AN EXPERIMENTAL AND THEORETICAL INVESTIGATION OF THE STATISTICS OF AVALANCHE BREAKDOWN IN SILICON

by

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ABSTRACT

Measurements are made on Si p - n junctions under reverse bias in the unstable avalanche breakdown region to determine the statistics of the current pulses, and their effect on the measured mean current - mean voltage characteristics.

It is found that the slope and shape of the measured mean current - mean voltage is not unique, but depends on the external resistance in series with the power supply. The mean current - mean voltage characteristics are also found to display turnover, with the voltage at turnover and the magnitude of the maximum negative resistance both increasing with increasing resistance R.

Measurements are also made to determine the relation between the current pulse amplitude I_1 and the voltage V_1 across the diode during the avalanche. The results show that over the range where data are available, a linear relation exists between reverse voltage and current pulse amplitude. The intersection of this line with the voltage axis is defined as the breakdown voltage V_B, and its slope is the conductance denoted by g.

A simple model consisting of a random switch in series with a two terminal device having the property that for $V_1 < V_B$, $I_1 = 0$ and for $V_1 > V_B$, $I_1 = g(V_1 - V_B)$ is proposed.

An avalanche initiation transition probability which increases monotonically with increasing voltage in excess of V_B, and an extinction transition probability which decreases monotonically with increasing excess voltage are postulated. The experimentally derived probability functions satisfy the chosen theoretical functional dependence on excess voltage to within experimental error.

Assuming that the "switching" process is Markoffian, an expression for the fraction of the time during which the avalanche occurs is derived. Using this relation and the pulse amplitude data, a theoretical mean current - mean voltage characteristic is obtained which is in good agreement with the experimental curve.

The predicted mean pulse rate curve is also experimentally verified.

Illumination experiments show that the initiation transition probability is directly proportional to the number of carriers entering the avalanche region; the discharge probability for an electron entering the avalanche region is independent of illumination and carrier density in the breakdown region.

The apparent predominance of a form of "surface leakage" in the reverse current makes it impossible to obtain data regarding the discharge probability per incoming electron from temperature variation experiments.

Measurements of the spectral density of the current fluctuations over a wide range of frequency confirms the simple form $S_I(\omega) = S_I(0)/(1+\omega^2\tau^2)$ associated with a Markoffian process.

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CHAPTER I

INTRODUCTION

A considerable amount of research has been done by various workers on "breakdown" in Si and Ge. It has been shown that breakdown in Si and Ge p-n junctions is avalanche in nature, and in many respects similar to the discharge phenomena encountered in gas tubes. Using current multiplication data, experimental plots of the dependence of the ionization rate α_i (F) for both holes and electrons on the electric field have been obtained. Fair agreement between these and theoretical predictions have been reported.

Observers have also reported the appearance of a peculiar form of "noise" in the form of apparently random current pulses in the avalanche breakdown region. It has been shown that the entire current at the onset of breakdown is due to these pulses, with no other contribution present. However, very little detailed work has been done to determine the statistics, and the exact nature of these current pulses as a function of the applied voltage, temperature, carrier injection by illumination, etc., and the combined effect of these on the macroscopically observed mean current - mean voltage characteristic. It is largely around this general topic that this work is centered.

The following are some of the problems which prompted the present investigation:

1. The slope of the mean current - mean voltage characteristic in the breakdown region varies from diode to diode. It is of some interest to know just what factors determine the slope and general shape of the mean current - mean voltage characteristic in the breakdown region.

2. General observations have been made by observers (See McKay 1954) regarding the dependence of the current pulse amplitude and the mean rate on the mean voltage and mean current, but few details have ever been presented. Since the current in the breakdown region is due entirely to the current pulses which are statistical in nature, it is important to know how the observed results are affected by purely circuital properties such as external resistance and capacitance. Before this is known, little reliable information can be obtained regarding the basic properties of avalanches in Si.

3. It is known that illumination of a p-n junction under reverse bias conditions causes an increase in the reverse saturation current by increasing the density of carriers in the depletion region. The effect of this increase in carrier density on the mean current - mean voltage characteristic and the statistics of the current pulses in the avalanche region has not been published. Some effort is made in the present investigation to determine this dependence.

4. A study of the effect of temperature on the statistics of the current pulses and the macroscopic mean current - mean voltage characteristic is also made.

5. Rose (1957), in a qualitative theoretical discussion suggests that the avalanche region in a semi-conductor diode consists of two negative resistance cathode fall regions separated by a low field positive resistance region. Because of the magnitude of the positive resistance in the low field region, no evidence of negative resistance is likely to be detected from the diode terminals. It is of some interest to see if negative resistance does in fact occur in the avalanche region.

6. Rose also made some qualitative suggestions regarding the initiation and extinction probabilities of avalanches. No experimental work has been done up to this time to determine the functional dependence of these probabilities on applied voltage, temperature, etc.

7. A model is needed to account for the statistical nature of the current pulses, and the shape of the observed mean current - mean voltage characteristic in the avalanche region.

This investigation is limited to the study of a small number of p-n junction Si diodes whose properties in the avalanche region are found to be representative of other p-n junctions under similar conditions - i.e., they all displayed a "sharp" increase of reverse current for small increase in reverse bias at breakdown, and displayed the typical current pulses at the onset of breakdown. The study was confined to current values which correspond to the first sequence of current pulses, and hence to discharges through a single patch in the p-n barrier.

CHAPTER II

SUMMARY OF PREVIOUS WORK ON BREAKDOWN IN P-N JUNCTIONS

Zener breakdown

When a reverse bias of sufficient magnitude is applied to a p-n junction, a phenomenon known as "breakdown" occurs. This condition is characterized by a large and abrupt increase in current with a small increase in reverse bias. The phenomenon is a reversible and reproducible one and does not result in any structural change in the crystal itself provided the current is limited to prevent excessive power dissipation.

Zener (1934) proposed a theory for the release of carriers from the valence band into the conduction band based on internal field emission to explain breakdown in certain dielectric solids. The theory indicated that electrons could be excited from the valence to the conduction bands by high electric fields. His formula giving the rate at which electrons pass from one energy band to the next because of excitation by the electric field F was:

$$\gamma = \frac{e a_1 F}{h} \exp - \left\{ \frac{\pi^2 m^* a_1 \mathcal{E}^2}{h^2 \, |eF|} \right\} \qquad (1)$$

where $\mathcal{E}$ is the energy gap between the two bands, a_1 is the lattice period, m^* the mass of an electron and e its charge. This expression shows that under the field emission mechanism, breakdown can not occur until a certain critical field is reached, and upon reaching this field, the electron density in the conduction band increases very rapidly with increasing electric field.

McKay et. al. (1951) extended Zener's theory so that it was applicable to semiconductors. If the field is uniform over a certain region in the junction, and the voltage drop across the region is V, the Zener current density is given by:

$$I = V \exp\left(\alpha - \frac{\beta}{F}\right) \text{ amps./cm}^2. \tag{2}$$

where F is the electric field in the junction and α and β are constants which depend on the properties of the crystal.

This theory was for many years thought to apply to Si and Ge p-n junctions (e.g. Pearson and Sawyer (1952)) until McKay and McAfee (1953) showed that charge multiplication took place in some p-n junctions at pre-breakdown voltages. Cascade multiplication is not possible under the Zener theory.

In a later study, Miller (1955) showed that for field emission to take place in step junctions, the breakdown voltage should be inversely proportional to the net impurity concentration on the higher resistivity side of the junction. In the range of resistivity where the mobility is essentially constant with respect to the electric field, the breakdown voltage V_B should be proportional to the resistivity. Miller found that this relation was not experimentally verified for any Ge junctions, even very narrow ones. McKay's (1954) data shows the same to be true for Si alloyed junctions. Knott's et. al. (1955) work on Ge step junctions was in substantial agreement with that of Miller down to resistivities of the order of 0.5 ohm-cm.

The observed impulsive nature of the current in p-n junctions at the onset of breakdown reported by McKay (1954) could not be understood in the light of Zener's theory either. Zener merely predicts a large smooth increase in the release and flow of carriers under high field conditions with no collective process such as this inferred.

Collision innization by electrons

Certain suggestions for another possible mechanism were envisaged in the work of Seitz (1949). He concluded that the Zener theory was not a likely one in typical dielectric solids because, first of all, it predicted a breakdown voltage which was about ten times too high, and secondly, it could not account for the fact that the breakdown field decreases with decreasing temperature in the range below room temperature in some dielectrics as first reported by Buehl and von Hippel (1939). He favoured a form of collision ionization where one initial electron would produce another by impact ionization; the pair then produce a second pair and thus going through n generations giving 2^n free electrons in the end. This mechanism was, however, inadequate to explain the "steepness" of the current - voltage characteristic at breakdown observed in Si p-n junction diodes.

Avalanche breakdown

Up to this time, no clearly defined regenerative mechanism was known that would give us this distinct breakdown characteristic.

However, in 1953, McKay and McAfee's work on charge multiplication in Si and Ge p-n junctions showed that electrons and holes can both ionize, and with approximately equal ionization rates. Using this fact, McKay (1954) applied to Si p-n junctions a modified version of the Townsend β theory in which positive ions can themselves produce electron - ion pairs by collision (See e.g. Loeb, 1939).

Using a simplified plane geometry for the junction, and assuming equal ionization rates for both holes and electrons, he arrives at an expression

$$1 - \frac{1}{M} = \int_0^W \alpha_i \, dx \qquad (3)$$

where M is the multiplication factor which is defined as the ratio of the number of electrons leaving the breakdown region to the number entering. Breakdown is defined as the point where M becomes infinite.

In applying this expression, three assumptions are implicit in the development:

1. The ionization rate α_i (F) is a function of F only.

2. Recombination while the carriers traverse the junction can be neglected, because the transit time is orders of magnitude less than the recombination time.

3. The space charge effects of the carriers in the junction are negligible, since the densities in the region are likely to be small compared with the fixed ionic space charge responsible for the field therein.

He solved equation (3) involving α_i for two idealized field configurations developed by Shockley (1949), namely, the step junction, and the linear gradient type junction. A step junction is one in which the impurity concentration varies abruptly from n to p type, and the linear type junction one in which we have a linear transition from n to p type material. The following relations exist between the field, voltage and junction widths for the two junction types. For the step junction we have

$$F = F_M \left(1 - \frac{X}{W}\right) \qquad F_M = 2\ V\ W^{-1}$$

$$\text{and} \quad W = W_1 V^{1/2} \tag{4}$$

where F_M is the maximum field in the junction, V is the voltage across the junction, W is the junction width and W_1 is the width constant defined by

$$W_1 = \left[\frac{1.317 \times 10^7}{N_D - N_A}\right]^{1/2} \text{ for a Si step junction.}$$

For the linear gradient junction,

$$F = F_M \left[1 - \left(\frac{2x}{W}\right)^2\right], \quad F_M = \tfrac{3}{2} V W^{-1}$$

$$W = W_1 V^{1/3} \tag{5}$$

where W_1 here is the width constant for the linear gradient type junction, the other symbols having the same meaning as for the step junction. The resulting expression which enables α_i (F) to be deduced from experimental observations of M comes out to be:

$$\alpha_i (F_M) = \frac{2}{W_1^2} \frac{d(1 - \frac{1}{M})}{dF_M} - \frac{4}{W_1^3} \left(1 - \frac{1}{M}\right) \frac{dW_1}{dF_M} \tag{6}$$

A similar expression was obtained for the linear gradient type junction. This expression yields two alternative methods of obtaining $\alpha_i (F_M)$, namely, measuring M vs F_M for the same junction, or measuring F_B, the breakdown field, vs the junction width constant for various junctions. The multiplication experiment was done by injecting carriers through alpha particle bombardment of the junction. A point by point plot of $\alpha_i(F_M)$ vs F_M was then made using junctions of both types. The results show that $\alpha_i(F)$ at first increases rapidly with the field F, but levels off for fields greater than about 500 K.V./cm. It should be noted that the values of $\alpha_i(F)$ obtained were only a result of a number of indirect measurements. The dependability of these values hinges largely upon the validity of the assumptions used.

Miller (1955) made some measurements similar to those of McKay on Ge p-n junctions. He assumed that electrons and holes will in general have different ionization rates. His work was done on p-n-p and n-p-n type transistors, and showed that the ionization rates for holes and electrons in Ge differed by about a factor of two, with holes having the higher rate. In a recent paper, Miller (1957) showed electrons have a higher ionization rate than holes in Si p-n junctions.

Theoretical calculations of the ionization coefficient

Wolff (1954) made some theoretical calculations of $\alpha_i(F)$ using methods similar to those applied to gases. His work consisted largely of solving Boltzmann's equation for the motion of carriers in a high field taking into consideration the effect of electron - phonon scattering and pair producing collisions on the distribution function. The modified distribution function is then used to calculate α_i, The final solution involves parameters such as the mean free path and the threshold energy which can not all be experimentally determined. However, the only one which had a significant effect on the solution was the mean free path of electron - phonon scattering under high field conditions. Wolff obtained this value by matching his curve to McKay's curve at a single point, and evaluating the theory on the basis of the fit obtained for the remainder of the curve. Reasonably good agreement with McKay's data was obtained for high values of F, but significant discrepancies occured at lower field values. Miller's (1955) data gives much better agreement with Wolff's theoretical curve than does McKay's.

In contrast to Wolff's theory based on the assumption that carriers interact largely with optical phonons in the crystal lattice, Groschwitz (1956) developed a relation for the rate of ionization, $\alpha_i(F)$, assuming that the chief mode of interaction was between electrons and acoustical phonons. He considered the semiconductor as a gaseous mixture of phonons and electrons, and using the simplifying assumptions that the energy exchange during each collision of an electron with the phonon was small compared to the average energy of the electrons, and that the scattering of electrons with phonons is essentially isotropic, (i.e. the effective electron mass is much less than the effective phonon mass) he arrives at an expression the two asymtotic forms of which are

$$\alpha_i (F) \simeq 2 \frac{C}{\sqrt{3\pi}} \left(\frac{M^*}{m^*} \right)^{3/2} \frac{e}{Fi} \exp \left(- \frac{3m^* Fi^2}{M^* F^2} \right) \qquad (7)$$

for low field F and

$$\alpha_i (F) \simeq \frac{C}{2} \frac{M^*}{m^*} eF \qquad (8)$$

for high fields. The function for α_i for low fields given by eqn. (7) shows a very rapid rise with increasing F. The symbols used are:

M^* is the effective mass of the acoustical phonon.

m^* is the effective mass of the electron.

l is the mean free path length of electrons.

F_i is the ionization field which is given by $\frac{U_i}{l}$ where U_i is the ionization potential.

C is a constant.

The value of Fi is difficult to determine from the properties of the lattice alone. He obtained a value of 5×10^5 V/cm for Si using McKay and McAffee's (1953) data. The constant C is again obtained by matching the theoretical curve to McKay's experimental curve at one point. The fit obtained is somewhat better than that of Wolff. He concluded that for a more exact treatment, scattering by both optical and acoustical modes would have to be considered simultaneously. Both Groschwitz and Wolff's theory do display the essential features of the α_i(F) dependence on F given by McKay, in spite of the fact that their approximations may have been rather crude.

Visible light emitted at breakdown

Another interesting aspect of avalanche breakdown was first observed by Newman (1955). He reported that for low voltage breakdown diodes, ($<$ 100 volts) visible light was emitted from the junction during breakdown. After a more careful investigation of the phenomenon, this light was found to come from small patches (about 10μ in d) at the edge of the junction. Also, different light spots appeared at different current levels, and the sequence was entirely reproducible. He also noted that when a freshly etched surface was scratched, the current - voltage breakdown characteristic became "softened" and light spots appeared where the scratch had taken place. This observation suggests that surface breakdown may occur before bulk breakdown, although it is not conclusive.

Chynoweth and McKay (1956) demonstrated that light was emitted from those regions where breakdown occurred. Their work was done on a specially prepared surface junction formed by polishing a slab of Si of known conductivity to optical quality and then diffusing an appropriate impurity into the surface of the crystal to a controlled depth. They found that the appearance of each spot of light was correlated with the appearance of a new sequence of characteristic current pulses. The emitted light was identified with radiative recombination and interband transitions of energetic carriers in the breakdown region.

Current Pulses during avalanches breakdown

These peculiar current pulses, the study of which this work is largely devoted to, have been known to appear at the onset of breakdown in certain Si p-n junctions for some time. (e.g. Pearson and Sawyer 1952). The pulses usually appear clipped and have a rather uniform spectrum. These features distinguish this form of "noise", as it is commonly called, from the form normally encountered in semiconductors. The noise in semiconductors usually has a noise spectrum which varies inversely as the frequency.

McKay (1954) made a more careful study of these current pulses and found the following facts to be true:

1. All the pulses were in the same direction.

2. The rise and decay times were shorter than .02 μsec., the rise time of the amplifier used.

3. The pulses were all of the same amplitude but seemed to vary in length in a random manner.

4. The tops of the pulses appeared flat, that is, the amplitude was essentially constant throughout the pulse lifetime.

5. The distance between pulses varied in a random manner.

His observations of the variation of the pulse character with applied voltage for a typical diode showed that when the pulses first appeared, they were very short ($\lesssim$.1 μsec.) and about 25 μamp. in amplitude. As the voltage was increased further, they rapidly increased in length, the time between pulses being correspondingly shortened, until eventually they appeared to be "on" most of the time. During this time, the aplitude of the pulses had increased to about 80 μamps. At this point, a new sequence of pulses appeared. This set evolved in the same way as the first. Several such sets of current pulses were observed in the same diode before currents of destructive proportions were reached. Another very significant result reported by McKay was that all the current in the breakdown region was carried by these current pulses.

McKay interpreted these observations as follows:

The first set of pulses represents a region in the junction breaking down. At this point, the breakdown is bistable, that is, either "on" or "off". Thermal fluctuations can switch it from one state to the other. As the voltage across the diode is increased, it prefers the "on" position, and eventually, if the voltage is high enough, it will remain "on" all the time. The next sequence corresponds to a new region in the junction breaking down, thus going through a similar cycle.

All the current in the breakdown region is accounted for by an increase in the current through the regions already broken down together with the current increase resulting from the successive breakdown of new regions.

<u>Rose's discussion of the avalanche mechanism</u>

Rose (1957) in a qualitative discussion of avalanche breakdown in Si shows that these very small regions in many respects resemble the plasma in a gas discharge tube. He refers to these regions as microplasmas. In Si diodes, these microplasmas consist of a high field ionization region at either side of the junction within the breakdown region, each of which closely resembles the cathode fall region in a gas tube. These are regions of negative resistance separated by a region of low field positive resistance. The effect of this positive resistance is to blank out any evidence of the negative resistance as seen from the diode terminals. He also suggests a mechanism which could account for the observed extinction of the microplasma. The statistical fluctuation of carrier densities in the microplasma occurs in such a way as to cause a decrease in the breakdown current through the region. This would require the voltage across it to rise, but the internal capacity of the diode prevents it from doing so resulting in a possible extinction of the microplasma. Rose's crude calculations of the statistical fluctuations of the ionization show that they occur at approximately the correct rate to account for the duration of the microplasmas.

Surface breakdown effects

Many Si and Ge p-n junction diodes have been known to break down at voltages lower than required for bulk breakdown and, moreover, the breakdown voltage was sensitive to surface treatment. Certain experiments (e.g. Miller 1955) have shown this to be the result of "surface breakdown".

In a theoretical discussion of the problem, Garrett and Brattain (1956) used a model consisting of an ideal p-n junction with the surface of the higher resistivity near the junction covered with surface charge having the same sign as the ionized body impurity. The origin of this charge is not clearly understood, but there is some evidence to show that it is chemical in origin. Because of these surface charges, the field strength near the surface of the junction will be greater than in the bulk material resulting in a narrowing of the junction width at that point. The consequence of this fact is a lowering of the breakdown voltage below that of bulk breakdown. Whether or not breakdown in any particular diode is of the bulk or surface type depends on the surface charge density, and the dielectric constant of the surrounding medium. Garrett and Brattain also showed that surface breakdown, like body breakdown is an avalanche phenomenon.

CHAPTER III

THE DEVICES STUDIED, APPARATUS AND TECHNIQUES USED

Devices investigated

The following types of commercially made Si p-n junction diodes were used in the present investigation:

a) Raytheon type IN303 (CK 738) diodes.

These diodes were made by bonding an Al wire to an n-type Si die of resistivity from 3 - 6 ohm-cm. The finished product was hermitically sealed in plastic. A considerable amount of work was done on these diodes, but all of the samples of this type were deemed unsuitable for the present investigation for the following reasons:

1) The current - voltage characteristic displayed a relatively high pre-breakdown current compared to other similar devices of different make (e.g. 0.1 microamps compared to 0.005 microamps for most Hughes 6005 series at room temperature.)

2) The reverse current also failed to saturate, but showed a gradual increase of current with increasing voltage until the breakdown voltage was reached.

3) The breakdown characteristic was not sharp.

4) It was not possible to obtain consistent data on the voltage-temperature relationship at constant current in the breakdown region. The measurements showed so much scatter that no reliable curve could be fitted.

5) There was evidence of current drift as well as the appearance of a peculiar form of hysteresis in the reverse current - voltage characteristic. Several attempts were made to eliminate these effects, but all proved ineffective. Their presence made accurate and reproducible experimental work impossible.

This peculiar form of hysteresis is thought to be a form of "surface leakage" similar to that reported by Eriksen and Statz (1957) of Raytheon. They showed that the presence of certain gases and water vapour on the surface of the diode caused a marked departure of the observed diode current - voltage characteristic from the ideal. They discarded the possibility of ionic conduction because an experiment designed to detect this failed to show any evidence of the same. They postulated an anomalous surface conductance due to absorbed molecules of the ambient which gave rise to a net charge on the surface. It appears that the plastic encapsulation is not adequate in guarding the junction against the penetration of such ambients as water vapour, etc.

b) Hughes HD6005 series diodes.

Of all the diodes tested, these were by far the most satisfactory for the present investigation. Like the Raytheon diodes, these were also made by fusing Al to n-type Si of known resistivity. As the temperature is raised during the fusion process, the Al wire melts disolving a small amount of Si upon which it has been carefully positioned. When the temperature is reduced,

Fig. 1

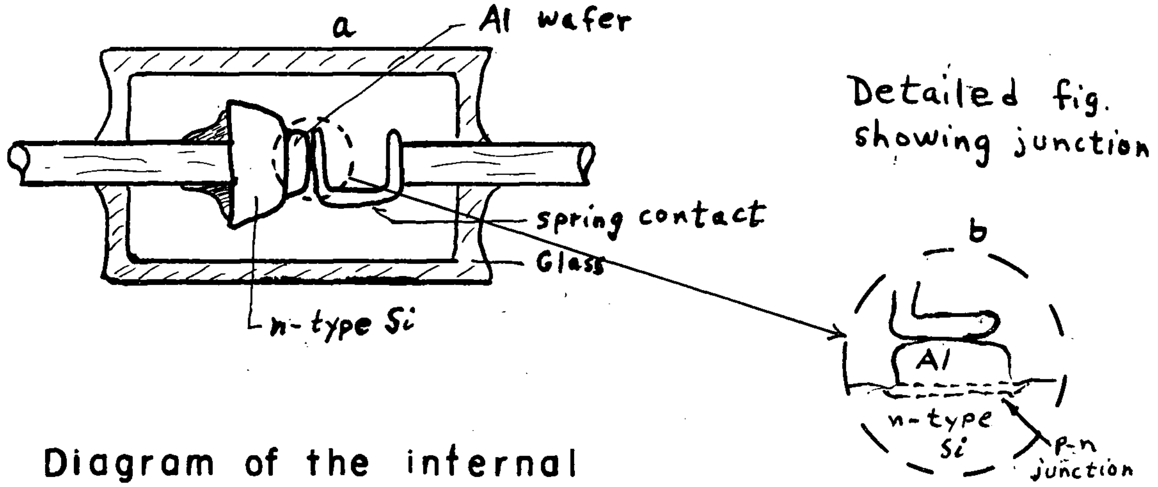


Diagram of the internal structure of the diode.

the molten solution freezes forming an Al - Si eutectic mixture whose freezing point is approximately 577°C. As the cooling process goes on, a small amount of Si together with enough Al to make it p-type is rejected from the solution forming a narrow re-grown region about 0.001" thick. This p-type layer on an n-type Si wafer forms a rectifying p-n junction. The junction is hermetically sealed in a small glass capsule which is impervious to moisture and gases. (See Fig. 1 a and b for detailed.)

Unlike the Raytheon diodes, these "saturated" well, and had a much sharper characteristic. The saturation current was about .005 microamps at room temperature for most of the diodes. The breakdown voltages ranged from about 40 - 70 volts. Using Miller's (1957) data for the net impurity center density vs V_B, and noting that the electron mobility in Si is about 1200 cm^2. per volt sec., it can be shown that resistivity of the n-type material in these diodes ranges from about 0.3 ohm-cm for V_B = 40 volts to 0.5 ohm-cm for V_B = 60 volts. No undesirable drift or hysteresis effects were encountered while using these diodes. The opaque black paint coating on the glass could be removed without difficulty making it possible to illuminate the junction. All of the experimental work was done on these diodes.

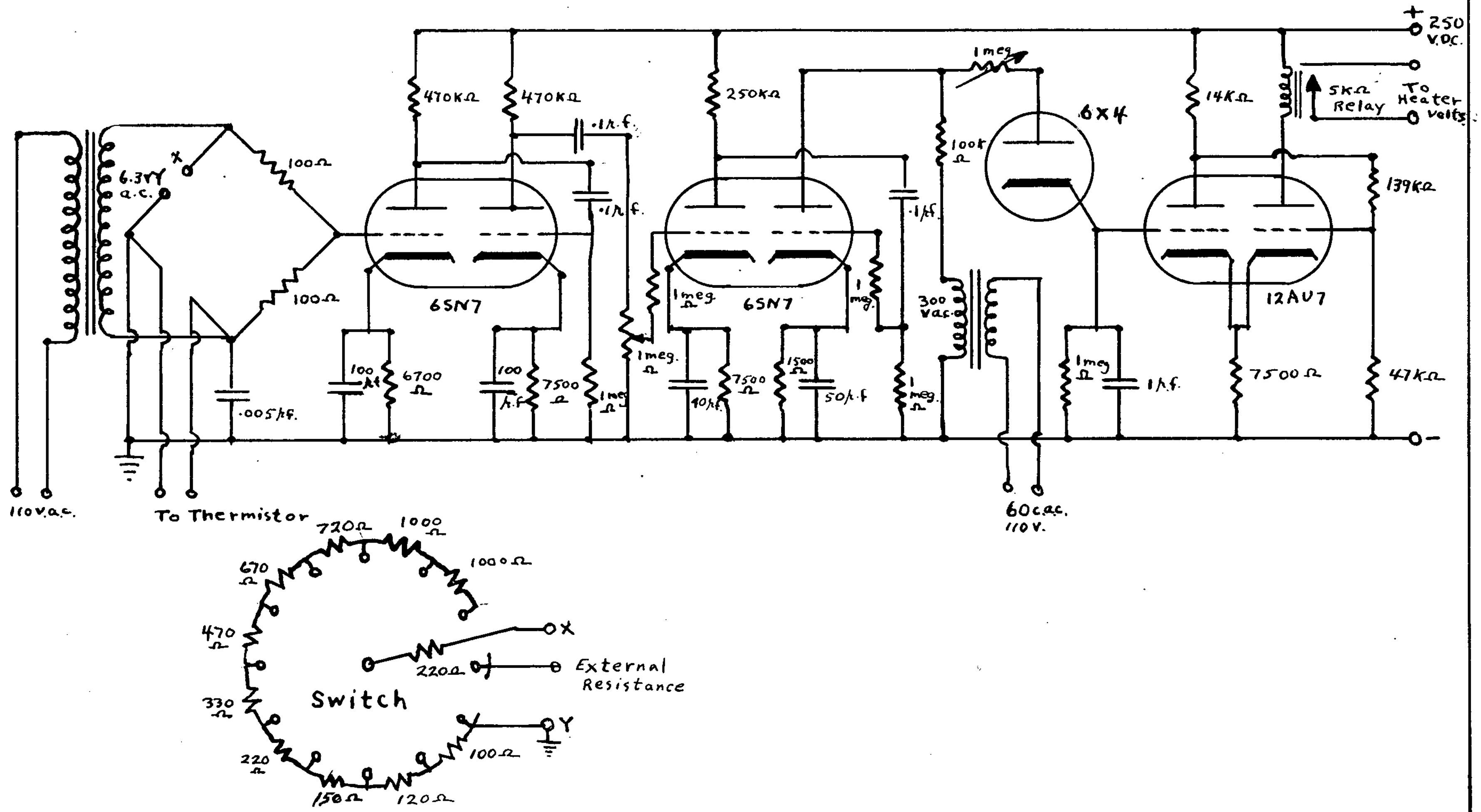
<u>Constant temperature bath</u>

Because of the sensitivity of the diode characteristics such as the saturation current and breakdown voltage to temperature change, it was necessary to find some way of keeping the temperature constant to a fairly high degree of accuracy over long periods of time. An example of this sensitivity is given by the fact that the breakdown voltage of most of the Hughes diodes increased by more than 0.06 volts per deg. C. temperature increase, and the pre-breakdown current by about 5% per deg. C. For this purpose, an electronic temperature controller was built. Because of its sensitivity and relative simplicity compared to continuous control methods, a bi-stable system was used.

The device consisted of an a.c. Wheatstone bridge with a 6.3 volt 60 cycle voltage source. In one arm of the bridge was a thermistor which acted as the temperature sensitive element with a large negative temperature coefficient of resistivity. The other arm of the bridge consisted of a series of fixed wire-wound resistances which could be step-wise adjusted to correspond to bridge balance against the thermistor at a number of fixed constant temperatures ranging from 25°C to 100°C. The stray reactance in the bridge was carefully balanced out so that a good null was obtained with a reversal of phase on passing from one side of balance to the other. The signal from the bridge was amplified and applied to a phase sensitive detector consisting of a triode with a 300 volt 60 cycle supply voltage. Both the bridge and detector plate voltages were derived from the a.c. mains. The output voltage across the detector tube depended on the amplitude of the input signal and its phase with respect to the plate supply voltage. A fraction of the detector plate current was rectified and filtered giving a variable positive bias having an average value of about 95 volts with a total excursion of about 8 volts on either side of this value. This bias was greater or less than the mean value depending on whether the thermistor was too cold or too hot. The positive bias controlled a bi-stable

PLATE I

TEMPERATURE CONTROLLER



Schmidt trigger circuit so adjusted that a bias change of about 1 volt would cause it to change from one stable state to the other. A relay as the load of one of the tubes in the trigger circuit turned the heater current on or off as necessary. Both the thermistor and the heater element were placed in a quart size thermos flask filled with transformer oil as a constant temperature bath. The normal power input varied from 1 to 1.2 watts, and the time required to raise the temperature of the bath 1°C was about 20 minutes. The thermistor and diode were mounted close together so that both were at the same temperature. In order to keep temperature fluctuations at the diode to a minimum, the heating element was placed as far from the diode as possible, the column of oil acting as a temperature smoothing filter. No agitation was used. For circuit detaile, refer to Plate I. By adjusting the circuit carefully, the controller could be made to switch from on to off in a thermistor resistance change of about 0.2 ohms in 3000 ohms. This corresponded to a temperature change of about $^1/_{500}$°C. The over-all temperature stability depended largely on how carefully the heating element power input was adjusted. For example, if too much heat was put into the system, we got a temperature overshoot which was many times larger than the temperature increment necessary to turn the controller from on to off. The control obtained was better than $\pm$ 1/50 of a degree C., which was adequate since no diode characteristic fluctuations due to temperature changes were observed. The long period stability was also considered adequate.

Measurement of the Macroscopic Current-Voltage Characteristics

Fig. 2 shows the circuit used to measure the mean current-mean voltage characteristics. The supply voltage source was a Dresden Barnes

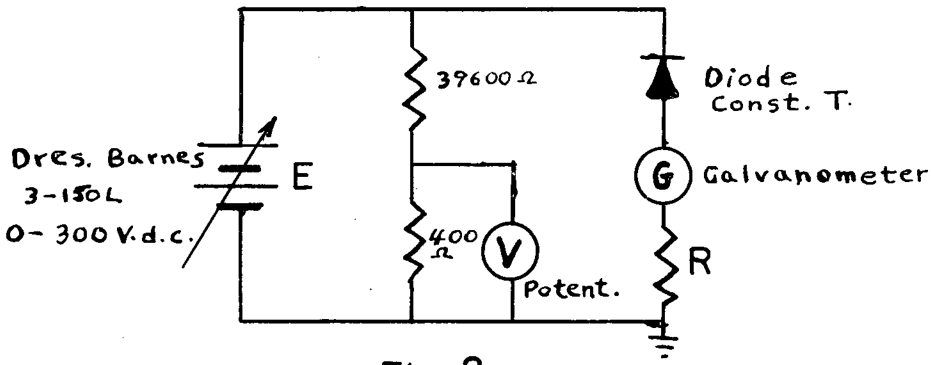


Fig. 2

Circuit for measuring the

$\bar{I}-\bar{V}$ chs.

Mod. 3 - 150 L regulated power supply. The supply voltage could be continuously varied from 0 - 300 volts. An additional 200 ohm variable resistance was placed in series with the control potentiometer in the power regulator circuit to act as a vernier voltage adjuster. The supply output voltage was measured using a potentiometer connected across a 100: 1 voltage divider. This made it possible to read the voltage to 0.01 volts without much difficulty. An additional feature which was very important for this work was that the supply voltage could be continually monitored and any voltage drift compensated for by manually adjusting the voltage control vernier. The supply voltage could thus be accurately held at a constant value over long periods of time. The main source of voltage drift was found to be aging voltage reference tubes. This problem could be alleviated by periodically replacing these tubes with new ones. The current meter was a Scalamp galvanometer having a resistance of 1250 ohms. When using large values of series resistance R, a Keithley electrometer (Model 210) with a high resistance decade shunt was used to measure the current.

Since all the current in the avalanche region is due to the current pulses with no other significant contribution present as was reported by McKay (1954), we must refer to the current - voltage characteristics in this region as the mean current - mean voltage characteristic. These are the voltages and currents which are measured using meters whose time constants are long compared to the mean pulse period.

After measuring the mean current $\bar{I}$ and the supply voltage, the mean voltage $\bar{V}$ was given by

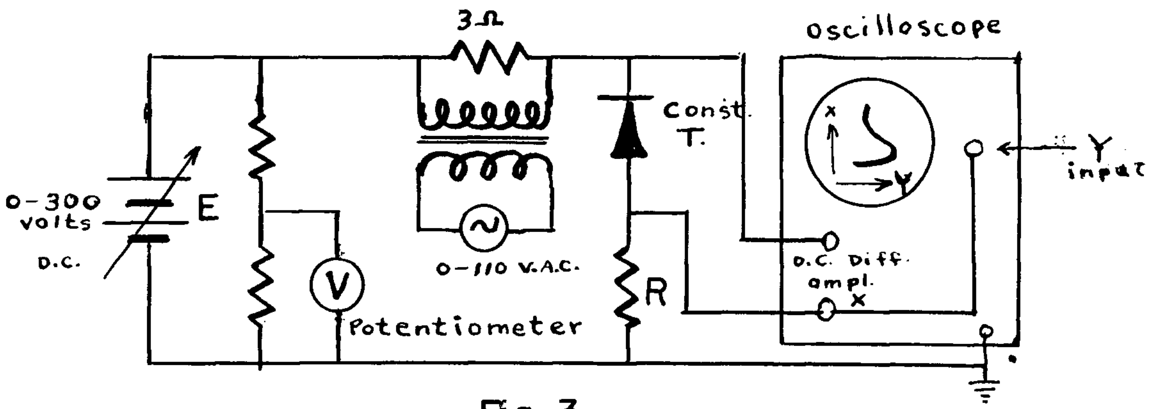


Fig. 3

Circuit for the display of the dynamic current-voltage chs.

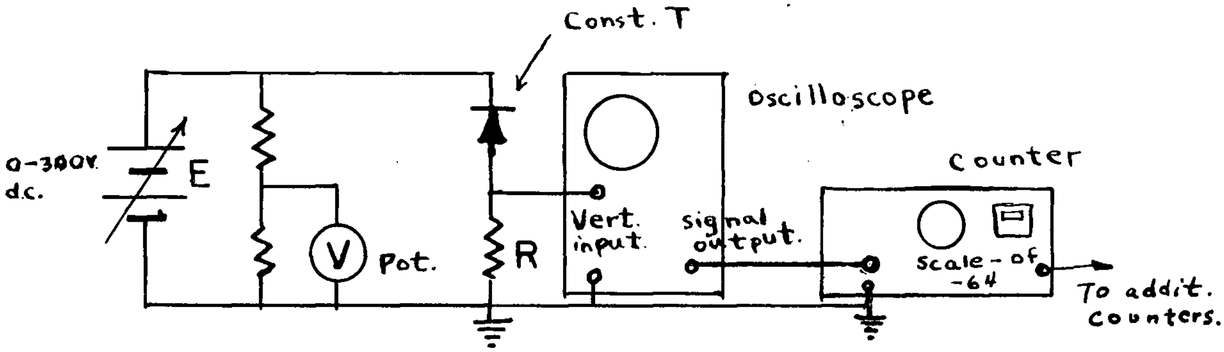


Fig. 4

Circuit for pulse counting and amplitude measurement

$$\overline{V} = E - \overline{I}\,(R + R_g) \tag{9}$$

where R_g is the galvanometer resistance. The mean current - mean voltage characteristics are plotted by drawing load lines corresponding to $R + R_g$ through various values of E and marking the intersection of these with the appropriate $\overline{I}$ value.

Because the current pulses were statistical in nature, and their "on" and "off" periods depended on the voltage across the diode, it was essential to keep all stray capacitance to a minimum so that circuital RC time constants would not influence this voltage significantly. As will be shown later, these time constants can have a marked effect on all the diode measurements. To minimize the RC circuital time constants, all leads were kept as short as possible. As is seen by equation (9), the total resistance in the circuit had to be known accurately as well. Care had to be taken to ensure a proper ground so as to keep the stray pickup of 60 c/s voltage to as low a value as possible because of the possibility that rectification by the diode under test could displace the operating conditions.

Dynamic display circuit

It was of some interest to compare the static current - voltage characteristic to the dynamic display. The experimental arrangement to give this display is shown in Fig. 3. A 60 c/s a.c. signal, the amplitude of which could be adjusted, was superimposed on the d.c. bias. The d.c. bias was raised to a point just below the onset of breakdown. To minimize reactance

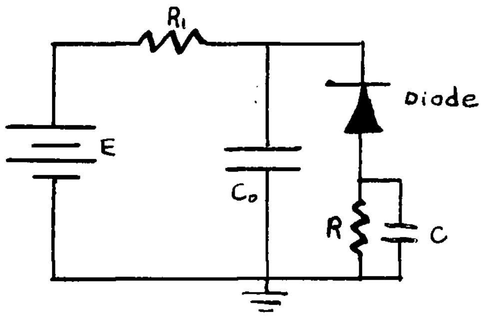


Fig. 5a

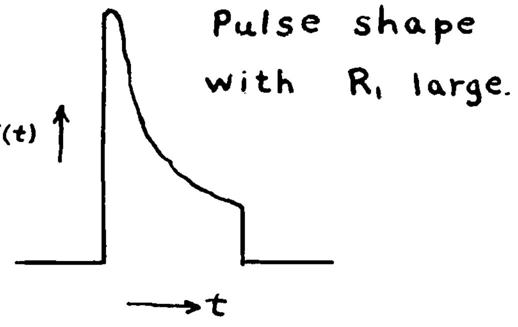


Fig. 5b

Distribution of stray capacitance.

effects due to the transformer, a low resistance load was placed across the secondary as is shown in Fig. 3. The capacitance across the load resistance R was again kept to a low value. The characteristic was displayed on a calibrated oscilloscope.

Measurement of the statistics of the avalanche pulses

Fig. 4 shows the experimental set up used to study the avalanche pulse amplitude characteristics and rates. A calibrated oscilliscope was used to study the pulse amplitude as a function of the supply voltage E. The observed pulse shape and mean recurrence rate was found to depend strongly on the capacitance in the circuit, and on the placement of the resistance R in the circuit. If R is split by putting part of it above and part below the diode as shown in Figure 5a, R_1 and C isolate the diode momentarily from the supply voltage E when the current pulse comes on. This causes the pulse to have a spike on the leading edge because of "voltage holding" effect of the capacitance C. See Fig. 5b for details. To make accurate amplitude and mean pulse frequency measurements as a function of E, all the resistance was placed in the R_2 position and the total shunt C was kept to a minimum. The pulses then had a characteristic flat top which is their true form when unmodified by the measuring circuit.

The mean current pulse rate was measured using an Atomic Instruments Scale - of - 64 counter coupled to the oscilloscope output cathode follower. For fast pulse rates, two counters were connected in series as shown in Figure 4. It was possible to make reliable counts up to about 20K counts/sec.

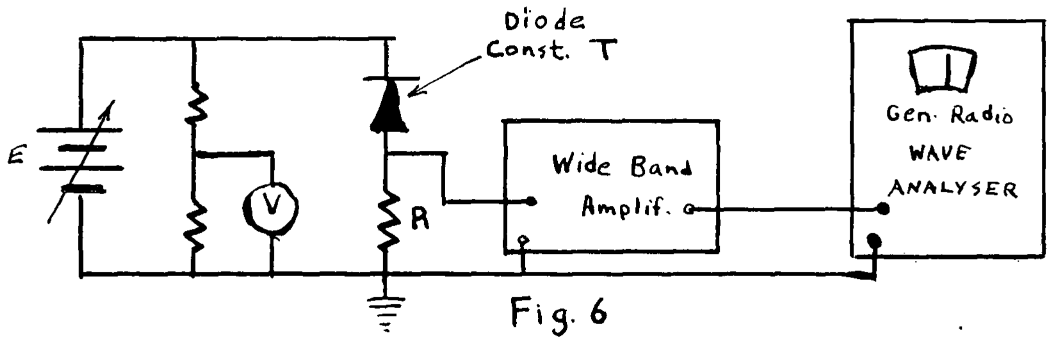


Fig. 6

Arrangement for measurement of current spectrum using this method. Beyond this, the counter began to miss pulses. For rates above 20K counts/sec., the oscilloscope was externally triggered at about 1 cycle/sec, and visual counts made directly from the oscilloscope screen noting the time taken for the beam to traverse the screen width. The sweep time had to be short enough so only a small number of pulses appeared during each sweep to permit visual counting. By taking a larger number of such counts, a good measure of the average pulse rate could be obtained. Throughout all counting experiments, the supply voltage E had to be monitored continually and kept at a constant value by adjustment of the vernier voltage control. Long period stability of 0.05% in voltage was quite easily obtained using the above experimental setup.

<u>Spectrum Analysis</u>

An analysis of the avalanche current spectrum over the range from 5 - 18 K cycles/sec was made using a General Radio Mod. 736-A Wave Analyser. Because of the violent fluctuations in the Wave Analyser meter reading, a large capacitance was connected across the meter terminals giving it a time constant of about 7 seconds. For higher frequencies, a Sierra Model 121 Wave Analyser was used. Adequate amplification of the signal was obtained by using a Technology Instruments Corp. Mod. 500-A wide band amplifier. Amplifier noise was small compared to the diode fluctuations being measured. Stray capacitance was once again kept to a minimum and the supply voltage held constant. See Fig. 6 for circuit details.

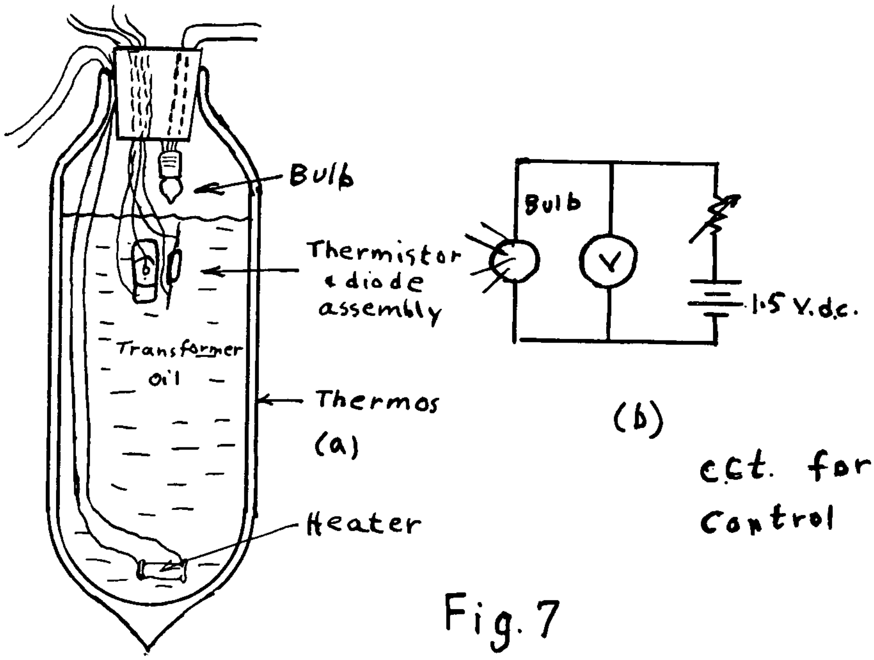


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Fig. 7

Arrangement for illumination
experiments.

Illumination Experiments

For diode illumination experiments, a small 2.2 volt filament penlight bulb was mounted inside the thermos as shown in Figs. 7 a and b. A small bulb was used so that excess heating was kept ot a minimum. The small lens on the bulb made it possible to focus the light directly onto the diode. The voltage drop V across the bulb was used as a parameter to fix the illumination intensity during mean pulse rate measurements. This was found to be a useful method because a given experiment could be performed in stages and repeated at will.

In this investigation, the pre-breakdown current I_0 and the pulse rate were measured as the illumination was varied. To do this, I_0 was first plotted as a function of the bulb voltage V. Then the mean pulse rate f was plotted as a function of V. Using these two curves, the parameter V could be eliminated giving the desired I_0 vs. f curve. The fact that the experiment could be repeated giving the same readings within experimental error was sufficient evidence to show that the approach was sound.

Temperature Experiments

To observe the effect of temperature change on the mean pulse rate, pulse amplitude and breakdown voltage of the diode, the temperature controller was set to a given temperature, allowed to come to equilibrium, and the desired measurements made under constant temperature conditions. The oil bath temperature was measured using a calibrated thermocouple. The same procedure was followed for a whole series of different temperatures ranging from room temperature to about 80°C.

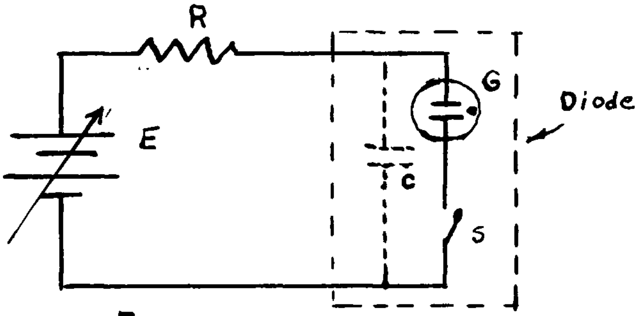


Fig. 8

Diode model.

CHAPTER IV

THEORETICAL DISCUSSION

Discussion of a model

In order to predict the measured mean current - mean voltage characteristics as well as the observed statistical nature of the current of a typical Si p-n junction diode in the avalanche breakdown region, consider a model having the following features:

1) A two terminal device G which breaks down at a voltage V_B such that when $V < V_B$, $I = 0$, and when $V > V_B$, $I = g(V - V_B)$, i.e., the current through the device alone is zero until the voltage across it reaches the critical voltage V_B above which the tube current increases linearly with increasing tube voltage. Such a device has the properties of an ideal gas discharge tube.

2) A random switch s connected in series with the ideal gas discharge tube. This system consisting of an ideal gas tube in series with a random switch is said to be in state 1 when the switch is closed, and in state 0 when the switch is open. The two elements, as shown in Fig. 8, shall serve as a model for the Si p-n junction in the unstable avalanche region.

Because the switch is a statistical element, we cannot in general be certain when we shall find the system in a given state, but can only speak in terms of a probability of finding it in that state. Thus, we define P

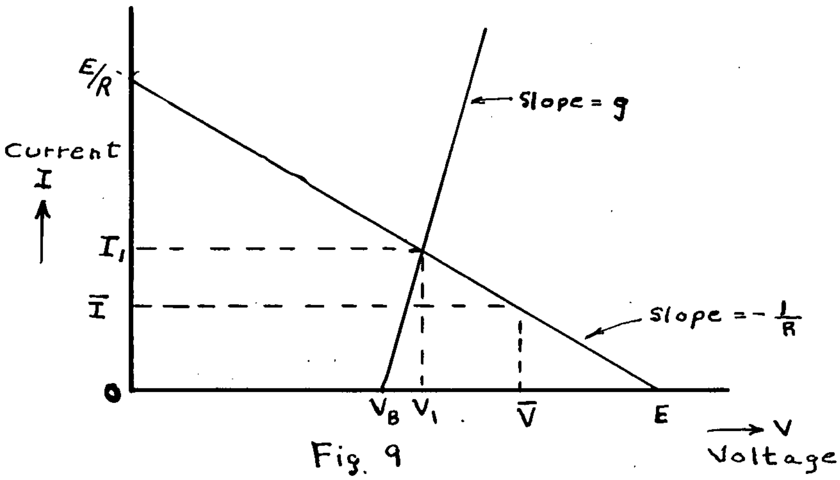


Fig. 9

Figure illustrating the parameters involved in the model.

as the probability that at any arbitrary moment of time, we shall find the system in state 1. Likewise, under the same conditions, 1 - P is the probability that we shall find the system in state O. We can also define P as the fraction of time the system spends in state 1, and 1 - P as the fraction of time the system spends in state O. The probability P will in general be a function of the excess voltage $E - V_B$.

From Fig. 9, we see that if the switch is open, the current I_O through the diode is O, and the voltage across it is E. However, in an actual Si diode, the current below breakdown, I_O, is never O, but has a small finite value which is small compared to the measured currents in the avalanche breakdown region, and can therefore be neglected.

When the switch is closed, or the diode is in state 1, the current I_1 is given by $I_1 = g\,(V_1 - V_B)$ where V_1 is the voltage across the diode while in state 1. In terms of E and V_B, these are given by

$$I_1 = \frac{g\,(E - V_B)}{1 + gR} \quad (10) \qquad \text{and} \qquad V_1 = \frac{E + gRV_B}{1 + gR} \quad (11)$$

Because the diode is switching back and forth between state 1 and O in a random fashion, the measured currents and voltages in this region are statistical averages. In terms of the probability P, these are given by

$$\overline{I} = \frac{gP\,(E - V_B)}{1 + gR} \quad (12) \qquad \text{and} \qquad \overline{V} = E - \frac{gP\,(E - V_B)}{1 + gR} \quad (13)$$

where R is the total circuital resistance, and E the supply voltage.

$\overline{I}$ and $\overline{V}$ are the mean current and mean voltage respectively such as are measured by d.c. meters in the circuit whose time constants are long compared to the mean switching period. The actual current through the diode will take the form of pulses of amplitude I_1 varying randomly in their on-off-periods. The slope resistance of the mean current - mean voltage characteristic can be found by differentiating both equation (12) and (13) with respect to the parameter E at constant R giving

$$\rho = \left(\frac{\partial \bar{V}}{\partial E}\right)_R \Bigg/ \left(\frac{\partial \bar{I}}{\partial E}\right)_R = \left(\frac{\partial \bar{V}}{\partial \bar{I}}\right)_R \qquad (14)$$

$$= \frac{1}{g} \left[\frac{1 + gR}{P + (E - V_B)\left(\frac{\partial P}{\partial E}\right)_R}\right] - R \quad ,$$

where ρ is the slope resistance.

Several important facts are immediately obvious upon studying these results obtained in the analysis of the model.

First, from equations (12) and (13), we see that the mean current - mean voltage characteristic is a function of the external circuital resistance which is an extremely unusual feature of a linear circuit element. This fact is not dependent on the presence of the non-linear gas tube as can be demonstrated by replacing the tube by a pure resistance $\frac{1}{g}$ and making V_B equal to 0. (See equations (12) and (13)). This is an important result since it means that the measured $\bar{I}$ - $\bar{V}$ curves are not unique, and to have any significance, they must be labelled by the appropriate series resistance R. It should be noted that failure to obey Thévenin's theorem is a property not encountered in time - invarient passive elements, whether linear or non-linear. The exact nature of the mean current - mean voltage characteristics depends on P as well as R.

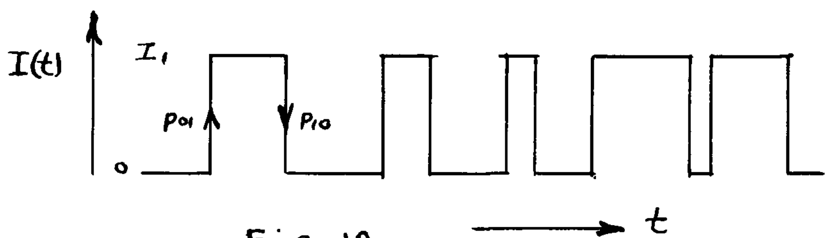


Fig. 10

Figure illustrating the general shape of the current pulses

Secondly, from equation (14), we see that it is possible for the slope resistance of the $\bar{I}$ - $\bar{V}$ curve to go through zero and become negative. The turnover point is given by the equation:

$$R = \frac{1 + gR}{g\left(P + (E - V_B)\frac{\partial P}{\partial E}\right)} \qquad (15)$$

Whether or not equation (15) can be satisfied depends on the exact functional dependence of P on E and R. A more comprehensive discussion of equations (12), (13) and (14) shall be given later when the exact form of P has been worked out.

Development and discussion of a probability function, P

Fig. 10 illustrates the general shape of the current pulses expected during avalanche breakdown under the random switch model. In order to facilitate the setting up of a suitable probability function P which has the correct dependence on E and R, the following parameters are introduced:

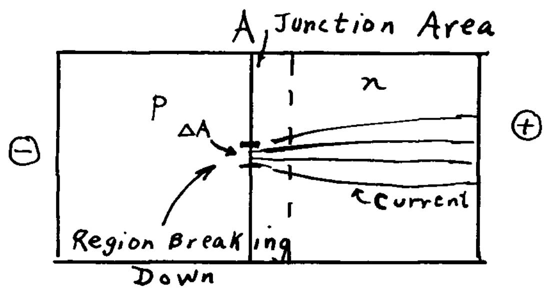
The function $p_{01}\,dt$ is the probability that if the diode system is found in state 0 at time t, it shall switch to state 1 in the time interval between t and t + dt. Similarly, $p_{10}\,dt$ is the probability that if the system is found in state 1 at time t, it shall traverse to state 0 in the time interval between t and t + dt. In applying these, assume that p_{01} and p_{10} are independent of time, and of the previous history of the system, that is, the switching process is Markoffian. Using statistical methods, it can be shown that P, in terms of p_{10} and p_{01}, is given by

$$P = \frac{p_{01}}{p_{01} \;\; p_{10}} \qquad (16)$$

Similarly, the mean current pulse rate f is given by

$$f = \frac{p_{10}\,p_{01}}{p_{10} + p_{01}} \qquad (17)$$

Fig. 11



SKETCH OF JUNCTION

From elementary intuitive considerations of the diode model, the probability functions p_{01} and p_{10} are expected to have the following properties:

1) p_{01} should be a monotonically increasing function of $E - V_B$, i.e., the excess of the voltage in state 0 over V_B so that the system is more likely to pass from state 0 to state 1 for increasing values of $E - V_B$. The function p_{01} should approach zero as E approaches V_B. The simplest general function having these desired properties is

$$p_{01} = a\,(E - V_B)^m \tag{18}$$

where m is a constant exponent to be determined experimentally and a is a coefficient depending on the pre-breakdown saturation current I_0 and the area of the region breaking down. In determining the dependence of a on I_0 and the area of the breakdown region, it is assumed that each avalanche current pulse is initiated by only one carrier entering the breakdown region in the junction. It is also assumed that many carriers enter the region per unit time, but only a small fraction of this number is statistically "lucky" enough to initiate an avalanche. Consider a junction of area A containing a small region of area ΔA where a discharge can occur. See Fig. 11. If ν_0 is the number of carriers entering the region ΔA per second, we can write

$$p_{01} = \nu_0 \, Pd \tag{19}$$

where Pd is the probability that a carrier will initiate a discharge upon entering the region, providing the region is not already broken down. In terms of I_0 and A, ν_0 is given by

$$\nu_0 = \frac{I_0 \, \Delta A}{e \, A} \ /\mathrm{sec} \tag{20}$$

Where e is the electron charge. The element of area ΔA is assumed to remain substantially constant with respect to E because the increase in voltage necessary for the diode to be completely broken down is a small fraction of the breakdown voltage V_B. Substituting (19) and (20) into (18) we get

$$p_{01} = \alpha' \frac{I_0 \Delta A}{e\ A} (E - V_B)^m \qquad (21)$$

The discharge probability is given by

$$Pd = \alpha' (E - V_B)^m \qquad (22)$$

2) The function p_{10} should depend only on $V_1 - V_B$ where V_1 is the voltage across the diode when the switch is closed, that is, in state 1. If the junction is in state 1 at a low value of $V_1 - V_B$, a large value of p_{10} is needed to correspond to the rapid reversion to state 0 that will occur. The function p_{10} should thus decrease monotonically with increasing $V_1 - V_B$ so that for large enough values of $V_1 - V_B$, the system would be very unlikely to return to state 0. Analogous to equation (18), a simple function having these properties is given by

$$p_{10} = b\,(V_1 - V_B)^{-n} \qquad (23)$$

where b is a constant coefficient, and n is another constant exponent to be determined experimentally. In terms of $E - V_B$, equation (23) becomes

$$p_{10} = \frac{b\,(1 + gR)^n}{(E - V_B)^n} \qquad (24)$$

The coefficient b is expected to depend on the statistical fluctuation of the avalanche multiplication in the breakdown region during the discharge.

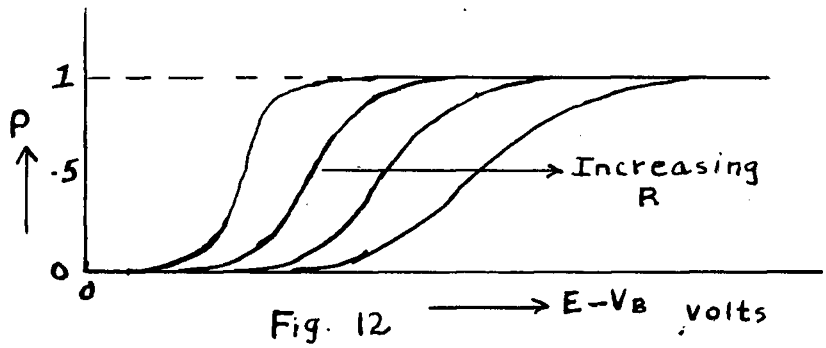


Fig. 12

Curve showing the expected P dependence on $E - V_B$

By substituting equations (18) and (24) into equation (16) we obtain

$$P = \left[1 + \frac{b(1+gR)^{n}}{a\,(E - V_B)^{n+m}}\right]^{-1} \qquad (25)$$

The function P so formed has the following properties: See Fig. 12 for details.

1) It is zero at $E = V_B$ and approaches 1 as $E - V_B \longrightarrow \infty$

2) The slope, $\left(\frac{\partial P}{\partial E}\right)_R$ becomes zero for $E = V_B$ and when $E - V_B$ approaches infinity.

3) The slope $\left(\frac{\partial P}{\partial E}\right)_R$ at a given P decreases as R is increased as is shown by the example for $P = 0.5$

$$\left(\frac{\partial P}{\partial E}\right)_{R,\;P=.5} = \frac{m+n}{4}\left(\frac{a}{b}\right)^{\frac{1}{n+m}}(1+gR)^{-\frac{n}{n+m}} \qquad (26)$$

Besides decreasing in slope as R increases, the value of $E - V_B$ at constant P also increases with increasing R as is shown by

$$E - V_B = \left(\frac{P}{1-P}\right)^{\frac{1}{m+n}}\left(\frac{b}{a}\right)^{\frac{1}{m+n}}(1+gR)^{\frac{n}{n+m}} \qquad (27)$$

The function P can be substituted back into the equations for $\overline{I}$ and $\overline{V}$ and a theoretical mean current - mean voltage characteristic curve plotted.

Using the fact that

$$\left(\frac{\partial P}{\partial E}\right)_R = \frac{P(1-P)(m+n)}{E - V_B} \qquad (28)$$

and substituting this value into equation (14), we obtain the result

$$\rho = \frac{1+gR}{Pg\left[1+(m+n)(1-P)\right]} - R \qquad (29)$$

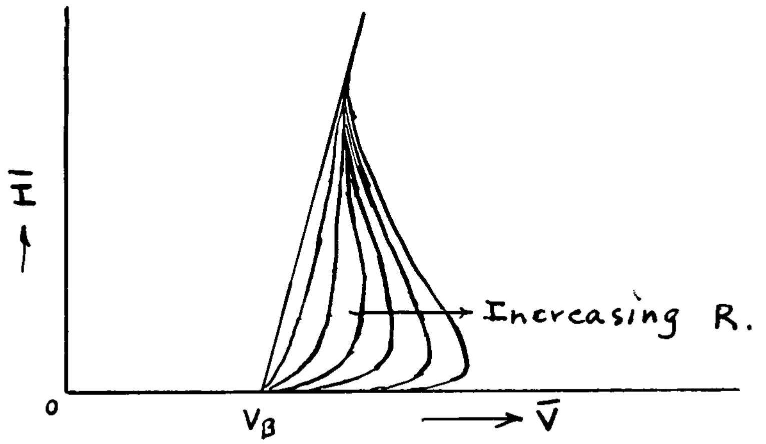


Fig. 13

EXPECTED SHAPE OF THE $\bar{I}$-$\bar{V}$ CURVES.

The turnover points between which the slope resistance is negative are given by

$$P_T = \tfrac{1}{2}\left(1+\frac{1}{m+n}\right) \pm \sqrt{\tfrac{1}{4}\left(1+\frac{1}{m+n}\right)^2 - \frac{\left(1+\frac{1}{gR}\right)}{m+n}} \tag{30}$$

The minimum value of R for which a negative slope resistance of the mean current - mean voltage characteristic can occur is given by the equation

$$R = \frac{4(m+n)}{g(m+n-1)^2} \tag{31}$$

By examining equation (12), and noting that the smaller P_T value decreases as R increases, we see that the mean current at turnover decreases with increasing R. Similarly, it can be shown that the mean voltage at turnover increases as R increases. The combined effect is to shift the turnover point towards higher $\overline{V}$ values and at the same time increase the magnitude of the negative resistance obtained for currents above turnover. Fig. 13 illustrates these properties.

By substituting p_{10} and p_{01} into f given by equation (17), we have

$$f = \frac{b(1+gR)^n}{(E-V_B)^n}\left[1+\frac{b(1+gR)^n}{a(E-V_B)^{n+m}}\right]^{-1} \tag{32}$$

where f is the mean current pulse rate. This function has a maximum at

$$(E-V_B) = \left(\frac{mb}{na}\right)^{\frac{1}{m+n}} (1+gR)^{\frac{n}{m+n}} \tag{33}$$

The maximum value of f at this point is given by

$$f_{max} = b\left(\frac{m}{m+n}\right)\left(\frac{na}{mb}\right)^{\frac{n}{n+m}} (1+gR)^{\frac{mn}{m+n}} \tag{34}$$

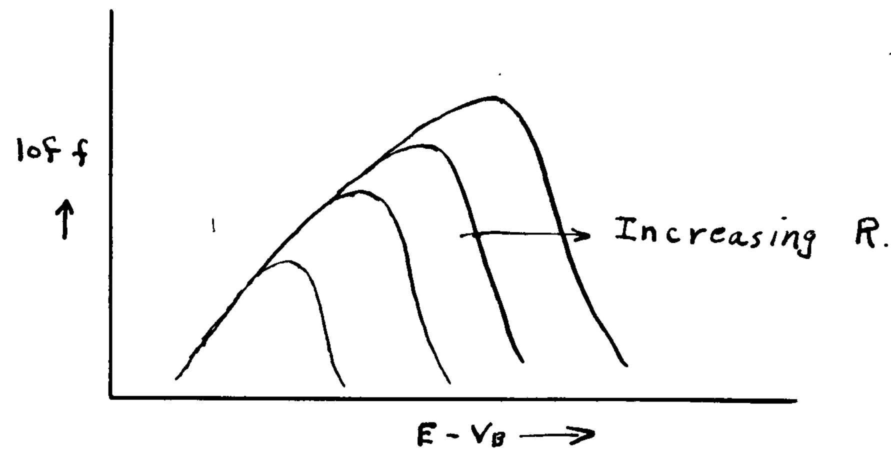


Fig. 14

EXPECTED SHAPE OF THE f vs $E - V_B$ CURVES.

These are both functions which increase monotonically with R. By inspecting equations (18) and (24), we see that for low enough values of $E - V_B$, p_{01} is much smaller than p_{10}, and for high enough values, p_{10} is much smaller than p_{01}. Physically, this means that low $E - V_B$, f is determined entirely by the probability p_{01}, and for high $E - V_B$, the pulse rate depends only on the extinction probability p_{10}. Applying these facts to equation (32) for the mean pulse rate, we have

$$f \simeq p_{01} = a\,(E - V_B)^m \text{ for low } E - V_B, \tag{35}$$

that is, for $E - V_B << \left[\frac{b}{a}(1 + gR)^n\right]^{\frac{1}{n+m}}$

$$\text{and } f \simeq p_{10} = \frac{b\,(1 + gR)^n}{(E - V_B)^n} \text{ for high values of } E - V_B, \tag{36}$$

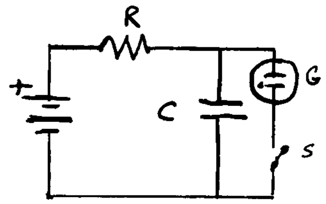
that is, for $E - V_B >> \left[\frac{b}{a}\ (1 + gR)^n\right]^{\frac{1}{m+n}}$

From equation (35), we see that f is independent of R for low $E - V_B$, that is, f has a common asymptotic form for all R as $E - V_B$ approaches zero. Fig. 14 illustrates the general shape of the mean pulse rate vs $E - V_B$ curves using R as a parameter.

Effect of stray capacitance

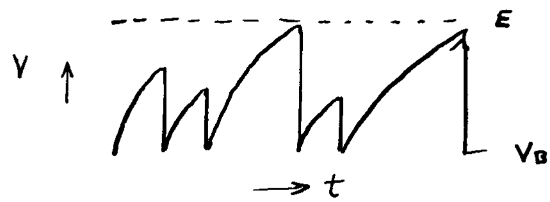
The effect of stray capacitance across the diode has been neglected in the analysis up to this point, because it was assumed to be small. Normally, it is the product of the resistance and the shunt capacitance RC that matters. Therefore, in such cases where this RC time constant is comparable to the mean pulse periods, its effect can not be neglected. An exact treatment of the dependence

Fig. 15



(a)

Diode model with capacitance.



(b)

Voltage pulse shape for low pulse amplitude.

of f on RC becomes difficult to calculate. A qualitative treatment, however, will clearly illustrate the trends involved by increasing the circuital RC time constants.

Assume for the sake of simplicity that when the system is in state 1, $V = V_B$, and that the ratio of the on to off periods, T_1/T_0 is much less than one. The voltage across the diode when the switch opens, or at $t = o$, is given by

$$V = E - (E - V_B) \exp - t/RC \tag{37}$$

See Fig. 15 for details.

First of all, consider the probability $p_{01} = a(E - V_B)^m$. It should be noted that the voltage across the diode while in state 0, i.e., the switch is open, is not E as for the case of $C = 0$, but instead, it becomes time dependent as is shown by equation (37). After substituting equation (37) into p_{01}, we obtain the equation

$$p_{01} = a \left[(E - V_B)(1 - \exp - t/RC) \right]^m \tag{38}$$

The effect of the RC time constant here is to reduce the average value of p_{01}. By examining equation (35), we see the observed effect of RC will be a reduction of the mean pulse rate for low $E - V_B$ valves. Since it is the product of R and C that matters, and not C alone, the effect can be made negligible by minimizing C and using low values of R.

The effect of C on p_{10} will be much less pronounced than on p_{01}, at least for low $E - V_B$, because the time constant C/g will in general be short compared to the "on" period of the pulse.

The presence of large circuital time constants is expected to distort the normal current and voltage pulse shape. The criterion for determining if the RC time constant is negligible is the appearance of a good rectangular current and voltage waveform.

A simple procedure is needed for determining p_{10} and p_{01} experimentally. To do this, note first of all that

$$P = \frac{f}{p_{10}} \tag{39}$$

and from the model,

$$P = \frac{\bar{I} - I_0}{I_1 - I_0} \tag{40}$$

By combining these equations and neglecting I_0 compared to I_1 and $\bar{I}$, the result

$$p_{10} = \frac{I_1 f}{\bar{I}} \simeq f \tag{41}$$

for high enough $E - V_B$, or $E - V_B >> \left[\frac{b}{a}(1 + gR)^n\right]^{\frac{1}{m+n}}$, is obtained.

Similarly, it can be shown that

$$p_{01} = \frac{I_1 f}{I_1 - \bar{I}} \simeq f \tag{42}$$

for $E - V_B << \left[\frac{b}{a}(1 + gR)^n\right]^{\frac{1}{m+n}}$. (See equations (35) and (36).)

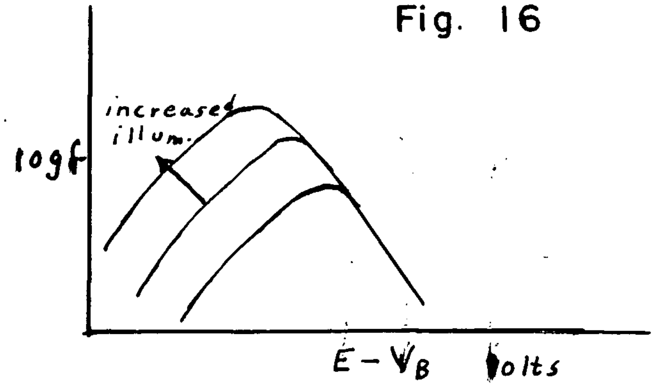
These two equations indicate an experimental method for determining the various parameters in p_{10} and p_{01}.

Illumination effects

Illumination of a p - junction is a simple method of injecting carriers into the ionization region. The immediate effect is to increase the pre-breakdown saturation current I_0. From equation (21), this would result in an increase in "a" because more carriers are available to initiate an avalanche. With no illumination, the rate at which carriers enter the region ΔA is given by

$$n = \frac{\Delta A \, I_{0D}}{A \, e} \tag{43}$$

where I_{0D} is the dark saturation current.



EXPECTED EFFECT OF ILLUMINATION ON THE PULSE RATE.

If P_d is the probability that a carrier will initiate an avalanche when it enters the breakdown region, then

$$p_{o1} = \frac{\Delta A}{A e} \; I_{oD} \; P_d \tag{44}$$

Under illumination, the saturation current increases giving

$$I_{oL} = I_{oD} + \Delta I_o \tag{45}$$

where I_{oL} is the saturation current under illumination. Because of the geometry of the junction, only a small unknown fraction $\frac{\delta A}{A}$ of the added saturation current will pass through the breakdown region.

Then,

$$(p_{o1})_L = \frac{\Delta A}{A \; e} \; I_{oD} \; P_d + \frac{\delta A}{A \; e} \; \Delta I_o P_d \tag{46}$$

where $(p_{o1})_L$ is the illuminated p_{o1} and $\frac{\delta A}{A}$ is the fraction of ΔI_o entering the region. By subtracting (44) from (46) and substituting (45) into the result, then

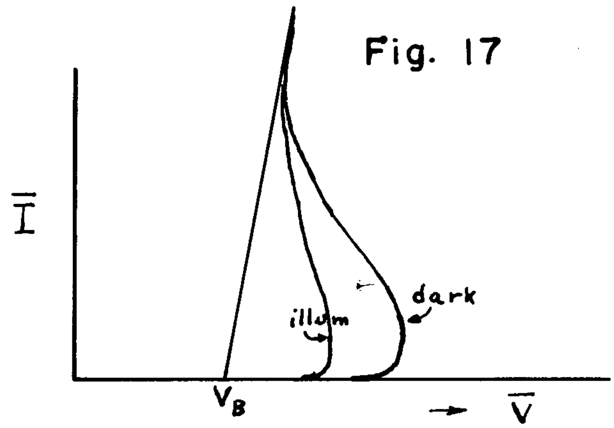
$$(p_{o1})_L - p_{o1} = \frac{P_d \, \delta A}{A \; e} \; (I_{oL} - I_{oD}) \tag{47}$$

The value of p_{o1} under both illuminated on dark conditions can be obtained by using the approximation of equation (42). Implicit in the above treatment are the assumptions that each avalanche is initiated by only one carrier entering the barrier region, and that the discharge probability is independent of illumination.

By examining equations (32) and (35), we see that illumination will result in an increase in f for low values of $E - V_B$ because of an increase in the coefficient, a, of p_{o1}. This effect is illustrated by Figure 16.

From a consideration of equation (25), we see that illumination has the same effect on f as a decrease in the circuital resistance R. Thus,

Fig. 17



EXPECTED EFFECT OF ILLUMINATION ON THE $\bar{I}-\bar{V}$ CHS.

the effect on the mean current - mean voltage characteristic will be to shift the turnover point towards lower values of $E - V_B$ as shown in Figure 17.

Temperature effects

Carriers introduced into the avalanche region by thermal ionization of the junction material are expected to increase the probability of discharge per unit time in much the same manner as carriers introduced through illumination. This is manifested by an increase in I_o in the equation for p_{o1}, namely,

$$p_{o1} = \frac{I_o \Delta A}{e \; A} \qquad P_d \tag{48}$$

Several factors which may influence the result must be considered:

1) The probability P_d of a discharge occurring when an electron enters the high field reginn may depend on the temperature of the lattice in some unknown way.

2) It is important to know if the pre-breakdown current I_o is made up largely of bulk current passing through the junction proper, or surface leakage current passing through a surface barrier region. The pre-breakdown current can be written as $I_o = I_{\ell} + I_j$ where I_{ℓ} is the surface leakage component, and I_j is the junction component, each of these having a different functional dependence on temperature. The position of the avalanche region, whether near the surface or in the bulk material, determines which component should be used. An exact analysis of the temperature effect on p_{o1} is hampered

by an uncertainty in the temperature dependence of the pre-breakdown current passing through the avalanche region. An additional experimental difficulty in determining the temperature dependence of p_{01} is the fact that V_B increases with increasing temperature making it difficult to obtain accurate $E - V_B$ readings.

In general, an increase in temperature results in an increase in the pre-breakdown current passing through the avalanche region which by equation (21) is expected to result in an increase in p_{01} for a given value of $E - V_B$.

The fluctuation of the avalanche multiplication is expected to be dependent on temperature, and if Rose's mechanism for the extinction of the avalanche pulse is correct, p_{10} should be temperature dependent as well. Because of the lack of information regarding this extinction mechanism, and the fact reproducible data for p_{10} is difficult to obtain as is discussed in the next chapter, no theoretical predictions as to the temperature dependence of p_{10} are made.

<u>Current pulse spectrum</u>

Another check on whether or not the random switch is an adequate model for a Si p-n junction in the unstable avalanche breakdown region is a measurement of the current spectrum. In developing the probability function P, it was assumed that p_{01} and p_{10} are independent of time and of the past history of the system, that is, the process was Markoffian. Also implicit in the development was the fact that the on - off periods were completely random. The spectrum for such a wave is simply that of a random telegraph signal as given by Machlup (1954). The a.c. component of the spectrum can be shown to be

$$S_I(\omega) = \frac{4\, p_{10}\, p_{01}}{p_{10} + p_{01}} \; \frac{I_1^{\,2}}{(p_{10} + p_{01})^2 + \omega^2} \tag{49}$$

which can be written as

$$S_I(\omega) = \frac{S(0)}{1 + \omega^2 \tau^2} \tag{50}$$

where

$$S(0) = \frac{4\, p_{10}\, p_{01}\, I_1^{\,2}}{(p_{10} + p_{01})^3} \quad \text{and} \quad \tau = \frac{1}{p_{10} + p_{01}}$$

The important point to note is that this spectrum has only one relaxation time τ in spite of the fact that we are dealing with two independent transition probabilities p_{01} + p_{10} .

PLATE 11A

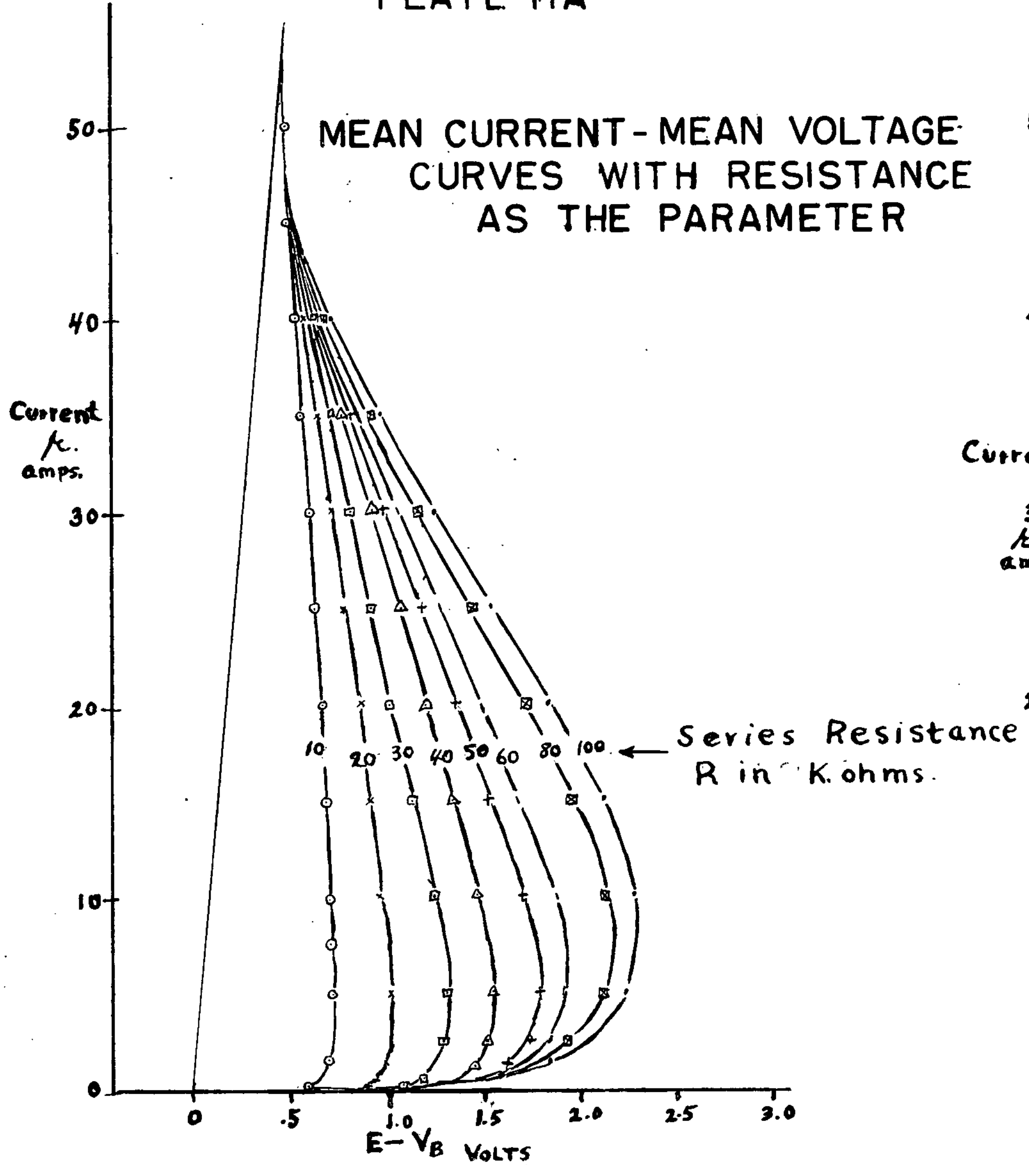
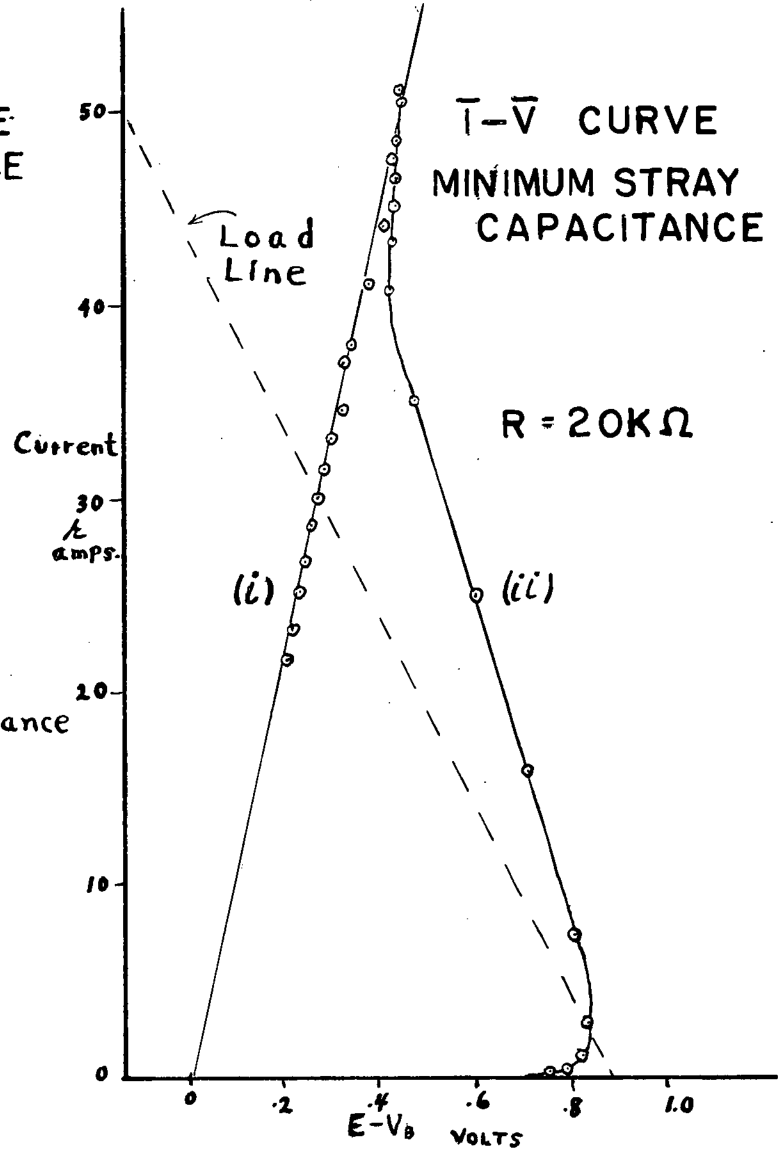


PLATE 11B



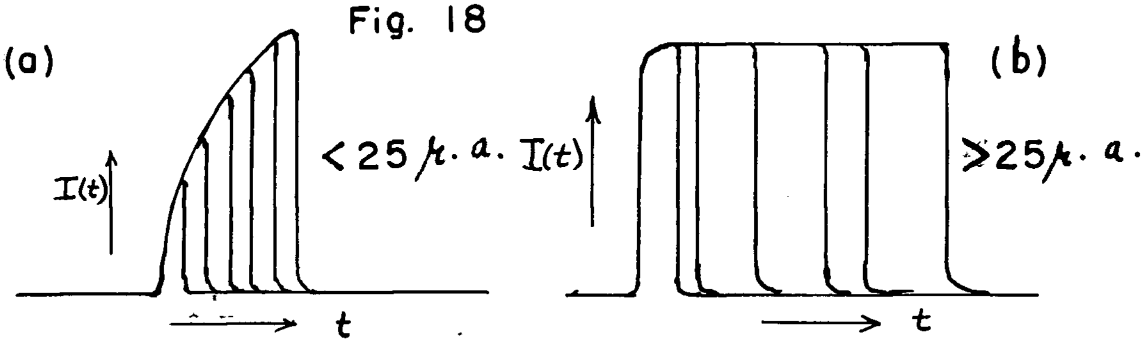
CHAPTER V

EXPERIMENTAL RESULTS AND INTERPRETATION

The mean current - mean voltage characteristics

Plate IIA gives examples of the mean current - mean voltage characteristics obtained for a typical p - n junction in the avalanche region for various values of R. These measurements were made under constant temperature conditions, and the curves plotted using the method described in Chapter III. All of the diodes studied displayed the same general features of the mean current - mean voltage characteristics, that is, each one had a turnover point with the mean voltage at turnover increasing with increasing series resistance, R. The mean current at turnover decreased with increasing R values up to about 50K ohms. For values of R above 50K ohms, there was a small net increase in the mean current at turnover, an effect attributed to the presence of shunt capacitance which causes an increase of the RC time constant perturbing the normal shape and statistics of the current pulses. The magnitude of the negative slope resistance for a given R varied from diode to diode. Plate IIB shows the $\bar{I} - \bar{V}$ characteristic of another diode with a total circuital resistance of 20K ohms. In this case, a considerable effort was made to reduce the value of stray capacitance to a minimum. The estimated value of capacitance across the diode was between 25 and 50 pf. at most.

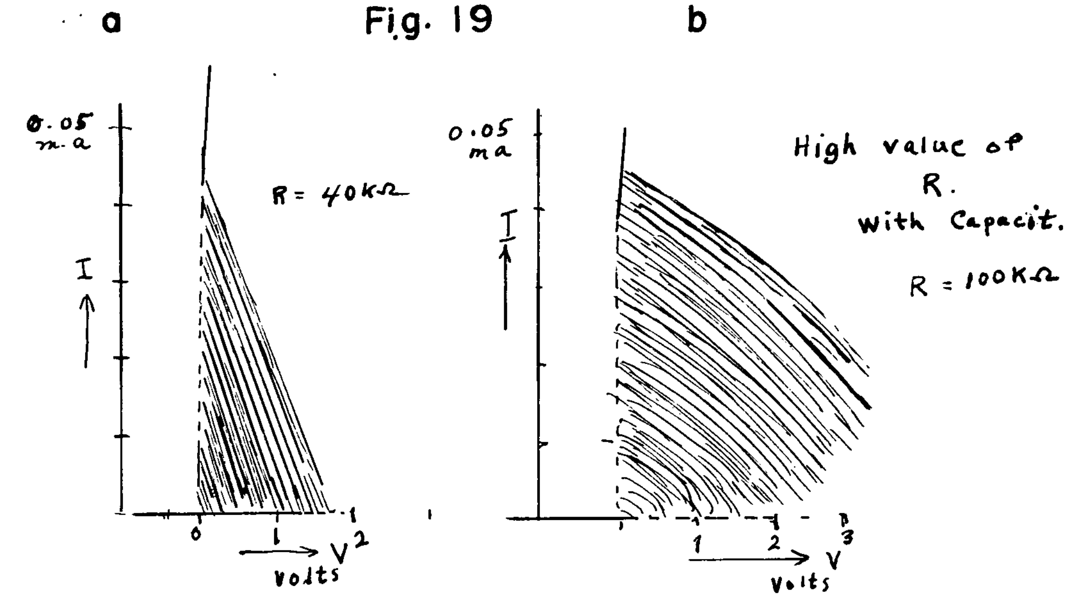
Fig. 18



PULSE SHAPE FOR VARIOUS CURRENT LEVELS.

Curve (i) in Plate IIB was obtained by measuring the current pulse amplitude in microamps by means of a calibrated oscilloscope as a function of V_1, the voltage across the diode when the current pulse was on, using the same method of plotting as for the $\bar{I}$ - $\bar{V}$ curves. For low current pulse amplitudes ($< 25\ \mu$a), the average pulse length was less than 1 micro sec. in length, and because of the circuital RC time constants, failed to reach the characteristic flat top observed at higher pulse amplitudes. Because the current pulses in this region never reached their true amplitude for a given voltage, it was not possible to obtain reliable data for amplitudes below $25\ \mu$a. Fig. 18 a illustrates the pulse shape for amplitudes less than $25\ \mu$a., and Fig. 18 b for pulses of amplitude greater than $25\ \mu$a. The trace was obtained by triggering the oscilloscope sweep with the incoming pulse. It should be noted that no minimum pulse length as was shown by McKay's (1954) data was observed. His results were apparently influenced by circuit capacitance which tended to hold the pulse on for a certain minimum period of time. The curves made by plotting pulse amplitude I_1 against the voltage across the diode when the pulse is on, V_1, were quite linear over the range where data were obtained. The constant slope conductance was of the order of 8×10^{-5} mhos for most diodes, and showed little significant change with temperature. See Plate IIB for an example. The I_1 - V_1 curve and the mean current - mean voltage curves converged at about $60\ \mu$a for most diodes, the point where the pulses remaind "on" most of the time.

Fig. 19



DYNAMIC CURRENT – VOLAGE CHARACTERISTICS.

The I_1 - V_1 curves were extended to zero current, and the point where they crossed the voltage axis was defined as the breakdown voltage V_B. It must be noted that the V_B used here is not necessarily the same as the one defined by McKay (1954) which was the voltage where the multiplication factor M became infinite, a point not easily located in practice. Throughout this work, V_B shall always refer to the intercept of the I_1 - V_1 curve with the voltage axis. The value of V_B was found to increase linearly with increasing temperature, the rate of increase for a typical diode being about 0.06 V/cm.

Both the mean current - mean voltage characteristics and the I_1 - V_1 curves are consistent with those expected from the random switch model.

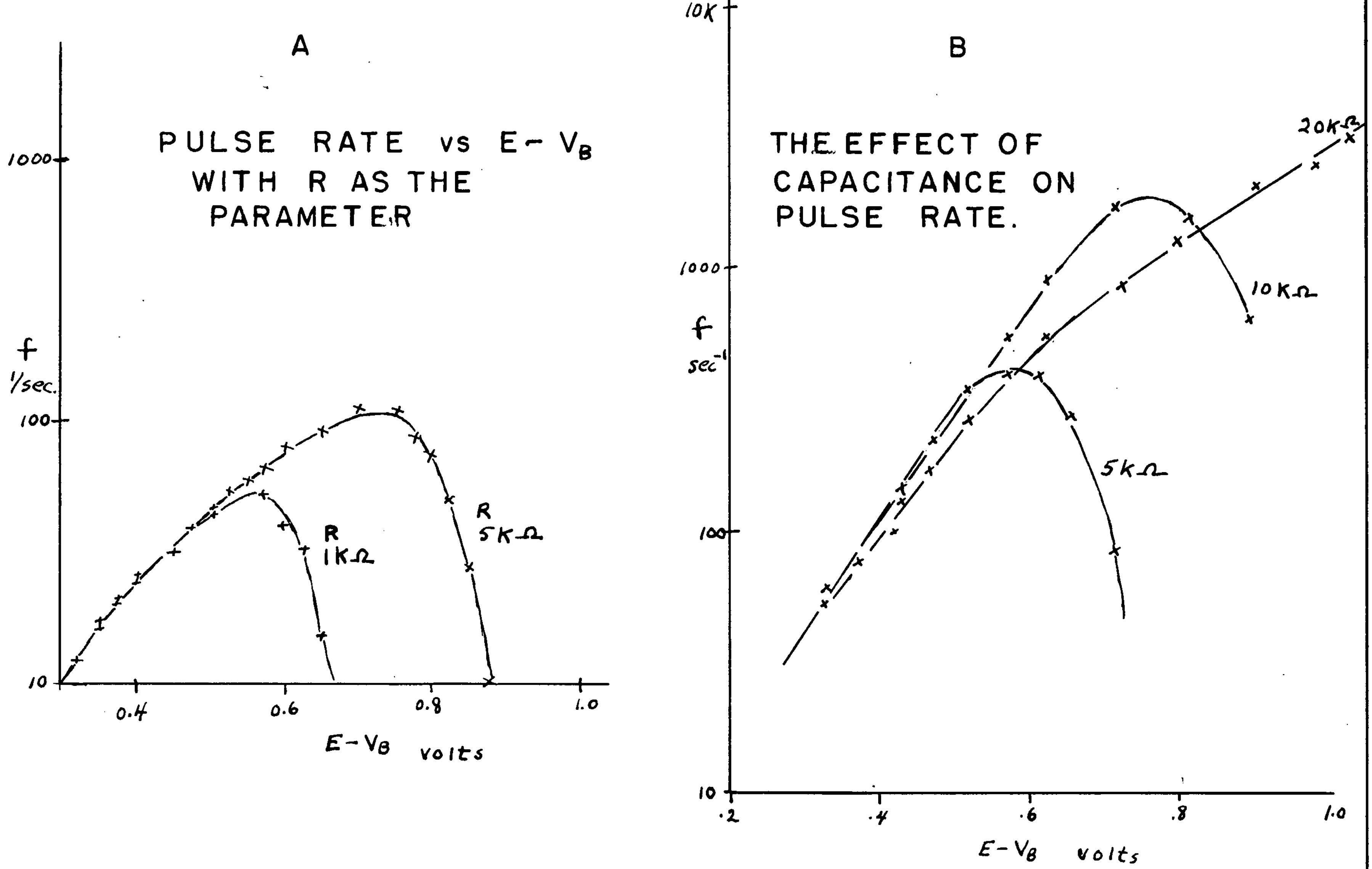
Dynamic display of the I - V characteristics

Fig. 19 a and b are examples of the dynamic display of the I - V characteristic on the oscilloscope for a given R. Several important results are apparent when examining such a display, some of which are:

1) Substantially all the current in the avalanche region of the reverse I - V characteristic is carried by the current pulses as was first reported by McKay (1954). This fact makes it obvious why the measured current - voltage characteristic should be referred to as the mean current - mean voltage characteristic.

2) The reverse voltage when the current pulses first appear is quite well defined on such a display. However, it is expected that current multiplication does take place at voltages below V_B, the point where instability begins to occur.

PLATE III


A
PULSE RATE vs E - V_B
WITH R AS THE
PARAMETER
1000
100
10
f
1/sec.
R
1KΩ
R
5KΩ
0.4
0.6
0.8
1.0
E - V_B volts
B
THE EFFECT OF
CAPACITANCE ON
PULSE RATE.
10K
1000
100
10
f
sec^{-1}
20KΩ
10KΩ
5KΩ
.2
.4
.6
.8
1.0
E - V_B volts

3) The dynamic current - voltage characteristic in the avalanche breakdown region shows all the features that are expected from the model discussed in the previous chapter. The current level for a given supply voltage switches back and forth between I_0 and the point of intersection of the $I_1 - V_1$ curve with the appropriate load line, the diagonal lines between these points having the slope of $-\frac{1}{R}$.

4) The visible effect of capacitance across the diode was to cause the diagonal lines on the display to become curved and shortened as shown in Fig. 19b. This experiment shows that the random switch model is a good approximation to a Si p - n junction in the unstable region at the onset of breakdown.

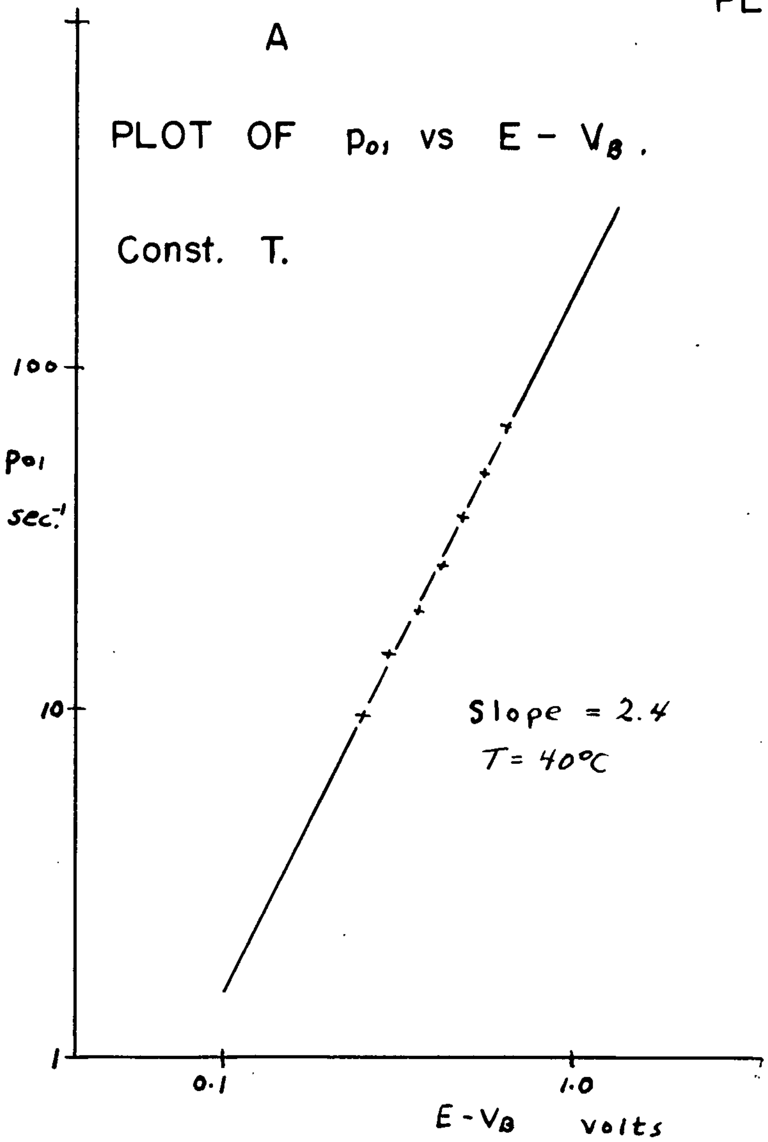
Mean current pulse rate

Plate IIIA gives an example of the mean current pulse rate plotted as a function of the supply voltage in excess to V_B with R as a parameter. For values of R below 10K ohms, the curves all converge to a common curve as $E - V_B$ approaches zero. For higher series circuital resistances R, the RC time constant of the circuit becomes large enough to cause a gradual lowering of the mean pulse rate to values below that expected under ideal measuring conditions, that is, with C = 0. See Plate IIIB. The maximum value of the mean pulse rate and the voltage at which this maximum occurs both increase as R increases.

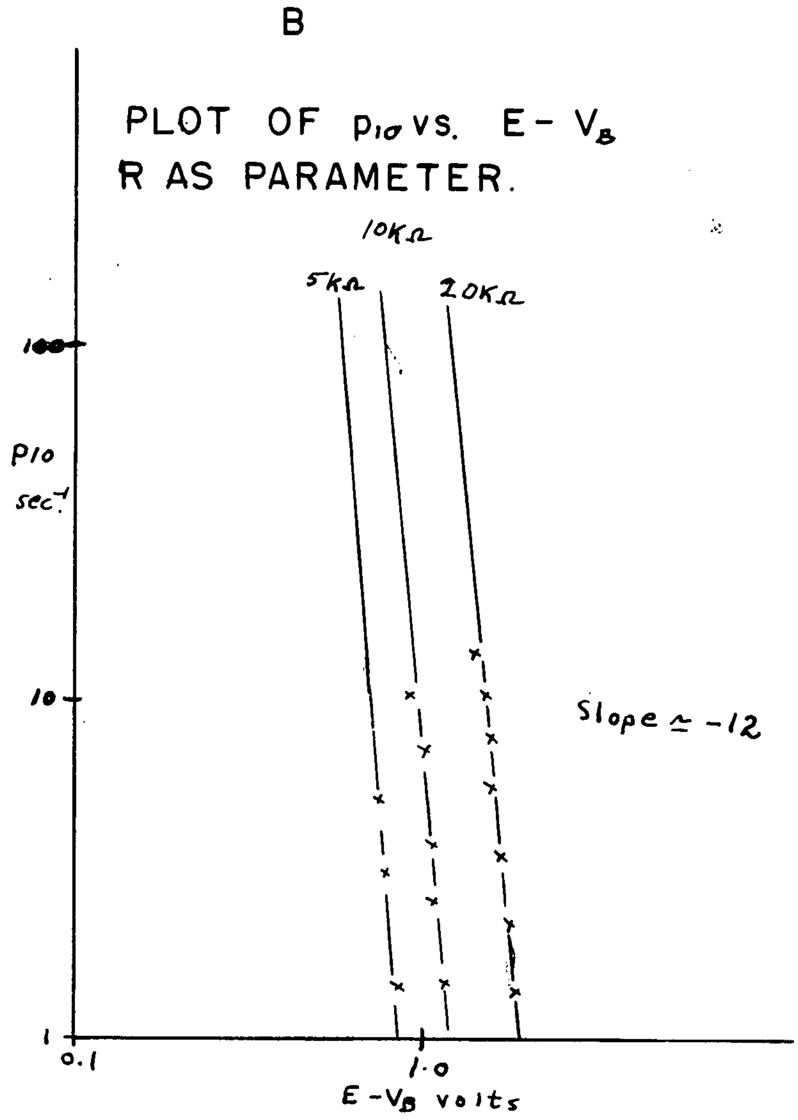
Experimental relations for p_{10} and p_{01}

In determining the functional dependence of p_{10} and p_{01} on the excess voltage $E - V_B$, equations (41) and (42) were used. These are

PLATE IV



A
PLOT OF p_{01} vs E − V_B.
Const. T.
100
10
1
p_{01}
sec.$^{-1}$
Slope = 2.4
T = 40°C
0.1
1.0
E − V_B volts



B
PLOT OF p_{10} vs. E − V_B
R AS PARAMETER.
10KΩ
5KΩ
20KΩ
100
10
1
p_{10}
sec.$^{-1}$
Slope ≃ −12
0.1
1.0
E − V_B volts

$$p_{10} = \frac{I_1 f}{\overline{I}} \simeq f \quad \text{for high enough } E - V_B \tag{41}$$

and

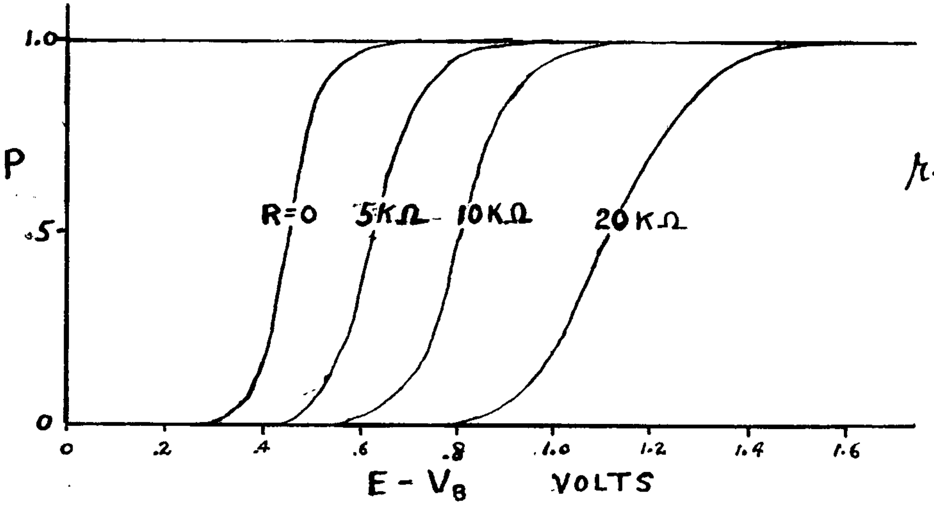
$$p_{01} = \frac{I_1 f}{I_1 - \overline{I}} \simeq f \quad \text{for low enough } E - V_B \tag{42}$$

The exact range of $E - V_B$ where these approximations are valid is determined by examining the mean current - mean voltage characteristic for a given R together with the $I_1 - V_1$ curve and seeing for what range of values of $E - V_B$ the coefficients $\frac{I_1}{I_1 - \overline{I}}$ and $\frac{I_1}{\overline{I}}$ are approximately equal to 1.

(See for example Plate IIB.).

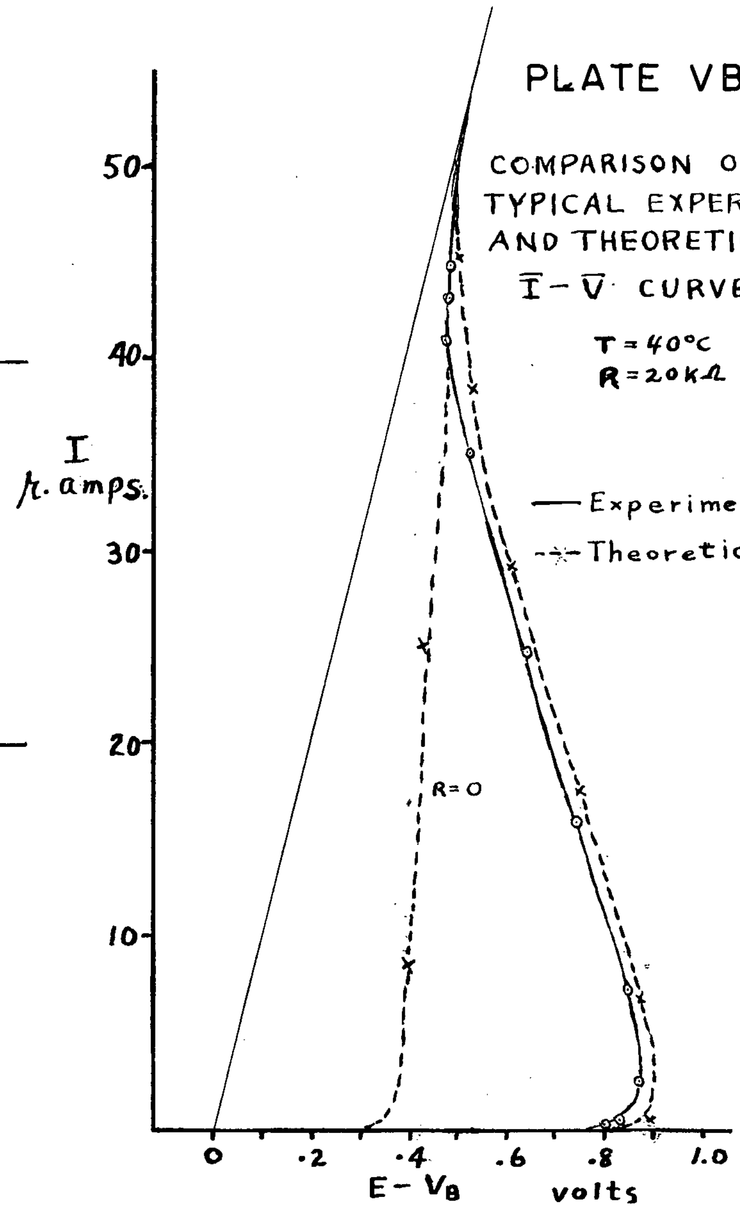
Plate IV A is an example of p_{01} curve, and Plate IV B is an example of the p_{10} curve with R as a parameter, both curves taken under constant temperature conditions. Over the range where the approximations to equations (41) and (42) are valid, the p_{01} and p_{10} curves both satisfy the forms given by equations (18) and (24) in the theoretical discussion. The experimental value of the exponent m of p_{01} given by equation (18) decreases slightly in value as the temperature is increased, the value for the particular diode of Plate IV A at 40 °C being about 2.5. A more detailed discussion of the temperature dependence of p_{01} will be given later. The measured value of the exponent n of p_{10} given by equation (24) was found to be about 12. Because of the rapid functional dependence of p_{10} on $E - V_B$, it was necessary to maintain extremely fine control of $E - V_B$ throughout the counting period which was not easily achieved in spite of the circuital refinements outlined in Chapter III. This difficulty made consistent data for p_{10} difficult to obtain, and for this reason, no measurements to determine the temperature dependence of p_{10} were attempted.

PLATE VA



PLOT OF THE PROBABILITY P vs $E-V_B$ WITH R AS A PARAMETER.

PLATE VB



COMPARISON OF A TYPICAL EXPERIMENTAL AND THEORETICAL $\bar{I}-\bar{V}$ CURVE.

T = 40°C
R = 20 KΩ

By examining the p_{10} curves, it is seen that the experimentally derived coefficient is consistent with the theoretically expected relation, namely,

$$p_{10} = b\,(1 + gR)^n\,(E - V_B)^{-n}$$

Since it was impossible to obtain plots of the mean current - mean voltage characteristic for low values of R because of the large statistical current fluctuations with time, the exact forms of equations (41) and (42) were not employed to obtain additional data for p_{10} and p_{01} beyond the ranges where the approximate forms were valid. Instead, the curves obtained using the approximations were extended as shown in Plate IV A and B assuming the theoretical relation of equation (18) and (24) were valid over the whole range of $E - V_B$ of interest.

Plate V A shows the form of the dependence of P on $E - V_B$ obtained by substituting the experimental values of p_{01} and p_{10} into equation (16). The values of P as a function of $E - V_B$ for R = 20K ohms together with the relation $\bar{I} = PI_1$ were used to obtain a theoretical mean current - mean voltage curve. Plate V B compares the experimentally measured mean current - mean voltage curve with the theoretical curve. The discrepancy between the curves is attributed to a small inaccuracy in V_B, and in the slope of the $I_1 - V_1$ curve as well as an uncertainty in the coefficient of p_{10} for the reasons mentioned before. The agreement obtained between the experimental and theoretical mean current - mean voltage curves is considered good evidence that the extension of the p_{10} and p_{01} curves is justified, and that the postulated dependence of p_{10} and p_{01} on $E - V_B$ is satisfactory.

PLATE VIA

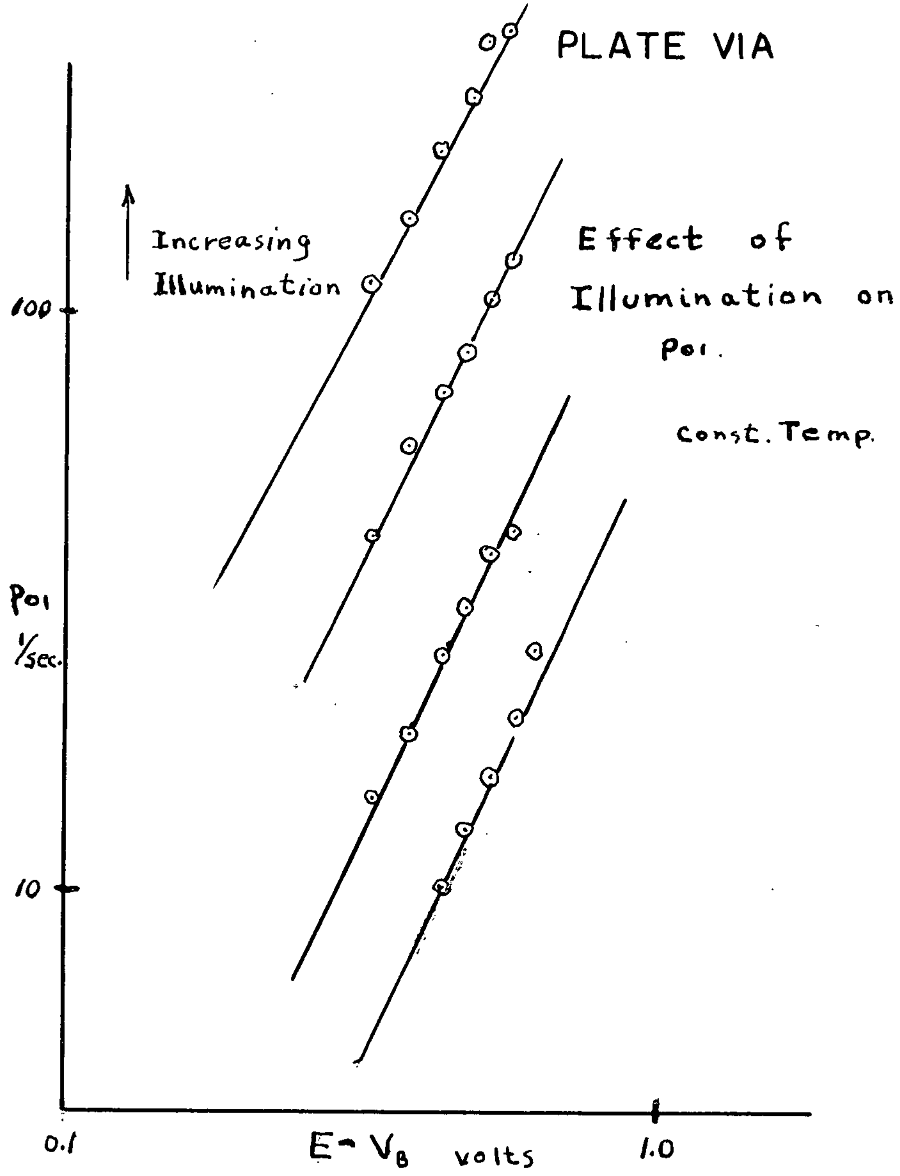
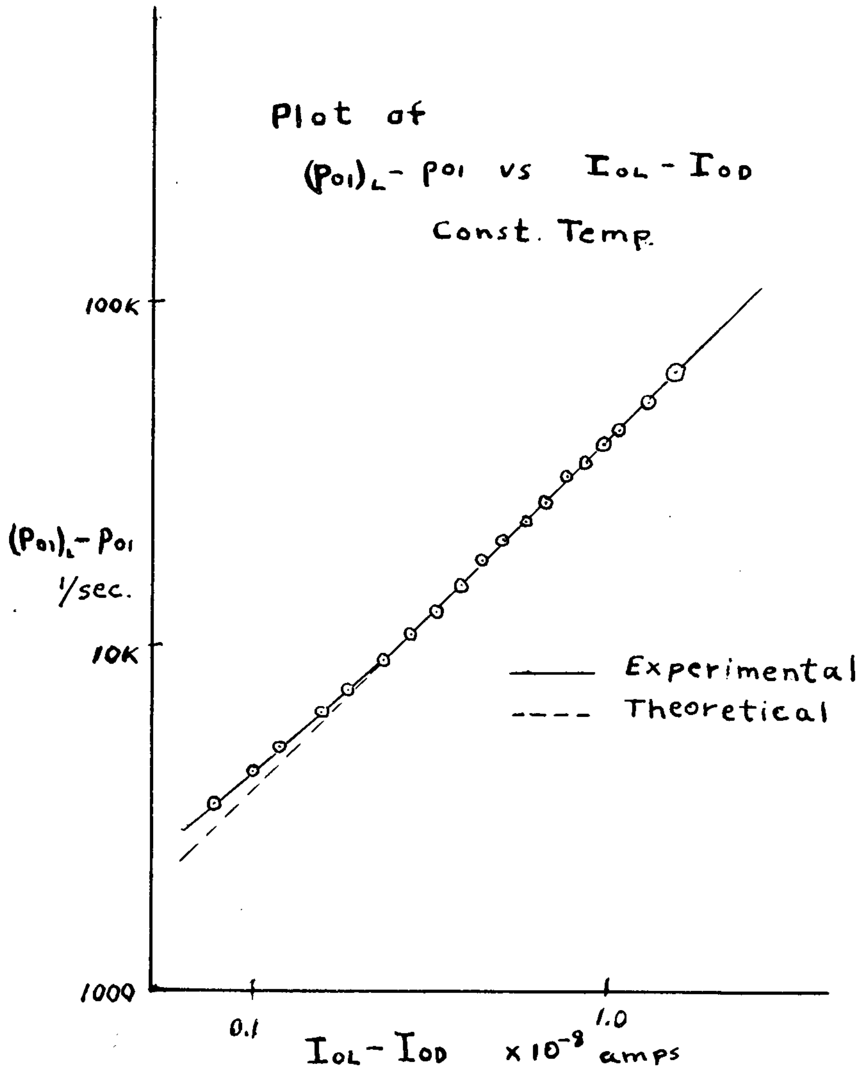


PLATE VIB



Illumination

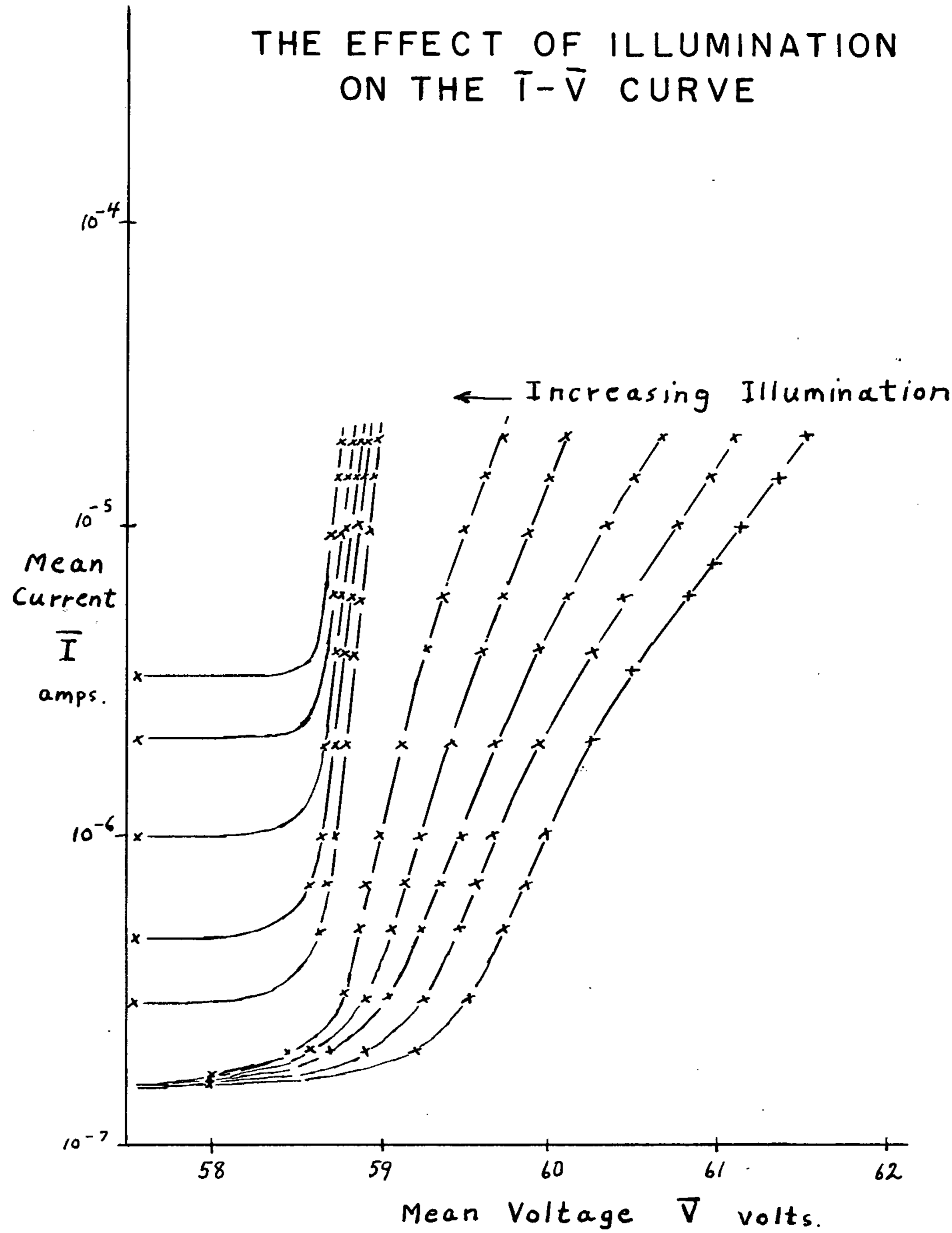
Illumination of a p - n junction results in an increase in the density of ionized carriers within the barrier region available for current conduction without actually causing any change in the diode operating conditions such as temperature and V_B. The increase in carrier density results in an increase in the pre-breakdown saturation current which in turn causes the coefficient of p_{01} to increase as predicted by equation (21). Plate VIA is an example of the effect of illumination on the coefficient, a, as observed in a typical p - n junction. The immediate effect on the mean pulse rate is to cause a net upward shift in the common asymtotic value for low $E - V_B$ as the intensity of illumination was increased. No effect on p_{10} was observed as expected.

Plate VIB is a plot of the difference between illuminated and dark values of p_{01} at constant $E - V_B$ against the excess pre-breakdown current resulting from the illumination. By examining the curve, it is found that the theoretically predicted linear relation given by equation (47) is experimentally verified. The small departure from linearity at low $I_{0L} - I_{0D}$ values is due to an uncertainty in I_{0D}. Since the electrometer with which I_{0D} was measured was only accurate to within 10%, this error was easily accounted for. The following conclusions can be drawn from an analysis of the results shown in Plate VIB:

1) Each avalanche is initiated by only one carrier entering the breakdown region.

2) The average probability of a discharge occuring in the time interval between t and t + dt under constant temperature and voltage is directly

PLATE VII



THE EFFECT OF ILLUMINATION ON THE $\bar{I}-\bar{V}$ CURVE
← Increasing Illumination
Mean Current $\bar{I}$ amps.
10^{-4}
10^{-5}
10^{-6}
10^{-7}
58
59
60
61
62
Mean Voltage $\bar{V}$ volts.

proportional to the number of carriers entering the patch breaking down.
If the operating conditions are not changed, but an increase in I_o is obtained through illumination of the junction, the increase in the rate that carriers enter the breakdown region is directly proportional to the increase in I_o. Thus, in accordance to equation (47),

$$(p_{o1})_L - p_{o1} = \frac{\delta A P_d}{A e}(I_{oD} - I_{oL}) \qquad (47)$$

The probability P_d is independent of illumination, and of the carrier density in the junction.

Plate VII shows the effects of illumination on the mean current - mean voltage characteristic. The slope of the $\bar{I}$ - $\bar{V}$ curves on a semi-log plot increases rapidly with increased illumination. More intense light results in the predicted increase in I_o.

Temperature effects

Plate VIII A is an example of the relation of p_{o1} vs $(E - V_B)$ with temperature as the parameter. These results were obtained by measuring the pulse rate f for $E - V_B$ voltages well below the voltage at which f_{max} occurs, taking into account the fact that V_B increases with temperature. The circuital resistance in each case was 5K ohms, a value for which the circuital RC time constants were negligibly small compared to the average pulse duration.
The curves show a gradual increase of the coefficient as well as a decrease in the exponent m as the temperature is increased. The value of m ranged from 2.05 at 74 °C to about 2.9 at 53 °C. Provided P_d at constant voltage

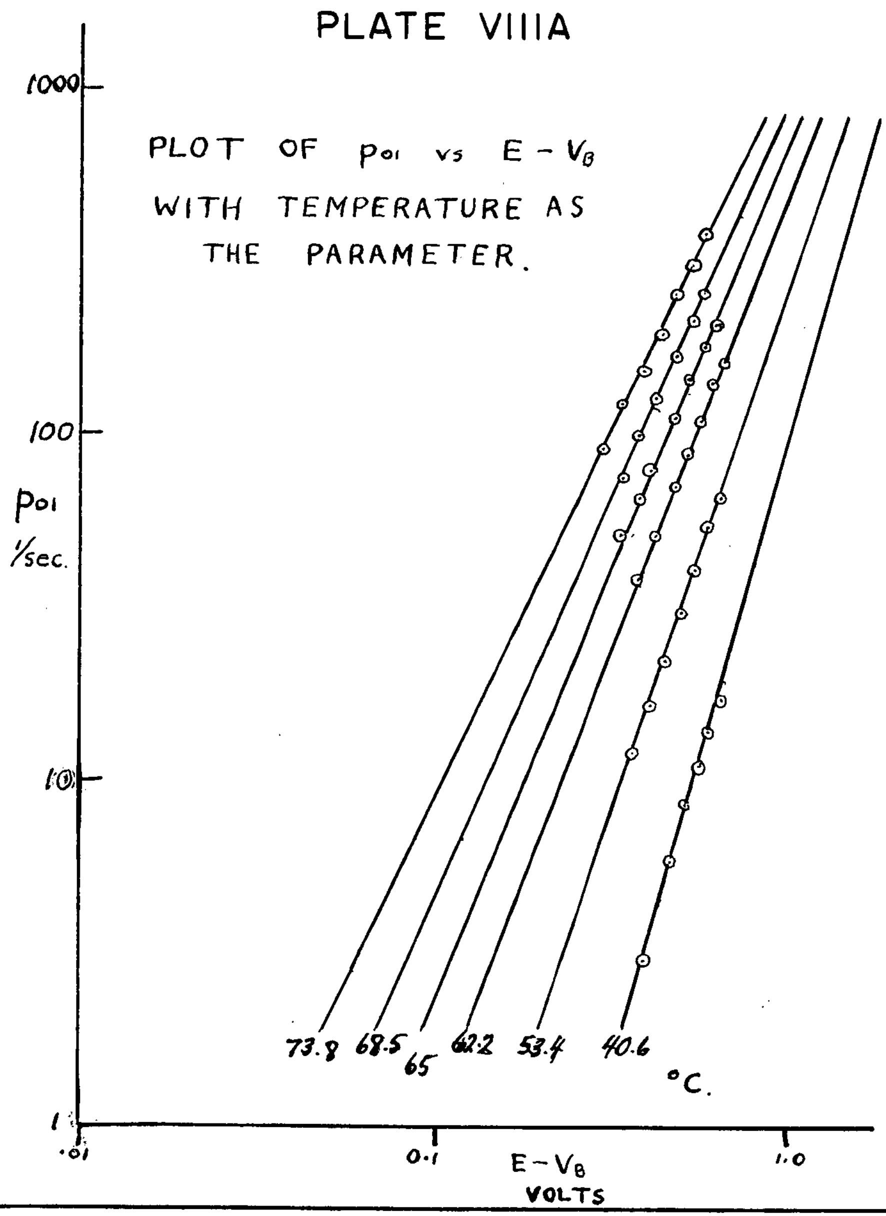


PLATE VIIIA
PLOT OF p_{01} vs $E - V_B$
WITH TEMPERATURE AS
THE PARAMETER.
1000
100
10
1
p_{01}
1/sec.
73.8
68.5
65
62.2
53.4
40.6
°C.
.01
0.1
1.0
$E - V_B$
VOLTS

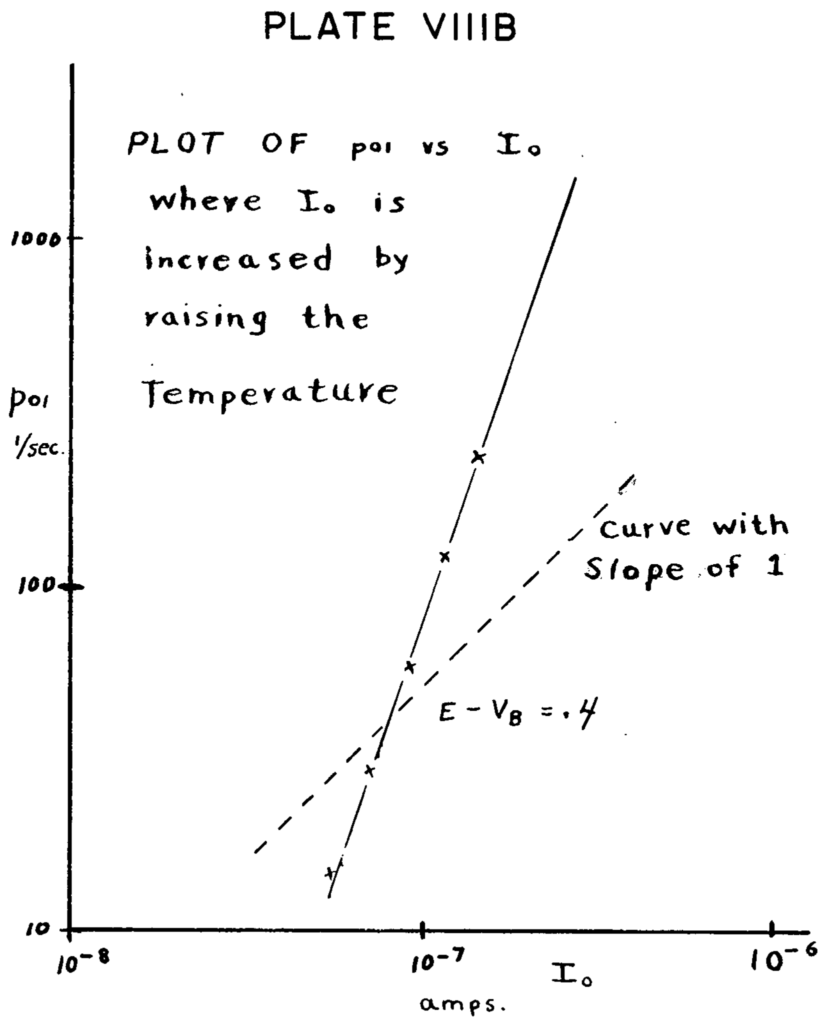


PLATE VIIIB
PLOT OF p_{01} vs I_0
where I_0 is
increased by
raising the
Temperature
1000
100
10
p_{01}
1/sec.
Curve with
Slope of 1
$E - V_B = .4$
10^{-8}
10^{-7}
10^{-6}
I_0
amps.

is substantially independent of temperature, p_{o1} is again expected to increase in proportion to the number of carriers entering the breakdown region. If the fraction of the pre-breakdown current I_0 entering the region remains constant with respect to temperature, we expect a linear relation between p_{o1} and I_0 as given by

$$p_{o1} = \frac{I_0 \Delta A}{e\ A} \quad P_d$$

By plotting p_{o1} vs the temperature T at constant $E - V_B$ together with I_0 vs T, and then eliminating the parameter T, we obtain the curve shown in Plate VIIIB. Instead of a slope of unity on a log plot which would result if the assumptions were valid, the curve has a slope of between 1.5 and 2.5 depending on the exact value of $E - V_B$ at which the p_{o1} values were obtained.

The following are two reasons to account for this deviation from the predicted result, either or both of which may be valid:

1) The probability that a carrier will initiate a discharge, P_d, is not independent of the temperature of the junction.

2) In the theoretical treatment, it was assumed that the measured pre-breakdown current I_0 was proportional to the current through the breakdown patch. However, much of the current I_0 is leakage current of some form having a functional dependence on temperature which in general differs from that through the junction, and thus a linear relation between p_{o1} and I_0 will not prevail.

Culter and Bath (1957) of the Hughes Aircraft Co., working with the same type of diodes used in this investigation, showed that the "saturation" reverse current was in fact composed of two components, namely, the junction component I_j, and the surface component I_ℓ. Their theoretical form of the current in a Si diode was

$$I = I_j \left(\exp \frac{eV}{kT} - 1 \right) \pm I_\ell \left(\exp \frac{eV}{kT} - \frac{eV}{kT} - 1 \right)^{1/2}$$

where the negative sign is used for the reverse bias case. Under high reverse bias, the leakage component becomes

$$I_\ell \left(\frac{-e|V|}{kT} - 1 \right)^{1/2}$$

Their experimental results showed that at room temperature, the leakage component could be as high as 10^4 times larger than the junction component. The activation energy they obtained for the junction component was about 1.2 e.V. which is close to the expected value, that is, the width of the forbidden energy gap of Si. On the other hand, the activation energy for the leakage component was about 0.62 e.V. Examination of the I_o vs $1/T$ plot for the diode used in this investigation shows that the slope corresponds to an activation energy of approximately 0.55 e.V. This implies that the surface leakage component in the diode studied is also much greater than the junction component. An increase in this slope towards the expected junction activation energy of 1.2 e.V. would have the desired effect on the slope of the log $p_{o\prime}$ vs log I_o, curve, that is, a decrease in its slope towards the value 1.

The conclusion is that the number of carriers entering the breakdown region is not proportional to the measured current I_o when varying the temperature of the junction, because I_o contains current components which do not pass through the junction, and have some unknown dependence on temperature. Because of this complex temperature dependence, little useful information regarding the avalanche breakdown mechanism can be derived from temperature variation measurements.

It is not understood why the functional dependence of the extinction probability p_{10} on the excess voltage $(E - V_B)$ should be so much more rapid than that of p_{01}. Rose's (1957) qualitative discussion of a possible extinction mechanism as outlined in Chapter II is of little value because no theoretical relation for p_{10} is given which can be checked experimentally. Also, there is no evidence of any significant fluctuation in the current during avalanche which is necessary if Rose's mechanism is operating. Very little therefore can be said about the exact nature of the avalanche extinction mechanism involved in Si p - n junctions.

Spectrum measurements

If the values of p_{01} and p_{10} are independent of the past history of the system, and independent of time, that is, the switching process is truly Markoffian, then for a constant value of $E - V_B$, the current spectral density should have the form

$$S_I(\omega) = \frac{S_I(0)}{1 + \omega^2 \tau^2}$$

where τ is a single relaxation time given by

$$(p_{10} + p_{01})^{-1} \text{ and } S(0) = \frac{4 p_{10} p_{01} I_1^2}{(p_{01} + p_{10})^3} = \frac{4 p_{10} \bar{I}^2}{p_{01}(p_{01} + p_{10})}$$

PLATE IX

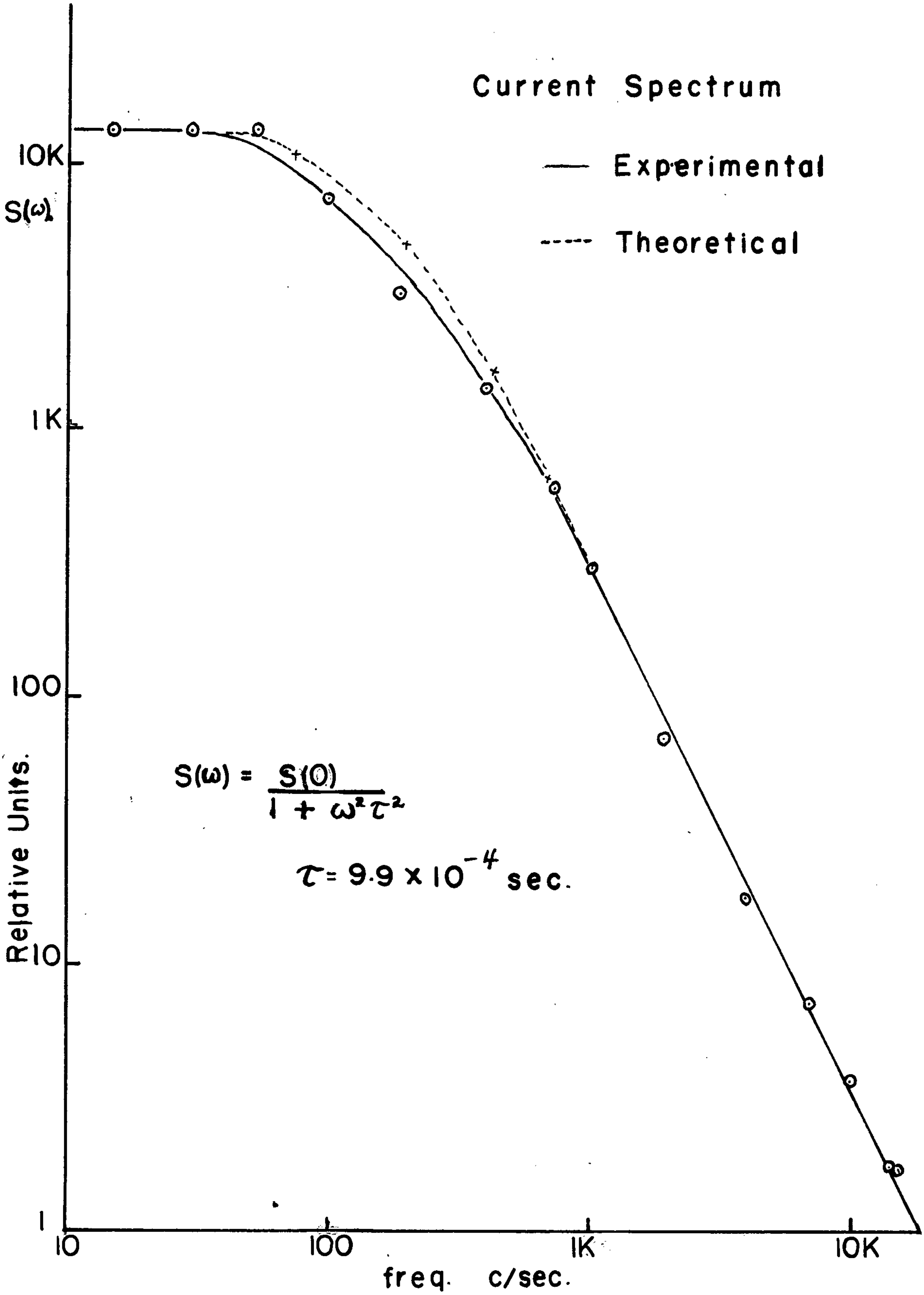

Current Spectrum
Experimental
Theoretical
10K
S(ω)
1K
100
10
1
Relative Units.
S(ω) = S(0) / 1 + ω²τ²
τ = 9.9 x 10⁻⁴ sec.
10
100
1K
10K
freq. c/sec.

Plate IX is an example of the spectrum obtained experimentally over the frequency range from 10 c/s to 18 Kc/s compared to an ideal theoretical curve indicating that the form for $S_I(\omega)$ is correct to within experimental error. No attempt was made to check experimentally the value of $S_I(0)$ against the measured values of p_{10}, p_{01} and I_1. The value of the relaxation time $\tau = (p_{10} + p_{01})^{-1}$ in this case was found to be 9.9×10^{-4} sec. It is interesting to note that both of the transition probabilities p_{01} and p_{10} can be determined from a measurement of the spectrum and the mean current at which the spectrum is taken. For example, it can be shown that

$$p_{01} = \frac{1}{\tau}\left[1 + \frac{S_I(0)}{4\bar{I}^2\tau}\right]^{-1}$$

and

$$p_{10} = \frac{1}{\tau}\left[1 + \frac{4\bar{I}^2\tau}{S_I(0)}\right]^{-1}$$

The values of p_{01} and p_{10} obtained using this method could be compared to those obtained from the mean pulse rate measurements.

Conclusions

Except for the temperature dependence of p_{01} on I_0, good agreement was obtained between the theoretical and experimental results. The following is a brief summary of the results:

1) The measured mean current - mean voltage characteristics depend on the external circuital resistance R, and if R is large enough, the characteristic will display turnover. In general, the apparent slope of the breakdown characteristic $(d\bar{I}/d\bar{V})$ is a function of the measuring circuit, a fact not appreciated by diode manufacturers.

2) The pulse amplitude I_1 increases linearly with increasing voltage to within experimental error, the slope for most diodes being of the order of 8×10^{-5} mhos.

3) There was no observed minimum pulse length as indicated by McKay's (1954) data. The pulses appeared to vary randomly in frequency and length.

4) The dynamic display showed that the random switch in series with an ideal discharge device was a good physical model to account for most of the observed characteristics of a Si p-n junction in the unstable avalanche region.

5) The measured values of the parameters p_{01} and p_{10} had the expected dependence on the excess voltage $E - V_B$. The measured value of the exponent m of p_{01} as given by equation (18) was about 2.5 at 40 °C varying somewhat with temperature, and the measured exponent n of p_{10} as given by equation (24) was about 12 for the diode on which measurements were made.

6) The resulting equation for P obtained by combining p_{01} and p_{10} gives the observed mean current - mean voltage curves to within experimental error.

7) The measured current pulse rate displayed the predicted dependence on the circuital resistance and the voltage, that is, the curves converged to a single curve for all R at low $E - V_B$ values, and had a maximum value of pulse rate which increased and shifted towards higher $E - V_B$ values as R was increased.

8) There was strong evidence that each avalanche current pulse was initiated by only one carrier entering the breakdown region. The probability of a given carrier initiating an avalanche does not depend on illumination or on the density of carriers present in the region of breakdown.

9) Thermally ionized carriers do not result in a linear dependence of p_{01} on I_0 because the current through the breakdown patch is a different function of temperature from the measured pre-breakdown saturation current. This is due to the predominance of "surface leakage" current which has a different temperature dependence from the true junction current.

10) The spectral density of the avalanche current displayed a single relaxation time indicating that p_{01} and p_{10} are stationary parameters, and thus that the process is Markoffian.

<u>Outstanding problems</u>

Some problems which require further investigation are:

1) Why are the $\bar{I}$ - $\bar{V}$ breakdown characteristics observed in Si junctions so much sharper than those in Ge junctions? It may be that Ge is much more sensitive to surface effects or other extraneous effects than is Si resulting in a "softening" of the breakdown $\bar{I}$ - $\bar{V}$ characteristics.

2) What mechanism is responsible for the extinction of an avalanche discharge once it is initiated, and why does the measured extinction probability p_{10} decrease rapidly with increasing excess voltage $E - V_B$?

3) What basic physical factors determine the exponents of the p_{10} and p_{01} curves as given by equations (18) and (24)?

These and other related problems require further study in order to gain a full fundamental understanding of the avalanche pulses in Si diodes.

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