THE REACTION $\text{He}^3(\text{He}^3,2p)\text{He}^4$ AND
THE DIPROTON STATE

by

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We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA
April, 1965
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Department of **Physics**

The University of British Columbia, Vancouver 8, Canada

Date **March 12, 1965**
ABSTRACT

The processes by which the three particle final state is formed in the \( \text{He}_3^3(\text{He}_3^3,2p)\text{He}_4^4 \) reaction were investigated by observing the angular distribution of coincidence events between the two protons as a function of the angle between the protons. The reaction mechanism was determined by comparing the experimental distribution with those predicted for the various possible processes obtained from kinematic and phase space arguments. The reaction was found to proceed predominantly by sequential decays through unbound intermediate states and to a lesser extent by a direct instantaneous three body breakup. The majority of the two stage decays passed through the ground state of Li\(^5\). The mean lifetime of this state was measured and found to be \((1.0 \pm 0.3) \times 10^{-21}\) sec. There was also good evidence of a sequential decay through the diproton state. In order to fit the shape of the observed distribution it was necessary to assume that a diproton system exists which is unbound by 600 keV and has a mean lifetime of \(1.5 \times 10^{-22}\) sec. However another possible interpretation is that a direct breakup occurs and the angular distribution of the protons is distorted by an attractive final state two proton interaction similar to the scattering interaction, although whether this interaction would be strong enough to produce the observed distribution is not known. A more quantitative three body decay theory is therefore necessary in order to draw any firm conclusions about the existence of the diproton state.
ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to Dr. J. B. Warren for his kind supervision of the work described in this thesis. I would also like to thank Dr. M. McMillan for his helpful discussions on the theoretical aspects of the problem.

I am especially grateful to Mr. B. A. Whalen for his assistance in setting up the electronics and in performing the measurements. The help of the other members of the Van de Graaff group is also greatly appreciated.

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CHAPTER 1

Introduction

In most nuclear reactions occurring at low energies only two nuclear particles emerge as final products as all other final states are forbidden by energy conservation. However, in some cases, in particular for some He$^3$ and H$^3$ induced reactions, the compound system has a sufficiently high excitation energy that breakup into three or more particles is energetically possible. In the past few years there has been an increased interest in the study, both experimentally and theoretically, of these multiparticle breakup reactions.

Of particular interest in these reactions is the reaction mechanism, that is the process or processes by which the multiparticle final state is formed. The problem is to determine whether the reaction proceeds via a single-stage, instantaneous breakup, by a series of two-body decays or by a combination of both these processes. If all the intermediate systems which would result from the sequential processes are unbound then this becomes a very difficult problem to solve experimentally. In this case the study of single particle spectra is usually inadequate as both breakup processes would result in similar continua in the energy spectra. In many of the early experiments involving multiparticle final states these continua were usually explained as due to a direct instantaneous breakup, as the elliptical shape predicted using phase space arguments for this process adequately fitted the observed continua. Any peaks in the continua were
attributed to final state interactions between two of the particles.

In 1960, Phillips and Tombrello (1960) introduced the cluster model which suggested that these three body decays could be treated as a time sequence of two body interactions. For instance the reaction

\[ a + A \rightarrow D^* \rightarrow b + C + c \]

could be considered as proceeding via the two reactions

\[ D^* \rightarrow B^* + b \]
\[ B^* \rightarrow C + c \]

The assumption that the reaction can be treated as a time sequence of two body decays requires that the decay of \( D^* \) occurs in such a manner that the constituents of \( B^* + c + G \) remain together as an excited localized system at least slightly longer than the time required for the emission of particle \( b \). Using the assumptions of this model the cross section for emission of particle \( b \) as a function of energy can be written in terms of the scattering phase shifts for the reaction

\[ c + C \rightarrow B^* \rightarrow C + c \]

and in terms of the interaction Hamiltonian for the reaction

\[ a + A \rightarrow D^* \rightarrow b + B^* \]

Experiments have been carried out to confirm the assumptions of this model. Beckner, Jones and Phillips (1961) have investigated the three body decay of \( ^{12}_C \) and \(^{10}_B \) by observing the alpha and deuteron particle spectra produced in the reactions \( ^{11}_B(p, \alpha)\)\(^{2}_He \) and
Be\(^9\)(p, d)\(^2\)He\(^4\). In both cases the spectra exhibited a low energy anomaly in the continua which could only be explained by assuming that the dominant mechanism in the reaction was a sequence of two body decays through states of Be\(^8\). This anomaly is present at the corresponding energy in \(\alpha - \alpha\) phase shifts and therefore enters into the energy spectrum of the breakup particles if the cluster model is used.

However the overall shape of the continuum in the alpha particle spectrum from the Be\(^9\)(He\(^3\), \(\alpha\))2He\(^4\) reaction has been fitted by Dorenbusch and Brown (1963) by assuming that the reaction proceeds primarily by the instantaneous breakup of the compound nucleus C\(^{12}\) into three alpha particles. These results are in common with most of the results of fitting single particle spectra in these multiparticle final state reactions. The overall continuum can usually be adequately fitted by assuming an instantaneous breakup and modulations on the continuum require the assumption of sequential decays.

It is evident that a different experimental approach is required to determine the reaction mechanism more precisely. Moazed, Etter, Holmgren and Waggoner (1964) have found that, in order to determine uniquely the reaction mechanism in a three particle final state, it is necessary to measure the energies \(E_A\), \(E_B\) and directions \(\Theta_A\), \(\Theta_B\) of two of the outgoing particles in coincidence. The outputs of the two particle detectors are analysed in a two dimensional kieksorter. Conservation of energy and momentum restrict all possible events to a kinematic curve \(E(E)\) which is determined by the two angles \(\Theta_A\) and \(\Theta_B\) and the reaction Q-values. Any reactions which proceed by sequential processes through discrete states of some intermediate system would then appear as points on this curve or as
segments of the curve in the case of broad states. This technique was used by Moazed et al to investigate the mechanism of the $\text{Be}^9(\text{He}^3, \alpha)\text{He}^4$ reaction. The energies of two of the alpha particles detected in coincidence were observed using two solid state counters mounted at fixed angles. Their conclusion from the results of these measurements was that the reaction proceeds almost entirely by sequential processes through the states of $\text{Be}^8$.

An extension of this technique would be to measure both the angular distribution and the energies of two of the outgoing particles detected in coincidence. It is possible in some cases that the angular distributions alone would be sufficient to give conclusive evidence about the reaction mechanism. If this is the case the use of a two dimensional kick-sorter would not be necessary. A theoretical angular distribution of coincidence events between two of the final state particles can be determined for both an instantaneous breakup and for the various possible sequential breakups if all final state interactions are neglected. These calculations then depend only on kinematic and phase space arguments. The distribution can be modified to account for coulomb interactions by considering the effect of the coulomb barrier on the energy distributions of the outgoing particles. The nuclear interactions or processes which predominate in the particular reaction can then be determined by comparing the predicted distributions for the various possible processes with the experimental angular distribution.

It was decided to use this technique in the investigation of the reaction mechanism and intermediate states present in the $\text{He}^3(\text{He}^3, 2p)\text{He}^4$ reaction. In this case the angular distribution of coincidence events between the two protons was measured. The main reason for choosing this particular reaction was that two of the final products are protons. Since
it has been found that in most cases three particle final state reactions proceed via sequential decays through intermediate states it was hoped that in this reaction one of the intermediate states might be the diproton state. The existence of the diproton state has been the topic of much interest and speculation in nuclear physics mainly because it is one of the three possible nuclear systems consisting of two nucleons. A knowledge of the properties of these nuclei, the deuteron, diproton and dineutron is of fundamental importance in nuclear physics as these properties are the starting point in the development of a nuclear theory. Of these nuclear systems only the triplet state (spins parallel) of the deuteron is stable, being bound by about 2.22 MeV. The diproton and dineutron are forbidden from having a triplet state because of the Pauli Exclusion Principle. The singlet states (spins antiparallel) of the three systems are unbound and therefore do not exist in nature although it is quite possible that they might exist for a very short time if produced during a nuclear reaction.

The first experiments carried out to investigate the existence of the diproton state were low energy p - p scattering studies. The scattering results showed conclusively that the diproton state was unbound but did not yield any information about the lifetime of the state or its binding energy. More recently the p + D and n + D reactions have received considerable attention in an effort to learn more about the p - p and n - n interactions. Wong et al (1959) observed the neutron spectra from the p + D reaction and Friedes and Brussel (1963) observed the proton spectra from the same reaction. The high energy peak in the neutron spectra and the low energy dip in the proton spectra were explained by assuming that some final state p - p interaction was present. However the effect of this
interaction was not thought sufficient to warrant calling the interaction
the diproton state. Likewise, Ilakovac et al (1961) observed the proton
spectra from the $n + D$ reaction and found a similar peaking indicating the
presence of a final state $n - n$ interaction. They stated that a more
thorough theoretical analysis was necessary to draw any conclusions about
the existence of the dineutron state.

The formation of the diproton state in the $p + D$
reaction and in proton - proton scattering is not particularly favoured
as the state must be formed during the reaction and the coulomb barrier
has to be penetrated by one of the protons from the outside. A more favour­
able situation would be to have the two protons initially together along with
another nucleon or stable nucleus and to have this extra particle removed
by the reaction. This would be the case in the $^{3}He(d, t)^{2}He$ or $^{3}He(\alpha)^{2}He$
reactions. Since the $^{3}He$ nucleus consists part of the time of two closely
lying paired protons plus a neutron, it is conceivable that this neutron
will be picked up by the incident deuteron or $^{3}He$ nucleus leaving the two
paired protons together in their paired configuration. The lifetime of this
system will determine whether it is purposeful to speak of a diproton state.
If the lifetime is of the order of a nuclear transit time then it would be
more appropriate to consider the system in terms of final state interactions.

In order to determine whether the diproton state can be
formed in this way Bilaniuk and Slobodrian (1963) observed the triton spectra
from the $^{3}He(d, t)^{2}He$ reaction and concluded that the broad peak they found
was evidence of a quasi-stable diproton state. In a later experiment
Artjomov et al (1964) studied the single particle spectra of both the
$^{3}He(d, t)^{2}He$ and $^{3}He(\alpha)^{2}He$ reactions but could not find definite
evidence of a diproton state and concluded that more precise measurements were necessary. After a more careful investigation of the $\text{He}^3(\text{d}, \text{t})\text{He}^2$ reaction Conzett, Shield, Slobodian and Yamabe (1964) announced that it was perhaps more proper to explain the broad triton peak in terms of a final state interaction between the two protons. By using the scattering length for the p- p interaction found from scattering experiments they were able to fit the shape of the triton peak very accurately.

In the light of this uncertainty in the interpretation of the energy spectra from the neutron pickup reactions on $\text{He}^3$ it was hoped that the technique of measuring the angular distribution of the coincidence events between the two outgoing protons would provide more definite evidence about the two proton interaction. Also as this experiment was carried out at lower energies than that used in the previous experiments it might be expected that the probability of the formation of the two proton cluster would be more favourable.

The study of the reaction using the coincidence technique is made somewhat easier by the fact that the outgoing particles are sufficiently energetic to make detection and identification simple. Also they are not complex nuclei so that only ground states have to be considered in the analysis of the reaction mechanism. The $\text{He}^3(\text{He}^3, 2\text{p})\text{He}^4$ reaction has not been studied to any great extent. Almquist, Allen, Dewan and Pepper (1953) and Good, Kunz and Moak (1954) found that the proton spectrum at $90^\circ$ to the incident beam consisted of a wide high energy peak corresponding to the transition to the ground state of $\text{Li}^5$ superimposed on a broad continuum which they attributed to the three body breakup. There was also a low energy peak from the decay in flight of the $\text{Li}^5$ nucleus into a proton and
an alpha particle. There was no evidence of well defined proton groups corresponding to transitions to an excited state of Li$^5$. The total reaction cross section was found to increase monotonically with incident energy from a value of 2.5 $\mu$b. at 200 keV to 2 mb. at 800 keV.

The present investigation of the He$^3$(He$^3$,2p)He$^4$ reaction was carried out using a He$^3$ beam from the U.B.C. Van de Graaff generator. This thesis describes the design of the He$^3$ gas target and solid state counter assembly and the results obtained from an observation of the angular distribution of coincidence events between the two final state protons.
CHAPTER 2

Choice of Target - Detector Arrangement

2-1. Preliminary Considerations

The bombardment of \( \text{He}^3 \) with \( \text{He}^3 \) leads to a three particle final state consisting of two protons and one alpha particle. The overall Q-value for the reaction is 12.859 MeV and therefore the maximum proton energy is about 11 MeV for low incident beam energies. The total reaction cross section for this reaction as found by Good, Kunz and Moak (1954) is shown in Figure 2-1. Possible contaminant reactions which could be present at low bombarding energies and their cross sections at representative energies are given in the following table.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Q-value MeV</th>
<th>Cross section mb. at ( E_{\text{He}^3} ) MeV</th>
</tr>
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<tbody>
<tr>
<td>( \text{D(He}^3, \text{p)He}^4 )</td>
<td>18.351</td>
<td>700</td>
</tr>
<tr>
<td>( \text{H}^3(\text{He}^3, \text{np)He}^4 )</td>
<td>12.095</td>
<td>180</td>
</tr>
<tr>
<td>( \text{C}^{12}(\text{He}^3, \text{p)N}^{14} )</td>
<td>4.767</td>
<td>0.5</td>
</tr>
<tr>
<td>( \text{N}^{14}(\text{He}^3, \text{p)C}^{16} )</td>
<td>15.242</td>
<td>0.5</td>
</tr>
</tbody>
</table>

As the cross sections for the first two contaminant reactions are considerably larger than the reaction to be studied, it is necessary to ensure that they are not present in the target chamber to any great extent. This requirement must be taken into consideration in the choice of the target arrangement to be used.
FIGURE 2-1. Total Reaction Cross Section for the $^{3}\text{He}^3(3\text{He}^3, 2p)^4\text{He}$ Reaction

x EXPERIMENTAL POINTS
2-2. Discussion of $^{\text{He}}_3$ Targets

Since $^{\text{He}}_3$ is available only as a gas unless extremely low temperatures are used, there are two target arrangements which are possible. The simplest technique is to produce a $^{\text{He}}_3$ target by bombarding a thin metal backing of copper or aluminum with the beam of $^{\text{He}}_3$ nuclei from the accelerator. This method requires high beam currents and therefore cooling of the metal backing is necessary to allow the $^{\text{He}}_3$ buildup to take place. There are several disadvantages in using this target arrangement:

a. The possibility of contaminant reactions being present is very high.
b. The number of target atoms cannot be determined accurately as the $^{\text{He}}_3$ distribution with thickness is not known.
c. Angular distribution measurements are difficult, if not impossible, because of the target backing.

d. Accurate cross section measurements and angular distributions can be carried out.
e. Target uniformity is inherent except for beam heating effects (Robertson et al, 1961).
f. Contamination can be held to a minimum by using collimators to determine the active volume of the target.
g. If solid state detectors are used to monitor the reaction products they
can be immersed in the gas target chamber.

Gas targets usually are of two types: those in which the beam enters from the high vacuum region through a thin window and those in which the beam enters through a differentially pumped capillary. The second type is preferable if accurate measurements are required as the window introduces larger energy straggling in the beam. However differential pumping requires a larger amount of target gas and a complex vacuum system so that unless a large supply of target gas is available, as is not the case with He\(^3\), it is necessary to use the thin foil for the beam entrance window.

2-3. Properties of Thin Foils

The choice of a suitable window material is very important as it usually is the limiting factor in gas targets as far as maximum beam current and target pressure are concerned. Possible choices are aluminum, nickel, molybdenum, tantalum and zirconium. Other materials than metals may also be used such as SiO or carbon films but the extra difficulties in preparing such films limits their application even though they can withstand higher beam currents. Nickel is the most commonly used window material because of its high mechanical strength and the availability of high quality, vacuum tight foils of thicknesses down to 0.010 mils. The nickel foils used in the target chamber in the experiment described here were obtained from Chromium Corporation of America, Waterbury, Conn.

In Table 2 the properties of nickel foils when used as a thin window for a beam of He\(^3\) particles are listed. The energy loss in the nickel foil was obtained from the stopping cross section data of Whaling (1958). The root mean square scattering angles were determined from Fermi's formula for multiple coulomb scattering, Fermi (1950). The maximum beam current and maximum pressure differential across the foil were obtained from
<table>
<thead>
<tr>
<th>Nickel foil thickness (10^{-3}) in mg/cm²</th>
<th>Energy of He(^2) beam MeV</th>
<th>Energy loss in foil keV</th>
<th>R.M.S. scat. angle degrees</th>
<th>Max safe beam curr. μA</th>
<th>Max press. diff. mm of Hg</th>
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<td>760</td>
<td>2.6</td>
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<tr>
<td></td>
<td>2.0</td>
<td>590</td>
<td>1.7</td>
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</table>
earlier experimental data and by tests carried out on the foils. These figures apply to the case of nickel foils mounted on 3/16 inch diameter holes and the incident beam limited to 0.10 inches in diameter.

The nickel foils were soft soldered to their brass holders using a high tenacity flux (Kester Soldering Salts). The technique found most satisfactory was first to put a thin layer of solder on the face of the brass holder, allow the holder to cool, and then apply a small amount of the flux to the soldered face. The foil was then carefully placed on the face of the holder and a large aluminum block was used to hold the foil in place and also to act as a heat sink. The aluminum block was then heated until the solder underneath the foil melted. When the foil and its holder were cool again, the excess flux was removed by soaking in methanol.

2.4. Detection of Reaction Products

Although there are many possible methods of detecting the charged reaction products from the \( \text{He}^3(\text{He}^3, 2p)\text{He}^4 \) reaction, the special properties of the solid state detector make it the most obvious choice in this particular experiment. The small size of these detectors makes it possible to mount the counters inside the reaction chamber. The detectors produce very fast rising pulses which are necessary for accurate timing in coincidence measurements.

The requirements on the detector for this particular use are a linear response for protons over a range of energies from 1 MeV to 12 MeV and a reasonably good intrinsic resolution (\(< 35 \text{ keV}\)). In order to stop the higher energy protons a depletion thickness of 1000 microns is required. This depletion thickness can be conveniently obtained in "surface barrier" detectors. These counters are made by evaporating a thin layer of
gold (100 - 2000 A.U.) onto high resistivity (10,000 ohm-cm) n-type silicon. A reverse bias of 300 - 400 volts gives the require depletion thickness.

These counters are available with an active area ranging from a few mm$^2$ to several hundred mm$^2$. The choice of the active area is governed mainly by the desired counting rate and the required resolution of the angular distribution measurements. An approximate estimate of the yield to be expected from this reaction is necessary before the choice of detector size can be made. The yield of emitted particles $Y$ is given by

$$Y = \frac{N n L \eta}{\text{particles/sec}}$$

where $N$ is the number of incident particles per second; $n$ is the number of target nuclei per cm$^3$; $L$ is the thickness of the target in cm; $\eta$ is the effective solid angle the detector subtends and $(\frac{d\sigma}{d\omega})_{\text{av}}$ is the laboratory cross section in cm$^2$ averaged over the solid angle and over the spread in incident beam energy.

For a gas target in which the target volume is fixed by a pair of defining slits of width 'a' at a distance 'r' from the beam centre line, the geometrical factor $L\eta$ can be expressed approximately by

$$G = L\eta \approx \frac{aA}{\pi R}$$

for a detector of area 'A' placed a distance 'R' from the beam and perpendicular to the incident beam direction. The effect of the beam width is assumed to be negligible.

Assuming isotropy of the reaction products the reaction
yield can be written

\[ Y = 16.7 \text{ PIG } \sigma_t \text{ particles/sec.} \]

where \( P \) is the target pressure in mm of Hg; \( I \) is the beam current in \( \mu \)amps; \( G \) is the geometrical factor determined from expression 2-2 and \( \sigma_t \) is the total reaction cross section in mb.

The product \( \text{PIG} \sigma_t \) should be made as large as possible but the parameters are adjustable only over a restricted range of values as other conditions must satisfied as well. The incident beam energy should be high to increase the reaction cross section and to decrease the percentage of energy lost in the entrance window. However it is desirable to have as small centre of mass motion as possible in order to simplify the kinematic calculations. The beam energy chosen was 1.60 MeV which meant that about 1.15 MeV was left after the beam passed through the entrance foil. The cross section corresponding to this energy is about 5 mb.

The gas pressure should be high to give a high counting rate and good cooling of the entrance window but it is limited by the strength of the foil. The He\(^3\) gas pressures used were between 50 and 100 mm of Hg. The beam current likewise was limited by the entrance foil and had to be kept below about 0.25 \( \mu \)a. The detectors chosen had an active area of 78.5 mm\(^2\) corresponding to a diameter of 10 mm. By placing the counter at a distance of 50 mm from the beam centre line and using a collimator of width 5 mm placed a distance of 12.5 mm from the centre line of the beam it was possible to obtain a single particle counting rate of
\[ Y = 16.7 \times 75 \left( \frac{5 \times 785}{125 \times 5} \right) \times 5 \]

\[ = 400 \text{ counts/sec/} \mu \text{a.} \]

for a target pressure of 75 mm of Hg. This gave a reasonable coincidence counting rate of about 10 counts per minute.
CHAPTER 3

Experimental Arrangement

3-1. General Outline of Experiment

A schematic diagram of the experimental arrangement is shown in Figure 3-1. Two solid state detectors were required for the coincidence measurements, both counters mounted at 90° to the beam direction. One counter was fixed and the other could be rotated at various angles with respect to the fixed counter in the plane perpendicular to the beam. In order to observe coincidence events between the two protons only it was necessary to discriminate against the alpha particles. This was done by placing an aluminum foil in front of each counter to stop completely the alpha particles but to allow most of the protons to pass through. The counters were mounted inside the He³ gas target and the incident He³ beam entered the target chamber through a thin nickel foil.

The experiment consisted of measuring the number of coincidence events between the two protons for various angles between the two counters from 180° to the minimum possible angle. The coincidence measurements were all normalized to the total number of proton counts in both counters. The energy spectrum of the protons were also observed.

3-2. Details of Gas Target Chamber

The gas target chamber is shown in Figure 3-2. The cylindrical slits which, together with the active area of the detector, define the target volume were arranged so that only particles from reactions occurring in the target gas could be detected. This eliminated any contamin-
FIGURE 3-1. Schematic Diagram of Experimental Arrangement

- FIXED SOLID STATE DETECTOR
- CYLINDRICAL SLIT TO DEFINE ACTIVE VOLUME OF TARGET GAS
- ROTATING SOLID STATE DETECTOR
- INCIDENT He³ BEAM
- BEAM DEFINING COLLIMATORS
K.OVARSSAL

To 'DETECTORS

CYLINDRICAL SLIT TO DEFINE TARGET VOLUME

FIXED SOLID STATE DETECTOR #1

KOVAR SEAL TO DETECTOR #2

ANGULAR SCALE

HANDLE TO ROTATE DETECTOR # 2

INSULATORS CURRENT COLLECTOR CUP

ROTATING SOLID STATE DETECTOR #2

FIGURE 3-2. Details of Gas Target Chamber

SCALE 1/2 = 1.
ant reactions from the nickel foil and the chamber walls. The nickel foils were typically 0.025 mils thick. The brass holder on which the foil was mounted also served to collimate the incident beam. The soldered connection helped to provide a good thermal contact with the rest of the chamber. As the intended beam currents were less than 0.5 μa, it was not necessary to cool the nickel foil. The heat resulting from the energy loss of the incident beam in the foil was dissipated partly by conduction through the foil to its mounting and partly by conduction through the target gas.

The target gas pressure was continuously monitored by a Wallace and Tiernan absolute pressure indicator Type FA - 160 (0 - 410 mm). The He³ gas used in the target chamber and in the ion source for the Van de Graaff generator was obtained from Monsanto Research Corporation, Mound Laboratories. The specified purity of the gas was greater than 99.8 % He³ in He⁴, greater than 99.99 % total helium and less than $3.6 \times 10^{-11}$ % tritium. This tritium content was considerably lower than that which would have given a significant contribution from the $^3\text{H}(^3\text{He}, \text{np})^4\text{He}$ reaction.

After passing through the active volume of the target gas the beam entered a collector cup so that the beam current could be monitored. As the Faraday cup was surrounded by the target gas it was found necessary to apply a negative bias of 90 volts to the cup to prevent collection of electrons from the ionization of the gas by the beam. The current was measured with an Eldorado Current Integrator Model CI - 110. The magnitude of the current was not considered reliable but it did provide a means of monitoring the current for focussing purposes. An accurate knowledge of the beam current was not necessary as all measurements were normalized to the total number of protons detected in both counters.
The solid state detectors used to detect the protons were Type PH 8-25-10 obtained from Nuclear Diodes. These counters had the required depletion thickness at an operating voltage of 300 volts. Their active diameter was 10 mm and their resolution was about 25 keV. The detectors were biased using the ORTEC Model 201, 0 - 1000 V Detector Control Unit. This bias supply also allowed the leakage current to be continuously monitored in either counter. The counters were mounted at a distance of 5 cm from the centre line of the incident beam and the width of the counters prevented measurements from being taken at angles less than 23°. However by tilting both counters at an angle to the direction of the incident particles it was possible to observe the coincidence events at lower angles. As this resulted in a decrease in the solid angle subtended by the detectors the angle could only be reduced to about 8° before the coincidence rate became prohibitively low.

3-3. Electronics

The block diagram of the amplification and coincidence system is shown in Figure 3-3. The pulses from the solid state counters were first amplified by similar charge sensitive preamplifiers. The circuit diagram for these preamplifiers, which used mulfistors in the input stage, is shown in Figure 3-4. These preamplifiers combined low noise (resolution less than 10 keV) with a fast risetime (< 10 nsec.) and a slow decay time (~ 100 μsec.). This shape of pulse is desirable for use in delay line clipped amplifiers. The main amplifiers were Cosmic Double Delay Line Clipped Linear Amplifiers, Model 901. These amplifiers could be operated in either the single clipped or double clipped mode and were compatible with the Cosmic Single Channel Analysers, Model 901 SCA. The single
FIGURE 3-3. Block Diagram of Amplification - Coincidence System
FIGURE 3-4. Nuvistor Preamplifier Circuit
channel analysers were used as discriminators to allow only pulses of a desired voltage range to produce a timing pulse coincident in time with the zero cross over point of the D.D.L. clipped pulse.

The outputs of both single channel analysers were then fed through variable delays and into the coincidence circuit. The timing pulses put out by the S.C.A. were about 200 nsec. long. In order to obtain better time resolution these pulses were shortened to 10 - 30 nsec. width at the input of the coincidence circuit using a delay line clipping technique (see Figure 3-5). The coincidence circuit is shown in Figure 3-6. In this circuit the input pulses were transformed into current pulses passing through a tunnel diode in its low voltage state. Whenever the two pulses arrived simultaneously the current was sufficient to switch the tunnel diode to its high voltage state producing a coincidence pulse at the output. The resolution time of the circuit was set at about 30 nsec. for this experiment. The probability of a random pulse was negligible with the counting rates used for this resolution time. The output of the coincidence circuit and the positive outputs of both single channel analysers were fed into similar scalers (UBC - NP11). Thus the number of coincidence events between the two protons for a given number of counts in both counters could be obtained.

The delayed output of one of the Cosmic D.D.L. clipped amplifiers was attenuated and put into the Nuclear Data Type 120 Kicksorter. The output of the coincidence circuit was used as a gating pulse in the kicksorter. When the coincidence circuit was operated in the singles mode, the single particle proton spectrum could be obtained. When operated in the doubles mode, only protons from coincidence events were analysed.
FIGURE 3-5. Pulse Shaper for Coincidence Circuit
FIGURE 3-6. Coincidence Circuit
3-4. Adjustment of S.C.A. and Delay for Coincidences

Figure 3-7 shows the response of the detector-amplifier system to the alpha particles from an Am$^{241}$ source. The major group of alpha particles from this source has an energy of 5.477 MeV. The sharp peaks correspond to pulses from a Datapulse 106A Pulse Generator applied to the test input. The input pulse height has been increased in equal intervals and the equal spacing of the output pulses indicates the linearity of the amplification system. The inset shows a greatly amplified response to the alpha particles indicating the presence of other alpha groups from the Am$^{241}$ source. This spectrum was obtained using a biased amplifier (ORTEC Model 201 Low Noise Biased Amplifier).

It is possible to determine the pulse height necessary to simulate a 5.477 MeV alpha particle and also the corresponding pulse height to simulate an 11 MeV proton (the maximum expected energy of protons from the He$^3$(He$^3$, 2p)He$^4$ reaction). Using the pulser to simulate the proton pulses, the gains of the two D.D.L. clipped amplifiers were adjusted to give an output 9.0 volts for an input corresponding to the maximum expected proton energy. The baseline of the two S.C.A. was set at 0.5 volts and the window was set at 9.5 volts, thereby allowing the protons of energy between 600 keV and 12 MeV to produce an output pulse from the S.C.A. The output of two pulsers, one synchronized from the output of the other to produce simultaneous pulses, was then fed into the two S.C.A. through the amplifiers. The delays were adjusted to obtain a coincidence output from the coincidence circuit operating in the doubles mode.

These delay settings had to be readjusted slightly to obtain a coincidence output when running the actual experiment to compe-
FIGURE 3-7. Calibration of Detection System with Am$^{241}$ alpha particles.
sate for unequal cable lengths or small differences in the response of the
two detector-amplifier systems to the protons. The curve of number of
coincidences vs. delay time should be a flat plateau of width equal to the
resolution time of the coincidence circuit. The delays were set at the
centre of this plateau.
4. Two Body Kinematics

A reaction leading to a three particle final state

\[ a + A \rightarrow D^* \rightarrow b + c + C + Q \text{ MeV} \]

can be considered as a sequence of two body decays

\[ D^* \rightarrow b + B^* + Q_1 \]
\[ B^* \rightarrow c + C + Q_2 \]

if the decay of $D^*$ occurs in such a manner that the intermediate system $B^*$ remains together sufficiently long to satisfy the conservation of energy and momentum. The momentum diagram for this process is shown below.

If the reaction $Q$-values for the sequential decays are $Q_1$ and $Q_2$ respectively then the energy and momentum relations are as follows:
\[ E_1 + E_2 = Q_1 \]
\[ E_3 + E_4 = Q_2 + E_2 \]
\[ \mathbf{p}_1 + \mathbf{p}_2 = 0 \]
\[ \mathbf{p}_3 + \mathbf{p}_4 = \mathbf{p}_2 \]

From these relations it follows that

\[ E_1 = Q_1 \frac{m_2}{m_1 + m_2} \]
\[ E_2 = Q_1 \frac{m_1}{m_1 + m_2} \]

Eliminating \( E_4 \) and solving for \( E_3 \) gives

\[ E_3 + E_2 \frac{m_3}{m_2} - Q_2 \frac{m_4}{m_2} = 2\sqrt{\frac{m_3}{m_2} E_2 E_3} \cos \Theta \]

Squaring gives a quadratic equation from which \( E_3 \) can be determined as a function of \( \cos \Theta \). To complete the energy determination \( E_4 \) can be found from the relation

\[ E_4 = E_2 + Q_2 - E_3 \]

From Appendix A, the two-particle density of states factor
which determines the probability of particle $c$ being emitted into solid angle $d\Omega$ at an angle $\Theta$ to the direction of $B^*$ is given by

$$\frac{d}{d\Omega} P_2(E) = \frac{d}{dE} \int P_3^2 dP_3$$

$$= \frac{m_3 m_4 P_3^2}{m_4 P_3^2 - m_3 P_3^4 + P_4^2}$$

$$= \frac{m_3 m_4 P_3^3}{(m_3 + m_4) P_3^4 - m_3 P_3^4 \cos \Theta}$$

In the $\text{He}^3 + \text{He}^3$ reaction the following sequential decays are energetically possible.

I a. \quad \text{He}^3 + \text{He}^3 \rightarrow p + \text{Li}^5 \rightarrow p + p + \alpha

$$Q_1 = 10.892 \text{ MeV}$$

$$Q_2 = 1.967 \text{ MeV}$$

$$m_1 = m_3 = m_p$$

$$m_2 = 5m_p$$

$$m_4 = 4m_p$$

b. \quad \text{He}^3 + \text{He}^3 \rightarrow p + \text{Li}^{5\alpha} \rightarrow p + p + \alpha

$$Q_1 = 10.892 - E_{\text{ex}} \text{ MeV}$$

$$Q_2 = 1.967 + E_{\text{ex}} \text{ MeV}$$

$$m_1 = m_3 = m_p$$

$$m_2 = 5m_p$$

$$m_4 = 4m_p$$
where $E_{\text{ex}}$ is the excitation energy of the first excited state in Li$_5$ which is thought to be 5 - 10 MeV. The width of this state is about 3 MeV. The calculations will be carried out for several values of $E_{\text{ex}}$.

II $\text{He}^3 + \text{He}^3 \rightarrow \alpha + \text{He}^2 \rightarrow \alpha + p + p$

$$Q_1 = 12.859 - Q_{2p} \text{ MeV}$$

$$Q_2 = Q_{2p} \text{ MeV}$$

$$m_1 = 4m_p$$

$$m_2 = 2m_p$$

$$m_3 = m_4 = m_p$$

where $Q_{2p}$ is the binding energy of the diproton. As the binding energy is not known the calculations will be carried out for several values of $Q_{2p}$.

In reaction I particles b and c are protons. If $\Theta$ is the angle between these protons the energy equation 4.7 can be written

$$E_3^2 + \frac{4}{5} E_3 \left[ E_2 \left(1 - 2 \cos^2 \Theta \right) - 4 Q_2 \right] + \frac{1}{25} \left(E_2^2 - 4Q_2^2\right) = 0$$

In terms of the energies $E_2$ and $E_3$, the phase space factor, equation 4.7, becomes

$$\frac{d}{dS_{\alpha}^2} \rho_2(E) = \frac{E_3(\Theta)}{\sqrt{E_3(\Theta)} + \sqrt{E_2^2 \cos \Theta}}$$

where $E_3(\Theta)$ is the solution of the energy equation corresponding to the angle $\Theta$.

The probability that the two protons will be detected in
coincidence in two detectors separated by an angle $\Theta$ is found by integrating this expression over the solid angles subtended by the two counters. This integration cannot be done explicitly and requires a Simpson's rule integration. The program for this calculation is listed in Appendix B. These calculations were carried out using the U.B.C. IBM 7040 computer. The theoretical angular distribution for the sequential decay through the ground state and 1st excited state of Li$^5$ is shown in Figure 4-la. The momentum diagram for this process is shown in the inset. The solid angles used in the integration correspond to detectors of diameter 10 mm positioned a distance of 50 mm from the centre line of the incident beam.

The aluminum foil in front of each counter (thickness 3.0 mg/cm$^2$), to prevent alpha particles from being detected and the lower level setting on the single channel analysers to discriminate against the electronic noise, result in the loss of coincidence events between the two protons in which one of the protons has an energy less than 1.5 MeV. The correction to the theoretical curves to take account of this effect is shown by the dotted line in Figure 4-la. This correction is only required for the distribution corresponding to the transition to the ground state of Li$^5$.

In reaction II particles c and C are protons. This is the sequential decay which proceeds through the diproton state. If $\Phi$ is the angle between the two protons the energy equation 4-6 becomes

$$E_3^2 - E_3 (E_2 + Q_2) + \left(\frac{E_2 - Q_2}{2 \cos \Phi}\right)^2 = 0$$

and the phase space factor becomes
FIGURE 4-1. Theoretical Angular Distribution for the Sequential Decays
\[
\frac{d}{d\Omega_3} \mathcal{R}_2(E) = \frac{E_3(\varphi)}{\sqrt{E_3(\varphi)} - \sqrt{E_4(\varphi)}} \cos \varphi
\]

where \( E_3(\varphi) \) and \( E_4(\varphi) \) are the solutions of the energy equation for \( E_3(\varphi) \geq E_4(\varphi) \). The results of the integration of this expression (see Appendix B for a listing of the computer program) over the solid angles of the two counters, normalized to the probability of the event occurring with \( \Theta = 0^\circ \), are shown in Figure 4-1b. The different curves correspond to the different \( Q \)-values for the breakup of the diproton. The momentum diagram for this sequential decay is shown in the inset. The correction for the loss of coincidence events for proton energies less than 1.5 MeV is indicated by the dotted line. The peak in the theoretical distribution occurs at the angle at which the two protons are emitted with equal energies. This angle increases as the energy available to the breakup protons increases. If the diproton system remains bound the peak would be expected to occur at \( 0^\circ \).

4-2. **Three Body Kinematics**

The three particle final state produced by a reaction of the type

\[
a + A \rightarrow D^* \rightarrow b + c + C
\]

can also be considered to be formed by a single-stage instantaneous breakup of the compound nucleus \( D^* \). The momentum diagram for this process is shown in the following diagram.
If the compound nucleus is assumed to be at rest when the breakup occurs, which is a reasonable assumption for low bombarding energies and for a high Q reaction, then the energy and momentum relations may be written as follows

\[ E_1 + E_2 + E_3 = Q \quad 4-11 \]

\[ \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0 \quad 4-12 \]

In the case of the breakup of the Be\(^{6+}\) system into two protons and one alpha particle, two of the final particles have equal masses so that \( m_1 = m_2 = m \). From the energy and momentum relations

\[ E_1 + E_2 + (E_1 E_2)^{1/2} \frac{2m \cos \theta}{m+m_3} = \frac{Q m_3}{m+m_3} \quad 4-13 \]

This is the equation of a family of ellipses in the two-dimensional energy space \( E_1 - E_2 \). The energy calculations can be simplified by making the following substitutions.

\[ a = \frac{2m}{m + m_3} \]
Equation 4-13 becomes on substitution of the above expressions

\[ b = Q \frac{m_3}{m + m_3} \]

\[ \mu = \cos \theta \]

\[ E_1 = \frac{x+y}{a} \quad E_2 = \frac{x-y}{a} \]

\[ y = \left\{ x^2 - \frac{b^2}{\mu^2} \left( 1 - \frac{2x}{ab} \right)^2 \right\}^{1/2} \]

The limits on \( x \) become

\[ \mu < 0 \quad \frac{ab}{2} \leq x \leq \frac{ab}{2 + \mu a} \]

\[ \mu > 0 \quad \frac{ab}{2 + \mu a} \leq x \leq \frac{ab}{2} \]

For the \( \text{He}^3(\text{He}^3,2p)\text{He}^4 \) reaction the overall Q-value is 12,859 MeV. The two-dimensional energy spectra for various angles between the two outgoing protons are shown in Figure 4-2. The probability of an energy division of particles 1 and 2 in a three particle breakup into energy intervals \( dE_1 \) and \( dE_2 \) is given in Appendix A by

\[ d^2 \rho_3(E) = 8\pi^2 m_1 m_2 m_3 dE_1 dE_2 \]

Using the substitutions listed previously this differential phase space factor becomes
FIGURE 4-2. Two Dimensional Energy Spectra
The probability of particle 1 being emitted at an angle between $\Theta$ and $\Theta + d\Theta$ to the direction of emission of particle 2 is determined from the expression

$$d^2P_3(E) \propto dx \, dy$$

by substituting for $\frac{dy}{d\mu}$ from equation 4-14 and then integrating over $x$.

On making the substitution

$$\frac{x}{ab} = \frac{2 - \mu a \sin \xi}{4 - \mu^2 a^2}$$

the integral becomes

$$\mu > 0 \quad \frac{d}{d\xi} r_3(E) \propto \int_{\sin^{-1} \frac{a}{2}}^{\frac{\pi}{2}} \frac{1}{(4 - \mu^2 a^2)^{\frac{5}{2}}} (2 \sin \xi - \mu a)^2 d\xi$$

$$\mu < 0 \quad \frac{d}{d\xi} r_3(E) \propto \int_{\frac{\pi}{2}}^{\sin^{-1} \frac{a}{2}} \frac{1}{(4 - \mu^2 a^2)^{\frac{5}{2}}} (2 \sin \xi - \mu a)^2 d\xi$$

The integrations give the following results:

$$\mu = 0 \quad \frac{d}{d\xi} r_3(E) = \frac{\pi}{32}$$

$$\mu > 0 \quad \frac{d}{d\xi} r_3(E) = \frac{(2 + \mu^2 a^2) \tan^{-1} \sqrt{\frac{4-\mu^2 a^2}{2}} - \frac{3}{2} \mu a \sqrt{4-\mu^2 a^2}}{(4-\mu^2 a^2)^{\frac{5}{2}}}$$

$$\mu < 0 \quad \frac{d}{d\xi} r_3(E) = \frac{(2 + \mu^2 a^2)(\pi - \tan^{-1} \sqrt{\frac{4-\mu^2 a^2}{2}}) - \frac{3}{2} \mu a \sqrt{4-\mu^2 a^2}}{(4-\mu^2 a^2)^{\frac{5}{2}}}$$
The probability of a coincidence event between the two protons occurring for an angle $\Theta$ between the detectors is found by integrating the above expressions over the solid angles subtended by the counters. This also was done by a Simpson's rule integration (see Appendix B). The results of the integrations normalized to the probability of the coincidence event occurring with $\Theta = 180^\circ$ is shown on Figure 4-3a. The correction for the loss of coincidence events for proton energies less than 1.5 MeV is shown by the dotted line assuming that the single particle energy distribution is elliptical. The theoretical distribution for a three particle breakup in which the masses of the three final state particles are equal is shown in Figure 4-3b. This would be the distribution expected from the photodisintegration of the $^3$He nucleus into two protons and one neutron assuming no final state interactions. These distributions do not depend on the energy available to the breakup particles but only on their relative masses. The momentum diagrams for both breakups are shown in the insets of Figure 4-3.
FIGURE 4-3. Theoretical Angular Distribution for the Direct 3-Body Breakup
EXPERIMENTAL RESULTS

5-1. Energy Spectrum of Protons

A beam energy of 1.60 MeV was used for all measurements. The energy loss in the entrance foil corresponding to this incident He$^3$ energy is 450 keV so that the beam energy inside the gas target was 1.15 MeV. This energy was found to be high enough to give a reasonable coincidence rate while the centre of mass motion due to the incident energy of the He$^3$ beam was sufficiently small that it could be ignored, making the comparison with the theoretical angular distributions easier.

The single particle proton spectrum observed at 90° to the incident beam direction is shown in Figure 5-1. The energy scale has been drawn using the proton peak from the He$^3$(d,p)He$^4$ reaction as a calibration point. The cross section for this reaction is sufficiently high that the small amount of HD$^+$ in the He$^{3+}$ beam, present in the ion source because of previous running with a deuteron beam, produced a significant proton peak for calibration purposes. This reaction produces protons of energy 14.7 MeV. The spectrum was taken with the front face of the solid state counter mounted at an angle of 60° to the direction of the protons being detected. This doubled the effective depletion thickness of the counter and therefore it had a linear response up to about 16 MeV.

Table 3 lists the reactions contributing to the spectrum and the proton energies expected from each reaction. The Q-values for the reactions have been taken from Lauritsen and Ajzenberg-Selove (1962). The proton energies at 90° to the beam direction corrected for the incident
FIGURE 5-1. Proton Spectrum from the He$^3(He^3,2p)He^4$ Reaction
TABLE 3

Reactions Contributing to Proton Spectrum

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$Q$ MeV</th>
<th>$E_p$(C.M.) MeV</th>
<th>$E_p$(LAB) MeV</th>
<th>$E_p$(MEAS) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>He$^3$(d,p)He$^4$</td>
<td>18.352</td>
<td>14.681</td>
<td>15.14</td>
<td>15.04</td>
</tr>
<tr>
<td>He$^3$(He$^3$,p)Li$^5$</td>
<td>10.892</td>
<td>9.077</td>
<td>9.46</td>
<td>9.30</td>
</tr>
<tr>
<td>He$^3$(He$^3$,2p)He$^4$</td>
<td>12.859</td>
<td>10.716</td>
<td>11.10</td>
<td>10.97</td>
</tr>
<tr>
<td>He$^3$(He$^2$,He$^2$)He$^4$</td>
<td>12.859</td>
<td>8.544</td>
<td>8.92</td>
<td>8.75</td>
</tr>
<tr>
<td>Li$^5$(p)He$^4$</td>
<td>1.967</td>
<td>1.573</td>
<td>3.87</td>
<td>3.57</td>
</tr>
</tbody>
</table>
beam energy are listed in the column $E_p^{(LAB)}$ and the laboratory energies corrected for the energy loss in the aluminum foil in front of the counter are listed in the column $E_p^{(MEAS)}$. The thickness of the aluminum foil used was 4.0 mg/cm$^2$. The proton energy listed for the three particle breakup $\text{He}_3(\text{He}_3^3,2p)\text{He}_4^4$ is the maximum expected energy. The spectrum for this instantaneous breakup would have an elliptical shape as shown by the dotted line in Figure 5-1. However the broad continuum found in the experimental spectrum is not necessarily due to this process. The protons from a sequential decay through either the 3 - 5 MeV wide first excited state of Li$^5$ or a broad state of the diproton would produce two wide overlapping peaks and therefore yield a similar continuum.

The high energy peak superimposed on this continuum lies at approximately 9.3 MeV and therefore corresponds to protons from the sequential decay through the ground state of Li$^5$. The lifetime of this state can be determined from the excess broadening of the proton peak (see Section 5-2). Immediately below this peak there are several hundred counts statistically above the continuum which are due to the two final state protons entering one counter simultaneously. This process would produce a peak at 8.75 MeV, the same energy as that expected from a bound diproton. Better statistics are necessary in order to separate this peak from the peak at 9.3 MeV.

The lowest proton energy analysed by the kicksorter was 1.5 MeV. This was determined by the thickness of the aluminum foil in front of the counter. The Q-value for the breakup of the Li$^5$ nucleus into a proton and an alpha particle is 1.967 MeV. The expected proton energy from this breakup would therefore be
\[ E_P = \frac{4}{3} (1.967) = 1.57 \text{ MeV} \]

However because of the recoil energy of the Li\(^5\) nucleus before breakup the protons can be emitted with energies up to \( E_{\text{pmax}} \) where \( E_{\text{pmax}} \) is the solution of the energy equation 4-7 for \( \Theta = 180^\circ \).

\[ E_P^2 - \frac{2}{3} E_P (E_{\text{Li5}} + 4Q) + \frac{1}{25} (E_{\text{Li5}} - 4Q)^2 = 0 \]

Substituting for \( E_{\text{Li5}} \) and \( Q \) and solving the quadratic equation gives

\( E_{\text{pmax}} = 3.87 \text{ MeV} \). This would correspond to a measured energy of \( 3.57 \text{ MeV} \) because of the aluminum foil. The rise in the number of counts at the low energy end of the spectrum is therefore due to the breakup protons from Li\(^5\).

5-2. **Lifetime of the Li\(^5\) Ground State**

The ground state of Li\(^5\) is unstable to breakup into a proton and an alpha particle. The lifetime of this state can be determined from the excess broadening of the proton peak at 9.3 MeV. The width of this peak is due to the following effects:

a. Statistical fluctuations in the incident He\(^3\) energy due to the entrance foil.

b. Statistical fluctuations in the proton energy due to energy straggling in the aluminum foil in front of the detector.

c. Geometry of the target-detector arrangement.

d. Resolution of the solid state detector and the amplification system.

e. Short lifetime of the Li\(^5\) ground state.
The magnitude of effects \( a, b, c, d \) will determine how much of the broadening arises from the lifetime of the \( \text{Li}^5 \) ground state. These effects will now be calculated. The energy loss of the incident beam in the entrance foil was 450 keV. For a \( \text{He}^3 \) beam of energy 1.5 MeV the energy straggling is approximately 3% (Marion, 1960). As the \( \text{He}^3 \) incident energy has a 4% effect on the proton energy the contribution to the width due to the nickel foil is

\[
\Delta E_{\text{Ni foil}} = 450 \times 0.03 \times 0.04 \sim 0.5 \text{ keV}
\]

The energy loss of the protons in the aluminum foil was 160 keV. Assuming an energy straggling of 5% (Sternheimer, 1960), the contribution to the width due to this effect is

\[
\Delta E_{\text{Al foil}} = 160 \times 0.05 \sim 8 \text{ keV}
\]

The geometrical width is due to the finite size of the active target volume as determined by the width of the cylindrical slit and the area of the detector. This width arises because the contribution to the measured proton energy due to the incident beam energy is angularly dependent. The proton energy \( E_3 \) at an angle \( \Theta \) to the direction of the incident beam of energy \( E_1 \) is given by the expression

\[
E_3(\Theta) = E_3(\Theta=0) + \frac{2(M_3 M_4)^{\frac{1}{2}}}{M_3 + M_4} \left( E_1 E_3(\Theta) \right)^{\frac{1}{2}} \cos \Theta
\]

where

\[
E_3(\Theta=0) = Q \frac{M_4}{M_3 + M_4} + E_1 \left( \frac{M_4-M_1}{M_3 + M_4} \right)
\]

\( Q \) is the corresponding \( Q \)-value.
\[ E_3(\Theta) \approx E_3(90^\circ) + \frac{2(M_3M_5)}{M_3 + M_4} \left( E_1E_3(90^\circ) \right)^{1/2} \cos \Theta \]

The detector - slit geometry is shown in the following diagram.

The geometrical factor for this arrangement can be approximated by

\[ G = \frac{\pi a^2 b}{R(R-r)} \]

and the angle \( \Theta \) is given by

\[ \Theta = \left( \frac{R-r}{\tan^{-1} \left( \frac{R-r}{x+y} \right)} \right) \]

The probability of a proton being detected at an angle \( \Theta \) to the direction of the incident beam relative to the probability of a proton being detected at an angle of \( \Theta = 90^\circ \) is very difficult to determine as the protons are not produced by a point source. An approximate calculation yields the values of 86° and 94° for the half-width angles. The proton
energy at 90° for the $\text{He}^3(\text{He}^3, \text{p})\text{Li}^5$ reaction is

$$E_3(\phi = 90^\circ) = \frac{8}{6} (10.892) + \frac{2}{6} (1.15) = 9.46 \text{ MeV}$$

and the proton width as determined from the half-width angles and expression 5-2 is approximately

$$\Delta E_{\text{Geom}} = \frac{4(3)^{1/2}}{6} (1.15 \times 9.46)^{1/2} \cos (86^\circ) \approx 270 \text{ keV}$$

The measured resolution of the solid state detector and the amplification system is approximately

$$\Delta E_{\text{Res.}} \approx 50 \text{ keV}$$

The total width of the proton peak due to these combined effects is therefore the square root of the sum of the squares of the statistical effects plus the geometrical effect

$$\Delta E_{\text{Total}} = 270 + \sqrt{50^2 + 8^2 + 0.5^2} \approx 321 \text{ keV}$$

This inevitable width resulting from geometry and straggling can also be determined from the broadening of the proton peak from the $\text{He}^3(d,\text{p})\text{He}^4$ reaction. As this reaction proceeds through the ground state of He$^4$ the width due to nuclear lifetimes is negligible. From Figure 5-1 the width of this proton peak is of the order of 3 channels or 450 keV. However the geometrical contribution to the width must be corrected for the higher proton energy. This results in a total width for protons of energy 9.46 MeV.
Thus the two methods of determining the width due to the geometrical and statistical effects in the detection system give approximately the same result. In order to determine the total width of the proton peak at 9.3 MeV in Figure 5-1 due to the $^{3}\text{He}(^{3}\text{He}, p)^{5}\text{Li}$ reaction, the contribution to this peak from the other possible processes must be evaluated. Protons in this energy range could result from either the instantaneous breakup or from the other possible sequential decays, the maximum contribution resulting if the instantaneous breakup predominate. Assuming this is the case the width of the proton peak would be 5 channels or 720 keV. If it is assumed that all the protons in the region of 9.3 MeV are from the transition to the ground state of Li$^{5}$ the width of the peak would be 8 channels or 1150 keV. By averaging these two extremes and by correcting the resultant width for the contributions from the geometrical and statistical effects, the width of the ground state of Li$^{5}$ was found to be

$$\Gamma_{\text{Li}^5_{g.s.}} = 660 (1 \pm .3) \text{ keV}$$

This corresponds to a mean lifetime of

$$\tau \approx \frac{1}{\Gamma} \approx \frac{6.58 \times 10^{-22}}{660} = (1.0 \pm .3) \times 10^{-21} \text{ sec}.$$  

5.3. Angular Distribution of Coincidence Events

The results of the measurement of the number of coincidence events between the two protons as a function of the angle
between the two detectors are listed in Tables 4 and 5. The first run was made using an angular width of the detectors of 11.5° and the second run which was made to measure the angular distribution at lower angles used an angular width of 4°, allowing the distribution to be observed for angles down to 8°.

The total number of proton counts in both counters was used to normalize the number of coincidence events observed at each angle. The ratio of the normalized number of coincidence events to the number at an angle of 180° is listed in the last column for each angle. For the second run the results were normalized to the first run results at an angle of 60°.

The distribution of coincidence events is shown in Figure 5-2 as a function of the angle between the detectors. The circles correspond to the measurements taken in the first run and the triangles to the measurements taken in the second run. The statistics on the points indicated by the circles are better than 10% and on the points indicated by the triangles, better than 15%. These statistics were obtained with approximately 20 minute runs per angle in the first case and one hour runs per angle in the second case.

A comparison of this experimental distribution to the predicted distributions (Chapter 4, Figures 4-1, 4-3) indicates that the coincidence events in the region of 180° are mostly due to the sequential decay through the ground state of Li⁵. The fact that the experimental distribution rises considerably more sharply at 180° than the phase space calculations predict, indicates the importance of final state interactions in determining the angular distribution of the reaction products. If the effect of the coulomb barrier on the energy distribution of the outgoing particles is considered (see Appendix A) both the angular distribution for the instantaneous
**TABLE 4**

**RUN #1**

<table>
<thead>
<tr>
<th>Angle between counters</th>
<th>Number of coincidence events</th>
<th>Number of protons counter #1</th>
<th>Number of protons counter #2</th>
<th>( \frac{N_{\text{coinc}}(\Theta)}{N_{\text{coinc}}(180)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>206</td>
<td>23930</td>
<td>25067</td>
<td>1.000</td>
</tr>
<tr>
<td>170</td>
<td>417</td>
<td>70463</td>
<td>75551</td>
<td>0.700</td>
</tr>
<tr>
<td>160</td>
<td>259</td>
<td>82406</td>
<td>88111</td>
<td>0.357</td>
</tr>
<tr>
<td>150</td>
<td>112</td>
<td>64532</td>
<td>67823</td>
<td>0.199</td>
</tr>
<tr>
<td>140</td>
<td>125</td>
<td>90288</td>
<td>95181</td>
<td>0.159</td>
</tr>
<tr>
<td>130</td>
<td>93</td>
<td>103165</td>
<td>109455</td>
<td>0.103</td>
</tr>
<tr>
<td>120</td>
<td>200</td>
<td>224680</td>
<td>236495</td>
<td>0.102</td>
</tr>
<tr>
<td>110</td>
<td>80</td>
<td>94978</td>
<td>100079</td>
<td>0.096</td>
</tr>
<tr>
<td>100</td>
<td>104</td>
<td>90688</td>
<td>92275</td>
<td>0.134</td>
</tr>
<tr>
<td>90</td>
<td>119</td>
<td>100100</td>
<td>104805</td>
<td>0.137</td>
</tr>
<tr>
<td>80</td>
<td>132</td>
<td>100922</td>
<td>105699</td>
<td>0.150</td>
</tr>
<tr>
<td>70</td>
<td>200</td>
<td>159532</td>
<td>167731</td>
<td>0.144</td>
</tr>
<tr>
<td>60</td>
<td>126</td>
<td>100484</td>
<td>105489</td>
<td>0.144</td>
</tr>
<tr>
<td>50</td>
<td>105</td>
<td>79592</td>
<td>80880</td>
<td>0.153</td>
</tr>
<tr>
<td>40</td>
<td>113</td>
<td>86811</td>
<td>90676</td>
<td>0.150</td>
</tr>
<tr>
<td>30</td>
<td>79</td>
<td>62748</td>
<td>64277</td>
<td>0.146</td>
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<td>27</td>
<td>213</td>
<td>155918</td>
<td>163440</td>
<td>0.157</td>
</tr>
<tr>
<td>25</td>
<td>71</td>
<td>49504</td>
<td>51249</td>
<td>0.166</td>
</tr>
<tr>
<td>23</td>
<td>210</td>
<td>135098</td>
<td>142475</td>
<td>0.178</td>
</tr>
</tbody>
</table>

**Beam energy** 1.60 MeV

**Beam current** .25 a.

**Target gas pressure** 75 mm Hg.

**Angular width of detectors** 11.5°.
TABLE 5

Run #2

<table>
<thead>
<tr>
<th>Angle between counters</th>
<th>Number of coincidence events</th>
<th>Number of protons counter #1</th>
<th>Number of protons counter #2</th>
<th>$N_{\text{coinc}}(\Theta) / N_{\text{coinc}}(180)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>54</td>
<td>96480</td>
<td>70839</td>
<td>.150</td>
</tr>
<tr>
<td>45</td>
<td>54</td>
<td>110962</td>
<td>82722</td>
<td>.130</td>
</tr>
<tr>
<td>40</td>
<td>44</td>
<td>80978</td>
<td>62271</td>
<td>.140</td>
</tr>
<tr>
<td>35</td>
<td>81</td>
<td>125010</td>
<td>92600</td>
<td>.173</td>
</tr>
<tr>
<td>30</td>
<td>71</td>
<td>111672</td>
<td>94876</td>
<td>.160</td>
</tr>
<tr>
<td>25</td>
<td>72</td>
<td>111602</td>
<td>84617</td>
<td>.171</td>
</tr>
<tr>
<td>20</td>
<td>91</td>
<td>142198</td>
<td>116037</td>
<td>.164</td>
</tr>
<tr>
<td>15</td>
<td>97</td>
<td>127469</td>
<td>93139</td>
<td>.162</td>
</tr>
<tr>
<td>10</td>
<td>61</td>
<td>120490</td>
<td>89473</td>
<td>.135</td>
</tr>
<tr>
<td>8</td>
<td>95</td>
<td>170088</td>
<td>123616</td>
<td>.150</td>
</tr>
</tbody>
</table>
FIGURE 5-2. Experimental Angular Distribution of Coincidence Events
breakup and for the sequential decay through the states of Li\(^5\) would rise more sharply at higher angles. It is extremely difficult to determine this effect exactly but it does not seem likely that its magnitude would be sufficient to correct the phase space curves to fit the experimental rise at 180°. This would mean that the p - \(\infty\) final state interaction causes the protons of higher energies to be emitted even more preferentially from the Li\(^5\) system than phase space theory with coulomb interactions predicts.

The sequential decay through the ground state of Li\(^5\) cannot contribute to the coincidence events observed at angles below 75° as kinematics restricts the second proton in the decay to energies less than 1.5 MeV. This can be determined by substituting \(E_p = 1.5\) MeV in the energy equation 4-7 which can be written

\[
E_p^2 + \frac{2}{5}E_p \left( E_{Li}^5 \left(1 - 2\cos^2\Theta\right) - 4Q\right) + \frac{1}{25}(E_{Li}^5 - 4Q)^2 = 0
\]

and solving for \(\Theta\).

However kinematics does not rule out the possibility that the events at small angles could be due to either the instantaneous breakup or the sequential decay through the excited state of Li\(^5\). The predicted angular distribution for both these processes are similar in shape. The maximum possible contribution from the instantaneous breakup is shown by the dotted line in Figure 5-2. It is assumed that all the coincidence events at the minimum in the experimental distribution are due to this process.

Since the predicted distributions for both processes mentioned above decrease monotonically as the angle decreases, it is evident that they cannot produce the observed rise in the distribution as the angle...
decreases below 90°. The modifications to these distributions to account for
coulomb interactions result in the distributions decreasing even more at lower
angles. Also it was noted previously that the effect of a final state \( p - \alpha \)
interaction must be to cause the protons to be emitted even more preferentially
in opposite directions. Therefore this increase in the number of coincidence
events with decreasing angle between the protons must be due to either a
sequential decay through a broad diproton state or, in other terms, a strong
final state \( p - p \) interaction which causes the two protons from the single
stage breakup to be emitted preferentially together.

By comparing the experimental distribution to the
predicted distributions for the various values of the binding energy of the
diproton (Figure 4-1b.), it is possible to determine the lifetime and the
binding energy of the diproton system. The distribution rises to a maximum
at 25° - 30° and drops off at lower angles. Assuming that this maximum
corresponds to the maximum predicted by the phase space curves, the diproton
state must be unbound by approximately 600 keV. The rise in the distribution
begins at 110° indicating that the diproton state must be very wide. The
width necessary to fit the general shape of the rise is

\[ \Gamma \approx 4 - 5 \text{ MeV} \]

corresponding to a mean lifetime of

\[ \tau \approx \frac{6.6 \times 10^{-22}}{4.5} \approx 1.5 \times 10^{-22} \text{ sec}. \]
Conclusions

The technique of measuring the angular distribution of coincidence events between two of the particles in a three particle final state has proved partially successful in identifying the various processes which take place in the $\text{He}^3(\text{He}^3,2p)\text{He}^4$ reaction. The reaction has been found to proceed predominantly by sequential processes through unbound intermediate states. This is in agreement with recent work by Beckner et al (1961) and Moazed et al (1964) who found that the dominant mechanism in the multiparticle breakup reactions $\text{Be}^9(p,d)2\text{He}^4$ and $\text{Be}^9(\text{He}^3,\alpha)2\text{He}^4$ is the sequential decay through the states of $\text{Be}^8$.

The intermediate state which is most evident from the experimental data is the ground state of $\text{Li}^5$. This state was found to have a width of $\Gamma = 660 (1.0 \pm .3) \text{ keV}$ corresponding to a mean lifetime of $\tau = (1.0 \pm .3) \times 10^{-21} \text{ sec}$. This width is significantly less than the width of $\Gamma = 1.5 \text{ MeV}$ quoted in Ajzenberg-Selove and Lauritsen (1959). This figure is an average of several values found previously for the width of this state (Likely, 1955 and Frost and Hanna, 1958). However none of these results were obtained using detectors of resolution comparable to that of a solid state detector and the widths quoted were not corrected for broadening from effects other than the short lifetime of the $\text{Li}^5$ ground state. The smaller value of the width is in better agreement with the width of the ground state of the mirror nucleus $\text{He}^5$ which is $\Gamma = 550 \text{ keV}$. It would be expected that the extra proton in $\text{Li}^5$ would increase the width of this state slightly.

There is also good evidence that a sequential decay through
a broad diproton state takes place. This state was found to be unbound by about 600 keV. The singlet state scattering length for the p – p interaction is \( a_s = -7.7 \times 10^{-13} \text{ cm} \). This would correspond to a value of

\[
\frac{\hbar^2}{m_p a_s^2} = 700 \text{ keV}
\]

for the energy of the "virtual" singlet state of the diproton as defined for proton-proton scattering. The negative sign of the scattering length indicates that the state is unbound. Therefore the experimental result of 600 keV for the binding energy of the diproton state formed in the He\(^3\)(He\(^3\),2p)He\(^4\) reaction is in good agreement with the scattering length for the p – p interaction. As the lifetime of the diproton system necessary to fit the experimental data is approximately \( \tau = 1.3 \times 10^{-21} \text{ sec} \), or about 10 nuclear transit times it might be considered questionable to speak of this system as a state. Because of this short lifetime the diproton system lies in a rather nebulous region between a "virtual" state and a strong final state interaction. It might prove useful to try to fit the shape of the angular distribution at low angles by assuming some form for the proton-proton interaction based on scattering theory and using this to modify the phase space predictions for the three particle breakup. Gonzett, Shield, Slobodrian and Yamabe (1964) found that they could adequately fit the shape of the triton spectrum from the He\(^3\)(d,t)2p reaction by using a 2p final state interaction. The choice of the method of considering the diproton system, either as a virtual state or a final state interaction depends mainly on which method yields the most useful information about the nuclear force involved.

The other processes which can take place in the He\(^3\)(He\(^3\),2p)He\(^4\)
reaction are the instantaneous three particle breakup and the sequential decay through the first excited state of Li$^5$. As the contribution to the angular distribution from both these processes is very similar it is not possible at present to determine whether these processes both occur to some extent in the reaction or whether one or the other is not present.

Two extensions to the experimental technique are possible in order to determine the reaction mechanism more completely. They should also give further proof that the sequential decay through the diproton state does occur and the relative probabilities with which the different processes take place. One method involves the use of a two dimensional kicksorter to analyse the energies of both protons taking part in the coincidence event for various angles between the detectors. This would give a two dimensional energy spectrum of $E_{P_1}$ as a function of $E_{P_2}$. The instantaneous breakup would appear as a curve in this spectrum and the sequential processes would appear as segments of this curve. The two dimensional spectra predicted for counters placed 180$^\circ$ apart and 30$^\circ$ apart are shown in Figure 6-1 for the various processes which take place. The size of the points for the sequential decays is proportional to the number of counts expected in the corresponding channel. For the instantaneous breakup a uniform distribution of counts along the kinematical curves is expected.

The other method involves the use of a conventional kicksorter to determine the energy spectrum of one of the coincident protons for various angles between the detectors. At an angle of 180$^\circ$ where the sequential decay through the ground state of Li$^5$ predominates, two proton groups would be expected, one corresponding to the proton from the transition to the Li$^5$ and the other to the proton from the breakup of Li$^5$. Similarly
Points correspond to coincidence events from:

- Sequential decay through ground state of Li$^5$
- Sequential decay through 1st excited state of Li$^5$
- Sequential decay through di-proton state
- Instantaneous three-body breakup.

**FIGURE 6-1.** Predicted Two Dimensional Energy Spectra for the He$^3$(He$^3$,2p)He$^4$ Reaction
at an angle of $25^\circ - 30^\circ$ the two wide proton groups from the decay through the
diproton state would be expected to overlap. The processes occurring at the
minimum in the distribution at $120^\circ$ would be determined from the energy
spectrum at this angle between the detectors.

These experiments should be carried out for several reasons. A knowledge of the reaction mechanism in these multiparticle breakup reactions
is very useful in our understanding of the forces between nucleons and the
formulation of a nuclear reaction theory. The fundamental nature of the
proton-proton interaction certainly makes it worthwhile to learn more about
the virtual diproton system. Moreover it should be possible to relate this
system directly to the appropriate $p - p$ phase shifts and hence determine
independently a value of the $p - p$ singlet state scattering length.
APPENDIX A

Phase Space Calculations

A-1. Introduction

In the study of strong interactions of elementary particles a great variety of different final state interactions have been detected by the observation that relative yields, momentum and effective mass distributions of the outgoing particles deviate from the expectations from phase space arguments. Similarly in low energy physics the existence of short lived resonances or of final state interactions can be determined by comparing the experimental energy and angular distributions with those predicted using phase space considerations. This method is especially useful in reactions leading to more than two final products and can be applied to determining the mechanism of these reactions.

The general phase space formula have been determined for relativistic particles by Block (1956), Williams (1961), and Skjeggested (1964). The purpose of this appendix will be to derive the useful non-relativistic expressions.

A-2. General Phase Space Formula

The concept of "phase space" is closely connected to the calculation of transition rates. From perturbation theory the general formula for the transition rate, or the probability that a transition from an initial state to a final state will occur in unit time, is given by

$$ \omega = \frac{2\pi}{\hbar} \left| \langle \psi_f | H' | \psi_i \rangle \right|^2 \rho(\mathcal{E}) \quad A-1. $$
where \( \langle \psi_f | H' | \psi_i \rangle \) is the matrix element for the transition caused by a perturbation of the Hamiltonian \( H' \); \( \mathcal{C}(E) \) is a function of the total energy of the system and the masses of the individual particles in the final state and is usually called the "density of states factor" or phase space factor.

As the matrix element is in general completely unknown, it is necessary to make certain assumptions about its behaviour in order to calculate the transition rate. The simplest assumption is that the matrix element is a constant, independent of the individual momenta of the particles in the final state. In this case the transition rates as well as the individual momenta in the final state are determined by the phase space factor alone.

For one particle a definite state of motion, i.e. specified position \((x, y, z)\) and momentum \((p_x, p_y, p_z)\), can be represented as a point in a 6-dimensional phase space. Classical mechanics places no restrictions on the density of the representation points. However quantum mechanics, through the Uncertainty Principle, requires that the representation points be separated by finite distances, each point being confined to an elementary cell of size \((2\pi\hbar)^3\). The number of final states \( N_1 \) available to one particle will therefore be finite and equal to the total volume of phase space divided by the size of an elementary cell.

\[
N_1 = \frac{1}{(2\pi\hbar)^3} \int d^3x \, d^3p
\]

The number of final states for a system of \( n \) particles will be the product of the number of final states for each particle,
This formula applies only for a system of \( n \) spinless particles. If the particle \( i \) has spin \( S_i \) the above expression should be multiplied by

\[
\prod_{i=1}^{n} (2S_i + 1)
\]

However, since these spin factors can be included in the final normalization of the phase space integral these factors will be neglected here.

Since all the particles are confined to the same geometrical volume \( V \), equation \( A-3 \) can be written

\[
N_n = \left[ \frac{V}{(2\pi \hbar)^3} \right]^n \int \prod_{i=1}^{n} d^3 p_i
\]

Given the total number of states the density in phase space is defined as the number of states per unit energy interval.

\[
p_n(E) = \frac{dN_n}{dE} = \frac{d}{dE} \int \prod_{i=1}^{n} d^3 p_i
\]

The constant factor \( \left[ \frac{V}{(2\pi \hbar)^3} \right]^n \) will be dropped to simplify handling of the expressions. In order to conserve total momentum \( \vec{p}_T \), the \( n \) particle momenta cannot be independent but are constrained by the equation

\[
\sum_{i=1}^{n} \vec{p}_i - \vec{p}_T = 0
\]
It is usual to indicate this restriction by putting

$$\rho_n(E) = \frac{d}{dE} \int \prod_{i=1}^{n-1} d^3p_i$$ \hspace{1cm} \text{(A-6)}$$

The calculation of the density of states factor is impossible for all but the simplest systems. The non-relativistic calculations for \( n = 2, 3 \) are of use in low energy nuclear physics.

A-3. Two Particle Final State

The energy and momentum relations for a two particle final state are given by

$$E = E_1 + E_2$$

$$\vec{P}_f = \vec{P}_1 + \vec{P}_2$$

Using equation (A-6) for \( n = 2 \)

$$\rho_2(E) = \frac{d}{dE} \int d^3p_1$$ \hspace{1cm} \text{(A-7)}$$

In polar coordinates this becomes

$$\rho_2(E) = \frac{d}{dE} \int p_1^2 dp_1 \int d\Omega_1$$ \hspace{1cm} \text{(A-8)}$$

Since

$$\frac{d}{dE} = \frac{d}{dp_1} \frac{dp_1}{dE} = \frac{1}{dE_1 + dE_2} \frac{d}{dp_1}$$

and

$$\frac{dp_2}{dp_1} = -\frac{\vec{P}_1 \cdot \vec{P}_2}{p_1 p_2}$$
the expression A-8 can be written

$$R_2(E) = \frac{P^2}{m_1 + \frac{P_2}{m_2}(-\frac{P_1 \cdot P_2}{P_1 P_2})} \int d\Omega_1$$

or

$$\frac{d}{d\Omega_1} R_2(E) = \frac{m_1 m_2 P_1^3}{m_2 P_1^2 - m_1 P_1 \cdot P_2}$$ \hspace{1cm} A-9

In terms of the total momentum $\vec{P}_T$ this becomes

$$\frac{d}{d\Omega_1} R_2(E) = \frac{m_1 m_2 P_1^3}{(m_1 + m_2) P_T^2 - m_1 \vec{P}_T \cdot \vec{P}_T}$$ \hspace{1cm} A-10

For the case of $\vec{P}_T = 0$ the angular distribution is isotropic as would be expected for a two particle final state.

A-4. Three Particle Final State

The energy and momentum relations extended to a three particle final state are given by

$$E^- = E_1 + E_2 + E_3$$

$$\vec{P}^- = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

Using equation A-6 for $n = 3$

$$R_3(E) = \frac{d}{dE} \int d\Omega_1 \int P_1^2 dP_1 \int d\Omega_2 \int P_2^2 dP_2$$ \hspace{1cm} A-11

The overall phase space factor is determined by evaluating all
the integrals in expression A-11. The integration proceeds from the right with momentum \( \vec{p}_1 \) held fixed during the integration over \( \vec{p}_2 \). However for low energy nuclear reactions the differential phase space factors with respect to energy or angle of one of the final state particles are more useful as they play a considerable role in determining the energy and angular distributions of the reaction products. In the evaluation of these differential phase space factors the appropriate integrations are omitted.

The phase space factor which determines the probability of particle 1 being emitted into solid angle \( d\Omega_1 \) and momentum interval \( dp_1 \) and particle 2 into solid angle \( d\Omega_2 \) is given by

\[
\frac{d^3}{d\Omega_1 d\Omega_2 dP_1} P_3(E) = \frac{d}{dE} \frac{P_1^2}{P_0} \int P_2^2 dp_2 \tag{A-12}
\]

Since

\[
\frac{d}{dE} = \frac{d}{dP_2} \frac{dP_2}{dE} = \frac{1}{\frac{dE_1}{dP_2} + \frac{dE_2}{dP_2} + \frac{dE_3}{dP_3}} \frac{d}{dP_2}
\]

and \( E_1 \) is fixed so that

\[
\frac{d}{dE} = \frac{1}{\frac{P_1}{m_2} - \frac{P_1 \cdot P_2}{m_3 P_2}} \frac{d}{dP_2}
\]

\[
\frac{d^3}{d\Omega_1 d\Omega_2 dP_1} P_3(E) = \frac{m_2 m_3 P_1^2 P_2^3}{(m_2+m_3)P_2^2 - m_2 P_2 
\cdot (\vec{P}_1 - \vec{P}_2)} \tag{A-13}
\]

For the case of zero initial momentum this simplifies to

\[
\frac{d^3}{d\Omega_1 d\Omega_2 dP_1} P_3(E) = \frac{4\pi m_2 m_3 P_1^2 P_2^2}{(m_2+m_3)P_2 + m_2 P_1 \cos \theta} \tag{A-14}
\]
where $\Theta$ is the angle between $\vec{p}_1$ and $\vec{p}_2$.

The probability of a certain energy division among the reaction products can be found from equation A-14 by transforming from momentum intervals to energy intervals. Upon making the following substitutions:

$$d\Omega = 2\pi \, d(\cos \Theta)$$

$$P_1 P_2 \, d(\cos \Theta) = \frac{-1}{m_z} \left[ P_z(m_z + m_3) + m_2 P \, \cos \Theta \right] \, dp_2$$

$$dE_1 = \frac{P_1}{m_1} \, dp_1 \quad \text{and} \quad dE_2 = \frac{P_2}{m_2} \, dp_2$$

equation A-14 becomes

$$\frac{d^2}{dE_1 dE_2} \rho_3(E) = 8 \pi^2 m_1 m_2 m_3$$

The energy distribution of particle 1 can be found by integrating this expression over $E_2$.

$$\frac{d}{dE_1} \rho_3(E) = 8 \pi^2 m_1 m_2 m_3 \left[ E_2^{\text{max}} - E_2^{\text{min}} \right]$$

From the energy and momentum relations

$$aE_1 + bE_2 + c(E_1E_2)^{\frac{1}{2}} = E$$

where

$$a = \frac{m_1 + m_2}{m_3}$$

$$b = \frac{m_2 + m_3}{m_3}$$

$$c = \frac{2}{m_3} \sqrt{m_1 m_2} \cos \Theta$$
Squaring the above expression gives

\[ b^2 E_2^2 - E_2 (c^2 E_1 + 2a (E - a E_1)) + (E - a E_1)^2 \]

The difference \( E_2^{\text{max}} - E_2^{\text{min}} \) is the difference between the solutions of this quadratic equation at \( \cos \Theta = -1 \) and \( \cos \Theta = +1 \).

\[ E_2^{\text{max}} - E_2^{\text{min}} = \frac{c}{b^2} (E_1)^{\frac{1}{2}} (E - E_1 \left( \frac{4ab - c^2}{4b} \right) \right)^{\frac{1}{2}} \]

Substituting for \( a, b, \) and \( c \) gives

\[ \frac{d}{dE_1} \rho_3(E) = \frac{4\pi (m_1 m_2 m_3)^{\frac{1}{2}}}{(m_2 + m_3)^{\frac{1}{2}}} \left( E_1 \right)^{\frac{1}{2}} \left( E - \left( \frac{m_1 + m_2 + m_3}{m_2 + m_3} \right) E_1 \right)^{\frac{1}{2}} \]

The energy distribution is therefore of the form

\[ f(E_1) dE_1 = N \left( E_1 \right)^{\frac{1}{2}} \left( E - \left( \frac{m_1 + m_2 + m_3}{m_2 + m_3} \right) E_1 \right)^{\frac{1}{2}} \]

This distribution is shown in Figure A-1. A more accurate determination of the single particle spectrum for a three particle breakup requires the addition of the final state interactions. The energy distribution can be approximately corrected for the coulomb barrier by adding the appropriate coulomb penetrabilities to the distribution predicted by phase space calculations. This modified distribution would be of the form

\[ \lambda_n = N \int f(E_n) P_1^{(1)} P_2^{(2)} P_3^{(3)} \]

where \( P_1^{(1)} \) is the penetrability for particle 1 of energy \( E_1 \).
DISTRIBUTION PREDICTED FROM PHASE SPACE CALCULATIONS

MODIFIED DISTRIBUTION TO ACCOUNT FOR COULOMB BARRIER

FIGURE A-1. Single Particle Spectrum from a 3-Body Breakup

\[ f(E_1) = \delta \]

\[ E = \frac{m_2 + m_3}{m_1 + m_2 + m_3} \]

ENERGY OF PARTICLE 1
The effect of this modification is to cause the distribution to be peaked at higher energies as shown in Figure A-1. It is also possible to qualitatively predict the effect that the coulomb interaction will have on the angular distributions of the final state particles. Since the probability of a particle penetrating the coulomb barrier increases with energy it is obvious that final state momentum configurations in which the particles have the highest possible energies will be preferred. Likewise final state configurations in which one of the particles has a very low energy would not be favoured. This is very similar to the effect that allowable phase space has on the distribution of the final state momentum configurations. Therefore it would be expected that any peaks in the angular distributions predicted by phase space arguments would be accentuated by the effect of the coulomb interaction.
APPENDIX B

Computer Programs for the Phase Space Calculations

The integration of the phase space functions over the solid angle subtended by the counters has to be done numerically because of the complexity of the functions involved. The most common method of numerical integration using a computer is the Simpson's Rule integration. A subroutine was written for the IBM 7040 computer for performing the necessary integrations. This subroutine, called by SIMP(A,B,RES), evaluated the integral of a function FUN(U) over the limits A to B and stored the result in RES. The function FUN(U) was evaluated in a second subroutine. A listing of the Simpson's Rule integration subroutine is given below:

```
SUBROUTINE SIMP(A,B,RES)
    N = 1
    FODD = 0.0
    FEVEN = FUN((A + B)/2.0)
    FAB = FUN(A) + FUN(B)
11
    N = 2 * N
    EN = N
    H = (B - A)/(2.0 * EN)
    FEVEN = FEVEN + FODD
    FODD = 0.0
    DO 10 I = 1, N
         DI = 2 * I - 1
    10    FODD = FODD + FUN(A + DI * H)
```
IF (N = 8) 11,11,8

8  RES = (H/3.0) * (FAB + 4.0 * FODD + 2.0 * FEVEN)

9  RETURN

END

For each of the various possible reaction processes a main program to read in the necessary data, evaluate the limits of integration and print the results, had to be written as well as the subroutine to evaluate the phase space function. For the sequential decays the input data consisted of the masses of the four particles present in the reaction denoted by AM1, AM2, AM3, AM4 and the Q-values for the sequential decays denoted by Q1 and Q2. For the instantaneous breakup only one parameter AA was necessary in the function subroutine corresponding to the variable 'a' in Section 4-2.

The main program for evaluating the angular distribution for the sequential decay through the states of Li\(^5\) is listed below. The programs for the other processes are similar except for the input data and the function subroutine. The subroutines FUN(U) for evaluating the phase space functions corresponding to each process are also listed. The output is printed in three columns ANGLE, INT. RES, and RATIO. In the last column the ratio of the integration result corresponding to the angle given in the first column divided by the result at $\Theta = 0^\circ$ is listed.

C ANGULAR DISTRIBUTION FOR SEQUENTIAL DECAY THROUGH Li\(^5\)

PRINT 1

READ 3, AM1, AM2, AM3, AM4, Q1, Q2

COMMON AM1, AM2, AM3, AM4, Q1, Q2, E1, E2
DTHETA = 0.20

DTHETA IS THE ANGULAR WIDTH OF COUNTER IN RADIANS

E1 = Q1*AM2/(AM1 + AM2)
E2 = Q2*AM1/(AM1 + AM2)

THETA = 0.0

A = COS(DTHETA)
B = 1.00

CALL SIMP(A, B, RES)

PZERO = RES * 4.0

PTHETA = PZERO

GO TO 15

10 IF (THETA = 180.0) 13, 12, 16

12 A = -1.00
B = -COS(DTHETA)

CALL SIMP(A, B, RES)

PTHETA = RES * 4.0

GO TO 15

13 TRAD = THETA * 0.017453

A = COS(TRAD + DTHETA)
B = COS(TRAD - DTHETA)

CALL SIMP(A, B, RES)

ST = SIN(TRAD)

DFI = 2.0 * ATAN(DTHETA/(2.0 * ST))

PTHETA = RES * DFI

15 RAT = PTHETA/PZERO

PRINT 2, THETA, RES, RAT
THETA = THETA + 10.0
GO TO 10
STOP

16

FORMAT (30H ANGLE INT. RES RATIO /
1
FORMAT (Fx, F10.0, 2F10.4)
2
FORMAT (4F5.0, 2F10.0)
END

FUNCTION FUN(U)
C
PHASE SPACE FUNCTION EVALUATION FOR LI5 DECAY
COMMON AM1, AM2, AM3, AM4, Q1, Q2, E1, E2
C = (AM3 * E2 / AM2 - AM4 * Q2 / AM2) * * 2
B = E2 * AM3 / AM2 * (1.0 - 2.0 * U * * 2) - AM4 / AM2 * Q2
IF (U) 5, 5, 6
5
E3 = - SQRT(B * * 2 - C) - B
GO TO 7
6
E3 = SQRT(B * * 2 - C) - B
7
FUN = E3 / (SQRT(E3 / AM3) - SQRT(E2 / AM2) * U)
RETURN
END

FUNCTION FUN(U)
C
PHASE SPACE FUNCTION EVALUATION FOR DIPROTON DECAY
COMMON AM1, AM2, AM3, AM4, Q1, Q2, E1, E2
SUM = E2 + Q2
DIFF = E2 - Q2
IF (U) 5, 5, 4
IF (SUM - DIFF/U) 5, 6, 6
FUN = 0.0
RETURN
E3 = 0.5 * (SUM + SQRT(SUM^2 - (DIFF/U)^2))
E4 = SUM - E3
FUN = E3/(SQRT(E3) - SQRT(E4) * U)
RETURN
END

FUNCTION FUN(U)
C PHASE SPACE FUNCTION EVALUATION FOR 3 BODY BREAKUP
COMMON AA
UA = U * AA
UA2 = UA * UA
FACT = SQRT(4.0 - UA2)
FACT5 = FACT * 5
IF (UA) 5, 6, 7
UA = ABS(UA)
FUN = ((2.0 + UA2) * (3.14159 - ATAN(FACT/UA)) + 1.5 * UA * FACT)/ FACT5
GO TO 8
FUN = 3.14159/32
GO TO 8
FUN = ((2.0 + UA2) * ATAN(FACT/UA) - 1.5 * UA * FACT)/FACT5
RETURN
END
BIBLIOGRAPHY


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