THE IMPACT OF UNITED STATES FINAL DEMAND ON CANADIAN PRODUCTION

AN INPUT-OUTPUT STUDY

by

LESLIE WILLIAM KEITH HORNER
B.A., University of British Columbia, 1963.

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

in the department

of

ECONOMICS

We accept this thesis as conforming to
the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
April, 1967

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Economics

The University of British Columbia
Vancouver 8, Canada

Date April, 1967

ABSTRACT

In this thesis, the impact of United States final demand on Canadian demand and production is investigated using an interregional input-output model.

First, the simple Leontief input-output model is considered. It is a disaggregated model of the production sector of an economy that allows a set of industry outputs to be expressed as a function of a corresponding set of industry final demands. It improves on other output determination models by admitting that industry outputs are interdependent. However, it requires the assumption of fixed production coefficients.

Next, the extension of the model to incorporate interregional trade is considered. Several models are described that determine the industry outputs of each of a group of regions as functions of the industry final demands in all regions.

A model is selected that differs from all of these, not in its essential algebraic structure, but in the method by which it is applied. In the simplified form in which it is used in this study, it requires that Canada's merchandise exports to the United States be reclassified according to the industry schemes of the Canadian and American input-output tables. The main advantage of the model over the other interregional models considered is that it allows the input-output tables of the individual regions to be used in their original form.

Using the model, two questions are investigated. First, how do equal expenditures on the various components of United States final demand - Consumption, Fixed investment, Federal Government purchases, and State and local government purchases - compare in their impact on Canadian demand and output ? Second, in the period 1956 to 1960, did variation in the level and pattern of United States final demand tend to aggravate fluctuations in Canadian demand, output, and net exports ?

Several results are obtained.

With reference to the first question, Investment expenditure is found to have considerably greater impact on Canadian demand and production than any of the other components of United States demand. The wide disparity in impact is largely explained by the concentration of Canadian exports to the United States on a few commodities.

Concerning the second question, it is concluded that variations in both the level and pattern of United States final demand helped to generate fluctuations in the growth of Canadian demand and output. By contrast, the fluctuation of United States final demand tended to damp fluctuations in Canadian net exports.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

The close economic relationship between Canada and the United States has long been a subject of discussion and concern in Canada. It has been argued that so many important economic decisions in Canada are dictated by policies or conditions existing in the United States that Canada has little economic or political autonomy. Two separate questions are involved in evaluating such a claim. First, does United States ownership of Canadian industry imply that citizens of the United States are responsible for many decisions that directly affect Canada's political posture or economic development ? Second, to what extent do the high levels of commodity and capital flows between the countries make the Canadian economy sensitive to changes in economic conditions in the United States ?

In this paper, some of the factors bearing on the second question are examined. A model is developed which will yield estimates of the amount of Canadian output generated by various levels and patterns of United States final demand. Thus it may be used to investigate the impact of cyclical variations in United States demand on Canadian production activity. The model may also be used to estimate changes in the levels of Canadian exports and imports attributable to a change in

United States final demand, and by subtraction, the primary effect of the change on the Canadian merchandise balance of trade. However, the model yields only a partial answer to the question of Canadian sensitivity to economic conditions in the United States. It can not be used to estimate changes in the levels of Canadian Consumption and Investment that result from changes in United States final demand; nor can it be used to predict changes in Canadian output attributable to the influence of Canadian-American capital flows on Canadian interest rates.

Within the area of enquiry limited by the nature of the model, several results are obtained.

1. Of equal aggregate expenditures on the four major components of United States final demand - Personal consumption, Investment, Federal Government expenditure, and State and local government expenditure - the Investment expenditure has much the strongest impact on Canadian aggregate demand and output. The prime reason for the wide variation in effect among these demand components is the concentration of Canadian exports to the United States on a small number of products.

2. For the period 1956 to 1960, the level of Canadian exports generated by United States final demand grew at a rate that fluctuated in phase with, but more widely than the growth rate of United States aggregate demand itself. Thus, shifts in the composition of United States demand acted to

exaggerate the impact of fluctuations in its level.

3. The growth rate of induced Canadian demand also varied in phase with the observed fluctuations in the growth of Canadian final demand, and therefore contributed to them. In other words, the dependence of Canadian aggregate demand on United States business activity had a cyclically destabilizing effect on Canadian economic growth.

4. This conclusion regarding the transmission of business cycles did not apply universally to the growth of output of important Canadian export industries.

5. Variations in United States final demand had a stabilizing effect on fluctuations in the Canadian balance of merchandise trade. This resulted from the fact that import fluctuations dominated export fluctuations in determining the growth of Canadian net exports.

The model used to obtain these results is an extension of the simple Leontief Input-Output model. That model determines the set of industry outputs required by a corresponding set of final demands. In doing so it recognizes explicitly the interdependence of industry output levels. It is based on the assumption that the production of a unit of an industry's output will require as inputs, fixed amounts of the outputs of other industries. In other words it assumes constant production, or input-output, coefficients.

By assuming, in addition, that exports from Canada constitute a fixed proportion of the total supply of each industry's output in the United States, the simple Leontief model is extended so that the set of Canadian industry outputs required by a set of United States final demands may be determined. This is essentially the same model as that proposed by R. J. Wonnacott[1] in which the sets of both Canadian and United States industry outputs are related simultaneously to the combined set of final demands. The advantage of this variant of the Wonnacott model is that it is easier to apply and update while sacrificing very little in precision.

The paper may be divided into two parts, the first being concerned with the development of the model, and the second with its application.

The nature of the simple Leontief model is elaborated in Chapter II. In Chapter III its extension to include foreign trade is discussed, and in Chapter IV the model to be used is described. The data and procedure used in applying the model are discussed in Chapter V. In Chapter VI the results are developed and in Chapter VII they are summarized and evaluated.

1. R. J. Wonnacott, Canadian-American Dependence: An inter-industry Analysis of Production and Prices, Amsterdam: The North-Holland Publishing Company, 1961.

CHAPTER II

INTRODUCTION TO INPUT-OUT ANALYSIS

The Leontief Input-Output model is an attempt to put some aspects of general equilibrium theory into computationally workable form. In its basic open construction it is concerned only with the production side of economic activity and does not deal with the determination of final demand.

The basis of the model is a set of accounting identities which describes the inter-industry flow of goods and services in a particular economy. The identities are transformed into equations with the aid of a critical assumption. Then the set of equations may be used as a disaggregated model of the technological structure of the economy. In particular, individual industry output levels may be simultaneously determined as functions of the industry final demands.

The model's main advantage over partial equilibrium analysis is that it recognizes and is capable of dealing with the interdependency of industry output levels. That is, it explicitly accounts for the effects of changes in the final demand for one product on the output levels of others. Similarly, its main advantage over aggregative analysis is that it admits that aggregate input and output levels are affected by the composition of final demand.

On the other hand, the model has definite limitations which restrict its power of prediction and range of application.

The nature of the model and its limitations will now be considered in more detail.

1. The Nature of the Open Leontief Model

Three stages of construction of an open Leontief model may be identified. They are (a) the transactions table, (b) the direct requirements matrix, and (c) the total requirements matrix. The discussion will follow these stages.

(a) The Transactions Table

The transactions table is built from a set of accounting identities which describe the pattern of inter-industry flows of goods and services for a certain time period. Two steps must be taken at the outset in building a set of such flows.

First, the multitude of industries in the economy must be classified into a workable number of sectors (also called <u>industries</u>). The number chosen is arbitrary from a theoretical standpoint and in practice will depend largely on what is desired of the model, what data is available, and what resources of time and money are available for the compilation of the table. Regardless of the number of sectors, the guiding principle of classification is that the industries within each sector should have, as far as possible, the same kinds and combinations of inputs and outputs. In this way a necessary assumption of the analytical model is approximated. In theory, each sector must produce a single homogeneous product to ensure that changes in a sector output, however caused, will always require the same combination of inputs.

The causes and effects of heterogeneous sector outputs will be discussed in the second part of this chapter.

Second, a common unit should be adopted to express the physical flows so that inputs and outputs of dissimilar goods may be combined. This step is not strictly necessary. Physical units may be used as long as the units within each equation are consistent. However, a common unit simplifies both the analysis and exposition. The unit chosen is a dollar's worth of product. The use of this value unit makes it important to express all subsequent flows in the prices prevailing in the period of application of the model. Otherwise, an increase of say fifty percent in the price of a product would appear to result in a fifty percent increase in the physical flow of that product.

With these steps taken, the two sets of accounting identities may be defined.

Let:

N = the number of producing sectors,

X_i = the output of sector i ,

x_{ij} = the output of sector i used by sector j ,

V_j = the primary inputs to sector j, and

Y_i = the output of sector i distributed directly to final users.

Then the sets of identities are:

$$X_j \equiv \sum_{i=1}^{N} x_{ij} + V_j \qquad (j = 1,\ldots,N) \qquad (2.1)$$

$$X_i \equiv \sum_{j=1}^{N} x_{ij} + Y_i \qquad (i = 1,\ldots,N) \qquad (2.2)$$

The first demands that the output of each sector be identical to the sum of the inputs to it from the N producing sectors, plus primary inputs. The latter, also called <u>value added</u>, are considered to come from sector $N + 1$. They generally include imports, indirect taxes, and depreciation, as well as payments to households in the form of wages and salaries, interest and dividends, and net profits of unincorporated businesses.

The second requires that the output of each sector be distributed either as inputs to other sectors or directly to final users. The final output, Y_i, is generally shown as the sum of outputs to the basic National Accounts categories of final demand:- Personal consumption expenditure, Gross private fixed capital formation, Net inventory change, Exports, and Government expenditures on goods and services.

The system of flows for the whole economy may be portrayed in a <u>transactions table</u> where the inputs to each section (identity 2.1) are shown as columns, and the outputs (identity 2.2) as rows. In Table I, an example is given for $N = 3$.

TABLE I

THE LEONTIEF TRANSACTIONS TABLE

Using Sectors

		1	2	3	4	
Producing Sectors	1	x_{11}	x_{12}	x_{13}	Y_1	X_1
	2	x_{21}	x_{22}	x_{23}	Y_2	X_2
	3	x_{31}	x_{32}	x_{33}	Y_3	X_3
	4	V_1	V_2	V_3	$V_4=Y_4$	$\sum_{j=1}^{N} V_j$
		X_1	X_2	X_3	$\sum_{i=1}^{N} Y_i$	

$V_4=Y_4$ represents inputs of value added directly required by final users. An example would be Government payments to civil servants.

The aggregates $\sum_{j=1}^{N+1} V_j$, $\sum_{i=1}^{N+1} Y_i$, and $\sum_{k=1}^{N} X_k$ (not shown) warrant explanation. In National Accounts terminology, $\sum_{j=1}^{N+1} V_j$ is Gross National Product ; $\sum_{i=1}^{N+1} Y_i$ is Gross National Expenditure. Thus the identical macro-economic variables, aggregate income and aggregate demand, are obtainable from the transactions table. The grand total of the sector outputs, $\sum_{k=1}^{N} X_k$, includes both final and intermediate outputs. For this reason it does not give a direct indication of the level of economic performance and has no counterpart in traditional aggregative analysis.

(b) Direct Requirements Matrix

As a first step in moving from the descriptive transactions table to a system capable of yielding predictions, the coefficients a_{ij} are introduced.

Let:

$$a_{ij} = \frac{x_{ij}}{X_j} \qquad (i,j = 1,\ldots,N) \qquad (2.3)$$

Thus the coefficient a_{ij} is defined as the amount of product i directly required in the production of one unit of product j. Introduction of the a_{ij} does not change the substance of the systems (2.1) and (2.2). They are still sets of identities that describe interindustry flows in the accounting period. However, if the nature of the a_{ij}, the V_j, and the Y_i are specified, two sets of simultaneous equations are produced. They are:

$$X_j = \sum_{i=1}^{N} a_{ij} X_j + V_j \qquad (j = 1,\ldots,N) \qquad (2.4)$$

and

$$X_i = \sum_{j=1}^{N} a_{ij} X_j + Y_i \qquad (i = 1,\ldots,N) \qquad (2.5)$$

Here the X_i are unknowns which depend on the a_{ij} and, in their respective systems, on the V_j and Y_i. It is also apparent that the systems represent a set of production functions. In (2.4), industry output levels are related to input levels. In a more general form, this set of functions would be

$$X_j = F_j(x_{1j}, x_{2j}, \ldots, x_{Nj}, V_j) \qquad (j = 1,\ldots,N) \ (2.6)$$

In equations (2.5) on the other hand, total industry outputs are shown as functions of output for final use.

The usual Leontief model is of the type (2.5). Nevertheless, in the specification of the a_{ij}, the requirements of the traditional production function, (2.4) or (2.6), are of importance. The only universal requirement is that it exhibits diminishing returns when any of its inputs are varied alone.[2] In addition to this requirement, the quality of linear homogeneity is usually attributed to production functions. This means that all terms in the function are of the same (first) order. In general,

$$F_j(\lambda x_{1j},\lambda x_{2j},\ldots,\lambda x_{Nj},\lambda V_j) = \lambda^r F_j(x_{1j},x_{2j},\ldots,x_{Nj},V_j) \quad (2.7)$$

for homogeneity, with $r = 1$ for linear homogeneity. Linear homogeneity thus implies that if the input quantities are all doubled, the quantity of output will double; that is, it implies constant returns to scale. This assumption is made for the sake of both simplicity and plausibility. Linear homogeneous functions are relatively easy to work with mathematically. Moreover, their implication of constant returns to scale is acceptable.[3]

2. In terms of equation (2.6) this means that (for continuous production functions) $\frac{\partial^2 X_k}{\partial x_{ik}^2} < 0 \quad (i = 1,\ldots,N)$.

3. This is discussed in section 2.b of this chapter.

Of all linear homogeneous production functions, that involving constant input coefficients is the simplest, and this is Leontief's critical assumption. In systems (2.4) and (2.5) the a_{ij} are taken as constant and the set of general production functions, (2.6), becomes

$$X_i = \text{minimum} \left(\frac{x_{1j}}{a_{1j}}, \frac{x_{2j}}{a_{2j}}, \ldots, \frac{x_{Nj}}{a_{Nj}}, \frac{V_j}{v_j}\right) \quad (j = 1,\ldots,N) \quad (2.8)$$

Here $v_j = \frac{V_j}{X_j}$ is a constant input coefficient for the $N + 1^{th}$ sector. It will be noted that since the functions are discontinuous, the condition of diminishing returns cannot be stated in its usual form. Instead a stronger condition holds. When the level of any input, x_{ij}, is increased so that the ratio $\frac{x_{ij}}{a_{ij}}$ is greater than the minimum of the other input ratios, say $\frac{x_{kj}}{a_{kj}}$, then the input of product i represented by $\frac{x_{ij}}{a_{ij}} - \frac{x_{kj}}{a_{kj}}$ produces no increase in output. With the Leontief assumption of constant a_{ij}, system (2.5) becomes a set of N linear equations in N unknowns. It may now be written as:

$$X_i - a_{i1}X_1 - a_{i2}X_2 - \ldots - a_{ii}X_i - \ldots - a_{iN}X_N = Y_i \quad (i = 1,\ldots,N)$$

or

$$-a_{i1}X_1 - a_{i2}X_2 - \ldots + (1 - a_{ii})X_i - \ldots - a_{iN}X_N = Y_i \quad (i = 1,\ldots,N) \quad (2.9)$$

The N × N array of the coefficients of this system is called the direct requirements matrix . Now, providing the equations are <u>independent</u> (no equation is a linear combination of the others),[4] the X_i can be simultaneously determined in terms of the Y_i.

To complete the derivation of this structural model of the economy it only remains to be shown that the resulting industry output levels will be meaningful. Specifically, any set of non-negative final demands must generate a set of non-negative industry outputs. It has been demonstrated by D. Hawkins and H.A. Simon that a set of conditions on the production coefficients are necessary and sufficient to ensure this result.[5] The Hawkins-Simon conditions are: $|a| > 0$ where the $|a|$ are all the principle minors of the array of coefficients in system (2.9). In the extreme case of single element minors, the requirement is that $1 - a_{ii} > 0$. These conditions require that in all industries together and in every sub-group of industries, the production of a unit of each product, i, will require, directly and indirectly, less than a unit of i. In other words, all industries and groups of industries must be <u>self-sustaining</u>. If these conditions were not met the system would be unstable since increasing the final demand for, say, product i would result in a proportionally greater deficiency of it.

4. Failure to meet this condition would mean that the outputs (or inputs) of an industry were of the exact pattern of another industry or combination of industries. Since the industries in the model are each defined as producing a single homogeneous product, the industry in question would be redundant.

5. D. Hawkins and H.A. Simon, "Note: Some conditions of Macroeconomic Stability", <u>Econometrica</u>, XXX (1963), pp, 90-110.

That the Hawkins-Simon conditions are in fact met by the production coefficients is easily demonstrated. For example, consider a system of two sectors (or second order principal minor of larger system). The conditions require:

(i) $1 - a_{11} > 0$,

(ii) $1 - a_{22} > 0$, and

(iii) $$\begin{vmatrix} (1 - a_{11}) & - a_{12} \\ -a_{21} & (1 - a_{22}) \end{vmatrix} > 0$$

i.e., $(1 - a_{11})(1 - a_{22}) > a_{12}a_{21}$.

Now, except in the improbable cases where there is a negative value added or where all values added are zero, the sum of the elements in each column of the production coefficients matrix will be no greater than one, and at least one sum will be less than one. Therefore,

$$a_{11} + a_{21} < 1 \quad \text{or} \quad a_{21} < (1 - a_{11})$$

and

$$a_{12} + a_{22} \leq 1 \quad \text{or} \quad a_{12} \leq (1 - a_{22})$$

From these inequalities it can be seen by inspection that (i), (ii), and (iii) are satisfied. The demonstration may easily be extended to systems of more than two sectors.

This discussion indicates a final attribute of the Leontief assumption of constant input coefficients. Apart from yielding a simple model with plausible production functions, it yields a system for which a stable solution exists.

(c) The Total Requirements Matrix.

At this point it will be convenient to represent the system and solution in terms of matrices and to continue the discussion in this form. The system described in (2.9) may be written as:

$$X - aX = Y$$

or

$$(I - a)X = Y \qquad (2.10)$$

Here X and Y are N-element column vectors of the X_i and Y_i, and a is an $N \times N$ matrix of the a_{ij}. I is an N^{th} order identity matrix (that is, a square matrix with ones on the diagonal and zeros elsewhere).

If the equations of (2.9) or (2.10) are independent, $(I - a)$ is non-singular and its inverse, $(I - a)^{-1}$, exists. This means that a unique solution of industry output levels may be found for each Y. The solution is written:

$$X = (I - a)^{-1}Y \qquad (2.11)$$

and since the Hawkins-Simon conditions are met, $X \geq 0$.

The inverse, $(I - a)^{-1}$, is the total requirements matrix. Its typical element, b_{ij}, represents the total amount of product i required directly and indirectly in the production of a unit of final output of product j. Since the nature of the total requirements coefficients is key to the application of the model, it will be discussed in more detail.

The inverse may be described by means of the identity:

$$(I - a)^{-1} \equiv I + a + a^2 + a^3 + \ldots \quad (2.12)$$

[7]

When this expression is used for $(I - a)^{-1}$, the coefficients may be expressed as follows:

$$b_{ij}(i \neq j) \equiv a_{ij} + \sum_{k=1}^{N} a_{ik}a_{kj} + \sum_{\ell=1}^{N} \sum_{k=1}^{N} a_{ik}a_{k\ell}a_{\ell j} + \ldots$$

$$b_{ii} \equiv 1 + a_{ii} + \sum_{k=1}^{N} a_{ik}a_{ki} + \sum_{\ell=1}^{N} \sum_{k=1}^{N} a_{ik}a_{k\ell}a_{\ell i} + \ldots \quad (2.13)$$

A total requirements coefficient, b_{ij}, can therefore be represented as the sum of the direct requirements coefficient, a_{ij}, and a series of cross-product terms of diminishing importance. The latter represent indirect flows whose degree of circuitousness is defined by the number of terms in the cross-product. For example, the terms $a_{ik}\, a_{kj}$ $(k = 1,\ldots,N)$ describe the requirement of product i embodied in all the direct inputs to industry j.

As an example, consider the coefficients of three industries in the Canadian input-output matrices for 1949.[8]

7. The identity depends on the convergence of the series since, multiplying both sides by $(I-a)$ yields:
Left Side: $(I - a)(I - a)^{-1} = I$
Right Side: $(I - a)(I + a + a^2 + \ldots) = I + a + a^2 + a^3 + \ldots$
$- a - a^2 - a^3 - a^4 - \ldots$

8. See D.B.S. Publication No. 13-513, Supplement to the Inter-Industry Flow of Goods and Services, Canada, 1949, Ottawa: Queen's Printer, 1960, Tables 1, 2, and 3.

The industries chosen are: (2) Forestry, (24) Paper products, and (25) Printing, publishing and allied industries. The relevant inter-industry transactions, direct coefficients, and inverse coefficients are tabled below.

TABLE II

SELECTED INPUT-OUTPUT FLOWS AND COEFFICIENTS

Industries / Flows and Coefficients	2,24	24,25	2,25
x_{ij}	215.6	68.2	-
a_{ij}	.223512	.195808	-
b_{ij}	.226139	.198423	.045350

Several observations may be made from this table. The output of forestry products used directly by the paper industry is 215.6 million dollar's worth or about 22.35 % of the paper industry inputs. The direct output of forestry to the printing industry is negligible or zero. The total requirements coefficients $b_{2,24}$ and $b_{24,25}$ are only slightly greater than the corresponding direct coefficients (about 0.0026 in each case). Thus, little forestry output is used <u>indirectly</u> by the paper industry, and little paper used indirectly by printing. However, $b_{2,25}$ is substantially greater than $a_{2,25}$ so there is a significant indirect requirement for forestry products by printing. One would suspect that this indirect flow would occur largely through inputs of paper to printing. This suggestion is supported by the fact that $b_{2,25} = .045350$ is little greater than $a_{2,24} \cdot a_{24,25} = (.223512)(.195808) = .043765$.

This example illustrates the nature of the inverse, or total requirements, coefficients. However, because of its very simplicity it can not illustrate the usefulness of a simultaneous solution for industry output levels. It is in sectors, such as chemicals or metal products, which have a much greater diversity of inputs and outputs that the indirect flows become important. When complex industries such as these are examined, the advantages of the Leontief simultaneous solution over partial equilibrium analysis become apparent.

As well as these advantages, the input-output model has several important limitations. These are considered next.

2. Limitations of the Open Leontief Model

Weaknesses in the Input-Output model will be discussed under three headings: (a) the problem of industry classification, (b) theoretical implications of the assumption of constant coefficients, and (c) the neglect of induced changes in final demand.

(a) The problem of industry classification

The aim of industry classification is to produce sectors whose outputs are _effectively_ homogeneous. This is achieved by constructing the sectors from industries whose input coefficients are identical or whose output levels vary in exact proportion in response to any change in final demand. Only in these two cases are heterogeneous sector outputs consistent with constant input coefficients.

In the attempt to approach this ideal, industries are classified by establishment, the smallest business unit for which the necessary statistics are generally available. Thus establishments are mills, factories, etc. and there may be many establishments within a firm. The establishments combined to form a sector are chosen so that their input levels or output levels are likely to vary in proportion. As a result, sectors commonly consist of establishments that either produce commodities with similar uses, or handle a particular material at successive stages of production. The Electrical apparatus industry is an example of the former type; the Metal mining, smelting and refining industry, an example of the latter.

However, success in producing effectively homogeneous sector outputs is limited in the end by the frequent impossibility of isolating single commodities that correspond to single production processes. Not only do most establishments produce several dissimilar products, but some products are produced in two or more different industries. Important examples of the latter case are fertilizer, which is produced in both chemical fertilizer and metal mining and smelting establishments, and advertising, which is an output of publishing, radio and T.V., and business services establishments.

This problem can not be solved by aggregation techniques because of its double-edged nature. A finer classification lightens the problem of multi-product industries but aggravates the problem of products that belong to more than one sector.

The resulting restriction on the ability to define sectors with effectively homogeneous outputs limits the plausibility of constant production coefficients. Further limitations are discussed in the following section.

(b) Implications of the Constant Coefficients Assumption

The critical Leontief assumption entails three assertions that contradict traditional mocroeconomic theory: constant returns to scale for industries, no substitution, and no technological change. In the main, these implications must be accepted as failures of the model and attention centred on the extent to which they damage its predictive power.

Constant returns. The most direct assertion is that industry output will vary proportionally with the level of inputs, providing input composition is not varied. In opposition to this, traditional theory asserts that in many instances, the indivisibility of some of the factors of production will lead to increasing returns to scale.

In defense of constant returns it may be noted that the argument for increasing returns applies less at the industry level than at the level of the firm since it depends on the existence of particular, partially-utilized factors. Also, the factors usually described as relatively indivisable are plant and machinery, and technical and managerial skills, which are not included among the direct inputs of the Leontief model.[9] For these reasons, the implication of constant returns to scale is not considered to be a significant weakness of the model.

No substitution. The second assertion implied in the assumption of constant production coefficients is that the methods of production will not change in the face of changes in the relative prices of inputs. One of the basic tenets of microeconomic theory, on the other hand, is that rational producers will try to employ inputs to the levels where their marginal products equal their respective marginal costs. If this is true, changing relative prices must certainly result in substitution among inputs and hence, changing input coefficients. Since relative price changes occur in response to changing final demands - the very changes analysed by the Leontief model - the denial of substitution could be a serious limitation.

The soundest defense of the input-output model's neglect of substitution was suggested by Leontief when he first advanced the model. He argued that it is the magnitude of the effect, rather than the fact of substitution that is important.

9. This introduces a more serious problem regarding the usefulness of the production coefficients. It is discussed in section (c).

He said:

> Insofar as the proportions in which the separate factors can be combined within the same production function . . . are variable, these proportions will most probably vary with every change in their relative prices. This theoretical proposition . . . is beyond dispute. It is, however, not the fundamental validity of the principle of substitution but its quantitative significance which is important from the point of view of empirical analysis.[10]

His conclusion with regard to the effect of relative price changes on his input coefficients was that the resulting errors "lie within relatively narrow limits".[11] Leontief's empirical conclusion may be rationalized by arguing that in capital-intensive, technologically sophisticated economies such as those of Europe and North America, production methods leave very little room for variation in input proportions, at least in the short run. This argument, while sufficient to justify empirical application of the model, does not deny the fact that substitution in response to relative price changes is a possibility which may limit its predictive power.

No technological change. The third assertion inherent in the assumption of constant coefficients is that of an unchanging technology. It is the most obviously violated but, at the same time, the easiest to deal with. An innovation which changes the nature of industrial processes or encourages the use of different raw materials clearly invalidates

10. W.W. Leontief, The Structure of the American Economy, 1919-1939, Second edition, New York, Oxford University Press, 1950, p. 201.

11. Loc. cit.

production coefficients derived before the change. This implies that Leontief matrices naturally tend to become less useful as time passes. Since these matrices typically take six or seven years to produce, their deterioration in accuracy presents a serious problem. For example, when 1956 industry outputs were estimated using the input-output coefficients for 1949, a weighted-average error of about eight percent was found.[12]

A partial solution to the problem of technological change has been found in updating the input-output matrices. Two approaches to updating have been used. The first consists of incorporating known technological changes into the direct requirements matrix. If, for example, product k replaces product ℓ as an input to the j^{th} industry, the coefficients a_{kj} and $a_{\ell j}$ are altered to account for the change. This simple correction in the direct coefficients matrix will, of course, result in several, possibly many, changed coefficients in the inverse matrix. The second approach is used to improve the accuracy of input-output results without investigation of particular technological changes. Production coefficients are altered in such a way that the resulting equations are made consistent with independently estimated vectors of both final demand and industry outputs for the year in question. This method may be applied in several ways, ranging from the multiplication of each row of a by a proportionality constant,[13]

12. See T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer, "L'Ajustement Periodique des Systemes de Relations Inter-industrielles, Canada, 1949-1958," Econometrica, XXXI (1963), p.94.

13. Ibid., p.96.

to employing a linear programme to minimize the sum of changes in individual coefficients.[14] In addition, the second approach may be used in conjunction with the first when a few changes in technology are outstanding. Matuszewski, Sawyer and Pitts have demonstrated that these techniques are successful in materially reducing forecast errors. They compared the forecasts of original and updated matrices to estimated final demands and industry outputs for a third year close to the year to which the matrices are updated. They found, for example, that updating the 1949 Canadian matrix to conform to to 1956 data reduced the weighted-average error of predictions of 1958 industry outputs from 11.69 % to 5.46 %.[15]

It may be concluded that while technological change is certainly an important source of forecast error, its effects can be at least partly accounted for by updating the Leontief matrices.

(c) The neglect of induced changes in final demand.

The final set of limitations is only indirectly concerned with the nature of input coefficients. The *open* Leontief model is based on the assertion that a change in final demand will result in predictable changes in the levels of the industry outputs and primary inputs. No consideration

14. T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer, "Linear Programming Estimates of Changes in Input Coefficients", *Canadian Journal of Economics and Political Science*, XXX (1964), pp. 203-210.

15. T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer, "L'Ajustement Periodique des Systemes de Relations Interindustrielles, Canada 1949-1958", *Econometrica*, XXXI (1963), p. 93, 99.

is given to the possibility that secondary changes in final demand itself may also result.[16] In macro-economic theory, on the other hand, it is concluded that significant consumption and investment expenditures will be induced by a change in aggregate demand.

To begin with, a stable relationship has been demonstrated to exist between aggregate personal consumption expenditure and aggregate income. Thus an increment of aggregate income is expected to result in a certain smaller increment of aggregate consumption. Since any change in aggregate final demand is at the same time a change in aggregate income, it follows that such a change will result in successive increments of consumption expenditure. This is called the multiplier effect.

Similarly it has been concluded that an increase in final demand will result in additional investment expenditure. This conclusion is based on the assumption that a stable relationship exists between the level of final output and the stock of capital necessary to produce it. If the necessary capital stock depends on the level of final demand, then increases in capital stock, or investment, will be required by changes in final demand. This is the accelerator effect.

In failing to recognize multiplier and accelerator effects, the open Leontief model neglects induced changes in final demand.

16. This is true only of the open Leontief model. Closed models have been developed to include precisely these effects.

Other things being equal, the changes in industry output levels obtained as solutions of the model will be under-estimates.

At the same time, without induced investment, it could also happen that some or all of the solution outputs would not be feasible. This possibility places an obvious limitation on the increases in final demand that may be examined using the model.

To sum up, this chapter has introduced the Open Leontief or Input-Output model, an extension of which is developed and used in this paper. The model is built from the actual physical transactions of the production units of an economy for a particular interval of time. These transactions are first expressed in value units and classified as inputs or outputs of a workable number of industries. Then the assumption of constant input coefficients is introduced to transform the descriptive scheme into a simple mathematical model that relates industry output levels to the level and pattern of final demands. The assumptions necessary to build the model in this form were seen, in several instances, to be imperfectly attained or to involve contradictions with expected economic behaviour. The effect of these problems on predictions obtained from the model were suggested. In the case of the model's denial of technological change, methods of improving the predictions were indicated.

The following chapter discusses the extension of the input-output model to analyse the structural effects of foreign trade.

CHAPTER III

INPUT-OUTPUT ANALYSIS AND FOREIGN TRADE

Foreign trade in goods and services plays an important part in determining the level and pattern of production in most countries. Accordingly, commodity trade is generally included in the Leontief model, and the effects of changes in trade patterns are often the subject of input-output analysis. This chapter first discusses the treatment of trade in the simple Leontief model and then the extension of the model to consider more than one country or region.

1. Analysis of Foreign Trade with the Simple Leontief Model.

In the simple input-output system, exports are considered as a category of final demand and therefore autonomous. Imports, on the other hand, may be treated in a variety of ways. If import levels are not desired as results of the model, imports may simply be classified as negative elements of final demand. Alternatively, they may be treated as inputs with constant input coefficients. In this case import levels are determined in the solution of the domestic activity levels.

In the latter case, where import levels are explained, there are two principal methods of defining the model. Since both of these will be observed later in the paper, it will be useful to outline them here.

First, two classifications of imported commodities must be defined. Competitive imports are those for which there is an equivalent commodity produced domestically. Non-competitive imports are those, like tropical foods in Canada, for which there is no domestic equivalent. In both models, non-competitive imports are distributed to using industries and final demand. They may be shown either as a row or matrix of inputs. In the latter case the rows of the import matrix identify their industries of origin. In the treatment of competitive imports, however, the two models differ.

In Model I, competitive imports are treated in the same way as non-competitive imports. The level of total competitive imports in each industry is related to the output of using industries by an N×N matrix of constant coefficients. The model is

$$X_i - \sum_{j=1}^{N} a_{ij} X_j = Y_i \qquad (i = 1,\ldots,N) \qquad (3.1)$$

$$M_i - \sum_{j=1}^{N} m_{ij} X_j = Y_i^M$$

where: X_i = the total domestic output of product i ,

Y_i = the final demand for domestically produced i ,

a_{ij}= the amount of domestically produced i required in the production of a unit of product j ,

M_i = the import level of product i ,

Y_i^M = the final demand for imports of product i and

m_{ij}= the amount of product i required in the domestic production of a unit of product j .

The m_{ij} are assumed constant. In matrix form, the systems are

$$\begin{aligned} (I - a)X &= Y \\ M - mX &= Y^M \end{aligned} \qquad (3.2)$$

where X,Y,M, and Y^M are N element column vectors whose typical elements are X_i , Y_i , M_i , and Y_i^M , and a and m are N^{th} order square matrices with typical elements a_{ij} and m_{ij} . Using the result shown in (2.12), the following solutions are obtained

$$\begin{aligned} X &= (I - a)^{-1}Y \\ M &= m(I - a)^{-1} + Y^M \end{aligned} \qquad (3.3)$$

In Model II, each competitive import is considered as an input to the domestic industry which produces the same product. Each of the basic equations of the model describes the distribution of the total supply (domestic and imported) of a product. Competitive import levels are determined by constant coefficients relating them to the total supply of each of the commodities. Using this method it is unnecessary to consider the imports of any product as being distinct from the domestic product.
At the same time, the ability to identify the amount of any imported product used by a particular industry is sacrificed.[1]

1. The distribution of imported products is available only under the artificial assumption that they are demanded by users in the same proportions as their domestic counterparts.

The equations of the model are

$$\hat{X}_i - \sum_{j=1}^{N} \hat{a}_{ij} \hat{X}_j = \hat{Y}_i \qquad (i = 1,\ldots,N) \qquad (3.4)$$
$$M_i = \bar{m}_i \hat{X}_i$$

where $\hat{X}_i = X_i + M_i$, the <u>total supply</u> of product i ,

$\hat{Y}_i = Y_i + Y_i^M$,

$\hat{a}_{ij} = (a_{ij} + m_{ij})$, known only in total, and

$\bar{m}_i = M_i/\hat{X}_i$, the share, assumed constant, of the total supply of product i accounted for by imports.

In matrix form the systems are

$$(I - \hat{a})\hat{X} = \hat{Y} \qquad (3.5)$$
$$M = \bar{m}\hat{X}$$

in which $\hat{X}$, $\hat{Y}$, and M are N-element column vectors, $\hat{a}$ is an N^{th} order square matrix of the $\hat{a}_{ij}$, and $\bar{m}$ is an N^{th} order diagonal matrix of the $\bar{m}_i$. The solutions are

$$\hat{X} = (I - \hat{a})^{-1} \hat{Y} \qquad (3.6)$$
$$M = \bar{m}(I - \hat{a})^{-1} \hat{Y}$$

In both of these methods of treating competitive imports, non-technical assertions are implicit in the assumption of constant input-output coefficients. In Model I, the assumption of constant production coefficients requires that the imported input of product i used by industry j be a constant proportion of industry j's total requirements of product i .

If M_{ij} is the flow of imported product i to industry j , then M_{ij}/X_{ij} must be constant. In Model II, on the other hand, the total imports of product i , M_i, must maintain a constant proportion of the total supply of i , $\hat{X}_i$.[2] Both these implications are unfortunate because the argument against input substitution is a technological one that does not apply to any question of the share of a market held by various suppliers.

Whether or not one of the methods is superior to the other has not been demonstrated conclusively. Backcast tests made with Canadian data showed that Model I yielded slightly more

2. A purely technical coefficient involving imports would show the total supply product i required for the domestic production of a unit of j . Thus,

$$\tilde{a}_{ij} = \frac{x_{ij} + M_{ij}}{X_j} .$$

The input-output coefficients of Model I and Model II may be related to the technical coefficients, $\tilde{a}_{ij}$, as follows:

Model I $\quad a_{ij} = \frac{x_{ij}}{X_i} = \tilde{a}_{ij} \cdot \frac{1}{1+\frac{M_{ij}}{x_{ij}}}$

Model II $\quad \hat{a}_{ij} = \frac{x_{ij} + M_{ij}}{X_j + M_j} = \tilde{a}_{ij} \cdot \frac{1}{1 + \frac{M_j}{X_j}}$

For further discussion see: C. P. Modlin and G. Rosenbluth, "The Treatment of Foreign and Domestic Trade and Transportation Charges in the Leontief Input-Output Table", Economic Activity Analysis (ed. O. Morganstern), New York: John Wiley and Sons, Inc., 1954. and T. I. Matuszewski, P.R. Pitts, and J. A. Sawyer, "Alternative Treatments of Imports in Input-Output Models - A Canadian Study," Journal of the Royal Statistical Society Series A, CXXVI (1963), pp. 410-432.

accurate predictions than Model II.[3] More detailed import statistics are required in Model I than in Model II so no strong preference can be indicated *a priori*.

With a simple Leontief model that incorporates foreign trade flows using an appropriate method, several questions may be investigated. For example, the effects of different levels and patterns of exports on industrial output may be examined. Again, using either of Model I or II, the *import content* of various categories of final demand may be estimated.[4] As well as having implications regarding the level and structure of domestic production activity, this question is of interest in investigating balance of payments determinants.

2. Extension of the Simple Leontief Model

The next logical step in input-output trade analysis is to try and *explain* the level and composition of the export vector. Since exports are imports of other counties, they can be related to foreign activity levels and foreign final

3. T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer, "Alternative Treatments of Imports in Input-Output Models: A Canadian Study", *Journal of the Royal Statistical Society*, Series A, CXXVI (1963), p. 425.

4. For discussion and examples of this type of analysis, see: (i) R. E. Caves, "The Inter-Industry Structure of the Canadian Economy", *Canadian Journal of Economics and Political Science*, XXIII (1957), pp. 313-330. (ii) T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer, "The Impact of Foreign Trade on Canadian Industries, 1956", *Canadian Journal of Economics and Political Science*, XXXI (1965), pp. 206-221.

demands by the methods of section 1.[5] This procedure allows changes in domestic activity levels to be related to changes in final demand in foreign countries. Such a model may therefore be used to examine the international transmission of business cycles.

This question has traditionally been investigated by aggregative analysis based on foreign trade multipliers. In such analysis, the aggregate level of imports is related to national income by an import function which recognises that the change in imports produced by a change in income may vary according to the initial income level, but does not recognise that the induced change in imports will vary with the composition of the income change. That is, a country's marginal propensity to import is assumed constant for any given level of income. Exports are either taken as autononous or related to the national income of another country (where the other country usually represents an amalgamation of all the first country's trading partners). The equilibrium national income of the first country is then determined by the necessary equality of exports and imports, if no capital imports or exports are allowed, or by the equality of domestic investment

5. The explanation of other autonomous vectors of final demand is also a natural step and results in a fully or partially closed model. However, such models are seldom used in empirical studies because the necessary assumptions of unchanging patterns of consumption and investment expenditures are less tenable than the assumptions of constant production or import coefficients.

plus exports and savings plus imports.[6] The comparative advantages and disadvantages of the input-output approach to the transmission of business cycles parallel those of simple Leontief analysis as compared to analysis of domestic aggregates. The input-output approach admits that the relationship of changes in import levels to changes in domestic final demand depends in part on the composition of the final demand change. On the other hand, it neglects the secondary effects of changes in income.

Several input-output models have been developed that may be used to analyse the transmission of national income changes through international trade. Three of these will be introduced here.

(a) Leontief's Interregional Analysis

Leontief's interregional input-output model is one example of such a model.[7] While it is described in terms of regions within a national economy, there are no theoretical objections to considering the regions as national economies and the overall system as the description of a world or trading bloc economy.

The basis of the model is a division of the sector outputs into regional and national goods. The former are products,

6. For a more detailed discussion of foreign trade multiplier models see:
C. P. Kindleberger, International Economics, Revised Edition, Homewood: R. D. Irwin, 1958, Chapter 10.

7. W. W. Leontief et al., Studies in the Structure of the American Economy. New York: Oxford University Press, 1953, pp. 93-115.

such as minerals and agricultural products, which are consumed in the region where they are produced. The latter are products which are traded among regions. These are predominantly durable, manufactured goods.

The structure of the <u>nation</u> and the regions is determined by: (1) the final demands for each product in each region, (2) a matrix of national production coefficients, which is assumed to apply to each region as well, and (3) a set of coefficients determining the proportion of the output of each interregional good produced in each region.

In Leontief's notation, m industries are defined with $\ell = 1,\ldots,h$ representing the regional ones and $g = h+1,\ldots,m$ representing the <u>national</u> ones. Next, n regions are defined with regional outputs identified by a prefixed subscript $j = 1,\ldots,n$. Thus for the m <u>national</u> outputs there correspond nm regional outputs ${}_jX_i$. Similarly, there are nm final demands, ${}_jY_i$.

The structural parameters are:

$$a_{ik} = \frac{x_{ik}}{X_k} \qquad (k = 1,\ldots,m) \text{ , and}$$

$$_jr_g = \frac{_jX_g}{X_g} \qquad (g = h+1,\ldots,m \text{ ; } j = 1,\ldots,n) \text{ .}$$

The activity levels are determined in three stages. First, the <u>national</u> output of each product is found by

$$X_i = \sum_{k=1}^{m} b_{ik} Y_k \quad (i = 1,\ldots,m). \tag{3.7}$$

where the b_{ik} are the total requirements coefficients - the elements of $(I - a)^{-1}$. Now, given X_i, the regional outputs of the national products are determined. Thus,

$$_jX_g = {_jr_g}\, X_g \qquad (g = h+1,\ldots,m \,;\; j = 1,\ldots,n) \quad (3.8)$$

Finally, the regional outputs of regional goods are determined using sub-matrices of both the direct requirements matrix a, and the total requirements matrix, $(I - a)^{-1}$. Thus,

$$_jX_\ell = \sum_{g=h+1}^{m} a_{\ell g} \cdot {_jX_g} + \sum_{k=1}^{h} b_{\ell k} \cdot {_jY_k} \quad (l = 1,\ldots,h;\; j = 1,\ldots,n) \quad (3.9)$$

Since the national final demands are totals of the regional final demands and since the national and regional input-output matrices are identical, the regional industry outputs of local and traded goods will be consistent.

The strongest point of Leontief's model is that it encompasses the production of a complete set of regions or nations. It takes another step towards the ideal general equilibrium model in recognizing interdependency in the output levels of all the regions in a trading group. In addition, once the regional final demand vectors are estimated and the class of traded goods defined, it is a relatively easy model to deal with. However, there are some drawbacks to the model which are particularly serious in the context of production for international trade. The application of a single classification of traded and local products to all regions is an oversimplification which should result in an underestimation of the amount of interregional trade. Second, there is only a partial explanation of interregional trade flows. The regional pattern of production for trade is determined by fixed supply patterns, and there is no means of predicting trade between particular regions. In a more realistic model, the quantities of traded goods supplied by any region would depend on the regional distribution of demand. Third, and perhaps the most important limitation of the model, is the uniform technology attributed to the nation and each of its regions. When input-output coefficients are estimated for each region individually, more realistic relations between regional final demand vectors and their input and import requirements may be derived. Like the others, this limitation would be especially serious if the model were used to investigate the effects on national outputs of international trade.

(b) The Interregional Models of Isard and Moses

In this section models designed by Walter Isard[8] and Leon N. Moses[9] are introduced. They are discussed together because they are essentially the same model. They differ only in the procedure used in deriving regional import coefficients. Both models follow Leontief's interregional model in using a uniform production matrix for all regions. Thus they are better

8. W. Isard, "Interregional and Regional Input-Output Analysis: A Model of a Space Economy," Review of Economics and Statistics, XXXIII, 1951, pp. 318 - 328.

9. L. N. Moses, "The Stability of Interregional Trading Patterns and Input-Output Analysis," The American Economic Review, XLV, 1955, pp. 803 - 832.

suited for intranational than international analysis. On the other hand, they differ from the Leontief model by relating the exports of each region to the import requirements of the other individual regions. For instance, instead of assuming that region one supplies sixty percent of the total iron and steel requirements of the nation, these models might assume that region one supplies fifty percent of region two's iron and steel requirements, eighty percent of region three's requirements, etc. The trade parameters are still rigid but they incorporate a locational factor and are therefore more realistic than Leontief's supply coefficients. Moreover, it is unnecessary in these models to define classes of traded and local goods.

The model is developed using the Isard procedure for treating regional imports. Afterwards the Moses variant will be discussed with reference to the same equations.

Consider a system of R regions, each with N industries.

Let: a_{ij} = the amount of domestically produced i required in the production of a unit of j (These parameters apply to all regions.),

X_i^k = the output of industry i in the k^{th} region, and,

Y_i^k = the final demand for the output of i produced in the k^{th} region.

First, the final demand of each region is partitioned into exports to the other regions and a residual. The residual includes any exports to regions outside the system.

Thus $$Y_i^k = \sum_{\ell=1}^{R} E_i^{k\ell} + F_i^k \ . \qquad (3.10)$$

where $E_i^{k\ell}$ represents the exports of product i to region ℓ, and F_i^k represents all other final demands for the output of i produced in region k. Next, the interregional export demands are related to production activity in the importing regions. In the Isard model, import coefficients, $S_{ij}^{k\ell}$, are defined by

$$E_i^{k\ell} = \sum_{j=1}^{N} S_{ij}^{k\ell} X_j^{\ell} + f^{k\ell} \qquad (3.11)$$

Thus, $S_{ij}^{k\ell}$ describes the amount of product i imported from region k that is required in the production of a unit of product j in region ℓ. Note that S_{ij}^{kk} will be zero. Similarly, $f^{k\ell}$ describes the amount of i, imported from region k, that is demanded directly by final users in region ℓ. For simplicity it is considered to be incorporated in F_i^k in (3.10). With these definitions, the distribution of the regional industry outputs may be expressed in the following set of equations.

$$X_i^k = \sum_{j=1}^{N} a_{ij} X_j^k + \sum_{\ell=1}^{R} \sum_{j=1}^{N} S_{ij}^{k\ell} X_j^{\ell} + F_i^k \quad (i = 1,\ldots,N; \quad k = 1,\ldots,R) \qquad (3.12)$$

Let: a = the N^{th} order matrix of the a_{ij}.

$S^{k\ell}$ = the N^{th} order matrix of the coefficients, that describe requirements of region k's products in region ℓ.

X^k = the N- element column vector of the X_i^k, and

F^k = the N- element column vector of the F_i^k.

Then, in matrix form, (3.12) is

$$X^k = aX^k + S^{k1}X^1 + \ldots + S^{kR}X^R + F^k \quad (k = 1,\ldots,R)$$

or

$$(I-a)X^k - S^{k1}X^1 - \ldots - S^{kR}X^R = F^k \quad (k = 1,\ldots,R) \qquad (3.13)$$

Taking $R = 3$ as an example, (3.13) may be written

$$\begin{bmatrix} (I-a) & -S^{12} & -S^{13} \\ -S^{21} & (I-a) & -S^{23} \\ -S^{31} & -S^{32} & (I-a) \end{bmatrix} \begin{bmatrix} X^1 \\ X^2 \\ X^3 \end{bmatrix} = \begin{bmatrix} F^1 \\ F^2 \\ F^3 \end{bmatrix}$$

Let L be the parameter matrix, which in general will have N^2R^2 elements, and X and F, column vectors with subvectors X^k and F^k.

Then $$LX = F \qquad (3.14)$$

Since L is square and non-singular its inverse exists. Therefore the industry outputs in each region may be written as linear functions of the regional final demands by

$$X = L^{-1}F \qquad (3.15)$$

Finally, as long as value added (excluding imports) is still non-negative for each regional industry, the Hawkins-Simon conditions will be met. For $F \geq 0$, the X_i^k will be positive.

The model developed by Leon Moses differs only in the derivation of the import coefficients, $S_{ij}^{k\ell}$. In Isard's model, they are derived under the assumption that the

i^{th} output of each region is distinct from the corresponding outputs of the other regions. That is, the technique described in Model I is used in relating import levels to levels of domestic output. In contrast, the $S_{ij}^{k\ell}$ in Moses' system involve an assumption that is very similar to that of Model II.[10] Imports are considered to be substitutable for domestic products and are distributed among using industries and final demand in the same proportions as the domestic outputs. Equation (3.11) becomes

$$E_i^{k\ell} = m_i^{k\ell} X_i^{\ell} = m_i^{k\ell} a_{ij} X_j^{\ell} + m_i^{k\ell} F_i^{\ell} \qquad (3.16)$$

Here the $m_i^{k\ell}$ are coefficients relating the imports of i from k to the domestic outputs of i in region ℓ.

Thus $S_{ij}^{k\ell} = m_i^{k\ell} a_{ij}$

in the Moses model. The $m_i^{k\ell} F_i^{\ell}$ are equivalent to the $f^{k\ell}$ in (3.11). An advantage of this model over the Isard model is that the $m_i^{k\ell}$ may be estimated with greater ease and accuracy than Isard's $S_{ij}^{k\ell}$.

The algebra of the Isard and Moses models (systems (3.13), (3.14), and (3.15)) is basic to the Wonnacott model discussed

10. In Model II imports are related to the total supply rather than the domestic supply of competing industry outputs. If this procedure were used in an Isard-Moses model the resulting predictions would be identical to those obtained in the Moses variant. To use the procedure of Model II, the X_i^k and a_{ij} would have to be redefined in terms of total supply. The $S^{k\ell}$ matrices would become diagonal matrices of coefficients relating the imports of any commodity to its total supply in the importing region.

next and to the model actually used in this paper. These models differ from the Isard-Moses model only in their methods of defining the trade and production matrices.

(c) The Wonnacott Model.

The third interregional model to be discussed was developed by R. J. Wonnacott.[11] Its main departure from the Isard-Moses model is that it incorporates different production matrices for each region.[12] For this reason it is a superior model with which to investigate the interdependence of regional outputs at an international level.

The model may be expressed by the equations (3.13) if the $(I - a)$ sub-matrices are changed to $(I - a^k)$ $(k = 1,\ldots,R)$. The $S_{ij}^{k\ell}$ in Wonnacott's model are derived using Moses' assumption that $S_{ij}^{k\ell} = m_i^{k\ell}\, a_{ij}^{\ell} + m_i^{k\ell}\, F_i^{\ell}$.

Wonnacott developed his model in terms of two regions, Canada and the United States. To represent the Canadian and American technologies in matrices that would fit into the Isard-Moses system, he aggregated the input-output matrices of each of the countries so as to make their sector definitions conform as closely as possible. Starting with a forty-two sector

11. R. J. Wonnacott, Canadian-American Dependence: An Inter-industry Analysis of Production and Prices, Amsterdam: The North-Holland Publishing Company, 1961.

12. Wonnacott also developed a method of handling capacitated industries in his model. See Chapter IV of his book.

Canadian input-output table and a four hundred and fifty sector United States table, he produced separate thirty-five sector tables. Thus while the Wonnacott model is very similar to an Isard-Moses model, the statistical procedure necessary to apply it is quite different.

The three interregional models introduced in this chapter have the common purpose of yielding simultaneous solutions for regional industry output levels as functions of regional final demands. They all recognize the interdependency of industry output levels in any group of regions which trade together. In Leontief's interregional model, the outputs of a region depend on its final demands and the aggregate final demands of the system of regions. In the Isard-Moses and Wonnacott models, a region's outputs depend on the final demands of each individual region.

As well as the basic Leontief assumption of constant production coefficients, the models assume fixed supply coefficients for regional imports. Depending on the nature of the actual regions under examination, the latter assumption is likely to be less realistic than the former. Finally, the interregional models parallel the simple Leontief model in their neglect of induced changes in the Investment and Personal consumption components of final demand.

The model to be used in this paper is elaborated in the next chapter. It is a variant of Wonnacott's model and therefore shares most of the strengths and weaknesses of the models described above.

CHAPTER IV

THE EXPORT RECLASSIFICATION MODEL

The aim of this paper is to investigate the dependence of Canadian industry outputs on American final demand. For the investigation a model is developed that is similar to the Wonnacott model. However, it is applied in a different way and has some simplifying restrictions placed on it. Section (1) of this chapter describes the model in its general form, and section (2) details the restrictions that are introduced to facilitate its empirical application.

1. The Export Reclassification Model

The purpose of the model is to relate the industry outputs of regions with separate input-output tables, without having to reconstruct those tables according to a uniform classification system. The method used is to build trade matrices that relate the exports of one region, classified according to its industry scheme, to the industry inputs of another, classified differently. If the production matrix of region k has M sectors and that of region ℓ has N, then the trade matrix $S^{k\ell}$ will have M rows and N columns.

When the production and trade matrices are arrayed in the manner of the Isard-Moses and Wonnacott systems (see the example following (3.13)), it is found that the coefficient matrix, L , is square. Moreover, L will normally be found to be non-singular and to obey the Hawkins-Simon conditions so the system will yield a positive solution vector of regional industry outputs for positive vectors of final demand.

This model has a distinct advantage over Wonnacott's model in that the task of applying it to actual data is easier. Greater accuracy is also to be expected since trade data are generally found in more detail than is available in input-output tables or the working papers used in building them. Clearly the advantages of the export reclassification model will be particularly noticable for systems involving three or more regions.

Before restating the model in a restricted form it will be convenient to express it in a slightly simpler notation. This is possible because only two regions are involved in the application. The notation defined here will be used throughout the rest of the paper.

Let: M = the number of sectors in the Canadian input-output table,

N = the number of sectors in the United States input-output table,

x = the vector of Canadian activity levels,

X = the vector of United States activity levels,

e = the vector of Canadian exports to the United States,

f = the vector of all other final demands for outputs produced in Canada,

y = e + f = the total final demand for Canadian outputs,

E = the vector of American exports to Canada,

F = the vector of all other final demands for products of United States industries (As in the Isard-Moses and Wonnacott models F will include any exports to Canada from the United States that are allocated directly to final demand. In this model, it will be assumed that there are no such exports. This assumption is made for the demand for Canadian outputs, f , as well),

Y = E + F = the total final demand for outputs of United States industries,

a = the M^{th} order matrix of Canadian input-output coefficients,

A = the corresponding N^{th} order input-output matrix for the United States,

J = the $M \times N$ matrix describing the pattern of Canadian exports required by unit activity levels of United States industries, and

K = the $N \times M$ matrix describing the pattern of United States exports required for unit output levels of Canadian industries.

With these definitions, (3.13) becomes

$$\begin{bmatrix} (I - a) & -J \\ -k & (I - A) \end{bmatrix} \begin{bmatrix} x \\ X \end{bmatrix} = \begin{bmatrix} f \\ F \end{bmatrix} \tag{4.1}$$

The solution ($X = L^{-1}F$ in the notation of chapter III) is

$$\begin{bmatrix} x \\ X \end{bmatrix} = \begin{bmatrix} (I - a) & -J \\ -K & (I - A) \end{bmatrix}^{-1} \begin{bmatrix} f \\ F \end{bmatrix} \tag{4.2}$$

2. A Restricted Form of the Model

In the application of this paper, only the solution values of the Canadian activity levels, x , are of interest. So that they may be isolated, the inverse of (4.2) is expressed in terms of its sub-matrices.[1] In this form, (4.2) is

$$\begin{bmatrix} x \\ X \end{bmatrix} = \begin{bmatrix} [(I-a)-J(I-A)^{-1}K]^{-1} & [(I-a)-J(I-A)^{-1}K]^{-1}J(I-A)^{-1} \\ [(I-A)-K(I-a)^{-1}J]^{-1}K(I-a)^{-1} & [(I-A)-K(I-a)^{-1}J]^{-1} \end{bmatrix} \begin{bmatrix} f \\ F \end{bmatrix} \tag{4.3}$$

Now only the first system of equations need be considered. The Canadian activity levels are expressed as functions of Canadian and American final demands by

$$x = [(I-a)-J(I-A)^{-1}K]^{-1}f + [(I-a)-J(I-A)^{-1}K]^{-1}J(I-A)^{-1}F \tag{4.4}$$

Such a solution recognizes that a change in the level or pattern of the final demand of either country has not only a direct effect on that country's activity levels, but an infinite chain of indirect effects on the activity levels of both countries.

In order to reduce the data requirements in the empirical application of the model, it is assumed that United States activity levels are not responsive to changes in Canadian industry output levels. That is, the effects on United States

---

1. If $\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} (I - a) & -J \\ -K & (I - A) \end{bmatrix}^{-1}$

then $\begin{bmatrix} (I - a) & -J \\ -K & (I - A) \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$

where the I are identify matrices of order M and order N respectively. From this system the B_{ij} may be easily obtained.

industry outputs of induced changes in Canadian imports are neglected. This assumption is incorporated in the model by postulating $K = 0$. System (4.4) may now be written

$$x = (I - a)^{-1}f + (I - a)^{-1}J\ (I - A)^{-1}F\ .$$

Furthermore, no changes will be postulated for the vector of Canadian final demands, f . Accordingly, the model in the restricted form desired is

$$x = (I - a)^{-1}J\ (I - A)^{-1}F \qquad (4.5)$$

It might be argued that the neglect of the indirect effects operating through Canadian imports is a serious omission. However, the effects omitted are less direct and therefore less likely to be significant than effects produced by the stimulation of third country activity levels.[2] A greater increase in accuracy would probably be obtained by generalizing the restricted model to more regions than by incorporating the indirect effects generated by Canadian imports.

The model found in (4.5) is stated in very general terms. In particular, the nature of J is largely unspecified. For

2. The restricted form of the export reclassification can be simply altered to include such effects.

Let: α = the third country's production matrix, of order P ,
G = the $M \times P$ trade matrix that relates Canadian exports to the third country to the latter's activity levels, and
H = the $P \times N$ trade matrix that relates third-country exports to United States activity levels.

Then $x = (I-a)^{-1}[J + G(I-\alpha)^{-1}H]\ (I-A)^{-1}F$

example, in the present formulation, imports could be treated according to either of Model I, Model II, or the Moses variant of Model II. The precise definition of the trade matrix, J , will depend largely on the nature of the input-output tables of the two economies. Therefore, it will be discussed in the next chapter which examines the data and procedure used in constructing the empirical model.

CHAPTER V

APPLICATION OF THE MODEL: DATA AND PROCEDURE

In this chapter, the sources and nature of the data are first examined. Then the application of the model to this data is described, and finally, the procedure followed in estimating the parameters and obtaining solutions is discussed.

1. The Data

Three basic groups of data are discussed here: the American and Canadian input-output tables, and Canadian exports to the United States.

(a) The United States Input-Output Tables

The United States input-output tables used in this paper are found on page 33 of the September 1965 issue of Survey of Current Business, a publication of the United States Department of Commerce, Office of Business Economics.[1]

The information contained in this reference is based on data for the year 1958 and consists of:

1. These tables - with imports treated not as primary inputs but as negative elements of final demand - also appear in an article by W. W. Leontief in the April 1965 issue of Scientific American.

(i) a transactions table constructed to agree with National Accounts data,

(ii) a production coefficients matrix,

(iii) a total requirements, or inverse, matrix, and

(iv) a table defining the production sectors of the preceding tables by reference to codes found in the 1957 edition of the United States Standard Industrial Classification Manual.

The transactions table and direct and total requirements matrices are constructed using eighty-two producing sectors. There are also dummy sectors (namely: (83) Scrap, Used and Secondhand Goods, (84) Government Industry, (85) Rest of the World Industry, (86) Household Industry, and (87) Inventory Valuation Adjustment), and a Value added row. The principal use of the dummy sectors is to record payments for services shown in the National Accounts as final demand which do not belong to any of the eighty-two industries. Examples are payments to domestic servants and, most important, payments to civil servants at all levels of government.

Final demand in the transactions table, is shown as the sum of six component vectors. These are:

(i) Personal consumption expenditures,

(ii) Gross private fixed capital formation,

(iii) Net inventory change,

(iv) Net exports,

(v) Federal Government purchases, and

(vi) State and local government purchases.

The flows shown in the transactions table are valued at producers' prices. This means that trade and transport margins are excluded from the sales of any sector. These margins are shown as payments by the purchasing sector directly to the trade and transport sector. Indirect taxes less subsidies are also excluded from sector outputs. They are included as primary inputs in the value added row.

Imports are shown as a productive sector - industry 80. Since imports are actually a primary input, industry 80 has outputs to all other sectors and final demand, but no inputs from other sectors. The output of the import industry is an aggregation of competitive and non-competitive imports which are treated in different ways.

Non-competitive imports are shown as inputs to using industries and final demand. No breakdown by originating industry is published so the input of non-competitive imports into any industry will typically be an aggregation of imports from several, unidentified foreign industries.

Competitive imports are allocated according to Model II in Chapter III. They are transferred to import-competing industries and there are no inputs of competitive imports routed directly to final demand. This means that the typical activity level, X_i, describes the total supply of output i in the United States. Similarly, the production coefficients A_{ij} in matrix $A = [A_{ij}]$ describe the direct requirements of domestic and imported product i per unit of total supply of product j. To emphasize this the United States input-output equations will be rewritten using the notation for total supply introduced in Chapter III. Thus, the system is written $\hat{X} = (I - \hat{A})^{-1}F$.

(b) The Canadian Input-Output Tables

The basic source for information on the nature of the Canadian input-output tables is Dominion Bureau of Statistics publication number 13-513, Supplement to the Inter-industry Flow of Goods and Services, Canada 1949. It contains, using 1949 data:

(i) a transactions table integrated with the National Accounts,

(ii) a direct requirements matrix,

(iii) an inverse or total requirements matrix, and

(iv) a table defining the producing sectors in terms of the codes found in the 1948 edition of the Canadian Standard Industrial Classification Manual.

Forty-two sectors are defined in the Canadian tables. As in the United States tables, all flows are expressed in producers' prices.

An updated inverse and import requirements matrix are also available. They were prepared by T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer for the Carter Royal Commission on Taxation.[2]

2. A copy of these updated matrices, on IBM punch cards, was supplied by Dr. G. Rosenbluth.

The method of updating involved a combination of the two approaches suggested in section 2.b of Chapter II. It is elaborated in Appendix C.

The updated inverse matrix applies to the year 1959. The import requirements matrix is a product of the updated inverse and an import coefficients matrix, $\bar{m}$, which is updated to 1956.[3]

(c) Canadian Exports to the United States, 1958

The principal sources of information on Canadian exports to the United States were Trade of Canada volumes I and II, published annually by the Dominion Bureau of Statistics, (publication number 65-202). Also used was The Canadian Balance of International Payments (Dominion Bureau of Statistics publication number 67-201).

Exports are listed in commodity groups in fair detail in Trade of Canada, volume I, and in much greater detail in volume II.

2. Application of the Model to the Canadian and United States Data

The statistical application of the model involves the selection of a base year and the derivation of the trade coefficients, J_{ij} , in terms of the rest of the data.

3. The import coefficients matrix follows Model I in allocating all imports to using industries.

An input-output model provides an accurate description of the relation between industry outputs and final demand only in the period for which the production coefficients are estimated. For this reason it is important that the input-output systems used to represent the Canadian and American technologies should apply to years as close together as possible. For this application, 1958 was chosen as the base year, and the updated Canadian production coefficients (base year 1959) were assumed to describe Canadian technology in 1958.

The first and most important step in deriving the trade coefficients was to decide how United States imports from Canada should be related to United States activity levels. It was assumed that all Canadian exports are substitutable for American domestic outputs. Accordingly, the level of exports of each commodity was related to the total supply, $\hat{X}_j$, of the equivalent output in the United States. This method was chosen by reason of its simplicity, and because the similarity of geography and technology in the two countries makes the assumption of substitutable outputs plausible.

A final problem in the application of the model was posed by the units in which the Canadian commodity flows were expressed. The Canadian output requirements in the updated inverse matrix were expressed in dollar's worths at 1949 prices. In contrast, the exports to the United States used to derive the J_{ij} were valued at 1958 prices. Two methods were available by which to standardize these units; the inverse coefficients could be adjusted so that they would be expressed in 1958 dollar's worths, or the export values could be deflated to 1949 prices. The second method was chosen because it was felt that export price relatives would be more accurate than domestic price relatives. Domestic price levels tend to be obscured by intra-firm transfers of intermediate products at non-market prices.

In the light of these decisions, Canadian exports may be represented as functions of United States activity levels by

$$e_i = \sum_{j=1}^{82} J_{ij} \hat{X}_j \qquad (i = 1,\dots,42) \qquad (5.1)$$

where J_{ij} = the amount of Canadian output i, in 1949 Canadian dollars' worths, that is included in a 1958 United States dollars' worth of the total supply of United States output j.

The J_{ij} are assumed to be constant over changes in the $\hat{X}_j$ and over changes in the rate of exchange of Canadian and United States dollars. The latter assumption is of particular importance if, as in this paper, the analysis is extended beyond the base year of the model. The J_{ij} are defined in terms of physical flows (or dollars' worths at base year prices) so they are not directly affected by a changing exchange rate. However, it is quite possible that a change in exchange rate would precipate changes in the physical coefficients. An example might be shifts in the proportion of the total supply of a product accounted for by imports. This possibility is neglected in the model.

3. Estimation of the Parameters and Solution of the Model

The procedure followed in estimating the J_{ij} and obtaining solutions of the model may be divided into four steps.

First, the list of 1958 Canadian exports to the United States was classified according to both the Canadian and United States industry schemes. The aim was to produce an array of elements, τ_{ij}, where

> τ_{ij} = the quantity of 1958 Canadian exports to the United States that belongs as outputs to both the i^{th} Canadian and j^{th} American sectors. They are expressed in Canadian dollar's worths at 1958 prices.

The method of classification is discussed fully in Appendix B. Briefly it involved matching the descriptions of commodity groups of exports with the coded descriptions in the Standard Industrial Classification Manual of each country. The coded exports were then allocated to pairs of industries using the classification tables provided in each set of input-output data. By aggregating the exports allocated to each pair of industries, the array of τ_{ij} was produced.[4]

4. From the τ_{ij}, the vector of 1958 exports (in 1958 Canadian dollars) conforming to either classification is easily obtained. Classified according to the Canadian scheme, the exports are $\tilde{e}_i = \sum_{j=1}^{82} \tau_{ij}$ $(j = 1,\ldots,42)$. and according to the United States scheme
$\tilde{\tilde{e}}_j = \sum_{i=1}^{42} \tau_{ij}$ $(j = 1,\ldots,82)$.
Vectors $\tilde{e} = [\tilde{e}_i]$ and $\tilde{\tilde{e}} = [\tilde{\tilde{e}}_j]$, and the non-zero τ_{ij} are shown in Appendix B.

Second, coefficients t_{ij} were obtained from

$$t_{ij} = \frac{\tau_{ij}}{\hat{X}_j} \qquad (i = 1,\ldots,42; \quad j = 1,\ldots,82)$$

Now Canadian exports valued at 1958 prices could be expressed in terms of United States activity levels by

$$\tilde{e} = \sum_{j=1}^{82} t_{ij} \hat{X}_j \qquad (i = 1,\ldots,42)$$

or in matrix form by $\tilde{e} = T\hat{X}$ where $T = [t_{ij}]$. (5.2)

The third step was to express the vector of exports, $\tilde{e}$, in terms of 1949 prices. A diagonal matrix, p , of elements, p_i , was estimated with

$$p_i = \frac{\text{the export price of Canadian output } i \text{ in 1949}}{\text{the export price of Canadian output } i \text{ in 1958}} .$$[5]

The vector of Canadian exports at 1949 prices, e , could therefore be found by

$$e = p\,\tilde{e} \qquad (5.3)$$

Moreover, combining (5.2) and (5.3) yields

$$e = pT\hat{X}$$

so the trade matrix J was obtained by estimating separately the parameters of p and T .

The final step in applying the model was to obtain solution vectors of Canadian outputs and imports for posultated vectors of United States final demand. The calculations were made with

5. The estimation of these export price relatives is described in Appendix D.

the IBM 7040 computer at the University of British Columbia Computing Centre. The relatively simple calculations

$e_i = \sum_{j=1}^{82} p_i \ t_{ij} \ \hat{X}_j$ were written into the programme so that it would be unnecessary for the entire p and T matrices to be stored in the computer. To calculate the Canadian imports generated by the postulated vectors of United States final demand, the Canadian inverse matrix was replaced by the import requirements matrix, $\bar{m}(I - a)^{-1}$.

CHAPTER VI

THE IMPACT OF UNITED STATES FINAL DEMAND ON CANADIAN PRODUCTION AND TRADE

The model developed in Chapter IV is used to investigate the impact on the Canadian economy of changes in the level and pattern of United States final demand. Of primary interest is the effect on Canadian final demand and the output of Canadian industries, but the effect on Canada's balance of merchandise trade is also examined.

The investigation is presented in two parts. The first part compares the effects of a billion dollars worth of each of the major components of United States final demand. An attempt is made to isolate the factors chiefly responsible for the differences in the levels of Canadian output and imports generated. The second part examines the effect of cyclical changes in United States final demand on the growth of Canadian final demand and output, and on the balance of merchandise trade.

1. Comparison of the Effects of One Billion Dollar Increases in United States Final Demand.

Table XIII in Appendix A presents vectors representing a billion dollars worth of each of the four major components of United States final demand: Personal consumption expenditure, C , Gross private fixed capital formation (Investment), I , Federal Government expenditure, G_F , and State and local government expenditure, G_S. It should be observed that not all of the billion dollars applies to the eighty-two producing sectors. This is particularly evident in the vectors of government expenditure where the payments of wages and salaries to government exployees is important.

Tables XIV, XV, XVI, and XVII in Appendix A exhibit the vectors of United States output, and Canadian exports, output, and imports that are generated by the one billion dollar final expenditures. The sum of the elements, or aggregate, of each of these vectors is shown in Table III below.

TABLE III

COMPARISON OF THE AGGREGATE EFFECTS OF INCREASES IN FOUR COMPONENTS OF UNITED STATES FINAL DEMAND, 1958

Increase in:	C	I	G_F	G_S
		(millions of dollars)		
United States final demand[a]	1000.00	1000.00	1000.00	1000.00
United States output[a]	1933.19	2281.27	1384.92	1142.57
Canadian exports[b]	4.18	8.55	4.29	4.09
Canadian output[b]	7.69	15.37	7.60	7.25
Canadian imports[b]	.37	.87	.44	.36

a. United States dollars; 1958 prices

b. Canadian dollars; 1949 prices.

Two interesting observations may be made from these results. First, Canadian demand and output are stimulated to a much greater degree by United States Investment expenditure than by any of the other expenditures. Second, the ratios between the aggregate values of induced Canadian exports (or output) and induced United States output vary considerably. In particular, a dollars' worth of United States output generated by an increase in Personal consumption expenditure requires a relatively small increase in Canadian exports and output.

Both these observations suggest that the industry composition of induced United States output is important in determining the extent to which Canadian output is stimulated. The reason for this may be investigated by observing the impact of the increases in United States final demand on particular Canadian industries.

Table IV shows the percent of total induced Canadian output that is accounted for by the five, and ten, most affected industries. In similar fashion, the industry concentration of exports is shown.

TABLE IV

THE CONCENTRATION OF INDUCED OUTPUT AND 1958 EXPORTS AMONG CANADIAN INDUSTRIES.

Percent of Total Accounted for by:	Top Five Industries	Top Ten Industries
	(%)	
Canadian Output Generated by U.S. Final Demand Component:		
C	53.4	74.5
I	56.3	75.7
G_F	57.4	77.6
G_S	60.8	79.2
Canadian Exports, 1958	72.5	86.8

It is apparent from these figures that the output generated by each type of United States final expenditure is concentrated on a relatively small number of Canadian industries. The top five of the forty-two industries produce between fifty and sixty percent of the total induced Canadian outputs, the top ten between seventy-five and eighty percent. The explanation may be found in the concentration of exports on a few products, for it is even greater.

In Table V the industries with high induced outputs and high 1958 exports are identified. From this table it may be observed that, despite the dissimilarity of the United States final demand vectors, the same few export industries are the ones most strongly stimulated.[1] Moreover, they are not the same industries that are stimulated most by over-all Canadian final demand. In the 1949 Canadian interindustry study[2] for example, five of the top ten industries ranked by value of output are not common to any of the rankings in Table V. The top ten are: Transportation, storage and trade (38), Service industries (42), Construction (37), Agriculture (1), Finance, insurance and real estate (41), Transportation equipment (29), Paper products (24), Iron and steel products, n.e.s. (28), Clothing (textile and fur) (21), and Meat products (7).

Consideration of the induced outputs in particular Canadian industries leads to the conclusion that the effect of an increase in United States final demand on Canadian aggregate output depends largely on the degree to which a certain few export industries are stimulated. At the same time, it explains why the level of aggregate induced Canadian output should be so sensitive to the pattern of United States output requirements and final demands.

1. A notable exception is the Transportation, storage and trade industry (number 38). For each vector of United States final demand it is ranked in the top five industries according to induced output; but it has no exports. Other industries with high induced outputs but low exports are: Forestry (2), Iron and steel products (28), Electric power, gas and water (40), and Products of petroleum and coal (34).

2. Dominion Bureau of Statistics, number 13-513, Supplement to the Inter-Industry Flow of Goods and Services, Canada, 1949 Table 1.

TABLE V

OUTPUT GENERATED IN PARTICULAR CANADIAN INDUSTRIES BY ONE BILLION DOLLAR INCREASES IN UNITED STATES FINAL DEMAND; 1958 EXPORTS TO THE UNITED STATES BY INDUSTRY

(thousands of dollars)

Rank	Type of United States Final Expenditure.								1958 Exports to the U.S.	
	C		I		G_F		G_S			
	Industry	Output Generated	Ind.	Out. Gen.	Ind.	Out. Gen.	Ind.	Out. Gen.	Ind.	Exports
1	(24)	1867	(4)	2783	(4)	2137	(24)	1120	(24)	846,235
2	(1)	742	(23)	2016	(24)	984	(4)	1090	(4)	620,664
3	(4)	532	(24)	1682	(38)	512	(23)	1074	(23)	273,157
4	(38)	530	(38)	1084	(35)	435	(2)	617	(1)	183,284
5	(2)	440	(2)	1081	(23)	375	(38)	509	(27)	90,335
6	(5)	353	(28)	724	(29)	367	(35)	318	(3)	88,080
7	(35)	352	(27)	712	(40)	339	(33)	271	(35)	80,284
8	(40)	297	(40)	523	(2)	317	(40)	265	(33)	79,780
9	(23)	285	(33)	516	(5)	273	(5)	264	(5)	75,744
10	(34)	236	(35)	504	(28)	265	(34)	216	(14)	68,290

Key to Industries: (1) Agriculture, (2) Forestry, (3) Fishing and trapping, (4) Metal mining and smelting and refining, (5) Coal, crude petroleum and natural gas, (14) Alcoholic beverages, (23) Wood products (except furniture), (24) Paper products, (27) Agricultural implements, (28) Iron and steel products, n.e.s., (29) Transportation equipment, (33) Non-metallic mineral products, (34) Products of petroleum and coal, (35) Chemicals and allied products, (38) Transportation, storage and trade, (40) Electrical power, gas and water.

2. The Cyclical Effect of United States Final Demand on Canadian Output and the Canadian Balance of Merchandise Trade

In this section the investigation is carried a step further. Using observed aggregate values of the components of United States final demand for a period of several years, the model is used to estimate the annual values of induced Canadian exports, induced output and induced net exports (induced exports less induced imports). Then the annual fluctuations in growth of the latter aggregates are compared with fluctuations in the growth of the corresponding Canadian aggregates to determine whether they were stabilizing or destablizing over the period. If, for instance, the annual growth of induced Canadian exports fluctuated in phase with the growth of total Canadian final demand, it could be concluded that variation in the growth of United States final demand had a destabilizing effect on Canadian final demand.

The accuracy of input-output predictions has been shown to decline fairly rapidly as years distant from the base year are considered. For this reason the analysis is limited to the five year period 1956 to 1960, centred on the base year, 1958.

Since the analysis covers a short span of time and makes use of annual final demand data, it is convenient to discuss annual rather than strictly cyclical fluctuations. Fortunately the years considered were characterized by pronounced fluctuations in the growth of demand. The values for 1956 to 1960 of the various components of United States Gross National Expenditure (hereinafter GNE) are presented in Table VI. From these values, the annual growth in GNE is calculated for 1957 to 1960. These growth rates are shown in Table VI and plotted in the appended figure as well. From the graph it is apparent that the rate of growth of final demand fluctuated over a cycle whose period almost exactly coincided with the four years 1957 to 1960. Between mid-points in the cycle in 1957 and 1960, the rate of growth of final demand declined to a low point in 1958 and rose to a peak in 1959. As a result it is probably fair to investigate the cyclical behavior of final demand using annual variations in its growth.

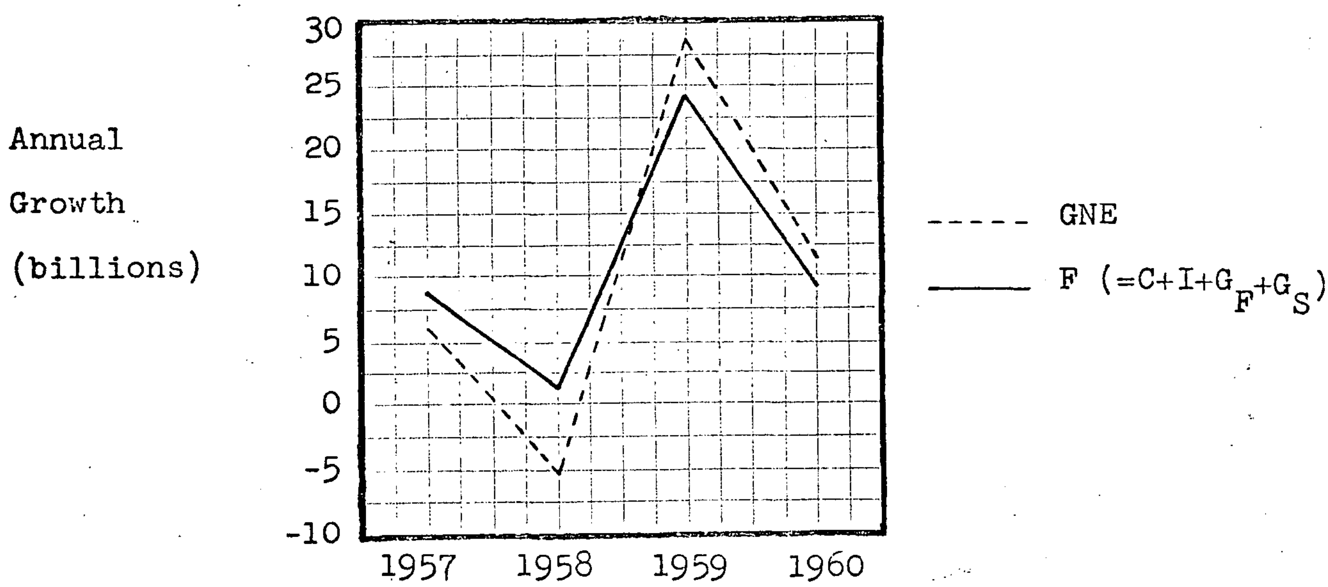
The cyclical behavior of United States final demand refers to variation in its level and industry composition. The latter is approximated here by considering the variation in the relative weights of major expenditure components. With the exception of Personal consumption expenditure (see below), no variations are considered in the pattern of demand within any of the expenditure classifications. This means that an increment of, say, Federal Government expenditure is assumed to have the same industry pattern as total Federal Government expenditure in the base year, 1958. This assumption admittedly weakens the predictive power of the model but should not be untenable if the increments considered are small in relation to total expenditure in the base year.

Table VI can be used to compare the annual increments of the various final demand components to their 1958 aggregate values. The condition that the changes be small in relation to the 1958 totals is met for all classifications but Net inventory change and Net exports. For these components, the annual increments are typically double or triple the 1958 totals.

TABLE VI

GROWTH IN UNITED STATES FINAL DEMAND, 1956-1960

	1956	1957	1958	1959	1960
	(billions of dollars; 1958 prices)				
Personal Consumption Expenditure					
- durables	41.0	41.5	37.9	43.7	44.9
- non-durables	136.2	138.7	140.2	146.9	149.7
- services	104.1	108.0	112.0	116.8	121.6
Total Personal Consumption Exp.	281.4	288.2	290.1	307.3	316.2
Gross Private Fixed Capital Exp.	69.5	67.6	62.4	68.8	68.9
Federal Government Expenditure	49.7	51.7	53.6	52.5	51.4
State and Local Government Exp.	35.6	37.6	40.6	42.2	43.5
SUBTOTAL: $(F = C+I+G_F+G_S)$	436.2	445.1	446.7	470.8	480.0
Net Inventory Change	4.8	1.2	-1.5	4.8	3.5
Net Exports	5.0	6.2	2.2	0.3	4.3
Gross National Expenditure (GNE)	446.1	452.5	447.3	475.9	487.8
Annual Growth in GNE		6.4	-5.2	28.6	11.9
Annual Growth in F		8.9	1.6	24.1	9.2



Source: Survey of Current Business, August, 1965, p.27.

Note: Figures do not always add to totals due to rounding.

It would not be reasonable to assume that the 1958 industry patterns of the expenditure would be good estimators of the impact on industry outputs of the changes in their levels. Therefore, these components of United States GNE are omitted from consideration.[3] For the purposes of this investigation, United States final demand is defined as the sum of Personal consumption expenditure, Gross private fixed capital formation, Federal Government expenditure, and State and local government expenditure. (i.e., $F = C+I+G_F+G_S$)

The result of neglecting Net inventory change and Net exports is to significantly understate the cyclical fluctuation of United States aggregate demand. The growth of final demand with and without these components is compared in Table VI and the appended chart. The annual fluctuation in growth rate is more pronounced for GNE than for final demand, F , at both the trough and peak of the cycle. The average change in annual growth rate,[4] $\bar{A}$, is used as a measure of the amplitude of each of the fluctuations. Comparing the degree or amplitude of the variations in this way shows that using F instead of

3. There is another argument for neglecting Net exports. It is composed to a large extent of exports to Canada which depend on Canadian activity levels, the solution variables of the model.

4. The average change in annual growth rate, $\bar{A}$, is an average of the declines in annual growth in 1957-1958 and 1959-1960 and the increase in annual growth in 1958-1959, summed without respect to sign. For GNE,

$$\bar{A} = \frac{|(-5.2) - 6.4| + |28.6 - (-5.2)| + |11.9 - 28.6|}{3}$$

$= 20.7$ billion dollars.

It is used to measure amplitude here because it does not require the definition of a trend in the rate of growth.

GNE understates the cyclical fluctuation in United States final demand by approximately thirty-eight percent. ($\bar{A}$ = \$20.7 billion for GNE and \$14.9 billion for F) Obviously, the impact of the cyclical variation in United States business conditions on the Canadian economy will also be significantly underestimated.

On the other hand, an improvement in the ability of the model to reflect the cyclical behavior of United States final demand is obtained by using three sub-classifications of Personal consumption expenditures. The sub-classes are expenditure on: durables, non-durables, and services. It is possible to separate Personal consumption expenditures into these three types because industry dissaggregations are available for them that conform to the sector definitions of the United States input-output matrices.[5] Unfortunately no such dissaggregations are published for the major components of Investment or government expenditures. In Table VII, the aggregate effects on Canadian exports, output, and imports of expenditures of one billion dollars on each of the three types of consumption are compared to the corresponding effects for 1958 total Personal consumption expenditure.

5. Survey of Current Business, October 1965, p.13, Table 3.

TABLE VII

COMPARISON OF AGGREGATE EFFECTS OF ONE BILLION DOLLAR INCREASES IN TYPES OF UNITED STATES PERSONAL CONSUMPTION EXPENDITURE

Increase in:	Durables	Non-durables	Services	1958 Total
	(millions of dollars)			
United States Personal Consumption Expenditure[a]	1,000.00	1,000.00	1,000.00	1,000.00
Canadian Exports[b]	5.68	5.63	1.85	4.18
Canadian Output[b]	10.28	10.44	3.38	7.69
Canadian Imports[b]	.57	.49	.16	.37

a. United States dollars; 1958 prices.

b. Canadian dollars; 1949 prices.

It was apparent from the results of section 1 of this chapter that the cyclical impact of shifts in the composition of United States final demand will depend to a great extent on shifts between Investment and any of the other three categories of expenditure. These results show that a strong cyclical effect might also arise from shifts between purchases of services and materials within the Personal consumption category of demand.

With this background, four questions will be investigated:

(a) How did shifts in the level and composition of United States final demand compare in their effects on the level of induced Canadian demand?

(b) Did variations in induced Canadian demand aggrevate or dampen Canadian business cycles?

(c) What effects did the fluctuations in United States final demand have on the growth of output in particular Canadian industries?

(d) Did variations in induced net exports aggravate or dampen fluctuations in Canadian net exports?

a. The relative effect on Canadian demand of cyclical shifts in the level and composition of United States final demand.

To isolate the effect of cyclical variation in the pattern of United States demand, it is only necessary to compare the fluctuations in the growth rate of induced Canadian final demand with the fluctuations in the growth of United States demand. If the industry composition of United States demand were unchanged over the period, the pattern and amplitude of the variations in the two growth rates would be identical. In order to compare changes in variables that are expressed in different units, the changes are first expressed as percentages. Table VIII shows the values of United States final demand and induced Canadian exports for 1956 to 1960, and the annual percentage growth in these two variables for 1957 to 1960. In the appended chart, the fluctuations in growth rates are compared graphically. It is immediately apparent that the percentage growth rate of induced Canadian exports varied more widely than that of United States final demand. Variation in the composition of United States aggregate demand therefore

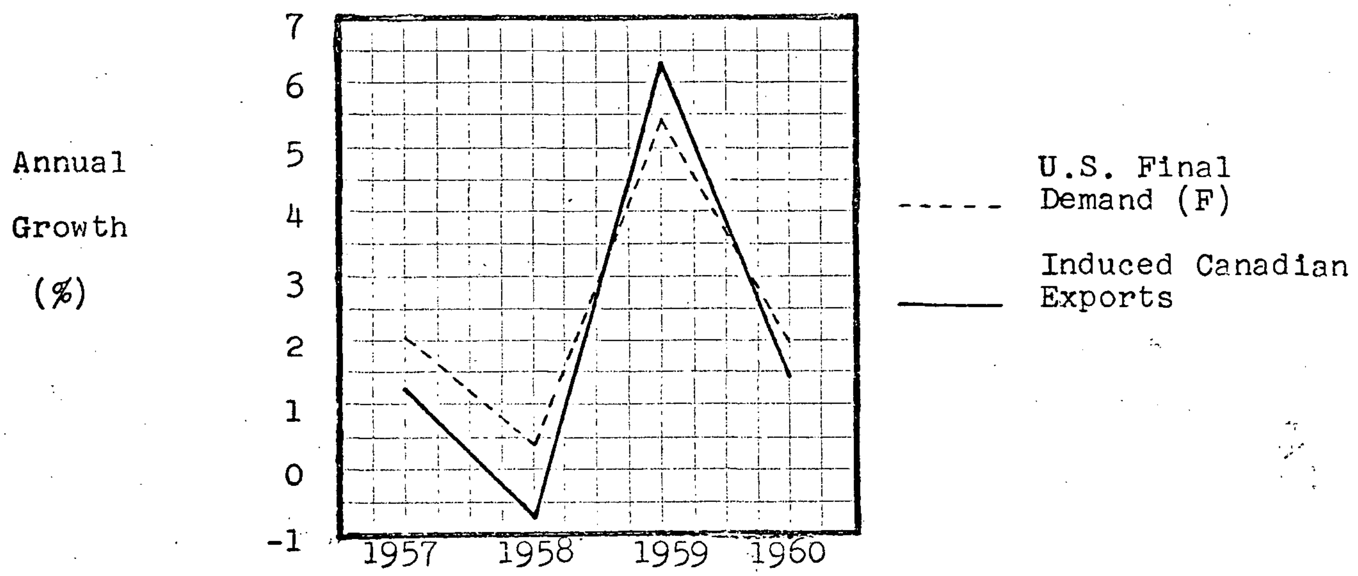
TABLE VIII

COMPARISON OF ANNUAL GROWTH RATES: UNITED STATES FINAL DEMAND AND INDUCED CANADIAN EXPORTS

	1956	1957	1958	1959	1960
United States Final Demand (F)[a]	436.2	445.1	446.7	470.8	480.0
Induced Canadian Exports[b]	2145	2171	2142	2277	2309
Growth in United States Final Demand (%)		2.04	.36	5.40	1.95
Growth in Induced Canadian Exports (%)		1.22	-.74	6.30	1.41

a. Billions of United States dollars; 1958 prices.

b. millions of Canadian dollars; 1949 prices.



reinforced the effects of variations in its level. Calculating the average change in annual growth, $\overline{A}$, for both variables yields 3.38% for United States final demand and 4.63% for induced exports. In other words, variation in the level of United States demand explained seventy-three percent, and variation in its composition twenty-seven percent, of the amplitude of fluctuations in induced Canadian exports.

Induced exports, and their annual growth rates, were also calculated using the industry pattern of total Personal consumption expenditures instead of the patterns of expenditures on durables, non-durables, and services. Thus estimated, induced exports varied with an amplitude of 4.38%. This means that shifts among the four major components of demand explained twenty-one percent of the variation in the growth of induced Canadian exports while six percent was accounted for by variation among the three types of Personal consumption expenditure.

b. The effect of variations in induced Canadian exports on the growth of Canadian aggregate demand.[6]

The annual growth rates of induced and total Canadian final demand are developed and compared in Table IX and its appended chart. The first question to be answered by this

6. The question of transmission of business cycles is discussed in terms of aggregate demand rather than aggregate output because the latter statistic was not readily available and is not generally used in discussing economic fluctuations.

comparison is whether or not the annual growth rates of the two aggragates varied in phase over the four years. Since they did it may be concluded that the fluctuations in induced exports reinforced the fluctuations in Canadian final demand. The cyclical behavior of final demand in the United States had a destablizing effect on Canadian economic growth in the period considered.

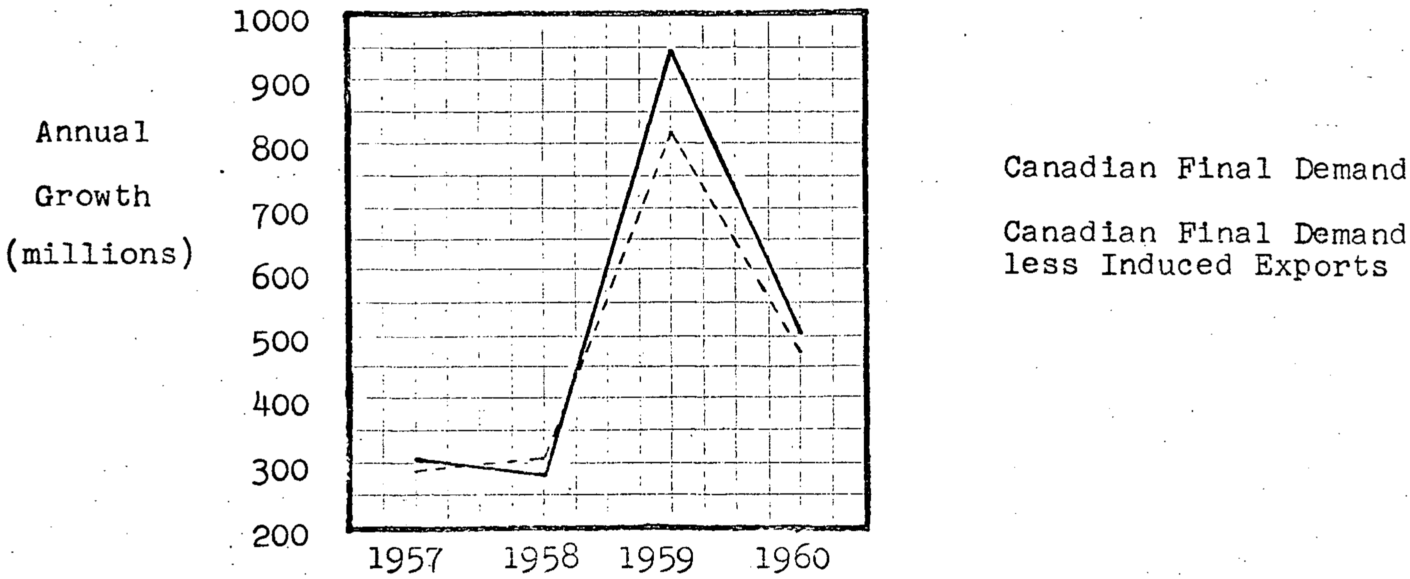
The strength of this destablizing effect can be estimated by comparing the amplitude of fluctuations in Canadian final demand - with and without the induced exports. The average change in annual growth, $\bar{A}$, was 376 million for actual Canadian final demand, and 292 million for Canadian final demand less induced exports. The fluctuation in Canadian exports generated by the fluctuation in the level and composition of United States final demand accounted for $84 million or twenty-two percent of the amplitude of the fluctuation in Canadian final demand. This is quite a remarkable conclusion for two reasons. First, the fluctuation in United States final demand is substantially understated. Second, averaged over the period considered, the induced exports accounted for only nine percent of Canadian aggregate demand.

c. A comparison of the growth in induced and total Canadian output for six export industries.

The validity of the conclusion that United States final demand had a destablizing effect on Canadian economic growth can be tested by comparing the growth patterns of induced and total output for particular industries. Six export industries are

COMPARISON OF ANNUAL GROWTH RATES: CANADIAN FINAL DEMAND WITH AND WITHOUT INDUCED EXPORTS

	1956	1957	1958	1959	1960
		(millions of dollars; 1949 prices)			
Induced Canadian Exports	2145	2171	2142	2277	2309
Canadian Final Demand	23,811	24,117	24,397	25,342	25,849
Canadian Final Demand less Induced Exports	21,666	21,946	22,250	23,065	23,540
Growth in Canadian Final Demand		306	280	945	507
Growth in Canadian Final Demand less Induced Exports		280	304	815	475



considered: Paper products (number 24), Wood products (23). Metal mining and smelting and refining (4), Iron and steel products, n.e.s. (28), Chemicals and allied products (35), and Agriculture (1). The growth of induced and total output is considered because estimates of the growth of industry output are available and because the output induced by a dollars worth of final demand varies considerably among industries. The values of induced and total output for 1956 to 1960 are shown in Table X. Also shown is the annual growth in output with and without the output generated by United States final demand. In Figure 1, the growth rates are compared graphically.

At first glance, the results of this investigation appear to detract from the strength of the conclusion reached in section (b). It is found that output generated by United States demand was destabilizing in three cases and stabilizing in three. However, in the three cases where induced output was stabilizing (Wood products, Chemicals, and Agriculture) it may be observed that the fluctuation in the growth of the industry output is out of phase with the fluctuation in both Canadian and United States final demand. The induced export demand therefore tended to aggravate variation in the growth of industries whose output varied cyclically, and dampen variation in those whose outputs varied counter-cyclically.

d. The impact of variation in United States final demand on Canada's balance of merchandise trade.

Canada's balance of merchandise trade is the difference between Total Merchandise Exports and Total Merchandise Imports

TABLE X

COMPARISON OF ANNUAL GROWTH RATES:
THE OUTPUT OF SELECTED EXPORT INDUSTRIES
WITH AND WITHOUT THE OUTPUT GENERATED
BY UNITED STATES FINAL DEMAND

(millions of dollars; 1949 prices)

i. Paper Products

	1956	1957	1958	1959	1960
Induced output	736	751	746	792	805
Total output[a]	1503	1478	1479	1579	1619
Net output (Total less induced)	767	727	733	787	814
Growth in total output		-25	1	100	40
Growth in net output		-40	6	54	27

ii. Wood products (except furniture)

	1956	1957	1958	1959	1960
Induced output	280	280	272	293	297
Total output[a]	871	802	832	861	857
Net output	591	522	560	568	560
Growth in total output		-69	30	29	-4
Growth in net output		-69	38	8	-8

TABLE X (Continued)

(millions of dollars; 1949 prices)

iii. Metal Mining and Smelting and Refining

	1956	1957	1958	1959	1960
Induced output	495	499	487	518	522
Total output[a]	972	1095	1161	1296	1273
Net output	477	596	674	778	751
Growth in total output		123	66	135	-23
Growth in net output		119	78	104	-27

iv. Iron and Steel Products, n.e.s.

	1956	1957	1958	1959	1960
Induced output	129	130	125	135	137
Total output[a]	480	461	424	486	453
Net output	351	331	299	351	316
Growth in total output		-19	-37	62	-33
Growth in net output		-20	-32	52	-35

v. Chemicals and Allied Products

	1956	1957	1958	1959	1960
Induced output	168	171	170	180	182
Total output[a]	477	500	541	568	599
Net output	309	329	371	388	417
Growth in total output		23	41	27	31
Growth in net output		20	42	17	29

TABLE X (Continued)
(millions of dollars; 1949 prices)

vi. Agriculture

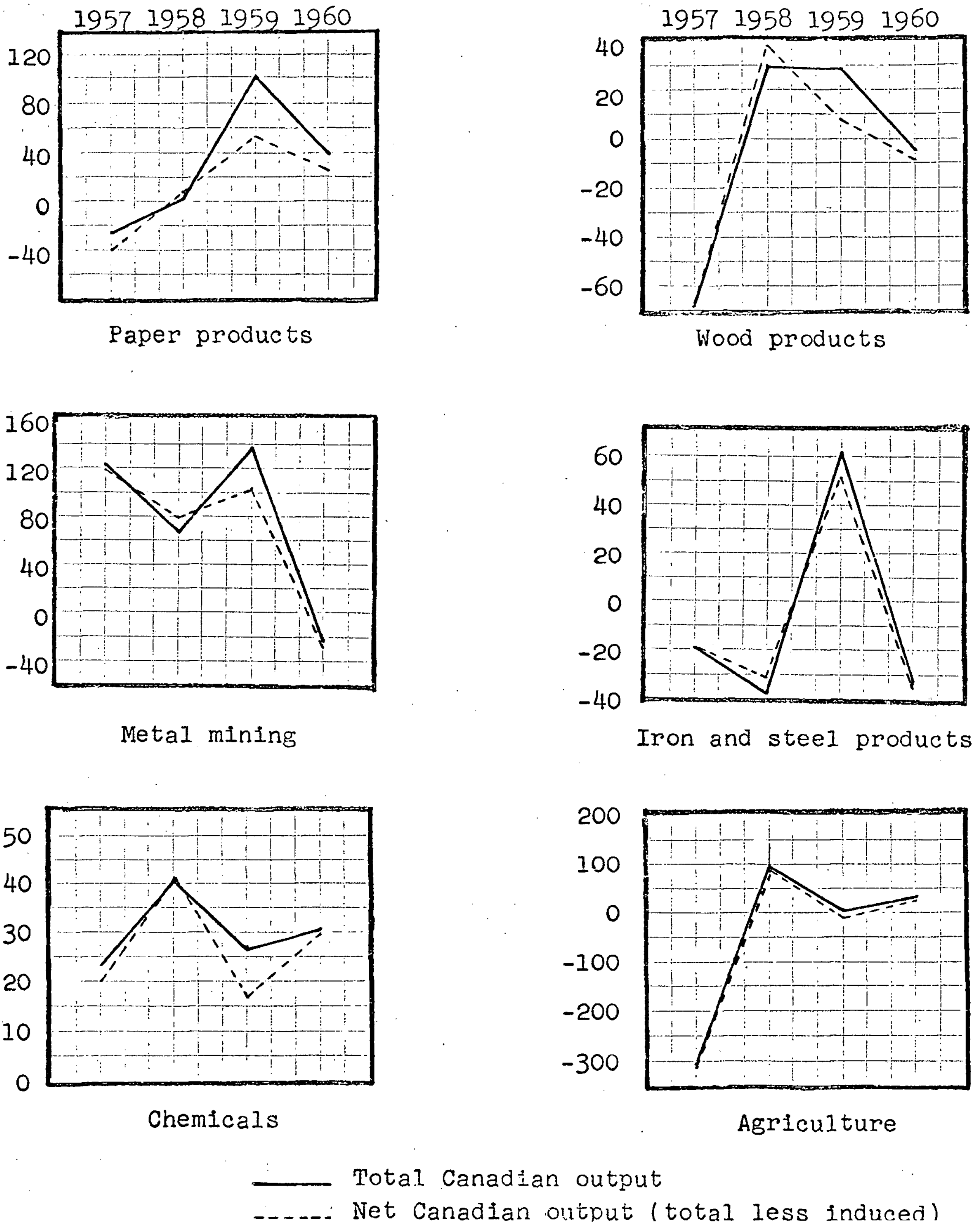
	1956	1957	1958	1959	1960
Induced output	233	238	239	251	256
Total output[a]	1785	1480	1577	1578	1612
Net output	1552	1242	1338	1327	1356
Growth in total output		-305	97	1	34
Growth in net output		-310	96	-11	29

a. Source: DBS 61-505, Indexes of Real Domestic Product by Industry of Origin, 1935-61, p.67.

FIGURE 1

THE EFFECT OF INDUCED OUTPUT ON THE ANNUAL GROWTH OF SELECTED EXPORT INDUSTRIES

(millions of dollars; 1949 prices)



for any period and will be referred to as net exports. Total Merchandise Exports includes the exports of most goods and services but differs from Total Current Receipts in the exclusion of items such as Gold production available for export, Travel expenditures, Interest and dividends, Freight and shipping, and Inheritances and immigrants funds.

The initial impact of United States final demand on the Canadian balance of trade is the generation of induced exports and induced imports, and by subtraction, the generation of induced net exports. The cyclical effect on the Canadian trade balance of dependence on United States final demand will be estimated by comparing the growth of Canadian net exports with the growth of net exports excluding induced net exports.

First, Table XI compares the growth of Canadian exports with and without induced exports. It may be observed that the effect of the induced exports was to make total Canadian exports fluctuate cyclically.

Table XII and the appended chart present the growth of Canadian net exports with and without induced net exports. It appears that the fluctuation in induced net exports dampened the fluctuation in net exports. The explanation of this result is that the amplitude of variation in the level of Canadian imports was much greater than the amplitude of export variation. ($\bar{A}$ was \$557 million for imports and \$64 million for exports.) Since the fluctuations in imports were cyclical, the fluctuations in net exports were counter-cyclical. As a result, they were

TABLE XI

GROWTH OF CANADIAN MERCHANDISE EXPORTS WITH AND WITHOUT INDUCED EXPORTS

	1956	1957	1958	1959	1960
		(millions of dollars; 1949 prices)			
Induced exports	2145	2171	2142	2277	2309
Total Merchandise Exports[a]	4117	4167	4189	4331	4527
Growth in induced exports		26	-29	135	32
Growth in total exports		50	22	142	196
Growth in:total exports less induced exports		24	51	7	164

a. Source: DBS 67-201, The Canadian Balance of International Payments (annual), Table 2, p. 8.

The figures are deflated to 1949 prices using indexes found in DBS 65-205, Review of Foreign Trade (annual), Table XX.

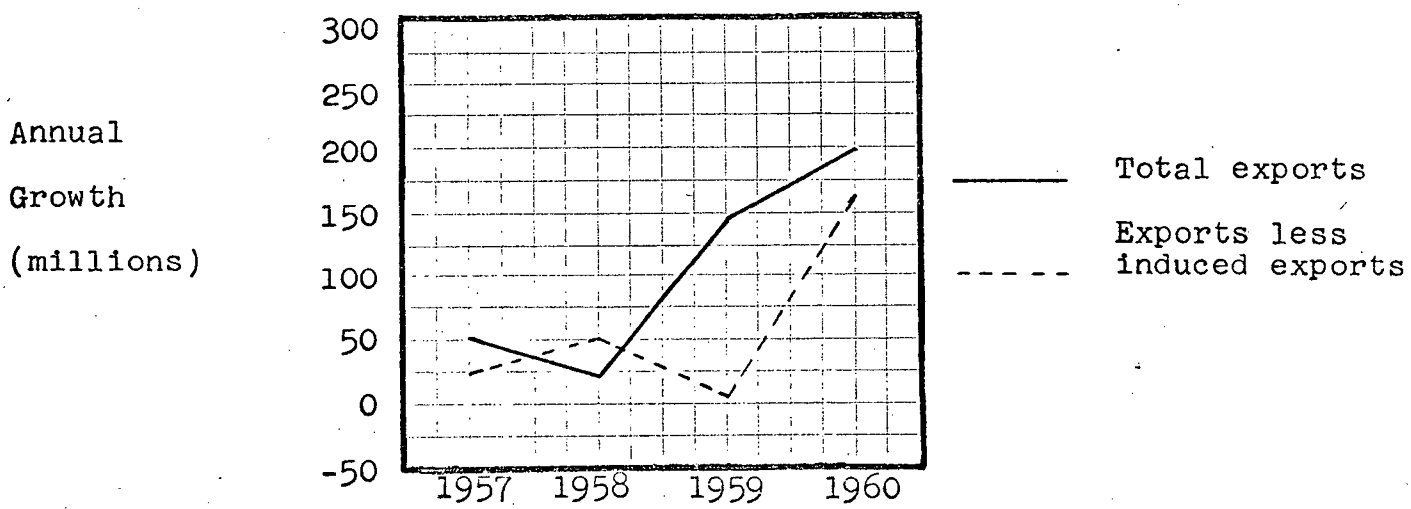


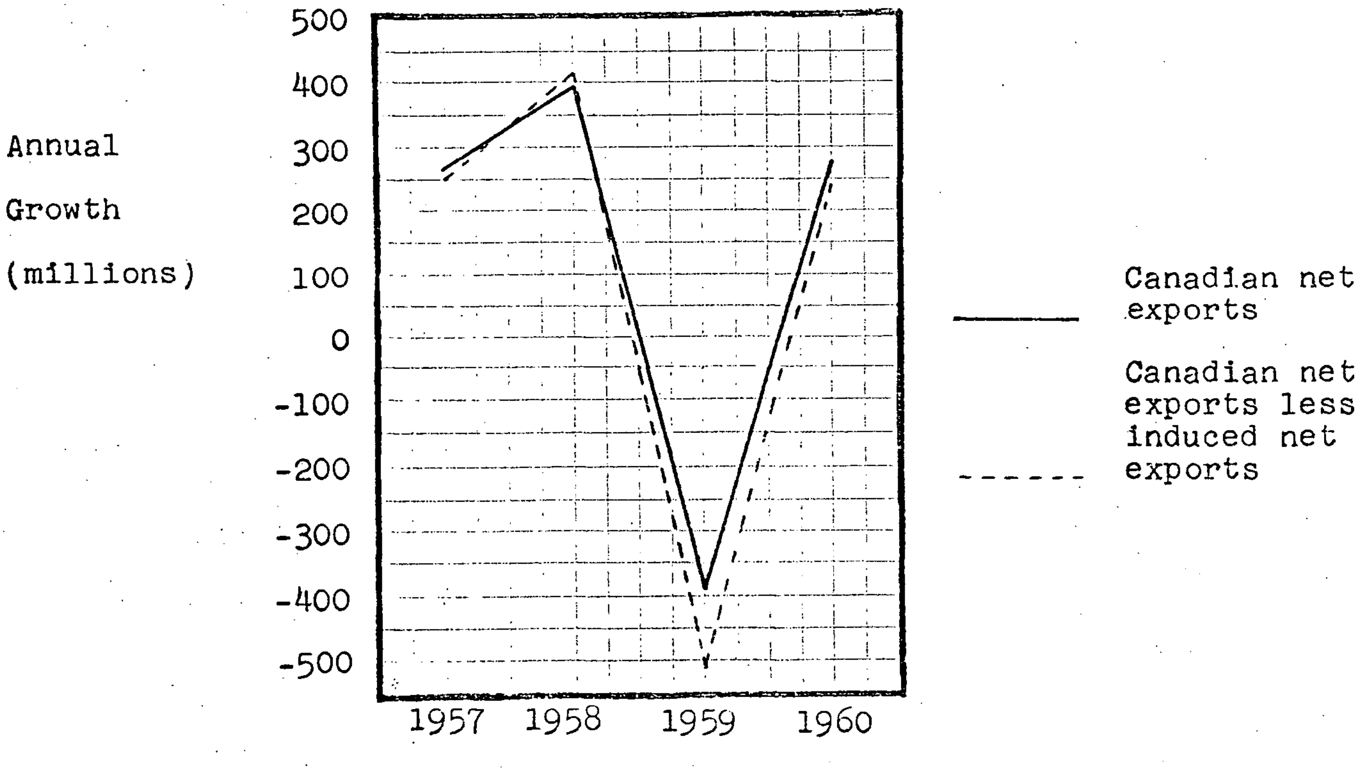
TABLE XII

GROWTH OF CANADIAN NET EXPORTS WITH AND WITHOUT INDUCED NET EXPORTS

	1956	1957	1958	1959	1960
	(millions of dollars; 1949 prices)				
Induced net exports[a]	1947	1966	1943	2065	2096
Canadian net exports[b]	-937	-668	-274	-666	-393
Growth in induced net exports		19	-23	122	31
Growth in Canadian net exports		269	394	-392	273
Growth in: Canadian net exports less induced net exports		250	417	-514	242

a. Induced net exports = induced Canadian exports less induced Canadian imports.

b. Net exports = total Canadian exports less total Canadian imports.



dampened by the cyclical fluctuations in induced net exports.

In the final chapter, these results are summarized and their probable bias discussed.

CHAPTER VII

CONCLUSIONS

The following conclusions were drawn from the results obtained in applying the input-output model to Canadian and American data for 1956 to 1960.

1. Of equal expenditures on the four major components of United States final demand, Gross private fixed capital formation, or Investment, had much the greatest impact on Canadian aggregate demand and production. There was relatively little difference in effect between expenditures on the other three components. The main factor contributing to the wide divergence between the impact of Investment and the other components was the concentration of Canadian exports on a relatively small number of products.

2. The effect on the Canadian economy of cyclical fluctuations in the level of United States final demand was reinforced by variation in the relative weights of its major components. In particular, the relative weight of Investment expenditure tended to decline during periods of demand contraction and rise during periods of expanding demand. Cyclical variation in the share of Personal consumption expenditure devoted to durable goods was also a contributing factor.

3. The rate of growth of Canadian exports generated by United States final demand fluctuated in phase with the rate of growth

of total Canadian final demand. Therefore, the fluctuations in induced exports contributed to the fluctuations in Canadian aggregate demand. Indeed, it was estimated that twenty-two percent of the amplitude of the annual fluctuations in Canadian demand was attributable to the cyclical behavior of United States final demand.

4. Fluctuations in the level and pattern of United States demand aggravated fluctuations in the growth of output of some Canadian industries and dampened those of others. The direction of effect depended on whether or not the industry growth rate varied in phase with the over-all growth of the Canadian and American economies. Fluctuations in growth were reinforced in the Paper products, Metal mining, and Iron and steel industries, and dampened in the Wood products, Chemicals, and Agriculture industries.

5. Cyclical fluctuations in the Canadian balance of merchandise trade were dampened by fluctuations in the exports and imports generated by United States final demand. The reason for this was that fluctuations in induced exports dominated the growth of induced net exports while import fluctuations dominated the growth of total Canadian net exports.

In the derivation of the model and in its application, several assumptions were introduced. In this final section the effect of relaxing these assumptions is suggested.

1. The neglected factor most directly affecting the conclusions is the impact of cyclical changes in the levels of Net inventory change and Net exports. It is certain that the neglect of these components of United States demand resulted in a substantial understatement of the transmission of business cycles from the United States to Canada.

2. Changes were not considered in the industry composition of the components of United States final demand. The effect of such changes could have been either cyclically stabilizing or destabilizing and could be quite important, particularly with regard to the outputs of individual Canadian industries.

3. Changes in the Canadian export share of the total United States supply of any industry output were neglected. Again, it is difficult to predict what the effect of relaxing this restriction might be.

4. Similarly, changes in industry input coefficients that might result from non-constant returns to scale, substitution, or technological change were neglected. Moreover, the possibility of error arising from non-constant production coefficients was enhanced by the fact that 1958, the base year of the study, was a year in which the rate of economic growth declined sharply. Any variability in production alternatives should have been fully exploited in such a year, which suggests that 1958 inter-industry transactions may have been poor data on which to base production coefficients for other years.

5. Finally, no changes were considered in Canadian Personal consumption and Investment expenditures in response to changes in the level of United States final demand. Again, this is a factor that would increase the sensitivity of the Canadian economy to changes in United States business conditions.

Consideration of these neglected factors leads to the conclusion that the impact of fluctuations in United States final demand on Canadian final demand and industry outputs is under-estimated in this investigation. Neglect of the first and last factors definitely imparted a strong conservative bias to the results, while the direction of errors due to neglect of the other factors was probably mixed.

BIBLIOGRAPHY

I. Unpublished Material

1. Updated Canadian inverse and import matrices, punched on IBM cards, with accompanying technical notes by T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer.

II. Government Publications

A. Publications of the Dominion Bureau of Statistics, Ottawa: Queen's Printer.

1. No. 12-501, Standard Industrial Classification Manual, 1948.
2. No. 13-513, Supplement to the Inter-Industry Flow of Goods and Services, Canada, 1949, 1960.
3. No. 62-002, Prices and Price Indexes (annual).
4. No. 65-201, Trade of Canada, Volume I, Summary, (annual).
5. No. 65-202, Trade of Canada, Volume II, Exports, (annual).
6. No. 65-205, Review of Foreign Trade (annual).
7. No. 67-201, The Canadian Balance of International Payments (annual).
8. Canada Year Book (annual).

B. Other

1. Executive Office of the President, Bureau of the Budget, Standard Industrial Classification Manual, 1957.
2. United States Department of Commerce, Office of Business Economics, Survey of Current Business:
   - "The National Income and Product Accounts of the United States: Revised Estimates, 1929-64," August, 1965, p.6.
   - "The Transactions Table of the 1958 Input-Output Study and Revised Direct and Total Requirements Data," September, 1965, p.33.
   - "Personal Consumption Expenditures in the 1958 Input-Output Study," October, 1965, p.7.

III. Books.

1. Allen, R.D.G., Mathematical Economics, Second Edition, London: MacMillan and Company Ltd., 1956.

2. Kindleberger, C.P., International Economics, Revised Edition, Homewood: R. D. Irwin, 1958.

3. Leontief, W.W, The Structure of the American Economy, 1919-1939, Second Edition, New York: Oxford University Press, 1951.

4. ____________, et al, Studies in the Structure of the American Economy, New York: Oxford University Press, 1953.

5. Morgenstern, O. (ed.), Economic Activity Analysis, New York: John Wiley and Sons, Inc., 1954.

6. National Bureau of Economic Research, Input-Output Analysis: An Appraisal, Studies in Income and Wealth, XVIII, Princeton University Press, 1955.

7. Wonnacott, R.J., Canadian-American Dependence: An Interindustry Analysis of Production and Prices, Amsterdam: The North-Holland Publishing Company, 1961.

IV. Journal Articles.

1. Caves, R.E., "The Inter-Industry Structure of the Canadian Economy," Canadian Journal of Economics and Political Science, XXIII (1957), pp. 313-330.

2. Degen, R.A., "U.S. Recessions and Selected Imports," Canadian Journal of Economics and Political Science, XXV (1959), pp. 180-189.

3. Dorfman, R., "The Nature and Significance of Input-Output," Review of Economics and Statistics, XXXVI (1954), pp. 121-133.

4. Hawkins, D. and Simon, H.A., "Note: Some Conditions of Macroeconomic Stability," Econometrica, XVII (1949), pp. 245-248.

5. Isard W., "Interregional and Regional Input-Output Analysis: A Model of a Space Economy," Review of Economics and Statistics, XXXIII (1951), pp. 318-328.

6. Leontief, W.W., "Output, Employment, Consumption, and Investment," Quarterly Journal of Economics, LVIII (1944), pp. 290-314.

7. Leontief, W.W., "Exports, Imports, Domestic Output and Employment," Quarterly Journal of Economics, LX (1946), pp. 175-193.

8. ______________, "The Structure of the U.S. Economy," Scientific American, April, 1965.

9. Matuszewski, T.I., Pitts, P.R., and Sawyer, J.A., "Alternative Treatments of Imports in Input-Output Models: A Canadian Study," Journal of the Royal Statistical Society, Series A, CXXVI (1963), pp. 410-432.

10. ______________, "Interindustry Estimates of Canadian Imports, 1949-1958," Canadian Political Science Association, Conference on Statistics, 1961 - Papers, University of Toronto Press, 1963, pp. 140-167.

11. ______________, "L'Ajustement Periodique des Systemes de Relations Inter-Industrielles, Canada, 1949-1958," Econometrica, XXXI (1963), pp. 90-110.

12. ______________, "Linear Programming Estimates of Changes in Input Coefficients," Canadian Journal of Economics and Political Science, XXX (1964), pp. 203-210.

13. ______________, "The Impact of Foreign Trade on Canadian Industries, 1956," Canadian Journal of Economics and Political Science, XXXI (1965), pp. 206-221.

14. Moses, L.N., "The Stability of Interregional Trading Patterns and Input-Output Analysis," The American Economic Review, XLV (1955), pp. 803-832.

15. Rosenbluth, G., "Changes in Canadian Sensitivity to U.S. Business Fluctuations," Canadian Journal of Economics and Political Science, XXXIII (1957) pp. 480-503.

16. ______________, "Changing Structural Factors in Canada's Cyclical Sensitivity, 1903-54," Canadian Journal of Economics and Political Science, XXIV (1958), pp. 21-43.

17. Sawyer, J.A., "Measurement of Inter-Industry Relationships in Canada," Canadian Journal of Economics and Political Science, XXI (1955), pp. 480-497.

APPENDIX A

DETAILED RESULTS

The postulated vectors of United States final demand are shown in Table XIII. They are obtained from vectors published on page 39 of Survey of Current Business, September 1965. The elements of each of the published vectors are multiplied by a constant factor to make them total one billion dollars. The vector of Personal consumption expenditure is represented by C ; I represents the vector of Gross private fixed capital formation, G_F the vector of Federal Government purchases, and G_S the vector of State and local government purchases.

Tables XIV, XV, XVI, and XVII show the vectors of United States activity levels, Canadian exports, Canadian activity levels, and Canadian imports generated by the postulated vectors of United States final demand.

In all tables, the figures may not add to the totals due to rounding.

TABLE XIII

POSTULATED INCREASES IN UNITED STATES FINAL DEMAND [F]

(thousands of United States dollars; 1958 prices)

INDUSTRY	C	I	G_F	G_S
1. LIVESTOCK+LIVESTOCK PRODUCTS	7278	-	-56	272
2. OTHER AGRICULTURAL PRODUCTS	8374	-	20020	667
3. FORESTRY+FISHERY PRODUCTS	969	-	-2557	-
4. AGRICULTURAL, FORESTRY+FISHERY SERVICES	-	-	840	-1675
5. IRON+FERROALLOY ORES MINING	-	-	-	-
6. NONFERROUS METAL ORES MINING	-	-	3582	-
7. COAL MINING	900	-	-	1505
8. CRUDE PETROLEUM+NATURAL GAS	-	-	-	-
9. STONE+CLAY MINING QUARRYING	59	-	187	-296
10. CHEMICAL+FERTILIZER MINERAL MINING	3	-	205	296
11. NEW CONSTRUCTION	-	592346	63216	297530
12. MAINTENANCE+REPAIR CONSTRUCTION	-	-	20169	82314
13. ORDNANCE+ACCESSORIES	545	-	42355	99
14. FOOD+KINDRED PRODUCTS	157755	-	989	6706
15. TOBACCO MANUFACTURES	14652	-	-	-
16. BROAD+NARROW FABRICS, YARN+THREAD	2455	-	933	223
17. MISC. FABRICATED TEXTILE PRODUCTS	2561	721	75	25
18. APPAREL	38491	-	746	2268
19. MISC. FABRICATED TEXTILE PRODUCTS	3796	-	1922	-
20. LUMBER+WOOD PRODS., EXCEPT CONTAINERS	514	96	-112	25
21. WOODEN CONTAINERS	-	-	37	-
22. HOUSEHOLD FURNITURE	8333	2020	466	1406
23. OTHER FURNITURE+FIXTURES	445	12790	485	3107
24. PAPER+ALLIED PRODS., EXCEPT CONTAINERS	2923	-	1343	147
25. PAPERBOARD CONTAINERS+BOXES	131	-	93	-
26. PRINTING+PUBLISHING	8429	-	1717	4267
27. CHEMICALS+SELECTED CHEMICAL PRODUCTS	734	-	13807	5967
28. PLASTICS+SYNTHETIC MATERIALS	34	-	93	-
29. DRUGS, CLEANING+TOILET PREPARATIONS	12783	-	2481	4414
30. PAINTS+ALLIED PRODUCTS	63	-	56	-
31. PETROLEUM REFINING+RELATED INDUSTRIES	25025	-	13545	9418
32. RUBBER+MISCELLANEOUS PLASTICS PRODUCTS	4513	833	2201	1850
33. LEATHER TANNING+INDUSTRIAL LEATHER PRODS.	-	-	-	-
34. FOOTWEAR+OTHER LEATHER PRODUCTS	8988	80	429	49
35. GLASS+GLASS PRODUCTS	448	-	37	-
36. STONE+CLAY PRODUCTS	738	-	93	99
37. PRIMARY IRON+STEEL MANUFACTURING	66	-	2107	25
38. PRIMARY NONFERROUS METALS MANUFACTURING	38	-	6156	-
39. METAL CONTAINERS	-	160	317	-
40. HEATING, PLUMBING+STRUCTURAL METAL PRODS.	241	11348	37	-
41. STAMPINGS, SCREW MACHINE PRODUCTS+BOLTS	858	-	1698	123
42. OTHER FABRICATED METAL PRODUCTS	1307	2661	2126	1135
43. ENGINES+TURBINES	434	9232	4459	74
44. FARM MACHINERY+EQUIPMENT	28	26767	93	420
45. CONSTRUCTION, MINING+OIL FIELD MACHINERY	-	21141	1493	519
46. MATERIALS HANDLING MACHINERY+EQUIPMENT	-	5642	2537	1234
47. METALWORKING MACHINERY+EQUIPMENT	107	18480	3190	123
48. SPECIAL INDUSTRY MACHINERY+EQUIPMENT	66	23529	560	741
49. GENERAL INDUSTRIAL MACHINERY+EQUIPMENT	-	16845	3731	123
50. MACHINE SHOP PRODUCTS	-	-	765	864
51. OFFICE, COMPUTING+ACCOUNTING MACHINES	200	16284	1399	2195
52. SERVICE INDUSTRY MACHINES	852	15307	1194	519
53. ELECTRIC INDUSTRIAL EQUIPMENT+APPARATUS	52	25917	3396	123
54. HOUSEHOLD APPLIANCES	8329	1491	392	25
55. ELECTRIC LIGHTING+WIRING EQUIPMENT	1079	401	317	197
56. RADIO, T.V.+COMMUNICATION EQUIPMENT	4699	16172	26066	1529
57. ELECTRONIC COMPONENTS ACCESSORIES	514	433	4366	-
58. MISCELLANEOUS ELECTRICAL MACHINERY+SUPPLIES	896	1330	1679	815
59. MOTOR VEHICLES+EQUIPMENT	31720	57300	5710	10799
60. AIRCRAFT+PARTS	93	5738	121264	-
61. OTHER TRANSPORTATION EQUIPMENT	2503	18881	12221	938
62. SCIENTIFIC+CONTROLLING INSTRUMENTS	1203	8527	10243	2121
63. OPTICAL, OPHTHALMIC+PHOTOGRAPHIC EQUIPT.	1613	2613	2556	371
64. MISCELLANEOUS MANUFACTURING	8743	4472	672	4414
65. TRANSPORTATION+WAREHOUSING	29851	8126	26849	9911
66. COMMUNICATIONS - EXCEPT RADIO+T.V.	13476	5802	3153	4685
67. RADIO+T.V. BROADCASTING	-	-	-	-
68. ELECTRIC, GAS, WATER+SANITARY SERVICES	27786	-	6492	11982
69. WHOLESALE+RETAIL TRADE.	212239	60056	12034	4512
70. FINANCE+INSURANCE	40737	-	19	4710
71. REAL ESTATE+RENTAL	137763	19378	2090	5745
72. HOTELS - PERSONAL+REPAIR SERVICES	32597	-	4590	2146
73. BUSINESS SERVICES	6509	-	9179	13683
74. RESEARCH+DEVELOPMENT	-	-	96597	-
75. AUTOMOBILE REPAIR+SERVICES	15125	-	2407	2407
76. AMUSEMENTS	11247	-	336	-1084
77. MEDICAL EDUCATION SERVICES	70492	-	2164	7668
78. FEDERAL GOVERNMENT ENTERPRISES	2179	-	1045	1653
79. STATE+LOCAL GOVERNMENT ENTERPRISES	1076	-	2108	148
80. GROSS IMPORTS OF GOODS+SERVICES	13290	256	50696	74
81. BUSINESS TRAVEL, ENTERTAINMENT+GIFTS	-	-	-	-
82. OFFICE SUPPLIES	-	-	1381	3255
DUMMY INDUSTRIES	8053	(13175)	368719	478755
TOTAL	1000000	1000000	1000000	1000000

TABLE XIV

INDUCED INCREASES IN UNITED STATES ACTIVITY LEVELS

$$\left[\hat{X} = (I - \hat{A})^{-1}F\right]$$

(thousands of United States dollars; 1958 prices)

INDUSTRY	C	I	G_F	G_S
1. LIVESTOCK+LIVESTOCK PRODUCTS	81663	5700	5350	5622
2. OTHER AGRICULTURAL PRODUCTS	61443	10725	25926	7350
3. FORESTRY+FISHERY PRODUCTS	3205	5887	-1465	3240
4. AGRICULTURAL, FORESTRY+FISHERY SERVICES	4341	955	1998	-1121
5. IRON+FERROALLOY ORES MINING	1379	7432	2964	2713
6. NONFERROUS METAL ORES MINING	1181	5632	8745	2311
7. COAL MINING	5117	6582	3195	5137
8. CRUDE PETROLEUM+NATURAL GAS	26979	15690	15679	15475
9. STONE+CLAY MINING QUARRYING	1355	12111	2476	6565
10. CHEMICAL+FERTILIZER MINERAL MINING	903	1063	1376	1175
11. NEW CONSTRUCTION	-	592346	63216	297530
12. MAINTENANCE+REPAIR CONSTRUCTION	35290	14709	29834	90583
13. ORDNANCE+ACCESSORIES	1124	2090	75507	504
14. FOOD+KINDRED PRODUCTS	211325	10551	7961	13877
15. TOBACCO MANUFACTURES	18530	485	295	213
16. BROAD+NARROW FABRICS, YARN+THREAD	33698	5324	6296	3884
17. MISC. FABRICATED TEXTILE PRODUCTS	6941	3753	2050	1419
18. APPAREL	48045	1309	1946	3252
19. MISC. FABRICATED TEXTILE PRODUCTS	6842	1312	2766	612
20. LUMBER+WOOD PRODS., EXCEPT CONTAINERS	8066	60991	10546	32851
21. WOODEN CONTAINERS	1134	897	497	398
22. HOUSEHOLD FURNITURE	8878	7009	2293	3529
23. OTHER FURNITURE+FIXTURES	730	15957	1369	4627
24. PAPER+ALLIED PRODS., EXCEPT CONTAINERS	24691	21749	12803	14668
25. PAPERBOARD CONTAINERS+BOXES	9280	6642	3925	3466
26. PRINTING+PUBLISHING	31875	20780	14027	21209
27. CHEMICALS+SELECTED CHEMICAL PRODUCTS	22248	23236	32765	22006
28. PLASTICS+SYNTHETIC MATERIALS	9349	8569	6149	4765
29. DRUGS, CLEANING+TOILET PREPARATIONS	18902	2443	5049	6330
30. PAINTS+ALLIED PRODUCTS	3371	5843	3129	6908
31. PETROLEUM REFINING+RELATED INDUSTRIES	43923	26685	27029	26184
32. RUBBER+MISCELLANEOUS PLASTICS PRODUCTS	14928	17711	12246	8805
33. LEATHER TANNING+INDUSTRIAL LEATHER PRODS.	2811	425	289	176
34. FOOTWEAR+OTHER LEATHER PRODUCTS	10246	372	662	202
35. GLASS+GLASS PRODUCTS	5188	5119	2618	2538
36. STONE+CLAY PRODUCTS	5241	62611	11782	32909
37. PRIMARY IRON+STEEL MANUFACTURING	21006	123031	45251	43491
38. PRIMARY NONFERROUS METALS MANUFACTURING	9830	52628	42053	20902
39. METAL CONTAINERS	6097	1285	1324	1170
40. HEATING, PLUMBING+STRUCTURAL METAL PRODS.	3305	77415	10483	35940
41. STAMPINGS, SCREW MACHINE PRODUCTS+BOLTS	6017	15690	12593	4522
42. OTHER FABRICATED METAL PRODUCTS	9625	32092	14477	13128
43. ENGINES+TURBINES	1603	16273	8085	1170
44. FARM MACHINERY+EQUIPMENT	1175	30040	1354	1091
45. CONSTRUCTION, MINING+OIL FIELD MACHINERY	882	28564	3526	2990
46. MATERIALS HANDLING MACHINERY+EQUIPMENT	256	10128	3513	3139
47. METALWORKING MACHINERY+EQUIPMENT	2398	28681	14065	2352
48. SPECIAL INDUSTRY MACHINERY+EQUIPMENT	1140	27404	2015	1700
49. GENERAL INDUSTRIAL MACHINERY+EQUIPMENT	1862	33685	10746	4135
50. MACHINE SHOP PRODUCTS	1604	5380	11667	2319
51. OFFICE, COMPUTING+ACCOUNTING MACHINES	1832	19911	3220	3617
52. SERVICE INDUSTRY MACHINES	1934	20783	2975	2634
53. ELECTRIC INDUSTRIAL EQUIPMENT+APPARATUS	2863	46361	16359	6032
54. HOUSEHOLD APPLIANCES	9398	6553	4664	2164
55. ELECTRIC LIGHTING+WIRING EQUIPMENT	2668	13473	6203	6602
56. RADIO, T.V.+COMMUNICATION EQUIPMENT	6727	21241	45928	2849
57. ELECTRONIC COMPONENTS ACCESSORIES	3266	8007	19184	1366
58. MISCELLANEOUS ELECTRICAL MACHINERY+SUPPLIES	2858	5054	4376	1941
59. MOTOR VEHICLES+EQUIPMENT	52342	91817	20288	18553
60. AIRCRAFT+PARTS	1715	11164	206098	1031
61. OTHER TRANSPORTATION EQUIPMENT	3993	23218	14737	2142
62. SCIENTIFIC+CONTROLLING INSTRUMENTS	3878	15462	20355	4731
63. OPTICAL, OPHTHALMIC+PHOTOGRAPHIC EQUIPT.	3296	3936	4636	1098
64. MISCELLANEOUS MANUFACTURING	14389	9788	4089	7931
65. TRANSPORTATION+WAREHOUSING	72786	74666	56416	44387
66. COMMUNICATIONS - EXCEPT RADIO+T.V.	24553	16370	9165	9964
67. RADIO+T.V. BROADCASTING	3515	3774	1928	2748
68. ELECTRIC, GAS, WATER+SANITARY SERVICES	55525	24180	21075	26070
69. WHOLESALE+RETAIL TRADE	264692	168750	52672	61455
70. FINANCE+INSURANCE	79154	25680	12176	18567
71. REAL ESTATE+RENTAL	190945	51997	22139	23555
72. HOTELS- PERSONAL+REPAIR SERVICES	38660	4899	7461	4375
73. BUSINESS SERVICES	55607	60998	30831	43973
74. RESEARCH+DEVELOPMENT	324	416	97070	171
75. AUTOMOBILE REPAIR+SERVICES	22906	8573	5684	6625
76. AMUSEMENTS	17289	1964	1622	-220
77. MEDICAL EDUCATION SERVICES	73847	2461	13343	8969
78. FEDERAL GOVERNMENT ENTERPRISES	10733	6065	4329	5080
79. STATE+LOCAL GOVERNMENT ENTERPRISES	12596	6351	6653	5165
80. GROSS IMPORTS OF GOODS+SERVICES	44085	34170	71030	16670
81. BUSINESS TRAVEL, ENTERTAINMENT+GIFTS	14916	17835	10960	7803
82. OFFICE SUPPLIES	2873	2373	2811	4562
TOTAL	1933187	2281273	1384924	1142569

TABLE XV

INDUCED INCREASES IN CANADIAN EXPORTS $[e = J(I-\hat{A})^{-1}F]$

(thousands of Canadian dollars; 1949 prices)

	INDUSTRY	C	I	G_F	G_S
1	Agriculture	539	57	103	47
2	Forestry	31	237	41	128
3	Fishing, hunting and trapping	146	267	-67	146
4	Metal mining & smelting & refining	492	2623	2033	1031
5	Coal, crude petroleum & natural gas	178	107	104	104
6	Non-metal mining & prospecting	26	69	25	40
7	Meat products	162	8	6	11
8	Dairy products	1	-	-	-
9	Fish processing	31	2	1	2
10	Fruit and vegetable preparations	3	-	-	-
11	Grain mill products	17	1	1	1
12	Bakery products	10	-	-	1
13	Carbonated beverages	-	-	-	-
14	Alcoholic beverages	205	10	8	13
15	Confectionery and sugar refining	2	-	-	-
16	Miscellaneous food preparations	17	1	1	1
17	Tobacco and tobacco products	-	-	-	-
18	Rubber products	5	6	4	3
19	Leather products	14	2	1	1
20	Textile products (except clothing)	18	8	5	3
21	Clothing (textile and fur)	6	1	1	1
22	Furniture	-	-	-	-
23	Wood products (except furniture)	247	1856	321	1000
24	Paper products	1564	1378	811	929
25	Printing, publishing & allied ind	7	4	3	4
26	Primary iron and steel	23	134	49	47
27	Agricultural implements	28	707	32	26
28	Iron & steel products, n.e.s.	48	177	67	41
29	Transportation equipment	18	43	196	7
30	Jewellery & silverware	2	1	1	1
31	Non-ferrous metal products, n.e.s.	42	223	177	89
32	Electrical apparatus & supplies	10	24	40	5
33	Non-metallic mineral products	42	404	80	216
34	Products of petroleum & coal	8	13	7	7
35	Chemicals and allied products	140	136	188	131
36	Misc. manufacturing industries	13	17	17	8
37	Construction	-	-	-	-
38	Transportation, storage & trade	-	-	-	-
39	Communication	-	-	-	-
40	Electric power, gas & water	84	36	32	39
41	Finance, insurance & real estate	-	-	-	-
42	Service industries	-	-	-	-
	TOTAL	4178	8556	4289	4086

TABLE XVI

INDUCED INCREASES IN CANADIAN ACTIVITY LEVELS

$[x = (I-a)^{-1}J(I-\hat{A})^{-1}F]$

(thousands of Canadian dollars; 1949 prices)

	INDUSTRY	C	I	G_F	G_S
1	Agriculture	742	171	164	118
2	Forestry	440	1081	317	617
3	Fishing, hunting and trapping	159	271	-66	149
4	Metal mining & smelting & refining	532	2783	2137	1090
5	Coal, crude petroleum & natural gas	353	439	273	264
6	Non-metal mining & prospecting	68	185	74	98
7	Meat products	195	24	19	21
8	Dairy products	2	1	-	-
9	Fish processing	33	3	2	3
10	Fruit and vegetable preparations	4	1	-	-
11	Grain mill products	89	17	16	12
12	Bakery products	10	1	-	1
13	Carbonated beverages	-	-	-	-
14	Alcoholic beverages	211	11	8	14
15	Confectionery and sugar refining	4	1	-	-
16	Miscellaneous food preparations	40	6	4	5
17	Tobacco and tobacco products	-	-	-	-
18	Rubber products	34	90	37	24
19	Leather products	23	11	5	4
20	Textile products (except clothing)	74	82	37	39
21	Clothing (textile and fur)	9	15	11	7
22	Furniture	4	8	5	4
23	Wood products (except furniture)	285	2016	375	1074
24	Paper products	1867	1682	984	1120
25	Printing, publishing & allied ind	32	49	27	27
26	Primary iron and steel	91	459	155	118
27	Agricultural implements	32	712	33	27
28	Iron & steel products, n.e.s.	202	724	265	197
29	Transportation equipment	134	306	367	123
30	Jewellery & silverware	4	5	2	3
31	Non-ferrous metal products, n.e.s.	66	292	224	116
32	Electrical apparatus & supplies	66	153	121	63
33	Non-metallic mineral products	88	516	135	271
34	Products of petroleum & coal	236	457	223	216
35	Chemicals and allied products	352	504	435	318
36	Misc. manufacturing industries	37	61	34	30
37	Construction	79	148	77	72
38	Transportation, storage & trade	530	1084	512	509
39	Communication	49	90	44	46
40	Electric power, gas & water	297	523	339	265
41	Finance, insurance & real estate	119	204	104	99
42	Service industries	98	187	101	89
	TOTAL	7689	15373	7600	7254

TABLE XVII

INDUCED INCREASES IN CANADIAN IMPORTS

$[m = \bar{m}(I-a)^{-1}J(I-\hat{A})^{-1}F]$

(thousands of Canadian dollars; 1949 prices)

	INDUSTRY	C	I	G_F	G_S
1	Agriculture	15	7	6	5
2	Forestry	4	22	4	12
3	Fishing, hunting and trapping	1	-	-	-
4	Metal mining & smelting & refining	6	31	17	10
5	Coal, crude petroleum & natural gas	76	133	67	67
6	Non-metal mining & prospecting	7	22	9	11
7	Meat products	5	2	2	1
8	Dairy products	-	-	-	-
9	Fish processing	1	1	-	-
10	Fruit and vegetable preparations	1	-	-	-
11	Grain mill products	-	-	-	-
12	Bakery products	-	-	-	-
13	Carbonated beverages	-	-	-	-
14	Alcoholic beverages	1	-	-	-
15	Confectionery and sugar refining	-	-	-	-
16	Miscellaneous food preparations	1	1	1	1
17	Tobacco and tobacco products	-	-	-	-
18	Rubber products	2	5	2	2
19	Leather products	1	1	-	-
20	Textile products (except clothing)	21	25	9	13
21	Clothing (textile and fur)	2	12	9	5
22	Furniture	-	1	-	-
23	Wood products (except furniture)	6	29	6	13
24	Paper products	19	22	12	14
25	Printing, publishing & allied ind	1	2	1	1
26	Primary iron and steel	15	72	24	15
27	Agricultural implements	4	4	1	-
28	Iron & steel products, n.e.s.	29	139	44	31
29	Transportation equipment	20	43	47	18
30	Jewellery & silverware	1	2	1	1
31	Non-ferrous metal products, n.e.s.	6	24	16	9
32	Electrical apparatus & supplies	5	12	10	5
33	Non-metallic mineral products	9	24	10	11
34	Products of petroleum & coal	14	42	28	19
35	Chemicals and allied products	43	63	46	35
36	Misc. manufacturing industries	4	8	4	4
37	Construction	-	-	-	-
38	Transportation, storage & trade	49	112	64	52
39	Communication	-	-	-	-
40	Electric power, gas & water	2	4	2	2
41	Finance, insurance & real estate	-	-	-	-
42	Service industries	-	-	-	-
	TOTAL	371	866	444	359

APPENDIX B

CLASSIFICATION OF 1958 EXPORTS FROM CANADA TO THE UNITED STATES ACCORDING TO THE CANADIAN AND UNITED STATES INDUSTRY SCHEMES

The appendix first discusses the derivation of allocated exports (i.e., exports that can be meaningfully assigned to export industries) from balance of payments data. Second, the procedure used in classifying the allocated exports is described.

As indicated in the text, the export data and information regarding industry classification are taken from seven sources. They are:

(i) D.B.S., 67-201, The Canadian Balance of International Payments, annual.

(ii) D.B.S., 65-201, Trade of Canada, vol. I, annual.

(iii) D.B.S., 65-202, Trade of Canada, vol. II, annual.

(iv) D.B.S., 12-501, Standard Industrial Classification Manual, 1948 edition.

(v) United States Standard Industrial Classification Manual, 1959.

(vi) D.B.S., 13-513, Supplement to the Inter-Industry Flow of Goods and Services, Canada 1949, p. 26, Table 10, "Industrial Classification For the 1949 Table of Inter-Industry Flow of Goods and Services".

(vii) "The Transactions Table of the 1958 Input-Output Study and Revised Direct and Total Requirements Data" Survey of Current Business, September, 1965, p. 83, Table: "Industry Numbering for the 1958 Input-Output Study".

1. Derivation of Allocated Exports.

The table on page 102 shows the relation of 1958 allocated exports to the United States to Total Merchandise Exports to the United States and to Total Current Receipts.

Following the procedure used in the Canadian input-output study of 1949,[1] the export values in the Agriculture and Fish processing industries were reduced by 10.6%. In all other industries the F.O.B. point of shipment valuation used in Trade of Canada was taken as equivalent to producers' prices.

On the other hand, two departures were made from the definition of allocated exports used in the 1949 study. Gold production available for export was excluded because it could not be assumed to vary with United States final demand. Its inclusion would have distorted output predictions for the important industry, Metal mining and smelting and refining (industry 4). Second, freight exports were excluded from allocated exports because they could not be meaningfully linked to a competitive United States industry.[2]

1. D.B.S., 13-513, Supplement to the Inter-Industry Flow of Goods and Services, Canada, 1949, p. 19.

2. If they were particularly desired in an application of the model, they could perhaps be related to total exports to the United States.

TABLE XVIII

ALLOCATED EXPORTS TO THE UNITED STATES, 1958

(millions of Canadian dollars; 1958 prices)

Total Current Receipts		6579
Total Current Receipts from the United States		4010
LESS:	Gold production available for export	160
	Travel expenditures	309
	Interest and dividends	100
	Freight and shipping	206
	Inheritances and immigrants funds	47
	All other current receipts	280
Total Merchandise Exports to the United States (per D.B.S. 67-201)		2908
LESS:	Re-exports	86
	Adjustment	-6
Total Merchandise Exports to the United States (per D.B.S. 65-201)		2828
LESS:	Unallocated items (settlers effects, gifts, contractors outfits, all other articles)	29
	Trade margins in Agriculture (1) and Fish Processing (9)	23
Total Allocated Exports to the United States		2776

2. The Industry Classification of Allocated Exports.

This section describes the procedure used to obtain the flows, τ_{ij}, ($i = 1,\ldots,42$; $j = 1,\ldots,82$), common to pairs of Canadian and United States industries.

First, 1958 exports to the United States were listed from Trade of Canada, vol. I (pp. 235-239). In this source they are broken down into nine major commodity groups, each of which is further refined into twenty or thirty sub-groups.

Second, each of the sub-groups was matched with one or more Standard Industrial Classification (or S.I.C.) code numbers for both the Canadian and United States classifications. This was the most difficult and most important operation. It was done by refering to the product index and the detailed industry descriptions in the S.I.C. manuals. At the same time, the classification tables (sources vi and vii) were used to allocate the exports to pairs of industries according to their assigned S.I.C. numbers.

Some of the sub-groups of commodities were found to belong to two or more industries in either or both of the input-output classifications. In these cases, Trade of Canada, vol. II was used to obtain a finer breakdown of the exports. These detailed export items were then assigned to industries in the same way as were the commodity sub-groups.

There were a few instances where either the commodity detail or industry descriptions were inadequate. Here it was necessary to make assumptions about the nature of the exports. Four examples are described below:

(i) Fur skins. This product could be an export of fur farms (part of industry 1) or of Fishing, hunting and trapping (3). The total value was assigned to industry 1.

(ii) Hides and Skins. They are not mentioned in either S.I.C. Manual. They were assumed to be products of the abbatoir and therefore of Meat processing (Canadian industry 7; United States industry 14).

(iii) Scrap. (mostly scrap metals). These were assumed to be products of the Primary iron and steel industries (Canadian, 28; United States 37).

(iv) Furniture. Here the export classification is not as detailed as the United States industry classification. All furniture exports were routed to the Household furniture industry (22), and none to Other furniture and fixtures (23).

Finally, with all the commodity exports allocated to Canadian and American industries, the elements τ_{ij} were calculated by aggregating the export values assigned to each pair of industries.

The non-zero elements τ_{ij} are listed in Table XIX following. In Tables XX and XXI the exports are shown classified according to the Canadian and United States industry schemes.

TABLE XIX

CANADIAN EXPORTS TO THE UNITED STATES, 1958
CLASSIFICATION INTO ELEMENTS, τ_{ij}

(Canadian dollars; 1958 prices)

Element (i,j)	Value
1,1[a]	106,012,793
1,2[a]	71,803,227
6,2	7,505,873
3,3	88,079,896
4,5	81,062,304
4,6	35,824,881
5,7	2,699,852
5,8	73,043,757
6,9	10,659,774
33,9	48,167,316
6,10	2,910,426
28,13	6,077
35,13	17,217
1,14[a]	5,467,774
7,14	61,480,513
8,14	505,899
9,14[a]	12,700,884
10,14	835,319
11,14	4,163,384
12,14	2,317,660
14,14	68,289,675
15,14	467,470
16,14	5,541,544
35,14	866,849
36,14	492,628
17,15	70,061
20,16	1,117,134
20,17	5,366,008
21,18	1,279,078
2,20	40,300,681
23,20	272,597,731
23,21	559,338
22,22	221,875
24,24	846,156,107
24,25	79,168
25,26	3,299,726
35,27	73,299,560
35,28	1,366,849
35,29	3,377,839
35,30	1,355,210
34,31	3,209,080
18,32	3,916,857
19,33	5,282,445
19,34	1,095,829
33,35	899,121
33,36	30,713,636
26,37	26,582,151
28,37	3,048,119
34,37	1,545,570
4,38	503,776,695
31,38	51,946,544
28,40	203,699
28,41	55,745
28,42	5,630,442
31,42	681,999
28,43	136,505
27,44	90,334,809
28,45	807,774
28,46	214,700
28,47	1,158,215
28,48	2,213,635
28,49	664,954
28,50	4,806,545
28,51	5,834,981
28,52	14,266
32,53	10,673
28,54	16,723
32,54	13,883
32,55	97,534
32,56	5,036,679
32,58	2,913,008
28,59	17,717,540
29,59	9,224,588
29,60	14,977,096
29,61	945,801
36,62	2,371,032
36,63	1,192,902
20,64	5,177
21,64	781,415
30,64	943,156
36,64	3,446,055
40,68	30,561,313
Total	2,776,398,208

a. Elements adjusted for trade margins.

TABLE XX

CANADIAN EXPORTS TO THE UNITED STATES, 1958

CANADIAN INDUSTRY CLASSIFICATION

(Canadian dollars; 1958 prices)

	INDUSTRY	VALUE
1	Agriculture	183,283,794
2	Forestry	40,300,681
3	Fishing, hunting and trapping	88,079,896
4	Metal mining & smelting & refining	620,663,880
5	Coal, crude petroleum & natural gas	75,743,609
6	Non-metal mining & prospecting	21,076,073
7	Meat products	61,480,513
8	Dairy products	505,899
9	Fish processing	12,700,884
10	Fruit and vegetable preparations	835,319
11	Grain mill products	4,163,384
12	Bakery products	2,317,660
13	Carbonated beverages	-
14	Alcoholic beverages	68,289,675
15	Confectionery and sugar refining	467,470
16	Miscellaneous food preparations	5,541,544
17	Tobacco and tobacco products	70,061
18	Rubber products	3,916,857
19	Leather products	6,378,274
20	Textile products (except clothing)	6,488,319
21	Clothing (textile and fur)	2,060,493
22	Furniture	221,875
23	Wood products (except furniture)	273,157,069
24	Paper products	846,235,275
25	Printing, publishing & allied ind	3,299,726
26	Primary iron and steel	26,582,151
27	Agricultural implements	90,334,809
28	Iron & steel products, n.e.s.	42,529,880
29	Transportation equipment	25,147,485
30	Jewellery & silverware	943,156
31	Non-ferrous metal products, n.e.s.	52,628,543
32	Electrical apparatus & supplies	8,071,777
33	Non-metallic mineral products	79,780,073
34	Products of petroleum & coal	4,754,650
35	Chemicals and allied products	80,283,524
36	Misc. manufacturing industries	7,502,617
37	Construction	-
38	Transportation, storage & trade	-
39	Communication	-
40	Electric power, gas & water	30,561,313
41	Finance, insurance & real estate	-
42	Service industries	-
	TOTAL	2,776,398,208

TABLE XXI

CANADIAN EXPORTS TO THE UNITED STATES, 1958

UNITED STATES CLASSIFICATION

(Canadian dollars; 1958 prices)

INDUSTRY	VALUE
1.LIVESTOCK+LIVESTOCK PRODUCTS	106012793
2.OTHER AGRICULTURAL PRODUCTS	79309100
3.FORESTRY+FISHERY PRODUCTS	88079896
4.AGRICULTURAL,FORESTRY+FISHERY SERVICES	-
5.IRON+FERROALLOY ORES MINING	81062304
6.NONFERROUS METAL ORES MINING	35824881
7.COAL MINING	2699852
8.CRUDE PETROLEUM+NATURAL GAS	73043757
9.STONE+CLAY MINING QUARRYING	58827090
10.CHEMICAL+FERTILIZER MINERAL MINING	2910426
11.NEW CONSTRUCTION	-
12.MAINTENANCE+REPAIR CONSTRUCTION	-
13.ORDNANCE+ACCESSORIES	23294
14.FOOD+KINDRED PRODUCTS	163129599
15.TOBACCO MANUFACTURES	70061
16.BROAD+NARROW FABRICS,YARN+THREAD	1117134
17.MISC.FABRICATED TEXTILE PRODUCTS	5366008
18.APPAREL	1279078
19.MISC.FABRICATED TEXTILE PRODUCTS	-
20.LUMBER+WOOD PRODS.,EXCEPT CONTAINERS	312898412
21.WOODEN CONTAINERS	559338
22.HOUSEHOLD FURNITURE	221875
23.OTHER FURNITURE+FIXTURES	-
24.PAPER+ALLIED PRODS.,EXCEPT CONTAINERS	846156107
25.PAPERBOARD CONTAINERS+BOXES	79168
26.PRINTING+PUBLISHING	3299726
27.CHEMICALS+SELECTED CHEMICAL PRODUCTS	73299560
28.PLASTICS+SYNTHETIC MATERIALS	1366849
29.DRUGS,CLEANING+TOILET PREPARATIONS	3377839
30.PAINTS+ALLIED PRODUCTS	1355210
31.PETROLEUM REFINING+RELATED INDUSTRIES	3209080
32.RUBBER+MISCELLANEOUS PLASTICS PRODUCTS	3916857
33.LEATHER TANNING+INDUSTRIAL LEATHER PRODS.	5282445
34.FOOTWEAR+OTHER LEATHER PRODUCTS	1095829
35.GLASS+GLASS PRODUCTS	899121
36.STONE+CLAY PRODUCTS	30713636
37.PRIMARY IRON+STEEL MANUFACTURING	31175840
38.PRIMARY NONFERROUS METALS MANUFACTURING	555723239
39.METAL CONTAINERS	-
40.HEATING,PLUMBING+STRUCTURAL METAL PRODS.	203699
41.STAMPINGS,SCREW MACHINE PRODUCTS+BOLTS	55745
42.OTHER FABRICATED METAL PRODUCTS	6312441
43.ENGINES+TURBINES	136505
44.FARM MACHINERY+EQUIPMENT	90334809
45.CONSTRUCTION,MINING+OIL FIELD MACHINERY	807774
46.MATERIALS HANDLING MACHINERY+EQUIPMENT	214700
47.METALWORKING MACHINERY+EQUIPMENT	1158215
48.SPECIAL INDUSTRY MACHINERY+EQUIPMENT	2213635
49.GENERAL INDUSTRIAL MACHINERY+EQUIPMENT	664954
50.MACHINE SHOP PRODUCTS	4806545
51.OFFICE,COMPUTING+ACCOUNTING MACHINES	5834981
52.SERVICE INDUSTRY MACHINES	14226
53.ELECTRIC INDUSTRIAL EQUIPMENT+APPARATUS	10673
54.HOUSEHOLD APPLIANCES	30606
55.ELECTRIC LIGHTING+WIRING EQUIPMENT	97534
56.RADIO,T.V.+COMMUNICATION EQUIPMENT	5036679
57.ELECTRONIC COMPONENTS ACCESSORIES	-
58.MISCELLANEOUS ELECTRICAL MACHINERY+SUPPLIES	2913008
59.MOTOR VEHICLES+EQUIPMENT	26942128
60.AIRCRAFT+PARTS	14977096
61.OTHER TRANSPORTATION EQUIPMENT	945801
62.SCIENTIFIC+CONTROLLING INSTRUMENTS	2371032
63.OPTICAL,OPHTHALMIC+PHOTOGRAPHIC EQUIPT.	1192902
64.MISCELLANEOUS MANUFACTURING	5175803
65.TRANSPORTATION+WAREHOUSING	-
66.COMMUNICATIONS - EXCEPT RADIO+T.V.	-
67.RADIO+T.V. BROADCASTING	-
68.ELECTRIC,GAS,WATER+SANITARY SERVICES	30561313
69.WHOLESALE+RETAIL TRADE.	-
70.FINANCE+INSURANCE	-
71.REAL ESTATE+RENTAL	-
72.HOTELS - PERSONAL+REPAIR,SERVICES	-
73.BUSINESS SERVICES	-
74.RESEARCH+DEVELOPMENT	-
75.AUTOMOBILE REPAIR+SERVICES	-
76.AMUSEMENTS	-
77.MEDICAL EDUCATION SERVICES	-
78.FEDERAL GOVERNMENT ENTERPRISES	-
79.STATE+LOCAL GOVERNMENT ENTERPRISES	-
80.GROSS IMPORTS OF GOODS+SERVICES	-
81.BUSINESS TRAVEL,ENTERTAINMENT+GIFTS	-
82.OFFICE SUPPLIES	-
TOTAL	**2,776,398,208**

APPENDIX C

THE TECHNIQUE USED IN UPDATING THE CANADIAN INPUT-OUTPUT MATRICES

The updating procedure used by T. I. Matuszewski, P. R. Pitts, and J. A. Sawyer in obtaining the 1959 direct requirements matrix is made more complex by the fact that the 1949 matrix was updated to apply to 1956 and the resulting matrix updated to 1959. The import matrix was updated to 1956 by a method quite similar to that used for the production matrix, so its updating will not be described.

Two methods were combined in updating the 1949 direct requirements matrix, a^{49}, to 1956.

The first method used was to re-estimate coefficients individually. This was done for several coefficients (eight in the production matrix) that described processes affected by two important technological changes. The changes corrected for were the shift from natural to synthetic fibres in textiles and clothing, and the shift from manufactured to natural gas.

The second method used by Sawyer was the theoretically less sound but nevertheless effective one of multiplying each row of a^{49} by a constant proportionality factor. The factor generally used is

$$d_i = \frac{x_i^1 - y_i^1}{\sum_{j=1}^{m} a_{ij}^o \, x_j^1}$$

where o and 1 are the base year and current year respectively. It represents the ratio between two estimates of the intermediate output of product i . The first estimate is made by direct National Accounts estimates of industry output and final demand. The second estimate is made by applying the old production coefficients, a^{o}_{ij} , to the new estimated total outputs, x^{1}_{j}

The proportionality factor actually used was a modification of the one shown, designed to avoid influencing the previously re-estimated coefficients. The flows to be represented by the independently estimated coefficients were temporarily set equal to zero and the factor applied to the remaining flows. Thus the coefficient that was applied to the i^{th} row of a^{49} was

$$d^{56}_{i} = \frac{x^{56}_{i} - y^{56}_{i} - C^{56}_{i}}{\sum_{j=1}^{42} a^{49}_{ij} x^{56}_{j}}$$

where the C^{56}_{i} represents the blocked flows in the i^{th} row.

The application of the two methods was completed by placing the independently estimated coefficients in $d^{56} a^{49}$ where d^{56} is the diagonal matrix of d^{56}_{i} .

Finally, a^{59} was formed using the second method alone so that

$$a^{59} = d^{59} a^{56}$$

where the principal elements of d^{59} are

$$d^{59}_{i} = \frac{x^{59}_{i} - y^{59}_{i}}{\sum_{j=1}^{42} a^{56}_{ij} x^{59}_{j}} \qquad (i = 1,\ldots,42)$$

APPENDIX D

PRICE DEFLATION OF THE CANADIAN EXPORT VECTOR

This appendix explains how the p_i , the diagonal elements of P , were obtained.

The primary source of information was D.B.S. Number 65-205, Review of Foreign Trade, annual, Table XX, "Prices of Domestic Exports by Groups and Selected Commodities". The prices in this table are indexes based on the year 1948. If P_k^{58} is the index for export commodity k for 1958, then

$$P_k^{58} = \frac{1958 \text{ price}}{1948 \text{ price}} \times 100.0 \quad .$$

Accordingly, the first step in deriving p_i was to combine the 1949 and 1958 export commodity price indexes obtaining

$$P_k = \frac{P_k^{49}}{P_k^{58}}$$

Next the commodity indexes, P_k , were allocated to Canadian industries. In most cases the index of a single commodity (or commodity group) was considered sufficient to represent the price experience of an export industry. For these industries, $p_i = P_k$. Such was the case, for instance, with whisky (industry 14), leather (19), and farm machinery (27). In other cases, commodity indexes were aggregated to obtain the industry index. Here, 1958 export values taken from Trade of Canada were used to weight the commodity indexes. For these industries $p_i = \sum_{k=1}^{\ell} P_k V_k$ where V_k is the 1958 export value of commodity k, and ℓ the number of commodity indexes aggregated.

The products of a few industries were not explicit in the Review of Foreign Trade table and, for most of these, domestic indexes were used. The source of these indexes was D.B.S. number 62-002, Prices and Price Indexes, annual, "Wholesale Price Index Numbers Showing Component Detail (1935-39 = 100.0)". It is Table IV is editions covering the years to 1952 and Table III in later editions. Again the ratio of two indexes was used to relate 1949 prices to 1958 prices.

Finally, for the products:- uranium (4), and electric power and natural gas (40), no price information could be found for years prior to 1958. In the first case this is because information on uranium exports was classified before 1958. In the second it is because exports before 1957 or 1958 were insignificant. For these commodities, a price deflator of 100.0 was assumed.

The values and derivation of the indexes p_i $(i = 1,..,42)$ are shown in Table XXII following.

TABLE XXII

DERIVATION OF EXPORT PRICE INDEXES FOR THE CANADIAN INDUSTRIES

Industry	Price Index,	Source	Component Commodities (and Indexes)
1	100.8	(a)	barley (140.0), oats (106.8), rye (142.4), wheat (126.2), dairy cattle (80.7), slaughter cattle (93.8), fur skins (93.6), eggs (142.9)
2	81.5	(a)	pulpwood
3	74.9	(a)	fish
4	80.4	(a) (c)	iron ore (77.7), aluminum (66.9), copper (83.7), lead (190.3), silver (85.2), platinum (133.7), zinc (138.9), nickel (52.6); uranium (100.0)
5	81.8	(a) (b)	coal (78.4); petroleum (81.9)
6	97.0	(a) (b)	asbestos (68.2); lime and cement (96.8), stone (99.4)
7	81.5	(a)	beef and veal, fresh
8	88.4	(a)	milk processed
9	74.9	(a)	fish
10	99.3	(b)	miscellaneous foods
11	127.5	(a)	wheat flour
12	127.5	(a)	wheat flour
13	-		no exports
14	93.0	(a)	whisky
15	94.5	(b)	sugar
16	99.3	(b)	miscellaneous foods
17	87.3	(a)	tobacco
18	64.2	(a)	rubber
19	71.2	(a)	leather
20	71.3	(a)	fibres and textiles

TABLE XXII (continued)

Industry	Price Index	Source	Component Commodities
21	71.3	(a)	fibres and textiles
22	96.2	(a)	planks and blocks
23	94.2	(a)	planks and blocks (96.2), shingles (72.4), plywood (100.6)
24	78.8	(a)	woodpulp (86.3), newsprint (76.0)
25	76.0	(a)	newsprint
26	79.5	(a)	pig iron, ferro-alloys
27	66.9	(a)	farm machinery
28	75.4	(a)	machinery (non-farm)
29	77.4	(a)	autos, trucks, and parts
30	85.2	(a)	silver
31	81.4	(a)	aluminum (66.9), copper (83.7), lead (190.3), zinc (138.9)
32	86.7	(a)	miscellaneous manufacturing
33	66.0	(a)	abrasives (61.4), asbestos (68.2)
34	80.4	(a)	coal (78.4)
35	91.5	(a)	chemicals and fertilizers
36	86.7	(a)	miscellaneous manufacturing
37	-		no exports
38	-		no exports
39	-		no exports
40	100.0	(c)	electric power and natural gas
41	-		no exports
42	-		no exports

Key to Sources

(a) Review of Foreign Trade
(b) Prices and Price Indexes
(c) No price information for years previous to 1958.