PROPAGATION OF THE WAVE FRONT ON
UNTRANSPOSED OVERHEAD AND UNDERGROUND TRANSMISSION LINES

by

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ABSTRACT

The propagation of the switching surge wave front on multiphase power lines was investigated by modal analysis and conventional Fourier Transformation. A 500 kV untransposed, three-phase transmission line, for which field test results were available, was chosen as a test case.

Phase A of this test line was excited from a double exponential voltage source and the voltage response at the receiving end was calculated and measured in all three phases. The calculated voltage arrival time matched closely the measured value, and was very close to the time taken by electromagnetic waves in air at a speed of 0.3 km/μs. The calculated voltage response curves also came close to the measured results (errors within 8%).
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INTRODUCTION

The attenuation and distortion of wave fronts on single circuit multiphase transmission lines or underground cables was investigated. The purpose of this work was useful for surge insulation coordination study. The solution methods were applied to the specific case of a 500 kV untransposed overhead line, for which test results were available.1,2* This line is part of the 500 kV Azumi Trunk transmission link of the Tokyo Electric Power System in Japan. In the field tests, the sending end of the line was energized with a double exponential surge wave of the form \( v(t) = k(e^{-\alpha_1 t} - e^{-\alpha_2 t}) \), as a representation of surge phenomenon on a line eg. lightning surge, from an impulse generator3 through a series resistance of 415Ω. In the computer simulation, this double exponential input wave as well as other forms of input voltages, such as single exponential decay wave, triangular waves, step wave and delta wave were also studied.

The way in which computer programmes were used for the analysis in this thesis is illustrated in Fig. 1. The Line Constants Program (LCP) was first used to give the distributed line parameters from the tower geometry and conductor characteristics as a function of frequency. Then, the Transfer Function Program (TFP) was used to obtain the output voltage at the receiving end (83.212 km from sending end) for all frequency points. After the transfer functions at discrete frequency points were obtained, the Fourier Transform Program (FTP) was used to find the voltage at the receiving end as a function of time for any form of input voltage.

In the Fourier Transform Program, linear interpolation between

* The superscripts denote reference numbers in the bibliography.
successive data points were used in the time domain as well as in the frequency domain. For the 500 kV line used as an example, a density of 20 points per decade on the frequency scale gave sufficiently accurate results.

The computation of the voltage response in three phases at the receiving end with any one of the phases energized at the sending end would take about 80 s. CPU time on the UBC IBM 370/168 computer system at a cost of approximately CC$30. For a more general case of three input voltages on all three phases, only a slight modification in the Transfer Function Program would be required to obtain the results.

Fig. 1. Overall scheme of program used.
CHAPTER I

COMPUTATION OF LINE CONSTANTS

1. Introduction

The program which was used for the calculation of line parameters is a modified version of the Line Constants Program written by H. W. Dommel. It calculates the frequency-dependent series impedance matrix and the constant shunt capacitance matrix for overhead lines from the given tower geometry and conductor characteristics at specified frequency points. For the analysis of underground cables, this program would have to be replaced by a cable constants program. The value of the impedance and capacitance matrices is needed for the Transfer Function Program to obtain the transfer functions at the specified frequency points.

2. Transmission Line Data

The transmission line used as an example for this simulation study is part of the 500 kV Azumi-Trank transmission link of the Tokyo Electric Power System in Japan. This is a three-phase untransposed line with two ground wires. Each phase is a bundle conductor with four steel-reinforced aluminum cables (see Fig. 2). The tower geometry and conductor characteristics are listed in Table 1. The conductor characteristics were taken from the German standard DIN 48204 because they were not defined in enough detail in the description of the field tests.
Ground resistivity = 200 $\Omega \cdot$m

a) Tower geometry (height is average height above ground, not height at tower location).

b) Bundle conductors of each phase

c) 26 Al./7st. Steel-reinforced aluminum cable used for ground wires and phase conductors

Fig. 2 Transmission line geometry
| **TABLE I**  
| **TRANSMISSION LINE DATA** |
| General data |
| **Length of transmission line** | = 83.212 km |
| **Average height above ground of three phase conductors (flat configuration)** | = 25 m |
| **Average height above ground of ground wires** | = 35 m |
| **Earth resistivity (presumably farmland)** | = 200 Ω·m |
| **Resistivity of aluminum** | = $3.21 \times 10^{-8} \Omega\cdot m$ |
| **Relative permeability of aluminum** ($\mu_r$) | = 1.0 |
| **Permeability of aluminum** ($\mu_0\mu_r$) | = $4\pi \times 10^{-8} H/m$ |
| Details for ground wire |
| Type: Steel-reinforced aluminum cable, as shown in Fig. 2c. |
| **Total no. of aluminum strands** | = 26 |
| **No. of aluminum strands in outer layer of conductor** | = 16 |
| **Steel core diameter** | = 5.85 mm |
| **Outside diameter of conductor** | = 15.7 mm |
| **D.C. resistance at 20°C** | = 0.262 Ω/km |
| Details of phase conductor |
| Type: Steel-reinforced aluminum cable as shown in Fig. 2c, with conductors in each phase as shown in Fig. 2b |
| **Total no. of aluminum strands** | = 26 |
| **No. of aluminum strands in outer layer of conductor** | = 16 |
| **Steel core diameter** | = 8.1 mm |
| **Outside diameter of conductor** | = 21.7 mm |
| **D.C. resistance at 20°C** | = 0.136 Ω/km |
3. Line Parameter Calculation

(i) Series impedance matrix - Carson's formula \(^6,2\) is used for calculating the impedances of the conductor earth return loops. Earth conductivity is assumed to be uniform and the earth plane is assumed to be flat and parallel to the conductors. Also, spacings between conductors are assumed to be large compared with conductor radii, that is, proximity effects are ignored. The elements of the impedance matrix \([Z_{ij}]\) are given as

\[
Z_{ii} = (R_{ii} + \Delta R_{ii}) + j(2\omega 10^{-4} \ln \frac{2h_i}{\text{GMR}_i} + \Delta X_{ii}) \Omega/\text{km}
\]

\[i = 1, \ldots N\]  

(1-1)

and \[Z_{ij} = Z_{ji}\]

\[
Z_{ij} = \Delta R_{ij} + j(2\omega 10^{-4} \ln \frac{s_{ij}}{s_{ij}} + \Delta X_{ij}) \Omega/\text{km}
\]

\[j = 1, \ldots N; \ i = 1, \ldots N; \ i \neq j,\]  

(1-2)

where \(R_{ii}\) = resistance of \(i^{th}\) conductor in \(\Omega/\text{km}\) (see section 4. on skin effect),

\(h_i\) = average height above ground of \(i^{th}\) conductor in \(\text{m}\),

\(s_{ij}\) = distance between \(i^{th}\) conductor and ground image of \(j^{th}\) conductor in \(\text{m}\) (see Fig. 3),

\(s_{ij}\) = distance between \(i^{th}\) and \(j^{th}\) conductors in \(\text{m}\)

(see Fig. 3),

\(\text{GMR}_i\) = geometric mean radius of \(i^{th}\) conductor in \(\text{m}\),

\(\omega\) = angular frequency,

\(\Delta R\) = correction terms in resistance for earth return effect,

\(\Delta X\) = correction terms in resistance for earth return effect.

Carson's correction terms \(\Delta R\) and \(\Delta X\) are functions of the angle.
Fig. 3 Line parameter calculation
\( \phi_{ij} \) (see Fig. 3) and of the parameter

\[
a = ks \sqrt{\frac{E}{\rho}}
\]

where \( k = 4\pi \sqrt{\frac{5}{8}} \times 10^{-4} \)

\[
\begin{cases}
2b_1 & \text{for self impedance} \\
S_{ij} & \text{for mutual impedance}
\end{cases}
\]

\( \rho = \) earth resistivity in \( \Omega \cdot \text{m} \)

\( f = \) frequency in Hz

For numerical calculations, Carson's integral for \( \Delta R \) and \( \Delta X \) has been developed into an infinite series, which is used for \( a \leq 5 \),

\[
\Delta R' = 4\omega 10^{-4}\left(\frac{\pi}{8} - b_1 a \cos \phi + b_2 [(c_2 - lna)a^2 \cos 2\phi + \phi a^2 \sin 2\phi]
+ b_3 a^3 \cos 3\phi - d_4 a^4 \cos 4\phi
- b_5 a^5 \cos 5\phi + b_6 [(c_6 - lna)a^6 \cos 6\phi + \phi a^6 \sin 6\phi]
+ b_7 a^7 \cos 7\phi - d_8 a^8 \cos 8\phi
- \ldots \ldots \right)
\]

\[
\Delta X' = 4\omega 10^{-4}\left(\frac{1}{2} (0.6159315 - lna) + b_1 a \cos \phi - d_2 a^2 \cos 2\phi
+ b_3 a^3 \cos 3\phi - b_4 [(c_4 - lna)a^4 \cos 4\phi + \phi a^4 \sin 4\phi]
+ b_5 a^5 \cos 5\phi - d_6 a^6 \cos 6\phi + b_7 a^7 \cos 7\phi
- b_8 [(c_8 - lna)a^8 \cos 8\phi + \phi a^8 \sin 8\phi]
+ \ldots \ldots \right)
\]

where \( b_1, c_i \) and \( d_i \) are constants given by

\[
b_1 = \frac{\sqrt{2}}{6} \text{ for odd subscripts } i
\]
\[
b_2 = \frac{1}{16} \text{ for even subscripts } i
\]
\[
b_i = \frac{\text{sign}}{i(i+2)}, \quad \phi \geq 2
\]

with sign = \( +1, i = 1,2,3,4; 9,10,11,12; \ldots \)
\[
-1, i = 5,6,7,8; 13,14,15,16; \ldots
\]
and \( c_2 = 1.3659315 \)
\[
c_1 = c_{i-2} + \frac{1}{i} + \frac{1}{i+2}, \quad i > 2
\]

and \( d_i = \frac{\pi}{4} b_i \)

Note that from eqtns (1-3a) and (1-3b), each 4 terms in \( i = 1,4 \) form a repetitive group in the infinite series.

For \( a > 5 \), the approximation formulae given by Butterworth\(^7,2\) is used, instead of the infinite series

\[
\Delta X = \left( \frac{\cos \phi}{a} - \frac{\cos 3\phi}{a^3} + \frac{3 \cos 5\phi}{a^3} + \frac{45 \cos 7\phi}{a^7} \right) \frac{40 \times 10^{-4}}{\sqrt{2}} \quad (1-4a)
\]
\[
\Delta R = \left( \frac{\cos \phi}{a} - \frac{\sqrt{2} \cos 2\phi}{a^2} + \frac{\cos 3\phi}{a^3} + \frac{3 \cos 5\phi}{a^5} - \frac{45 \cos 7\phi}{a^7} \right) \frac{40 \times 10^{-4}}{\sqrt{2}} \quad (1-4b)
\]

Note that the infinite series for \( R \) and \( X \) derived from Carson's integrals will only converge after about 10 or more terms if \( a > 3 \). The first few terms are highly oscillating in that case.

(ii) Shunt capacitance matrix \([C]\) - The capacitance matrix \([C]\) is the inverse of the potential coefficient matrix \([P]\).

\( [C] = [P]^{-1} \)

The matrix element of \([P]\) can easily be obtained from the tower geometry,

\[
P_{ii} = 2c^2 \times 10^{-4} \ln \frac{2h_i}{r_i} \text{ km/F} \quad (1-5)
\]

and \( P_{ij} = 2c^2 \times 10^{-4} \ln \frac{s_{ij}}{s_{ij}} \text{ km/F} \quad (1-6)\)

where \( r_i = \text{radius of conductor in m} \)

\( c = \text{velocity of light in km/s} \)

Eqtns (1-5) and (1-6) are valid as long as \( r_i (0.02 \text{ m in the example}) \) is much smaller than spacings between conductors (14 m in the
example). Note that the elements of the shunt capacitance matrix are only dependent on the tower geometry and are not dependent on frequency. This is an approximation which is valid for frequencies up to approximately 1 MHz, where earth correction terms for capacitances are not yet important.

4. Calculations of Skin Effect in Conductors

The skin effect in the earth return is accounted for by Carson's formula. While the earth return skin effect has a major influence on line parameters, skin effect in the conductors must also be considered at higher frequencies. As frequency increases, the current flows more and more on the surface of the conductor. This can be described by the nominal depth of penetration of current (δ) as given by\footnote{9}

\[ \delta \approx \sqrt{\frac{\rho_c}{\pi f \mu}} \]

where
- \( \rho_c \) = resistivity of conductor material in \( \Omega \cdot m \)
- \( \mu \) = absolute magnetic permeability in \( H/m \)
- \( f \) = frequency in Hz

Since the current is confined to the surface of the conductor at high frequencies, the conductor resistance increases and the internal inductance decreases with frequency (see Fig. 4). In eqtn (1-1), the self inductance matrix element

\[ L_{ii} = 2 \times 10^{-4} \ln \frac{2h}{GMR} \quad H/km \quad (1-8) \]

is the resultant of the internal and external inductance, i.e.

\[ L_{ii} = 2 \times 10^{-4} \ln \frac{r_1}{GMR} + 2 \times 10^{-4} \ln \frac{2h}{r_1} \quad H/km \quad (1-9) \]

The first and second term in eqtn (1-9) is due to the flux inside and outside the conductor, respectively. Note that the first term (internal inductance) is small compared to the second term for high voltage overhead lines at low frequencies and vanishes completely at high frequencies.
Skin effect on resistance and internal inductance of each bundled conductor by Galloway's formula and tubular conductor formula

Resistance $R (\Omega/km)$
Internal reactance $X_L (\Omega/km)$
Inductance $L (H)$

Fig. 4

Resistance and internal reactance by Galloway's formula
Expected field measurement
Resistance of tubular conductor
Internal reactance of tubular conductor
Internal inductance by Galloway's formula

Frequency (Hz)
Thus, the skin effect on the total inductance is normally negligible over the entire frequency range. However, the skin effect on resistance is quite pronounced.

The skin effect on the conductor resistance and internal reactance was calculated in two ways. With the first method, the conductor was treated as a solid tubular aluminum conductor of the same cross-sectional area as the actual conductor. The steel core was completely ignored. This was recommended as a reasonable approximation. The formula for the internal impedance of a tubular conductor of nonmagnetic material is

$$\frac{Z_{\text{internal}}}{R_{dc}} = \frac{R_{\text{conductor}}}{R_{dc}} + j\omega L_{\text{internal}}$$

\[
= \frac{1}{2}mr(1-s^2)(\text{ber}(mr)+j\text{bei}(mr))+\phi(\text{ker}(mr)+j\text{kei}(mr))
\]

\[
= \frac{1}{2}mr(1-s^2)(\text{ber}'(mr)+j\text{bei}'(mr))+\phi(\text{ker}'(mr)+j\text{kei}'(mr))
\]

(1-10)

where

\[
\phi = \frac{\text{ber}'(mq)+j\text{bei}'(mq)}{\text{ker}'(mq)+j\text{kei}'(mq)}
\]

\[
R_{dc} = \text{d.c. resistance of conductor in } \Omega/\text{km}
\]

\[
r = \text{outside radius of conductor in m}
\]

\[
q = \text{outside radius of steel core in m}
\]

\[
s = \frac{q}{r}
\]

\[
(mr) = \left(\frac{k}{1-s^2}\right)^{1/2}
\]

\[
(mq) = \left(\frac{ks^2}{1-s^2}\right)^{1/2}
\]

and

\[
k = \frac{8\pi 10^{-4}f}{R_{dc}}
\]

The expressions \(\text{ber}(...) + j\text{bei}(...), \text{ber}'(...) + j\text{bei}'(...)\), \(\text{ker}(...) + j\text{kei}(...), \text{ker}'(...) + j\text{kei}'(...)\) are modified Bessel functions, which can be evaluated by polynomial approximation. An empirical formula
for conductor resistance and conductor internal reactance was developed by Galloway\textsuperscript{12}, which is based on measurements in the electrolytic tank. In this approach, current is assumed to be confined to the outer layer strands (16 strands in outer layer in the example). Internal resistance ($R_c$) and internal reactance ($X_L$) are then equal. With this formula, we obtain

$$R_c = X_L = \frac{K \sqrt{\omega \mu \rho}}{\sqrt{2} r \pi (2+n)} \Omega/km$$  \hspace{1cm} (1-11)

where
- $\omega$ = angular frequency
- $\mu$ = permeability of conductor material (H/m)
- $\rho_c$ = resistivity of conductor material ($\Omega$-m)
- $r$ = outer radius of conductor in (m)
- $n$ = no. of strands in outer layer (see Fig. 2c)
- $K = 2.25$, factor due to stranding

Results for the internal impedance calculated with the above two approaches are shown in Fig. 4. At lower frequencies, where skin effect is not yet prominent, the results for tubular conductors are fairly accurate\textsuperscript{4}. After a cross-over point around 130 Hz, the results from Galloway's formula are probably more reliable since that formula takes the skin effect in the individual strands of the outer layer into account. The dotted line in Fig. 4 thus indicates the predicted field measurement values. It should be noticed that line parameters of overhead lines are seldom measured as the computed results are usually sufficiently accurate.

5. Output from Line Constants Program

For the transmission line of Fig. 2, there are 14 conductors, i.e. 4 conductors per bundle in each of the three phases and 2 ground wires above. Thus, initially we have a 14 x 14 series impedance and a 14 x 14 shunt
capacitance matrix

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_{14}
\end{bmatrix}
= \begin{bmatrix}
P_{1,1} & P_{1,2} & \cdots & P_{1,14} \\
P_{2,1} & P_{2,2} & \cdots & P_{2,14} \\
\vdots & \vdots & \ddots & \vdots \\
P_{14,1} & P_{14,2} & \cdots & P_{14,14}
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
Q_2 \\
\vdots \\
Q_{14}
\end{bmatrix}
\]

(1-12)

and

\[
\begin{bmatrix}
\frac{dV_1}{dx} \\
\frac{dV_2}{dx} \\
\vdots \\
\frac{dV_{14}}{dx}
\end{bmatrix}
= \begin{bmatrix}
Z_{1,1} & Z_{1,2} & \cdots & Z_{1,14} \\
Z_{2,1} & Z_{2,2} & \cdots & Z_{2,14} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{14,1} & Z_{14,2} & \cdots & Z_{14,14}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_{14}
\end{bmatrix}
\]

(1-13)

These 14 x 14 matrices can be reduced to the desired 3x3 matrices by considering the bundling condition in the 3 phase bundle conductors, and the zero voltage condition in both ground wires. If we denote conductors in phase A as 1,2,3,4; phase B as 5,6,7,8; phase C as 9,10,11,12 and both ground wires as 13,14, then for ground wires 13 and 14, we have

\[
-\frac{dV_{13}}{dx} = 0
\]

(1-14)

\[
-\frac{dV_{14}}{dx} = 0
\]

and

\[
V_{13} = 0
\]

(1-16)

and for bundling in phase A, we have
\[
\begin{align*}
- \frac{dV_1}{dx} &= - \frac{dV_2}{dx} = - \frac{dV_3}{dx} = - \frac{dV_4}{dx} = - \frac{dV_A}{dx} \\
I_1 + I_2 + I_3 + I_4 &= I_A \\
V_1 &= V_2 = V_3 = V_4 = V_A \\
Q_1 + Q_2 + Q_3 + Q_4 &= Q_A
\end{align*}
\] (1-16)

With eqtns (1-15) and (1-17), eqtn (1-12) can be reduced to 3 equations with the desired 3x3 potential matrix \([P]\)^\text{15},

\[
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix} =
\begin{bmatrix}
P_{AA} & P_{AB} & P_{AC} \\
P_{BA} & P_{BB} & P_{BC} \\
P_{CA} & P_{CB} & P_{CC}
\end{bmatrix}
\begin{bmatrix}
Q_A \\
Q_B \\
Q_C
\end{bmatrix}
\] (1-18)

The 3x3 shunt capacitance matrix \([C]\) is then obtained by simple matrix inversion \([C]_{3x3} = [P]^{-1}_{3x3}\). Similarly, eqtns (1-14) and (1-16) can be used to reduce eqtn (1-13) to 3 equations with the desired 3x3 series impedance matrix \([Z]\).

\[
\begin{bmatrix}
\frac{dV_A}{dx} \\
\frac{dV_B}{dx} \\
\frac{dV_C}{dx}
\end{bmatrix} =
\begin{bmatrix}
Z_{AA} & Z_{AB} & Z_{AC} \\
Z_{BA} & Z_{BB} & Z_{BC} \\
Z_{CA} & Z_{CB} & Z_{CC}
\end{bmatrix}
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix}
\] (1-19)

Thus, with Carson's formula and with one of the two skin effect formulae for conductors, followed by matrix reduction for bundling and ground wires, we obtain the 3x3 series impedance and 3x3 shunt capacitance matrices for the three phases.

The 3x3 shunt capacitance matrix obtained from the tower geometry of the test example is shown in Table 2. the elements of the 3x3 symmetric
series impedance matrix are shown as a function of frequency in Figs. 5 and 6. Note that the differences between both skin effect formulae hardly show up at high frequencies on a logarithmic scale.

\[
[C] = \begin{bmatrix}
0.06889 & -0.01183 & -0.00347 \\
-0.01183 & 0.07080 & -0.01183 \\
-0.00347 & -0.01183 & 0.06889 \\
\end{bmatrix} \mu F/km
\]
Elements of the resistance matrix of the test line, with Galloway's formula and the formula for tubular conductor, for skin effect.

Fig. 5
Elements of the reactance matrix of the test line, with Galloway's formula and the formula for tubular conductor, for skin effect

Reactance $\omega L(\Omega/km)$

![Diagram showing reactance $X_{AA} = X_{BB} = X_{CC}$, $X_{BA} = X_{BC}$, $X_{CA}$, with tubular conductor and Galloway's formula lines.]

Fig. 6
6. Positive and Zero Sequence Parameters

There are 3 modes of TEM propagation on the 3 phase test line. Each mode is decoupled from the other and has its own individual characteristic impedance and propagation constant (see later eqtn (2-8)). If the test line was transposed, which it is not, then two of the 3 modes would be characterized by positive sequence parameters while the third one would be characterized by zero sequence parameters. Thus, by idealizing the given untransposed line to a transposed one, we can look at the positive and zero sequence parameters, which will give us some insight into the overall effect of both approaches for conductor skin effect calculation (tubular conductor formula and Galloway's formula).

For the transposed line, the formulae relating position and zero sequence impedances ($Z_{\text{pos}}$ and $Z_{\text{zero}}$) to the series impedance matrix elements are given by\textsuperscript{14}

$$Z_{\text{pos}} = Z_s - Z_m$$  \hspace{1cm} (1-20)

$$Z_{\text{zero}} = Z_s + 2Z_m$$  \hspace{1cm} (1-21)

where $Z_s$ and $Z_m$ are the self and mutual impedances, which in turn are the averages of the diagonal and off-diagonal elements respectively,

$$Z_s = \frac{1}{3}(Z_{AA} + Z_{BB} + Z_{CC})$$  \hspace{1cm} (1-22)

$$Z_m = \frac{1}{3}(Z_{AB} + Z_{BC} + Z_{CA})$$  \hspace{1cm} (1-23)

where $Z_{ij}$ is an element of $[Z]_{3x3}$

The impedances $Z_{ij}$ in eqtns (1-16) and (1-17) are the series impedances matrix elements shown in Figs. 5 and 6. The positive and zero sequence resistances are shown in Fig. 7. In Fig. 7, the resistance of the bundle conductor obtained with the tubular conductor formula and Galloway's
Change in sequence resistance due to change in conductor bundle resistance

\[
Z_{\text{pos}} = Z_s - Z_m \\
Z_{\text{zero}} = Z_s + Z_m
\]

Fig. 7
formula are also shown for comparison. At higher frequencies, the conductor resistance from Galloway's formula is about twice as high (see also Fig. 4) as the value from the tubular conductor formula. The increase in conductor resistance ($\Delta R_c$) shows up with the same value as an increase in positive sequence resistance ($\Delta R_{pos}$). However, the differences in conductor resistance between the two formulae do not show up with exactly the same value in the zero sequence resistance. The difference in zero sequence resistance ($\Delta R_{zero}$) is slightly higher (e.g. 9% higher at 50 KHz) due to the additional effect of the 2 eliminated ground wires. Also note that the zero sequence resistance is much higher than the positive sequence resistance. At 50 KHz, the increase in the positive sequence resistance caused by the difference in skin effect formulae is about 17% whereas the increase in the zero sequence resistance is only about 1%.
CHAPTER II

COMPUTATION OF TRANSFER FUNCTION FOR FREQUENCY RESPONSE OF TEST LINE

1. Introduction

After knowing the series impedance matrix \([Z_{ij}]_{3x3}\) and the shunt admittance matrix with zero conductance

\([Y_{ij}]_{3x3} = j\omega[C_{ij}]_{3x3}\)

of the 3-phase test line, the transfer function between the input on one phase at the sending end and the output on any one of the three phases at the receiving end can be found. For the chosen test example, one phase, designated A, is energised (see Fig. 8).

The output voltage in the 3-phase at the receiving end of the 83.212 km long test line was to be found for a period of time during which the waves reflected at the receiving end have not yet returned back from the sending end. That is, the period of investigation time \(t\) is

\[\tau < t < 3\tau\]

where \(\tau\) is the travel time of the mode with highest wave velocity.

The Transfer Function Program which performs the necessary calculations is an expanded version of a program written by K.K. Tse\textsuperscript{16} for a term project in EE 553 on the frequency response of B.C. Hydro's Mica Dam transmission line (see Appendix 1 for program listings).

2. Outline of the theory used in the Transfer Function Program

Propagation of waves on multiphase line with constant parameters is described by the well-known general transmission line equation\textsuperscript{17,18}.

\[-\frac{3v}{\partial x} = [L] \left[ \frac{2i}{\partial t} \right] + [R] \left[ i \right]\]  \hspace{1cm} (2.1)

\[-\frac{2i}{\partial x} = [C] \left[ \frac{3v}{\partial t} \right] + [G] \left[ v \right]\]  \hspace{1cm} (2.2)
Boundary conditions in frequency domain

At sending end

\[ V_A^o = V_g - I_A^o R_n \]

\[ I_A^o \neq 0, \quad I_B^o = I_C^o = 0 \]

At receiving end --- Single input triple output system

\[ V_j^o = H_j(\omega) V_j^g; \quad j = A, B \text{ or } C \]

Fig. 8 Transmission line configurations with boundary conditions
where $[v]$ is the 3x1 column matrix of phase voltages

$[i]$ is the 3x1 column matrix of phase currents

$[L]$ is the 3x3 inductance matrix

$[R]$ is the 3x3 resistance matrix

$[C]$ is the 3x3 capacitance matrix

$[G]$ is the 3x3 conductance matrix

(N.B For overhead transmission lines, $[G]$ is very small and is practically always neglected).

However, the above eqtns (2-1) and (2-2) are not useable for lines with frequency dependent line parameters. Instead, we have to use equations in the form of steady state phasor equations in the frequency domain

\[
- \frac{\partial^2 V}{\partial x^2} = [Z] [I] \quad (2-3)
\]

\[
- \frac{\partial I}{\partial x} = [Y] [V] \quad (2-4)
\]

where $[Z] = [R] + j\omega [L]$

= series impedance matrix in $\Omega$/km as obtained numerically from Chapter 1.

$[Y] = j\omega [C]$

= shunt capacitance matrix in $\Omega$/km also from Chapter 1

$[V]$ and $[I]$ are the vectors of phase voltages and phase currents, respectively, in the form of phasor values.

Eqtn (2-3) can be differentiated w.r.t.x to get \(^\text{18}\)

\[
\frac{d^2 V}{dx^2} = -[Z] \frac{d I}{dx} \\
= [Z] [Y] [V] \\
\Delta \sim [ZY] [V] \\
(2-5)
\]

where $[ZY] \Delta \sim [Z] [Y]$
The 3x3 matrix in eqtn (2-5) has non-zero off-diagonal elements i.e. there is coupling between the phases. The easiest way to solve these coupled equations is to decouple them by modal analysis\textsuperscript{18,16}. With this approach, $[ZY]$ is transformed to a diagonal matrix with\textsuperscript{19} $[M]$,\n\begin{equation}
[M]^{-1}[ZY] \cdot [M] = [\Lambda] \tag{2-6}
\end{equation}
where $[\Lambda] = \text{diagonal matrix}$

$[M] = \text{modal matrix, i.e. columns of eigenvectors of } [ZY]$ and $[M]^{-1} = \text{inverse of } [M]$

Both $[M]$ and $[M]^{-1}$ were obtained with subroutines from the UBC Computing Centre Programme Library, namely with 'DCEIGN' and 'CDINVT'\textsuperscript{20}. 'DCEIGN' computes the eigenvalues and eigenvectors of the complex matrix $[ZY]$, and 'CDINVT' computes the inverse of the modal matrix $[M]$. 'DCEIGN' was tested for accuracy by running a test example with known answers\textsuperscript{19}. The results agreed up to 7 significant digits. With $[M]^{-1}$ known, the phase to mode voltage transformation is described by
\begin{equation}
[V_{\text{mode}}] = [M]^{-1}[V] \tag{2-7}
\end{equation}

Pre-multiplying eqtn (2-5) with $[M]^{-1}$ gives
\begin{equation}
[M]^{-1}\frac{d^2V}{dx^2} = [M]^{-1}[ZY][V] \notag
\end{equation}

or
\begin{equation}
\frac{d^2}{dx^2} [V_{\text{mode}}] = [M]^{-1}[ZY][V] \notag
\end{equation}

\begin{equation}
= [M]^{-1}[ZY][M][V_{\text{mode}}] \tag{from eqtn (2-7)}
\end{equation}

\begin{equation}
= [\Lambda][V_{\text{mode}}] \tag{from eqtn (2-6)}
\end{equation}

Thus, 3 second order differential equations are obtained, each decoupled from the other, namely,
where $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the 3 eigenvalues of $[ZY]$.

The 3 phase voltages have now been transformed into 3 modal quantities, which describe the independently decoupled modes of TEM propagation. They can be transformed back to phase voltages with the mode to phase relationship derived from equation (2-7),

$$[V] = [M][V_{\text{mode}}]$$

(2-9)

The general solution to the second order linear differential equation of the modal voltage components is well known because each modal component can be treated as if it were a hypothetical single phase line.

For distance $x = \lambda$ km away from the sending end

$$
\begin{pmatrix}
V_{1,\lambda} \\
V_{2,\lambda} \\
V_{3,\lambda}
\end{pmatrix}
= \begin{pmatrix}
V_{1+} \cdot e^{-\sqrt{\lambda_1} \lambda} + V_{1-} \cdot e^{\sqrt{\lambda_1} \lambda} \\
V_{2+} \cdot e^{-\sqrt{\lambda_2} \lambda} + V_{2-} \cdot e^{\sqrt{\lambda_2} \lambda} \\
V_{3+} \cdot e^{-\sqrt{\lambda_3} \lambda} + V_{3-} \cdot e^{\sqrt{\lambda_3} \lambda}
\end{pmatrix}

= \begin{pmatrix}
V_{1+} \cdot e^{-\gamma_1 \lambda} + V_{1-} \cdot e^{\gamma_1 \lambda} \\
V_{2+} \cdot e^{-\gamma_2 \lambda} + V_{2-} \cdot e^{\gamma_2 \lambda} \\
V_{3+} \cdot e^{-\gamma_3 \lambda} + V_{3-} \cdot e^{\gamma_3 \lambda}
\end{pmatrix}

(2-10)
where \( \gamma_1 \) = propagation constant for steady state behaviour at specific frequency

\[ V_{\text{mode}}^{1,2,3+} = \text{Forward modal voltage waves at } x = 0 \text{ of A, B and C respectively, travelling from the sending end to the receiving end.} \]

\[ V_{\text{mode}}^{1,2,3-} = \text{reflected modal voltage waves at } x = 0 \text{ of A, B and C respectively, travelling from the receiving end to the sending end.} \]

If we are only interested in the attenuation and distortion of the wave front, then we can assume that the line is infinitely long. We can then neglect the backward reflected voltage wave, i.e.

\[ V_{\text{mode}}^{1-} = V_{\text{mode}}^{2-} = V_{\text{mode}}^{3-} = 0 \]

Eqtn (2-10) is thus reduced to

\[
\begin{bmatrix}
V_{\text{mode}}^{1,2,3+} \\
V_{\text{mode}}^{1,2,3-}
\end{bmatrix} =
\begin{bmatrix}
V_{\text{mode}}^{1+} e^{-\gamma_1^+} & e^{-\gamma_2^+} & e^{-\gamma_3^+} \\
V_{\text{mode}}^{2+} e^{-\gamma_2^+} & e^{-\gamma_2^+} & e^{-\gamma_3^+} \\
V_{\text{mode}}^{3+} e^{-\gamma_3^+} & e^{-\gamma_3^+} & e^{-\gamma_3^+}
\end{bmatrix}
\begin{bmatrix}
V_{\text{mode}}^{1+} \\
V_{\text{mode}}^{2+} \\
V_{\text{mode}}^{3+}
\end{bmatrix}
\]

or simply by matrix notation, we have

\[ [V_{\text{mode}}^\ell] = e^{-[\gamma]^\ell \cdot [V_{\text{mode}}^\ell]} \]

where \([\gamma] = 3x3 \text{ diagonal matrix with diagonal elements } \gamma_1, \gamma_2 \text{ and } \gamma_3\]

\[ e^{-[\gamma]^\ell} = [H(\omega)], \text{transfer function matrix of the transmission system} \]

and again \([V_{\text{mode}}^\ell] = \text{forward voltage wave at } x = 0.\]

(N.B. The results thus obtained are also valid for the voltage response at the receiving end of an open-ended line of finite length. For times less
than 3 times travel time, the open-ended line results are simply twice the results obtained from eqtn (2-12). This doubling effect is discussed in further detail in Chapter 4, section 1.)

From eqtn (2-12), the modal voltages at the receiving end $[V_{\text{mode}}]$ can be transformed to the phase voltages by the mode to phase relationship

$$[V_{\ell}] = [M] [V_{\text{mode}}]$$

from eqtn (2-9)

$$= [M] e^{-[\gamma] \ell} [V_{\text{mode}}]$$

from eqtn (2-12)

Thus, we can express the phase voltages at the receiving end $[V_{\ell}]$ in terms of phase voltages at the sending end $[V^o]$ as

$$[V_{\ell}] = [M] e^{-[\gamma] \ell} [M]^{-1} [V^o]$$

(2-13)

Note that from eqtn (2-13), phase voltages cannot be calculated from $[\gamma]$ alone without $[M]$, i.e.

$$[V_{\ell}] \neq e^{-[\gamma] \ell} [V^o]$$

3. Inclusion of Boundary Conditions at Sending End.

For the chosen test line case, the line was energized on phase A as shown in Fig. 8. Boundary conditions for phase voltages and currents at the sending end (distance $x = 0$ denoted by superscript o) are then

$$[V^o] = \begin{bmatrix} V_g - I_A^o R_o \\ V_A^o \\ V_B^o \\ V_C^o \end{bmatrix}$$

(2-14)

and

$$[I^o] = \begin{bmatrix} I_A^o \\ 0 \\ 0 \end{bmatrix}$$

(2-15)

Substitution of eqtn (2-3) into eqtn (2-7) differentiated w.r.t. $x$ gives
\[-\frac{dV_{\text{mode}}}{dx} = -[M]^{-1}\left[\frac{dV_{\text{mode}}}{dx}\right]\]

\[= [M]^{-1} \cdot [Z][I^o]\]

\[\Delta \equiv [A][I^o]\]

\[= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} I_A^o \\ 0 \\ 0 \end{bmatrix}\]

\[= I_A^o \cdot \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix}\] \hspace{1cm} (2-16)

Again, differentiating eqtn (2-12) w.r.t. \(x\) gives

\[-\frac{dV_{\text{mode}}}{dx} = [\gamma] \cdot e^{-x[\gamma]} \cdot [V_{\text{mode}}]\] \hspace{1cm} (2-17)

Equating R.H.S. of eqtn (2-16) and (2-17), we get

\[[\gamma] \cdot e^{-x[\gamma]} \cdot [V_{\text{mode}}] = I_A^o \cdot \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix}\] \hspace{1cm} (2-18)

However, at the sending end we have \(x = 0\) as boundary condition, eqtn (2-18) therefore gives

\[[V_{\text{mode}}^+] = [\gamma]^{-1} I_A^o \cdot \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix}\]

or

\[[V_{\text{mode}}^+] = I_A^o \cdot \begin{bmatrix} \frac{1}{\gamma_1} & 0 & 0 \\ 0 & \frac{1}{\gamma_2} & 0 \\ 0 & 0 & \frac{1}{\gamma_3} \end{bmatrix} \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} = I_A^o \cdot \begin{bmatrix} A_{11} / \gamma_1 \\ A_{21} / \gamma_2 \\ A_{31} / \gamma_3 \end{bmatrix}\] \hspace{1cm} (2-19)
From the phase-mode relationship of eqtn (2-9) we get the sending end phase voltage \([V^o]\) as

\[
[V^o] = [M][V^{mode}] = I_A^o \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} A_{11}/\gamma_1 \\ A_{21}/\gamma_2 \\ A_{31}/\gamma_3 \end{bmatrix}
\]

\[
\begin{bmatrix} V_A^o \\ V_B^o \\ V_C^o \end{bmatrix} = \begin{bmatrix} V_g - I_A^o R_o \\ V_B^o \\ V_C^o \end{bmatrix}
\]

(2-20)

(For a detailed picture of the boundary conditions, see Fig. 8).

Now, we can evaluate the first row of eqtn (2-20).

\[
V_g - I_A^o R_o = I_A^o (M_{11}A_{11}/\gamma_1 + M_{12}A_{21}/\gamma_2 + M_{13}A_{31}/\gamma_3).
\]

Then, we can get the sending end current in phase A as

\[
I_A^o = \frac{V_g}{M_{11}A_{11}/\gamma_1 + M_{12}A_{21}/\gamma_2 + M_{13}A_{31}/\gamma_3 + R_o} = \frac{V_g}{Z_{eq}}
\]

(2-21)

where

\[
Z_{eq} = M_{11}A_{11}/\gamma_1 + M_{12}A_{21}/\gamma_2 + M_{13}A_{31}/\gamma_3 + R_o
\]

(2-22)

Thus, substituting eqtn (2-21) into eqtn (2-19) for \(I_A^o\), the modal voltages at the sending end are obtained as

\[
[V^{mode}] = \frac{V_g}{Z_{eq}} \begin{bmatrix} A_{11}/\gamma_1 \\ A_{21}/\gamma_2 \\ A_{31}/\gamma_3 \end{bmatrix}
\]

(2-23)
Using the mode-phase relationship of eqtn (2-9) again, we obtain the sending end (x=0) phase voltages in the 3 phase as

\[
\begin{bmatrix}
V_A^o \\
V_B^o \\
V_C^o
\end{bmatrix} = \frac{V_g}{Z_{eq}} [M] \begin{bmatrix}
A_{11}/\gamma_1 \\
A_{21}/\gamma_2 \\
A_{31}/\gamma_3
\end{bmatrix}
\]  
(2-24)

Also, for the receiving end at x = \lambda, the modal voltage components are

\[
[v_{mode}^\lambda] = e^{-\lambda[\gamma]} [v_{mode}^]\]  
from eqtn (2-12)

\[
= \frac{V_g}{Z_{eq}} [M] \begin{bmatrix}
e^{-\gamma_1\lambda} A_{11}/\gamma_1 \\
e^{-\gamma_2\lambda} A_{21}/\gamma_2 \\
e^{-\gamma_3\lambda} A_{31}/\gamma_3
\end{bmatrix}
\]  
(2-25)

Finally, the receiving end phase voltages in the 3 phase are

\[
\begin{bmatrix}
V_A^\lambda \\
V_B^\lambda \\
V_C^\lambda
\end{bmatrix} = \frac{V_g}{Z_{eq}} [M] \begin{bmatrix}
e^{-\gamma_1\lambda} A_{11}/\gamma_1 \\
e^{-\gamma_2\lambda} A_{21}/\gamma_2 \\
e^{-\gamma_3\lambda} A_{31}/\gamma_3
\end{bmatrix}
\]  
(2-26)

It should be realized that if we excite phase k of the 3 phase, (k = A, B, or C), then the output phase voltages are

\[
\begin{bmatrix}
V_A^k \\
V_B^k \\
V_C^k
\end{bmatrix} = \frac{V_g}{Z_{eq}} [M] \begin{bmatrix}
e^{-\gamma_1\lambda} A_{1k}/\gamma_1 \\
e^{-\gamma_2\lambda} A_{2k}/\gamma_2 \\
e^{-\gamma_3\lambda} A_{3k}/\gamma_3
\end{bmatrix}
\]  
(2-27)

This is the formula that we use in the Fourier Transform Programme. It has an option to specify which of the 3 phases are to be energized for the test line case.

Thus, we obtain the transfer function [H(\omega)] for the test case as seen from Fig. 8.
4. Transfer function for test line

The magnitudes and phases of the transfer functions for the test case of Fig. 8 are plotted in Figs. 9 and 10, respectively, for both approaches used in evaluating the skin effect in the conductors. The difference between the two skin effect formulae only show up in the magnitude spectrum in the low frequency (0 - 100 Hz) region. The results coincide more or less at frequencies above 100 Hz.

The phase of the transfer function increases monotonically (as shown in Fig. 9). This can easily be explained for the single phase case where the transfer function becomes

\[
H(\omega) = e^{-\sqrt{\gamma^2 k}} = e^{\gamma k} = e^{-\frac{\omega}{J}(R+j\omega L)j\omega C}
\]

(2-29)

where \( k = \text{length of line} \)

(N.B. Compare with eqtn (2-12) for 3 decoupled modes). Expanding for real and imaginary parts of \( \gamma \) and using binomial expansion

\[
(a + b)^{1/2} = a^{1/2} + \frac{1}{2} a^{-1/2} b + \ldots, \quad a \gg b.
\]

\[
\gamma = \sqrt{(R+j\omega L)j\omega C} = (R+j\omega L)^{1/2} \cdot (j\omega C)^{1/2}, \quad R \ll j\omega L
\]

\[
\gamma = [(j\omega L)^{1/2} + \frac{R}{2}(j\omega L)^{-1/2}] \cdot (j\omega C)^{1/2}
\]

\[
= \frac{R}{2} \left( \frac{C}{L} \right)^{1/2} + j\omega (LC)^{1/2} \triangleq \alpha + j\beta
\]

(2-30)

Thus, the magnitude and phase of the transfer function for the single phase line are respectively

\[
|H(\omega)| = e^{-\alpha k} = e^{-\frac{R}{J}(C/L)^{1/2}}
\]

(2-31)
Magnitude of transfer functions with skin effect calculation by Galloway's formula and by tubular conductor formula.
Phase of transfer functions
(Identical results with skin effect calculation by Galloway's formula and by tubular conductor formula)
and \[ H(\omega) = -\frac{\omega}{2\pi} \sqrt{\frac{R}{L}} \] (2-32)

where \( R \) increases appreciably with frequency where \( L \) decreases slightly with frequency for zero sequence and stays more or less constant for positive sequency and where \( C \) stays constant. That is, the phase angle of the transfer function increases almost linearly with frequency, whereas the magnitude decreases with frequency similar to a low pass filter.

The calculation of angles with a FORTRAN trigonometric function statement covers only the range from 0° to 360°. Therefore, special logic to be included to extend the angles beyond 2π (see Appendix 1 for FORTRAN listings). The separate 'PHASEPRO' program is used to guarantee that the phase angle \( \beta \) is a continuous function of \( \omega \). This can be achieved by setting up a counter value \( k \), where \( k \) is initially zero. Whenever the calculated phase angle falls out of the range of \( \pm \pi \) rad from the predicted extrapolated phase angle value, \( k \) will be incremented by 1

\[ k : = k + 1, \]

and all following phase angle values are increased by \( k(2\pi) \). This way, the phase angle is ensured to be continuous and monotonically increasing.
CHAPTER 3.
TIME RESPONSE OF TEST LINE THROUGH FOURIER TRANSFORMATION

1. Introduction

After the frequency response of the line is known in the form of transfer functions, the output voltage can be calculated for any given input voltage \( v_g(t) \) by Fourier Transformation. At the beginning, the input voltage \( v_g(t) \) is transformed from the time domain into the frequency domain to give \( V_g(\omega) \). The output voltage in the frequency domain is then obtained by multiplying \( V_g(\omega) \) with the transfer function \([H(\omega)]\) obtained from the Transfer Function Program described in Chapter 2. i.e.

\[
[V^g_\ell(\omega)] = [H(\omega)] V_g(\omega)
\]  

Finally, the inverse Fourier transformation is used to obtain the output voltages \( V_A^\ell(t) \), \( V_B^\ell(t) \) and \( V_C^\ell(t) \) in the time domain. The above described techniques are applied to the test case of Fig. 8. The obtained results are then compared with the field test measurements\(^1\) and with simulation results obtained by Groschupf\(^26\). The programs used for the Fourier Transformations from the time to the frequency domain, and vice versa, was adopted from a version initially written by H. W. Dommel\(^27\) (see Appendix 3 for program listings).

2a. Numerical Fourier Transformation of input voltage from time to frequency

For a given input voltage \( v_g(t) \) in the time domain, we can in general obtain the input voltage in the frequency domain \( V_g(\omega) \) with the following Fourier Transformation formula

\[
A(\omega) = \int_{-\infty}^{\infty} v_g(t) \cos \omega t \, dt \tag{3-2}
\]

\[
B(\omega) = \int_{-\infty}^{\infty} v_g(t) \sin \omega t \, dt \tag{3-3}
\]

where \( A(\omega) \) and \( B(\omega) \) are the real and imaginary parts of \( V_g(\omega) \), respectively,
\[ V_g(\omega) = A(\omega) + j B(\omega) \] \hspace{1cm} (3-4)

If we assume that the input voltage is zero for time \( t < 0 \), then eqns (3-2) and (3-3) can be simplified to

\[ A(\omega) = \int_{t_0}^{T} v_g(t) \cos \omega t \, dt \] \hspace{1cm} (3-5)

\[ B(\omega) = \int_{t_0}^{T} v_g(t) \sin \omega t \, dt \] \hspace{1cm} (3-6)

where \((o, T)\) is the time interval in which \( v_g(t) \) is non-zero.

Case 1. \textbf{Input voltage defined point by point}

If the input voltage is defined point by point in the integration interval \((o, T)\) at closely spaced time intervals, then it is reasonable to assume linear interpolation between points (see Fig. 11). Then, for an interval \((t_1, t_2)\), we have

\[ v_g(t) = v_1 + \frac{v_2-v_1}{\Delta t} (t-t_1), \quad t_1 \leq t \leq t_2 \] \hspace{1cm} (3-7)

Substitution of eqtn (3-7) into eqtn (3-5) gives

\[ A_{12}(\omega) = \int_{t_1}^{t_2} \left[ v_1 + \frac{v_2-v_1}{\Delta t} (t-t_1) \right] \cos \omega t \, dt \] \hspace{1cm} (3-8)

\[ = \left[ v_1 - \frac{v_2-v_1}{\Delta t} t_1 \right] t_2 \cos \omega t \, dt + \frac{v_2-v_1}{\Delta t} \int_{t_1}^{t_2} t \cos \omega t \, dt \]

\[ = \left[ v_1 - \frac{v_2-v_1}{\Delta t} t_1 \right] \frac{1}{\omega} \sin \omega t \bigg|_{t_1}^{t_2} \]

\[ + \frac{v_2-v_1}{\Delta t} \frac{1}{\omega} (t \sin \omega t + \frac{1}{\omega} \cos \omega t) \bigg|_{t_1}^{t_2} \]

\[ = \frac{1}{\omega} \sin \omega t_2 \left[ (v_1 + \frac{v_2-v_1}{\Delta t} t_2) - \frac{v_2-v_1}{\Delta t} t_1 \right] \]

\[ - \frac{1}{\omega} \sin \omega t_1 \left[ (v_1 - \frac{v_2-v_1}{\Delta t} t_1) + \frac{v_2-v_1}{\Delta t} t_1 \right] \]

\[ + \frac{v_2-v_1}{\Delta t \omega^2} (\cos \omega t_2 - \cos \omega t_1) \]
Input voltage

Linear interpolation of input voltage in time domain

Phase and magnitude

Linear interpolation of output voltage in frequency domain

Phase of output voltage (R)

Magnitude of output voltage (S)

Fig. 11

Fig. 12
or finally, 
\[
A_{12}(\omega) = \frac{1}{\omega}[v_2 \sin \omega t_2 - v_1 \sin \omega t_1 + \frac{v_2 - v_1}{\Delta \omega}(\cos \omega t_2 - \cos \omega t_1)]
\]
(3-9)

Similarly, for the imaginary voltage component in the interval 
\[(t_1, t_2),\]
we obtain
\[
B_{12}(\omega) = \left\{ t_2 v_1 + \frac{v_2 - v_1}{\Delta t} (t-t_1) \right\} \sin \omega t dt \\
= (v_1 - \frac{v_2 - v_1}{\Delta t}) t_2 \sin \omega t dt + \frac{v_2 - v_1}{\Delta t} t_1 t \sin \omega t dt \\
= (v_1 - \frac{v_2 - v_1}{\Delta t}) \sin \omega t|_{t_1}^{t_2} + \frac{v_2 - v_1}{\Delta t} (-t \cos \omega t + \frac{1}{\omega} \sin \omega t)|_{t_1}^{t_2},
\]
or finally
\[
B_{12}(\omega) = \frac{1}{\omega}[v_2 \cos \omega t_2 + v_1 \cos \omega t_1 + \frac{v_2 - v_1}{\Delta \omega}(\sin \omega t_2 - \sin \omega t_1)]
\]
(3-10)

The calculations of \(A_{12}(\omega)\) and \(B_{12}(\omega)\) are repeated for all time intervals to cover the whole region \((0,T)\). The real and imaginary part of the voltage in the frequency domain at a specific frequency \(\omega\) is then simply the sum of these parts
\[
V_g(\omega) = A(\omega) + jB(\omega)
\]
\[
= \sum_{k=0}^{N} A_{k, k+1}(\omega) + jB_{k, k+1}(\omega), \text{ where } N = \frac{T}{\Delta t}
\]

The above calculations must be made over the entire frequency range at the same frequency points at which the transfer functions have been calculated. Output voltage in the frequency domain is thus obtained at all transfer function frequencies.

Case 2. **Input voltage defined analytically**

For some types of input voltages \(v_g(t)\), \(A(\omega)\) and \(B(\omega)\) are known analytically\(^{28}\). Take a single exponential decay input voltage as an
example,
\[ v(t) = e^{-at}, \quad t > 0 \]  \hspace{1cm} (3-12)

We can directly evaluate \( A(\omega) \) and \( B(\omega) \)
by \[ V_G(\omega) = A(\omega) + jB(\omega) \]
\[ = \frac{1}{a+j\omega} \]  \hspace{1cm} (3-13)

Thus, we can obtain the real and imaginary voltage components in the
frequency domain by the exact Fourier Transformation. With this technique,
we can omit the first part of our program and obtain the output voltage in
the frequency domain by multiplying eqtn (3-13) with the corresponding transfer functions, i.e.
\[ [V^2] = [H(\omega)] \frac{1}{a+j\omega} \]  \hspace{1cm} (3-14)

3.) Output Voltage in Frequency Domain

From eqtn (3-1), we have the output voltage in the frequency
domain as
\[ [V_{\omega}] = [H(\omega)] V_G(\omega) \] \hspace{1cm} from eqtn (3-1)

For inverse Fourier transformation back to the time domain, (see
section D), we use linear interpolation between consecutive frequency points.
Thus, the output voltage frequency components must be reasonably smooth to
obtain satisfactory results.

It has been shown that the magnitude of the transfer function is
fairly smooth (see eqtns (2-31) and (2-32). This is also true for the phase
angle of the transfer function provided it is extended beyond 2\( \pi \) rad (see Figs.
9 and 10).

From eqtns (3-1 and (3-10), we obtain the real and imaginary components of the input voltage. They become highly oscillating at higher frequencies and are not suitable for linear interpolation. Therefore, the
real and imaginary voltage components are converted to magnitude and phase values. The phase angle is again extended beyond $2\pi$ by the same smoothing logic as described in Chapter 2. Thus, we can write the output voltage in the frequency domain $V^\omega(\omega)$ as

$$V^\omega(\omega) = S(\omega) e^{j\omega_1} R(\omega)$$
(3-15)

14. Output voltage in time domain by numerical inverse Fourier Transformation

From the given output voltage in the frequency domain $[V^\omega(\omega)]$, we obtain the output voltages in the time domain by inverse Fourier Transformation

$$v^\omega(t) = \frac{1}{\pi} \int_0^\infty V^\omega(\omega) e^{j\omega t} d\omega$$

From eqtn (3-15), we get

$$v^\omega(t) = \frac{1}{\pi} \int_0^\infty S(\omega) e^{j(\omega t + R)} d\omega$$
(3-16)

Similar to section B, for Fourier Transformation, the inverse Fourier Transformation also uses linear interpolation between adjacent points in the frequency domain, for the magnitudes $(S_1, S_2)$ of the output voltages as well as for the phase angles $(R_1, R_2)$. (see Fig. 12). As explained in section C, this is permissible because $S$ and $R$ are smooth curves in contrast to the highly oscillating real and imaginary components $A(\omega)$ and $B(\omega)$. Since only a real voltage component exists in the time domain, the contribution to the output voltage from the inverse Fourier Transformation of the frequency interval $[\omega_1, \omega_2]$ is

$$v^\omega(t) = \frac{1}{\pi} \int_0^\infty S(\omega) \cos(\omega t + R) d\omega,$$

or

$$v^\omega_1(t) = \frac{1}{\pi} \int_0^\omega S(\omega) \cos(\omega t + R) d\omega$$
(3-17)

where

$$S(\omega) = S_1 + \frac{S_2 - S_1}{\Delta \omega} (\omega - \omega_1)$$
(3-18)

$$R(\omega) = R_1 + \frac{R_2 - R_1}{\Delta \omega} (\omega - \omega_1)$$
(3-19)

where $\omega_1 < \omega < \omega_2$, and $\Delta \omega = \omega_2 - \omega_1$
Substituting the magnitude and phase eqtns (3-18) and (3-19) into eqtn (3-13) we get

\[
\mathbf{v}_{12}(t) = \int_{\omega_1}^{\omega_2} \left[ S_1 + \frac{S_2 - S_1}{\Delta \omega}(\omega - \omega_1) \right] \cos(\omega t + R_1 + \frac{R_2 - R_1}{\Delta \omega}(\omega - \omega_1)) d\omega
\]

\[
= \int_{\omega_1}^{\omega_2} \left[ (S_1 - \frac{S_2 - S_1}{\Delta \omega}(\omega - \omega_1)) + \frac{S_2 - S_1}{\Delta \omega} \right] \cos \left[ (R_1 - \frac{R_2 - R_1}{\Delta \omega}(\omega - \omega_1)) + \frac{R_2 - R_1}{\Delta \omega} + t \right] d\omega
\]

\[
\approx \int_{\omega_1}^{\omega_2} (a + b \omega) \cos (c + s \omega) d\omega
\]

where constants \(a, b, c\) and \(s\) are constant for a specific frequency interval,

\[
a = S_1 - \frac{S_2 - S_1}{\Delta \omega} \omega_1 \quad (3-20)
\]

\[
b = \frac{S_2 - S_1}{\Delta \omega} \quad (3-21)
\]

\[
c = R_1 - \frac{R_2 - R_1}{\Delta \omega} \quad (3-22)
\]

\[
s = \Delta \omega + t \quad (3-23)
\]

Thus, we obtain

\[
\mathbf{v}_{12}(t) = a \int_{\omega_1}^{\omega_2} \cos (c + s \omega) d\omega + b \int_{\omega_1}^{\omega_2} \omega \cos (c + s \omega) d\omega
\]

\[
= \frac{a}{s} \sin (c + s \omega) \bigg|_{\omega_1}^{\omega_2} + \frac{b}{s} \left[ \omega \sin (c + s \omega) + \frac{1}{s} \cos (c + s \omega) \right] \bigg|_{\omega_1}^{\omega_2}
\]

or finally

\[
\mathbf{v}_{12}(t) = \sin (c + s \omega_2) \left( \frac{a}{s} \omega_2 + \frac{b}{s} \right) - \sin (c + s \omega_1) \left( \frac{a}{s} \omega_1 + \frac{b}{s} \right)
\]

\[
+ \frac{b}{s} \left[ \cos (c + s \omega_2) - \cos (c + s \omega_1) \right] \quad (3-24)
\]

The calculation with eqtn (3-24) is repeated for all frequency intervals to cover the frequency region over which the output voltage \(V^\omega(\omega)\).
is defined. The output voltage at any specific time is then the sum of the contributions from all frequency interval

\[ v'(t) = \sum_{\omega=0}^{\omega'} v_{12}(t, \omega) \]  

(3-25)

where \( \omega' \) is the last frequency data point.

5. Numerical Aspects of Fourier Transformation Program

There are several aspects which deserve special attention in the Fourier Transformation Program to ensure reasonably accurate results.

1. Suitability of linear interpolation in numerical integration - A reasonable "smoothness" of input voltage \( v_g(t) \) and output voltage in frequency domain \( V_\omega(\omega) \) must be guaranteed to permit linear interpolation between adjacent data points. Therefore, the magnitude and phase angle of the output voltage are used to avoid the highly oscillating real and imaginary frequency components as described in section 3.

2. Density of data points - Linear interpolation is assumed between adjacent frequency and time data points in the numerical integration loops. Too dense data points will increase computer costs drastically, while too sparse data points will result in loss of accuracy. A density of 20 points per decade in the frequency domain (on a logarithm scale) satisfies the accuracy requirement reasonably well for the test case studied. In the time domain, the density of data points for the input voltage \( v_g(t) \) depends on its wave shape and can readily be determined by the program user.

3. Number of decades in frequency domain over which \( H(\omega) \) and \( V_g(\omega) \) must be defined - It is easy to judge the required no. of decades as the transfer function magnitudes decrease substantially at high frequencies. Thus 7 to 8 decades of frequency data points, starting at \( f_{\text{start}} = 1 \) Hz will
ensure reasonable accuracy without increasing computer costs too much for the test case studied. Integration between \( f = 0 \) and \( f_{\text{start}} \) where the frequency data points start is done separately, again assuming linear interpolation between 0 and \( f_{\text{start}} \). Therefore, we can start our frequency data at any decade. This is allowable as long as the output voltage \( \bar{V}(\omega) \) remains fairly constant and linear interpolation from zero to the starting frequency \( f_{\text{start}} \) does not cause appreciable deviations.

3. Input voltage wave form - An efficient and simple way to check the accuracy of the Fourier Transformation Program is to run it in a test mode where the transfer function is set to 1,

\[ H(\omega) = 1 \]

and to check how closely the output voltage in the time domain agrees with the input voltage \( v_g(t) \). In our test case, the known input voltage \( v_g(t) \) is a double exponential of the form

\[ v_g(t) = e^{-\alpha_1 t} - e^{-\alpha_2 t} \]  

(3-26)

where \( \alpha_1 = 0.17 \times 10^3 \text{s}^{-1} \) and \( \alpha_2 = 3.27 \times 10^6 \text{s}^{-1} \)

This input voltage matches exactly the output voltage thus obtained from our transformation program (see Figs. 13 and 14). In Fig. 13, in the time interval from 0 to 7\( \mu \)s step widths of

\[ \Delta t = 0.05 \mu \text{s} \]

and

\[ \Delta \omega = 20 \text{ pts/decade (log scale)} \]

were chosen. In Fig. 14, for time >10\( \mu \)s, the input voltage \( v_g(t) \) is essentially a single exponential decay, for which step widths of

\[ \Delta t = 0.1 \text{ ms} \]

and

\[ \Delta \omega = 20 \text{ pts/decade (log scale)} \]

were chosen.
Input voltage and calculated output voltage with $H(\omega) = 1.0 \angle 0^\circ$

$\Delta t = 0.05 \mu s$
$\Delta \omega = 20$ points/decades
$f_{\text{start}} = 1$ Hz
$f_{\text{end}} = 10$ MHz
Same test as in Fig. 13 from 0.1 to 15 ms

- $\Delta t = 0.1 \text{ ms}$
- $\Delta \omega = 20 \text{ points/decades}$
- $f_{\text{start}} = 1 \text{ Hz}$
- $f_{\text{end}} = 10 \text{ MHz}$

**Fig. 14**
4. Numerical problems with step function inputs - No problem of numerical instability were encountered when the field tests of the test line were simulated. The input voltage \( v_g(t) \) in this case is a double exponential wave (see eqtn 3-26). The computations were numerically stable for large values of decay constants, namely \( \alpha_1 \) and \( \alpha_2 > 10 \). However, many cases were run with step function inputs for checking purposes to debug the programme and to gain confidence before the duplication of field tests could be attempted. Serious numerical instability problems were encountered with pure step function inputs, which were then overcome by replacing the step function with an exponentially decaying function \( e^{-\alpha t} \). The decay parameter \( \alpha \) is chosen in such a way that this function is practically equal to a step function over the time span of interest.

For an input voltage step function
\[
v_g(t) = 1
\]
the voltage in the frequency domain is
\[
V_g(\omega) = \int_0^\infty 1 \cdot e^{-j\omega t} \, dt
= \frac{-1}{j\omega} \left|_0^\infty e^{-j\omega t} \right|
= \frac{1}{j\omega} - \frac{1}{j\omega} \lim_{t \to \infty} e^{-j\omega t} \quad (3-28)
\]
For time \( t \to \infty \), the second term of eqtn (3-28) is highly oscillating and is non-zero, which causes numerical instability in the Fourier Transformation Programme. However, this problem can be remedied by introducing a slow decay into the input voltage \( v_g(t) \) as in eqtn (3-12), namely
\[
v_g(t) = 1 \cdot e^{-\alpha t}, \quad t > 0
\]
from eqtn (3-12)
The input voltage in the frequency domain now becomes
\[
V_g(\omega) = \int_0^\infty e^{-\alpha t} e^{-j\omega t} dt
\]
\[
= -\frac{1}{\alpha + j\omega} e^{-(\alpha+j\omega)t} \bigg|_0^\infty
\]
\[
= \frac{1}{\alpha - j\omega} - \frac{1}{\alpha + j\omega} \lim_{t \to \infty} e^{-\alpha t} e^{-j\omega t}
\]

Now, for time to \( \rightarrow \infty \), the second term is no longer oscillating due to the presence of the decay factor \( e^{-\alpha t} \) in it, and goes to zero as \( t \to \infty \), or

\[
V_g(\omega) = \frac{1}{\alpha + j\omega}
\]

Furthermore, the single exponential decay voltage is better than a cut-off step function voltage (rectangular pulse) inasmuch as \( 1 \cdot e^{-\alpha t} \) has a smoother amplitude and phase angle spectrum than a rectangular pulse. Thus, fewer data points per decade are required to achieve the same degree of accuracy.

The problem of numerical instability with a step function voltage is therefore easily solved by introducing the decay factor \( \alpha \). Numerical experiments showed that \( \alpha > 10 \) will be good enough to ensure numerical stability. Note that for the case \( \alpha = 10 \), the deviation of the exponentially decaying input voltage

\[
v_g(t) = e^{-\alpha t}
\]

from the ideal step voltage is negligible for the time span of interest here. For \( t_{\text{max}} = 10\mu s \)

\[
v_g(t) = e^{-10 \times 10^{-5}} = 0.9999
\]

That is, the maximum deviation is less than 0.01% from the step input voltage at the upper limit \( t_{\text{max}} \) of the study.
CHAPTER IV

DUPLICATION OF FIELD TESTS

1. Doubling effect on open-ended line.

In the analysis leading to eqtn. (2-12), we have neglected the reflected voltage wave and obtained the modal voltage for the infinite line at a distance $x$ from the sending end.

$$V_{\text{mode}}^x (\omega) = V_{+}^x (\omega) e^{-\gamma x}$$  \hspace{1cm} (4-1)

This expression is not directly usable for the test case since we now have a transmission line of finite length which is open-ended at the receiving end terminal (see Fig. 8). The equations for the decoupled modal quantities are analogous to the equation of a single phase line, where the comparison between the infinite line and the open-ended finite line is well known. Thus, we can use the well known solution for the single phase case for voltages and currents \cite{14,24,29} in the modal domain,

$$V_{\omega}^x = V_{\text{mode}}^x \cosh \gamma x + Z_o I_{\text{mode}}^x \sinh \gamma x$$ \hspace{1cm} (4-2)

$$I_{\omega}^x = \frac{V_{\text{mode}}^x}{Z_o} \sinh \gamma x + I_{\text{mode}}^x \cosh \gamma x$$ \hspace{1cm} (4-3)

where $I_{\omega}^x$ and $V_{\omega}^x$ are the modal voltage and current at the sending end $x = 0$, and $I_{\text{mode}}^x$ and $V_{\text{mode}}^x$ are the modal voltage and current at the receiving end $x = l$.

For an open-ended line, $I_{\text{mode}}^l = 0$. Then we obtain from (4-2) and (4-3)

$$V_{\omega}^x + Z_o I_{\omega}^x = V_{\text{mode}}^x \cosh \gamma x + \sinh \gamma x$$

$$= V_{\text{mode}}^x e^{\gamma x}.$$  \hspace{1cm} (4-4)

i.e. $V_{\text{mode}}^x = e^{-\gamma x} (V_{\omega}^x + Z_o I_{\omega}^x)$.
From \( t = 0 \) to \( t < 2\tau \), no reflection has yet come back from the receiving end, and the conditions at the sending end are therefore the same as those of an infinite line during this time period,

\[
V_0^{\text{mode}} = Z_0 I_0^{\text{mode}}
\]  

(4-5)

(This relationship is no longer true at the sending end after \( t > 2\tau \), and is no longer true at the receiving end for \( t > 3\tau \).)

With substitution of eqtn (4-5) into eqtn (4-4), we get for the receiving end,

\[
V_+^{\text{mode}} = 2e^{-\gamma \lambda} V_0^{\text{mode}} = 2 V_+^{\text{mode}} e^{-\gamma \lambda}
\]

This is twice the obtained receiving end voltage for the infinite line at location \( x = \lambda \). Thus, there is a doubling effect in the receiving end voltage of the open-ended line in comparison with the infinite line.

Comparison with field measurements and other simulation results

The output voltage at the receiving end is plotted in Fig. 15 for the test case with the voltage doubling effect taken into account.

The arrival time of the first part of the voltage wave coincides closely with the time taken by electromagnetic waves (TEM propagation) in air, i.e. 277 \( \mu \text{s} \) for 83.212 km at a wave velocity of 3 km/\( \mu \text{s} \). On a transposed line, this first part of the wave would be associated with the positive sequence parameters, and the second part of the wave would correspond to the zero sequence wave. It can be observed that the wave velocity of the zero sequence mode is slower than that of the positive sequence mode.

Also the skin effect calculation with Galloway's formula gave slightly higher resistances (eg., \( \Delta R_{\text{pos}} \approx 0.67 \Omega \) and \( \Delta R_{\text{zero}} \approx 0.73 \Omega \) at 50 KHz, see chapter 1, section E) than the formula for tubular conductors. The
output voltage based on Galloway's formula will be slightly smaller than that obtained with the tubular conductor formula. This can be seen from Fig. 15. Galloway's formula gives results closer to field measurements than the tubular conductor formula, as expected. This is because the double exponential wave front contains high frequency components where Galloway's formula is more accurate. (See Chapter 1, Sections 4 and 5).

For comparison purposes, the field measurement results from Ametani and simulation studies by Groschupf are included in Fig. 16. The simulation results obtained with the methods described in this thesis compare favorably with the field measurement results (within 8%). Some probable causes of discrepancies between simulation and field measurements may be due to the following phenomena:

1) Assumption of uniform earth resistivity (200 Ω·m) -- An increase in earth resistivity will increase the zero sequence parameters and thereby increase attenuation and decrease wave velocity of the zero sequence voltage wave. Also, a homogenous earth is only an approximation of a stratified earth which will again cause some differences in impedance line parameter calculations.

2) Temperature of conductor -- We assumed a conductor temperature of 20°C. An increase in temperature will increase conductor resistance appreciably (e.g. 40% rise for increase of temperature to 120°C).

However, a difference between numerical and measured values of less than 8% is well within acceptable accuracy criteria for these types of studies.
Output voltage at receiving end of transmission line

Output voltage (p.u.)

Phase A:
A-- first part of voltage wave (pos. sequence on transposed line)
B-- second part of voltage wave (zero sequence on transposed line)

--- tubular conductor
------- Galloway's formula

Phase B

Phase C

Fig. 15

Time (μs)
Output voltage at receiving end of transmission line with field measurement and Groschupf’s simulation results

Output voltage (p.u.)

![Graph showing output voltage over time for different phases A, B, and C with various simulation results.]

Fig. 16
CHAPTER V

CONCLUSIONS

The attenuation and distortion of wave fronts on multiphase overhead transmission lines or underground cables was studied. A specific case of a Japanese 500 kV three-phase overhead line was chosen as a test example because field measurements were available for this line. The voltage response at the open-ended receiving end was simulated. The simulation results agreed very well with the field measurement results and with simulation results of another investigator.

Results obtained with their technique developed in this thesis will be useful for switching surge insulation co-ordination studies in power transmission systems. For instance, the technique could be used to calculate the wave front which could hit a transformer at the receiving end of the line. It should be realized, however, that this wave front would be modified by the transformer itself. This wave front modified by the transformer would be a worthwhile topic for future research. Depending on the rise time of the incident voltage wave, the transformer insulation may be more or less stressed. This is a problem of current concern in the electric utility industry.
## Appendix 1

**Transfer Function Program Listings**

### Main

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1001</td>
<td><code>MAIN</code></td>
<td>Program Start</td>
</tr>
<tr>
<td>1002</td>
<td><code>CALL</code></td>
<td>Subroutine Call</td>
</tr>
<tr>
<td>1003</td>
<td><code>END</code></td>
<td>Program End</td>
</tr>
</tbody>
</table>

### Laplace Transform

- **Program Listings**
  - **Transfer Function**
  - **Program Listings**
    - **Main Program**
      - **Subroutine Calls**
        - **Real Part**
        - **Phase**
      - **Input Data**
        - **Frequency Input**
      - **Output Data**
        - **Step Response**
      - **Program Comments**
        - **Complex Part**
        - **Impedance Matrix**
      - **Program Variables**
        - **Initialization**
        - **Iteration Loop**
      - **Conditional Statements**
        - **Frequency Range**
        - **Error Handling**
      - **Conclusion**
        - **Program Termination**

### Mathematical Expressions

- **Variables**
  - **Real**
  - **Imaginary**
- **Equations**
  - **Transfer Function**
  - **Input Data**
  - **Output Data**
- **Algorithms**
  - **Subroutine Calls**
  - **Program Flow**

### Notes

- **Program Structure**
- **Algorithm Efficiency**
- **Error Handling**
- **Documentation**

### References

- **Related Literature**
- **Previous Work**
- **Contributions**

---

**APPENDIX 2**

**Transfer Function Program Listings**

### Subroutine Listings

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td><code>SUBROUTINE DET</code></td>
<td>Determinant Calculation</td>
</tr>
<tr>
<td>2002</td>
<td><code>SUBROUTINE INV</code></td>
<td>Matrix Inversion</td>
</tr>
<tr>
<td>2003</td>
<td><code>SUBROUTINE DECAY</code></td>
<td>Decaying Exponential Function</td>
</tr>
</tbody>
</table>

### Additional Notes

- **Subroutine Calls**
- **Matrix Operations**
- **Functionality Expansion**

---

**APPENDIX 3**

**Transfer Function Program Listings**

### Iteration Listings

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3001</td>
<td><code>DO 10 J = 1, N</code></td>
<td>Loop Initialization</td>
</tr>
<tr>
<td>3002</td>
<td><code>DO 20 I = 1, M</code></td>
<td>Loop Initialization</td>
</tr>
</tbody>
</table>

### Conclusion

- **Program Summary**
- **Future Directions**
- **Acknowledgments**

---

**APPENDIX 4**

**Transfer Function Program Listings**

### Conclusion

- **Program Insights**
- **Technical Implications**
- **Application Scenarios**

---

**APPENDIX 5**

**Transfer Function Program Listings**

### Conclusion

- **Program Outcomes**
- **Impact Analysis**
- **Reproducibility**

---

**APPENDIX 6**

**Transfer Function Program Listings**

### Conclusion

- **Program Evaluation**
- **Feedback Mechanism**
- **Future Improvements**
FORMAT (/,'THE IMAGINARY PART OF IMPEDANCE MATRIX $Z_i^*$',/)

DO 10 J=1,N

PRINT 6, $(Z(I,J) \times Y(I,J))$

CALL IMULT $(A, Z, X, Y) = -Z^* X$

DO 90 J=1,N

PRINT 9, CH (I, J), J=1,3

CALL DCEIGN $(21 \times Y(I,J))$

PRINT 12, $(Y(I,J))$, J=1,3

PRINT 13, $(Z(I,J))$, J=1,3

PRINT 14, $(X(I,J))$, J=1,3

C *** PROPAGATION CONSTANT ES

C * VECTOR $E$ IS PURELY IMAGINARY

DO 10 J=1,N

PRINT 9. $(Z(I,J))$

CALL IMULT $(A, X, Y, Z) = -Z^* X$

DO 90 J=1,N

PRINT 9, $(Z(I,J))$

CALL DCEIGN $(21 \times Z(I,J))$

PRINT 12, $(Z(I,J))$

PRINT 13, $(Z(I,J))$

PRINT 14, $(Z(I,J))$

C COMPLEX EIGENVALUES $ER+JEI$ & COMPLEX EIGENVECTORS $VR+JVI$

C OBTAINED FROM SUBROUTINE DCEIGN

PRINT 11, $(E(I))$

PRINT 12, $(E(I))$

PRINT 13, $(E(I))$

PRINT 14, $(E(I))$

C *** PROPAGATION CONSTANT ES

C ** EIGENVALUES $ER$ & EIGENVECTORS $VR$ OR $VR$
<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0070</td>
<td>DO 120 J=1,3</td>
</tr>
<tr>
<td>0080</td>
<td>SUMA=0,0</td>
</tr>
<tr>
<td>0090</td>
<td>DO 120 J=1,3</td>
</tr>
<tr>
<td>0092</td>
<td>SUMA=SUMA+V(J,J)*V(J,1)*V(1,1)*V(1,1)</td>
</tr>
<tr>
<td>0093</td>
<td>121 CONTINUE</td>
</tr>
<tr>
<td>0094</td>
<td>122 CONTINUE</td>
</tr>
<tr>
<td>0096</td>
<td>DO 120 IH=1,5</td>
</tr>
<tr>
<td>0097</td>
<td>IH=1,5</td>
</tr>
<tr>
<td>0100</td>
<td>V{J,J}=V{J,J}/</td>
</tr>
<tr>
<td>0101</td>
<td>A{J,J}=A{J,J}/</td>
</tr>
<tr>
<td>0102</td>
<td>A{J,J}=A{J,J}/</td>
</tr>
<tr>
<td>0103</td>
<td>IH=1,5</td>
</tr>
<tr>
<td>0104</td>
<td>CALL CDEMM (KMI,1,3,Det,Cond)</td>
</tr>
<tr>
<td>0105</td>
<td>DO 590 J=1,3</td>
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<tr>
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**Note:** The code snippet appears to be a part of a FORTRAN program, possibly for a simulation or analysis related to terminal systems.
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*OPTIONS IN EFFECT: SOURCE, NOLINE, NOLIST, NUNDEF, NNUM, NMAP*
*STATISTICS: SOURCE STATEMENTS = 159, PROGRAM SIZE = 832K*
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**OPTIONS IN EFFECT**
- IFAC, NOSUB, NOSLICE, NODIME, NODMIME, NODMIME
- NOSLICE, NODMIME

**STATISTICS**
- NO STATEMENTS EXECUTED
- 60 STATEMENTS PERFORMED
- 620 PROGRAM SIZE

**NO ERRORS IN MHYMLT**

**NO STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.**

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**EXECUTION TERMINATED**

**END OF FILE ON ISDATAJARAN. CAUSES A RETURN TO MTS.**

**EXECUTION TERMINATED**

---

**SL FPREIMJARAN(1,40)**

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C T32

0016 | 110 FORMAT(//1X,'PHASE EXCITED',13) | 9.500 |        |
| 0017 | 111 FORMAT(13) | 9.600 |        |
| 0018 | C '1' SPECIFIED WHICH PHASE TO BE SMOOTHED | 10.000 |        |
| 0019 | A = TFRE(1) | 11.000 |        |
| 0020 | G = TFIM(1) | 12.000 |        |
| 0021 | 1 = 0.000 | 12.200 |        |
| 0022 | 2 = 0.000 | 12.400 |        |
| 0023 | 3 = 0.000 | 12.600 |        |
| 0024 | 4 = 0.000 | 12.800 |        |
| 0025 | 5 = 0.000 | 13.000 |        |

C T32

0026 | HP = HP | 20.000 |        |
| 0027 | AP = AP | 21.000 |        |
| 0028 | S1 = 28318530717958D5 | 22.000 |        |
| 0029 | STEG = 52 | 23.000 |        |
| 0030 | A1 = 2 | 24.000 |        |
| 0031 | A2 = 2 | 25.000 |        |

C T32

0032 | WRITE(7,13)FRE, TXT(1), PTF(1), PTF(2), PTF(2), PTF(3), PTF(3) | 32.000 |        |
| 0033 | S4 = Formula S5,2E-12, S5, S5, S5, S5, S5, 11S, S5, S5, S5, S5, | 32.200 |        |
| 0034 | GO TO 3 | 33.000 |        |
| 0035 | END | 35.000 |        |

ADDITIONS IN EFFECT: IN EQUATE, SOURCE, HULLIST, NODECK, LOAD, NOMAR

NO ERRORS IN MAIN

TRUE IN EFFECT: NAME = MAIN, LINECAT = NO, SOURCE STATEMENTS = 35, PROGRAM SIZE = 3522

NO ERRORS IN MAIN

EXECUTION TERMINATED

$R = LOAD 7TFJAPAN, 2*SOURCE = TFREIMJAPAN(2) 5 = SINK*

EXECUTION BEGINS

PHASE EXCITED 1
## MICHIGAN TERMINAL SYSTEM FORTRAN G(+)360

### MAIN

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### APPENDIX 3

### FOURIER TRANSFORMATION PROGRAM LISTINGS

```fortran
C PROGRAM FOURIER INPUT
C THIS INPUT IS FOR TRANSFER FUNCTION
Pr(r) = O.OO
C
C INPUT FOR 1.000 MS INCIDENT WAVE
C
C READ IN INCIDENT WAVE
C
C CHANGE OF RING (RPN) STATEMENTS IF DIMENSION IS CHANGED **************
C
C READING AMPLITUDE AND PHASE SPECTRA OR PRESETTING THEM **************
C
C T.F. IS READ IN
C
C PT(1) = 0.00
C THIS INPUT THE TRANSFER FUNCTION EXP(-GAMMA t) AND RETAR
```

### PAGE 001

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```plaintext
MICHIGAN TERMINAL SYSTEM FORTRAN G(133X)

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<th>Line</th>
<th>Statement</th>
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</thead>
<tbody>
<tr>
<td>0045</td>
<td>18. READ (12.13)M</td>
</tr>
<tr>
<td>0046</td>
<td>IF (blank,11.1,0) GO TO 16</td>
</tr>
<tr>
<td>0047</td>
<td>GO TO 18</td>
</tr>
<tr>
<td>0051</td>
<td>15 IF (10,11,0) GO TO 24</td>
</tr>
<tr>
<td>0052</td>
<td>C THIS IS SKIPPED FOR OPTION 1</td>
</tr>
<tr>
<td>0053</td>
<td>C $1 IS FIRST INPUT VOLTAGE VALUE</td>
</tr>
<tr>
<td>0054</td>
<td>N=1</td>
</tr>
<tr>
<td>0055</td>
<td>DTIN=.05040</td>
</tr>
<tr>
<td>0056</td>
<td>T=0.05040</td>
</tr>
<tr>
<td>0057</td>
<td>RT%=SIN (ALPHA1+ALPHA2) / (ALPHA1-ALPHA2)</td>
</tr>
<tr>
<td>0058</td>
<td>C ALPHA1=INPUT VOLTAGE IS SET TO POSITIVE</td>
</tr>
<tr>
<td>0059</td>
<td>N1=1</td>
</tr>
<tr>
<td>0060</td>
<td>IF (10,11,0) GO TO 24</td>
</tr>
<tr>
<td>0061</td>
<td>DTIN=</td>
</tr>
<tr>
<td>0062</td>
<td>T</td>
</tr>
<tr>
<td>0063</td>
<td>RT%=SIN(ALPHA1+ALPHA2) / (ALPHA1-ALPHA2)</td>
</tr>
<tr>
<td>0064</td>
<td>K%=S2</td>
</tr>
<tr>
<td>0065</td>
<td>R1=S2*EXP(ALPHA1+ALPHA2) / (ALPHA1-ALPHA2)</td>
</tr>
<tr>
<td>0066</td>
<td>C INPUT VOLTAGE 92 IS PRESENT IN PLACE OF READ-IN</td>
</tr>
<tr>
<td>0067</td>
<td>IF (10,11,0) GO TO 24</td>
</tr>
<tr>
<td>0068</td>
<td>C S=FLOATN/TIN/0.0</td>
</tr>
<tr>
<td>0069</td>
<td>C S2=EXP(-ALPHA1+ALPHA2)</td>
</tr>
<tr>
<td>0070</td>
<td>GO TO 18</td>
</tr>
<tr>
<td>0071</td>
<td>N1=</td>
</tr>
<tr>
<td>0072</td>
<td>N2=</td>
</tr>
<tr>
<td>0073</td>
<td>GO TO 18</td>
</tr>
<tr>
<td>0074</td>
<td>N1=</td>
</tr>
<tr>
<td>0075</td>
<td>WRITE (6,812) (TIN=</td>
</tr>
<tr>
<td>0076</td>
<td>WRITE (6,812) (TARRAY(11,1),N1=1,200)</td>
</tr>
<tr>
<td>0077</td>
<td>WRITE (6,812) (TIME=</td>
</tr>
<tr>
<td>0078</td>
<td>WRITE (6,812) (VOUT=</td>
</tr>
<tr>
<td>0079</td>
<td>WRITE (6,812) (REAL V, IMAGINARY AMPLITUDE=ABSOLUTE)</td>
</tr>
</tbody>
</table>
```

C change following statement if dimension is changed ***************

IF (AT,GT,1500) GO TO 97

IF (AT,GT,1500) GO TO 97

IP = IP + 1

ISTORE = ISTORE + 1

M = M + 1

S1 = S1 + 1

IF (1P,GT,18) GO TO 70

GO TO 68

M = M + 1

GO TO 68

THE STEP IN OUTPUT IS SANK AS INPUT DELAYED BY T, E11

IF (I, LE, MAX), GO TO 63

WRITE (I, 72) (I(II), I(1), KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

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WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

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WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

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WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

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WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

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WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

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WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)

WRITE (I, 71) (I(II), 11, KT)
<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>COOP</td>
<td>-0.7</td>
<td>Change Order Processing</td>
</tr>
<tr>
<td>NOF</td>
<td>-0.6</td>
<td>No Output File</td>
</tr>
<tr>
<td>ROMAP</td>
<td>-0.7</td>
<td>Read Only Mode Access Point</td>
</tr>
<tr>
<td>NOSTMT</td>
<td>0.1</td>
<td>No Statement Flagged</td>
</tr>
<tr>
<td>NUMFD</td>
<td>0.0</td>
<td>Number of Main File</td>
</tr>
<tr>
<td>PRG</td>
<td>0.5</td>
<td>Program Size</td>
</tr>
<tr>
<td>EXECUTION</td>
<td>BEGINS</td>
<td>Execution Begins</td>
</tr>
<tr>
<td>LOAD</td>
<td>2.5</td>
<td>Load Constant</td>
</tr>
<tr>
<td>SOURCE</td>
<td>6.5</td>
<td>Source Constant</td>
</tr>
<tr>
<td>EXECUTION</td>
<td>REGIONS</td>
<td>Execution Regions</td>
</tr>
<tr>
<td>TIME</td>
<td>0.1</td>
<td>Time Constant</td>
</tr>
<tr>
<td>INPUT TIME</td>
<td>0.5</td>
<td>Input Time</td>
</tr>
<tr>
<td>AREA</td>
<td>0.9</td>
<td>Area of Input</td>
</tr>
</tbody>
</table>

**Statistics**
- Source Statements: 229
- Program Size: 74864
- No Errors in Main

No statements flagged in the above compilations, execution terminated.


