A D.C. AMPLIFIER AND REFERENCE VOLTAGE SUPPLY SUITABLE FOR USE IN A MAGNET CURRENT REGULATOR

by

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ABSTRACT

The requirements imposed upon a d.c. amplifier and reference voltage supply, suitable for measurement and control of signals of the order of 1 volt to an accuracy of 10 parts per million are investigated. A basic block design is proposed to achieve this goal using a transistor differential amplifier of new design and a highly regulated reference voltage supply. The detailed design procedure for these building blocks is then presented. Exact analysis of important characteristics of several relevant circuit configurations is presented.

The unit under discussion was constructed and the measured performance was found to be in good agreement with the predicted performance.
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1. INTRODUCTION

A d.c. amplifier and reference voltage supply suitable for use in a magnet current control, or precision potentiometer, was designed and built.

In the past\(^1\),\(^2\) such systems have usually employed amplifiers based on some modulation technique, such as a chopper-stabilized amplifier. This technique usually requires an auxiliary a.c. amplifier to extend the pass-band out to the required few kilocycles. The modulation system is very susceptible to pickup or noise at the modulator frequency. Electronic choppers can work at much higher frequencies than mechanical or magnetic choppers or modulators. However zero-drift again becomes a problem.

Magnetic modulators and amplifiers were investigated. It appeared highly unlikely that the stability required could be achieved with these devices. Consequently, a new transistor differential amplifier was designed which offered superior performance.

The new amplifier has good zero-stability, a low noise figure and provides a d.c. to 20 kilocycles per second bandwidth with inexpensive devices. Since only one amplifier is used instead of separate low and high frequency amplifiers, the gain is constant and the phase shift small up to the upper cutoff frequency. The new device has operated for several days with its input shorted. Under this condition the peak-to-peak voltage drifts were of the order of three microvolts, referred to the input.
An independent reference voltage supply (which is extremely stable under varying line voltage conditions) is included in the system. In practice, the stability of the reference voltage is limited only by that of the break-down-diode used as a reference.
2. OUTLINE OF SYSTEM

2.1 General

The block diagram of the system is shown in Figure 2-1. The system consists of a d.c. amplifier, temperature control and power amplifier, a regulated power supply with pre-regulator and a reference supply with pre-regulator. The specifications required for measurement and control of signals of about 1 volt to ± 10 ppm follow.

2.2 The D.C. Amplifier

The d.c. amplifier must have a zero stability of ± 10 microvolts or better. It should have an available output of several volts. The gain should be great enough, that any subsequent amplifier need not be an ultra-low drift type. Input impedance should be of the order of 100K ohms or greater to avoid loading the source. Since such a unit would be extremely sensitive, a high degree of common-mode rejection is essential to reduce the effects of stray pickup. The output impedance should be low enough to drive a medium impedance meter or recorder directly. A well defined stable voltage gain would allow direct use for low voltage level measurements.

2.3 The Temperature Control System

A unit with the sensitivity and stability of that described above would be extremely difficult to temperature compensate over a temperature range of any appreciable extent.
It was therefore felt that the more direct approach of enclosing all critical components in an oven at constant temperature would be more fruitful. It was felt that the oven temperature should be held to ± 0.1°C or better if practicable.

2.4 Regulated Power Supply

It is reasonable to expect equipment of the nature of that described above to be sensitive to power supply fluctuations.

While it might be possible to compensate the units for supply fluctuations, the temperature changes caused by changing power dissipation and the changes of component parameters with voltage would be expected to cause problems more severe than that of the construction of a well regulated power supply. It was therefore decided to design a power supply with regulation against line voltage changes of 0.001% or better. This degree of regulation presents similar, but much less severe, problems than mentioned above in connection with the amplifier. It was therefore decided to divide the regulator into two units: a rough pre-regulator (1%) and a main regulator inside the oven. The power supplies were designed to tolerate line voltage variations from 90 to 135 volts rms.

2.5 Reference Voltage Supply

A reference voltage supply was incorporated to allow differential measurements or control. The design is very similar to the supply described above. To avoid limiting the accuracy of the system, this reference supply should have a stability of 0.0001% or better.
Figure 2-1. Block Diagram of D.C. Amplifier and Reference Supply
3. MODULE DESIGN

3.1 The Two Transistor Compound Block

Appendix 5 gives the gain of a common-emitter transistor amplifier with an external emitter impedance, $Z_E$, and load impedance $Z_L$, as

$$A_v = -G \left( \frac{h_{fe}}{1+h_{fe}} - \frac{h_{ob}}{1-h_{rb}} \right) \frac{1-h_{rb}}{1+q} \quad \ldots 3-1$$

where

$$G = \frac{Z_L}{Z_E}$$

$$q = \frac{h_{ib}}{Z_E} + G h_{ob} Z_E + G \left( \frac{h_{fe} h_{rb}}{1+h_{fe}} + \frac{h_{ib} h_{ob}}{1-h_{rb}} \right)$$

In the usual design of common-emitter amplifiers the major departure of the gain from the ideal value, $-G$, is due to the terms $h_{fe}/(1+h_{fe})$ and $h_{ib}/Z_E$. The major source of non-linearity is the term $h_{ib}/Z_E$, since $h_{ib}$ is inversely proportional to the emitter current.

A great improvement in both the predictability and stability of the gain can be achieved by lowering the value of $h_{ib}$ and raising the value of $h_{fe}$. The well known Darlington compound raises $h_{fe}$ to the product of $h_{fe1}$ and $h_{fe2}$ of the two transistors used. Unfortunately, however, the effective value of $h_{ib}$ is actually increased somewhat and is still proportional to the reciprocal of the emitter current.

The two transistor compound suggested here, and shown
in Figure 3-1 also raises the current gain to approximately the product of the current gains of the individual units. However, $h_{ib}$ is reduced greatly (typically one order of magnitude) over either a single transistor or the Darlington compound. In addition, the "base-emitter" voltage drop of the Darlington compound is the sum of the two base-emitter voltage drops with attendant temperature coefficients. The "base-emitter" voltage drop of the compound suggested here is only that associated with one transistor.

In the circuit used in this work the collector current of the first transistor is several times larger than the collector current of the second transistor divided by its current gain. Thus the collector current of the first transistor varies only slightly over the operating range of the compound. Since $h_{ib}$, for the compound, is primarily determined by $h_{ib1}$, $h_{ib}$, as well as being small, will be subject to much less percentage variation over the operating range. The compound circuit is analyzed in detail in Appendix 6 and only the approximate values of the parameters will be reproduced here.

$$h_{ib} \approx \frac{h_{ib1}}{h''_{fe2}} \quad \text{where} \quad h''_{fe2} \approx \frac{h_{fe2}}{1 + \frac{h_{fe2} h_{ib2}}{R_B} \frac{h_{ib1}}{h_{fe2}}} \quad \ldots 3-2$$

$$h_{rb} \approx h_{rb1} + h_{ib1} h_{ob2}$$

$$h_{ob} \approx h_{ob1} + \frac{h_{ob2}}{h_{fe1}}$$

$$h_{fe} \approx \frac{h_{fe1} h''_{fe2}}{1 + h_{fe1} h''_{fe2} h_{ob1} h_{ib2}}$$

$$I_{cbo} = I_{cbol} + \frac{I_{cbo2}}{h_{fe1}} \; ; \quad V_{BE} = V_{BE1}$$
3.2 A Practical Common-Emitter Amplifier Suitable For Use In a Differential Amplifier Employing the Two Transistor Compound

The technique of using an emitter resistor in a common-emitter amplifier to stabilize gain, raise input impedance, and make the amplifier properties less dependent on transistor parameters is well known. This technique is especially effective with the two transistor compound described above. The amplifier herein described employs differentially-connected common-emitter units incorporating the two-transistor compound with emitter resistors. This basic building block is shown in Figure 3-2.
Appendix 5 gives the voltage gain and input impedance of a common emitter amplifier with emitter resistor as

\[ A_{ve} = -G \left( \frac{h_{fe}}{1+h_{fe}} \frac{h_{ob} R_E}{1-h_{rb}} \right) \frac{1-h_{rb}}{1+q} \]...

where \( A_{ve} \) = common emitter voltage gain

\[ G = \frac{R_L}{R_E} \]

\[ q = \frac{h_{ib}}{R_{EO}} \left( \frac{R_E}{R_{EO}} \right) + G \left( \frac{h_{fe} h_{rb}}{1+h_{fe}} + \frac{h_{ib} h_{ob}}{1-h_{rb}} \right) \]

\[ R_{EO} = \sqrt{\frac{h_{ib}}{G h_{ob}}} \]

and

\[ Z_{IE} = (1+h_{fe}) R_E \frac{1+q}{l-h_{rb} + (1+G)(1+h_{fe}) h_{ob} R_E} \]

\[ = \frac{1+q}{1-h_{rb} \frac{1}{R_E} + (1+G) h_{ob}} \]

3.3 Considerations in the Amplifier Design

It was decided to design a three stage amplifier with an overall gain of 1,000, and an input impedance of about 100K ohms. The gain figure of 1,000 was chosen so that an auxiliary amplifier used to increase the gain further could be
relatively simple without adversely effecting the zero stability.

The three stages were to consist of:

1. A high-gain, very low-drift input stage with good common-mode rejection
2. A medium-gain, low-drift second stage
3. An emitter-follower output stage with provisions for common-mode feedback to the current-source transistor of the input stage.

3.4 The Input Stage of the D.C. Amplifier

The input stage consists of two amplifiers of the type shown in Figure 3–2 connected differentially with a transistor as a high-impedance current source in the emitter leads. To reduce problems associated with leakage current, the transistors used in the two amplifier units were all silicon. The circuit is shown in Figure 3–3. Q401 and Q402 are bonded together, cap-to-cap with Dow Corning 731 Silicone rubber. This assembly is then wrapped with a layer of thin insulating tape, followed by two layers of copper tape separated by insulating tape. The whole assembly is then wrapped with another layer of insulating tape. The wrappings act as a thermal filter to keep the two transistors very nearly at the same temperature even when the ambient temperature is changing. The other pairs of transistors in the amplifier are treated similarly.

The compound block consisting of Q402 and Q404 will now be examined. The base-emitter voltage of a silicon transistor operating at 1 ma at room temperature is about 0.6 volts. The common-emitter input impedance, $h_{ie}$, of such a transistor is
Figure 3-3. The Input Stage of the D.C. Amplifier

approximately \( h_{fe} h_{ib} \), where \( h_{fe} \) is the common-emitter current gain and \( h_{ib} \), the common-base input impedance, is approximately 30 ohms. If a resistor \( R_B \), is placed between the base and the emitter, the input current to the combination (see Figure 3-4) will be

\[
I_B'' = I_B + \frac{V_{BE}}{R_B}
\]

\[
= \frac{I_c}{h_{FE}} + \frac{V_{BE}}{R_B}
\]
Thus

\[ h_{FE}'' = \frac{I_C}{I_B''} \]

\[ h_{FE}'' = h_{FE} \frac{1}{1 + \frac{V_{BE}}{I_C R_B} h_{FE}} \]

...3-5

where \( I_{B''}, I_B, I_C, V_{BE}, h_{FE} \) are d.c. large-signal parameters.

However,

\[ i_{B''} = \frac{R_B + h_{ie}}{R_B} i_B = \frac{R_B + h_{fe} h_{ib}}{R_B} i_e \]

\[ h_{fe}'' = \frac{i_c}{i_{B''}} = h_{fe} \frac{1}{1 + \frac{h_{fe} h_{ib}}{R_B}} \]

...3-6

where \( i_{B''}, i_B, i_c, h_{fe}, h_{ib} \) are small-signal parameters.

Figure 3-4. Transistor With Base-Emitter Resistor
If the above mentioned parameters are substituted into 3-5 and 3-6, 3-7 and 3-8 are obtained.

\[
\frac{1}{1 + \frac{600}{R_B} h_{FE}} \quad \ldots 3-7
\]

\[
\frac{1}{1 + \frac{30}{R_B} h_{fe}} \quad \ldots 3-8
\]

If \( h_{fe} \) and \( h_{FE} \) are assumed to be approximately equal, as is usually the case, it is seen that \( h_{FE} \) may be considerably reduced without serious reduction of \( h_{fe} \). This permits larger current in the input transistor of a compound for a given compound collector current. If the current gains of Q404 are taken as \( h_{FE} = h_{fe} = 40 \) and R408 is taken as 5.6K ohms, then \( h_{FE}'' = 7.5 \) and \( h_{fe}'' = 33 \). Q404 is actually operated at a little less than 1 ma. However, the results quoted above are still approximately correct. OC-201's were chosen for Q403 and Q404. Q402 is operated at approximately 100 \( \mu \)a. From the data sheet for a 2N697 at \( I_c = 100 \mu \)a,

\[ h_{ib} \approx 300 \text{ ohms} \]

\[ h_{fe} \approx 50 \]

\[ h_{rb} \approx 3 \times 10^{-5} \quad \ldots 3-9 \]

\[ h_{ob} \approx 2 \times 10^{-7} \text{ mhos} \]

From the data for the OC-201, \( h_{fe} \) is as quoted above, 40 and \( h_{ob} = .5 \) \( \mu \) mho. The presence of R408 leads to \( h_{fe}'' = 33 \) as
stated above. Combining this with 3-9 and 3-2 gives as the parameters of the compound Q402, Q404 and R408:

\[ h_{ib} \approx 9 \text{ ohm} \]
\[ h_{rb} \approx 1.8 \times 10^{-4} \]
\[ h_{ob} \approx 2.1 \times 10^{-7} \]
\[ h_{fe} \approx 2000 \]

From 3-3 and 3-10 and taking \( G \approx 60 \)

\[ R_{E0} = \sqrt{\frac{h_{ib}}{G h_{ob}}} \approx 850 \text{ ohms} \]
\[ q \approx 0.011 + 0.01 \left( \frac{R_{E0}}{R_E} + \frac{R_E}{R_{E0}} \right) \]
\[ \frac{h_{fe}}{1 + h_{fe}} \approx 1 - 0.5 \times 10^{-3} \approx 1 \]

and

\[ A_{VE} \approx -G \left( \frac{1}{1+q} \right) \approx -G (1-q) \]
\[ A_{VE} \approx -G ((1 - 0.011 - 0.01 \left( \frac{R_{E0}}{R_E} + \frac{R_E}{R_{E0}} \right))) \quad \ldots 3-11 \]

and

\[ Z_{IE} \approx \frac{1 + 0.01 \left( \frac{R_{E0}}{R_E} + \frac{R_E}{R_{E0}} \right) + 0.011}{50 \frac{R_E}{R_{E0}} + 0.021 (G + 1)} \times 10^5 \text{ ohms} \]
\[ \ldots 3-12 \]
It was decided to use $R_E = 100$ ohms because a larger value would require a correspondingly higher value of $R_L$ for the same gain and the stage would then require a higher voltage to maintain the desired operating current. Lower operating currents would give a higher value of $h_{ib}$ and lower values of $h_{fe}$ and thus degrade the stage. Higher operating voltage would raise power dissipation and aggravate the thermal drifts. With $R_E = 100$ ohms, $A_{VE} \cong -0.9G$ and $Z_{IE} \cong 100K$ ohms/$(0.45 + 0.019(G+1))$. If $R_E$ was set equal to $R_{E0}$, only about 6 per cent more gain would be obtained. If it is assumed that the second stage input impedance is very high compared to $R_L$, the gain of the first stage will be

$$A_{VE} = -0.9 \quad \frac{R_{406}}{R_{404}} = -0.9 \quad \frac{R_{406}}{100}$$

Since $A_{VE} \cong 60$ was desired, $R_{406}$ was chosen as 6.8K ohms to give $G = 68$ and $A_{VE}$ just over 60. These values give $Z_{IE} = 57K$ ohms. If $R_{401}$ and $R_{402}$ are chosen as 150K ohms, the input impedance would be approximately 82K ohms between differential inputs.

$R_{401}$ supplies base current to the compound transistor. If $R_{410}$ is correctly chosen the voltage drop across $R_{402}$ will be zero at the desired operating point and zero input-voltage.

Two of the above described elements are connected together to form a differential amplifier. $Q_{403}$ is the same type as $Q_{404}$, selected for matched $h_{fe}$. $Q_{401}$ is the same type as $Q_{402}$, selected for matched $h_{fe}$ and $I_{CBO}$.

The effect of the collector-base leakage current, $I_{CBO}$, of the compound is as follows. The difference between the $I_{CBO}$ of
Q401 and that of Q402 flows through the source resistance and creates an offset voltage, $\Delta I_{\text{CBO}} R_S$. It is therefore desirable to have these $I_{\text{CBO}}$'s as small as possible and as closely matched as possible. The effect of the $I_{\text{CBO}}$ of Q403 is similar except that the effective value is the actual value divided by the current gain of the input transistor. Thus, if these leakages are of the same order as the leakage of the input transistor their effect will be negligible. The $I_{\text{CBO}}$ of Q401 and Q402 was approximately $10^{-9}$ amp.

R403 and R404 were hand wound with advance resistance wire on a silicone rubber form and matched to within 0.05%. R405 and R406 were selected from available 6.8K ohm, 1% deposited carbon resistors. A pair was found that matched to within 0.1%.

With the input shorted and the load resistors matched, zero output voltage will be obtained from the differential amplifier if the collector current of the two compound "transistors" are exactly equal. The emitter currents will then be very nearly equal since the compound transistors have very large, nearly matched, current gains. This implies that the voltage drops across the emitter resistors will be very nearly equal. This is only possible if the base-emitter voltages of the two compound "transistors" are equal for equal collector current. Fortunately this parameter is variable in the compound-transistor circuit used. If $R_B$ (Figure 3-4) is varied while $I_C$ is held constant, $I_B$" (in the compound circuit $I_B$" is collector current of the input transistor, Figure 3-1) must be varied. When $I_B$" is varied the base emitter voltage will vary. Thus $R_B$ may be varied to change the base-emitter voltage of the compound at constant
collector current. Since the base-emitter voltage of a modern silicon transistor is already a quite predictable parameter, only a small correction is necessary. This can be easily accomplished without upsetting other parameters of the circuit.

For the reasons set out above R407 was made somewhat larger (6.8K ohms instead of 5.6K ohms) than R408 and shunted by a fine-adjust network that could vary the resulting resistance from somewhat above to somewhat below 5.6K ohms. To accomplish this R415 was made 10K ohms and R416 experimentally selected to place the zero near the center of the adjustment. This adjustment should be made with the input short-circuited.

R409, R402, R413 and R414 are a similar arrangement used to compensate for differences in base current required to establish the desired operating point. The adjustment of R412 should be made, after the adjustment of R415, with the input open-circuited. The zero adjustment should then be independent of source impedance.

Q411 is a high-impedance current source for the emitter circuit. The common-mode feedback signal, B, (see Figure 3-3) is generated from a source of low impedance compared with R411. Q411 is thus operating in the common-base mode, and has an output conductance approximately equal to $h_{ob}$. The actual current that flows will be approximately equal to the voltage at the base of Q411 divided by R411.

3.5 The Second Stage of the D.C. Amplifier

The voltage gain required of the second stage is approximately $\frac{1000}{60} \approx 17$. The stage was designed for a somewhat
greater gain which could be adjusted. The circuit is shown in Figure 3-5.

The transistors used in the second stage are germanium transistors. The base-emitter voltage of a germanium transistor at 1 ma collector current and room temperature is typically 0.1 volt compared with 0.6 volt for silicon. In order that the technique of using a resistor between the base and the emitter, to reduce $h_{FE}$ without seriously affecting $h_{fe}$, work effectively, it is necessary to increase the ratio of $V_{BE}/h_{ib}$. A silicon diode has a voltage-current characteristic very similar to the base-voltage-emitter-current characteristic of a silicon transistor. Thus the forward voltage drop at 1 ma current is about 0.6 volt, and the dynamic impedance is about 30 ohms.

Q406, Q408, D402 and R418 form a p-n-p compound transistor complimentary to that shown in Figure 3-1. D402 raises the effective base-emitter voltage of Q406 from 0.1 volt to 0.7 volt at 1 ma collector current, while raising the effective $h_{ib}$ from
30 ohms to 60 ohms. Using a 2N1305 for Q408, a 2N1304 for Q406, and a 1N465A for D402 gives the following parameters for the compound transistor thus formed.

With the following values for transistor parameters,

\[ h_{fe1} \approx 60 \]
\[ h_{ib1} \approx 300 \text{ ohms} \]
\[ h_{rb1} \approx 7 \times 10^{-4} \]
\[ h_{ob1} \approx 0.12 \times 10^{-6} \]
\[ h_{fe2} \approx 120 \]
\[ h_{ib2} \approx 30 \text{ ohms} \]
\[ h_{rb2} \approx 5 \times 10^{-4} \]
\[ h_{ob2} \approx 0.34 \times 10^{-6} \]
\[ h_{fe2} \approx 80 \]

\[ I_{c1} \approx 0.1 \text{ ma} \]
\[ I_{c2} \approx 1 \text{ ma} \]

Equations 3-2 give

\[ h_{ib} \approx 2.5 \text{ ohms} \]
\[ h_{rb} \approx 8 \times 10^{-4} \]
\[ h_{ob} \approx 0.13 \times 10^{-6} \]
\[ h_{fe} \approx 4700 \]

From 3-3 and 3-13 taking \( G \approx 20 \)

\[ R_{Eo} \approx \sqrt{\frac{h_{ib}}{G h_{ob}}} \approx 1 \text{K ohm} \]

\[ q = 0.016 + 2.5 \times 10^{-3} \left( \frac{R_E}{R_{Eo}} + \frac{R_{Eo}}{R_E} \right) \]

\[ \frac{h_{fe}}{1 + h_{fe}} \approx 1 - 2 \times 10^{-4} \approx 1 \]

\[ A_{VE} \approx -G \left[ 1 - 0.016 - 2.5 \times 10^{-3} \left( \frac{R_E}{R_{Eo}} + \frac{R_{Eo}}{R_E} \right) \right] \]

\[ \ldots 3-14 \]
\[
Z_{IE} = \frac{1 + .016 + 2.5 \times 10^{-3} \left( \frac{R_E}{R_{E0}} + \frac{R_{E0}}{R_E} \right)}{10^{-5} + (1+G) .13 \times 10^{-6}}
\]

For reasons similar to those outlined regarding the choice of R404, R420 was set equal to 100 ohms. With \( R_{420} = 100 \) ohms, equations 3-14 and 3-15 become

\[
A_{vE} = -0.96 \, G
\]

and

\[
Z_{IE} = \frac{100K}{.20 + .013 (1+G)}
\]

Taking

\( G \approx 20, \)

\[
Z_{IE} \approx 220K
\]

Setting

\( R_{422} = 3.3K \)

and

\( R_{423} = 10K \)

\[
R_L = \frac{R_{423}}{2} // R_{422} \approx 2K
\]

Thus \( G = 20 \)

and

\[
A_{vE} \approx -19
\]

An identical unit is used for the other half of the second stage of the differential amplifier with a 2K ohm resistor (R424) as an emitter current source.
3.6 The Emitter-Follower Third Stage of the D.C. Amplifier

The third stage of the differential amplifier is a differential emitter-follower. Its purpose is to provide a low output-impedance and thus make the gain more independent of loading at the output. The circuit is shown in Figure 3-6.

![Diagram of the Emitter-Follower Third Stage](image)

Figure 3-6. The Emitter Follower Third Stage

Q409 and Q410 are 2N1309 high-gain germanium transistors. R425 and R426 are 8.2K ohms resistors, used to establish the desired 1 mA operating current with the common-mode A output at +4 volts.

The output impedance of an emitter-follower is given by (Appendix 3)

\[ Z_{oc} \approx -\frac{h_{ib} + \frac{R_g}{h_{fe}}}{1 + h_{ob}R_g} \]  \( \ldots 3-16 \)
For a 2N1309 at $I_c = 1$ mA:

$$h_{fe} \approx 190$$

$$h_{ob} \approx 0.4 \times 10^{-6} \text{ mhos}$$

$$h_{ib} \approx 30 \text{ ohms}$$

$R_g$ varies, depending on the setting of $R423$, between 0 and 2K ohms. Thus the output impedance will be between 60 ohms and 82 ohms.

3.7 The Oven Temperature-Control Servo

The base-emitter voltage of a silicon or germanium transistor operated at constant collector current decreases approximately 2 Millivolts for each centigrade degree rise in temperature over a wide temperature range. This voltage and its temperature coefficient appear stable with time. This phenomenon is utilized to control the temperature of the oven. The circuit for the temperature control servomechanism is shown in Figure 3-7.

Basically the unit is a d.c. amplifier which is not compensated for the effect of the change of the base-emitter voltage of the input transistor with temperature. In fact the amplifier is designed to emphasize this effect.

A germanium transistor is used for Q301. A silicon transistor would have about six times as large a base-emitter voltage with about the same value of $\frac{\delta V_{BE}}{\delta T}$ of approximately $-2 \text{ mV/}^\circ C$. The base-circuit voltage divider, R301 and R302,
Figure 3-7. The Oven Temperature-Control System

would then have to have about six times the ratio \((1 + \frac{R_{302}}{R_{301}})^{-1}\)
and the sensitivity of the circuit to power supply voltage changes would be increased by the same factor.

Transistor Q301 is operated in the common emitter mode. The impedance of the base-circuit voltage divider must be much less than the common-emitter input impedance to maximize the voltage sensitivity of Q301.

The performance of the circuit may be analysed by replacing Q301 with a hypothetical transistor with \(\frac{dV_{BE}}{dT} = 0\)
and having a voltage generator of 2 mv \((T_o - T)\) in series with its base lead.

Using equations A5-9, A5-12, and A5-13 of Appendix 5, and A4-9 of Appendix 4 the gain of the amplifier consisting of Q301, Q302 and Q303 may be shown to be

\[ A_{VE} = 1.5 \times 10^4 \]
The emitter-follower, Q304 will provide an output-impedance of approximately 250 ohms. R310 in the collector circuit of Q304 serves to limit the current through Q304 in the event of overload or short circuit.

The voltage at the point marked $V_T$ in Figure 3-7 will have a temperature coefficient of

$$\frac{\partial V_T}{\partial T} = 1.5 \times 10^4 \times (-2) \text{ mv/}^\circ C$$

$$\frac{\partial V_T}{\partial T} = 30 \text{ volts/}^\circ C$$

and will be generated by a source of approximately 250 ohms impedance.

As well as being sensitive to temperature changes, the unit is sensitive to supply voltage variations. The unit that was constructed gave an operating point of $35^\circ C$ with $R302 = 15K$ and $R301 = 100$ ohms. The divider ratio for $R302$ and $R301$ is then $6.7 \times 10^{-3}$. When this is multiplied by the amplifier gain, $1.5 \times 10^4$, the result is

$$\frac{\partial V_T}{\partial V_S} = 100 \text{ volts/volt}$$

where

$V_T = \text{voltage at test point } V_T \text{ (figure 3-7)}$

$V_S = \text{supply voltage}$
The ratio \( \frac{\partial V_T}{\partial V_S} / \frac{\partial V_T}{\partial T} \) = 3.1 C°/volt is the error in operating point that would be caused by a change of 1 volt in supply voltage. The power supplies used in this equipment are capable of more than sufficient regulation to keep this source of error below a few millidegrees for line voltages between 90 and 135 volts r.m.s.

The effect of the change of base-emitter voltage of Q302 with temperature is suppressed by the product of the voltage gain of Q301 and one plus the ratio of output impedance of Q301 to the input impedance of Q302. This quantity is approximately 330. The temperature sensitivity of the base-emitter voltage of Q302 only changes the overall sensitivity by about 1/3%. Transistors further along in the unit will have even less effect.

The effect of the leakage current of Q302 is much more serious. If the \( I_{CBO} \) of Q302 changes by 1 \( \mu \)A, this will appear as a 1 \( \mu \)A change in current through the output impedance of Q301 which, including the effect of its load resistor, R303, is 36K ohms. The 1 \( \mu \)A change in \( I_{CBO} \) of Q302 then appears as a 36 millivolt change at the output of Q301 which is equivalent to about 1/3 mv at the input or about 1/6 C°. It would therefore be desirable to replace Q302 with a low leakage silicon transistor.

Self heating of Q301 is another potential source of error and instability. Q301, which is required to dissipate about 1/4 mw, is mounted in a finned heat-sink which gives an overall collector junction-to-ambient thermal resistance of about 0.2 C°/mw.
The junction of Q301 would then be expected to be at about 0.05°C above ambient. Since this quantity should be stable to within about 10%, the instability caused should be negligible.

The heater power amplifier is a common-emitter power stage, with a voltage gain of about 5, driven by an emitter-follower. R311, R312 and R315 provide adequate thermal stability. R313 and R314 limit the power dissipation in the collector circuit of Q305. The output voltage to the heater is then about 150 volts/C°. The oven will then be turned from full on to full off by an internal temperature change of about 0.15°C.

For the unit constructed, the value of R302 required for 35°C operation was approximately 15K ohms and \( \frac{\partial R_{302}}{\partial T} \) was found to be 250 ohms/C°.

3.8 The Common-Collector Regulator

A very common regulator is the so-called emitter-follower, or common-collector, regulator shown schematically in Figure 3-8.

![Common-Collector Regulator Diagram](image-url)
The performance of this regulator will be analysed by evaluating $g$ and $r_o$ as discussed in Appendix 1, where

$$g = \frac{\partial I}{\partial V_i} \bigg|_{V_o = \text{const}}$$

Let $V_i$ be changed by $\Delta V_i$ while $V_o$ is held constant by changing the load as much as is necessary. The input to $Q2$ remains constant since $V_o$, and thus the output of amplifier, $A$, remains constant. The base-emitter voltage of $Q1$ will change slightly when $V_i$ is changed. This small change will cause a negligible change in the collector current of $Q2$. Thus the collector current of $Q2$ may be regarded as constant. Any change in $V_{BE1}$ will be much less than $\Delta V_i$. As $V_i$ is increased $I$ will increase due to the increase in collector-emitter voltage of $Q1$ and due to the increase of base current through $R$.

$$\Delta I = \frac{\Delta V_i}{R} \left( h_{fe1} + h_{oe1} \Delta V_i \right) \quad \ldots 3-19$$

The current supplied to the base of $Q1$ through resistor $R$ under the condition that $V_i = V_{i \text{ min}}$ must be $\frac{I_{\text{max}}}{h_{FE1}}$. Therefore

$$R \leq h_{FE1} \frac{V_{i \text{ min}} - V_o}{I_{\text{max}}} \quad \ldots 3-20$$

Since a small value of $R$ will degrade the performance of the supply, we choose the largest acceptable value and substitute it into Equation 3.19 for $\Delta I$
\[ \Delta I = \left( \frac{I_{\text{max}}}{V_{i \text{ min}} - V_o} \right) h_{\text{fe}} \left( h_{\text{fe}1} + h_{\text{fe}1} h_{\text{obl}} \right) \Delta V_i \quad \ldots 3-21 \]

If \( h_{\text{fe}} \ll h_{\text{FE1}} \),

\[ g = \frac{\Delta I}{\Delta V_i} = \left( \frac{I_{\text{max}}}{V_{i \text{ min}} - V_o} + h_{\text{fe}} h_{\text{obl}} \right) \quad \ldots 3-22 \]

To find \( r_o \) the output resistance, let \( V_i \) be held constant and examine the change in \( I, \Delta I \), due to a small change in \( V_o, \Delta V_o \). The change in voltage at the base of \( Q_2, \Delta V_{B2} \), due to \( \Delta V_o \) will be:

\[ \Delta V_{B2} = aA \Delta V_o \quad \ldots 3-23 \]

This will cause a change in base current of \( Q_2 \) which will be amplified by \( Q_2 \) and \( Q_1 \) to give

\[ \Delta I = (1 + h_{\text{fe}}) h_{\text{fe}2} \frac{aA \Delta V_o}{h_{\text{ie}2}} \quad \ldots 3-24 \]

\[ \ll h_{\text{fe}} h_{\text{fe}2} \frac{aA \Delta V_o}{h_{\text{ie}2}} \text{ if } h_{\text{fe}} \gg 1 \]

using

\[ h_{\text{ie}2} \ll h_{\text{fe}} h_{\text{ib}2} \text{ and } r_o = -\frac{\Delta V_o}{\Delta I} \]

\[ r_o = -\frac{\Delta V_o}{\Delta I} = \frac{h_{\text{ib}2}}{a (-A) h_{\text{fe}}1} \quad \ldots 3-25 \]
Equation 3-25 neglects the fact that resistor, $R$, shunts the input to $Q_1$. $r_o$ will in practice be somewhat higher than predicted by 3-25.

3.9 The Common-Emitter Regulator

An alternative regulator circuit, the common-emitter regulator, is shown in Figure 3-9.

![Figure 3-9. The Common-Emitter Regulator](image)

The parameters $g$ and $r_o$ are evaluated as in 3-8. If $V_i$ is increased by $\Delta V_i$ while $V_o$ is held constant, $I$ will increase due to the increased base-emitter voltage on $Q_1$. $I$ will increase further due to increased collector emitter voltage on $Q_2$ causing more base current to flow into $Q_1$. If it is assumed that:

$$\Delta V_{BE1} \ll \Delta V_i$$

and

$$\frac{1}{g_{o2}} \gg h_{ie1}$$

where

$$g_{o2} = \text{output conductance of } Q_2$$
then

\[ g = \frac{\Delta I}{\Delta V_i} = h_{fe1} (h_{ob1} + g_{o2}) \] ...3-26

If \( V_o \) is held constant and the change in \( I, \Delta I \), due to a change, \( \Delta V_o \), in \( V_o \) is examined, then the same argument used to obtain Equation 3-25 yields:

\[ r_o = \frac{h_{ib2}}{a(-A) h_{fe1}} \] ...3-27

where \( A \) is the gain of the amplifier, \( A \), in Figure 3-9.

3.10 Modification of Regulator Output-Resistance by Interaction of Output Voltage and Reference Voltage

If a passive reference element (e.g., a breakdown diode) is driven by a regulator, the reference voltage, \( V_{R1} \) will, in general, change with a change of output voltage, \( V_o \). The analysis for \( r_o \) must be modified to account for this effect. The input to the amplifier unit becomes \( (a - \frac{\partial V_R}{\partial V_o}) \Delta V_o \) and thus:

\[ r_o = \frac{h_{ib2}}{h_{fe1}} \frac{1}{A( a - \frac{\partial V_R}{\partial V_o})} \] ...3-28

3.11 Comparison of Common-Emitter and Common-Collector Regulators

It is seen that, for equivalent supplies, the output resistances are the same. The input conductances differ.
\[ g_{cc} = \frac{I_{\text{max}}}{V_{1 \text{ min}} - V_o} + h_{fe1} h_{obl} \] ...3-22

and

\[ g_{ce} = h_{fe1} (g_{o2} + h_{obl}) \] ...3-26

It may be seen from Appendix 1, A1.6 that the line regulation factor, \( K \), is given by

\[ K = g \cdot r_o \] ...3-29

Since \( r_{oc1} \) and \( r_{oce} \) are approximately equal if loading due to the drive resistor, \( R \), may be neglected, \( \frac{K_{ce}}{K_{cc}} \approx \frac{g_{ce}}{g_{cc}} \).

The operating conditions of the regulated supply for the d.c. amplifier will be taken as an example.

\[ I = 50 \text{ ma} \]
\[ V_{1 \text{ min}} - V_o = 5V \]
\[ h_{fe1} = 70 \]
\[ h_{obl} = 3 \times 10^{-6} \text{ mho} \]

\( g_{o2} \) is the output conductance of the common-emitter amplifier, \( Q2 \), and is given by A-4-15 in Appendix 4 as

\[ g_{o2} \approx \frac{1}{2} \left( h_{ob2} h_{fe2} + h_{rb2}^2 \right) + \frac{1}{2} \left( h_{ob2} h_{fe2}^2 - \frac{h_{rb2}}{h_{ib2}} \right) \left( 1 - \frac{h_{fe2} h_{ib2}^2}{Z_g} \right) \]
\[ \frac{1 + \frac{h_{fe2} h_{ib2}^2}{Z_g}}{} \]

...3-30
if

\[ h_{fe2} \gg 1 \text{ and } h_{rb2} \ll 1 \]

Typical values for Q2

\[ h_{ob2} = 0.4 \times 10^{-6} \text{ mho} \]
\[ h_{rb2} = 7 \times 10^{-4} \]
\[ h_{ib2} = 30 \text{ ohms} \]
\[ h_{fe2} = 50 \]

These with 3-29 give

\[ g_{o2} = 22 \times 10^{-6} - 1.5 \times 10^{-6} \frac{1 - \frac{1500}{Z_g}}{1 + \frac{1500}{Z_g}} \]

\[ \ldots 3-31 \]

\[ g_{o2} \approx 22 \times 10^{-6} \]

Using this value in 3-21 and 3-25 yields

\[ g_{CE} \approx 1.8 \times 10^{-3} \]
\[ g_{CC} \approx 10^{-2} \]

Clearly the common-emitter regulator will perform more than five times as well as the common-collector regulator in this application.
3.12 The Temperature Coefficient of Breakdown-Diodes

It is well known that low-voltage breakdown-diodes have a negative temperature coefficient of voltage and high-voltage diodes have a positive temperature coefficient. A very small temperature coefficient is observed in diodes which break down at about 5.5 volts. It is also well known that the temperature coefficient is a function of current, becoming more positive at higher current. These phenomena were investigated and several salient features noted.

The breakdown voltage at 10 ma and the operating current that produced zero average temperature coefficient between 0°C and 100°C were measured on fourteen diodes. The resulting distribution is shown in Figure 3-10. The voltage-temperature characteristics between -50°C and +100°C of these diodes were measured at the current which gave zero average temperature coefficient. The results were fitted to the equation

\[ V = V(50°C) \left[ 1 + a(T - 50) + b(T - 50)^2 + c(T - 50)^3 + d(T - 50)^4 \right] \]

...3-32

Within the accuracy of available test equipment (.01%), it was found that

\[ a = 0 \] (by choice of operating point) ...3-33
\[ b \approx .7 \times 10^{-6} \]
\[ c \approx 0 \]
\[ d \approx 0 \]
Diode Voltage at 10mA

\[ V = 6.21 - 0.777 \log \frac{I}{1ma} \]

Figure 3-10. Breakdown-diode Voltage at 10ma vs. Diode Current for Zero Temperature Current
3.13 General Power Supply Considerations

There are four regulators in the system. Two serve as rough pre-regulators and two as regulators. These units are all common-emitter regulators. The main regulators have a differential amplifier stage. The pre-regulators do not have their own reference diodes but use the output of the main regulators as references. The pre-regulators also use small amounts of open-loop feedback to improve their regulation.

3.14 The Regulated Power Supply for the D.C. Amplifier

The circuit of the regulated power supply for the d.c. amplifier is shown in Figure 3-11. It is of the common-emitter type with a differential amplifier with a gain of about 5.

![Diagram of the regulated power supply for the D.C. Amplifier](image)

Figure 3.11 The Regulated Power Supply for the D.C. Amplifier

R208 is present to improve the thermal stability of Q204 and also
to raise the gain of Q203 by increasing the collector current of Q203 and thus reducing its $h_{ib}$. R207 and R205 are protection resistors to limit the currents in paths that would otherwise contain only forward biased diodes and transistors, which could be momentarily overloaded by transients. R206 serves to increase the collector current of Q202 and improve the thermal stability of Q203. (See 3-5 and 3-6). D202 raises the base-emitter voltage of Q203 as discussed in 3.5.

Q201, Q202, and R204 form a conventional "long-tailed-pair" differential amplifier. D201 is the reference diode supplied with current from the regulated output through R203. R202 and R201 form the sampling divider. The resistance of R201 and R202 in parallel should be much less than the input impedance of Q201 to avoid loss of gain. R203 is chosen such that it passes the correct current for zero temperature coefficient to the particular reference diode used.

The quantity $1 + R202/R201$ is made equal to the desired value of $V_{out}/V_{ref}$ and R204 is adjusted to make the base voltage of Q201 equal to the base voltage of Q202. C201 keeps the supply impedance low at high frequencies and also reduces the loop gain of the regulator to below unity at frequencies below which phase shifts become excessive. R209 is necessary to turn the unit on and reduce the power dissipation in Q204.

The performance of the supply was estimated on the basis of the following typical parameters.
<table>
<thead>
<tr>
<th></th>
<th>Q204</th>
<th>Q203</th>
<th>Q201 and Q202</th>
</tr>
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<tbody>
<tr>
<td>$h_{fe}$</td>
<td>70</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$h_{ob}$</td>
<td>$3 \times 10^{-6}$</td>
<td>$4 \times 10^{-6}$</td>
<td>$2 \times 10^{-6}$ mho</td>
</tr>
<tr>
<td>$h_{rb}$</td>
<td>$4 \times 10^{-4}$</td>
<td>$7 \times 10^{-4}$</td>
<td>$7 \times 10^{-4}$</td>
</tr>
<tr>
<td>$h_{ib}$</td>
<td>1</td>
<td>60</td>
<td>150 ohms</td>
</tr>
<tr>
<td>$h_{oe}$</td>
<td>$2 \times 10^{-4}$</td>
<td></td>
<td>mho</td>
</tr>
<tr>
<td>$h_{ie}$</td>
<td>70</td>
<td></td>
<td>ohms</td>
</tr>
</tbody>
</table>

**D201**

$r_z$ | 250 ohms |

$V_z$ | 6 volts |

From 3-30 $g_0$ of Q202 $\cong 5 \times 10^{-6}$ mho. These values are substituted into 3-26 to obtain

$$g = 1.2 \times 10^{-3} \text{ mho}$$

...3-34

To the above value of $g$ must be added $1/820$ mho due to the presence of R209 (820 ohms) to yield

$$g = 2.4 \times 10^{-3} \text{ mho}$$

The impedance at the base of Q203 is approximately $h_{fe}(h_{ib} + r_d)/(1 + h_{fe} h_{ob} R_L)$ in parallel with R206, that is, about 1.5K.
The gain of the amplifier is then

\[ A = \frac{1}{2} \frac{R_L}{r_{ib}} \approx 5 \]

The factor \( \frac{1}{2} \) appears due to the fact that a single-ended output is being taken from a differential amplifier.

The attenuation of the voltage divider is

\[ a = 0.45 \]

\[ \frac{\partial V_R}{\partial V_0} = \frac{r_R}{r_R + R 203} \approx 0.05 \ll a \]

Substitution of these values into 3-25 yields

\[ r_0 = 0.4 \text{ ohm} \quad \ldots 3-36 \]

and since \( K = g r_0 \)

\[ K \approx 10^{-3} \quad \ldots 3-37 \]

3.15 The Reference Supply

The design of the reference supply is similar to that of the regulated supply for the d.c. amplifier. The parameter \( a \) was chosen to be \( \frac{1}{2} \). Similar calculations to those of 3.14 show

\[ K \approx 10^{-3} \quad \ldots 3-37 \]

The reference voltage output is taken directly from the reference diode. Any change in output voltage of the regulator is then
further attenuated by

\[ \frac{\partial V_R}{\partial V_0} \approx 40 \times 10^{-3} \]

Thus

\[ K_{ref} \approx 40 \times 10^{-6} \] ...3-38

3.16 The Pre-regulators

The pre-regulators are simpler two-transistor regulators using the output voltage of the main regulators as references. The voltage dividers are in the emitters of the driver transistors. This results in the driver transistors operating in the common-base mode with \( g_{o2} = h_{ob2} \). The detailed design calculations are given in Appendix 2 and the circuit diagrams in Figure 3-13.

3.17 The Oven

The oven is a 6 inch by 3 inch by 5 inch box fabricated of \( \frac{1}{4} \) inch thick aluminium. The top and bottom are insulated with about 1 inch of styrofoam. The sides are wrapped with fiberglass tape on which is wound a 25 ohm nichrome heater. The heater is held in place with another layer of tape and outer panels of fibre board. The fibre board panels also provide some thermal insulation. The gain of the temperature control system is set at about \( \frac{1}{2} \) the value required to sustain oscillations. The period of the damped oscillations which occur when first
switched on is about 6 minutes. After about 2 hours of operation the temperature appears to have steadied down to within a few hundredths of a centigrade degree from the equilibrium value.
Figure 3-12. Complete Schematic of D.C. Amplifier
Figure 3-13. Complete Schematic of Power Supplies
<table>
<thead>
<tr>
<th>R 301</th>
<th>100 ohm</th>
<th>C301</th>
<th>.1 µfd</th>
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</thead>
<tbody>
<tr>
<td>R 302</td>
<td>see fig. 4-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 303</td>
<td>47 K</td>
<td>Q 301</td>
<td>2N 1303</td>
</tr>
<tr>
<td>R 304</td>
<td>3.3 K</td>
<td>Q 302</td>
<td>2N 1305</td>
</tr>
<tr>
<td>R 305</td>
<td>330</td>
<td>Q 303</td>
<td>2N 1304</td>
</tr>
<tr>
<td>R 306</td>
<td>8.2 K</td>
<td>Q 304</td>
<td>2N 1305</td>
</tr>
<tr>
<td>R 307</td>
<td>560</td>
<td>Q 305</td>
<td>2N 1305</td>
</tr>
<tr>
<td>R 308</td>
<td>1.5 K</td>
<td>Q 306</td>
<td>2N 250</td>
</tr>
<tr>
<td>R 309</td>
<td>6.8 K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 310</td>
<td>2.7 K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 311</td>
<td>3.3 K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 312</td>
<td>330</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 313</td>
<td>270</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 314</td>
<td>680</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 315</td>
<td>5 ohm/5 watt</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

see Figure 3-7.

Figure 3-14. Parts List for Temperature Controller
4. PERFORMANCE OF EQUIPMENT

4.1 The D.C. Amplifier

The amplifier output was connected to a stripchart recorder and the system run for several days with the input shorted. During the first day a steady drift of about 1 microvolt per hour (referred to the input) was observed. After the first 24 hours the drift remained below ± 2 microvolts. Source resistances up to about 1,000 ohms made no appreciable difference. A source resistance of 10K ohms gave rise to about 6 microvolts peak-to-peak of very low-frequency noise. 100K ohms source resistance gave about 40 microvolts. The frequency response of the amplifier was down 1 db at 10 kc, 3 db at 20 kc, and 6 db at 40 kc. The broad-band noise figure was found to be about 3 db with a 10K ohm source impedance and about 6 db with a 1K ohm source impedance.

The input impedance was measured as 120K with the voltage gain set at 1,000.

4.2 The Temperature Regulator System

The temperature control unit was placed in an oven with a thermometer. The temperature was slowly raised while the output of the control unit was maintained constant at 1 volt by varying R302. The resulting data is plotted in Figure 4-1.

From the slope of the plot of R302 vs. temperature, it was determined that a 250 ohm change in R302 corresponds to a change of 1°C. R302 was then varied at constant temperature and it was found that 10 ohms change in R302 caused the output of temperature
Figure 4-1. Value of R302 vs. Oven Temperature
control amplifier ($V_T$ of Figure 3-7) to change 1 volt. It is thus seen that

$$\frac{\partial V_T}{\partial T} = \frac{250 \text{ ohms}/^\circ C}{10 \text{ ohms}/V} = 25 \text{ volts}/^\circ C \text{ measured}$$

This value compares favorably with the estimated value (3-17) of 30 volts/$^\circ C$.

The heater current required to raise the oven temperature from $25^\circ C$ to $35^\circ C$ was measured and found to be 0.6A. The heater control and power amplifier can supply about 1.4 times this amount of current or about twice the power. It is therefore expected that the oven and temperature control will function properly in ambient temperatures from about $15^\circ C$ to just under $35^\circ C$.

4.3 Power Supply Performance

The output impedance and line regulation factor of the regulated power supply were measured and compared to the predicted values.

A change in load current of 11 millamps was found to cause a change in output voltage of 5 millivolts from which $r_o = 0.45$ ohm. The output voltage changed 6 millivolts for an input change of 5 volts. The line regulation factor, $K$, is then $1.2 \times 10^{-3}$. Predicted and measured values are compared in table 4-1.
Predicted Measured

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0$</td>
<td>0.4 ohm</td>
<td>0.45 ohm</td>
</tr>
<tr>
<td>$K$</td>
<td>$10^{-3}$</td>
<td>$1.2 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 4-1. Predicted and Measured Parameters of the Regulated Power Supply

Similar tests performed on the reference supply gave the results of Table 4-2.

Predicted Measured

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$10^{-3}$</td>
<td>$1.4 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\frac{\partial V_R}{\partial V_o}$</td>
<td>$4 \times 10^{-2}$</td>
<td>$2 \times 10^{-2}$</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>$40 \times 10^{-6}$</td>
<td>$&lt; 50 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 4-2.

The discrepancy in $\frac{\partial V_R}{\partial V_o}$ was due to the fact that reference diode used had a dynamic resistance of only 27 ohms instead of the 50 ohms anticipated from the data sheet.

The value of $K_{ref}$ obtained by multiplying the measured values of $K$ and $\frac{\partial V_R}{\partial V_o}$, namely

$$K_{ref} = 28 \times 10^{-6}$$

is probably more accurate than the value actually measured, since the change in reference voltage caused by a 5 volt change in input voltage was only a few times the threshold of detectability on the measuring equipment used.
4.4 Pre-regulator Performance

It was found possible to adjust the pre-regulators to give less than 15 millivolts output change for 15 volts input change, that is, $K_{\text{pre-reg}} \approx 10^{-3}$.

The overall line regulation factor for the reference system should then be

$$K_{\text{ref total}} \approx 30 \times 10^{-9}$$

The reference supply and pre-regulator operate at a nominal input of 22 volts and a nominal output of 5.6 volts. A 10% change in line voltage would then be expected to produce about 0.07 microvolt change in 5.6 volts or about 0.0013 ppm. A change of a.c. line voltage from 90 to 135 volts ($\pm 20\%$) caused a change in reference voltage of less than the limit of resolution of the measuring system used (about 2 µv).

The use of high quality shielded line isolation transformers would greatly reduce the problems of 60 cycle coupling when working with high impedance sources.

A voltage divider, consisting of a 1000 ohm precision resistor and a decade resistance box, was placed across the output of the reference supply. A standard cell was connected in series-opposing to the voltage across the 1000 ohm resistor and the resultant voltage was fed into the amplifier. The resistance box was adjusted for zero output voltage from the amplifier and the amplifier output was connected to a recorder.
During several days of operation the drift never exceeded 5 microvolts referred to the input, that is, about 5 parts per million.
APPENDIX 1. LINE AND LOAD REGULATION OF REGULATED POWER SUPPLIES

The regulator is regarded as a four-terminal network (Figure Al-1) with characteristics described by Equations Al-1 and Al-2.

\[
\begin{align*}
\frac{dI_o}{dV_o} + g_i dV_i & \quad \ldots \text{Al-1} \\
\end{align*}
\]

where

\[
\begin{align*}
    r_o = - \frac{\partial V_o}{\partial I_o} \\
    | V_i \\
\end{align*}
\]

and

\[
\begin{align*}
    g_f = \frac{\partial I_o}{\partial V_i} \\
    | V_0 \\
\end{align*}
\]

\[
\begin{align*}
    \frac{dI_i}{dV_o} + g_i dV_i & \quad \ldots \text{Al-2} \\
\end{align*}
\]

where

\[
\begin{align*}
    r_r = - \frac{\partial V_o}{\partial I_i} \\
    | V_i \\
\end{align*}
\]
The line regulation factor, $K$, is defined by Equation Al-3.

$$K = \frac{dV_o}{dV_i} \quad \ldots \text{Al-3}$$

If the input voltage is changed, both the output voltage and current will change. These changes are related by:

$$dI_o = \frac{1}{R_L} \ dV_o \quad \ldots \text{Al-4}$$

Equation Al-4 is substituted into Equation Al-1, and the resulting equation solved for $K$ as defined by Equation Al-3.

$$K = g_f \ \frac{r_o R_L}{R_L + r_o} \quad \ldots \text{Al-5}$$

Since a supply well regulated against load variations, will have $r_o \ll R_L$, the approximation Al-6 is justified.

$$K \approx g_f r_o \quad \ldots \text{Al-6}$$

The output resistance, $r$, is defined by Equation Al-7.

$$r = -\ \frac{dV_o}{dI_o} \quad \ldots \text{Al-7}$$

and

$$g_i = \frac{\partial I_i}{\partial V_i} \bigg|_{V_0}$$
If the output current is changed, both the output voltage and input voltage will change. The input voltage change and the input current change are related by Equation Al-8.

\[
\frac{1}{r_s} = g_s = - \frac{dI}{dV_i} \quad \text{(definition of } g_s \text{ and } r_s) \quad \ldots \text{Al-8}
\]

Equation Al-8 is substituted into Equation Al-2 and the result is solved for \( dV_i \) in terms of \( dV_o \).

\[
dV_i = \frac{1}{r_o (g_i + g_s)} dV_o \quad \ldots \text{Al-9}
\]

Equation Al-9 is substituted into Equation l-1 and the result is solved for \( r \) using Equation Al-7 to yield:

\[
r = r_o \frac{1}{1 - \frac{g_f r_o}{(g_i + g_s) r_r}} \quad \ldots \text{Al-10}
\]

In the regulators considered here \( dI_i \sim dI_o \). Thus \( g_i \approx g_f \) and \( r_r \approx r_o \). If these approximations are made in Al-10, the output impedance is found to be:

\[
r \approx r_o (1 + g_f r_s) \quad \ldots \text{Al-11}
\]

Since \( g_f r_s \ll 1 \) in the regulators and power supplies considered:

\[
r \approx r_o \quad \ldots \text{Al-12}
\]
APPENDIX 2. DERIVATION OF EQUATIONS PREDICTING POWER SUPPLY PERFORMANCE

1. Pre-regulator

![Figure 1. Pre-regulator](image)

First, consider the simplified circuit of Figure 2.

![Figure 2. Simplified Circuit of Pre-regulator](image)
Calculation of \( g' \) and \( r_o \) for circuit of Figure 2. We set \( \Delta V'_2 = 0 \), assume \( \Delta V'_R = 0 \) and calculate \( g' \). The output current will increase when \( V_1 \) is increased for two reasons:

(a) \( V_{CE1} \) increases and causes a small increase in collector current of \( Q_1 \).

(b) \( V_{CB2} \) increases and causes a small increase in collector current of \( Q_2 \) which is multiplied by the current gain of \( Q_1 \).

When a transistor is operated with non-zero impedance in both emitter and base circuits, it cannot, strictly speaking, be said to be operating either common-base or common-emitter. However, if the emitter-circuit impedance is much larger than the base circuit impedance divided by \( h_{fe} \), the circuit may be regarded as common-base with negligible error. Similarly, if the emitter-circuit impedance is much larger than the base-circuit impedance divided by \( h_{fe} \), the circuit may be said to approximate a common-emitter.

The voltage reference, \( V_R \), will normally be a low-impedance source. Thus \( Q_2 \) may be regarded as a common-base configuration and its output conductance will be \( h_{ob2} \). The base-circuit impedance for \( Q_1 \) is the output impedance of \( Q_2 \), which is \( 1/h_{ob2} \). The emitter-circuit impedance seen by \( Q_1 \) is the internal impedance of the power source. \( h_{ob2} \), for the supplies considered here will be \( \leq 1 \) micro mho. If we assume \( h_{fe1} \leq 100 \), \( Q_1 \) will perform as a common-emitter as long as \( r_{source} \ll 10K \) ohms which will certainly be the case for a reasonable transformer-rectifier-filter system. Therefore the
output admittance of Q1 will be $h_{oe1}$.

Thus:

$$\begin{align*}
dI &= h_{oe1}dV_1 + h_{fe1}h_{ob2}dV_1 \\
&\approx h_{fe1}(h_{ob1} + h_{ob2})dV_1 : h_{fe1} \gg 1, h_{rb} \ll 1
\end{align*}$$

$$g' = h_{fe1}(h_{ob1} + h_{ob2}) \quad \ldots A2-2$$

If we now assume a change in output voltage $\Delta V_2$, with $V_1$ held constant, we can calculate $\Delta I$ and thus $r_o$. The junction of R6 and R7 may be regarded as a source of voltage, $V''_2 = \frac{R_6}{R_6 + R_7} V'_2$ with internal impedance, $R' = \frac{R_6 R_7}{R_6 + R_7}$ driving the emitter of Q2.

$$\begin{align*}
\Delta I_{E2} &= \frac{\Delta V''_2}{R' + h_{ib2}} \\
&= \frac{R_6}{R_6 + R_7} \Delta V'_2 1 \\
&= \frac{R_7}{R_7 + h_{ib2} \left(1 + \frac{R_7}{R_6}\right)} \\
&= \frac{\Delta V'_2}{R_7 + h_{ib2} \left(1 + \frac{R_7}{R_6}\right)}
\end{align*}$$
\[ \Delta I_{c2} = \left| h_{fb2} \right| \Delta I_{E2} \approx \Delta I_{E2} \quad \cdots \text{A2-4} \]

\[ \Delta I = h_{fe1} \Delta I_{c2} \quad \cdots \text{A2-5} \]

\[ = \frac{h_{fe1} \Delta V'_2}{R_7 + h_{ib2} (1 + \frac{R_7}{R_6})} \]

\[ r'_o = \frac{\Delta V'_2}{\Delta I} = \frac{R_7 + h_{ib2}(1 + \frac{R_7}{R_6})}{h_{fe1}} \quad \cdots \text{A2-6} \]

\[ K' = g'r'_o = (h_{ob1} + h_{ob2}) \left[ R_7 + h_{ib2}(1 + \frac{R_7}{R_6}) \right] \quad \cdots \text{A2-7} \]

Returning to the circuit of Figure 1, we set \( R_3, R_4 \) and \( R_5 = \infty \) and examine the effect of \( R_8 \).

\[ V_2 = V'_2 - IR_8 \quad \cdots \text{A2-8} \]

\[ V'_R = V_R + IR_8 \quad \cdots \text{A2-9} \]

From A2-8

\[ r_o = -\frac{\partial V_2}{\partial I} = -\frac{\partial V'_2}{\partial I} \left|_{V'_R, \frac{dV'_R}{dI}} + R_8 \right. \quad \cdots \text{A2-10} \]
and from A2-9

\[ \frac{dV_R'}{dI} = R_8 \tag{A2-11} \]

combining A2-10 and A2-11 we have

\[ r_o = r_o' - R_8(\frac{\partial V'}{\partial V_R'} - 1) : r_o' = -\frac{\partial V'}{\partial I} \bigg|_{V_R'} \tag{A2-12} \]

If I is held constant, \( I_{c2} \) must remain constant and therefore \( I_{E2}, I_{B2} \) and \( V_{BE2} \) remain constant. Summing currents at the junction of \( R_7 \) and \( R_6 \) gives

\[ \frac{V'}{R_7} - \frac{(V_R' - V_{BE2})}{R_7} + I_{E2} = \frac{V_R' - V_{BE2}}{R_6} \tag{A2-13} \]

Taking differentials in A2-13 with \( I = \text{constant} \) yields

\[ \frac{dV'}{R_7} - \frac{dV_R'}{R_6} = \frac{dV_R'}{R_6} \]

\[ dV' = dV_R' \left( \frac{1}{R_6} + \frac{1}{R_7} \right) = dV_R' \left( \frac{R_6}{R_6} + \frac{R_7}{R_6} \right) \tag{A2-14} \]
thus

\[
\frac{\partial V'_2}{\partial V'_R} \bigg|_I = \frac{R_6 + R_7}{R_6} \quad \ldots A2-15
\]

Substituting A2-15 into A2-12 yields

\[ r_0 = r'_o - R_8 \frac{R_7}{R_6} \quad \ldots A2-16 \]

Let us now examine the effect of \( R_1, R_2 \) and \( R_3 \). If \( R_1 \)
and \( R_2 \) are chosen such that at the nominal value of \( V_1 \),

\[
V_1(\text{nom}) \frac{R_1}{R_1 + R_2} = V_{E2}(\text{nom}) \quad \ldots A2-17
\]

then the current through \( R_3 \) is zero at nominal supply voltage.
If \( V_1 \) increases (at constant \( I \)) current will flow through \( R_3 \)
and cause \( V_{E2} \) to rise thus tending to reduce \( V_{BE2} \) and therefore
\( I_{E2} \). This will, in turn, tend to reduce \( I \). If the values are
chosen correctly it should be possible to just offset the
tendency of \( I \) to increase with increasing \( V_1 \). The current
through \( R_3 \) is \( \Delta V'_1 / R'_3 \)

\[
\Delta V'_1 = \Delta V_1 \frac{R_1}{R_1 + R_2} \quad \ldots A2-18
\]

\[
\Delta V_1 = V_1 - V_1(\text{nom})
\]

\[
R'_3 = (R_1 // R_2 + R_3 + h_{ib2} // R_6)
\]
This current causes

$$\Delta I_{c2} = h_{fb2} \Delta I_{E2} \approx + \Delta I_{E2} \approx - I_{R3} \quad \text{...A2-19}$$

$$\Delta I = h_{fe1} \Delta I_{c2} = -h_{fe1} I_{R3} = -\Delta V \frac{R_1}{R_1 + R_2} \frac{h_{fe1}}{R'_3} \quad \text{...A2-20}$$

$$\frac{\partial I}{\partial V_1} = - \frac{R_1}{R_1 + R_2} \frac{h_{fe1}}{R'_3} = g''$$

$$g = g' + g''$$

$R_5$ is necessary to start the supply since with $Q_1$ non-conducting there is no power to the reference section. $R_5$ lowers $g$ to $(g' + \frac{1}{R_5}) + g''$ and $R_4$ improves the thermal stability of $Q_1$ but lowers the effective value of $h_{fe1}$ to

$$h_{fe1} \frac{R_4}{R_4 + h_{ile}} \cdot$$
APPENDIX 3. DERIVATION OF THE EACT PARAMETERS FOR AN EMITTER-FOLLOWER

It has been found desirable to work in terms of "mixed h-parameters", i.e., $h_{fe}$, $h_{ib}$, $h_{rb}$, $h_{ob}$ since these are usually specified and $h_{ib}$, $h_{rb}$ and $h_{ob}$ are quite predictable.

The important mid-band parameters of an amplifier are its voltage gain, $A_v$, its current gain, $A_i$, its input impedance, $Z_i$, and its output impedance, $Z_o$.

![Emitter-Follower Circuit Diagram](image)

Figure A3-1. Emitter-Follower

In terms of common collector h-parameters

\[ \text{voltage gain} = A_{vc} = \frac{1}{h_{rc} - \frac{h_{ic}}{Z_L} \left( \frac{1 + h_{oc}Z_L}{h_{fc}} \right)} \]  \[ \ldots A3-1 \]

\[ \text{current gain} = A_{ic} = \frac{h_{fc}}{1 + h_{oc}Z_L} \]  \[ \ldots A3-2 \]

\[ \text{input impedance} = Z_{ic} = h_{ic} - \frac{h_{fc}h_{rc}Z_L}{1 + h_{oc}Z_L} \]  \[ \ldots A3-3 \]
output impedance \( Z_{oc} = \frac{1}{h_{oc}} \)  
\[ h_{oc} = \frac{h_{fc} h_{rc}}{h_{ic} + Z_{g}} \]  

where \( Z_L = \) load impedance, and \( Z_g = \) generator impedance.

These will be converted to mixed \( h \)-parameters.

\[ h_{fe} = \frac{-h_{fb} (1 - h_{rb}) - h_{ob} h_{ib}}{(1 + h_{fb})(1 - h_{rb}) + h_{ob} h_{ib}} \]  
\[ A3-5 \]

\[ 1 + h_{fe} = \frac{1 - h_{rb}}{(1 + h_{fb})(1 - h_{rb}) + h_{ob} h_{ib}} \]  
\[ A3-6 \]

\[ (1 + h_{fb})(1 - h_{rb}) + h_{ob} h_{ib} = \frac{1 - h_{rb}}{1 + h_{fe}} \]  
\[ A3-7 \]

\[ 1 + h_{fb} = \frac{1}{1 + h_{fe}} - \frac{h_{ob} h_{ib}}{1 - h_{rb}} \]  
\[ A3-8 \]

\[ h_{ic} = \frac{h_{ib}}{(1 + h_{fb})(1 - h_{rb}) + h_{ob} h_{ib}} \]  
\[ A3-9 \]

Substituting A3-7 into A3-9 yields

\[ h_{ic} = \frac{h_{ib}}{h_{ib}} \frac{1 + h_{fe}}{1 - h_{rb}} \]  
\[ A3-10 \]
\[ h_{rc} = \frac{1 + h_{fb}}{(1 + h_{fb})(1 - h_{rb}) + h_{ob}h_{ib}} \quad \ldots A3-11 \]

Substitute A3-7 and A3-8 into A3-11 to obtain

\[
h_{rc} = \left(\frac{1}{1 + h_{fe}} - \frac{h_{ob}h_{ib}}{1 - h_{rb}}\right) \frac{1 + h_{fe}}{1 - h_{rb}} \quad \ldots A3-12
\]

\[ = \frac{1}{1 - h_{rb}} \left(1 - \frac{1 + h_{fe}}{1 - h_{rb}} h_{ob}h_{ib}\right) \]

\[ h_{fc} = -(1 + h_{fe}) \quad \ldots A3-13 \]

\[ h_{oc} = \frac{h_{ob}}{(1 + h_{fb})(1 - h_{rb}) + h_{ob}h_{ib}} \quad \ldots A3-14 \]

Substitute A3-7 into A3-14 to obtain

\[ h_{oc} = \frac{1 + h_{fe}}{1 - h_{rb}} h_{ob} \quad \ldots A3-15 \]

Substituting A3-10, A3-12, A3-13, and A3-15 into A3-1 gives

\[ A_v = \frac{1}{\frac{1}{1 - h_{rb}} \left[1 - (1 + h_{fe}) \frac{h_{ob}h_{ib}}{1 - h_{rb}}\right] + \frac{h_{ib}}{Z_L} \frac{1 + h_{fe}}{1 - h_{rb}} \left[1 + \frac{1 + h_{fe}}{1 - h_{rb}} h_{ob}Z_L\right]} \quad \ldots A3-16 \]
which simplifies to

\[ A_v = \frac{1 - h_{rb}}{h_{ib}} \frac{1}{1 + \frac{h_{rb}}{Z_L}} \]  \hspace{1cm} \ldots A3-17

Substitute A3-13 and A3-15 into A3-2 to get

\[ A_{ic} = -\frac{1 + h_{fe}}{1 + \frac{1 + h_{fe}}{h_{ob}Z_L}} \]  \hspace{1cm} \ldots A3-18

Substituting A3-10, A3-13 and A3-15 into A3-3 gives

\[ Z_{ic} = h_{ib} \frac{1 + h_{fe}}{1 - h_{rb}} + \frac{(1 + h_{fe})}{1 - h_{rb}} \left( 1 - \frac{1 + h_{fe}}{l - h_{rb}} h_{ob}h_{ib} \right) \frac{Z_L}{1 + \frac{1 + h_{fe}}{h_{ob}Z_L}} \]  \hspace{1cm} \ldots A3-19

which simplifies to

\[ Z_{ic} = \frac{1 + h_{fe}}{l - h_{rb}} \frac{h_{ib} + Z_L}{1 + \frac{1 + h_{fe}}{h_{ob}Z_L}} \]  \hspace{1cm} \ldots A3-20

Substituting A3-10, A3-12, A3-13 and A3-15 into A3-4 gives

\[ Z_{oc} = \frac{1}{\frac{1 + h_{fe}}{l - h_{rb}} h_{ob} + \frac{1 + h_{fe}}{l - h_{rb}} \left( 1 - \frac{1 + h_{fe}}{l - h_{rb}} h_{ib} h_{ob} \right) \frac{1 + h_{fe}}{l - h_{rb}} h_{ib} + Z_g} \]  \hspace{1cm} \ldots A3-21
Simplifying,

\[ Z_{oc} = \frac{h_{ib} + \frac{1-h_{rb}}{1+h_{fe}} Z_g}{l + h_{ob} Z_g} \]  \( \ldots A3-22 \)

In summary:

\[ A_{vc} = \frac{1-h_{rb}}{l + \frac{h_{ib}}{Z_L}} ; \quad \ldots A3-17 \]
\[ A_{ic} = -\frac{1+h_{fe}}{l + \frac{1+h_{fe}}{1-h_{rb}} h_{ob} Z_L} \]  \( \ldots A3-18 \)

\[ Z_{ic} = \frac{1+h_{fe}}{1-h_{rb}} \frac{h_{ib} + Z_L}{l + \frac{1+h_{fe}}{1-h_{rb}} h_{ob} Z_L} \]  \( \ldots A3-20 \)

\[ h_{ib} + \frac{1-h_{rb}}{1+h_{fe}} Z_g \]
\[ Z_{oc} = \frac{l+h_{ob} Z_g}{l+h_{ob} Z_g} \]  \( \ldots A3-22 \)
APPENDIX 4. DERIVATION OF THE EXACT PARAMETERS FOR A COMMON-EMITTER AMPLIFIER

Fig. A4-1. Common-Emitter Amplifier

Pursuing the line of reasoning used in Appendix 3 we start from the equations

\[ A_{VE} = \frac{l}{h_{re} + \frac{h_{ie}}{Z_L} + \frac{l+h_{oe}Z_L}{h_{fe}}} \] \hspace{1cm} \ldots \text{A4-1} \\

\[ A_{IE} = \frac{h_{fe}}{l+h_{oe}Z_L} \] \hspace{1cm} \ldots \text{A4-2} \\

\[ Z_{IE} = h_{ie} - \frac{h_{fe}h_{re}Z_L}{l+h_{oe}Z_L} \] \hspace{1cm} \ldots \text{A4-3} \\

\[ Z_{OE} = \frac{l}{h_{oe} - \frac{h_{fe}h_{re}}{h_{ie} + Z_g}} \] \hspace{1cm} \ldots \text{A4-4}
where $Z_L$ = load impedance and $Z_g$ = generator impedance

$$h_{ie} = h_{ic} = h_{ib} \frac{1+h_{fe}}{l-h_{rb}} \text{ from Appendix 3 } \ldots A4-5$$

$$h_{re} = 1-h_{rc} = \frac{1}{1-h_{rb}} \left[ \frac{1+h_{fe}}{l-h_{rb}} h_{ob} h_{ib} \right] \text{ using } h_{rc} \text{ from Appendix 3 } \ldots A4-6$$

$$h_{oe} = h_{oc} = \frac{1+h_{fe}}{l-h_{rb}} h_{ob} \text{ from Appendix 3. } \ldots A4-7$$

$h_{fe}$ is used as is.

Equations A4-5, A4-6, and A4-7 are substituted into A4-1, giving

$$A_{VE} = \frac{1}{\frac{1}{1-h_{rb}} \left[ \frac{1+h_{fe}}{l-h_{rb}} h_{ob} h_{ib} \right] - \frac{1+h_{fe}}{l-h_{rb}} h_{ib} \frac{1+1+h_{fe}}{l-h_{rb}} h_{ob} Z_L}$$

$$\ldots \ A4-8$$

Simplifying we have

$$A_{VE} = -\frac{Z_L}{h_{ib}} \frac{h_{fe}}{1+h_{fe}} \frac{1-h_{rb}}{1+\left( \frac{h_{fe}}{1+h_{fe}} h_{rb} + \frac{h_{ob}}{l-h_{rb}} \right) Z_L}$$

$$\ldots \ A4-9$$
Equation 7 is substituted into equation 2 giving

$$\begin{align*}
A_{IE} &= \frac{h_fe}{l + \frac{1+h_fe}{1-h_{rb}} h_{ob} Z_L} \\
\text{...A4-10}
\end{align*}$$

Equations 5, 6, and 7 are substituted into equation 3

$$\begin{align*}
Z_{IE} &= h_{ib} \frac{1+h_fe}{1-h_{rb}} \frac{h_fe}{1-h_{rb}} - \frac{1+h_fe}{1-h_{rb}} (\frac{h_fe}{1-h_{rb}} h_{ob} h_{ib} - h_{rb}) Z_L \\
&\quad \frac{1 + \frac{1+h_fe}{1-h_{rb}} h_{ob} Z_L}{1 + \frac{1+h_fe}{1-h_{rb}} h_{ob} Z_L} \\
\text{...A4-11}
\end{align*}$$

Simplifying yields

$$\begin{align*}
Z_{IE} &= \frac{1+h_fe}{1-h_{rb}} \frac{h_fe}{h_{ib}} \frac{1 + \frac{h_fe}{1+h_fe} h_{rb} + \frac{h_fe}{1-h_{rb}} h_{ob}}{1 + \frac{1+h_fe}{1-h_{rb}} h_{ob} Z_L} \\
\text{...A4-12}
\end{align*}$$

Equations 5, 6, and 7 are substituted into equation 4, to give,

$$\begin{align*}
Z_{0E} &= \frac{1}{\frac{1+h_fe}{1-h_{rb}} h_{ob} - \frac{h_fe}{1-h_{rb}} \frac{1+h_fe}{1-h_{rb}} h_{ob} h_{ib} - h_{rb}} \frac{1+h_fe}{1-h_{rb}} h_{ib} + Z_g \\
\text{...A4-13}
\end{align*}$$
Simplified 13 yields

\[
Z_{OE} = \frac{1-h_{rb}}{h_{ob} + \left( h_{ob} \cdot \frac{h_{rb}}{Z_g} \right) \cdot \frac{h_{fe}}{1 + l+h_{fe} \cdot \frac{h_{ib}}{Z_g}}} 
\]

\[\ldots A4-14\]

Also,

\[
\frac{1}{Z_{OE}} = g_{OE} = \frac{h_{ob}}{1-h_{rb}} + \frac{1}{2} \left( \frac{h_{ob} \cdot h_{fe}}{1-h_{rb}} + \frac{h_{fe} \cdot h_{rb}}{1+h_{fe} \cdot h_{ib}} \right) + \frac{1}{2} \left( \frac{h_{ob} \cdot h_{fe}}{1-h_{rb}} - \right.
\]

\[
\frac{h_{fe}}{1+h_{fe}} \cdot \frac{h_{rb}}{h_{ib}} \left( \frac{l-h_{rb}}{1-h_{rb}} \cdot \frac{h_{ib}}{Z_g} \right) \frac{l}{1} + \frac{1+h_{fe} \cdot \frac{h_{ib}}{Z_g}}{1-h_{rb}} 
\]

\[\ldots A4-15\]
APPENDIX 5. DERIVATION OF THE EXACT PARAMETERS FOR A COMMON-EMITTER AMPLIFIER WITH EXTERNAL EMITTER-IMPEDANCE.

Figure A5-1. Common-Emitter Amplifier with External Emitter-Impedance

The amplifier of Figure A5-1 differs from a common-emitter amplifier only in having an external emitter-impedance. Therefore if a set of $h'$-parameters (the $h'$-parameters) is found for a device consisting of a transistor with a resistor in the emitter lead, the $h'$-parameters may be used with the results of Appendix 4 to predict the performance of the amplifier.

$h'$-parameters

Figure A5-2. Small-signal Hybrid Equivalent Circuit of Common-Base Transistor with External Emitter-Impedance
From Figure A5-2 it is clear that

\[ h_{ib}' = h_{ib} + Z_E \]  \[ \ldots A5-1 \]

\[ h_{rb}' = h_{rb} \]  \[ \ldots A5-2 \]

\[ h_{fb}' = h_{fb} \]  \[ \ldots A5-3 \]

\[ h_{ob}' = h_{ob} \]  \[ \ldots A5-4 \]

It only remains to determine \( h_{fe}' \) to complete the set of mixed \( h \)-parameters.

If \( Z_L \) in Figure A5-1 is reduced to zero an emitter-follower results, with load \( Z_E \). The current gain of this emitter-follower is just \( h_{fc}' = -(1+h_{fe}') \).

From A3-18 in Appendix 3

\[ h_{fc}' = -\frac{1+h_{fe}}{1 + \frac{1+h_{fe}}{1-h_{rb} h_{ob} Z_E}} \]  \[ \ldots A5-5 \]

Therefore,

\[ h_{fe}' = \frac{1+h_{fe}}{1 + \frac{1+h_{fe}}{1-h_{rb} h_{ob} Z_E}} - 1 \]  \[ \ldots A5-6 \]

\[ h_{fe} = \frac{1+h_{fe}}{1 - h_{rb} h_{ob} Z_E} \]

\[ = \frac{1+h_{fe} h_{ob} Z_E}{1 + \frac{1+h_{fe}}{1-h_{rb} h_{ob} Z_E}} \]
In Equations A4-9, A4-10, A4-12 and A4-14, $h_{fe}$ appears in the forms $h_{fe}$, $h_{fe}/(1+h_{fe})$ and $(1+h_{fe})/(1-h_{rb})$.

Hence,

$$\frac{h'_{fe}}{1+h'_{fe}} = \frac{h_{fe}}{1+h_{fe}} - \frac{h_{ob}Z_{E}}{1-h_{rb}} \quad \ldots A5-7$$

and,

$$\frac{1+h'_{fe}}{1-h'_{rb}} = \frac{1+h_{fe}}{1-h_{rb} + (1+h_{fe})h_{ob}Z_{E}} \quad \ldots A5-8$$

**Amplifier Performance**

If Equations (1) through (4) and (6) through (8) are substituted into Equation A4-9 of Appendix 4, the result may be expressed as shown in Equation (9).

$$A'_{VE} = -G \left( \frac{h_{fe}}{1+h_{fe}} - \frac{h_{ob}Z_{E}}{1-h_{rb}} \right) \frac{1-h_{rb}}{1+g} \quad \ldots A5-9$$

where $G = Z_{L}/Z_{E}$,

and

$$g = \frac{h_{ib}}{Z_{E}} + Gh_{ob}Z_{E} + G \left( \frac{h_{fe}h_{rb}}{1+h_{fe}} + \frac{h_{ib}h_{ob}}{1-h_{rb}} \right)$$

It is noteworthy that $g$ may be minimized for given values of $G$ and $h$-parameters by a proper choice of $Z_{E}$ thus,
$$\frac{\partial g}{\partial Z_E} = -\frac{h_{ib}}{Z_E^2} + G h_{ob} \quad \ldots A5-10$$

Setting $\partial g/\partial Z_E = 0$ and designating the value of $Z_E$ which results $Z_{E0}$ yields

$$Z_{E0} = \sqrt{\frac{h_{ib}}{G h_{ob}}} \quad \ldots A5-11$$

and

$$g = \frac{h_{ib}}{Z_{E0}} \left( \frac{Z_{E0}}{Z_E} + \frac{Z_E}{Z_{E0}} \right) + G \left( \frac{h_{fe} h_{rb}}{1+h_{fe}} + \frac{h_{ib} h_{ob}}{1-h_{rb}} \right) \quad \ldots A5-12$$

If Equations (1) through (4) and (6) through (8) are substituted into Equation A4-12 of Appendix 4, the result may be expressed as shown in (13).

$$Z'_{IE} = (1+h_{fe})Z_E \frac{1+g}{1-h_{rb} + (1+G)(1+h_{fe})h_{ob} Z_E} \quad \ldots A5-13$$

$$= \frac{1+g}{\frac{(1-h_{rb})}{(1+h_{fe})Z_E} + (1+G) h_{ob}}$$

where $g$ and $G$ are as previously defined. Similarly $A'_{IE}$ is
given by Equation (14) as,

$$A'_{IE} = \frac{(1-h_{rb})h_{fe}-(1+h_{fe})h_{ob}Z_E}{1-h_{rb}+(1+G)(1+h_{fe})h_{ob}Z_E} \quad \cdots A5-14$$

and $Z'_{0E}$ by Equation (15),

$$Z'_{0E} = \frac{1-h_{rb}}{h_{ob} + (h_{ob} + \frac{h_{rb}}{Z_g}) \frac{(1-h_{rb})h_{fe} - (1+h_{fe})h_{ob}Z_E}{1-h_{rb}+(1+h_{fe})(h_{ob}Z_E + \frac{h_{ib}+Z_E}{Z_g})}}$$

$$\cdots A5-15$$
If two transistors are connected as shown in Figure A6-1, they can be regarded as one device. A set of $h$-parameters (the $h''$-parameters) may be derived in terms of the $h$-parameters of the individual transistors to describe the operation of the compound pair.

![Figure A6-1. Two-Transistor Compound in the Common-Emitter Configuration](image)

The common-emitter $h''$-parameters are defined by Equations (1) and (2).

\[ v''_b = h''_{ie}i''_b + h''_{re}v''_c \quad \cdots A6-1 \]

\[ i''_c = h''_{fe}i''_b + h''_{oe}v''_c \quad \cdots A6-2 \]

Q1 is connected in the common-emitter configuration and can be described by Equations (3) and (4).
\[ v''_b = h_{ie} i''_b + h_{re} v_{cl} \quad \ldots A6-3 \]

\[ i_{cl} = h_{fe} i''_b + h_{oe} v_{cl} \quad \ldots A6-4 \]

Q2 is connected in the common-collector mode and is therefore described by Equations (5) and (6).

\[ v_{cl} = h_{ic2} i_b + h_{rc2} v''_c = -h_{ic2} i_{cl} + h_{rc2} v''_c \quad \ldots A6-5 \]

\[ i''_c = h_{fc2} i_b + h_{oc2} v''_c = -h_{fc2} i_{cl} + h_{rc2} v''_c \]

Solving Equation (3) for \( v_{cl} \) gives

\[ v_{cl} = \frac{1}{h_{re}} v''_b - \frac{h_{ie}}{h_{re}} i''_b \quad \ldots A6-7 \]

which when substituted into Equation (4) gives the result

\[ i_{cl} = (h_{fe} - \frac{h_{ie} h_{oe}}{h_{re}}) i''_b + \frac{h_{oe}}{h_{re}} v''_b \quad \ldots A6-8 \]

Equations (7) and (8) are combined with Equation (5) and the result is solved for \( v''_b \).

\[ v''_b = (h_{ie} - \frac{h_{fe} h_{re} h_{ic2}}{1+h_{oe} h_{ic2}}) i''_b + \frac{h_{re} h_{rc2}}{1+h_{oe} h_{ic2}} v''_c \quad \ldots A6-9 \]
Equation (9) is substituted into Equation (8) to yield,

\[
i_{cl} = \frac{h_{fe}}{1+h_{oel}h_{ic2}} i''b + \frac{h_{oel}h_{rc2}}{1+h_{oel}h_{ic2}} v''c \quad \ldots A6-10
\]

Equation (10) is substituted into Equation (6) to give,

\[
i''c = -\frac{h_{fc2}h_{fe}}{1+h_{oel}h_{ic2}} i''b + (h_{oc2} - \frac{h_{oel}h_{rc2}h_{fc2}}{1+h_{oel}h_{ic2}}) v''c \quad \ldots A6-11
\]

If Equations (9) and (11) are now compared with Equations (1) and (2), it is clear that,

\[
h''ie = h_{iel} - \frac{h_{fe}h_{rel}h_{ic2}}{1+h_{oel}h_{ic2}} \quad \ldots A6-12
\]

\[
h''re = \frac{h_{rel}h_{rc2}}{1+h_{oel}h_{ic2}} \quad \ldots A6-13
\]

\[
h''fe = \frac{-h_{fe}h_{fc2}}{1+h_{oel}h_{ic2}} \quad \ldots A6-14
\]

\[
h''oe = h_{oc2} - \frac{h_{oel}h_{rc2}h_{fc2}}{1+h_{oel}h_{ic2}} \quad \ldots A6-15
\]

These equations are now converted to give the \( h'' \)-parameters in terms of the mixed \( h \)-parameters by substituting (see Appendices 3 and 4):

\[
h_{ie} = h_{ic} = h_{ib} \frac{1+h_{fe}}{1-h_{rb}} \quad \ldots A6-16
\]

\[
h_{re} = 1-h_{rc} = \frac{1}{1-h_{rb}} \left( \frac{1+h_{fe}}{1+h_{rb}} h_{ob} h_{ib} - h_{rb} \right)
\]
\[ h_{oe} = h_{oc} = \frac{1+h_{fe}}{1-h_{rb}} h_{ob} \]

\[ h_{fc} = -(1+h_{fe}) \]

These substitutions yield the results

\[ h_{fe}^{*} = B \left[ h_{ib1} \frac{1-h_{rb2}}{1+h_{fe2}} + h_{ib2} \left( \frac{h_{obl} h_{ib1}}{1-h_{rb1}} + \frac{h_{fel}}{1+h_{fel}} h_{rb1} \right) \right] \]

...A6-17

\[ h_{re}^{*} = B \left( \frac{h_{obl} h_{ib1}}{1-h_{rb1}} - \frac{h_{rb1}}{1+h_{fel}} \right) \left( \frac{1}{1+h_{fe2}} - \frac{h_{obl} h_{ib2}}{1-h_{rb2}} \right) \]

...A6-18

\[ h_{fe}^{*} = \frac{h_{fel} (1+h_{fe2})}{1 + \frac{1+h_{fel}}{1-h_{rb1}} \frac{1+h_{fe2}}{1-h_{rb2}} h_{obl} h_{ib2}} \]

...A6-19

\[ h_{oe}^{*} = B (h_{obl} + \frac{1-h_{rb1}}{1+h_{fel}} h_{ob2}) \]

...A6-20

where

\[ B = \frac{1+h_{fel}}{1-h_{rb1}} \frac{1+h_{fe2}}{1-h_{rb2}} \]

...A6-21

It is usually the case that \( h_{fe} \gg 1 \) and \( h_{rb} << 1 \). Making the approximations \( 1+h_{fe} \approx h_{fe} \) and \( 1-h_{rb} \approx 1 \) in Equations (16) through (21) one obtains,
\[ h_{ib} \approx \frac{h_{ie}}{h_{fe}} \]

\[ h_{ob} \approx \frac{h_{oe}}{h_{fe}} \]

\[ h_{rb} \approx \frac{h_{oe} h_{ie}}{h_{fe}} - h_{re} \]

and

\[ h_{re}^{\prime} \approx h_{fe}^{\prime} \left( \frac{h_{ib}}{h_{fe}^{2}} + h_{ib2} (h_{obl} h_{ibl} + h_{rbl}) \right) \]

\[ h_{re}^{\prime} \approx h_{fe}^{\prime} (h_{obl} h_{ibl} - \frac{h_{rbl}}{h_{fe}^{2}}) \left( \frac{1}{h_{fe}^{2}} - h_{ob2} h_{ib2} \right) \]

\[ h_{re}^{\prime} \approx h_{fe}^{\prime} \left( h_{obl} + \frac{h_{ob2}}{h_{fe}^{2}} \right) \]

\[ h_{fe}^{\prime} \approx \frac{h_{fel} h_{fe}^{2}}{1 + h_{fel} h_{fe}^{2} h_{obl} h_{ib2}} \]

Equations (23) are substituted into Equations (22)

\[ h_{ib}^{\prime} \approx h_{ib} \left( h_{obl} h_{ibl} + h_{rbl} \right) \]

\[ h_{rb}^{\prime} \approx h_{rbl} + h_{ibl} h_{ob2} + h_{fe} h_{ibl} h_{ib2} (h_{obl})^{2} \]

\[ h_{ob}^{\prime} \approx \frac{h_{ob2}}{h_{fel}} \]

\[ h_{fe}^{\prime} \approx \frac{h_{fel} h_{fe}^{2}}{1 + h_{fel} h_{fe}^{2} h_{obl} h_{ib2}} \]
REFERENCES

