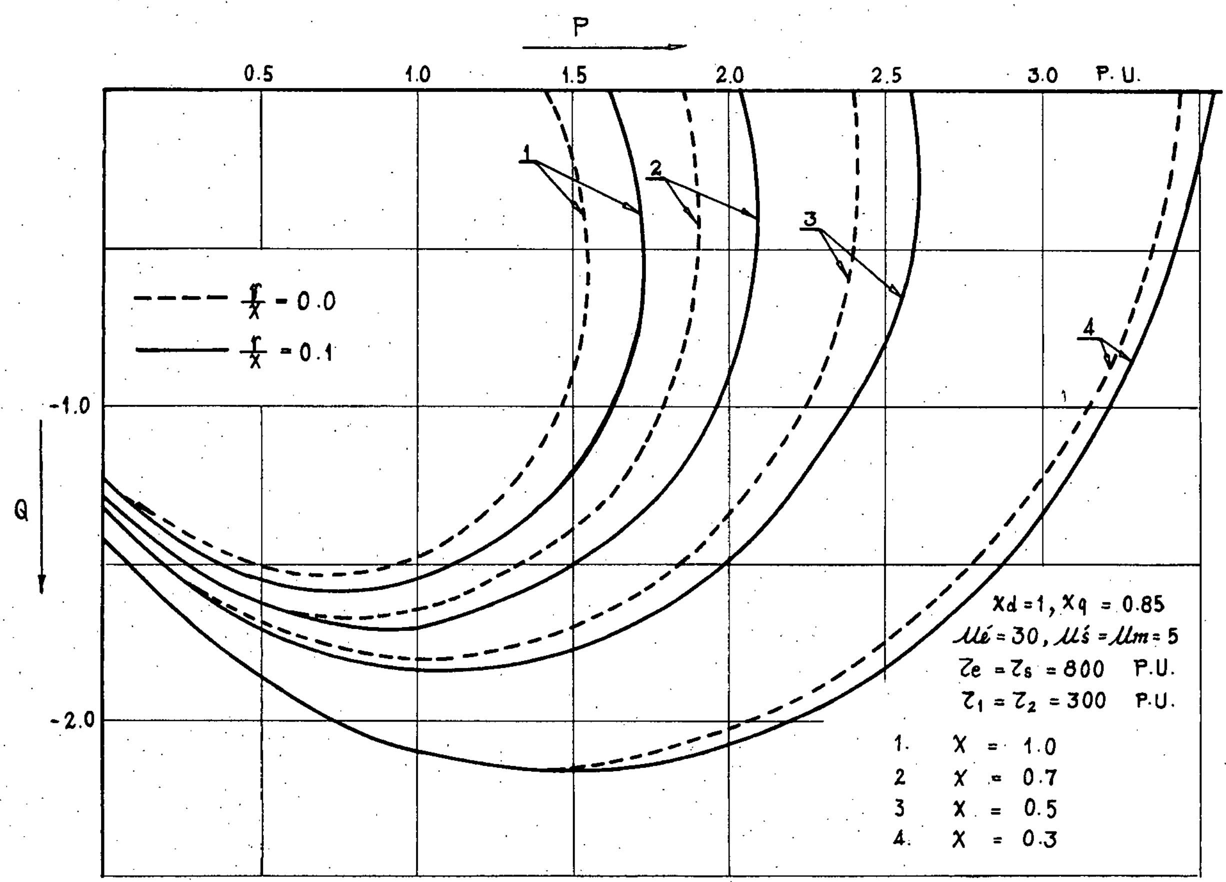


Fig. 5-7 Tie Line Reactance Effect on Power Limits.

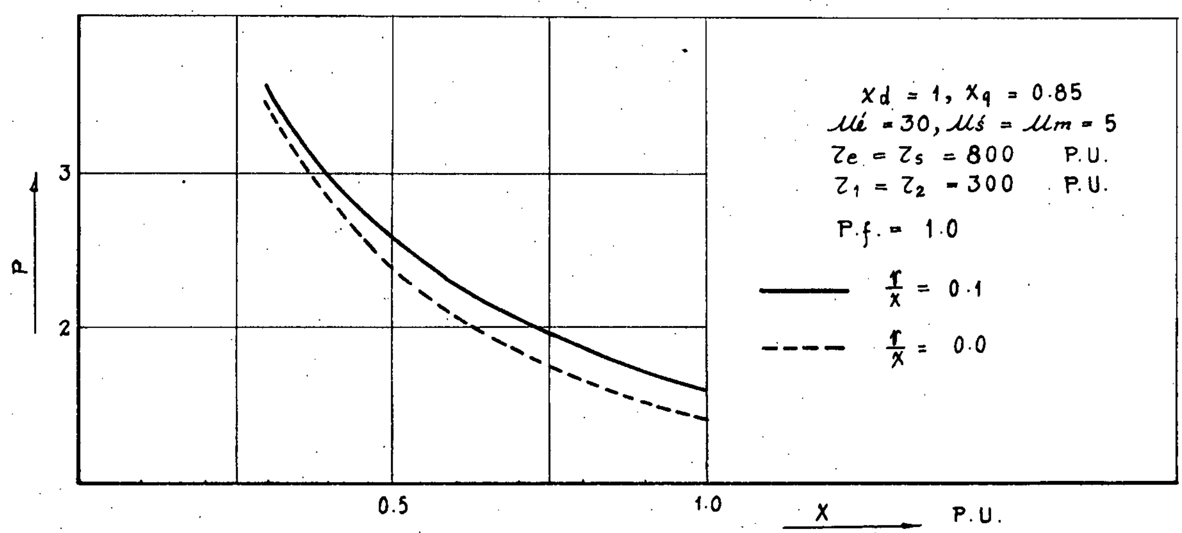


Fig. 5-8 Tie Line Reactance Effect on Power Limits at Unity Power Factor.

5.4 Effect of Exciter Time Constant

The range of conventional exciter time constants varies from about 0.5 to 3 seconds. Fig. 5-9 shows the effect of varying exciter time constant while the other parameters of the regulator and governor are kept constant. It can be seen that for the particular gain $\mu'_e = 30$ the stability limit tends to increase with increasing exciter time constant.

Fig. 5-10 shows the results of two other studies. By varying the voltage-regulator time τ_e and keeping the gain $\mu'_e = 30$, the increase in the value of τ_e seems to have a favourable effect on stability limit. However, the maximum limit depends on both μ'_e and τ_e. Fig. 5-10 also shows that for a fixed τ_e of 800 p.u. the maximum power limit occurs in the region of low voltage-regulator gain.

5.5 Effect of the Governor Time Constants on the Dynamic Stability Limit

5.5.1 Governor with one time constant

When the speed governor has a single time constant, the stability limit of a synchronous generator can be improved considerably by making the time constant small. Fig. 5-11 shows the stability limit of the system under unity and leading power factor operation as affected by the value of the governor time constant. The solid lines are for the round-rotor machine and the dotted lines are for the salient-pole machine. Both sets of curves show that fast acting governors are desirable for the improvement of the system dynamic stability.

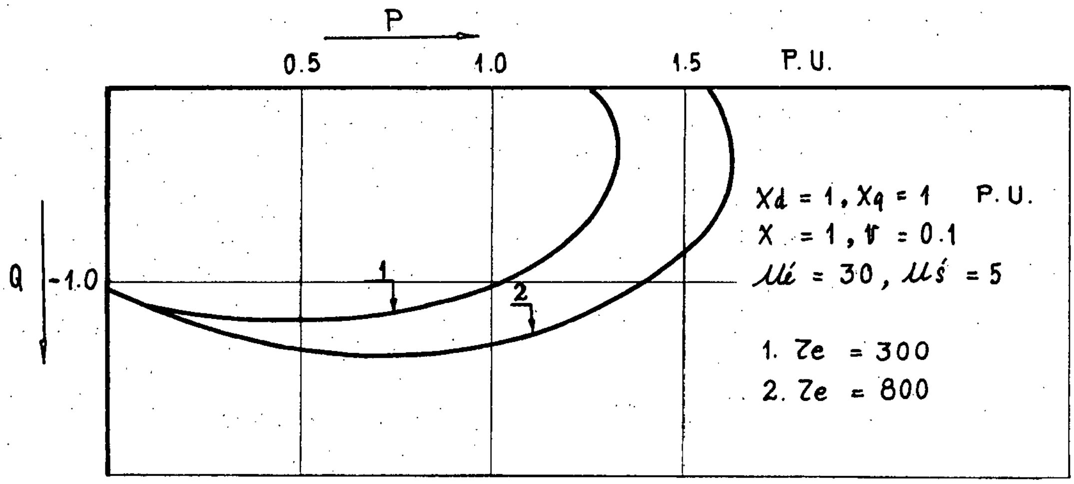


Fig. 5-9 Exciter Time Constant Effect on Power Limits.

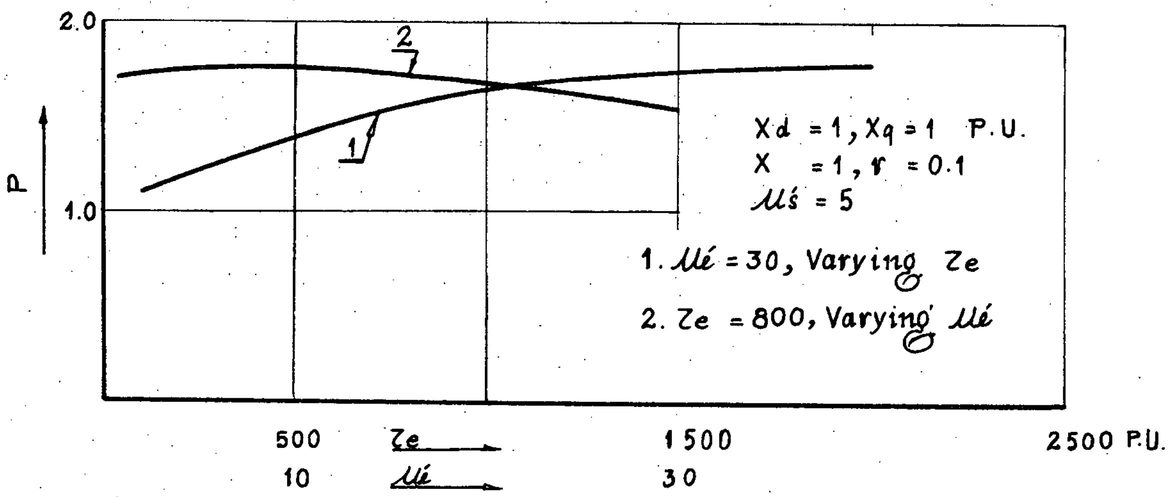


Fig 5-10 Exciter Time Constant and Gain Effect on Power Limits.

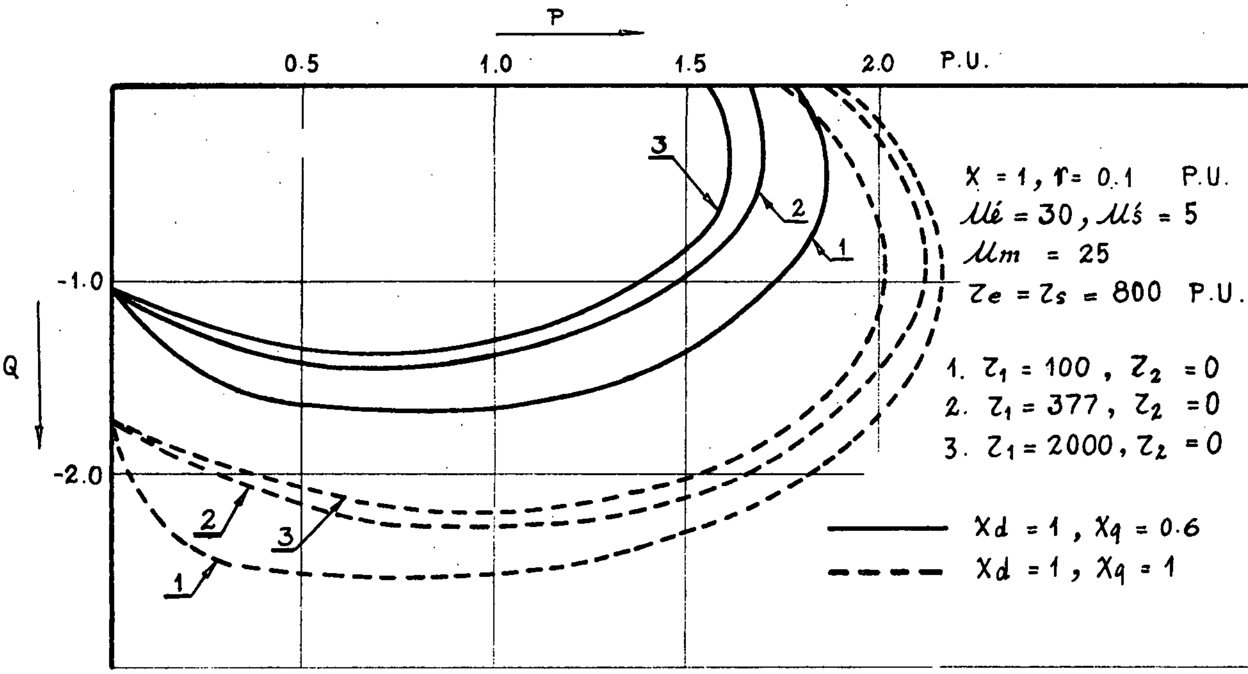


Fig. 5-11 Effect of the One-time-constant Governor on Power Limits.

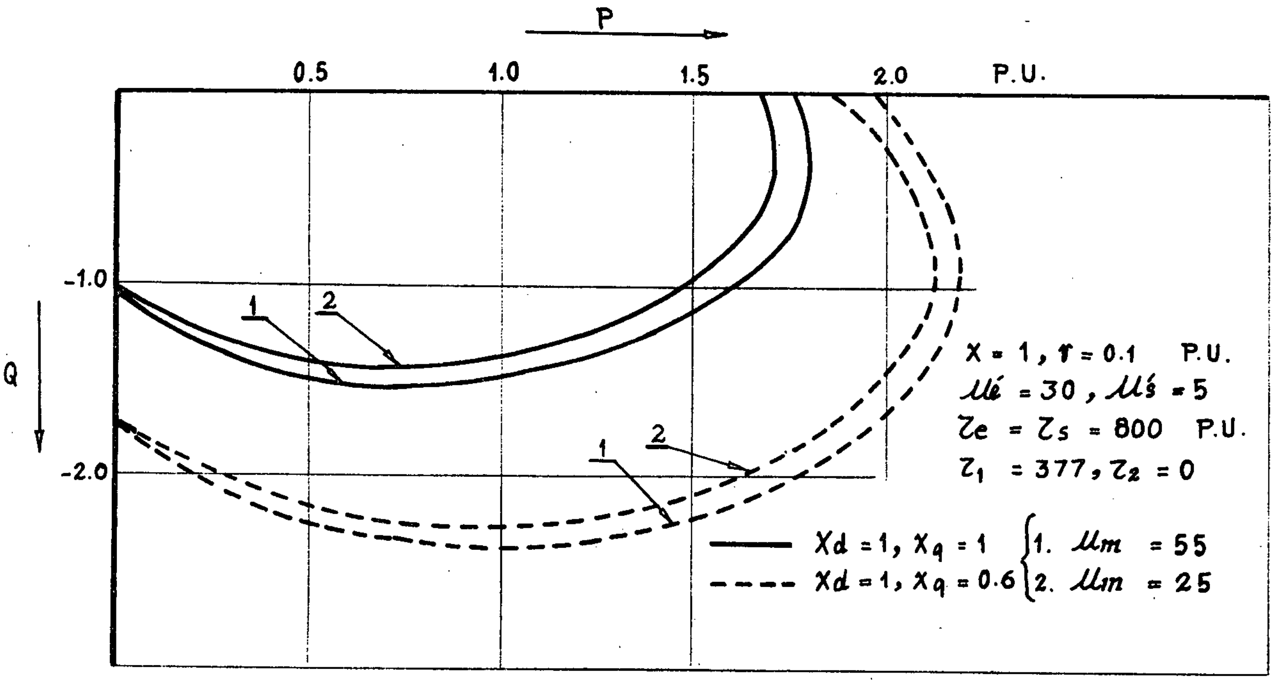


Fig. 5-12 Effect of the Gain of a One-time-constant Governor on Power Limits.

Fig. 5-12 shows the effect of governor gains on the stability limit. It can be seen that the larger the gain the larger is the stability limit. For a governor with a large time constant it will have little effect on the stability limit. This can be seen by comparing curve 1 of Fig. 5-1 with curve 1 of Fig. 5-11.

5.5.2 Governor with two time constants

When the speed governor has two equal time constants its effect on the system dynamic stability changes. The improvement in the stability limit is very small even when the governor time constants are decreased from 1000 to as low as 100 p.u. Moreover, there is a value of governor time constant, between zero and the highest possible value, that gives the smallest stability limit. Curve 3 (solid line) in Fig. 5-13 shows that the lowest stability limit, among the four computations, occurs for a governor time constant value of 200 p.u. While the solid lines in Fig. 5-13 indicate the stability limit of the round-rotor synchronous machine , the dotted lines in the same figure indicate those of the salient-pole machine. Very little improvement in stability limit is obtained by reducing the governor time constant of a salient-pole machine from 1000 p.u. to zero.

Fig. 5-14 shows the slight decrease obtained instead of an expected increase of stability limit with an increase in governor gain. This is contradictory to the results obtained from the study of the system with a governor with one time constant.

The above conclusions suggest that the governor effect on the dynamic stability limit must be included if the governor

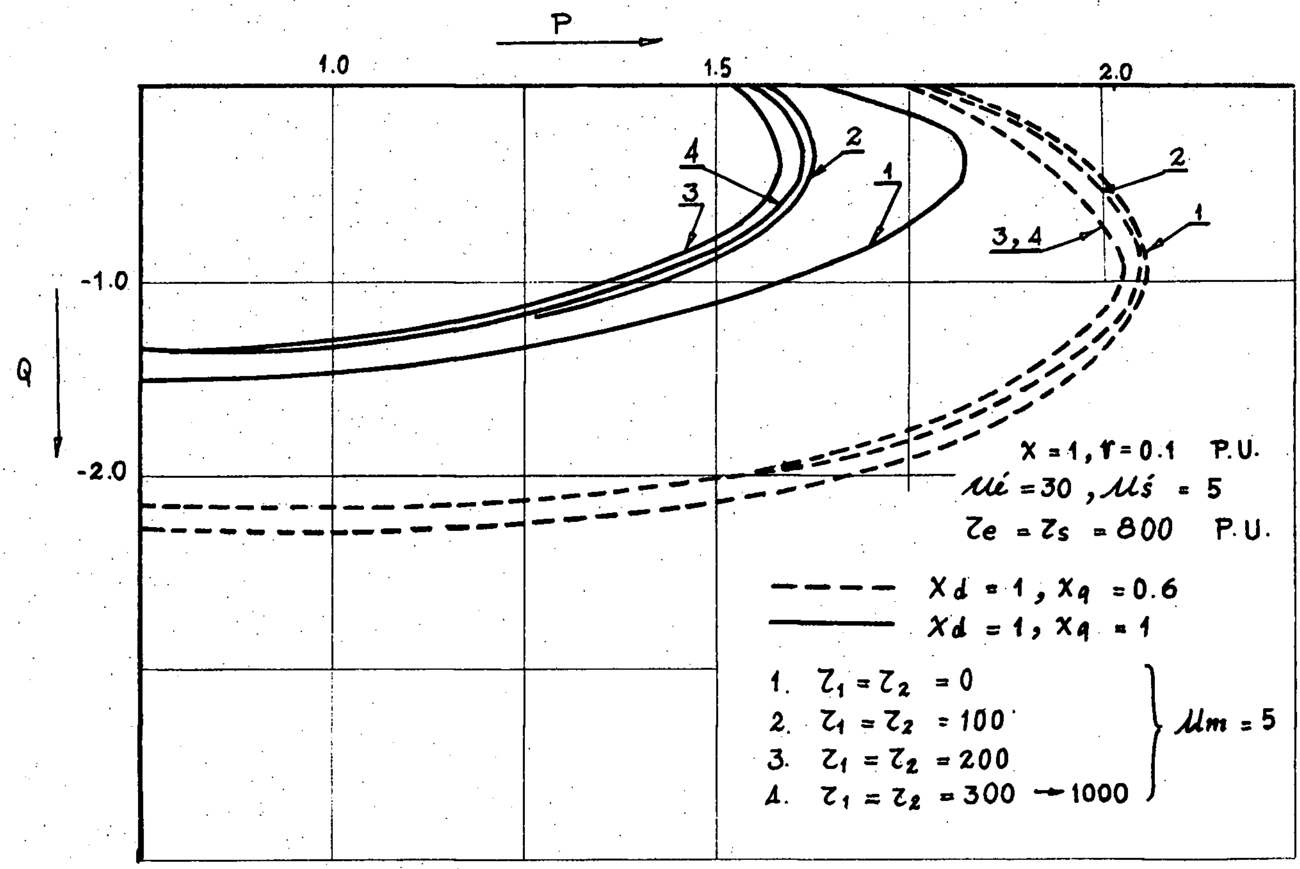


Fig. 5-13 Effect of the Two-time-constant Governor on Power Limits.

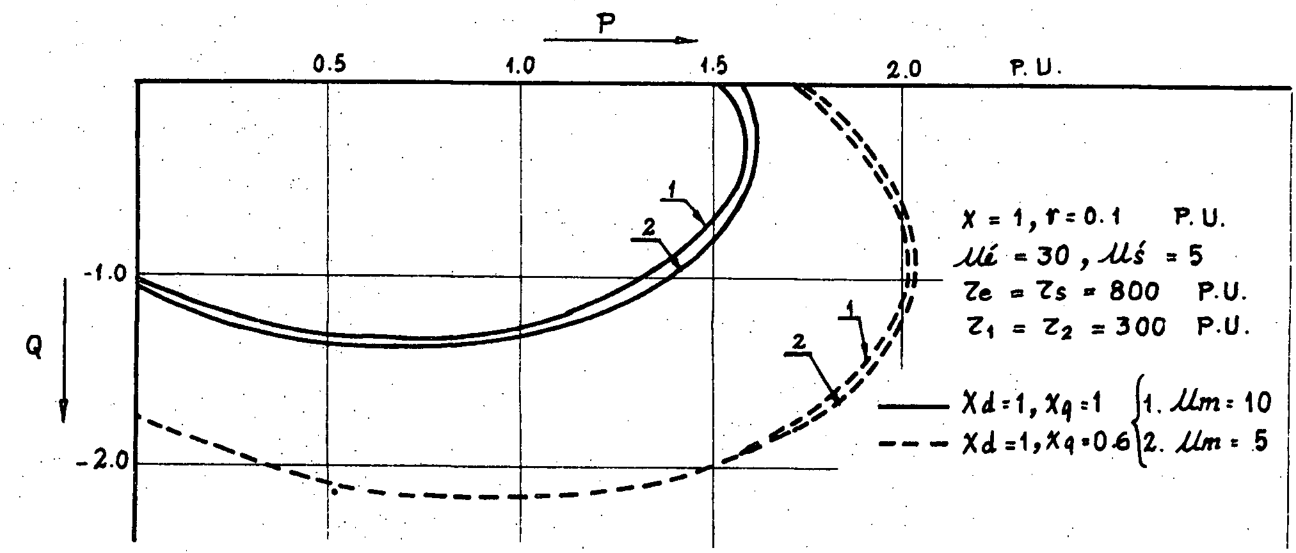


Fig. 5-14 Effect of the Gain of a Two-time-constant Governor on Power Limits.

is represented by a single time constant. On the other hand, the effect of the governor is rather small and could therefore be neglected if the governor is represented by two equal time constants.

6. DYNAMIC STABILITY STUDIES USING THE D-PARTITION METHOD

For a given operating load condition, the boundary of the dynamic stability of a system can be found in the $(\mu_e'-\mu_s')$ plane by using the D-partition Method.

An ω value ranging from 0 to 0.02 is found to be sufficient for finding all the results of practical interest. Usually $\omega = 0$ to 0.001 yields the boundary of maximum permissible μ_s', and $\omega = 0.001$ to 0.011 yields the boundary of the maximum permissible μ_e'.

The studies are carried out on the system by varying system operating conditions and also some of the system parameters.

In general, the stability region of interest in each study is shown with two boundaries in the $(\mu_e'-\mu_s')$ plane. The upper-left boundary will be referred to hereafter as the high-μ_s' bound and the lower-right boundary as the high-μ_e' bound.

6.1 The Effect of Saliency on the Stability Boundaries

Fig. 6-1 shows the effect of saliency on the stability boundaries in the $(\mu_e'-\mu_s')$ plane. The governor action is excluded by setting $\mu_m = 0$. Curves 1, 2 and 3 correspond to $x_q =$ 0.6, 0.85, and 1.0 respectively while x_d is kept constant at unity.

It is observed that for the portion of the curves shown, the presence of saliency increases the stability region at the high-μ_e' bound, but decreases it in the high-μ_s' bound. However, in the very low μ_s' region the presence of saliency decreases the

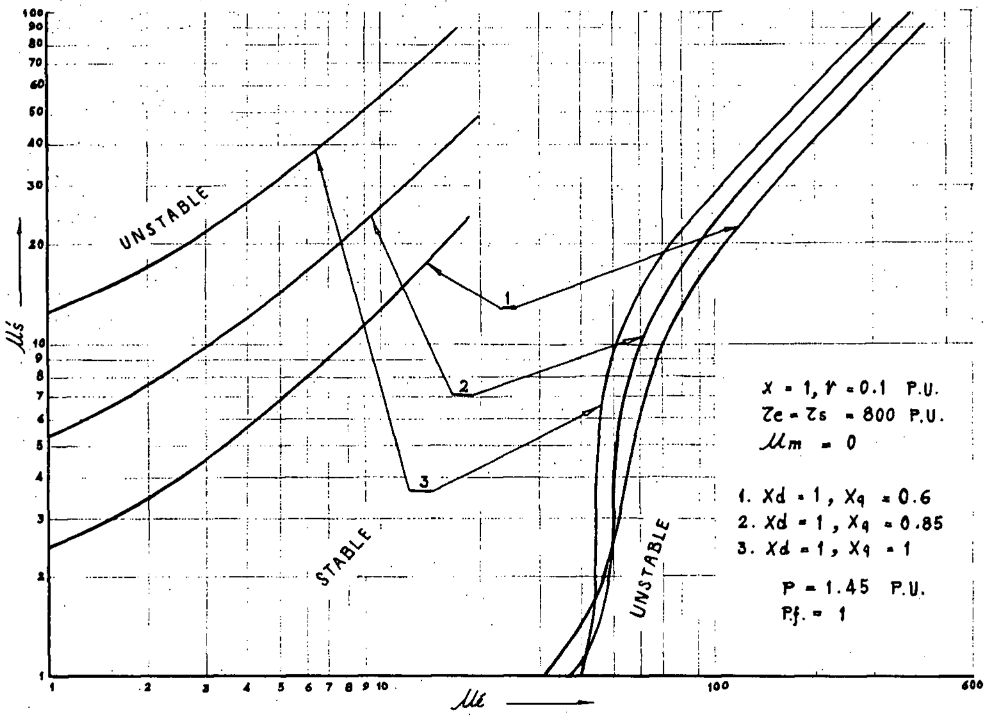


Fig. 6-1 Saliency Effect on the Stability Region

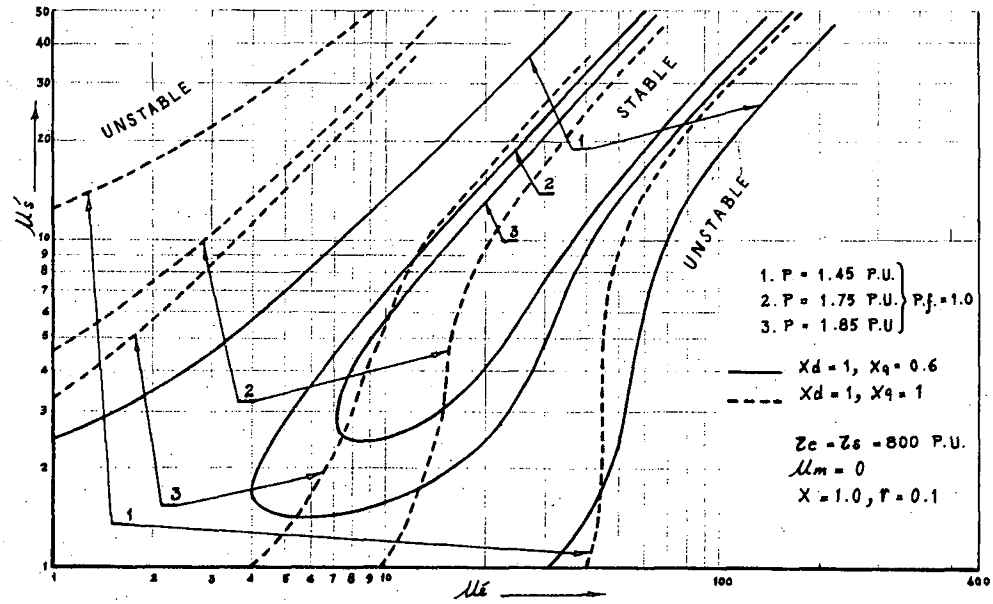


Fig 6-2 Saliency Effect on the Stability Region
for Various Power limits.

stability region at the high-μ'_e bound.

Curves 3 and 2 in Fig. 6-1, which correspond respectively to the stability region of a round-rotor machine with and without rotor saliency, clearly indicate that the saliency yields a rather pessimistic result for the stability region at the high-μ'_e bound and a rather optimistic result for the stability region at the high-μ'_s bound.

Fig. 6-2 shows that for the same power limit a salient-pole machine must be operated with a higher μ'_e gain than for the round-rotor machine.

6.2 The Effect of the Tie Line Resistance and Reactance

Fig. 6-3 shows the effect of the tie line resistance on the stability region of the system. The line reactance is set at 1.0 p.u., and the $\frac{r}{x}$ ratio is varied.

The stability region is increased considerably by an increase in the $\frac{r}{x}$ ratio from zero. By comparing curves 1 and 2 in Fig. 6-3, it can be seen that a very pessimistic result in the stability region would be obtained if the tie line resistance is completely ignored. This is rather misleading and might result in an inappropriate coordination of the voltage regulator parameters.

Curves 1 and 2 of Fig. 6-4 show the effect of tie line reactance on the stability region. It can be seen that the decrease in the tie line reactance gives a larger stability region.

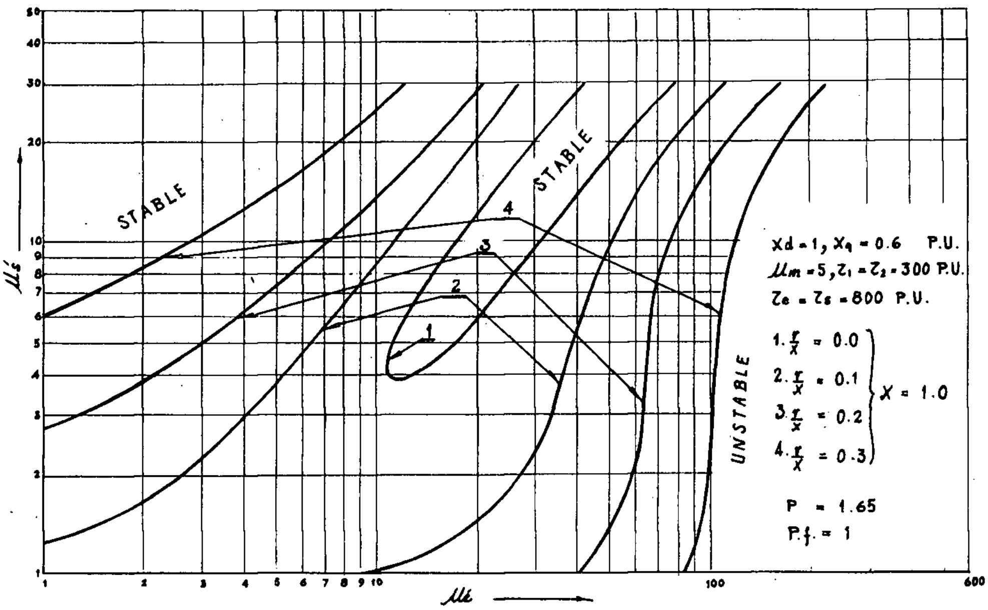


Fig. 6-3 Tie Line Resistance Effect on the Stability Region.

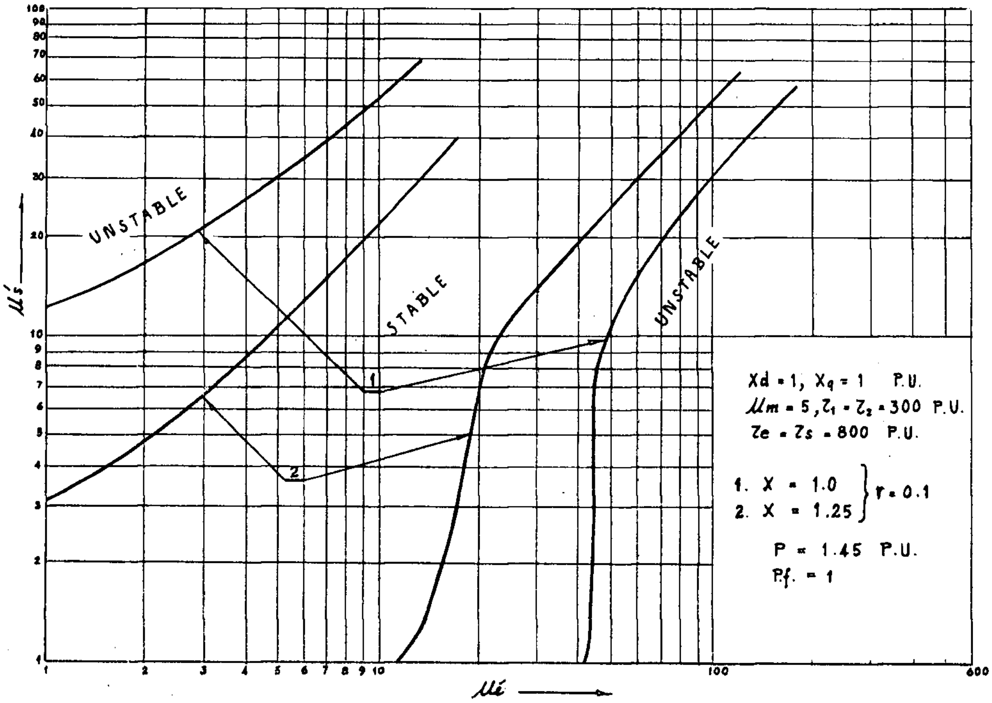


Fig. 6-4 Tie Line Reactance Effect on the Stability Region

6.3 Neglecting Both the Tie Line Resistance and the Saliency in a Round-rotor Machine

Fig. 6-5 shows the effect of both tie line resistance and saliency on the stability region of a round-rotor machine operating at unity power factor for various power values. It can be seen that the neglect of both line resistance and saliency gives a smaller stability region at the high-μ'_e bound. For the stability region at the high-μ'_s bound, the effect of neglecting both line resistance and saliency becomes very great when x_d is large or the short circuit ratio is small. This can be seen from Fig. 6-6.

6.4 The Effect of Short Circuit Ratio

Fig. 6-6 also shows that the stability region at the high-μ'_e bound decreases with short circuit ratio. As for the stability region at the high-μ'_s bound, the decrease in short circuit ratio usually increases the stability region. The solid lines show the stability region of the round-rotor machine with a saliency $\frac{x_q}{x_d} = 0.85$, and the dotted lines are those of the round-rotor machine with saliency and tie line resistance neglected.

6.5 The Effect of the Exciter Time Constant

Fig. 6-7 shows the stability region of a round-rotor machine, $x_d = x_q$, operating at unity power factor subjected to the variation of τ_e. Noticeable effects are observed only in the stability region at the high-μ'_e bound, especially for low μ'_s values.

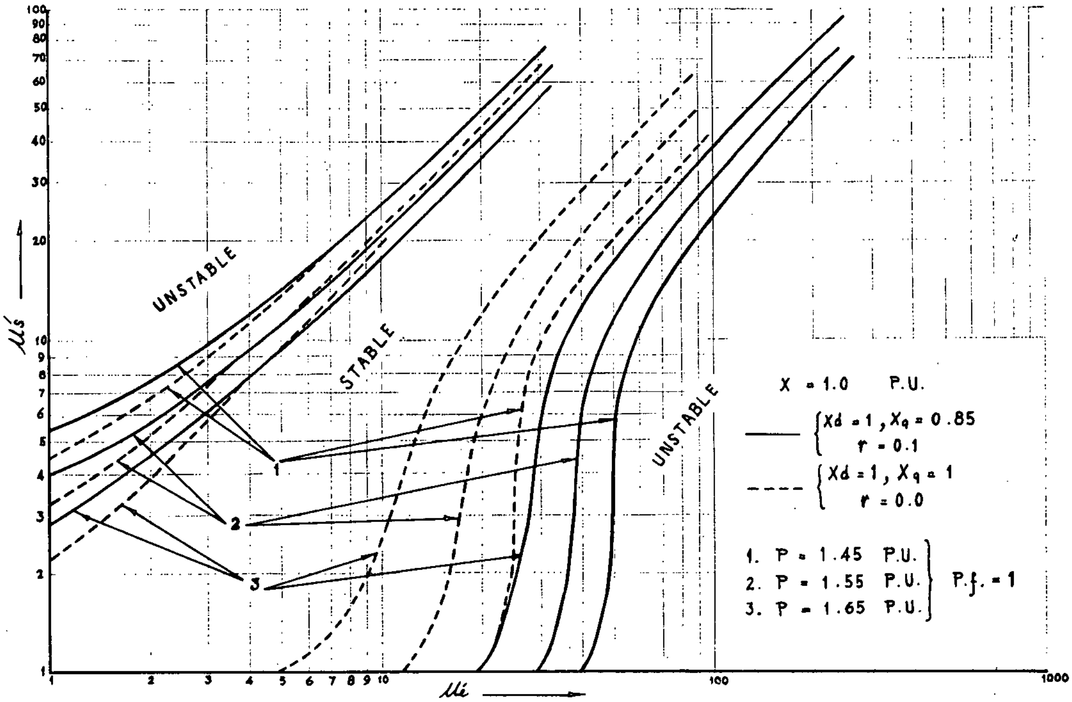


Fig 6-5 Effect of Neglecting Tie Line Resistance and Saliency on the Stability Region.

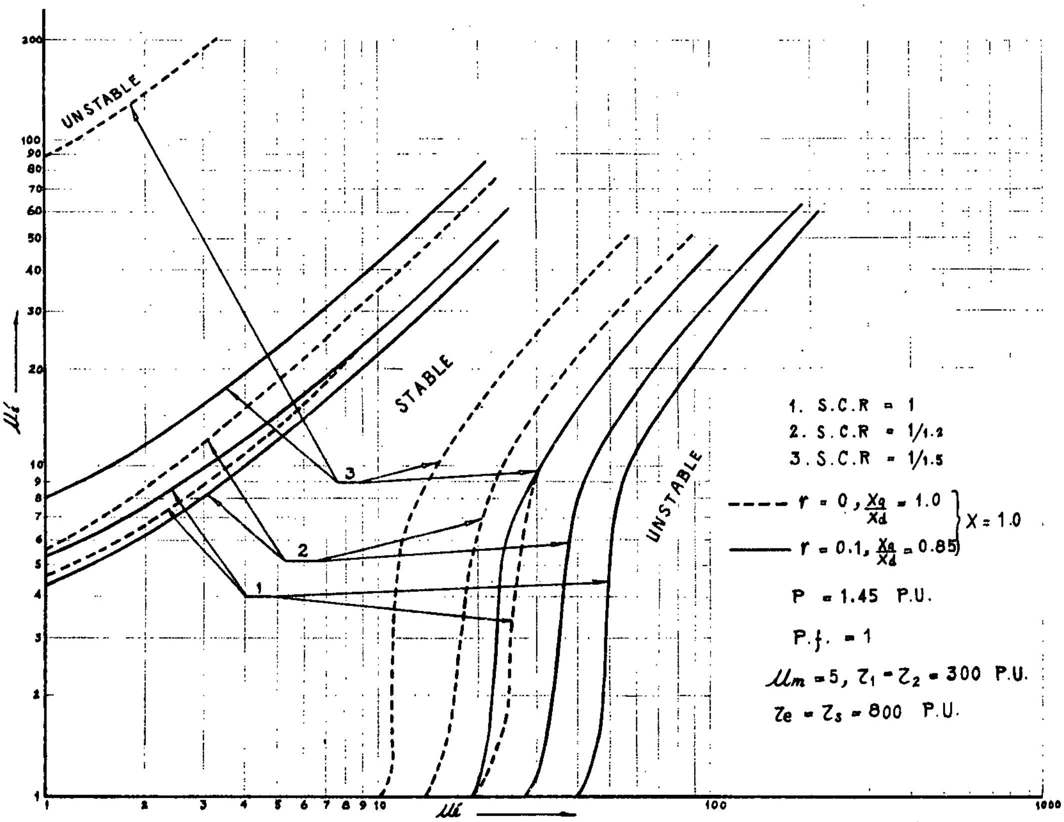


Fig. 6-6 Effect of Short Circuit Ratio on the Stability Region.

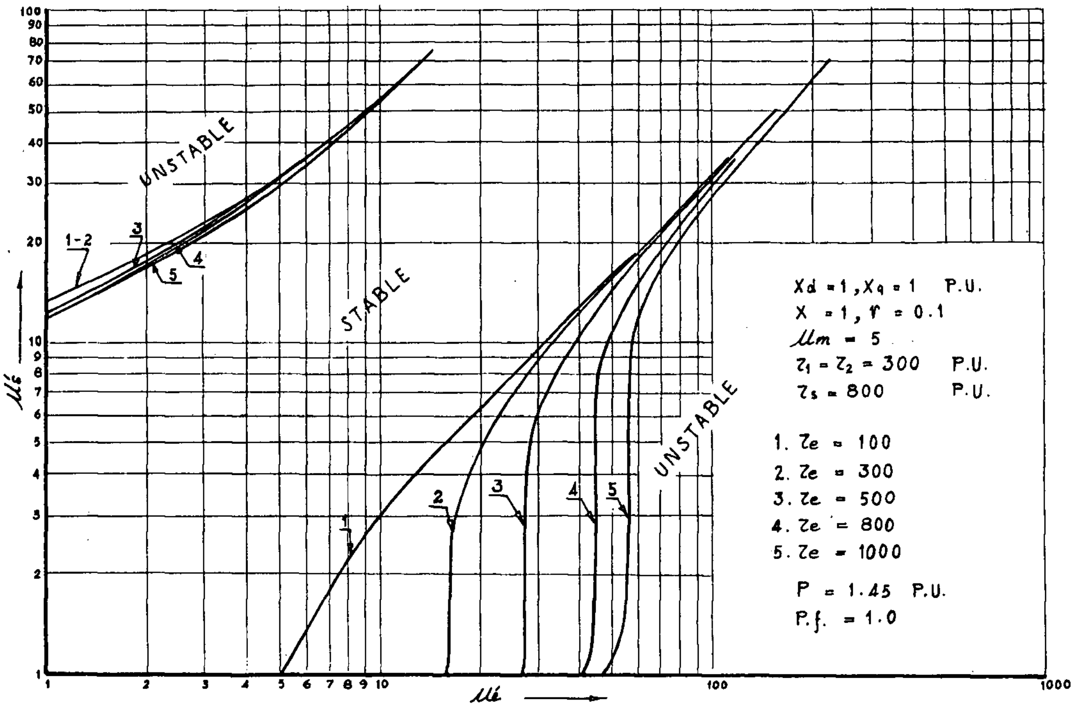


Fig. 6-7 Exciter Time Constant Effect on the Stability Region of a Round-rotor Machine.

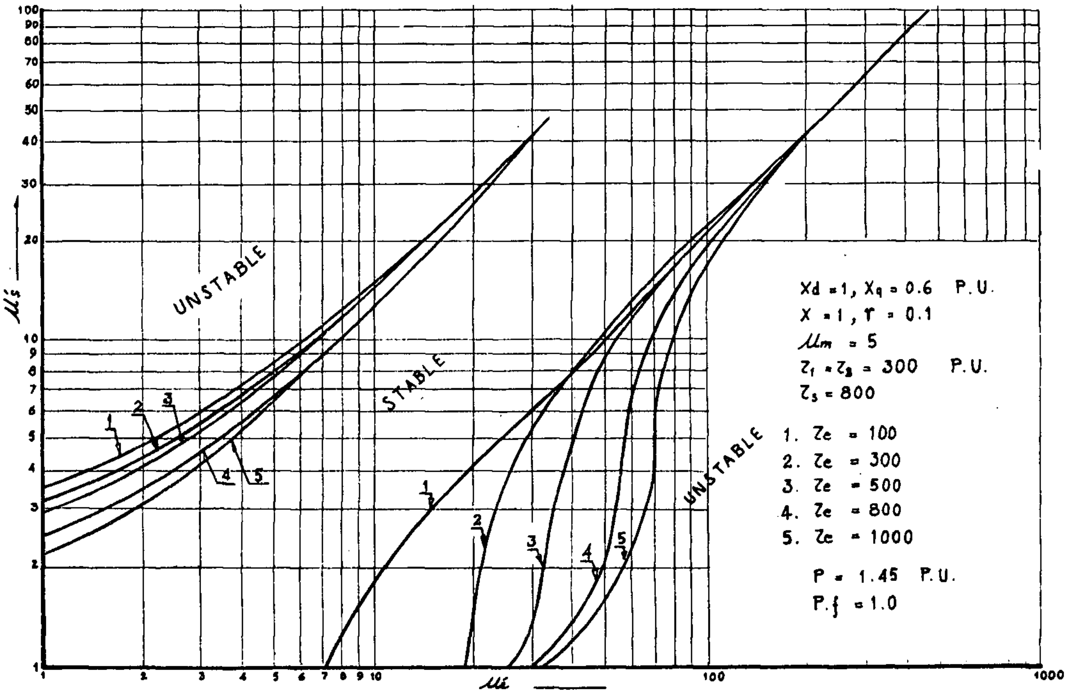


Fig 6-8 Exciter Time Constant Effect on the Stability Region of a Salient-pole Machine.

Similar computations are repeated for a salient-pole machine instead of the round-rotor machine. The results are given in Fig. 6-8 which shows that the same conclusion can be drawn for both types of machine, namely, for low μ'_s values the smaller the exciter time constant, the smaller the stability region at the high-μ'_s bound.

6.6 The Effect of the Stabilizer Time Constant

The decrease in τ_s usually results in an increase in the stability region. The stability region at the high-μ'_e bound and the stability region at the high-μ'_s bound increase as τ_s decreases which can be seen from Fig. 6-9 and 6-10. However, for a salient-pole machine, $x_d = 1$ and $x_q = 0.6$, a decrease in τ_s may cause a decrease in the stability region at the high-μ'_e bound for low μ'_s values, as is seen in Fig. 6-10.

6.7 The Effect of the Governor Time Constants

Fig. 6-11 shows the effect of a governor with two time constants on the stability region of a round-rotor machine, $x_d = x_q = 1.0$. It can be seen that the two-time-constant governor affects the stability region at the high-μ'_e bound but not the high-μ'_s bound.

For high μ'_s values, the smaller the governor time constants, the larger would be the stability region at the high-μ'_e bound. However, there exists a case, $\tau_1 = \tau_2 = 200$ p.u., that yields the smallest stability region.

The above conclusion applies also to the case of a salient-pole machine, $x_d = 1$, $x_q = 0.6$, as seen in Fig. 6-12.

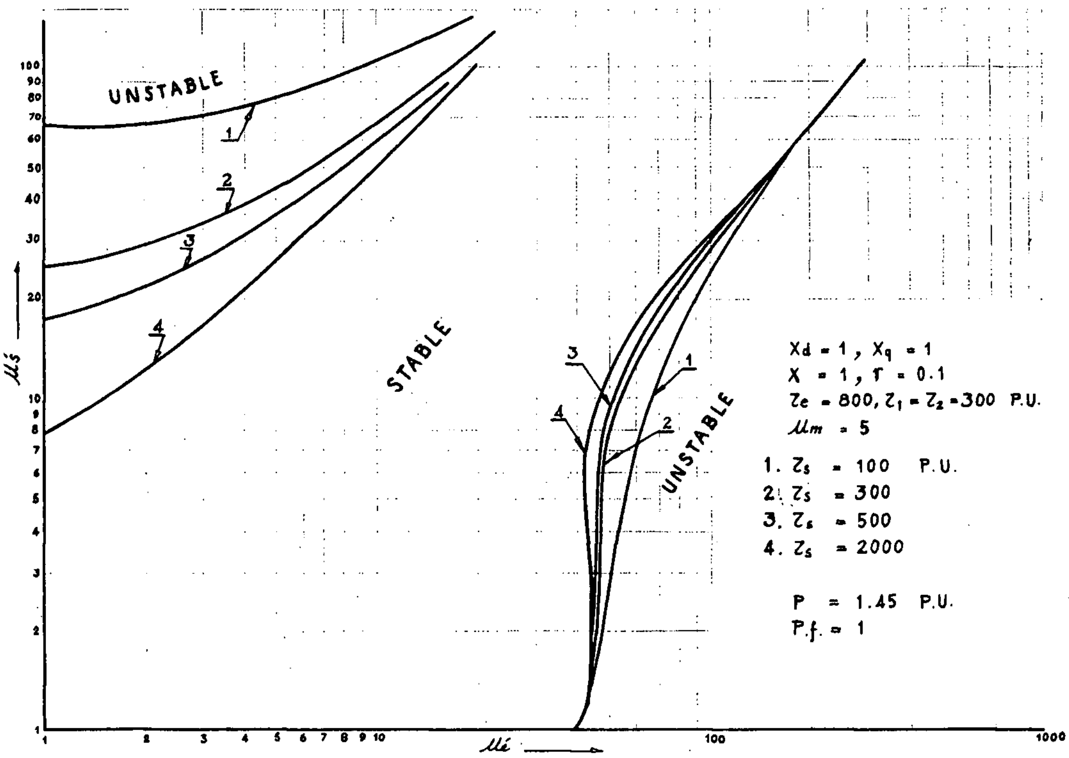


Fig. 6-9 Stabilizer Time Constant Effect on the Stability Region of a Round-rotor Machine.

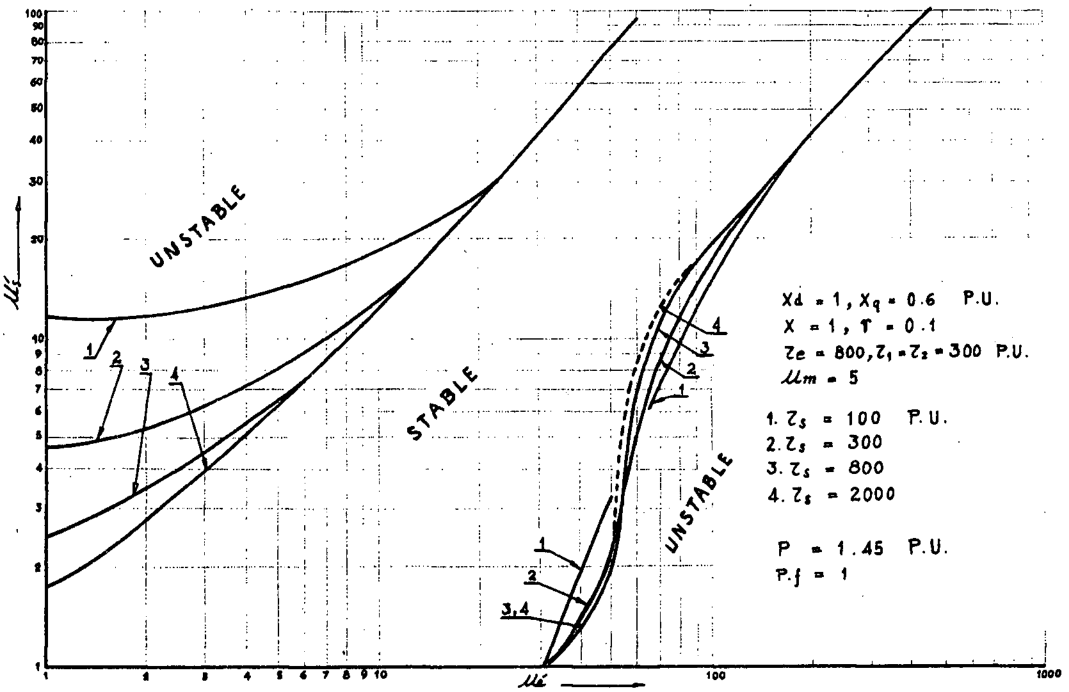


Fig. 6-10 Stabilizer Time Constant Effect on the Stability Region of a Salient-pole Machine.

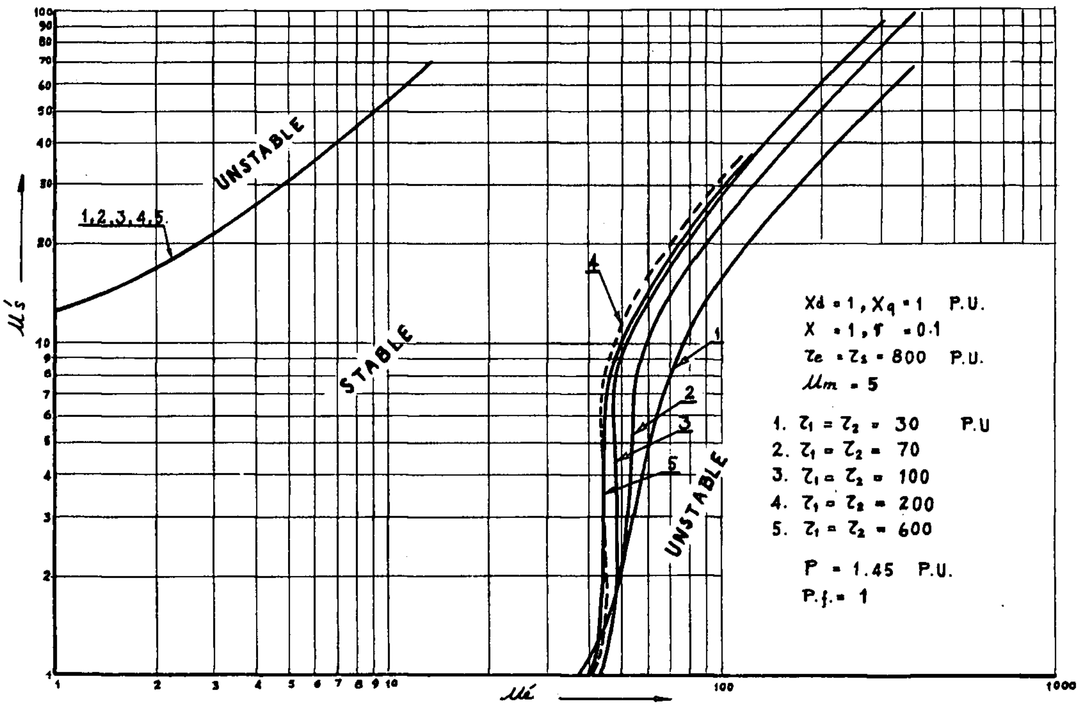


Fig. 6-11 Effect of the Two-time-constant Governor on the Stability Region of a Round-rotor Machine.

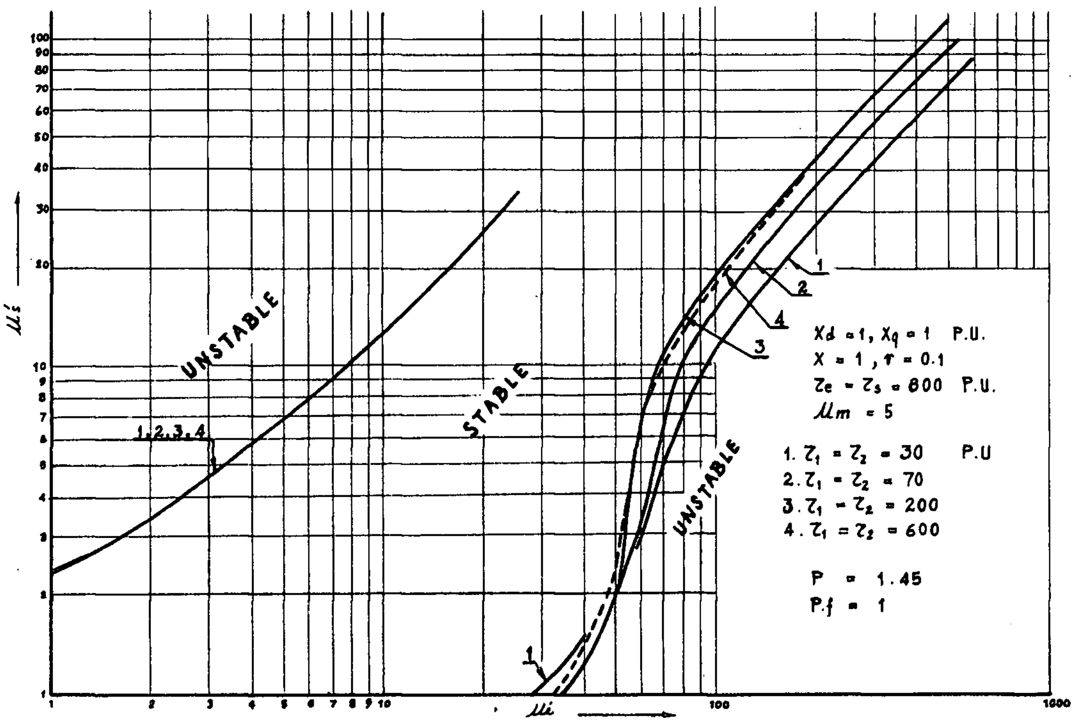


Fig. 6-12 Effect of the Two-time-constant Governor on the Stability Region of a Salient-pole Machine.

The effect of a governor with one time constant on the stability region is shown in Fig. 6-13. The time constant changes the stability region considerably. A faster governor usually gives a larger stability region at the high-μ_e' bound, except for low μ_s' where a very fast acting governor may cause a smaller stability region at the high-μ_e' bound.

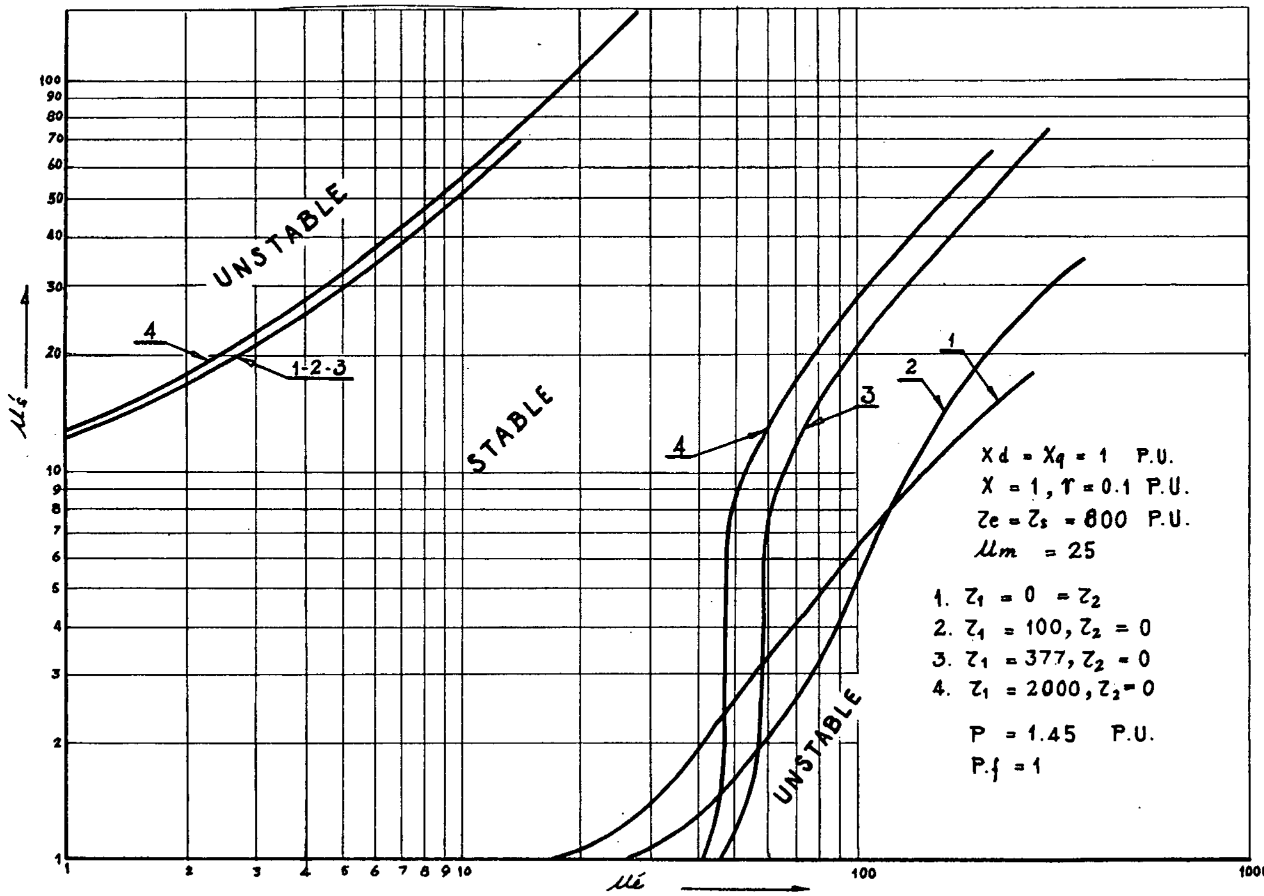


Fig. 6-13 Effect of the One-time-constant Governor on the Stability Region.

7. CONCLUSIONS

From the results of the studies in this thesis, the principal conclusions that can be drawn are as follows:

1. It has been found from Routh-Hurwitz Criterion study that for fixed settings of control parameters the saliency of a synchronous machine considerably increases the dynamic stability limit of a power system and from the D-partition study that for a fixed power limit the saliency of a synchronous machine increases the stability region at the high-μ_e' bound except for low μ_s' values, but decreases it at the high-μ_s' bound considerably.

2. The short circuit ratio of a synchronous machine has effects similar to that of $\frac{1}{x_d}$. The larger the short circuit ratio, the larger is the dynamic stability limit. It also increases the stability region at the high-μ_e' bound and decreases it at the high-μ_s' bound.

3. The tie line impedance has a significant effect upon the dynamic stability. Both the stability limit in the Routh-Hurwitz Criterion study and the stability region in the D-partition study increase with the tie line resistance, but decrease with an increase in the line reactance. For a fixed ratio of $\frac{r}{x}$, the effect of tie line resistance on stability limit is rather small for a small tie line reactance but becomes larger for a larger tie line reactance.

4. Both the stability limit and the stability region are greatly reduced if the saliency and the tie line resistance are neglected. Actually a larger, rather than a smaller, μ_e'

gain is permissible for the machine when the saliency and the tie line resistance are considered.

5. It has been found that a larger exciter time constant gives a larger stability limit for a particular setting of parameters, or a larger stability region for a particular power. However, in order to obtain a maximum stability limit, the time constants and the gain of an exciter must be appropriately coordinated.

6. A decrease in the stabilizer time constant increases the stability region at the high-μ'_e bound and also at the high-μ'_s bound. However, for low μ'_s values, decreasing the stabilizer time constant may result in a decrease of the stability region at the high-μ'_e bound.

7. The governor has comparatively less effect on the dynamic stability. For a governor with two equal time constants, its effect on the dynamic stability limit is small and in some cases it can even decrease the stability limit. For a governor with one time constant, its effect on stability is comparatively large . For high-μ'_s values at the high-μ'_e bound, the smaller the governor time constant, the larger is the stability region. For the low μ'_s part of the high-μ'_e bound, however, a small time-constant governor may decrease the stability region. On the other hand, the governor time constant has little effect on the stability region at the high-μ'_s bound.

The study in this thesis has been confined to the dynamic stability in the small without considering nonlinearities. The dynamic stability in the large of a power system including nonlinearities is left for future studies.

APPENDIX I. SYMBOLS AND UNITS

Subscript o denotes an initial condition

Prefix Δ denotes a small change about the initial operating value

$p = \frac{d}{dt}$ = time derivative

i_d, i_q = armature currents in d- and q-axes respectively

v_d, v_q = armature voltages in d- and q-axes respectively

v_t = armature terminal voltage

v_{fd} = applied voltage in field winding

v_o = infinite bus-bar voltage

ψ_d, ψ_q = armature flux linkages in d- and q-axes respectively

$E = \frac{x_{ad}}{R_F} v_{fd}$ = armature open circuit voltage

x_d, x_q = synchronous reactance in d- and q-axes respectively

x_{ad} = mutual reactance between the stator and rotor in d-axis

x = tie-line reactance between the generator and the bus-bar

R_F = field winding resistance

r_a = armature winding resistance in d- or q-axis circuit

r = tie-line resistance between the generator and the bus-bar

δ = power angle

T_m = mechanical input torque to the rotor

T_e = energy conversion torque

P = real power output of the machine

Q = reactive power output of the machine

H = inertia constant

J = moment of inertia

α = damping coefficient

θ = instantaneous angular position of rotor

$p\theta$ = angular velocity of machine

ω = rated angular velocity

f = rated system frequency

τ'_{do} = direct-axis transient open-circuit time constant

τ'_{d} = direct-axis transient short-circuit time constant

τ''_{do} = direct-axis subtransient short-circuit time constant

τ''_{qo} = quadrature-axis subtransient open-circuit time constant

τ''_{q} = quadrature-axis subtransient short-circuit time constant

τ_{Dl} = direct-axis damper leakage time constant

τ_{e} = exciter time constant

τ_{s} = stabilizer time constant

τ_{1}, τ_{2} = governor time constants

μ_{ex} = exciter gain

μ_{r} = converter gain

μ_{a} = amplifier gain

μ_{st} = stabilizer gain

$\mu_{e} = \mu_{ex}\mu_{a}\mu_{r}$ = regulator gain

$\mu'_{e} = \frac{x_{ad}}{R_{F}} \mu_{e} \triangleq$ over all regulator gain

$\mu'_{s} = 1 + \mu_{ex}\mu_{st} \triangleq$ over all stabilizer gain

Throughout the thesis all calculations are made using the per-unit system based on M.K.S. units.

The unit of time is 1 radian; at $f = 60$ Hz, 1 second = $2\pi f = 377$ radians. Moment of inertia $J = 4\pi fH$, and p.u. power and p.u. torque are numerically identical, i.e. $P = T_{eo}$ p.u.

APPENDIX II. VOLTAGE REGULATOR AND SPEED GOVERNOR TRANSFER FUNCTIONS

a. Voltage Regulator Transfer Function[4,8]

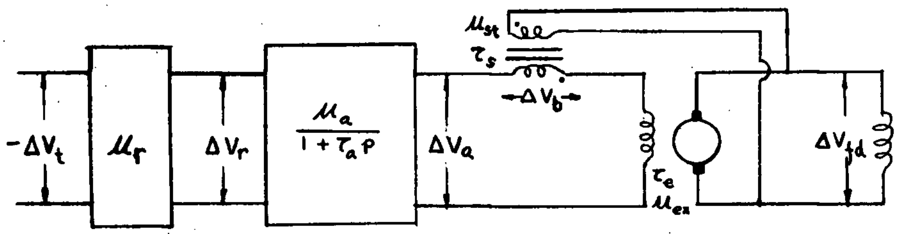


Fig. I. Voltage Regulator

The voltage regulating system under consideration is shown in Fig. I. It has the following parts:

1. A means of measuring the terminal voltage error.

2. A device to convert the voltage error to a suitable signal (Δv_r). The converter may be defined by an amplification factor (μ_r) and zero time constant.

3. A static or rotating amplifier with a large gain (μ_a) and a small time constant (τ_a).

4. An exciter supplying the field current to a synchronous machine.

5. A stabilizing transformer feeding back into the regulator circuit a signal proportional to the rate of change of the field current.

Since

$$\Delta v_r = -\mu_r \Delta v_t, \text{ converter}$$

$$\Delta v_a = \frac{\mu_a \Delta v_r}{1+\tau_a p}, \text{ amplifier}$$

$$\Delta v_b = \frac{\mu_{st}\tau_s p}{1+\tau_s p}\,\Delta v_{fd}, \text{ stabilizer}$$

and

$$\Delta v_{fd} = \frac{\mu_{ex}}{1+\tau_e p}\,(\Delta v_a - \Delta v_b), \text{ field}$$

the voltage regulator transfer function

$$g(p) \triangleq \frac{\Delta v_{fd}}{\Delta v_t} = -\frac{\mu_e (1+\tau_s p)}{(1+\tau_a p)\left[1+\{\tau_e+\tau_s(\mu_s+1)\}p+\tau_e\tau_s p^2\right]}$$

where

$$\mu_e = \mu_{ex}\mu_a\mu_r, \quad \mu_s = \mu_{st}\mu_{ex}$$

b. <u>Speed Governor Transfer Function</u>[18,19]

The speed governor system, including the time lags due to steam or water and some intermediate actuating member can be specified by two time constants, and one gain.

Thus the transfer function of the governor

$$f'(p) \triangleq \frac{-\mu_m}{(\tau_1 p+1)(\tau_2 p+1)}$$

If one of the time constants can be neglected the transfer function becomes

$$f'(p) \triangleq \frac{-\mu_m}{(1+\tau_1 p)} \quad .$$

APPENDIX III. DERIVATION OF THE CHARACTERISTIC EQUATION

From equation (3-14), the following characteristic equation is obtained

$$(1+\tau'_{do}p)(1+\tau_1 p)(1+\tau_2 p)\left\{1+(\tau_e+\tau_s\mu'_s)p + \tau_e\tau_s p^2\right\}\Delta$$

$$= \left[J(p)(1+\tau_1 p)(1+\tau_2 p)\right]\left[(1+\tau'_{do}p)(1+\tau_1 p)(1+\tau_2 p)\left\{1+(\tau_e+\tau_s\mu_s)p + \tau_e\tau_s p^2\right\}h(p)\right.$$

$$\left.+(A_4 p + A_5)\right]+(1+\tau'_{do}p)(1+\tau_1 p)(1+\tau_2 p)\left\{1+(\tau_e+\tau_s\mu'_s)p+\tau_e\tau_s p^2\right\}x_d(p)(A_6 p+A_7)$$

$$+(1+\tau'_{do}p)(1+\tau_1 p)(1+\tau_2 p)\left\{1+(\tau_e+\tau_s\mu_s)p+\tau_e\tau_s p^2\right\}(A_8 p+A_9)$$

$$= 0$$

To expand, let

$$B_1 = A_2 x_d \tau'_d + A_3 \tau'_{do}$$

$$B_2 = A_2 x_d + A_3$$

$$B_3 = A_7 x_d + A_9$$

$$B_4 = A_6 x_d \tau'_d + A_8 \tau'_{do} + B_1\left(\alpha - \frac{T_{eo}}{\omega}\right) + B_2 J$$

$$B_5 = (A_6 + A_7\tau'_d)x_d + A_8 + A_9\tau'_{do} + B_2\left(\alpha - \frac{T_{eo}}{\omega}\right)$$

$$B_6 = A_1\left(\alpha - \frac{T_{eo}}{\omega}\right) + A_4$$

$$T_a = \tau_1\tau_2\tau_e\tau_s$$

$$T_b = \tau_1+\tau_2+\tau_e+\tau_s\mu'_s$$

$$T_c = \tau_1\tau_2+\tau_e\tau_s+(\tau_1+\tau_2)(\tau_e+\tau_s\mu'_s)$$

$$T_d = \tau_1\tau_2(\tau_e+\tau_s\mu'_s)+(\tau_1+\tau_2)\tau_e\tau_s$$

$$T_E = \tau_1\tau_2+\tau_2\tau_s+\tau_1\tau_s$$

$$T_F = \tau_1+\tau_2+\tau_s$$

Now

$$J(p)(1+\tau_1 p)(1+\tau_2 p)(1+\tau'_{do}p)\left\{1+(\tau_e+\tau_s\mu'_s)p+\tau_e\tau_s p^2\right\}\left\{h(p)A_1+x_d(p)A_2+A_3\right\}$$

$$= p^7 T_a JB_1 + p^6\left[T_a\left\{B_1(\alpha-\frac{T_{eo}}{\omega})+JB_2\right\}+T_d\,JB_1\right]$$

$$+p^5\left[T_a B_2(\alpha-\frac{T_{eo}}{\omega})+T_c JB_1+T_d\left\{B_1(\alpha-\frac{T_{eo}}{\omega})+JB_2\right\}-\mu'_e JA_1\tau_1\tau_2\tau_s\right]$$

$$+p^4\left[T_b JB_1+T_c\left\{B_1(\alpha-\frac{T_{eo}}{\omega})+JB_2\right\}+T_d B_2(\alpha-\frac{T_{eo}}{\omega})\right.$$

$$\left.+\mu_m B_1\tau_e\tau_s-\mu'_e A_1\left\{JT_E+(\alpha-\frac{T_{eo}}{\omega})\tau_1\tau_2\tau_s\right\}\right]$$

$$+p^3\left[T_b\left\{B_1(\alpha-\frac{T_{eo}}{\omega})+JB_2\right\}+T_c B_2(\alpha-\frac{T_{eo}}{\omega})+\mu_m\left\{B_1(\tau_e+\tau_s\mu'_s)+B_2\tau_e\tau_s\right\}\right.$$

$$\left.-\mu'_e A_1\left\{JT_F+T_E(\alpha-\frac{T_{eo}}{\omega})\right\}+JB_1\right]$$

$$+p^2\left[T_b B_2(\alpha-\frac{T_{eo}}{\omega})+\mu_m\left\{B_1+B_2(\tau_e+\tau_s\mu_s)-\mu'_e A_1\tau_s\right\}\right.$$

$$\left.-\mu'_e A_1\left\{(\alpha-\frac{T_{eo}}{\omega})T_F+J\right\}+B_1(\alpha-\frac{T_{eo}}{\omega})+JB_2\right]$$

$$+p\left[\mu_m(B_2-\mu'_e A_1)-\mu_e A_1(\alpha-\frac{T_{eo}}{\omega})+B_2(\alpha-\frac{T_{eo}}{\omega})\right]$$

Next consider

$$h(p)(1+\tau'_{do}p)(1+\tau_1 p)(1+\tau_2 p)\left\{1+(\tau_e+\tau_s\mu_s)p+\tau_e\tau_s p^2\right\}(A_4 p+A_5)$$

$$= -\mu'_e\left\{p^4 A_4\tau_1\tau_2\tau_s + p^3(A_4 T_E+A_5\tau_1\tau_2\tau_s)+p^2(A_4T_F+A_5T_E)+p(A_4+A_5T_F)\right.$$

$$\left.+A_5\right\}$$

$$(1+\tau'_{do}p)(1+\tau_1 p)(1+\tau_2 p)\left\{1+(\tau_e+\tau_s\mu'_s)p+\tau_e\tau_s p^2\right\}x_d(p)(A_6p+A_7)$$

$$=\Big[p^6 T_a A_6\tau'_d + p^5\left\{T_a(A_6+A_7\;{}_d)+T_d A_6\tau'_d\right\}$$

$$+\,p^4\left\{T_a A_7+T_c A_6\;{}_d+T_d(A_6+A_7\tau'_d)\right\}$$

$$+\,p^3\left\{T_b A_6\tau'_d+T_c(A_6+A_7\tau'_d)+T_d A_7\right\}$$

$$+\,p^2\left\{T_b(A_6+A_7\tau'_d)+T_c A_7+A_6\tau'_d\right\}$$

$$+\,p\left\{T_b A_7+A_6+A_7\tau'_d\right\}+A_7\Big]x_d$$

$$(1+\tau'_{do}p)(1+\tau_1 p)(1+\tau_2 p)\left\{1+(\tau_e+\tau_s\mu'_s)p+\tau_e\tau_s p^2\right\}(A_8p+A_9)$$

$$=p^6 T_a A_8\tau'_{do}+p^5\left\{T_a(A_8+A_9\tau'_{do})+T_d A_8\tau'_{do}\right\}$$

$$+\,p^4\left\{T_a A_9+T_c A_8\tau'_{do}+T_d(A_8+A_9\tau'_{do})\right\}$$

$$+\,p^3\left\{T_b A_8\tau'_{do}+T_c(A_8+A_9\tau'_{do})+T_d A_9\right\}$$

$$+\,p^2\left\{T_c A_9+T_b(A_8+A_9\tau'_{do})+A_8\tau'_{do}\right\}$$

$$+\,p\left\{T_b A_9+A_8+A_9\tau'_{do}\right\}+A_9$$

Hence the final form of the characteristic equation is written as equation (3-15).

APPENDIX IV. DERIVATION OF THE D-PARTITION EQUATIONS

Let $T_c' = \tau_1\tau_2 + \tau_e\tau_s + (\tau_1+\tau_2)\tau_e$

equation (3-15) becomes

$$
\begin{aligned}
&\mu_s'\Big[p^6 JB_1\tau_1\tau_2\tau_s + p^5\left\{JB_1(\tau_1+\tau_2)\ {}_s + B_4\tau_1\tau_2\tau_s\right\} + p^4\left\{JB_1\tau_s + B_4(\tau_1+\tau_2)\tau_s\right. \\
&\qquad \left. + B_5\tau_1\tau_2\tau_s\right\} + p^3\left\{B_3\tau_1\tau_2\tau_s + B_4\tau_s + B_5(\tau_1+\tau_2)\tau_s + \mu_m B_1\tau_s\right\} \\
&\quad + p^2\left\{B_3(\tau_1+\tau_2)\tau_s + B_5\tau_s + \mu_m B_2\tau_s\right\} + pB_3\tau_3\Big] \\
&-\mu_e'\Big[p^5 JA_1\tau_1\tau_2\tau_s + p^4\left\{T_E JA_1 + B_6\tau_1\tau_2\tau_s\right\} + p^3\left\{T_E B_6 + T_F JA_1 + A_5\tau_1\tau_2\tau_s\right\} \\
&\quad + p^2\left\{T_E A_5 + T_F B_6 + JA_1 + \mu_m A_1\tau_s\right\} + p\left\{T_F A_5 + B_6 + \mu_m A_1\right\} + A_5\Big] \\
&\quad + \Big[p^7 T_a JB_1 + p^6\left\{T_a B_4 + T_E JB_1\tau_e\right\} + p^5\left\{T_a B_5 + T_c JB_1 + T_E B_4\tau_e\right\} \\
&\quad + p^4\left\{T_a B_3 + T_c' B_4 + T_E B_5\tau_e + T_F JB_1 + \mu_m B_1\tau_e\tau_s\right\} \\
&\quad + p^3\left\{T_c' B_5 + T_E B_3\tau_e + T_F B_4 + B_1 J + \mu_m(B_1 + B_2\tau_s)\tau_e\right\} \\
&\quad + p^2\left\{T_c' B_3 + T_F B_5 + B_4 + \mu_m(B_1 + B_2\tau_e)\right\} \\
&\quad + p(T_F B_3 + B_5 + \mu_m B_2) + B_3\Big] = 0
\end{aligned}
$$

The characteristic equation can be expressed as

$$\mu_s' P(p) - \mu_e' Q(p) + R(p) = 0$$

Let $p = j\omega$, we have

$$\mu_s'\left\{P_1(\omega) + jP_2(\omega)\right\} - \mu_e'\left\{Q_1(\omega) + jQ_2(\omega)\right\} + \left\{R_1(\omega) + jR_2(\omega)\right\} = 0$$

where

$$P_1(\omega) = -\omega^6 JB_1\tau_1\tau_2\tau_s + \omega^4\left\{JB_1\tau_s + B_4(\tau_1+\tau_2)\tau_s + B_5\tau_1\tau_2\tau_3\right\}$$
$$-\omega^2\left\{B_3(\tau_1+\tau_2)\tau_s + B_5\tau_s + \mu_m B_2\tau_s\right\}$$

$$P_2(\omega) = \omega^5\left\{JB_1(\tau_1+\tau_2)\tau_s + B_4\tau_1\tau_2\tau_s\right\} - \omega^3\left\{B_3\tau_1\tau_2\tau_s + B_4\tau_s\right.$$
$$\left. + B_5(\tau_1+\tau_2)\tau_s + \mu_m B_1\tau_s\right\} + \omega\, B_3\tau_3$$

$$Q_1(\omega) = \omega^4\left\{T_E JA_1 + B_6\tau_1\tau_2\tau_s\right\} - \omega^2\left\{T_E A_5 + T_F B_6 + JA_1 + \mu_m A_1\tau_s\right\} + A_5$$

$$Q_2(\omega) = \omega^5 JA_1\tau_1\tau_2\tau_s - \omega^3\left\{T_E B_6 + T_F JA_1 + A_5\tau_1\tau_2\tau_s\right\} + \omega\left\{T_F A_5 + B_6 + \mu_m A_1\right\}$$

$$R_1(\omega) = -\omega^6\left\{T_a B_4 + T_E JB_1\tau_e\right\} + \omega^4\left\{T_a B_3 + T_c' B_4 + T_E B_5\tau_e + T_F JB_1\right.$$
$$\left. + \mu_m B_1\tau_e\tau_s\right\} - \omega^2\left\{T_c' B_3 + T_F B_5 + B_4 + \mu_m(B_1 + B_2\tau_e)\right\} + B_3$$

$$R_2(\omega) = -\omega^7 T_a JB_1 + \omega^5\left\{T_a B_5 + T_c' JB_1 + T_E B_4\tau_e\right\}$$
$$-\omega^3\left\{T_c' B_5 + T_E B_3\tau_e + T_F B_4 + JB_1 + \mu_m(B_1 + B_2\tau_s)\tau_e\right\}$$
$$+\omega\left\{T_F B_3 + B_5 + \mu_m B_2\right\}$$

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