THEO SMYRNAEUS

ON

ARITHMETIC

by

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B.A., London University, 1948
B.Ed., University of British Columbia, 1958

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS

in the Department

of

CLASSICS

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

August, 1969
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ABSTRACT

The purpose of this thesis is to examine the arithmetical portion of Theon of Smyrna's work entitled: τὰ κατὰ τὸ μαθηματικὸν χρήσιμα εἰς τὴν Πλάτωνος ἀνάγνωσιν and to determine its significance in the study of the theory of numbers.

The thesis comprises three main parts. The first is a brief introductory discussion of the biography of Theon with an attempt to establish his identity and works. Very little scholarly work has been devoted to Theon; what little could be found was dated for the most part in the second half of the nineteenth century, and in Greek mathematical works his activities have attracted little more than a passing mention. In my introductory chapter I have drawn exclusively from this secondary material. The second part of the study is a literal translation of the appropriate arithmetical section of the work. The third part consists of a commentary amounting to a simple exposition of the mathematical content.

In a concluding chapter I have attempted to assign to Theon his place in the history of arithmetic and have given some indications of the reasons for his relative unimportance.
ABBREVIATIONS

Dupuis : Théon de Smyrne, philosophe Platonicien, Exposition des connaissances mathématiques utiles pour la lecture de Platon, par J. Dupuis.


FW : Pauly-Wissowa, Realencyclopaedie der klassischen Altertumswissenschaft.
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ACKNOWLEDGMENT

I would like to express my thanks to Professor E. Bongie and Professor H.A. Thurston for their helpful suggestions, to Mr H.G. Radunz and Dr G.A. Lieben of Vancouver City College for their aid with German translation, and to Professor J. Russell for his considerate guidance and assistance throughout.
CHAPTER ONE

INTRODUCTION

I CHRONOLOGY AND IDENTITY

We have very little precise information about the date and identity of Theon of Smyrna, Platonic philosopher and astronomer, but some clues may be elicited from certain authorities he mentions in the course of his writings. On five occasions, for instance, he quotes Thrasyllus, the court astronomer of Tiberius, while he also makes extensive use of the Peripatetic philosopher Adrastus in the second portion of his work, quoting him on a variety of points in music and astronomy. On the other hand, his failure to men-

1 Fritz, in PW s.v. "Theon" 14), VA2, 2067, 18ff.

2 Hiller, 47.17, 85.8, 93.8, 205.5. Thrasyllus (d. 36 A.D.) made Tiberius' acquaintance in Rhodes and remained in close contact with him until his death. He wrote two serious works on astronomy, and was also responsible with Dercyllides for the division of Plato's works into tetralogies (PW s.v. "Thrasyllus").

3 Nothing very certain is known about Adrastus of Aphrodisias. He was a Peripatetic of the middle of the second century A.D., a teacher of Lucius Verus, the colleague of Marcus Aurelius in the principate. He is mentioned by Galen together with Aspasius, another Peripatetic dated in the first half of the second century. (PW s.v. "Adrastus").
tion the Almagest of Ptolemy—a strange omission for anyone writing on an astronomical topic after the date of its publication—suggests that Theon's astronomical treatise at least antedates that famous work. This argumentum ex silentio, moreover, appears to find some confirmation in the discovery of a bust of Theon of Smyrna, found in Smyrna bearing the inscription Θέωνα Πλατωνίκον φιλόσοφον ὁ ἱερεὺς Θέων τὸν πατέρα and clearly dated by style to the reign of Hadrian (117 - 138 A.D.). Any attempt to be more specific, however, encounters difficulties.

The first complication arises in Ptolemy's mention of a Theon, whom he terms μαθηματικός in the Almagest in reference to some astronomical observations made in the years 127, 129, 130 and 132 A.D. Theon of Alexandria, in his commentary on Ptolemy, refers to this Theon as τὸν παλαιὸν Θέωνα on

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4 Claudius Ptolemaeus has a floruit between 121 and 151 A.D. His greatest work, the μαθηματική σύνταξις or μεγάλη σύνταξις, termed al Majisti by the Arabs (and hence Almagest) has no firm date.

5 This bust, made out of one piece together with its pedestal, is now kept in the "Hall of Philosophers" in the Capitoline Museum (N.25). For photographs see The Museo Capitolino, (Oxford 1912) Plate 57; Bernouilli, Griech. Ikonographie, II, Plate 29; Schuster, Über die Erhaltenen Portraits griech. Philosophen, Plate 2, 6; Gisela M. A. Richter, The Portraits of the Greeks, (London 1965), III, p.285, fig. 2038.

6 Almagest, IX, 9; X, 1, 2.

7 PF, loc. cit., 2067, 50. Theon of Alexandria lived towards the end of the fourth century A.D. and wrote a commentary on Ptolemy's Syntaxis in eleven books in addition to editing Euclid's Elements (Heath, H.G.M., ii, 526-7).
several occasions, but in one instance he adds the words "the mathematician". This raises the awkward question of identifying the Theons. Is Theon the mathematician to be identified with Theon of Smyrna (perhaps τὸν παλαιὸν θεόν) or are they distinct persons?

Martin, in his edition of the second (astronomical) part of Theon of Smyrna's work, argues against the two Theons having a common identity. He points out that Theon of Smyrna gives the greatest angular distance of Mercury from the sun as $20^\circ$, a figure that agrees with that of Cleomedes, whereas Ptolemy, quoting Theon "the mathematician", gives $26^\circ 15'$, a much more accurate figure. The existence of two different observations each championed by an authority named Theon certainly suggests that the Theons concerned were different persons.

Nonetheless, an alternative explanation may well provide the grounds for considering the two Theons mentioned to be in fact the same person. To postulate a common identity for the Theon of Smyrna and his mathematical namesake one must assume that his preserved work had been written and published before the date of the astronomical observations mentioned by Ptolemy, and so has retained the earlier and less accurate

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9Cleomedes (probably ca. 150 - 200 A.D.), astronomer, was an author of the popular work κυκλικὴ θεωρία μετεώρων (De Motu Circulari Corporum Caelestium) largely founded upon Posidonius (O.C.D. s.v. "Cleomedes").

Further grounds for this theory of a common identity may be sought in the internal contradictions within the work itself. The writer indeed, despite his original claims, seems more enthusiastic about the Platonic ideals of education than about the fundamentals of mathematics, a study in which he appears to be too singularly inept and unreliable to be considered a true mathematician. On the other hand, it is generally agreed that the astronomical part of the work is far better than the mathematical part, a quality amply emphasised by the beautifully simple proof he advances to supersede the faulty proof of Adrastus, showing that the epicyclic movement of the inner planets must necessarily be considered eccentric. With this in mind, it is possible to argue that Theon may have undergone a change of interests, beginning from philosophy and ending with exact mathematics and astronomy. Such a change, if continued after the publication of the extant work, would at least account for the dual character of Theon and reconcile the variant astronomical observations.

As for the Theons mentioned by Ptolemy and Theon of Alexandria, one may justifiably suppose in the absence of any more specific distinguishing factors that they all refer to the same person and that that person is Theon of Smyrna.

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11 PW, loc. cit., 2068, 5.
12 PW, loc. cit., 2068, 9.
If this hypothesis is correct it may be useful to consider a more specific date for Theon of Smyrna within the period already proposed. The failure to take account of Ptolemy's astronomical writings in his extant work suggests a date before the earliest possible date of publication for the *Almagest* (no later than 127 A.D.). The more accurate astronomical figures attributed by Ptolemy to Theon "the mathematician" would of necessity date to a later period of Theon of Smyrna's career, perhaps indicating continued activity for a further decade or two. There is, however, one serious obstacle to an early dating for the extant treatise, viz. the frequent references to Adrastus in the work. Unfortunately the little that is known of Adrastus seems to indicate a date in the middle of the second century A.D. Therefore, in the face of this, a *floruit* date for Theon of Smyrna ca. 125 - 150 A.D. is as close as conflicting evidence permits.

II LOST WORKS

There are two works not now extant that have been assigned by tradition to Theon of Smyrna.

1. A commentary on Plato's *Republic*, ὑπομνήματα τῆς πολιτείας, quoted by Theon himself in his *χρήσιμα*, but not mentioned elsewhere.

2. A second work dealing with the order in which one should read Plato's works, discussing their titles, and known only

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Hiller, 146.4
from Arabic tradition.

In the latter work Theon presumes a tetralogical order for Plato's writings, according to a certain an-Nadim,\textsuperscript{14} an eighth century Arabian writer. But the catalogue of writings he lists as Theon's bears little resemblance to the canonical tetralogies given by Thrasyllus.\textsuperscript{15} This may be considered a strange anomaly in view of the strong dependence of Theon upon Thrasyllus elsewhere in his work.\textsuperscript{16} Lippert believes that Theon in his work first listed the dialogues of Plato in a non-Thrasylillian catalogue and subsequently re-ordered them mainly according to Thrasyllus.\textsuperscript{17}

A further possible reference should be mentioned that may have some bearing on one or other of these lost works of Theon. The issue arises from a discussion by Proclus\textsuperscript{18} of Plato's genealogy. In his \textit{Commentary on Plato's Timaeus}\textsuperscript{19}

\begin{itemize}
\item \textsuperscript{14}Abi Ja'kub an-Nadim, \textit{Kitab al-Fihrist}, Leipzig, F.C. W.Vogel, 1871-2, 255, 12.
\item \textsuperscript{15}an-Nadim, \textit{op. cit.}, 246, 4ff.
\item \textsuperscript{16}PW, \textit{loc. cit.}, 2069, 24.
\item \textsuperscript{17}J. Lippert, \textit{Studien auf dem Gebiet der Arabischen Übersetzungs litteratur}, Braunschweig, 1894, 2, 45ff. The slight divergencies from Thrasyllus' ordering are also found in another Greek ordering, that of Diogenes Laertius (\textit{Diog. Laert.}, III, 62.).
\item \textsuperscript{18}Proclus (410-485 A.D.) was a competent and industrious mathematician and teacher of the Neoplatonic school, even a poet. He wrote a famous commentary on Euclid (Book 1) and also commentaries on the \textit{Republic}, the \textit{Timaeus} and other dialogues of Plato. (Heath, \textit{H.G.M.} 11, 529-37)
\item \textsuperscript{19}Proclus, \textit{Comm. on Plato's Timaeus}, 26A.
\end{itemize}
Proclus reports that Theon considered the (older) Glauccon to be the son of Callaischrus and the brother of the famous Critias, while he himself considers him to be a brother of Callaischrus and the son of the older Critias. This information undoubtedly belongs to an extensive genealogy of Plato which, on the side of the father, reaches back to the kings Codrus and Melanthus and, on the side of the mother, to Solon. Hiller assigns the genealogy to Theon's Commentary on the Republic, whereas Lippert considers that it must be assigned to the treatise upon the serial sequence of the Platonic writings. But as no confirmation of this appears in Nadim, sole authority for the latter work attributed to Theon, it is not possible to assign this information with any certainty to any specific work.

III EXTANT WORKS

1) The χρήσιμα of Theon

The sole extant work of Theon of Smyrna is entitled: τὰ κατὰ τὸ μαθηματικὸν χρήσιμα εἰς τὴν Πλάτωνος ἀνάγνωσιν.

This work is a somewhat curious medley, valuable not particularly for its own intrinsic worth as a scientific treatise, but rather for the large number of historical notices it contains chiefly in the second part. The title with its claim

20 Lippert, loc. cit.
to present the mathematics useful for the reading of Plato is indeed a little too pretentious. It was no doubt an elementary introduction or *vade mecum* for students of philosophy, but little in it has particular reference to the mathematical questions posed by Plato.

There is a long introduction pointing out the paramount importance of mathematics in the training of the philosopher and demonstrating the relationship between the five branches of mathematics, arithmetic, geometry, stereometry (or solid geometry), astronomy and music.

Theon promises to present the mathematical theorems in arithmetic, music and geometry and the applications of stereometry and astronomy most necessary for students of Plato, but the promise is by no means fulfilled as regards geometry and stereometry.

The work in effect comprises two separate halves:

Part 1
a) Introduction (Hiller 1.1 - 16.24)
b) Arithmetic (Hiller 16.24 - 46.19)c) Music i) instrumental (Hiller 46.20 - 72.20)
   ii) based on numbers (Hiller 72.21 - 119.21)

Part 2 Astronomy (Hiller 120.1 - 205.6)

ii) Summary of content of Theon's *χρήσιμα*

a) Arithmetic This section comprises broadly what is termed Pythagorean arithmetic. It deals with the classification of numbers into odd and even and their subdivision further into prime numbers, composite numbers with equal and unequal factors, invoking a distinction between those with a
difference of 1, and of 2 and more than 2.

Plane numbers are divided into square, oblong, triangular and polygonal numbers, with their respective "gnomons" and their properties as the sums of successive terms of arithmetical progressions beginning with 1 as the first term. Circular numbers are defined, as are spherical numbers, solid numbers with three factors, pyramidal numbers and truncated pyramidal numbers, perfect numbers and the over-perfect and defective kinds. All this is found in the *Introductio Arithmetica* of Nicomachus of Gerasa (floruit ca. 100 A.D.) where it is treated in still greater detail. Of particular interest however, is Theon's exposition of side and diagonal numbers, a treatment which does not have an equivalent in Nicomachus, for it demonstrates without producing a proof a method for finding an approximation of the value of $\sqrt{2}$ to any required degree of accuracy.

b) Music Theon first deals with instrumental music (\(\muουσική\ \varepsilonν \ οργάνοις\)) i.e., the teaching of the sounds and the intervals in music. The intervals which provide harmony, notes and scales, the tones and semitones and the octave are discussed, and the writer furnishes substantial quotations from Thrasyllus and Adrastus, and in addition refers to the views of Aristoxenus, Hipparus, Archytas, Eudoxus and Plato.

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23 Hiller, 46.20 - 72.20
The second section, music based on numbers, (μουσική ἐν ἀριθμοῖς), develops into a general discussion of ratios, proportions and means. There are further quotations from Plato, followed by a presentation of Thrasyllus' divisio canonis. Although Theon promises to apply this musical analysis later to the "harmony of the universe", he proceeds without apology to a discussion of the δέκα (decad) and the τετρακτύς (quaternary) with its eleven applications and the mystic and curious properties of the numbers from 2 to 10, part of the theologumena of arithmetic. 

Astronomy: The most voluminous part of Theon's work is that dealing with astronomy. Here Theon is mainly dependent on the work of Adrastus whom he quotes regarding the sphericity of the earth. By using Eratosthenes' figure of 252,000 stades for the circumference of the earth and the Archimedean value of 22/7 for π he obtains a figure of 80,182 stades for its diameter.

He then describes the principal astronomical divisions of the heavens and gives the maximum deviations in latitude of the planets and the sun. This is followed by an explanation of the orbits of the sun, moon and planets on the hypothesis of a geocentric universe. He observes that "some of the Pythagoreans" made the order, reckoning outward from the earth,

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24 Hiller, 72.21 - 119.21

to be: the moon, Mercury, Venus, the sun, Mars, Jupiter and Saturn. Now this is the "Chaldaean" order not found in Greece until the second century B.C. so that "some of the Pythagoreans" will refer to the later Pythagoreans, of whom Nicomachus was one. Plato and the earlier Pythagoreans placed the sun next to the moon and interchanged the positions of Venus and Mercury. Then assigning to each of these planets, to the earth, and to the sphere of the fixed stars, one note each he proceeds to arrange the nine to form an octave by comparing the distances separating them to musical intervals. Thus by their rotation he demonstrates that the "symphony of the stars" is produced. According to Heath, the whole of this passage is possibly intended as the promised account of the "harmony of the universe", although at the end of the work Theon implies that he has still to give a brief account of what he and Thrasyllus think of this subject.

Next Theon deals with the movements and apparent stationary points and retrogradations of the five planets, and with the "saving of the phenomena" by the hypotheses of eccentric circles and epicycles. There follows an allusion to the systems of Eudoxus, Callippus, and Aristotle and a descrip-

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27 Hiller, 205.
28 Eudoxus (408-355 B.C.), a mathematician of the highest rank; to Cicero 'in astrologia judicio doctissimorum hominum facile princeps'. He was famous for his ingenious "theory of concentric spheres" and his brilliant use of "the method of exhaustion" to obtain the volume of a pyramid.
tion of a system in which "carrying" spheres, termed "hollow" have "solid" spheres attached to them which, by their motion roll the "carrying" spheres in the opposite direction.\textsuperscript{30} The figures of 20° and 50° given by Theon for the maximum arcs separating Venus and Mercury\textsuperscript{31} agree with Cleomedes' observations.

The final pages, quoted from Adrastus, deal with conjunctions, transits, occultations and eclipses. The work concludes with a considerable extract from Dercyllides,\textsuperscript{32} a Platonist with Pythagorean leanings.

iii) Theon's "harmony of the universe"

The manuscript has come down to us in two separate halves and in view of Theon's vague and at times contradictory statements regarding the scope of the work we may well wonder whether the work as we have it is complete.

Right at the beginning\textsuperscript{33} Theon defines the scope of his work to be a description of arithmetic, music, geometry,

\textsuperscript{29}Callippus of Cyzicus (ca. 370-300 B.C.) was perhaps the greatest astronomer of his time. He corrected and added to Eudoxus' theory of concentric spheres designed to account for the movements of the sun, moon and the planets (Simplicius, \textit{On Arist. De Caelo}, 493, 5-8, Heib.)

\textsuperscript{30}Hiller, 181.12ff.

\textsuperscript{31}Hiller, 187.10-13

\textsuperscript{32}See Note 2. Dercyllides' date is difficult; he wrote a book on Plato's philosophy, according to Heath before Tiberius or perhaps even before Varro (116-27 B.C.), \textit{(H.G.M.} 11, 244)

\textsuperscript{33}Hiller, 1.15
stereometry and music in so far as they are important for the understanding of the Platonic writings, an enterprise which he tries to justify by quoting numerous passages from Plato that emphasise the importance of mathematics in the pursuit of philosophy. Then, as if to limit the scope of his work, he pleads that it is not really his objective to make perfect mathematicians. Plato did not teach that one should waste time upon mathematics until one's old age; rather should mathematics be considered a preparatory training for philosophy. Furthermore, to compound his inconsistency he considers it desirable that his readers should have some elementary knowledge of geometry already, in order to make the understanding of his dissertation the easier; yet his style and exposition are to be such that even a mind completely untrained in mathematics will understand him.34 There would appear to be from the very outset some inconsistency of objectives here.

On specific issues too, Theon may be convicted of contradiction. For instance, he states that he intends to describe arithmetic and that this will be followed only by "music based on numbers"; "instrumental music" need not be presented at all since Plato himself declares this to be superfluous.35 Yet he contradicts himself in this resolve for, at the conclusion of the section on arithmetic he now states in his intro-

34 Hiller, 16.21
35 Hiller, 16.26. Plato preferred theoretical music (see Note 15, p. 74.)
duction to the second part, that he does wish after all to speak about μουσικὴ ἐν ὀργάνοις as well as μουσικὴ ἐν ἀριθμοῖς and at the end will deal with yet a third category, ἀρμονία ἐν κόσμῳ. And while the former two are duly presented as promised, the third, "harmony of the universe", is apparently missing.

Now if we examine his introductory remarks to the section on arithmetic, Theon appears to take great pains to justify his intention of treating "music based on numbers" next after arithmetic, "for in the natural order should have come geometry followed by stereometry and astronomy with music fifth". Then he points out that Plato's music was the "harmony of the universe" and cannot be understood unless one first learns "music based on numbers"; wherefore, he is prepared to take the drastic step of presenting it next after arithmetic. Yet the most cogent reason for "music based on numbers" to follow arithmetic—that it is closely allied to the study of numbers "pure and simple"—is added as an afterthought. The reasoning is altogether vague and unconvincing.

36 Hiller, 47.6
37 Hiller, 17.14
38 The "natural" order presumably according to Plato, who gives this order for the branches of mathematics in Rep., 522C, 526C, 527D, 528B and 530D.
39 Hiller, 17.8
40 Hiller, 17.11
Now despite all this elaborate presentation of excuses for departing from Plato's recognised order of priority in the branches of mathematics, what we actually find after the treatment of arithmetic is a digression on "instrumental music" after all, a topic that Theon had apparently already decided to reject on Plato's authority.\textsuperscript{41} And moreover "music based on numbers", when it eventually does appear, is a confusing complex of quite unrelated topics. There is a section on "means",\textsuperscript{42} followed by one on "proportions";\textsuperscript{43} then, a treatment of the divisio canonis\textsuperscript{44} of Thrasyllus which would surely more fittingly have been presented along with "instrumental music"; next, the subject of "means" is resumed only to be interrupted by two other topics, the δεκάς and the τετρακτύς,\textsuperscript{45} and the treatment of figures (περὶ σχημάτων).\textsuperscript{46} The insertion of these two subjects seriously mars the continuity of the discussion of "means" and could have been incorporated with much greater relevance in other sections of the work. The δεκάς and τετρακτύς, for instance, is related only superficially to music and perhaps would have found a more

\textsuperscript{41}Hiller, 16.26

\textsuperscript{42}Hiller, 22.21ff.

\textsuperscript{43}Hiller, 85.6ff.

\textsuperscript{44}Hiller, 87.4ff.

\textsuperscript{45}Hiller, 93.17ff.

\textsuperscript{46}Hiller, 111.13ff.
natural context in the discussion of the four dimensions
i.e., point, line, surface and solid, and that of the four
regular solids. The same might be said for the positioning
of the περὶ σχημάτων, for it contains a few elementary def-
initions of plane and spatial figures. The first half of
the work concludes with a proof of the "golden mean"—the
only proof in the first half of the work.47

In retrospect we are conscious of a failure to fulfil
the plan proposed at the beginning of the section. In no
sense can Theon’s piecemeal treatment of unrelated topics be
regarded as the promised description of geometry and stereo-
metry. All the more astounding therefore are the words with
which the author concludes Part One of his work:

τὰῦτα μὲν τὰ ἀναγκαίατά τα ἄριστα ἔν τοῖς προ-
είρημενοῖς μαθήμασιν ὡς ἐν κεφαλαίωδει παραδοσεῖ πρὸς τὴν
τῶν Πλατωνίκων ἀνάγνωσιν. λείπεται δὲ μνημονέυσαι
στοιχειώδως τῶν κατ’ ἀστρονομίαν.48

--just as if there were nothing amiss and he were checking off
the list of topics so elaborately enumerated earlier. Also
missing is the "harmony of the universe;" another topic prom-
ised not only in the general introduction,49 but again in the
introduction to music,50 and yet again at the beginning of
the exposition of the τετρακτύς.51

47 Hiller, 117.12
48 Hiller, 119.16
49 Hiller, 17.24
50 Hiller, 47.8ff.
51 Hiller, 93.9
Moreover the treatment of "harmony of the universe" still remains unpresented in Part Two, devoted in its entirety to astronomy, where it could well have been discussed in a suitable context as we learn from the closing words of the treatise:

ἐπεὶ δ' ἐφαμεν εἶναι μουσικὴν καὶ ἀρμονίαν τὴν μὲν ἐν ὀργανοῖς, τὴν δὲ ἐν ἀρίθμοῖς, τὴν δὲ ἐν κόσμῳ, καὶ περὶ τῆς ἐν κόσμῳ τἀναγκαία παντα ἐξής ἐπηγγειλάμεθα μετὰ τὴν περὶ ἀστρολογίας παράδοσιν . . . . . . . .' καὶ περὶ τούτων ἐν κεφαλαίοις παραδείκνυσιν ὁ θεάσυλλος σὺν οἷς καὶ αὐτοὶ προεξειργασμεθα δηλωτέον. 52

From this it would appear that Theon's intention remains unshaken, to finish the work with the "harmony of the universe". Its absence leaves a gap of some importance.

Regarding the missing sections on geometry and stereometry, it is not easy to make a definitive judgment. Tannery 53 and, in partial agreement with him, Heath 54 are of the opinion that the work of Theon was not mutilated at the time of its division into two parts. Tannery believes that the section on triangular numbers, square numbers, pyramidal numbers and the like in the initial part of the work that deals with number theory represents all that Theon intended to offer as geometry and stereometry, for Theon's conception of real geometry is one of a purely abstract science differing from Euclid's geometry of spatial figures. And he supp-

52 Hiller, 204.23ff.
oses that the second section devoted to "music based on numbers" contains several later additions as, for instance, the portion concerning figures (περὶ σχημάτων)\textsuperscript{55} and that on "means" (μεσοτητες);}\textsuperscript{56} these he believes to be Byzantine interpolations.

The passage about the δεκας and the τετρακτυς which rather forcibly interrupts the discussion about "means" is considered by Tannery to be a part out of the section on "harmony of the universe", which he supposes to have formed the last part of the work and from which he believes several portions were removed and replaced in different positions elsewhere in the work. This was Tannery's original explanation of the location of "harmony of the universe" in the work although he himself later abandoned his hypothesis about the origin of the δεκας and the τετρακτυς and declared this portion too to be a Byzantine interpolation.\textsuperscript{57}

However, if we survey the whole work and consider the detailed statements of Theon about the sequence to be followed in his work, it is indeed difficult to believe that the section on polygonal numbers and pyramidal numbers appearing in the section dealing with number theory, as Tannery believes, or the chapter "concerning figures", as Heath supposes, were all that Theon intended to offer in the field of geometry and

\textsuperscript{55}Hiller, 111.13ff.
\textsuperscript{56}Hiller, 113.9 - 119.21
\textsuperscript{57}Tannery, \textit{op. cit.}, 126ff.
stereometry and that, therefore, only the portion dealing with "harmony of the universe" at the end is missing. It is true that Theon cannot have intended to present a treatment of pure geometry and solid geometry after the manner of Euclid, as is shown most clearly by his treatment of the theory of proportions. His presentation would be expected to consist of definitions, explanations and examples and not to comprise rigorous proofs and constructions. Nevertheless, an explanation is still required for the fact that, when bringing to a close his treatment of "means," Theon announces astronomy as the next subject to be discussed instead of geometry and stereometry as his original arrangement required.

But if one considers the nature and content of the sections which interrupt the continuity of the first part, and further if one notes that a section in the second part of the work may best be explained as having some connection with the "harmony of the universe," then an expansion of Tannery's earlier hypothesis seems to offer the most likely explanation of the condition of the work as it has come down to us.

It seems, then, that the work was edited in two halves and that in both cases only parts were published complete, on the one hand, arithmetic and music, and on the other, astro-
nomy, and that sections of the parts omitted were then, either immediately or later, inserted into the published parts in a rather unrelated fashion. Nonetheless, none of these conclusions accounts for the particularly involved statements made by Theon regarding the arrangement of his work, and the question whether the work ever did appear in the final complete form intended by the author certainly remains open.

60 PW, loc. cit., 2074, 57.
CHAPTER TWO

ON ARITHMETIC: TEXT AND APPARATUS

Theon's work has come down to us divided into two parts, which have been derived respectively from two manuscripts, designated A and B. Codex A is an 11th/12th century parchment manuscript (307) of the St. Mark's Library, Venice. Codex B is a 14th/15th century paper manuscript of the same library. All other manuscripts of either both halves of the work or of one complete half are either directly or indirectly dependent upon the above-mentioned manuscripts.

The following are the editions of the work which have been published to date:

1. I. Bullialdus, first part only, Paris, 1644.
2. J. J. de Gelder, first part only, Lyons, 1827.
5. J. Dupuis, both parts, with French translation, Hachette, Paris, 1892.

Edition 4 by Hiller, noted above, contains all the Greek text of what we have of Theon's work and is carefully annotated with the numerous variant readings of the several manuscripts. This text has been used for the translation and the appropriate portion (pp. 1-46) is here presented,
preceded by a summary of Hiller's accompanying notes.

1. All manuscripts have been derived from those designated A and B.

2. To obviate an undue burdening of the notes, manuscripts derived from A and B are given the common designation 'apographi'.

3. The passage 46.20 - 57.6 has an archetype (Z) different from A.

4. Ms. 203 Venet. Marc. used for the correction of the "verses of Alexander" is designated C.

5. < > brackets enclose suggested additions to fill clearly indicated lacunae.

6. [ ] brackets enclose what should be erased.

7. A¹ is used to mark readings in A changed by A².

8. * indicates Martin's corrections of B.

9. Corrections not assigned above are Hiller's own.
"Ότι μὲν οὖν ο tablename; τοιούτοις μαθηματικῶς λεγομένοις παρὰ Πλάτωνος μὴ καὶ αὐτὸν ἱκεχεῖν ἐν τῇ θεωρίᾳ ταύτῃ, παρὰ οὖν ἀπολογήσειν ὡς δὲ οὐδὲ τὰ ἄλλα ἄνωθεν οὐδὲ ἀνάνητος ἢ περὶ ταύτα ἐμπειρία, διὰ πολλῶν αὐτὸς ἐμφανίζει δοκικὴ. τὸ μὲν 5 οὖν συμπάθης γεωμετρίας καὶ συμπάθης μουσικῆς καὶ ἀστρονομίας ἐμπειρον γενόμενον τοῖς Πλάτωνος συγ-
γραφέων μεταχέαν μακροηγίαν μὲν εἰ τῷ γένοιτο, ὥστε μὲν ἐμφανον οὐδὲ ὄρδον ἄλλες πάνυ πολλοῦ του ἐκ παθῶν πάνοιρον δεόμενον. ὅτε δὲ τοὺς δημοφίλης 10 τα τοῦ ἐν τοῖς μαθηματίσιν ἀσχόηθηναι, ἄφησαμένους δὲ τῆς γυναικῶν τῶν συγγραφέων αὐτοῦ μὴ παντελῶν ὅπως ποιοῦσε διαμαρτυρίαν, κεφαλαίῳ καὶ σύντομον ποιησόμεθα τῶν ἀναγκαίων καὶ ὧν δὲ μᾶλλον τοῖς ἐντυπώσαντες Πλάτωνος μαθηματικῶν θεωρημάτων παρὰ- 15 δοσιν, ἀφθινηθηκὸν τε καὶ μουσικὸν καὶ γεωμετρικὸν τῶν τε κατὰ στερεουμετρίαν καὶ ἀστρονομίαν, ὧν χαρίζ

Inscr. Θέωνος Σμαραγδίου Πλατωρικοῦ τῶν κατὰ τὸ (τὸ μῆκος τοῦ) μαθηματικῶν χρησμῶν εἰς τὴν Πλάτωνος ἀναγραφεῖν. 1 Inscr. τοῖς ἀναγραφεῖ τὰ μαθήματα Α 2 ήκεχεῖν: οὖν κατ' αὐτὴν Α' 4 οὐδὲ τὰ αὐτῶν τοῖς Α 5 τὸν κατὰ τὸν Α' Β γεωμετρίας καὶ ἀφθινηθηκὲς add. recentior manus in apogr. fort. recte Β 10 σχοτόν τοῦ βιβλίου;

Theo Smyth. 1
οὐχ οἷόν τε εἶναι, φάσι τυχένι τοῦ εὐρύτοιο βίου, διὰ πολ. λόν πάνω δηλάτας ὡς οὐ χρὴ τῶν μαθημάτων ἐμελείαν.

'Εραστεύεται μὲν γὰρ ἐν τῷ ἐπιστημονικῷ Πλατ.
tονικῷ φήμην ὅτι, Ἀθήναι τοῦ θεοῦ χρήσαντος ἐπὶ σπειλαγγύλου λοιμοῦ βομβοῦ τοῦ ἄστος διπλάσιον μετα.
sκευάζει, πολλὴν ἐφικτέτοιον ἐπικεφαλῆν ἀποφθέγματι ἐπι.
tοτέσσαρα ἐπός χρῆ στερεόν στεφεῖν γενέσθαι διπλάσιον, ἐφαίνειν τε πεισομένους περὶ τούτου Πλάτωνος, τῶν ὃς χάνει αὐτοὺς, ὃς ἀρα ὑπὸ διπλασίον βομβοῦ ὁ θεὸς ἐν θεομένη τούτῳ Ἀθήναις ἐφαινόμενον, προφέρον δὲ καὶ ὀνομαίζον τοῖς Ἐλληνισὶ ἐμελεύσαται μαθημάτων καὶ γεω.

μετρίας ἠλίσθων. ἀκολούθει τὸ τοῦ Πυθίου περιελεύθερον πολλὰ καὶ αὐτὰς διδάσκουσιν ἕλπι τοῦ ἐν τοῖς μαθημάτισι χρησίμου. ἐν ἔν τε γὰρ τῷ Ἑλληνικῷ προφέροντός ἐπὶ τὰ μαθηματικά φήμην ὡς χρὴ ἀνευ τούτον πολὺ τις ἐν πολλὲς ἐνδομονάς γενέσθαι φύσις, ἄλλα ὁτους ὁ τρόπος, ἄλλη ἡ τροφή,

ταυτά τὰ μεθύματα, εἰτέ χαλέπα εἰτε ὀξύθυς, διὰ ταὐτ.

τῆς ἑτοῦ ἐμελήματι δὲ ὡς θερμοῦ ἔστι θείων, καὶ ἐν ἐν τοῖς ἐφεξῆς τοῖς τοιούτων φήμην ἐν πελλών ἐνα γεγονότα εὐθαμονάς τε ἐξεῦθεν καὶ συνάξατον ἐς καὶ μακάριον.

ἐν δὲ τῷ Πολλείς φημὶ ἐκ τῶν ἑτῶν οἱ προ-
κριθέντες τιμάς τε τῶν Ἐλλήνων μείζονος οἰδώτατί, τα τέ


(ἡ ἀδ. Ast) τροφή, ταυτά τὰ μεθύματα, εἰτέ χαλέπα εἰτε ὀξύθυς, ταυτά τοιούτων (ἔστιν Nicom.) ἐμελήματι δὲ ὡς θερμοῦ ἔστι θείων ἐνδομονάς τε ἐξεῦθεν καὶ συνάξατον ἐς καὶ μακάριον.

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Plato, cf. Schneider 23 μείζονος ταῖς Ἐλλήνων Plato
ΜΑΘΗΜΑΤΙΚΑΕ.

χύδην μαθήματα πάσιν ἐν τῇ παιδείᾳ γενόμενα τούτοις συνεπεῖ τοῖς σύνοιοι ὁδεγήτητοι τοῖς ἀλλήλοις τῶν μαθημάτων καὶ τῆς τοῦ ὦτος φύσεως. περιμετέ τε πρῶτον μὲν ἐμπειρίᾳ γενέωθαι ἀριθμητικής, ἔπειτα γεωμετρικῆς, τρίτον δὲ στροφῆς, τέταρτον ἀστρονομίας μίκας, ἢν φησίν εἶναι ἑπεραίον φερομένου στερεοῦ, πέμπτον δὲ μονοσκῆς. τὸ τῇ χρήσιμῳ παραδεικτῆς τῶν μαθημάτων φαιν' ἢς εἶ, ὅτι ἐνοικα διδέειναι, μὴ ἀφροτία τὰ μαθήματα προστάττομι. τὸ δ' ἔστων οὖ πάντων φαίλοις, ἀλλὰ πᾶσι χαλατον πιστεύον, ὅτι ἐν τούτοις τοῖς μαθήμασι ἐκόστον ὁδὸν ὕγιας τὸ ψυχῆς ἐκκαθαίρεται καὶ ἀναζωοπυρεῖ ὁμα τυφλοῦμενον καὶ ἀποσβαγμένον ὑπὸ τῶν ἄλλων ἑποτερεμάτων, κρείττων ὑν σωθήμει μυρίων ὁμίλεται· μόνῳ γὰρ αὐτῷ ἀληθεία ὑπάτη.

ἐν δὲ τῷ ἐβδόμῳ τῆς Πολιτείας περὶ ἀριθμητικῆς λέγων ὅτι ἔστων ἀνεφακιοιτήτη πασῶν φησίν, ἔπειτα ἕς


fort. add. ab Α

DE UTILITATE

dei pásais méν téxnais, pásais dé diανoiais kai ε̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣̣...
MATHÆMATICÆ.

αλλ' ὡς ἢν ἐπὶ θέαν τῆς τῶν ἀρίθμων φύσεως ἑρίκον-
tαι τῇ νοήσει, οὔτε πράξεως χάριν ἐκπόρφων ἡ κατή-
λου μελετώντες, ἀλλ' ἔνεκα ψυχῆς τῆς ἐπὶ ἀλήθειαν
καὶ συναίν ὦδον. τούτο γὰρ ἄνω ἰδεῖ τὴν ψυχήν καὶ
περὶ αὐτῶν τῶν ἀρίθμων ἀναγκάζει διαλέγεσθαι. οὕτως
ἀποδεχόμενον, ἢν τις αὐτῷ σώματα ἢ αὖ τὰ ἔρατα
ἲχνα ἀρίθμοις προσφέρομενος διαλέγεσθαι. καὶ πάλιν
ἐν τῷ αὐτῷ φησίν ἐτι οἱ λογιστικοὶ εἰς ἀπαντα νὰ
μαθήματα ὑδεῖς φύσεις, οὗ τε βραδεῖς εἰς τὸ ὄβευ-
θεὶν αὐτοῦ αὑτῶν γνῶσθαι. ἐτι ἐν τῷ αὐτῷ φησίν καὶ ἢ
ἐν πολεμῷ ὥς ἢν χρήσιμον πρὸς τὰς στρατευτεῖσιν
καὶ καταλήψεις χαριῶν καὶ ἱκανοτέρας καὶ ἐξεύθες
στρατιῶς. ἐν τέ τοὺς ἐξῆς ἐπαινῶν τὴν ἐνεργεῖ
tὰ σοφιᾶν μαθῆμα ἀνακάθεν, γεωμετρία μὲν, φήσιν, ἐτι
περὶ τῆς τοῦ ἑπιτέου θεωρικῆς, ἀντικομία ἢ περὶ τῆς τοῦ
ἰσοτεροῦ φοράς αὐτῆς ὥς ἢν ἀναγκάζει εἰς τὸ ἄνω ὀρᾶν καὶ
ἀπὸ τῶν εὐθέλει ἐκεῖσε ἀγεῖ. καὶ μὲν δὴ περὶ μονα-
χίας ἐν τῷ αὐτῷ φησίν, ὅτι δυνὴν θέται ἡ τῶν ἄνων

ἀλλ' ἐνεκα πολέμου τε καὶ αὐτῆς τῆς ψυχῆς ὑποτάνης τε (το
ομ. codd. multi) μεταστροφῆς ἀπὸ γενέσθαι ἐτι ἢ θείειν τε ἢ
ἐτι ἀλι. codd. complures) καὶ σύστην 4 Civ. 525 D τοῦτο
γε — ὡς σφόδρα ἢν ποι ἰδεῖ τὴν ψυχὴν καὶ π. ἢ αὐτῶν τῶν
ἀρίθμων ἀναγκάζει διαλέγεσθαι, ἀδέναι ἀποδεχόμενον, ἐτι
τῆς αὐτῇ ὑπάτη ἑ ὑπάτα (ἡ ἀπὸ τοῦ ἀλ. καί, ἡ τούτου ἀλλή
σωματα ἰχνος (ἐχον τατα codd. tis) ἀρίθμους προστείρουν το διαλέγεται
8 Civ. p. 525 B τὸ ἐν ἡ ἐπονίβα, ὡς ἢ τε φυσει λογιστικο-
τε σανα τὰ μαθήματα ὥς ἐποτ εἰς ἐς ἔδεις φύσεις, φεὶ τε
βραδεῖς, ἢν ἐν τούτῳ ποιεῖν ωθάς καὶ γεγομένως, καὶ ἄλλοι
ἐπικείμενοι, ἠμος εἰς τὸ βραδεῖα αὐτῶν τῶν γέρουν-
τος πάντες ἐπιστέανοι; 10 Civ. p. 526 δὲς καὶ ἐρή-
ποδ τὰ πολεμικά αὐτοῦ τείνει, δήλων θεία προσφέραντες, ἂν γαρ
τὰς στρατευτεῖσιν καὶ καταλήψεις χαριῶν καὶ ἱκανοτέρας καὶ
ἐξεύθες στρατιῶς. — 10 Civ. p. 529 δὲν γαρ μας ἔθελεν
δήλων, ὅτι αὐτὴ γαρ ἀναγκάζει φυσών εἰς τὰ ἄνω ὀρᾶν καὶ ἢν ἐκ
τῶν ἐνθέλει ἐκεῖσε ἀγεῖ 18 δεικν οὐτ. εὐκ δυνὴν Δ.
DE UTILitate

6


thetuarii, aestuornumias et ommunias: et ait aedilem
at exstitit, acsi si Pheagorismi. o si uin uin tis
apermonemis siphonias at cal phaoroyales allhemi oua-
murportes ahpnata ponoua. telios pererepologetes
o tis atis, oin eis geitonon faaun xeperomneoi, o si uin
faain akein en meo tie ighan kai miinamatos estai
diastem tuto, o metropenoi, o de amphiomioi uos
omiau he phaoroyalos, tis idia tou tou prostateta-
muoi. tais xohdes prologeta porisouou epi tais kolal-
etao baino stergpologete. o de agraphoi ephidmiuxoi xetofen
epakopoiynites, tines aipwnaio aipmodi aipmodis kai
tines oin. kai tuto xohmiou xoris tis tou aghmou

1 Civ. p. 530 D — kai aitai alliwn aedeplai eteis at epie-
statemis estai, ois ois te Pheagorismi xoan —. 2 Civ. p. 531 A
tis xer akopomeneos aip aipmodi kai phaoroyalos alliwn aip-
mpougetes ahpnates aestaia aiporoi kai aipmodi poion,
eis tis xhoxou, xrep, kai geitwos te, kai naxwmeto aitai aipomizeites kai
paragpololites tis uin, oin eis geitonon faaun xeperomneoi,
oi uin faain eis (eis om. codd. tei) xapatikes en meo tie
ighan kai miinamatos estai toto diastem, o metropenoi, o
de aipromioi uos omoni he phaoroyalos (phaoroyalos
codd. duo), aipomipon oitai tou tou prostatetamis. ois, mi, ton
-o, eis, ton xohdes aiploges tais tais xohdes prologeta por-
erfontes kai bapoiexites, epi tais kolalwos (kolametis, keleto-
pai, kolamwos ali: ci. Tim. lex.) stergpologetes 4 aitai-
ete u coff. ex ei A 5 en coff. ex en A . 1 metropenoi: e
coff. ex e A . 11 Civ. p. 531 C tais xer eis teutatas tais
aipmodi mias tais akopomeneos aipmodi xetofen, allis aip
problumatet alainas epakopoiynites, tines xeperomneoi aipmodi
kai tines oin, kai diei ti ekmete aipmodios xer, eis, pro-
lagies. xohmenei uin oin, eis o, eis, proq tis tis tou
kole te kai aghmou xetofen, allis de metaxonemias aipmodios. e
-mpis o, eis, oimai de xer, de o, eis, kai h toutoun paiw-
en dielbathineis methodos eis mi eis tis allhmi xoh-
wos ahpnates kai xepesointa, kai xetofeni touto o estin
allhmi xhelia, pasi eis auton eis a portiowpi tis progra-
meti. ci. Schneider
καὶ κελοῦς ἐξήσῃ τοῖς Άλλοις δὲ ἔχρηστον. καὶ τοὺτον παίτων ἡ μέθοδος ἢν μὲν ἐπὶ τὴν ἀλλήλους ἀφήκη ταχεύομαν καὶ ἀπολογισθῇ ἢ ἐστὶ ἀλλήλους οἴκεια, φέρει αὐτῶν ἡ προφυτεία κατοικίων. οἱ δὲ ταῦτα δεινῶλ διαλέπτομεν, οὐ γὰρ ἐκεῖνοι λαβεῖν τε καὶ ἀπο- σέβασθαι λόγουν. οὐχ οὖν τε δὲ τούτο μὴ δὲ ἐκεῖνον ἐλθοῦν τῶν μαθημάτων ὅσος γὰρ ἐστὶ δὲ αὐτῶν ἐπὶ τὴν τῶν ὑπὸν θέαν ἐν τῷ διαλέγεσθαι.

πάλιν τε ἐν τῷ Ἑπικόμῳ πολλὰ μὲν καὶ ἄλλα ὑπὲρ ἀριθμητικῆς διεξέρχεται, θεοῦ δόρου εὐθὺς λέγοντα, καὶ 19 οὐκ οἷον τε ἐνεν ταύτης σπουδαίως γενέσθαι τινα, ὑποβάς δὲ εὐτυχῶς φαίνει εἴπερ γὰρ ἀριθμῶν ἐκ τῆς ἀνθρωπίνης φύσεως ἔξελειμένη, οἷς ἄν ποιήσῃς φρόνιμοι γενομένα, οὔτε ἢν ἐτὶ ποτὲ τούτων τοῦ ἔρμου, φρονίμη, ἡ ψυχὴ πίσεων ἐρετὴν λάβης σχεδὸν ὁ τούτων λόγος ἐστὶν. ἔρχον δὲ δὲ τοῖς μὲν γενόσκοι δίῳ καὶ τρίῳ μιμὸς περιττὸν τοῖς δέ ἐρεθίστω, ἀγνοοῖ δὲ τὸ παράχω αὐτοῖς, οἷς ἂν ποτὲ διδόναι λόγον, περὶ Ἐπικόμου καὶ μνήμης

4 Σιτ. της 531 D οὐ γὰρ ποι ἐνεν ταύτης διεξέρχεται, οὐ μὲ τῶν ἄλλων, ἀλλὰ μὲ τὸν Β', εἴπερ, εἰ τὸ μή μᾶλλον γῆ τεντονίᾳ, ἐξ ἄλλας ὅρας, εἰπον μὲν δύνασθαι τιμῆς ἄντεις δοθῶ ἐν τοῖς ἀποδέχεσθαι λόγουν ἐπειδὴ τοτὲ ταῦτα διὰ καὶ ἀποδέχεσθαι λόγουν εἰπὸν ταῦτα ἐν διαλέγεσθαι, 6 εἰς αὐτὸν ρήτο ἐκεῖνων Α, οὐδὲ διαλέγεσθαι λόγουν εἰς διαλέγεσθαι λόγουν εἰπὸν ταῦτα ἐν διαλέγεσθαι, ἐπειδὴ τοτὲ ταῦτα διαλέγεσθαι, καὶ ἀποδέχεσθαι λόγουν εἰπὸν ταῦτα ἐν διαλέγεσθαι.
μόνον εἰς κατημένον· συνόμενον δὲ ἀληθοῖς λόγοις ὁφεὶ οἷς ἔν ποτε γένετο. οὐ μὴν οὖθε τὰ τῶν ἄλλων τεχνῶν λεγόμενα, ὥν δὴ διηλθομεν, οὐδὲποτε τοιῶν οὐδὲν μέναν, πάντα δὲ ἀπολείπεται τὸ περάπαν, ὅταν ἐρεθιστικῆς τις ἀμελή, δόξεις δ' ἂν ἰδια τιαὶ βραχεῖα ἀρίθμοι δείσθη τὸ τῶν ἀνθρώπων γένος, ὡς εἰς τὰς τέχνες ἀποβλέψας· κατόι μέγα μὲν καὶ τούτο. εἰ δὲ τίς Ἰδος τὸ θεῖον τὴν γενέσεα καὶ τὸ θυσίαν, ἐν δὲ καὶ τὸ θεοσεβὴς γνωρισθήσεται καὶ ὁ ἀριθμὸς ὑπός, οὐχ οὐν οἱ οὐκέτι γνωκαὶ ὑμῖν ὑπομαντὰ ἀριθμοῖς, οὐχ ἢ μὴν διονύσεως εἰςτὸς ἦν εἰς συγγνωμήνοις, ἐπεὶ καὶ μουσικὴν πάσαν δι᾽ ἀριθμοῦ μετὰ κυκλεσαία τε καὶ φθόγγον ὁδὸν ὑπὲρ καὶ τὸ μέγεθον, ἐργαθοῦν ὡς πάντων ἀετιοὺς· ὅτι δὲ κακῶν οὐδενὸς ἐστὶ, τούτῳ γνωστῶν. σχε- δον δὲ ἀλόγιστος, ἀπακος, σιρήνων τε καὶ ἐρφαθρὸς ἀνάρχοστος τε σφόνδυλα καὶ πάνθη δὲν κακοῦ κεκοινοβε- νηχε ντινος, διότι λέειται παντοῦ ἀριθμοὶ. ἐν δὲ τοῖς ἐσεξῆς ζῆσαι· ἐστὶν ἢν οἷον μηδεὶς ἡμῖς ποτε πειθέτω τῆς εὐσεβείας εἰναι τῷ θυρεῷ γένει. ἐκ μᾶς τοῦτον γραφεῖσθαι καὶ τὰς ἄλλας ὅρεται τῷ μαθῶν κατὰ τρόπον. 2 Εριν. p. 977 D καὶ ο ὑπὸν (λόγον) ὁφεὶς ὁθεδεται, ὅτι καὶ τῶν ἄλλων τεχνῶν λεγόμενα, δ ὑπὸ διετήσεις ἐκεῖσε εἶναι πάσας τις τέχνης, οὔτε τοιῶν ἐν οὕτωι μένοι (μεταθεμάτισ, πάντα δὲ ἀπολείπεται τὸ παράπαν, ὅταν ἐρεθιστικὴν τις ἀνάλη, δόξεις δ' ἂν ἵκανος τιαὶ βραχεῖαν ἐν γονο ἀριθμοῦ δείσθη τὸ τῶν ἀνθρώπων γένος, ἐκεῖνος τὴς τέχνης ἀποβλέποντι κατοι πλ. ῥ ὁ οὐκ οἰκεῖται αὐ πλ. 11 ἐπί καὶ τὰ ματα ὅρα- νη τίποτε διερθομένους κυκλεσαία τε πλ. 15 καὶ(κενοι) εἰς Παλλαδοὺς ἐγκαθίσθων μ. 14 καὶ δὲ κακῶν οὐδενὸς, εἰ τοῦτο γνωστοῖ, ὁ καὶ τάχα γένεις· ἂν, ἂλλ' ὁ σχεδὸν ἀλόγιστος τε καὶ ἀπακος κατοι πλ. 16 σφόνδυλος φησί πλ. ὀποία πλ. κε- κοινοβενηχε ντινος Α 17 ῥ ὁτις λέειταις ἐπιπέδηταις πλ. Εριν. p. 998 B μεν εὐελξιον καὶ γὰρ ἀρετῆς μηδεὶς ἡμᾶς ποτε πειθὴ τῆς εὐσεβείας εἶναι τῷ θυρεῷ γένει. 19 εἰς. Εριν. p. 998 D
1 Επι. p. 399 Ε — Θεορήματα ὁπιν τρόπῳ τῆς τῶν μνήμης.

2 Επι. p. 399 Α ἀργοτάτον ἀνάγκη τῶν ἀλλοποιῶν ὅπως οὕτως εἶναι, μή τὸν καθ' ἶδιον στροφομοιοῦντα καὶ πάντες τοὺς τοιούτους, οἷς ὑποκείμενα ἐκ τῆς ἀνάλογης ἑπεξεργασίας, ἀλλὰ τῶν τῶν οὐκέτι περίοδον τῶν ἑκάτερον, διεξεργασία τὴν αὐτὸν κόσμον ἐπέστη, ὡς ἀπὸ αὐτὸ ἡ ἄλοιπαι πολλά ἐκεῖνης ἑκάτου τοῦ θεοφάνους.

3 Επι. p. 400 C ἔτη δὲ ταῦτα παρακείμενοι σὺν ὅπως, δὲ γε (οὐδὲ δὲ οὖν ὅπως ἀλλὰ ἀνετάτην εἶναι χρονὸν πολὺ προδιδόκατο καὶ ἐξήντα ἅξιον μεταλλωθῆναι πάλιν ὅπως καὶ ἑκατέρως, διὸ μεθηματάρην ἄλοιπον ὅπως ἐν εἴτε τὸ δὲ μέγατον τε καὶ προϊόν εἰσοδῶν αὐτῶν, ᾧ τὸ αὐτὸν ἔχοντα, ἐὰν γὰρ τῆς τῶν περίπου τε καὶ αὐτῶν γενέσεως τε καὶ δυνάμεως, ὅπως προέλθαι πρὸς τὴν τῶν ἄνων μνήμην ταῦτα δὲ μαθάσατε τούτους ἵππως ἔστιν ὁ καλὸς ἢν σφάδα γενάτων ὅπως γενετομενοί, τῶν οὐκ ὅπως ὅπως ἄλλως φησὶ ἢν οὕτως ἐπιθέτικας πρὸς τὴν τῶν ἑπτάδες καὶ τῶν γενετομενοί ἢ ὅπως οὐκ ἑπεξεργάσθησαν ἀλλὰ γένοις θείων ἑπικρίνῃ ἡ μεγάλη τὸ δυναμικὸν ἑξανότου, μετὰ δὲ τούτων τοῖς τρεῖς (ταῖς πρώταις) ἔξεργασίας καὶ τῇ στρεφεῖσθαι ὑποδοθῆναι, τοὺς δὲ ἀνομοίως καὶ γεγονότα ἐπικρίνῃ τῆς τρεῖς τοῦτος έκ τῆς γεγονότας γεγονότας ἢ δὲ θείων τέ εκτα καὶ δυναμικῶν —
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αριθμών ἐπιστήμων αὐτῶν, ἀλλ' οὖ σώματα ἐχόντων, καὶ αὐτῆς τῆς τοῦ περιτευτοῦ τε καὶ ἑρτίου χειρότερον τε καὶ δυναμείου, δόσον παρέχεται πρὸς τῷ τῶν ὅντων σφόντων τรามος ὡς ἐρχείης μιαθήμεθα μὲν καλοῦσι, φησὶ, ἵνα ἄφθαση γελοιοῦν ὄνομα γεωμετρίαν ἐστι δὲ τῶν ὅντων ὅμοιον ἀλλήλους φύει αριθμῶν ὁμοίωσις πρὸς τῷ τῶν ἐπιπέδων μοῖραν. λέγει δὲ τίνα καὶ ἐπέρθη ἐμπεσείθης καὶ τέχνην, ἥν ὅς στερεομετρίαν κατέληξεν, εἰ τις, φησὶ, τούτου τρεῖς ἀριθμοὺς ἔξων τὰ ἐπιτέθη εἶναι αὐξημένοις τείνοις ὑπὸ τῶν ὕποτα σοὶ τειχεῖας, ὡς προείπον, στερεά ποιεῖ σῶματν τούτο δὲ θείον τε καὶ θεαματικόν ἐστι.

καὶ ἐν Πολιτείᾳ δὲ περὶ σχηματίζως τῆς κατ' ὑμνίαν ψηφιν' καλλίτητα καὶ μεγίστη τῶν περὶ πολλῶν σχηματιῶν ἐστίν ἡ σοφία, ἥς ὅς μὲν κατὰ λόγον ἐκεῖνον μέτοχον, δὲ ἀπολειτόμενον ὑποστῆθαι καὶ περὶ πολλῶν συνεργῶν συνεργίας, ἐτὰ τὰ μέγατα ἀρκεταίων

καὶ ἐν τῷ τρίτῳ δὲ τῆς Πολιτείας, διδάσκων ὅτι μάρτυς μουσικὸς ὁ σχῆμον, ψηφιν' ἠδ' οὖν πρὸς θεόν οὐτοίς οὐδὲ μουσικὸς πρότερον ἐσώμεθα, οὔτε εἰτε οὔτε 

τοις ψηφιν' ἡμῖν παιδείτενα εἶναι τοῖς θείοις, πρὶν

12 Λεγ. ΙII p. 689 Ο ἠδ' ἡ καλλίτητα καὶ μεγίστη τῶν ἐγκαταστάσεων μεγίστη διεκείσθη τε σφήν τοῦ σωμάτος, ἵνα μὲν κατὰ λόγον ἐκεῖνον καὶ μέτοχον, δὲ ἀπολειτόμενον οἰκουθάνοις καὶ περὶ πολλῶν συνεργῶν συνεργίας ἑλθέτων ἐπὶ τῶν ἐπιτεθῶν ἐπιτεθέντες 18 Στ. ΙII p. 402 οὕς σωμάτων, ἐπὶ τῶν πολλῶν περὶ πολλῶν ἐκείσθη, οὕτως εἰτε οὔτε τοῖς ψηφιν' ἐπὶ τῶν πολλῶν ἐπιτεθῶν ἐπὶ τῶν καθήκων, πρὶν ἐκεῖ 

τῷ τῶν συνεργῶν ἐπὶ τῷ σφήν ταὐτῷ ἐπιτεθέντες καὶ τῇ τῶν αὐτῶν καὶ τῷ τῶν σφην ἐπιτεθέντες, καὶ ἐπὶ τῶν τών ἐπιτεθέντες, καὶ τῷ τῶν σφήν ἐπιτεθέντες, καὶ τῷ τῶν σφῆν ἐπιτεθέντες (οὗ Π. 12, 1) καὶ ἐντὸς ἐν οἷς ἔστεις αὐτοτρόπως καὶ χεῖρα καὶ κατὰ σκευοῦσαν καὶ ἑστὶν ἐν σχῆμα πάντων σφην ἐπιτεθέντες, ἐλλά 

τῆς καλλίτητος ἀξίωμα θεῖος εἶναι καὶ καλλίτητα, ἐφ' ἑπτ. ἐκ
MATHEMATICA.

11

διού ἁπαντα τὰ τῆς σοφοσύς: εἰδὴ καὶ ἀνθρισίς καὶ μεγαλεότητος καὶ μεγαλοπροσές καὶ ὅσα τοιτον ἀδέλφη καὶ τὰ τοιτον ἀπεκτικα περιστρό-

μενα χαρῖσμαν καὶ ἐννοία ἐν ὑπ' ἐστιν αἰσθανάμεθα καὶ αὐτὰ καὶ ἐκόπῃς αὐτών καὶ μήτε ἐν μικροῖς μήτε ἐν μεγάλοις ἀπομάκρυμεν, ἀλλὰ τὰς αὐτῶς οἰκομέθα τέχνης ἐναι καὶ μελέτης; διὰ γὰρ τετελω καὶ τῶν πρὸ αὐτῶν τό τε ὀφελος ἐν μονακής δῆλος, καὶ ὁτι μόνον ὄντως μονακής ὁ φιλοσοφός, ὁμοιός δὲ ὁ καθός. τῇ γὰρ εὐθείᾳ ὄντως, ἤτοι ἡτίν ἄρῃ τὸ εὐ τῇ ἡθῇ κατε- οισκεύασμά ἔχειν, ἐπιστεῖν φητε εὐλογίαν, τούτεστι τὸ ἐν λόγῳ χρηστᾷ, τῇ δὲ εὐλογιᾷ τὴν εὐσχημοσύνην καὶ εὐφυμίαν καὶ εὐφυμιστίαν εὐσχημοσύνην γὰρ περὶ μέλος, εὐφυμιστίαν δὲ περὶ ἔμοιον, εὐφυμίαν δὲ περὶ ᾳν᾽ ἑὐ μὲν τῇ δὲ κακοθείᾳ τούτεστι τῷ κακῷ ἦθελε, ἐπεὶ ἐν εὐ λογίαν καὶ κακολογίαν, τούτεστι κακὸν λόγον χρη-

σαι, τῇ δὲ κακολογίᾳ ἀσχημοστίαν καὶ ἀφυμίαν καὶ ἀειφυμιστίαν περὶ πάντα τὰ γενέμα καὶ μοιομένα: ὅστε μόνος ἐν εἰς μονακής ὁ κυρίος εὐθείᾳ, ἀπὸς εἰς ἐν ὁ φιλόσοφος. ὁμοιὸ δὲ καὶ τῇ εὐθείᾳ ἐπειδῆ δὲ τὸν εὐθείᾳ οὐ εὔρηκαν καὶ εὐφυμίαν δὲ τῷ ψυχῇ ἐν πέτει εὐθείᾳ διὰ τῇ ἀφελείᾳ μεμιγμένην ἔχειν ἀβλαβὴ ἱδεσία, ἀδύνατον φησὶ τέλος μονακῆν γενέσθαι μὴ εἰδότα λόγον καὶ τὰς εὐσχημοσύνης καὶ ἀειφυμιστίας καὶ σοφοσύ-

1 ἐν supra vs. A 5 ενίθετ' ἐνακ vs. A 9 cf. Civ. p. 10D—101 A 12 εὐ ἑγὼς εὐθείᾳ A 13 ad εὐθείαν in mg. A adnotatum erat τὸ μήτε ἐξὶν ἐγουθθαν, ἀλλ' ἔτεκτον φησὶ τέλος μονακῆν γενέσθαι μὴ εἰδότα λόγον καὶ τὰς εὐσχημοσύνης καὶ ἀειφυμιστίας καὶ σοφοσύ-

17 ἄφιδιάν A 20 τὰ εἰσημένα: p. 10, 18

sqq. an scr. τὰ προφημένα;?
DE UTILITATE

igitur multis genere saeptur, sicut etiam in eis idem. Hæc tamen

eiusmodi, ut est certe, in illa philosophia — sicut etiam in eis idem

eiusmodi, ut est certe, in illa philosophia — sicut etiam in eis idem

eiusmodi, ut est certe, in illa philosophia — sicut etiam in eis idem

etiam in eis idem
καὶ τάδε ἐφη ἀραθής δὲ ἀνή όρις διασώζει τὴν ὁρθὴν ὁμόνω τὸν ἑκατέριον ἐργανομένου ὑπὲρ λύπας καὶ ἦδονας καὶ ἐπιθυμίας καὶ σφάλμας καὶ ἰνής ἐκβάλλει. οὐ δὲ μοι δοκεῖ ὅμοιον εἶναι, σέλε ἀπεκώλυτο. οἱ γὰρ βαφεῖς, ἐπείδαιν βουλήθηδα βέβαια ἐστὶν ὅτι εἶναι ἀληφρία, πρὸτότο καὶ ἐκλείπονται ἐκ τοσοῦτον χρωματῶν μίαν φύσιν τὴν τῶν λευκῶν, ἐπεὶ προκατα-
σκέυασθαι αὐτήν δύνη περαιτέρω ἑπαρπασάντες, ὅπως δέχηται ὁ τι μᾶλλον τὸ ἐνθοῦς, καὶ αὐτοὺς ῥάπτοντο· καὶ ὅ μὲν ἄν τούτῳ τῷ τρόπῳ βαφῇ, ὅμοιο τὸ το βαφὲν καὶ τὸν ἵνα ἂν τοῦτῳ τῷ τρόπῳ βαφῇ, δεισιδεῦσον γίγνεται τὸ βαφέν, καὶ ἡ πλεῖον οὐτὶ ἀντὶ ὁμοίων τῶν ὁμοίων ἀποκεφαλίζεται αὐτὰν τὸ ἄνθρωπος ἀπο-
κεφαλίζεται. οὐ δὲ ἐν μόνῳ σέλε ὅτι ὅψηται, έν τίς τις ἄλλοι ὀφθάλμοι μᾶς ἔκτροπαπέσασθαι, τίνα, ἐφη, ὅτι ἐκπέλεται καὶ γελαῖος. τροφίσμον (τροφίσμα codd. δευτ. τοιοῦτον) τὴν ἐν ἑρμάβη κατὰ σύνομα ἐγγυήσεσθαι καὶ ἦδος, ὅτι εἰσὶν ἄλλα τοῖς τετράτοις καὶ ἐπειδῶν (ἐν add. codd. Stobaei) μοῦνας καὶ γραμματικὰ μὴ ὄντος ἀλλὸ ἀποκεφαλίζεσθαι, οὐ όποιο τί ὁ τι καλλίστα τοῖς νόμοις πειθότας δέχοντο ὅσιον βεβήν. ἔνα δεισιδεύον εὐταίρης ὁ δέχατο ἄντων καὶ πρὸτὸ θειόν καὶ πρὸ τῶν ἄλλων, διὰ τὸ τὴν τούς θρίαμβος καὶ τὴν κοινὴν ἐπιτεθεῖσθαι διογκᾶναι, καὶ τῆς αὐτῶν ἐκπέλεσθαι τὴν βαφὴν τοῖς φύσισι τοιοῦτοι, διότι ὅτα ἐκκλητέν, ἐν τῇ ἠκολούθωσι τεῖναι διαστρεμένοις ὁμοίως καὶ τῇ τοῦτο ὅρθεν καὶ συνεῖς, λύπης καὶ τοῦτο, καὶ ἐπικεφαλεῖ, παντὸς ἄλλον ὃμορφως.
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θείας, καὶ οὕτω ἐκείνη γνωμήτων οὕτε μετὰ γνωμήτων
δύναι τῶν τό ἐνθος ἐφαρμοσθῆναι. οὔ δ' ἐν μή,
οἴσθα δ' ὅρω, ἐν μή γίνεται, ᾧ μή προθερμητέος βάπτης,
ἐκπληκτα καὶ εὐτῆλα καὶ οὐ δευσοποιεῖ. τοιοῦτο δὲ κατὰ
δύναμιν ἐφαρμοσθῆναι γενεῦται χρή καὶ ἡμῖν: παιδίωμεν
γάρ τούς παιδαὶς ἐν μορφῇ τε καὶ γνωτιστῇ καὶ
γνώμαις καὶ γυμνοῖς καὶ ἐν ἴδιοθητικῇ, οὐδὲν ἀλλὰ
μηχανόμενοι, ἢ ἅπασα ἡμῖς προσεκχειρίσαντες καὶ προ-
θερμητέος ἐβάπτος τοῦ σωτηρίου τοῦ μαθήματος
10 τούτοσι, τοῖς περὶ ἀγάπης ἐρετῆς ἦν αὐτοὶ ἐκπαιδεύσασιν
ὑπότον λόγους ἐνδείξασιν ἄμεσος βασιλέως, ὡσα νευσ-
ποιῶς αὐτῶν ἡ ὅρω γίνεται, διὰ τὸ τὴν ἑαυτοῦ καὶ
tροφὴν ἐπιτεθεῖν ἐσχηκέναι, καὶ μή ἐκπλήνη αὐτῶν
τήν βασίλεια τὰ δύναμες ταῦτα, δεινὰ ὅποτα ἐκκαλὼν.
15 ἢ τε ἡ ἐδούνη, παντὸς στρεβλοῦ διευθετεῖ σύστα καὶ κοινω-
νίας, λήπη τε καὶ φόβος καὶ ἐπιθετία, παντὸς ἐλλο
δύναμες.
καὶ γὰρ αὕτη τήν γυμνοστρέφειν μήθην σαίτι τῷ ἐν
ἀλήθειας τελεῖς καὶ ποὺ ὄρει ἔλθε, μυστηρίων
20 παραδόσει. μυθισταὶ δὲ μερὶς πέτεις. τὸ μὲν προθερ-
μενον καθαρόν οὕτω ἤτοι ἢ ποὺ διὰ τῶν ρούμων μετ-
συνάλη μυστηρίων ἐστίν, ἀλλ' εἶδον οὗτος αὐτῶν εἰρη-
νικαὶ προσβάλλεται, οἷον τῶν χεῖρος μὴ καθαρής καὶ
φυσικῶν ἐξενεκτέν ἐχοντες, καὶ αὐτοῖς δὲ τοῖς μὴ ἐγχο-
νυμοὺς ἀπελευθεροῦσιν τοὺς πρότερον τυχών. μετὰ
dὲ τῆς καθαρόν δευτέρα ἐστίν τῇ τῆς τελεῖς παράδοσις
9 συντηκόλπα: ο in sac. trium aut quattuor litterarum A
12 γένοιο A 18 cf. Plat. Phaed. p. 69 D 20 κ. A in
22. Schoemann opusc. II p. 351 28 β mg. A
MATHEMATICAE. 15

tρίτη δὲ ή ἐπονομαζομένη ἐποπτεία: τετάρτη δὲ, δὲ ή καὶ τέλος τῆς ἐποπτείας, ἀνάδειξι καὶ σεμείατον ἐπίθεσι, ωστε καὶ ἕτεροι, ἐς τις περίπατε τελετάς, παραδοθέντα δύνασθαι, διδοχάζοντα τυχόντα ή ἱεροφανίας ή τινος ἄλλης ἱερασίας, πέμπτη δὲ ή εἰς αὐτῶν περί-
γενοµένη κατὰ τὸ θεοφιλεῖς καὶ θεοῖς συνδείταιν εὐδαι-
µονία. κατὰ ταύτα δὲ καὶ ή τῶν Πλάτωνικῶν λόγων περί-
δοσις τὸ μὲν πρῶτον έχει καθύποιν τινα, οἷον τήν ἐν τοῖς
προσφέρειν µαθήµαταν εἰς παιδόν συγγραµµατίαν. ο ἡν
µὲν ἑκτὸς ἐµπεδοκλῆς ἱεροµοιοικάτον πείν' ἀναµιστήρας ἀτεί-
φεῖ χαλαρφ δείν απορρίφτεται. ο δὲ Πλάτων ἐπὶ πέντε
µαθηµέταν δείν φιάς ποιεόθαι τὴν κάθαρσιν ταύτα
δ' ἐστὶν ἀριθµητική, γεωµετρία, στερεοµετρία, µουσική,
ἀστρονοµία. τῇ δὲ τελετῇ ἐοίκην ή τῶν κατὰ φιλοσοφίας
θεοµοικῶν περίδοσις, τῶν τε λογικῶν καὶ πολιτικῶν ἐς
καὶ φυσικῶν. ἐποπτείας δὲ ὀνοµάζει τὴν περὶ τὰ
νοµατα καὶ τὰ ὑπαρχόντα καὶ τὰ τῶν ἑδέων πραγ-
µατείαν. ἀνάδειξι δὲ καὶ κατοίκων ἡµικόν τὸ εἰς
ἀυτοὶς τῇ κατέµαθεν οἷον τε πενεύσει καὶ ἕτεροις
εἰς τὴν ἑαυτήν θεωρίαν καταστίχησαι. πέριπτον δ' εἰν Ἔ
καὶ τελεστάτων ή εἰς τούτων περιγενοµένη εὐδαιµονία

1 γε et δ' mg. A  ἦ adi. Lobeck Aglaoph. p. 39 5 πεπρὶ δὲ ή δὲ ή δὲ ς' A 6 εὐδαιµονίαν A, em. Bullialdus 7 ταύτα Λ Πλάτωνικῶς πολιτείαν Λ 8 α mg. A 9 τῆ—
συγγραµµατική C ή — συγγραµµατική Λ 10 Εμπεδοκλῆς τν. 122
Kasten, 412 Stein, 452 Mullach. cf. Aristot. Poet p. 1457 b
ἀνιώκτα: αὐτὸ καὶ αὐτὸ ex corr. i in ras. Α ἀτείφει corr. ex ἀνιώκτα.
inter qu et i una lit. er. A 11 χαλαρφ δείν απορρίφτεται:
καὶ δείν et pr. q in ras. Α 13 στερεοµετρίας ο corr. ex ὧ Λ
β mg. A 16 cf. Phaedrus p. 250 C γ mg. A 17 τὰ
τῶν Ἔλλησ] τῆν τῶν Λ 18 δ mg. A 20 δ
mg. A
16 DE CONSILIO SCRIPTORIS:

καὶ κατ’ αὐτὸν τὸν Πλάτανον ὑμοίωσες θεῷ κατὰ τὸ δύνατόν.

πολλὰ μὲν οὖν καὶ ἄλλα ἔχει τις ἐν λέγειν παρα-

dεικνύσι τὸ τῶν μαθημάτων χρήσειν καὶ ἀναγκαῖον.

τοῦ δὲ μὴ δοξέω ἀνειροκάλας διετέθην <ἐν> τῷ τῶν

μαθημάτων ἐπαίνῳ τριπτῖον ἣδη πρὸς τὴν περαθέοις

tῶν ἀναγκαίοις κατὰ τὰ μαθήματα θεωρημάτων, οὔχ

όποι δύνατο ἂν τὸν ἐντυγχάνοντα ἡ ἀριθμητικὴν τελεῖς

ἡ γεωμετρίᾳ ἡ μουσικήν ἡ ἀστρονόμον ἔπορθηναί οὔδὲ

tὸ γὰρ ἐστι τούτῳ προσεχόμενον ἢ προεξέμενον ἑποτά

tοῖς Πλάτωνι ἐντυγχάνονοι, μόνα δὲ ταῦτα παραδώσα-

μεν, οὔσα ἔξοχαί πρὸς τὸ δυνατόν συνείναι τῶν

συγγραμμάτων αὐτοῦ, οὔδὲ γὰρ αὐτὸς ἐξελεῖ εἰς ἐσχικὸν

γῆς ἀριθμάσαι διαγράφωμα γράφοντα καὶ μελετάν, ἵνα

παράδοξα ἀπέκται ταῦτα τὰ μαθήματα, προσωρι-

νασιστικὰ καὶ καθαραῖα ἢ ὅποις ἢ ἐς τὸ ἐπαισθεῖν

αὐτὴν πρὸς ἡμῶν μαθησάμεθα. δέλτα μὲν οὖν καὶ

χρῆ τοῦ μελλονταί οἷς τῇ ἡμεῖς παραδώσουμεν οἷς τῇ

Πλάτων υἱοφόρως ἐνετείλαθα διὰ γενὸς τῆς προτεῖ-

ς χρηματίζῃ στοιχειόδους ἀναγραφέαται. διὸν γὰρ ἐν

ἐξερέτειοι οἷς παραδώσουμεν. ἐστὶ δὲ ὅπως τοιοῦτα

καὶ τὰ παρ’ ἴμων, ὅς καὶ τῷ παντάπασιν ἐμιήρῳ τῶν

μαθημάτων γνώριμα γενέθηκαν.

πρὸ τοῦ δὲ μημονεύσομεν τῶν ἐφιθημικῶν θεωρη-

μάτων, οἷς συνείχαται καὶ τὰ τῆς ἐν ἁρμονίᾳ μουσικής

tῆς μὲν γὰρ ἐνδοξάσθαι προσεδιδῆθη, καθά

καὶ αὐτὸς ὁ Πλάτων ἐγγύηται λέγων ὅς οὐ χρῆ ἐπεφ.

1 Theaet. p. 170 B 3 ἐν λέγειν ἀργοτ.] ἀναλήγει A
21 inser. ἐσεὶ ἐφιθημικῶς Α, β in mg. 27 Cir. VII p. 531 A, cf. p. 6, 5 sqq.
DE DISCIPLINARUM MATH. ORDINE.

17

ex reitònon phònèn Θηρηνομένονος πράγματα παρεξελα

taìς χορδαῖς. δ' ὑγείαμεθα δὲ τὴν ἐν κόσμῳ ἁρμονίαν καὶ
tίνι ἐν τούτῳ μουσικῇ κατανοήσαι· ταύτῃ δὲ οὐχ

οἷς τὸ καταλεῖν μὴ τῆς ἐν ἀριθμοῖς πρότερον Θεωρητικοῦ γενομένος.

καὶ καὶ περίπτην ὁ Πλάτων φησὶν εἶναι τὴν μουσικήν, τὴν ἐν κόσμῳ λέγουν, ἤτε ἔστιν ἐν

tῇ κυνήγει καὶ τάξει καὶ συμφωνιᾷ τῶν ἐν αὐτῷ κυνο-

μένων ἄστοιν. ἦμιν δ' ἀναγκαῖον δευτέραν αὐτὴν
tάξειν μετὰ ἀριθμητικὴν καὶ κατ' αὐτὸν τὸν Πλάτωνα, ἐπεὶ ὁδὸ 

ἡ ἐν κόσμῳ μουσικῇ λειτή ἄνευ τῆς ἔξοδος· τὸ μουσικῆς καὶ νουμανίας μουσικῆς. οὕτοι εἰ μὲν ὀνε

ζενταὶ τῇ περὶ φιλοῦν ἀριθμοῦς θεωρία ἢ ἐν ἀριθμοῖς

μουσικῇ, δευτέρα ἄν ταχθεῖν πρὸς τὴν τῆς ἡμετέρας

θεωρίας εὑμέρειν. πρὸς δὲ τὴν μουσικὴν τὰξὶν προέτε 

μὲν ἐν εἰς ἡ περὶ ἀριθμοὺς θεωρία, καλουμένη ἀριθ-

μητικῇ· δευτέρα δὲ ἡ περὶ τὰ ἐπίπεδα, καλουμένη γεω-

μετρία· τρίτη δὲ ἡ περὶ τὰ στερεα, ἤτε ἐστὶ στερεο-

μετρῶν· τετάρτη <δ' ἡ> περὶ τὰ κυνόμενα στερεα, ἤτε 

ἔστιν ἀναφορομα· ἡ δὲ τῆς τῶν κυνήγεων καὶ διαστη-

μάτων ποιὰ σχέσεις ἔστιν μουσική, ἤτε σοι οὐ 

τὸ ἐστὶ πολὺ λιπθήναι μὴ πρότερον ἢμον αὐτὴν ἐν ἀριθμοῖς κατα-

νοημάτων· διὸ πρὸς τὴν ἡμετέραν θεωρίαν μετὰ ἀριθ-

μητικῆς τέταρτος ἢ ἐν ἀριθμοῖς μουσικῆ ἡ δὲ πρὸς τὴν

φύσιν πέμπτη <ἡ> τῆς τοῦ κόσμου ἐρµοµας θεωρη-

τικῆ μουσικῆ. κατὰ δὴ τοὺς Πυθαγορικούς προσλέπεται τοῦ

4 τῆς συν. εἰ τοῦτο Α 5 Πλάτων. cf. Civ. π, 530 D

6 τὴν ἐν κόσμῳ λέγουν τῶν ἐν κόσμῳ λάγους Α, cf. τα 2 et 10

7 αὐτῷ Bull. αὐτὴ Α 10 autib. vid. ἀνεύ τῆς ἐν ἀριθ-

μοῖς (τοι ἐν ἀριθμοῖς) κατανοομένη. cf. τα. 21 15 δὴ άδδ.

Bull. 19 sc. vid. πέμπτη δὲ ἡ τῆς τῶν κιν. καὶ διαστ. πρὸς

ἄλλα σχέσεως θεωρητικῆ μουσικῆ 21 αὐτή] τῆς?

23 μέχρι τοῦτον ing. Α προσλέπεται Α

Theo Smyt.
18 DE UNO

τὰ τῶν ἀριθμῶν ὡς ἀρχῇ καὶ περὶ καὶ βίβα τῶν πάντων.

ἀριθμὸς ἦστε σύστημα μονάδας, ἡ προσοδιαμέτρησις πλῆθους ἀπὸ μονάδος ἀριθμοῦ καὶ ἀναποδιαμέτρησις εἰς τὰ μονάδα καταλήξον. μονάς δὲ ἦστε περιήγοντά ποσότητα, ἢ ποσότητα τῶν ἀριθμῶν], ἢ ταῖς μειονεύμονος τοῦ πλῆθος κατὰ τὴν ὑφαίσθεν τῷ παντὸς ἀριθμοῦ στηριχθέας μονάς τε καὶ στάσεως λαμβάνειν, οὐ γὰρ οἶδον τε περαιτέρω γενέσθαι τὴν τομήν καὶ γὰρ οὐκ εἰς μέρια εἰς διαμέτρομεν τὸν ἐν ἐν αἰσθητοῖς, ἐμπείρον πλῆθος γενήσεται τὸν καὶ πολλὰ, καὶ καταλήξει εἰς ἐν κατὰ τὴν ὑφαίσθεν ἐκάστος τῶν μορίων καὶ ἐκατόν πάλιν εἰς μέρια διαμέτρομεν, πλῆθος τὸ τὰ μόρια γενήσεται καὶ ἡ κατάληξις καθ' ὑφαίσθεν ἐκάστος τῶν μορίων εἰς ἐν.

δὲ ὅστε αἱρέσιν καὶ ἀναποδιαμέτρησις τὸν ἐν ἐν ἐν. καὶ γὰρ ὃ μὲν ἀλλὰ ἀριθμὸς διαμετρούμενος ἐπαντοῦται καὶ διαμέτρεται εἰς ἐλάττων αὐτοῦ μόρια, οἷον τὰ ἐπὶ τὰ γ' καὶ γ' καὶ 'τ' καὶ β' ἤ καὶ α'. τὸ δὲ ἐν ἐν μὲν ἐν αἰσθητοῖς διαμετρήται, ὡς μὲν σῶμα ἐπαντοῦται καὶ διαμετρείται εἰς ἐλάττων αὐτοῦ μόρια τῆς τομῆς γενετέντως, ὡς δὲ ἀριθμὸς αἰρείται ἀκτίνις ἑνὸς πίνακα πολλὰ, ὅστε καὶ ἐν τούτῳ ἀμετρῆ πολλά, αὐθεντικώς ἐπερείται εἰς μεῖζον ἕνωτος μόρια διαμετρεῖται τὸ ἐν <ἐν>
ET UNITATE.

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diarioitmenon καὶ εἰς μείζωνα τοῦ ὅλου μόρια ὡς ἐν ἀριθμοῖς διαιρεῖται καὶ (εἰς) ἴσα τῷ ὅλῳ οἷον τὸ ἐν τῷ ἀριθμῷ ἀν εἰς ἕξ διαιρεθῇ, εἰς ἱδὰ μὲν τῷ ὅλῳ ὡς ἀριθμὸς διαιρεθῆσαι α' α' α' α' α' α', εἰς μείζωνα δὲ τοῦ ὅλου ὡς ἀριθμὸς εἰς δ' καὶ β' τὰ γάρ β' καὶ δ' ὡς ἀριθμοὶ πλείονα τοῦ ἐνός. ἀδιαιρετοὶ ἄρα ἡ μονάς ὡς ἀριθμὸς, καλεῖται δὲ μονάς ἢτοι ἀπὸ τοῦ μένου ἀριθμοῦ καὶ μὴ ἐξισοταθεὶς τῆς ἑνής φύσεως· ὀσπὶς γὰρ ἐν ἐφ', ἐν τῆν πολλαπλασιάζομεν τὴν μονάς, μὲντε μονάς· καὶ γάρ ἑπάξ ἐν ἐν, καὶ μέχριον ἄπειρον ἐὰν πολλαπλασιάζομεν τὴν μονάδα, μὲνε μονάς. ἡ ἀπὸ τοῦ διαιρεθέντος καὶ μεμοιχθῆθαι ἀπὸ τοῦ λοιποῦ πληθύνος τῶν ἀριθμῶν καλεῖται μονάς. ἢ δὲ διεισνικόντων ἀριθμῶν καὶ ἀριθμητῶν, τεῖται καὶ μονάς καὶ ἐν. ἀριθμὸς μὲν γὰρ ἐπὶ τὸ ἐν νομοτός ποιόν, ἵνα οἶνον ἅτα ε' καὶ ἄντα ε', οὗ σώματα τινα οὐδὲ αὐθητή, ἀλλὰ νοητόν ἀριθμήτων δὲ τὸ ἐν αὐθητοῖς ποιόν, ἀν ἢπιον ε', βότες ε', ἀνθρωποι ε'. καὶ μονάς τοῖς ἔτισιν ἦταν ἢ τοῖς ἐνός ὑπὲρ ἡ νοητή, ἢ ἂτεν ἄκομος ἢν δὲ τὸ ἐν αὐθητοῖς καὶ ἐν τὸ ἐνοτότι λεγόμενον, οἷον εἰς ἢπιος, εἰς ἐκ ἀνθρώπος. ὡςτε εἶδον ἂρχη τῶν μὲν ἀριθμῶν ἡ μονᾶς, τῶν δὲ ἀριθμητῶν τὸ ἐν· καὶ τὸ ἐν ὡς ἐν αὐθητοῖς

7 ρεμπ Stob. 1 ὡς μονάς ἤτοι ἀπὸ τοῦ ἑσταίνει καὶ κατὰ ταύτα ὕπατος ἀριθμητός καὶ κατὰ ταύτα ὤκεντος ἀριθμητοῦ καὶ κατὰ ταύτα μεμοιχθηνῦ τοῦ πληθύνος εὐλογῆς ἐκλήθη 11 fort. add. ἐπ' ἑσταίνει 19 ἢ 0 A 21 inscr. τῷ ἀρχή ἀριθμοῦ ἢ A, δι in mag. 22 Stob. ecl. 1 1, 9 τοῦ τῶν ἀριθμῶν ἄρχην ἀντικατατο τῆς μονάδας, τῶν δὲ ἀριθμητῶν τὸ ἐν· τοῦ δὲ ὅλου τειχάθεν τοις ἐπείρος ὡςτε τὰς ἀριθμητὰς τῶν ἀριθμῶν τειχή διαλέγεται ἢ διαφέρει τά σώματα τῶν ἀριθμῶν των εὐθανατῶν, εἴθαι δὲ καὶ τούτο χρῆ ὅτι τῶν ἀριθμῶν ἀντικαταστάτο τῆς ἀρχῆς οἷον μὲν νοητός τῆς εἰς μονάδας καὶ τῆς ὑμάδας, οὗ δὲ Πυθαγόρεια τάσσει ποιότο τὸ εἰς τῆς τῶν ὀρῶν ἐκθέσεις,
τέμνεσθαις φασόν εἰς ἀπειρον, οὐχ οἶς ἀριθμοὺς οὐδὲ ὁς ἀρχὴν ἀριθμοῦ, ἀλλ' ὁς ἀναθητόν. ἂντε ἢ μὲν μονής νοστὴ ἀνά ἀναθητός, τὸ δὲ ἐν ὁς ἀναθητοὶ εἰς ἄπειρον τιμῶν. καὶ τὰ ἀριθμήματα τῶν ἀριθμῶν εἰς ἀν διαφό
ροντα τὸ τὰ μὲν σώματα εἶναι, τὰ δὲ ἀσώματα. ἀπλάς
dὲ ἀρχὴς ἀριθμῶν οἱ μὲν ύποτρόφοι φασί τὴν το μονάδα
cαὶ τὴν ὀνομάζετο, οἱ δὲ ἀπὸ Πυθαγόρου πάσης κατὰ τὸ
ἐξής τὰς τῶν ὀρθῶν ἐκθέσεις, δ’ ἢ ἐν ἀρτιοὶ τι καὶ περί
τοι νοοῦνται, οἴνον τῶν ἐν ἀλάθμητος τριῶν ἀρχῆς τὴν
τιν τριάδα καὶ τῶν ἐν αἰσθητοῖς τεσσαράν πάνων ἀρχῆς
tὴν τετράδα καὶ ἑκατὸν τῶν ἄλλων ἀριθμῶν κατὰ τοῦτο.
οἱ δὲ καὶ αὐτῶν τούτων ἀρχῆς τὴν μονάδας φασί καὶ
tὸ ἐν πάσης ἀπηλλαγμένων διαφοράς ὁς ἐν ἀριθμοῖς,
μόνον αὐτὸ ἐν, οὐ τὸ ἐν, τουτεστὶν οὐ τὸδε τὸ ποιὸν
cαὶ διαφοράν τινα πρὸς ἔτερον ἐν προσελήφθη, ἀλλ’
αὐτὸ καθ' αὐτὸ ἐν. οὕτω γὰρ ἐν ἀρχῆς τε καὶ μέγερον
eἰς τῶν ὑπ’ ἐκατὸ ὑπονέων, καθὸ ἐκατόν τῶν ὑπονέων ἐν
λέγεται, μεταξὺ τῆς πρῷτης τοῦ ἑνὸς συνήθες τε καὶ
ἰδιαίς. Ἀρχίτικος δὲ καὶ Φαίλοις ἀδιαφοροῖς τὸ ἐν καὶ
τὸ μονάδα καλοῦσι καὶ τὴν μονάδαν ἐν. οἱ δὲ πλεῖστοι
προστίθεσι τῇ μονάδα αὐτῆς τὴν πρῴτην μονάδα, ὡς
οὔδες τινὸς οὐ πρῴτης μονάδος, ἢ ἐστὶ νοοτρέσων καὶ
αὐτὴ μονάδα καὶ ἐν — λέγοντοι δὴ καὶ τὸ ἐν —, τοῦτο
Zeller die Philos. d. Gr. 1 p. 318. 335, 1. 339, 4. 5 ἀπλάς
corr. ex ἄπλων A 11 ταῦτα A 14 μόνον ἢτα ἄτω ἢτα?
sαῦτα τοῦ ἐν κατὰ A 16 καθ’
κατὰ τὸ ἐν Bull. 17 ἐκατὸ: ὁ κατὰ ex ἄτα A 19 ἐνθέτεις:
Mullach fragm. philos. Gr. II p. 117. Φαίλοις: Boeckh Philo-
τά τῶν μονάδας] scrib. vid. aut τάς μονάδας aut τά μονάδα
tῶν μονάδας) 22 μονάδας] μ’ Α 23 αὐτῆς A καὶ τὸ ἐν] aut
ET UNITATE.

21

οὔτε τινὰ τοῦ ἕνως, ἐκάστου τῶν

προσεχτῶν παράξουσα ἐν μετοχῇ γὰρ αὐτῶς ἔκαστον

ἐν καλείται. διὸ καὶ τούτων ἀυτοῦ οὐδὲν παρεμφαίνει

ti δι καὶ τῶν γένους, κατὰ πάντων δὲ κατηγορεῖται,

[ἐστι καὶ ἡ μονᾶς καὶ ἐν ἐστὶ] καὶ τὰ μὲν νοεῖ καὶ οὐ

παραδείγματα μηδὲν ἀλλήλων διαφέροντα, τὰ δὲ εἰσθήτα.

ἐνοι δὲ ἐτέρων διαφοράν τῆς μονάδος καὶ τοῦ ἑνώς

παράξουσα. τὸ μὲν γὰρ ἐν οὔτε κατ’ οὐσίαν ἀλλοιούται,

οὔτε τῇ μονάδι καὶ τοῖς περιτοῖς αὐτῶν ἐστὶ τοῦ μὲν

ἀλλοιούσθαι κατ’ οὐσίαν, οὔτε κατά ποιότητα, αὐτὸ λάγος

μονῶς ἐστὶ καὶ οὐχ ὀπέρ αἱ μονάδες πολλαὶ, οὔτε

κατὰ τὸ ποιόν· οὔδε γὰρ συνείδεται ὡσπερ αἱ μονάδες

ἀλλὰ μονάδι· ἐν γὰρ ἐστὶ καὶ ὁ πολλὸς, διὸ καὶ ἐνικῶς

καλέται ἐν. καὶ γὰρ εἰ πορείᾳ Πλάτωνι ἐνάδες εἴρηται

ἐν Φιλήμβω, οὗ παρὰ τὸ ἐν ἐλέξθησαν, ἄλλα παρὰ τὴν ἡ

ἐνάδα, ἦτις ἐστὶ μονᾶς μετοχῆ τοῦ ἑνός. κατὰ πάντα

δὴ ἀμεβάλλοντο τὸ ἐν τὸ ἀφισμένον τούτο ἐν τῇ μονάδι.

ἐστὶ διαφέροι αὐτό τὸ ἐν τῇ μονάδας, ὅτι τὸ μὲν ἔστιν

ἀφισμένον καὶ πέρας, αἱ δὲ μονάδες ὑπεροῦν καὶ ἀφιστοῖ.

τῶν δὲ ἐφήμων ποιοῦνται τὴν πρῶτην τοιοῦτοι ἐς τὸ

ὅτι τοῖς μὲν γὰρ αὐτῶν ἄρτιώς, τοὺς δὲ περιτοῖς

ποιοῦν. καὶ  ἔστιν μὲν ἐστὶν ὁ ἐπιδικήμονα τῆς ἐν ἑα

διαφέρεισιν, ὡς ἡ ὑμᾶς, ἡ τετράς· περισσοῦ δὲ ὁ ἐς ἐνάδα

dιαφέροντο, οἷον ὁ τῇ, τῇ τῇ· πρῶτην δὲ τῶν περισσῶν

ἐστὶ ἑξατάξει τῆς μονάδας. τὸ γὰρ θρόνον τῷ περισσῶς ἐς

ἐναντίων· ἡ δὲ μονᾶς ὑποὺ περιτῶν ἐστὶν ἡ ἄρτιώς καὶ

1 ἐστι; 9 καὶ τοῖς ἔχονται καὶ περιτοῖς; μὴ del. Bull. 12 συνείδεται Α 13 τον συνείδετα Α 15 τον Φιλ. Βοι. p. 15 Α 16 μετοχή συνείδετα Α 18 το μὲν ἔχον (ἐν) 29 οἰκ. πάντων περιτοῦ καὶ περιτοῦ Α, i in mg. 22 ἐν δὲ νῦν Gell., sed cf. p. 25, 21 sqq. 70, 16, 19, 71, 3, 72, 20
DE NUM. PARIBUS ET IMPARIBUS.

22 DE NUM. PARIBUS ET IMPARIBUS.

αρτίον μέν οὐκ ἐν εἰθ' οὐ γὰρ ἄπως εἰς ἓκα, ἀλλ' οὐδὲ ὧλος διαμετέταται περιττή ἢ καὶ ἡ μονάς. καὶ ἀρτίον ἄρτιον προσθῇ, τὸ πέντε γίνεται ἀρτίον' μονάς δὲ ἀρτίον προσθετεῖν τὸ πέντε περιττὸν ποιεῖ' οὐκ ἂν ἀρτίον ἡ μονάς ἀλλὰ περιττόν. Ἀριστοτέλης δὲ ἐν τῷ Πεθανωρίῳ τὸ ἐν φημι ἀριστότης μετέχει τῆς φύσεως· ἀρτίον μέν γὰρ προστεθέν περιττὸν ποιεῖ, περιττὸ δὲ ἀρτίον, ὥς οὐκ ἂν ἦν ἡμέτατο, εἰ μὴ ἐγγὺς τοῖς φυσικοῖς μετέχει· διὸ καὶ ἀριστοπέπτον καλεῖται τὸ ἐν. συμπέ-10 ρέται δὲ τούτοις καὶ άριστεῖς. περιττόν μὲν οὐν πρώτη ἢδα ἦν τῆς μονῆς, καθάπερ καὶ ἐν κόσμῳ τῷ ἀριστέρῳ καὶ τετραμέτρῳ τὸ περιττὸν προσαρμοζοῦντα· ἀρτίον δὲ πρώτη ἢδα τῆς ἀριστοτος διάς, καθά καὶ ἐν κόσμῳ τῷ ἀριστέρῳ καὶ ἀριστεῖς καὶ ἀτέτατο τὸ ἀρτίον.

ψηφομότοσκν. διὸ καὶ ἀριστοτος καλεῖται ἡ διάς, ἐπειδὴ οὐκ ἦν τῶν ὀσπερ ἡ μονῆς ὁμομείην. οὔτε δὲ ἢδα ἐπάπνευτοι τούτοις ὥραι ἀπὸ μονάδος ἐπειθέως τοὺς εἰτέ-15 αυτοὺς μὲν τῇ ἕκα ἐπερχῷ μονάδι τὰ ἐκαστὸι εὐ-τῶν τῶν προτέρων πλεονάζει· εἰσέλθετο δὲ τούς λόγους τῆς πρὸς ἐλλήκουσιν ὁδίκοις τούτοις μενοῦσιν, οἷον ἐπε-θέτον ἐριστοῖς εἰς ὧλος ἡ μονάς πρὸς τὴν μονάδα· ἢτοι διπλασίοις· ὁ δὲ τῆς τριάδος πρὸς τὴν δυάδα ἡμιόλοις, ὁ δὲ τῆς τετράδος πρὸς τὴν τριάδα ἐπίτοιχος, ὁ δὲ τῆς πεντάδος πρὸς τὴν τετράδα ἐπίτε-20 τοίχας, ὁ δὲ τῆς ἕκαστος πρὸς τὴν πεντάδα ἐπίπεμπτος. ἢτοι δὲ ἐλάττων λόγος ὁ μὲν ἐπίπεμπτος τοῦ ἐπιτετάρτου,

DE NUMERIS PRIMIS.

23

ο δὲ ἔτειτερος τοῦ ἑτερίου, ὁ δὲ ἑταῖρος τοῦ ἐπιτρίτου, ὁ δὲ ἠττός τοῦ ἡμιλίου, ὁ δὲ ἡμιλιός τοῦ διπλαίου καὶ ἐπὶ τῶν λοιπῶν δὲ ἀριθμῶν ὁ ἄρτος λόγος. ἐναλλάξας δὲ εἰδῶν ἐλλήλοις οἱ τε ἄρτοι καὶ οἱ περίττοι παρ' ἕνα θεωροῦμενοι.

τῶν δὲ ἀριθμῶν οἱ μὲν πρῶτοι καλοῦνται ἀπλῶς καὶ ἀσύνθετοι, οἱ δὲ πρὸς ἄλληλους πρῶτοι καὶ οἱ χ' ἀπλῶς, οἱ δὲ σύνθετοι ἀπλῶς, οἱ δὲ πρὸς αὐτούς σύνθετοι. πρῶτοι μὲν ἀπλῶς καὶ ἀσύνθετοι οἱ ὑπὸ μηδενῶς μὲν ἀριθμοῦ, ὑπὲρ μόνης δὲ μονάδος μετροῦμενοι, ἅμα ὧν ε' ε' ε' ε' ε' ε' καὶ οἱ τούτοις δημοσιοὶ. λέγονται δὲ οἱ αὐτοὶ οὕτωι γραμμικοὶ καὶ εὐθυμετρικοὶ διὰ τὸ καὶ τὰ μέχρι καὶ τὰς γραμμὰς κατὰ μὲν διάστασιν θεωροῦντες καλοῦνται δὲ καὶ περισσότερος περισσότεροι ὡστε ὄνομα-μάζεσθαι αὐτοὺς πενταχῶς, πρῶτος, ἀσύνθετος, γραμμικὸς, εὐθυμετρικὸς, περισσότερος περισσότερος. μόνης δὲ οὕτως καταμετροῦμεν. τὰ γὰρ τρία οὐκ ἐν ἑνὶ ἄλλῳ καταμετρηθείν οὐκ ἐνενθηκόνται ἐκ τοῦ πολλαπλασιασμοῦ αὐτῶν, ἡ ὑπὸ μόνης μονάδος· οὗτος γὰρ τρία τρία. ὑμοῖος δὲ καὶ ἀπαξ ε' ε', καὶ ἀπαξ ε' ε'. καὶ ε' ἀπαξ ἀ' ἀ'. δυὸ καὶ περισσάκοις περισσότερος χάλκηται· οὗ τε γὰρ καταμετροῖμεν οἰκεῖοι, ἡ τε καταμετροῦσα αὐτοῦς μονάδος περισσότερος. διὸ καὶ πρῶτοι καὶ ἀσύνθετοι μόνου οἱ περιοῦσι. οὗ γὰρ ἄρτοις οὔτε πρῶτοι οὔτε ἀσύν-θετοι οὔτε ὑπὸ μόνης μονάδος μετροῦμεν, ἀλλὰ καὶ ὡς·

1 τοῦ add. apogr. 2 καὶ supra va. add. A 6 inscr. περὶ πρῶτου καὶ ἀσύνθετου A; ε' ac max ad significanda quattuor genera ε' β' γ' δ' in mg. 8 sec. vid. οἱ ὑπὸ σύνθετος ἀπλῶς καὶ πρὸς αὐτοὺς, οἱ ὑπὸ πρὸς ἄλληλους σύνθεται. 9 ε' mg. A 11 ὡς τοῦ A 15 μόνοι A 18 πολλαπλασιασμοῦ A, em. apogr.
Δέκα Αριστείων εἶχαν τετράγωνα μὲν ὑπὸ δυάδων, δίς γάρ β', δ', ἐξὸς δὲ ὑπὸ δυάδος καὶ τριάδος, δίς γάρ γ', ζ' καὶ τρίς β', ε'. καὶ οἱ λοιποὶ ὀρθοὶ κατὰ τὰ αὐτὰ ὑπὸ τῶν μεταξύνασθαι τῆς μονάδος ἀριθμῶν καταμετροῦνται, πλὴν τῆς δυάδος. ταύτη γὰρ μόνη συμβρέχει, ὅπερ καὶ ἐνὶ τῶν περισσών, τὸ ὑπὸ μονάδος μετροῦσθαι μόνον ἦπαξ γὰρ β' β' διὸ καὶ περισσοεἰδῆς εἰρηκτεῖται ταυτὸ τοῖς περισσοεἰδοῖς πεπονθηκέ. πρὸς ἀλλήλους δὲ λέγονται πρῶτον ἀριθεῖον καὶ οὕ ἀριθμὸς καὶ τὸ μέτρον μετροῦμενον τῇ μονάδι, καὶ ὑπ’ ἄλλαν τινῶν ἀριθμῶν ὡς πρὸς ἑκατοντοφόρος καταμετρώντα. οὖν ἐν ἥ μετρείται μὲν καὶ ἐπὶ τῶν β' καὶ δ', καὶ ὁ θ' ὑπὸ τῶν γ', καὶ ὁ ι' ἐπὶ τῶν δ', καὶ ε' ἔχοντα δὲ καὶ κοινῶν μέτρουν καὶ πρὸς ἀλλήλους καὶ πρὸς τοὺς καὶ ἀριθμοὶ πρῶτον τὴν μονάδα καὶ ἕφορεν ἦπαξ γ' γ' καὶ ἦπαξ γ' γ' καὶ ἦπαξ θ' θ' καὶ ἦπαξ ι' ι'.

σύνθετοι δὲ εἰσὶ πρὸς ἑκατοντοφόροι οἱ ὑπὸ τῶν ἑκατοντοφόρων ἀριθμοὺς μετροῦσιν, ὡς ὁ ε' ὑπὸ δ' ὑπὸ τριάδος. πρὸς ἀλλήλους δὲ σύνθετοι οἱ κοινοὶ ὀρθοὶ μέτρον μετροῦσιν, ὡς ὁ η' καὶ ὁ ε' [καὶ ὁ θ'] κοινῶν γὰρ ἔχοντα μέτρουν ἡ τριάδος καὶ ἡ τριάδος β' β' καὶ τρίς γ' γ'. οὕτω δὲ ἡ μονάς ἀριθμὸς, ἀλλὰ ἀριθμῷ, οὕτω ἡ ἀριθμοῦ δυάς, πρῶτῃ οὐκ ἄρτῳς ἀριθμοῦσιν δυάς ἤ τε ἄρτῳς ἄρηκτος ἔχοντα. τῶν δὲ συνεθέστων τοὺς μὲν ὑπὸ δυαδῶν περισσοεἰδῶν καλοῦσιν ἐπιπέδους, ὡς κατὰ δύο διαστάσεως θεωρου-
mēnous καὶ οἱον ὑπὸ μῆχους καὶ πλάτους περιεχομένους, 
τοῖς δὲ ὑπὸ τριῶν στεφομένοις, ὡς καὶ τὴν τρίτην διάστασιν προσελθούσα. περιοχὴν δὲ καλούσιν ἀριθμὸν τῶν 
δὲ ἀλλήλων αὐτῶν πολλαπλασιασμὸν.

tōn δὲ ἀρτίων οἱ μὲν εἶσαι ἀρτιάκες ἄρτιοι, οἱ δὲ τοι 
περιτικάς ἄρτιοι, οἱ δὲ ἀρτιοπεριτικοί. ἀρτιάκες μὲν ἄρτι 
τοῖς τοιούτοις ἑστιν] οῖς τριὰ αὐθεντημένοι, ἐν 
tό ὑπὸ δόο ἀρτίων ἐπ’ ἀλλήλους πολυπλασιασθέντων 
γεγενηθῆςαί, δεύτερον τὸ πάντα ἄρτια ἔχειν τὰ μέρη 
μέχρι τῆς εἰς μονάδα καταληκτικοίς, τρίτον τὸ μηδὲν ἀν- 
τὸν μέρος ὡμοίωμον εἶναι περιτεθ’ ὅποιοι εἶσιν ὁ λβ’ 
ξδ’ φιλ’ καὶ οἱ ἀπὸ τούτων ξῆς κατὰ τὸ διπλάσιον 
λαμβανόμενοι. τὰ γάρ λβ’ γένοντε μὲν ἐκ τʼ ἵνα 
τῇ ἑστιν ἄρτια: μέγη δὲ αὐτῶν πάντα ἄρτια, ἡμῶν ἵδ’, 
tέταρτον ὁ η’, ὁγίδου ὁ δ’ αὐτὰ τὰ μόρα ὡμοίωμα εἰς 
ἄρτιας, το ὁ ημίσιν ὡς ἐν δυάδες θεωροῦμεν καὶ 
tέταρτον καὶ ὁγίδου. ὁ δὲ αὐτὸς λόγος καὶ ἐπὶ τῶν 
λοιπῶν ὁμοίως ἄρτιμον.

ἄρτιοπεριτικοί δὲ εἶσαι οἱ ὑπὸ δυσόδος καὶ περιττῶν 
συντιμοῦσιν μετορίμουσιν, οὔτενεπὶπαντὸςπεριττὰ 
μέρος 10 ἔχουσίν τὰ ἡμίσις κατὰ τὴν εἰς ἑαυτὸς διαφέρειν ὡς τὰ 
δίς 13 ιδ’. ἄρτιας μὲν γὰρ οἴτοι καλοῦσιν περιττοῖς, ἐπὶ 
ὑπὸ τῆς δυσόδος ἄρτιας οὔθεν μετορίμουσι καὶ περισσοῦ 
tιμοῦσι, ὁ μὲν δόο τοῦ ἑνος ὁ δὲ ζ’ τοῦ γ’, ὁ δὲ ι’ τοῦ 
ε’, ὁ δὲ ιδ’ τοῦ ζ’. διεισοῦσιν δὲ οἴτοι τὴν προτήν 13

6 inscr. περὶ τῆς τῶν ἀρτίων διασφορᾶς Α, ἦ in mg. 
6 inscr. περὶ τῶν ἀρτιάκες ἄρτιών Α, cf. Zeller I p. 366 
mēn ἄρτων Δ] mēn, ἀρτίων αἰσχρ. 7 αἰ] ἡ Α 9 τὸ 
apog.) τὸν Α 11 ὡμοίωμοι: οὐ corr. ex ο Δ 11 
11 corr. ex ou Α 14 ιδ corr. ex ι] ἡ Α 18 inscr. περὶ 
ἀρτιοπεριτικῶν Α, τ in mg. 20 μεθ. Del. Hultsch
DE NUM. QUADRATIS

5 perissàmís dé ártoioi elánav óv ó pollapliasíaomóz
ék duévón óntivnvoúnon periísoú kai ártoín gíneTai, kai
pollapliasíaóntes eíz ísa mév árta méreì díka dia-
foreúnta, kara dé táis pleiòus diairésiás é mév árta-
mégh, ò dé perissà éxounai óws ó iβ' kai k' trís mév
10 ə' iβ', kai pevntáis ə' k' kai tá mév iβ' díkhì diairé-
tai (eis) ə' kai ə', toúkh dè eis ə' kai ə' kai ə', tetafgh
ðè eis tetýakis ə' tá dè k' díkhì mév eis ə', tetafgh dè
eis ə', penvtachì dè eis ə'.

5 inscr. perì perisadas ártoion Λ, Í in mg. 14 inter.
perì isákis Ísoun Λ, Í in mg. 15 kai èpíseoi fort. del.
ýmì post ɛpíseis add. Λ2 15 ãnynhèis: òc corr. ex Í 2
18 inscr. perì tavn ånysakes ånìsoún Λ, Íy in mg.
21 sqq. cf. Cantor mathemat. Beitr. zum Culturleben der
Völker p. 105 sqq. inscr. perì ãtëromakón (corr. ex ãtë-
romakón) Λ 23 tòv periísoú åtëromou wal. in tòv periísoú
åtëromou Λ.
ET ALTERA PARTE LONGORIBUS.

υ τοῦ ἄρτιον οἱ ἐπερομίχεουσ. ἐ γάρ ἀρχὴ τῶν ἀριθμῶν, τοῦτοις οὐδὲ τὴν ἐπερματικὴν τυχαία ἀπειραιομενή ἑποίησε, καὶ διὰ τοῦτο ἡ δύνας τῆς μοναδὸς ἐπερομίχεουσ. οὐδὲ καὶ μοναδῆ ἐπερέχομαι τοὺς ἄρτιον οἱ ἀριθμοὶ τῶν περισσῶν ἐπερομίχεως ποιεῖ μονάδι ἐπερ- ἐχομαι. γεννάων τε δὲ διχωθεῖ, ἐκ τε πολλαπλασιασμοῦ καὶ ἐπισυνθέσεως. ἐκ μὲν ἐπισυνθέσεως οἱ ἄρτιον τοὺς ἐφεξῆς ἐπισυνθεῖσαν τοῖς ἐπογεννημένοι ποιοῦν ἐπερομίχεως. οὐν ἐπεκλείσασθεν ἄρτιον κατὰ τὸ ἔξης β' δ' ε' η' ι' ι' ε' τ' ρ' τ' ρ', μίας δὲ καὶ ἐπισυνθέσεως β' καὶ δ' ζ', ε' καὶ ε' ι' ι' ι', καὶ ι' ι' ι', ὁμίας ἐν ὑπὸ τοὺς γεγενημένους ἐπερομίχεως ζ' ι' ι' ι' ι', ὁ δὲ αὐτὸς λόγος καὶ ἐπὶ τῶν ἔξης. κατὰ δὲ πολλαπλασια- σμον οἱ αὐτοὶ ἐπερομίχεως γεννάων τῶν ἐφεξῆς ἄρτιον 15 τε καὶ περιττῶν τοῦ πρώτου ἐπὶ τῶν ἔξης πολλαπλασια- ταιμένοι οὖν α' β' γ' δ' ε' ζ' η' ι' ἐ' ἀποτελεί μὲν γὰρ β', δ' ί' ε' τρ' τρ' δ' η' ι' τετράων δὲ δ' η', πεντάων δὲ ζ' λ' καὶ ἐπὶ τῶν ἔξης ὁ αὐτὸς λόγος. ἐπερομίχεως δὲ οἱ τουτοῦτοι τέλειωται, ἐπειδὴ πρώτην ἐτε- ροῦται τῶν πλευρῶν ἡ προσθήκη τῇ ἑτέρᾳ πλευρᾷ τῆς μοναδὸς ποιεῖ.

παραλληλογράμμοι δὲ εἰσιν ἀριθμοὶ οἱ δυάδι η' καὶ μεῖον αὐθοῦν ἡ τῆς ἑτέρᾳ πλευρᾷ τῆς ἑτέρας

DE NUM. QUADRATIS

ἐπερέχονσαν ἐχοντες, ὡς ὁ δες θ' καλ ὁ τετράγωνος ζ' καλ ὁ ἐξάκις η' καλ ο ὀκτάκις ε' ὡς ὁ η' καθ' μη' π'.

τετράγωνοι εἶναι ὁ ἐκ τῶν κατὰ τὸ εξῆς περὶσσὸν ἐκάνταθεμέναν ἀλλήλοις γεννᾶμενοι. οἷον ἐκαθένας

σαν ἐφεξῆς περισσὸν α' γ' ε' ζ' θ' κατ' ἐν καλ γ' θ', ὡς ἐστι τετράγωνος, ἴσας γὰρ ἔστιν ἴσος, τοιοῦτό δις β'

θ' καλ ε' θ', ὡς καὶ αὐτὸς τετράγωνος ἔστι γάρ τρις γ' θ' ε' θ' καλ κατ' ζ' ε', ὡς καὶ αὐτὸς τετράγωνος ἔστι τετράκις

γέφο δ' ε' ζ' καὶ θ' κατ' ὁ λάθος, ὡς καὶ πεντάκις ε' κατ' ἑκάστης ἔστις ἐν γάρ τοις τυχαίς τετράγωνοι, τῶν ἐφεξῆς περισσῶν τῷ γεννῷ ἔφορον νομοδέον τῇ πολλαπλασιασμῷ δε, ἐπειδήν ὡστὶ διὸν ἄρθυμος εὑ' ἑαυτὸν πολλα-

πλασιασθῇ, οἰον δες β' θ', τρις γ' θ', τετράκις δ' ε'.

οὶ μὲν οὖν τετράγωνοι πάντες τοὺς ἐπερέχομενες περιλαμβάνουσιν κατὰ τὴν γεωμετρικὴν ἀναλογίαν καὶ

μέσους αὐτοὺς ποιοῦσι [τοιοῦτοι τοῖς μονάδι μείζονας τὴν ἐτέραν πλευρὰν τῆς ἐτέρας ὑπερέχοισαν] οἱ δὲ ἐν ἐπερέχομενες αὐτὲ ἐν τοῖς τετραγώνοις περιλαμβάνονται ὡς μέσους εἶναι κατὰ ἀναλογίαν. οἰον α' β' γ' θ' ε'.

οὐσι τῷ μὲν ὁδῷ πληθῦνε τοὺς πολλαπλασιασμοῦντας ποιοῦσι τετραγώνοις ἀπὸ τὸ γάρ α' ε' καὶ δις β' θ' καὶ τρις γ' θ' καὶ τετράκις δ' ε' καὶ πεντάκις ε' καὶ οὐκ εἴπαινοι τῶν ἂν ὄροις ὁmega ὑποεν

3 inscr. προὶ τετραγώνων δεθήμον Α, ἵνα ἐν μγ. τε-

τράγωνοι (δε') εἶναι?  δ' θ' ε' καὶ ἵνα ἑι ἐν συμφωνίᾳ σε-

ρίμας ἀναγρ. Α ὁ δε ἀναγρ. Α 16 inscr. ὁτι ὁ τετράγω-

νος μέσος τοὺς ἐπερεχόμενας καταλαμβάνοις Α, ἵνα ἐν μγ.

16 μέσων Gelder 19 ὑπερέχοντας ἑναγρ. ἀναγρ.

22 οὕτωι ὁμοὶ οἱ Α 25 τὸν ἑδον ὅφον Α²
ET ALTERA PARTE LUNIOERIDUS.

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ēdīcēs καὶ ἡ τρίτη ἐκείνη ἐτράβηκεν, ὡστε εἰλέν ἐν τετράγωνοις οἱ ἐξῆς ο' δ' θ' ἐς ξ'· μέσος δέ ἔχουσι τοῖς ἐτερωμένοις οὕτως. τετράγωνοι δύο ἐπεξής ὃ τ' ἐς καὶ δ'· τοῖς μέσος ἐτερωμένος ο' β'· κείθουσαν δὴ α' β' δ'· μέσος γίνεται ὃ β', τῷ αὐτῷ λόγῳ τῶν ἕξων τοὺς μὲν ἐπεξής, ὡς εὖ δὲ ὑπερεχέμενος· τοῦ μὲν γὰρ ἕνως τὰ β' διπλάσια, τῶν δὲ β' τὰ δ'. πάλιν τετράγωνοι μὲν ὃ β' καὶ θ'· μέσος δὲ αὐτών ἐτερωμένος ο' ε'· κείθουσαν δὴ ε' ε' θ'· μέσος ὃ ε', τῷ αὐτῷ λόγῳ τῶν ἕξων τοὺς μὲν [κεφα] ἐπεξής, ὡς εὖ δὲ ὑπερεχέμενος· τοῦ μὲν γὰρ δ' τὰ ε' δ' τῆς ἑτερωμένης, τῶν δὲ ε' τὰ θ'. ὃ δὲ αὐτός· έγος καὶ εἴπ τῶν ἔξω. οἱ δὲ ἐτερωμένοις. ἕπο τοὺς τ'· μονότα ὑπερεχέμενοι πολλαπλασιαζόμενοι, οὔτε μένων τίνι ἐν τοῖς ἑδώς ὅροις οὔτε περιεχοῦσι τοὺς τετραγώνοις· οὖν τὰ δὲ ε' γεννήθην τὸν σ' καὶ τὸ τρίς ος δ'· γενικά τίνι ο' β' καὶ τὰ τετράγωνα ἐγενή τῶν α'· καὶ οὐδὲις· οὔτε ἐν μένει ἐν τῷ ἑαυτῷ ὅρῳ, ἀλλὰ μετασάρκευσ τῷ πολλαπλασιαζόμενοι. οὖν δένες ἐπὶ τρίς· καὶ τρίς· ἐπὶ τετράγωνοι καὶ τετράγωνοι· ὃ τοῖς ἐτερωμένοις· οὓς τετραγώνοις· ὃ τοῖς ἐτερωμένοις· οὐκ ἠμήκαντε δὲ αὐτῶν ὃ τῇ τετράγωνος ο' δ'· ἀλλὰ κατ' οὐδὲν ἀναλήγην περιεχοῦσι τῇ εἴπτων ὡστε ἐν τῷ αὐτῷ λόγῳ πρὸς τὰ ἕξων εἶναι· ἐκείνης γὰρ β'· δ'· ε'· ἡ τετράγωνος ἐν διαφόροις· λόγοις· πρὸς τὰ ἕξων· γενήσεται· τῶν μὲν γὰρ β' τὰ δ' διπλάσια, τῶν δὲ δ'· 9 μέσος· ὃ τέρα τ' Θ'· 10 ἔχρα· om. apogρ. 20 ὑπά τοὺς ἐτερωμένοις· del. vid. 23 συνέ σιν α'· A· om. ἐκείνης· 24 αὐτός· τερα· add. Α· ἐκείνης· γὰρ· ἐκείνης· om. apogr. 26 τοῦ τοῦ πλάσμα· α' om. ἐκείνης·
3 το τρίτον ἀρχεῖ η 57 τον τρίτον Α ἀνάλογος μέσον ἐλείά. ὦ αἰτί ὑπότος μέσον εἶναι, ὥστε ὃν ἔχει λόγον το πρῶτον πρὸς το μέσον, τούτον το μέσον πρὸς το τρίτον. πάλιν τῶν εκεῖ τῇ μείζον τῇ ἔκμιθοι μέσος τῇ τέτευξε τετράγωνον ἐκ θυ' ἐπεξεργάσαι τὸ τοιοῦ ὁ πρὸς τὸ ὑπὸ τούτον μέσον τῷ μὲν γέρο εἴ τῇ θῇ ἠμιδεῖς, τῶν δὲ θῇ τῇ ἐπίτρεπτα, ὦ δὲ αὐτῶς καὶ ἐπὶ τῶν ἤδη λόγος.

προμήχησα δὲ ἐστὶν ἀριθμὸς ὃ ὑπὸ δύο ἀνίσων ἀριθμῶν ἐποτελεῖμενος ὀνοματοῦν, ἡ μονάδι ἡ δεύδι 10 ἡ καὶ πλέον τοῦ ἐτέρου τοῦ ἐτέρου ἐπερχόμενος, ὡς ὁ κα', ἐστὶ μὲν ἐξεικές ὅ', καὶ οἱ τοιοῦτοι. ἐστὶ δὲ τρία μέρη τῶν προμήχων. καὶ γὰρ πέλες ἐπομήχης προμήχης, καθὼς μεῖζον τὴν ἐτέραν πλευρὰν τῆς ἑτέρης ἔχει. ὡστε εἰ μὲν τῆς ἐπομήχης, ὦτος καὶ προμήχης οὖ οὐ μὴν ἀνέσαλλον ὁ γὰρ μεῖζον πλέον ἡ μονάδι τῆς ἑτέρας ἔχον πλευρὰν προμήχης μὲν. οὐ μὴν ἐπομήχης· εἰς ἐπομήχης ὁ μονάδι μεῖζον τῆς ἑτέρας ἔχον πλευρὰν, ὡς ὁ ε' ἐστι μὲν δὴς γ' 'ε'. ἔστι προμήχης καὶ ὁ κατὰ διαφορὰν πολλαπλασιασμοῦ ποτὲ μὴν μονάδι 20 μεῖζον τὴν ἑτέραν πλευρὰν ἔχον, ποτὲ δὲ πλέον ἡ μονάδις ὡς ὁ ε' ἐστι μὲν τρὶς δ' καὶ δὶς ε' ἡμεῖς κατὰ μὲν τὸ τρὶς δ' εἰς οἱ ἐπομήχης, κατὰ δὲ τὸ δὺς ε' προμήχης ἐτί προμήχης ἐστὶν οἱ κατὰ πάσας τῶν ὀχιδίας τῶν πολλαπλασιασμῶν πλέον ἡ μονάδι μεῖζον εἰς τὴν ἑτέραν ἔχον πλευρὰν ὡς ὁ μ' καὶ γὰρ τετράγωνον τῇ

3 το τρίτον ἀρχεῖ, τὸν τρίτον Α ἀνάλογος μέσον ἐλείά. 8 εἰς θ᾽ ἐβ' ἀρχεῖ. 9 ἐκ τῆς ἀρχῆς ἢ 12 μέρις.] ἢ ἡ 11 ὅθεν ἢ κατὰ προμήχης ὡς ὁ μ' καὶ γὰρ τετράγωνον τῇ
DE NUM. PLANORUM GENERATIONE. 31

καὶ πεντάς ή καὶ δίς ξ', ὡστις καὶ μόνος ἂν ἐγ' προ-
μῆς. ἐτερομῆς γὰρ ἔστιν ὁ ἐκ τῶν ἱδὼν ἁρμήμων
τὴν πρώτην λαμβάνων ἐτερότητα· ἢ δὲ τῆς μονάδος τὸ
ἐτέρο ἁρμήμη προσθήκη προφίτην ποιεῖ ἐτερότητα· διὸ
οἱ ἐκ τούτων ἱππός ἀπὸ τῆς πρώτης τῶν πλευρῶν 5
ἐτερότητος ἐτερομῆν. οἱ δὲ πλέον ἡ μονάδι τὴν ἐτέ-
ραν πλευρὰν μείζονα ἐχοντες διὰ τῶν ἐπὶ πλέον προ-
βιβασμὸν τοῦ μέγας προφίτης κέκληται.

εἰσὶ δὲ τῶν ἁρμήμων οἱ μὲν ἐπίπεδοι, ὅσοι ὑπὸ
διὸ ἁρμήμων πολλαπλασιάζονται, οἱ δὲ μέγας καὶ πλα-
τοὶ τοὺς, τούτον δὲ οἱ μὲν τρίγωνοι, οἱ δὲ τετράγωνοι, οἱ
δὲ πεντάγωνοι καὶ κατὰ τὸ ἐξῆς πολύγωνοι.

gεννᾶται δὲ οἱ τρίγωνοι τῶν τριῶν τούτων.
[οὖσαι] οἱ ἐφεξῆς ἄρτιοι ἀγαθοὶ ἐπίσεως ἑπισυνεδριακοὶ
κατὰ τὸ ἐξῆς ἐτερομήνης ἀρμήμων πολλά.

τὸ σχῆμα αὐτὸν ἔσται ἐτερομῆς κατὰ μὲν γὰρ τὸ
μέγας ἐστὶν ἐπὶ διὸ, κατὰ δὲ τὸ πλάτος ἐρ' ἐν. μετὰ
τὰ διὸ ἔστιν ἄρτιος ὁ δ'. "Αἱ ἐστὶν προσθήκαι τοῦς πρῶτος 25 .

1 μόνος 3 Λαμβάνων Λ, em. progr. 4 πρῶτην corr. 7 Λαμβάνων ut vid., antea una litt. erasa Λ 9 inscr. περὶ 13 inscr. περὶ τρι-
ἐπιτίθεον ὑποθεῖν Λ, ἰδ. in mg. 11 Λαμβάνων 13 Λαμβάνων, ἁρμήμων ποιοθεῖ τοῖς χρωτεῖται καὶ περὶ τῶν ἐξῆς
πολυγώνων Λ, x in mg. 23 χρωτεῖ τοῖς compendium eius-
dem voculæ erasum in Λ. 24 ἐρ' τῷ Λ 25 ἐν Λ
DE NUM. PLAN. GENERATIONE.

32

δύο ἀληθείας: [εventing περιθομένον τὸ δ' τοὺς β', γίνεται ἑτερόμορφος τὸ τῶν ε' σχῆμα: κατὰ μὲν γὰρ τὸ μήκος γίνεται ἐπὶ τοῖς, κατὰ δὲ τὸ πλάτος ἐπὶ β'. ἦταν ἐρατος μετὰ δ' ὧν ἐν προσθήκῃ ταῦτα τοὺς πρῶτοις ε', γίνεται ὁ ε', κἂν περιθομένος εἰτέ τοὺς πρῶτος, ἦταν σχῆμα ἑτερόμορφος ὥς ἦρεν ταῦτα κατὰ τὸ μήκος μὲν δ', κατὰ πλάτος δὲ γ'. καὶ μήκος ἐπείγου ὁ αὐτὸς λόγος κατὰ τὴν τῶν ἐρετοὺς ἐπιτείχειν.

πάλιν δὲ οἱ ἐξής περισσοὶ ἀληθείαις ἐπισυνειδημέναι ἦν τετεράγωνοι ποιοῦσιν ἑράμοις. εἰς δὲ οἱ ἐφεξής περισσοὶ α' γ' ε' θ' ὡς. ταῦτα δὲ ἐφεξής συνεπεῖς ποιήσεις τετεράγωνος ἑράμοις. οἶον τὸ ἐν πρῶτον τετεράγωνον ἦτοι μᾶρ ἄπαξ ἐν ἐν. εἶτα περίσσος ὁ γ' τοὐτὸν ἐν προσθήκῃ τοῖς γνώμονα τῷ ἐνὶ, ποιήσεις ἰσοτετράγωνον ἴσως ἢν· ἦταν γὰρ κατὰ μήκος β' καὶ κατὰ πλάτος β'. ἐφεξής περίσσος ὁ ε' τοῦτον ἐν περιθομής τῶν γνώμονα τῷ δ' τετεράγωνο, συνήθως πάλιν τετεράγωνον ὁ θ', κατὰ μήκος ἦρεν γ' καὶ κατὰ πλάτος γ'. ἐφεξής περίσσος ὁ ε' τοῦτον ἐν προσθήκῃ τῷ θ', ποιεῖς τῶν ε', καὶ κατὰ μήκος δ' καὶ κατὰ πλάτος δ'. ὁ δὲ αὐτὸς λόγος μέχρις ἐπείγου.

κατὰ ταῦτα δὲ ἂν μὴ μόνον τοὺς ἐφεξής ἐρετοὺς.

2 2ος supp. vs. A 5 περιθομένος Bull. προσθήκῃ A τοὺς πρῶτος τοὺς ε' ε' cf. vs. 1 5 ταῦτα del. vid. 8 fig. semper lineas circumscir. A 14 περιθομένος Gelder 22 ταῦτα corr. ex tauta Α
καὶ ἔ μονον τοὺς ἐ δεῖξε περὶ σαφένες, ἀλλὰ καὶ ἱστορίας καὶ περίσσως ἥδη ἔφαινε ἐπίσκινθαι, τρίγωνες ἦμιν ἐπιμελεῖς γενέσθαι. Ἐκκείσθομεν μὲρ ἐ δεῖξε περὶ σαφένες καὶ ἐφεύρεται, α' β' γ' δ' ε' ς' η' θ' ι'. γίνονται κατὰ τὴν τούτων ὀψιθέαν οἱ τρίγωνοι. πρὸ τοῦ μὲν ἡ μορφής οὔ ἐπιγενής, εἰ καὶ μὴ ἐπεκείνει, δυνάμει πάντα ἐπιτη, ἄμως πάντων ἐφθακόν ὤσα. τῇ δὲ ἐξ ἑαν αὕτη δυνάμει προστιθέντος γίνεται τρίγωνος θ' γ' εἶτα πρόσθες γ', γίνεται ε' εἶτα πρόσθες δ', γίνονται ε' εἶτα πρόσθες ε', γίνονται ε' εἶτα πρόσθες ζ', γίνονται κα' εἶτα ιο πρόσθες η', γίνονται θ' εἶτα πρόσθες δ', γίνονται με' εἶτα πρόσθες ι', γίνονται νε' καὶ μέχρις ἀπείρου οἱ αὐτῶς λόγοι. δὴ λόγοι δὲ ὅτι τρίγωνοι αὐτῶν οἱ ἰδιότηται κατὰ τὸν σχηματισμὸν, τοῖς προίκοις ἐφημοί τοῦ ἐ δεῖξες γρόνοις προστιθέ-ις μένων' καὶ εἶν αὐτοὶ ἐν οἷς οἱ τῆς ἐπισκίνθεως ἐπορευμέ-μενοι τρίγωνοι ὦδεις γ' ζ' ι' ε' κα' η' λζ' με' νε'. καὶ οὕτως ἐπὶ τῶν ἐξ ι' τῶν με' καὶ νε'.

4 γίνονται Α' 6 ἐπισκίνθεα εὐθ. ex ἐ δεῖξεις Α'
19 γίνονται κα' γίνονται compositio eur. A ut in sequentibus
18 τῶν με' καὶ νε' del. vid.

Thes Smyræ.
DE NUM. MULTIANG. GENERATIONE.

οί δὲ τετράγωνοι γεννώνται μὲν, ὡς προείρηται, ἐν τοῖς ἐξεχθές ἀπὸ μονάδος περίττων ἀλλήλων ἐπίσωτερον· συμβέβηκε δὲ αὐτοῖς ὅτι ένεκεί παρ᾽ ἕνα ἐφήθη εἰς καὶ περίττος, διότι ὁ παῖς ἑφθαζεν παρ᾽ ἕνα ἐφήθη ἕστατον ἡ περίττος, οἷον δ᾽ θ᾽ εξ' ἕξ' μοῦ' εἴς' πά' ᾗ δὲ ἀπὸ μονάδος κατὰ τὸ ἐξής ἔλησε, τῶν ἐφήθη τε καὶ περίττων ἑρμῆν συμβέβηκε, τοὺς γνοίζοντας τοὺς ἀνάλογα ἀλλήλους ὑπερέχοντες ἐν τῇ συνθέσει τετραγώνων ἐποτελεῖ, ὡς ἐπάνω ἐποδέδειται. 10 ὑπερέχοντοι γὰρ ἄναλοι ἀλλήλου ἀπὸ μονάδος ἐφημενοί (οι) περίττοι. ὑμεῖς δὲ οἱ τριάδες ἐλάλημεν ὑπερέχοντες ἐν τῇ συνθέσει ἀπὸ μονάδος πενταγώνων ἐποτελουόμενον, ἔξεργον δὲ οἱ τετράδες. οἳ τοῖς νυνιμώς 15 ὑπ’ ἑνὸς ἐποτελοῦνται οἱ πελάγηνοι ὑφάδες ἐστὶ πλῆθος τῶν ἐποτελουμένων γυαλῶν.

ἔτην δὲ πάνω ἐστὶν ἄλλῳ ἐν τοῖς πολυγώνωσιν τούς ἀπὸ μονάδος πολυελθεῖσιν ἀκριβωθεὶς. τῶν γὰρ ἀπὸ μονάδος πολυελθεῖσιν, λέγοι δὲ διπλασία τριπλασίον καὶ τῶν ἐξασ, εἰ μὲν ἐνα παρ᾽ ἑνα δισελέπτοντας ἐφημαί τοι τετράγωνοι πάντες εἶναι, οἱ δὲ δύο δισελέπτοντες κιβοῦ πάντες, οἱ δὲ πεντε δισελέπτοντες κιβοῦ ἦμα τοῖς τετράγωνοι εἶναι καὶ τῷ μὲν πλευρῷ ἔχοντος τετραγώνοι

1 inser. περὶ τῶν ἑξάς πολυγώνων Α, ἐν μιν.
αổα τὸν θὸν ἐν αἰ τοῖς.
5 in μιν. sup. cod. A hace scritta sint:

δ᾽ θ᾽ εξ' ἕξ' μοῦ' εἴς' πά' ᾗ ἀπὸ ἀνάλος πολυελθεῖσιν.

12 πενταγώνων corr. ex τετράγωνοι A 13 ἡ πυρὰ ντ. A
DE NUM. QUÆDATIS ET CURIÉS.

5 în mēn tois dīplēiōnōn ďlōn oûtōs? 6 pro hac numero-
tum serie Gelderas hanc posuit: a' b' d' η' ε' ι' ι' ι' ε' ι' x' x' x' x' propôs dīplēiōnōn 6 b' eîta 6 d' 6 ἐστὶ tētrāγωνων' ēîta 6 η', 6 ἐστὶ κυῖδος'. ēîta 6 ε' 6 ἐστὶ tētrāγωνων' ēîta 6 λβ' me̊θ' 6n 6 ξ', 6 ἐστὶ tētrāγωνων ὅμα καὶ 6 κυῖδος' 6 ἐστὶ Ṿκη' 6 me̊θ' ὅν συζ', 6 ἐστὶ tētrāγωνων' καὶ μέχρις ἀπείρου 6 autǒs lógoς. καὶ ἐν τῷ τριπλέων ἐνεργήσατο ὅν περ' ἐν τῷ tētrāγωνων, καὶ ἐν τῷ πεν-
tapλεων, καὶ κατὰ τῶν ἐξῆς πολλαπλασίων. ὅμως ὃ ἐν ἐνεργήσατο καὶ ὃ δὴ διαλείποντες ἐν τῶν πολλ-

10 πλεώνων κυῖδοι πάντες, καὶ ὃς ἐδιαλείποντες κυῖδοι ἐμα-

DE NUM. SIMILIBUS.

hwv τοιον μήτε τέτερον μήτε τέταρτον έρωτε μονέδος ἀγαθοκριτεύς και τρίτον ἤλθεν και τέταρτον, οὔ τι λέει.

ἐπὶ τὸν ἐρυθρὸν οἱ μὲν ἡςάρχα ἡσυχασμένοι οὖν, οὐ δὲ ἀναρχόμενα ἐπερμηνεύεις καὶ περιθηκῶς,

καὶ ἀπλοῦσι οἱ δίχως πολλαπλασιώτεροι ἐπικεφαλος, οὐ δὲ τρικῶς στεφείτο. λέγονται δὲ ἐπικάθους ἐρυθροὶ καὶ τρίτως καὶ τέταρτον καὶ στέφειτο καὶ τάλαν τὸν νυ-

νίῳ ἀλλὰ επίκαθως τοῖς χαλῖαν ἐκατεμιτρο-

σεῖν' ὁ γὰρ δ', ἐπεὶ τετράγωνον περίον κετεμιτροῖ, ἐκ' ἰν' αὐτοῦ καλεῖται τετράγωνος, καὶ ὁ 'ε διὰ τὰ αὐτὰ ἐπερ-

μηχνής.

ὁμοιοὶ δ' εἰσὶν ἐρυθροὶ ἐν μὲν ἐπικάθους τετράγω-

νοι οἱ πλέον πάλιν, ἐπερμηνεύεις δὲ ὅσον ἐδὲ πλευραῖς, πολλαπλασίας αὐτοὺς ἐρυθροὶ, ἐναλόγοιον

ν έσειν'. οἱν ἐπερμηνεύης ἢ τὰ 'γ' πλευραὶ δὲ αὐτοῦ μή-

χος 'γ', πλάτος 'β' ἐπερμηνεύης τὸ κάθ' πλευραῖς δὲ εὐτειχὸς μὴν 'γ', πλάτος δὲ δ'. καὶ τότε πρῶς ἡ τοιχαίναι πρὸς τῷ μή-

χος 'γ' πρὸς 'γ', στεφείτο δ' πρὸς 'β'. ὁμοίοι οὖν ἔριζο-

ται ἐπικάθους τὰ τὰ 'ε' καὶ τὰ 'κα'. σχηματιζόμενοι δὲ οἱ ἐπικάθους ἐρυθροὶ ἐν μὲν εὐς πλευραῖς ἡ τοιχαίναι τοιχαίναι, ἢ τὸν καὶ πρὸς

τοιχαίναι οὔσωσι λειψανόντες, ὅτε δὲ εὐς ἐπικάθους, ὅταν ἐκ πολλαπλασιώμενον δύο ἐρυθροῖ γεννηθοῦσιν, ὅτε

1 Α' 3 Α 3 in loc. περὶ ἐσέκεις ἡσέταν καὶ ἐπι-

τείχους ἠμέρως Α, καὶ εἰς μν. τοιχαίναι επικαθοί Α. Β τετράγωνον προτερ. ἕ γ. ε. τετράγωνον) Α. 12 in loc. περὶ ἐσέκεις ἡσέκεις ἡσέκεις Α, καὶ εἰς μν., διοργανών Α. Β

14 αὐτοῦ) αὐτὸς Α, cf. p. 21, 29 19 δ' πρὸς Β ἀποφρ. Β πρὸς Β Α 21 ἡ τοιχαίναι Καὶ ἡ τοιχαίναι Β πρὸς Β Α 22 ὡς μήκος καὶ πλάτη καὶ ἡ τοιχαίναι; Β δ' Α ἀποφρ.] Β δυσ Α
DE NUM. TRIANGULIS.

37
dé eis stereóiōs, ὅτιν ἐκ πολλάκισσι ὁμοιοί τριῶν λη-
θοδώσιν ἀριθμοὺς. Ἐν δὲ τοῖς στερεόις πάλιν οἱ μὲν κύ-
βοι πίντες πεδίων εἰσίν ὁμοίοι, τῶν δὲ ἔλλοι οἱ τέσ-
τε πλευραὶ ἐχούσες ἀνάλογον· ὥς ἡ τοῦ μέγιστος πρὸς τὴν
tоῦ μικρούς, οὕτως ἡ τοῦ πλέονος πρὸς τὴν τοῦ πλα-
tους καὶ ἡ τοῦ ὄψους πρὸς τὴν τοῦ ὄψους.

tὸν δὲ ἐπιπέδων καὶ πολυγώνων ἀριθμῶν πρὸς τὸ τρίγωνον,
ὅς καὶ τῶν ἐπιπέδων εὐθυγραμμῶν σχη-
mατῶν πρὸς τὸ τρίγωνον ἦτο τὸ τρίγωνον. πῶς δὲ γεγονότεκα
προσέρχεται, ὅτι τὸ πρῶτο ἀριθμὸ τοῦ ἕξις ὀρθῶν καὶ τὸ
περιττὸ προσετθεμένῳ· πάντες δὲ οἱ ἐπιθῆς ἀριθμοὶ,
ἀπορχενιότες τριγώνοις ἡ τετραγώνως ἡ πολυγώνως,
γνώμονες καλοῦνται. τοσοῦτον δὲ μονάδων ἑκάστον
τρίγωνον ἦτο πλευρὰς πάντως, ὡσιν καὶ μοῦς ἦσιν
ὁ προσαλεμβάνων ἀριθμόν. οὐν ἐστι τρίγων ἤ ἡ
μοῦς, λειτομίνη τρίγωνον οὐ κατενελέξκων, ὡς
προπερίκειαν· ἀλλὰ κατὰ δύναμιν· ἐπεὶ γάρ αὐτῆ ὁ ὀνὸν
στέρα πλέκων ἦσιν ἀριθμοῖς, ἐλεί ἐν αὐτῇ καὶ τρι-
γώνων ὅμως. προσαλεμβάνοις γοῦν τὴν ἀνάδω
ἀποτελεῖ τρίγωνον, ἢ ἔχουν πλευρὰς τοσοῦτον μοῦδαν, δι
ὅσον ἦσιν ὁ προσαλεμβανῶν γνώμον τῆς ὀνάθες. τὸ δὲ
ὅλον τρίγωνον τοσοῦτον ἦσιν μοῦδαν, ὡσιν καὶ οἱ
συντεθεμένως γνώμονες, ὁ τε γάρ τοῦ ἐνός καὶ ἡ τῶν
ὀυδέν γνώμον τὰ γ' ἐποίησαν, ὡστε καὶ τὸ τρίγωνον

7 incer. περὶ τριγώνων ἠγιασμένων Α, ἐκ ἔκ τινς.
10 προτείσχθηται: p. 37, 1 11 cf. Nesselmann p. 203 14 τὰς
πλευρὰς αὐτῶν ἔθη Α1 πάντως συν. εἰς πάντως Α. 17 προ-
τεθεμένως p. 33, 6 24 τοσοῦτον, τίνι ἐκ τοῦ του πλή-
πρὸ τοῦ τριγώνου, ὡσιν μοῦδαν τὸν ἠ προσαλεμβανῶν ἀρι-
θμόν τοσοῦτον ἦσιν αἱ τῶν γνώμων μοῦδαν, ὡσιν 
εἶναι οἱ γνώμονες οἱ εἰς τὰ τρίγωνα εὐθεῖότες έκ τοῦ.
38 DE NUM. CYCLICIS.

δέται μὲν τριῶν μονάδων, ἐξεῖ δ' ἐκάστην πλευρὰν τῶν δικών, ὡσι καὶ οἱ γνώμονες συνετήθησαν. εἶτα τὸ γ' τρίγωνον προσλαμβάνει τῶν τῶν γ' γνώμονα, δ' μονάδι ὑποδέχεται τις δυάδος, καὶ γίνεται τὸ μὲν ὀλον τρίς γωνιῶν ε' πλευρῶς δ' ἔξει τοσοῦτον μονάδων καὶ τούτῳ τὸ τρίγωνον, ὡσι γνώμονες συνετήθησαν· ἐκ γὰρ τοῦ ἐνὸς καὶ γ' καὶ γ' συνετήθη δ' ε'.

εἶτα θ' προσλαμβάνει τῶν δ' γίνεται τοῦ τοῦ ε' τρίγωνον, ἐκάστην πλευρὰν ἔχουν δ' μονάδων· ὁ μέσος προσλαμβάνει τῶν ε', καὶ γίνεται τοῦ τοῦ ε' τρίγωνον, πλευρὰν ἔχουν ἐκάστην μονάδων ε', καὶ τοῖς ε' γνώμονα συνετήθη. όμοιος καὶ οἱ εξ γνώμονες .

15 τοῖς γνωριμοικοῖς ἐφιμοῦν· ἀποτελοῦσι.

λέγουσα δ' τινὲς καὶ καλλομείδες καὶ συγκαλλομείδες καὶ ἀποκαλλομείδες ἐφιμοῦν· οὕτω δ' εἰδίν οἴτινες· ἐν τῷ πολιτικομείδειον ἡ ἐπιτείνοις ἡ στερεώμεις, ποιμένικη· κατὰ τό διόν ἀπακτήσας κατὰ τρεῖς, ἀρ' οὐ ἐν τῷ ἐξομοίωσι ἐφιμοῦν· ἐπὶ τοσοῦτον ἀποκαλλομείδεις· τοιοῦτον δὲ ἐστὶ καὶ οἱ κύκλοι· ἀρ' οὐ ἐν ἐφιμοῖσι σημεῖον,
DE NUM. QUADRATIS ET PENTAGONIS. 33

ēpī toūtō ἀποκαθίσταται: ὑπὸ ἑρὸ μιᾶς γραμμῆς περί-
εύομενος ἀπὸ τοῦ αὐτοῦ ὀρθοῦ καὶ τῆς ταύτης καταλή-
γει. τοιαῦτη δὲ καὶ ἐν στεφάνῳ ἢ σφαιρᾷ κύκλων γρά-
κατά πλευράν περιμετρῶν ἢ ἀπὸ τοῦ αὐτοῦ ἢ τὸ
αὐτό ἀποκαθίστατος σφαιρᾶς γράφει. καὶ ἀριθμὸι δὴ ἢ
οἱ ἐν τῷ πολλαπλασιασμῷ ἐφ’ ἐαυτοῦ καταλήγουν. τὸ
κυκλικὸ τε καὶ τοῦτο καὶ σφαιροειδεῖς ὃν εἴσαν ὃ τε
ἐ’ καὶ ὃ ἢ’ πεντάκες γράφες ἢ κε’, πεντάκες κέ’ ὀκε’, ἐξή-
κις ἢ’ λζ’, καὶ ἐξάκις ὧν ἢ’.

tῶν δὲ τετραγώνων ἢ μὲν γένεσις, ὅς ἐπον, ἢ τὸ
tῶν περισσῶν ἀλλήλους ἐπισυνετεθηκέντων, τοντεῖ
tῶν ἐπὸ μονάδος διὰ ἀλλήλου ὑπερεχόμενον ἐν γράφε
καὶ ἢ’ δ’, καὶ δ’ καὶ ὧ’ θ’, καὶ θ’ καὶ ἢ’ ęż’, καὶ  Jazeera
καὶ θ’ καὶ.

πεντάγωνοι δὲ εἴσαν ἀριθμοὶ οἱ ἐκ τῶν ἢ ὑπὸ μοιά-
δος κατὰ τὸ ἢ’ δ’ τριάδι ἀλλήλους ὑπερεχόμενοι ὥν
τεθηκέντο. ὃν εἴσαν οἱ μιὰν γραμμήν τε ἢ’ ἢ’ ęż’ ἢ’ ęż’ 
τεθηκέντο. εὐτὸ καὶ δὲ οἱ πενταγώνοι ἢ’ ἢ’ ęż’ ἢ’ ęż’ καὶ ἢ’ ęż’
ἄροις. σχηματίζονται δὲ πενταγωνίων οὔτας.

α’  ι’  ιβ’  ιθ’  λθ’
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α
α  α  α  α  α  α

10 inscr. περὶ τετραγώνων ἀριθμῶν Α. εἶπον: p.
23, 3, 32, 9, 34, 1 11 inscr. περὶ πενταγώνων ἀριθμῶν
Α. ἦς in mg.
DE NUM. MULTIANULIS.

ēxāgonoi de eisān ērīmīos oi ēk tōn kata το ēxēs ἀπὸ μονάδος τετράδι μᾶλλον ὑπερεχθῶν συντιθήμε-

νος' δι' τὸν γνώμων εἰς α' ζ' θ' η' ι' κα' κε' ὁ δὲ ēk tōu toun ēxēgonoi oide' α' ζ' η' με' ζ' η' ι'. σχη-

μαζίζονται δὲ οὖτως·

a' ζ' η' ε' με' ζ'

ἐπιέγονοι δὲ εἰσὶν οἱ ἀπὸ μονάδας πεντάδι μᾶλλον ὑπερεχθῶν συνιστάμενοι' δι' τὸν γνώμων μὲν α' ζ'

ζ' μὲ ζ' κα' κε'. οἱ δὲ ēk tōu toun συντιθήμενοι α' ζ' ι' λ' ν' π'. ὁμοίως δὲ καὶ σχημάτων ὁ<ο> ἀπὸ

τοῦ μονάδος ἐξέδεις ὑπερεχθῶν συνιστάμενοι' ἐν


καὶ τῶν πολυμόρφων καθώς ὅσιώμος ἢν λέγεται μὲν ἀριθμός, ψευδών δεοῦσαν μονάδον τοῦ πλήθους τῶν

1 inscr. περὶ ἔξαγων ἐφιεμῶσιν A 2 inscr. ὀρνίατον ἀνὴρ καὶ ἐπὶ τῶν λοιπῶν πολυμών Α

ἐκ τῶν hic et in illo quae sequuntur negligenter omissum 11 μονάδος Λ ἐπὶ τῶν λοιπῶν πολυγώνος Λ

13 ἀνὴρ κάτων ἀνὴρ καὶ ἀνὴρ καὶ ἀνὴρ A 15 ὀρνίατον ἐπὶ τῶν λοιπῶν τοῦ πλήθους τῶν 62
DE NUM. SOLIDIS.

41

γωνίων ἢ ὑπεροχὴ τῶν ἀριθμῶν λαμβάνεται, ἐξ ὧν οἱ πολλάκιοι συντιθένται.

ἐκ δύο τριγώνων ἀποτελεῖται τετράγωνον' α' καὶ γ', δ', γ' καὶ ε', ε' καὶ κα' λ. κ., κα' κα' νθ. η′ καὶ λο' λε' λε' καὶ με' πα', καὶ οἱ εἴδη τῶν ὀμοίως συνδυαζόμενοι τρίγωνοι τετράγωνος ὀπτελεῖσθαι, ὡς καὶ ἐπὶ τῶν γραμμικῶν τριγώνων σύνθεσις τετράγωνον σχήμα ποιεῖ.

ἐκ τῶν στερεών ἀριθμῶν οἱ μὲν ἱσας πλευρὰς ἔχουσιν, [ὡς ἀριθμοὶ τρεῖς θρόων· ἦλ θρόων πολλάκιοι συνδυαζόμενοι], οἱ δὲ ἐνδύσουσι τούτων δ' οἱ μὲν πάσας ἐνιαυτοῖς ἔχουσι, οἱ δὲ τὰς δύο ἱσας καὶ τὴν μιὰν ἱσάντια, πάλιν τε τῶν τὰς δύο ἱσας ἔχοντων οἱ μὲν μεῖξινα τὴν τρίτην ἔχουσι, οἱ δὲ ἐλάττονα. οἱ μὲν οὖν ἱσας ἔχουσι πλευρὰς, ἱσαὶς ἱσοὶ ἱσοὶ ὀντες, κύβοι καλοῦται οἱ δὲ πάσας ἐνδύσουσι τὰς πλευρὰς, ἐνδύςας ἑνίσθοι ἐνιαυτοῖς ἱσαῖς κυβοῖς καλοῦσι· οἱ δὲ δύο μὲν ἱσας, τὴν δὲ τρίτην ἐκτέταρτος τῶν διέτης ἱσάσιν, ἱσαὶς ἱσοὶ ἑλάττονικαι, πλευρίδες ἐκλήθησαν· οἱ δὲ δύο μὲν ἱσας,
DE NUM. PYRAMIDALIBUS.

τὴν δὲ τρίγων ἑπτάγων τῶν δύον μεῖζον, ἵνα τοιαύτα, δοξίδες καλοῦνται.

εἰς δὲ καὶ πυραμιδεῖς ἀριθμοὶ πυραμίδας κατα-
μετροῦντες καὶ χοροφοπυραμίδας. κόλουρος δὲ πυρα-
κὼς μέγις ἔστω ὑπὸ τὴν κορυφὴν ἀποστρεμένη. τινὲς δὲ
[κόλουρον] τὸ τοιοῦτον τριαγών προσγράμματαν ὧπο
τῶν ἑπτάγών τριαγών τριαγών γὰρ λέγεται, ὡμεν
τριγώνον ἢ κορυφή ὑπὸ περισσότερον τῇ βάσει εὐθεῖας
ἀπομεμιήθη.

10 ἀδὲπερὶ δὲ τρίγωνικοὺς καὶ τετραγωνικοὺς καὶ πεν-

2 ad figuras, quae eis descritae sunt, pertinent, haec adeo talio marginis λι: το ἑπτάγων σχεδίων ἐτοι ἐσσον,
tὸ ὑποκάτω μεῖζον. 3 inscr. περὶ πυραμιδῶν ἀριθμῶν
Α, κῆ in mg. πυραμίδα Α 4 κολοφοντομαῖδα: sc corr.
ex τι Α 10 inscr. περὶ πλευρικῶν καὶ διαμετρικῶν
ἀριθμῶν Α, κηθ in mg. cf. Nesselmann p. 223 sqq.
DE NUM. LATERALIBUS ET DIAGONIS.

43
temnonicoque cal matra tis loipta, schimata logou exouni
unetemei ois aerismi. outheis kai pleuricois kai diame-
troikoi logous eufoimae apati tais spemeratikoi
logouis epiphanemounous touis aerismoi. ex gar tois
phonizetai tis schimata.宽容 ouv panthei tais schimais
oun kata tais anovatous kai spemeratikous logous h
mounas
exhrei, outheis kai tis diameprou kal tis pleurois logous
ev tis monadies einofisketai. ouv ekpistetai duo monai
dun tis mepi theinei diaimetrou, tis de pleurois,
epihe tis monai, pantoun outhei orxhyn. det dunemey
kai pleurois einai kal diameprous. kal prostatgetai tis
mepi pleuroi diaemetrou, tis de diameprou duo pleuroi,
epihe ouon h pleuroi deix dunamei, h diaimetrou epixe.
epetheto ouv meizoun mepi h diaimetrou, elattous de h
pleuroi. kal eti mepi tis prophoris pleurois te kal dia-
15 metrou eu ev tis apo tis monaidos diaemetrou tevar SOUND
monadis me elattov h diplaisioun tov apo tis mona-
dou pleuroi tevarous h ev doutei chei el monai
to o en tais evos monadis elattov h diplaisioun. pros-
theimeni deti tis mepi pleuroi diaemetrou, toutietai tis
monadis eli h pleuroi oras duo monaidoun. tis de dia-
metrou prosstheimeni duo pleurois, toutietai tis
monadis duo monaidoun. eli tis

4 12 de metrou A 15 y mona-
dedra (ex corr.) 16 monadou diamegrou B
17 elatov h corr. ex elattov A
18 monaidos progr. 19 elattov A
20 diaemetrou 21 monaidous progr. 22 monaides Bull. 23 monaidos B
24 nota vocabuli oras in ras. nota voc. 25 monaides
DE NUM. LATERALIBUS ET DIAGONIS.

μὲν ἀπὸ τῆς δυᾶς πλευρᾶς τετράγωνον ὑ', τι δ' ἀπὸ τῆς τριάδος διαμέτρου τετράγωνον θ', τὸ θ' ἀρα μονάδι μεῖζον ἦ διπλάσιον τοῦ ἀπὸ τῆς β' πλευρᾶς. πάλιν προσθέμεν τῇ μὲν β' πλευρᾷ διαμέτρου τὴν τρίαδα.

εἶτα ἡ πλευρὰ ε'. τῇ δὲ τρίαδι διαμέτρῳ β' πλευρᾷ, τοιτέοτι δὲ τὰ β', εἶται ε' εἶται τὸ μὲν ἀπὸ τῆς ε' πλευρᾶς τετράγωνον ξ', τὸ δὲ ἀπὸ τῆς ε' <διαμέτρου> μθ', μονάδι ἐλασσον ἦ διπλάσιον τοῦ ξ' ἀρα τὸ μθ'.

πάλιν ἂν τῇ <ε'> πλευρᾷ προσθέζῃ τὴν ξ' διαμέτρου, ἢ ἐτέλεσαι ιβ' πᾶν τῇ ξ' διαμέτρῳ προσθῆς δὲ τὴν ε' πλευρὰν, ἢ ἐτέλεσαι ε' καὶ τοῦ ἀπὸ τῆς ιβ' τετράγωνον τὸ ἀπὸ τῆς ε' μονάδι πλέον ἦ διπλάσιον, καὶ κατὰ τὸ ἐξῆς τῆς προσθήσεως ὧμειος μνημονεύων, ἢ ἐτέλεσαι ἐναλλάξ' ποτὲ μὲν μονάδι ἐλεπτον, ποτὲ δὲ μονάδι πλέον ἢ ἦ διπλάσιον τὸ ἀπὸ τῆς διαμέτρου τετράγωνον τοῦ ἀπὸ τῆς πλευρᾶς· καὶ ὡς ταῦτα καὶ πλευραὶ καὶ μέγειραι.
DE NUM. PERFECTIS.

μὲν ουδὲς μεῖζον ἡ διπλάσια δινόμει, ποτὲ δὲ μονάδι ἐλάττωσιν ἡ διπλάσια ἡ ὁμάλος· πάσας οὖν αἱ διέμεροι ποσῶν τῶν πλευρῶν γενόμεναι δυνάμει διπλάσια, τοῦ ἐναλλάξ πλεύνος καὶ ἐλάττωσι τῇ αὐτῇ μεταξὺ ἐν πάσαις ὁμάλος τεθεμένη λαύτητα ποιοῦντοι εἰς τὸ μῆτε ἐλλείπειν μῆτε ὑπερβάλλειν ἐν ἑπάσῳ τὸ διπλάσιον· τὸ γὰρ τῇ πρωτῆς διαμέτρῳ λεπτὸν δυνάμει τῇ ἐγκαζῇ ὑπερβάλλει.

ὅτε τὸ τῶν ἐριθῶν οἷον τινες τέλειοι λέγονται, οἱ δ’ ἐπετέλεσθοι, οἱ δ’ ἐλλείπεις. καὶ τέλειοι μὲν εἶσαι ἐν οἷον τοῖς αὐτῶν μέρεσιν ἔσοι, ὡς ὁ τῶν ε’ μέρῃ γὰρ αὐτοῦ ἡμῖν ρ’, τρίτου β’, ἦκτον α’, ἕτερας συντεθέμενα ποιεῖ τὸν ε’. γεννώτατοι δὲ οἱ τέλειοι τούτοι τὸν τρόπον. ἢν γὰρ ἡμῶν τοὺς ἀπὸ μονάδος διπλάσιον καὶ συντεθέμενα αὐτοῖς, μέρης οὐ ἐν γένειται πρῶτος καὶ ἰσ ἐν ὑμῖν ὁ αὐτὸς ὁ θρός, καὶ τὸν ἐν τῇ συνθέσις ἐπὶ τὸν ἑξάχθον τῶν συντεθέμενον πολλαπλασίασμαν· ὁ ἀπογεννηθεὶς ἐστιν τέλειος. οὗν ἐστεθάσασθαι διπλάσιοι α’ β’ δ’ ε’ γίνεται· γ’ καὶ τὸν ε’ ἐπὶ τὸν ὑστερόν τὸν ἐν τῇ συνθέσις πολλαπλασίασμαν, τουτέστιν ἐπὶ τῶν β’ γίνεται ε’. ὡς ἐστι πρῶτος τέλειος. ἐν πάλιν τρεῖς τοῖς ἐγκαζῇ διπλάσιοι συντεθέμεν. α’ καὶ β’ καὶ δ’, ἐστιν ε’ καὶ τούτων ἐπὶ τὸν ἑξαχθόν τῶν τῇ συνθέσις πολλαπλασίασμαν· τὸν ε’

46 DE NUM. SUPERFLUIS ET DEMINUTIS.

εις τοὺς δ' οὖσαν ὁ αὐτ' ὡς ἡταί δεύτερος τέλειος εὑρεθέν
tαι εκ τοῦ ἡμίσεως τοῦ ἰδ' τετέρατον τοῦ ζ', ἡμίσιον τοῦ
d', τετσαφακαιδέκατον τοῦ β', ἐλικοῦν ὁ ἐρδόυ τοῦ α'.

ὑπερτελεῖοι δὲ εἶσιν ὡς τὰ μέρη συντεθέντα μεταξὺ τὸ ὑπερτελεῖον.

εἰς τοὺς δ' εὐθὺς, εἰς τὸν ἵππον γάρ ἡμῖν ἰσίν εὖν,

τρίτου δ', τέταρτου γ', ἔκτου β', δωδέκα τ' ἐτης

συντεθέντα γίνεται εὖν, ὡς ἡταί μεταξὺ τοῦ ἰδ' ἐρδόν,
tούτεοι τὸν ἱππόν.

Εὐλογεῖς δ' εἶσιν ὡς τὰ μέρη συντεθέντα εὐπτομα.

τὸν ἐφικτὸν ποιεῖ τοῦ ἰδ' ἐρδόν προτεθέντος ἐφικτοῦ,

οἰνον ὃ τὸν γ' τουτοῦ γὰρ ἡμῖν δ', τετέρατον β', ἡμίσιον

ἐν. τὸ κατά δὲ καὶ τῷ συνβεβηκέναι, ὡς καθ' εἶναι

λόγον τέλειον ἐρασάν οἱ Πυθαγόρειοι, περὶ οὐ καὶ τῆς

οὐδεῖς ὁχήμα τοῦ ἀποδώσομεν. λέγεται δὲ καὶ ὁ γ'

τέλειος, ἐπειδὴ πρῶτος ἐρχόμεν καὶ μέοι καὶ πάροι ἔχει

ὁ δ' κατὰ καὶ γραμμὴ ἐστὶ καὶ ἐπίπεδον, τρίγωνον γὰρ

τὸ ἑσπεριοῦ ἐκαθημην πλευρὰς δυναιμον ἐξαίροντο,

καὶ πρῶτος διαμέρισκα ταῖς δύναμισιν ἐν γὰρ τοιαὶ δια-

στάθη στὸ στεφάνον τοῦτον.

ἐτέλει δ' καὶ συμφωνοῦσιν τινὲς σχεῖν ἐφικτοῖς, καὶ

ὁ περὶ συμφωνοῦντος λόγος οὐκ ἐν εὐρεθεῖ ἐκεῖν ἐφι-
CHAPTER THREE
TRANSLATION

A presentation by the Platonic philosopher Theon of mathematics useful for the reading of Plato

1.1 Everyone would agree, I suppose, that it is not possible to understand the mathematical discourses of Plato, unless one is oneself thoroughly practised in this branch of speculation. That skill in this subject is not useless nor without profit in other respects also he seems to make clear by many remarks. Fortunate, then, is any man if it happens that he studies the writings of Plato after becoming conversant with the whole of geometry and the whole of music and astronomy—a skill which is not readily or easily acquired, but one demanding a very great deal of toil from youth onwards.

1.11 To ensure that those who have missed a training in mathematics yet aspire to an understanding of his writings may not entirely fail in their objective, I shall present a concise summary of the requisite fields of knowledge and an exposition of those mathematical theorems especially needed by those who would become acquainted with Plato, namely arithmetic, music and geometry as well as solid geometry and

1 Θέωνος Σπυριδίου Πλατωνικοῦ τῶν κατὰ τὸ μαθηματικὸν χρησίμα εἰς τὴν Πλατωνοῦ ἄναγνωσίν is the inscription on A.
astronomy. Without these, according to him, it is not possible to attain the best life, for he has demonstrated on many occasions that one ought not to disregard mathematics.

2. 3 Eratosthenes, in his work entitled *Platonicus*, says that the god, in an oracle on the question of their being freed from a pestilence, directed the Delians to set up an altar double the size of the existing one. Whereupon, the builders were afflicted with great perplexity, as they sought the way a solid could become doubled; that was how they came to consult Plato concerning this problem; and he told them that the god had proclaimed this oracle to them, not really because he wanted a double-sized altar, but as an objection and a reproach to the Greeks for their disregard of mathematics and their contempt for geometry.

2.13 In conformity with the advice of the Pythian oracle Plato himself also discourses in great detail upon the usefulness of mathematics; for instance, in the *Epinomis*, urging man to mathematical studies, he says:

> Never, without these studies, will any nature be happy in the State; no, this is the way, this the nurture; these are the studies, whether they be difficult or easy, and by this way must we go, for it is not lawful to disregard the gods.

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2This book, not now extant, was maybe a sort of commentary on the *Timaeus* of Plato, and evidently dealt with the fundamental notions of mathematics underlying Plato's philosophy. Accordingly, it was probably an important source for Theon's writing and Theon cites the book twice by name.

3See Mathematical Note 1 (p. 122)

4cf. Plato, *Epin.*, 992A. Plato has εὐδαιμονία, Theon
Subsequently, he says:

Such a man (the mathematician) alone of many will be the one to become blest by fortune and at the same time most wise and happy.\(^5\)

2.22 In the Republic, he says:

Those who are given preference from the twenty-five year-old age-group will receive greater honours than the others and they must bring together the studies haphazardly pursued in their education as children to provide a comprehensive picture of the interrelations of these studies and of the nature of things.\(^6\)

He advises men first to become practised in arithmetic, then in geometry, thirdly in solid geometry, fourthly in astronomy, a subject which according to him is the study of the solid in motion, and fifthly in music.

\(\epsilon\upsilon\delta\alpha\iota\mu\omicron\nu\nu\nu. \epsilon\upsilon\delta\alpha\iota\mu\omicron\nu\nu\nu\seems easier to translate and has the authority of one ms, and this is the reading I have assumed.\)

This is the first of many quotations from Plato that Theon makes in his proem emphasising the importance of mathematics in education. Like many of the Neoplatonists of his day, Theon had a vast knowledge of Plato, but his quotations are carelessly presented and are rarely textual, being quoted no doubt largely from memory.

\(^5\)cf. Plato, Epin., 992B.

\(^6\)cf. Plato, Rep., 537B,C where Plato outlines his curriculum for education. \(\epsilon\kappa\, \tau\omicron\nu\, \kappa\varepsilon\, \hat{\iota}\tau\omicron\nu\) may be a copyist's error for \(\epsilon\hat{\iota}\kappa\omicron\nu\, \hat{\iota}\tau\omicron\nu\), i.e., the twenty year-old age-group. Preparatory studies were to take until the seventeenth or eighteenth year; then was to follow three years of military service, which would delay higher studies proper until the age of twenty or twenty-one when the pupil entered the Academy. The direction of the mind from the more concrete to the more abstract -- and the pursuit of the nature of Being -- was reserved for the years of dialectic. The study of mathematics prepared the way for such enquiry and it was also necessary to consider the importance of the other disciplines and their relationship to one another.
Demonstrating the utility of mathematics, he says:

You are naive in appearing to fear that I would prescribe a useless study. That the eye of each man's soul, blinded and dimmed by other pursuits, is cleansed and kindled anew by these studies as if by instruments, is a conviction that is with difficulty brought home not only to second-rate minds but to all men; yet it is a faculty whose preservation is far more precious than, a thousand eyes, for through it alone is truth perceived.

3.16 In the seventh book of the Republic, speaking about arithmetic, he says:

Of all the branches of knowledge, it is the most necessary and it is therefore needed by all the arts, all forms of thought and all sciences, and by the art of war itself. Palamedes, at any rate, in tragedy is always depicting Agamemnon as a comical commander; for he says that, after he had discovered numbers, he marshalled his army in camp at Troy and counted his ships and everything else as if, prior to this, they had not been counted, and Agamemnon apparently did not even know how many feet he had, if indeed he did not know how to count.

4. 8 It seems likely then that arithmetic is one of those studies that is naturally conducive to thought, but none use it to lead them to the pursuit of true Being and to summon them to think. All those objects that merely set our sense

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7cf. Plato, Rep., 527D, E.

8cf. Plato, Rep., 522C, D. Palamedes, like Prometheus, was a "culture" hero, who personifies in Greek tragedy the inventions and discoveries that produced civilisation. See P. Shorey, Plato, The Republic, ii, p.151(note) in the Loeb series.

9cf. Plato, Rep., 523A. Plato believed in mathematics for its own sake, as a puzzle in abstract logic; but Jowett indicates that Plato anticipates modern educational theory by advocating concrete aids (μηλαν τε διανομας και στεφάνων, Laws, 819B); the modern doctrine "learning is fun" seems to echo this passage.
of perception in motion as, for instance, the sight of a finger seen as thick or thin, long or short, are not capable of stimulating or arousing the thought processes. On the other hand, those objects that set our sense of perception going in opposite directions are capable of stimulating and arousing our mental processes, as when the same object appears to us large and small, light and heavy, single and manifold.

4.17 Unity then and number are capable of arousing and stimulating thought, for unity sometimes appears to be many. The science of calculation and arithmetic is what attracts and leads us to truth. And one must come to grips with the art of calculation in no amateur fashion, but by pure thought strive to attain to the contemplation of the nature of numbers, practising it not for business reasons as do the dealers and merchants, but in order to assist the soul in its journey towards truth and reality. For it is this that directs the soul upward and compels one to discourse concerning pure numbers without accepting any person's reference in the discourse to numbered objects that are tangible or visible.10

5. 7 And again in the same book, he says:

Furthermore, those who are good reckoners are naturally quick at all their studies, and those who are slow themselves become quicker than they were before.11

10 cf. Plato, Rep., 525B,D.
11 cf. Plato, Rep., 526B.
He says further in the same place:

"In war too the art of calculation is useful for encampments, for capturing places and for the assembling and disposition of the army." 12

Furthermore, in praising the earnest study of such sciences, he says:

"While geometry deals with the study of the plane surface, astronomy deals with the movement of the solid, and this compels one to look upward and leads one away from things here to those higher visions." 13

In the same work, on the subject of music, he says:

"The contemplation of the universe requires two sciences, astronomy and harmony, and these are sister sciences according to the Pythagoreans." 14

6. 2 Some make a futile effort of measuring the harmony of sounds they hear by measuring them against one another. By assiduously laying the ear alongside as if they were trying to catch a sound in the neighbourhood, some claim that they can detect an intermediate note and that this is the smallest interval and should be the unit of measurement, while others disagree and claim that it is the same as the note already sounded, setting a greater value on the judgment of ear than of mind and persecuting the strings as they rack them on the pegs. 15

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12 cf. Plato, Rep., 526D.
13 cf. Plato, Rep., 529A.
14 cf. Plato, Rep., 530D.
15 cf. Plato, Rep., 531A. Plato satirised the empirical methods of the μουσικοί who made the quarter-tone their unit, while he was all for simplicity in music, as J. Adam contends,
6.10 But the good arithmeticians by their reflections seek to find out which numbers are harmonious with other numbers and which are not.\textsuperscript{16} And this search is useful for the pursuit of the good and the beautiful; for other purposes the search is useless. And if all this line of enquiry leads to the mutual connections of these numbers and an assessment of their relationships to each other, the study of them proves fruitful. Those who are clever in these matters are the dialecticians, else would they be unable to exact and render an account of their opinions in discussion and it is impossible to do this unless one has proceeded by way of those studies, for the path to the contemplation of the universe lies by way of them in reasoned dialectic.\textsuperscript{17}

7.9 Again, in the \textit{Epinomis}, Plato has many other points to make concerning arithmetic, calling it a gift of God,\textsuperscript{18}

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\textsuperscript{16} cf. Plato, \textit{Rep.}, 531C.

\textsuperscript{17} cf. Plato, \textit{Rep.}, 531D.

\textsuperscript{18} cf. Plato, \textit{Epin.}, 976D,E.
without which it would not be possible for anyone to become
excellent. Then, moving further on, he says:

If we were to remove number from human nature we would
no longer have any intelligence at all, and moreover the
soul of that living creature would no longer be capable
of achieving complete virtue, for his reason would
scarcely exist. The creature which did not know Two
and Three nor Odd and Even but was totally ignorant
of number would never be able to tell of those things
concerning which he had acquired merely sensations
and memories; deprivéd of true reasoning he would
never become wise.\textsuperscript{19}

8. 2 And yet, with regard to the attributes of the other arts
which we just now reviewed, not a single one can abide but
all will be utterly destroyed whenever the science of numb­
ers is neglected. And perhaps it might seem to some who
merely glance at the arts that the race of mankind has but
small need of number, and yet even that need is a very great
matter. For if a person were to perceive the divine and the
mortal element in man's generation, wherein will be observed
reverence for the gods and number in its real sense, still no
seer could understand the magnitude of the power that number
in its entirety produces for us, since it is clear that all
music, through its association with notes and movement, is
created with the aid of numbers;\textsuperscript{20} and, most important of all,
as a blessing it is the cause of all good things and it should

\textsuperscript{19}cf. Plato, Epin., 977C. Theon uses Plato's expression,
διδόναι λόγον here, where Lamb, Epin., (Loeb), points to the
curious play on the two meanings of λόγος, "reckoning" and
"description"; the English "tale" and "account" are similar.

\textsuperscript{20}cf. Epin., 978A. The Greek is somewhat difficult here.
As Theon is obviously trying to quote Plato, some such word as
γενέσθαι may perhaps be understood to present Plato's opinion
on the subject.
be understood that it is the cause of no evil thing. Virtually devoid of reason, without order and without grace, wholly bereft of harmony and rhythm and utterly possessed of the qualities of an evil person is that man who is altogether bereft of number.

8.18 Further on, he adds:

Let nobody try to persuade us that there is any greater part of virtue for the human race than piety, for it is from this that the other virtues are engendered in the man who has studied in a methodical way.  

9.1 Next he demonstrates how a man may learn reverence for the gods. He says that one must first learn astronomy, for if it is a dreadful condition to be in error concerning human affairs, it is much more dreadful to be so in relation to the Divine. And the man in error would be the one who holds false opinions concerning the gods, and the man who holds false opinions concerning the gods is the man who has never examined the nature of the observable gods, i.e., astronomy. He says:

It is a fact not known by the majority of men that the man who is truly an astronomer must necessarily be the wisest, not he who practises his astronomy in the sense understood by Hesiod, i.e., the sort of person who has merely studied the risings and settings of the stars, but the man who has studied the orbits of the seven planets, something that a person of common disposition would never readily be capable of contemplating.

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22 cf. Plato, *Epin.*, 989E.

and also:

The person who is preparing natures for these pursuits must give preliminary instruction to as many as possible, training them in childhood and youth in the mathematical sciences. The greatest of these studies is to be scientifically versed in pure numbers, without reference to bodily substance, and in the absolute origin of Odd and Even and the magnitude of their influence on the nature of Reality. 24

10. 4 He proceeds:

Next after this comes the study to which they give the utterly ridiculous name of geometry, but it rather consists in establishing resemblances between numbers not naturally alike by having recourse to the province of planes. 25

He mentions also another scientific skill, which he calls solid geometry:

whereby a person, by multiplying together three numbers comprising plane surfaces, albeit like and unlike as mentioned before, produces a solid body -- and this is a divine and wondrous thing. 26

10.12 In the Republic, he says, speaking of musical harmony:

The finest and greatest form of political harmony is wisdom, and he who leads a rational life has a share in this wisdom, whereas the man lacking it is a ruin to his house and no saviour of the State at all, in as much as he is ignorant of things of the greatest import. 27

10.17 And in the third book of the Republic, where he teaches that the philosopher alone is musical, he asks:

24 cf. Plato, Epin., 990C.
25 ibid.
26 ibid.
27 cf. Plato, Laws, 689D. Theon mistakenly gives the source of this quotation as the Republic.
Then, by heaven, shall we thus never be true musicians, neither we nor those whom we say we must instruct as guardians, until we can differentiate between all the forms of temperance and bravery, generosity and highmindedness and all those qualities kindred with them and opposite to them in all their combinations everywhere, and until we can recognise the presence of these qualities in certain cases, both the qualities themselves and their likenesses, disregarding them neither in small affairs nor in great, but believing them to form part of the same art and discipline?28

11. 7 By arguments such as these and the earlier ones, he shows what benefit is derived from music and demonstrates that the philosopher alone is truly musical, whereas the evil man has no communion with the Muses. For, according to him, true goodness of heart, namely the virtue that consists of having a well-ordered character, is attended by good speech, i.e., the ability to employ words properly, and this good speech, in turn, is attended by elegance, rhythm and concord, elegance in the tune, rhythm in the measure and concord in harmony. 29

11.15 By contrast, evil temper, i.e., the evil character is, according to him, attended by evil speech, i.e., the use of the evil word and this evil speech is attended by an absence of grace and a lack of rhythm and harmony in connection with all that one does or imitates. 30 Consequently, only that man could be musical who is absolutely good at heart, and he would be our philosopher. This has been shown by what has

28 cf. Plato, Rep., 402B.
29 cf. Plato, Rep., 400D,E.
30 cf. Plato, Rep., 401A.
been said. For since music enters the soul from an early age because of the harmless pleasure it affords blended with its usefulness, and implants there rhythm, harmony and elegance it is impossible, according to Plato, to become an accomplished musician if one has no comprehension of that which is seemly in everything and if one does not recognise the forms of elegance, of liberality and temperance, i.e., their Ideas. At any rate, he contends they are present everywhere, i.e., their forms, and one must not disregard them either in small things or in large things, for the knowledge of Ideas is the concern of the philosopher. For nobody could possibly understand propriety, temperance and elegance who is himself unseemly and intemperate. The elements of life which are elegant and endowed with rhythm and harmony are images of elegance and of rhythm and harmony in the absolute, that is to say, objects of perception are reflections of thoughts and ideas. 31

12.10 Now the Pythagoreans, whom Plato follows in many respects, claim that music is a combination of opposites, a oneness of many and a concord of discordants, for music does not merely compose rhythm and tune but harmonises completely every interval; for its object is to unify and bring into harmony. For God is also a bringer of harmony to the discordant and herein lies His greatest task, by means of music and

by means of medicine to reconcile things which are hostile
to one another. Upon music, they say, depends the harmony of
things and moreover, the excellent governing of the universe;
for this naturally takes the form of harmony in the universe,
good order in the State, and moderation in the home; for it
has the power to bring together and unify the many. The operation and practice of this science, according to Plato, occurs
in four human attributes, the soul, body, home and State; for
these four spheres have need of ordering and arrangement.

12.26 In the Republic Plato also spoke as follows on the
subject of mathematics: 32

The good man is he who in the face of pains and pleasures, desires and fears, preserves and does not reject the right belief of those at his disposal as a result of his education. I would like to illustrate my meaning with a simile.

Now when dyers of the present day wish to dye wool purple, first of all they choose out from among the various colours the one nature of the white; then, they make their preliminary preparations with no small care in order that the material may take the hue in the best way. In such fashion do they dye it; if one dyes anything by this method, the natural and the dyed colour become one, and neither without nor with washing-soaps can the colour be removed. If they do not follow this method and do not take precautions in the dyeing, you know what transpires; the wools have a washed-out appearance, lose their colour and are not fast-dyed. And you must believe that this is the kind of thing that we too are doing to the best of our ability. We train our children in music and gymnastics, letters, geometry and arithmetic with the express object of giving them a preliminary cleansing and preparation with these studies acting as astringents. And the purpose of this is that they may accept like a dye the arguments concerning virtue in general which they may later learn, so that their opinions may be fast-dyed through having had a suitable nurturing of their innate capacity and that the dye of their opinions may not be washed out of them by such

32 cf. Plato, Rep., 429D
soaps as these, which are dreadful washing-agents, namely
pleasure, which is more deadly than any clothes-press or
running of the colours, or pain, fear and desire, which
are more deadly than all other detergents.

14.18 On the other hand, one might describe philosophy as
the initiation into a mystic rite and as a transmission of
truly genuine mysteries. There are five stages to the initia-
tion: 1) The first is the preliminary purification; for a
share in the mysteries is not open to all who desire it, but
there are some who are publicly forbidden access, for insta-
ence, those with unclean hands and a voice devoid of under-
standing; and those not forbidden access must undergo some
preliminary purification. 2) After the purification comes
the transmission of the mystery. 3) The third stage is
what is called the mystic vision. 4) The fourth stage, which
in fact completes the mystic vision, is the binding of the head
and the laying on of the garland, so as to be able to hand
on to others the mysteries that one has received, either in
the role of torchbearer or hierophant or some other priest.
5) The fifth stage, far surpassing the previous ones, is the
complete happiness which arises from being loved by the gods
and having fellowship with them.

15.7 In exactly the same way, the transmission of Plato's
thoughts involves in the first place a certain purification,

33 There is some difficulty here. παντὸς στρεβλοῦ δεινο-
στέρα καὶ κοινωνίας as hendiadys may mean "more deadly than
any wicked perversion;" but I have taken Professor Russell's
suggestion of στρεβλή = clothes-press and κοινωνία = running
of the colours. Again, κοινωνίας may be a copying error for
καὶ κοινὰς (lye).
namely a training from youth in the appropriate mathematical studies. For, according to Empedocles:

He who seeks to draw water from the five fountains must cleanse himself with the tireless bronze.\(^{34}\)

But Plato says the purification must be obtained from the five branches of mathematics; and these are arithmetic, geometry, solid geometry, music and astronomy. The transmission of the various philosophical theories, namely logic, politics and physics, resembles the process of initiation. The mystic vision is the name he gives to the diligent concern with the intelligible, with true existence and the world of ideas.\(^{35}\)

The binding of the head and the crowning one must interpret as the ability to benefit from one's own studies and set up others in the same contemplation. The fifth and most perfect stage would be the complete, surpassing happiness which ac-

\(^{34}\) It is not certain whether the "bronze" is a "cup" or a "blade", but the passage refers to Empedocles' ritual of purifying the spirit. The exact form of the verse of Empedocles is perhaps irrecoverable. Diels/Kranz, *Fragmente der Vorsokratiker*, i, B143 has: κρηνάων ἀπὸ πέντε ταμόντα <ἐν> ἀτείρει χαλκῷ ... but it is not difficult to see, as pointed out by Bywater (Ingram Bywater, *Aristotle and the Art of Poetry*, Oxford, 1909, p. 283), that Theon's ἀνιμώντα (Hiller 15, 10) is a prosaic substitute for ταμόντα, which is preserved by Aristotle in Poetics, 1457b, 10. Here Aristotle appears to use Empedocles' quotation to illustrate what he calls the transference of meaning from one species of metaphor to another. Thus as examples he gives χαλκῷ ἀπὸ ψυχῆν ἀρώγας and ταμών ἀτείρει χαλκῷ, the first of which must be translated "drawing off his life with the bronze" and the second, "severing with the tireless bronze", where "bronze" in the first instance will be a "blade" and in the second a "cup".

\(^{35}\) cf. Plato, *Phaedrus*, 250C.
ording to Plato himself consists in becoming as much like God as possible.\textsuperscript{36}

16. 3 There are many other things one might say to demonstrate the usefulness of and necessity for mathematics. Lest I should seem to display a lack of good taste in wasting time upon a eulogy of mathematics I must turn forthwith to the presentation of the requisite theorems in mathematics, not every one which could make the average person the perfect arithmetician or geometrician or musician or astronomer, for this is not the prescribed objective of those who would become acquainted with Plato, but I shall present only as much as suffices to enable the reader to understand his writings. For not even Plato himself requires that one should continue into extreme old age drawing diagrams and writing songs, but he regards these as the studies of youth that are calculated first to prepare and purify the soul for the express purpose of rendering it capable of assimilating philosophy. Therefore it is especially necessary for the man who intends to study both my presentation and Plato's writings to have worked through at least the elementary steps of geometry; for then would he more easily follow my presentation. Nevertheless, what I have to say will be of such a kind as to be intelligible even to a person completely unversed in mathematics.

16.24 First I shall mention the theorems of arithmetic that are closely associated also with those theorems of music \textsuperscript{36}

\textsuperscript{36}cf. Plato, \textit{Theaetetus}, 176B.
expressed in numbers. For we have no need at all of instrumental music, even as Plato himself implies when he says that we must not trouble the strings (straining our ears as if we were) trying to catch a sound in the neighbourhood.\textsuperscript{37} We yearn to understand the harmony of the universe and the music therein, but it is not possible to perceive this music unless we first contemplate the music expressed in numbers. And this is the reason Plato says music should take fifth place,\textsuperscript{38} for he is referring to the music of the universe that deals with the movement, order and symphony of the stars which move in the universe. But in my opinion it is necessary to place the music based on numbers next after arithmetic also in accordance with Plato's opinion, since the music of the universe is not to be comprehended without the music based on numbers and thought. So then, since the principles of music based upon numbers are associated closely with the study of numbers pure and simple, they should be placed second for the convenience of our study.

17.14 Following the natural order on the other hand, first there should be the study of numbers which is called arithmetic, second the study of surfaces called geometry, third

\textsuperscript{37} See Hiller 6. 5. Plato in \textit{Republic}, 531A has \textit{kai paraballoves ta ota} and some such phrase must be understood to make sense of this passage.

\textsuperscript{38} cf. Plato, Rep., 530D. Plato's order of priority given in the \textit{Republic} to the branches of mathematics was arithmetic(522C), geometry(526C), solid geometry(527D), astronomy(528B) and music(530D).
the study of solids called solid geometry, fourth the study of moving bodies which is astronomy. As for the music that deals with movements and intervals and their mutual relations, this cannot be apprehended without our first understanding the music that is based on numbers.

17. 22 Wherefore, for the purpose of this study of mine, the music based upon numbers should follow arithmetic; but, following the natural order, fifth place should go to that musical study which deals with the harmony of the universe. Indeed, according to the Pythagoreans, the study of numbers should take first place as being the starting-point, the fount and root of all things.

18. 3 A number is a collection of units or a progression of many units beginning with the number One and finishing off by returning to One. As for Unity, it is the terminating quantity—(the initial element of the numbers) which, when a large number is decreased by subtraction and is deprived of all other numbers, still maintains its permanent and fixed position for, you see, it is impossible for the dissection of the number to proceed further.

18. 9 For if, in the case of material objects, we partition the One into parts, then the One on the contrary will become a number of many parts and manifold and, by the process of subtracting each of its component parts we will finish up

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39 Stobaeus credits Moderatus with this definition. Stobaeus, Eclogues, 1, pr., 8.
40 Hiller believes these words should be deleted.
with One. And if this One we again partition into parts, it will become a number of parts and the result of the subtraction of each of these parts will be One; consequently, One in respect of its oneness has no parts and is indivisible.\footnote{See Mathematical Note 2 (p. 125)}

18.16 Now every other number, when partitioned, is diminished and dissected into parts smaller than itself; as, for example, 6 into 3 and 3, or 4 and 2, or 5 and 1. But the One, whenever it is partitioned in the realm of the senses, is diminished if regarded as a body and, as a result of the process of dissection, is partitioned into parts smaller than itself, but on the other hand, if regarded as a number, it increases, for in place of one there are many; so it is in this respect that Unity is indivisible. For nothing can be partitioned and in the process change into parts greater than itself.

18.23 Just as One, when partitioned, is dissected into a greater number of parts than has the whole, so it is partitioned after the manner of numbers into parts which are in sum equal to the whole. For example, whenever one material object is partitioned into six, as a number it will divide into \(x \times x \times x \times x \times x\), each part having the same number of parts as the whole (i.e., one), but if partitioned as a number into 4 and 2, each will have more parts than the whole; for 2 and 4 as numbers are greater than one. Unity then, considered as a number, is without parts and indeed it is called Unity from its abiding unmoved and not deserting its inherent nature.
For, however many times we multiply Unity by itself, it remains One and one times one always gives One, and if we keep on multiplying one to infinity we still get One. Assuredly, it is called Unity from its having been singled out and separated from the rest of all the numbers.

19.13 Just as a number differs even from that which is numbered, in the same way does Unity differ from One. The number is, in fact, a quantity conceived in the mind as, for instance, are the numbers Five and Ten, which, by themselves, have no perceptible substance but are concepts. A quantity that may be perceived can be numbered, as, for instance, 5 horses, 5 oxen, 5 men but Unity, however, is the conceptual form of One and is indivisible. As for the One which may be perceived such as one horse or one man, it is termed One absolutely.

19.21 Unity then will be the principle of the numbers and One the first of the numbered articles. And One, they say, in so far as it is the object of perception, can be dissected to infinity, not in as much as it is a number or the principle of number but in so far as it is perceptible. Consequently, Unity, being a concept, is not divisible while One, being perceptible, can be divided to infinity. And things numbered will differ from numbers in as much as the former are corporeal bodies while the latter have no substance. To put it generally, men of later times term Unity and the number Two the principal elements of the numbers, but the followers of
Pythagoreans claim that the principal elements consist of the series of successive terms by which the Odd and Even numbers are conceived; for example, the prime quality of Three in the realm of perception is the Triad and that of Four in every instance is the Tetrad; and so on for all other numbers. Further, they also state that the unit is the principle of all these very numbers, and that the One is free of all the distinctions that other numbers apparently have, being only One itself, not a particular one; that is, not being the specific one which denotes this quality and admits a certain difference as compared to another, but being simply One, considered in its own right. For thus it would become the principle and measure of those objects subject to itself, in so far as each of the things that exist is called one and has a share in the principal essence and form of One.

20.19 Archytas and Philolaus employ the terms unit and One without distinction and term the unit, one. The majority

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43In connection with the definition of the principle of the numbers we may detect a progressive narrowing of the definition by three stages. At first, according to Stobaeus, all numbers were considered to be principles; then the Pythagoreans (οι μὲν ὄρθορροι) considered One as the principle of numbers and Two as the principle of the Even numbers. Heath (H.G.M. 1, 71) believes that this view of Two is implicit in Nicomachus, Intro. Arith., i, 7, 4. By the time of Plato, Two is considered to be an even number, cf. Parmenides, 143D.

44Owing to the secrecy which bound the early Pythagoreans, some of the fragments ascribed to Philolaus (second half of fifth century) assume a greater significance. Thus
give to the unit itself the epithet "prime" on the understanding that there is a certain unit which is not prime but a more general term, embracing both the unit itself and one; for they also call it One, that is, the prime intelligible essence of the one, which gives oneness to every single thing. By sharing in this essence, every single thing is called One. Wherefore the epithet "one" gives no indication as to which one or of what kind of one, but is applied to all things.

21. 5 Although it is not the intelligible concepts and patterns that would differ the one from the other; it is the perceptible objects that differ. But some assign another distinction between the unit and One; for One does not change in its essence neither is it responsible for the failure of the unit and the odd numbers to change in their essence, and also it does not change in its quality, for it is the unit in itself and not like a multitude of units as it were; nor does it change in quantity either, for it is not composed like units to which another is added, being one and not many units, and for that reason it is called One uniquely. For if, in Plato, reference has been made to the units in the Philebus, they have not been called after the One but after the number

Stobaeus preserves the following fragment: "All things that can be known have number; for it is impossible for a thing to be known or conceived without number" (Diels, Vors, 44B, 111). Archytas (first half of fourth century) greatly influenced Plato and was largely instrumental in establishing mathematics in the curriculum of the Academy.

45 cf. Plato, Philebus 15A.
One which is the unit by virtue of its share in the one. So this One which is defined by the bounds of the unit is in all respects unchangeable, so that the One would differ from the unit in so far as it is defined and limited, whereas the units are boundless and undefined.

21.20 Numbers may first be classified into two kinds, called Even and Odd. The even numbers such as the number Two and the number Four are those which admit of division into equal parts, while the odd numbers such as Five and Seven are those which may be divided only into unequal parts. Some called the first of the odd numbers Unity, for even is the opposite of odd and Unity is assuredly either odd or even. Now, it could not be even, for it cannot be divided into equal parts and indeed it cannot be partitioned at all; therefore, Unity is odd. If you add an even number to an even number, the total is even; but Unity added to an even number gives a total that is odd, so assuredly Unity is not even but odd.

22.5 Aristotle in the Pythagorean states that One partakes of the nature of both odd and even for, when added to an even number, it makes it odd, and when added to an odd number, it makes it even, which it could not do if it did not share in the nature of both. Wherefore, One is called "odd-even" and Archytas agrees with this.

46 Probably one of Aristotle's lost scientific treatises.

47 cf. Arist., Metaph., A5, 986a, 19. Aristotle's reasoning for this definition does not seem convincing, for any odd number would meet such conditions. Thus, take 7; then
The first form of the odd then is Unity, just as in the universe men apply the term odd to that which is defined and organised; but the first form of the even is the undefined number Two, just as also in the universe men apply the term even to that which is unbounded, unknown and not ordered. Therefore the number Two is also called indefinite since it is not, as is Unity, defined.\(^\text{48}\) As for the numbers which follow, set out in succession after Unity, these increase by an equal amount, for each of them is greater than the preceding by one; but as they increase the ratio of the one to the other decreases.\(^\text{49}\)

Take, for example, the numbers 1, 2, 3, 4, 5, 6; the ratio of the number 2 to unity is double; that of the number 3 to the number 2 is one and one-half to one; that of the number 4 to the number 3 is one and one-third to one; that of the number 5 to the number 4 is one and one-fourth to one; and the ratio of the number 6 to the number 5 is one and one-fifth to one. Now the ratio of one and one-fifth to one is

\[^{7+6 = \text{odd}, 7+9 = \text{even}, \text{and } 7 \text{ can hardly be termed "odd-even". But Heath(H.G.M. 1, 71) points to a fragment of Philolaus which mentions many forms of the even and the odd and a third form "even-odd" which refers to a number which is the product of an even number and an odd number, so that "even" in the same passage must refer to numbers which are powers of 2, i.e., of form } 2^n. \text{ Theon is here being influenced by Philolaus-- his first two even numbers are 2, 4 (Hiller 21.23).}\]

\[^{48}\text{The term άριστος (undefined) is applied here and again in 24.24 to the number Two; for whereas the odd numbers by addition, } 1+3+5+\ldots \text{ form squares with constant side ratios (ή μονάς άριστευρί), the even numbers starting from Two, i.e., } 2+4+6+\ldots \text{ form rectangles (termed heteromecic) and the ratio of their sides constantly changes (άριστος ή τυάς).}\]

\[^{49}\text{Thus } 6/5 < 5/4 < 4/3 < 3/2 < 2/1\]
less than the ratio of one and one-fourth to one; that of one and one-fourth to one less than that of one and one-third to one; the ratio of one and one-third to one is less than that of one and one-half to one; and the ratio of one and one-half to one is less than double. And for the rest of the numbers the ratios follow the same pattern. And the numbers are seen to be alternatively even and odd with the exception of One.  

23. 6 Among numbers some are called absolutely prime or in-composite, some are called prime with respect to each other but not absolutely so, some are absolutely composite and others composite with respect to each other.  

23. 9 Those numbers are absolutely prime and in-composite which cannot be divided by any other number except unity al-one, such as 3, 5, 7, 11, 13, 17 and numbers such as these. These same numbers are called linear and euthymetric by reason of the fact that the lengths and the lines can be visualised in only one dimension; they are thus called "oddly-odd". Consequently they are named in five ways, prime, in-composite, linear, euthymetric and "oddly-odd"; this is the only way in which they can be divided, for 3 could not be divided by any other number so as to be the result of the multiplication of

50 For One has been termed "odd-even" (Hiller 22. 9)

51 The word used here for "divided by" is μετρούμενοι, and it is interesting to note that this word still survives in English in G.C.M.--"greatest common measure", the largest number that can be divided into two or more numbers. A more modern mathematical term is modulus, from Latin modulus, a small measure. See Mathematical Note 9 (p. 134).
the factors except by unity alone; for one times 3 is 3. Likewise, one times 5 is 5, one times 7 is 7 and one times 11 is 11. This is the reason these numbers are called "oddly-odd"; for they are odd measures and the unity which divides them is odd also. Wherefore, only odd numbers can be prime and incomposite.

Indeed, even numbers are neither prime nor incomposite and are not divisible by unity alone but by other numbers; as, for instance, the number 4 is divisible by 2, for 2 times 2 is 4; the number 6 is divisible by 2 and 3, for 2 times 3 and 3 times 2 are 6. And, in the same way, all the rest of the even numbers with the exception of the number 2 are divisible by numbers greater than unity. This number alone of the even numbers is in exactly the same case as are also a number of the odd numbers; namely being divisible by unity alone. For one times 2 is 2; wherefore, it is said to "have an odd-form" and is treated the same as the odd numbers.

Numbers are defined as prime to each other but not absolutely so, when they have unity as a common factor yet are also divisible by certain other numbers when taken by themselves; as, for instance 8 which is divisible by 2 and 4, 9 which is divisible by 3, and 10 which is divisible by 2 and 5. But they also have unity as a common factor, both with respect to each other and to their prime factors; for one times 3 is 3 and one times 8 is 8, one times 9 is 9 and one times 10 is 10.
24.16 Wholly composite numbers are those divisible by a number less than themselves, such as 6 which has factors 2 and 3. Numbers composite with respect to each other are those divisible by some common factor, as are 8 and 6 which have a common factor 2, for 2 times 3 is 6 and 2 times 4 is 8. Also composite with respect to each other are 6 and 9, their common factor being 3; for 3 times 2 is 6 and 3 times 3 is 9. Now Unity is not a number but the "principle of number" and neither is the undefined number Two, for it is the first number different from Unity and contains nothing more basic than Unity among the even numbers. 53

24.25 Of the composite numbers, those produced by two numbers are called "plane", being envisaged as having two dimensions, i.e., being the product of a length and a width; while those composite numbers made from three numbers are termed "solid", for they have a third dimension. 54 The product is the name given to the result of the multiplication of two numbers with each other.

25. 5 Among even numbers, some are called "evenly-even", others "oddly-even" and others "even-odd". Those numbers are

52 Thus 3 and 7 would be absolutely prime; 9 and 8 prime with respect to each other, but not absolutely prime, for 9 = 3·3 and 8 = 4·2, "when taken by themselves", but 3 and 7 are 3·1 and 7·1, absolutely prime, "when taken by themselves". Dupuis deletes ἀπαξ γυναι, for 3 is "absolutely prime" (Dupuis, 38.15).

53 "nothing more basic" sc. as a factor. It has no even factors.

54 See Mathematical Note 4 (p. 129)
"evenly-even" which meet three conditions, first they are formed by the multiplication of two even numbers; second, they have all their parts even as far as the final factor unity; and third, none of their factors could go by the name of "odd" number. Such numbers are 32, 64, 128 and those obtained next after these by the process of doubling. For 32 results from the factors 4 and 8 which are even, and all its fractions are even; one-half of it is 16, one-fourth is 8, one-eighth is 4—so the fractions themselves are called even. For one-half is considered as belonging to the number Two, and so are one-fourth and one-eighth, and the same reasoning applies equally to the rest of these numbers.

25.19 "Even-odd" numbers are those numbers which are the products of 2 and some other number which is odd; all such numbers without exception upon division into equal parts have halves which are odd. Such a number is twice 7, or 14. "Evenly-odd" they are then called because they are divisible by the even number Two and by some odd number; as is 2 divisible by the number One, 6 by 3, 10 by the number 5 and 14 by the number 7. By the first division these numbers are rendered odd, and after the first division they are no longer divisible into equal parts. For the half of 6 is 3, but 3 is divisible no further into equal parts—for Unity is indivisible.

26.5 The numbers termed "oddly-even" are those resulting from the multiplication of two other numbers, an odd number and an even number, and when multiplied their products are

55 Thus the "evenly-even" numbers are of form $2^n$. See note 47.
divisible into two equal parts that are even; but, using larger divisors, they produce quotients which are sometimes even and sometimes odd.\textsuperscript{56} Such numbers are 12 and 20; for 3 times 4 is 12, and 5 times 4 is 20; and 12 divides into two parts 6 and 6, and 3 parts 4 and 4 and 4, and into 4 parts of 3.

In the same way 20 divides into two parts of 10, and four parts of 5, and five parts of 4.

26.14 Further, of the composite numbers some are "equally-equal" and so square and plane, when some number is formed by the multiplication of an equal by an equal;\textsuperscript{57} as, for instance, 4 which is 2 times 2, and 9 which is 3 times 3.

26.18 On the other hand, numbers are "unequally-unequal" when unequal numbers are multiplied together, as in the case of 6 which is 2 times 3.

26.21 Those of the numbers that have one side larger than the other by unity are heteromeric.\textsuperscript{58} Now a number larger than an odd number by unity is also even; wherefore, heteromeric numbers can only be even, for the principle of the numbers, i.e., Unity is odd and, by its tendency to effect a change by

\textsuperscript{56}Theon has classified numbers: (1) "even", of form $2^n$ (note 47), (2) "even-odd", of form $2x$ ($x$ is odd), (3) "oddly-even", of form $2^n x$ ($x$ is odd). Type (3) yields a "quotient that is odd" upon division only by $2^n$.

\textsuperscript{57}Hiller suggests the deletion of ο̇ γεννη̱ςε̱ις ισάκις ις ιςος και τετραγωνος ἐστίν, probably on the grounds of redundancy, but in the light of the tedious repetitive detail of this portion of the work, this measure hardly seems justified.

\textsuperscript{58}ἐπερομήκης in Plato and Aristotle has the more general sense of "any number with two unequal factors", i.e., oblong. Nicomachus however and, here, Theon use the word for the
the doubling of itself produces the number Two which is heteromecic. And for this reason the number Two, being heteromecic and exceeding Unity by one unit, makes heteromecic numbers out of the even numbers and the odd numbers that they exceed by one.\textsuperscript{59}

27. 7

They are formed in two ways: by multiplication and by addition--by addition, when even numbers are added to the succeeding even numbers taken in order and the numbers produced are heteromecic. Take, for instance, in order the even numbers 2, 4, 6, 8, 10, 12, 14, 16, 18. The result of the addition of 2 plus 4 is 6, of 6 plus 6 is 12, of 12 plus 8 is 20, and of 20 plus 10 is 30; so the sums resulting, namely 6, 12, 20, and 30 would be heteromecic and the same applies to the sums of succeeding even numbers.

27.14

The same heteromecic numbers are produced by the multiplication of successive even and odd numbers, by multiplying the first number by the one following it.\textsuperscript{60} Thus, take 1, 2, special case of a number whose factors differ by one; and they use \textit{προμήκης} for the general case of "oblong" numbers (See Hiller 30.8). Thus, for translating Theon's \textit{ἐτερομήκης} Dupuis uses \textit{hétéromèque}, while some mathematical writers in English have coined the term "heteromecic". On the other hand, where they use "oblong" for \textit{ἐτερομήκης} they use "prolate" for \textit{προμήκης}.

\textsuperscript{59}The reasoning is: Two is the first heteromecic number, for it exceeds unity by one; Two makes all even numbers even; therefore, Two makes the even numbers, when taken with their odd counter-parts (the numbers preceding them), into heteromecic numbers.

\textsuperscript{60}See Mathematical Note 6 (p. 132)
3, 4, 5, 6, 7, 8, 9, 10. Now 1 times 2 is 2, 2 times 3 is 6, 3 times 4 is 12, 4 times 5 is 20, 5 times 6 is 30, and so on for the succeeding numbers. Such numbers are termed heteromecic immediately the addition of unity to one of the sides causes a difference between the sides.  

27.23 Parallelogram-numbers are those numbers which have one side greater than the other by 2, as have the numbers 8, 24, 48, and 80, which are 2 times 4, 4 times 6, 6 times 8, and 8 times 10.  

28.3 The square numbers are those produced by the addition of the odd numbers in succession. Now, take in order the odd numbers 1, 3, 5, 7, 9, 11. One plus 3 makes 4, which is a square.

Or possibly, "causes a change in the sides", namely from being both odd or both even, to one odd and one even. The Greek here, I think, gives an indication of the practical way in which numbers were studied, i.e., by diagrams composed of pebbles or counters. Thus:  

• • • = 9, and the first change in one side • • • produces • • • •  the heteromecic number 12; cf. τὴν ἕτερότητα ἰητούσα, the same idiom in Hiller 27.3.

I have followed Dupuis (44.23 note) in deleting the words ἕκαι μείγων; ἄριθμο; otherwise, parallelogram-numbers would not differ from the oblong (προμήκης) numbers already defined. In addition, Theon quotes as examples, numbers whose factors differ only by 2. It is apparent that Theon is attempting to classify numbers according to their factors as follows:  
a) τετράγωνος (square): factors equal, of form \( n \cdot n = n^2 \)  
b) προμήκης (oblong or prolate): factors differing by 1, 2, or more, of form \( n(n + a) \)  
c) ἕτερομήκης (heteromecic): factors differing by 1, of form \( n(n + 1) \)  
d) παραλλαγόγραμμος (parallelogram): factors differing by 2, of form \( n(n + 2) \)
square for it is "equally-equal", i.e., 2 times 2 is 4; 4 plus 5 makes 9, which again is also a square, for 3 times 3 is 9; 9 plus 7 is 16, which is also a square for 4 times 4 is 16; 16 plus 9 makes 25, which is a square and "equally-equal", being 5 times 5; and one could continue the same procedure to infinity. Such, then, is the method of producing square numbers by the process of addition, the next odd number being added to the square obtained by summing the preceding odd numbers starting from unity. 63

28.13 And square numbers are also produced by means of multiplication, when any number at all is multiplied by itself; as, for example, 2 times 2 gives 4, 3 times 3 gives 9, and 4 times 4 gives 16.

28.16 Now all the square numbers have heteromecic numbers as means in a geometric proportion, but heteromecic numbers, for their part do not enclose square numbers as means in a proportion. 64 Thus, take the numbers 1, 2, 3, 4, 5. Each of

63 See Mathematical Note 5 (p. 137)

64 "Consecutive" or "adjacent" square numbers are meant. Thus, 4²(16) and 5²(25) have the heteromecic number 20 as a geometric mean, for 16:20 = 20:25. The Pythagorean patterns would demonstrate this relationship with classic simplicity.

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Heteromecic numbers do not enclose square numbers as means in a "geometric proportion", but they do have square numbers as "arithmetical" means. See Mathematical Note 7 (p. 132)
these multiplied by itself gives a square number; for 1 times 1 is 1, 2 times 2 is 4, 3 times 3 is 9, 4 times 4 is 16, 5 times 5 is 25. Now these numbers are produced by identical factors, for the 2 only doubles itself, the 3 triples itself, so the square numbers would be in order 1, 4, 9, 16, 25 and they have heteromecic numbers for means in the following way. Two successive squares are 1 and 4, and their mean is the heteromecic number 2. Now set down the numbers 1, 2, 4, and the mean becomes 2, exceeding the one extreme in the same ratio as it is exceeded by the other, for 2 is the double of 1, and 4 is the double of 2. Again, take the square numbers 4 and 9; their mean is the heteromecic number 6. Set down 4, 6, 9; then the mean 6 exceeds the former extreme in the same ratio as it is itself exceeded by the latter extreme, for the ratio of 6 to 4 is one and one-half to one, as is the ratio of 9 to 6. The same reasoning applies to the rest of the successive square numbers.

29.12 On the other hand, the heteromecic numbers, products of factors which differ by unity, do not keep the same factors nor enclose square numbers in a proportion. Thus 2 times 3 gives 6, 3 times 4 gives 12, and 4 times 5 gives 20, none of the numbers keeping the same factors but changing it in the

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65 ὁρος can mean the "term" of a proportion, but I think it is more likely to have its original meaning of "limit" here; hence the literal meaning is: "do not change their own limits", cf. οὐδεὶς αὐτῶν μένει ἐν τῷ ἐαυτοῦ ὁρῷ (Hiller, 29.17) But Aristotle uses ὁρος in the sense of Theon's γνώμαν in Phys, iii, 203a, 13; this meaning would not however seem possible here, for Theon has just expressed the squares as "products".
process of multiplication for, you see, 2 is multiplied by 3, and 3 by 4, and 4 by 5. But the resulting heteromecic numbers do not enclose the square numbers in a geometric proportion; for take the successive heteromecic numbers 2 and 6; in the position between them is the square number 4, but it is not enclosed by them in any proportion which has the same ratio to its extremes. Set down 2, 4, 6 and 4 will have a different ratio to the extremes; for the ratio of 4 to 2 is double but the ratio of 6 to 4 is one and one-half to one. For the mean to form a geometrical proportion, it would be necessary for it to be such that the ratio of the first term to the mean should be equal to that of the mean to the third term. Again, although the square number 9 is set in position between the heteromecic numbers 6 and 12, it will not be found to bear the same ratio to these extremes; for the ratio of 9 to 6 is one and one-half to one, but the ratio of 12 to 9 is one and one-third to one. The same applies to the rest of the successive heteromecic numbers.

30. 8 An oblong number is a number produced by any two unequal numbers, where one exceeds the other perhaps by 1, perhaps by 2 or even more; as, for instance, the number 24 and

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66 Theon is somewhat inconsistent, interpreting a geometric proportion differently, here and in 30. 1 above.

67 See note 57. Theon has shown that square numbers result from the addition of the odd numbers in series (see Mathematical Note 5), and heteromecic numbers from the addition of the even numbers (see Mathematical Note 6). Heath (H.C.M. 1, 82) suggests that we possibly have here an explanation for the
such numbers as it, for it is 6 times 4. There are three classes of oblong number. Every heteromecic number is oblong in as much as it has one side longer than the other, so if a number is heteromecic it is also oblong; but the converse is not true, for a number which has one side larger than the other by more than unity is oblong, but assuredly not heteromecic, for the heteromecic number is that having one side greater than the other by unity, as does 6; for it is 2 times 3.

30.18 Furthermore, a number is also oblong when it can be obtained by a variety of possible multiplications; in the first instance, with one side greater than the other by unity and in the second, greater by more than unity; as, for instance, the number 12, for it is 3 times 4, and 2 times 6, so that it would be heteromecic on the first count and oblong on the second.

30.23 Again, a number is oblong if, as a result of resorting to all possible combinations of multiplication the one side is still greater than the other by more than unity; as, for instance, is the number 40, for it is 4 times 10, and 5 times 8, and 2 times 20. Whatever number follows this pattern can...
only be oblong. For the heteromecic number is that number which first results from changing a number formed of equal sides—and the addition of unity to one of these sides first effects this change. Wherefore, numbers resulting from this first change in a side are rightly termed heteromecic; but those numbers that have one side greater than the other by more than unity are called oblong by reason of the greater extent in the length of one side.

31.9 Those numbers are plane numbers that are produced by the multiplication of two numbers representing, as it were, a length and a width. Of these numbers, some are triangular, some are square, some pentagonal and of succeeding polygonal forms.

31.13 Triangular numbers are produced in the following way. Successive even numbers added to one another in series make heteromecic numbers. For instance, the number 2 is the first even number and it is also heteromecic, for it is one times 2. Then, if you add 4 to 2 the result is 6, which again is heteromecic, for it is 2 times 3. The same process may be repeated to infinity and, to put it more plainly that my proposition may be manifest to all, it may also be demonstrated in this way. First let the number 2 be represented by these two x's

\[ x \quad x \]

The figure that they make will be heteromecic, for it is 2 in length and 1 in width. After the number 2 comes the even

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See Mathematical Note 4 (p. 129)
number 4; if we should add these four units to the first two units, and if we place the 4 around the 2, the result is a heteromecic figure of 6 units, with a length of 3 and a width of 2. Next after 4 comes the even number 6; if we add this to the first 6, the result is 12 and the figure will be heteromecic if we place these units around the first 6 units, for it will have a length of 4 units and a width of 3. And the same pattern may be obtained with the addition of successive even numbers to infinity.

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32. 9 Again, the addition of successive odd numbers produces square numbers. The odd numbers in order are 1, 3, 5, 7, 9, 11. If you add these in order you will construct square numbers. Thus the number One is the first square number, for 1 times 1 is 1. Then comes the odd number 3; if you add this as a gnomon to 1, you will make a square "equally-equal", for it will be 2 in length and 2 in width. The next odd number is 5; if you should take this as gnomon and add it to your square, once again a square number 9 will be produced, with a length of 3 and a width of 3. The next odd number is 7; if you add this to 9 you will get 16, which has a length of 4 and a width of 4. The same procedure can be followed to infinity.

\[69\text{See Mathematical Note 5 (p.137)}\]
And in the same way, if we should add together not solely the series of even numbers nor solely the series of odd numbers, but both the even numbers and the odd numbers, we shall have triangular numbers. 70 Let us take the odd numbers and the even numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. The triangular numbers are formed by the addition of these numbers. The first triangular number is Unity; which, if not triangular in actuality, is so by reason of the universality of its power, for it is the "principle" of all the numbers. If the number 2 is added to it, the triangular number 3 is produced. Then by adding 3, 6 is produced, and by adding 4, 10 is produced; then by adding 5, 15 is produced; by adding 6, 21 is produced; by adding 7, 28 is produced; then 8 gives 36, 9 gives 45, 10 gives 55, and so on to infinity.

Now it is obvious that these numbers are triangular in accordance with the figures obtained by the addition of successive gnomons to the numbers already obtained. Thus the triangular numbers resulting from these additions will be 3, 6, 10, 15, 21, 28, 36, 45, 55 and in this way do they follow after 45 and 55.

70 See Mathematical Note 10 (p. 135)
34. 1  The square numbers are produced, as has been said before, by the addition of the successive odd numbers starting from unity and, with the exception of one, they happen to be alternatively even and odd, just as are the natural numbers, viz. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

34. 6  By setting out in order from one the even numbers and the odd numbers, it happens that the gnomons exceeding each other by 2 produce by the process of addition the square numbers as has been shown above, for the numbers that increase by 2 from unity are the odd numbers.\(^7\)

34.10  In the same way the numbers increasing by 3 from unity by addition form pentagons; those increasing by 4 produce hexagons; and furthermore, the increase in the gnomons from which the polygonal numbers are produced is always less by 2

\(^7\) One is excepted, because it has already been termed "odd-even" cf. Hiller 18.25.

\(^7\) See Mathematical Note 3 (p. 127)
than the number of angles in the figure produced.  

34.16 Among the polygonal numbers, there is another set of numbers formed from multiplication by a certain number starting from unity. From these numbers that are a result of multiplication starting at unity, namely by multiplying by 2, or by 3 or the numbers that follow, all those form squares which have an interval of 1, all those form cubes that have an interval of 2, while those with an interval of 5 form cubes and squares at the same time; for they are either cubes that have sides which are square numbers or squares that have sides which are cubic numbers. This will demonstrate how, of the numbers that are powers starting from 1, every other one is a square, every third one is a cube, and every sixth is both a square and a cube at the same time. In the series of numbers obtained by multiplying by 2, some go like this: 1, 2, 4, 8, 16, 32, 64, 128, 256. The first double is 2; then comes 4 which is a square; then 8 which is a cube; then 16 which is a square; then 32, and after that 64 which is at the same time a square and a cube. Then there is 128, and after it 256 which is a square; and so on, to infinity.

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73 See Mathematical Note 11 (p. 136)
74 See Mathematical Note 8 (p. 133)
75 I have read a ά β ά δ ά η ά ι έ Α ά ξ ά ρκη έ σνγή the series of numbers conjectured by Gelder, as this is the only series that seems to make sense following ἐν μεν τοῖς διπλασίοις in 35. 5.
35.12 In the series of numbers obtained by multiplying by 3, it will also be found that every other one is a square number, and in the series of powers of 5 and in those derived from other numbers, the same will be the case. And in the same way too among these powers, all those that jump two terms will be found to be cubes, and those that jump five terms will also be found to be cubes and squares at the same time.

35.17 In the case of the square numbers, they have the peculiar property of having the factor 3, or of being exactly divisible by 3 after the subtraction of 1; or again, they have the factor 4 or are exactly divisible by 4 upon the subtraction of 1. Again, the even square that has 3 as a factor after the subtraction of 1, is also exactly divisible by 4, as in the case of the number 4; while the square that is divisible by 4 upon the subtraction of 1, is exactly divisible by 3 as, for instance, is the number 9; or a square may be exactly divisible by both 3 and 4, as is 36; or the square that has neither 3 nor 4 as a factor becomes divisible by these numbers after the subtraction of 1, as is the case with 25.

76 See Mathematical Note 9 (p.134). I have taken Dupuis' conjecture <αρτίον> to follow και τὸν μὲν in 35.20; for there are two types of "square numbers divisible by 3 after the subtraction of 1", of types (A) and (D), but of these only the even one, of form (A), is exactly divisible by 4. There is a further difficulty; for while there are two types of "squares divisible by 4 after the subtraction of 1", of forms (B) and (D), only that of form (B) is exactly divisible by 3. Theon lacks a method that affords the generality obtained by algebra.
36. 3 Again among the numbers, those "equally-equal" are squares, those "unequally-unequal" are heteromecic or oblong; or, to put it simply, those that are the product of two factors are plane and those that are the product of three factors are solid.

36. 6 Numbers are called plane, and triangular, and square, solid and other similar names, not in the proper sense but by reason of their similarity to the spaces that they measure. Thus 4, since it measures a square space, is called a "square" number, and 6 is called "heteromecic" for the same reason. 77

36.12 Among the plane numbers, the square numbers are all similar to each other; and those heteromecic numbers are also similar whose sides, i.e., whose numerical factors are in proportion. Take, for instance, the heteromecic number 6; its sides are of length 3 and of width 2. Again another plane number is 24, with sides of length 6 and of width 4. Now the ratio of the length of one to the length of the other is equal to the ratio of the width of the one to the width of the other; for the ratio of 6 to 3 is equal to the ratio of 4 to 2; wherefore, the plane numbers 6 and 24 are similar.

36.20 Sometimes the same numbers may be represented in three different ways; first, as sides when taken as lengths for the composition of other numbers and, secondly, as plane numbers, when they are the product of the multiplication of two other

77i.e., because it measures a "heteromecic" space.
numbers; and thirdly, when they are taken as solid numbers, produced as a result of the multiplication of three numbers.

37. 2 Among the solid numbers again, all the cubic numbers are similar to each other; of the rest, those rectangular solids are similar that have their sides in proportion, for the ratios of the length of one to the length of the other, the width of one to the width of the other, and the height of the one to the height of the other, are equal.

37. 7 Of the plane polygonal numbers, the first is the triangular number, even as the triangle is the first of the rectilinear figures. The method of formation of the triangular numbers has been told already, namely by adding to the first number the even numbers and odd numbers in succession.

37.11 All the numbers in series which produce triangular, square and polygonal numbers are called gnomons, and for the triangular numbers each triangle has sides of exactly as many units as has the single gnomon last added. Take first Unity, which is considered as a triangular number, not in the proper sense, as we have already said, but potentially. Since it is, as it were, the germ of all numbers it holds within itself in addition the capacity to assume the triangular form. At any rate, when it takes the number 2, it completes a triangle with sides of as many units as has the gnomon of 2 just added. Then the whole triangle consists of as many units as the sum of the gnomons, for the sum of the gnomons 1 and 2 is 3 so that the

\[ \text{See Mathematical Note 3 (p. 127)} \]
triangle will consist of 3 units and will have each side consisting of 2 units, i.e., of as many units as the number of gnomons added.

38. 2 Next, the triangle 3 receives in addition the gnomon 3, which exceeds the number 2 by 1 and the whole triangle produces 6. This triangle too, will have sides of as many units as the number of gnomons added, for 6 is the sum of 1, and 2 and 3.

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x\times \\
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38. 8 Then the number 6 takes the number 4, and a triangle of 10 is formed with each side of 4 units, for the gnomon 4 has been added and the total is now made up of 4 gnomons, i.e., 1, 2, 3, and 4.

38.11 Next, to the number 10 is added 5 and a triangle of 15, comprising the sum of 5 gnomons, is produced with each side of 5 units; and in like manner do the successive gnomons produce the triangular numbers.

38.16 Some numbers are called circular, spherical or recurrent. These are those which, in being squared or cubed, that is, in being multiplied in two or three dimensions, return to the number from which they started. A circle also describes such a figure for it returns to its starting-point and comprises a single line starting and finishing at the same point.

79 I have read ὅμως καὶ οἱ ἐξ ὑπὸ γνώμονας τοὺς τριγώνους ἀριθμοὺς ἀποτελοῦσι for this passage, as conjectured by Dupuis (64.6)
39. 3 Among the solids the sphere has the same property, for when a circle is rotated about its diameter its return from one position to the same position describes a sphere. And so we see that those numbers which, by being multiplied, finish with themselves are called circular or spherical. Both 5 and 6 produce such numbers. For 5 times 5 is 25, and 5 times 25 is 125; and 6 times 6 is 36, and 6 times 36 is 216. 80

39.10 Now as to the formation of the square numbers, as I have said, they result from the addition of the odd numbers, that is, those increasing by 2 from unity; for 1 plus 3 is 4, and 4 plus 5 is 9, 9 plus 7 is 16, and 16 plus 9 is 25.

39.14 The pentagonal numbers are those which are the result of the addition of the series of numbers that start from unity and increase by 3; so the terms on the one hand are 1, 4, 7, 10, 13, 16, 19 while the pentagonal numbers themselves are 1, 5, 12, 22, 35, 51 and so on. They are sketched in pentagonal form as follows:

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80 Theon says 5·5 (25) is circular, 5·5·5 (125) is spherical because they end in 5, the number they started from. This definition is obviously based on notation, i.e., the decimal notation, and therefore departs from the principles of number classification we have seen so far, depending as they do upon a graphical arrangement of units. I would suspect
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40. 1 Hexagonal numbers are those which are a result of the addition of the series of numbers that increase by 4, starting from unity. Their gnomons are 1, 5, 9, 13, 17, 21, 25, from which are derived these hexagonal numbers: 1, 6, 15, 28, 45, 66, 91. They are sketched in this way:

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that this definition is not Pythagorean at all, but a later addition. Note that 1 would also give circular and spherical numbers, and 0 too if permitted; in short, all numbers ending in 0, 1, 5 or 6 would. Theon's failure to quote some such numbers as 15·15 (225 - circular), 15·15·15 (3375 - spherical) or 11·11 (121 - circular), 11·11·11 (1331 - spherical) seems to be strong evidence that Theon does not take a "mathematically" critical approach to his presentation.
40. 6 The heptagonal numbers are those that are composed from numbers that increase in fives from unity; their gnomons are 1, 6, 11, 16, 21, 26, and the heptagonal numbers that are formed from them are 1, 7, 18, 34, 55, 81.

40. 9 The octagonal numbers are also formed in like manner by the addition of numbers that increase in sixes from unity; the nonagonal numbers are composed from numbers that increase in sevens from unity, while decagonal numbers are formed by the addition of numbers that increase from unity in eights. Thus in general for all polygonal numbers, the increase of the numbers in the series from which they are formed is obtained by subtracting two units from the number of angles that gives the polygonal number its name.

41. 3 From two triangular numbers a square number is produced. Thus 1 plus 3 equals 4, 3 plus 6 equals 9, 6 plus 10 equals 16, 10 plus 15 equals 25, 15 plus 21 equals 36, 21 plus 28 equals 49, 28 plus 36 equals 64, 36 plus 45 equals 81; and in like manner succeeding triangular numbers taken in order two at a time produce square numbers, even as the joining together of the sketches of their triangles makes a square figure.

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81 Hiller (praef. vi.) criticises the diagrams of the ms. as "figuris negligentissime factis". Here the diagram appears to indicate that a square number may be divided into two "equal" triangular numbers. This is clearly incorrect; see diagram illustrating $6 + 3 = 9$. 

Moreover among the solid numbers some have equal sides and some have unequal sides. In the latter case, some have all sides unequal and some have two sides equal and one unequal. Again, in the case of those having two sides equal, some have the third side longer, and some have the third side shorter.

Now those having equal sides, being equal in three dimensions, are called cubes; those having all sides unequal, being unequal in three dimensions are called "blocks"; those that have two sides equal and the third less than the other two, being "equally-equal-lesser" are called "tiles"; and those that have two sides equal and the third side greater than the other two, being "equally-equal-greater" are called "rods".

I have read καὶ τὴν μίαν ἁνίσον for ἤττονα, as conjectured by Bullialdus (see Hiller 41.11, note).
42. 3 Pyramidal numbers are those that measure exactly pyramids and truncated pyramids. A truncated pyramid is a pyramid with its top cut off. Some have called such a truncated solid a trapezoidal solid after the plane trapezium; a triangle is called a trapezium whenever its top is cut off by a straight line drawn parallel to the base.

42.10 Just as some numbers are invested with power to make triangles, squares, pentagons and the other figures, so also we find the side and diagonal ratios being revealed by numbers in accordance with the generative principles, for from these ratios do the figures receive their proportions. For just as the unit, according to the supreme generative principle, is the starting-point of all figures, so also in the

The terms used by Theon to define his classification of solid numbers are βωρίσκος (little altar), πλινθίς (plinth, wedge), and δοξίς (beam) and these have been translated block, tile and rod respectively. However his criteria for the classification are unconvincing. It would appear at any rate that one should postulate in addition that the three dimensions should be prime numbers and the factors remain invariable. For consider: \(60 = 3 \cdot 4 \cdot 5\) (block) = \(15 \cdot 2^2\) (rod)
\(72 = 8 \cdot 3^2\) (rod) = \(2^6\) (tile)
\(147 = 3 \cdot 7^2\) (tile) invariable; prime factors. Yet the diagrams given by Theon representing a block, a tile and a rod depict the composite numbers 64, 32 and 40 respectively and these have alternative forms: 64 (rod, \(16 \cdot 2^2\)), 32 (rod \(8 \cdot 2^2\)) and 40 (block, \(2 \cdot 4 \cdot 5\)).

See Mathematical Note 12 (p. 140)
unit will be found the ratio of diagonal to side. For example, let two units be taken, of which we set one to be a diagonal and the other a side since the unit, as the beginning of all things, must potentially be both side and diagonal. Now a diagonal is added to the side and to the diagonal two sides; for as often as the square on the diagonal is taken once, so often is the square on the side taken twice. Then the diagonal has become the greater and the side the lesser. Now in the case of the first side and diagonal, the square on the unit diagonal will be less by one unit than twice the square on the unit side; for units are equal, and 1 is less by one unit than twice 1. Indeed let us add to the side a diagonal, i.e., to the one unit let us add one unit; therefore the (second) side will be two units. To the diagonal let us now add two sides, i.e., to the one unit let us add two units; the (second) diagonal will therefore be three units. Now the square on the side of two units will be 4, while the square on the diagonal of three units will be 9; and 9 is greater by one unit than twice the square on the side 2.  

44. 3  In the same way, let us add to the side 2 the diagonal 3; the (third) side will be 5. To the diagonal 3 let us add two sides, i.e., twice 2; the third diagonal will be 7. Now the square from the side 5 will be 25, while that from the diagonal 7 will be 49; and 49 is less by one unit than twice 25. Again if you add to the side 5 the diagonal 7, the result will be 12; and if you add twice the side 5 to the diagonal 7, the

85 See Mathematical Note 13 (p. 142)
result will be 17. And the square of 17 is greater by one unit than twice the square of 12. And in like manner for subsequent calculations there will be the same alternating relationship; the square on the diagonal first being greater by one unit, and then less by one unit than twice the square on the side; and such sides and diagonals are alike rational.

\[ \begin{array}{c|c|c|c|c}
  80 & 2 & 3 & 4 & 9 \\
  4 &  &  & 25 & 49 \\
\end{array} \]

44.18 The squares on the diagonals compared with double the squares on the sides, are alternately greater by one and then less by one. Thus the square on each diagonal will become double the square on each side, as the addition of the very same unit alternately restores the equality, preventing the double square on the side being in excess or deficient in each case. For the amount lacking in the preceding diagonal is found in excess in the subsequent one.

45.9 Moreover some numbers are called perfect, some over-perfect, and some defective. Perfect numbers are those that are equal to their own parts, as is the number 6; for its half part is 3, its third part is 2 and its sixth is 1, and these added together give 6.

45.13 The perfect numbers are produced in the following way.

---

\(^{86}\pi(80)\) and 6(4) given for the first diagonal and side in the diagram are probably copying errors for \(\alpha(1)\).  

\(^{87}\) See Mathematical Note 14 (p.144)
If we take the double numbers starting from unity and if we add them until a prime and incomposite number results, and if we then multiply the sum obtained by the last term which was added, the result of the multiplication will be a perfect number. For example, let the double numbers be 1, 2, 4, 8, 16; then let us add 1 and 2, which give 3; next we must multiply it by the last of the terms added, that is, by 2 and the result is 6, which is the first perfect number.88

45.22 Again, if we add three of the series of doubles, 1 and 2 and 4, the result will be 7. Then must we multiply it by the last of the numbers added, that is, we multiply 7 by 4; the result will be 28, which is the second perfect number. It is composed of its half 14, its fourth part 7, its seventh part 4, its fourteenth part 2, and its twenty-eighth part 1.

46. 4 Over-perfect numbers are those whose fractional parts added together are greater than their wholes as, for example, 12: for the half of this number is 6, the third is 4, the fourth is 3, the sixth is 2, and the twelfth is 1, all of which added together produce 16, which is larger than the original number 12.

46. 9 Defective numbers are those whose fractional parts, when added, produce a number less than the number originally given, such as 8; for the half of this number is 4, the fourth 2, and the eighth is 1. The same property applies to the number 10, a number that the Pythagoreans call perfect for a

88 See Mathematical Note 14 (p. 144)
different reason, and this we shall discuss in the proper place. 89

46.14 The number 3 is also called perfect, because it is the first number which has a beginning, a middle and an end. 90 It is, moreover, both a line and a surface for it is an equilateral triangle in which each side is two units, and it is the first bond and power of the solid, for it is in three dimensions that the concept of the solid lies.

89 Theon omits to mention the "friendly" or "amicable" numbers in his treatment of perfect numbers. Two numbers are friendly when each is the sum of the aliquot parts of the other, e.g. 284 and 220 (for 284 = 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110, while 220 = 1 + 2 + 4 + 71 + 142). Iamblichus (in Nicom., p.33, 20-23) infers from a story he recounts about Pythagoras that the latter knew of these numbers and, consequently, of perfect numbers too.

90 For a treatment of the mystical properties assigned by the Pythagoreans to particular numbers, see J. Gow, A Short History of Greek Mathematics (Cambridge 1884) p.69.
Note 1 - Doubling the cube (cf. Hiller 2. 5)

Theon's story about the Delians consulting the oracle at Delphi and being told to construct an altar of double size is probably a reference to the intricate problem of doubling the cube which taxed the ingenuity of the Greek mathematicians of the sixth century B.C.

Of course, it must be presumed that the altar called for was to be cubic in shape with length, width and height all equal; for to construct a similar parallelepiped of double size, preserving the ratios between corresponding sides, would be an impossible task.

Eutocius gives a list of the authors of solutions to this problem of doubling the cube as follows: Plato, Heron, Philon, Apollonius, Diocles, Pappus, Sporus, Menaechmus (two solutions), Archytas, Eratosthenes and Nicomedes.

He identifies Hippocrates of Chios\(^1\) as the first to see that, if two mean proportionals could be constructed in continued proportion between two straight lines, one of which was double the other, the smaller of the proportionals found would produce a cube double the volume of that cube derived

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\(^1\)Eutocius, Comm. on Archimedes' Sphere and Cylinder ii, Archim. ed. Heiberg iii, 88, 4ff.
from the smaller of the two straight lines. This may be shown by simple algebra as follows:

Let the two straight lines have lengths $a$ and $2a$, so that we need to find a length $x$, such that $x^3 = 2a^3$.

Suppose two means, $x$ and $y$, are inserted between $a$ and $2a$, so as to form the continued proportion $a, x, y, 2a$.

Then $\frac{a}{x} = \frac{x}{y} = \frac{y}{2a}$

Thus $\frac{a}{x} = \frac{x}{y}$, whence $y = \frac{x^2}{a}$

And $\frac{a}{x} = \frac{y}{2a}$, whence $y = \frac{2a^2}{x}$

Hence $\frac{x^2}{a} = \frac{2a^2}{x}$

Therefore $x^3 = 2a^3$

Thus the first of the two mean proportionals found ($x$) will produce a cube of volume twice that of a cube of the smaller ($a$) of the initial two straight lines.

Eutocius ascribes to Plato a beautifully simple practical solution for finding the lengths of two mean proportionals between any two straight lines, although it is virtually certain that this solution is wrongly ascribed to Plato. Thomas points out that Eutocius alone mentions it, and that if it had been known to Eratosthenes he could hardly have failed to quote it along with the solutions of Archytas, Menaechmus and Eudoxus. Furthermore, Plato according to

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2 Eutocius, ibid., 56, 13.

3 Ivor Thomas, Greek Mathematical Works, Loeb, 1939, 1, 262 (note).
Plutarch told the Delians that Eudoxus or Helicon of Cyzicus would solve the problem for them; he did not apparently propose to tackle it himself. And Plutarch twice says that Plato objected to mechanical solutions on the grounds that they destroyed the good of geometry, a statement which is consistent with his known attitude towards mathematics. The solution is here explained, and gives an insight into the facility with which the Greeks, although lacking the analytical methods of algebra, brought geometry into play.

A rectangular framework PQRS is constructed with grooves which permit the length TU to move in such a way as to be always parallel with QR, while an additional piece comprising two straight arms fixed together at right-angles may be clamped on the framework in any position.

Fig.1 - Apparatus for finding two mean proportionals

Suppose a and b are the two lengths between which the two means are to be inserted in continued proportion;

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4 Plutarch, de genio Socratis, c.7, 579 C,D.
5 Plutarch, Quaest. Conviv., 8. 2. 1., 718 E,F; Vita Marcelli, c.14.5.
then take BC of length \(a\) and BA of length \(b\), and adjust the position of the arm-piece in such a way that A lies on QR, C lies on TU, and AB produced and CB produced pass through U and R respectively (See Fig.1). There is only one unique position in which this can be effected.

Then, if \(BC = a\), \(AB = b\), \(BU = x\), \(BR = y\) the two required mean proportionals are of lengths \(x\) and \(y\).

For in the right-angled triangles CBU, UBR, RBA

\[
\angle CUB = \angle URB = \angle RAB
\]

Therefore, the three triangles are equiangular and their sides are proportional.

\[
\frac{a}{x} = \frac{x}{y} = \frac{y}{b}
\]

So that, if \(b = 2a\),

\[
\frac{a}{x} = \frac{x}{y} = \frac{y}{2a}
\]

and, by proof on p. 123 \(x^3 = 2a^3\)

Thus, if BC is marked off equal to \(a\) and BA equal to \(2a\), BU will give the length of the side of the double cube.

**Note 2 - One is without parts and indivisible** (cf. Hiller 18.15)

Theon's reasoning of course appears fallacious. He argues that, if the One is divided into many parts and if each of these is taken away one at a time, we finally have one left; if then, that one is divided into many parts and each of these is again subtracted, then only one will remain again; and, this result will always be obtained however many times the division and subtraction take place. One is always
left ὁμοίως ὁμοίως καὶ ἄνδρῳ τὸ ἐν ὡς ἐν (in so far as its oneness is concerned, One has no parts and is indivisible).

The Greeks of the sixth century B.C. were greatly puzzled by fractions, and Plato was continually fascinated by the character of One. The *Parmenides* contains phrases corresponding to what we find in Euclid's preliminary matter. Thus Plato speaks of something which is a "part" but not "parts" of the One. Heath observes that this reminds one of Euclid's distinction between a fraction which is a "part", i.e., an aliquot part and one which is "parts", i.e., some number more than one of such parts, e.g., 3/7.

Indeed, Plato points out that Zeno of Elea wrote a book whose object was to defend the system of Parmenides by attacking the common conception of things. Parmenides held that only the One exists, whereupon it had been pointed out that many contradictions and absurdities would follow if this were admitted. Zeno replied that, if the popular view that the Many exist be accepted, still more absurd results would follow and he accordingly advanced the paradoxes for which he is well known.

Heath believes that there is no justification for Tannery's contention that the arguments of Zeno refuting the

---

6 cf. Plato, *Parmenides*, 153D.
divisibility of magnitudes and times were especially directed against the Pythagorean view that bodies, surfaces and lines are made up of mathematical points. The Pythagorean definition of a point was "a unit having position" (μονάς θέσιν ἔχουσα) but according to Aristotle the Pythagoreans maintained that the units and numbers do have magnitude.

Theon is here, I believe, rather pointing out the uniqueness of the number One. Aristotle observes that the One is reasonably regarded as not being itself a number, because a measure is not the things measured, but the measure or the One is the beginning or "principle" (ἀρχή) of number.

This doctrine may be Pythagorean in origin; Nicomachus has it and Theon himself re-echoes it in his treatment of the μονάς.

Note 3 - The term γνώμων

It is clear from Aristotle's allusions to "gnomons" placed around 1, which "now produce different figures every time" (oblong figures each dissimilar from the preceding one), "now preserve one and the same figure" (as is the case with

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9 Heath, H.G.M., i, 283. Heath also draws attention to a most comprehensive series of papers by Florian Cajori, The History of Zeno's arguments on Motion, American Mathematical Monthly, 1915; while most helpful of the vast literature on Zeno's paradoxes may be recommended W.D. Ross, Aristotle's Physics, pp. 655 -666, H.P.D. Lee, Zeno of Elea, and Heath, H.G.M., i, 271 - 283.

10 Arist, Metaph., M 6, 1088 b 19, 32.

11 Ibid., N 1, 1088 a 6.

12 Nicomachus, Introd. arithm., ii, 6.3, 7.3.
triangular and square numbers), that these gnomons are the successive terms which are added to produce the patterned numbers.\textsuperscript{13}

Heath gives some interesting historical information concerning the term \( \gammaνώμον \).\textsuperscript{14}

a) It was originally an astronomical instrument for measuring time, and consisted of an upright stick which cast shadows on a plane or hemispherical surface. Following this use of the word, gnomon becomes "marker" or "pointer" -- a means of marking off or "knowing" something, and we find Oenopides defining a straight line drawn from an external point, perpendicular to a straight line, as drawn \( \kappaατά γνώμονα \) (gnomon-wise).

b) Next the term was used to denote an instrument used for drawing right-angles, as is shown in Fig. 2

\begin{figure}[h]
\centering
\includegraphics[width=0.2\textwidth]{gnomon.png}
\caption{Gnomon for drawing right-angles}
\end{figure}

Subsequently, gnomon was used to denote the figure which remained of a square after a smaller square had been


\textsuperscript{14} Heath, H.G.M., i, 78.
cut out of it -- or the figure which, as Aristotle says, "when added to a square, preserves the shape and makes up a larger square". Probably, as Boeckh says, the connection between the gnomon and the square to which it is added was regarded as symbolical of union and agreement, and Philolaus used the idea to explain the knowledge of things, making the "knowing" embrace the "known" as the gnomon does the square.

d) In Euclid the meaning is still further expanded (II Def.2) to cover the figure which serves the same complementary function to a parallelogram, as the figure in c) serves for the square.\(^1^5\)

![Fig 3 - Gnomon of parallelogram](image)

\(e\) Later still, Heron defines the "gnomon" in general as that which, when added to anything, number or figure, makes the whole similar to that to which it is added.\(^1^6\)

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**Note 4 - Figurate (or figured) numbers**

The part which geometry played in Greek mathematics is admirably illustrated by the manner in which the Greeks applied it to the study of the composition of numbers and their properties. The theory of figured numbers seems to go

\(^1^5\) cf. Scholium 11, Bk II in Euclid, ed. Heib., v, 225

\(^1^6\) Heron, Def. 58 (Heron, iv, Heib., p.225)
back to Pythagoras himself.

Nicomachus enunciates the principles upon which the
Greeks studied the composition of numbers as follows:

"Εστιν οὖν σημεῖον ἀρχὴ διαστήματος, οὐ διάστημα δέ, τὸ δ' αὑτὸ καὶ ἀρχὴ γραμμῆς, οὐ γραμμῆ δέ καὶ γραμμὴ ἀρχὴ ἐπιμελείας, οὐκ ἐπιμελεία δε, καὶ ἀρχὴ τοῦ δική διαστατοῦ, οὐ δική διαστατοῦ. καὶ εἰκότως ἡ ἐπιφάνεια ἀρχὴ μεν σωμάτος, οὐ σώμα δε, καὶ ἡ αὑτῇ ἀρχῇ μεν τοῦ τρικὴ διαστατοῦ, οὐ τρικὴ διαστατοῦ, οὕτως δὲ καὶ ἐν τοῖς ἀρίθμοις οὐ μεν μονας ἀρχὴ παντος ἀρίθμου ἐφ' ἐν διάστημα κατὰ μονάδα προβιβαζομένου, ὁ δὲ γραμμικὸς ἀρίθμος ἀρχὴ ἐπιπέδου ἀρίθμου ἐφ' ἐτερον διάστημα ἐπιπέδως πλατυνομένου, ὁ δὲ ἐπιπέδου ἀρίθμος ἀρχὴ στερεοῦ ἀρίθμου ἐπί τρίτον διάστημα προς τα ἐξ ἀρχῆς βαθος τι προσκυμνέων οἷον καθ' ὑποδιαίρεσιν γραμμικοί μεν εἰσιν ἀρίθμοι ἀπλῶς ἀπαντες οἱ ἀπὸ δυάδος ἀρχομένοι καὶ κατὰ μονάδος προσθέσιν ἐπ' ἐν καὶ τὸ αὑτὸ προχωροῦντες διάστημα, ἐπιπέδοι δὲ οἱ ἀπὸ τριάδος ἀρχομένοι ἀρχαὶ κωτατῆς βίζης καὶ διὰ τῶν ἔξις συνεχῶς ἀρίθμους προϊόντες, λαμβανόντες καὶ τὴν ἐπανωμαγκατα τὴν αὐτὴν ταξιν πρωτοταῖο γαρ τριγώνοι, εἴτα μετ' αὐτούς τετράγωνοι, εἴτα μετ' αὐτοὺς πεντάγωνοι, εἴτα ἐπὶ τούτοις ἕξαγωνοι καὶ ἕπταγωνοι καὶ ἐπ' ἅπειρον. 17

Point is therefore the principle of dimension, but is not dimension, while it is also the principle of line, but is not line; and line is the principle of surface, but is not surface, and is the principle of the two-dimensional, but is not two-dimensional. Naturally also surface is the principle of body, but is not body, while it is the principle of the three-dimensional, but is not three-dimensional. Similarly among numbers the unit is the principle of every number set out by units in one dimension, while linear number is the principle of plane number broadened out in another dimension in the manner of a surface, and plane number is the principle of solid number, which acquires a certain depth in a third dimension (at right angles) to the dimensions of the surface. For example, by subdivision linear numbers are all numbers without exception beginning from two and proceeding by the addition of a unit in one and the same dimension, while plane numbers begin from three as their fundamental root and advance through an orderly series of numbers, taking their designation according to their order. For first come triangles, then after them are squares, then after these are pentagons, then succeeding these are hexagons and heptagons and so on to infinity.

17 cf. Nicomachus, Intro. Arithm., ii, 7 1-3, (ed. Hoche, 86.9 - 87.6)
This attempt to present a rational explanation of the three dimensions is in every respect comparable to the modern concept. What is of more interest to us is the manner in which the point becomes associated with the unit (μονάς).

Most of the Greeks' discoveries in the realm of ἄριθμον μουθήματι are based upon the figures which these points can be made to represent. We may remark upon the importance of the graphical approach as an aid in most branches of mathematics even today.

Thus, a point or dot is used to represent 1; two dots are used to represent the number 2, which also represents a straight line; three dots represent the number 3 and form the first plane number (a triangle); four dots represent the number 4 and define the first solid number (a tetrahedron). Theon follows Nicomachus of Gerasa in his presentation of the composition and properties of polygonal numbers.

Note 5 - Formation of square numbers (cf. Hiller 32.10)

If the odd numbers are taken in series and added, square numbers result, as is shown by Theon's diagrams. A general proof that the sum of n of these odd numbers will be \( n^2 \) may be demonstrated as follows:

The nth odd number may be denoted \((2n-1)\), so that if S denotes the sum of these n terms, we have

\[
S = 1 + 3 + 5 + \ldots + (2n-3)+(2n-1)
\]

also \( S = (2n-1)+(2n-3)+(2n-5)+ \ldots + 3 + 1 \)
whence \( 2S = 2n + 2n + 2n + \ldots \ldots \ldots \ldots -2n + 2n \)
\[ = 2n \cdot n \]
Therefore \( S = n \cdot n \)
\[ = n^2 \]

Note 6 - Formation of heteromecic numbers (cf. Hiller 27.7)

Theon points out that the sums of the even numbers taken in order produce heteromecic numbers, i.e., numbers having one side larger than the other by unity.

Algebraically, we may denote the nth even number by \( 2n \), and denoting the sum of the n even numbers by \( S \), we have
\[ S = 2 + 4 + 6 + \ldots \ldots + (2n-2) + 2n \]
also \( S = 2n + (2n-2) + (2n-4) + \ldots \ldots + 4 + 2 \)
Thus \( 2S = (2n+2)+(2n+2)+(2n+2)+\ldots+(2n+2)+(2n+2) \)
and \( S = (n+1)+(n+1)+(n+1)+\ldots+(n+1)+(n+1) \)
\[ = \text{the sum of n terms equal to} \ (n+1) \]
\[ = n(n+1) \]
and this is the general form of the heteromecic number and, as pointed out by Theon, it may be obtained either by the addition of the even numbers in succession or by the multiplication of two adjacent numbers.

Note 7 - Means between heteromecic and square numbers (cf. Hiller 28.20)

If we consider two consecutive heteromecic numbers \( n(n-1) \) and \( n(n+1) \), i.e., \( (n^2-n) \) and \( (n^2+n) \), it will be obvious that \( n^2 \) will be their arithmetic mean, for \( (n^2-n), n^2 \)
and \((n^2 + n)\) form an arithmetic progression.

Note that the geometric mean between these two numbers will be less than their arithmetic mean.

For, if \(x\) denotes their geometric mean,

we have \[
\frac{n(n-1)}{x} = \frac{x}{n(n+1)}
\]

\[
x^2 = n^2(n^2-1)
\]

\[
x = n\sqrt{n^2-1}
\]

Thus \(x < n^2\), for \(\sqrt{n^2-1} < n\).

Note 8 – Powers which are squares and cubes (cf. Hiller 34.16)

While Theon considers these numbers as being obtained from the number 1, by multiplying by 2 and by 3 \(\tau\epsilon\nu\tau\epsilon\nu\), we naturally recognise these numbers as powers of 2, 3 etc.

and may consider the general case as follows:

<table>
<thead>
<tr>
<th>Powers of (a)</th>
<th>1</th>
<th>(a)</th>
<th>(a^2)</th>
<th>(a^3)</th>
<th>(a^4)</th>
<th>(a^5)</th>
<th>(a^6)</th>
<th>(a^7)</th>
<th>(a^8)</th>
<th>(a^9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squares ((S))</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cubes ((C))</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Squares/Cubes (SC)</td>
<td>SC</td>
<td>SC</td>
<td>SC</td>
<td>SC</td>
<td>SC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is clear that those numbers will be squares and cubes which are powers of \(a^6\), i.e., \(a^6\), \(a^{12}\), \(a^{18}\), \(a^{24}\) ... or, in general, those of the form \(a^{6n}\) which is equivalent to either \((a^{2n})^3\) denoting cubes having squares as sides or \((a^{3n})^2\) denoting squares having cubes as sides.
Note 9 - Divisibility of Squares (cf. Hiller 35.17 ff.)

Theon contents himself with simple illustrations of his proposition. A proof may be derived simply by algebra.

Any natural number may be written in the form 6n, 6n+1, 6n+2 6n+3, 6n+4, 6n+5.

But 6n+5 = 6n+6-1 = 6(n+1) - 1, i.e., of form (6n -1)
and 6n+4 = 6n+6-2 = 6(n+1) - 2, i.e., of form (6n -2)

Hence, the natural numbers may be represented by 6n, 6n ± 1, 6n ± 2, 6n ± 3 and their squares by

\[ 36n^2, \quad 36n^2 ± 12n + 1, \quad 36n^2 ± 24n + 4, \quad 36n^2 ± 36n + 9 \]

(C) (D) (A) (B)

and the factors of these types may be tabulated as follows:

<table>
<thead>
<tr>
<th>Square</th>
<th>Divisibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Divisible by 4, but not by 3; but the subtraction of 1 leaves a remainder ( 36n^2 ± 24n + 3 ) divisible by 3.</td>
</tr>
<tr>
<td>B</td>
<td>Divisible by 3 but not by 4; but subtraction of 1 leaves a remainder ( 36n^2 ± 36n + 8 ) divisible by 4.</td>
</tr>
<tr>
<td>C</td>
<td>Divisible by 3 and 4; but subtraction of 1 leaves a remainder ( 36n^2 - 1 ) divisible neither by 3 nor 4.</td>
</tr>
<tr>
<td>D</td>
<td>Divisible by neither 3 nor 4; but subtraction of 1 leaves a remainder ( 36n^2 ± 12n ) divisible by 3 and 4.</td>
</tr>
</tbody>
</table>

It has been noted by Thomas that this may be expressed in modern mathematical terminology as "all square numbers are congruent to 0 or 1, modulus 3 or congruent to 0 or 1, modulus 4". 18

18 Ivor Thomas, Greek Mathematical Works, (Loeb, 1939) 1, p.105. He observes that the above is the first appearance of any work on congruence which is fundamental in the modern theory of numbers,
Note 10 - Triangular numbers (cf. Hiller 32.22, 37.11)

It was probably Pythagoras who discovered that the sum of any number of successive terms of the series of natural numbers forms a triangular number. This follows readily from the Pythagorean representation of numbers by means of dots which form equilateral triangles. Theon sketches these triangular numbers up to 36 but, to consider the general case, the nth number may be denoted n and then the sum of these n numbers (S) may be found thus:

\[ S = 1 + 2 + 3 + \ldots + (n-1) + n \]
and \[ S = n + (n-1) + (n-2) + \ldots + 2 + 1 \]
then \[ 2S = (n+1) + (n+1) + (n+1) + \ldots + (n+1) + (n+1) \]
\[ = n \text{ terms of } (n+1) \]
\[ = n(n+1) \]
whence \[ S = \frac{n(n+1)}{2} \]

From the above result it may be observed that any triangular number is equal to one-half of the corresponding heteromecic number.

Theon, in a rather laborious and circuitous exposition of these numbers, has two points to emphasise, namely that the number of units comprising the side of the equilateral triangle is equal to a) the number of units in the last term (or gnomon) added, and to b) the number of terms (or gnomons) comprising the number.
Note 11 - Polygonal numbers (cf. Hiller 37.11:ff.)

Two methods of representing numbers geometrically were used by the Greeks. The method used by Plato was to depict numbers by lines proportional in length to their magnitude. The Pythagorean method was to use dots or alphas patterned in various forms to represent numbers. As the Pythagoreans have been credited also with the discovery of the "irrational", it is no doubt likely that they also used the former method.

"The unit", as Theon says, "is the fount and source of all number", hence the "principle" (δρχή) of number, and it is instructive to observe the ingenious method by which series of numbers could be obtained, based upon regular polygons developed ἀπὸ μονάδος.

By means of algebra it is possible to generalise upon the method of formation of these polygonal numbers and investigate some of the common properties which they have and the relationships which exist between them.

In Fig. 4 are shown the square number 16, the triangular number 10, the pentagonal number 12 and the hexagonal number 15, together with the gnomons which are used in their composition. It may be observed that the increase which occurs in the gnomon and which serves to preserve the original pattern of the number also serves to indicate the number of angles composing the polygon, for it is less by 2 in each case.
To consider it algebraically, let the gnomons be

\[ 1, 1 + d, 1 + 2d, 1 + 3d \ldots \ldots \ldots \ldots 1 + (n-1)d \]  

(i)

Then the polygonal numbers will be the sums of 1, 2, 3 ... n terms of this series. Thus, adding the successive gnomons, the polygonal numbers will be given by

\[ 1, 2 + d, 3 + 3d, 4 + 6d \ldots \ldots \ldots \text{nth polygonal number} \]  

(ii)

and the nth polygonal number will be given by

\[
1 + (1 + d) + (1 + 2d) + (1 + 3d) + \ldots + (1 + \frac{n-1}{2})d \\
= n + (1 + 2 + 3 + \ldots + n-1)d \\
= n + \frac{(n-1)n}{2}d
\]  

(iii)
where \( d \) gives the increase in the gnomon and \((d+2)\) denotes the order of the polygon number.

Thus, applying different values of \( d \) in (i) we get

\[
\begin{align*}
\text{d} = 1 & \quad \text{gnomons are } 1, 2, 3, 4 \ldots. \\
giving & \quad 1, 3, 6, 10 \ldots \text{ (triangular numbers)} \\
\text{d} = 2 & \quad \text{gnomons are } 1, 3, 5, 7 \ldots. \\
giving & \quad 1, 4, 9, 16 \ldots \text{ (square numbers)} \\
\text{d} = 3 & \quad \text{gnomons are } 1, 4, 7, 10 \ldots. \\
giving & \quad 1, 5, 12, 22 \ldots \text{ (pentagonal numbers)} \\
\text{d} = 4 & \quad \text{gnomons are } 1, 5, 9, 13 \ldots. \\
giving & \quad 1, 6, 15, 28 \ldots \text{ (hexagonal numbers)}
\end{align*}
\]

From (iii) above, we have

\[
\text{nth polygonal number} = n + \frac{(n-1)n}{2} d
\]

but, as Dupuis points out

\[
\frac{(n-1)n}{2} = (n-1)\text{th triangular number}
\]

Hence, nth polygonal number = \( n + d \) times \((n-1)\)th triangular number.

Wherefore \( d = 2 \) gives polygons of \((d+2)\) sides (square numbers) and nth square number = \( n + n \) times \((n-1)\)th triangular number

Thus first square number = \( 1 + 2(0) = 1 \)

second " \( = 2 + 2(1) = 4 \)

third " \( = 3 + 2(3) = 9 \)

fourth " \( = 4 + 2(6) = 16 \)

-- a property which may be admirably illustrated by the

\[19\] Dupuis, 338. 4
dissection of the square numbers 1, 4, 9, 16 and 25 shown in Fig. 5.

\[ 1 + 2(0) \quad 2 + 2(1) \quad 3 + 2(3) \quad 4 + 2(6) \quad 5 + 2(10) \]

**Fig 5 - Dissection of square numbers**

The principle is further demonstrated in Fig. 6, where \( d = 4 \) gives hexagonal numbers, and the \( n \)th hexagonal number = \( n + 4 \times (n-1) \)th triangular number.

\[ 1 + 4(0) \quad 2 + 4(1) \quad 3 + 4(3) \quad 4 + 4(6) \]

**Fig 6 - Dissection of hexagonal numbers**
It may further be noted that the triangular numbers are 1, 3, 6, 10, 15, 21, 28 and that the hexagonal numbers 1, 6, 15, 28 are the odd-numbered triangular numbers.

In effect, nth hexagonal number = (2n-1)th triangular number for, using formula (iii),

\[ \text{nth hexagonal number} = n + \frac{4(n-1)n}{2} \]

\[ = 2n^2 - n \]

and (2n-1)th triangular number = \( \frac{(2n-1)2n}{2} \)

\[ = 2n^2 - n \]

Therefore, all hexagonal numbers are also triangular numbers.

Note 12 - Pyramid and truncated pyramid numbers (cf. Hiller 42.3)

Pyramid numbers are formed upon triangular or square bases and consist of the sums of the series of triangular or square numbers starting from 1.

Thus, the triangular numbers are

1, 3, 6, 10, 15, \ldots \ldots \frac{n(n+1)}{2} \]

and the triangular pyramid numbers are then

1, 4, 10, 20, 35 \ldots \ldots \text{nth number}

So, nth triangular pyramid number = \( \sum_{i=1}^{n} \frac{n(n+1)}{2} \]

\[ = \frac{1}{2} \sum_{i=1}^{n} n^2 + \frac{1}{2} \sum_{i=1}^{n} n \]

\[ = \frac{2n^3 + 4n}{3} \]

\[ = \frac{n(n^2 + 2)}{3} \]
Now the square numbers are 1, 4, 9, 16, 25... $n^2$

Thus, the square pyramid numbers will be

1, 5, 14, 30 .... nth number

Thus, nth square pyramid number = \[ \sum_{i=1}^{n} i^2 \]
\[ = \frac{n(n+1)(2n+1)}{1 \cdot 2 \cdot 3} \]

Theon makes reference also to truncated pyramid numbers. These are the numbers which comprise the units remaining after the top portion of a given pyramid has been removed. These numbers may therefore be obtained by finding the difference between any two pyramid numbers, or by summing any adjacent terms of the series of triangular or square numbers.

Thus, take the following triangular numbers

\[ \begin{align*}
B & \quad \underline{3}, \underline{6}, \underline{10}, \underline{15}, \underline{21} \ldots \ldots \\
A & \quad 1, \quad \underline{6}, \quad \underline{10}, \quad \underline{15}, \quad 21 \ldots \ldots \\
C & \quad \underline{1}, \quad \underline{3}, \quad \underline{6}, \quad \underline{10}, \quad 15 \ldots \ldots
\end{align*} \]

Then, examples of truncated triangular pyramid numbers are

A (9), B (34), C (46)
And, taking the square numbers

\[1, 4, 9, 16, 25, 36 \ldots \]

the following will be truncated square pyramid numbers

\[D(13), E(54), F(77)\]

and they are composed diagrammatically as shown in Fig. 8.

Fig 8 - Truncated square pyramid number - D (13)

Note 13 - Side and diagonal numbers (cf. Hiller 43.2)

The discovery of the right-angled property of the 3-4-5 triangle probably originated in Egypt, although Pythagoras or the Pythagoreans are usually credited with discovering a proof of it. A simple construction would show that this relationship also existed for isosceles right-angled triangles but, as no integral values were readily at hand, the Greeks were here confronted with the incommensurability of the length of the diagonal of a square with its side and with the irrationality of \(\sqrt{2}\).

With his exposition of side and diagonal numbers, Theon reveals the method by which the Pythagoreans obtained an infinite series of ratios, which approximated closely to
the value of $\sqrt{2}$.

The reasoning went as follows. The unit, being the principle of all number, is first taken as the side and diagonal of the first isosceles right-triangle, and the first crude (integral) approximation (1) is obtained for $\sqrt{2}$. Then, applying the findings of Pythagoras' Theorem, a diagonal is added to the side but two sides are added to the diagonal, and a second triangle is obtained; and we have 3 as the next integral approximation for $\sqrt{50}$. The process is repeated for the next triangle and 7 is obtained as an approximation for $\sqrt{288}$, and again repeated for subsequent triangles as is shown in Fig. 9.

\[
\begin{align*}
1 & \sim 1 & 2 & \sim 1+2(1) & 5 & \sim 3+2(2) & 12 & \sim 7+2(5) \\
\sqrt{2} & \sim \sqrt{50} & & & & & & \sqrt{288}
\end{align*}
\]

Fig 9 - Approximations for $\sqrt{2}$

By this method, integral values of the side and diagonal numbers are obtained as follows:

\[
\begin{align*}
1, & 1, & 1 \\
2, & 2, & 3 \\
5, & 5, & 7 \\
12, & 12, & 17
\end{align*}
\]

and the ratios of diagonal to side furnish values which become progressively more accurate approximations of $\sqrt{2}$.

Thus $\sqrt{2} \approx 1/1 \approx 3/2 \approx 17/12 \approx 41/29 \ldots$. 
Now it may be observed that the square on the diagonal, compared with the double of the square on the side, is first 1 less than it (1), then 1 more than it (9), then 1 less than it (49), then 1 more than it (289) and so on, so that the process, as Heath indicates, amounts to finding all the integral solutions of the indeterminate equations
\[ 2x^2 - y^2 = \pm 1 \]
for the formula, if true, yields two larger numbers \((x+y)\) and \((2x+y)\) such that
\[
2(x+y)^2 - (2x+y)^2 = \pm 1
\]
i.e., \((2x+y)^2 - 2(x+y)^2 = \pm 1\)
which is demonstrably true, for
\[
(2x+y)^2 - 2(x+y)^2 = 4x^2 + 4xy + y^2 - 2x^2 - 4xy - 2y^2
\]
\[= 2x^2 - y^2\]
\[= \pm 1\]

Note 14 - Perfect numbers (cf. Hiller 45.9)

Theon gives the same definition of a perfect number as does Euclid, "one which is equal to (the sum of) its parts", i.e., equal to the sum of all its factors including 1.

Thus 6 = 1 + 2 + 3

28 = 1 + 2 + 4 + 7 + 14

and 496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248

and these numbers are accordingly perfect.

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21 Euclid, VII, Def. 22
Euclid also gives the law of formation for these numbers together with a proof and, in effect, it states that, if any number of the terms of the series \(1, 2, 2^2, 2^3, \ldots\)
\[2^{n-1}(S_n)\] is prime,
then, \(S_n \cdot 2^{n-1}\) is a "perfect" number.\(^{22}\)
The algebraic proof is rather long, but is given by Heath.\(^{23}\)

But the early Pythagoreans called 10 the perfect number, and Theon draws attention to this also.\(^{24}\) They called 10 the τετραχωρίς as it was composed of the sum of the numbers 1, 2, 3 and 4. For them it was "their greatest oath", symbolising all that existed and mystically embracing the point (1), the line (2), the plane (3) and the solid pyramid (4), while these numbers also included the ratios corresponding to the musical intervals discovered by Pythagoras, namely 4:3 (the fourth), 3:2 (the fifth) and 2:1 (the octave).

\(^{22}\) Euclid, *Elements* IX, Prop. 36.


\(^{24}\) Hiller, 94. lff.
CHAPTER FIVE

CONCLUSION

In concluding this study of Theon of Smyrna's work on arithmetic some appraisal of his contribution to the study of number in the abstract (ἀριθμητική) may perhaps be considered appropriate. Since it is impossible to offer this evaluation without taking some account of Theon's arithmetical antecedents, I shall preface my remarks by giving a brief résumé of the development of Greek arithmetic as it pertains to the topics examined by Theon.

The scientific study of number by the Greeks begins with Pythagoras. The enquiries generally attributed to him or to his school represent the first switch in the focus of philosophy from the study of matter to the study of the structure of things—an attitude succinctly summarised in the established dictum of the Pythagoreans, "Number rules the Universe".¹ Despite the disparaging remarks of Dantzig and others that their lore was essentially mystic,² few scholars now seriously dispute the claim of Pythagoras and his followers to be the first to have studied "numbers pure and simple". This opinion moreover accords with the conviction of the ancients. Aristotle, for instance, credits Pythagoras with being

¹See note 44 (p. 89).
the first to work on mathematics and arithmetic, while, according to Stobaeus, he anticipated Plato in divorcing arithmetic from the realm of commercial utility.

One of the main difficulties in appraising the work of Pythagoras is the lack of written evidence, probably as a result of the oral transmission in the teaching of the school rather than of the alleged oath of secrecy that bound the members of the school. Nonetheless despite this absence of definite evidence some idea of the state of Pythagorean arithmetic may be deduced from later writers. From an excerpt of Speusippus' work On the Pythagorean Numbers, for instance, it would appear that polygonal numbers were known and by inference many of the associated ideas on the number, the unit, odd and even numbers, composite and prime numbers. Square, triangular and oblong numbers would also have been adopted and, certainly by Plato's time if not before, these concepts were commonplace.

While Heath believes that Pythagoras had not probably developed any theory of irrationals, he contends that he would have been aware of the incommensurability of the diagonal of a square with its side and have developed a method of approximating the value of \( \sqrt{2} \), after the manner described by Theon.

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3In a lost work, On the Pythagoreans, Apollonius, Hist. mirabil. 6 (Vors, i3, p.25, 5).
4Stobaeus, Ecl., i, proem., 6 (Vors, i3, p.346, 12).
6Heath, H.G.M., i, 91.
Three kinds of proportion, the arithmetical, geometrical and harmonic were certainly known. Yet how much of this was discovered by Pythagoras himself, or what precisely must be ascribed to his followers Philolaus, Archytas and Plato is obscure and the absence of certain basic concepts should be assumed. There is no trace, for example, in the fragments of Philolaus, Plato or Aristotle of an exposition of the perfect number (τέλειος); for the Pythagoreans, according to Theon, 10 was the perfect number.

By the end of the fourth century the study of arithmetic had advanced considerably from its Pythagorean beginnings. This progress is conveniently recorded in the propositions gathered together and published about 300 B.C. by Euclid as a portion of his famous Elements. Prime numbers, composite numbers, square, plane and solid numbers are all dealt with; the nomenclature is Pythagorean, but Euclid uses a deductive method of proof and proceeds further in the realm of generality.

During the succeeding generations geometry enjoyed some amazing successes under the direction of Archimedes and Apollonius, but the study of numbers languished. Indeed virtually nothing was done in the field of arithmetic for nearly four centuries although the studies of the astronomers Hypsicles (floruit circa 150 B.C.) and Eratosthenes (floruit

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7Iamblichus, in Nicom., p.100, 19-24.
8Hiller, 46.13.
230 B.C.) are of some relevance to the development of arithmetic and had a considerable influence on Theon in particular. The former is the reputed author of an astronomical tract entitled *De Ascensionibus*, still extant, besides other works not surviving, dealing with the harmony of the universe and polygonal numbers. Eratosthenes is commonly remembered for his "sieve" which comprised a method of "sifting out" composite numbers in order to obtain the successive primes; indeed he was a mathematician of great ability—Archimedes himself dedicated *The Method* to him—and was also author of a work entitled *Platonicus*, not now extant, but one which may well have been one of Theon's major sources. There are remarkable parallels one could point to in the two works, such as their opening with the "Delian problem", the theme of "mathematics for the reading of Plato", the disquisitions on proportion, the treatment of geometrical and arithmetical definitions and the discussion of the principles of music.

With the first stirrings of Neoplatonism at the beginning of the second century attention was once more focused on the study of numbers and we find a revival of interest in the neglected systems of Pythagoras and Plato. It is at this point that Theon wrote his dissertation upon "mathematics useful for the reading of Plato" and the arithmetical section with which we are concerned.

To this period also belongs the work of Nicomachus

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of Gerasa whom Theon follows closely in his writings. Nicomachus' treatise entitled *Introductio Arithmetica*\(^{10}\) enjoyed a high reputation, if one may judge from the number of commentaries written upon it, the most important being that of Iamblichus. This is surprising when one considers that Nicomachus does not appear to be a genuine mathematician and that little of his presentation appears to be original.

The work broadly comprises a publication of the old Pythagorean arithmetic in the form in which it had become established by about Euclid's time. There is a philosophical introduction and a classification of numbers into odd and even, prime and incomposite, perfect, over-perfect and defective, a full treatment of polygonal numbers, pyramidal numbers, circular and spherical numbers. The treatment, however, is not in the Euclidean manner by means of deductive proofs; Nicomachus writes a continuous narrative with some attempt at rhetoric and allusions to philosophy and history.\(^{11}\)

Heath's judgments are critical in the extreme: \(^{12}\) "no proofs in the proper sense of the word", "cumbrous circumlocutions", "not really a mathematician", "a popular treatment calculated to awaken in the beginner an interest in the theory of numbers by making him acquainted with the most noteworthy

\(^{10}\) Nicomachus, *Introductio Arithmetica*, Hoche, Leipzig 1866.

\(^{11}\) J. Gow, *A Short History of Greek Mathematics* (Cambridge 1884) p. 95.

results obtained up-to-date", "the properties of numbers appear marvellous or miraculous", "little is original", "in essence it evidently goes back to the early Pythagoreans". He supposes that the work was adequate as a mathematical compendium for the philosophers of the day, especially as there were few mathematicians of any stature at that time.

Theon of Smyrna was contemporary with, or certainly not much later than, Nicomachus. The material he presents on arithmetic is practically the same as that given by Nicomachus, and in his treatment and arrangement Theon appears equally remiss. Heath indeed considers Theon to be even less systematic than Nicomachus.\(^{13}\) Certainly his work does not seem well integrated; similar topics are dealt with in widely separated chapters; much of his thought is repetitious, and much of his language redundant.

Nonetheless in spite of his close dependence on Nicomachus, certain topics seem to belong exclusively to Theon's treatise. In Nicomachus, for instance, we find no treatment of i) the theory of side and diagonal numbers, or of ii) the divisibility of squares. On the other hand Theon can hardly be credited with either discovery. The former was clearly familiar to the early Pythagoreans,\(^{14}\) and the treatment of squares, though assuredly not found elsewhere until Iamblichus discussed the subject, is also believed to be an

\(^{13}\)Heath, H.G.M., i, 112.

\(^{14}\)Heath, H.G.M., i, 91.
element in Pythagorean arithmetic.

Basically, however, the arithmetical writings of Theon and Nicomachus suffer from the same weakness. Each labours under a crushing handicap of a language ill-suited for the presentation of number theory. Further progress could not be made without the invention of an appropriate mathematical symbolism. An analytical method involving an "unknown" had been used as early as Plato, to be sure, but it was left for Diophantus to provide a developed symbolism and lay the foundations of algebra. "With Diophantus", writes Gow, "the history of Greek arithmetic comes to an end; no original work was done by anyone after him".

Was Theon then philosopher, mathematician or astronomer? His introduction to the work, copiously illustrated with passages from Plato apparently quoted from memory, would certainly indicate a man with an avid interest in

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15 cf. Plato, Charmides, 168E.

16 This brilliant mathematician is given a floruit of 250 B.C. by Heath and by O.C.D. His most important work, the Arithmetica, deals with the solution of equations using algebraic symbols. His chief problems were indeterminate and semi-indeterminate equations and the method is strictly analytical, although a pamphlet On Polygonal Numbers was also written after the manner of Euclid.

17 Gow, op. cit., p. 121. But this assertion is not completely accurate. Pappus (ca. 300 A.D.) and Iamblichus (probably a little later) made some further contributions to the theory of numbers, the former in treating Apollonius' "tetrads" and establishing ten means, and the latter on the properties of numbers of the form 3n-2, 3n-1, 3n.
Platonic philosophy. The narrative moreover is noticeably more alive here than in many of his disquisitions on numbers where his style is turgid and the content repetitive. But Theon never displays the least inclination to handle his material with any originality, being content merely to use Plato as an authority for a succession of platitudes on the value of mathematics. In no sense can he be considered anything more than an amateur philosopher.

The place of Theon amongst a field of mathematical giants such as Pythagoras, Euclid, Eratosthenes and Diophantus is even less difficult to assess. If he could be credited at least with the original discovery of the properties of squares mentioned earlier, his reputation would be redeemed. But his work in large measure comprises what had been known for six hundred and fifty years, namely Pythagorean arithmetic. Furthermore, as has been noted, his work closely parallels that of Nicomachus, even to the philosophical introduction, and is believed to have a similar form and content as had the Platonicus of Eratosthenes. His contribution can hardly have been greater than that of Nicomachus, and that was little enough. It is then difficult not to accept his own confession that he is presenting arithmetic "for the layman".

The verdict of modern mathematical writers on the originality of Theon of Smyrna has been consistently unfavorable. Heath believes his work is of value not primarily for its intrinsic worth, but by virtue of the numerous hist-
orical notices that it contains. To Cajori his work is ill-
arranged and has little merit. It is generally agreed that
his work is a compilation and Dupuis in the preface of his
edition of the work attaches importance to it only in the
realm of the history of science:

Si les mathématiques n'ont rien à gagner à la publica-
tion de cette traduction, l'histoire des sciences peut y
trouver du moins quelques renseignements utiles.

This leaves only his claim to be an astronomer. Here
alone perhaps is there some justification for his reputation.
His section on astronomy is generally considered to be super-
ior to the section on arithmetic. If only we could be sure
that the Theon referred to with respect by Ptolemy for his
astronomical observations was in fact Theon of Smyrna, we
might with confidence reserve at least this distinction for
him.

18 Heath, H.G.M., ii, 239.
20 F. E. Robbins, Posidonius and Pythagorean Arithmology,
Class. Phil. vol. 15 (1920), p. 309.
21 Dupuis, Pref. viii.
22 Fritz, in PW s.v. "Theon".
23 p. 2ff. in my Introduction.
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