PROPAGATION OF THE WAVE FRONT ON UNTRANSPOSED OVERHEAD AND UNDERGROUND TRANSMISSION LINES

by

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ABSTRACT

The propagation of the switching surge wave front on multiphase power lines was investigated by modal analysis and conventional Fourier Transformation. A 500 kV untransposed, three-phase transmission line, for which field test results were available, was chosen as a test case.

Phase A of this test line was excited from a double exponential voltage source and the voltage response at the receiving end was calculated and measured in all three phases. The calculated voltage arrival time matched closely the measured value, and was very close to the time taken by electromagnetic waves in air at a speed of 0.3 km/μs. The calculated voltage response curves also came close to the measured results (errors within 8%).

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Finally, I am endebted to my parents and my wife for their tireless encouragement and patience.

INTRODUCTION

The attenuation and distortion of wave fronts on single circuit multiphase transission lines or underground cables was investigated. The purpose of this work was useful for surge insulation coordination study. The solution methods were applied to the specific case of a 500 kV untransposed overhead line, for which test results were available.[1,2*] This line is part of the 500 kV Azumi Trank transmission link of the Tokyo Electric Power System in Japan. In the field tests, the sending end of the line was energized with a double exponential surge wave of the form $v(t)=k(e^{-\alpha_1 t} - e^{-\alpha_2 t})$, as a representation of surge phenomenom on a line eg. lightning surge, from an impulse generator[3] through a series resistance of 415Ω. In the computer simulation, this double exponential input wave as well as other forms of input voltages, such as single exponential decay wave, triangular waves, step wave and delta wave were also studied.

The way in which computer programmes were used for the analysis in this thesis is illustrated in Fig. 1. The Line Constants Program (LCP) was first used to give the distributed line parameters from the tower geometry and conductor characteristics as a function of frequency. Then, the Transfer Function Program (TFP) was used to obtain the output voltage at the receiving end (83.212 km from sending end) for all frequency points. After the transfer functions at discrete frequency points were obtained, the Fourier Transform Program (FTP)was used to find the voltage at the receiving end as a function of time for any form of input voltage.

In the Fourier Transform Program , linear interpolation between

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* The superscripts denote reference numbers in the bibliography.

successive data points were used in the time domain as well as in the frequency domain. For the 500 kV line used as an example, a density of 20 points per decade on the frequency scale gave sufficiently accurate results.

The computation of the voltage response in three phases at the receiving end with any one of the phases energized at the sending end would take about 80 s. CPU time on the UBC IBM 370/168 computer system at a cost of approximately C$30. For a more general case of three input voltages on all three phases, only a slight modification in the Transfer Function Program would be required to obtain the results.

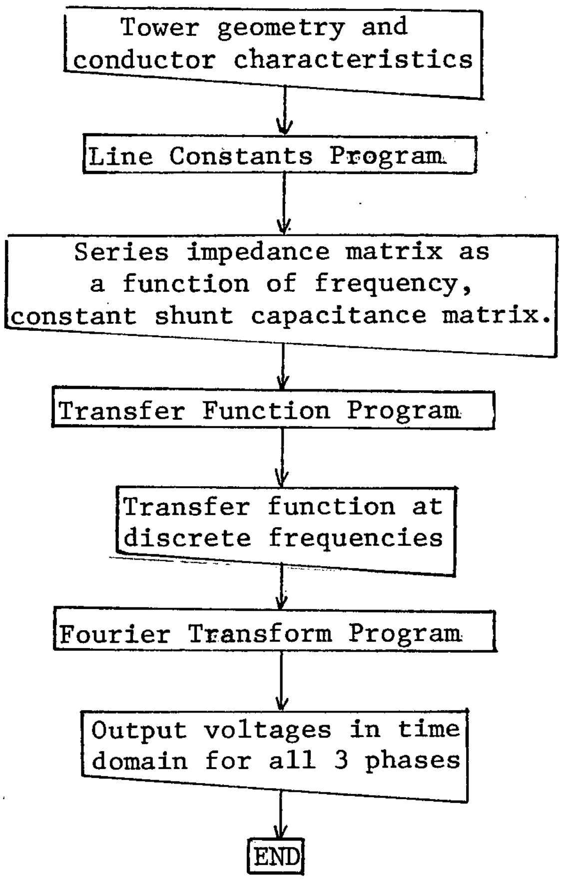


Fig. 1. Overall scheme of program used.

CHAPTER I

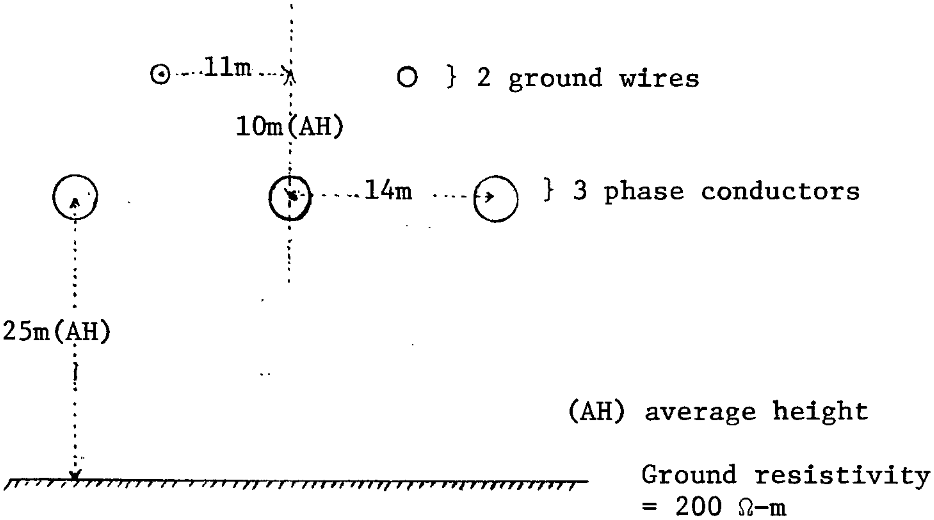
COMPUTATION OF LINE CONSTANTS

1. Introduction

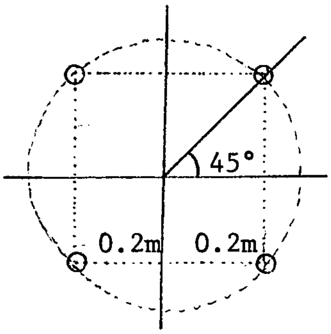
The program which was used for the calculation of line parameters is a modified version of the Line Constants Program written by H. W. Dommel[4,5]. It calculates the frequency-dependent series impedance matrix and the constant shunt capacitance matrix for overhead lines from the given tower geometry and conductor characteristics at specified frequency points. For the analysis of underground cables, this program would have to be replaced by a cable constants program. The value of the impedance and capacitance matrices is needed for the Transfer Function Program to obtain the transfer functions at the specified frequency points.

2. Transmission Line Data

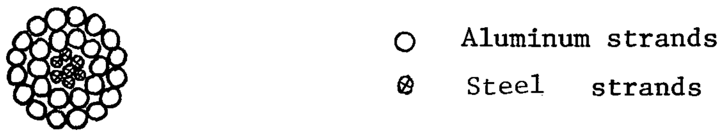
The transmission line used as an example for this simulation study is part of the 500 kV Azumi-Trank transmission link of the Tokyo Electric Power System in Japan[1,2]. This is a three-phase untransposed line with two ground wires. Each phase is a bundle conductor with four steel-reinforced aluminum cables (see Fig. 2). The tower geometry and conductor characteristics are listed in Table 1. The conductor characteristics were taken from the German standard DIN 48204 because they were not defined in enough detail in the description of the field tests[4,5].



a) Tower geometry (height is average height above ground, not height at tower location).



b) Bundle conductors of each phase



c) 26 Al./7st. Steel-reinforced aluminum cable used for ground wires and phase conductors

Fig. 2 Transmission line geometry

TABLE I

TRANSMISSION LINE DATA

General data

Length of transmission line	= 83.212 km
Average height above ground of three phase conductors (flat configuration)	= 25 m
Average height above ground of ground wires	= 35 m
Earth resistivity (presumably farmland)	= 200 Ω.m
Resistivity of aluminum	= 3.21×10^{-8} Ω.m
Relative permeability of aluminum (μ_r)	= 1.0
Permeability of aluminum ($\mu_o \mu_r$)	= $4\pi \times 10^{-8}$ H/m

Details for ground wire

Type: Steel-reinforced aluminum cable, as shown in Fig. 2c.

Total no. of aluminum strands	= 26
No. of aluminum strands in outer layer of conductor	= 16
Steel core diameter	= 5.85 mm
Outside diameter of conductor	= 15.7 mm
D.C. resistance at 20°C	= 0.262 Ω/km

Details of phase conductor

Type: Steel-reinforced aluminum cable as shown in Fig. 2c, with conductors in each phase as shown in Fig. 2b

Total no. of aluminum strands	= 26
No. of aluminum strands in outer layer of conductor	= 16
Steel core diameter	= 8.1 mm
Outside diameter of conductor	= 21.7 mm
D.C. resistance at 20°C	= 0.136 Ω/km

3. Line Parameter Calculation

(i) Series impedance matrix - Carson's formula[6,2] is used for calculating the impedances of the conductor earth return loops. Earth conductivity is assumed to be uniform and the earth plane is assumed to be flat and parallel to the conductors. Also, spacings between conductors are assumed to be large compared with conductor radii, that is, proximity effects are ignored. The elements of the impedance matrix $[Z_{ij}]$ are given as

$$Z_{ii} = (R_{ii} + \Delta R_{ii}) + j(2\omega 10^{-4} \ln \frac{2h_i}{GMR_i} + \Delta X_{ii}) \Omega/\text{km}$$

$$i = 1, \ldots N \quad (1\text{-}1)$$

and $Z_{ij} = Z_{ji}$

$$= \Delta R_{ij} + j(2\omega 10^{-4} \ln \frac{S_{ij}}{s_{ij}} + \Delta X_{ij}) \Omega/\text{km}$$

$$j = 1, \ldots N;\ i = 1, \ldots N;\ i \neq j, \quad (1\text{-}2)$$

where R_{ii} = resistance of i^{th} conductor in Ω/km (see section 4. on skin effect)

h_i = average height above ground of i^{th} conductor in m,

S_{ij} = distance between i^{th} conductor and ground image of j^{th} conductor in m (see Fig. 3),

s_{ij} = distance between i^{th} and j^{th} conductors in m (see Fig. 3),

GMR_i = geometric mean radius of i^{th} conductor in m,

ω = angular frequency,

ΔR = correction terms in resistance for earth return effect,

ΔX = correction terms in resistance for earth return effect.

Carson's correction terms ΔR and ΔX are functions of the angle

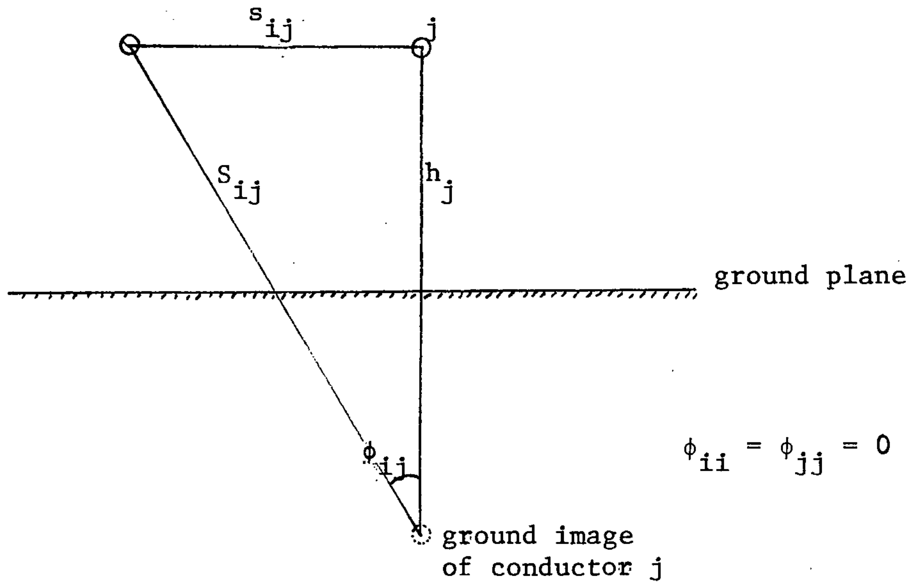


Fig. 3 Line parameter calculation

ϕ_{ij} (see Fig. 3) and of the parameter

$$a = ks\sqrt{\frac{f}{\rho}}$$

where $k = 4\pi\sqrt{5} \times 10^{-4}$

$$s = \begin{cases} 2h_i & \text{for self impedance} \\ S_{ij} & \text{for mutual impedance} \end{cases}$$

ρ = earth resistivity in Ω.m

f = frequency in Hz

For numerical calculations, Carson's integral for ΔR and ΔX has been developed into an infinite series[4], which is used for $a \leq 5$,

$$\begin{aligned} \Delta R' = 4\omega 10^{-4}\{\frac{\pi}{8} &- b_1 a \cos\phi + b_2[(c_2 - \ln a)a^2 \cos 2\phi + \phi a^2 \sin 2\phi] \\ &+ b_3 a^3 \cos 3\phi - d_4 a^4 \cos 4\phi \\ &- b_5 a^5 \cos 5\phi + b_6[(c_6 - \ln a)a^6 \cos 6\phi + \phi a^6 \sin 6\phi] \\ &+ b_7 a^7 \cos 7\phi - d_8 a^8 \cos 8\phi \\ &- \ldots\ldots \} \end{aligned} \tag{1-3a}$$

$$\begin{aligned} \Delta X' = 4\omega 10^{-4}\{\frac{1}{2} &(0.6159315 - \ln a) + b_1 a \cos\phi - d_2 a^2 \cos 2\phi \\ &+ b_3 a^3 \cos 3\phi - b_4[(c_4 - \ln a)a^4 \cos 4\phi + \phi a^4 \sin 4\phi] \\ &+ b_5 a^5 \cos 5\phi - d_6 a^6 \cos 6\phi + b_7 a^7 \cos 7\phi \\ &- b_8[(c_8 - \ln a)a^8 \cos 8\phi + \phi\, a^8 \sin 8\phi] \\ &+ \ldots\ldots \} \end{aligned} \tag{1-3b}$$

where b_i, c_i and d_i are constants given be

$$b_1 = \frac{\sqrt{2}}{6} \text{ for odd subscripts } i$$

$$b_2 = \frac{1}{16} \text{ for even subscripts } i$$

$$b_i = b_{i-2}\frac{\text{sign}}{i(i+2)}, \quad i > 2$$

$$\text{with sign} = \begin{cases} +1, & i = 1,2,3,4;\ 9,10,11,12;\ \ldots \\ -1, & i = 5,6,7,8;13,14,15,16;\ \ldots \end{cases}$$

and $c_2 = 1.3659315$

$$c_1 = c_{i-2} + \frac{1}{i} + \frac{1}{i+2}, \quad i > 2$$

and $d_i = \frac{\pi}{4} b_i$

Note that from eqtns (1-3a) and (1-3b), each 4 terms in i = 1,4 form a repetitive group in the infinite series.

For a > 5, the approximation formulae given by Butterworth[7,2] is used, instead of the infinite series

$$\Delta X = \left(\frac{\cos\phi}{a} - \frac{\cos 3\phi}{a^3} + \frac{3\cos 5\phi}{a^3} + \frac{45\cos 7\phi}{a^7}\right)\frac{4\omega 10^{-4}}{\sqrt{2}} \tag{1-4a}$$

$$\Delta R = \left(\frac{\cos\phi}{a} - \frac{\sqrt{2}\cos 2\phi}{a^2} + \frac{\cos 3\phi}{a^3} + \frac{3\cos 5\phi}{a^5} - \frac{45\cos 7\phi}{a^7}\right)\frac{4\omega 10^{-4}}{\sqrt{2}} \tag{1-4b}$$

Note that the infinite series for R and X derived from Carson's integrals will only converge after about 10 or more terms if a > 3. The first few terms are highly oscillating in that case.

(ii) Shunt capacitance matrix [C] - The capacitance matrix [C] is the inverse of the potential coefficient matrix [P].

$$[C] = [P]^{-1}$$

The matrix element of [P] can easily be obtained from the tower geometry,

$$P_{ii} = 2c^2 \times 10^{-4} \ln \frac{2h_i}{r_i} \text{ km/F} \tag{1-5}$$

and
$$P_{ij} = 2c^2 \times 10^{-4} \ln \frac{S_{ij}}{s_{ij}} \text{ km/F} \tag{1-6}$$

where r_i = radius of conductor in m

c = velocity of light in km/s

Eqtns (1-5) and (1-6) are valid as long as r_i(0.02 m in the example) is much smaller than spacings between conductors (14 m in the example). Note that the elements of the shunt capacitance matrix are only dependent on the tower geometry and are not dependent on frequency. This is an approximation which is valid for frequencies up to approximately 1 MHz, where earth correction terms for capacitances are not yet important[8].

4. Calculations of Skin Effect in Conductors

The skin effect in the earth return is accounted for by Carson's formula. While the earth return skin effect has a major influence on line parameters, skin effect in the conductors must also be considered at higher frequencies. As frequency increases, the current flows more and more on the surface of the conductor. This can be described by the nominal depth of penetration of current (δ) as given by[9]

$$\delta = \sqrt{\frac{\rho_c}{\pi f \mu}}$$

where ρ_c = resistivity of conductor material in Ω.m

μ = absolute magnetic permeability in H/m

f = frequency in Hz

Since the current in confined to the surface of the conductor at high frequencies, the conductor resistance increases and the internal inductance decreases with frequency (see Fig. 4). In eqtn (1-1), the self inductance matrix element

$$L_{ii} = 2 \times 10^{-4} \ln \frac{2h_i}{GMR_i} \quad \text{H/km} \tag{1-8}$$

is the resultant of the internal and external inductance, i.e.

$$L_{ii} = 2 \times 10^{-4} \ln \frac{r_i}{GMR_i} + 2 \times 10^{-4} \ln \frac{2h_i}{r_i} \quad \text{H/km} \tag{1-9}$$

The first and second term in eqtn (1-9) is due to the flux inside and outside the conductor, respectively. Note that the first term (internal inductance) is small compared to the second term for high voltage overhead lines at low frequencies and vanishes completely at high frequencies.

Skin effect on resistance and internal inductance of each bundled conductor by Galloway's formula and tubular conductor formula

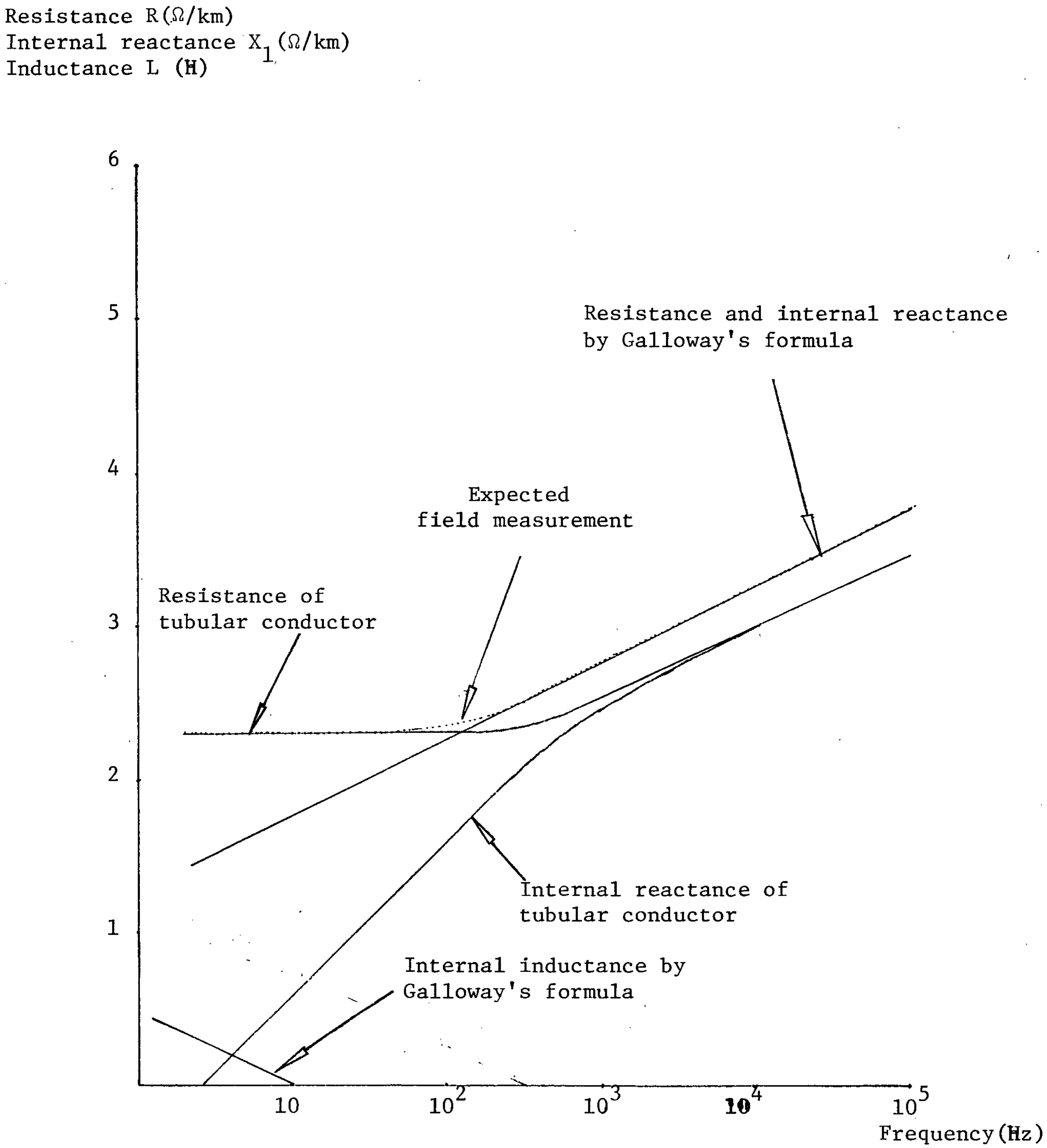


Fig.4

Thus, the skin effect on the total inductance is normally negligible over the entire frequency range. However, the skin effect on resistance is quite pronounced.

The skin effect on the conductor resistance and internal reactance was calculated in two ways. With the first method, the conductor was treated as a solid tubular aluminum conductor of the same cross-sectional area as the actual conductor. The steel core was completely ignored. This was recommended as a reasonable approximation. The formula for the internal impedance of a tubular conductor of nonmagnetic material is[10,4]

$$\frac{Z_{internal}}{R_{dc}} = \frac{R_{conductor} + j\omega L_{internal}}{R_{dc}}$$

$$= j\frac{1}{2}\, mr(1-s^2)\frac{(ber(mr)+jbei(mr))+\phi(ker(mr)+jkei(mr))}{(ber'(mr)+jbei'(mr))+\phi(ker'(mr)+jkei'(mr))} \tag{1-10}$$

where $\phi = -\dfrac{ber'(mq)+jbei'(mq)}{ker'(mq)+jkei'(mq)}$

R_{dc} = d.c. resistance of conductor in Ω/km

r = outside radius of conductor in m

q = outside radius of steel core in m

$s = \dfrac{q}{r}$

$(mr) = \left(\dfrac{k}{1-s^2}\right)^{1/2}$

$(mq) = \left(\dfrac{ks^2}{1-s^2}\right)^{1/2}$

and $k = \dfrac{8\pi 10^{-4} f}{R_{dc}}$

The expressions ber (...) + j bei(...), ber'(...) + j bei'(...), ker(...)+j kei(...) and ker'(...) + j kei'(...) are modified Bessel functions, which can be evaluated by polynomial approximation[11,4]. An empirical formula for conductor resistance and conductor internal reactance was developed by Galloway[12], which is based on measurements in the electrolytic tank. In this approach, current is assumed to be confined to the outer layer strands (16 strands in outer layer in the example) . Internal resistance (R_c) and internal reactance (X_L) are then equal. With this formula, we obtain

$$R_c = X_L = \frac{K\sqrt{\omega\mu\rho_c}}{\sqrt{2}\ r\ \pi\ (2+n)}\ \Omega/\text{km} \tag{1-11}$$

where ω = angular frequency

μ = permeability of conductor material (H/m)

ρ_c = resistivity of conductor material (Ω-m)

r = outer radius of conductor in (m)

n = no. of strands in outer layer (see Fig. 2c)

K = 2.25, factor due to stranding

Results for the internal impedance calculated with the above two approaches are shown in Fig. 4. At lower frequencies, where skin effect is not yet prominent, the results for tubular conductors are fairly accurate[4]. After a cross-over point around 130 Hz, the results from Galloway's formula are probably more reliable since that formula takes the skin effect in the individual strands of the outer layer into account. The dotted line in Fig. 4 thus indicates the predicted field measurement values. It should be noticed that line parameters of overhead lines are seldom measured as the computed results are usually sufficiently accurate.

5. Output from Line Constants Program

For the transmission line of Fig. 2, there are 14 conductors, i.e. 4 conductors per bundle in each of the three phases and 2 ground wires above. Thus, initially we have a 14 x 14 series impedance and a 14 x 14 shunt capacitance matrix

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_{14} \end{bmatrix} = \begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,14} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,14} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ P_{14,1} & P_{14,2} & \cdots & P_{14,14} \end{bmatrix}_{14\times 14} \cdot \begin{bmatrix} Q_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Q_{14} \end{bmatrix} \tag{1-12}$$

and

$$\begin{bmatrix} dV_1/dx \\ dV_2/dx \\ \cdot \\ \cdot \\ \cdot \\ dV_{14}/dx \end{bmatrix} = \begin{bmatrix} Z_{1,1} & Z_{1,2} & \cdots & Z_{1,14} \\ Z_{2,1} & Z_{2,2} & \cdots & Z_{2,14} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ Z_{14,1} & Z_{14,2} & \cdots & Z_{14,14} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_{14} \end{bmatrix} \tag{1-13}$$

These 14 x 14 matrices can be reduced to the desired 3x3 matrices by considering the bundling condition in the 3 phase bundle conductors, and the zero voltage condition in both ground wires. If we denote conductors in phase A as 1,2,3,4; phase B as 5,6,7,8; phase C as 9,10,11,12 and both ground wires as 13,14, then for ground wires 13 and 14, we have

$$\begin{aligned} -\frac{dV_{13}}{dx} &= 0 \\ -\frac{dV_{14}}{dx} &= 0 \end{aligned} \tag{1-14}$$

and

$$\begin{aligned} V_{13} &= 0 \\ V_{14} &= 0 \end{aligned} \tag{1-16}$$

and for bundling in phase A, we have

$$-\frac{dV_1}{dx} = -\frac{dV_2}{dx} = -\frac{dV_3}{dx} = -\frac{dV_4}{dx} = -\frac{dV_A}{dx}$$

$$I_1 + I_2 + I_3 + I_4 \doteq I_A \tag{1-16}$$

$$V_1 = V_2 = V_3 = V_4 = V_A$$

$$Q_1 + Q_2 + Q_3 + Q_4 = Q_A \tag{1-17}$$

With eqtns (1-15) and (1-17), eqtn (1-12) can be reduced to 3 equations with the desired 3x3 potential matrix [P][15],

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} \doteq \begin{bmatrix} P_{AA} & P_{AB} & P_{AC} \\ P_{BA} & P_{BB} & P_{BC} \\ P_{CA} & P_{CB} & P_{CC} \end{bmatrix}_{3x3} \begin{bmatrix} Q_A \\ Q_B \\ Q_C \end{bmatrix} \tag{1-18}$$

The 3x3 shunt capacitance matrix [C] is then obtained by simple matrix inversion $[C]_{3x3} = [P]^{-1}_{3x3}$. Similarly, eqtns (1-14) and (1-16) can be used to reduce eqtn (1-13) to 3 equations with the desired 3x3 series impedance matrix [Z].

$$-\begin{bmatrix} dV_A/dx \\ dV_B/dx \\ dV_C/dx \end{bmatrix} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix}_{3x3} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}. \tag{1-19}$$

Thus, with Carson's formula and with one of the two skin effect formulae for conductors, followed by matrix reduction for bundling and ground wires, we obtain the 3x3 series impedance and 3x3 shunt capacitance matrices for the three phases.

The 3×3 shunt capacitance matrix obtained from the tower geometry of the test example is shown in Table 2. the elements of the 3×3 symmetric series impedance matrix are shown as a function of frequency in Figs. 5 and 6. Note that the differences between both skin effect formulae hardly show up at high frequencies on a logarithmic scale.

TABLE 2.

Capacitance Matrix of Three Phase Test Line

$$[C] = \begin{bmatrix} 0.06889 & -0.01183 & -0.00347 \\ -0.01183 & 0.07080 & -0.01183 \\ -0.00347 & -0.01183 & 0.06889 \end{bmatrix} \mu F/km$$

Elements of the resistance matrix of the test line, with Galloway's formula and the formula for tubular conductor, for skin effect

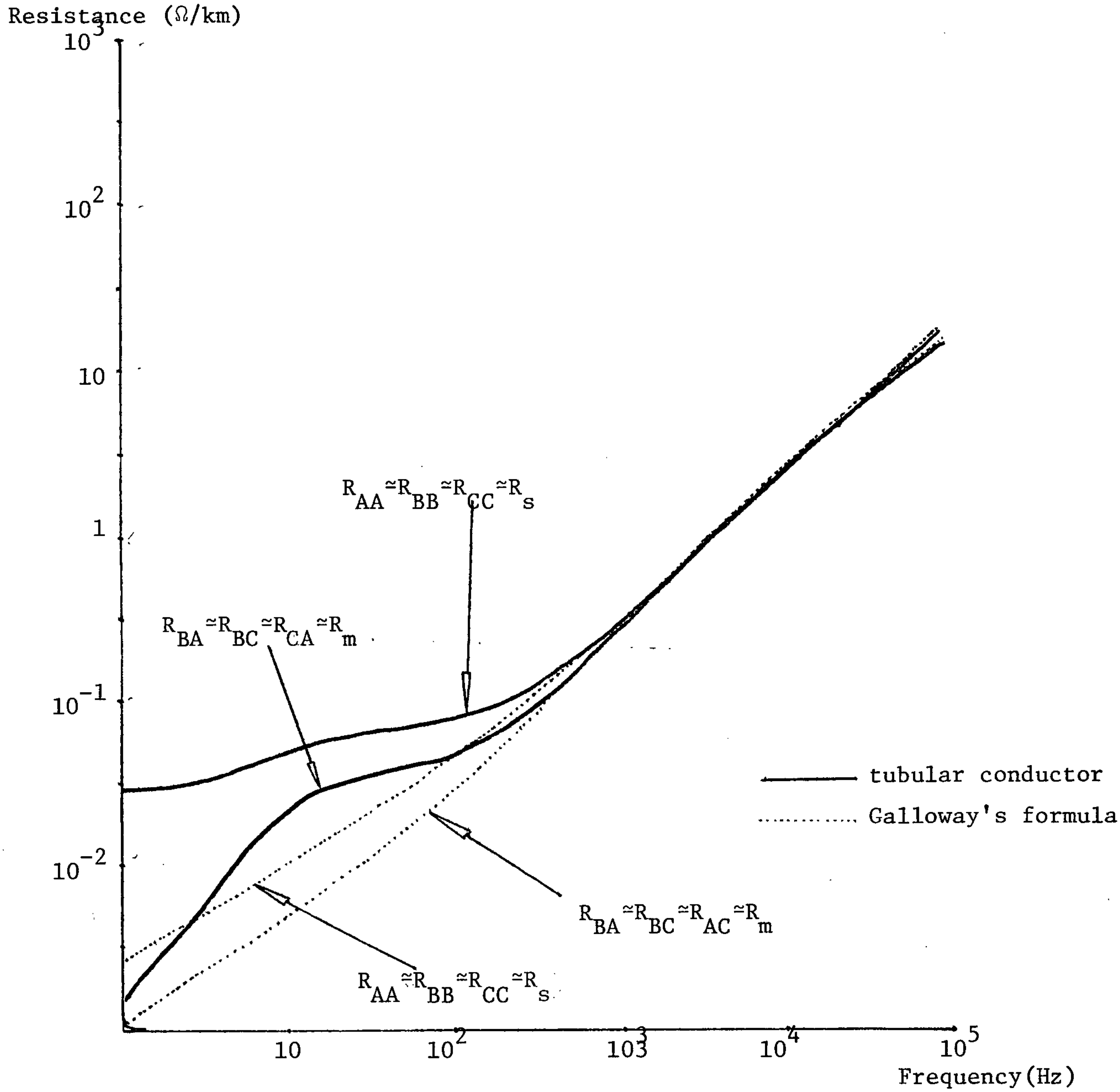


Fig.5

Elements of the reactance matrix of the test line, with Galloway's formula and the formula for tubular conductor, for skin effect

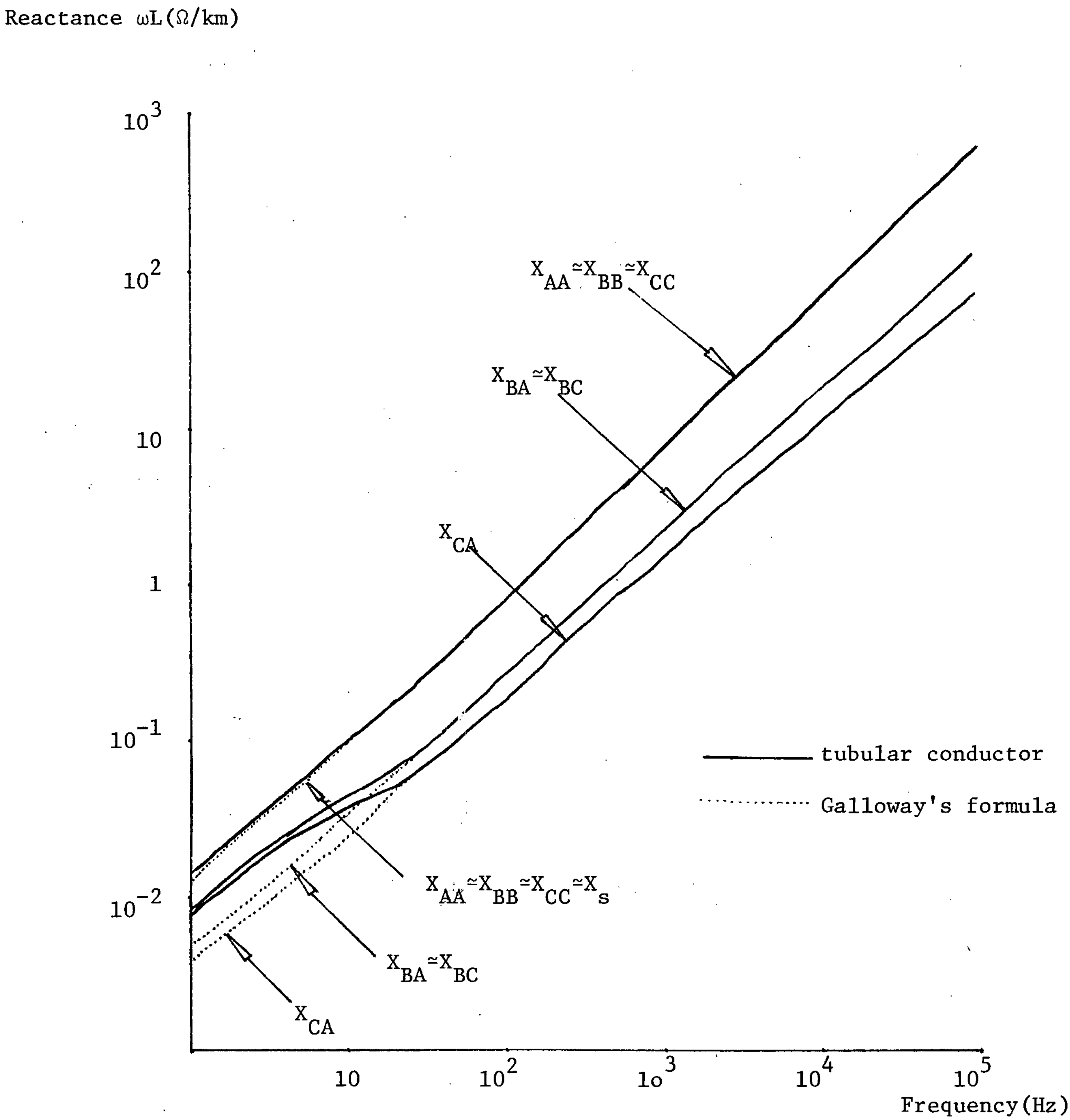


Fig.6

6. Positive and Zero Sequence Parameters

There are 3 modes of TEM propagation on the 3 phase test line. Each mode is decoupled from the other and has its own individual characteristics impedance and propagation constant (see later eqtn (2-8)). If the test line was transposed, which it is not, then two of the 3 modes would be characterized by positive sequence parameters while the third one would be characterized by zero sequence parameters. Thus, by idealizing the given untransposed line to a transposed one, we can look at the positive and zero sequence parameters, which will give us some insight into the overall effect of both approaches for conductor skin effect calculation (tubular conductor formula and Galloway's formula).

For the transposed line, the formulae relating position and zero sequence impedances (Z_{pos} and Z_{zero}) to the series impedance matrix elements are given by[14]

$$Z_{pos} = Z_s - Z_m \tag{1-20}$$

$$Z_{zero} = Z_s + 2Z_m \tag{1-21}$$

where Z_s and Z_m are the self and mutual impedances, which in turn are the averages of the diagonal and off-diagonal elements respectively,

$$Z_s = 1/3(Z_{AA} + Z_{BB} + Z_{CC}) \tag{1-22}$$

$$Z_m = 1/3(Z_{AB} + Z_{BC} + Z_{CA}) \tag{1-23}$$

where Z_{ij} is an element of $[Z]_{3x3}$

The impedances Z_{ij} in eqtns (1-16) and (1-17) are the series impedances matrix elements shown in Figs. 5 and6. The positive and zero sequence resistances are shown in Fig. 7. In Fig. 7, the resistance of the bundle conductor obtained with the tubular conductor formula and Galloway's

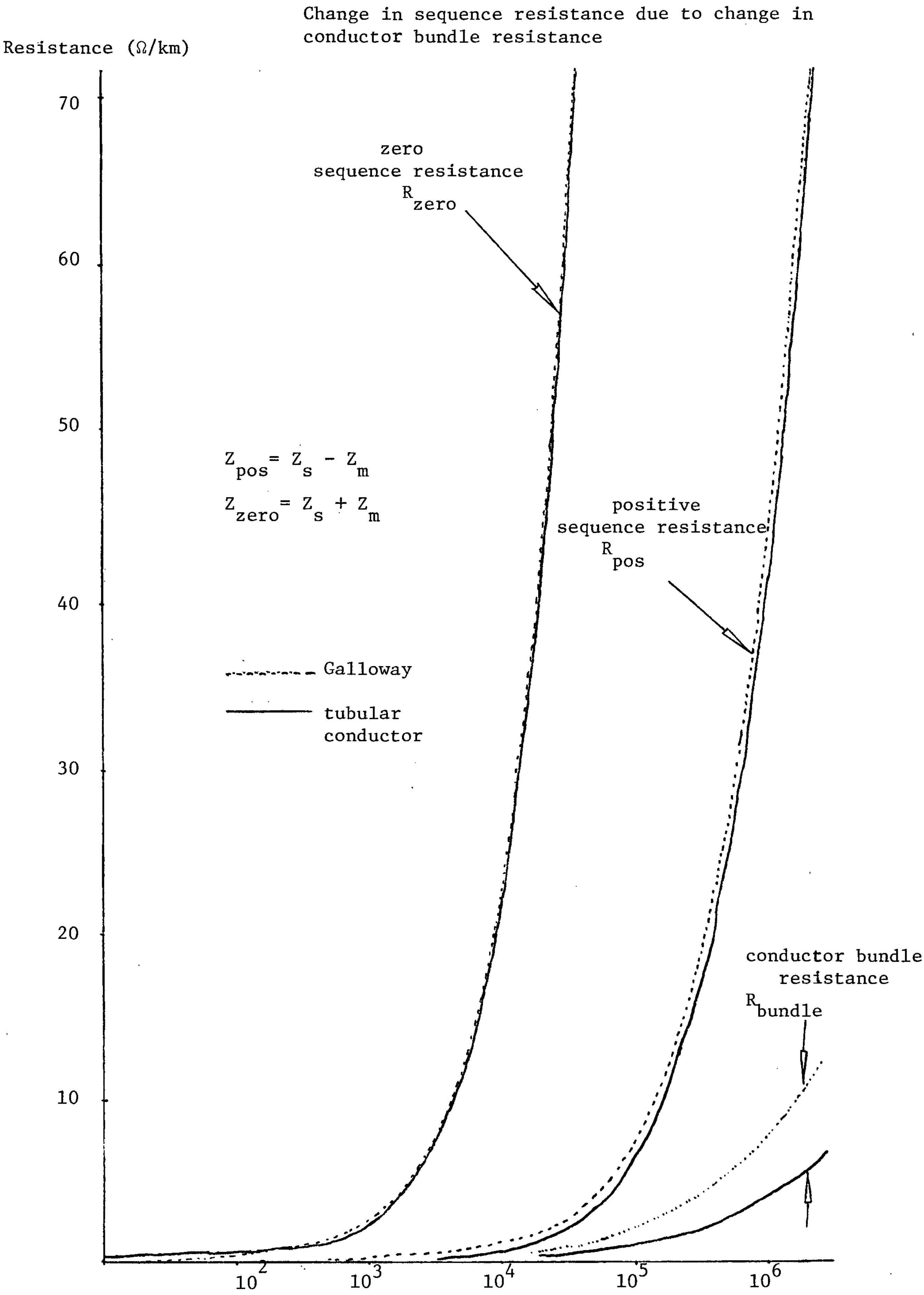

Change in sequence resistance due to change in conductor bundle resistance
Resistance (Ω/km)
70
60
50
40
30
20
10
zero sequence resistance R_{zero}
$Z_{pos} = Z_s - Z_m$
$Z_{zero} = Z_s + Z_m$
positive sequence resistance R_{pos}
Galloway
tubular conductor
conductor bundle resistance R_{bundle}
10^2
10^3
10^4
10^5
10^6

Fig. 7

formula are also shown for comparison. At higher frequencies, the conductor resistance from Galloway's formula is about twice as high (see also Fig. 4) as the value from the tubular conductor formula. The increase in conductor resistance (ΔR_c) shows up with the same value as an increase in positive sequence resistance (ΔR_{pos}). However, the differences in conductor resistance between the two formulae do not show up with exactly the same value in the zero sequence resistance. The difference in zero sequence resistance (ΔR_{zero}) is slightly higher (e.g. 9% higher at 50 KHz) due to the additional effect of the 2 eliminated ground wires. Also note that the zero sequence resistance is much higher than the positive sequence resistance. At 50 KHz, the increase in the positive sequence resistance caused by the difference in skin effect formulae is about 17% whereas the increase in the zero sequence resistance is only about 1%.

CHAPTER II

COMPUTATION OF TRANSFER FUNCTION FOR FREQUENCY RESPONSE OF TEST LINE

1. Introduction

After knowing the series impedance matrix $[Z_{ij}]_{3x3}$ and the shunt admittance matrix with zero conductance

$$[Y_{ij}]_{3x3} = j\omega[C_{ij}]_{3x3}$$

of the 3-phase test line, the transfer function between the input on one phase at the sending end and the output on any one of the three phases at the receiving end can be found. For the chosen test example, one phase, designated A, is energised (see Fig. 8).

The output voltage in the 3-phase at the receiving end of the 83.212 km long test line was to be found for a period of time during which the waves reflected at the receiving end have not yet returned back from the sending end. That is, the period of investigation time t is

$$\tau < t < 3\,\tau$$

where τ is the travel time of the mode with highest wave velocity.

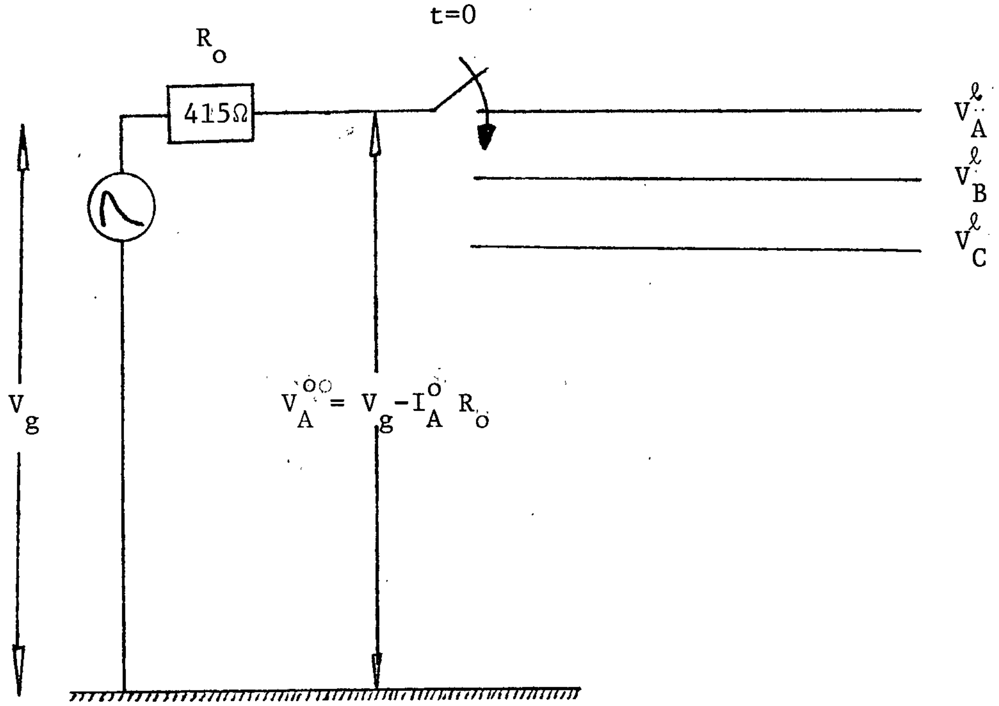
The Transfer Function Program which performs the necessary calculations is an expanded version of a program written by K.K. Tse[16] for a term project in EE 553 on the frequency response of B.C. Hydro's Mica Dam transmission line (see Appendix 1 for program listings).

2. Outline of the theory used in the Transfer Function Program

Propagation of waves on multiphase line with constant parameters is described by the well-known general transmission line equation[17,18].

$$-\left[\frac{\partial v}{\partial x}\right] = [L]\left[\frac{\partial i}{\partial t}\right] + [R]\,[i] \qquad (2.1)$$

$$-\left[\frac{\partial i}{\partial x}\right] = [C]\left[\frac{\partial v}{\partial t}\right] + [G]\,[v] \qquad (2.2)$$



Boundary conditions in frequency domain

At sending end

$$V_A^o = V_g - I_A^o R$$

$$I_A^o \neq 0, \quad I_B^o = I_C^o = 0$$

At receiving end --- Single input triple output system

$$V_j^\ell = H_j(\omega) V_g \; ; \quad j = A, B \text{ or } C$$

Fig. 8 Transmission line configurations with boundary conditions where [v] is the 3x1 column matrix of phase voltages

[i] is the 3x1 column matrix of phase currents

[L] is the 3x3 inductance matrix

[R] is the 3x3 resistance matrix

[C] is the 3x3 capacitance matrix

[G] is the 3x3 conductance matrix

(N.B For overhead transmission lines, [G] is very small and is practically always neglected).

However, the above eqtns (2-1) and (2-2) are not useable for lines with frequency dependent line parameters. Instead, we have to use equations in the form of steady state phasor equations in the frequency domain

$$-\left[\frac{\partial V}{\partial x}\right] = [Z]\ [I] \tag{2-3}$$

$$-\left[\frac{\partial I}{\partial x}\right] = [Y]\ [V] \tag{2-4}$$

where $[Z] = [R] + j\omega[L]$

= series impedance matrix in Ω/km as obtained numerically from Chapter 1.

$[Y] = j\omega[C]$

= shunt capacitance matrix in Ω/km also from Chapter 1

[V] and [I] are the vectors of phase voltages and phase currents, respectively, in the form of phasor values.

Eqtn (2-3) can be differentiated w.r.t.x to get[18]

$$\begin{aligned}\left[\frac{d^2V}{dx^2}\right] &= -[Z]\ \left[\frac{d}{dx}I\right] \\ &= [Z]\ [Y]\ [V] \\ &\triangleq [ZY]\ [V] \qquad (2\text{-}5)\end{aligned}$$

where $[ZY] \triangleq [Z]\ [Y]$

The 3x3 matrix in eqtn (2-5) has non-zero off-diagonal elements i.e. there is coupling between the phases. The easiest way to solve these coupled equations is to decouple them by modal analysis[18,16]. With this approach, [ZY] is transformed to a diagonal matrix with[19] [M],

$$[M]^{-1}[ZY] \cdot [M] = [\Lambda] \qquad (2\text{-}6)$$

where $[\Lambda]$ = diagonal matrix

$[M]$ = modal matrix, i.e. columns of eigenvectors of [ZY]

and $[M]^{-1}$ = inverse of [M]

Both [M] and $[M]^{-1}$ were obtained with subroutines from the UBC Computing Centre Programme Library, namely with 'DCEIGN' and 'CDINVT'[20]. 'DCEIGN' computes the eigenvalues and eigenvectors of the complex matrix [ZY], and 'CDINVT' computes the inverse of the modal matrix [M]. 'DCEIGN' was tested for accuracy by running a test example with known answers[19]. The results agreed up to 7 significant digits. With $[M]^{-1}$ known, the phase to mode voltage transformation is described by

$$[V^{mode}] = [M]^{-1}[V] \qquad (2\text{-}7)$$

Pre-multiplying eqtn (2-5) with $[M]^{-1}$ gives

$$[M]^{-1}[\frac{d^2V}{dx^2}] = [M]^{-1}[ZY][V]$$

or

$$[\frac{d^2}{dx} V^{mode}] = [M]^{-1}[ZY][V]$$

$$= [M]^{-1}[ZY][M][V^{mode}] \qquad \text{from eqtn (2-7)}$$

$$= [\Lambda][V^{mode}] \qquad \text{from eqtn (2-6)}$$

Thus, 3 second order differential equations are obtained, each decoupled from the other, namely,

$$\begin{bmatrix} \dfrac{d^2V_1^{mode}}{dx^2} \\ \dfrac{d^2V_2^{mode}}{dx^2} \\ \dfrac{d^2V_3^{mode}}{dx^2} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} V_1^{mode} \\ V_2^{mode} \\ V_3^{mode} \end{bmatrix} \tag{2-8}$$

where λ_1, λ_2 and λ_3 are the 3 eigenvalues of [ZY].

The 3 phase voltages have now been transformed into 3 modal quantities, which describe the independently decoupled modes of TEM propagation. They can be transformed back to phase voltages with the mode to phase relationship derived from equation (2-7),

$$[V] = [M][V^{mode}] \tag{2-9}$$

The general solution to the second order linear differential equation of the modal voltage components is well known[22] because each modal component can be treated as if it were a hypothetical single phase line.[23,24,25] For distance x = ℓ km away from the sending end

$$\begin{bmatrix} V_{1,\ell}^{mode} \\ V_{2,\ell}^{mode} \\ V_{3,\ell}^{mode} \end{bmatrix} = \begin{bmatrix} V_{1+}^{mode} \cdot e^{-\sqrt{\lambda_1}\,\ell} + V_{1-}^{mode} \cdot e^{\sqrt{\lambda_1}\,\ell} \\ V_{2+}^{mode} \cdot e^{-\sqrt{\lambda_2}\,\ell} + V_{2-}^{mode} \cdot e^{\sqrt{\lambda_2}\,\ell} \\ V_{3+}^{mode} \cdot e^{-\sqrt{\lambda_3}\,\ell} + V_{3-}^{mode} \cdot e^{\sqrt{\lambda_3}\,\ell} \end{bmatrix}$$

$$\triangleq \begin{bmatrix} V_{1+}^{mode} \cdot e^{-\gamma_1 \ell} + V_{1-}^{mode} \cdot e^{\gamma_1 \ell} \\ V_{2+}^{mode} \cdot e^{-\gamma_2 \ell} + V_{2-}^{mode} \cdot e^{\gamma_2 \ell} \\ V_{3+}^{mode} \cdot e^{-\gamma_3 \ell} + V_{3-}^{mode} \cdot e^{\gamma_3 \ell} \end{bmatrix} \tag{2-10}$$

where γ_i = propagation constant for steady state behaviour at specific frequency

$V^{mode}_{1,2,3+}$ = Forward modal voltage waves at x = 0 of A, B and C respectively, travelling from the sending end to the receiving end.

$V^{mode}_{1,2,3-}$ = reflected modal voltage waves at x = 0 of A, B and C respectively, travelling from the receiving end to the sending end.

If we are only interested in the attenuation and distortion of the wave front, then we can assume that the line is infinitely long. We can then neglect the backward reflected voltage wave, i.e.

$$V^{mode}_{1-} = V^{mode}_{2-} = V^{mode}_{3-} = 0$$

Eqtn (2-10) is thus reduced to

$$\begin{bmatrix} V^{mode}_{1,\ell} \\ V^{mode}_{2,\ell} \\ V^{mode}_{3,\ell} \end{bmatrix} = \begin{bmatrix} V^{mode}_{1+} & e^{-\gamma_1 \ell} \\ V^{mode}_{2+} & e^{-\gamma_2 \ell} \\ V^{mode}_{3+} & e^{-\gamma_3 \ell} \end{bmatrix} = \begin{bmatrix} e^{-\gamma_1 \ell} & 0 & 0 \\ 0 & e^{-\gamma_2 \ell} & \\ 0 & 0 & e^{-\gamma_3 \ell} \end{bmatrix} \begin{bmatrix} V^{mode}_{1+} \\ V^{mode}_{2+} \\ V^{mode}_{3+} \end{bmatrix} \tag{2-11}$$

or simply by matrix notation, we have

$$[V^{mode}_{\ell}] = e^{-[\gamma]\ell} \cdot [V^{mode}_{+}] \tag{2-12}$$

where $[\gamma]$ = 3x3 diagonal matrix with diagonal elements γ_1 γ_2 and γ_3

$e^{-[\gamma]\ell}$ = $[H(\omega)]$, transfer function matrix of the transmission system

and again $[V^{mode}_{+}]$ = forward voltage wave at x = 0.

(N.B. The results thus obtained are also valid for the voltage response at the receiving end of an open-ended line of finite length. For times less than 3 times travel time, the open-ended line results are simply twice the results obtained from eqtn (2-12). This doubling effect is discussed in further detail in Chapter 4, section 1.)

From eqtn (2-12), the modal voltages at the receiving end $[V_{\ell}^{mode}]$ can be transformed to the phase voltages by the mode to phase relationship

$$[V^{\ell}] = [M]\,[V_{\ell}^{mode}] \qquad \text{from eqtn (2-9)}$$
$$= [M]\, e^{-[\gamma]\ell}[V_{+}^{mode}] \qquad \text{from eqtn (2-12)}$$

Thus, we can express the phase voltages at the receiving end $[V^{1}]$ in terms of phase voltages at the sending end $[V^{o}]$ as

$$[V^{\ell}] = [M]\, e^{-[\gamma]\ell}\, [M]^{-1}[V^{o}] \tag{2-13}$$

Note that from eqtn (2-13), phase voltages cannot be calculated from $[\gamma]$ alone without $[M]$, i.e.

$$[V^{\ell}] \neq e^{-[\gamma]\ell}\, [V^{o}]$$

3. Inclusion of Boundary Conditions at Sending End.

For the chosen test line case, the line was energized on phase A as shown in Fig. 8. Boundary conditions for phase voltages and currents at the sending end (distance x = 0 denoted by superscript o) are then

$$[V^{o}] = \begin{bmatrix} V_g - I_A^{o}R^{o} \\ V_B^{o} \\ V_C^{o} \end{bmatrix} \tag{2-14}$$

and
$$[I^{o}] = \begin{bmatrix} I_A^{o} \\ 0 \\ 0 \end{bmatrix} \tag{2-15}$$

Substitution of eqtn (2-3) into eqtn (2-7) differentiated w.r.t.x gives

$$-\left[\frac{dV_{\ell}^{mode}}{dx}\right] = -[M]^{-1}\left[\frac{dV^{\ell}}{dx}\right]$$

$$= [M]^{-1} \cdot [Z][I^{o}]$$

$$\triangleq [A][I^{o}]$$

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} I_A^o \\ 0 \\ 0 \end{bmatrix}$$

$$= I_A^o \cdot \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} \tag{2-16}$$

Again, differentiating eqtn (2-12) w.r.t.x gives

$$-\left[\frac{dV_{\ell}^{mode}}{dx}\right] = [\gamma]\ e^{-x[\gamma]}\ [V_o^{mode}] \tag{2-17}$$

Equating R.H.S. of eqtn (2-16) and (2-17), we get

$$[\gamma]\ e^{-x[\gamma]}[V_o^{mode}] = I_A^o \cdot \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} \tag{2-18}$$

However, at the sending end we have x = 0 as boundary condition, eqtn (2-18) therefore gives

$$[V_+^{mode}] = [\gamma]^{-1}\ I_A^o \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix}$$

or

$$[V_+^{mode}] = I_A^o \begin{bmatrix} \frac{1}{\gamma_1} & 0 & 0 \\ 0 & \frac{1}{\gamma_2} & 0 \\ 0 & 0 & \frac{1}{\gamma_3} \end{bmatrix} \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} = I_A^o \begin{bmatrix} A_{11}/\ \gamma_1 \\ A_{21}/\ \gamma_2 \\ A_{31}/\ \gamma_3 \end{bmatrix} \tag{2-19}$$

From the phase-mode relationship of eqtn (2-9) we get the sending end phase voltage $[V^o]$ as

$$[V^o] = [M][V_+^{mode}]$$

$$= I_A^o \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} A_{11}/\gamma_1 \\ A_{21}/\gamma_2 \\ A_{31}/\gamma_3 \end{bmatrix}$$

$$= \begin{bmatrix} V_A^o \\ V_B^o \\ V_C^o \end{bmatrix}$$

$$= \begin{bmatrix} V_g - I_A^o R_o \\ V_B^o \\ V_C^o \end{bmatrix} \tag{2-20}$$

(For a detailed picture of the boundary conditions, see Fig. 8).

Now, we can evaluate the first row of eqtn (2-20).

$$V_g - I_A^o R_o = I_A^o (M_{11}A_{11}/\gamma_1 + M_{12}A_{21}/\gamma_2 + M_{13}A_{31}/\gamma_3).$$

Then, we can get the sending end current in phase A as

$$I_A^o = \frac{V_g}{M_{11}A_{11}/\gamma_1 + M_{12}A_{21}/\gamma_2 + M_{13}A_{31}/\gamma_3 + R_o} = V_g / Z_{eq} \tag{2-21}$$

where $$Z_{eq} = M_{11}A_{11}/\gamma_1 + M_{12}A_{21}/\gamma_2 + M_{13}A_{31}/\gamma_3 + R_o \tag{2-22}$$

Thus, substituting eqtn (2-21) into eqtn (2-19) for I_A^o, the modal voltages at the sending end are obtained as

$$[V_+^{mode}] = \frac{V_g}{Z_{eq}} \cdot \begin{bmatrix} A_{11}/\gamma_1 \\ A_{21}/\gamma_2 \\ A_{31}/\gamma_3 \end{bmatrix} \tag{2-23}$$

Using the mode-phase relationship of eqtn (2-9) again, we obtain the sending end (x=0) phase voltages in the 3 phase as

$$\begin{bmatrix} V_A^o \\ V_B^o \\ V_C^o \end{bmatrix} = \frac{V_g}{Z_{eq}} [M] \begin{bmatrix} A_{11}/\gamma_1 \\ A_{21}/\gamma_2 \\ A_{31}/\gamma_3 \end{bmatrix} \tag{2-24}$$

Also, for the receiving end at x = ℓ, the modal voltage components are

$$[V_\ell^{mode}] = e^{-\ell[\gamma]} [V_+^{mode}] \qquad \text{from eqtn (2-12)}$$

$$= \frac{V_g}{Z_{eq}} \begin{bmatrix} e^{-\gamma_1 \ell} A_{11}/\gamma_1 \\ e^{-\gamma_2 \ell} A_{21}/\gamma_2 \\ e^{-\gamma_3 \ell} A_{31}/\gamma_3 \end{bmatrix} \tag{2-25}$$

Finally, the receiving end phase voltages in the 3 phase are

$$\begin{bmatrix} V_A^{\ell} \\ V_B^{\ell} \\ V_C^{\ell} \end{bmatrix} = \frac{V_g}{Z_{eq}} [M] \begin{bmatrix} e^{-\gamma_1 \ell} A_{11}/\gamma_1 \\ e^{-\gamma_2 \ell} A_{21}/\gamma_2 \\ e^{-\gamma_3 \ell} A_{31}/\gamma_3 \end{bmatrix} \tag{2-26}$$

It should be realized that if we excite phase k of the 3 phase, (k = A,B, or C), then the output phase voltages are

$$\begin{bmatrix} V_A^{\ell} \\ V_B^{\ell} \\ V_C^{\ell} \end{bmatrix} = \frac{V_g}{Z_{eq}} [M] \begin{bmatrix} e^{-\gamma_1 \ell} A_{1k}/\gamma_1 \\ e^{-\gamma_2 \ell} A_{2k}/\gamma_2 \\ e^{-\gamma_3 \ell} A_{3k}/\gamma_3 \end{bmatrix} \tag{2-27}$$

This is the formula that we use in the Fourier Transform Programme. It has an option to specify which of the 3 phases are to be energized for the test line case.

Thus, we obtain the transfer function [H(ω)] for the test case as seen from Fig. 8.

$$[H(\omega)] = \begin{bmatrix} H_A(\omega) \\ H_B(\omega) \\ H_C(\omega) \end{bmatrix} = \begin{bmatrix} V_A^{\ell} \\ V_B^{\ell} \\ V_C^{\ell} \end{bmatrix} \frac{1}{V_g} \tag{2-28}$$

4. Transfer function for test line

The magnitudes and phases of the transfer functions for the test case of Fig. 8 are plotted in Figs. 9 and 10, respectively, for both approaches used in evaluating the skin effect in the conductors. The difference between the two skin effect formulae only show up in the magnitude spectrum in the low frequency (0 - 100 Hz) region. The results coincide more or less at frequencies above 100 Hz.

The phase of the transfer function increases monotonically (as shown in Fig.9). This can easily be explained for the single phase case[9] where the transfer function becomes

$$H(\omega) = e^{-\sqrt{YZ}\ell} = e^{\gamma\ell} = e^{-\ell\sqrt{(R+j\omega L)j\omega C}} \tag{2-29}$$

where ℓ = length of line

(N.B. Compare with eqtn (2-12) for 3 decoupled modes). Expanding for real and imaginary parts of γ & using binomial expansion

$$(a+b)^{1/2} = a^{1/2} + \frac{1}{2} a^{-1/2} b + \ldots, \quad a >> b.$$

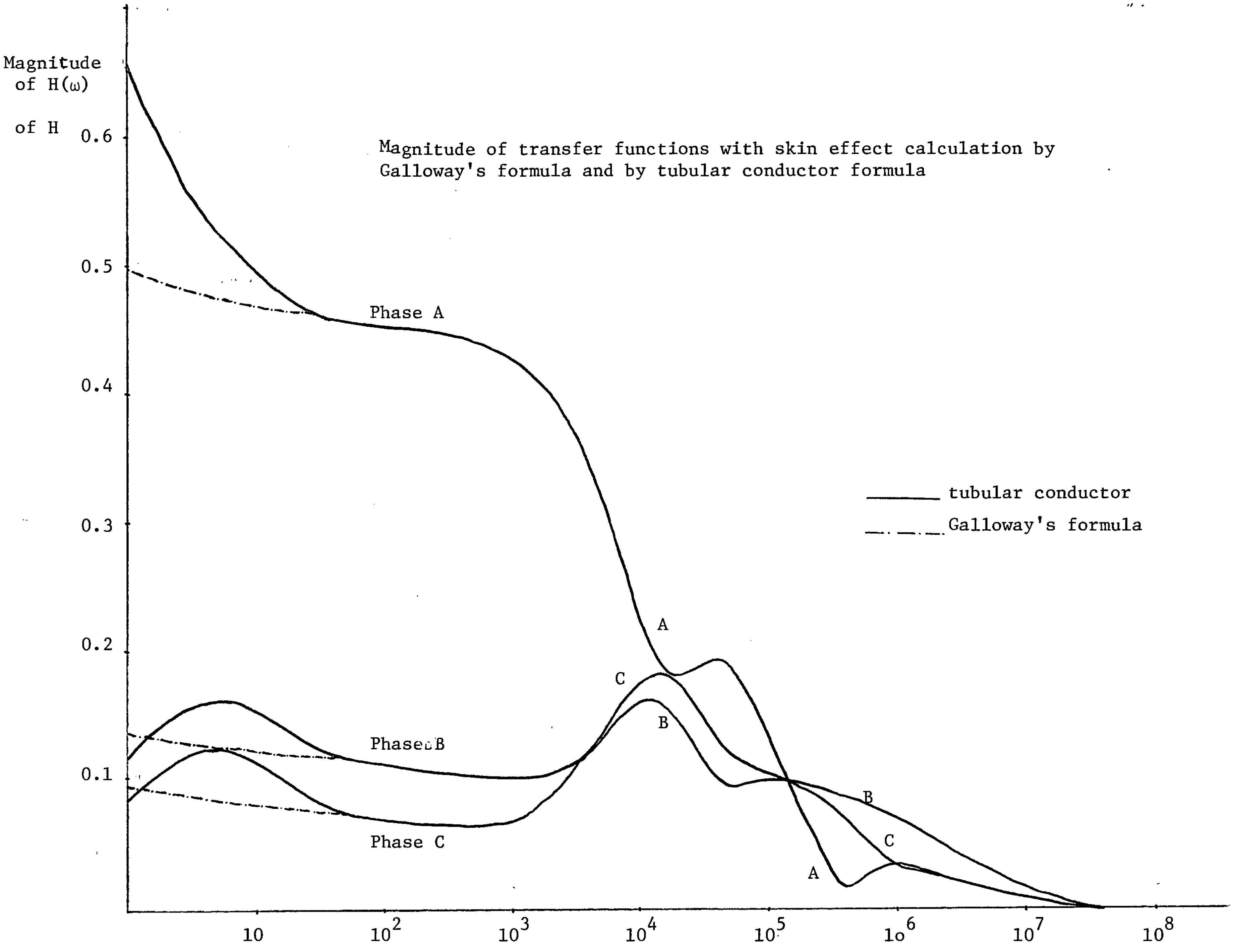
$$\gamma = \sqrt{(R+j\omega L)j\omega C} = (R+j\omega L)^{1/2} \cdot (j\omega C)^{1/2}, \quad R << j\omega L$$

$$\gamma \simeq [(j\omega L)^{1/2} + \frac{R}{2}(j\omega L)^{-1/2}] \cdot (j\omega C)^{1/2}$$

$$= \frac{R}{2}\left(\frac{C}{L}\right)^{1/2} + j\omega(LC)^{1/2} \triangleq \alpha + j\beta \tag{2-30}$$

Thus, the magnitude and phase of the transfer function for the single phase line are respectively

$$|H(\omega)| = e^{-\alpha\ell} = e^{-\ell\frac{R}{2}\left(\frac{C}{L}\right)^{1/2}} \tag{2-31}$$



Magnitude of transfer functions with skin effect calculation by Galloway's formula and by tubular conductor formula
Magnitude of $H(\omega)$
of H
0.6
0.5
0.4
0.3
0.2
0.1
Phase A
Phase B
Phase C
A
B
C
tubular conductor
Galloway's formula
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
Frequency (Hz)

Fig.9

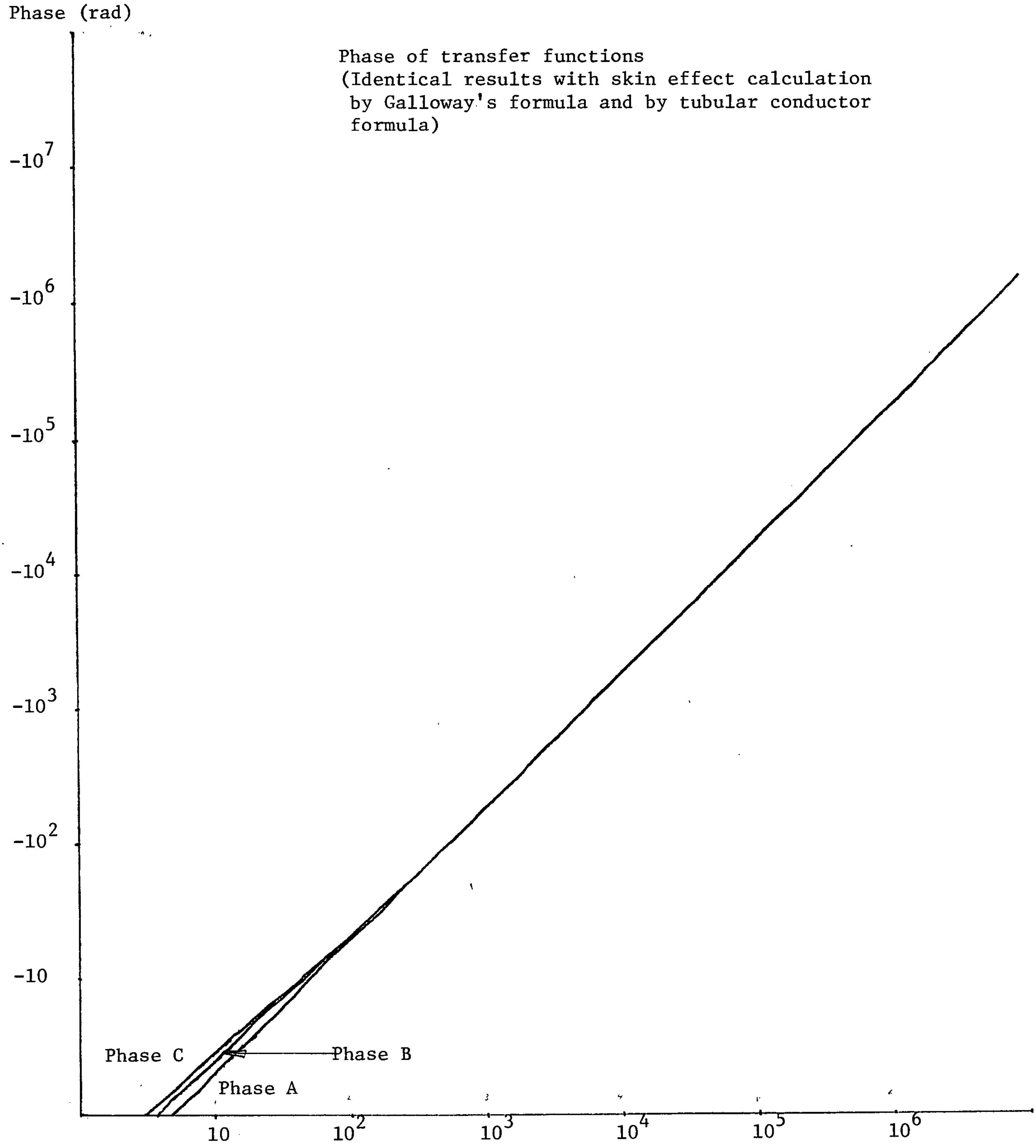

Phase (rad)
Phase of transfer functions
(Identical results with skin effect calculation by Galloway's formula and by tubular conductor formula)
-10^7
-10^6
-10^5
-10^4
-10^3
-10^2
-10
Phase C
Phase B
Phase A
10
10^2
10^3
10^4
10^5
10^6
Frequency(Hz)

Fig.10

and $$\angle H(\omega) = -\ell\beta = -\ell\omega(LC)^{1/2} \quad (2\text{-}32)$$

where R increases appreciably with frequency where L decreases slightly with frequency for zero sequence and stays more or less constant for positive sequency and where C stays constant. That is, the phase angle of the transfer function increases almost linearly with frequency, whereas the magnitude decreases with frequency similar to a low pass filter.

The calculation of angles with a FORTRAN trigonometric function statement covers only the range from 0° to 360°. Therefore, special logic to be included to extend the angles beyond 2π (see Appendix 1 for FORTRAN listings). The separate 'PHASEPRO' program is used to guarantee that the phase angle $\ell\beta$ is a continuous function of ω. This can be achieved by setting up a counter value k, where k is initially zero. Whenever the calculated phase angle falls out of the range of $\pm\,\pi$ rad from the predicted extrapolated phase angle value, k will be incremented by 1

$$k := k + 1,$$

and all following phase angle values are increased by $k(2\pi)$. This way, the phase angle is ensured to be continuous and monotonically increasing.

CHAPTER 3.

TIME RESPONSE OF TEST LINE THROUGH FOURIER TRANSFORMATION

1. Introduction

After the frequency response of the line is known in the form of transfer functions, the output voltage can be calculated for any given input voltage (v_g) by Fourier Transformation. At the beginning, the input voltage $v_g(t)$ is transformed from the time domain into the frequency domain to give $V_g(\omega)$. The output voltage in the frequency domain is then obtained by multiplying $V_g(\omega)$ with the transfer function $[H(\omega)]$ obtained from the Transfer Function Program described in Chapter 2. i.e.

$$[V^{\ell}(\omega)]=[H(\omega)]\ V_g(\omega) \tag{3-1}$$

Finally, the inverse Fourier transformation is used to obtain the output voltages $\bar{v}_A^{\ell}(t)$, $\bar{v}_B^{\ell}(t)$ and $\bar{v}_C^{\ell}(t)$ in the time domain. The above described techniques are applied to the test case of Fig. 8. The obtained results are then compared with the field test measurements[1] and with simulation results obtained by Groschupf[26]. The programs used for the Fourier Transformations from the time to the frequency domain, and vice versa, was adopted from a version initially written by H. W. Dommel[27] (see Appendix 3 for program listings).

2. Numerical Fourier Transformation of input voltage from time to frequency

For a given input voltage $v_g(t)$ in the time domain, we can in general obtain the input voltage in the frequency domain $V_g(\omega)$ with the following Fourier Transformation formula

$$A(\omega) = \int_{-\infty}^{\infty} v_g(t)\cos\omega t\, dt \tag{3-2}$$

$$B(\omega) = \int_{-\infty}^{\infty} v_g(t)\sin\omega t\, dt \tag{3-3}$$

where $A(\omega)$ and $B(\omega)$ are the real and imaginary parts of $V_g(\omega)$, respectively,

$$V_g(\omega) = A(\omega) + j\,B(\omega) \tag{3-4}$$

If we assume that the input voltage is zero for time $t \leqslant 0$, then eqtns (3-2) and (3-3) can be simplified to

$$A(\omega) = \int_o^T v_g(t) \cos \omega t \; dt \tag{3-5}$$

$$B(\omega) = \int_o^T v_g(t) \sin \omega t \; dt \tag{3-6}$$

where (o,T) is the time interval in which $v_g(t)$ is non-zero.

Case 1. Input voltage defined point by point

If the input voltage is defined point by point in the integration interval (o,T) at closely spaced time intervals, then it is reasonable to assume linear interpolation between points (see Fig. 11). Then, for an interval (t_1, t_2), we have

$$v_g(t) = v_1 + \frac{v_2 - v_1}{\Delta t}(t - t_1), \qquad t_1 \leqslant t \leqslant t_2 \quad \& \; \Delta t = t_2 - t_1 \tag{3-7}$$

Substitution of eqtn (3-7) into eqtn (3-5) gives

$$A_{12}(\omega) = \int_{t_1}^{t_2} [v_1 + \frac{v_2-v_1}{\Delta t}(t-t_1)] \cos \omega t \; dt \tag{3-8}$$

$$= (v_1 - \frac{v_2-v_1}{\Delta t} t_1) \int_{t_1}^{t_2} \cos \omega t \; dt + \frac{v_2-v_1}{\Delta t} \int_{t_1}^{t_2} t \cos \omega t \; dt$$

$$= (v_1 - \frac{v_2-v_1}{\Delta t} t_1) \frac{1}{\omega} \sin \omega t \Big|_{t_1}^{t_2}$$

$$+ \frac{v_2-v_1}{\Delta t} \frac{1}{\omega} (t \sin \omega t + \frac{1}{\omega} \cos \omega t) \Big|_{t_1}^{t_2}$$

$$= \frac{1}{\omega} \sin \omega t_2 [(v_1 + \frac{v_2-v_1}{\Delta t} t_2) - \frac{v_2-v_1}{\Delta t} t_1]$$

$$- \frac{\sin \omega t_1}{\omega} [(v_1 - \frac{v_2-v_1}{\Delta t} t_1) + \frac{v_2-v_1}{\Delta t} t_1]$$

$$+ \frac{v_2-v_1}{\Delta t \omega^2} (\cos \omega t_2 - \cos \omega t_1)$$

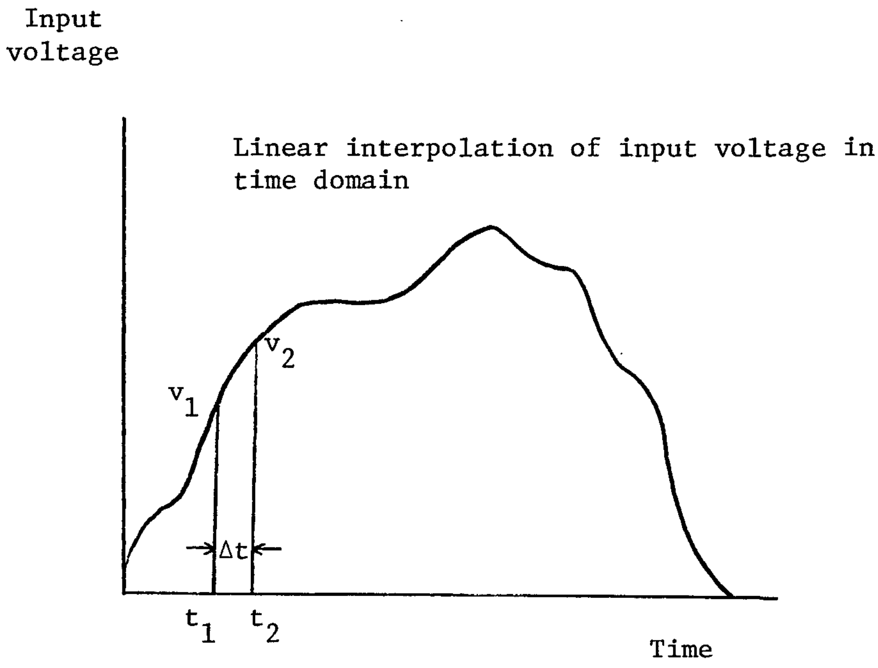

Input
voltage
Linear interpolation of input voltage in
time domain
v_1
v_2
Δt
t_1
t_2
Time

Fig.11

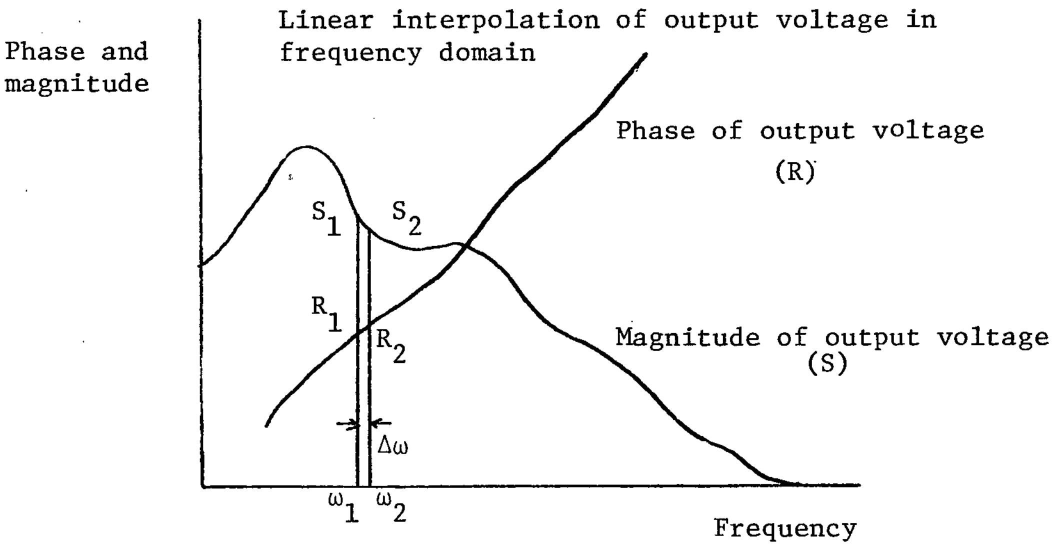

Phase and
magnitude
Linear interpolation of output voltage in
frequency domain
Phase of output voltage
(R)
S_1
S_2
R_1
R_2
Magnitude of output voltage
(S)
$\Delta\omega$
ω_1
ω_2
Frequency

Fig.12

or finally, $$A_{12}(\omega) = \frac{1}{\omega}[v_2 \sin \omega t_2 - v_1 \sin \omega t_1 + \frac{v_2 - v_1}{\Delta t\omega}(\cos \omega t_2 - \cos \omega t_1)] \qquad (3\text{-}9)$$

Similarly, for the imaginary voltage component in the interval (t_1, t_2), we obtain

$$\begin{aligned} B_{12}(\omega) &= \int_{t_1}^{t_2}[v_1 + \frac{v_2 - v_1}{\Delta t}(t - t_1)] \sin \omega t \, dt \\ &= (v_1 - \frac{v_2 - v_1}{\Delta t})\int_{t_1}^{t_2} \sin \omega t \, dt + \frac{v_2 - v_1}{\Delta t}\int_{t_1}^{t_2} t \sin \omega t \, dt \\ &= (v_1 - \frac{v_2 - v_1}{\Delta t})\frac{1}{\omega} \cos \omega t \Big|_{t_1}^{t_2} + \frac{v_2 - v_1}{\Delta t}(-t \cos \omega t + \frac{1}{\omega} \sin \omega t)\Big|_{t_1}^{t_2}, \end{aligned}$$

or finally $$B_{12}(\omega) = \frac{1}{\omega}[-v_2 \cos \omega t_2 + v_1 \cos \omega t_1 + \frac{v_2 - v_1}{\Delta t\omega}(\sin \omega t_2 - \sin \omega t_1)] \qquad (3\text{-}10)$$

The calculations of $A_{12}(\omega)$ and $B_{12}(\omega)$ are repeated for all time intervals to cover the whole region (o,T). The real and imaginary part of the voltage in the frequency domain at a specific frequency ω is then simply the sum of these parts

$$\begin{aligned} V_g(\omega) &= A(\omega) + jB(\omega) \\ &= \sum_{k=0}^{N} A_{k,k+1}(\omega) + jB_{k,k+1}(\omega), \text{ where } N = \frac{T}{\Delta t} \end{aligned}$$

The above calculations must be made over the entire frequency range at the same frequency points at which the transfer functions have been calculated. Output voltage in the frequency domain is thus obtained at all transfer function frequencies.

Case 2. <u>Input voltage defined analytically</u>

For some types of input voltages $v_g(t)$. $A(\omega)$ and $B(\omega)$ are known analytically[28]. Take a single exponential decay input voltage as an example,

$$v_g(t) = e^{-at},\ t \geq 0 \tag{3-12}$$

We can directly evaluate $A(\omega)$ and $B(\omega)$

$$\text{by } V_g(\omega) = A(\omega) + jB(\omega)$$
$$= \frac{1}{a+j\omega} \tag{3-13}$$

Thus, we can obtain the real and imaginary voltage components in the frequency domain by the exact Fourier Transformation. With this technique, we can omit the first part of our program and obtain the output voltage in the frequency domain by multiplying eqtn (3-13) with the corresponding transfer functions, i.e.

$$[V^{\ell}] = [H(\omega)]\ \frac{1}{a+j\omega} \tag{3-14}$$

3.) Output Voltage in Frequency Domain

From eqtn (3-1), we have the output voltage in the frequency domain as

$$[V^{\ell}_{(\omega)}] = [H(\omega)]\ V_g(\omega) \qquad \text{from eqtn (3-1)}$$

For inverse Fourier transformation back to the time domain, (see section D), we use linear interpolation between consecutive frequency points. Thus, the output voltage frequency components must be reasonably smooth to obtain satisfactory results.

It has been shown that the magnitude of the transfer function is fairly smooth (see eqtns (2-31) and (2-32). This is also true for the phase angle of the tranfer function provided it is extended beyond 2π rad(see Figs. 9 and 10).

From eqtns (3-1 and (3-10), we obtain the real and imaginary components of the input voltage. They become highly oscillating at higher frequencies and are not suitable for linear interpolation. Therefore, the real and imaginary voltage components are converted to magnitude and phase values. The phase angle is again extended beyond 2π by the same smoothing logic as described in Chapter 2. Thus, we can write the output voltage in the frequency domain $V^{\ell}(\omega)$ as

$$V^{\ell}(\omega) = S(\omega)\angle R(\omega) \tag{3-15}$$

14. Output voltage in time domain by numerical inverse Fourier Transformation

From the given output voltage in the frequency domain $[V^{\ell}(\omega)]$, we obtain the output voltages in the time domain by inverse Fourier Transformation

$$v^{\ell}(t) = \frac{1}{\pi}\int_0^{\infty} V^{\ell}(\omega)\, e^{j\omega t}\, d\omega$$

From eqtn (3-15), we get

$$v^{\ell}(t) = \frac{1}{\pi}\int_0^{\infty} S(\omega)\, e^{j(\omega t + R)} d\omega \tag{3-16}$$

Similar to section B, for Fourier Transformation, the inverse Fourier Transformation also uses linear interpolation between adjacent points in the frequency domain, for the magnitudes (S_1, S_2) of the output voltages as well as for the phase angles (R_1, R_2). (see Fig. 12). As explained in section C, this is permissible because S and R are smooth curves in contrast to the highly oscillating real and imaginary components $A(\omega)$ and $B(\omega)$. Since only a real voltage component exists in the time domain, the contribution to the output voltage from the inverse Fourier Transformation of the frequency interval $[\omega_1, \omega_2]$ is

$$v^{\ell}(t) = \frac{1}{\pi}\int_0^{\infty} S(\omega)\cos(\omega t + R)\, d\omega,$$

or
$$v^{\ell}_{12}(t) = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} S(\omega)\cos(\omega t + R)d\omega \tag{3-17}$$

where
$$S(\omega) = S_1 + \frac{S_2 - S_1}{\Delta\omega}(\omega - \omega_1) \tag{3-18}$$

$$R(\omega) = R_1 + \frac{R_2 - R_1}{\Delta\omega}(\omega - \omega_1) \tag{3-19}$$

where $\omega_1 < \omega < \omega_2$, & $\Delta\omega = \omega_2 - \omega_1$

Substituting the magnitude and phase eqtns (3-18) and (3-19) into eqtn (3-13) we get

$$v_{12}^{\ell}(t) = \int_{\omega_1}^{\omega_2} [S_1 + \frac{S_2-S_1}{\Delta\omega}(\omega-\omega_1)] \cos [\omega t + R_1 + \frac{R_2-R_1}{\Delta\omega}(\omega-\omega_1)]d\omega$$

$$= \int_{\omega_1}^{\omega_2} [(S_1 - \frac{S_2-S_1}{\Delta\omega}\omega_1) + \frac{S_2-S_1}{\Delta\omega}\omega] \cdot$$

$$\cos [(R_1 - \frac{R_2-R_1}{\Delta\omega}\omega_1) + (\frac{R_2-R_1}{\Delta\omega} + t)\omega]d\omega$$

$$\triangleq \int_{\omega_1}^{\omega_2} (a + b\omega) \cos (c + s\omega)d\omega$$

where constants a,b,c and s are constant for a specific frequency interval,

$$a = S_1 - \frac{S_2-S_1}{\Delta\omega} \omega_1 \tag{3-20}$$

$$b = \frac{S_2-S_1}{\Delta\omega} \tag{3-21}$$

$$c = R_1 - \frac{R_2-R_1}{\Delta\omega} \tag{3-22}$$

$$s = \frac{R_2-R_1}{\Delta\omega} + t \tag{3-23}$$

Thus, we obtain

$$v_{12}^{\ell}(t) = a\int_{\omega_1}^{\omega_2} \cos (c + s\omega)d\omega + b\int_{\omega_1}^{\omega_2} \omega\cos(c + s\omega)d\omega$$

$$= \frac{a}{s} \sin (c + s\omega) \Big|_{\omega_1}^{\omega_2} + \frac{b}{s} [\omega\sin(c + s\omega) + \frac{1}{s} \cos (c+s\omega)]\Big|_{\omega_1}^{\omega_2}$$

or finally

$$v_{12}^{\ell}(t) = \sin(c+s\omega_2)(\frac{a}{s}+\omega_2 \frac{b}{s}) - \sin(c+s\omega_1)(\frac{a}{s} + \omega_1 \frac{b}{s})$$

$$+ \frac{b}{s} [\cos(c+s\omega_2)-\cos(c+s\omega_1)] \tag{3-24}$$

The calculation with eqtn (3-24) is repeated for all frequency intervals to cover the frequency region over which the output voltage $V^{\ell}(\omega)$

is defined. The output voltage at any specific time is then the sum of the contributions from all frequency interval

$$v^{\ell}(t) = \sum_{\omega=0}^{\omega'} v_{12}^{\ell}(t,\omega) \tag{3-25}$$

where ω' is the last frequency data point.

5. Numerical Aspects of Fourier Transformation Program

There are several aspects which deserve special attention in the Fourier Transformation Program to ensure reasonably accurate results.

1. Suitability of linear interpolation in numerical integration - A reasonable "smoothness" of input voltage $v_g(t)$ and output voltage in frequency domain $V_{\omega}^{\ell}(\omega)$ must be guaranteed to permit linear interpolation between adjacent data points. Therefore, the magnitude and phase angle of the output voltage are used to avoid the highly oscillating real and imaginary frequency components as described in section 3.

2. Density of data points - Linear interpolation is assumed between adjacent frequency and time data points in the numerical integration loops. Too dense data points will increase computer costs drastically, while too sparse data points will result in loss of accuracy. A density of 20 points per decade in the frequency domain (on a logarithm scale) satisfies the accuracy requirement reasonably well for the test case studied. In the time domain, the density of data points for the input voltage $v_g(t)$ depends on its wave shape and can readily be determined by the program user.

3. Number of decades in frequency domain over which $H(\omega)$ and $V_g(\omega)$ must be defined - It is easy to judge the required no. of decades as the transfer function magnitudes decrease substantially at high frequencies. Thus 7 to 8 decades of frequency data points, starting at f_{start}=1 Hz will

ensure reasonable accuracy without increasing computer costs too much for the test case studied. Integration between $f = 0$ and f_{start} where the frequency data points start is done separately, again assuming linear interpolation between 0 and f_{start}. Therefore, we can start our frequency data at any decade. This is allowable as long as the output voltage $\dot{V}^{\ell}(\omega)$ remains fairly constant and linear interpolation from zero to the starting frequency f_{start} does not cause appreciable deviations.

3. Input voltage wave form - An efficient and simple way to check the accuracy of the Fourier Transformation Program is to run it in a test mode where the transfer function is set to 1,

$$H(\omega) = 1 \angle 0$$

and to check how closely the output voltage in the time domain agrees with the input voltage $v_g(t)$. In our test case, the known input voltage $v_g(t)$ is a double exponential of the form

$$v_g(t) = e^{-\alpha_1 t} - e^{-\alpha_2 t} \tag{3-26}$$

where $\alpha_1 = 0.17 \times 10^3 s^{-1}$

and $\alpha_2 = 3.27 \times 10^6 s^{-1}$

This input voltage matches exactly the output voltage thus obtained from our transformation program (see Figs. 13 and 14). In Fig. 13, in the time interval from 0 to 7μs step widths of

$$\Delta t = 0.05\mu s$$

and $\Delta\omega = 20$ pts/decade (log scale)

were chosen. In Fig. 14, for time >10μs, the input voltage $v_g(t)$ is essentially a single exponential α decay, for which step widths of

$$\Delta t = 0.1 \text{ ms}$$

and $\Delta\omega = 20$ pts/decade (log scale)

were chosen.

Input voltage and calculated output voltage with

$H(\omega)=1.0\angle 0^{\circ}$

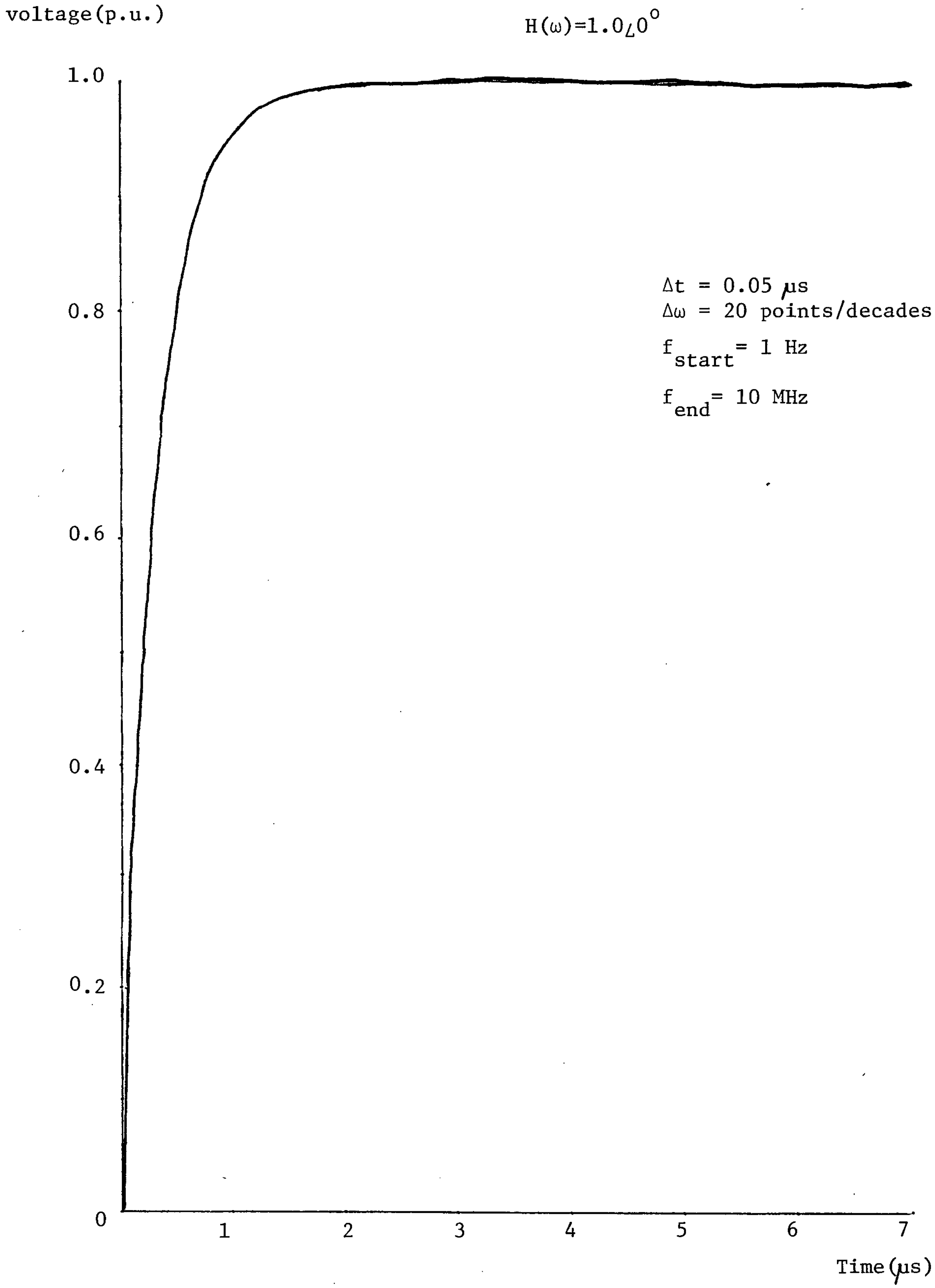
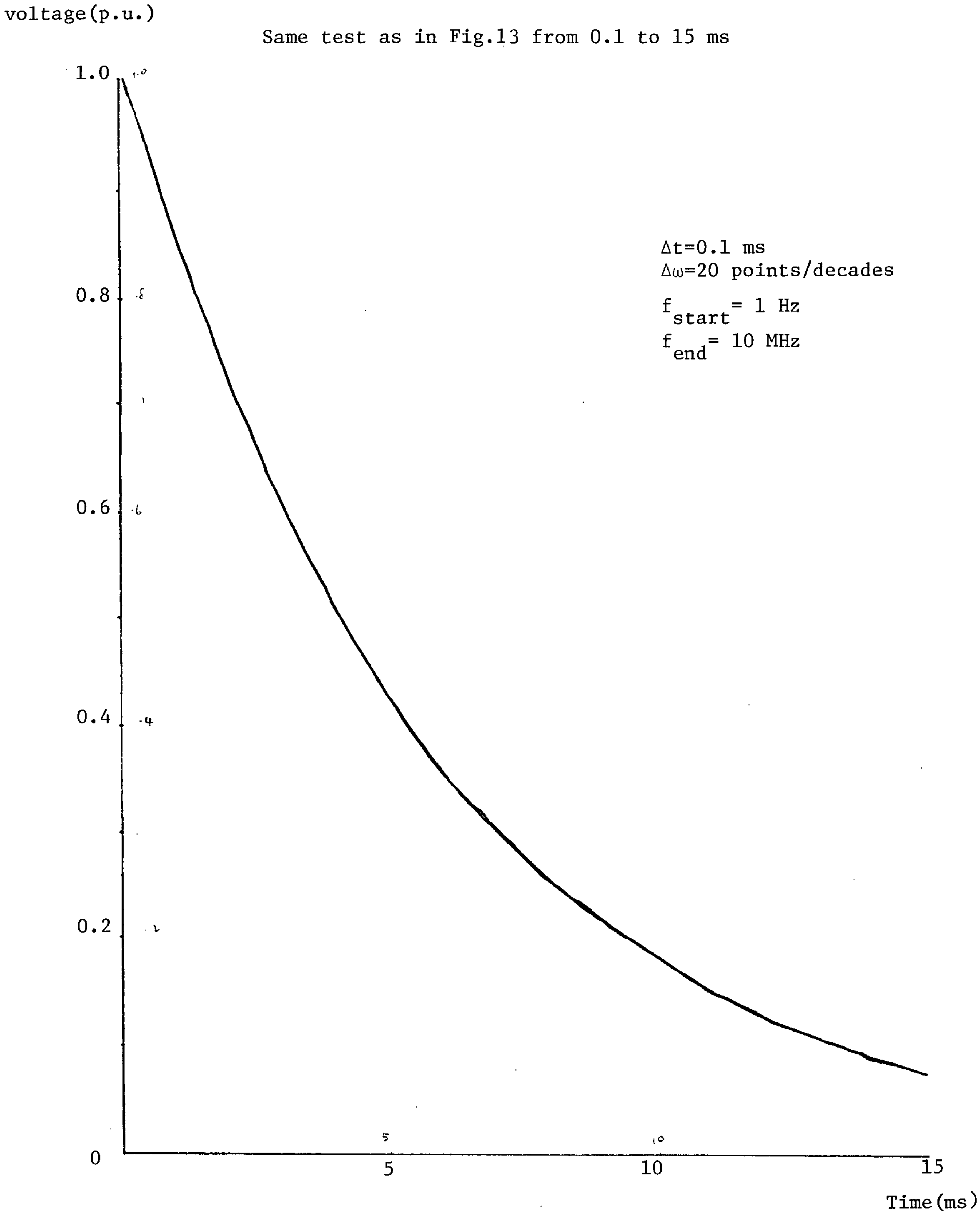


Fig.13



voltage(p.u.)
Same test as in Fig.13 from 0.1 to 15 ms
1.0
0.8
0.6
0.4
0.2
0
5
10
15
Time(ms)
Δt=0.1 ms
Δω=20 points/decades
f start = 1 Hz
f end = 10 MHz

Fig.14

4. Numerical problems with step function inputs - No problem of numerical instability were encountered when the field tests of the test line were simulated. The input voltage $v_g(t)$ in this case is a double exponential wave (see eqtn 3-26). The computations were numerically stable for large values of decay constants, namely α_1 and $\alpha_2 \geq 10$. However, many cases were run with step function inputs for checking purposes to debug the programme and to gain confidence before the duplication of field tests could be attempted. Serious numerical instability problems were encountered with pure step function inputs, which were then overcome by replacing the step function with an exponentially decaying function $e^{-\alpha t}$. The decay parameter α is chosen in such a way that this function is practically equal to a step function over the time span of interest.

For an input voltage step function

$$v_g(t) = 1 \tag{3-27}$$

the voltage in the frequency domain is

$$V_g(\omega) = \int_0^\infty 1\, e^{-j\omega t}dt$$

$$= \frac{-1}{j\omega} e^{-j\omega t}\Big|_0^\infty$$

$$= \frac{1}{j\omega} - \frac{1}{j\omega} \lim_{t\to\infty} e^{-j\omega t} \tag{3-28}$$

For time $t \to \infty$, the second term of eqtn (3-28) is highly oscilating and is non-zero, which causes numerical instability in the Fourier Transformation Programme. However, this problem can be remedied by introducing a slow decay into the input voltage $v_g(t)$ as in eqtn (3-12), namely

$$v_g(t) = 1 \cdot e^{-\alpha t},\ t > 0 \qquad \text{from eqtn (3-12)}$$

The input voltage in the frequency domain now becomes

$$V_g(\omega) = \int_0^{\infty} e^{-\alpha t} - e^{-j\omega t} dt$$

$$= \frac{-1}{\alpha+j\omega} e^{-(\alpha+j\omega)t} \Big|_0^{\infty}$$

$$= \frac{1}{\alpha+j\omega} - \frac{1}{\alpha+j\omega} \lim_{t\to\infty} e^{-\alpha t} e^{-j\omega t}$$

Now, for time to $\to\infty$, the second term is no longer oscillating due to the presence of the decay factor $e^{-\alpha t}$ in it, and goes to zero as $t\to\infty$, or

$$V_g(\omega) = \frac{1}{\alpha+j\omega}$$

Furthermore, the single exponential decay voltage is better than a cut-off step function voltage (rectangular pulse) inasmuch as $1\cdot e^{-\alpha t}$ has a smoother amplitude and phase angle spectrum than a rectangular pulse. Thus, fewer data points per decade are required to achieve the same degree of accuracy.

The problem of numerical instability with a step function voltage is therefore easily solved by introducing the decay factor α. Numerical experiments showed that $\alpha > 10$ will be good enough to ensure numerical stability. Note that for the case $\alpha = 10$, the deviation of the exponentially decaying input voltage

$$v_g(t) = e^{-\alpha t}$$

from the ideal step voltage is negligible for the time span of interest here.

For $\quad t_{max} = 10\mu s$

$$v_g(t) = e^{-10x10^{-5}}$$

$$= 0.9999$$

That is, the maximum deviation is less than 0.01% from the step input voltage at the upper limit t_{max} of the study.

CHAPTER IV

DUPLICATION OF FIELD TESTS

1. Doubling effect on open-ended line.

In the analysis leading to eqtn. (2-12), we have neglected the reflected voltage wave and obtained the modal voltage for the infinite line at a distance ℓ from the sending end.

$$V_{\ell}^{mode}(\omega) = V_{+}^{mode}(\omega)\, e^{-\gamma\ell} \tag{4-1}$$

This expression is not directly usable for the test case since we now have a transmission line of finite length which is open-ended at the receiving end terminal (see Fig. 8). The equations for the decoupled modal quantities are analogous to the equation of a single phase line, where the comparison between the infinite line and the open-ended finite line is well known. Thus, we can use the well known solution for the single phase case for voltages and currents[14,24,29] in the modal domain,

$$V_{o}^{mode} = V_{\ell}^{mode} \cosh \gamma\ell + Z_o\, I_{\ell}^{mode} \sinh\gamma\ell \tag{4-2}$$

$$I_{o}^{mode} = \frac{V_{\ell}^{mode}}{Z_o} \sinh \gamma\ell + I_{\ell}^{mode} \cosh \gamma\ell \tag{4-3}$$

where I_o^{mode} and V_o^{mode} are the modal voltage and current at the sending end x = o, and I_{ℓ}^{mode} and V_{ℓ}^{mode} are the modal voltage and current at the receiving end x = ℓ.

For an open-ended line, $I_{\ell}^{mode} = o$. Then we obtain from (4-2) and (4-3)

$$V_o^{mode} + Z_o\, I_o^{mode} = V_{\ell}^{mode} (\cosh \gamma\ell + \sinh\gamma\ell)$$

$$= V_{\ell}^{mode}\, e^{\gamma\ell}.$$

$$\text{i.e.}\quad V_{\ell}^{mode} = e^{-\gamma\ell} (V_o^{mode} + Z_o\, I_o^{mode}). \tag{4-4}$$

From $t = o$ to $t < 2\tau$, no reflection has yet come back from the receiving end, and the conditions at the sending end are therefore the same as those of an infinite line during this time period,

$$V_o^{mode} = Z_o \; I_o^{mode} \tag{4-5}$$

(This relationship is no longer true at the sending end after $t \geq 2\tau$, and is no longer true at the receiving end for $t \geq 3\tau$.)

With substitution of eqtn (4-5) into eqtn (4-4), we get for the receiving end,

$$V_\ell^{mode} = 2e^{-\gamma\ell} V_o^{mode} = 2 \; V_+^{mode} \; e^{-\gamma\ell}$$

This is twice the obtained receiving end voltage for the infinite line at location $x = \ell$. Thus, there is a doubling effect in the receiving end voltage of the open-ended line in comparison with the infinite line.

2. Comparison with field measurements and other simulation results

The output voltage at the receiving end is plotted in Fig. 15 for the test case with the voltage doubling effect taken into account. The arrival time of the first part of the voltage wave coincides closely with the time taken by electromagnetic waves (TEM propagation) in air[3], i.e. 277 μs for 83.212 km at a wave velocity of 3 km/μs. On a transposed line, this first part of the wave would be associated with the positive sequence parameters, and the second part of the wave would correspond to the zero sequence wave. It can be observed that the wave velocity of the zero sequence mode is slower than that of the positive sequence mode. Also the skin effect calculation with Galloway's formula gave slightly higher resistances (eg., $\Delta R_{pos} \simeq 0.67\ \Omega$ and $\Delta R_{zero} \simeq 0.73\ \Omega$ at 50 KHz, see chapter 1, section E) than the formula for tubular conductors. The output voltage based on Galloway's formula will be slightly smaller than that obtained with the tubular conductor formula. This can be seen from Fig. 15. Galloway's formula gives results closer to field measurements than the tubular conductor formula, as expected. This is because the double exponential wave front contains high frequency components where Galloway's formula is more accurate. (See Chapter 1, Sections 4 and 5).

For comparison purposes, the field measurement results from Ametani[12] and simulation studies by Groschupf[26] are included in Fig. 16. The simulation results obtained with the methods described in this thesis compare favorably with the field measurement results (within 8%). Some probable causes of discrepancies between simulation and field measurements may be due to the following phenomena:

1) Assumption of uniform earth resistivity (200 $\Omega \cdot m$) -- An increase in earth resistivity will increase the zero sequence parameters[30,31] and thereby increase attenuation and decrease wave velocity of the zero sequence voltage wave. Also, a homogenous earth is only an approximation of a stratified earth which will again cause some differences in impedance line parameter calculations.

2) Temperature of conductor -- We assumed a conductor temperature of 20°C. An increase in temperature will increase conductor resistance appreaciably (eg. 40% rise for increase of temperature to 120°C).

However, a difference between numerical and measured values of less than 8% is well within acceptable accuracy criteria for these types of studies.

Output voltage at receiving end of transmission line

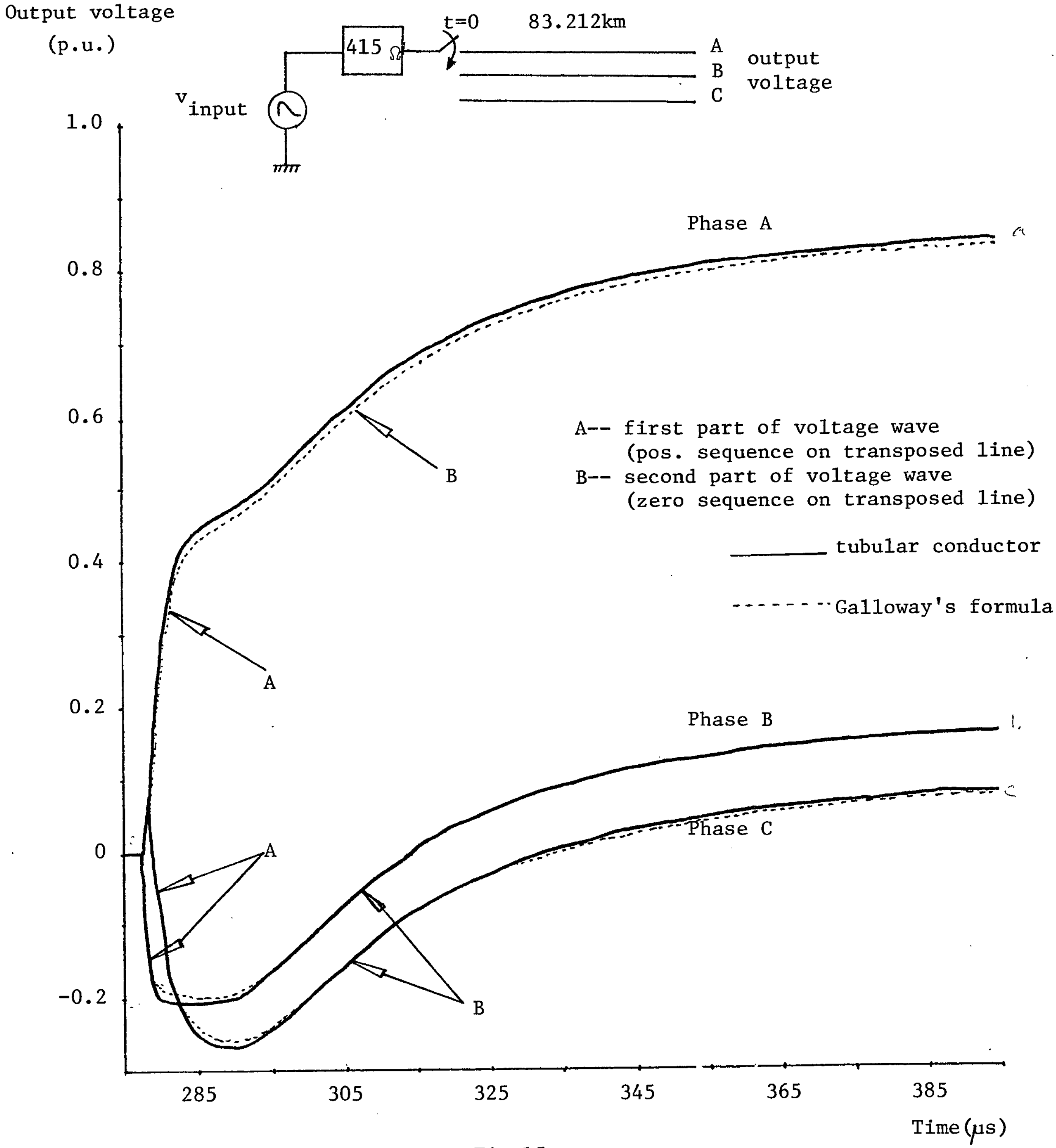


Fig.15

Output voltage at receiving end of transmission line with field measurement and Groschupf's simulation results

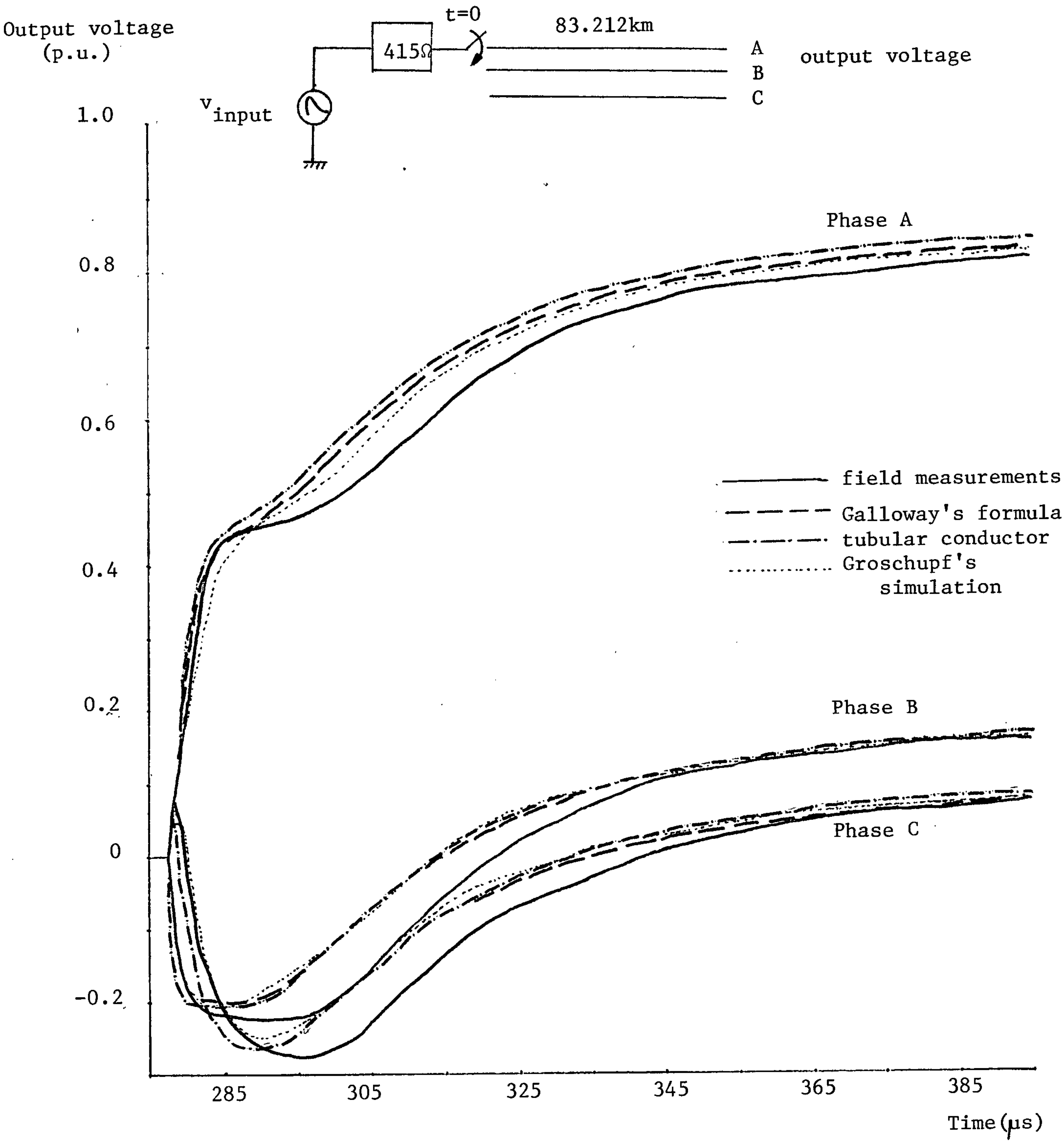


Fig.16

CHAPTER V

CONCLUSIONS

The attenuation and distortion of wave fronts on multiphase overhead transmission lines or underground cables was studied. A specific case of a Japanese 500 kV three-phase overhead line was chosen as a test example because field measurements were available for this line. The voltage response at the open-ended receiving end was simulated. The simulation results agreed very well with the field measurement results and with simulation results of another investigator.

Results obtained with their technique developed in this thesis will be useful for switching surge insulation co-ordination studies in power transmission systems. For instance, the technique could be used to calculate the wave front which could hit a transformer at the receiving end of the line. It should be realized, however, that this wave front would be modified by the transformer itself. This wave front modified by the transformer would be a worthwhile topic for future research. Depending on the rise time of the incident voltage wave, the transformer insulation may be more or less stressed. This is a problem of current concern in the electric utility industry[32].

APPENDIX 1

TRANSFER FUNCTION PROGRAM LISTINGS

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)        MAIN        03-23-77        11:37:42        PAGE P001

              COMPLEX*8 DET,COND,VEXCIT
              COMPLEX*16 CU(3)
              COMPLEX*16 CURR1,CURR2,CURR3
              COMPLEX*16 A1(3,3),B1(3,3)
              REAL*8 VPF(3),DTAUF,FREQ,TAU,VPT(3),RMAG,PHASE1,PHASE
              COMPLEX*16 CZ(3,3),VO(3),VP(3),X,CURR
              REAL*8 VOI,VOR,VEXRF,VEXIM,Z1(3,3),ZI1(3,3)
              REAL*8 FR(3),DSURT,SUM,ONORM(3),EI(3),VR(3,3),VI(3,3)
              COMPLEX*16 VP0(3,1),VP30(3,1),VM30(3,1),VM0(3,1)
              COMPLEX*16 EC(3),ES(3),RM(3,3),RMI(3,3)
              REAL*8 A(3,3),B(3,3),C(3,3),D(3,3)
              REAL*8 OMEGA,DECAY,A2K2,Z(3,3),ZI(3,3),Y(3,3)
              REAL*8 M
C ****  FREQUENCY INPUT = M
              N=3
              DECAY=10.D0
               RINPUT=415.
C VEXCIT IS INPUT STEP PHASE VOLTAGE
123           READ (2,3)M
3             FORMAT (E14.6)
C  IMPEDENCE MATRIX Z IS COMPLEX AND ADMITTANCE MATRIX Y IS PURELY
C IMAGINARY
C ****  Z  X  Y READ IN
              READ (2,1)Y(1,1),Y(2,1),Y(2,2),Y(3,1),Y(3,2),Y(3,3)
              READ (2,1)Z(1,1),Z(2,1),Z(2,2),Z(3,1),Z(3,2),Z(3,3)
1             FORMAT (6D13.5)
              READ (2,1)ZI(1,1),ZI(2,1),ZI(2,2),ZI(3,1),ZI(3,2),ZI(3,3)
              PRINT 7,M
            7 FORMAT (///1H1,'***THE FOLLOWING ARE COMPUTING AT FREQUENCY',E13.6
             1,' HZ')
C             Y(2,1)=0.
C             Y(3,1)=0.
C             Y(3,2)=0.
C             Z(2,1)=0.
C             Z(3,1)=0.
C             Z(3,2)=0.
C             ZI(2,1)=0.
C             ZI(3,1)=0.
C             ZI(3,2)=0.
C DECOUPLED CONDITION IS SET IN
              IF (M.EQ.999999) GO TO 99
C
C
C CREATE FULL  Z  X Y  FROM INPUT LOWER TRIANGULAR MATRIX
C
C
C
              DO 10 I=1,N
              DO 10 J=1,N
              Z(I,J)=Z(J,I)
              ZI(I,J)=ZI(J,I)
              CZ(I,J)=DCMPLX(Z(I,J),ZI(I,J))
              Y(I,J)=Y(J,I)
           10 CONTINUE
              PRINT 2
2             FORMAT (/,'THE REAL PART OF IMPEDANCE MATRIX Z',/)
              DO 20 I=1,N
```

```
0037       4    FORMAT (3(5X,D13.5))
0038            PRINT 5
0039       5    FORMAT (/,'THE IMAGINARY PART OF IMPEDANCE MATRIX Z',/)
0040            DO 30 I=1,N
0041      30    PRINT 4,(ZI(I,J) ,J=1,3)
           C
           C PRINTOUT OF Y
0042            PRINT 6
0043       6    FORMAT (/,'THE SUSCEPTANCE MATRIX Y',/)
0044            DO 40 I=1,N
0045      40    PRINT 4,(Y(I,J),J=1,3)
           C
           C
           C  PRODUCT OF Z & Y
           C
0046            DO 50 I=1,N
0047            DO 50 J=1,N
0048            A(I,J)=Z(I,J)
0049      50    B(I,J)=Y(I,J)
0050            CALL MYMULT (A,B,C,D)
0051            DO 60 I=1,N
0052            DO 60 J=1,N
0053            ZI1(I,J)=D(I,J)
0054      60    A(I,J)=ZI(I,J)
0055            CALL MYMULT (A,B,C,D)
0056            DO 70 I=1,N
0057            DO 70 J=1,N
0058      70    ZI(I,J)=-D(I,J)
           C
           C ********      Y IS PURELY IMAGINARY
0059            PRINT 9
0060       9    FORMAT (/,'THE REAL PART OF Z*Y MATRIX',/)
0061            DO 90 I=1,N
0062      90    PRINT 4,(ZI(I,J),J=1,3)
0063            PRINT 8
0064       8    FORMAT (/,'THE IMAGINARY PART OF Z*Y MATRIX',/)
0065            DO 80 I=1,N
0066      80    PRINT 4,(ZI1(I,J),J=1,3)
0067            CALL DCEIGN (ZI,ZI1,N,N,ER,EI,VR,VI,IERROR,1,0)
           C
           C COMPLEX EIGENVALUES ER+JEI  & COMPLEX EIGENVECTORS  VR+JVI
           C   OBTAINED FROM SUBROUTINE DCEIGN
           C
0068            PRINT 11
0069      11    FORMAT (/,'THE EIGENVALUES OF Z*Y MATRIX',/)
0070            DO 100 I=1,N
0071            EC(I)=DCMPLX(ER(I),EI(I))
0072     100    PRINT 12,ER(I),EI(I)
0073      12    FORMAT (5X,'REAL',D13.5,5X,'IMAGINARY',D13.5)
0074            PRINT 13
0075      13    FORMAT (/,'THE PROPAGATION CONSTANT  IS THE SQUARE ROOT',/)
0076            DO 110 I=1,N
           C
           C ****** PROPAGATION CONSTANT ES
0077            ES(I)=CDSQRT(EC(I))
           C
           C
0078     110    PRINT 12,ES(I)
```

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)          MAIN          03-23-77          11:37:42          PAGE P003

 0079              DO 122 I=1,3                                                    114.000
 0080              SUM=0.D0                                                        115.000
 0081              DO 121 J=1,3                                                    116.000
 0082                SUM=SUM+VR(J,I)*VR(J,I)+VI(J,I)*VI(J,I)                       117.000
 0083          121 CONTINUE                                                        118.000
 0084              DNORM(I)=DSQRT(SUM)                                             119.000
 0085          122 CONTINUE                                                        120.000
 0086              DO 120 I=1,N                                                    121.000
 0087              DO 120 J=1,N                                                    122.000
 0088              VR(I,J)=VR(I,J)/DNORM(J)                                        123.000
 0089              VI(I,J)=VI(I,J)/DNORM(J)                                        124.000
             C MODAL TRANSFORMATION MATRIX M IS NORMALIZED                         125.000
 0090              RM(I,J)=DCMPLX(VR(I,J),VI(I,J))                                 126.000
 0091        120   RMI(I,J)=RM(I,J)                                                127.000
             C  MODAL MATRIX RM INVERTED TO RMI.                                   128.000
             C                                                                     129.000
 0092              CALL CDINVT (RMI,3,3,DET,COND)                                  130.000
 0093              DO 502 J=1,3                                                    131.000
 0094              DO 502 L=1,3                                                    132.000
 0095              A1(J,L)=DCMPLX(0.D0,0.D0)                                       133.000
 0096              DO 502 I=1,3                                                    134.000
 0097              B1(J,L)=RMI(J,I)*CZ(I,L)                                        135.000
             C *****                                                               136.000
             C                                                                     137.000
             C IEXCIT IS PHASE EXCITED                                             138.000
             C                                                                     139.000
 0098              IEXCIT=1                                                        140.000
             C                                                                     140.100
             C                                                                     140.200
             C                                                                     140.300
             C                                                                     140.400
 0099        502   A1(J,L)=A1(J,L)+B1(J,L)                                         141.000
 0100              CURR1=RM(IEXCIT,1)*A1(1,IEXCIT)/ES(1)                           142.000
             C *** NOTE FOR CHANGE FROM LINE 85 TO 92 IN ISE'S PROGRAMME????       143.000
             C ***                                                                 144.000
 0101              CURR2=RM(IEXCIT,2)*A1(2,IEXCIT)/ES(2)                           145.000
 0102              CURR3=RM(IEXCIT,3)*A1(3,IEXCIT)/ES(3)                           146.000
             C VEXCIT COMPUTED FROM STEP INPUT OF TIME=20 MICROSEC,OR TAU=10 M-SEC 147.000
             C VEXCIT OBTAINED FROM FOURIER TRANSFORM FORMULA                      148.000
 0103              TAU=10.0D-6                                                     149.000
 0104              FREQ=H                                                          150.000
 0105              OTAUF=2.0D0*3.1415926535*TAU*FREQ                               151.000
             C     VEXRE=DSIN(OTAUF)/OTAUF*TAU                                     152.000
             C     VEXIM=-(1.D0-DCOS(OTAUF))/OTAUF*TAU                             153.000
             C EXPONENTIAL VOLTAGE INPUT OF DECAY RATE = 'DECAY'                   154.000
             C    VOLTAGE IN FREQUENCY SPECTRUM  1./(DECAY+ J OMEGA)               155.000
 0106               OMEGA=6.283185307D0*FREQ                                       156.000
 0107               A2W2=DECAY*DECAY+OMEGA*OMEGA                                   157.000
 0108               VEXRE=DECAY/A2W2                                               158.000
 0109               VEXIM=-OMEGA/A2W2                                              159.000
             C RPA VOLTAGE INPUT OF 0 TO 1.00 RISE TIME OF 4 MICROSEC              160.000
             C    VOLTAGE IN FREQ SPECTRUM .25*E06/OMEGA**2                        161.000
             C      VEXRE=-0.25D06/(OMEGA*OMEGA)                                   162.000
 0110               VEXRE=1.D0                                                     162.200
 0111               VEXIM=0.D0                                                     163.000
 0112              VEXCIT=DCMPLX(VEXRE,VEXIM)                                      164.000
             C                                                                     165.000
             C INPUT RESISTANCE BETWEEN GENERATOR & T.L. IS ADDED                  165.200
```

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MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)          MAIN          03-23-77        11:37:42         PAGE P004

              C    IN EXPRESSION FOR CURRENT IN EXCITED PHASE                                  165.400
              C                                                                                166.000
 0113               CURR=VEXCIT/(CURR1+CURR2+CURR3+RINPUT)                                     167.000
              C CURR1,CURR2,CURR3 ARE TERMS IN IN EQTN. 10 OF TSE'S PAPER                      168.000
 0114               DO 210 I=1,3                                                               169.000
 0115         210   CU(I)=CURR*A1(I,IEXCIT)/ES(I)                                              170.000
              C                                                                                171.000
              C                                                                                172.000
              C ****** GET RESPONSE VOLTAGE IN 3 PHASES                                        173.000
              C                                                                                174.000
 0116               DO 220 I=1,3                                                               175.000
 0117           220 VO(I)=RM(I,1)*CU(1)+RM(I,2)*CU(2)+RM(I,3)*CU(3)                            176.000
 0118               WRITE(6,51)                                                                177.000
 0119            51 FORMAT(/' THE A MATRIX = M MATRIX INVERSE * PHASE IMPEDENCE MATRIX         178.000
                   1')                                                                         178.000
 0120               WRITE(6,1)A1                                                               179.000
 0121               X=(51.7037,0.0)                                                            180.000
              C THIS IS DISTANCE FROM SENDING END                                              180.200
 0122               PRINT 14                                                                   181.000
 0123         14    FORMAT (/,'THE FOLLOWING IS THE RESPONSE AT 30 MILES',/)                   182.000
 0124               PRINT 18                                                                   183.000
 0125         18    FORMAT (/,'THE VOLTAGE IN COMPLEX FORM',/)                                 184.000
 0126               DO 140 I=1,N                                                               185.000
              C PHASE VOLTAGE DECAY EXPONENTIALLY FROM INITIAL VOLTAGE VO                      186.000
              C                                                                                187.000
              C                                                                                188.000
              C TSE'S ERROR IN ASSUMING PHASE VOLTAGE IS DECOUPLED                             189.000
              C     VP(I)=VO(I)*CDEXP(-ES(I)*X)                                                190.000
              C     VPR(I)=DREAL(VP(I))                                                        191.000
              C     VPI(I)=DIMAG(VP(I))                                                        192.000
 0127               VOR=DREAL(VO(I))                                                           193.000
 0128               VOI=DIMAG(VO(I))                                                           194.000
 0129               VP0(I,1)=DCMPLX(VOR,VOI)                                                   195.000
              C     VP0(I,1)=VO(I)                                                             196.000
              C 3 BY 1 PHASE VOLTAGE MATRIX IS FORMED                                          197.000
              C                                                                                198.000
 0130           140 CONTINUE                                                                   199.000
              C MODAL VOLTAGE MATRIX  VM0 = M..INVERSE.. * VP0                                 200.000
 0131               CALL CDMULT(RMI,VP0,VM0,3,3,1,3,3,3)                                       201.000
 0132               DO 150 I=1,3                                                               202.000
 0133               VM30(I,1)=VM0(I,1)*CDEXP(-ES(I)*X)                                         203.000
 0134           150 CONTINUE                                                                   204.000
              C PHASE VOLTAGE MATRIX VP30 = M * VM30                                           205.000
 0135               CALL CDMULT(RM,VM30,VP30,3,3,1,3,3,3)                                      206.000
              C                                                                                207.000
 0136               ES1RE=DEXP(-DREAL(ES(1))*30.D0)                                            208.000
 0137               ES1IM=-DIMAG(ES(1))*30.D0                                                  209.000
 0138               ES2RE=DEXP(-DREAL(ES(2))*30.D0)                                            210.000
 0139               ES2IM=-DIMAG(ES(2))*30.D0                                                  211.000
 0140               ES3RE=DEXP(-DREAL(ES(3))*30.D0)                                            212.000
 0141               ES3IM=-DIMAG(ES(3))*30.D0                                                  213.000
 0142               VO1RE=DREAL(VO(1))                                                         214.000
 0143               VO1IM=DIMAG(VO(1))                                                         215.000
 0144               VO2RE=DREAL(VO(2))                                                         216.000
 0145               VO3RE=DREAL(VO(3))                                                         217.000
 0146               VP1RE=DREAL(VP30(1,1))                                                     218.000
 0147               VP1IM=DIMAG(VP30(1,1))                                                     219.000
 0148               VP2RE=DREAL(VP30(2,1))                                                     220.000
```

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MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)          MAIN          03-23-77          11:37:42          PAGE P005

 0149              VP2IM=DIMAG(VP30(2,1))                                          221.000
 0150              VP3RE=DREAL(VP30(3,1))                                          222.000
 0151              VP3IM=DIMAG(VP30(3,1))                                          223.000
 0152              VO2IM=DIMAG(VO(2))                                              224.000
 0153              VO3IM=DIMAG(VO(3))                                              225.000
             C THIS IS OUTPUT TO FILE                                              226.000
 0154              WRITE(6,333)FREQ,ES1RE,ES1IM,ES2RE,FS2IM,ES3RE,ES3IM,VO1RE,VO1IM,V 227.000
                  1O2RE,VO2IM,VO3RE,VO3IM,VP1RE,VP1IM,VP2RE,VP2IM,VP3RE,VP3IM       227.000
 0155              WRITE(7,333)FREQ,VP1RE,VP1IM,VP2RE,VP2IM,VP3RE,VP3IM             227.200
             C333    FORMAT(2X,E13.6,6(E13.6,5X)/15X,6(E13.6,5X)/15X,6(E13.6,5X))  228.000
 0156        333   FORMAT(2X,E13.6,6(E13.6,5X))                                    228.200
             C150   CONTINUE                                                       229.000
             C     PRINT 23                                                        230.000
             C23    FORMAT (/,'CHECK THE CONDITION OF THE INVERSION OF MATRIX')    231.000
             C     PRINT 22,DET,COND                                               232.000
             C22    FORMAT (/,'DETERMINANT',2D13.5,5X,'CONDITION NUMBER',2D13.5,/) 233.000
             C     PRINT 26,IERROR                                                 234.000
             C26    FORMAT ('IERROR =',I4)                                         235.000
 0157              GO TO 123                                                       236.000
 0158        99    STOP                                                            237.000
 0159              END                                                             238.000
    *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
    *OPTIONS IN EFFECT*  NAME = MAIN    , LINECNT =       60
    *STATISTICS*    SOURCE STATEMENTS =       159,PROGRAM SIZE =      8324
    *STATISTICS*  NO DIAGNOSTICS GENERATED
NO ERRORS IN MAIN
```

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MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)          MYMULT          03-23-77          11:37:46          PAGE P001

     0001                SUBROUTINE MYMULT (A,B,C,D)                                         239.000
     0002                REAL*8 A(3,3),B(3,3),C(3,3),D(3,3)                                  240.000
     0003                DO 501 J=1,3                                                        241.000
     0004                DO 501 L=1,3                                                        242.000
     0005                D(J,L)=0.                                                           243.000
     0006                DO 501 I=1,3                                                        244.000
     0007                C(J,L)=A(J,I)*B(I,L)                                                245.000
     0008          501   D(J,L)=D(J,L)+C(J,L)                                                246.000
     0009                RETURN                                                              247.000
     0010                END                                                                 248.000
      *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
      *OPTIONS IN EFFECT*  NAME = MYMULT   , LINECNT =      60
      *STATISTICS*    SOURCE STATEMENTS =       10,PROGRAM SIZE =       620
      *STATISTICS*  NO DIAGNOSTICS GENERATED
     NO ERRORS IN MYMULT

     NO STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.

     NAME        NUMBER OF ERRORS/WARNINGS   SEVERITY

     MAIN                   0                    0
     MYMULT                 0                    0
     EXECUTION TERMINATED


     $R -LOAD+*NUMLIB 6=*A  7=TFREIMJAPAN 2=TSEDATAJAPAN
     EXECUTION BEGINS

     END OF FILE ON TSEDATAJAPAN CAUSES A RETURN TO MTS.
     EXECUTION TERMINATED

     $L TFREIMJAPAN(1,40)
         1     0.100000E-15 0.999845E+00   -0.643220E-04   0.896559E-01  -0.574022E-05   0.370580E-01  -0.235636E-05
         2     0.100000E+00 0.594561E+00   -0.566883E-01   0.144887E+00   0.683947E-02   0.106917E+00   0.644839E-02
         3     0.112202E+00 0.590340E+00   -0.559751E-01   0.145424E+00   0.606116E-02   0.107401E+00   0.557660E-02
         4     0.125892E+00 0.586186E+00   -0.552438E-01   0.145862E+00   0.527251E-02   0.107812E+00   0.470039E-02
         5     0.141254E+00 0.582084E+00   -0.544934E-01   0.146272E+00   0.447600E-02   0.108134E+00   0.381465E-02
         6     0.158489E+00 0.578039E+00   -0.537236E-01   0.146561E+00   0.367124E-02   0.108378E+00   0.292890E-02
         7     0.177828E+00 0.574060E+00   -0.529400E-01   0.146822E+00   0.286725E-02   0.105543E+00   0.204737E-02
         8     0.199526E+00 0.570146E+00   -0.521405E-01   0.146989E+00   0.206480E-02   0.108626E+00   0.117102E-02
         9     0.223872E+00 0.566293E+00   -0.513250E-01   0.147085E+00   0.126842E-02   0.108633E+00   0.305765E-03
        10     0.251189E+00 0.562508E+00   -0.504983E-01   0.147104E+00   0.480995E-03   0.108559E+00  -0.545248E-03
        11     0.281838E+00 0.558787E+00   -0.496574E-01   0.147057E+00  -0.293398E-03   0.108407E+00  -0.137966E-02
        12     0.316228E+00 0.555137E+00   -0.488066E-01   0.146940E+00  -0.105273E-02   0.108185E+00  -0.219140E-02
        13     0.354813E+00 0.551557E+00   -0.479454E-01   0.146755E+00  -0.179329E-02   0.107892E+00  -0.297919E-02
        14     0.398107E+00 0.548048E+00   -0.470753E-01   0.146505E+00  -0.251229E-02   0.107529E+00  -0.373926E-02
        15     0.446684E+00 0.544612E+00   -0.461970E-01   0.146192E+00  -0.320811E-02   0.107099E+00  -0.447078E-02
        16     0.501187E+00 0.541240E+00   -0.453172E-01   0.145826E+00  -0.387580E-02   0.106615E+00  -0.516887E-02
        17     0.562341E+00 0.537946E+00   -0.444325E-01   0.145405E+00  -0.451601E-02   0.106070E+00  -0.583428E-02
        18     0.630957E+00 0.534731E+00   -0.435439E-01   0.144924E+00  -0.512769E-02   0.105469E+00  -0.646219E-02
        19     0.707946E+00 0.531592E+00   -0.426553E-01   0.144399E+00  -0.570863E-02   0.104818E+00  -0.705368E-02
        20     0.794328E+00 0.528526E+00   -0.417709E-01   0.143832E+00  -0.625573E-02   0.104125E+00  -0.760805E-02
        21     0.891251E+00 0.525538E+00   -0.408921E-01   0.143221E+00  -0.677090E-02   0.103392E+00  -0.812162E-02
        22     0.100000E+01 0.522627E+00   -0.400207E-01   0.142575E+00  -0.725191E-02   0.102624E+00  -0.859802E-02
```

APPENDIX 2

PHASE SMOOTHING PROGRAM LISTINGS

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)          MAIN          03-23-77          11:45:13          PAGE P001

        CC    PROGRAM FOURIER(INPUT,OUTPUT)
              REAL*8 A(1500),PHI(1500),U(1000),FREQ,TEXT(10)
              REAL*8 C2,S2,OMEGA,PHINEW,PHIOLD,H,S1,C1,FLOATN,T
              REAL*8 OMDEC,STEIG,UOLD,UI,AW,AWD,BW,ANEW,AOLD,DIFF,S,A1,W1
               REAL*8 TFRE(3),TFIM(3)
               REAL*8 XTF(3),PTF(3)
              REAL*8 DECAY,VOUTO,DT,TRISE,DTIN,POUTO
              STEIG=0.D0
              W1=0.D0
              A1=0.D0
                READ(2,111)I
                WRITE(6,110)I
            3 CONTINUE
                 READ(3,33)FREQ,TFRE(1),TFIM(1),TFRE(2),TFIM(2),TFRE(3),TFIM(3)
              OMEGA=6.28318530700*FREQ
           33 FORMAT(2X,E13.6,6(E13.6,5X))
        C       I=2
          110 FORMAT(///10X,'PHASE EXCITED',I3)
          111 FORMAT(I3)
        C 'I' SPECIFIED WHICH PHASE TO BE SMOOTHED
               AW=TFRE(I)
               BW=TFIM(I)
              S1=DSQRT(AW*AW+BW*BW)
              S2=DARCOS(AW/S1)
              IF(BW.LT.0.D0) S2=6.2831853071795B-S2
              C1=OMEGA-W1
              AKP=(STEIG*C1-S2+A1)/6.283185307
              AKP=AKP+SIGN(0.5,AKP)
              KP=AKP
              AKP=KP
              S2=AKP*6.28318530717958D0+S2
              STEIG=(S2-A1)/C1
              A1=S2
              W1=OMEGA
        C      WRITE(7,33)HERTZ,XTF(1),PTF(1),XTF(2),PTF(2),XTF(3),PTF(3)
                WRITE(7,33)FREQ,TFRE(I),TFIM(I),S1,S2
           54 FORMAT(E13.5,2E12.3,E15.5,E18.5,E17.5,E18.5)
               GO TO 3
              END
     *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
     *OPTIONS IN EFFECT*  NAME = MAIN    , LINECNT =      60
     *STATISTICS*    SOURCE STATEMENTS =        35,PROGRAM SIZE =    33226
     *STATISTICS*  NO DIAGNOSTICS GENERATED
   NO ERRORS IN MAIN

   NO STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.
   EXECUTION TERMINATED

   $R -LOAD 7=TFJAPAN1  2=*SOURCE*   3=TFREIMJAPAN(2)   6=*SINK*
   EXECUTION BEGINS

             PHASE EXCITED  1
```

APPENDIX 3

FOURIER TRANSFORMATION PROGRAM LISTINGS

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)          MAIN          03-23-77          12:03:25          PAGE P001

CC    PROGRAM FOURIER(INPUT,OUTPUT)
      REAL*8 A(1500),PHI(1500),U(1000),FREQ(200),TEXT(10)
      REAL*8 C2,S2,OMEGA,PHINEW,PHIOLD,H,S1,C1,FLOATN,T
      REAL*8 OMDEC,UOLD,UI,AW,AWD,BW,ANEW,AOLD,DIFF,S,A1,W1
      REAL*8 DECAY,VOUTO,DT,TRISE,DTIN,POUTO,TFRE(1500),TFIM(1500)
      REAL*8 ALPHA,ALPHA1,ALPHA2,A12,VT1(1000)
      REAL*8 TOUTS,TARRAY(1000),DTOUT2,DTIN2
      REAL*8 AIN,BIN,TTIN,TFIN,TINPUT,FLOATN
      CALL FTNCMD('SET MINUSZERO=ON', 16)
       TRISE=5.D-06
    5 READ(2,10) KOPT,IB,IZ,OMIN,DT,TMAX,(TEXT(I),I=1,10)
      IEXP=1
      WRITE(6,10)KOPT,IB,IZ,OMIN,DT,TMAX
   10 FORMAT(3I3,1X,3E10.3,10A4)
      IF(IB.EQ.0) STOP
      WRITE(6,6)(TEXT(I),I=1,10)
    6 FORMAT(1H0,10A4)
      IW=IB*IZ+1
C INPUT FOR 1/4000 MS INCIDENT WAVE
       AIN=.6800
       BIN=3.2700
       TFIN=1.D-06
       TTIN=4000.D-06
       ALPHA1=AIN/TTIN
       ALPHA2=BIN/TFIN
       WRITE(6,800)ALPHA1,ALPHA2
  800 FORMAT(8X,'INPUT VOLTAGE TIME CONSTANTS: ALPHA1=',E13.5,' ALPHA2='
     1,E13.5)
      IF(IEXP.EQ.1)ALPHA=ALPHA1
      IF(IEXP.EQ.2)ALPHA=ALPHA2
C CHANGE FOLLOWING TWO STATEMENTS IF DIMENSION IS CHANGED **************
       WRITE(6,816)ALPHA
  816 FORMAT(//'***TIME CONSTANT IN DECAY = ',E13.5,'*****')
      IF(IW.GT.1500) GO TO 97
      IF(IZ.GT.200) GO TO 97
      S1=IZ
      H=0.D0
      S2=2.302585092994D0/S1
      DO 11 K=1,IZ
      FREQ(K)=DEXP(H*S2)
   11 H=H+1.D0
C READING AMPLITUDE AND PHASE SPECTRA OR PRESETTING THEM ***************
      IF(KOPT.EQ.1) GO TO 14
C
C
C T.F. IS READ IN
C
C
C
      DO 12 K=2,IW
      READ(3,33) A(K),PHI(K)
C      A(K)=1.D0
C      PHI(K)=0.D0
C THIS INPUT THE TRANSFER FUNCTION EXP(-GAMMA*L) AND -BETA*L
   33 FORMAT(51X,2(E13.6,5X))
      IF(IBLANK(A(K)).LT.0) GO TO 200
   13 FORMAT(2E10.0)
   12 CONTINUE
```

```
0045      18 READ (2,13)H
0046         IF(IBLANK(H).LT.0) GO TO 16
0047         GO TO 18
0048      14 DO 15 K=1,IW
0049         A(K)=1.D0
0050      15 PHI(K)=0.D0
        C READING INPUT FUNCTION ***********************************************
0051      16 H=0.D0
0052         IF(KOPT.EQ.3) GO TO 24
        C THIS IS SKIPPED FOR OPTION 3
0053         S1=0.D0
        C S1 IS FIRST INPUT VOLTAGE VALUE
        C    U(1)=S1
0054         N=1
0055         DTIN=.05D-06
0056         TINPUT=-DTIN
0057         WRITE(6,809)DTIN
0058     809 FORMAT(' INPUT TIME STEP FROM 0 TO 5 MS =',E13.4)
0059      17 IF(N.GT.200)GO TO 22
0060         FLOATN=N
0061         TINPUT=TINPUT+DTIN
0062         TARRAY(N)=TINPUT
0063         S2=(ALPHA1+ALPHA2)/(ALPHA1-ALPHA2)
        C ALPHA12 INPUT VOLTAGE IS SET TO POSITIVE
0064         A12=-S2
0065     819 FORMAT(//'(ALPHA1+ALPHA2)/ALPHA1-ALPHA2)= ',E14.5)
0066         S2=S2*(DEXP(-ALPHA1*TINPUT)-DEXP(-ALPHA2*TINPUT))
        C INPUT VOLTAGE S2  IS PRESET INSTEAD OF READ-IN
        C    S2=S2*DEXP(-ALPHA*TINPUT)
        C    S2=FLOATN*DTIN/TRISE
        C    N=N+1
0067         IF(KOPT.EQ.7)GO TO 97
0068         H=(S1+S2)*DTIN/2.+H
0069         S1=S2
0070         U(N)=S2
0071         N=N+1
0072         GO TO 17
0073      22 IF(N.LT.2) GO TO 95
0074         TMAXIN=TINPUT
0075         N=N-1
0076         TIN2=TMAXIN/2.
0077         WRITE(6,801)H
0078     801 FORMAT('  TIME-VOLTAGE AREA OF INPUT = ',E15.5)
        C FIRST 100 ENTRY OF INPUT VOLTAGE IS PRINTED OU
        C THE VOLTAGE INPUT AT ZERO FREQ IS F(W=0)=TIME OF STEP = H
0079         WRITE(6,819)A12
0080         WRITE(6,72) (U(I),I=1,N)
        C    VOUT0=1.D0/DECAY
0081         WRITE(6,811)
0082     811 FORMAT(///'TIME OF INPUT VOLTAGE')
0083         WRITE(6,812)(TARRAY(II),II=1,200)
0084     812 FORMAT(10E11.3)
0085      55 FORMAT(////'OUTPUT VOLTAGE IN FREQ DOMAIN AT FREQ=0, ',2E15.5)
0086         H=1.0/H
        C TRANSFORM INPUT FUNCTION FROM TIME TO FREQUENCY DOMAIN ***************
0087      24 WRITE(6,32)
0088      32 FORMAT('0',24X,'INPUT FUNCTION IN FREQUENCY DOMAIN',19X,'TRANSFER
            1FUNCTION',/,' FREQUENCY(HZ) REAL V    IMAGINARY AMPLITUDE(ABSOLUTE
```

```
                   2) PHASE(RADIANS) AMPLITUDE(ABSOLUTE) ANGLE(RADIANS)',3X,'OUTPUT')
0089          STEIG=0.
0090          W1=0.D0
0091          A1=0.D0
0092          OMDEC=OMIN*6.283185307179600
0093          IP=0
        C PRINTOUT OF FREQUENCY ZERO ENTRY
0094          C1=1.0D0
        C     WRITE(6,54)W1,C1,H,W1,A(1),PHI(1)
0095          ISTORE=2
0096       36 IP=IP+1
0097          IF(IP.GT.IB) GO TO 57
0098          K=0
0099       38 K=K+1
0100          IF(K.GT.IZ) GO TO 44
0101          OMEGA=FREQ(K)*OMDEC
0102          HERTZ=OMEGA/6.283185307179600
0103          C1=1.0D0
0104          IF(KOPT.EQ.3) GO TO 220
0105          S1=0.D0
0106          AW=0.D0
0107          BW=0.D0
0108          IF(KOPT.EQ.1)GO TO 815
0109          IOLD=1
0110          UOLD=U(1)
0111          DO 48 I=2,N
0112          UI=U(I)
0113          T=IOLD
        C     C2=OMEGA*T*DT
0114           C2=OMEGA*TARRAY(I)
0115          S2=DSIN(C2)
0116          C2=DCOS(C2)
0117          T=(UI-UOLD)/(DT*OMEGA)
0118          AW=(C2-C1)*T+UI*S2-UOLD*S1+AW
0119          BW=UI*C2-(S2-S1)*T-UOLD*C1+BW
0120          S1=S2
0121          C1=C2
0122          UOLD=UI
0123       48 IOLD=I
0124          AW=AW/OMEGA
0125          BW=BW/OMEGA
        C
        C
        C AW AND BW ARE NOW OUTPUT VOLTAGE IN FREQ DOMAIN
        C
        C     READ(3,34)HERTZ
0126      815 CONTINUE
        C
        C
        C
0127          AW=A12*ALPHA1/(ALPHA1*ALPHA1+OMEGA*OMEGA)
0128           AW=AW-A12*ALPHA2/(ALPHA2*ALPHA2+OMEGA*OMEGA)
0129          BW=-A12*OMEGA/(ALPHA1*ALPHA1+OMEGA*OMEGA)
0130           BW=BW+A12*OMEGA/(ALPHA2*ALPHA2+OMEGA*OMEGA)
0131          OMEGA=6.283185307179600*HERTZ
        C     READ(3,33)AW,BW
0132       34 FORMAT(2X,E13.6)
0133          S1=DSQRT(AW*AW+BW*BW)
```

```
0134            S2=DARCOS(AW/S1)
0135            IF(BW.LT.0.D0) S2=6.28318530717958-S2
0136            C1=OMEGA-W1
0137            AKP=(STEIG*C1-S2+A1)/6.283185307
0138            AKP=AKP+SIGN(0.5,AKP)
0139            KP=AKP
0140            AKP=KP
0141            S2=AKP*6.28318530717958D0+S2
0142            STEIG=(S2-A1)/C1
0143            A1=S2
0144            W1=OMEGA
0145            C1=S1/TMAXIN
          C     C1=S1
          C  AMPLITUDE OF VOLTAGE OBTAINED BY FORMULA AND PROGRAMME ARE PRINTED
          C OUT AS C1 AND S1
          C  53 ISTORE=ISTORE+1
          C ISTORE INCREMENTAL SHIFTED TO LINE 147
          C     WRITE(6,54) HERTZ,AW,BW,S1,S2,A(ISTORE),PHI(ISTORE)
          C A & PHI ARE STILL TRANSFER FUNCTION AS READ IN FROM INPUT FILE
0146            XTF=A(ISTORE)
0147            PTF=PHI(ISTORE)
0148        54 FORMAT(E13.5,2E12.3,E15.5,E13.5,E17.5,E13.5,3X,2E13.5)
0149            A(ISTORE)=S1*A(ISTORE)
0150            PHI(ISTORE)=S2+PHI(ISTORE)
          C A & PHI NOW BECOMES OUTPUT VOLTAGE & PHASE AS AFTER #171
0151            WRITE(6,54)HERTZ,AW,BW,S1,S2,XTF,PTF,A(ISTORE),PHI(ISTORE)
0152        53   ISTORE=ISTORE+1
          C
0153            GO TO 38
0154        44 OMDEC=OMDEC*10.D0
0155            GO TO 36
          C  TRANSFER  OUTPUT FUNCTION FROM FREQUENCY TO TIME DOMAIN *************
0156        57 CONTINUE
          C
          C DTOUT IS DELTA T OF OUTPUT
0157            DTOUT=.25D-06
0158            TOUT1=260.E-6
0159            T=TOUT1
0160            WRITE(6,58) DTOUT,T
0161        58 FORMAT('0OUTPUT FUNCTION V2(T), DELTAT=',E12.5,',INITIAL TIME = ',
               1E12.5)
          C      TF0=0.999803
          C
          C
          C T.F. AT ZERO FREQ
          C
          C
0162             TF0=.0896522
          C      TF0=.0370565
          C GALLOWAY'S FORMULA ON 'SKIN' DECREASE T.F.
          C
          C
0163             A(1)=TF0*A12*(1.D0/ALPHA1-1.D0/ALPHA2)
0164            PHI(1)=0.D0
0165             WRITE(6,55)A(1),PHI(1)
0166            KT=0
0167        63 AW=0.D0
0168            BW=0.D0
```

```
0169          KT=KT+1
        C CHANGE FOLLOWING STATEMENT IF DIMENSION IS CHANGED *******************
0170          IF(KT.GT.1500) GO TO 97
0171          S1=DSIN(PHI(1))
0172          C1=DCOS(PHI(1))
0173          IP=0
0174          ISTORE=1
0175          OMDEC=OMIN*6.283185307179586
0176          AOLD=A(ISTORE)
0177          PHIOLD=PHI(ISTORE)
0178       64 IP=IP+1
0179          IF(IP.GT.IB) GO TO 70
0180          K=0
0181       66 K=K+1
0182          IF(K.GT.IZ) GO TO 68
0183          OMEGA=FREQ(K)*OMDEC
0184          ISTORE=ISTORE+1
0185          PHINEW=PHI(ISTORE)
0186          ANEW=A(ISTORE)
0187          DIFF=1.D0/(OMEGA-BW)
0188          S=1.D0/(T+(PHINEW-PHIOLD)*DIFF)
0189          C2=OMEGA*T+PHINEW
0190          IF(C2.GT.0.8E06)C2=C2-628318.530717959D0
0191          IF(C2.GT.0.8E06)C2=C2-628318.530717959D0
0192          IF(C2.GT.0.8E06)C2=C2-628318.530717959D0
0193          S2=DSIN(C2)
0194          C2=DCOS(C2)
0195          BW=ANEW*S2-AOLD*S1+(ANEW-AOLD)*(C2-C1)*DIFF*S
0196          AW=BW*S+AW
0197          S1=S2
0198          C1=C2
0199          BW=OMEGA
0200          PHIOLD=PHINEW
0201          AOLD=ANEW
0202          GO TO 66
0203       68 OMDEC=OMDEC*10.D0
0204          GO TO 64
0205       70 IF(KT.GT.100)DTOUT=DTOUT
0206           KT1=KT+1
0207          I=I+DTOUT
0208          AW=AW*0.318309886
0209          U(KT)=AW
0210          IF(I.LE.IMAX) GO TO 63
        C     WRITE(6,72) (U(I),I=1,KT)
0211           WRITE(6,807)TOUT1
0212      807 FORMAT(///'  TIME STEP IN OUTPUT IS SAME AS INPUT DELAYED BY ',E13
             1.3,' SEC')
0213          WRITE(7,72)(U(I),I=1,KT)
0214       72 FORMAT(1X,10E12.4)
0215          GO TO 5
0216       95 WRITE(6,96)
0217       96 FORMAT('0INPUT FUNCTION MISSING OR WRONG NUMBER OF CARDS FOR SPECT
             1RA.JOB TERMINATED')
0218          GO TO 100
0219       97 WRITE(6,98)
0220       98 FORMAT('0STORAGE LIMIT EXCEEDED FOR SUBSCRIPTED VARIABLE. JOB TERM
             1INATED')
0221      100 STOP
```

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(41336)            MAIN          03-23-77        12:03:25        PAGE P006

      0222          200 DO 210 I=K,IW                                                    265.000
      0223                A(I)=0.D0                                                      266.000
      0224          210 PHI(I)=0.D0                                                      267.000
      0225                GO TO 16                                                       268.000
      0226          220 S1=1.D0                                                          269.000
      0227                S2=0.D0                                                        270.000
      0228                GO TO 53                                                       271.000
      0229                END                                                            272.000
    *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
    *OPTIONS IN EFFECT*  NAME = MAIN    ,  LINECNT =      60
    *STATISTICS*    SOURCE STATEMENTS =        229,PROGRAM SIZE =     79864
    *STATISTICS*  NO DIAGNOSTICS GENERATED
  NO ERRORS IN MAIN

  NO STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.
  EXECUTION TERMINATED


  $R -LOAD 2=*SOURCE* 6=*SINK*  7=ENDALPHA128            3=TFJAPAN2
  EXECUTION BEGINS
   2  9 20  0.100E+00 0.500E-06 0.450E-03

    JAPAN LINE 1/4000 MS INPUT
           INPUT VOLTAGE TIME CONSTANTS: ALPHA1=  0.17000E+03 ALPHA2=  0.32700E+07


  ***TIME CONSTANT IN DECAY =   0.17000E+03*****
  INPUT TIME STEP FROM 0 TO 5 MS =    0.5000E-07
   TIME-VOLTAGE AREA OF INPUT =     -0.96361E-05


  (ALPHA1+ALPHA2)/ALPHA1-ALPHA2)=     0.10001E+01
   -0.0         -0.1508E+00 -0.2789E+00 -0.3877E+00 -0.4801E+00 -0.5585E+00 -0.6251E+00 -0.6816E+00 -0.7296E+00 -0.7704E+00
   -0.8050E+00 -0.8344E+00 -0.8594E+00 -0.8806E+00 -0.8986E+00 -0.9139E+00 -0.9269E+00 -0.9379E+00 -0.9472E+00 -0.9552E+00
   -0.9619E+00 -0.9676E+00 -0.9725E+00 -0.9766E+00 -0.9801E+00 -0.9831E+00 -0.9856E+00 -0.9878E+00 -0.9896E+00 -0.9911E+00
   -0.9924E+00 -0.9935E+00 -0.9945E+00 -0.9953E+00 -0.9960E+00 -0.9965E+00 -0.9970E+00 -0.9974E+00 -0.9978E+00 -0.9981E+00
   -0.9983E+00 -0.9985E+00 -0.9987E+00 -0.9989E+00 -0.9990E+00 -0.9991E+00 -0.9992E+00 -0.9992E+00 -0.9993E+00 -0.9994E+00
   -0.9994E+00 -0.9994E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00
   -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00
   -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9995E+00 -0.9994E+00 -0.9994E+00 -0.9994E+00
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   -0.9993E+00 -0.9993E+00 -0.9993E+00 -0.9993E+00 -0.9993E+00 -0.9993E+00 -0.9993E+00 -0.9993E+00 -0.9993E+00 -0.9993E+00
   -0.9993E+00 -0.9992E+00 -0.9992E+00 -0.9992E+00 -0.9992E+00 -0.9992E+00 -0.9992E+00 -0.9992E+00 -0.9992E+00 -0.9992E+00
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   -0.9988E+00 -0.9988E+00 -0.9988E+00 -0.9988E+00 -0.9988E+00 -0.9988E+00 -0.9988E+00 -0.9988E+00 -0.9988E+00 -0.9988E+00
   -0.9987E+00 -0.9987E+00 -0.9987E+00 -0.9987E+00 -0.9987E+00 -0.9987E+00 -0.9987E+00 -0.9987E+00 -0.9987E+00 -0.9987E+00
   -0.9987E+00 -0.9987E+00 -0.9986E+00 -0.9986E+00 -0.9986E+00 -0.9986E+00 -0.9986E+00 -0.9986E+00 -0.9986E+00 -0.9986E+00
   -0.9986E+00 -0.9986E+00 -0.9986E+00 -0.9985E+00 -0.9985E+00 -0.9985E+00 -0.9985E+00 -0.9985E+00 -0.9985E+00 -0.9985E+00
   -0.9985E+00 -0.9985E+00 -0.9985E+00 -0.9985E+00 -0.9985E+00 -0.9984E+00 -0.9984E+00 -0.9984E+00 -0.9984E+00 -0.9984E+00


  TIME OF INPUT VOLTAGE
   0.0         0.500E-07  0.100E-06  0.150E-06  0.200E-06  0.250E-06  0.300E-06  0.350E-06  0.400E-06  0.450E-06
```

BIBLIOGRAPHY

1. T. Ametani, "Single Propagation on a 500kV Untransposed Line: Field Test Results", CIGRE WG 13-05 13-74 (W.G. 05) 36 IWD, Oct. 1974.

2. T. Ametani et al, "Surge Propagation on Japanese 500 kV Untransposed Transmission Line", Proc. IEE, Vol. 121, pp. 136-138, 1974.

3. A. Greenwood, "Electrical Transients in Power Systems" Wiley Interscience, 1971.

4. H.W. Dommel, "Line constants of Overhead Lines: User's Manual", Method Analysis Group - BPA U.S. Department of Interior, June 1974.

5. H.W. Dommel, private communication, 1976.

6. J.R. Carson, "Wave Propagation in Overhead Wires with Ground Return", Bell System Technical Journal, Vol. 5, pp. 539-554, 1926.

7. S. Butterworth, "Electrical Characteristics of Overhead Lines", Electrical Research Association, Techn. Ref. O/T4, 1954.

8. D.E. Hedman, "Propagation on Overhead Transmission Lines II-Earth Conduction Effects and Practical Results", IEEE Trans. PAS, Vol. 84, pp. 205-211, March 1961.

9. W.C. Johnson, "Transmission Lines and Networks", McGraw-Hill, 1950.

10. A.E. Kennedy et al, "Experimental Research on Skin Effect in Conductor" AIEE Trans., pt. II, Vol. 34, pp. 1953-2018, 1915.

11. "Handbook of Mathematical Functions", Edited by M. Abramowitz and I.A. Stegun, published by U.S. Dept. of Commerce, pp. 384-385, 1964.

12. R.H. Galloway et al, "Calculation of Electrical Parameters for Short and Long Polyphase Transmission Lines", Proc. IEE, Vol. 111, No. 12, pp. 2051-2059, 1964.

13. H.W. Dommel, J.H. Sawada, "Report on the Calculation of Induced Voltages and Currents on Pipeline Adjacent to A.C. Power Lines" B.C. Hydro and Authorities CEA Contract No. 75-02, Aug. 1976.

14. H.W. Dommel, E.E. 463 and E.E. 553 Class Notes, U.B.C., 1976-77.

15. M.H. Hesse, "Electromagnetic and Electrostatic Transmission Line Parameter by Digital Computer", IEEE Trans. PAS, Vol. 82, pp. 282-290, 1963.

16. K.K. Tse, Private Communication, 1976.

17. D.F. Hedman, "Propagation on Overhead Transmission Lines I - Theory of Modal Analysis", IEEE Trans., PAS, Vol. 84, pp. 200-205, March 1965.

18. A. Budner, "Introduction of Frequency - Dependent Line Parameters into an Electromagnetic Transient Programme", IEEE Trans. PAS, Vol. 89, pp. 88-97, Jan. 1970.

19. W.G. Campbell, "Linear Algebra with Applications", Appleton Century Crofts, 1971.

20. U.B.C. Computing Centre Programme Library. "UBC MATRIX", Sept, 1976.

21. R.P. Gregory, "A Collection of Matrices for Testing Computational Algorithm", Wiley - Interscience, 1969.

22. F. Ayres, "Theory and Problems of Differential Equations", McGraw-Hill (Schaum Series)1952.

23. E.W. Kimbark, "Electrical Transmission of Power and Signal", Wiley, 1958.

24. W.D. Stevenson, "Elements of Power System Analysis", McGraw-Hill, 1975.

25. R.K. Moore, "Travelling Wave Engineering", McGraw-Hill, 1960.

26. E. Groschupf "Simulation Transienter Vorgäange auf Leitungssystemen der Hochspannungs - Gleichstrom - und- Drehstrom - Übertragung", Ph.D. Thesis, Technischen Hoschschule Darmstadt, 1975.

27. H.W. Dommel, "User's Instruction for Fourier Transformation Programme", Sept. 1976.

28. L. Lathi, "Signals, Systems and Communication", John Wiley & Sons, 1968.

29. W.S. Meyer, H.W. Dommel, "Numerical Modelling of Frequency-Dependent Transmission Line Parameters in an Electromagnetic Transient Programme", IEEE Trans. PAS, Vol. 74, pp. 1401-1409, 1974.

30. G.W. Swift, E.E. 571 Class Notes, 1977.

31. G.E.C. Measurements, "Protective Relays Application Guide", Tinlings Limited, 1976.

32. P.T.B. Adams et al. "Lightning Surges Propagations in 550 kV SF6 Switchgear and Bus at B.C. Hydro Mica Generating Station", presented at the CEA Spring Meeting, Montreal, Quebec, March, 1977.