ENERGY DISSIPATION IN PAPER TEARING AS TIME-DEPENDENT PHENOMENON

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ABSTRACT

The nature of ballistic-type internal paper tear test methods has been reviewed. The kinetic energy of the tester sector is considered to be the prime contributor to paper rupture. In agreement with energy dissipation concepts and the principle of energy conservation, a mathematical model expressing tearing energy was derived based on kinetic energy variations in paper during tearing.

It is shown that this mathematical model can be used to calculate the net energy of the tester sector, which is available for tearing paper, and the residual energy. Consequently, the difference between net and residual energy, or tearing energy, is that portion expended in the rupture process. Furthermore, the mathematical model relates tearing energy to velocity, hence can be used to examine the effect of tear rate and time-dependent properties of paper subjected to tearing stress.

A method was devised for measuring the time required to tear standard samples. From an oscilloscope trace, the tear distance and time relationship was measured and represented by a quadratic equation. From this equation, sector swing and tearing velocities were calculated for computing various energy factors and their variation at any instant of the tearing process.

Results have shown that ballistic-type tear test methods are time-dependent, in that time required to tear paper varies with the sample condition. The higher the number of plies torn simultaneously, the longer was the time required to tear a paper sheet. The energy required to tear paper was also time-dependent, increasing with decreasing tear rate.

It was found that the direct relationship between tearing strength and number of plies torn simultaneously does not always hold, but that a constant direct relationship exists between tearing strength and tearing energy.

Although the ballistic-type tear test is time-dependent, inherent specimen properties may have a profound effect on results.

Test results with an Elmendorf tear tester on five paper grades varying in tearing strength from 14 to 156 g/sheet have confirmed that the energy dissipation concept is adequate.

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INTRODUCTION

Since advancement of the Elmendorf internal tear test method about half a century ago, it has proved valuable in measuring a basic paper strength property. This method has become widely accepted as a useful means for quality control evaluations and research studies. Adoption of standard test procedures has led to an important point of communication in the pulp and paper industry.

Considerable work has been done in the past on evaluating effects of various tear test variables. Several authors have published their findings on instrument calibration, as well as tearing principles and mechanism, which have contributed to the recommended standards. Yet, there still remains one variable, the effect of number of plies torn simultaneously on value obtained, which has been treated theoretically but has not been satisfactorily explained by experimental evidence. Hence, some authors continue to doubt value of this test method, continually pointing to the number of plies effect.

It has been constantly reported that Elmendorf tear test results increase as the number of plies torn simultaneously is increased. This positive relationship (in a few cases, negative relationship) may be so large as to double the tear strength value, when compared with results obtained under rigorous conditions. Previous investigators have reasoned or given evidence to explain this deviation in terms of several test factors. These include, clamp design, "spread-out" or "fan-out" of the plies, frictional binding between the torn paper edges, stiffness differences, basis weight and the degree of splitting or "skinning".

Recent advances in paper testing have emphasized the stress-strain relationship, and in some few cases rheological behavior has been studied. Most work in these fields has been concentrated on tensile strength. Sometimes paper tearing resistance has been examined under constant rate of strain or stress.

The application of rheological principles to paper has explained some properties which were unknown in the past. In a similar way, examination of the Elmendorf tear test method in rheological terms could provide some basic information about the test not revealed previously.

Two very obvious and basic phenomena, which have been neglected or ignored in paper tear testing, are the time required to fracture the piece and the rate at which this occurs. Researchers agree that the total time required is relatively short compared to other paper strength tests. Furthermore, one can easily distinguish a time difference between tearing a single paper ply and ten plies of the same material over the same distance. The longer time required to tear the ten plies demonstrates the slower rate effect. Combining this simple fact with the understanding that energy for the test instrument originates from position of the tester sector pendulum, suggests that a change in tearing time changes tear rate, and consequently changes the work done in tearing which affects the final test results.

From this it is possible to hypothesize that:

(1) The time required to tear paper varies with sample condition, which can be generalized in terms of tear resistance;

(2) Work required to tear paper is time-dependent, and increases with decreasing rate of tear;

(3) The increase in tear strength value as number of plies torn simultaneously is increased results from the longer time and slower rate of tear, a condition which requires more energy; and

(4) Results obtained from ballistic tear testers, such as the Elmendorf instrument, are time-dependent.

Thereby, the present study was designed to investigate time-dependent behavior of the paper tear test as it occurs with one standard instrument.

LITERATURE REVIEW

Numerous studies have been done on the effect of variables on paper tearing strength. These variables are known to originate from the morphological, physical and chemical characteristics of wood, changes occurring during pulping and pulp purification processes, as well as those introduced during papermaking processes. As an example, it has been reported that fiber length, cell wall thickness or other fiber density terms, the ratio of various fiber morphological characters, individual fiber strength, fragmentation or weakening of the fiber due to pulping and beating or refining processes, microfibril orientation and exposure, resultant degree of cellulose polymerization, distribution of the degree of polymerization and amount as well as kinds of hydrophylic hemicelluloses and residual lignin are important factors. The literature of this field has been discussed and recently reviewed by Casey (6), Dinwoodie (10) and Rydholm (21). All these factors are only indirectly related to the present study and are not further discussed.

Since the first report on determining paper tearing strength with the Elmendorf tester was published in 1920 (11), the method has been much discussed. Carson and Snyder (5) described theoretical aspects of the design and calibration of ballistic-type tear testers. Clark (7) used Thwing and

Marx-Elmendorf tear testers as examples to describe a simple method for calibrating a dynamic tear tester from which he determined a correction factor to be applied to each observed value. Bergea (3) proposed another calibration method, which was based upon adjustment of the tearing distance of sample sheets, so that the instrument yields approximately constant values over the ordinary working range.

Cohen and Watson (8) reviewed definitions and described the internal tearing resistance with paper as a group of forces acting on the tearing zone. The resultant of these forces was proposed as acting equally in opposite directions. By assuming that each resultant force was equal to the internal tearing resistance, the widely accepted definition for internal tearing resistance arose as the resistance opposed to the force exerted on either of the two portions of the paper area adjacent to the tear zone or line. This definition is expressed as:

$$R = \frac{W}{2d} \qquad [1]$$

where: R = internal tearing resistance, and

W = work done in tearing across a piece of paper having a certain length, d.

The definition has been adopted in calibration of Thwing-Elmendorf and Poller-Elmendorf tear testers.

In addition to this definition, Mallett and Marx (19) defined internal tearing resistance in terms of work done in tearing the paper divided by the tearing distance as:

$$R = \frac{W}{d} \qquad [2]$$

Clark (7) has shown, however, that the Marx-Elmendorf tear tester was graduated according to Equation [1].

Cottrall (9) studied the mechanism of tearing and its relationship with other paper strength parameters, concluding that the tear test itself is of little or no value and might even be harmful, if included as part of paper specifications. His explanation is that burst, tensile strength and folding endurance might easily be sacrificed by putting too much emphasis on obtaining high tearing strength. In fact, some paper grades receive more emphasis on burst and tensile strength than tear resistance. For other papers, tear resistance is preferred over burst and tensile strength. Otherwise, these same three strength properties are considered equally important for a large range of paper grades.

In order to account for both tearing and burst strengths, Fanselow and Fanselow (12) suggested using the product of both values as an index for evaluating fibre and particularly qualities developed during refining. They indicated advantages of characterizing pulp strength potential, removing complications involved in comparing pulps with divergent tendencies in developing burst or tensile strength versus retaining resistance to tear, evaluating performance characteristics of different refiners, evaluating pulp strength and other purposes. By similar calculations, various "beater values" have appeared in the trade.

Variations Caused by Conduct of the Test

Operation of the tearing test has very important effects on results. The specimen includes several, but a variable number of plies torn together. Varying the number of plies has been reported constantly as affecting tear test values (7, 8, 14, 16, 17, 24, 28 and 29). The relationship can be either positive or negative, seemingly dependent upon properties of a particular paper. Standard methods specify the range of acceptable tester scale values with number of plies adjusted so as to remain within the working range.

Winterbottom and Minor (29) showed that the number of plies torn simultaneously in the Elmendorf tear tester has a profound effect on the final test value. They recommended that adjusting number of plies to provide a scale reading between twenty to forty grams allows too large a working range. In order to overcome this disadvantage, they suggested using a single sheet as long as results were not too low for accuracy, while in no case should sheets be doubled to give scale readings over 30.

Swartout and Setterholm (24) explained cause of test value variations due to increasing the number of plies torn simultaneously in terms of "spread-out" of the sheet at the top of the specimen, bulking in the clamp and deviation from a straight line projected from the initial slit. They found that variation due to increasing the number of plies was not constant in that thicker papers had higher tear values than thinner ones, even at similar density. The tendency for spreading at the specimen top when securing in the clamps, and the tendency for the failure line to deviate from a straight line projected from the initial slit were associated with increasing number of plies.

Increasing bulk by placing separators between individual sheets increased the tear value, but the increase in bulk was accompanied by increased degree of spreading. By comparing different paper grades with 2-mil and 3-mil aluminum foil specimens, they concluded that relationship between tear value and number of plies torn is not an inherent feature of the tear tester, but relates to the type of material being tested.

Wink and Van Eperen (28) examined the effect of number of plies, as well as different clamping methods by changing the clamp design. They found that the method of clamping could introduce 10% variation in test results, while varying the number of plies could cause 100% variation. The degree of variation depended upon the type of material. These variations were described as due to change in the nature of tearing such as splitting and "spreading-out" of sheets. Change in the nature of tearing is further complicated by the degree of interfibre bonding. Accordingly, they compared the nature of tearing by the Elmendorf tear tester with tear failures in the usual paper applications and questioned the value of the Elmendorf tear test method.

Jones and Gallay (17) reported the positive relationship between tear factor (the ratio between tearing strength and basis weight) and the number of plies torn simultaneously as mainly caused by degree of paper splitting. Rate of increase was affected by basis weight, type of pulp and degree of beating. They concluded that increase in stiffness with increasing basis weight was not an explanation for the different behavior of their papers. Instead, they mentioned that higher basis weight sheets have a higher tendency to split than the lower basis weight sheets. Relationship between basis weight and splitting was also confirmed by results of Hardacker and Van den Akker (14).

In contrast to the Jones and Gallay idea (17), Wahlberg (27) emphasized a linear relationship between tear factor and stiffness expressed as flexural rigidity. He suggested that this occurs because stiffness is the most important phenomenon changed by increasing basis weight.

For the purpose of eliminating the effect of number of plies torn simultaneously, standard procedures for Thwing-Elmendorf and Marx-Elmendorf tear testers specify that several plies are to be torn together in order to give the instrument scale reading within certain defined limits. Cohen and

Watson (8) presumed that these limits are introduced to ensure a fairly uniform rate of tear. They further considered that these limits and the tear rate have a more significant influence on tear value than that caused by number of plies. The importance of tear rate was also considered by members of the Institute of Paper Chemistry (16).

Sample width has been reported to affect tear values. Cohen and Watson (8) found no significant effect on tear values when width variation is within 50 ± 12 mm, but greater widths, such as 100 mm, produced significantly higher values. This was related to higher bending resistance offered by wider specimens.

The effect of non-symmetry of the tear fracture or line in specimens has been studied by Cohen and Watson (8). They concluded that this could cause different distribution of stresses and consequently increase friction between torn edges, giving higher tear values.

Principle and Methods for Calibrating Ballistic-type Tear Testers

A ballistic-type tear tester such as the Elmendorf instrument used in this study consists of a sector pendulum (hereafter referred to as sector) suspended from a stationary post by means of a ball bearing. A jaw for holding one part of the specimen is attached to the stationary post, while another jaw having the same function is attached to the right radial edge of the sector.

Moving the sector from its equilibrium free swing position simultaneously raises the sector center of mass. Consequently, the potential energy changes as a function of the sector mass and of the vertical distance through which its center of mass is raised. If the sector is allowed to complete one half-swing from its raised position, theoretically, the angle (θ_1) (all angles employed to illustrate the tearing principle are presented graphically in Fig. 1) should be of the same magnitude as before release. However, very small amounts of energy are absorbed by friction at the ball bearing, and as scale pointer friction and air resistance. These result in a slightly smaller angle (θ_2) which expresses the net amount of potential energy available to do work on tearing the paper specimen.

Determining the swing angle, or net energy available for tearing the specimen enables locating the zero point on the instrument scale. During the tearing operation, part of the net energy is absorbed in fracturing the specimen and a still smaller angle (θ_3) is obtained. This angle (θ_3) expresses residual energy in the system after the specimen has been failed. The difference between the net energy and residual energy is that energy portion represented as work done to overcome the tear resistance of a specimen. Based on Equation [1], the average force required to tear a certain distance through a specimen can be calculated. In practice, a cosine scale is fixed on the sector to represent the amount of energy that has been used to tear a specimen. This scale enables the tearing force to be read directly in grams from the tester.

Most tear testers are calibrated by using 68.8 cm as tear distance. The distance over which the force acts is by definition, 2 X 68.8 cm = 137.6 cm. Apparently, it is not practical to tear a specimen 68.8 cm long. By assuming that the work required in tearing is directly proportional to tearing distance, and that the force required is linearly related to the number of plies torn simultaneously, the practice is to tear a booklet of sixteen sheets across 4.3 cm. Results read from the scale when tearing a different number of plies are adjusted to the sixteen-sheet basis. The equation used for calculating tearing strength (26) under these conditions is:

$$\text{Average tearing force} = \frac{16 \times \text{average scale reading}}{\text{number of plies}} \qquad [3]$$

where the average tearing force is expressed in grams per sheet.

Tear Test Theory

Very little attention has been given to development of tear test theory. Brecht and Imset (4) in 1934 considered the tearing zone as extensive, instead of as a point. The elemental forces involved in the tearing stress were considered as producing a moment of force related to a reference point. The tearing force is then calculated by dividing this moment by the perpendicular distance between the reference point to the line of action of the tearing force. This theory emphasized influence of stress concentration in the tearing zone and how fibre length and sheet extensibility affect size of the tearing zone and tearing strength.

Members of the Institute of Paper Chemistry (16) have pointed out weaknesses of the Brecht and Imset theory in that it attempts to deal with the forces arising between the fibres during tear failure and assumes a uniform distribution of steady forces over the tearing zone.

Another theory was adopted by members of the Institute of Paper Chemistry (16) in 1944. This is based on the mechanism of tear failure and energy dissipation, i.e., energy expended in tearing a sheet of paper is dissipated within the sheet. In tearing a specimen, the energy expended is transformed mainly into two parts:

(1) fractional drag work, pulling individual fibres out of the fibre network, and

(2) rupture work caused by stressing individual fibres until they break in tensile failure.

Based on analysis of stress-strain diagrams of fibres stretched to failure, and force versus displacement curves for pulling fibres from the network, they concluded that the <u>work</u> required to rupture a fibre is very much less than that required to extract it unbroken from the fibre mesh, although the force required to rupture a fibre is greater than that needed to drag out an intact fibre.

This theory has been used to explain some phenomena associated with tearing strength. Among pulp preparation and papermaking processes, beating may be considered an important factor affecting paper strength. In general, the burst, tensile strength and folding endurance increase with beating within the commercial refining range. Tearing strength behaves differently. After slight increase in the earliest beating stages, coniferous pulp tearing strength decreases progressively with further beating. Hardwood pulps require longer periods of beating to reach maximum tearing strength before further beating starts to decrease values.

Initial rise in the tearing strength-beating time curve is explained by the theory that there is a rapid increase in the fractional drag work per component in the very early stages of beating. However, as beating proceeds the number of ruptured fibres increases and fewer of them are pulled intact from the tighter mesh. This change in the failure mechanism is caused by increasing the amount of interfibre bonding and lowering of intrinsic fibre strength as a result of the beating treatment. Since the fractional drag work per individual fibre is greater than the rupturing work, the energy required to tear the sheet is reduced. This same explanation has been used to account for the effect of fibre length and strength on tearing strength, since the fractional drag work increases with increasing fibre length and the stronger the fibre the higher the resistance to rupturing forces. This, in turn, provides more members to be pulled intact from the mesh rather than ruptured by tearing forces.

Giertz and Helle (13) have recently reviewed the theory adopted by members of the Institute of Paper Chemistry and have tested it with a series of laboratory experiments. They were in general agreement with the theory, but recommended slight modification by adding another term to include the effect of strain for some distance on both sides of the failure. They also pointed out the importance of fibre strength and fibre length, and agree with Brecht and Imset (4) concerning the importance of sheet extensibility in determining tearing strength.

Paper Rheological Properties

Recent studies on paper strength properties based on the stress-strain-time relationship have provided new understanding of the nature of paper strength, as well as information about test methods. Paper has been shown to exhibit both elastic and time-dependent flow properties. It displays a non-linear load-elongation (stress-strain) curve. This curve is time or rate-dependent, i.e., at high rate of loading, breaking elongation decreases and breaking load increases which results in a smaller rupturing work. The time-dependent behavior of paper has been illustrated by Rance (20), using a Schopper tensile tester and by hanging various weights onto paper specimens.

Frequently, investigators have examined the stress-strain relationship instead of unidimensional paper strength properties. Special attention has been given to tensile strength (1, 20, 22), but little has been done regarding tearing strength. Members of the Institute of Paper Chemistry have examined the relationship in further support of their theory mentioned above.

The failure mechanism with ballistic-type tear testers has been indicated by members of the Institute of Paper Chemistry (16), Cohen and Watson (8) and Balodis (2) as an energy dissipation phenomenon, but no detailed discussion has been given. The effect of tear rate on tear test values has been mentioned as well, but unfortunately no evidence has been advanced supporting this idea.

In contrast to the assumed tear rate effect, Higgins (15) indicated that rate of loading was not the basic factor which contributes speciality to the conventional tear test. Higgins (15) also claimed that the load-tear distance relationship was a phenomenon caused by need to increase load to a maximum in order to start tearing, followed by need for gradually decreasing load as tearing progresses. The same phenomenon was reported by Anderson and Falk (1), who suggested that the dėcrease in load requirement resulted from the release of elastic energy during the increase in tear distance.

ENERGY DISSIPATION AND RATE OF TEAR IN THE TEAR TEST

Recent advances in materials science refocuses attention on paper from concern with rupture strength to the study of pre-rupture phenomena. This has involved studies on the influence of time factors on the stress-strain history prior to final rupture. For example, attention is focused on work-to-failure in creep under dead loads and in testing with various strain rates to failure. Some investigations also reveal that rupture can occur in stress relaxation tests conducted at constant strain. Use of viscoelastic models, as have been applied to many polymeric materials, deserves increased attention of paper technologists. Network models consisting of springs and dashpots have been used to visualize mathematical analyses involved in stress-strain-time relationships. Basically it is necessary to differentiate between:

(1) Elastic deformation (immediate response in phase with an applied load),

(2) Viscoelastic deformation (time dependent, but recoverable when the applied force is removed), and

(3) Flow deformation (not recoverable upon removal of the exterior force).

(1) and (3) seem to be predominant where the mechanical behavior of paper is concerned (Fig. 2).

Fundamental studies of paper strength based upon such stress-strain-time relationships have not only provided knowledge on paper strength properties, but also have helped to examine adequacy of present testing methods and standard test requirements. The latter is important because some of the limiting variables in prevalent standards have been arbitrarily assigned without sound foundation.

The ballistic-type tear tester has been widely used in the pulp and paper industry and related research activities as a means for evaluating sheet tear resistance. According to TAPPI Standard T414 ts-64 (26), one arbitrary requirement is making up a specimen with a certain number of plies which, when torn together, will give an instrument scale reading near 40. No literature discussing reasons for this limitation has been found. Cohen and Watson (8), and members of the Institute of Paper Chemistry (16), simply mention the effect of tear rate on tear test values. This concept can be easily reasoned by estimating the total time required to tear two specimens from the same source, but with widely different number of plies. Apparently, the one with higher number of plies requires the longer time to tear through the whole distance. In addition, it is reported that the tear test value increases with the number of plies torn simultaneously. Observing these two restrictions disallows meaningful comparison between paper grades of different characteristics.

Tensile testing has shown that paper exhibits both elastic and time-dependent flow properties. Non-linear stress-strain behavior from tests conducted with constant rate of strain can indicate:

(1) Non-linear elastic behavior,

(2) Linear viscoelastic behavior, or

(3) Non-linear viscoelastic behavior.

The stress-strain curve can be divided into elastic and post-yield regions. In the elastic region paper performs according to Hooke's law. After reaching the elastic limit, the curve begins to represent a plastic region by deviation from the almost straight line established within the elastic region. The material begins to exhibit flow. This flow is time or rate-dependent. At high rate of loading, breaking elongation decreases and the breaking load increases. Total energy required to produce failure is also affected by rate of loading.

The breaking or failure energy difference between fractional drag work and fibre rupture has been adopted in support of theory explaining some paper tear phenomena. Breaking energy as determined from the stress-strain curve is much affected by the time-dependent plastic flow region.

The energy stored in the tear tester is obtained by raising the sector center of mass. The sector is allowed to swing at the time of tearing. The potential energy (P.E.)

of the sector is transformed to kinetic energy (K.E.). This kinetic energy is capable of doing work, overcoming friction at the sector bearing, pointer bearing and air resistance, as well as tear through a specimen if the residual force from the sector is larger than the specimen resistance. Otherwise, the specimen will not be torn through and the sector will be stopped. When tearing a specimen within capacity of the tester (the Dynamic Tear Tester designed in Australia has three different, interchangeable weight sectors to provide different capacities) part of the energy is expended to tear the paper and to overcome frictions, and part of the energy will be left as residual energy. Since net and residual energies are partially kinetic energy, they are highly affected by the tearing velocity and consequently are time-dependent.

If energy dissipation phenomenon is expressed in mathematical form, this can be used to evaluate:

(1) the amount of energy dissipated in tearing,

(2) the amount of residual energy, and

(3) the increase in energy required as tearing distance is increased.

If the energy dissipation concept is correct, then the energy expended to tear a specimen can be calculated from the difference between net energy and residual energy in the form of kinetic energy. Results calculated from this relationship would then be useful in examining the energy dissipation concept, as regards tear rate effects and other factors of paper tearing.

The principle of conservation of energy describes that no energy is lost, but that it is converted from one to another form. Thereby, the total energy of a system is constant and can be expressed in the following form:

$$E_{tot} = E_{kin} + E_{pot} + E_{converted} \qquad [4]$$

where: E_{tot} = total energy of a system,

E_{kin} = kinetic energy = $\frac{1}{2}mv^2$,

E_{pot} = potential energy = mgy,

$E_{converted}$ = energy converted to forms other than E_{kin},

m = sector mass,

g = gravitational acceleration,

y = vertical displacement of sector center of mass, and

v = (tangential) velocity at time t.

The energy conversion in a tearing process may be illustrated by diagram (Fig. 3). This shows that when the sector is raised from its lowest position to the top position, potential energy is maximum, and this is converted into kinetic energy upon releasing the sector. The kinetic energy of the sector is converted to other energy forms, friction ($E_{friction}$) and to do paper tearing ($E_{tearing}$).

A shorter time is required to travel the distance between the beginning and finish of tearing when no tearing work is done, and longer time is required as more energy is expended to tear a paper specimen.

When the sector is allowed to swing without tearing a specimen (Fig. 4a) (hereafter referred to as zero-swing in contrast to a frictionless system), it swings with zero-swing velocity (v_1) at time t_1 at vertical gravitational position (y_1). The total energy components (Equation [4]) take the following form at time t_1:

$$E_{total} = \frac{1}{2}mv_1^2 + mgy_1 + f(v_1, t_1) \qquad [5]$$

where: $f(v_1, t_1)$ = various forms of friction as a function of zero-swing velocity and time.

The first two terms in Equation [5] represent net energy which is directly and indirectly available for tearing the specimen at time t_1.

When tearing specimens (Fig. 4b) a part of the kinetic energy is dissipated. The sector, therefore, swings with slower tangential velocity (v_2) at gravitational position y_1. Then, in a tearing swing the total energy components take the form at time t_2 when a test specimen is torn a distance L:

$$E_{total} = \frac{1}{2}mv_2^2 + mgy_1 + f(v_2,t_2) + T.E. \qquad [6]$$

where: $f(v_2,t_2)$ = various forms of friction as a function of tearing velocity and time.

Tearing energy (T.E.) is that part of the net energy which has been dissipated in tearing a specimen.

There are two kinds of friction forces in the system, namely, sliding and rolling friction of the bearing, and viscous friction between the sector and air. The sliding and rolling friction force is assumed independent of velocity and hence contributes no difference between zero-swing and tearing swing. The viscous friction force is directly proportional to the velocity, when velocity is not too high (23). In ballistic-type tear testers the velocity is considered low, therefore, viscous friction difference between the zero-swing and tearing swing can be considered as negligible. Hence:

$$f(v_1,t_1) = f(v_2,t_2)$$

Since Equations [5] and [6] are equal according to the law of conservation of energy, they may be written as:

$$\frac{1}{2}mv_2^2 + mgy_1 + f(v_2,t_2) + T.E. = \frac{1}{2}mv_1^2 + mgy_1 + f(v_1,t_1) \qquad [7]$$

and transposed as:

$$T.E. = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 + mgy_1 - mgy_1 + f(v_1,t_1) - f(v_2,t_2) \qquad [8]$$

Equation [8] then becomes:

$$T.E. = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}m(v_1^2 - v_2^2) \qquad [9]$$

The mass (m) of the sector is a constant. The only term contributing to T.E. is the difference between squared velocities $(v_1^2 - v_2^2)$. The zero-swing velocity (v_1) for sector position y_1 is a constant for one specific instrument operated under the same conditions, but tearing velocity (v_2) for sector position y_1, varies with the specimen. Therefore, T.E. can be calculated if both zero-swing velocity and tearing velocity can be measured.

Velocities can be calculated by finding the time (Δt) required to tear a certain distance (ΔL), i.e., rate of tear. An equation can be fitted to express tear distance as a function of time:

$$\text{tear distance} = f(\text{tear time}) \qquad [10]$$

By definition, the instantaneous velocity will be the first derivative of Equation [10], while the second derivative is the acceleration in the zero-swing and tearing swing. Preliminary tests showed that the regression function can be best expressed in a second degree or quadratic form:

$$L = a + bt + ct^2 \qquad [11]$$

where: L = tear distance,

t = tear time,

a, b, and c = constants or regression coefficients.

The first derivative of Equation [11] is velocity (v):

$$\frac{dL}{dt} = b + 2ct \qquad [12]$$

The second derivative of Equation [11] is acceleration:

$$\frac{d^2L}{dt^2} = 2c \qquad [13]$$

By using this mathematical model, both rate of tear and tearing energy may be related to time, enabling evaluation of time effects in the tear test or time-dependent phenomenon as regards paper tearing strength.

MATERIALS AND METHODS

Materials

Tear distance and its required time are directly related to the ballistic-type tear test method. More specifically, this tear distance-time function is another expression of ballistic-type tear test results. Specimens of the same tearing strength should show the same tear rate.

The purpose of this study was to evaluate time-dependent phenomena of paper tearing. Consequently, papers displaying a wide tearing strength range would meet this purpose better than a series concerned with minor adjustment of properties by treating a single pulp in various ways.

Five commercial papers were selected to cover a wide tearing strength range. They included unglazed onion skin, newsprint, 30-lb. n & m bag paper, Island 55.5-lb. wrapper and parcel wrap (hereafter referred to as onion skin, newsprint, bag paper, 55.5-lb. wrapper and parcel wrap, respectively) arranged in the order of increasing tearing strength. (All grade weights refer to 500 sheets, 24-in. X 36-in.). Cross-machine paper direction tearing values ranging from 14 to 156 g/sheet are listed in Table 1 together with caliper and basis weight measurements.

Methods

Tearing procedures

A Thwing-Albert Co. Elmendorf tearing tester was used in this study. All papers were conditioned according to TAPPI Standard T 402 m-49 (25) at 50 ± 2% relative humidity and 73 ± 3.5°F temperature until equilibrium conditions were reached. Papers were cut into standard 7.6 X 6.3 cm segments within allowable variation by a shear-type paper cutter. The long dimension always represented machine direction. TAPPI Standard T 414 ts-64 (26) on tear testing procedures and instrument adjustment was closely followed, except as regards adjustment of number of plies, and that only the cross-machine paper direction was tested. The tear tester was bolted tightly onto a $\frac{3}{4}$-in. plywood sheet, which was in turn clamped onto a rigid table to prevent any possible movement of the instrument base during swing of the sector. Test results were read from the tester scale and adjusted to average tearing strength according to Equation [3] . The tearing strength results are presented in Table 2 and are plotted versus number of paper plies in Fig. 5.

At the same time, tear distance-time data were obtained on an oscilloscope storage scope. Traces were permanently recorded by photographic means. Design and arrangement of equipment for simultaneous measurement of tear distance and time, as well as data handling will be described in subsequent sections.

Experimental design

As stated, five kinds of paper were examined in the study. Each paper type required preparation of several samples with different number of plies. Choice of number of plies depended upon individual paper sheet tearing strength as related to the sector scale reading between 5 and 75. Approximately equal spacings were made between consecutive specimens by varying the numbers of plies. The assignments were as follows:

PAPER GRADE	NUMBER OF PLIES (TREATMENT)
Onion Skin	10, 20, 40, 60, 70, 80
Newsprint	5, 10, 15, 20, 25
Bag paper	2, 6, 10, 14, 18, 22
55.5-lb. wrapper	2, 4, 6, 8
Parcel wrap	1, 3, 5, 7

Statistical analysis showed that three replications were a proper number for both tearing strength and tear time. This replication number was further confirmed as adequate by further test results which showed, with 95% probability at ± 5% of tearing strength and time mean values, that the required replication number was one, except for a few cases which required replications between one and three.

Rate of tear measurement

Precise measurement of the tear distance-time relationship is the center of the experimental part of the study. Papers are usually torn over very short time with high tear rate by a ballistic-type tester. Electronic instruments, such as an oscilloscope equipped with proper plug-in units, can measure time events down to micro-seconds accurately over short intervals of time. With lower accuracy measurements can be made over longer time intervals. The time required to tear across a 4.3 cm specimen of several paper plies was found to range from 0.08 seconds to 0.4 seconds. This range well fits the oscilloscope capacity.

A Tektronix Inc. type 564 storage oscilloscope can register voltage change with respect to time in a system by coordinate methods. When properly regulated, changes in tear time with respect to tear distance can be recorded on a storage oscilloscope for reading out data useful in analysis and calculation. If an electrically conductive sheet is connected as part of an oscilloscope circuit, tearing this sheet simultaneously alters the current passing through the sheet creating a voltage change, since the amount of current that can pass through a conductor is proportional to its cross-sectional area. This voltage difference shown on the oscilloscope increases as tearing of a resistor proceeds (Appendix 1) and this can be registered along the y-axis on the oscilloscope. Simultaneously, the time required to tear through the sheet is registered as the x-axis. These x,y coordinates locate tearing distance at any specific time allowing subsequent tear rate calculations.

Ordinary papers at low moisture content have low electrical conductivity. Under such conditions it is useful to incorporate a conductive material, operated as a part of a circuit, as part of the specimen.

Several conductivity methods were studied. The general principle was to add or modify one specimen sheet as conductive material. Advantages and disadvantages of several methods are discussed below.

(1) Electrically conductive paper is available on the market. Its uniform conductivity fitted well the need of this study, and tests gave a continuous voltage-time curve on the oscilloscope. Tearing distances and corresponding times were obtained by calibrating this voltage-time curve against a tear distance-voltage curve obtained by measuring voltage variation when certain tear distance was reached by cutting. This provided a voltage calibration scale over time.

A major advantage of this method is that no elaborate material preparations are needed. A disadvantage is that the conductive paper has a different tear resistance than the specimen under test, which requires a correction. The correction factor itself may not be constant, but will vary with total specimen resistance if the tear test is time-dependent.

(2) Thin aluminum foil is essentially homogeneous and has negligible tearing strength. Its very good electrical conductivity, however, results in little voltage change with respect to change in cross-sectional area as tearing proceeds. This small voltage change lowers accuracy in reading oscilloscope traces.

(3) Metal particles may be deposited on a paper surface. Theoretically, a uniform layer of silver or other metal can be deposited on a paper surface by using a vacuum evaporator, which is a basic accessory for electron microscopy. This method was tried and given up immediately. The high temperature needed could drastically change paper properties. Too thin a layer provided poor conductivity, while thicker layers required long time at high temperature, which is not desirable. For example, about two minutes exposure was needed for preparing one 7.6 cm by 6.3 cm specimen. Further, uniformity of the metal layer deposited may be affected by the microscopically rough paper surface.

(4) Electrical conductivity paper was perforated along the median line in an attempt to reduce tear resistance. One set of results showed that there was no difference in tearing strength (55.3 g/sheet) of this combination and the tearing strength (55.5 g/sheet) of specimens without treated conductive paper. The disadvantage is that it requires careful work to introduce perforations on electrically conductive paper and frequently the tear-path of a whole specimen booklet does not

follow exactly the direction of perforations introduced in the center ply.

Introducing perforations produced a stepwide curve (Fig. 6a), reflecting spacings of perforations. The end of each step on the curve indicates when tearing has reached a certain distance. This enables reading tear time by projection to the time-base x-axis.

(5) Graphite may be used as a conductive material, since it has a rather low electrical conductivity in comparison with aluminum foil. Applying ladder-like graphite lines to a paper surface gave the same advantage as Method 4, but not its disadvantage. Furthermore, applying graphite directly onto a member of the specimen booklet avoids the variable tear resistance effect introduced by including a foreign material into the body of the specimen. A stepwise curve is obtained as in Method 4, but with better vertical step lines (Fig. 6b), which increases accuracy in reading tear time. Transfer of graphite from a 6B pencil to the specimen surface is very time consuming and sometimes inaccurate. This is because quite a thick, wide line is required to form a satisfactory conductor.

(6) As reported by Andersson and Falk (1), conducting ink may be used as conductive material. Instead of using graphite, commercial silver paint used for drawing electric circuits has been found to be a good conductor that can be transferred

easily and reproducibly to a specimen surface. The coarseness of cross-lines can be controlled as desired by using a drafting pen. It was found that finer cross-lines provided finer tear distance-time stepwise curves. Yet, using too fine lines introduced discontinuities, causing one or several steps to be missed. Coarse cross-lines did not have this disadvantage. They resulted in coarser and less accurate tear distance-time curves, and also reduced sensitivity by reducing the step height (Fig. 6c) which accompanied small overall voltage change.

All six methods have advantages and disadvantages, as noted. Methods 4, 5 and 6 are compared in Table 3.

The final method adopted combined advantages of both Methods 5 and 6, yielding results as shown in Fig. 6d. Silver paint was used to draw all the cross-lines and one of the two main vertical lines which connect all the cross-lines. A fine to medium line coarseness was adopted. The continuity of each line was checked by voltmeter to ensure conductivity. Thereafter, another vertical main line was applied as graphite (Fig. 7). This was made thick enough to give good continuity and also good conductivity. Manipulating the graphite addition allowed adjusting the height of each step on the curve.

Distances between cross-lines are determined by the operator. For these experiments tear distances of 0.2, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 and 4.3 cm were used. Each cross-line was drawn by hand with the help of a straight-ruler and drafting pen. The final edge of each cross-line was located exactly at the tear distance mentioned above, except the last one, with beginning located at the 4.3 cm position in order to facilitate a final time reading. It is also possible to vary line spacings and thereby represent some mathematical function.

At testing, the sheet of paper with conductive material (hereafter referred to as the conductive ply) becomes one of the many plies to be torn and is placed at the specimen center. This conductive ply is connected to a circuit (Fig. 8) including battery, decade box and oscilloscope (Fig. 8 and 9). Voltage calculations are shown in Appendix 1. The two main vertical conductive lines are led out from the specimen to the circuit by two half-pieces of razor blade. Razor blades have advantages of being thin, stiff, available and very good conductors. Wires are connected to the razor blades by two alligator clips.

The wire leading from the sector clamp was connected to the anode of a battery and suspended freely in the air (Fig. 8) to reduce possible resistance due to the wiring arrangement. TAPPI Standard T 414 ts-64 (26) requires that the sector should make at least 20 complete oscillations before the edge of the sector which engages the sector stop, no longer passes to the left of a pencil line located one inch to the right of the edge of the sector stop. By suspending the wire freely in the air, the sector made more than 50 oscillations before it no longer passed to the left of the pencil line. Thereby, this arrangement is considered as contributing no significant effect on the basic test method.

The time required to tear a specimen is relatively short when compared with other paper strength test procedures. A slight time variation could cause serious errors in results. A synchronous system is necessary for starting the tear test and oscilloscope simultaneously. To do this, a micro-switch was mounted underneath the sector stop in such a way that pressing down the sector stop started both the tearing process and the oscilloscope recording. The tear distance-time stepwise curve was stored on the oscilloscope. A polaroid picture was then taken for easier reading and permanent record of the oscilloscope traces.

When tearing across a certain known distance breaks a conductive line, the total current passing through the conductive sheet grid is reduced, voltage changes and simultaneously a step is registered on the distance-time diagram. Ten steps can be obtained if ten conductive lines are painted on the conductive ply, and ten sets of tear distance-time data can be collected according to the description in Method 4. These data can then be used for calculating tear rate.

Zero-swing tear distance-time measurement

According to Equation [9], two velocity measurements are required for calculating tearing energy at sector position y_1, namely, zero-swing velocity, v_1, and tearing velocity, v_2. Tearing velocity can be measured according to the method mentioned above. Theoretically, zero-swing velocity cannot be measured by the same method. In practice, it is convenient to simulate zero-swing behavior by tearing one sheet of a material which has extremely low tear resistance, and hence, contributes no significant adjustment to the true zero-swing velocity. Cellophane, because of its high degree of coherence to concentrated tearing forces (6), molecular homogeneity and characteristic low tear resistance once tear is started (15), would seem to be an ideal material for this purpose. In keeping with the regular procedure, conductive silver paint was applied as spaced lines to cellophane samples. Contrary to the expected behavior, the tear distance-time curves obtained displayed lower velocity than had for paper. This suggests that cellophane, as a non-fibrous material with extremely low bending resistance, responds differently to tear force than paper. Perhaps a similar effect with aluminum foil led Swartout and Setterholm (24) to their conclusion on the positive relationship between tear value and number of plies torn simultaneously as originating from the type of material being tested. Another material, perforated tracing paper, prepared with ladder-like silver paint lines, gave the same response to tear force as paper and registered no tear resistance on the sector scale. This method is assumed to closely approximate the true zero-swing velocity and was adopted for measuring response for the velocity (V_1) calculation at sector position y_1. Certainly other systems could be devised for deriving zero-swing velocity if greater accuracy is required. For example, a properly perforated scale fixed to the sector base and passed before an electric eye might be used.

Handling of data and curve fitting

From each test, ten sets of tear distance-time data were made available for defining tear rate. For example, the tear distance-time relationship for one of the five papers, bag paper, is plotted as Fig. 10. Similar relationships existed for all five kinds of paper.

It is observed that tear distance (L) relates to tear time (t) as a second degree or quadratic relationship (Fig. 10).

$$L = a + bt + ct^2 \qquad [11]$$

A least squares method was used to fit the relationship as to mathematical form. Second degree curves were fitted by regression analysis, programmed on an IBM 7044 computer.

Some sets of measurements were studied as third degree equations. The second degree expression was proved to be adequate, although statistical comparisons showed in some instances that the third degree term was significant. Further analysis (Table 4) according to the procedures described in Li's Statistical Inference (18) was done to find reasons for the unexpected significance of the third degree term.

When calculating the regression sum of squares contributed by each term, t, t^2, t^3, and residual variance, it was always found that the residual variance was of the order of 1.5 to 3.5 X 10^{-2}. This extremely small residual variance rendered the variance ratio test extremely sensitive. When considering the very small multiple correlation coefficient difference, 1 to 20 X 10^{-5}, and standard error of estimate difference, 1 to 10 X 10^{-2} mm, between second degree and third degree expressions, together with their extremely small residual variance, it may be concluded that the second degree equation best describes all sets of measurements. Further argument for this choice will be given in the Discussion.

The tear distance-time relationships for zero-swing and the five paper grades were measured. Each paper grade comprised several treatments (6, 5, 6, 4 and 4 treatments for onion skin, newsprint, bag paper, 55.5-lb. wrapper and parcel wrap, respectively), for the different number of plies. Three replications were used for zero-swing and each treatment, except that one replication only was used for 70 plies of onion skin paper. These gave a total of 76 tests for the entire study. Each test was analyzed at ten positions.

Final expressions could have been found by any of three ways. First, average values from the three replications may be used to solve a single equation. This method is no better than a second approach, which utilizes all three individual sets of data by the least squares method to fit a single equation. The disadvantage here is the higher standard error of estimate due to dispersion of tear times (in all tests, tear distance (L) is fixed). Third, each individual set of data can be used to fit an equation. Three replications can result in three different equations. Equations developed by the third method are presented in Table 5 together with their multiple correlation coefficients and standard errors of estimate. By the third method, tear velocity for each replication can be calculated by substituting tear time into its corresponding first derivative equation, and an average of these tear velocities can be used as the final result. Advantages of this method are that all the original measurements contribute directly to the final result and also that an individual equation for tear distance-time measurement has a smaller standard error of estimate.

For these reasons this method was adopted for the main study, i.e., for a certain number of paper plies, three regression equations were fitted. Substituting corresponding tear-times into the first derivative of the regression equation provided three tearing velocities which were averaged to find the sector net energy and residual energy. By such means, tearing energy for each specimen can be calculated according to Equation [9] . Since the mass (m) term in Equation [9] is a constant for a particular sector, all tearing energy values presented are made a fraction of total tearing energy by removing the sector mass factor. Tearing energy required per ply can be obtained by dividing the tearing energy for each specimen by its number of plies. Tearing energy as presented in Table 2 is the energy required to tear across 4.3 cm through a single sheet. These tearing energies are further presented in Fig. 11a to 11e to show tearing energy variation with respect to rate of tear.

Summary of methods

Test specimen booklets with different numbers of plies were tested with an Elmendorf tear tester. Positioned at the center of each booklet was a single ply of the test material to which ladder-like silver paint cross-lines had been added. At its outer edges, vertical-lines, one of which was graphite, connected the cross-lines. By this arrangement, the center ply rendered the specimen as part of an electric circuit which allowed measurement of times required to tear through pre-assigned distances. A synchronous mechanism was incorporated to record the tear distance-time relationship, beginning as paper tearing started.

Tear distance-time relationships were registered on an oscilloscope and were permanently recorded as polaroid pictures. Each test as done here provided ten tear distance-time measurements which were read from a polaroid picture.

These data were fitted as second degree equations. Velocity was calculated from the first derivative of the second degree equation.

The zero-swing time required to tear through distances on an imagined blank sheet was simulated by tearing a single sheet of perforated tracing paper treated in the same way as for tearing test specimens.

Three velocity values arose from replications of each sample: these were averaged for calculating tearing energy according to Equation [9]. Tearing energy was then presented in terms of fractional energy per ply for comparison between materials.

For comparison purposes, readings were taken from the tester sector scale and adjusted to the standard average tearing strength according to TAPPI Standard T 414 ts-64 (26).

DISCUSSION

The background of factors affecting tear test results has been reviewed in former sections.

According to the energy dissipation concept, energy expended in tearing a sheet of paper is dissipated within the sheet. In the present study a mathematical model has been derived for demonstrating the nature of ballistic-type tear testers. This model can be used to calculate the net energy of a tear tester sector which is capable of doing work, residual energy after tearing paper and overcoming friction, and the tearing energy which is that part of the energy absorbed by paper during the tearing process. Furthermore, this tearing energy concept is related to the dynamic nature of the tear test by using paper tear velocity as the basis for calculation. Since velocity is calculated from distance and time, an opportunity exists for relating tear test data with time, thus providing means for evaluating the effect of tear rate. Hopefully, this examination of tear rate will clarify some problems associated with the paper tear test. For example, showing tear progress and energy variation as tearing proceeds, and exploring the relationship between tearing energy and tear rate, provides new information on conduct of the test.

Tearing velocity is an expression of the time required to tear across a certain distance. In order to measure the distance-time relationship, a precise method had to be developed. The effectiveness and reproducibility of this method is demonstrated by the very small variation had between replications. In most cases, it has been shown that only one measurement is necessary for representing the specimen condition under study.

Reproducibility of the basic method could be further improved by applying the silver paint lines with some mechanical device which regulates distances more precisely than can be done by hand drawing.

Curve Fitting

Although there is no technical problem in measuring tear distance-time data, use of these measurements needs some further discussion. That is, decision is required on the best form for these data, i.e., expression as a linear, quadratic or cubic equation. All three types of equations can be used to express the tear distance-time relationship. The difference in degree, however, affects evaluation of the energy distribution. Since the tearing energy calculation is based on velocity, velocity is calculated by substituting tear time into the first derivative of the equation of any degree.

If a linear equation as:

$$L = a + bt \qquad [14]$$

is used, the first derivative, velocity, will be a constant

$$\frac{dL}{dt} = b \qquad [15]$$

with no acceleration, so the sector swings and tears with a constant velocity and residual tearing velocity.

If a quadratic equation as:

$$L = a + bt + ct^2 \qquad [11]$$

is used, its first derivative, velocity, will have a linear form

$$\frac{dL}{dt} = b + 2ct \qquad [12]$$

with a constant acceleration

$$\frac{d^2L}{dt^2} = 2c \qquad [13]$$

If a cubic equation as:

$$L = a + bt + ct^2 + dt^3 \qquad [16]$$

is used, its first derivative, velocity, will be of parabolic form,

$$\frac{dL}{dt} = b + 2ct + 3dt^2 \qquad [17]$$

with acceleration

$$\frac{d^2L}{dt^2} = 2c + 6dt \qquad [18]$$

Using all three forms among data of a single study would cause difficulty in comparisons. This is because the different forms of equation refer to basic differences in the nature of tearing or nature of the applied force. In fact, all tearing energy originates from the sector, so all the applied force will be the same, causing tear to proceed as common behavior. Hence, the replicated zero-swing tear distance-time measurements can logically only be expressed by equations of the same degree.

The tear distance-time relationships could be expressed as different degree equations for various tests. The linear relationship is true if the total paper tear resistance increment is slightly smaller than the net energy increment of the sector. Therefore, paper is torn at a state approaching linear relationship. Preliminary tests confirmed this phenomenon for the last part of tear distance-time curve when a specimen with high number of plies, or more properly a specimen with high tear resistance, was torn.

Preliminary tear distance-time test data with 25 plies of bag paper is plotted as part of Fig. 10 with other final test results. It can be observed that starting with 0.5 to 3.5 cm depth, the tear distance-time relationship of the 25 plies curve is of linear form. After 3.5 cm the effect of decreasing tear velocity becomes more and more pronounced and the curve direction starts to change into a concave form. This linear relationship can always be identified with sector scale readings over 75 or 80.

In order to avoid the complication caused by using the linear relationship, this study was designed to use only a limited capacity of the tear tester. Actually, this consideration is not necessary, because the initial part of the tear distance-time curve is always of curvilinear form and a linear expression ignores this fact.

A regression analysis program eliminating least important variables successively showed the second degree term, t^2, as always the most important single variable representing the relationship. Therefore, it is definitely incorrect to use the linear form.

There is a possibility of using the cubic as an expression for tear distance-time data. As above, the bag paper specimen with 25 plies is a good example for this case. This curve reflexes twice, from curvilinear at the beginning to linear, and then it appears as another curvilinear form, concave facing the x-axis. These data can be best expressed mathematically by a cubic equation. The experimental design for this study purposely avoided this case by using only a limited part of sector capacity. Hence, the cubic relationship should not exist within range of the present study.

The tear distance-time relationships collected for the study can be observed easily as quadratic forms (Fig. 10) best expressed by second degree equations. Statistical analyses showed the quadratic relationship as correct for a majority of the trials, but that a third degree term, t^3, could be retained in some equations. A further analysis was conducted to find reasons for this. It was discovered that all equations with significant third degree terms possessed an extremely small residual variance 0.1 to 8.5 X 10^{-2}. This level of residual variance made the variance ratio test extremely sensitive.

Table 4 contains an analysis of variance model used to test significance of the partial regression coefficient for measurements on a 10-ply newsprint specimen. This analysis showed that the t^3 term should be retained. Variance ratio of the t^3 term was 6.610, which was significant at the 5% level. When comparing this variance ratio to the total variance ratio value 16,739.9, including t, t^2, and t^3, the t^3 variance contributed only about 1/2500 of the total variance. Furthermore, the multiple correlation coefficient for an equation containing t, t^2, and t^3 was 0.99994, which was only 7 X 10^{-5} larger than the 0.99987 for an equation containing only t and t^2 terms. The difference of standard error of estimate between these two equations was 6.66 X 10^{-2} mm from 22.5 mm of the average tear distance. Such difference was indistinguishable in the graphic form. Consequently, it can be concluded that the t^3 term is not necessary, hence the quadratic equation may be used to express all tear distance-time relationships.

Interpretation of Results

The zero-swing velocity and tearing velocity were calculated from the first derivatives of Equation [11], as given in Equation [12] . From these two velocities, the sector net energy and paper tearing residual energy were calculated. The difference between these two energy terms, when divided by the number of plies in the specimen, is the total tearing energy per ply sought in this study.

By these methods, total tearing energy per ply at any instant of tear distance or time can be calculated. Although ten sets of tear distance-time data have been measured for each test specimen, tearing energy at any point of distance or time could have been calculated. Only total tearing energies per ply at 0.2, 1.0, 2.0, 3.0, 4.0 and 4.3 cm distances across tear specimens and their corresponding times were included. These average tearing energy per ply values together with average tear distance, time, velocity, conventional cross machine-direction tearing strength and their identification are presented in Table 2.

Tearing energy per ply variations with respect to tear distance and relative tear time for each kind of paper are further presented in Fig. 11a to 11e. The relative tear times used in these figures are defined as time differences between those required for the sector to swing to a certain tear distance without a paper specimen load and that required for the sector to swing over the same distance when tearing a paper specimen. The value is obtained by subtracting tearing swing time from zero-swing time at corresponding tear distance.

There is one common phenomenon appearing with all the five materials (Fig. 11a to 11e), namely, that the lowest ply specimen never behaved the same as other specimens having greater number of plies. It is also important to note that all of the lowest ply specimens from each kind of paper had a sector scale reading below 15. The exact scale readings were 11.7, 10.5, 7.5, 14.5 and 10.0 for 10-ply onion skin, 5-ply newsprint, 2-ply bag paper, 2-ply 55.5-lb wrapper and 1-ply parcel wrap, respectively. These specimens, having comparatively low tear resistance, were torn at relatively high velocities. High tearing velocity did not allow the specimens to respond completely to the advancing force, hence the behavior was different from other specimens of a series which were torn at lower tearing velocity. Since these lowest ply specimens behaved differently from other specimens, they are excluded from the data contributing to further discussion.

Figures 11a, for onion skin, 11b, for newsprint and 11d for 55.5-lb. wrapper all showed the same variation in total tearing energy per ply. That is, the total tearing energy required to tear a single sheet of paper increased as the number of plies torn simultaneously and tear time increased.

Increase in the rate of total tearing energy per ply was more pronounced when only a short time was required to tear through the paper and also when tear resistance of the paper sample was low. Tear resistance can be considered as another expression of number of plies within one kind of paper. The increase in the rate of total tearing energy per ply decreased as tearing time was prolonged by higher paper tear resistance.

One of the advantages of the method used is that it reports the tearing energy required to tear over any distance. Also shown in Fig. 11a to 11e are tearing energy profiles and their relationship with tear time at certain tear distances. Onion skin and newsprint showed the same tearing energy-time relationship at any tear distance throughout the test.

The 55.5-lb. wrapper presented another pattern in that, at the beginning of the tear, the tearing energy per ply decreased then increased as tearing time increased (Fig. 11d). This was because one of the three 4-ply specimens was torn at a disproportionately slower velocity. For example, 12.42 cm/sec compared to 13.67 and 14.08 cm/sec at 0.2 cm tear distance. This slower velocity resulted in less calculated residual energy and in turn higher tearing energy. Consequently, the average higher tearing energy of 4-ply specimens altered the tearing energy and time relationship as shown in Fig. 11d. Otherwise, the tearing energy values of 4-ply specimens were smaller rendering Fig. 11d with the same pattern as Fig. 11a and 11b.

These three cases (Fig. 11a, 11b and 11d) demonstrate that total tearing energy per ply is directly related to the time required to tear through a paper. When the incident force is the same, the longer time required to tear through a sheet of paper of definite length means that rate of tear is slower. Conversely, the relationship can be described as more total energy required to tear through a single sheet when the rate of tear is slower.

Bag paper presented another variation, as shown in Fig. 11c. This pattern can be divided into two parts. The first part is from the beginning of tear to a certain distance between 1.0 to 2.0 cm. In this region, total tearing energy per ply increased as number of plies and tear time increased. In the second region, which continued until failure, total tearing energy per ply decreased as number of plies and tear time increased, and then started to increase as number of plies and tear time was increased.

No unusual experimental errors occurred in measuring tearing energy for the bag paper. Otherwise, tearing energy values calculated from individual specimens with replications would not have provided such smooth curves. The different tearing energy variation with respect to time is thought to be inherent in the paper sample. The particular paper property involved is unknown.

It has been pointed out that a negative relationship may occur between tearing strength and number of plies torn simultaneously. The bag paper showed this behavior in part (Fig. 5).

Another paper, parcel wrap, presented an irregular tearing energy distribution (Fig. 11e) with no special tendencies. It was found that this parcel wrap had very poor formation, irregular distribution of fibres and fibre bundles and thin spots over some parts of the sheet. Such irregular fibre distribution can affect usefulness of all strength tests.

When the relative times (excluding the tear times for the lowest ply specimens for each paper grade) required to tear 4.3 cm through booklets of sheets containing different numbers of plies are divided by, and plotted versus, number of plies, a positive curvilinear relationship is obtained for all five kinds of paper (Fig. 12). This finding confirms as expected, that longer time is required to tear per unit sheet of paper when higher number of plies are torn simultaneously. In addition, this finding also confirms the time-dependent behavior of the tear test. Furthermore, this can be described as rate of tear being time-dependent.

The present study was designed according to the energy dissipation concept of paper tearing properties. This concept has been widely accepted in evaluating paper tearing strength and for calibrating the Elmendorf tear tester. Development of Equation [9] for calculating tearing energy in this study is based on transformation between tear tester sector gravitational potential energy and kinetic energy. In the course of tearing, the gravitational potential energy is transformed into kinetic energy to do work in tearing paper. Kinetic energy is obtained by evaluating velocity at instantaneous time. By the definition of velocity, there is no doubt about the time-dependent property of velocity and hence kinetic energy is itself time-dependent. The tearing energy calculated as in this study is the difference between two kinetic energy terms, therefore it is also kinetic energy and should be time-dependent as well.

Results with onion skin, newsprint and 55.5-lb. wrapper confirmed the above concept by showing positive relationship between total tearing energy per ply and tear time. This positive curvilinear relationship is also affected by specimen tear resistance.

Tear time was directly related to test specimen resistance, i.e., the higher the tear resistance the longer the time required to tear through a specimen. Within one kind of paper, tear resistance increased directly with number of plies. Hence, it can be said that with a given paper more time is required to tear through specimens with higher number of plies. This is shown in Fig. 12 (excluding all lowest ply specimens for each paper grade) as a curvilinear relationship. Since more total tearing energy per ply is required when tearing over longer time, or at a lower rate, it can then be concluded that more total tearing energy per ply is required when tearing a specimen with higher number of plies.

The results with bag paper and parcel wrap show that it is necessary to slightly modify the conclusion described above.

That is, while tearing energy is still time-dependent, and more tearing energy is required to tear a single sheet when higher number of plies are torn simultaneously, some inherent sheet properties may have profound effects. In fact, tearing energy calculated in this study is the total amount of energy required to tear a specimen. Any factor which can alter specimen tear resistance also alters tearing energy. The results shown in Fig. 13 confirmed this direct relationship. Taking parcel wrap for example, the irregular fibre distribution noted in the paper sheet caused irregular tear resistance, therefore caused the same total tearing energy variation no matter what time was required to tear a given specimen. Bag paper also showed the same effect. For unknown reasons the tearing strength of bag paper first decreased then increased as the number of plies torn simultaneously was increased (Fig. 5). The same tendency is shown for tearing energy with the same material (Fig. 11c). These modifications reaffirm the conclusion of Winterbottom and Minor (29), that the positive relationship between tear value and number of plies torn simultaneously is not inherent in the tear tester, but in the type of material being tested.

The tearing energy and tear distance relationship for onion skin paper as shown in Fig. 14 is linear in that the increase of energy dissipated per ply is a linear function of tear distance. The longer the tear distance, proportionally, the more tearing energy required per ply. This tearing energy per ply and tear distance relationship is further affected by the number of plies torn simultaneously. The higher the number of plies, the more tearing energy per ply is required to tear through the same distance compared to fewer plies. This may be because specimens having higher number of plies were torn at slower rate, and as a result required more tearing energy. The tearing energy and distance relationship of the 10-ply onion skin specimen in Fig. 14 further demonstrates that this specimen, because of its low tear resistance, behaved differently from other specimens comprising higher number of plies.

The acceleration of each tearing velocity, 2c (Equation [13]), was obtained from the second derivative of the tear distance-time, Equation [11] . When the average of three replications was plotted versus number of plies torn simultaneously, a linear relationship was obtained for newsprint, bag paper, 55.5-lb. wrapper and parcel wrap, respectively (Fig. 15). Onion skin maintained a straight line up to about 40 plies before changing to a curvilinear form (Fig. 15). It should be noted that all five lines intersected the y-axis at almost the same point, which represented the acceleration of the sector zero-swing velocity. This presents evidence that all data were measured with small range of experimental error. Linear relationships between tear acceleration and number of plies means proportional decrease in tearing velocity with respect to increase in number of plies. Onion skin was the weakest member among the five kinds of paper tested. The change in relationship from linear to curvilinear at about 40 plies may be because specimens with too many plies had the same bulk effect as demonstrated by Swartout and Setterholm (24). In addition, displacement of outer plies due to specimen bending when tearing is expected to be more serious when specimen gross thickness is large. This displacement of outer plies can be expected to produce some serious effect on the distribution of stresses, hence slow down tearing velocity by reducing acceleration.

The negative linear relationship between tearing acceleration with respect to number of plies within a certain range can be used to reason that, when the total specimen thickness is not too large, friction between neighboring plies, displacement of outer plies and increase in bending resistance due to increase in the number of plies or specimen bulk do not have a significant effect on tearing energy requirements. Otherwise, there would be a curvilinear instead of linear relationship.

Review of Present Tear Test Knowledge

(1) Energy dissipation concept

The energy dissipation concept has been adopted for calibrating the Elmendorf tear tester. Members of the Institute of Paper Chemistry (16) have applied it to explain some paper tearing phenomena.

The methods and mathematical model of this study are derived according to the concept of energy dissipation. Kinetic energy difference bet@een sector net energy and residual energy are used to calculate tearing energy. This tearing energy is then an expression of the amount of energy expended in tearing a sheet of paper; or it can be considered as the amount of energy dissipated within the sheet. The prevalent tearing strength is measured according to the energy dissipation concept as well. Hence, tearing strength should relate proportionally to tearing energy. Tearing strength results, shown in Fig. 5 with respect to number of plies, are further plotted in Fig. 13 with respect to the total tearing energy per ply of the corresponding number of plies. All lowest ply specimens for each kind of paper have been excluded from this figure because of their special behavior. Positive relationships were obtained for all five kinds of paper. All showed that higher tearing strength is always accompanied by higher energy requirement for tearing a single sheet. This direct evidence confirmed that the present prevalent tearing strength is a measure of the amount of energy dissipated in tearing a particular sheet of paper, hence it is a measure of the tear resistance of a paper against the incident tearing force.

(2) Dynamic property of paper

In order to determine whether tear resistance, measured in the conventional way, was basically a dynamic property of paper which reflected the response to a high rate of loading, Higgins (15) established good correlation between conventional tear factors and maximum load measured during very slow tearing test at a rate of strain of 0.64 mm/min. by using a D.F.P. (Division of Forest Products) rheometer (22). He concluded that the rate of loading was not the basic factor which contributes the speciality in the conventional tear test.

As this study has shown, the conventional tear test is basically affected by rate of tear or rate of loading. The difference between Higgins conclusion and this study arose from the different approaches used. Higgins used very slow tearing at constant rate of strain. This study used a high rate of loading and specimen strain was not considered. The latter approach is considered to relate better to the conventional tear test.

(3) Rate of tear effect and limitation of scale reading

It has been proved in this study that tear rate has an effect on the final test value. Figures 11a, 11b, and 11d illustrate that the variation of tearing energy is less pronounced when longer time is required to tear through a specimen, or when specimen tear resistance is large. It has also been found that variation in tearing energy with time becomes less pronounced when the sector scale reading is around and more than 50. Hence, it is preferable for the standard test method to have a specimen which will give a sector reading around or more than 50, rather than the 40 recommended in TAPPI Standard T 414 ts-64 (26). This is because at scale reading 40, tearing energy variation is still not stable.

Specimens used in this study covered a wide range of tear values. All five kinds of paper showed that tearing behavior at scale readings between 0 to 15 is different from that with scale readings above 15. Hence, it is not adequate to compare test results which are obtained from two different tearing behaviors. Winterbottom and Minor (29) suggested using single samples as long as these did not give values too low for accuracy. If results, and the interpretation of the present study are correct, Winterbottom and Minor's suggestion should not be adopted, since tearing energy variation over the low tear resistance range is not stable.

CONCLUSIONS

The following conclusions can be drawn from results of these tearing energy studies.

1. The energy required to tear papers of the study was time-dependent. More tearing energy was needed per unit ply when the tear rate was slower. Variation in total tearing energy per ply was affected more when total time was short. This effect was reduced by prolonging tear time.
2. Longer time was required per unit sheet when higher number of plies were torn simultaneously, and more energy was required per ply when tearing a specimen having higher number of plies.
3. Although the energy required to tear through a specimen was time-dependent, and more energy was required when higher number of plies were torn simultaneously, inherent material properties, particularly sheet discontinuities, may have profoundly effected some tearing energy results.
4. The conventional internal tear distance-time relationship was found to be a parabolic form which can be expressed in terms of a second degree equation.
5. The linear increase in total tearing energy as tear distance is increased is positive. The rate of increase is further affected by number of plies torn simultaneously.

6. The linear relationship between tear acceleration and number of plies torn simultaneously implies that friction between neighbouring plies, displacement of outer plies and increase in bending resistance due to increase in number of plies did not significantly change tearing energy requirements.
7. The positive relationship found between conventional tearing strength and tearing energy proves that the energy dissipation concept is adequate.
8. From findings of this study, it is suggested that the standard test method should treat a specimen giving a sector scale reading around or more than 50, and that in no case should sector scale readings lower than 15 be considered acceptable.

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Table 1

Parameters of the five paper grades used in the study

PAPER GRADE	CALIPER* mil	BASIS WEIGHT** g/m^2	TEARING*** STRENGTH g/sheet
Onion skin	1.70	28.5	14.1
Newsprint	3.41	52.1	42.7
Bag paper	2.88	48.8	57.0
55.5-lb. wrapper	5.90	90.3	128.0
Parcel wrap	6.65	97.6	156.0

*Data were measured according to TAPPI Standard T411 m-44.

**Basis weight is defined as the weight of paper in grams per square meter. Data were measured according to TAPPI Standard T410 os-61.

***Cross machine-direction tearing strengths measured according to TAPPI Standard T414 ts-64.

Table 2

Average tear distance, time, velocity, energy and strength values for five paper grades

PAPER GRADE	NO. OF PLIES	AVERAGE TEAR DISTANCE (cm)	TEAR TIME (sec)	ZERO-SWING VELOCITY (cm/sec)	SECTOR NET ENERGY (erg)	TEARING* STRENGTH (g/sheet)
zero-swing	0	0.2	0.078	15.13	114.43	
		0.5	0.098			
		1.0	0.127	19.84	196.73	
		1.5	0.150			
		2.0	0.171	24.19	292.53	
		2.5	0.190			
		3.0	0.209	27.83	387.20	
		3.5	0.226			
		4.0	0.243	31.20	486.68	
		4.3	0.254	32.20	518.53	0.00

PAPER GRADE	NO. OF PLIES	AVERAGE TEAR DISTANCE (cm)	TEAR TIME (sec)	TEARING VELOCITY (cm/sec)	TEARING ENERGY (erg)	TEARING STRENGTH (g/sheet)
onion skin	10	0.2	0.084	14.48	0.96	
		0.5	0.104			
		1.0	0.133	18.92	1.77	
		1.5	0.157			
		2.0	0.179	23.21	2.32	
		2.5	0.200			
		3.0	0.220	26.91	2.51	
		3.5	0.239			
		4.0	0.255	30.16	3.20	
		4.3	0.264	31.01	3.95	11.70

* These are cross-maching direction tearing strength

Table 2 cont'd

PAPER GRADE	NO. OF PLIES	TEAR DISTANCE (cm)	AVERAGE TEAR TIME (sec)	AVERAGE TEARING VELOCITY (cm/sec)	AVERAGE TEARING ENERGY (erg)	AVERAGE TEARING STRENGTH (g/sheet)
onion skin	20	0.2	0.087	14.60	0.39	
		0.5	0.108			
		1.0	0.137	18.78	1.02	
		1.5	0.161			
		2.0	0.184	22.76	1.68	
		2.5	0.204			
		3.0	0.225	26.23	2.16	
		3.5	0.245			
		4.0	0.262	29.36	2.79	
		4.3	0.270	30.06	3.33	11.80
	40	0.2	0.094	13.57	0.56	
		0.5	0.114			
		1.0	0.146	17.19	1.23	
		1.5	0.174			
		2.0	0.198	20.76	1.93	
		2.5	0.221			
		3.0	0.243	23.85	2.57	
		3.5	0.263			
		4.0	0.284	26.62	3.31	
		4.3	0.294	27.33	3.63	13.00
	60	0.2	0.096	12.56	0.59	
		0.5	0.120			
		1.0	0.157	15.43	1.30	
		1.5	0.186			
		2.0	0.214	18.28	2.09	
		2.5	0.240			
		3.0	0.263	20.67	2.89	
		3.5	0.288			
		4.0	0.312	23.05	3.68	
		4.3	0.323	23.59	4.01	14.00
	70*	0.2	0.097	12.45	0.53	
		0.5	0.122			
		1.0	0.160	14.64	1.28	
		1.5	0.191			
		2.0	0.223	16.84	2.15	
		2.5	0.250			
		3.0	0.276	18.68	3.04	
		3.5	0.302			
		4.0	0.330	20.57	3.93	
		4.3	0.344	21.05	4.24	14.30

*result of a single measurement without replication

Table 2 cont'd

PAPER GRADE	NO. OF PLIES	AVERAGE TEAR DISTANCE (cm)	TEAR TIME (sec)	TEARING VELOCITY (cm/sec)	TEARING ENERGY (erg)	TEARING STRENGTH (g/sheet)
onion skin	80	0.2	0.097	11.53	0.60	
		0.5	0.124			
		1.0	0.167	13.02	1.40	
		1.5	0.204			
		2.0	0.238	14.54	2.34	
		2.5	0.270			
		3.0	0.300	15.86	3.27	
		3.5	0.331			
		4.0	0.365	17.25	4.22	
		4.3	0.382	17.60	4.55	15.30
news-print	5	0.2	0.084	14.31	2.40	
		0.5	0.105			
		1.0	0.134	18.73	4.26	
		1.5	1.158			
		2.0	0.181	22.91	6.01	
		2.5	0.202			
		3.0	0.221	26.42	7.66	
		3.5	0.240			
		4.0	0.258	29.66	9.39	
		4.3	0.267	30.51	10.62	33.60
	10	0.2	0.086	14.35	1.15	
		0.5	0.106			
		1.0	0.138	18.07	3.36	
		1.5	0.164			
		2.0	0.187	21.51	6.13	
		2.5	0.208			
		3.0	0.229	24.54	8.60	
		3.5	0.250			
		4.0	0.270	27.43	11.05	
		4.3	0.279	28.09	12.39	41.60
	15	0.2	0.093	13.46	1.59	
		0.5	0.114			
		1.0	0.147	16.64	3.89	
		1.5	0.176			
		2.0	0.200	19.75	6.50	
		2.5	0.225			
		3.0	0.247	22.49	8.95	
		3.5	0.270			
		4.0	0.292	25.08	11.48	
		4.3	0.302	25.66	12.62	42.70

Table 2 cont'd

PAPER GRADE	NO. OF PLIES	AVERAGE TEAR DISTANCE (cm)	TEAR TIME (sec)	TEARING VELOCITY (cm/sec)	TEARING ENERGY (erg)	TEARING STRENGTH (g/sheet)
news-print	20	0.2	0.094	12.98	1.51	
		0.5	0.118			
		1.0	0.153	15.29	3.99	
		1.5	0.185			
		2.0	0.213	17.69	6.80	
		2.5	0.240			
		3.0	0.264	19.72	9.64	
		3.5	0.290			
		4.0	0.315	21.75	12.51	
		4.3	0.328	22.28	13.52	45.10
	25	0.2	0.102	11.74	1.82	
		0.5	0.130			
		1.0	0.168	13.51	4.22	
		1.5	0.204			
		2.0	0.237	15.34	7.00	
		2.5	0.268			
		3.0	0.297	16.95	9.74	
		3.5	0.326			
		4.0	0.356	18.51	12.62	
		4.3	0.372	18.93	13.57	46.60
bag paper	2	0.2	0.083	14.86	2.04	
		0.5	0.102			
		1.0	0.130	19.00	8.14	
		1.5	0.155			
		2.0	0.177	23.11	12.70	
		2.5	0.198			
		3.0	0.217	26.59	16.90	
		3.5	0.236			
		4.0	0.254	29.76	21.91	
		4.3	0.263	30.51	26.50	60.00
	6	0.2	0.087	14.65	1.18	
		0.5	0.107			
		1.0	0.138	18.81	4.54	
		1.5	0.163			
		2.0	0.186	21.94	8.65	
		2.5	0.208			
		3.0	0.227	24.98	12.52	
		3.5	0.248			
		4.0	0.267	27.91	16.19	
		4.3	0.277	28.67	17.93	58.20

Table 2 cont'd

PAPER GRADE	NO. OF PLIES	AVERAGE TEAR DISTANCE (cm)	TEAR TIME (sec)	TEARING VELOCITY (cm/sec)	TEARING ENERGY (erg)	TEARING STRENGTH (g/sheet)
bag paper	10	0.2	0.089	14.23	1.32	
		0.5	0.110			
		1.0	0.142	17.35	4.61	
		1.5	0.169			
		2.0	0.194	20.46	8.32	
		2.5	0.217			
		3.0	0.238	23.08	12.09	
		3.5	0.260			
		4.0	0.281	25.62	15.84	
		4.3	0.293	26.32	17.22	57.00
	14	0.2	0.091	13.53	1.64	
		0.5	0.113			
		1.0	0.147	16.15	4.73	
		1.5	0.177			
		2.0	0.203	18.78	8.29	
		2.5	0.229			
		3.0	0.253	21.10	11.75	
		3.5	0.275			
		4.0	0.298	23.22	15.51	
		4.3	0.312	23.87	16.69	57.00
	18	0.2	0.095	12.63	1.93	
		0.5	0.119			
		1.0	0.155	14.62	4.99	
		1.5	0.190			
		2.0	0.220	16.78	8.43	
		2.5	0.246			
		3.0	0.273	18.53	11.97	
		3.5	0.301			
		4.0	0.326	20.29	15.60	
		4.3	0.343	20.86	16.72	57.60
	22	0.2	0.096	12.24	1.80	
		0.5	0.123			
		1.0	0.163	13.20	4.98	
		1.5	0.201			
		2.0	0.236	14.25	8.68	
		2.5	0.268			
		3.0	0.300	15.17	12.37	
		3.5	0.333			
		4.0	0.365	16.11	16.22	
		4.3	0.387	16.44	17.43	59.40

Table 2 cont'd

PAPER GRADE	NO. OF PLIES	AVERAGE TEAR DISTANCE (cm)	TEAR TIME (sec)	TEARING VELOCITY (cm/sec)	TEARING ENERGY (erg)	TEARING STRENGTH (g/sheet)
55.5-lb. wrapper	2	0.2	0.080	13.32	12.85	
		0.5	0.101			
		1.0	0.132	17.88	18.42	
		1.5	0.158			
		2.0	0.182	22.35	21.34	
		2.5	0.202			
		3.0	0.223	25.92	25.68	
		3.5	0.241			
		4.0	0.259	29.12	31.30	
		4.3	0.270	30.06	33.31	116.00
	4	0.2	0.084	13.39	6.21	
		0.5	0.108			
		1.0	0.140	17.16	12.38	
		1.5	0.167			
		2.0	0.192	20.68	19.67	
		2.5	0.214			
		3.0	0.235	23.60	27.18	
		3.5	0.255			
		4.0	0.277	26.41	34.50	
		4.3	0.289	27.23	36.94	122.00
	6	0.2	0.092	13.17	4.62	
		0.5	0.115			
		1.0	0.149	15.98	11.52	
		1.5	0.180			
		2.0	0.207	18.77	19.40	
		2.5	0.231			
		3.0	0.254	21.09	27.46	
		3.5	0.278			
		4.0	0.300	23.33	35.74	
		4.3	0.315	24.06	38.17	127.10
	8	0.2	0.097	12.26	4.91	
		0.5	0.123			
		1.0	0.161	14.30	11.82	
		1.5	0.195			
		2.0	0.226	16.39	19.78	
		2.5	0.254			
		3.0	0.281	18.16	27.80	
		3.5	0.308			
		4.0	0.334	19.87	36.15	
		4.3	0.352	20.46	38.65	133.10

Table 2 cont'd

PAPER GRADE	NO. OF PLIES	AVERAGE TEAR DISTANCE (cm)	TEAR TIME (sec)	TEARING VELOCITY (cm/sec)	TEARING ENERGY (erg)	TEARING STRENGTH (g/sheet)
parcel wrap	1	0.2	0.083	14.21	13.48	
		0.5	0.104			
		1.0	0.134	18.67	22.37	
		1.5	0.158			
		2.0	0.181	22.79	32.94	
		2.5	0.201			
		3.0	0.221	26.34	40.18	
		3.5	0.240			
		4.0	0.258	29.52	51.05	
		4.3	0.268	30.39	56.72	160.00
	3	0.2	0.090	14.22	4.44	
		0.5	0.111			
		1.0	0.143	17.67	13.54	
		1.5	0.170			
		2.0	0.193	20.95	24.36	
		2.5	0.215			
		3.0	0.236	23.75	35.04	
		3.5	0.258			
		4.0	0.277	26.42	45.89	
		4.3	0.290	27.25	49.11	164.00
	5	0.2	0.094	12.63	6.95	
		0.5	0.121			
		1.0	0.156	15.82	14.31	
		1.5	0.184			
		2.0	0.211	18.73	23.44	
		2.5	0.236			
		3.0	0.261	21.31	32.02	
		3.5	0.283			
		4.0	0.305	23.58	41.71	
		4.3	0.320	24.40	44.16	157.90
	7	0.2	0.103	11.94	6.16	
		0.5	0.131			
		1.0	0.170	13.73	14.65	
		1.5	0.205			
		2.0	0.237	15.51	24.61	
		2.5	0.267			
		3.0	0.296	17.07	34.50	
		3.5	0.325			
		4.0	0.354	18.62	44.77	
		4.3	0.372	19.09	48.03	163.80

Table 3

Comparison between three different methods of preparing conductive plies used in the study

METHOD	CONDUCTIVE MATERIAL	PREPARATION TIME	SHARPNESS OF STEPS	TEARING STRENGTH* (g/sheet)
4	perforated conductive paper	medium	worst	55.3
5	graphite lines	longest	medium	55.5
6	silver paint lines	shortest	best	54.4
Control	no conductive material added			55.5

*These are cross machine-direction tearing strengths (n = 5).

Table 4

Test of significance for partial regression coefficient for the newsprint ten-ply specimen

SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE	F
Regression due to t, t^2, and t^3	1,890.2742	3	630.0914	16,739.9**
(1) Due to t and t^2, ignoring t^3	1,890.0254	2	945.0127	25,106.6**
(2) Addition due to t^3	0.2488	1	0.2488	6.6*
Residual	0.2259	6	0.0376	
Total	1,890.5000	9		

*significant at 5% level

**significant at 1% level

Table 5

Tear distance-time relationships for five paper grades with different numbers of plies and replications

PAPER GRADE	NO. OF PLIES	REPLI-CATION	REGRESSION EQUATIONS n = 10 a	b (T)	c (T^2)	R*	SE_E
zero swing	0	1	-6.3734	64.8480	524.552	0.99988	0.2504
	0	2	-7.1554	77.4525	476.396	0.99987	0.2600
	0	3	-7.6369	82.6602	460.313	0.99983	0.3030
onion skin	10	1	-6.6601	65.1748	464.815	0.99989	0.2435
	10	2	-6.9184	70.1631	450.124	0.99984	0.2952
	10	3	-7.2590	66.7583	462.635	0.99996	0.1442
	20	1	-7.2360	72.3810	418.749	0.99986	0.2782
	20	2	-7.6428	71.7888	423.554	0.99989	0.2402
	20	3	-8.0094	72.1515	427.279	0.99989	0.2420
	40	1	-7.2567	65.2621	356.484	0.99998	0.1005
	40	2	-7.6026	67.1902	351.563	0.99993	0.2004
	40	3	-8.1862	81.3997	322.430	0.99993	0.1931
	60	1	-8.3481	82.2631	238.983	0.99990	0.2284
	60	2	-7.4723	77.4058	249.292	0.99988	0.2576
	60	3	-8.0355	77.0044	240.137	0.99983	0.3024
	70	1	-8.6412	90.6479	174.260	0.99985	0.2835
	80	1	-7.9450	90.5993	120.855	0.99987	0.2639
	80	2	-8.3707	96.0691	99.866	0.99964	0.4396
	80	3	-8.9828	96.7433	100.187	0.99978	0.3424
news-print	5	1	-6.8220	69.4849	440.333	0.99991	0.2233
	5	2	-7.1175	69.6755	441.026	0.99994	0.1824
	5	3	-7.0784	67.6351	443.757	0.99995	0.1704
	10	1	-8.0350	92.1140	327.827	0.99972	0.3882
	10	2	-8.0386	82.7237	355.897	0.99987	0.2609

*All R values are highly significant at 1% level, all exceeding 0.999.

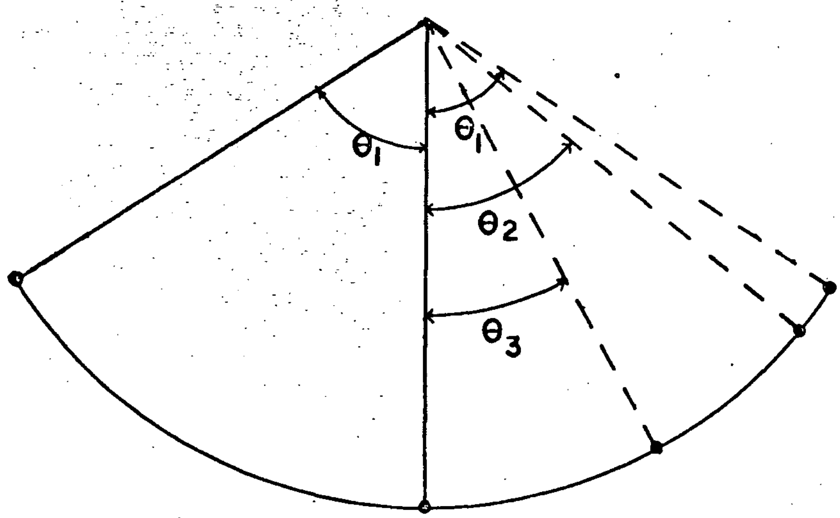
Table 5 cont'd

PAPER GRADE	NO. OF PLIES	REPLI-CATION	REGRESSION EQUATIONS n = 10 a	b (T)	c (T^2)	R	SE_E
news print	10	3	-7.3408	71.7685	383.236	0.99996	0.1535
	15	1	-7.6375	73.2894	314.879	0.99992	0.2099
	15	2	-8.2155	84.7818	271.433	0.99985	0.2860
	15	3	-8.2148	83.5157	289.588	0.99989	0.2469
	20	1	-8.8950	99.4156	184.703	0.99980	0.3262
	20	2	-8.8636	91.8325	197.206	0.99994	0.1855
	20	3	-8.0376	85.4665	214.333	0.99992	0.2141
	25	1	-8.7982	92.2542	122.220	0.99985	0.2805
	25	2	-8.8402	88.0074	150.664	0.99992	0.2123
	25	3	-8.7746	90.6555	127.307	0.99987	0.2647
bag paper	2	1	-8.6775	95.8356	377.100	0.99993	0.1876
	2	2	-7.2409	74.8557	437.617	0.99993	0.1960
	2	3	-6.2072	60.3486	488.641	0.99995	0.1672
	6	1	-7.6722	80.4961	378.226	0.99991	0.2264
	6	2	-8.8059	91.8116	338.037	0.99987	0.2698
	6	3	-7.7453	74.4592	391.204	0.99990	0.2371
	10	1	-8.3260	97.2049	262.674	0.99982	0.3102
	10	2	-8.2116	82.9565	320.323	0.99997	0.1328
	10	3	-8.6947	87.5043	308.978	0.99991	0.2262
	14	1	-8.5775	91.9893	237.750	0.99994	0.1777
	14	2	-9.2317	103.4080	207.965	0.99994	0.1813
	14	3	-7.7378	83.0536	254.740	0.99990	0.2365
	18	1	-9.2403	98.5485	158.527	0.99988	0.2552
	18	2	-7.8305	93.1304	169.119	0.99964	0.4439
	18	3	-8.8837	93.0997	169.829	0.99983	0.3049
	22	1	-8.8406	104.3110	76.548	0.99970	0.4050
	22	2	-9.6838	114.3930	65.638	0.99973	0.3827
	22	3	-9.5859	106.7370	74.125	0.99990	0.2300

Table 5 cont'd

PAPER GRADE	NO. OF PLIES	REPLI-CATION	REGRESSION EQUATIONS n = 10 a	b (T)	c (T^2)	R	SE_E
55.5-lb. wrapper	2	1	-5.5841	63.2721	441.483	0.99976	0.3579
	2	2	-6.3464	68.5575	424.470	0.99987	0.2676
	2	3	-5.6081	55.9263	458.187	0.99994	0.1808
	4	1	-7.3089	81.2928	333.736	0.99970	0.4017
	4	2	-7.9274	88.5774	306.911	0.99958	0.4787
	4	3	-6.1280	61.6303	372.181	0.99980	0.3258
	6	1	-8.0230	84.3801	241.810	0.99978	0.3442
	6	2	-8.2676	89.8457	238.169	0.99984	0.2904
	6	3	-8.3716	86.7671	250.907	0.99984	0.2933
	8	1	-8.4925	91.1334	154.389	0.99966	0.4305
	8	2	-8.8739	92.5650	159.350	0.99991	0.2150
	8	3	-8.5037	90.0730	169.091	0.99974	0.3745
parcel wrap	1	1	-6.7607	59.2432	462.346	0.99996	0.1434
	1	2	-8.0221	84.1428	398.174	0.99989	0.2473
	1	3	-6.0168	65.8681	451.687	0.99996	0.1488
	3	1	-8.3449	87.7845	314.328	0.99986	0.2711
	3	2	-8.1078	80.8438	336.730	0.99984	0.2901
	3	3	-8.4429	82.6506	326.161	0.99978	0.3441
	5	1	-7.5136	75.7751	268.025	0.99981	0.3175
	5	2	-8.9070	83.0102	247.169	0.99980	0.3249
	5	3	-7.4411	72.7527	266.158	0.99966	0.4306
	7	1	-9.1375	90.1060	130.316	0.99953	0.3561
	7	2	-9.1205	88.9178	140.065	0.99980	0.3249
	7	3	-9.3193	96.9541	128.945	0.99983	0.3209

Fig. 1. Schematic diagram illustrating angles involved in the ballistic-type tearing principle.



Angles are defined by the vertical line and the line connecting the sector center of mass and axis of rotation.

θ_1 = initial angle before swinging or tearing a specimen,

θ_2 = angular displacement of sector center of mass representing sector net energy, and

θ_3 = angular displacement of sector center of mass representing sector residual energy.

Fig. 2. Model consisting of springs and dashpots used to illustrate stress-strain-time relationships for polymeric materials.

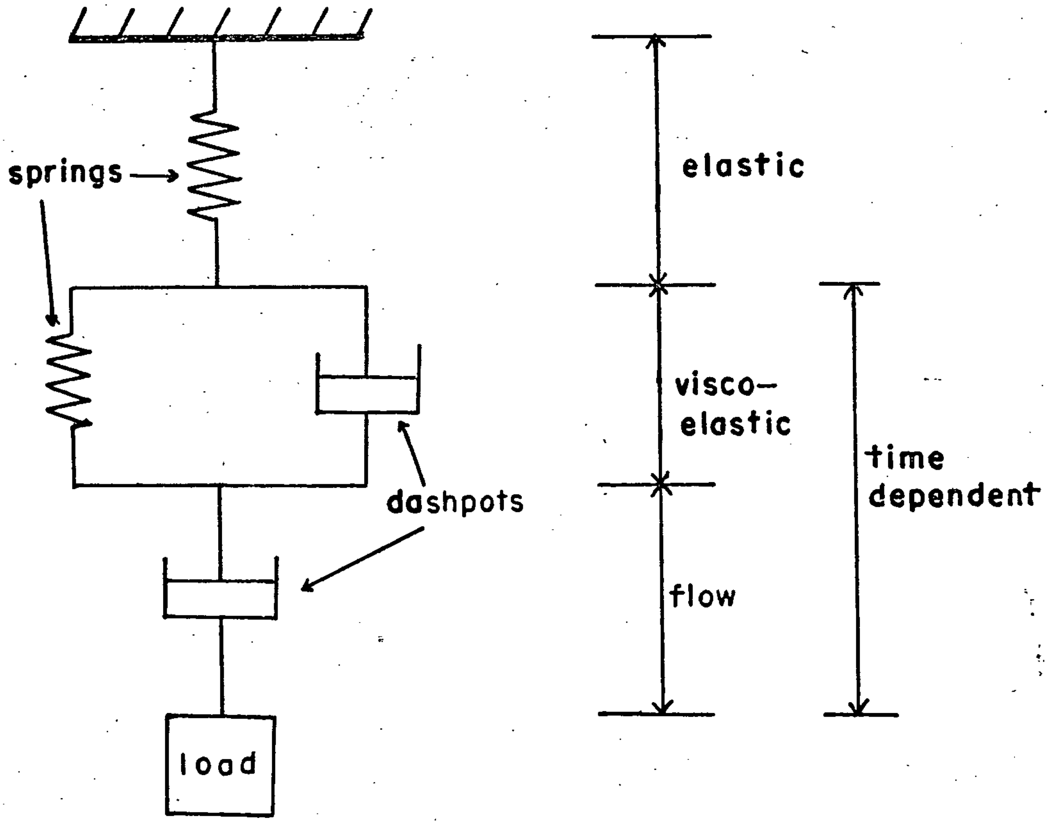


Fig. 3. Conversion of energies in the ballistic-type tearing process.

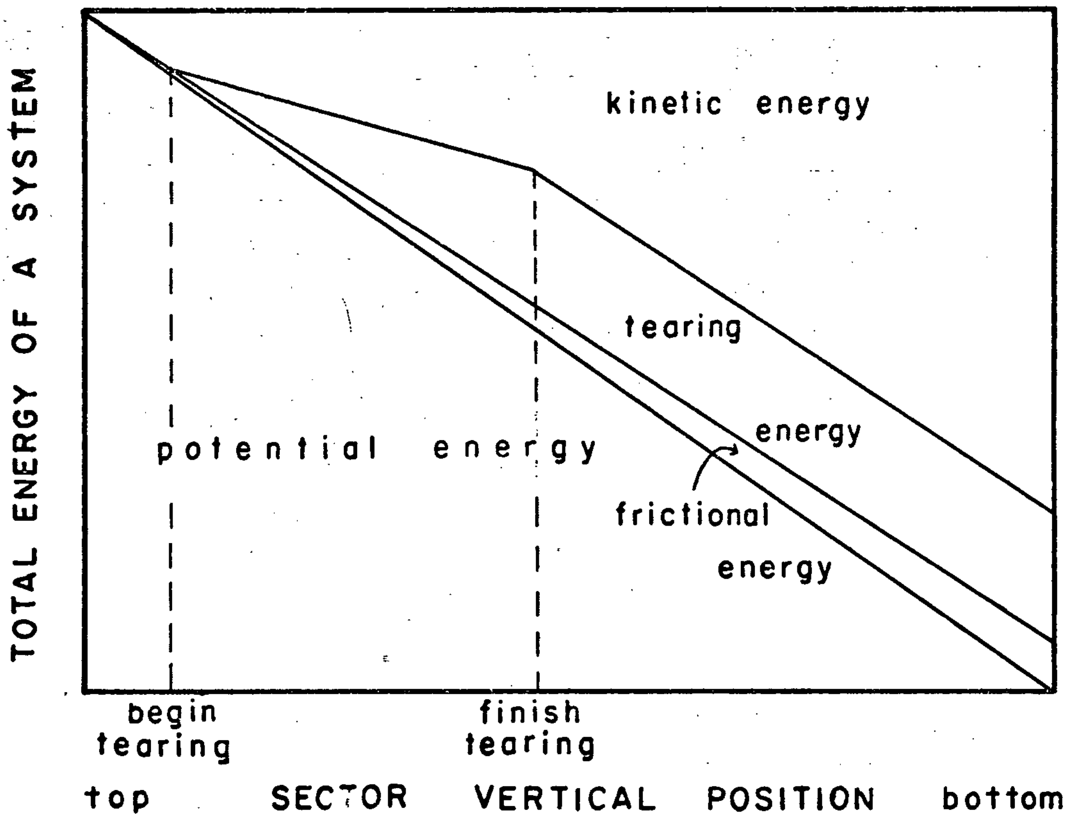
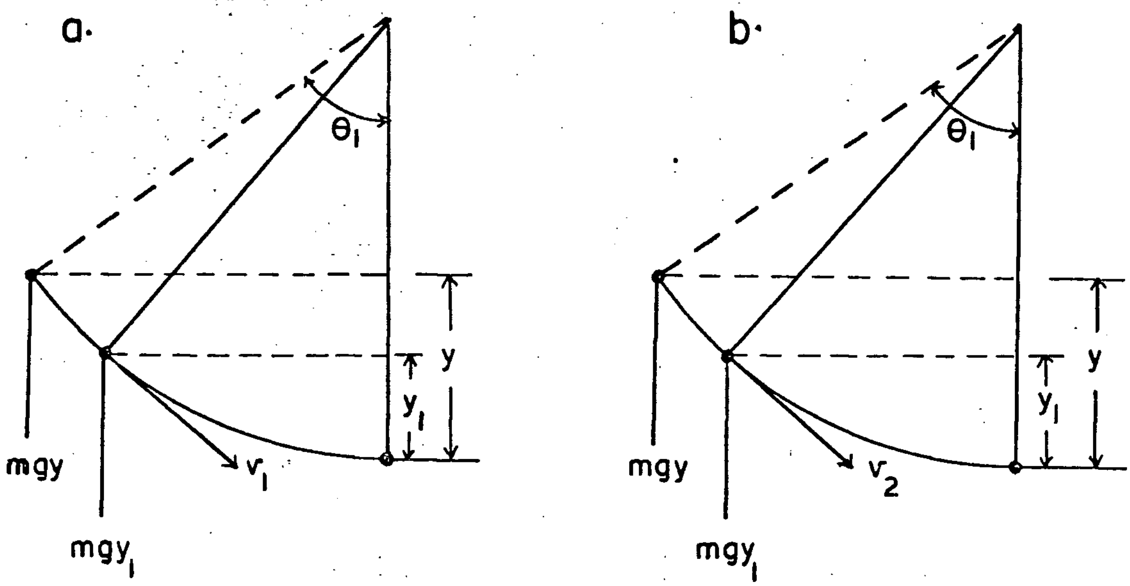


Fig. 4. Schematic diagrams illustrating the conversion of energies in, a. sector swing without specimen, b. sector swing when tearing a specimen.



$$E_{Tot} = mgy = \frac{1}{2}mv_1^2 + mgy_1 + f(v_1,t_1)$$

$$E_{Tot} = mgy = \frac{1}{2}mv_2^2 + mgy_1 + f(v_2,t_2) + T.E.$$

E_{Tot} = total energy of a system,

m = sector mass,

g = gravitational acceleration,

y = vertical position of the sector center of mass before swing,

y_1 = vertical displacement of the sector center of mass,

v_1 = tangential velocity of time t_1, at zero-swing,

v_2 = tangential velocity at time t_2, at tearing-swing, and

$f(v_1,t_1)$ & $f(v_2,t_2)$ = friction terms in case a and b., respectively.

Fig. 5. Relationships between cross machine-direction tearing strength and number of plies torn simultaneously for five paper grades.

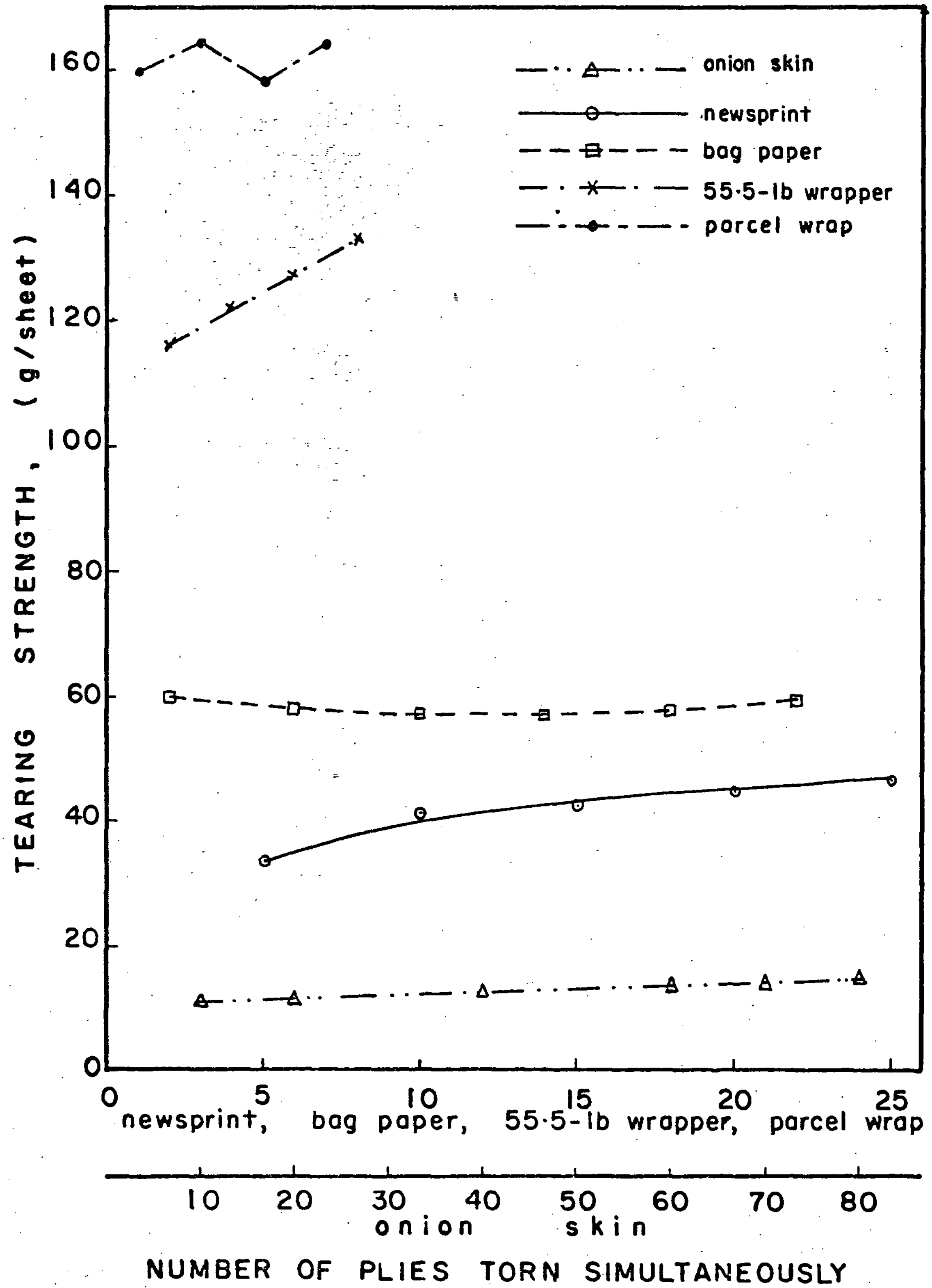
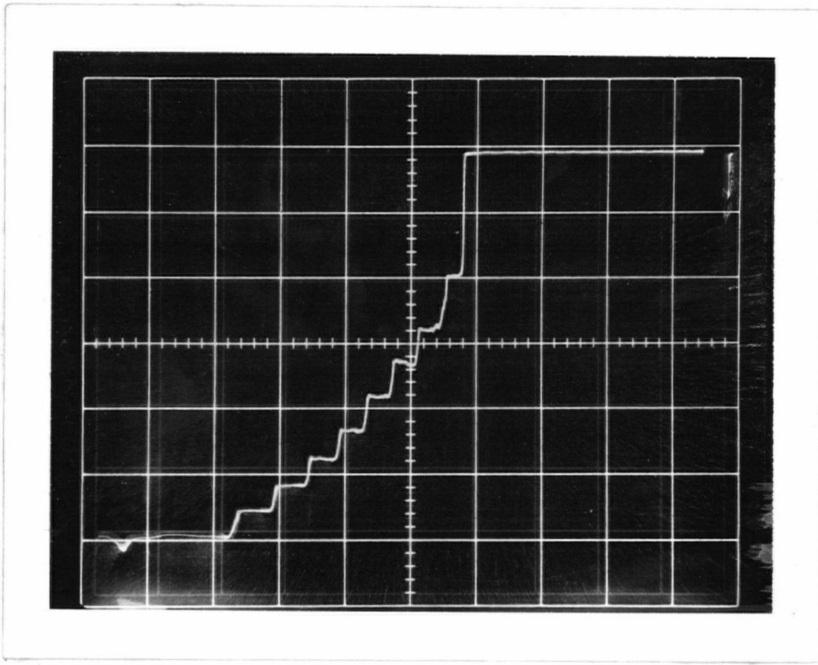
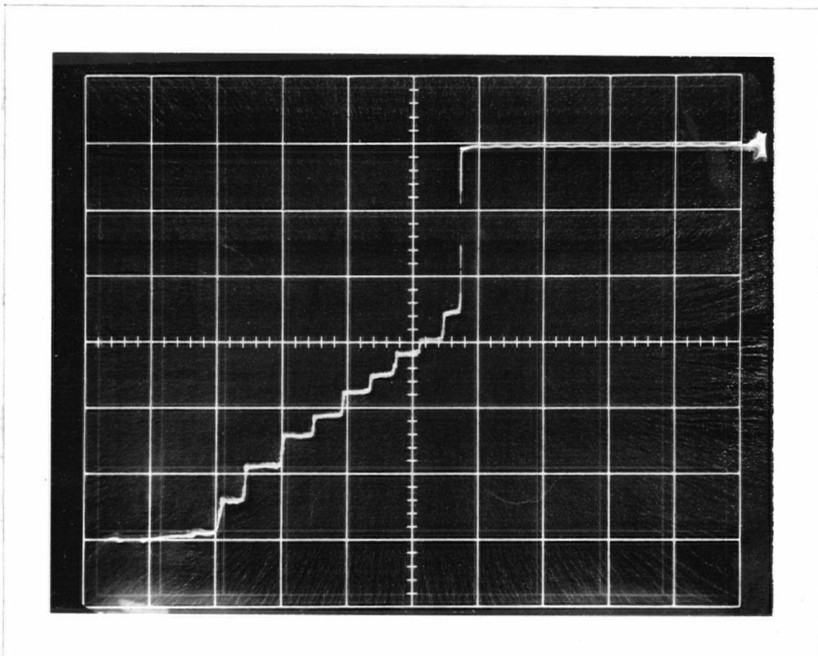


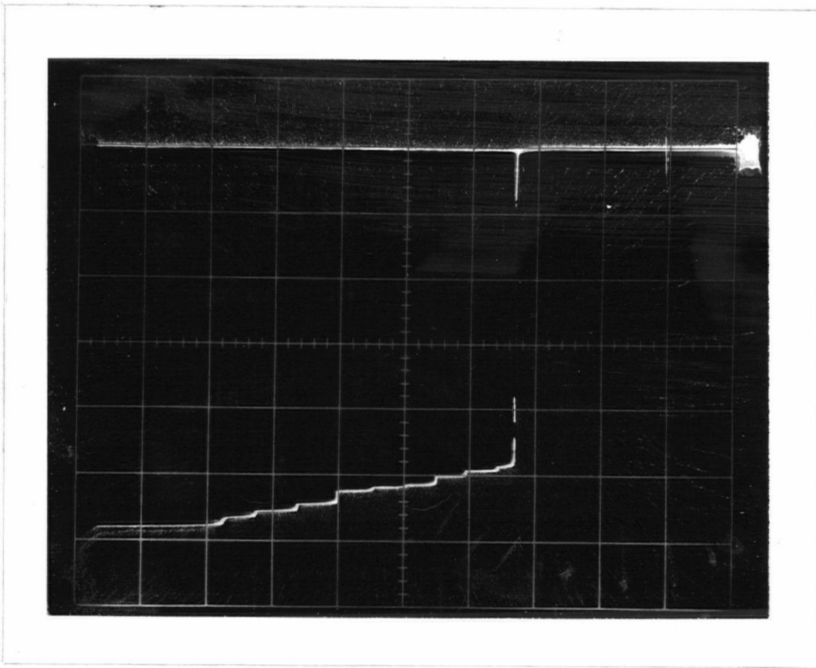
Fig. 6. Oscilloscope traces had with four different conductive materials.



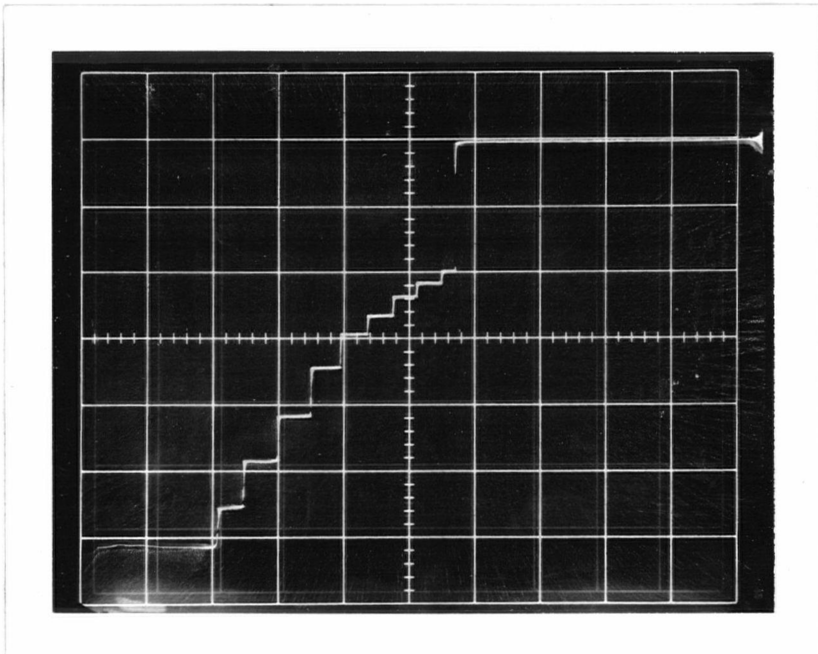
a. Perforated electrical conductivity paper as center ply.



b. Graphite conductive lines applied to the center ply.



c. Silver conductive lines applied to the center ply.



d. Combined silver and graphite lines applied to the center ply. (see Fig. 7).

Fig. 7. Pattern of conductive lines used for the study. Light and dark lines are silver and graphite conductive materials, respectively.

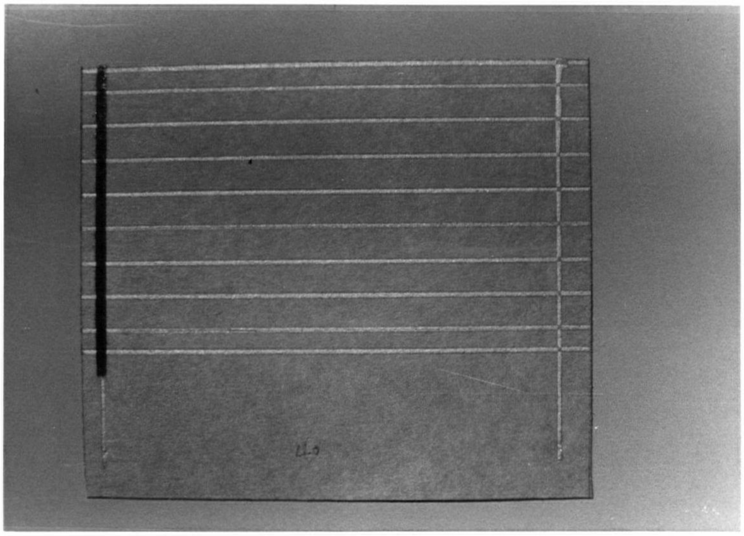
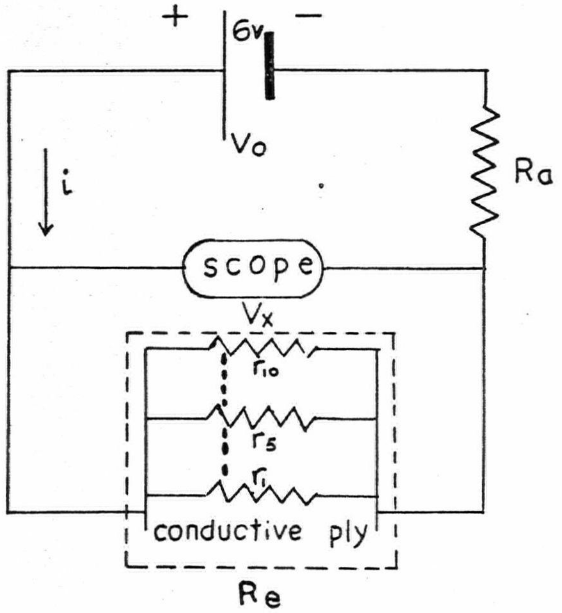
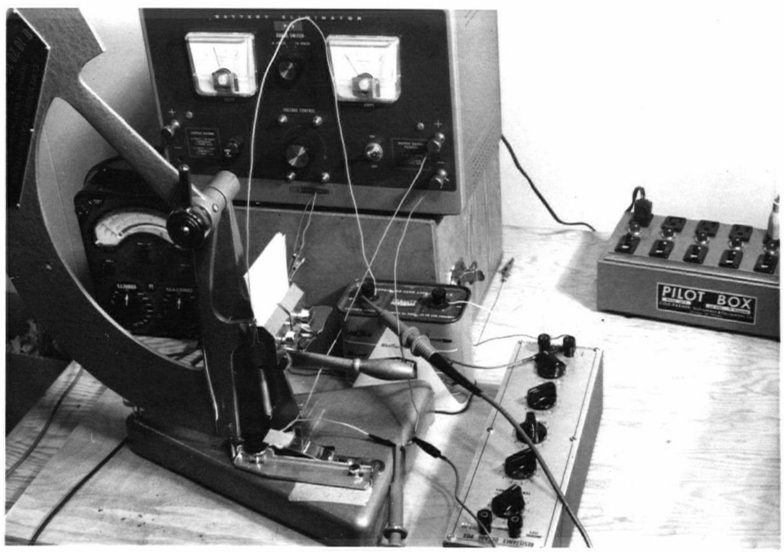


Fig. 8. Electrical circuit used for measuring the tear distance-time relationship.

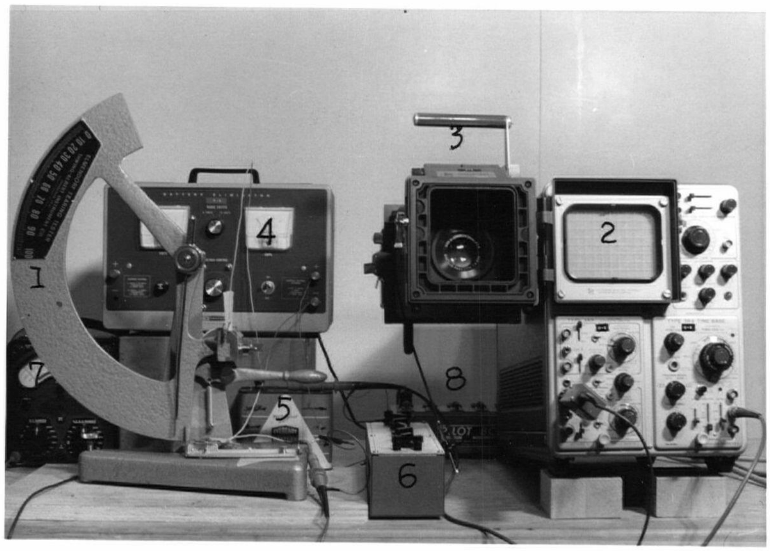


(a)



(b)

Fig. 9. Set-up used for the study



1. Thwing-Albert Instrument Co. No. 60-100 Elmendorf tearing tester.

2. Tektronix, In., type 564 storage oscilloscope equipped with 3B4 time base and 3A3 dual trace differential amplifier plug-in units.

3. Type C-27 Polaroid camera with camera mounting frame attached to the oscilloscope.

4. Battery eliminator.

5. 6-volt battery.

6. 6-decade resistance kit.

7. Voltmeter.

8. Pilot box.

Fig. 10. Tear distance-time relationships for 30-lb. n & m bag paper. Number of paper plies is marked on each curve.

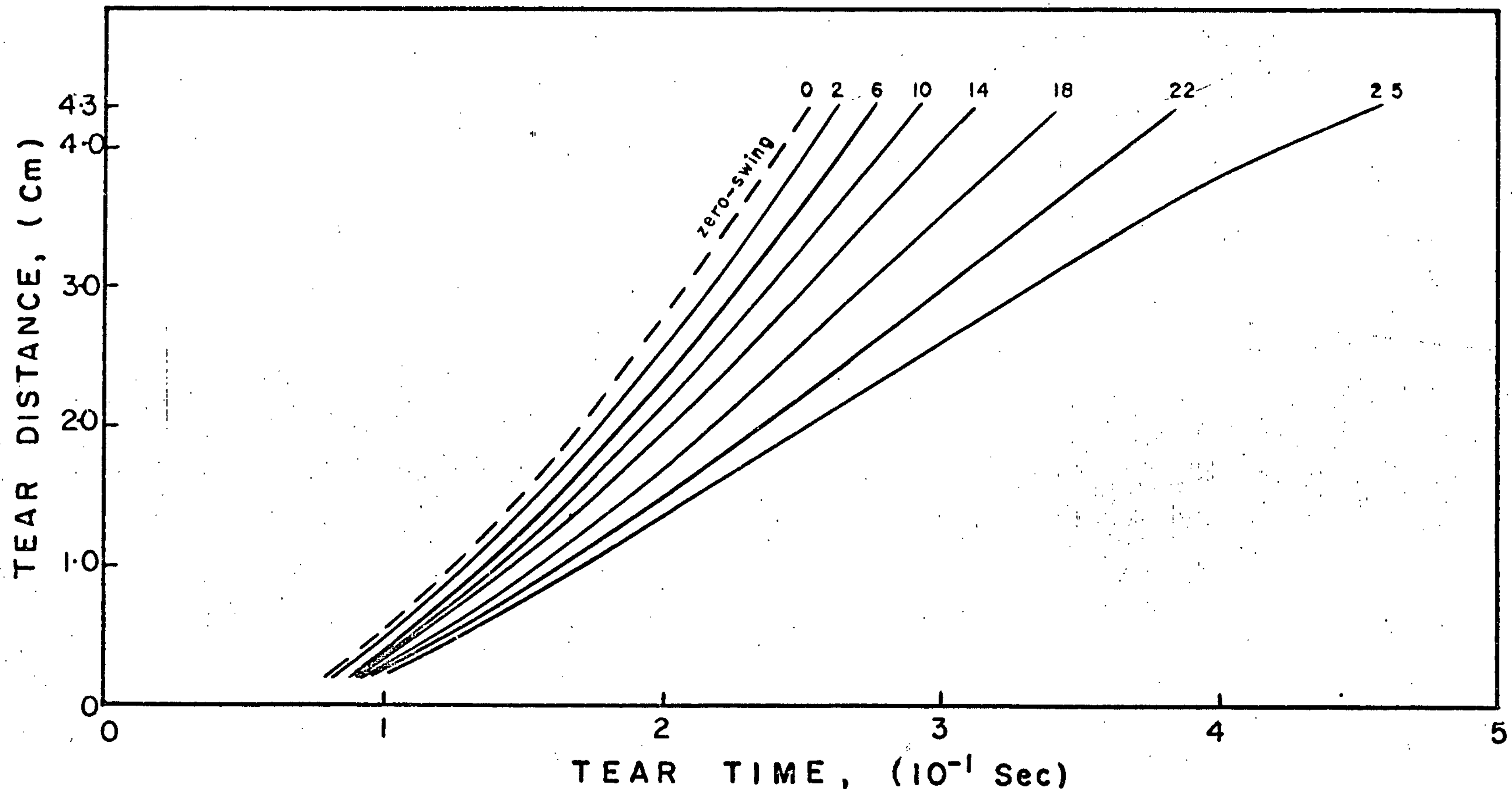


Fig. 11a. Tearing energy, distance and time relationships for unglazed onion skin. Broken and solid lines are number of paper plies and tear distances, respectively.

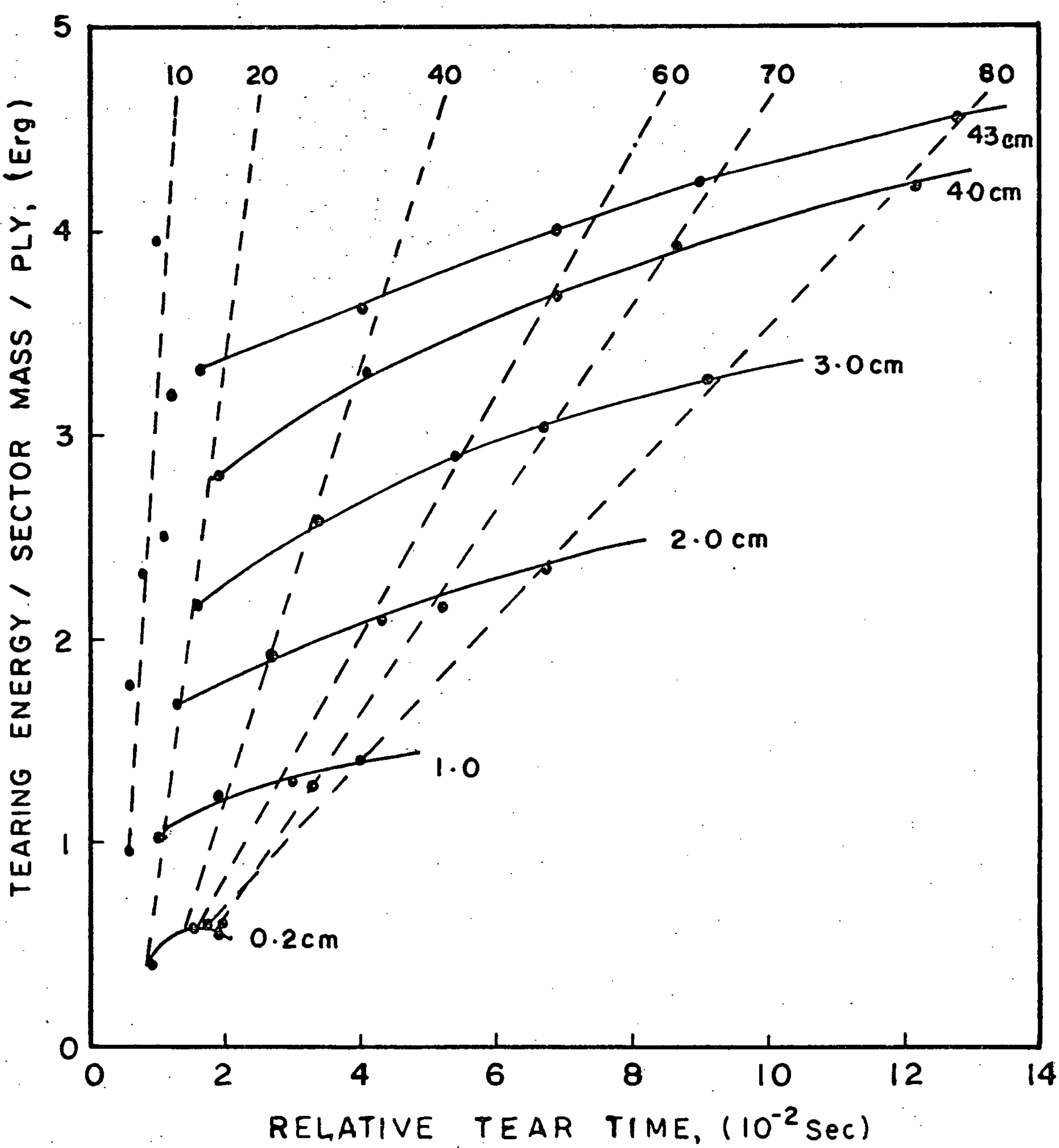


Fig. 11b. Tearing energy, distance and time relationships for newsprint. Broken and solid lines are number of paper plies and tear distances, respectively.

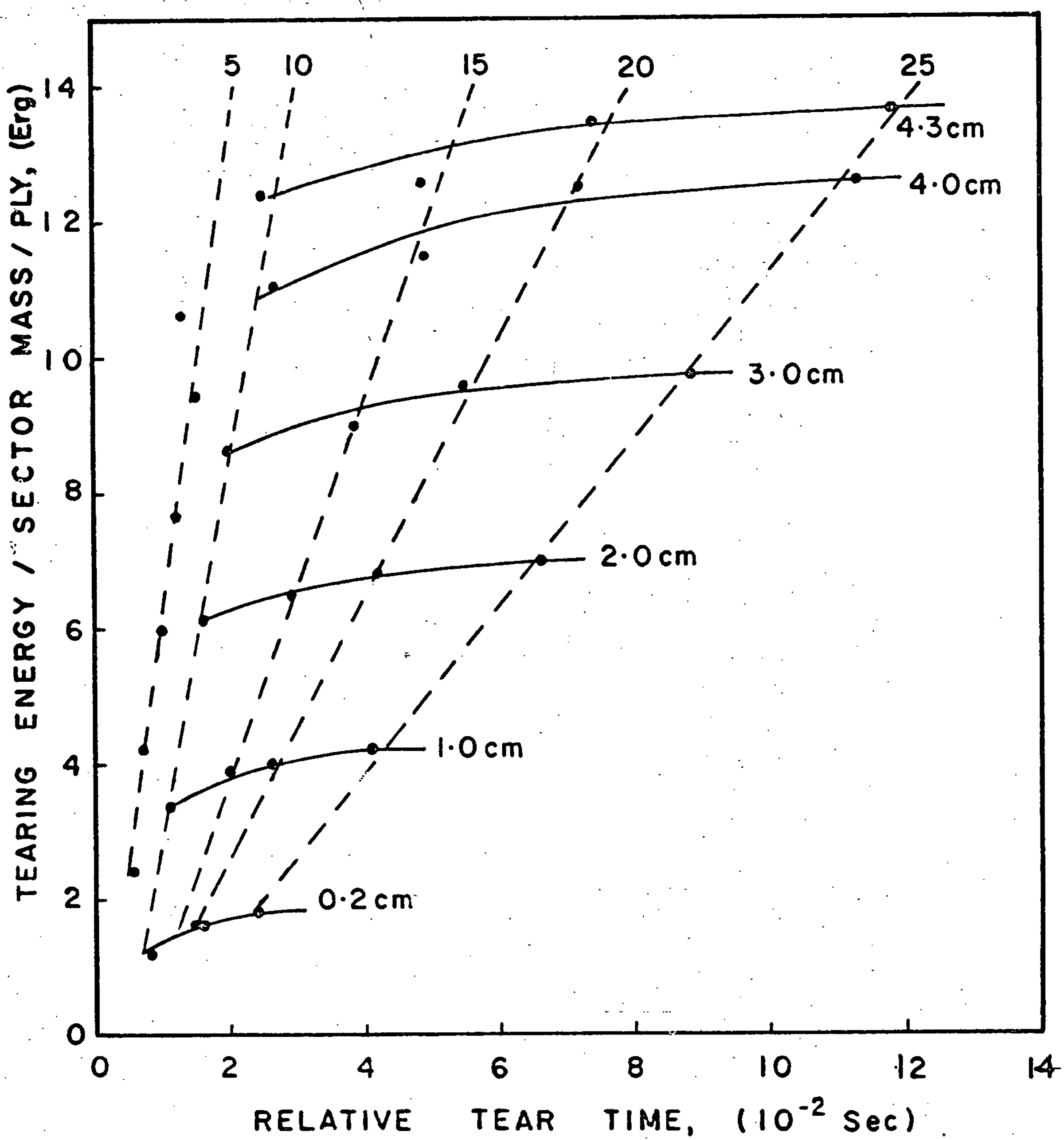


Fig. 11c. Tearing energy, distance and time relationships for 30-lb. n & m bag paper. Broken and solid lines are number of paper plies and tear distances, respectively.

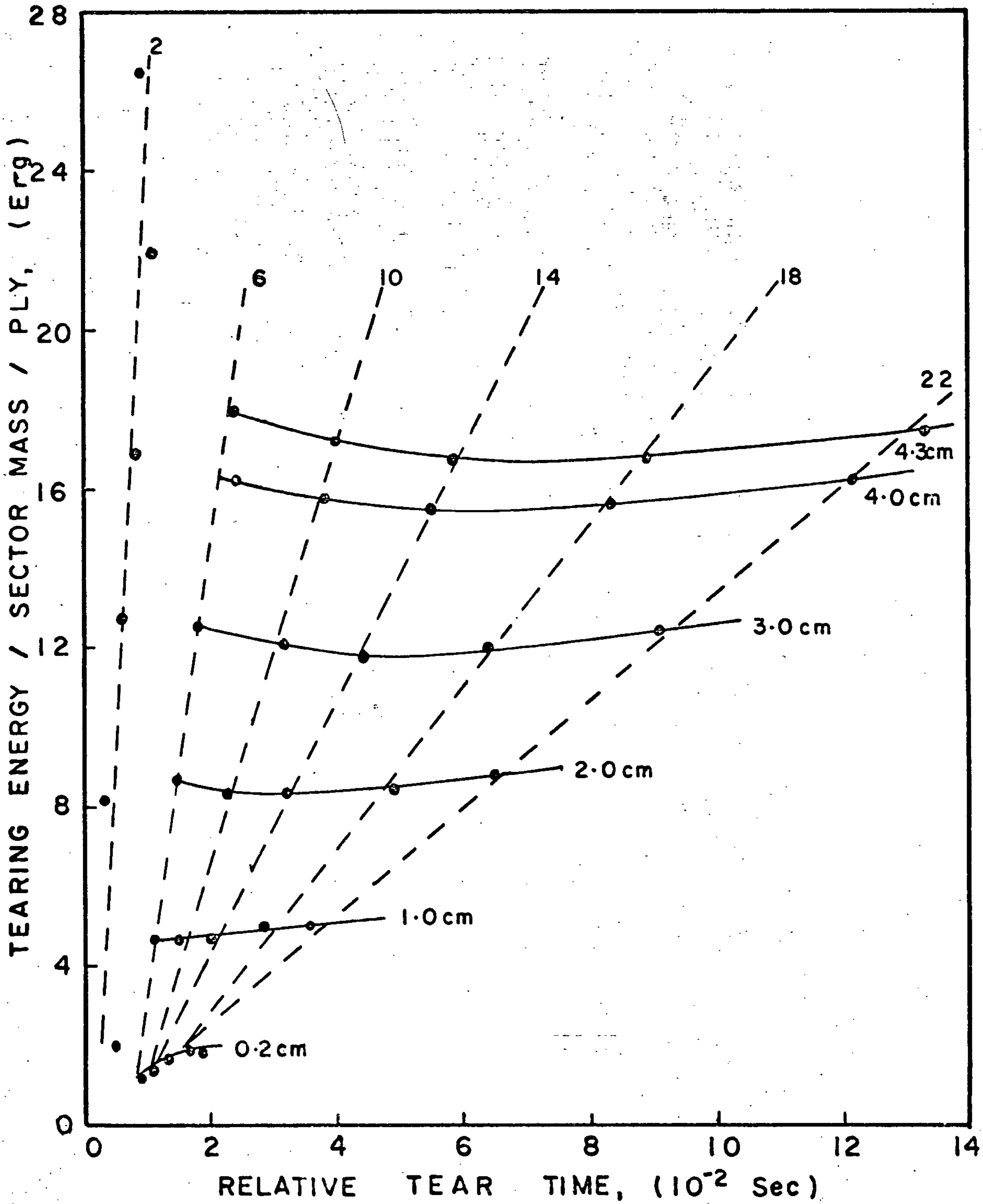


Fig. 11d. Tearing energy, distance and time relationships for Island 55.5-lb. wrapper. Broken and solid lines are number of paper plies and tear distances, respectively.

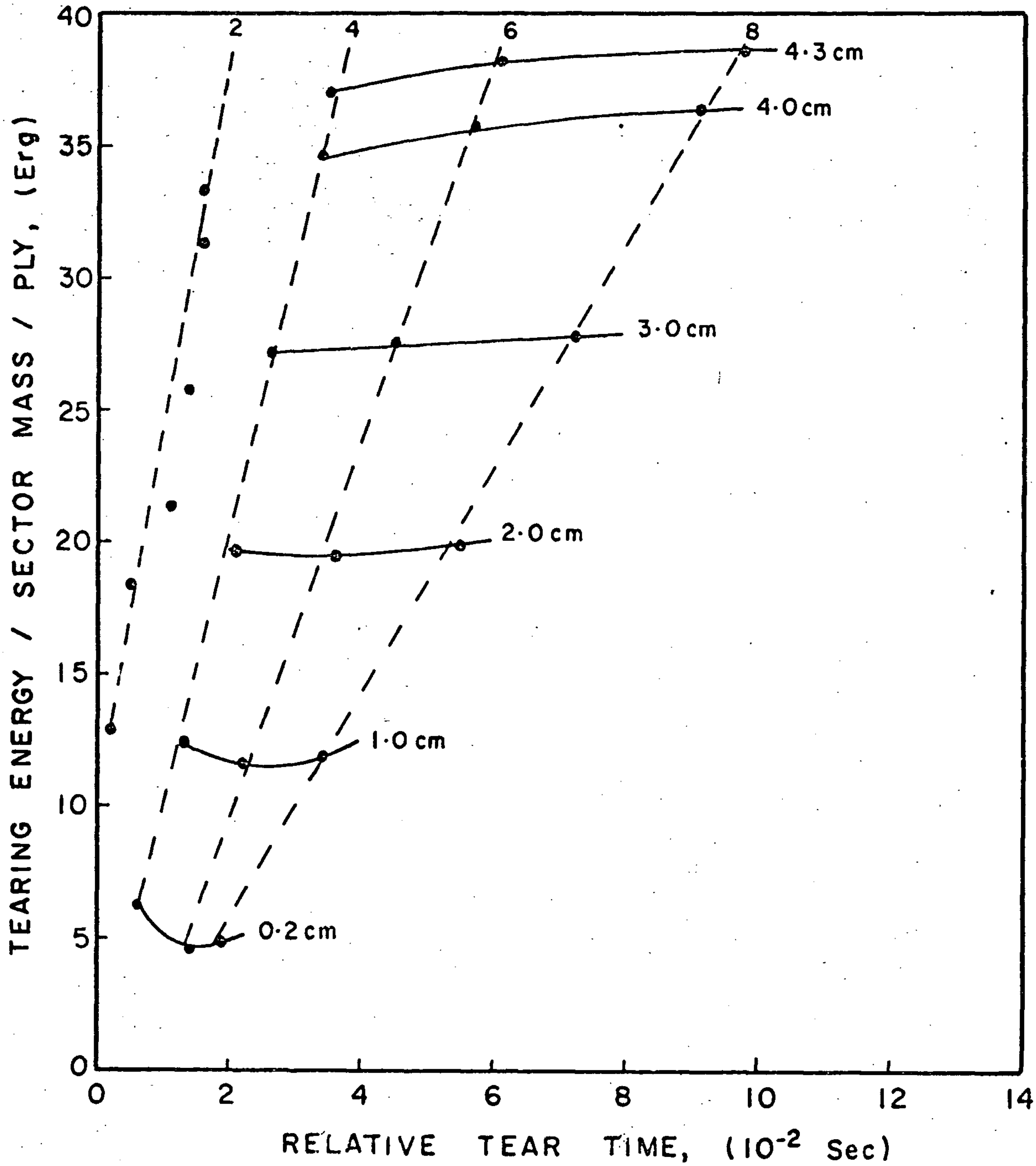


Fig. 11e. Tearing energy, distance and time relationships for parcel wrap. Broken and solid lines are number of paper plies and tear distances, respectively.

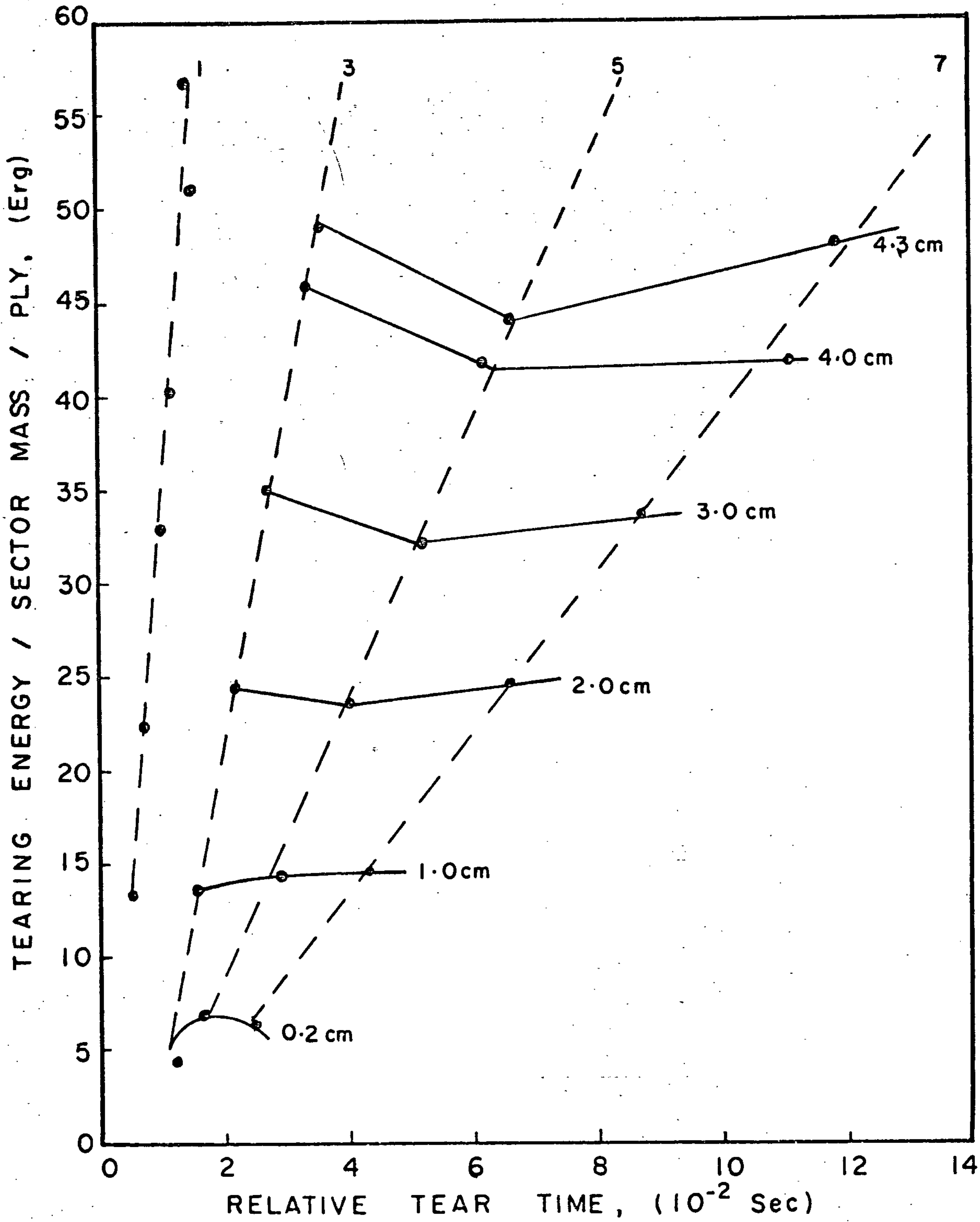


Fig. 12. Relationships between relative tear time per unit ply and number of plies torn simultaneously.

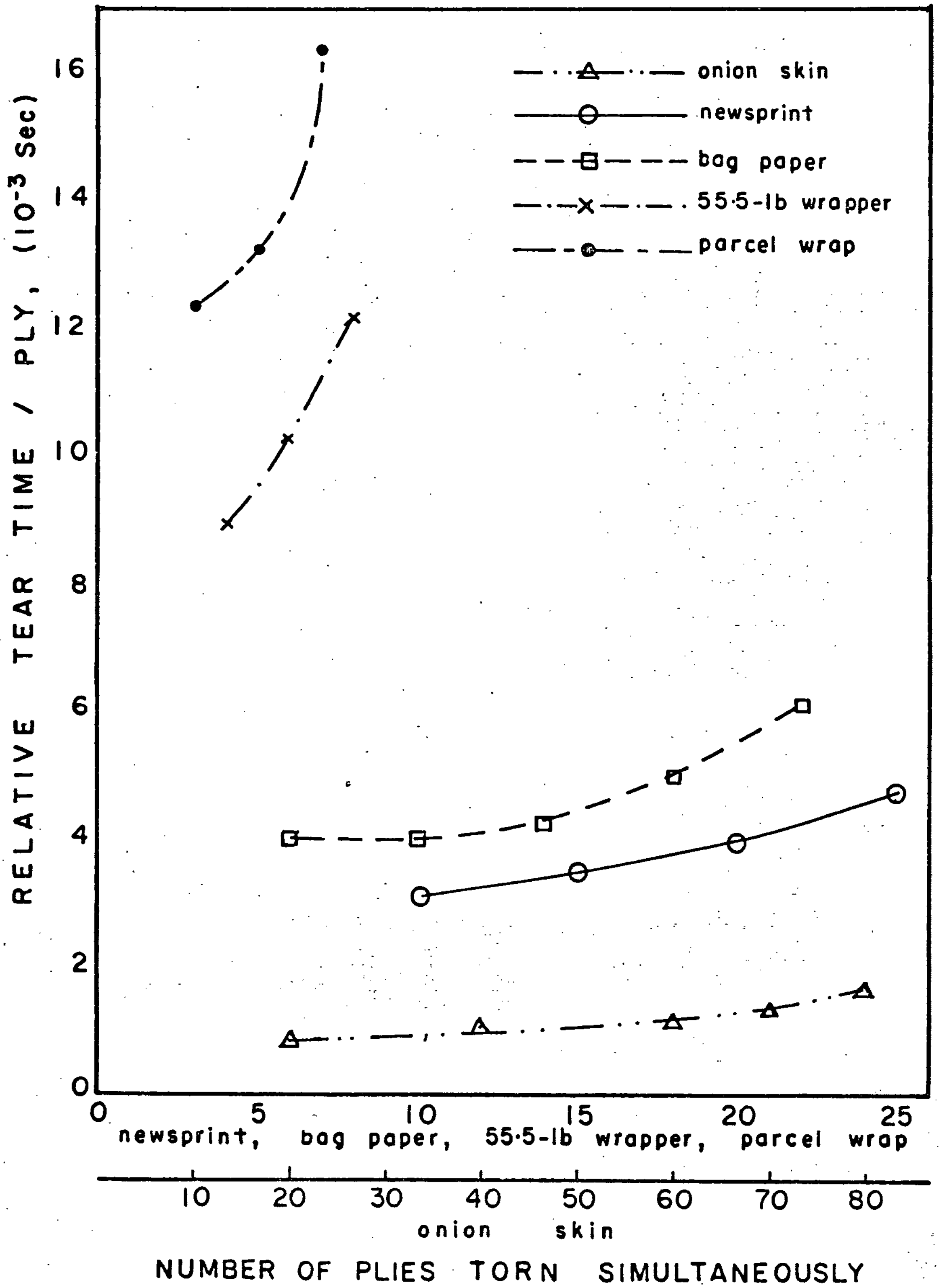


Fig. 13. Relationships between tearing strength and tearing energy for five paper grades.

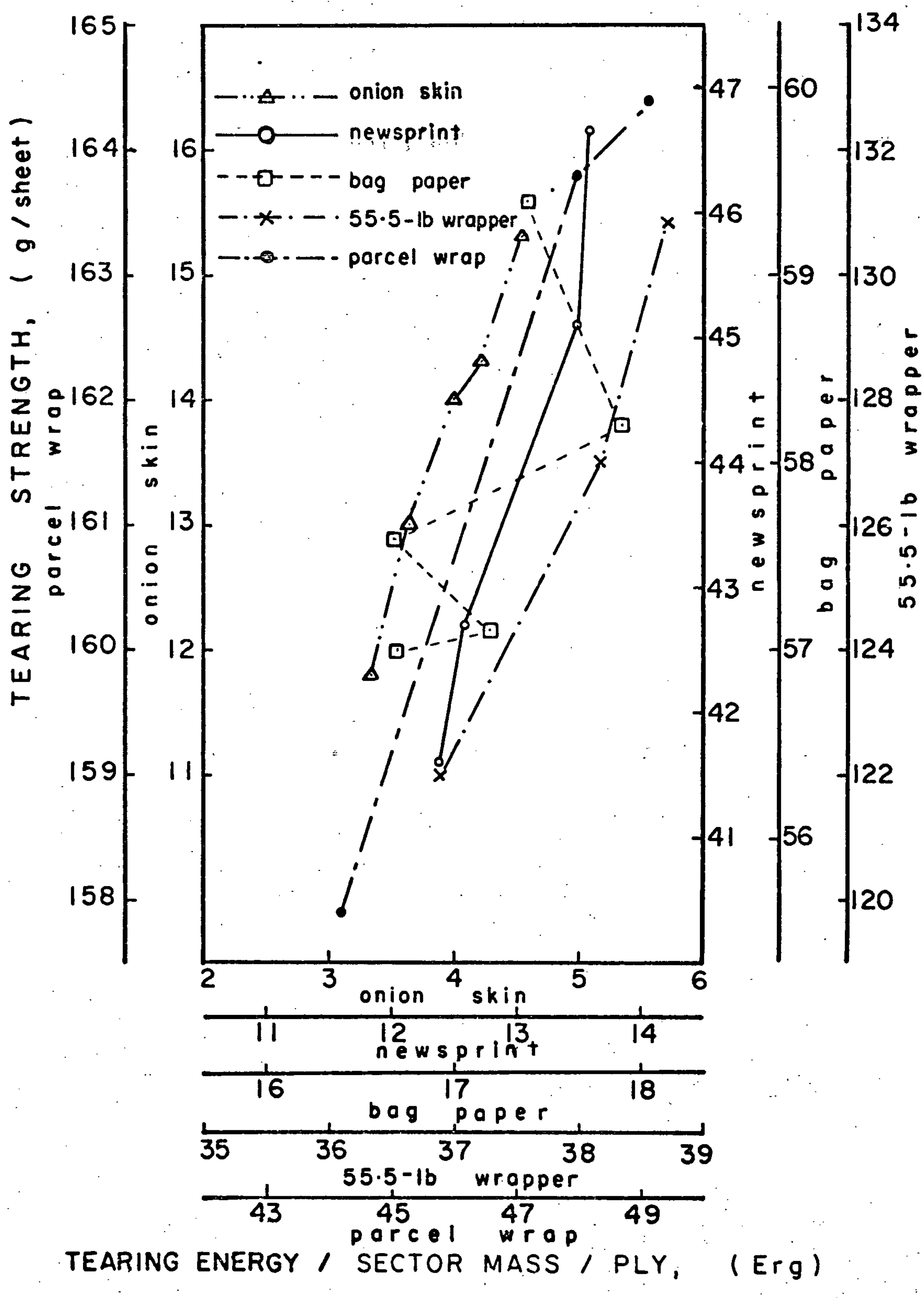


Fig. 14. Tearing energy and distance relationships for unglazed onion skin.

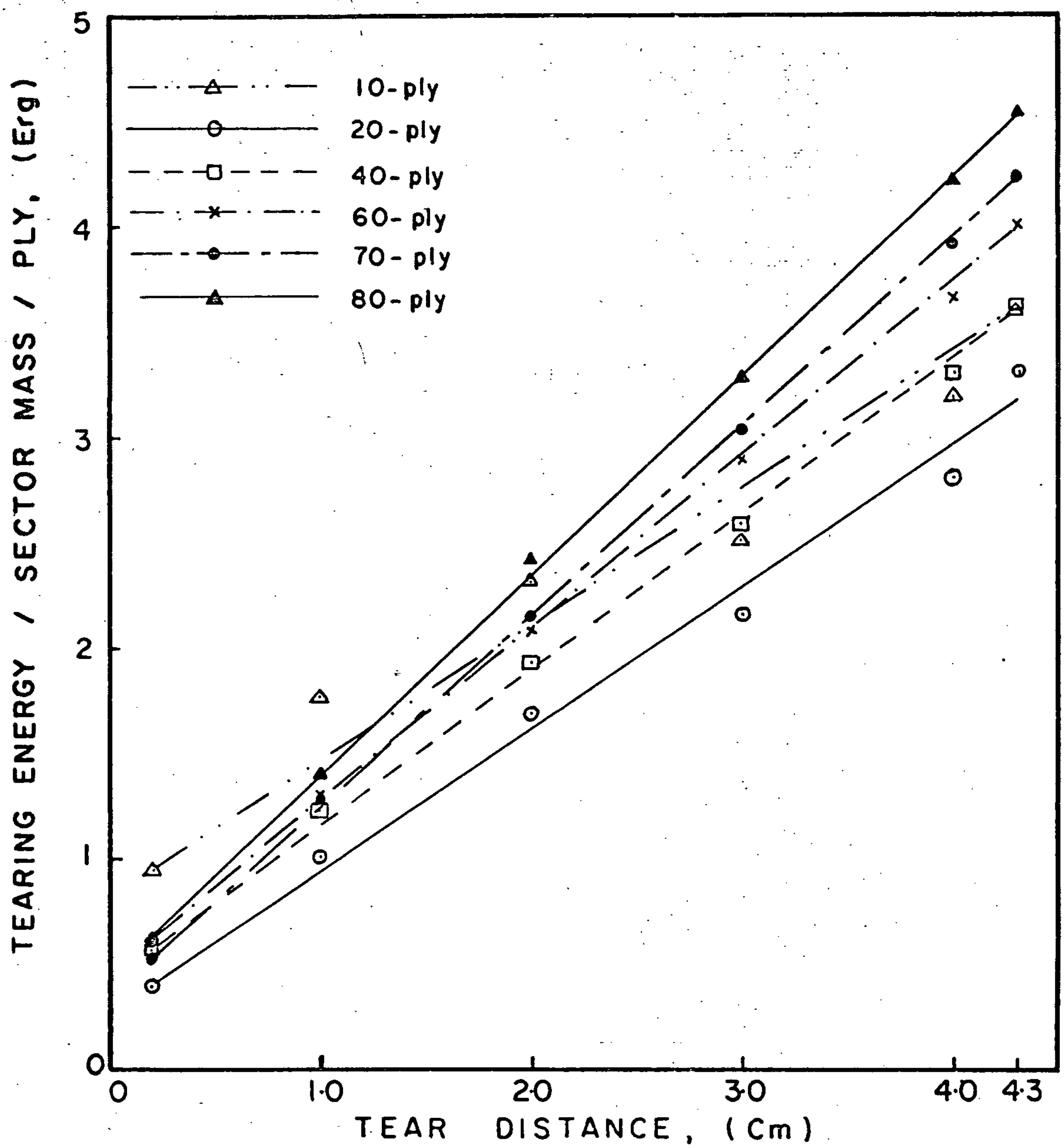
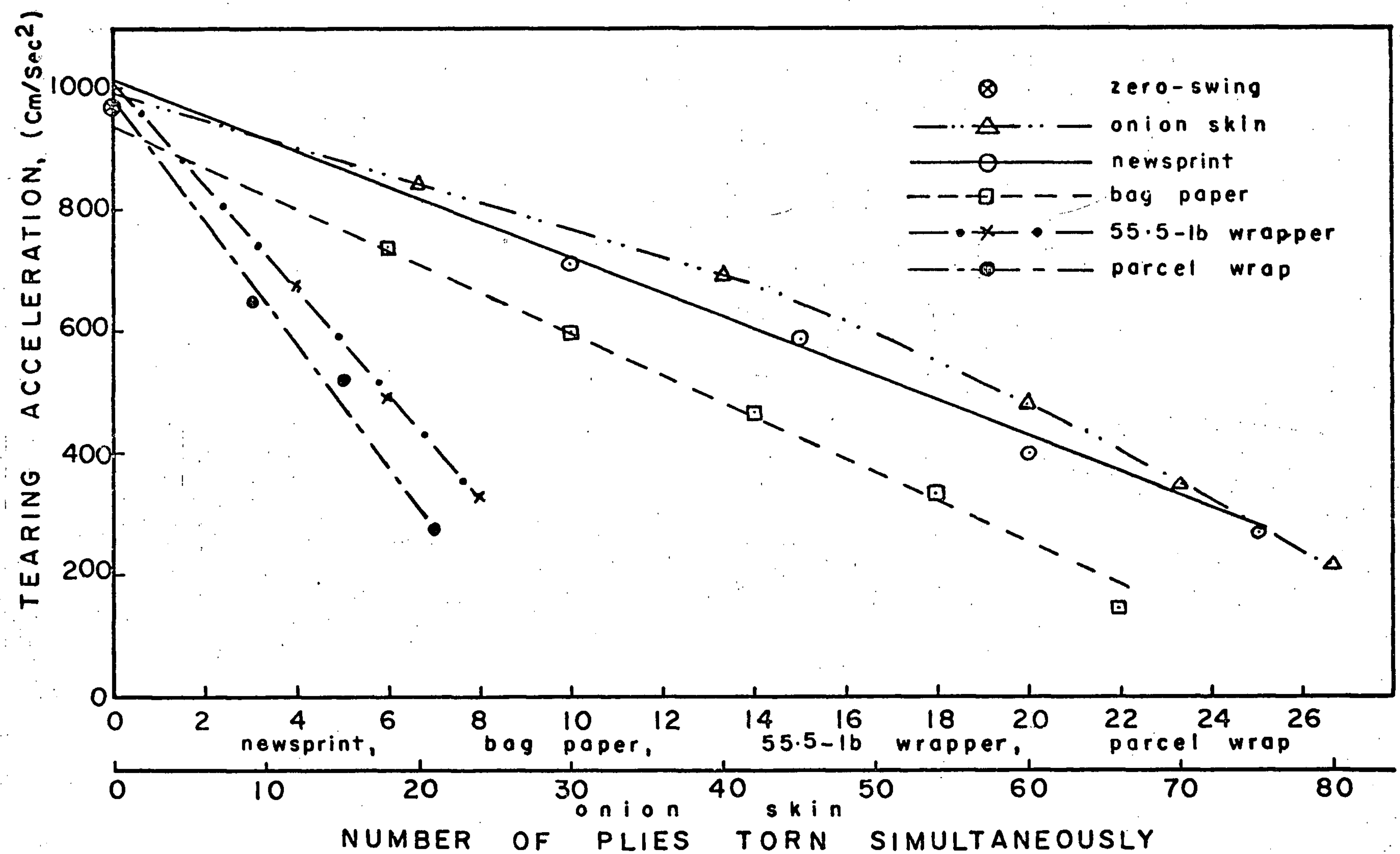


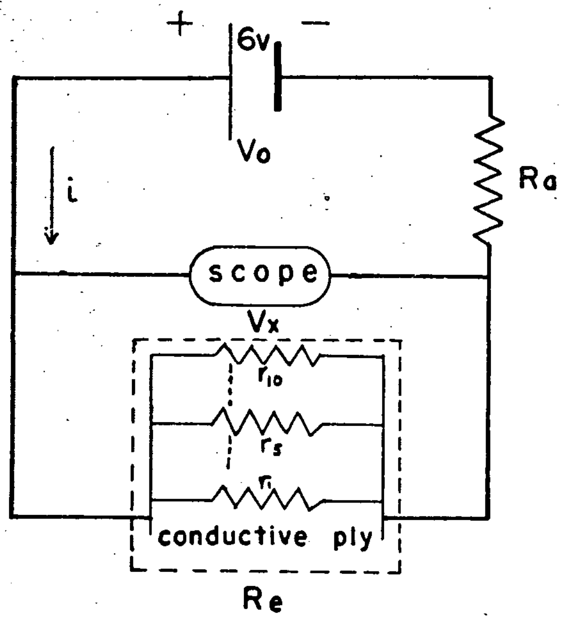
Fig. 15. Relationships between tearing acceleration and number of plies torn simultaneously for five paper grades.



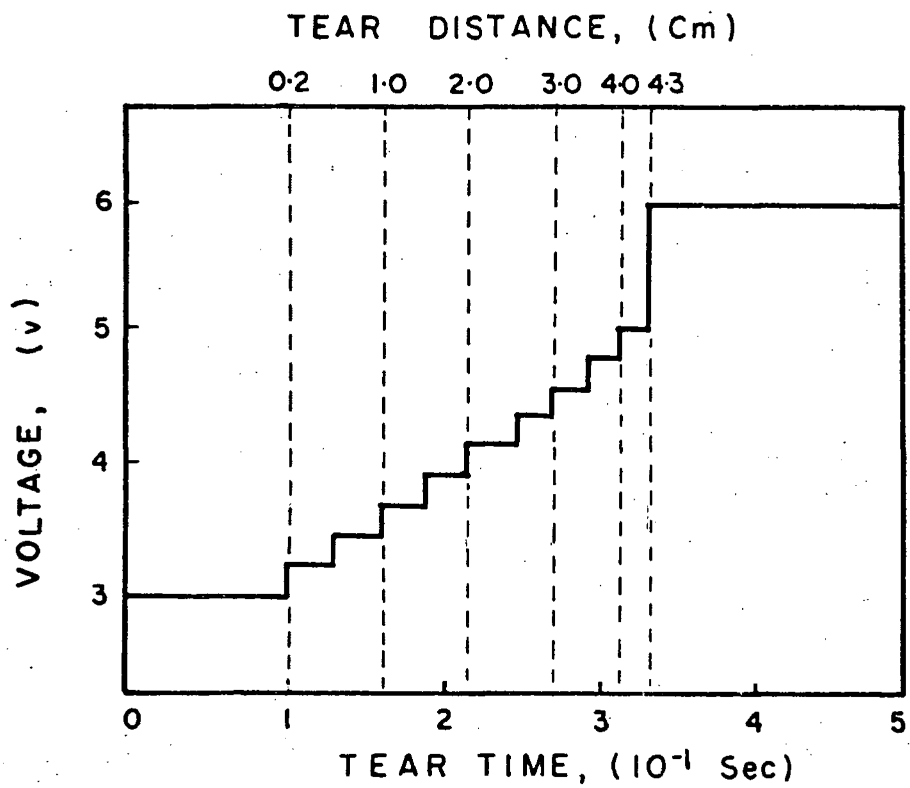
APPENDIX 1

Electrical Circuit Used for the Study and Voltage Variation in the Tearing Process.

a. Circuit for measuring the tear distance-time relationship.



b. Trace of the voltage variation on the oscilloscope.



c. Voltage variation in the conductive ply during the tearing process.

The voltage difference (V_x) across the conductive ply is:

$$V_x = iR_e \quad [19]$$

where: i = current (Amp)

R_e = equivalent resistance, the combination of resistors r_1 to r_{10} in parallel, of the specimen conductive ply.

$$\frac{1}{R_e} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots + \frac{1}{r_{10}}$$

A 6 volt battery is used in the circuit (Appendix 1a), hence

$$V_o = 6 = i\ (R_a + R_e) \quad [20]$$

where: R_a = resistance chosen to be equal to the equivalent resistance of the conductive ply.

Therefore,

$$V_x = \frac{6iR_e}{i(R_a+R_e)} = \frac{6R_e}{R_a+R_e} = 3 \text{ volt}$$

This is the voltage difference before tearing starts. The equivalent resistance, R_e, increases as tearing is in progress. Hence, V_x increases as tearing progresses, until the conductive ply is completely torn. Then, V_x reaches its maximum of 6 volt, as shown in Appendix 1b.

APPENDIX 2

Inter-relationship Between Tear Distance, Time, Velocity and Energy in the Ballistic-type Tearing Process. The relationship between zero-swing distance (L_1) and time (t_1) is:

$$L_1 = a_1 + b_1 t_1 + c_1 t_1^2 \qquad c_1 \neq 0 \qquad [11a]$$

The relationship between tear distance (L_2) and time (t_2) is:

$$L_2 = a_2 + b_2 t_2 + c_2 t_2^2 \qquad c_2 \neq 0 \qquad [11b]$$

Both Equations [11a] and [11b] show that the tear distance-time relationship is curvilinear.

The relationship between zero-swing velocity (v_1) and time (t_1) is:

$$v_1 = b_1 + 2c_1 t_1 \qquad c_1 \neq 0 \qquad [12a]$$

The relationship between tearing velocity (v_2) and time (t_2) is:

$$v_2 = b_2 + 2c_2 t_2 \qquad c_2 \neq 0 \qquad [12b]$$

Both Equations [12a] and [12b] show that the velocity-time relationship is linear.

The relationship between zero-swing velocity (v_1) and distance (L_1) can be obtained from Equations [11a] and [12a,] where:

$$v_1 = b_1 + 2c_1 t_1 \qquad [12a]$$

becomes:

$$t_1 = \frac{v_1 - b_1}{2c_1} \qquad [21]$$

Substituting Equation [21] into Equation [11a] gives:

$$L_1 = a_1 + b_1\left(\frac{v_1 - b_1}{2c_1}\right) + c_1\left(\frac{v_1 - b_1}{2c_1}\right)^2$$

$$= a_1 + \frac{2b_1v_1 - 2b_1^2}{4c_1} + c_1\frac{v_1^2 - 2b_1v_1 + b_1^2}{4c_1^2}$$

$$= a_1 - \frac{b_1^2}{4c_1} + \frac{v_1^2}{4c_1} \qquad [22]$$

Transposition of Equation [22] gives:

$$v_1^2 = 4c_1L_1 - 4a_1c_1 + b_1^2 \qquad [23a]$$

hence:

$$v_1 = \pm\sqrt{4c_1L_1 - 4a_1c_1 + b_1^2} \qquad [23b]$$

By the same approach, it can be shown that the relationship between tearing velocity (v_2) and distance (L_2) is:

$$v_2^2 = 4c_2L_2 - 4a_2c_2 + b_2^2 \qquad [24a]$$

$$v_2 = \pm\sqrt{4c_2L_2 - 4a_2c_2 + b_2^2} \qquad [24b]$$

Hence, the velocity-distance relationship can be expressed as branchs (where: $v_1 > 0$ and $v_2 > 0$) of the hyperbolas, Equations [23a] and [24a].

The sector net energy is:

$$E_{net} = \frac{1}{2}mv_1^2 = \frac{1}{2}m\,(4c_1L_1 - 4a_1c_1 + b_1^2) \qquad [25]$$

The sector residual energy is:

$$E_{residual} = \frac{1}{2}mv_2^2 = \frac{1}{2}m\,(4c_2L_2 - 4a_2c_2 + b_2^2) \qquad [26]$$

The difference between sector net and residual energies is tearing energy (T.E.).

$$T.E. = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}m\ (v_1^2 - v_2^2)$$
$$= \frac{1}{2}m\ \Big[4(c_1L_1 - c_2L_2) + 4(a_2c_2 - a_1c_1) + (b_1^2 - b_2^2)\Big] \qquad [27]$$

Hence, tearing energy relates to tear distance linearly.

The sector net energy and time (t_1) relationship can be obtained by substituting Equation [12a] into the net energy equation ($E_{net} = \frac{1}{2}\ mv_1^2$).

$$E_{net} = \frac{1}{2}mv_1^2 = \frac{1}{2}m\ (b_1 + 2c_1t_1)^2$$
$$= \frac{1}{2}m\ (b_1^2 + 4b_1c_1t_1 + 4c_1^2t_1^2) \qquad [28]$$

The sector residual energy and tear time (t_2) relationship can be obtained by substituting Equation [12b] into the residual energy equation ($E_{residual} = \frac{1}{2}mv_2^2$)

$$E_{residual} = \frac{1}{2}mv_2^2 = \frac{1}{2}m\ (b_2 + 2c_2t_2)^2$$
$$= \frac{1}{2}m\ (b_2^2 + 4b_2c_2t_2 + 4c_2^2t_2^2) \qquad [29]$$

Hence, the tearing energy-time relationship is:

$$T.E. = E_{net} - E_{residual}$$
$$= \frac{1}{2}m\ \Big[(b_1^2 - b_2^2) + 4\ (b_1c_1t_1 - b_2c_2t_2) + 4\ (c_1^2t_1^2 - c_2^2t_2^2)\Big] \qquad [30]$$

Which shows that tearing energy relates to tear time curvilinearly.