A CALCULATION OF THE TRITON BINDING ENERGY USING SOFT-CORE POTENTIALS

by

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We accept this thesis as conforming to the required standard

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ABSTRACT

In this thesis we calculate the symmetric S-state contribution to the binding energy of the triton \( (H^3) \) using the nucleon-nucleon potentials recently developed by Wong (1965).

Towards this end we employ a modification of the equivalent two-body method used by Feshbach-Rubinow (1955).
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ACKNOWLEDGEMENTS

I am indebted to Dr. J. M. McMillan for suggesting the problem and for generous assistance with it.
CHAPTER I INTRODUCTION

In this thesis we shall calculate the binding energy of the triton \((H^3)\) using recently developed nucleon-nucleon potentials.

In focussing our attention on the triton, we are immediately faced with trying to solve a three-body problem. Major steps in this direction were taken by Derrick and Blatt (1958, 1960a, 1960b) who have constructed a complete set of states in terms of which the triton wave function may be expanded, and derived a set of sixteen partial differential Schrödinger equation in three variables for the expansion coefficients. That work, while providing a great simplification of the nuclear problem, still clearly leaves one with an extremely difficult mathematical problem. Indeed relatively little has been done in finding its solution. Instead, the common procedure has been to compute triton wave functions and energies using a variational parameter approach. That is, a form of the wave function containing various parameters has been assumed, and the expectation value of the Hamiltonian of the system has been computed and minimized with respect to these parameters. This approach has obvious disadvantages: there is
no guarantee that the assumed function approximates the actual wave function, and there is no direct correspondence between the wave function parameters and the parameters appearing in the nucleon-nucleon potential.

Some work on a non-variational parameter approach for finding a triton wave function and energy has been done, in particular, by Feshbach and Rubinow (1955) and by McMillan (1965). Feshbach and Rubinow assumed that the symmetric S-state component of the triton wave function (which component is known from the work of Blatt et al (1962) to be strongly dominant) is a function of the sum of internucleon distances only, and derived an ordinary differential equation based on this assumption. McMillan (1965) discussed some properties of the Feshbach-Rubinow equation and solved it numerically using potentials that fitted low energy data.

In this thesis we are interested in using the Feshbach-Rubinow method with the Wong (1965) "soft core" potentials which give a good fit to the two-nucleon data to 250 MeV and which have been used successfully in nuclear matter calculations. These potentials have a strong repulsive core but not a "hard core". We have found that when the Wong potentials are used directly in the Feshbach-Rubinow equation, no bound three-nucleon state is found.
The reason for this is that the Feshbach-Rubinow approximation makes no special concession to the form of the triton wave function for small interparticle distances and, as a result, is inadequate for potentials with strong repulsive cores. Indeed, the repulsive core in the two-nucleon potential requires that the triton wave function be small for small interparticle distances, and we must build this into our approximate form, in the same way that Feshbach and Rubinow have required that for hard core potentials the triton wave function vanish inside the core. In the remainder of this thesis, attempts will be made to show the applicability of the Wong potential to the triton problem.
II. 1 The Variational Principle

As stated in the Introduction, we shall in this theses use the Feshbach-Rubinow (1955) method for finding an approximate triton energy. The essential step in this method is the restatement of the more customary Schrodinger differential equations in terms of a variational principle. In this subsection we shall outline the steps leading to the variational principle used by Feshbach and Rubinow and by McMillan (1965).

The set of coupled differential equations for the components of the $J=1/2, T=1/2$ wave function of the bound three-nucleon system can be found from Derrick (1960). When all but the symmetric S-state component $f_1$ is neglected, the following equation results

\[ \sum_{\text{cyclic}} \left( \frac{\hbar^2}{m} \left( \frac{\partial^2}{\partial r_1^2} + \frac{2}{r_1 \partial r_1} + \frac{r_2^2 + r_3^2 - r_1^2}{2r_2 r_3} \frac{\partial^2}{\partial r_2 \partial r_3} \right) + V(r) \right) f_1 = E f_1 \quad (2.1) \]

where $E$ is the energy in the centre-of-mass system, the $r_1$ denote the internucleon distances, and where in terms of the usual nucleon-nucleon potentials

\[ V = \frac{1}{2} (V_{\text{singlet-even}} + V_{\text{central-triplet-even}}) \quad (2.2) \]
We use, with Derrick and Blatt (1958), the internucleon distances as generalised coordinates. Indeed, there are six generalised coordinates needed in the centre-of-mass system; Derrick and Blatt use the three \( r_1 \) which specify the size and the shape of the triangle formed by the three particles, and the three Euler angles which specify the orientation in space of this triangle, relative to some standard orientation.

The function \( f_1(r_1, r_2, r_3) \) is symmetrical under the interchange of any two nucleons; it will be taken to be normalised as follows

\[
\int_0^\infty \int_0^\infty \int_0^{r_1+r_2} \left( r_1 r_2 r_3 \left| f_1(r_1, r_2, r_3) \right|^2 \right) \, dr_1 \, dr_2 \, dr_3 = 1 \quad (2.3)
\]

The essential step now is to replace (2.1) by the following variational principle

\[
\delta \int_0^\infty \int_0^\infty \int_0^{r_1+r_2} r_1 r_2 r_3 \left[ \sum_{\text{cyclic}} \left\{ -\frac{\hbar^2}{2m} \frac{\partial^2 f_1}{\partial r_1^2} + \frac{r_2^2 + r_3^2 - r_1^2}{2r_2 r_3} \frac{\partial f_1}{\partial r_2} \frac{\partial f_1}{\partial r_3} \right\} - V(r_1)f_1^2 + Ef_1^2 \right] = 0 \quad (2.4)
\]

from which equation (2.1) can be obtained using the Euler-Lagrange equation.

Feshbach and Rubinow (1955) and McMillan (1965) proceed at this point to assume that

\[ f_1 = f_1(R) \quad (2.5) \]
where

\[ R = \frac{1}{2}(r_1 + r_2 + r_3) \]

which approximation may be regarded as a generalization of the trial exponential function. When (2.5) is used in (2.4), some of the integrals may be performed, and subsequent use of the Euler-Lagrange equation yields the Feshbach-Rubinow ordinary differential equation. McMillan (1965) has solved this differential equation numerically using potentials that fitted low energy data. In this thesis we are interested in using the Feshbach-Rubinow method with the Wong (1965) "soft core" potentials which give a good fit to the two-nucleon data to 200MeV and which have been used successfully in nuclear matter calculations. These potentials have a strong repulsive core; we have found that when they are used directly in the Feshbach-Rubinow equation, no bound three-nucleon state occurs. The reason for this is as follows: approximation (2.5) makes no special concession to the form of the triton wave function for small interparticle distances and, as a result, is inadequate for our purpose here. Indeed, the repulsive core in the two-nucleon potential requires that the triton wave function be small in small interparticle distances, and we must build this into our approximate form, in the same way that Feshbach and Rubinow have required that for hard core potentials the triton wave function vanish inside the core.
In the remainder of this Chapter, we describe two attempts made to find a simple wave function which would mark an improvement to the trial wave function used by Feshbach and Rubinow (1955) and McMillan (1965) and which would be suitable with the Wong potentials.

II.2 First attempt: The polynomial factor

The first simple function proposed which is symmetrical under the interchange of any two particles and tends to zero as any one of the $r_i$'s tends to zero, is

$$f(r_1, r_2, r_3) = \psi(R) (r_1r_2r_3)^\alpha$$

where

$$R = \frac{1}{2}(r_1 + r_2 + r_3)$$

and $\alpha$ is a parameter to be determined.

When assumption (2.6) is used in (2.4) we obtain

$$\int_0^\infty dR \left( \frac{C_1(\alpha)}{8R} 6\alpha + 5 \frac{\partial^2}{\partial R} + \frac{2C_2(\alpha)}{4R} 6\alpha + 4 \psi \frac{\partial \psi}{\partial R} + \frac{\alpha^2 C_3(\alpha)}{2R} 6\alpha + 3 \right)$$

$$+ \frac{m}{h^2} U_{\text{eff}}(R) C_1(\alpha) R^{6\alpha + 5} + \frac{mE}{h^2} C_0(\alpha) R^{6\alpha + 5} \psi^2 = 0$$

(2.7)
where \( U_{\text{eff}} = \frac{3}{C_1} \int_0^1 \int_{-x}^1 dy \left[ V(Rx) \left( (2-x-y)xy \right)^{2\alpha+1} \right] \) \( (2.8) \)

and \( C_0(\alpha) = \int_0^1 \int_{-x}^1 dy \left( (2-x-y)xy \right)^{2\alpha+1} \)

\( C_1(\alpha) = \int_0^1 \int_{-x}^1 dy \left( (2-x-y)xy \right)^{2\alpha} (-8+16x-x^2) + (16-8x)y-8y^2 \)

\( C_2(\alpha) = \int_0^1 \int_{-x}^1 dy \left( (2-x-y)xy \right)^{2\alpha-1} (-16x+40x^2-32x^3+8x^4) + (16-64x-60x^2+16x^3)y+(40-60x+24x^2)y^2 + (-32+16x)y^3+8y^4 \)

\( C_3(\alpha) = \int_0^1 \int_{-x}^1 dy \left( (2-x-y)xy \right)^{2\alpha-1} (-16+32x-8x^2-8x^3+2x^4) + (32-48x+8x^2+4x^3)y+(-8+8x+6x^2)y^2 + (-8+4x)y^3+8y^4 \)

Application of the Euler-Lagrange Equation yields

\[ \frac{h^2}{m R^{6\alpha+5}} \frac{d}{dR} \left( R^{6\alpha+5} \frac{d\psi}{dR} \right) - \frac{h^2 \beta}{m R} \frac{d\psi}{dR} + \frac{h^2}{m} C_1 \left( 4 \psi + 8 U_{\text{eff}} \psi \right) = \frac{8c_0}{C_1} E \psi \]

\( (2.9) \)

where \( \beta = \frac{C_2}{C_1} \)

\( (2.10) \)

Now defining \( u(R) \) by

\[ \psi(R) = R^{-(3\alpha+\frac{5}{2} + \frac{\beta}{2})} u(R) \]

\( (2.11) \)
Then (2.9) may be written as

\[
\frac{\hbar^2}{m} \frac{d^2u(R)}{dR^2} + V_{\text{eff}}(R) u(R) = \frac{8c_0(\alpha)}{c_1(\alpha)} E u(R)
\]

(2.12)

where

\[
V_{\text{eff}}(R) = \frac{\hbar^2}{m} \left\{ \frac{1}{4} \alpha^2 + (3\alpha + 5/2)(3\alpha + 3/2 - \beta) + 4\alpha c_3 / c_1 \right\} \frac{1}{R^2} + \frac{24}{c_1^2} V_{\text{eff}}(R)
\]

(2.13)

After transformation (2.11), normalization integral (2.3) now becomes

\[
\int_0^\infty dR \left[ u^2(R) R^{6\alpha - \beta - 5} \right] = \frac{1}{2\nu_0}
\]

(2.14)

Equation (2.12) is the reduced Schrödinger Wave Equation and \( u(R) \) is uniquely determined once the two-body potential is given. We note that setting \( \alpha = 0 \) in (2.13) for a range of \( \alpha (\alpha = 0, 0.1, 0.5, 1.0, 5.0) \) for the potentials used by McMillan (1965) and for the Wong (1965) potentials. In all cases the minimum values of \( V_{\text{eff}}(R) \) occurred when \( \alpha = 0 \) (see, for example, Figure 1 which shows \( V_{\text{eff}}(R) \) for the PBBK potential used by McMillan (1965); further, \( V_{\text{eff}}(R) \) was positive for all values of \( R \) and \( \alpha \) for the Wong potentials, showing then that no bound state was possible. The reason
for our failure is that while the polynomial factor in (2.6) makes the three-nucleon function small for small interparticle distances, it becomes large for large interparticle distances. As a result the kinetic energy term become large and dominates the potential integral to the extent that no bound state is possible. Thus approximation (2.6) is inferior to the original Feshbach-Rubinow approximation.

II.3 Second attempt: The exponential factor

In view of the inadequacy of the polynomial factor built into the triton wave function (2.5), we repeated the procedure with a more complicated factor which is still completely symmetrical, which goes to zero more quickly as the interparticle distance vanishes and which goes to unity for large interparticle distances. More specifically, the triton wave function we propose in this section is

\[ f(r_1, r_2, r_3) = \phi(R) \exp\left(\frac{\alpha}{r_1^n} - \frac{\alpha}{r_2^n} - \frac{\alpha}{r_3^n}\right) \quad (2.15) \]

where \( \alpha \) and \( n \) are parameters to be determined.

For illustrative purposes, we have plotted in Figure 2 the function \( \exp\left(-\frac{\alpha}{r^n}\right) \) for \( \alpha = 0.5 \) and \( n = 1, 2, 3 \).

When equation (2.15) is used in (2.4) the Euler-
Figure 2

\[ f = \exp \left( -\frac{\alpha}{h^n} \right) \]

\( \alpha = 0.5 \)
Lagrange Equation in $\phi$ yields the following reduced Schroedinger equation

$$-\frac{\hbar^2}{m} \frac{d^2 y}{dR^2} + V_{\text{eff}}(R) \ y = \frac{8F_0(R)}{F_1(R)} \ E \ y$$

(2.16)

where $y(R)$ is defined by

$$\phi(R) = \frac{y(R)}{R^{5/2}(F_1)^{1/2}}$$

(2.17)

The effective potential $V_{\text{eff}}(R)$ is found to be

$$V_{\text{eff}}(R) = \frac{\hbar^2}{m} \left\{ \left( \frac{F_3}{F_1} - \frac{16F_6}{F_1} - \frac{F_4}{F_1} + \frac{4F_5}{F_1} \right) \alpha R^{2n+2} + \frac{(3-2n)F_4}{F_1} \right\}$$

$$- \frac{4(4-n)F_2}{F_1} \alpha \ln R^{n+2} + \frac{15}{4R^2}$$

$$+ \frac{24}{F_1} \int dx \int dy \left\{ V(R)(2-x-y)xy \exp\left(-\frac{2\alpha P(x,y)}{R^n}\right) \right\}$$

(2.18)

where the $F$'s are functions of $R$ and are defined by

$$F_0(R) = \int_0^1 \int_{1-x}^1 \left( (2-x-y)xy \exp\left(-\frac{2\alpha P(x,y)}{R^n}\right) \right)$$

(2.19)
\[ F_1(R) = \int_0^l \int_{l-x}^l \{a_1(x,y) \exp\left(-\frac{2aP(x,y)}{R^n}\right)\} \]

\[ F_2(R) = \int_0^l \int_{l-x}^l \{a_2(x,y) \exp\left(-\frac{2aP(x,y)}{R^n}\right)\} \]

\[ F_3(R) = \int_0^l \int_{l-x}^l \{a_3(x,y) \exp\left(-\frac{2aP(x,y)}{R^n}\right)\} \]

\[ F_4(R) = \int_0^l \int_{l-x}^l \{P(x,y)G_1(x,y)\exp\left(-\frac{2aP(x,y)}{R^n}\right)\} \]

\[ F_5(R) = \int_0^l \int_{l-x}^l \{[P(x,y)]^2 G_1(x,y)\exp\left(-\frac{2aP(x,y)}{R^n}\right)\} \]

\[ F_6(R) = \int_0^l \int_{l-x}^l \{P(x,y)G_2(x,y)\exp\left(-\frac{2aP(x,y)}{R^n}\right)\} \]

where the \( G(x,y) \)'s and \( P(x,y) \) are defined as follows

\[ G_1(x,y) = z(6x+6y-4) - z(x-y)^2 \]

\[ G_2(x,y) = \sum_{cyclic\ x,y,z} \left\{xyz \left[ \frac{1}{x^{n+1}} + \frac{1}{y^{n+1}} + \frac{1}{z^{n+1}} \right] \right\} \]

\[ G_3(x,y) = \sum_{cyclic\ x,y,z} \left\{xyz \left[ \frac{x^2+y^2-z^2}{(xy)^{n+2}} + \frac{1}{z^{2n+2}} \right] \right\} \]
\[ P(x,y) = \sum_{cyclic} \frac{1}{x^n} \]  

wherein \( z = 2-x-y \).

Normalisation integral (2.3) now becomes

\[ \int_0^\infty \frac{\left( \frac{2F_0(R)}{F_1(R)} y^2(R) \right)}{F_1(R)} \, dR = 1 \]  

Finally, we note that setting \( \alpha=0 \) in (2.16) yields the Feshbach-Rubinow (1955) equation.

Some rather general remarks can be made about equations (2.16) and (2.18). We note first that the functions \( F_1(R) \) \((i=0,1,2,3,4,5,6)\) approach non-zero constants as \( R \to \infty \), and vanish as \( R \to 0 \). Further, one can see that since the two-nucleon potential is short ranged, the integral in (2.18) is \( O(R^{-3}) \) as \( R \to \infty \). (That a short-range two-nucleon potential leads to this long-range behaviour results from the fact that large values of \( R \) do not preclude the occurrence of small interparticle distances as was pointed out by McMillan.) Thus one sees that as \( R \to \infty \), \( V_{\text{eff}}(R) \) is dominated by the term \( 15/4R^2 \), as in the original Feshbach-Rubinow case. The behaviour of \( V_{\text{eff}}(R) \) at the origin is more difficult to see analytically (since all of
the $F_1$ vanish). Our numerical results indicate however that $F_1$ vanishes the quickest, so the $V_{\text{eff}}$ has a repulsive core near the origin which is steeper than that occurring with the original Feshbach-Rubinow equation. This will be demonstrated numerically in Chapter III.
CHAPTER III NUMERICAL RESULTS

The eigenvalue problem (2.16) has been solved numerically for various phenomenological Yukawa-like potential which fit the two nucleon data. The first two potentials used were the relevant part of the g=0 Feshbach-Pease (1952) potential and the PBBK2 potential used by McMillan (1965) i.e.

\[ V_{\text{singlet}} = V_{\text{central-triplet}} = -46.48 \frac{\exp(-r/1.184F)}{r/1.184F} \text{ MeV} \] (3.1)

and

\[ V_{\text{singlet}} = -59.7 \frac{\exp(-r/1.05F)}{r/1.05F} \text{ MeV} \] (3.2)

\[ V_{\text{central-triplet}} = -48.96 \frac{\exp(-r/1.1928F)}{r/1.1928F} \text{ MeV} \]

The combination of potentials (3.2) will be labelled PBBK2-potential following McMillan. Potentials (3.1) and (3.2) fit the low energy two-nucleon data.

For each potential used, we calculated the three body effective potential (2.18) for \( \alpha = 0, 0.25, 0.50, 0.75, 1 \) and \( n = 0, 0.25, 0.50, 1.0, 2.0, 3.0, 4.0 \).

It was found for both FP g=0 potential and PBBK2 potential that the lowest value of \( V_{\text{eff}}(R) \) occurred when \( \alpha = 0.5 \).
Figure 3: Effective 3-body potentials and the corresponding wave function v.s. $\alpha$

$\alpha = 0.5 \quad n = 0.5$
and \( n = 0.5 \). The three body effective potentials and the corresponding wave functions that satisfy the reduced Schrödinger equation (2.16) for this case are plotted on Figure 3. The eigenvalues are given in Table 1.

<table>
<thead>
<tr>
<th>Potential</th>
<th>Eigenvalue ( E ) of Feshbach-Rubinow Equation ( (\alpha=n=0) ) (MeV)</th>
<th>Eigenvalue ( E ) of (2.16) ( (\alpha=n=0.5) ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feshbach-Pease</td>
<td>-2.47</td>
<td>-3.55</td>
</tr>
<tr>
<td>( g=0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PBBK2</td>
<td>-4.19</td>
<td>-5.39</td>
</tr>
</tbody>
</table>

Table 1: Comparison of results: the computed eigenvalues. The experimental value is \( E = -8.492 \text{MeV} \).

From Table 1, we see that the eigenvalue \( E \) of the reduced Schrödinger Equation (2.16) is roughly 1 MeV lower than that of the Feshbach-Rubinow equation, thus indicating an improvement of our trial wave function over the Feshbach-Rubinow wave function.

The calculations were repeated with the more recent phenomenological Wong (1965) potentials:

\[
V(r) = v_1 \frac{\exp(-\mu_1 r)}{r} + v_2 \frac{\exp(-\mu_2 r)}{r} - g_\pi^2 \frac{m_\pi}{2\pi} \frac{m_\pi c^2 \exp(-\mu r)}{r}
\]  

(3.3)
FIG. 4 VARIATION OF $V_{\text{eff}}$ WITH $n$ FOR WONG'S $Y_{\text{np1}}$ POTENTIAL $\alpha = 0.5$
where \( \mu_\pi = mc/h \) is the inverse range for the pion of average mass \( m_\pi \).

\( M \) = average nucleon mass.

\( g^2_\pi \) = the OPEP coupling constant.

\( v_1, u_1 \) = parameters specifying a short range repulsive potential.

\( v_2, u_2 \) = parameters specifying a longer range attractive potential.

The masses that Wong used for fitting the softcore potential are listed in Table 2 (Table 4 of Wong (1965)). The resulting potential parameters are shown in Table 3 for different choice of the coupling constant \( g^2_\pi \) from 14 to 10 (Table 5 of Wong).

For each potential (3.4) used, we again calculated \( V_{\text{eff}}(R) \) for \( \alpha = 0, 0.25, 0.50, 0.75, 1.0 \), and \( n = 0, 0.25, 0.50, 1.0, 2.0, 3.0, 4.0 \). It was interesting to find that for all Wong potentials, the lowest value of \( V_{\text{eff}}(R) \) occured when \( \alpha = 0.5 \) and \( n = 2.0 \).

The manner that effective potential (2.18) varies with a change in \( n \) and \( \alpha \) is demonstrated on Figure 4 for Wong's Ynp2 potential. As is shown there, \( V_{\text{eff}}(R) \) changes quite markedly when \( n \) is varied. We note also that \( V_{\text{eff}}(R) \) is everywhere positive when \( \alpha = 0 \) (the Feshbach-Rubinow case).
which indicates no bound state in this case. We further note the steep repulsive core in the effective potentials, as noted in the previous section.

<table>
<thead>
<tr>
<th>System</th>
<th>$m_{\pi}$</th>
<th>$m_{\pi^0}$</th>
<th>$\frac{1}{3}(m_{\pi^0} + 2m_{\pi^+})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2M</td>
<td>$2M_p$</td>
<td>$M_p + M_n$</td>
<td></td>
</tr>
</tbody>
</table>

$m_{\pi^0} = 134.97$ MeV
$m_{\pi^+} = 139.58$ MeV
$M_n = 939.505$ MeV
$M_p = 938.211$ MeV

Table 2: The masses used in the soft core potential (eq.(3.4)).

<table>
<thead>
<tr>
<th>Potential</th>
<th>Ypp1</th>
<th>Ypp2</th>
<th>Ypp3</th>
<th>Ynp1</th>
<th>Ynp2</th>
<th>Ynp3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\pi}^2$</td>
<td>14</td>
<td>12</td>
<td>10</td>
<td>14</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>$m_{\pi}$(meV)</td>
<td>134.97</td>
<td>134.97</td>
<td>134.97</td>
<td>134.97</td>
<td>134.97</td>
<td>134.97</td>
</tr>
<tr>
<td>$\mu_2$(fm$^{-1}$)</td>
<td>3.25</td>
<td>2.63</td>
<td>2.25</td>
<td>2.95</td>
<td>2.48</td>
<td>2.20</td>
</tr>
<tr>
<td>$v_2$(MeV-fm)</td>
<td>-4622.8</td>
<td>-1751</td>
<td>-989.8</td>
<td>-2949.4</td>
<td>-1435.3</td>
<td>-951.7</td>
</tr>
<tr>
<td>$v_3$(MeV-fm)</td>
<td>14280</td>
<td>7930</td>
<td>5620</td>
<td>11100</td>
<td>7200</td>
<td>5650</td>
</tr>
<tr>
<td>$v_1$(MeV-fm)</td>
<td>-446.1</td>
<td>-447.9</td>
<td>-449.3</td>
<td>-460.1</td>
<td>-461.3</td>
<td>-462.2</td>
</tr>
</tbody>
</table>

Table 3: Parameters for Wong's $^1S_0$ state soft-core potentials.
In Figures 5 and 6, the equivalent three-body potentials (2.18) are plotted for Wong's nucleon-nucleon potentials. And in Figures 7 and 8, the wave functions \( y(R) \) corresponding to Wong's potentials are plotted for \( \alpha=0.5 \), \( n=2.0 \). The corresponding energy eigenvalues are tabulated in Table 4.

<table>
<thead>
<tr>
<th>Wong Potential (Two-body)</th>
<th>Depth of ( V_{\text{eff}}(R) ) (2.18) (MeV)</th>
<th>Energy Eigenvalue ( E ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y^{pp1} )</td>
<td>(-115.8 \ (R=1.7F) )</td>
<td>(-23.8 )</td>
</tr>
<tr>
<td>( Y^{pp2} )</td>
<td>(-101.2 \ (R=1.7F) )</td>
<td>(-19.5 )</td>
</tr>
<tr>
<td>( Y^{pp3} )</td>
<td>(-92.1 \ (R=1.7F) )</td>
<td>(-16.6 )</td>
</tr>
<tr>
<td>( Y^{np1} )</td>
<td>(-111.2 \ (R=1.7F) )</td>
<td>(-23.0 )</td>
</tr>
<tr>
<td>( Y^{np2} )</td>
<td>(-99.6 \ (R=1.7F) )</td>
<td>(-19.7 )</td>
</tr>
<tr>
<td>( Y^{np3} )</td>
<td>(-90.9 \ (R=1.7F) )</td>
<td>(-17.7 )</td>
</tr>
</tbody>
</table>

Table 4: Energy eigenvalue \( E \) for Wong potentials
CHAPTER VI SUMMARY AND CONCLUSIONS

We have calculated the symmetric S-state contribution to the binding energy of triton ($H^3$) for the Wong (1965) potentials using Feshbach-Rubinow (1955) method. The results found are tabulated in Table 4.

In view of the fact we found that when the Wong potentials were used directly in the Feshbach-Rubinow equation, no bound three nucleon state occurs, two attempts were made to find a simple symmetrical wave function which would mark an improvement to the approximation used by Feshbach and Rubinow and which would be suitable with Wong's soft core potentials.

In the first attempt, the trial wave function with a polynomial factor failed to support a bound state. The reason for the failure is that while the polynomial factor makes the three-nucleon function small for small interparticle distances, it becomes large for large interparticle distances. As a result the kinetic energy term becomes large and dominates the potential integral to the extent that no bound state is possible.

In the second attempt we then repeated the procedure with a more complicated factor which is still completely
symmetrical, which goes to zero more quickly as the interparticle distance vanishes and which goes to unity for large interparticle distances. With this function we found a strongly bound state as is clear from Table 4. In fact, the binding energy may even be bigger than we found since we have not made an extensive parameter search. Indeed, the binding energies we found are so large that the results seem to indicate the Wong potentials are also not completely successful nucleon-nucleon potentials.
FIGURE 5
EFFECTIVE POTENTIALS FOR TRITON USING WONG'S $Y_{pp}$-NUCLEON-NUCLEON POTENTIALS
FIGURE 6
EFFECTIVE POTENTIAL FOR TRITON USING WONG'S Ynp - POTENTIALS
FIG. 7: WAVE FUNCTION CORRESPONDING TO WONG'S YPP-POTENTIALS (FIG. 5)
FIG. 8: WAVE FUNCTION CORRESPONDING TO WONG'S Ynp-POTENTIALS (FIG. 6)
APPENDIX

The numerical computation was carried out on the IBM 7040 computer of the UBC computing centre.

We calculated $V_{\text{eff}}(R)$ for $\alpha=0, 0.25, 0.50, 0.75, 1$, $n=0, 0.25, 0.50, 1.0, 2.0, 3.0, 4.0$. For a given $\alpha$ and $n$ we computed $V_{\text{eff}}(R)$ from $R=0.05F$ to $R=8.0F$ with $0.05F$ interval. The time taken to compute each point is 13 second. All the double integration were carried out by the Gaussian double integration subroutine of the UBC computing centre library with minor modifications to enable it carrying out repetitive integrations. Eight point Gaussian was found to give sufficient accuracy.

Solving the reduced Schrodinger differential equation involves solution to the eigenvalue problem (2.16). Numerical integration of the differential equation was done by the Runge-Kutta method. The eigenvalue $E$ was found as follows:

1) an arbitrary value of $E$ was chosen and the d.e. was integrated forward from $R=0.1F$ to $3.0F$ at $0.1F$ interval and the logarithmic derivative of $R=3.0F$ was calculated.

2) the d.e. (2.16) was integrated backward from $R=8.1F$ to $R=3.0F$ at $0.1F$ interval with the same value of
E as in 1) and the logarithmic derivative at \( R=3.0F \) calculated again.

3) The procedure was repeated with different value of \( E \) with the logarithmic derivative calculated in 1) and 2) differed by \( 10^{-3}F^{-1} \) (Successive values of \( E \) were chosen using a linear approximation.) The corresponding value of \( E \) is the eigenvalue.

The wave function obtained is then scaled by the normalization integral (2.30).
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