A LASER-EXCITED OPTICAL RESONATOR FOR ELECTRON DENSITY MEASUREMENTS ON A Z-PINCH PLASMA

by

SIDNEY SYLVESTER MEDLEY

B.Sc., University of British Columbia, 1963
M.Sc., University of British Columbia, 1965

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

in the department of PHYSICS

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

May, 1968
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and Study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Physics

The University of British Columbia
Vancouver 8, Canada

Date May 9, 1968.
ABSTRACT

A technique which employs a laser-excited optical resonator has been developed to measure the electron density distribution, both temporal and spatial, in the collapse stage of a fast Z-pinch discharge.

The resonator device has an experimentally determined time resolution of better than 0.05 μsec and is suitable for measuring electron densities in excess of $5 \cdot 10^{16} / L \lambda \text{ cm}^{-3}$, where L is the length of the plasma in cm and $\lambda$ is the wavelength of the laser in microns. A novel feature of this instrument is the use of an unstable optical resonator.

The technique is applied to a discharge in argon at filling pressures of 100 and 1000 μHg. The temporal and radial electron density distributions obtained exhibit several interesting features which are discussed from the point of view of the collapse process.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>i</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>vii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>PART I. THE LASER-EXCITED OPTICAL RESONATOR DEVICE</td>
<td>8</td>
</tr>
<tr>
<td>CHAPTER I. THE RESONATOR DEVICE - THEORY</td>
<td></td>
</tr>
<tr>
<td>1.0 Introduction</td>
<td>9</td>
</tr>
<tr>
<td>1.1 The Optical Resonator</td>
<td>11</td>
</tr>
<tr>
<td>1.2 The Modulation Effect</td>
<td>16</td>
</tr>
<tr>
<td>1.3 Maximum Modulation Rate</td>
<td>19</td>
</tr>
<tr>
<td>1.4 Coupling between the Laser and the Resonator</td>
<td>22</td>
</tr>
<tr>
<td>1.5 Deduction of Electron Density from the Modulation Effect</td>
<td>23</td>
</tr>
<tr>
<td>CHAPTER II. THE RESONATOR DEVICE - APPARATUS</td>
<td></td>
</tr>
<tr>
<td>2.0 Introduction</td>
<td>27</td>
</tr>
<tr>
<td>2.1 Description of the Resonator Device</td>
<td>27</td>
</tr>
<tr>
<td>2.2 Frequency Response and Sensitivity</td>
<td>33</td>
</tr>
<tr>
<td>2.3 Elimination of Optical Pick-up</td>
<td>39</td>
</tr>
<tr>
<td>PART II. THE PLASMA</td>
<td>41</td>
</tr>
<tr>
<td>CHAPTER III. THE Z-PINCH DISCHARGE</td>
<td></td>
</tr>
<tr>
<td>3.0 Discharge Apparatus</td>
<td>42</td>
</tr>
<tr>
<td>3.1 Measurement of the Discharge Current</td>
<td>47</td>
</tr>
</tbody>
</table>
CHAPTER IV. DYNAMICS OF THE Z-PINCH

4.0 Introduction 51
4.1 Plasma Symmetry and Electrode Effects 51
4.2 Collapse Curves 55

CHAPTER V. EFFECT OF RADIAL REFRACTIVITY GRADIENTS IN THE PLASMA

5.0 Introduction 57
5.1 Deflection of the Laser Beam in the Resonator due to a Radial Refractivity Gradient 57
5.2 Reduction of the Loss of the Laser Beam due to Deflection 63
5.3 Effect of the Reduction Procedure on the Validity of the Electron Density Measurement 66

PART III. EXPERIMENTAL RESULTS 74

CHAPTER VI. ELECTRON DENSITY MEASUREMENTS USING THE RESONATOR TECHNIQUE

6.0 Introduction 75
6.1 Experimental Procedure 75
6.2 Temporal and Radial Electron Density Distributions 84
6.3 Limitations of the Measurements and Sources of Error 102

CHAPTER VII. DISCUSSION OF RESULTS

7.0 Introduction 110
7.1 The Resonator 111
7.2 The Plasma 115
7.3 Future Work 120

APPENDIX. SPECTROSCOPIC MEASUREMENTS 122
REFERENCES 129
LIST OF TABLES

TABLE I Specifications of the Components of the Resonator Device 28

TABLE II Discharge Apparatus Specifications 43

TABLE III Scaling Check (100 μHg Argon) 105
| Figure 1.1 | Discharge Configuration | 2 |
| Figure 1.2 | Schematic Electron Density Profiles | 5 |
| Figure 1.3 | Schematic of the Experimental Arrangement | 12 |
| Figure 1.4 | Illustrating Variation of the Power Transmission Coefficient | 19 |
| Figure 2.1 | Schematic of the Laser-excited Optical Resonator Arrangement | 29 |
| Figure 2.2 | External Mirror Mount | 30 |
| Figure 2.3 | Photomultiplier Circuit Diagram | 31 |
| Figure 2.4 | Photomultiplier Risetime | 32 |
| Figure 2.5 | Rotating Quartz Plate | 34 |
| Figure 2.6 | Calibration of the Rotating Quartz Plate | 35 |
| Figure 2.7 | Experimental Arrangement for Measurement of Resonator Frequency Response | 37 |
| Figure 2.8 | Frequency Response of the Optical Resonator System | 38 |
| Figure 3.1 | Photograph of the Experimental Arrangement | 42 |
| Figure 3.2 | Z-pinch Discharge Circuit | 44 |
| Figure 3.3 | Discharge Vessel | 46 |
| Figure 3.4 | Integrating Circuit | 48 |
| Figure 3.5 | Discharge Current Waveforms | 49 |
| Figure 4.1 | Plasma Symmetry and Electrode Effects | 52 |
| Figure 4.2 | Collapse Curves | 56 |
| Figure 5.1 | Notation for the Beam Deflection Calculation | 58 |
| Figure 5.2 | Graphical Representation of the Conditions for Containment of the Laser Beam | 62 |
| Figure 5.3 | Glass Tube Mounting Assembly | 65 |
| Figure 5.4 | Beam Motion across Mirror Surface | 67 |
| Figure 5.5 | Effect of the Glass Tubes on the Plasma | 69 |
| Figure 6.1 | Typical Fringe Patterns | 81 |
| Figure 6.2 | Interpretation of the Fringe Pattern | 82 |
| Figure 6.3 | Electron Density Plots for 100 μHg Argon | 87 |
| Figure 6.4 | Electron Density Plots for 1000 μHg Argon | 95 |
| Figure 6.5 | Representative Beam Path Lengths | 103 |
| Figure 6.6 | Sensitivity of the Electron Density Measurements | 104 |
| Figure 7.1 | Illustrating Discussion of 100 μHg Results | 116 |
| Figure 7.2 | Illustrating Discussion of 1000 μHg Results | 118 |
| Figure A.1 | Experimental Arrangement for Stark-broadening Measurements | 124 |
| Figure A.2 | Typical Monochromator-Photomultiplier Signal | 125 |
| Figure A.3 | Stark-broadening Electron Density Results | 127 |
ACKNOWLEDGMENTS

I wish to thank Dr. F. L. Curzon for suggesting this project and for the supervision I received while carrying out this investigation.

The technical assistance of Mr. J. Dooyeweerd, Mr. D. Sieberg, Mr. J. Lees, Mr. P. Hass, Mr. T. Knopp and Mr. D. Stonebridge in the construction of the apparatus is gratefully acknowledged. Also, I thank the members of the plasma group, in particular Dr. J. H. Williamson, Dr. A. Folkierski and Mr. H. D. Campbell, for their interest and suggestions during the course of this work. Miss Jo-Anne McQuatt's assistance in preparing the manuscript is greatly appreciated.

I gratefully acknowledge the financial assistance provided by the National Research Council of Canada.
INTRODUCTION

The dynamics of the Z-pinch discharge has been investigated at this laboratory using magnetic probe and photographic techniques. For further understanding of the pinch mechanism, it is desirable to know the distribution of the electron density in the collapse stage of the discharge.

In this thesis the distribution, both temporal and spatial, of the number density of free electrons was measured in the collapse stage of a fast Z-pinch discharge in argon. The primary diagnostic technique for measurement of the electron density employed an optical resonator excited by CW He-Ne laser operated at a wavelength of 0.63μ. This technique was supplemented by photographic and spectroscopic measurements.

A Z-pinch discharge provides a highly ionized plasma which, in the collapse stage, has a well defined geometry and is very reproducible. The plasma is generated by discharging energy from a capacitor bank through a volume of argon. The gas is contained in a discharge vessel which consists, essentially, of a Pyrex tube with an electrode inserted at each end (Figure 1.1). The plasma takes the shape of a cylindrical shell axisymmetric with the discharge vessel. The shell initially forms at the wall of the discharge vessel and subsequently collapses radially under the Lorentz force \( \mathbf{F}_r = \mathbf{J}_z \times \mathbf{B}_\theta \). Here \( \mathbf{B}_\theta \) represents the
Figure 1.1 Discharge Configuration

R = Radius of discharge vessel
r = Radius of plasma column
s = Length of plasma on which measurement is performed

$J_z$ = Axial current density

$B_{\theta}$ = Azimuthal magnetic field

$F_r = \text{Radial (} J_z \times B_{\theta} \text{) force}$
azimuthal magnetic field associated with the axial current density \( J_z \). When the plasma is constricted to a minimum radius it is said to be 'pinched'.

The time interval between initiation of the discharge current and the maximum constriction of the plasma, hereafter called the 'time to pinch', becomes longer as the density of the filling gas is increased. In order to provide a suitably long time to pinch in which to perform measurements, a discharge in argon was used, rather than in hydrogen or helium. Also, a discharge in argon produces a highly luminous plasma which facilitates photographic investigations.

In the present experiment, a capacitor bank energy of 3.6 kJ and a charging voltage of 12 kV were used. The discharge vessel was 62 cm in length and 15 cm in diameter. At argon filling pressures of 100 and 1000 \( \mu \text{Hg} \), this gave times to pinch of 5 and 13 \( \mu \text{sec} \) respectively.

Several techniques are known for the measurement of electron density in a plasma. In order to determine the spatial and temporal distribution of electron density in the collapse stage of a fast discharge, the measuring technique used must provide sub-microsecond time resolution and spatial resolution an order of magnitude smaller than the thickness of the plasma shell (estimated to be approximately 2 cm by magnetic probe measurements). The laser-excited optical resonator technique satisfies both of these
requirements and furthermore possesses several other advantages: for example, the technique is not dependent on thermal equilibrium conditions in the plasma. In view of these considerations, the laser-excited optical resonator technique was chosen as the primary method for electron density measurement. Electron densities were also obtained by measurement of the Stark-broadening of selected plasma emission lines.

In brief, the laser-excited optical resonator technique depends on changes in the condition for resonance of the laser radiation in the optical resonator brought about by the presence of the plasma situated in the resonator. The modulation of the resonant condition is monitored by using a photomultiplier to detect the radiation transmitted through the resonator.

Measurement on a single discharge yields the time variation in the spatial average of the electron density along the path of the laser beam which propagates parallel to the discharge axis. The spatial resolution in the radial direction depends on the diameter of the laser beam and is typically 2 mm or less. Since the thickness of the plasma shell is of the order of 2 cm, the measurement provides spatial resolution in the radial direction but not in the axial direction. Because the collapse stage of the discharge is very reproducible, it is possible to combine measurements at different radii to obtain the radial distribution of electron density at a specified time after
current initiation. Figure 1.2 illustrates schematically measurements at only a few radial positions. However, this is sufficient to demonstrate that plotting the points of intersection of the curves with any $t = \text{constant}$ plane yields the radial electron density distribution in the discharge at that time.

Figure 1.2  Schematic Electron Density Profiles

\[ n_e = \text{number density of free electrons} \]

\[ r = \text{radial position in the discharge vessel} \]

\[ t = \text{time after initiation of the discharge current} \]
It is perhaps worth remarking that electron density measurement after the time of pinch would be complicated by the onset of instabilities which destroy the regular geometry of the plasma. Such measurements were not attempted in the present investigation.

The most serious difficulty associated with these measurements was due to refraction of the laser beam by electron density gradients transverse to the direction of propagation. In a first approximation, such gradients act like a prism to deflect the laser beam out of alignment in the resonator. Since the angle of deflection increases with the length of the region in which the gradient, presumed constant in the axial direction, exists, the deflection can be reduced to a tolerable level by shortening the path of the beam through the plasma. This was accomplished by inserting two glass tubes into the plasma with their axes parallel to the axis of the discharge as shown in Figure 1.1. The laser beam thus propagated within the tubes and was shielded from the plasma except in a region of variable length, s, separating the ends of the tubes. The effect of the tubes on the plasma is discussed in section 5.3 but it can be stated now that for most conditions the measurements can be regarded as valid.

Using the above technique, the distribution in time and space of the electron density was measured in the collapse stage of a Z-pinch discharge in argon for initial
filling pressures of 100 and 1000 μHg. Electron densities at 1000 μHg were also measured using Stark-broadening of the Hα and AII 5062 Å emission lines. The results obtained using the two methods agree within the limits of experimental error in the common region of measurement.

The laser-excited optical resonator technique was introduced by Ashby and Jephcott in 1963 and has since been modified by others. In this investigation a further modification of the device is made by the use of an unstable optical resonator which facilitates measurements in the presence of radial gradients of electron density.

In the past, the technique has been applied primarily to the measurement of electron densities which are either high and slowly varying (e.g. afterglow plasmas) or low and rapidly changing (e.g. shock tubes). However, the present author has successfully applied the technique to the measurement of high and rapidly varying electron density in the collapse stage of a fast Z-pinch discharge. The results obtained contribute to the study of the dynamics of the Z-pinch undertaken by others using magnetic probe and photographic techniques.
PART I

THE LASER-EXCITED OPTICAL RESONATOR
CHAPTER I. THE RESONATOR DEVICE - THEORY

1.0 Introduction

This chapter describes the principle of operation of the laser-excited optical resonator device used in this investigation. As mentioned earlier, the device has undergone several modifications since its introduction in 1963. Whereas previous versions of the device employed stable optical resonators, a further modification is introduced here through the use of an unstable resonator.

The theory of unstable resonators is apparently not well developed and extension of the theory is beyond the scope of this thesis. In describing the operation of the optical resonator the existing theory of unstable optical resonators will be quoted where applicable and extensive use will be made of the close analogy between the present device and another version introduced by Wheeler and Dangor\textsuperscript{6} and further developed by Gerardo and Verdeyen.\textsuperscript{7}

It is of interest to note that in the literature the laser-excited optical resonator is usually called a "laser interferometer". When the device was first introduced, this terminology was appropriate. At that time, the device was used to measure plasma refractive indices which varied relatively slowly in time. In these applications, the beam fed back from the resonator modified the field distribution in the laser cavity and this interference gave rise to changes in the output intensity of the laser. However, when
the refractive index of the plasma changes rapidly in time, as is the case here, the laser is unable to respond to the optical feedback and its output intensity remains effectively constant. Therefore, in the present investigation, it is more appropriate to describe the device as an optical resonator which is excited by laser radiation of constant intensity.

In general, the laser-excited optical resonator device may be used to measure time variations in the index of refraction of any optically transparent medium. Here it is used to measure the time varying refractive index of a pulsed plasma.

The technique depends on the effect of the plasma on the condition for resonance of the laser radiation in the optical resonator. Briefly, the essential features of the experimental arrangement are as follows. The radiation from the laser is transmitted through the resonator to a photodetector. The discharge is situated inside the resonator with the discharge and resonator axes parallel. In this application, the resonator has an active volume defined by the laser beam and the separation of the mirrors but not by the dimensions of the mirrors. During the collapse stage of the discharge, the plasma enters the active volume of the resonator, thereby changing the optical length and thus the resonant condition for the laser radiation. A photomultiplier is used to monitor changes in the resonant condition by detecting the intensity modulation of the radiation transmitted through
the resonator. With the aid of simple theory, this information permits measurement of the electron density.

A more detailed description of the operation of the device requires a discussion of the relevant properties of the optical resonator.

1.1 The Optical Resonator

The optical resonator is defined by two partially reflecting dielectric coated mirrors $M_1$ and $M_2$ as shown in Figure 1.3. The flat mirror ($M_2$) is also part of the laser while the spherical mirror ($M_1$) is mounted externally to the laser. These mirrors and the other laser mirror ($M_3$) in effect form two coupled optical resonators. However, as will be explained subsequently, the coupling back to the laser is negligible in the present application. Therefore, the arrangement can be regarded simply as an optical resonator which is excited by the laser radiation.

The properties of an optical resonator that are of interest in this investigation are the resonant modes, the quality factor ($Q$), and the stability. A resonant mode is defined as a field distribution that reproduces itself in spatial distribution and phase as the wave bounces back and forth between the two mirrors, but whose amplitude decays exponentially in time. The condition for resonance of a mode is that the phase shift for a double transit must be an integral multiple of $2\pi$ radians. $Q$ is a function of the length and the energy losses (e.g. due to diffraction, output coupling,
Figure 1.3 Schematic of the Experimental Arrangement
absorption, etc.) of the resonator. Stability is defined in terms of the behaviour of paraxial rays launched within the structure. As these rays are periodically redirected by the mirrors, they either remain close to the resonator axis or tend to diverge and escape from the transversely finite system. When the resonator confines the rays it is termed 'stable', otherwise it is called 'unstable'. The stability condition is given by:\(^{13}

\[ 0 < q_i, q_2 < 1 \]

where

\[ q_i = 1 - \frac{d}{b_i}, \quad i = 1 \text{ or } 2 \]

Here \( b_i \) is the radius of curvature of the \( i \)th mirror and \( d \) is the axial length of the resonator. Since for the resonator used here \( b_2 = \infty \), the stability condition is simply:

\[ 0 < 1 - \frac{d}{b_1} < 1 \]

The resonator is stable if this condition is satisfied, otherwise it is unstable.

Stability as defined above is a geometrical property of the resonator in the context of ray optics. In the context of the modes of a resonator, the terms stable and unstable are interpreted with respect to the distribution of the fields in the resonator. The fields of modes in stable resonators are more concentrated near the axis than those in unstable resonators. Consequently, the diffraction losses at the
mirrors are much higher for unstable resonators than they are for stable resonators.

Consider the optical resonator used here for the case where \( d < b_1 < \infty \): that is, the resonator is stable. For the special case where the laser beam is coincident with the axis of the resonator, the modes that can be supported in the resonator have been described by Boyd et al.\(^{14,15}\) They obtained the following condition for resonance of the TEM\(_{p,1,q}\) mode:

\[
\frac{2 \pi d}{\lambda} = q + \frac{j}{2\pi} \left[ 2p + l + 1 \right] \cos^{-1} \left[ 1 - \frac{2d}{b_1} \right]
\]

where

- \( d \) = resonator length
- \( b_1 \) = radius of curvature of mirror \( M_1 \)
- \( n \) = refractive index of the medium in the resonator
- \( \lambda \) = wavelength of the exciting radiation
- \( p, l, q \) = mode indices for the cylindrical TEM\(_{p,l,q}\) mode.

Here the modes are described in cylindrical coordinates in view of the fact that the apertures of the resonator mirrors are circular. The cylindrical mode indices \((p, l, q)\) specify the number of nodes in the field distribution in the radial, azimuthal and axial directions respectively. For a resonator whose length is large compared to the wavelength, the mode number 'q' is equal to the length of the resonator axis expressed in half-wavelengths.

If required, the diffraction loss for a stable
resonator of the type under discussion must be obtained numerically as an analytical expression is not available.

For the case where $b < d < \infty$ (unstable resonator), the modes apparently have not been previously studied except for a few numerical computations\textsuperscript{16} where the resonators are nearly stable. However, in order to utilize the present technique, it is only necessary to establish which modes are in fact predominant in the resonator. Consequently, if the modes can be identified, a general expression for the condition of resonance of a mode is not required.

In the present investigation, it has been assumed that only the TEM\textsubscript {00\text{\scriptsize{q}}} modes are resonant. This assumption is reasonable in view of the fact that the laser beam is coincident with the axis of the resonator to better than the diameter of the beam. It has been shown by others\textsuperscript{17,18} that radiation from a laser operating in the TEM\textsubscript{00\text{\scriptsize{q}}} modes will only excite transverse $(p, \lambda \neq 0)$ modes in an optical resonator if the exciting radiation is not exactly axial. In the present case, visual inspection of the intensity distribution of the laser beam showed the presence of only the TEM\textsubscript{00\text{\scriptsize{q}}} modes. The beam was aligned with the resonator axis to better than $10^{-4}$ radians. Thus transverse modes could not be significantly excited. Furthermore, this assumption is justified experimentally in section 2.2.

Kahn\textsuperscript{19} and Siegman\textsuperscript{20} have shown that the loss of an unstable resonator due to the finite size of the mirrors is, to a first approximation, independent of the mirror
aperture (either size or shape) for the lowest order (TEM\textsubscript{00}) mode. The formula for the loss, which is based on the ray theory of optical resonators, is

\[ \alpha = 1 - \left| \frac{1 - \sqrt{1 - g_1^2 g_2^2}}{1 + \sqrt{1 - g_1^2 g_2^2}} \right| \]

where \( \alpha \) is the fractional energy loss per double pass and \( g_1, g_2 \) are defined in Equation 1.2.

In concluding this section, two remarks should be made concerning the laser and the external mirror. First, the coherence length of the radiation exciting the resonator must be at least twice the resonator length, which in turn must be about 2 meters in order to accommodate the Z pinch discharge (Figure 1.3). A laser is the only source of optical radiation which provides such coherence lengths and is therefore indispensable to the operation of the resonator device. Secondly, the use of a spherical (as opposed to a plane) external mirror is essential in the technique used to minimize the loss of the laser beam from the resonator due to deflection by transverse gradients in the electron density. This point is discussed further in Chapter V.

1.2 The Modulation Effect

A fraction of the laser radiation entering the resonator emerges through the partially reflecting external mirror and is detected with a photomultiplier. When the plasma is pulsed, the detector sees a modulation in the
intensity of the radiation. This 'modulation effect' is
simply and adequately described by examining the power trans-
mission coefficient of the resonator as a function of the
optical length along its axis.

Since the radius of curvature of the external
mirror is large and the diameter of the laser beam is small
compared with the mirror aperture, the resonator used here
can be approximated by a plane parallel resonator or Fabry-
Perot etalon for the purpose of investigating the transmis-
sion coefficient. For a resonator having lossless plane
parallel mirrors of unequal reflectivity, the power trans-
mission coefficient \( T \) as a function of the 'optical length'
\( \Theta \) of the resonator is given by the well-known Airy
formula\(^{21,22}\)

\[
T(\Theta) = T_{\text{max}} \frac{1}{1 + F \sin^2 \Theta}
\]

where

\[
T_{\text{max}} = \frac{T_1 T_2}{(1 - r_1 r_2)^2}
\]

\[
F = \frac{4 + r_1 r_2}{(1 - r_1 r_2)^2}
\]

\[
\Theta = \frac{2\pi}{\lambda} nd + \frac{1}{2} (\Phi_1 + \Phi_2)
\]

Here the subscripts 1 and 2 refer to mirrors \( M_1 \) and \( M_2 \),
respectively, and

-17-
\[ T_{1,2} = \text{mirror power transmission coefficients} \]
\[ r_{1,2} = \text{mirror amplitude reflection coefficients} \]
\[ \varnothing_{1,2} = \text{phase angle associated with reflection at the mirrors} \]
\[ d = \text{axial length of the resonator} \]
\[ n = \text{refractive index of the medium inside the resonator} \]
\[ \lambda = \text{vacuum wavelength of the laser radiation} \ (0.63\mu) \]

It is necessary to make two remarks concerning the validity of Airy formula in the present case. First, diffraction losses at the mirrors, which are significant for the resonator used here, are not accounted for in the formula. Secondly, while the angles \( \varnothing_{1,2} \) are constant, their values are not known and here will be arbitrarily be put equal to zero. Nevertheless, Equation 1.6 is still valid for the purpose of examining the relative variation of the resonator transmission coefficient with optical length. This property is illustrated in Figure 1.4. Two conclusions can be drawn from this curve. First, and most important, the power transmission coefficient goes through one complete cycle for each half-wavelength \( (\lambda/2) \) change in the optical length of the resonator. Consequently, the intensity of the laser radiation transmitted through the resonator also varies cyclically for each \( \lambda/2 \) change in optical length. Secondly, the intensity modulation is not sinusoidal, but assumes the shape of the transmission curve.

For convenience in writing, one complete cycle of modulation of the laser intensity is called a 'fringe'.

-18-
From Equation 1.6, the transmission coefficient \( T \) is a maximum when the phase shift \( \theta \) from one mirror to the other is an integral multiple of \( \pi \) radians. For a double pass the phase shift is an integral multiple of \( 2\pi \) radians. Since this is the condition for resonance stated earlier, it can be concluded that power transmission through the resonator is maximum at resonance.

1.3 Maximum Modulation Rate

The maximum modulation rate depends on the maximum rate at which power transmission through the resonator can be altered. This varies inversely with the resonator \( Q \).
A convenient formulation of the problem is to consider the maximum rate at which the energy in the resonator can be altered. If the source of excitation were suddenly removed, the energy in the resonator would decay with time as

$$e^{-t/\tau}$$

where by analogy with a passive two port network

$$\tau = \frac{Q}{\omega}.$$  \hspace{1cm} 1.11

Here $\omega$ is the angular frequency of the laser radiation and, by definition,

$$Q = \omega \frac{\text{maximum energy stored}}{\text{energy dissipated per second}}.$$  \hspace{1cm} 1.12

In order to express $Q$ in terms of the resonator parameters, let $\mathbf{W}$ and $\mathbf{\mathbf{W}}$ be the line densities of the energy travelling to the right and the left, respectively, in the resonator. Using the notation introduced earlier, the total energy at resonance is

$$(\mathbf{W} + \mathbf{\mathbf{W}}) d$$

and the rate of loss of energy is

$$c (\mathbf{W} T_1 + \mathbf{\mathbf{W}} T_2) + c L (\mathbf{W} + \mathbf{\mathbf{W}}).$$

In the first term, $T_1, T_2$ account for mirror transmission loss and in the second term, $L$ represents the fractional power loss per transit due to diffraction, absorption and other
effects. In the approximation that $\bar{W} = \bar{W}$ in an interval of
time corresponding to a single transit, the $Q$ of the resonator
is given by

$$Q = \frac{2\eta}{\lambda} \cdot \frac{d}{\left[\frac{1}{2}(T_1 + T_2) + L\right]} \quad 1.15$$

The approximation used introduces an error of about 10% in
the value of $Q$ for the present case. Substituting this expres-
sion into Equation 1.11 gives

$$T = \frac{d}{c\left[\frac{1}{2}(T_1 + T_2) + L\right]} \quad 1.16$$

For the resonator used here, $T_1 = 0.4 \gg T_2$ and
typically $d = 250$ cm where the external mirror used had a
radius of curvature $b_1 = 200$ cm. The value of $L$ is not well
known, but a rough estimate can be given. The significant
contributions to $L$ are reflection loss at the windows of the
discharge vessel and the loss due to the finite size of the
resonator mirrors (Equation 1.5). The reflection loss is
estimated to be 20%, based on the observation that the ampli-
tude of the laser intensity modulation decreased by about 20%
when the windows in question were installed. From Equation 1.5,
the value of $a$ is 0.6. This is the fractional loss for a
double pass. If it is assumed that the loss for a single
pass is $a/2$, an estimated value of $L = 0.5$ is obtained. Sub-
stituting these values into Equation 1.11 yields a time con-
stant of the order $10^{-8}$ sec.
As is well known, a system which has a time constant \( \tau = Q/\omega \) for decay or build-up acts as a filter. Thus for modulating frequencies \( f > 1/\tau \) full transmission given by Equation 1.6 will not be realized. Physically one can consider that the transmission can change from \( T_{\text{max}} \) to half-\( T_{\text{max}} \) in time \( \tau \) and back to \( T_{\text{max}} \) in another time \( \tau \) so that the minimum time between two maxima is of the order \( 2\tau \). In this way, the maximum modulation rate of the power transmission coefficient is estimated to be \( 1/2\tau \) or about 50 MHz.

1.4 Coupling Between the Laser and the Resonator

In the preceding discussion it was assumed that the output intensity of the laser was constant. This assumption requires justification.

The mirrors \( M_1 \) and \( M_2 \) (Figure 1.3) may be regarded as a 'compound' mirror which terminates one end of the laser cavity. Since absorption by the mirrors is negligible, the power reflection \( (R) \) and transmission \( (T) \) coefficients are related by \( R + T = 1 \). Therefore, the power reflection coefficient of the compound mirror varies cyclically with each \( \lambda/2 \) change in optical length in the same manner as the power transmission coefficient discussed earlier.

It is well known that the lasing intensity is a sensitive function of the reflectivity of the laser mirrors. Hence a cyclic variation in the reflection coefficient of one of the mirrors may produce a modulation of the output intensity. However, the maximum rate at which the lasing intensity can
be altered is the order of 1 MHz for the $\lambda = 0.63\mu$ transition\textsuperscript{23}. This is a consequence of the lifetimes of the atomic processes contributing to this particular transition.\textsuperscript{24}

In this investigation, structural vibrations produced a modulation of the laser intensity with a period of roughly 100 $\mu$sec. The electron density measurements were performed in an interval of time of 5 $\mu$sec or less, so that during the measurement the lasing intensity was effectively constant. However, the rate of change of the reflection (transmission) coefficient in this time interval due to the plasma was in excess of 1 MHz. This justifies the earlier statement that the intensity of the laser radiation exciting the resonator is effectively constant.

1.5 Deduction of the Electron Density from the Modulation Effect

As demonstrated earlier, changes in the optical length of the resonator produce a modulation on the intensity of the laser radiation transmitted through the external mirror. Here the change in optical length is due to a variation in time of the refractive index of the pulsed plasma contained in the resonator.

In a plasma medium the index of refraction is determined by all constituents of the plasma: i.e. atoms, ions, and electrons. In practice, however, the contribution to the refractivity by the free electrons dominates over the contribution by all other constituents except perhaps the ground state atoms. If the laser frequency nearly coincides with a
transition frequency of the ground state atoms, they may give a large contribution to the refractivity due to the phenomenon of anomalous dispersion. However, in the case of argon discharges the resonance wavelengths occur in the far-ultraviolet so this effect will not be encountered here.

Several plasma parameters can contribute to the refractivity due to the free electrons. To each parameter a characteristic frequency can be assigned: for example, the cyclotron frequency or the electron-atom collision frequency. These parameters affect the refractivity through the ratio of the characteristic frequency to the laser frequency. The laser frequency is sufficiently high to neglect all parameters except the plasma frequency so that the refractive index of the plasma is given by \(^{25,26}\)

\[
n = \left[ 1 - \left( \frac{\omega_p}{\omega} \right)^2 \right]^{1/2}
\]

1.17

where

\[
\omega_p^2 = \frac{e^2}{\epsilon_0 m_e} n_e.
\]

1.18

Here \(n_e\) is electron density, \(\epsilon_0\) is the permittivity constant, \(m_e\) and \(e\) are the electronic mass and charge respectively; MKS units are used. Even in this case, \(\omega_p \ll \omega\), so the refractive index may be approximated by

\[
n = 1 - \frac{1}{2} \left( \frac{\omega_p}{\omega} \right)^2.
\]

1.19

Substituting Equation 1.18 into the above expression gives
the following relation between the free electron density and the plasma refractive index:

\[ n = 1 - \frac{1}{2} \frac{e^2 \lambda^2}{(2\pi)^2 c^2 m_e \epsilon_0} n_e \]  \hspace{1cm} 1.20

The contribution to the optical length of the resonator due to the plasma is the line integral of this expression:

\[ \beta = \int_{0}^{s} n \, dl = s - \frac{1}{2} \frac{e^2 \lambda^2}{(2\pi)^2 c^2 m_e \epsilon_0} s \, \overline{n_e} \]  \hspace{1cm} 1.21

The integration limits are determined by the length \( s \) of the laser beam path in the plasma and \( \overline{n_e} \) is the electron density averaged over the path. Only the optical length of the plasma needs to be considered since other contributions are invariant. There is a change from one fringe maximum to the next when the optical length of the resonator changes by \( \lambda/2 \). The change in electron density corresponding to one fringe is

\[ \Delta \overline{n_e} = \frac{(2\pi)^2 c^2 m_e \epsilon_0}{e^2 \lambda} \frac{1}{s} \]  \hspace{1cm} 1.22

For \( \lambda = 0.63 \mu \) and \( s \) in meters

\[ \Delta \overline{n_e} = 1.78 \frac{s}{S} \times 10^{21} \text{ m}^{-3} \]  \hspace{1cm} 1.23

or, for \( s \) in centimeters,

\[ \Delta \overline{n_e} = \frac{1.78}{S} \times 10^{17} \text{ cm}^{-3} \]  \hspace{1cm} 1.24
If $\bar{n}_e$ is initially zero, the electron density at time $t$ after initiation of the discharge is given by

$$\bar{n}_e(t) = 1.78 \times 10^7 \frac{N(t)}{s} \text{ cm}^{-3}$$

provided the electron density increases monotonically during this time. $N$ is the number of fringes at time $t$ after current initiation and $s$ is the length in centimeters of the path of the laser beam through the plasma.

In this experiment, the measured electron density did not vary monotonically with time. However, the electron density increased monotonically from zero to a maximum value and thereafter monotonically decreased. Hence the above equation could be applied to each of the monotonically varying regimes in such a manner as to give an uninterrupted time history of the electron density. Further discussion of this matter is deferred to section 6.1.
CHAPTER II. THE RESONATOR DEVICE - APPARATUS

2.0 Introduction

In the previous chapter, the principle of operation of the laser-excited optical resonator device was described without considering the details of its construction. This chapter, therefore, describes the construction of the device and the associated measuring equipment. In addition, the experimentally determined performance of the device is discussed.

2.1 Description of the Resonator Device

A photograph of the laser-excited optical resonator device with the Z-pinch discharge in situ is shown in Figure 3.1, page 42. The basic components of the device are a gas laser, a dielectric-coated spherical mirror, and a photodetector. Table I provides the important specifications for the components used in this investigation. A schematic diagram showing typical dimensions of the experimental arrangement is given in Figure 2.1.

The device is of simple construction. The laser and the spherical (or external) mirror are mounted on an aluminum channel which is 16' long, 8" wide, 3" deep and of 3/8" thickness. On one end of the channel the laser is mounted, and on the other is attached an optical bench 6' long on which the external mirror is mounted. This assembly is supported independently of the discharge tube which is situated between the laser and the external mirror. The
## TABLE I
### SPECIFICATIONS OF THE COMPONENTS OF THE RESONATOR DEVICE

#### LASER

*Spectra-Physics Model 116 CW He-Ne laser with Model 250 RF-Dc power supply*
- Operating wavelength: 6328 Å
- Output power: 25 milliwatts
- Beam diameter at exit aperture: 1.0 mm
- Beam divergence: 1.0 milliradian

#### EXTERNAL MIRROR

<table>
<thead>
<tr>
<th>Type</th>
<th>substrate</th>
<th>surface coating</th>
<th>radius of curvature</th>
<th>diameter</th>
<th>reflectivity at 6328 Å</th>
</tr>
</thead>
<tbody>
<tr>
<td>spherical</td>
<td>fused quartz</td>
<td>multilayer dielectric</td>
<td>1.0 or 2.0 m</td>
<td>3.8 cm</td>
<td>60%</td>
</tr>
</tbody>
</table>

#### LASER INTENSITY MEASUREMENT

*Phillips 150-CVP photomultiplier and Fluke Model 412B high voltage power supply*
- Risetime: 8 nsec

*Baird-Atomic narrow band interference filter*
- Peak wavelength: 6330 Å
- Peak transmission: 60%
- Total width at 1/2 peak transmission: 42 Å

*Tektronix Type 555 dual time base oscilloscope with Type 1A1 plug-in unit*
- Risetime: 10 nsec
Figure 2.1 Schematic of the Laser-excited Optical Resonator Arrangement
channel is attached at its ends to two tables by means of lateral travelling-screw arrangements and each table is vertically adjustable. Thus the optical assembly as a whole could be moved at will relative to the discharge vessel. This enabled relatively easy alignment of the laser beam path in the discharge vessel without disturbing the alignment of the optical system.

The external mirror is held in the mount shown in Figure 2.2. The mount permits coarse vertical and horizontal

![Figure 2.2 External Mirror Mount](image-url)
adjustment of the mirror as well as fine angular adjustment in the vertical plane by means of a 3-point support provided with spring-loaded differential screws. This arrangement enables alignment of the external mirror to reflect the laser beam back on itself. A 3/4 inch diameter hole at the back of the mount allows the laser radiation transmitted through the mirror to be received by the photodetector.

Two photodetectors were used to monitor the laser intensity. An EMI 9558B photomultiplier having a measured risetime of 16 nsec was used to monitor the alignment of the laser beam (section 6.1) in the resonator by detecting the intensity fluctuations in the beam emerging from mirror $M_3$ of the laser (Figure 2.1). No special requirement is placed on this detector, so it will not be discussed further.

A Phillips 150-CVP photomultiplier with a rated risetime of 3 nsec was used to measure the fluctuations in intensity of the laser beam emerging from the external mirror of the resonator. The circuit diagram is given in Figure 2.3.

![Figure 2.3 Photomultiplier Circuit Diagram](image)

$V_0$

100K $r$ $r$ $r$ $r$ $r$ $r$ $r$ $r$ $r$ $r$

$1K$

$V_0$

$1000 \text{v DC}$ $r = 47K$ $c = .05 \mu F$

Figure 2.3 Photomultiplier Circuit Diagram
The risetime of the 150-CVP was measured using a Tektronix Type 661 sampling oscilloscope, which had a rated risetime of ≤ .35 nsec, and a pulsed nanosecond barium titanate light source. This source delivered light pulses with a rated width at half peak intensity of 2 nsec at a repetition rate of 10 kHz. The rise and fall of the light intensity about the peak value was symmetrical to a good approximation. Risetime measurement was made using a 2 meter length of RG58 A/U transmission cable terminated by 50 ohms at the oscilloscope. The effective load resistance of the photomultiplier was therefore 50 ohms. In this way, the trace shown in Figure 2.4 was obtained, indicating a risetime of 8 nsec or a frequency response of approximately 60 MHz.

![Figure 2.4 Photomultiplier Risetime](image)

It was estimated that the plasma would produce fluctuations with frequencies up to 50 MHz (or risetimes of the order of 10 nsec) in the intensity of the laser beam emerging from the resonator. The 150-CVP with a measured risetime of 8 nsec will be able to follow these fluctuations.
2.2 Frequency Response and Sensitivity

Measurement of the frequency response of the combined laser-excited optical resonator and related detection equipment is discussed here. 'Frequency' refers to the number of fringes per second or, equivalently, the number of cycles of intensity modulation per second of the laser radiation transmitted through the resonator. The sensitivity: that is, the change in optical length of the resonator corresponding to one fringe, follows from the response measurement.

To measure the response, the optical length of the resonator was varied by rotating a fused-quartz plate in the path of the laser beam. The plate was rectangular of dimensions 1.25 cm x 2.50 cm x 3.00 cm with the large faces polished flat to \( \lambda/20 \) and parallel to one second. A 1/4 horsepower router motor with a rated maximum speed of 25,000 rpm was used to rotate the plate.

First, however, the fringing rate corresponding to a specified plate velocity is calculated under the subsequently justified assumption that one fringe is produced for each \( \lambda/2 \) change in optical length. If the laser beam lies in the plane described by the normal to the large face of the rotating plate, then the optical length \( \delta \) between two arbitrary points \( z \) and \( z' \) is

\[
\delta = \left[ (z'-z) - \frac{w \cos(\Theta_i - \Theta_r)}{\cos \Theta_r} \right] + \frac{n_g w}{\cos \Theta_r} \tag{2.1}
\]

The notation is given in Figure 2.5.
Figure 2.5 Rotating Quartz Plate

The rate of change of the optical length is given by

$$\frac{d\xi}{dt} = w(A + B) \omega$$

where

$$A = \frac{1}{1 - \xi^2} \frac{\cos \Theta_i}{\cos \sin^{-1} \xi}$$

$$B = \frac{\sin (\Theta_i - \Theta_r)}{\cos \Theta_r} \left[1 - \frac{1}{n_g^2} \cos \Theta_i \cos \Theta_r \right] - \frac{\cos (\Theta_i - \Theta_r) \cos \Theta_i}{n_g^2} \frac{1}{1 - \xi^2}$$

and from Snell's law

$$\xi = \frac{\sin \Theta_i}{n_g} = \sin \Theta_r.$$
Here $n_0$ is the refractive index of the quartz plate and $\omega = \frac{\pi}{180} \frac{d\phi}{dt}$ is the angular velocity. Since each $\lambda/2$ change in $\phi$ produces a fringe, the number of fringes per unit time is

$$\frac{dN}{dt} = \frac{2w}{\lambda} (A + B) \omega$$

In Figure 2.6, the fringe rate is plotted as a function of the angular velocity of the plate for the case where $\phi_1 = 70^\circ$.

---

**Figure 2.6 Calibration of the Rotating Quartz Plate**
Figure 2.7 illustrates the experimental arrangement used in the following procedure for measuring the frequency response. The resonator device was aligned with the quartz plate in situ but stationary at an angle of 70°. A 2 mm aperture placed between the plate and mirror M₁ served to limit the angles at which the detector could receive light to 70 ± 5°. Rotating the mirror at a known velocity then produced the fringe pattern displayed on oscilloscope 1 in Figure 2.7. The amplitude of the fringe pattern shown is largest at the 'peak' since then θ₁ = 70° and the light intensity received by the detector is maximum. The amplitude and frequency of the fringes were measured by displaying the region around the peak on oscilloscope 2 at a faster sweep speed. This oscilloscope was a Tektronix Type 585A with a Type 82 plug-in unit and had a bandpass of about 90 MHz. A third oscilloscope was used to monitor the amplitude of the laser intensity which was maintained constant during the course of measurement by adjusting the alignment of the laser mirrors (if required).

By varying the motor speed, the peak-to-peak amplitude of the fringes as a function of fringing frequency was obtained as plotted in Figure 2.8. Here the fringe amplitude is normalized at 1 MHz. The high frequency cut-off occurred around 50 MHz. At this frequency, the amplitude of the fringes was about one millivolt for a vertical deflection sensitivity of 5 mvolts/cm on the oscilloscope.
Figure 2.7 Experimental Arrangement for Measurement of Resonator Frequency Response
Figure 2.8 Frequency Response of the Optical Resonator System
In addition to providing the frequency response, the measurements described above served to calibrate the sensitivity of the device. As mentioned earlier, the curve shown in Figure 2.6 was calculated assuming that one fringe was produced for each $\lambda/2$ change in the optical length. The points plotted on this figure correspond to the fringing rates measured at the relevant plate velocities. Since they agree with the calculated values, it can be concluded that one fringe corresponds to a change in resonator length of $\lambda/2$. Furthermore, this justifies the earlier statement that only the lowest order (TEM$_{ooq}$) modes are resonant, since if higher order modes were present this correlation would not occur.$^{18}$

It should be noted that throughout this chapter, the approach to the resonator time response is 'quasi-stationary' in the following sense: the double transit time of the laser beam through the resonator is assumed to be small compared with the fringe period. In this investigation, fringing rates below 20 MHz and hence fringe periods in excess of 50 nsec are encountered whereas the double transit time for typical resonator lengths ($\sim$250 cm) is 16 nsec. Thus the quasi-stationary approach is adequate in the present case.

2.3 Elimination of Optical Pick-up

The photomultiplier signal due to modulation in the laser intensity is swamped by the signal due to radiation from the plasma unless steps are taken to reject the latter. An interference filter placed directly in front of the photocathode
gave considerable improvement. The filter had a 42 Å total halfwidth centered at 6330 Å with broad-band blocking from the infrared to the ultra-violet to remove optical harmonics. At the center of the passband, the transmission was 60%. The plasma radiation was further reduced by placing an 8 mm aperture inside the resonator as shown in Figure 2.7. Its function was to prevent plasma radiation emitted in the near axial direction from reaching the photomultiplier. A further improvement relied on the fact that the plasma radiation, being isotropic, dropped off as the inverse square of the distance from the plasma whereas the laser radiation was nearly parallel. The photomultiplier was placed approximately 8 meters from the plasma and the laser beam was guided to it by means of two plane front-surface mirrors. With this arrangement, the optical pick-up in the absence of the laser beam was less than 10% of the fringe amplitude, even at the time of pinch when the plasma was most luminous.

Further reduction of the optical pick-up was not required in the present investigation. However, it should be pointed out that since the laser radiation is linearly polarized, the above quoted figure could be reduced to 5%, if necessary, simply by placing a suitably orientated polarizer in front of the photomultiplier.
PART II  THE PLASMA
CHAPTER III. THE Z-PINCH DISCHARGE

3.0 Discharge Apparatus

This chapter contains a description of the Z-pinch discharge apparatus and specifications of the components. A photograph of the complete apparatus is shown in Figure 3.1. The basic specifications of the important components are listed in Table II.

Figure 3.1 Photograph of the Experimental Apparatus
TABLE II
DISCHARGE APPARATUS SPECIFICATIONS

CAPACITOR BANK AND LEADS

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total capacity (N.R.C. Type 203 low inductance)</td>
<td>51.5 µF</td>
</tr>
<tr>
<td>Inductance of capacitor bank and current leads</td>
<td>0.21±0.1 µH</td>
</tr>
<tr>
<td>Width of leads</td>
<td>10 cm</td>
</tr>
<tr>
<td>Length of leads</td>
<td>1.3 m</td>
</tr>
<tr>
<td>Separation of leads (polyethylene)</td>
<td>2 mm</td>
</tr>
</tbody>
</table>

DISCHARGE VESSEL

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge vessel</td>
<td>Pyrex</td>
</tr>
<tr>
<td>Electrodes</td>
<td>Brass</td>
</tr>
<tr>
<td>Electrode separation</td>
<td>61.6 cm</td>
</tr>
<tr>
<td>Inside tube diameter</td>
<td>15 cm</td>
</tr>
<tr>
<td>Outside tube diameter</td>
<td>17 cm</td>
</tr>
</tbody>
</table>

VACUUM SYSTEM

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 17 Balzers oil diffusion pump</td>
<td></td>
</tr>
<tr>
<td>Hyvac 14 Cenco backing pump</td>
<td></td>
</tr>
<tr>
<td>Base pressure</td>
<td>1 µHg</td>
</tr>
<tr>
<td>Leak rate</td>
<td>8 µHg/hr</td>
</tr>
</tbody>
</table>

CURRENT MEASUREMENT

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Rogowski' coil (RG65 A/U delay line core</td>
<td></td>
</tr>
<tr>
<td>with passive RC integrator)</td>
<td></td>
</tr>
<tr>
<td>Integration time constant</td>
<td>1.1 msec</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>5.0±0.5 kA/volt</td>
</tr>
<tr>
<td>Maximum discharge current</td>
<td>200 kA</td>
</tr>
<tr>
<td>Discharge current frequency</td>
<td>50 kc/s</td>
</tr>
</tbody>
</table>

VOLTAGE MEASUREMENT

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simpson microammeter in series with A.V.O multiplier</td>
<td></td>
</tr>
<tr>
<td>Charging voltage</td>
<td>25 kV D.C., 500M</td>
</tr>
<tr>
<td></td>
<td>12.0±2.2 kV</td>
</tr>
</tbody>
</table>

PRESSURE MEASUREMENT

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type GP-110 Pirani vacuum gauge</td>
<td></td>
</tr>
<tr>
<td>Vacustat</td>
<td>min. 1 µHg, max. 10 mmHg</td>
</tr>
</tbody>
</table>
The plasma source used in this experiment is a conventional linear Z-pinch discharge. The circuit diagram of the discharge apparatus is given in Figure 3.2.

![Circuit Diagram](image)

**Figure 3.1 Z-pinch Discharge Circuit**

A. Isolated ultra-violet trigger pulse  
B. Main spark gap trigger generator  
C. Trigger generator charging pick-off  
D. Main spark gap switch  
E. Discharge vessel  
F. Z-pinch capacitor bank  
G. High voltage supply

The capacitor bank and triggering arrangement are conventional in design. The discharge vessel, however, is specifically constructed to accommodate the diagnostic techniques employed in this experiment and is therefore described in detail.
A scale diagram of the discharge vessel is given in Figure 3.3. A Pyrex tube (17 cm O.D., 15 cm I.D., and 75 cm in length) with plane brass electrodes inserted at the ends is used. The vessel can be evacuated and then filled to the required pressure with argon.

The discharge vessel is designed for use with photographic, spectroscopic and laser diagnostics. Provision for photographic and spectroscopic observation in directions either normal or parallel to the discharge axis is made in the following manner. A fine mesh brass screen is used as the return conductor for the discharge current (Figures 3.1 and 3.3), the ends of the discharge vessel are sealed with 20 cm diameter windows and the electrodes are made of perforated brass.

For laser diagnostics, the perforated electrodes are replaced by 1/8" thick brass discs with a diametrical slot 3/8" wide extending to within 1/16" of the disc edge. The slots are aligned horizontally parallel in the discharge vessel. This arrangement allows the laser beam to pass through the discharge vessel. Furthermore, the slots permit two glass tubes to be inserted into the plasma; one through each electrode as illustrated in Figure 3.3. The glass tube set-up, which serves to adjust the length (s) of the laser beam path through the plasma, is described in section 5.2. A discussion of the effect on the discharge of the slots and the glass tubes is deferred to sections 4.1 and 5.3, respectively.

Two 1/4" diameter ports, one approximately 3" from each electrode and both on the same side of the discharge
Figure 3.3 Discharge Vessel
vessel, permit insertion of two graduated rods across the
diameter of the vessel and parallel to the electrode slots.
The rods, graduated in .025" divisions, are used to align
the laser beam parallel to the axis of the discharge vessel.

3.1 Measurement of the Discharge Current

The discharge current (I) is measured with a form
of Rogowski coil constructed from a 12 cm length of RG63 A/U
delay line with the outer conductor removed. The coil is
placed between the flat copper leads carrying the discharge
current.

The output from the coil must be integrated to
obtain a signal proportional to the discharge current. Inte­
gration is performed by feeding the signal from the coil to
two balanced, passive RC integrators shown in Figure 3.4.
The output from the integrators is fed to a Tektronix Type G
differential plug-in unit and displayed on an oscilloscope.
This integration arrangement permits rejection of common mode
or 'push-push' noise signal which accompanies the coil signal
due to capacitative coupling between the coil and the dis­
charge circuit. The integrators are provided with a
variable resistance which is adjusted to give an optimum
common mode rejection of approximately $10^2:1$ for frequencies
ranging from $10^2$-10$^5$ cps.

For reliable integration, the time constant (RC) of
the integrator must be large compared with the period (T) of
the signal to be integrated. Since the time constant of the
Figure 3.4 Integrating Circuit

integrator is 1.1 msec and the period of the discharge current is approximately 10 µsec, this condition is satisfied. Furthermore, the gain-frequency product for the coil and integrator must be constant for all frequencies to be measured. The self-resonant frequency of the coil is the order of 10 MHz which is much greater than the frequencies of interest (\(< .05\) MHz) so that the gain-frequency product is constant in the relevant frequency range.

Oscillograms showing the current waveforms for filling pressures of 100 and 1000 µHg argon are given in Figure 3.5. These waveforms were used to calibrate the current measuring system in the manner described below.

The oscillograms were enlarged by projection and the areas bounded by the waveform and the zero-current baseline were measured with a planimeter. The graticule area
The discharge was in argon at a pressure of 100 μHg for the upper trace and 1000 μHg for the lower trace. Both traces have sweep speed 10 μsec/cm and vertical deflection 20v/cm where a 1 volt deflection corresponds to 5 kA.

was used to calibrate the planimeter readings in μsec-volts. The measured area must be equivalent to the total charge on the capacitor bank

\[ \int I \, dt = Q = CV \]  

3.1
where 

\[ Q = \text{total charge on the capacitor bank} \]

\[ I = \text{discharge current} \]

\[ C = \text{total capacitance} \]

\[ V = \text{charging voltage} \]

In this way it was established that a 1 volt deflection on the graticule was equivalent to a discharge current of 5,000 amperes. The error associated with the calibration procedure is about 10%, the main source of error being the uncertainty in the rated capacitance of the capacitor bank.

The discharge current rises in 3.8 \( \mu \text{sec} \) to a peak value of 140 kA for 100 \( \mu \text{Hg} \) and in 4.4 \( \mu \text{sec} \) to a peak value of 150 kA for 1000 \( \mu \text{Hg} \).
CHAPTER IV. DYNAMICS OF THE Z-PINCH DISCHARGE

4.0 Introduction

A Barr and Stroud high-speed framing camera was used to study the dynamics of the Z-pinch discharge. In this camera, the film remains stationary and an air-turbine driven rotating mirror is used to pass light from the discharge through a series of 60 fixed lenses. A small light source inside the camera is focused onto the rotating mirror and at a certain mirror position the reflection is detected by a photomultiplier. The pulse delivered by the photomultiplier is shaped and then used to trigger the discharge. The rotating mirror is provided with a magnetic pick-up device which delivers one pulse per complete revolution of the mirror. A counter is used to record the period (T) between these pulses at the time of firing of the discharge. The manufacturers state that a mirror rotation of 5.5 kc/sec corresponds to a time interval between successive frames of .125 μsec. Thus the time interval (e) between successive frames can be determined from the period (T) according to

\[ e = 0.69 \times 10^{-3} T \]

where e and T are measured in microseconds. Typically, T is the order of 350 μsec and thus e is approximately .24 μsec.

4.1 Plasma Symmetry and Electrode Effects

The photographs shown in Figure 4.1 illustrate the cylindrical symmetry of the plasma column for an argon
The film strips show the collapse stage of the Z-pinch discharge in 500 μHg of argon.

A. End-on photograph at the cathode: time interval between frames, .28 μsec.

B. Top-on photograph with cathode position as indicated; time interval between frames, .30 μsec.

The time sequence zigzags, increasing from the bottom to the top of the page.
filling pressure of 500 μHg. Film strip A shows the collapse viewing end-on through a perforated electrode. Film strip B gives a side-on view of about 1.3 of the full length of the plasma column. In both cases, the discharge is viewed at the cathode. These photographs and others not included here showed that for filling pressures from 100 to 1000 μHg, the plasma column was uniformly cylindrical along the length of the discharge except within about 3 cm of the electrodes. However, the departure from symmetry near the electrodes presented no difficulty in this investigation since electron density measurements were made in the central region of the discharge.

To facilitate electron density measurement using the resonator device, it was convenient to use slotted electrodes as described in section 3.0. The effect of the slotted electrodes on the cylindrical symmetry of the plasma column was checked by taking side-on photographs of the discharge viewing simultaneously in directions parallel and normal to the slots. From these photographs, measurement of the outer luminous diameter of the column showed it to be slightly elliptical with the major axis parallel to the slots. However, the ellipticity never exceeded 3% and decreased as the column collapsed. Therefore it can be stated that the slots in the electrodes do not significantly perturb the cylindrical symmetry of the plasma column. Consequently, the electron density can be measured at any azimuthal position without loss of generality.
4.2 Collapse Curves

The radial position of the plasma as a function of time after current initiation was measured for filling pressures of 100 and 1000 µHg argon using the procedure described below.

As discussed in section 2.3, an interference filter is used to reject the plasma radiation received by the photomultiplier detecting the laser beam transmitted through the resonator. While this radiation was a disadvantage for the measurement of electron density, it provided a convenient method for determining the radial position of the plasma shell as a function of time after initiation of the discharge current. The radiation arose from emission in the near-axial direction when the plasma traversed a pencil-shaped region centered at the radial position of the laser beam. The 'pencil' dimensions were determined by a 5 mm diameter aperture at the discharge vessel and a similar aperture 8 meters away at the photodetector. With the interference filter removed, the detector provided a pulse whose peak occurred when the 'center' of the shell coincided with the axis of the pencil. The radius-vs-time history of the shell was determined by measuring the time after current initiation of the peak for several radial positions of the pencil. For these measurements, the laser beam served only to align the pencil at the selected radial positions.

The collapse curves obtained in this way are shown in Figure 4.2. These curves provided assistance in interpreting the fringe patterns obtained using the resonator technique for electron density measurement.
Figure 4.2 Collapse Curves
CHAPTER V. EFFECT OF RADIAL REFRACTIVITY GRADIENTS IN THE PLASMA

1.0 Introduction

In the present application of the resonator device, the axes of the laser beam, the resonator, and the Z-pinch discharge are aligned parallel but not necessarily coincident. However, the radial gradient $\frac{\partial n}{\partial r}$ of the refractive index (n) of the plasma causes the beam to be deflected from the intended path by an angle $30,31$

$$\epsilon = \int_0^L \frac{1}{n} \frac{\partial n}{\partial r} \, dz.$$  \hspace{1cm} 5.1

Here the integral is taken over the beam path length through the plasma in the z (axial) direction. If the radial gradient is sufficiently large, the laser beam will be deflected out of the transversely finite structure, thereby terminating measurement. In this chapter, the deflection of the laser beam due to a radial refractivity gradient in the resonator medium is examined and on the basis of the results obtained, two methods are devised to reduce the loss of the laser beam from the resonator due to deflection. One of the methods perturbs the plasma and the effect of this perturbation on the validity of the electron density measurements is discussed.

5.1 Deflection of the Laser Beam in the Resonator due to a Radial Refractivity Gradient

The deflection of the laser beam is to be investigated for the situation illustrated in Figure 5.1. Analysis is
carried out in the plane defined by the resonator and discharge axes in a coordinate system \((r,z)\) defined by the plane mirror and the resonator axis.

A slab of plasma of axial length \((s)\) is situated in the resonator and has a 'radial' refractivity gradient as indicated. Consider a beam which is launched from the plane mirror at an angle \(\theta_0\) and with a displacement \(r_0\) relative to the resonator axis (hereafter called the axis). The subsequent history of the beam, using the approximation of geometrical optics for paraxial rays and neglecting refraction at the plasma boundaries, is as follows. On passing through the plasma, the beam undergoes an angular deflection \(\epsilon\) (away from
the axis for negative $\frac{\partial n_0}{\partial r}$) and consequently a radial displacement from the axis. The beam is incident on the spherical mirror at an angle

$$\Theta' = \Theta_0 + \epsilon$$  

5.2

with a displacement

$$r' = r_0 + \Theta_0 d + \epsilon \left( \frac{s}{2} + d_2 \right)$$  

5.3

relative to the axis. Here d is the length of the resonator and $d_2$ is the axial distance from the spherical mirror to the nearer boundary of the plasma slab. The reflected beam is again deviated and displaced and returns to the plane mirror at an angle

$$\Theta_1 = \frac{2R}{R} + \left( \frac{2d}{R} - 1 \right) \Theta_0 + 2 \left( \frac{s}{2} + d_2 \right) - 1 \epsilon$$  

5.4

with a displacement

$$r_1 = (1 - \frac{2d}{R}) r_0 + 2d \left( 1 - \frac{d}{R} \right) \Theta_0 + 2d \left( 1 - \frac{s}{2} + d_2 \right) \epsilon$$  

5.5

where R is the radius of curvature of the spherical mirror. The details of the derivation of these equations are straightforward and will not be given here, except to remark that in the calculation of the radial displacement, the curvature of the beam path in the plasma is taken into account to a first order in the refractivity gradient.

Equations 5.4 and 5.5 may be iterated, if desired.
However, it is sufficient for the present purpose to consider only a double pass through the resonator. In this case, the equations can be simplified by putting $r_0 = 0$ and $\theta_0 = 0$. Dropping the irrelevant subscripts then gives

\[ \Theta = 2 \left( \frac{\frac{S}{2} + d_2}{R} - 1 \right) \varepsilon \quad \text{5.6} \]

\[ r = 2d \left( 1 - \frac{\frac{S}{2} + d_2}{R} \right) \varepsilon. \quad \text{5.7} \]

For $R \to \infty$, these equations reduce to the obvious result for a resonator with plane-parallel mirrors.

From Equation 5.6 and 5.7 it can be seen that the laser beam returns to the plane mirror with its initial trajectory when

\[ R = \frac{S}{2} + d_2 \quad \text{5.8} \]

This result has a simple physical interpretation. When the above equation is satisfied, the radius of curvature of the spherical mirror coincides with the middle of the plasma slab. As a result, the deflected beam is to a good approximation normally incident at the mirror and consequently returns on the same path. This argument assumes that the beam does not miss the spherical mirror on the first pass. Therefore, the following restriction must be placed on the maximum angular deflection:
\[ \mathcal{E}_{\max} < \frac{a}{\sqrt{a^2 + R^2}} \]  
\text{5.9}

where 'a' is the radius of the mirror aperture. Thus if the conditions specified by Equations 5.8 and 5.9 are satisfied, the laser beam will remain in the resonator in spite of a radial gradient in the refractivity of the resonator medium.

Recalling the condition for stability of the resonator under discussion (Equation 1.2 with the change in notation \( b_1 = R \)),

\[ 0 < 1 - \frac{d}{R} < 1 \]  
\text{5.10}

it is clear that in order to satisfy the beam containment condition of Equation 5.8, it is necessary to operate the resonator in the unstable regime. This justifies the earlier statement that the resonator must be operated in the unstable regime due to the effects of radial refractivity gradients in the resonator medium.

Equations 5.8 and 5.9 (hereafter referred to as the containment conditions) can be conveniently represented in the graphical form shown in Figure 5.2. Here the angular deviation is plotted as a function of the beam path length (s) in the plasma with the radial gradient in electron density as a parameter. The absolute value of the gradient is denoted by \( \nabla n_e \) and is of the order of \( 10^{18} \text{ cm}^{-4} \), corresponding to near-maximum gradients encountered in the present investigation. The maximum deflection, \( \mathcal{E}_{\max} \), is set by the specifications.
of the spherical mirror which in this case has a radius of curvature of \( R = 200 \) cm and an aperture radius of \( a = 1.9 \) cm. On the same plot, the axial distance \( d_2 \) from the mirror to the nearer boundary of the plasma is given as a function of the beam path length, \( s \).

\[ \varepsilon_{\text{max}} = 0.6 \]

\[ d_2 = R - \frac{s}{2} \]

\[ \nabla n_e = 5 \cdot 10^{-4} \text{ cm} \]

\[ \nabla n_e = 1 \cdot 10^{-4} \text{ cm} \]

\[ s \text{ in cm} \]

\[ \varepsilon \text{ in degrees} \]

\[ d_2 \text{ in cm} \]

Figure 5.2 Graphical Representation of the Conditions for Containment of the Laser Beam
To illustrate the application of Figure 5.2, suppose the radial gradient of the electron density is estimated to be
\[ \nabla n_e = 5 \cdot 10^{18} \text{ cm}^{-4} \]. Following the corresponding curve shown in the above figure, it is seen that the beam deflection will exceed the limit \[ \epsilon_{max} = 0.6^\circ \] for path lengths greater than 12 cm. If a path length of 10 cm is chosen, then the spherical mirror is to be placed at a distance \[ d_2 = 195 \text{ cm} \] from the point where the beam exits from the plasma and a maximum deflection of \[ \epsilon = 0.5^\circ \] may be expected.

5.2 Reduction of the Loss of the Laser Beam due to Deflection

In the previous section, deflection of the laser beam due to a radial refractivity gradient of the resonator medium was examined in the approximation of geometrical optics for paraxial rays. Two conditions were obtained for containment of the beam in the resonator:

\[ d_2 = \frac{R - s/2}{\epsilon} \]  \hspace{1cm} (5.8)

\[ \epsilon < \frac{a}{\sqrt{a^2 + R^2}} \]  \hspace{1cm} (5.9)

where
- \( s \) = path length of the laser beam through the plasma
- \( d_2 \) = axial distance between the spherical mirror and the exit point of the beam from the plasma
- \( R \) = radius of curvature of the external mirror
- \( a \) = aperture radius of the external mirror.

These 'containment conditions' impose certain requirements on the design of the experimental apparatus. To satisfy
Equation 5.8, it is necessary that the resonator length be greater than the radius of curvature of the external mirror. As stated earlier (Equation 5.10), this means the resonator must be operated in the unstable regime.

For small angular deflections, Equation 5.0 can be approximated by

\[ \varepsilon = \frac{1}{n} \frac{\partial n}{\partial r} s \]

and hence Equation 5.9 can be expressed in the form

\[ \varepsilon = \frac{1}{n} \frac{\partial n}{\partial r} s < \frac{a}{\sqrt{a^2 + R^2}} \]

To satisfy this equation for a specified radial gradient, either the aperture radius (a) of the external mirror or the path length (s) of the beam through the plasma must be adjustable. Since the former is impractical, the second requirement imposed by the containment conditions is that the beam path length through the plasma be variable.

In this investigation, provision to adjust the length of the beam path was made by inserting two glass tubes into the plasma: one through each discharge electrode as illustrated in Figure 3.3, page 46. The tubes were mounted in separate holders which in turn were mounted externally to the plasma behind the discharge electrodes as shown in Figure 5.3. Each holder was provided with a rack-and-pinion assembly which allowed the tubes to be moved to any radial position in the discharge along a diameter defined by the electrode slots (section 3.0).
One tube was inserted a fixed distance into the discharge vessel (usually about \(1/3\) the full length of the plasma) while the other tube was adjusted to provide the desired path length. To determine the path length, the distance between a band fastened onto the movable tube and the face of the tube clamp was measured with a vernier calliper (Figure 5.3). Previously, the band was placed at a known, convenient distance from the face of the clamp with \(s = 0\) (i.e. with the tubes butted together).

Figure 5.3 Glass Tube Mounting Assembly
The tubes were open-ended lengths of ordinary silica glass tubing (8 mm O.D., 6 mm I.D.) with the ends polished square. The length of the tube nearer the spherical mirror was such that the beam deflection (Equation 5.9) was limited by the aperture of the mirror rather than the tube.

The axes of the tubes were aligned coincident with that of the laser beam which in turn was parallel to (but in general not coincident with) the discharge axis. Thus the laser beam propagated within the tubes and was shielded from the plasma except over a path of variable length, s.

5.3 Effect of the Reduction Procedure on the Validity of the Electron Density Measurement

The procedures used to reduce loss of the beam from the resonator: namely, alignment of the center of curvature of the external mirror at the midpoint of the laser beam path in the plasma and control of the path length by means of the glass tube innovation, are discussed here from the point of view of their effect on the electron density measurements.

In the case of the mirror alignment, the problem of interest is whether or not motion of the deflected beam across the face of the external mirror causes the beam path length in the resonator to change. Such a change would give an extraneous contribution to the fringe count.

Consider the simplified situation shown in Figure 5.4. Here the beam is deflected through a small angle $\theta$ at a distance $z$ from the center of curvature of the external mirror. The quantity of interest is the difference in the path lengths.
From the cosine law

\[ c^2 = R^2 - z^2 + 2zc \cos \varepsilon \]  

Using these equations it is easy to show that in the approximation \( z \ll R \approx c \), the difference \( \Delta \) in path length is

\[ \Delta = \frac{z \varepsilon^2}{2} \]  

From section 1.5, it is recalled that a fringe is produced for each \( \lambda/2 \) change of the beam path length in the resonator. Hence the change in path length \( \Delta \) will produce \( \delta N \) fringes where

\[ \delta N = \frac{\Delta}{\lambda/2} = \frac{z \varepsilon^2}{\lambda} \]
If the contribution of $S$ to the total fringe count $N$ (typically about 10) is not to exceed 5%, the following condition must be satisfied:

$$Z \epsilon^2 < \frac{\lambda}{2}$$

This result provides a criterion for the precision to which the center of curvature of the external mirror must be set at the midpoint of the beam path in the plasma. Furthermore, if measurements are made with the center of curvature displaced from the path midpoint in question, the above equation imposes a more severe limitation on the maximum admissible angular deflection than that specified by Equation 5.9.

The second effect to be considered is perturbation of the plasma due to the presence of the glass tubes used to adjust the length of the beam path in the plasma.

It has been demonstrated by Folkierski and Daughney that an obstacle at one axial position in a Z-pinch discharge produces a negligible disturbance on the formation of the plasma elsewhere along the axis in the collapse stage. This behaviour was verified photographically in the present experiment using the high-speed framing camera described earlier. The photographs shown in Figure 5.5 were taken with a 2.5 cm gap between the tube ends. In film strip A the tubes were 2.5 cm from the discharge axis and became visible when the plasma shell arrived at their radial position. The pinch subsequently formed in the usual manner along the entire length of the discharge axis. However, when the tubes were aligned to within
Figure 5.5 Effect of the Glass Tubes on the Plasma

The collapse stage of the Z-pinch in argon is shown viewing vertically with the glass tubes inserted. In all cases, the ends of the tubes are separated by 2.5 cm. Below, (e) is the time interval between successive frames which zigzag.

A. Tubes 2.5 cm from the discharge axis.
   Left: 100 μHg, e = 0.34 μsec
   Right: 1000 μHg, e = 0.42 μsec

B. Tubes nearly coincident with the discharge axis.
   Left: 100 μHg, e = 0.36 μsec
   Right: 1000 μHg, e = 0.46 μsec
their radius of the discharge axis, the pinch formed in the usual manner only along the axial segment not occupied by the tubes. Therefore, on the basis of photographic evidence, it can be stated that the presence of the glass tubes inside the discharge vessel produced no apparent disturbance of the plasma in the region where measurement was performed.

A second check on the effect of the tubes on the electron density measurement relies on the fact that the total number of fringes observed should scale directly with the path length of the beam through the plasma (Equation 1.25). For example, the total number of fringes obtained should double if the measurement is repeated with a path length twice as long, other conditions being constant. This check is valid provided the axial variation of the electron density is below the sensitivity limit of the measurement for the path length used. In the present investigation, the fringe count scaled in the prescribed manner in most cases. The data was rejected if the scaling check failed.

Electron density measurement may be affected by introduction of impurities from the tubes (e.g., Si ions). Since measurement is performed in the region not occupied by the tubes, this effect is not expected to be significant. Another effect to be considered is flow of the plasma into the open ends of the tubes. For 'off-axis' measurements, this effect should be negligible because the plasma shell has a high velocity (~2 cm/μsec in the direction normal to the tube axes. However, this argument does not apply at the time of pinch since the plasma is then
'stationary'. The photographic evidence and scaling check indicate that this effect is negligible in this investigation, even in this case.

From the foregoing discussion, it may be concluded that the procedure used to reduce loss of the laser beam from the resonator due to deflection by radial refractivity gradients does not significantly affect the validity of the electron density measurements obtained.
PART III

EXPERIMENTAL RESULTS
CHAPTER VI. ELECTRON DENSITY MEASUREMENTS USING THE RESONATOR TECHNIQUE

6.0 Introduction

The object of this investigation was to measure the temporal and radial electron density distributions in the collapse stage of a Z-pinch discharge using the laser-excited optical resonator technique. To achieve this end, it was first necessary to develop a modification of the resonator technique previously used by others.

The modified resonator technique was described earlier. This part of the thesis continues with the description of the experimental procedure followed by the presentation and discussion of the electron density measurements obtained.

An independent measurement of the electron density for a filling pressure of 1000 μHg was obtained using the Stark-broadening method. However, this measurement is of secondary importance in this investigation and is therefore presented as an appendix (page 122).

6.1 Experimental Procedure

Before describing the alignment of the apparatus, the salient features of the experimental arrangement will be reviewed. The laser and the external mirror are mounted at opposite ends of a 16' long aluminum channel whose ends are attached to vertically adjustable tables by means of horizontal-travel screw arrangements. Another vertically adjustable table supports the discharge vessel between the laser and
external mirror. Incorporated into the discharge vessel are two glass tubes which are inserted axially at opposite ends of the vessel through diametrical slots in the electrodes. It is significant to note that the optical system (laser, external mirror, etc.) and the discharge vessel are independently adjustable. This feature greatly simplifies the alignment procedure.

The experimental apparatus is aligned so that the axes of the resonator, the glass tube arrangement and the laser beam are coincident. The axis of the discharge vessel is parallel to (but generally not coincident with) the others. All of the axes are contained in a plane which passes centrally through the horizontal slot in each of the discharge electrodes.

The important parts of the alignment procedure will now be discussed. The laser is aligned with the output beam parallel to the optical bench on which the external mirror is mounted. It should be stated that all alignment is carried out with respect to the path of the laser beam which is readily visible due to scattering of the 0.63μ radiation from dust particles, etc. Alignment of the resonator is a simple operation which requires only that the external mirror be positioned in the path of the laser beam so as to reflect the incident beam back on itself. This operation is facilitated by the mount for the mirror (section 2.1) which permits three degrees of translational and rotational adjustment. Rough alignment is performed by visual superposition of the incident and reflected beams to within the beam diameter (~2mm) with the external mirror approximately 2.5 meters from the nearer laser mirror.
At this point, small amplitude fringes are detected with the photomultiplier-oscilloscope systems used to monitor the intensity of the beams emerging from either end of the laser (Figure 2.1, page 29). These fringes are caused by changes in the resonator length due to structural vibrations in the optical system. Fine alignment of the resonator is achieved by further adjusting the external mirror to maximize the fringe amplitude.

The next step is to align the discharge vessel and the laser beam so that their axes are parallel and in a horizontal plane which passes centrally through the mutually parallel and horizontal slots in the discharge electrodes. The important point in this operation is the accuracy to which the laser beam and discharge vessel axes could be aligned parallel and coincident. The axis of the discharge vessel is determined to within ± .025" by means of the arrangement described in section 3.0. This arrangement allows two rods graduated in .025" divisions to be inserted diametrically into the discharge vessel, parallel to the slots in the electrodes and at axial positions separated by approximately 50 cm. This enables the laser beam to be aligned coincident with the discharge axis to within ± .050" and parallel to within ± 10^-3 radians. The lateral position of the aluminum channel corresponding to coincidence of the laser beam and discharge axes is read from two scales calibrated in 1 mm divisions and incorporated into the travelling screw arrangements. Thus the optical system
could be translated to any specified radius in the discharge vessel with the optical and discharge axes parallel to within the quoted tolerance.

The axial position of the external mirror (i.e., the distance from the external mirror to the nearer laser mirror) depends on the radial position at which measurements are made. For radii greater than 2 cm, the external mirror was placed 250 cm from the laser mirror and the path length (s) of the laser beam in the plasma was adjusted to satisfy the previously developed criterion

$$|z| \epsilon^2 < \frac{\lambda}{2}$$  \hspace{1cm} 5.16

Within 2 cm of the discharge axis, the radial gradient of the electron density sharply increased. Consequently, for these radii it is necessary to use the optimum arrangement to minimize loss of the beam from the resonator. From the considerations of section 5.2, this requires that the following containment conditions be satisfied:

$$d_2 = R - s/2$$  \hspace{1cm} 5.8

$$\epsilon < \frac{a}{R}, \quad a \ll R$$  \hspace{1cm} 5.9

Recall that for small angular deflections

$$\epsilon = \frac{1}{n} \frac{\partial n}{\partial r} S$$  \hspace{1cm} 5.11
where
\[ n = 1 - \frac{\lambda}{\hbar} n_e \]
\[ \lambda = \frac{1}{2} \frac{e^2 \lambda^2}{(\pi \hbar)^2 c^2 m_e \varepsilon_0} \]

Since \( \lambda n_e \ll 1 \) for the electron densities encountered in this investigation, the angular deflection can be approximated by:
\[ \varepsilon = -\frac{\lambda}{\hbar} \frac{dn_e}{dr} s \]

where \( \lambda = 1.8 \cdot 10^{-22} \text{ cm}^3 \) for the radial gradient and the path length in cgs units.

The radial gradient of the electron density is not known, hence the greatest path length \( s \) for which \( \varepsilon \) given by Equation 6.1 satisfies the inequalities 5.9 and 5.16 must be found by trial and error. The maximum value of \( s \) is desired because the sensitivity of the electron density measurement increases with path length. In the present investigation, the path length ranged from the full discharge length (61.6 cm) for measurements near the vessel wall to about 4 cm for near-axial measurements.

The quantity measured directly is the electron density at a specified radius as a function of time after initiation of the discharge current. Before each shot, the discharge vessel was evacuated to 5 \( \mu \text{Hg} \) and then filled to the desired pressure with commercial grade argon. A dual-beam Tektronix Type 555 oscilloscope and a Tektronix C-27 polaroid camera were used to record the time history of the...
electron density (hereafter called the fringe pattern) and
the discharge current simultaneously. Recall that the fringe
pattern is the signal from the photomultiplier detecting the
laser radiation transmitted through the resonator. This
signal was fed to a Tektronix Type 1A1 plug-in unit while
the discharge current signal was fed to a Tektronix Type D
differential plug-in unit. Typical oscillograms are shown in
Figure 6.1. In most cases, the current waveform was displayed
at a sweep speed of 2 μsec/cm while the fringe pattern was
displayed at 0.2 μsec/cm over an interval of time correspond­­
ing to the intensified segment on the current trace.

To obtain the electron density at a specified radius
as a function of time after current initiation (or the "temporal
profile"), the following relation is required:

\[
\bar{n}_e(t) = \frac{1.78 \times 10^{17}}{s} N(t)
\]

Here \(N(t)\) is the number of fringes at time \(t\) after current
initiation. The manner in which the fringes were counted
is illustrated in the self-explanatory diagram given in
Figure 6.2. The term "turnover" appearing in this figure
will be explained subsequently.

In the derivation of Equation 1.25, it was assumed
that the electron density varied monotonically in time.
In this investigation, the electron density does not vary
monotonically over the time interval in which measurements
are made; hence the use of this equation here requires
further explanation.
Figure 6.1 Typical Fringe Patterns

A - 100 µHg  B - 1000 µHg

Upper traces: fringe patterns displayed at 0.2 µsec/cm

Lower traces: discharge current waveforms displayed at 2 µsec/cm

The fringe patterns are displayed over a time interval corresponding to the intensified segment on the current waveforms.
During the collapse stage of the Z-pincher discharge, the electron density at any specified radius is expected to increase from zero to a maximum value, called the turnover, and thereafter decrease. Furthermore, on the basis of magnetic probe measurements of the current density distribution obtained by others, it is reasonable to assume that the electron density varies monotonically in time on either side of the turnover. In this case, Equation 1.25 can be applied to determine the electron density as a function of time simply by counting the fringes in the manner illustrated in Figure 6.2. The interpretation of the fringe pattern is therefore relatively straightforward, the only difficulty being perhaps in identification of the turnover.
In general, the turnover is characterized by one of two features on the fringe pattern, depending on the optical length of the resonator at the occurrence of the turnover. If the optical length is such that the transmission of the laser radiation through the resonator is near-maximum or near-minimum, the turnover will occur at an extrema of the fringe amplitude. In this case, a flat segment usually appears at the 'center' of the fringe pattern as illustrated in Figure 6.1A. On the other hand, if turnover occurs when the transmission is somewhere between the extrema, a sudden reversal in the direction of the intensity variation results as illustrated in Figure 6.1B. This behaviour is readily understood by examining the transmission curve given in Figure 1.4, page 19, for a reversal in the direction of change of the resonator optical length.

For measurements obtained at radii greater than 2 cm, the number of fringes on either side of the turnover was usually equal to within ± 1/2 of a fringe. This means that the electron density at these radii is effectively zero before and after passage of the collapsing plasma shell. However, for radii less than 2 cm, effects peculiar to the axial region arise. For example, the electron density does not fall to zero after the turnover within the time interval of interest: i.e., the time to pinch.

The distance on the oscillograms between successive maxima (or minima) of the fringe amplitude were easily measured
with a scaled eyepiece. The fringe pattern starts at time $t_f$ after current initiation corresponding to the start of the intensified segment on the current waveform (Figure 6.2). It was convenient to determine $t_f$ by measuring the time interval $t_d$ between the start of the intensified segment and the first current reversal. The time of current reversal, $t_r$, was previously determined to be $11.6 \pm 0.2 \mu \text{sec}$ for 100 $\mu \text{Hg}$ and $10.0 \pm 0.2 \mu \text{sec}$ for 1000 $\mu \text{Hg}$ argon. The fringe trace therefore starts at a time after current initiation given by $t_f = t_r - t_d$. Time scale measurements made in this way enabled the time at which turnover occurred to be determined to within $\pm 0.2 \mu \text{sec}$.

An uncertainty of $\pm 1/2$ of a fringe arises in the total fringe count for reasons discussed in section 6.3. However, it can be stated here that the total number of fringes on either side of turnover was usually greater than 10, so the uncertainty in the peak electron density was typically $\leq 5\%$.

6.2 Temporal and Radial Electron Density Distributions

The procedure used to obtain the experimental data was described in the previous section. An IBM 7040 computer was used to process the data and plot the electron density at specified radii as a function of time after current initiation. These plots, referred to as the temporal profiles, consisted of the experimental points connected by a smooth curve. In general, temporal profiles were obtained at 2 mm intervals over most of the discharge vessel radius (75 mm).

The peak-electron density points of the temporal profiles fall on a smooth radius-vs-time curve (called the
density collapse curve) to within the experimental accuracy of the time scale measurement (± .2 µsec). However, a jitter of ± .2 µsec in the time scale of the temporal profiles produces a large scatter (typically ± 20%) in the electron density values of the subsequently determined radial profiles (i.e. the radial distributions of electron density at specified times after current initiation). Explanation of the manner in which this scatter arises is given by way of the following example.

Typically, the temporal profiles exhibit a rate of change of electron density of the order of \(10^{-3} \text{ cm} \cdot \mu\text{sec}^{-1}\). Consequently, in reading the electron density from these profiles at a specified time, a jitter of ± .2 µsec results in a fluctuation of \(± .2 \times 10^{17} \text{ cm}^{-3}\) in the value of the electron density. Since the electron densities measured were the order of \(10^{17} \text{ cm}^{-3}\), the scatter in the values amounts to about ± 20%. To circumvent this difficulty, the temporal profiles were shifted in time by an amount necessary to make the peaks coincide with the density collapse curve. In most cases, the time shifts required were the order of ± .1 µsec. The radial profiles were then obtained by plotting the electron densities from the shifted temporal profiles corresponding to a specified time after current initiation.

Two other plots presented in this section remain to be discussed. One of the plots shows the radius-vs-time curves corresponding to an electron density of \(10^{16} \text{ cm}^{-3}\).
on the rising and falling sides of the temporal profiles. In this plot (referred to as constant density curves), the density collapse curve is included as a dashed line. The remaining plot provides the peak electron density as a function of radius as obtained from the temporal profiles. For a consistency check, the peak densities from the radial profiles are also plotted.

A discussion of the experimental error and the limitations of the measuring technique is presented in the following section.
Figure 6.3 Electron Density Plots for 100 μHg Argon
Temporal Profiles

100 \( \mu \text{Hg} \) Argon

- \( 7.0 \ \text{cm} \)
- \( 6.0 \ \text{cm} \)
- \( 5.0 \ \text{cm} \)
- \( 4.0 \ \text{cm} \)
- \( 3.0 \ \text{cm} \)
- \( 2.0 \ \text{cm} \)
- \( 1.0 \ \text{cm} \)
Peak Density Collapse Curve
100 μHg Argon

- o radial profiles
- x temporal profiles
Constant Density Curves

- $n_e = 1.0 \cdot 10^{16} \text{ cm}^{-3}$
- $n_e = 0.3 \cdot 10^{16} \text{ cm}^{-3}$
Peak Electron Density as a Function of Radius

100 μHg Argon

\[ n_e \cdot 10^{17} \text{ cm}^{-3} \]

- o radial profiles
- x temporal profiles

r, cm
Radial Profiles
100 μHg Argon

\[ n_e \cdot 10^7 \text{ cm}^{-3} \]

\( r, \text{ cm} \)

\[ \times - 1.0 \mu\text{sec} \]
\[ \circ - 2.0 \mu\text{sec} \]
\[ \triangle - 3.0 \mu\text{sec} \]
\[ \square - 4.0 \mu\text{sec} \]
\[ + - 5.0 \mu\text{sec} \]
Radial Profiles

100 μHg Argon

- 1.5 μsec
- 2.5 μsec
- 3.5 μsec
- 4.5 μsec
Radial Profiles

100 μHg Argon

\[ n_e \cdot 10^{17} \text{ cm}^{-3} \]

- \( \times \) - 4.2 μsec
- \( \circ \) - 4.4 μsec
- \( \triangle \) - 4.6 μsec
- \( \square \) - 4.7 μsec
- \( + \) - 4.8 μsec

r, cm
Figure 6.4 Electron Density Plots
for 1000 \( \mu \text{Hg Argon} \)
Temporal Profiles

1000 µHg Argon

- x - 7.0 cm
- o - 6.0 cm
- Δ - 5.0 cm
- n - 4.0 cm
- + - 3.0 cm
- φ - 2.0 cm

\[ n_e \cdot 10^{17} \text{ cm}^{-3} \]

\[ t, \mu\text{sec} \]
Peak Density Collapse Curve

1000 µHg Argon

-97-
Constant Density Curves

1000 μHg Argon

\[ n_e = 10^{16} \text{ cm}^{-3} \]
Peak Electron Density as a Function of Radius

$1000 \mu \text{Hg Argon}$

- $n_e \cdot 10^{17}$ cm$^{-3}$
- $r$, cm

- o radial profiles
- x temporal profiles
Radial Profiles

1000 \( \mu \text{Hg Argon} \)

- \( \times \) - 2.0 \( \mu \text{sec} \)
- \( \circ \) - 4.0 \( \mu \text{sec} \)
- \( \triangle \) - 6.0 \( \mu \text{sec} \)
- \( \square \) - 8.0 \( \mu \text{sec} \)
- \( + \) - 10.0 \( \mu \text{sec} \)
- \( \phi \) - 12.0 \( \mu \text{sec} \)

\( n_e \cdot 10^{17} \text{ cm}^{-3} \)

\( r, \text{ cm} \)
Radial Profiles

1000 µHg Argon

- 3.0 µsec
- 5.0 µsec
- 7.0 µsec
- 9.0 µsec
- 11.0 µsec
- 13.0 µsec

$n_e \cdot 10^{17}$ cm$^{-3}$

$r$, cm
6.3 Limitations of the Resonator Technique and Sources of Error

In this investigation, the most serious difficulty associated with obtaining the desired electron density measurements arose from deflection of the laser beam due to radial gradients of the electron density. While the modification of the resonator technique developed here was generally successful in circumventing this problem, the technique fell short of expectations on two counts: the immediate axial region defied measurement at the time of pinch and reduction of the beam deflection entailed a significant loss of sensitivity.

In order to limit the angular deflection

$$\epsilon = -\int \frac{\partial n_e}{\partial r} s$$

it was necessary to shorten the path length (s) of the beam in the plasma. From the relation

$$\bar{n}_e = \frac{1.78 \times 10^7}{s} N$$

it is evident that the sensitivity (i.e. the minimum change in electron density required to produce a fringe) decreases with path length. Representative path lengths used in this investigation are shown in Figure 6.5 as a function of the discharge vessel radius. As will be discussed shortly, the minimum change in electron density that could be measured reliably corresponded to 1/2 of a fringe. In Figure 6.6,
Figure 6.5 Representative Beam Path Lengths

The sensitivity is plotted as a function of radius for $N = 1/2$ and the representative path lengths given in the above figure. The maximum sensitivity was $1.5 \cdot 10^{-3}$ cm$^{-1}$, corresponding to $s = 62$ cm or the full discharge length. In general, the sensitivity was about 10% of the peak electron density obtained at each radial position of measurement.

Measurements were obtained to within 4 mm of the discharge axis for 100 µHg and to within 14 mm for 1000 µHg. For smaller radii than quoted, the path length required to prevent beam loss was less than 4 cm. However, for path lengths
below 4 cm, the electron density measurements did not scale in the manner described in section 5.3.

Failure of the scaling check is demonstrated in Table III for 100 μHg argon filling pressure. Recall that the scaling check requires the ratio N/s, and hence the electron density measurement (Equation 1.25, page 102), to be independent of the plasma length (s) used. From the table
TABLE III. SCALING CHECK (100 µHg ARGON)

<table>
<thead>
<tr>
<th>Discharge radius r (in cm)</th>
<th>Path length s (in cm)</th>
<th>Maximum Fringe Number, N</th>
<th>N/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>60</td>
<td>9</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>6</td>
<td>.15</td>
</tr>
<tr>
<td>4.0</td>
<td>42</td>
<td>10</td>
<td>.24</td>
</tr>
<tr>
<td></td>
<td>31</td>
<td>7</td>
<td>.23</td>
</tr>
<tr>
<td>3.0</td>
<td>31</td>
<td>12</td>
<td>.39</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>10</td>
<td>.40</td>
</tr>
<tr>
<td>2.0</td>
<td>15</td>
<td>10</td>
<td>.67</td>
</tr>
<tr>
<td></td>
<td>12.5</td>
<td>8</td>
<td>.64</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7</td>
<td>.70</td>
</tr>
<tr>
<td>1.0</td>
<td>6.5</td>
<td>11</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>9</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>3.8</td>
<td>6</td>
<td>1.6</td>
</tr>
<tr>
<td>0.0</td>
<td>3.0</td>
<td>4</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>5</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>4</td>
<td>4.0</td>
</tr>
</tbody>
</table>

It can be seen that the ratio N/s is independent of s to within the experimental accuracy (10%) at each of the listed radii except \( r = 0 \). For on-axis measurements, this ratio increases with decreasing values of s: i.e., the scaling check fails. Similar behaviour is observed for the case of 1000 µHg at radii less than 14 mm. Data for which the scaling check failed were regarded as unreliable and therefore rejected in this investigation. Although photographic study of the pinch did not reveal any disturbance of the plasma for the tubes on-axis, their presence is presumably the cause for these measurements being unreliable.

The possible error in the measured electron density
arising from error in the fringe count (N) and the path length (s) is considered next.

Evidence based on the scaling check indicates that, except perhaps for the near-axial measurements discussed above, the plasma is not significantly disturbed in the region between the tube ends. The length of the plasma through which the laser beam travelled was therefore taken to be the distance separating the tube ends. This length was measured to within 1 mm in the manner described in section 5.2. The uncertainty entering this measurement never exceeded 3% and was typically <1%. Consequently, the error introduced into the electron density measurement due to uncertainty in the path length was regarded as negligible.

Several effects may introduce error into the fringe count. In certain cases, a change in the effective optical length of the resonator may occur due to motion of the deflected beam across the mirror surface. A simplified calculation presented in section 5.3 showed that an error of less than 5% in the total fringe count was introduced by this effect provided the following condition was satisfied:

$$|z| \epsilon^2 < \lambda/2.$$  \hspace{1cm} 5.16

In this investigation, for measurements made under conditions where beam motion needed to be considered (z ≠ 0), this error was kept below 2% by limiting \( \epsilon \) to the order of 10\(^{-3}\) radians.
For all measurements, the total fringe count was always subject to an error of 1/2 of a fringe due to the fact that structural vibrations in the optical system caused a continual variation in the optical length of the resonator. The period of the 'structural' fringes was long (> 50 μsec) compared with the time interval of the measurement (< 5 μsec). Hence the structural vibrations did not contribute to the fringe count directly. However, due to such vibrations, the start of the fringe pattern is known only to within an accuracy of 1/2 of a fringe. For most measurements the total fringe count was greater than 10 and therefore accurate to within 5%.

Another source of error to be considered is the reproducibility of the plasma. The reproducibility of the discharge current was examined by superimposing several current waveforms on the same oscillogram: two for the glass tubes inserted on-axis in the discharge and two for the tubes removed. The waveforms coincided to within the width of the oscilloscope beam. Therefore it can be stated that the discharge current is very reproducible and furthermore, the presence of the glass tubes did not significantly affect the inductance associated with the pinched plasma column.

A further check on the plasma reproducibility was provided by obtaining several fringe patterns for the same experimental conditions. Direct comparison of the patterns by superposition was not possible due to the randomness of the fringe starting point discussed above. Instead, the corresponding temporal profiles were compared. For profiles obtained
at radii greater than 2 cm, the peak electron densities agreed to within 10% and coincided in time to within .2 μsec. However, at radii less than 2 cm the occurrence of the peaks coincided to within .2 μsec but the density values agreed only to within 20%.

The final source of error to be considered here concerns refractivity changes other than those due to the free electron density of the plasma. Since measurement is performed in an interval of time less than 5 μsec, changes in refractivity due to motion of the optical components etc. can be regarded as negligible. The effect of a change in the neutral gas density (for example, due to passage of a non-ionizing shock front) merits further consideration.

The refractivity of argon as a function of density and wavelength is given by

$$n_a - 1 = \left[ 1.03 \cdot 10^{-2.3} + \frac{0.58 \cdot 10^{3.3}}{\lambda^2} \right] N_a$$

where λ is in cm and the neutral gas density $N_a$ is in cm$^{-3}$. Recalling that a change of $\lambda/2$ in the optical length of the resonator produces one fringe, the number of fringes resulting from a change $\Delta n_a$ in the refractive index of the neutral gas is therefore

$$N = \frac{\Delta n_a \cdot S}{\lambda/2}$$

Consider the case of a strong, non-ionizing shock propagating in 100 μHg at room temperature. The limiting density ratio across the shock front is 4 and the corresponding change...
in refractive index is $\Delta n_a = 10^{-7}$ from Equation 6.2. Here, application of the ideal gas law gives $N_a = 3.8 \cdot 10^{15} \text{ cm}^{-3}$ for the stated conditions. For measurement using the full length of the discharge ($s = 62 \text{ cm}$), this change in refractive index produces $1/5$ of a fringe. Since the sensitivity of the measuring technique is the order of $1/2$ of a fringe, it can be concluded that a strong non-ionizing shock would be only detectable and certainly the concomitant change in refractivity would not introduce significant error to the electron density measurements obtained in this investigation.

In view of the preceding discussion, it can be stated that the principal sources of error were reproducibility of the plasma and uncertainty in the fringe count due to structural vibrations of the optical system. In general, measurements obtained at radii greater than 2 cm were accurate to within 10% while those obtained at smaller radii were accurate to within 20%.
CHAPTER VII. DISCUSSION OF RESULTS

7.0 Introduction

This chapter summarizes the important features of the resonator technique developed in this investigation and continues with a discussion of the electron density measurements presented in section 6.2.

The radial and temporal profiles exhibit several interesting features which are examined phenomenologically from the point of view of the dynamics of the pinch. Of particular interest is an apparent high acceleration of the collapsing shell near the time of pinch for the discharge in 100 µHg argon. This effect was also observed at this laboratory by Daughney\(^1\) who used framing camera diagnostics to measure the rate of collapse of the luminous plasma shell. His measurements were made on an argon Z-pinch discharge very similar to that used in this investigation.

The electron density measurements obtained in the present investigation suggest a mechanism to account for the 'accelerated pinch' effect. This mechanism also explains another effect observed at this laboratory: namely, an on-axis current flow preceding the arrival of the plasma shell. This effect was revealed by magnetic probe measurements of the current density distribution obtained by Tam\(^2\) for Z-pinch discharges in helium and in argon.
7.1 The Resonator

A technique which employs a laser-excited optical resonator has been developed to measure the time-dependent radial distribution of electron density in the collapse stage of a Z-pinch plasma. A description of the resonator device and a discussion of the principle of operation were presented in Part I of the thesis.

In the collapse stage, the Z-pinch plasma is characterized by large radial gradients of electron density. These gradients complicate the present application of the resonator technique since they cause the laser beam to be deflected out of the optical system, thereby terminating the measurement.

In Chapter V, two conditions were obtained for containment of the beam in the presence of an arbitrary radial refractivity gradient in the resonator medium:

\[ d_2 = R - \frac{s}{2} \]  \hspace{1cm} 5.8

\[ \varepsilon = \frac{1}{n} \frac{\partial n}{\partial r} s < \sqrt{\frac{q}{a^2 + R^2}} \]  \hspace{1cm} 5.12

where \( \varepsilon \) is the angular deflection due to the radial gradient \( \frac{\partial n}{\partial r} \) of the refractive index \( n \)

\( s \) is the axial length of the laser beam path in the plasma

\( d_2 \) is the axial distance from the spherical mirror to the nearer boundary of the plasma

\( R \) is the radius of curvature of the spherical (or external) mirror

\( a \) is the radius of the external mirror aperture.

These 'containment conditions' are valid for small angular
deflections and beam path lengths that are short compared with the radius of curvature of the external mirror.

The containment conditions have a simple physical interpretation. When Equation 5.8 is satisfied, the radius of curvature of the external mirror coincides with the middle of the beam path length \( s \). As a result, the deflected beam is to a good approximation normally incident at the mirror and consequently returns on the same path. Equation 5.12 simply means that unless the angular deflection is sufficiently small, the beam will miss the external mirror altogether. In choosing the external mirror, it is therefore advantageous to use a small radius of curvature and a large aperture in order to minimize this constraint.

For Equation 5.8 to hold, the axial length of the resonator must be greater than the radius of curvature of the external mirror. This leads to a novel feature of the present technique: namely, the use of an unstable optical resonator. To the author's knowledge, previous applications of similar resonator devices have employed only stable optical resonators. An interesting result of this investigation, therefore, has been to demonstrate that an unstable resonator can be employed for measurement of electron density. However, it should be clearly stated that the present technique does not rely on any property that is peculiar to an unstable resonator but not to a stable resonator. Rather, the technique relies on the fact that an unstable resonator exhibits the same cyclic variation of the power transmission coefficient as the stable resonator.
(section 1.2) and at the same time enables the containment condition of Equation 5.8 to be satisfied.

For equation 5.12 to hold, the path length of the laser beam through the resonator must be made variable since altering the other parameters is impractical. This was accomplished in the present investigation by inserting two glass tubes axially into the discharge vessel. With this arrangement, the laser beam propagated within the tubes and was shielded from the plasma except over a variable distance (s) separating the adjacent ends of the tubes. Photographic investigations and a 'scaling' test (section 5.3) showed that in most cases the tube arrangement did not significantly disturb the plasma in the region between the tube ends during the collapse stage.

The resonator device as used here has a spatial resolution of the order of 2 mm and a time resolution of 0.05 \(\mu\)sec or better (experimentally determined). The technique is suitable for measurement of electron densities in excess of \(5 \cdot 10^{16}/\lambda s \text{ cm}^{-3}\), where \(s\) is the beam path length through the plasma in cm and \(\lambda\) is the wavelength of the laser in microns. In the present application, most electron density measurements were accurate to within 10% (section 6.3).

For the purpose of electron density measurement in the collapse stage of a Z-pinch discharge, the present technique possesses several advantages over other commonly used methods. First, the resonator technique does not depend on other plasma properties as is the case with spectroscopic
methods, for example, which require local thermal equilibrium of the plasma. Secondly, for most measurements, the plasma is not significantly perturbed by this technique. With the Langmuir probe, on the other hand, the interaction of the probe and the plasma is often not negligible. Thirdly, measurements are made in the central region of the discharge, thereby avoiding the ever-present electrode effects at the end regions of the discharge. With holographic and conventional interferometric techniques, however, the full discharge length is used so electrode effects must be considered.

The general applicability of the technique as used in this investigation is limited, however, by several factors. First, the fact that the electron density is an average over the path length of the beam through the plasma may be troublesome in some applications. However, as the Z-pinch is axisymmetric (section 4.1), this does not present a limitation here. Second, although the present technique successfully circumvented the problem of beam deflection in most cases, the immediate axial region remained inaccessible to measurement at the time of pinch. Third, a meaningful interpretation of the measurement is possible only if the electron density either varies monotonically with time or its direction of change is known from other sources. Suggestions for relaxing the last two limitations are presented in section 7.3.
7.2 The Plasma

The electron density measurements presented in section 6.2 exhibit several interesting features which are discussed here from the point of view of the collapse process. The discussion is divided into two parts according to the argon filling pressures used: 100 and 1000 µHg. For convenience, the salient features of the experimental results are illustrated schematically in Figures 7.1 and 7.2 of this section. The reader may wish to refer to section 6.2 for more detail.

The term 'inner edge' will be used to denote that side of the radial profile nearer the discharge axis. In reference to the temporal profiles, the inner edge therefore corresponds to the side of increasing electron density with time. In this context, the meaning of 'outer edge' is obvious.

100 µHg:

The results obtained for 100 µHg exhibit two particularly interesting features. First, an extended region of relatively low electron density \( n_e \sim 2 \cdot 10^{15} \text{ cm}^{-3} \) precedes the inner edge of the high density \( n_e \sim 5 \cdot 10^{16} \text{ cm}^{-3} \) collapsing shell. The inner edge of the low density region (hereafter referred to as the ionizing front) advances toward the axis with a velocity of the order of 2 cm/µsec. The origin of the ionizing front can be ascribed to several mechanisms: for example, a shock wave, electron diffusion, or photo-ionization. However, as this investigation does not provide
Figure 7.1 Illustrating Discussion of 100 µHg Results
Electron density \( (n_e) \) in units of \( 10^{17} \text{ cm}^3 \)
sufficient information to determine the responsible mechanism, the nature of the ionizing front remains a subject of conjecture. Second, at all radii less than approximately 2 cm the electron density attains maximum value practically at the same time ($4.8 \pm 0.2 \mu\text{sec}$). On the assumption of phenomena which propagate radially inward, an event occurring simultaneously over a radial length of 2 cm is an unexpected result. According to the peak density curve (Figure 7.1), the shell appears to be suddenly accelerated at about 4.8 $\mu\text{sec}$. It is interesting to note that the discharge current waveform exhibits a 'kink' at this time, as illustrated in Figure 7.1.

Effects similar to those described above have also been observed at this laboratory as well as at Los Alamos and the Imperial College of London (by private communication). The distribution of electron density in the axial region near the time of pinch suggests a switching of the current flow from the shell to the axial region. The mechanism of the current switching, however, remains unclear except that it is apparently triggered by the ionizing front preceeding the shell.

1000 $\mu\text{Hg}$:

The most striking feature of the electron density results for 1000 $\mu\text{Hg}$ is a pronounced steepening of the inner edge of the density profiles. Below, it will be argued that the steepening is related to two other features: namely, the 'shift' appearing on the peak density collapse curve.
Figure 7.2 Illustrating Discussion of 1000 \( \mu \text{Hg} \) Results
Electron density \( (n_e) \) in units of \( 10^{17} \text{ cm}^{-3} \)
and an 'explosion' of the outer edge of the radial density distribution, both of which occur about 9 μsec after current initiation (Figure 7.2).

A ready explanation of the above effects follows from the observation that the discharge current undergoes reversal of flow at 10 μsec. The magnetic pressure confining the plasma at the outer edge of the shell will be small around the time of current reversal. Furthermore, the gas pressure in the region between the shell and the wall of the discharge vessel is low compared with the kinetic pressure within the shell. This leads to the observed explosion of the outer edge of the shell.

The explosion of the outer edge of the shell tends to decrease the electron density in this region. On the other hand, due to the kinetic energy of the collapsing shell, the electron density at the inner edge continues to increase. The combined effect is a steepening of the inner edge of the electron density distribution. Further, the peak of the density distribution tends toward the inner edge, thereby producing the observed shift in the peak density collapse curve.

Information received by private communication indicates that current switching should also occur at 1000 μHg. The fact that this effect is not observed at 1000 μHg in this investigation could be explained as follows. The electron density produced by the ionizing front may be below the sensitivity limit of the present device for the case of 1000 μHg. Furthermore, measurement was terminated 14 mm from the discharge.
axis for 1000 μHg while for 100 μHg results were obtained to within 4 mm of the axis. Thus current switching may in fact occur but is not observed here due to lack of measurements sufficiently close to the discharge axis.

7.3 Future Work

In the present application of the resonator technique, the interpretation of the fringe pattern relied on information obtained previously by other methods to deduce the direction of the change in electron density. It would be desirable to modify the resonator device so that the direction of electron density change may be inferred from the fringe pattern itself. A promising technique involves modulating the optical length of the resonator at a known rate using the piezoelectric effect to vibrate the external mirror.\textsuperscript{12}

Reliable measurements in the near-axial region at the time of pinch were not obtained in the present investigation. Apparently, the poor accuracy in this region is due to the short beam path lengths (<4 cm) required to keep the angular deflection below the present limiting value (ε≤ 0.6°, page 62). However, the present limit on ε could be relaxed considerably by reducing the overall length of the discharge. This would enable a spherical mirror with smaller radius of curvature to be used (see page 112).

Knowledge of the electron density is not enough to uniquely specify such plasma parameters as conductivity, recombination coefficients, and diffusion rates. Therefore, in
the future it would be worthwhile to combine the density measurements with spectroscopic measurements of the temperature or magnetic probe measurements of the current density distribution.
APPENDIX. SPECTROSCOPIC MEASUREMENTS

Introduction

The plasma electron density was determined for a discharge vessel filling pressure of 1000 μHg by measuring the half-widths of the Stark-broadened AII 5062 Å plasma line and the Hα impurity line. This measurement provided the electron density as a function of time for the first 20 μsec following current initiation and was a spatial average over the diameter of the discharge.

The half-widths were determined from time-resolved line profiles obtained by step-wise scanning the relevant line using a Spex f/6.8 grating monochromator (Model 1700-II) equipped with an EMI 9558B photomultiplier. This monochromator had a 75 cm focal length and an 11.0 cm wide grating with 1200 grooves/mm which provided a reciprocal dispersion of 10 Å/mm and a resolution of 0.1 Å in the first order. The experimentally determined risetime of the photomultiplier was 0.2 μsec.

In order to select an argon line suitable for scanning, the plasma spectrum was identified over a wavelength range of 3400 - 6700 Å. For this purpose a time-integrated spectrum of the plasma against an iron arc reference spectrum was obtained using a Hilger El prism spectrograph equipped with a quartz optical system. The analysis revealed a strong AII 5062 Å line which was conveniently separated by 10 Å from the nearest neighbouring lines. The distribution of the emission lines in the argon spectrum (predominantly AII) provided a rough
estimate of 25,000 °K for the electron temperature of the plasma. 26

The electron density \( n_e \) was calculated from the total half-widths (\( \Delta \lambda \)) according to Griem 27 by means of the relation

\[
n_e = \frac{n_e \Delta \lambda}{2 \omega}
\]

A.1

for the AlII 5062 results and by means of the relation

\[
n_e = C(n_e,T) (\Delta \lambda)^{3/2}
\]

A.2

for the H\( \alpha \) results. Here \( n'_e = 10^{17} \) electrons/cc, \( \omega = 0.13 \) Å is the Stark-broadening parameter for the relevant argon line 28,29 and \( C(n_e,T) = 3.88 \times 10^{15} \) Å\(^{-3/2}\) cm\(^{-3}\) is the hydrogenic Stark-broadening coefficient corresponding to an electron density of \( 10^{17} \) cm\(^{-3}\) and an electron temperature of 20,000 °K as given in Griem's tables. 37

Stark-broadening Results

The experimental arrangement used to obtain the line profiles is shown in Figure A.1. The monochromator collected radiation over a roughly rectangular volume approximately 3 mm in width, 15 mm in height and extending across the diameter of the discharge vessel at a position midway between the electrodes. Consequently, the measured electron density is a spatial average over this volume. For a single discharge, the photomultiplier mounted at the exit aperture of the monochromator gave the time history of the radiation intensity contained in a 0.16 Å wavelength interval centered at the monochromator
Figure A.1 Experimental Arrangement for Stark-broadening Measurements
wavelength setting. The photomultiplier signal was recorded simultaneously with the discharge current as shown in Figure A.2. For both the AII 5062 and the Hα lines, approximately five such oscillograms were taken at each of 30 different wavelength settings over the width of the profile. The oscillograms were analyzed in the manner described below.

![Figure A.2 Typical Monochromator-Photomultiplier Signal](image)

**Figure A.2 Typical Monochromator-Photomultiplier Signal**

Upper trace: discharge current for 1000 μHg argon

Lower trace: signal from monochromator-photomultiplier system for a wavelength near the center of the AII 5062 line

Both traces were triggered simultaneously: sweep speed 2 μsec/cm.

A digital X-Y readout system was used to measure the amplitude of the photomultiplier signal at 0.5 μsec steps over a time interval of roughly 10 μsec on either side of the first current zero after initiation of the discharge. This current zero served as the reference point for time measurement and
occurred at $10.0 \pm 0.2$ μsec after current initiation. An IBM 7040 computer was programmed to plot the amplitudes as a function of wavelength (i.e. the line profiles) with time after current initiation as a parameter. For each profile, the height relative to the background radiation level beyond the line wings was determined and the total half-width ($\Delta \lambda$) at half-height was measured. Together with Equations A.1 and A.2, these half-widths yielded the spatially averaged electron density as a function of time after current initiation as shown in Figure A.3.

The Stark-broadening measurements served a two-fold purpose. Together with the collapse curves for the plasma shell (Figure 4.2, page 56), this information permitted an estimate of the rate of change of electron density to be anticipated using the resonator technique. In addition, comparison of the peak radial profile densities with the Stark-broadening values provides a rough independent check on the resonator results. It should be pointed out that direct comparison of the measurements is complicated by the fact that the Stark-broadening results are radially-averaged while the resonator measurements are radially-resolved. From Figure A.3, it can be seen that agreement is poor for the initial stage of the discharge. However, for electron densities in excess of $10^{17}$ cm$^{-3}$, the values agree to within the experimental accuracy ($\pm 10\%$) in the region common to both measurements. As expected, the peak radial profile densities are greater than the Stark values in all cases.
Figure A.3  Stark-broadening Electron Density Results

Spatially-averaged electron density is plotted as a function of time after current initiation for the discharge in 1000 μHg argon.

- electron density from Hα half-widths
- electron density from AII 5062 half-widths
- optical resonator measurements
REFERENCES


