A POLARITY COINCIDENCE SPECTRUM ANALYZER FOR INPUTS WITH A WIDE DYNAMIC RANGE

by

KENNETH WAYNE BOWERING

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Department of ELECTRICAL ENGINEERING

The University of British Columbia
Vancouver 8, Canada

Date 12 JULY 1968

ABSTRACT

In the past, spectral analysis of Arctic Sea ambient noise has been carried out by octave band pass filtering and linear rectification, followed by analogue integration. To relate this integral to the power spectral density of the noise, the amplitude distribution of the noise has been assumed to be Gaussian, giving rise to certain errors.

A system is proposed which consists of a stage of variable gain followed by a polarity coincidence statistical wattmeter and measures the power spectral density of ambient noise after band pass filtering. The wattmeter will handle an input signal dynamic range of at least 20 dB and does so regardless of the statistical nature of the noise. This dynamic range is extended dynamically by controlling the gain of the driving stage. The gain level is automatically adjusted during a one minute "adaptive" time interval so that the noise delivered to the wattmeter is over the region of optimal system operation. Measurement of the power spectral density of the ambient noise is then made in the subsequent four minute interval.

A prototype wattmeter has been constructed and tested. The gain level is determined by requiring that the noise not exceed fixed levels more than a certain percentage of the time. This automatic adjustment is carried out during a one minute adaptive time interval, and a relatively accurate measure of the mean square value of the noise is determined during the four

minutes that follow.

For purposes of testing the prototype, d.c. inputs and sinusoidal inputs of wide frequency and amplitude ranges were used. The actual root mean square value of the inputs was measured with a thermal milliammeter and a precision voltage divider. The results of these tests show the region of operation where the input-output relationship of the wattmeter is linear. From these results, suggestions are made as to how the proposed system could be modified to replace the analogue system used for Arctic Sea ambient noise spectral analysis.

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1. INTRODUCTION

1.1 Requirement for Arctic Sea Spectral Analysis

In recent years, the Defence Research Establishment Pacific, (D.R.E.P.), has carried out a statistical analysis of Arctic Sea ambient noise. One of the reasons for determining these statistics lies in the design of sonar equipment. For active systems, in order to optimize the signal-to-noise ratio, the frequencies of transmission must lie in a range where the ambient noise power spectral density is relatively low. In passive systems, the *a priori* knowledge of the power spectral density would aid in the detection and classification of submarine life.

1.2 Characteristics of the Arctic Sea [1]

Oceanographic knowledge of the Arctic Sea has developed considerably since the 1950's. The sea consists of a Canadian basin with a depth of about 2000 fathoms and a Eurasian basin with a depth of about 2300 fathoms. These are separated by a ridge that is approximately 700 fathoms below sea level. In the Canadian basin the upper water circulates in a clockwise direction; in the Eurasian basin the flow is by the most direct path towards Greenland, and is of the order of 1.5 inches per second.

The melting and freezing of ice strongly influences the salinity of the Arctic Sea water, which varies between 28.0 and 33.5 grams of salt per kilogram of water (°/₀₀). The sea temperature is also controlled by the melting and freezing of ice and, as a consequence, remains close to freezing (-1.5 °C at 28.0 °/₀₀ to -1.8 °C at 33.5 °/₀₀). These values are typical of the upper

30 fathoms of the entire sea and are subject to seasonal variations of 0.2 C° in temperature and 2.0 °/oo in salinity.

Ice in the Arctic Sea predominantly originates from the freezing of sea water, first developing as slush and eventually becoming pancake and sheet ice. At -30 °C, 3 centimeters of ice can form in one hour and 30 centimeters in three days, the rate of ice formation decreasing because ice is a rather poor conductor of heat.

The polar ice cap covers about 70 % of the sea and is always present though annually about one-third is carried off in the East Greenland current. The average thickness is 8 feet in summer and 11 feet in winter with an occasional build up of up to 30 feet.

Outside of the ice cap is pack ice covering about 25 % of the sea, the coverage being greatest in May and least in September because some of the ice melts or is carried away. Shore ice reaches a maximum thickness of 6 feet in winter but completely melts during the summer months.

1.3 Summary of Results of Arctic Sea Ambient Noise Research [2], [3]

Noise measurements by D.R.E.P. have been made from aboard ship when the sea was reasonably clear of ice and at other times from stations set up on the ice. The ambient noise was recorded, using a lowered hydrophone, and power spectral densities calculated. It was found that the water flow was sufficient to cause the hydrophone cable to vibrate and thereby introduced erroneous noise levels which resulted in ambiguous results.

The results of these experiments did, however, give some indication as to the nature of the ambient noise by showing that there are two distinct components. The first component is entirely impulsive and is caused as a result of the surface ice cracking. The second component has a Gaussian amplitude distribution and results when the wind, blowing over a large surface area, causes oscillation of the surface. The power spectral density of this component varies in proportion to the wind speed.

During decreasing surface air temperatures the impulsive noise component is the dominant contributor to the ambient noise, while during increasing surface air temperatures both components are present. The wind driven noise component is limited to frequencies below 1000 hertz and the impulsive noise component is limited to frequencies below 16 kilohertz. Typical power spectra for these two components, and the variation of surface air temperature with time of day are shown in Figures 1 and 2.

The Arctic Sea ambient noise has been shown, in the D.R.E.P. experiments, to be stationary for analysis periods of between two and five minutes and nonstationary over larger periods. During the stationary periods, the mean square values of the two noise components can differ by as much as 40 dB, and over different five minute intervals, the mean square value of the additive noise can vary by an additional 35 dB. These short and long term dynamic ranges imply a total dynamic range of 75 dB.

To overcome some of the difficulties in obtaining data, retrievable instrumentation packages (R.I.P.), capable of recording a measure of the ambient noise power spectral density over

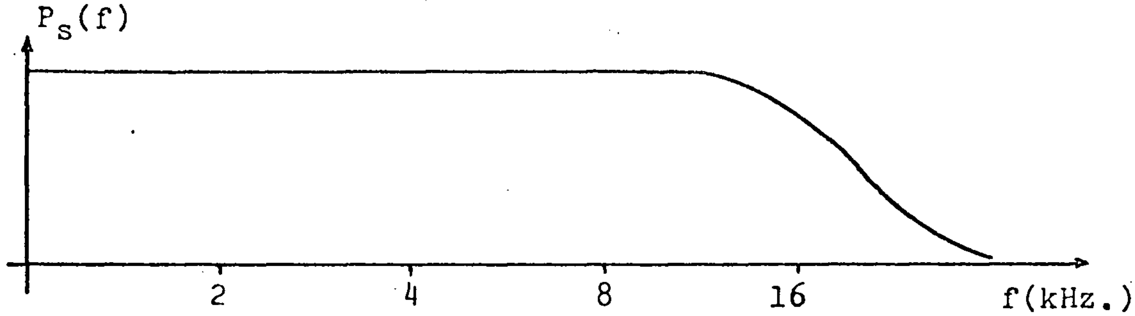


Figure 1 (a) Typical Power Spectrum of Impulsive Noise Component

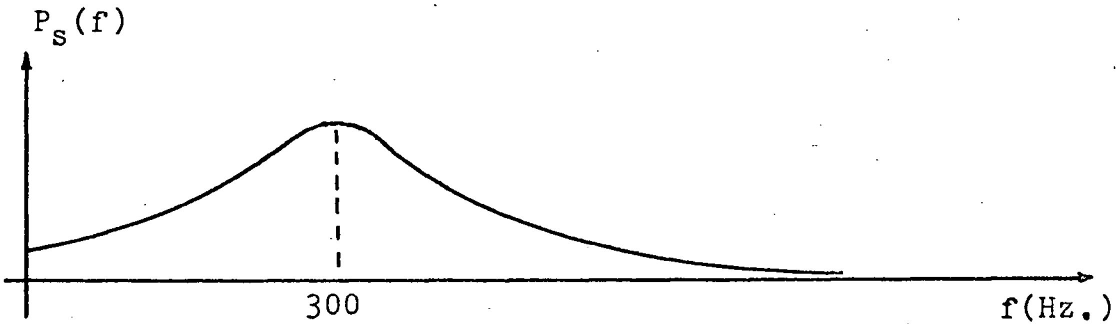


Figure 1 (b) Typical Power Spectrum of Wind Driven Noise Component

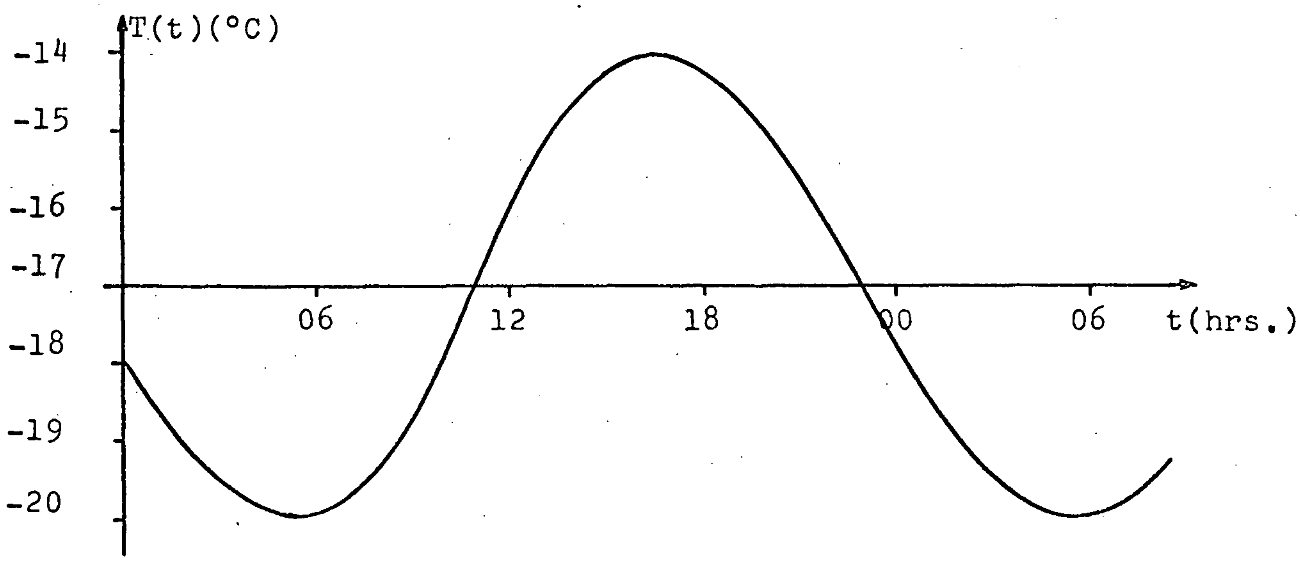


Figure 2. Typical Variation of Surface Air Temperature With Time of Day

periods in excess of one year, have been placed by D.R.E.P. at several locations in the Arctic Sea. Each package consists of six octave band pass filters, switched by means of an "Accutron" watch movement, to provide inputs to a wattmeter (to be described later) in such a manner that outputs of the entire bank of filters are scanned in a twenty-four minute time interval. The wattmeter outputs appear in digital form and are recorded in "IBM" compatible format on magnetic tape for processing upon recovery of the package. The locations of the pass bands are as shown in Table 1.

CHANNEL	1	2	3	4	5	6
PASS BAND	10-20 Hz.	40-80 Hz.	0.15-0.30 kHz.	0.50-1.0 kHz.	2.0-4.0 kHz.	8.0-16.0 kHz.
CENTRE FREQUENCY	15.0 Hz.	60.0 Hz.	0.225 kHz.	0.75 kHz.	3.0 kHz.	12.0 kHz.

Table 1. Pass Band Locations

The basic operation of the R.I. packages after band pass filtering of the input is linear rectification followed by analogue integration for four minutes and digital conversion. This method of analysis is based on the premise that the noise essentially has a Gaussian amplitude distribution and is stationary over the time of analysis. In addition, the system operates "wide open" at all times so that there is no change in gain level to accommodate the dynamic range of the ambient noise.

It appears worthwhile to use a method of analysis which does not assume that the ambient noise has a Gaussian amplitude distribution and that can accommodate the dynamic range of the noise. A wattmeter based on polarity coincidence techniques is suitable for this purpose and a laboratory verification of this concept is reported in the pages to follow.

2. THEORETICAL STUDY OF ARCTIC SEA AMBIENT NOISE

2.1 The Power Spectral Density Estimate of Arctic Sea Noise [4]

The Arctic Sea ambient noise can be considered to be a zero-mean stationary random process s(t), for which the autocorrelation function is

$$R_s(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T s(t)s(t+\tau)dt. \qquad \ldots\ldots\ldots \qquad (1)$$

In practice, the time interval T is finite or the integration is replaced by sampling and summation so that estimates of (1) are calculated from

$$\phi_s(\tau) = \frac{1}{T}\int_0^T s(t)s(t+\tau)dt,$$

and

$$\phi_s(\tau) = \frac{1}{N}\sum_{n=0}^{N-1} s(t+nT_s)s(t+nT_s+\tau), \qquad \ldots\ldots\ldots \qquad (2)$$

where T_s is the time interval between consecutive samples, and N is the number of samples taken over the time interval T.

The power spectral density function is used to describe the frequency composition in terms of the mean square spectral amplitude and, provided the random process is stationary, is the Fourier Transform of the autocorrelation function

$$P_s(f) = \int_{-\infty}^{+\infty} R_s(\tau)e^{-j2\pi f\tau}\, d\tau, \qquad \ldots\ldots\ldots \qquad (3)$$

where

$$R_s(\tau) = \int_{-\infty}^{+\infty} P_s(f)e^{+j2\pi f\tau}df.$$

When the ambient noise, s(t), is the input to a filter with transfer function H(f) and the filter output is u(t)

$$s(t) \longrightarrow \boxed{H(f)} \longrightarrow u(t)$$

then the power spectral density function of u(t) is related to that of s(t) by

$$P_u(f) = P_s(f)H(f)H^*(f), \qquad \ldots\ldots\ldots \qquad (4)$$

where * means the complex conjugate. For the case where H(f) represents an octave band pass filter

$$H(f) = \begin{cases} 1 & f_1 \leq |f| \leq 2f_1 \\ 0 & \text{elsewhere.} \end{cases}$$

For any one value of f_1, knowledge of $P_u(f)$ does not impart any knowledge of $P_s(f)$ outside the pass band defined by H(f), but, if several such bands are considered, then an estimate, $\hat{P}_s(f)$, of $P_s(f)$ can be produced from $P_u(f)$. Over the pass band, the average value of $P_u(f)$ is

$$\overline{P_u(f)} = \frac{1}{f_1}\int_{f_1}^{2f_1} P_u(f)df = \frac{R_u(0)}{2f_1} = \frac{\sigma_u{}^2}{2f_1},$$

where σ_u^2 is the variance of u(t). The estimate of $P_s(f)$ evaluated at f_c, the centre frequency of the pass band, is

$$\hat{P}_s(f_c) = \frac{3\sigma_u^2}{4f_c}, \qquad \ldots\ldots\ldots\ldots \quad (5)$$

and by measuring σ_u^2 for several values of f_c, the original power spectrum of s(t) can be reproduced.

The approximation that $P_u(f)$ is sufficiently flat (or smooth) over the pass band so that $\hat{P}_s(f_c)$ is a good estimate of $P_s(f)$ is assumed in the D.R.E.P. system as well as in the proposed system. Errors caused by such an approximation can be reduced by considering pass bands of smaller bandwidths than those of octave band pass filters.

2.2 An Approximation to the Autocorrelation and Power Spectral Density Functions of the Arctic Sea Ambient Noise

Mention was made in Section 1.3 of the two distinct components of the Arctic Sea ambient noise and their typical power spectra shown in Figure 1. For use in Chapter 3, it is desirable to know the class of autocorrelation and power spectral density functions to which these components belong.

The power spectrum of the wind driven noise component has the same shape as that of the output of a second order LCR underdamped circuit when the input is white noise, i.e.

$$P_s(f) = K\sigma_s^2 \left[\frac{1}{K^2 + 4\pi^2(f+f_o)^2} + \frac{1}{K^2 + 4\pi^2(f-f_o)^2}\right],$$

where 2K is the half power bandwidth. An estimate of K may be determined by assuming that

$$P_s(f = 0) \approx P_s(f = 500),$$

which, for f_o equal to 300 hertz, gives

$$K = 140 \text{ hertz.}$$

In the calculation of $\hat{P}_s(f_c)$, it is assumed that the power spectrum is flat over the pass band so that the autocorrelation function of u(t) can be determined by the approximation

$$R_u(\tau) = \int_{-\infty}^{+\infty} P_u(f)e^{+j2\pi f\tau}\,df \approx 2\overline{P_u(f)} \int_{f_1}^{2f_1} \cos(2\pi f\tau)df$$

$$= \frac{2\overline{P_u(f)}}{\pi\tau} \sin(\pi f_1\tau)\cos(3\pi f_1\tau).$$

Because $R_u(0)$ is equal to σ_u^2, the resulting approximate autocorrelation and power spectral density functions of the wind driven noise component are

$$R_u(\tau) = \frac{\sigma_u^2}{\pi f_1\tau}\sin(\pi f_1\tau)\cos(3\pi f_1\tau),$$

and

$$P_u(f) = \begin{cases} \frac{\sigma_u^2}{2f_1} & f_1 \leq |f| \leq 2f_1 \\ 0 & \text{elsewhere.} \end{cases} \qquad \text{. . . . . (6)}$$

The functions given by (6) are also assumed to be the autocorrelation and power spectral density functions for the additive noise

$$u(t) = u(t)_{\text{impulsive}} + u(t)_{\text{wind driven}}$$

because the two components, resulting from two entirely different processes, are statistically independent and the power spectrum of the impulsive noise component, as shown by Figure 1, is flat, as is the approximation to the power spectral density of the wind driven noise component after band pass filtering.

2.3 D.R.E.P. Spectrum Analysis and Associated Errors[2]

A system functionally similar to that used by D.R.E.P. in the analysis of Arctic Sea ambient noise is shown in Figure 3.

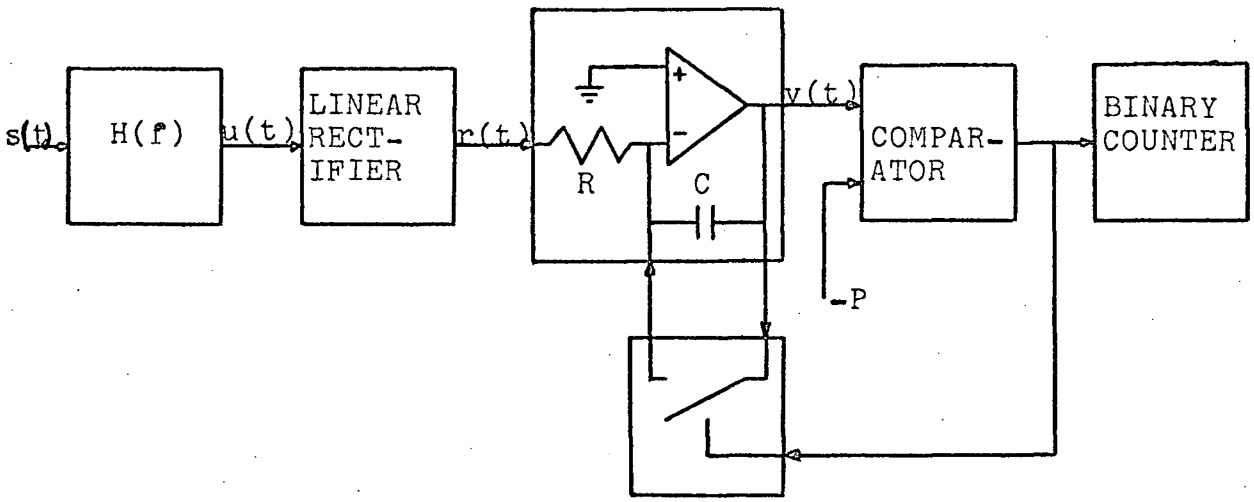


Figure 3. Functional Equivalence to D.R.E.P. System

To produce the estimate of $P_s(f)$ it is assumed that s(t), and therefore u(t), is a stationary Gaussian random process, and by measuring the mean value of r(t), the linearly rectified version of u(t), a value of σ_u can be obtained. For the case of full wave linear rectification, the probability density function of r(t) is

$$p_r(\gamma) = 2p_u(\gamma)\ \delta_{-1}(\gamma), \qquad \ldots\ldots\ldots\ldots \quad (7)$$

where $\delta_{-1}(\gamma)$ is the unit step function

$$\delta_{-1}(\gamma) = \begin{cases} 0 & \gamma<0 \\ 1 & \gamma\geq 0, \end{cases}$$

and $p_u(\gamma)$ is the probability density function of u(t) which, for a Gaussian random process, is

$$p_u(\gamma) = \frac{1}{\sigma_u\sqrt{2\pi}} \exp(-\gamma^2/2\sigma_u{}^2).$$

The mean value of r(t) is obtained by integrating (7)

$$\overline{r(t)} = \int_{-\infty}^{+\infty} \gamma p_r(\gamma)d\gamma = \sqrt{\frac{2}{\pi}}\sigma_u,$$

and the estimate of $P_s(f)$ is obtained using (5) as

$$\hat{P}_s(f_c) = \frac{3\pi}{8f_c}\{\overline{r(t)}\}^2. \qquad \text{. . . . . (8)}$$

With reference to Figure 3, the system functionally similar to that used by D.R.E.P. integrates r(t) until the integrator output reaches a level -P volts. This causes the comparator to switch to the high output state, which in turn, by means of a switch, short circuits the capacitor C. The integrator output rises to zero volts and in doing so, causes the comparator output to return to the low state. This high-to-low transition of the comparator states increments the binary counter by one and the integration of r(t) continues, so that after a time T

$$\overline{r(t)} = \frac{1}{T}\int_0^T r(t)dt = \frac{1}{T}\left[\int_0^{T_1} r(t)dt + \int_{T_1}^{T_2} r(t)dt + \ldots\right.$$

$$+ \int_{T_{n-1}}^{T_n} r(t)dt + \int_{T_n}^{T} r(t)dt \Bigg].$$

At time T_i the integrator output is zero volts and at time T_{i+1} the output is

$$v(t) = -\frac{1}{RC} \int_{T_i}^{T_{i+1}} r(t)dt = -P, \qquad \begin{array}{l} i = 0, 1, 2, \ldots, n-1 \\ T_o \equiv 0 \end{array}$$

so that

$$\overline{r(t)} = \frac{PRC}{T} n + \frac{1}{T} \int_{T_n}^{T} r(t)dt, \qquad \ldots\ldots\ldots\ldots (9)$$

where n is the count appearing in the binary counter.

The second term in (9) is a "round off" term and is neglected because it is assumed to be small when compared to $\frac{PRC}{T} n$, so that the D.R.E.P. estimate of $P_s(f)$ is

$$\hat{P}_s(f_c) = \frac{3\pi}{8f_c} \left[\frac{PRC}{T}\right]^2 n^2. \qquad \ldots\ldots\ldots\ldots (10)$$

Despite the apparent simplicity of this method the results obtained with the use of (10) may be in considerable error. This error arises in two ways: (i) the "round off" term of (9) is not small compared to $\frac{PRC}{T} n$, and, (ii) the ambient noise does not always have a Gaussian amplitude distribution. To ascertain the error which may occur in these measurements, the "round off error" is determined for the case when u(t) is a sinusoid. The error arising because u(t) does not have a Gaussian amplitude distribution is then calculated for non-Gaussian inputs:

(i) sinusoidal input,

(ii) level input,

(iii) input with a Laplacian probability density function, and

(iv) input with a uniform probability density function.

2.3.1 Round Off Error

For an input

$u(t) = B\sin(2\pi ft)$,

the rectified signal r(t) is

$r(t) = B|\sin(2\pi ft)|$,

and the mean value of r(t), averaged over one period of r(t) is

$$\overline{r(t)} = 2f\int_0^{1/2f} B\sin(2\pi ft)dt = \frac{2B}{\pi}\ .$$

Using the D.R.E.P. method of analysis, because T_1 is the time required for the integrator output to reach -P volts,

$T_n = nT_1$,

and

$$\overline{r(t)} = \frac{n}{T}\int_0^{T_1} r(t)dt + \frac{1}{T}\int_{nT_1}^{T} r(t)dt\ .$$

Evaluating this first integral gives

$$P = \frac{1}{RC}\int_0^{T_1} r(t)dt = \frac{B}{RC}\int_0^{T_1} |\sin(2\pi ft)|dt$$

$$= \frac{mB}{\pi RCf} + \frac{B}{\pi RCf}\sin^2(\pi fx), \qquad \text{. . . . . (11)}$$

where

$$T_1 = \frac{m}{2f} + x\,,$$

and m is the largest integer such that

$$0 \leq x < \frac{1}{2f}\,.$$

Then

$$T_1 = \frac{PRC\pi}{2B} + x - \frac{\sin^2(\pi f x)}{2f}\,, \qquad \ldots\ldots\ldots\ldots \quad (12)$$

where $\sin^2(\pi fx)$ is the fractional part of $\frac{\pi PRCf}{B}$.

The "round off" term of (9) is

$$\frac{1}{T}\int_{nT_1}^{T} r(t)dt = \frac{2B}{\pi} - n\frac{PRC}{T}\,,$$

and corresponds to a percentage error in the measurements of

$$\%\ \text{error} = 100\frac{\pi}{2B}\frac{1}{T}\int_{nT_1}^{T} r(t)\ dt = 100 - 50\pi\frac{PRC}{BT}n\,,$$

$$\ldots\ldots\ldots\ldots \quad (13)$$

where n is the integral part of the ratio of T to T_1. The percentage error given by (13) has been evaluated for frequencies the same as those of the centre frequencies of the octave pass bands as listed in Table 1 and for a range of amplitudes B. In the absence of detailed information about the D.R.E.P. system, values of constants used in these calculations were:

P = 5.0 volts, and

RC = 1.0 seconds.

The value of T was that used in the D.R.E.P. system, i.e., 240.0

seconds. The results of these calculations show that the percentage error is almost independent of the frequency of the sinusoid and a "percentage error bound" is illustrated in Figure 4.

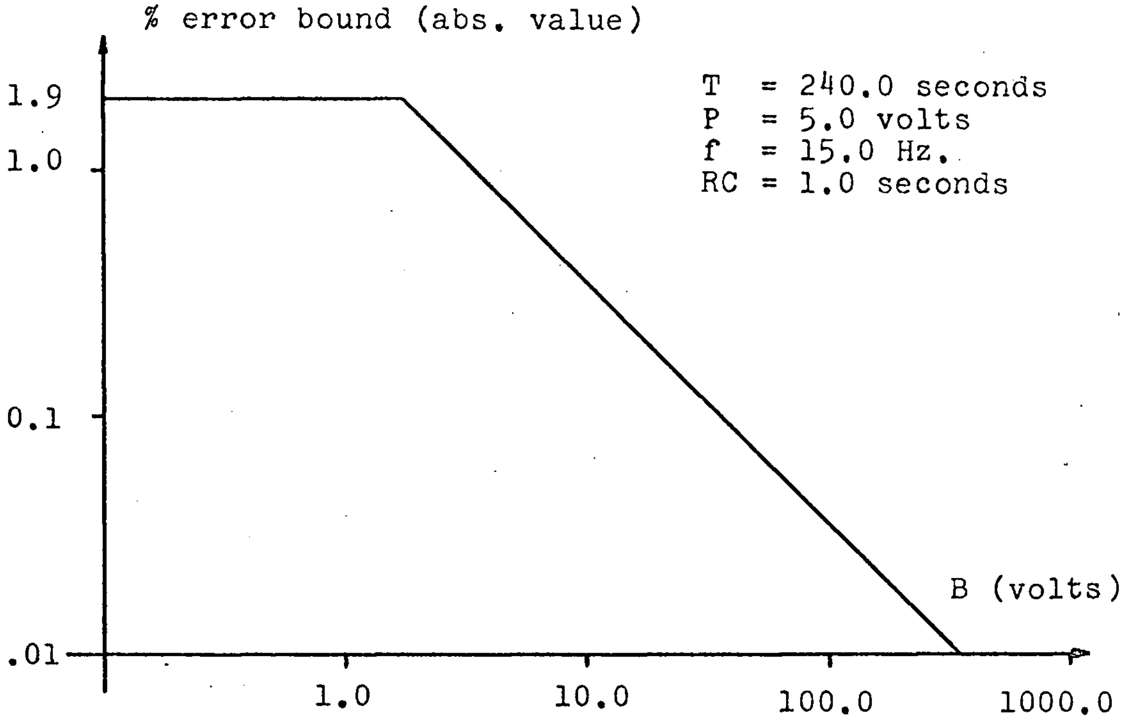


Figure 4. Round Off Error of D.R.E.P. System

For sinusoidal inputs a limitation of this method will occur if the time T_1 is greater than the time T so that a count of zero will be obtained, i.e.

$$T_1 \simeq \frac{PRC\pi}{2B} \geq T .$$

Corresponding to this limitation, the minimum amplitude of the sinusoidal input is

$$B_{min} = \frac{PRC\pi}{2T} . \qquad \ldots\ldots\ldots\ldots (14)$$

2.3.2 Error Resulting when the Input is Non-Gaussian

When s(t) is a zero-mean random process the estimate of $P_s(f)$ is as given by (5)

$$\hat{P}_s(f_c) = \frac{3\sigma_u^2}{4f_c} .$$

However, if s(t) is not a zero-mean random process the estimate of $P_s(f)$ is

$$\hat{P}_s(f_c) = \frac{3}{4f_c} \overline{u^2(t)} . \qquad (15)$$

(i) $u(t) = B\sin(2\pi ft)$

The mean square value of u(t), averaged over one period is

$$\overline{u^2(t)} = f \int_0^{1/f} B^2\sin^2(2\pi ft)\, dt = \frac{B^2}{2} ,$$

and the mean value of r(t) is

$$\overline{r(t)} = \frac{2B}{\pi} .$$

The actual estimate of $P_s(f)$ is

$$\hat{P}_s(f_c)_{act} = \frac{3B^2}{8f_c} ,$$

and that given by the D.R.E.P. system is

$$\hat{P}_s(f_c)_{DREP} = \frac{3B^2}{2\pi f_c} .$$

The difference in the estimates then gives rise to a percentage error of

$$\% \text{ error} = 100 \frac{\hat{P}_s(f_c)_{act} - \hat{P}_s(f_c)_{DREP}}{\hat{P}_s(f_c)_{act}}$$

$$= 100(1 - \frac{4}{\pi}) = -27.3 \% .$$

(ii) $u(t) = B$

The mean square value of u(t) is

$$\overline{u^2(t)} = B^2,$$

and the mean value of r(t) is

$$\overline{r(t)} = B.$$

The actual estimate of $P_s(f)$ is

$$\hat{P}_s(f_c)_{act} = \frac{3B^2}{4f_c},$$

and that given by the D.R.E.P. system is

$$\hat{P}_s(f_c)_{DREP} = \frac{3\pi B^2}{8f_c}.$$

This difference in the estimates gives rise to an error

$$\% \text{ error} = 100(1 - \frac{\pi}{2}) = -57.1 \% .$$

(iii) u(t) has a probability density function given by

$$p_u(\gamma) = \frac{1}{2b} \exp(-|\gamma|/b).$$

The variance of u(t) is

$$\sigma_u^2 = 2b^2,$$

and the mean value of r(t) is

$$\overline{r(t)} = b .$$

The actual estimate of $P_s(f)$ is

$$\hat{P}_s(f_c)_{act} = \frac{3b^2}{2f_c} ,$$

and the D.R.E.P. estimate is

$$\hat{P}_s(f_c)_{DREP} = \frac{3\pi b^2}{8f_c} .$$

The difference in the estimates gives rise to an error of

$$\% \text{ error} = 100(1 - \frac{\pi}{4}) \doteq 21.5 \% .$$

(iv) u(t) has a probability density function given by

$$p_u(\gamma) = \begin{cases} 0 & |\gamma| > b \\ \frac{1}{2b} & |\gamma| \leq b . \end{cases}$$

The variance of u(t) is

$$\sigma_u^2 = \frac{b^2}{3} ,$$

and the mean value of r(t) is

$$\overline{r(t)} = \frac{b}{2} .$$

The actual estimate of $P_s(f)$ is

$$\hat{P}_s(f_c)_{act} = \frac{b^2}{4f_c} ,$$

and the D.R.E.P. estimate is

$$\hat{P}_s(f_c)_{DREP} = \frac{3\pi b^2}{32f_c} .$$

The difference in the estimates gives rise to an error of

$$\% \text{ error} = 100(1 - \frac{3\pi}{8}) = -17.8 \%.$$

From these examples it is seen that when the ambient noise, s(t), does not have a Gaussian amplitude distribution, very significant errors can arise and an incorrect power spectral density estimate will be made.

2.4 Arctic Sea Ambient Noise Dynamic Range

The large dynamic range of the Arctic Sea ambient noise as discussed in Section 1.3 may be used to determine the relative magnitudes of the range of $\sigma_u{}^2$ that can be encountered in the analysis. The dynamic range is

$$\text{dynamic range} = 10 \log_{10} \frac{P_{umax}}{P_{umin}} = 10 \log_{10} \frac{\sigma^2_{umax}}{\sigma^2_{umin}} \text{ dB.}$$

For a dynamic range of 75 dB then $\sigma_u{}^2$ will lie in the range

$$\sigma^2_{umin} \leq \sigma_u{}^2 \leq \sigma^2_{umax} = 10^{7.5}\, \sigma^2_{umin}. \qquad \ldots . \quad (16)$$

If an estimate of $\sigma_u{}^2$ were known prior to commencing the analysis, the instrument could be adjusted so as to carry out the analysis with minimum error. This suggests the desirability of an adaptive period during which the estimate of $\sigma_u{}^2$ is measured, and the instrument is adjusted accordingly.

3. THE PROPOSED SYSTEM

3.1 Sampling Frequency [4]

The two main operations in digital systems are sampling and quantization. It is necessary that the sampling frequency be large enough so that a sufficient number of samples are taken to accurately describe the random process and also low enough so that consecutive samples are uncorrelated. When power spectral density calculations are to be made from single records of length T seconds, Bendat and Piersol suggest a sampling interval

$$T_s = \frac{T}{N} = \frac{2}{5f_m} ,$$

where N is the sample size and f_m is the maximum frequency present in the sample. In the Arctic Sea ambient noise analysis, T is equal to 240.0 seconds and f_m is equal to 16.0 kilohertz so that the sampling interval and sample size are

$$T_s = 25.0 \times 10^{-6} \text{ seconds, and}$$
$$N = 9.6 \times 10^{6} \text{ samples.}$$

3.2 Polarity Coincidence Statistical Wattmeter [5]

The estimate of $P_s(f)$ as given by (5) can be made with the use of a statistical wattmeter which operates on the techniques of polarity coincidence. Such a meter is shown in Figure 5.

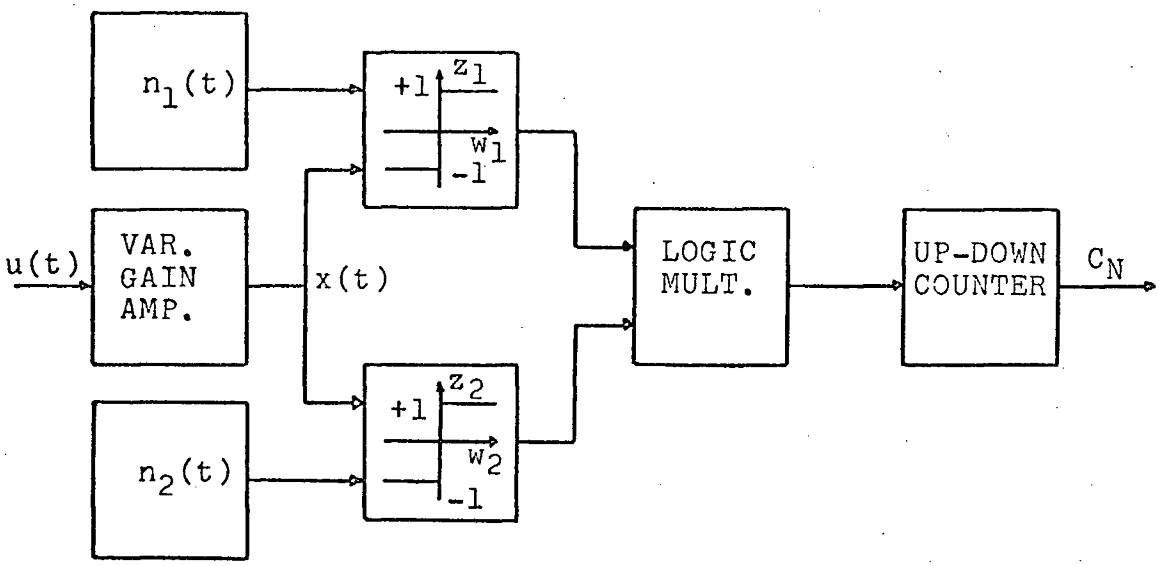


Figure 5. Polarity Coincidence Spectrum Analyzer with Adjustable Gain

The operation of this meter is based upon a theorem which is reproduced without proof in Section 3.3. A proof of this theorem is contained in reference (5).

3.3 Theorem

Let $x_1, x_2, \ldots, x_q$ be continuous random variables with each x_i bounded, $|x_i| \leq A_i$. Let $n_1, n_2, \ldots, n_q$ be continuous random variables, independent of each other and of the x_i and each with a probability density function given by

$$p_{n_i}(\gamma) = \frac{1}{2A_i}\left[\delta_{-1}(\gamma + A_i) - \delta_{-1}(\gamma - A_i)\right].$$

Define the random variables

$$w_i = x_i - n_i, \text{ and}$$

$$z_i = \mathrm{sgn}(w_i),$$

where

$$\mathrm{sgn}(w_i) = \begin{cases} +1 & w_i \geq 0 \\ -1 & w_i < 0 . \end{cases}$$

Then

$$\overline{x_1 x_2 \ldots x_q} = A_1 A_2 \ldots A_q \, \overline{z_1 z_2 \ldots z_q} . \qquad (17)$$

3.4 Realization of Polarity Coincidence Statistical Wattmeter

To determine the mean square value of a random variable, let $q = 2$ in the theorem of Section 3.3, and with

$$x_1 = x_2 = x(t) = -au(t),$$

where "a" is the gain of the amplifier shown in Figure 5, then

$$\overline{x^2(t)} = A_1 A_2 \, \overline{z_1 z_2} , \qquad (18)$$

where

$$\begin{aligned} z_1 &= \mathrm{sgn}\{x(t) - n_1(t)\}, \text{ and} \\ z_2 &= \mathrm{sgn}\{x(t) - n_2(t)\} . \end{aligned} \qquad (19)$$

Using digital circuits it is straightforward to measure $\overline{z_1 z_2}$ because the function $z_1 z_2$ can assume only the values +1 and -1. If $z_1 z_2$ is sampled at intervals of T_s seconds so that a total of N samples are taken, then

$$\overline{z_1 z_2} = \frac{1}{N} \sum_{n=0}^{N-1} z_1(t + nT_s) z_2(t + nT_s) = \frac{C_N}{N} , \qquad (20)$$

where C_N is the count that appears in an up-down counter where, upon the occurrence of the sampling pulse, the count is increased or decreased by one if z_1 and z_2 are of the same or opposite signs respectively. If U_N and D_N represent the number of up and down counts respectively, then after N samples have been taken

$$C_N = U_N - D_N, \text{ and}$$
$$N = U_N + D_N,$$

so that

$$C_N = 2U_N - N.$$

The up-down counter may be replaced by an up counter which, upon the occurrence of the sampling pulse, increases U_N by one only if z_1 and z_2 are of the same polarity. The mean square value of x(t) is then given by

$$\overline{x^2(t)} = A_1A_2\left[\frac{2U_N - N}{N}\right]. \qquad \ldots\ldots\ldots\ldots(21)$$

If Y_1 and Y_2 represent the Boolean algebraic outputs of the two quantizers, then the LOGIC MULTIPLY realization is

$$Y_3 = Y_1Y_2 + \overline{Y_1}\,\overline{Y_2},$$

which is the "exclusive OR complement".

3.5 Provision of Large Dynamic Range of Input Signals in Prototype Design

The implications of the dynamic range of the Arctic Sea ambient noise as discussed in Section 2.4 result in a wide variation of $\sigma_u{}^2$ and, in the proposed system of measuring $\sigma_u{}^2$, two critical cases exist. The first case corresponds to one of the limitations of the D.R.E.P. method of analysis and occurs for very small inputs ($|u(t)| \ll A_i$). For level inputs to the quantizers of Figure 5, it is shown in Section 3.6, that the mean square error of the estimate of $\overline{u^2(t)}$ is least if x(t), being representative of u(t), is also a level input equal to $\pm(A_1A_2)^{1/2}$. The second case occurs if $\sigma_u{}^2$ is large and therefore $|u(t)| > A_i$ for a large portion of the time of analysis, thereby invalidating the use of the theorem quoted in Section 3.3.

Both of these cases can be handled to any desired degree of accuracy by incorporating a scale change between u(t) and the input to the wattmeter. To provide such a scale change a variable gain amplifier is used, the gain of which is determined by some means during the adaptive time interval immediately preceding the actual computation of $\overline{x^2(t)}$. For small input signals a large gain is used and vice versa. The range of gain is directly related to the range of $\sigma_u{}^2$ and also to the percentage of time that $|x(t)| > A_i$ will be allowed. The gain steps are discrete and finite in number so that the choice of gain is made by choosing the largest gain such that the percentage of clipping is always greater than some fixed amount and less than some other fixed amount. In this manner each gain step will accommodate the analysis of u(t) over a small range of $\sigma_u{}^2$ and the relative errors will be the same over each range.

In the proposed system an operational amplifier is employed as a scale changer and

$$x(t) = -au(t)$$

where the gain "a" is variable and controlled by the ratio of feedback (variable) to input (constant) impedances. With the introduction of such a scale change, equations relating $P_u(f)$ to $P_x(f)$ contain the factor a^2

$$P_x(f) = a^2 P_u(f),$$

and the estimate of the power spectral density of the ambient noise is

$$\hat{P}_s(f_c) = \frac{3A_1A_2}{4a^2 f_c}\left[\frac{2U_N}{N} - 1\right]. \qquad \qquad (22)$$

The proposed instrument measures $\hat{P}_s(f_c)$ using (22) as an average over a pass band of which f_c is the centre frequency. Values of f_c, "a", and U_N must be known for each sample period but f_c, being one of a fixed set of frequencies, need not be recorded. U_N and "a" are recorded in binary form on magnetic tape.

3.6 Statistical Errors due to Sampling

Based on digital measurements, $\phi_x(0)$ is the expected value of $R_x(0)$ and ideally the expected value of $\phi_x(0)$ is equal to $R_x(0)$. If so, $\phi_x(0)$ is an unbiased estimate of $R_x(0)$. For fixed N, the fact that $\phi_x(0)$ is unbiased does not imply that $\phi_x(0)$ will be close to $R_x(0)$ and there may be considerable deviation. To study these cases the mean square error, $\overline{\varepsilon_x{}^2}$, of the estimate is determined

$$\overline{\varepsilon_x{}^2} = E[\{\phi_x(0) - R_x(0)\}^2]$$

$$= E[\{\phi_x(0) - E[\phi_x(0)] + E[\phi_x(0)] - R_x(0)\}^2]$$

$$= E[\{\phi_x(0) - E[\phi_x(0)]\}^2] + E[\{E[\phi_x(0)] - R_x(0)\}^2]$$

$$+ 2E[\{\phi_x(0) - E[\phi_x(0)]\}\{E[\phi_x(0)] - R_x(0)\}]$$

$$= E[\{\phi_x(0) - E[\phi_x(0)]\}^2] + E[\{E[\phi_x(0)] - R_x(0)\}^2]$$

$$= \text{Var}\ \phi_x(0) + E[\{E[\phi_x(0)] - R_x(0)\}^2],$$

where

$$\text{Var}\ \phi_x(0) = E[\{\phi_x(0) - E[\phi_x(0)]\}^2]$$

$$= E[\{\phi_x(0)\}^2] - [E[\phi_x(0)]]^2,$$

and

$$\phi_x(0) = \frac{1}{N}\sum_{n=0}^{N-1} x^2(t + nT_s) .$$

The expected value of $\phi_x(0)$ is

$$E[\phi_x(0)] = \frac{1}{N}\sum_{n=0}^{N-1} E[x^2(t + nT_s)] = \frac{1}{N}\sum_{n=0}^{N-1} R_x(0) = R_x(0),$$

so that $\phi_x(0)$ is an unbiased estimate of $R_x(0)$ and the mean square error of the estimate is

$$\overline{\varepsilon_x^{\,2}} = E[\{\phi_x(0)\}^2] - R_x^{\,2}(0).$$

$\phi_x(0)$ is measured using (18)

$$\phi_x(0) = A_1A_2\overline{z_1z_2} = \frac{A_1A_2}{N}\sum_{n=0}^{N-1} z_1(t + nT_s)z_2(t + nT_s),$$

and

$$E[\{\phi_x(0)\}^2] = \left[\frac{A_1A_2}{N}\right]^2 \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} E[z_1(t+nT_s)z_2(t+nT_s) \\ z_1(t+mT_s)z_2(t+mT_s)] .$$

From the theorem of Section 3.3 and because $z_i(t)$ is either +1 or -1 then

$$E[z_1(t+nT_s)z_2(t+nT_s)z_1(t+mT_s)z_2(t+mT_s)]$$
$$= \begin{cases} 1, \ m = n \\ \\ \dfrac{1}{(A_1A_2)^2} E[x^2(t+nT_s)x^2(t+mT_s)], \ m \neq n. \end{cases} \quad \ldots\ldots\ldots \quad (24)$$

The discussion of this fourth order moment requires knowledge of or assumptions about the statistics of x(t); three cases are considered:

(i) x(t) a level input,

(ii) x(t) a sinusoidal input, and

(iii) x(t) a stationary Gaussian random process.

The first case is included to verify the statement, made in Section 3.5, that the mean square error of the estimate can be minimized if the input is a constant. To verify that the statistical analysis using polarity coincidence techniques produces a linear output when the input varies over a wide dynamic range, a laboratory prototype has been constructed. This instrument was tested with sinusoidal and d.c. inputs and a region of linearity of input to output determined. The theoretical mean square error can then be compared to the actual mean square error. Results of the D.R.E.P. experiments showed that the Arctic Sea ambient noise very often has a Gaussian amplitude distribution so that the evaluation of $\overline{\epsilon_x{}^2}$ for a Gaussian random process is meaningful. In addition, the fourth order moment of (24) is easily evaluated when x(t) is a Gaussian random process.

3.6.1 Mean Square Error Evaluation

(i) $x(t) = -a\mu(A_1A_2)^{1/2}$, μ = constant multiplying factor.

The second and fourth order moments required for (23) and (24) are

$$R_x(0) = \overline{x^2(t)} = a^2\mu^2 A_1A_2,$$

and

$$\overline{x^2(t+nT_s)x^2(t+mT_s)} = a^4\mu^4(A_1A_2)^2.$$

The mean square error is

$$\overline{\varepsilon_x{}^2} = \frac{(A_1A_2)^2}{N} - a^4\mu^4(A_1A_2)^2 + \frac{1}{N^2}\sum_{\substack{n=0\\ m\neq n}}^{N-1}\sum_{m=0}^{N-1} a^4\mu^4(A_1A_2)^2$$

$$= \frac{(A_1A_2)^2}{N} + \frac{N-1}{N}a^4\mu^4(A_1A_2)^2 - a^4\mu^4(A_1A_2)^2$$

$$= \frac{(A_1A_2)^2}{N}(1 - a^4\mu^4). \quad \ldots\ldots\ldots\ldots \quad (25)$$

Therefore, with a level input u(t) equal to $\mu(A_1A_2)^{1/2}$, the mean square error of the estimate is minimized by choosing "a" such that

$$\mu a = \pm 1.$$

(ii) $x(t) = -aB\sin(2\pi ft)$.

To determine the second and fourth order moments required by (23) and (24), the respective products are integrated over one period of the sinusoid.

$$R_x(0) = a^2B^2f\int_0^{1/f} \sin^2(2\pi ft)dt = \frac{a^2B^2}{2},$$

and

$$E[x^2(t+nT_s)x^2(t+mT_s)]$$

$$= a^4B^4f\int_0^{1/f} \sin^2(2\pi f(t+nT_s))\sin^2(2\pi f(t+mT_s))dt$$

$$= \frac{a^4B^4}{4} + \frac{a^4B^4}{8}\cos(4\pi f(n-m)T_s).$$

The mean square error is given by

$$\overline{\varepsilon_x{}^2} = \frac{(A_1A_2)^2}{N} - \frac{a^4B^4}{4} + \frac{a^4B^4}{4N^2}\sum_{n=0}^{N-1}\sum_{\substack{m=0\\m\neq n}}^{N-1}\left[1 + \frac{\cos(4\pi f(n-m)T_s)}{2}\right]$$

$$= \frac{(A_1A_2)^2}{N} - \frac{a^4B^4}{4N} + \frac{a^4B^4}{8N^2}\sum_{n=0}^{N-1}\sum_{\substack{m=0\\m\neq n}}^{N-1}\cos(4\pi f(n-m)T_s).$$

It is shown (4) that such double summations may be written as a single summation provided that the function being summed is an even function of its argument.

$$\sum_{n=0}^{N-1}\sum_{\substack{m=0\\m\neq n}}^{N-1}\cos(4\pi f(n-m)T_s) = 2\sum_{n=1}^{N-1}(N-n)\cos(4\pi fnT_s).$$

Jolley (6) provides closed form solutions for the summations

$$\sum_{n=1}^{N-1} \cos(n\theta) = \cos(\frac{N\theta}{2})\sin((N-1)\frac{\theta}{2})\csc(\frac{\theta}{2}) ,$$

and

$$\sum_{n=1}^{N-1} n\cos(n\theta) = \frac{N}{2}\sin((2N-1)\frac{\theta}{2})\csc(\frac{\theta}{2}) - \frac{(1-\cos(N\theta))}{4}\csc^2(\frac{\theta}{2}) .$$

Then

$$2\sum_{n=1}^{N-1} (N-n)\cos(4\pi fnT_s) = 2N\cos(\frac{N\theta}{2})\sin((N-1)\frac{\theta}{2})\csc(\frac{\theta}{2})$$

$$-N\sin((2N-1)\frac{\theta}{2})\csc(\frac{\theta}{2}) + \frac{(1-\cos(N\theta))}{2}\csc^2(\frac{\theta}{2})$$

$$= -N + \frac{(1-\cos(N\theta))}{2}\csc^2(\frac{\theta}{2}) ,$$

where

$$\theta = 4\pi fT_s .$$

The expression for the mean square error can then be written as

$$\overline{\varepsilon_x^{\;2}} = \frac{(A_1A_2)^2}{N} - \frac{3a^4B^4}{8N} + \frac{a^4B^4}{8N^2}\left[\frac{\sin(2\pi fNT_s)}{\sin(2\pi fT_s)}\right]^2 . \quad . \quad . \quad . \quad (26)$$

Provided $f \neq 0$, an upper bound to the sinusoidal term of (26) is

$$\left[\frac{\sin(2\pi fNT_s)}{\sin(2\pi fT_s)}\right]^2 < \frac{1}{(2\pi fT_s)^2} ,$$

so that an upper bound to the mean square error of the estimate is

$$\overline{\varepsilon_x^{\;2}} < \frac{(A_1A_2)^2}{N} + \frac{a^4B^4}{8N}\left[\frac{1}{4\pi^2f^2NT_s^{\;2}} - 3\right] . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

(iii) x(t) is a stationary Gaussian input

When x(t) is a stationary Gaussian input, the fourth order moment of (24) is [3]

$$E[x^2(t+nT_s)x^2(t+mT_s)] = R_x{}^2(0) + 2R_x{}^2((n-m)T_s),$$

and the double summation in the expression for $E[\phi_x{}^2(0)]$ is equivalent to a single summation given by[4]

$$\sum_{n=0}^{N-1}\sum_{\substack{m=0\\ n\neq m}}^{N-1} E[x^2(t+nT_s)x^2(t+mT_s)] = N(N-1)R_x{}^2(0) + 4\sum_{n=1}^{N-1}(N-n)R_x{}^2(nT_s),$$

and the expression for the mean square error reduces to

$$\overline{\varepsilon_x{}^2} = \frac{(A_1A_2)^2}{N} - \frac{R_x{}^2(0)}{N} + \frac{4}{N^2}\sum_{n=1}^{N-1}(N-n)R_x{}^2(nT_s). \qquad \ldots\ldots\ldots \quad (28)$$

In Section 2.2 an approximate expression for the autocorrelation function of the Arctic Sea ambient noise, after band pass filtering, was found to be

$$R_u(\tau) = \frac{\sigma_u{}^2}{\pi f_1\tau}\sin(\pi f_1\tau)\cos(3\pi f_1\tau),$$

so that

$$R_x(\tau) = \frac{a^2\sigma_u{}^2}{\pi f_1\tau}\sin(\pi f_1\tau)\cos(3\pi f_1\tau) = \frac{\sigma_x{}^2}{\pi f_1\tau}\sin(\pi f_1\tau)\cos(3\pi f_1\tau).$$

The summation in (28) may be expressed as

$$\frac{4}{N^2}\left[\frac{\sigma_x{}^2}{\pi f_1 T_s}\right]^2 \sum_{n=1}^{N-1} \frac{(N-n)}{n^2} \sin^2(\pi f_1 n T_s)\cos^2(3\pi f_1 n T_s)$$

$$= \left[\frac{\sigma_x{}^2}{\pi f_1 N T_s}\right]^2 \sum_{n=1}^{N-1} \frac{(N-n)}{n^2} \left[1-\cos(n\alpha)-\frac{1}{2}\cos(2n\alpha)+\cos(3n\alpha) - \frac{1}{2}\cos(4n\alpha)\right],$$

where

$$\alpha = 2\pi f_1 T_s .$$

To evaluate these sums, the upper limit, N-1, was assumed to approach infinity because N is very large ($N = 9.6 \times 10^6$), except in the case

$$\sum_{n=1}^{N-1} \frac{1}{n} ,$$

which is without bound if the upper limit of the summation index approaches infinity.

These summations can then be handled using the following relationships(6)

$$\sum_{n=1}^{10^6} \frac{1}{n} = 14.39273 ,$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} ,$$

$$\sum_{n=1}^{\infty} \frac{\cos(n\alpha)}{n} = -\ln(2\sin(\frac{\alpha}{2})) ,$$

and

$$\sum_{n=1}^{\infty} \frac{\cos(n\alpha)}{n^2} = \frac{1}{4}(\alpha - \pi)^2 - \frac{\pi^2}{12} ,$$

provided

$$0<\alpha<2\pi .$$

The sum

$$\sum_{n=1}^{N-1} \frac{1}{n} ,$$

is approximated by

$$\sum_{n=1}^{N-1} \frac{1}{n} \approx \sum_{n=1}^{10^6} \frac{1}{n} + \int_{10^6}^{9.6\times10^6} \frac{dn}{n}$$

$$= 14.39273 + \ln(9.6) = 16.7 .$$

Substitution into (28) gives

$$\overline{\varepsilon_x{}^2} = \frac{(A_1A_2)^2}{N} - \frac{\sigma_x{}^4}{N} + \sigma_x{}^4\left[\frac{(1 - 2f_1T_s)}{f_1NT_s}\right]$$

$$- \frac{\sigma_x{}^4}{(\pi f_1NT_s)^2}\left[17.4 - \frac{1}{2}\ln\left[\frac{\sin^2(\frac{3\alpha}{2})}{\sin^2(\frac{\alpha}{2})\sin\alpha\sin(2\alpha)}\right]\right] .$$

The sinusoidal term in this expression will be infinite when 2α is equal to $k\pi$ where k is an integer. The case when k is equal to zero is excluded as this implies that f_1 is equal to zero. The lowest value of f_1 which does give an infinite result corresponds to k equal to 1, and this value for f_1 is given by

$$2\alpha = 4\pi f_1 T_s = \pi, \text{ and}$$

$$f_1 = \frac{1}{4T_s} = 10^4 \text{ hertz.}$$

In the analysis of the Arctic Sea ambient noise, the largest value of f_1 is equal to 8.0 kilohertz so that the sinusoidal term will never be infinite. Then, neglecting terms divided by N^2, the mean square error of the estimate is

$$\overline{\varepsilon_x{}^2} = \frac{(A_1A_2)^2}{N} + \frac{\sigma_x{}^4}{N}\left[\frac{1}{f_1T_s} - 3\right]. \qquad (29)$$

3.6.2 Constraint of Mean Square Error Estimates

Expressions (25), (27), and (29) give the estimates of the mean square error for the classes of inputs considered as a constant term, $\frac{(A_1A_2)^2}{N}$, plus a term which is proportional to the square of the mean square value of the input. For the case of the level input, as shown, the mean square error can be minimized. The mean square error for a sinusoidal input is also minimized if the gain "a" is such that

$$a^4 = \frac{8}{3B^4}(A_1A_2)^2,$$

and if the frequency can be written as

$$f = \frac{k}{480}, \qquad k = 1,2,3,\ldots,N-1.$$

The mean square error estimate for the Gaussian input cannot, by inspection of (29), be minimized. Because the gain levels are discrete and finite in number, it is more convenient to constrain the mean square error to be some finite, non-zero value. This can be accomplished by choosing "a" such that σ_x is maintained at some fixed value.

3.7 Error Resulting from the Variable Gain Option

Errors encountered in the system because of the variable gain option may be determined from the probability density functions of the random variables, $\{w_i(t)\}$. Using the standard notation of conditional probability [7]

$$p_{w_i}(\gamma|n_i=\beta) = p_x(\gamma+\beta|n_i=\beta) = p_x(\gamma+\beta), \text{ and}$$

$$p_{w_i,n_i}(\gamma,\beta) = p_{w_i}(\gamma|n_i=\beta)p_{n_i}(\beta) = p_x(\gamma+\beta)p_{n_i}(\beta).$$

Then

$$p_{w_i}(\gamma) = \int_{-\infty}^{+\infty} p_{w_i,n_i}(\gamma,\beta)d\beta = \int_{-\infty}^{+\infty} p_x(\gamma+\beta)p_{n_i}(\beta)d\beta$$

$$= \int_{-A_i}^{+A_i} \frac{p_x(\gamma+\beta)}{2A_i}d\beta. \qquad \ldots\ldots\ldots \qquad (30)$$

In Section 1.3 it was mentioned that the Arctic Sea ambient noise often has a Gaussian amplitude distribution so that x(t) has a Gaussian probability density function

$$p_x(\gamma) = \frac{1}{\sigma_x(2\pi)^{1/2}} \exp(-\gamma^2/2\sigma_x^2),$$

and the resulting probability density function of $w_i(t)$ is

$$p_{w_i}(\gamma) = \frac{1}{2A_i\sigma_x(2\pi)^{1/2}} \int_{-A_i}^{+A_i} \exp(-(\gamma+\beta)^2/2\sigma_x^{\ 2})d\beta$$

$$= \frac{1}{2A_i}\left[Q\left(\frac{\gamma-A_i}{\sigma_x}\right) - Q\left(\frac{\gamma+A_i}{\sigma_x}\right)\right], \quad \ldots\ldots\ldots \quad (31)$$

where the well-tabulated "Q" function is defined as

$$Q(z) = \frac{1}{(2\pi)^{1/2}} \int_z^{\infty} \exp(-t^2/2)dt,$$

with

$$Q(0) = \frac{1}{2}, \quad Q(-z) = 1 - Q(z) \quad z \geq 0 .$$

The probability that $|w_i(t)| \geq v$ is given by the integration of (31) and to evaluate this integral, a bound on Q(z) is used[8]

$$Q(z) \leq \frac{1}{2}\exp(-z^2/2),$$

and

$$P[|w_i| \geq v] = \int_{-\infty}^{-v} p_{w_i}(\gamma)d\gamma + \int_{+v}^{+\infty} p_{w_i}(\gamma)d\gamma$$

$$\leq \frac{\sigma_x(2\pi)^{1/2}}{2A_i}\left[Q\left(\frac{v-A_i}{\sigma_x}\right) - Q\left(\frac{v+A_i}{\sigma_x}\right)\right]. \quad \ldots\ldots \quad (32)$$

There are two values of v for which the probability given by (32) is important. The input signals, x(t) and $n_i(t)$, are applied to a differential comparator, the maximum differential input voltage of which is 5.0 volts, corresponding to v = 5.0. In Section 3.3 it was assumed that $|x(t)| \leq A_i$ which is not true when x(t) has a Gaussian probability density function, and because the maximum value of $|w_i(t)|$ is $2A_i$ for exact results of the theorem of Section 3.3, the probability given by (32) for v equal to $2A_i$ should be constrained to be less than some fixed number.

The probability that $|x(t)| \geq A_i$ is

$$P[|x(t)| \geq A_i] = \int_{-\infty}^{-A_i} p_x(\gamma)d\gamma + \int_{+A_i}^{+\infty} p_x(\gamma)d\gamma$$

$$= 2Q(\frac{A_i}{\sigma_x}), \quad \ldots\ldots\ldots \quad (33)$$

and in the polarity coincidence statistical wattmeter, the maximum value of $\sigma_x{}^2$ is given by (21) with U_N equal to N, i.e.

$$\sigma^2_{xmax} = A_1A_2.$$

In order to constrain the probabilities as given by (32) and (33), σ_x is required to lie within a fixed range, or at a fixed value. The probability that $|x(t)| \geq A_i$ can be determined by the proposed system, and is directly related to the count, U_{N1}, that appears after a time interval T_1, when $n_i(t)$ is equal to $(-1)^i A_i$, $i=1,2$. During T_1, a count is made for each occurrence of $|x(t)| \geq A_i$ and

$$P[|x(t)| \geq A_i] = \frac{U_{N1}}{N1} = \frac{T_s}{T_1} U_{N1}. \quad \ldots\ldots\ldots \quad (34)$$

3.8 Required Gain Levels and Gain Selection Methods

There are several methods for determining which level of gain is to be used during the analysis of the Arctic Sea ambient noise, two of which are discussed in this section. The choice of which method to use ultimately depends upon the accuracy required of the results and upon instrumentation limitations (e.g., minimum power consumption). In the two cases considered here, use is made of an adaptive time interval T_1 during which the level of gain required is decided, followed by an analysis time interval T; over the total time interval $T_1 + T$ the input is assumed stationary.

3.8.1 Required Gain Levels

The levels of gain that must be available are determined by the following criteria: x(t) is assumed to be a Gaussian random variable and the voltage levels A_1 and A_2 are equal. The probability that $|x(t)| \geq A$ is given by (33) so that for n levels of gain and the requirement that there be at least W_m and at most W_M percentage of clipping, σ_x must satisfy

$$W_m \leq 200Q(\frac{A}{\sigma_x}) \leq W_M . \qquad (35)$$

Each level of gain is such that, for a range of σ_u, where σ_u is equal to $\frac{\sigma_x}{a}$, this inequality is satisfied. Taking the inverse of the "Q" function gives

$$\sigma_{xmin} \leq \sigma_x \leq \sigma_{xmax} ,$$

where

$$\sigma_{xmin} = \frac{A}{Q^{-1}(\frac{W_m}{200})} , \qquad (36)$$

and

$$\sigma_{xmax} = \frac{A}{Q^{-1}(\frac{W_M}{200})} . \qquad (37)$$

The maximum and minimum "effective" gain levels are

$$a_{max} = \frac{\sigma_{xmax}}{\sigma_{umin}},$$

$$a_{min} = \frac{\sigma_{xmin}}{\sigma_{umax}}.$$

These are "effective" gain levels because they correspond to W_m % clipping for σ_{umin} and W_M % clipping for σ_{umax}. For n levels of gain, σ_u is divided into n logarithmically equal intervals and the gain at the geometric mean of the k^{th} such interval is

$$a_k = a_{max}\left[\frac{a_{min}}{a_{max}}\right]^{\frac{k}{n+1}} \qquad k = 1,2,\ldots,n .$$

By this division into n such intervals

$$\sigma_x = (\sigma_{xmin}\sigma_{xmax})^{1/2}, \qquad \ldots\ldots\ldots\ldots (38)$$

$$\sigma_{xmin} = \sigma_{xmax}\left[\frac{a_{min}}{a_{max}}\right]^{\frac{1}{n+1}} = \sigma_{xmax}\left[\frac{\sigma_{umin}}{\sigma_{umax}}\right]^{\frac{1}{n}}, \qquad \ldots (39)$$

and

$$a_k = a_1\left[\frac{\sigma_{umin}}{\sigma_{umax}}\right]^{\frac{k-1}{n}} = a_{k-1}\left[\frac{\sigma_{umin}}{\sigma_{umax}}\right]^{\frac{1}{n}}, \qquad \ldots\ldots\ldots (40)$$

where

$$a_1 = \frac{\sigma_{xmax}}{\sigma_{umin}}\left[\frac{\sigma_{umin}}{\sigma_{umax}}\right]^{\frac{1}{n}}.$$

3.8.2 Gain Selection Method 1

If the maximum percentage of clipping is W_M % and if during T_1 a count is made for every occurrence of $|x(t)| \geq A$ then

$$100\frac{T_s}{T_1}U_{N1} \leq W_M,$$

so that the normalized count must be less than $\frac{W_M}{100}$. By setting the gain to the maximum level, a_1, and observing U_N, then if at any time T_1' during T_1 the count reaches

$$\frac{T_1' W_M}{100 T_s}$$

the gain is reduced to a_2 and the process continued. After the time T_1 has elapsed, a gain level will have been reached such that the normalized count is always less than $\frac{W_M}{100}$. In practice, it is not the count normalized to T_1' but the count normalized to T_1 that is constrained. The inaccuracy so introduced will have little effect if n is small as each gain level will cover a wide range of σ_u.

3.83 Gain Selection Method 2

Throughout the time interval T_1 a nominal gain, a_{nom}, is used to determine an estimate of $\sigma_u{}^2$

$$\hat{\sigma}_u{}^2 = \frac{A^2}{a_{nom}^2}\left[\frac{2U_{N1}}{N1} - 1\right],$$

and the k^{th} gain level is used if $\hat{\sigma}_u$ lies in the range

$$\sigma_{umin}\left[\frac{\sigma_{umax}}{\sigma_{umin}}\right]^{\frac{k-1}{n}} \le \hat{\sigma}_u < \sigma_{umin}\left[\frac{\sigma_{umax}}{\sigma_{umin}}\right]^{\frac{k}{n}},$$

or if U_{N1} lies in the range

$$\frac{N1}{2}\left[1 + \frac{a_{nom}^2}{A^2}\sigma_{umin}{}^2\left[\frac{\sigma_{umax}}{\sigma_{umin}}\right]^{\frac{2k-2}{n}}\right] \le U_{N1} < \frac{N1}{2}\left[1 + \frac{a_{nom}^2}{A^2}\sigma_{umin}{}^2\left[\frac{\sigma_{umax}}{\sigma_{umin}}\right]^{\frac{2k}{n}}\right].$$

. . . . (41)

3.9 Prototype System Parameters

In the prototype system, four levels of gain were available and the first method of gain selection was used. In this instrument a maximum of 5 % clipping of x(t) was allowed and A was chosen to be equal to 2.0 volts. Corresponding to these parameters are: the maximum root mean square value of x(t)

$$\sigma_{xmax} = 1.02 \text{ volts},$$

the minimum root mean square value of x(t)

$$\sigma_{xmin} = 0.118 \text{ volts},$$

and the geometric mean, root mean square value of x(t)

$$\sigma_{x} = 0.348 \text{ volts}.$$

These values result in consecutive gain levels

$$a_k = a_{k-1}\left[\frac{\sigma_{umin}}{\sigma_{umax}}\right]^{\frac{1}{n}} = 0.115a_{k-1}.$$

4. INSTRUMENTATION OF PROTOTYPE SYSTEM

4.1 Basic System

A prototype instrument has been constructed to enable the method of power spectrum analysis using polarity coincidence techniques to be verified as being suitable for Arctic Sea ambient noise. The output of this instrument must be linear over a root mean square input signal range of 40 dB.

It was assumed that the input was band-limited so that the instrumentation of the octave band pass filters was not considered. This resulted in a system composed of five main units:

(i) timing circuits,

(ii) random noise generators,

(iii) variable gain amplifier,

(iv) quantizers and logical multiplication, and

(v) system output.

Each period of analysis consists of a 58-second adaptive interval, used to determine the amplifier gain desired, followed by 4 minutes spent measuring the mean square value of the system input. At the start of the adaptive interval, the gain is set to maximum and correspondingly reduced by one step whenever the number of samples for which $|x(t)| \geq A$ reaches a predetermined number.

4.2 Timing Circuits

In order to set the system in the appropriate state at the appropriate time five control orders needed are produced as voltage levels appearing when required at the various output terminals of the timing circuit. This circuit consists of two binary counters, of six and three bits each, counting seconds and minutes respectively. Initially all flip-flops in these counters are set to the one state (positive logic) which corresponds to a time of 7 minutes and 63 seconds. Thereafter the six-bit counter is toggled every second and upon the occurrence of 60 such toggle pulses the six-bit counter flip-flops are reset to the zero state which in turn toggles the three-bit counter. The control orders are

TIME MINUTES	TIME SECONDS	CONTROL ORDER
07	63	$a = a_1$
		$n_1 = -A$
		$n_2 = +A$
		Commence sampling
00	00	Clear output counter
		Allow gain determination to be made
		Record gain
00	58	$n_1 = n_1(t)$
		$n_2 = n_2(t)$
00	59	Clear output counter
04	59	Stop sampling

At the start of the analysis, the resetting of the control order circuit so that all flip-flops are in the one state sets the amplifier gain to the maximum level, the random noise generators to produce voltage levels of +A and -A, and starts sampling the LOGIC MULTIPLY output. One second later, the output counter is cleared and over the next 58 seconds the gain desired is determined by requiring that, at the selected gain, the output counter will contain a count of less than 2^{17} (corresponding to approximately 5 % clipping at a sampling interval of 25×10^{-6} seconds and a sampling time of 58 seconds). Upon completion of the adaptive time, the random noise generators switch from level outputs to time-varying outputs, and at second 59, the output counter is cleared and the four minute analysis of the input signal commences. The output is read at a time of 4 minutes and 59 seconds when sampling of the LOGIC MULTIPLY output has been terminated. The next analysis commences when the control order circuit flip-flops are reset to the one state, this being a manual operation.

Control order voltage levels are led from the control order circuit to either JK flip-flops or to electronic switching circuits and then to other main units of the system. A block diagram of the prototype is shown in Figure 6.

4.3 Random Noise Generators

If a deterministic triangular wave, with maximum and minimum of +A and -A respectively, is sampled at a frequency which is not commensurate with that of the deterministic waveform, the resulting signal will be pseudo-random with a uniform probability density function over [-A,+A]. Such a conjecture has been extensively investigated by Warman(9), where it was shown that if the frequencies of the two deterministic waveforms were related by a relatively prime integer ratio and separated from frequencies of interest to the analysis, the error introduced by replacing the random noise generators with pseudo-random noise generators is minimal.

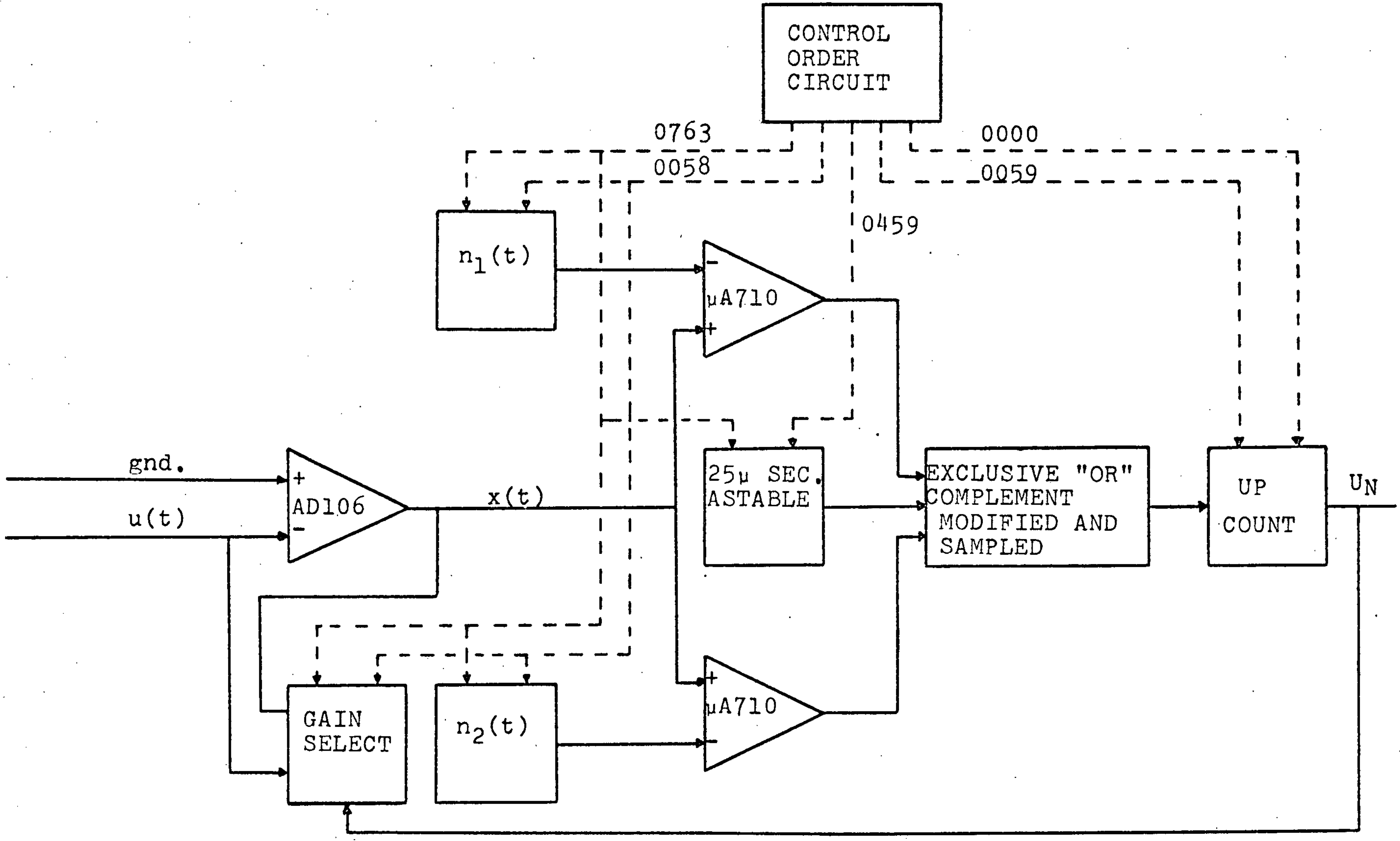


Figure 6. Prototype System

In the present application, because averages are computed over reasonably large pass bands and other approximations are made, it is considered that the error introduced by the use of pseudo-random noise generators will be negligible. The triangular wave generator proposed by Warman has, with modifications, been used to produce a triangular wave, $n_i(t)$, which varies between -2 and +2 volts.

Two switches, each using two field-effect transistors and with inputs $n_i(t)$ and $(-1)^i A$, were used to provide an input to each quantizer. The reset (0763) and 0058 control orders were used to drive these switches. The other quantizer input (common for both quantizers) was the variable gain amplifier output.

4.4 Variable Gain Amplifier

The variable gain amplifier was constructed using an "Analog Devices Model 106" operational amplifier employed in a scale changing configuration. Four levels of gain were produced by resistor switching in the feedback loop. When a particular count in the output counter was reached, a two-bit counter was toggled and the gain level determined by the combined states of the two flip-flops. These states were made available visually, as well as used to drive the field-effect transistor switches of the feedback resistors.

4.5 Quantizers and Logical Multiplication

The amplified signal, x(t), and the deterministic triangular waveforms $n_i(t)$ were compared using "Fairchild μA710" differential comparators, the output being a representation of the sign of $x(t) - n_i(t)$. As mentioned in Section 3.4 the LOGIC MULTIPLY realization is an "exclusive OR complement" and to prevent the output counter from being incorrectly incremented, a modification to the "exclusive OR complement" circuit was made. The "exclusive OR complement" and 25 microsecond clock pulse were combined in such a manner so that during the clock pulse, the "modified exclusive OR complement" was maintained at the level of the "exclusive OR complement" at the beginning of the clock pulse. The positive-going edge of the clock pulse was used to trigger a monostable designed to have a pulse width of 0.25 microseconds, and using an AND configuration, was multiplied with the "modified exclusive OR complement". This realization is shown in Figure 7 with typical circuit waveforms shown in Figure 8.

4.6 System Output

The output counter consisted of twenty-four bits, and because timing problems would not be encountered, was constructed as a ripple counter of which any particular flip-flop would change its state if and only if the preceding flip-flop had undergone a high-to-low transition. The toggle waveform for the first flip-flop was the negative-going edge of the 0.25 microsecond monostable and, as explained in Section 4.5, occurred only if the "exclusive OR complement" had been in the high state at the start of the clock pulse. The counter output, U_N, was read visually in binary form from the counter flip-flop states but is suitable for recording on magnetic tape as required in the D.R.E.P. system.

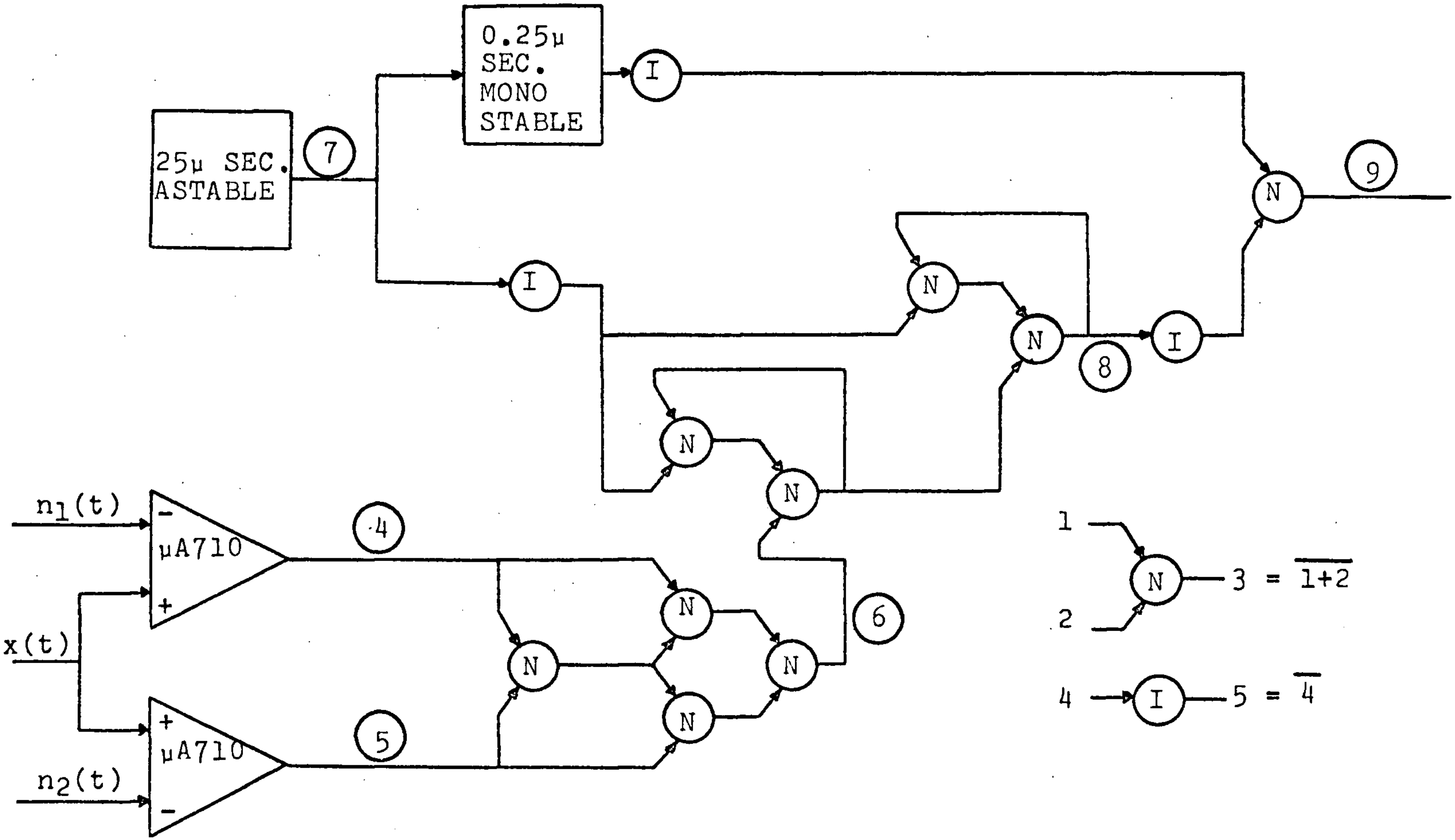
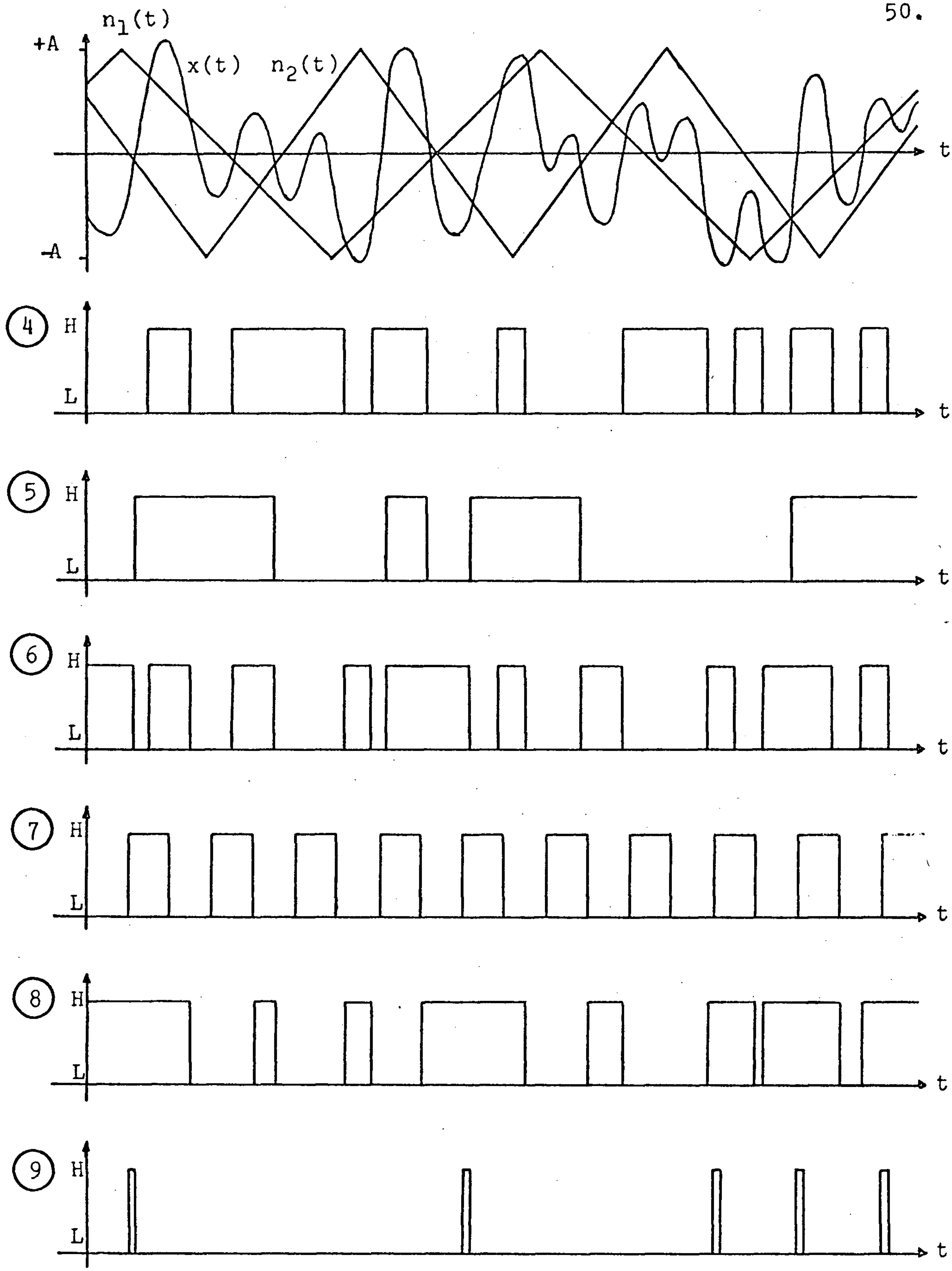


Figure 7. Realization of Modified Exclusive OR Complement (Encircled Numbers Refer to Figure 8)



<u>Figure 8.</u> Typical Circuit Waveforms

4.7 Prototype Operation Modes

To provide various modes of testing for the prototype system, several redundant stages were incorporated. Power supplies used were those available "on the shelf" and there was no attempt to minimize power consumption (this is a requirement in the D.R.E.P. specifications). A set of seven mode control switches enable system operation as follows.

BUTTON/SWITCH		OPERATION
1		Resets entire system to start of adaptive mode
2	ON	1 second pulse is applied to timing circuit
	OFF	Timing circuit is stopped
3	ON	Allows automatic gain changing during T_1
	OFF	Allows manual gain changing during T_1
4		Changes gain one step when 3 is OFF
5	ON	Counter toggled as per normal operation
	OFF	Counter toggled by clock pulse
6	ON	Counter toggled as per 5
	OFF	Counter not toggled
7	ON	x(t) is amplifier output
	OFF	x(t) is held at ground.

5. PROTOTYPE TESTS

5.1 Test Procedure

To determine the region of operation of the prototype system for which the output was linearly related to the mean square value of the input, sinusoidal signals of a large amplitude and frequency range were used. These inputs were applied to the system described in Chapter 4, with the use of the circuit shown in Figure 9.

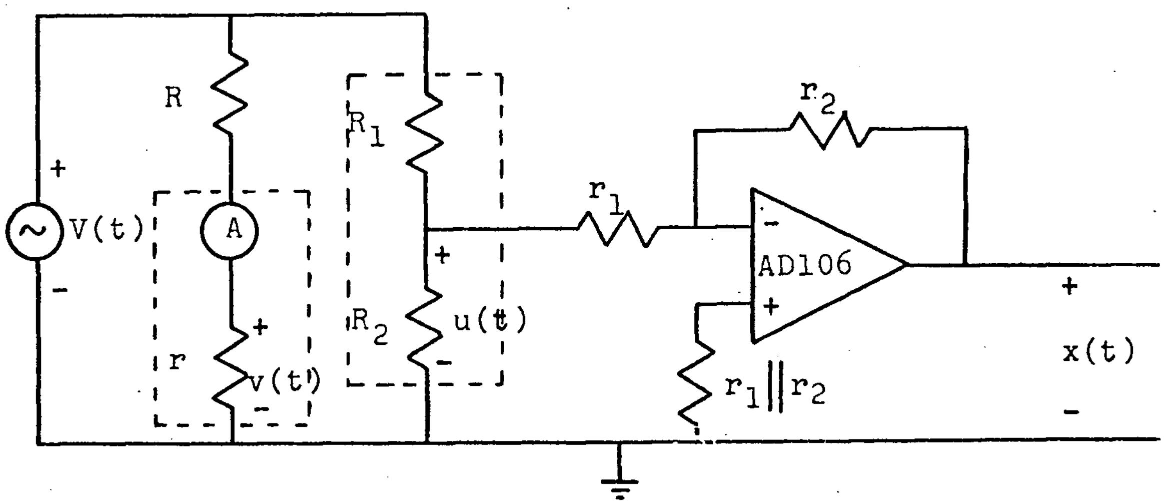


Figure 9. Prototype Input Signal Circuit

A "Unipivot Thermal Milliammeter" (A in Figure 9) was used to measure the root mean square current, I, through the milliammeter internal resistor, r. The R_1 and R_2 resistor combination was a "General Radio Voltage Divider" with a tolerance of ±0.05 %. The input signal V(t) was a fixed amplitude sine wave

$$V(t) = B\sin(2\pi ft),$$

and the value of B was calculated from the value of I as read from the thermal milliammeter. The input to the prototype, as seen from Figure 9, was

$$u(t) = \frac{V(t)r_1R_2}{r_1R_2+R_1(r_1+R_2)},$$

and in the G.R. voltage divider

$$R_1 + R_2 = 10^4 \text{ ohms, and}$$

$$\frac{R_2}{R_1+R_2} = m,$$

where m is the G.R. box coefficient and is variable. Using the thermal milliammeter current I

$$B^2 = 2(R+r)^2I^2,$$

so that the mean square value of the input, u(t), is

$$\overline{u^2(t)} = (IR+Ir)^2\left[\frac{r_1m}{r_1+m(1-m)10^4}\right]^2. \qquad \dots\dots\dots\dots (42)$$

5.2 Test Equipment Calibration

5.2.1 Thermal Milliammeter

Because the value of r was not known exactly, it was necessary to calibrate the thermal milliammeter to enable its use as an accurate root mean square voltmeter. This calibration was made with the use of a "John Fluke Differential D.C. Voltmeter" and led to relationships

$$Ir = (\overline{v^2(t)})^{1/2} = \begin{cases} (1.6327)I-0.017143 \text{ volts.} & I<0.780 \text{ ma.} \\ (1.7381)I-0.098095 \text{ volts.} & I\geq 0.780 \text{ ma.} \end{cases}$$

$$\dots\dots\dots\dots (43)$$

5.2.2 Prototype System

In the generation of the deterministic triangular waves, symmetry about zero volts may not have been achieved. In addition, the operational amplifier, used to produce x(t) from u(t), may have had a d.c. offset voltage. These deviations from the desired comparator inputs have the effect of altering the input-output relationship of the statistical wattmeter, as given by (21), to a relationship

$$\overline{x^2(t)} = \frac{(2A_1-\delta_1)(2A_2-\delta_2)(2U_N-N)}{4N} + \frac{(\delta_1+\delta_2)}{2}\overline{x(t)} - \frac{\delta_1\delta_2}{4}, \qquad \text{.......... (44)}$$

where

$$x(t) = -au(t) + b,$$

and "b" is the offset voltage of the operational amplifier when gain level "a" is used. In (44), the deterministic triangular waves are bounded by $[-(A_1-\delta_1), A_1]$ and $[-(A_2-\delta_2), A_2]$. Because u(t) is a zero mean input, the mean square value of u(t) is

$$\overline{u^2(t)} = \frac{1}{a^2}\left[\frac{(2A_1-\delta_1)(2A_2-\delta_2)(2U_N-N)}{4N} + \frac{(\delta_1+\delta_2)}{2}b - \frac{\delta_1\delta_2}{4} - b^2\right]. \qquad \text{.......... (45)}$$

To calibrate the prototype, u(t) was made equal to zero volts and the values of U_N and N recorded, so that

$$\frac{(\delta_1+\delta_2)}{2}b - \frac{\delta_1\delta_2}{4} - b^2 = -\frac{(2A_1-\delta_1)(2A_2-\delta_2)(2U(0)-N(0))}{4N(0)}. \qquad \text{.......... (46)}$$

The resulting input-output relationship of the statistical wattmeter is

$$\overline{u^2(t)} = \frac{(2A_1-\delta_1)(2A_2-\delta_2)}{2a^2}\left[\frac{U_N}{N} - \frac{U(0)}{N(0)}\right]. \quad \ldots\ldots \quad (47)$$

In the ideal wattmeter, the ratio of U(0) to N(0) will be $\frac{1}{2}$ and, by choosing b, or either of δ_1 or δ_2, such that

$$b = \frac{\delta_1}{2}, \qquad \text{or} \qquad b = \frac{\delta_2}{2},$$

then the left hand side of (46) is identically zero and the ideal ratio has been achieved. However, x(t) must satisfy

$$-(A_1-\delta_1),-(A_2-\delta_2) \leq x(t) = -au(t) + b \leq A_1,A_2$$

for the use of the theorem of Section 3.3 to be valid.

5.3 Test Results

Seven series of tests were conducted, six using sinusoidal inputs of the centre frequencies as listed in Table 1, and one using a d.c. input. The results of these tests have been plotted, using logarithmic scales, and are shown in Figures 10 through 16. In each of these figures, three plots of the ratio $\frac{C_N}{N}$ were made as functions of the actual value of $\overline{u^2(t)}$ which correspond to :

Curve 1 $\quad \frac{C_N}{N} = 2\left[\frac{U_N}{N} - \frac{1}{2}\right]$,

Curve 2 $\quad \frac{C_N}{N} = 2\left[\frac{U_N}{N} - \frac{U(0)}{N(0)}\right]$, and

Curve 3 $\quad \overline{u^2(t)} = A_1A_2\frac{C_N}{N}$.

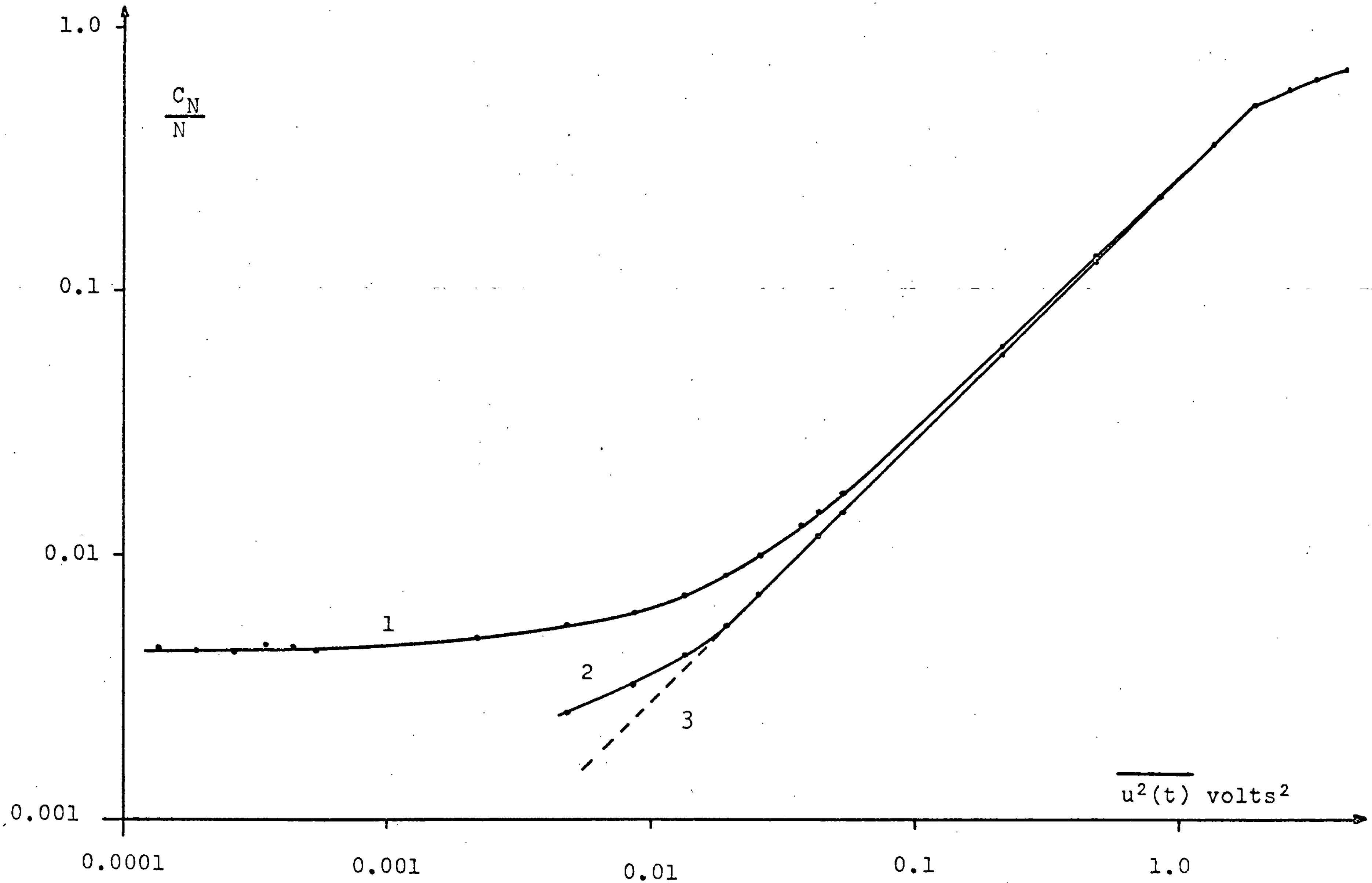


Figure 10. Sinusoid Test Results f = 15.0 Hz.

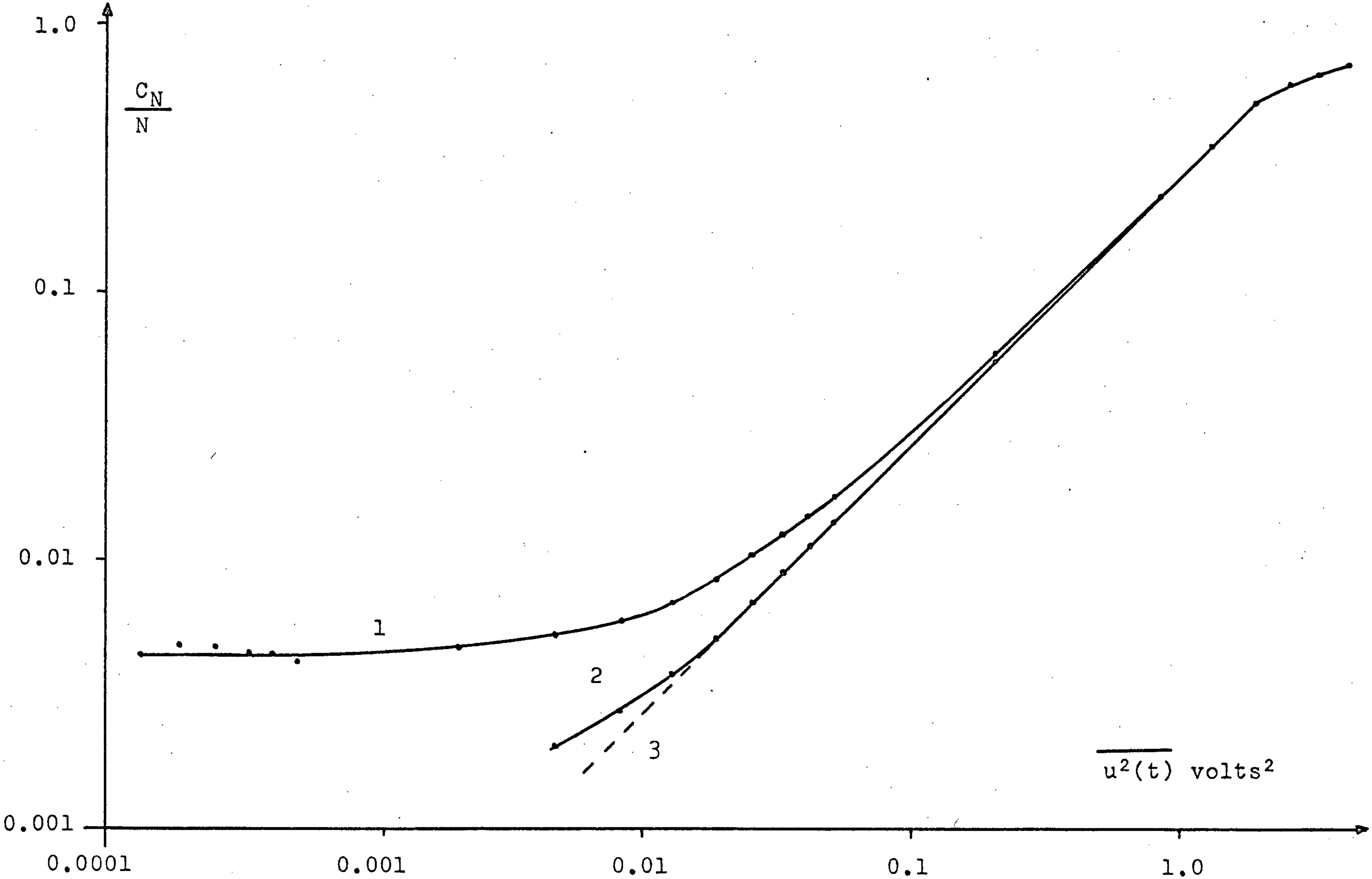


Figure 11. Sinusoid Test Results f = 60.0 Hz.

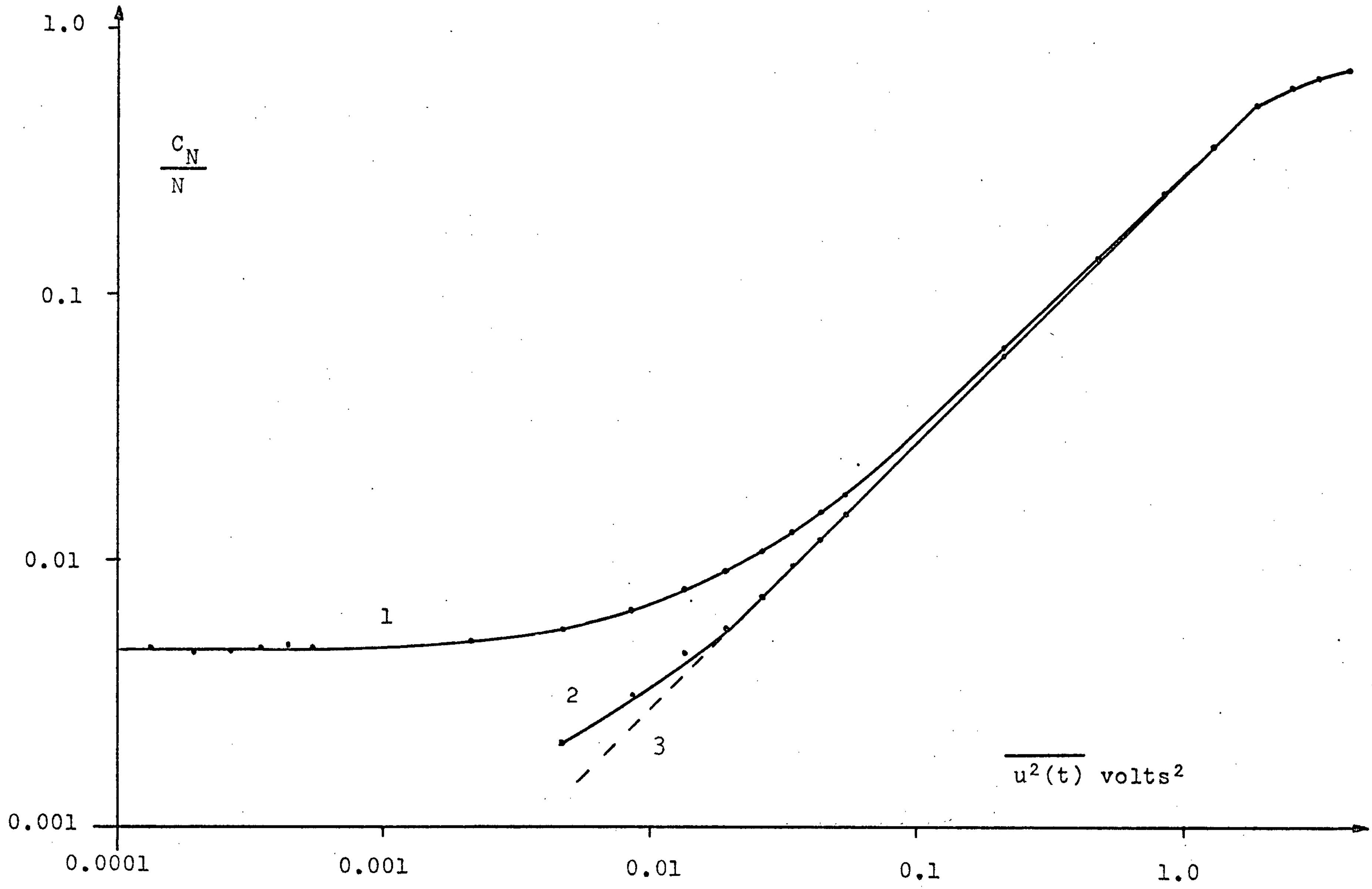


Figure 12. Sinusoid Test Results f = 225.0 Hz.

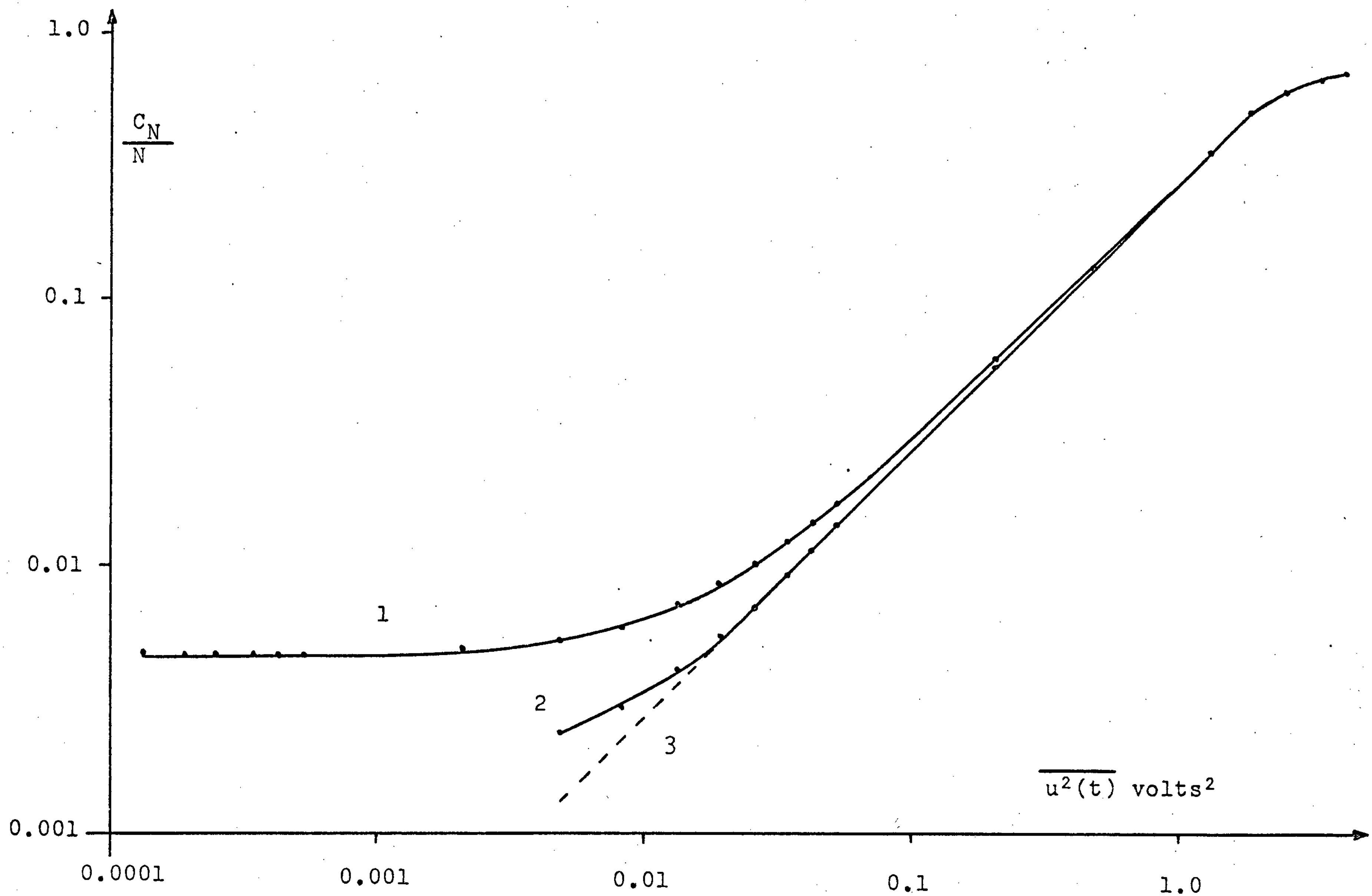


Figure 13. Sinusoid Test Results f = 750.0 Hz.

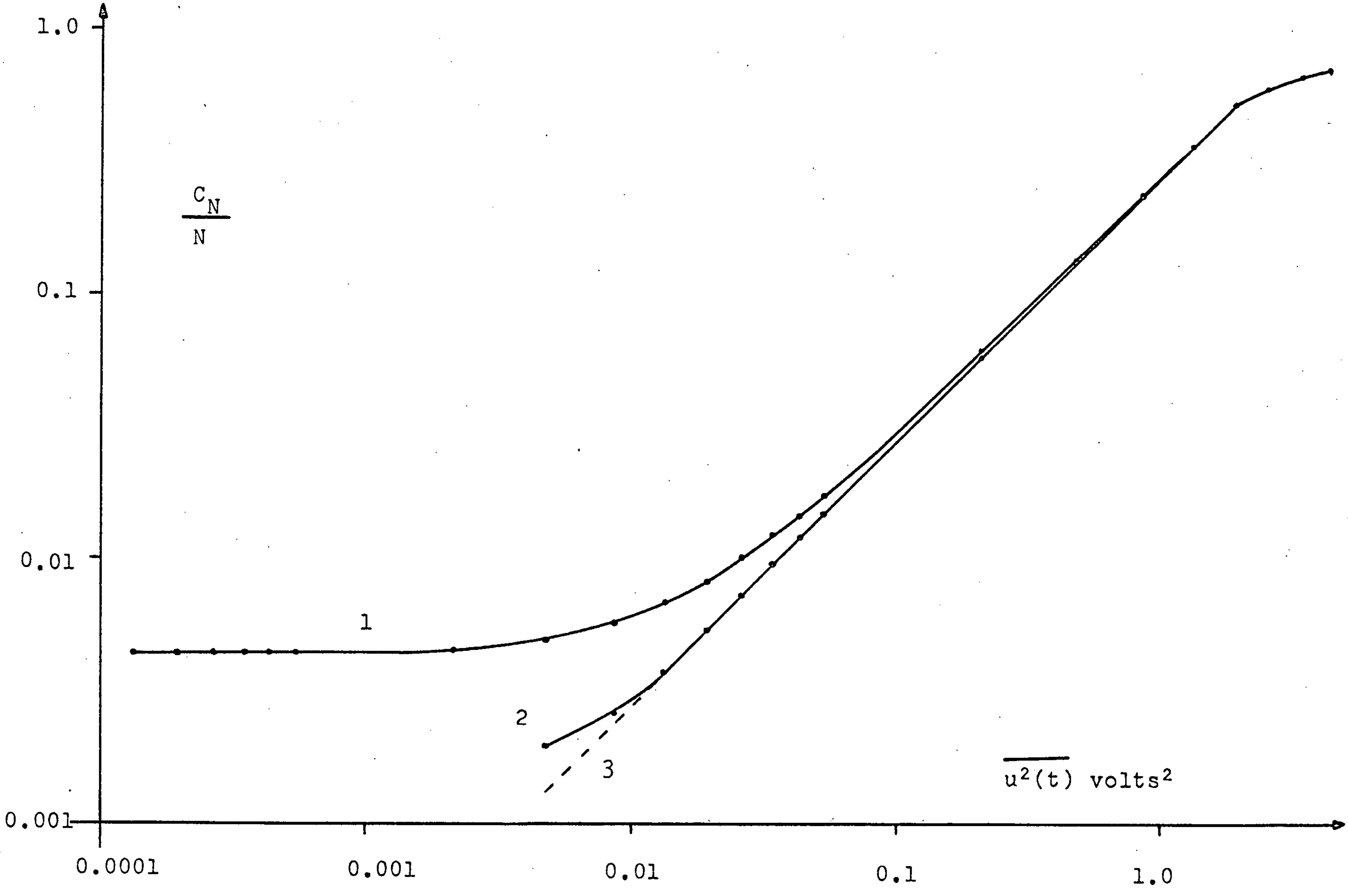


Figure 14. Sinusoid Test Results f = 3.0 kHz.

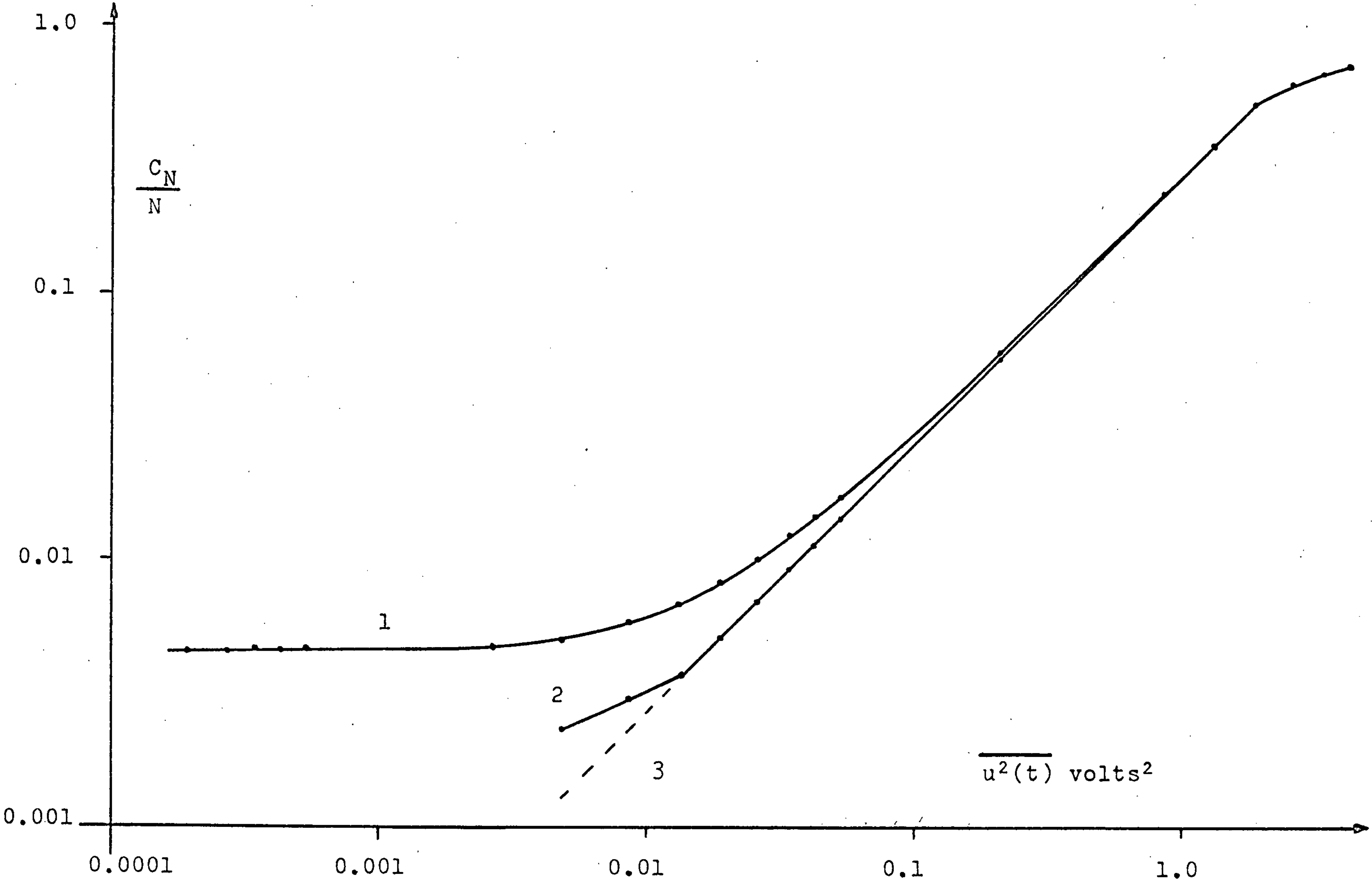


Figure 15. Sinusoid Test Results f = 12.0 kHz.

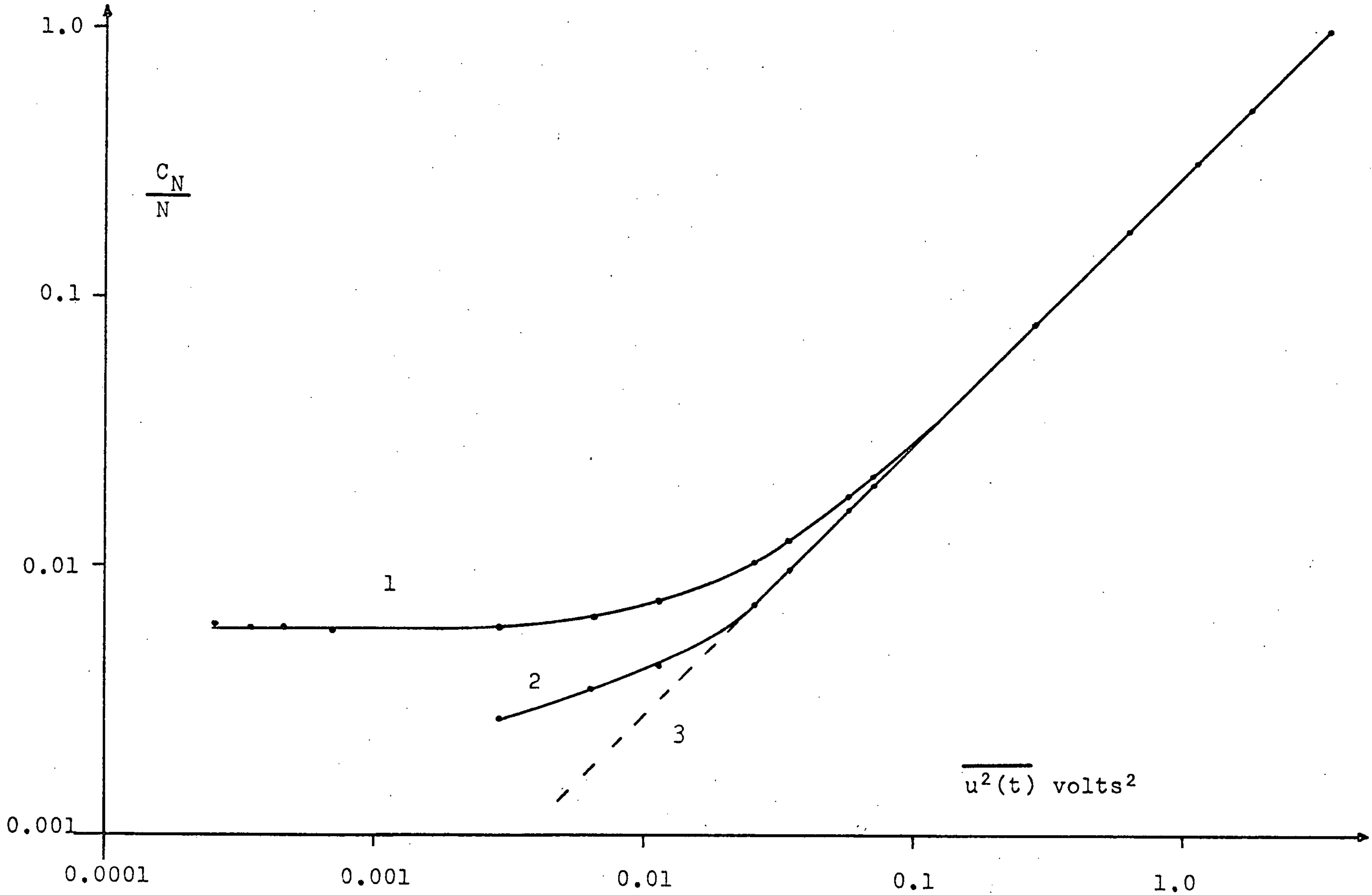


Figure 16. D.C. Test Results

From the results shown graphically in Figures 10 through 16, there does not appear to be any input signal frequency dependence upon the system output. The range of linearity of the input-output relationship is seen to be over a mean square input of 0.01 to 2.0 volts^2, which is less than the desired range of 40 dB. Because the maximum input signal was restricted by the maximum differential voltage of the comparators, the minimum mean square signal to provide the desired 100:1 amplitude range was comparable to the system noise. The calculated average of the ratio $\frac{U(0)}{N(0)}$ was 0.5015, and for mean square inputs between 0.0001 and 0.01 volts^2, the ratio $\frac{U_N}{N}$ was also of this magnitude, so that the data shown in these regions are unreliable. The non-linearity at the upper range of input signals is a result of excessive clipping of the input over the reference levels. The percentage of clipping varies up to 50 %. As indicated by curves 3, the slope of curves 2 should be equal to 1.0. The deviation in the slope occurred in the lower region of the linear input-output relationship and it was felt that this deviation was caused by the use of an average value for $\frac{U(0)}{N(0)}$ which was of the same order of magnitude as the ratio of $\frac{U_N}{N}$. If, in the place of the "Fairchild μA710" comparators, comparators with a larger maximum differential input voltage were used, it is felt that the input-output relationship would be linear over the desired range, thereby making this statistical method of analysis suitable for Arctic Sea ambient noise.

6. SUMMARY AND CONCLUSIONS

A polarity coincidence statistical wattmeter has been proposed whereby the Arctic Sea ambient noise power spectral density could be measured while no assumptions need be made concerning the statistics of the noise. By variation of the input mean square value, using a variable gain amplifier, a range, somewhat greater than 20 dB, of input signals can be analyzed. By such a variation of gain, the analysis of the input noise is always over the same range of input mean square value and is over a region where system operation is optimal. The gain levels required have been calculated and two methods of gain level determination proposed. A prototype wattmeter, constructed to determine the range of linearity of the input-output relationship, has shown that the range of linearity was not as large as desired, but it is expected that, by appropriate instrumentation changes, the desired range could be achieved.

In order to produce a system capable of replacing the R.I. packages (discussed in Chapter 1) as used in the analysis of Arctic Sea ambient noise, the two major instrumentation changes are (i) extension of the range of linearity of the input-output relationship by methods as outlined in Chapter 5, and (ii) reduction in instrument power consumption. It is felt that, with the advent of low power integrated circuit configurations, this latter change could be made such that the D.R.E.P. system requirements are met.

The statistical errors of the proposed system have been formulated for three classes of input signals. The prototype instrument was calibrated, using one of these classes, to eliminate the effect of d.c. voltages which may have appeared as a result of the amplifier offset or as a result of pseudo-random noise generators not having a symmetric probability density function. It had been hoped that actual Arctic Sea ambient noise records would be available to enable comparison of the prototype output to the mean square value as measured using some independent instrument. This comparison has not been made so that the adaptive feature has not been tested with random inputs, and errors other than the non-linearity of the input-output relationship in the prototype tests, have not been determined.

In conclusion, the statistical wattmeter proposed for the spectral analysis of the Arctic Sea ambient noise would require modifications to meet the requirements of the Defence Research Establishment Pacific. At a fixed gain, the wattmeter does enable analysis independent of the class of input but for a dynamic range of the input rather less than the desired 40 dB. By including several gain levels the total dynamic range of 75 dB of the input should be achieved. The output of the proposed system is a binary count, and together with the variable gain, is available for recording on magnetic tape and further analysis.

To ensure the suitability of the proposed system for Arctic Sea ambient noise analysis, further testing of the prototype, with the modifications as noted, must be made with random inputs.

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