A STUDY ON A DIELECTRIC LOADED
RESONANT LINEAR ELECTRON ACCELERATOR

by

ALLAN EARLE CREELMAN
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Department of Electrical Engineering

The University of British Columbia, Vancouver 8, Canada.

Date July 18, 1963
The pre-accelerator and the dielectric loaded slow-wave structure are examined in detail.

The fields in the pre-accelerator are expressed as an infinite series which converges rapidly. It was found that by considering only the first term of the series, good results were obtained for the stored energy, the power dissipated and the cavity Q. Low power tests on the pre-accelerator gave confirmation of theoretical calculations.

The fields in the dielectric loaded slow-wave structure were determined experimentally by means of a perturbation method. The results compare favourably with previous theoretical work.

An electron beam was introduced into the pre-accelerator cavity using a 2 KV electron gun and measurements were made of the gain in electron energy. Peak microwave power levels of up to 0.19 MW were used and electron energies up to 340 KEV were obtained. It was concluded that the pre-accelerator can provide adequate initial conditions for the capture of electrons in the main accelerator.
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A STUDY ON A DIELECTRIC LOADED RESONANT LINEAR ELECTRON ACCELERATOR

1. INTRODUCTION

A linear electron accelerator is a device in which electrons gain energy by interaction with an electromagnetic wave. A necessary condition is that the phase velocity of the wave is approximately equal to the electron velocity and the central problem in accelerator design is to devise a suitable slow-wave structure. Most accelerators in current use employ a waveguide loaded at regular intervals with metal discs. It has been realized for some time that dielectric loading offers certain advantages.

It was first shown by Fry and Walkinshaw\(^{(1)}\) that a resonant accelerator would be more efficient if it were operated at the $\pi$-mode, but this mode of operation was found to be unsatisfactory from a mode separation point of view. It was later shown by Walker and West\(^{(2)}\) that it is possible, by using dielectric loading, to eliminate the stop band between the first and second pass bands so that these pass bands are then confluent. Under these conditions the mode separation at the $\pi$-mode is greatly increased. The energy output which can be expected from this structure for a given power input is greater than could be obtained from a metal loaded structure.

The work to be described relates to a resonant dielectric loaded accelerator under construction at The University of British Columbia. The accelerator consists of three major components.
These are the electron gun, a pre-accelerator, and the main accelerator. Such a machine is shown diagrammatically in Figure 1. The electron gun is the source of a low energy, well collimated electron beam. The energy is raised to an intermediate level by the pre-accelerator which also bunches the electrons. The electrons then enter the main accelerator with the correct velocity and bunching characteristics to enable interaction to take place with the travelling wave.

A pre-accelerator and a dielectric loaded slow-wave structure were constructed prior to the beginning of the work at The University of British Columbia. The design of the dielectric loaded structure was based on the theoretical investigation carried out by West\(^\text{(3)}\). The purpose of this work is to determine experimentally the characteristics of the two structures, particularly their field patterns, in relation to the operation of the machine as a whole.
Fig. 1 The Linear Electron Accelerator System.
2. LOW POWER MEASUREMENTS

2.1 The Pre-accelerator Cavity

2.1.1 Description of the Pre-accelerator

In order to provide the correct energy and bunching characteristics to the electrons before they are injected into the main cavity, a pre-accelerator is required. In this study, the pre-accelerator consists of the re-entrant microwave cavity shown in Figure 2.

In design the pre-accelerator is similar to a high power klystron cavity, the object being to produce a high intensity electric field for interaction with electrons. The parameters of major importance are the cavity Q, the shunt resistance and the transit angle of the electrons in traversing the cavity.

For the optimum utilization of input power it is necessary to have a high shunt resistance. It has been shown\(^4\) that the maximum shunt resistance is obtained in a spherical cavity, but such a cavity would not be practical because the interaction path would be too long and because of the difficulty of manufacturing a spherically shaped cavity. To decrease the length of the interaction path while maintaining the same resonant frequency, a cavity with re-entrant noses is used, although the presence of the re-entrant noses lowers the shunt resistance by increasing the power dissipated in the cavity.

A high shunt resistance implies a high cavity Q which is not desirable since the narrow bandwidth of a cavity with a high Q would require that the frequency of the RF source be very stable.
Fig. 2 The Pre-accelerator Cavity.
Also, the coupling mechanism between the waveguide and the cavity is much more difficult to construct if the cavity has a high $Q$. For machines operating at a fixed frequency, the cavity $Q$ is not usually troublesome, but it is necessary to check to ensure that it is of the correct order. If necessary, the cavity can be made slightly lossy to reduce the $Q$.

2.1.2 Tuning Range

It is necessary that the resonant frequencies of the pre-accelerator and the main accelerator be equal and the RF power for both structures is supplied from the same magnetron. The resonant frequency of the pre-accelerator cavity is made equal to the resonant frequency of the main cavity by two means. The position of the noses is adjusted until the frequency is approximately correct and then the noses are fixed in this position. A further control of the frequency is provided by a tuning probe. The range of frequency variation provided by the tuning probe is shown in Figure 3.

2.1.3 The Fields in the Cavity

One representation which can be used to approximate the fields within the cavity is to consider the gap region between the noses to be a parallel plate capacitor and the region outside the gap as a radial transmission line\(^{(5)}\). In this approximation, the electric field intensity in the gap region is assumed to be constant over the region. Denoting this value of electric field intensity by $E_b$, the electric and magnetic field intensities in the region outside the gap may be expressed in terms of $E_b$ as
Fig. 3 Tuning Range of Pre-accelerator Cavity.
follows:

\[ E_z = E_b \sqrt{\frac{J_0^2(kr) + N_0^2(kr)}{J_0^2(kb) + N_0^2(kb)}} \left( \frac{\sin (\Theta - \Theta_a)}{\sin (\Theta_b - \Theta_a)} \right) \] ...

\[ H_\phi = \frac{E_b}{\gamma} \sqrt{\frac{J_1^2(kr) + N_1^2(kr)}{J_0^2(kb) + N_0^2(kb)}} \left( \frac{\cos (\Psi - \Theta_a)}{\sin (\Theta_b - \Theta_a)} \right) \] ...

where \( J_0, N_0 \) are Bessel functions of zero order of the first and second kind respectively.

\( J_1, N_1 \) are Bessel functions of order one of the first and second kind respectively.

\[ \Theta = \tan^{-1} \left( \frac{N_0(kr)}{J_0(kr)} \right) \]

\[ \Theta_b = \tan^{-1} \left( \frac{N_0(kb)}{J_0(kb)} \right) \]

\[ \Theta_a = \tan^{-1} \left( \frac{N_0(ka)}{J_0(ka)} \right) \]

\[ \Psi = \tan^{-1} \left( \frac{J_1(kr)}{-N_1(kr)} \right) \]

\[ k = 2\pi f \sqrt{\frac{\mu}{\varepsilon}}, \quad \gamma = \sqrt{\frac{\mu}{\varepsilon}} \]

\( f \) is the operating frequency, \( \mu \) is the permeability of free space, \( a \) is the radius of the cavity, and \( b \) is the radius of the nose.
In the above, $E_b$ is a function of time. The fields in the two regions are matched along their common boundary. This boundary is the cylindrical surface in the gap region at $r = b$. This approximation permits the longitudinal electric field to have a value at $r = b$. This is obviously not true over part of the surface at $r = b$ since the electric field would be tangential to a metal wall. Despite this inaccuracy, the approximation gives a surprisingly accurate value for the stored energy, the power dissipated and the cavity $Q$. Figure 4 shows the cavity used for the calculations which are carried out in the next section. The actual cavity shown in Figure 2 differs from this in the shape of the noses.

In Appendix A, a more accurate representation of the fields in the cavity is given. This representation follows closely one given by Hahn\(^{(6)}\) in which the fields are expressed in the form of an infinite series. It was found that only the first three terms of the series are required. The remaining terms of the series make a very small contribution to the total field. The difference between the value of the magnetic field given by the series and that given by the radial transmission line approximation was found to be of the order of 5% of the total field.

The series approximation was used to calculate the resonant frequency. The value obtained agreed with the measured value to within 3%.
2.1.4 Stored Energy

The stored energy of the cavity is made up of the sum of the energy stored in the gap region and the energy stored in the outer region. Both of these energies are calculated from the field intensity in each region by assuming that all of the energy of the electromagnetic field is stored in the electric field at the instant in time when it is a maximum. The energy stored in the gap region is given by

\[ U_g = \frac{1}{2} \varepsilon E_b^2 \pi b^2 x \quad \ldots (2-3) \]

where \( x \) is the gap length. On substitution of the values of \( b \) and \( x \) from Figure 4,

\[ U_g = 0.103 \times 10^{-16} E_b^2 \text{ Joules} \quad \ldots (2-4) \]

The energy stored in the region outside the gap is given by

\[ U_r = \frac{1}{2} \mu \int_V |H_\theta|^2 \, dv \quad \ldots (2-5) \]
\[ U_r = \frac{1}{2} \mu \int_0^\pi \int_0^a \int_0^h r E_b^2 D \frac{d\phi}{A \ b} \ dr \ dZ \quad \ldots(2-6) \]

where \( A, B, D \) are defined as

\[ A = \left[ J_0^2(kb) + N_0^2(kb) \right] \pi^2 \]
\[ B = \sin^2 (\phi_b - \phi_a) \]
\[ D = C_1 N_1^2(kr) + C_2 N_1(kr) J_1(kr) + C_3 J_1^2(kr) \]

and

\[ C_1 = \cos^2 \phi_a \]
\[ C_2 = -2 \sin \phi_a \cos \phi_a \]
\[ C_3 = \sin^2 \phi_a \]

Hence, by integration

\[ U_r = \mu \pi h E_b^2 G \frac{E_b}{A \ b} \quad \ldots(2-7) \]

where

\[ G = \left[ \frac{C_1 r^2}{2} (N_1^2 - J_0 J_2) + \frac{C_2 r^2}{4} (2J_1 N_1 - J_0 N_2 - J_2 N_0) \right. \]
\[ + \frac{C_3 r^2}{2} (J_1^2 - J_0 J_2) \left. \right]^a \]
\[ \left. \right|_b \]
Evaluation of this expression yields

$$U_r = 2.58 \times 10^{-16} E_b^2 \text{ Joules} \quad \ldots(2-8)$$

The total energy stored in the cavity, $U$, is given, therefore, by

$$U = 2.683 \times 10^{-16} E_b^2 \text{ Joules} \quad \ldots(2-9)$$

2.1.5 Power Dissipated

The power dissipated per unit area in the walls of the cavity

$$W = \frac{1}{2} \sqrt{\frac{\omega u}{2g}} \left| H_t \right|^2 \quad \ldots(2-10)$$

where $g$ is the conductivity of the metal and $H_t$ is the tangential component of the magnetic field at the metal surface.

The total power dissipated in the cavity can be considered to be the sum of several components which may be calculated as follows:

1) The power dissipated in the outer cylindrical wall of the cavity, $P_w$. In this case $H_t = H_\phi a$ where $H_\phi a$ is the value of $H_\phi$ evaluated at a radius $r = a$. The conductivity of brass is taken as $1.57 \times 10^7$ mhos/meter.

$$P_w = \frac{1}{2} \sqrt{\frac{\omega u}{2g}} \left| H_\phi a \right|^2 2\pi a h \quad \ldots(2-11)$$
Hence,

\[ P_w = 30.6 \times 10^{-11} \frac{E_b^2}{E_b} \text{ Watts} \quad \text{(2-12)} \]

2) The power dissipated in the end walls of the cavity, \( P_{ew} \). In this case \( H_t = H_\phi^* \).

\[ P_{ew} = 2 \times \frac{1}{2} \sqrt{\frac{\omega \mu}{2g}} \int_0^2 \int_b^a |H_\phi|^2 r \, dr \, d\phi \quad \text{(2-13)} \]

Hence,

\[ P_{ew} = 28.6 \times 10^{-11} \frac{E_b^2}{E_b} \text{ Watts} \quad \text{(2-14)} \]

3) The power dissipated in the curved portion of the noses, \( P_n \).

The cavity noses were plated with platinum to help prevent field emission from taking place. The conductivity of platinum is taken to be \( 0.9 \times 10^7 \) mhos/meter. \( H_t = H_\phi^* \) where \( H_\phi^* \) is the value of \( H_\phi \) evaluated at radius \( r = b \).

\[ P_n = \frac{1}{2} \sqrt{\frac{\omega \mu}{2g}} \left| H_\phi^* \right|^2 2 \pi b (h - x) \quad \text{(2-15)} \]

Hence,

\[ P_n = 14.3 \times 10^{-11} \frac{E_b^2}{E_b} \quad \text{(2-16)} \]
4) The power dissipated in the flat portion of the noses, $P_{en}$.

In this case $H_t = H_0n$ where $H_0n$ is assumed to vary linearly from a value of zero at $r = 0$ to a value of $H_0b$ at $r = b$.

\[ P_{en} = 2 \times \frac{1}{2} \sqrt{\frac{ou}{2g}} \int_0^{2\pi} \int_0^b \left| H_{on} \right|^2 \frac{r^3}{b^2} \, dr \, d\phi \]  \hspace{1cm} (2-17)

where $g$ is again taken as $0.9 \times 10^7$ mhos/meter.

Hence,

\[ P_{en} = 4.1 \times 10^{-11} E_b^2 \text{ Watts} \]  \hspace{1cm} (2-18)

The total power dissipated, $P$, given by the sum of the above components is

\[ P = 77.6 \times 10^{-11} E_b^2 \text{ Watts} \]  \hspace{1cm} (2-19)

2.1.6 Calculated $Q$

The unloaded $Q$ of the pre-accelerator cavity may be calculated from the total stored energy and the total power dissipated in the cavity. Using equations (2-9) and (2-19) the unloaded $Q$ is found to be

\[ Q_u = \frac{ou}{P} = 6500 \]  \hspace{1cm} (2-20)

2.1.7 Measured $Q$

The $Q$ of the pre-accelerator was measured by the standing
wave method described by Ginzton(7). The values of the loaded and unloaded $Q$'s were found to be the following:

a) Unloaded $Q$

$$Q_u = 4900 \quad \ldots (2-21)$$

b) Loaded $Q$

$$Q_1 = 2720 \quad \ldots (2-22)$$

In the measurement of the $Q$, the presence of losses in the coupling mechanism has been neglected. The coupling mechanism in this case consists of an iris in the wall of the cavity adjusted in size to give approximately unity coupling. Strong fields exist in the vicinity of the iris giving rise to power dissipation which, in the method of measurement, must appear as an additional power loss within the cavity. For this reason the measured value of the cavity $Q$ will always be lower than the calculated value.

The calculation given above was based on values of conductivity of brass and platinum given in The Microwave Engineers Handbook. These are D.C. values and are known to be affected by impurities. At microwave frequencies the state of the metal surface plays an important part in determining the power dissipated. Surface roughness and surface contamination both reduce the effective conductivity and for the type of surface used a reduction by as much as 20% is in accordance with experience. No
attempt was made to make a direct measurement of the state of the cavity surfaces since the results obtained were acceptable and within reasonable expectation.

2.1.8 Input Power

An estimate of the magnitude of the electric field strength in terms of input power can be obtained indirectly from the unloaded $Q$ of the cavity. Assuming that the total input power is dissipated as losses in the walls of the cavity, the relationship of the various quantities to one another is given by

$$Q_u = \frac{\omega U}{P} \quad \text{...}(2-23)$$

where

- $U$ is total stored energy given by equation (2-9)
- $Q_u$ is the unloaded $Q$ given by equation (2-21)
- $P$ is the power dissipated in the cavity.

Substitution of these values into equation (2-23) produces the following expression for $E_b$:

$$E_b = 7.80 \times 10^4 \sqrt{P} \quad \text{V/M} \quad \text{...}(2-24)$$

An experimental check on the accuracy of (2-24) was obtained by measurement of the energy gain of electrons in traversing the gap. This is discussed in a later section.

2.1.9 Output Energy of Pre-accelerator

Since the phase velocity in the main cavity is constant and equal to the velocity of light, the electrons must be injected
into the main cavity with a substantial initial energy. West\textsuperscript{(3)} has calculated the minimum injection energy required for capture, using a method described by Slater, and the injection energies required for the capture of 25\% and 50\% of the electrons as a function of field strength in the main cavity. His calculations were made on the basis that electrons are entering the main cavity in a continuous stream. The results of these calculations are shown in Figure 5 from which it can be seen that, for a field strength of 10 MV/M, the minimum injection energy required for capture is approximately 70 KEV. Whether or not a particular electron is captured depends not only on its energy when injected but also on the injection phase angle, that is, the phase of the accelerating field when the electron is injected. Electrons having the minimum energy will be captured only if they are injected at the time when the field is just entering the accelerating phase. Electrons with higher injection energies will be captured over a range of phase angles.

It has been shown\textsuperscript{(8)} that the electrons which gain the maximum energy in the main cavity are those which are injected at a phase angle of zero degrees, assuming that the electrons are all injected with the same energy. Hence, in order to have electrons emerging from the main cavity with maximum energy and minimum energy spread, the input to the cavity should be in the form of the shortest possible bunch, injected at a phase angle of zero degrees.

In order to investigate the range of output energies from the pre-accelerator which could be expected for a given microwave
Fig. 5 Initial Energy Required for Capture in the Main Accelerator.
input power, a series of calculations was carried out using the I.B.M. 1620 digital computer.

The energy gained by an electron as it passes through the pre-accelerator can be calculated from

\[ T = \frac{m_0 c^2}{e} \left( \sqrt{1 + (K - \alpha \cos U)^2} - 1 \right) \]  \hspace{1cm} \ldots (2-25)

where

- \( T \) is the energy gain in electron volts
- \( m_0 \) is the rest mass of the electron
- \( c \) is the velocity of light
- \( e \) is the charge on an electron

\[ \alpha = \frac{eE}{m_0 c} \]

\( E \) is the peak value of the electric field intensity in the pre-accelerator gap region.

\( U \) is the phase angle of the electric field when the electron leaves the cavity

\[ K = \frac{\beta_0}{\sqrt{1 - \beta_0^2}} + \alpha \cos U_0 \]

\[ \beta_0 = \frac{v_0}{c} \]

\( v_0 \) is the velocity of the electron on entering the pre-accelerator

\( U_0 \) is the phase angle of the electric field when the electron enters the cavity
The final phase angle, $U$, is found in terms of the initial phase angle, $U_0$, from the following expression.

$$g = \frac{c}{\omega} \int_{U_0}^{U} \frac{K - \alpha \cos U}{\sqrt{1 + (K - \cos U)^2}} \, dU \quad \ldots (2-26)$$

where $g$ is the gap length of the pre-accelerator. The integration was carried out numerically, using the computer, for a large range of values of $U_0$ and a fixed $E$. The corresponding values of $U$, found from the integration, were then substituted into equation (2-25) and from this expression the energy gain was calculated for each value of $U$. The maximum energy for each value of electric field strength was selected and then a new value of electric field strength was substituted into equation (2-26) and the calculation was repeated.

The values of electric field strength which were used in the above calculation were also substituted into equation (2-24). The results obtained from this equation were combined with those from equation (2-25) to produce a relationship between the microwave power input to the pre-accelerator and the maximum energy gain of an electron in the cavity. This relationship is shown in Figure 6.

A complete derivation of equations (2-25) and (2-26) is given by Vermeulen (8). The calculation of energy gain follows the method that he used. The calculation was carried out for a range of values of electric field strength from zero up to a value of $20 \times 10^6$ volts per meter.
Fig. 6 Maximum Electron Energy Gain in Pre-accelerator.
2.2 The Main Accelerator

2.2.1 Description of the Main Cavities

The main accelerator consists of two identical cavities, coupled by a drift tube, each cavity being one-half meter long. The cavities are supplied with RF power from the magnetron through a magic-T junction. The length of the waveguide from the magic-T to one cavity is made one-quarter of a wavelength longer than the corresponding waveguide connected to the second cavity. The reflected power from the two cavities during the build up process does not return to the magnetron. Instead, since the two reflected waves arrive at the magic-T in anti-phase, the reflected power is dissipated in a matched load located in the fourth arm of the magic-T. Hence, the use of the magic-T permits the magnetron to operate into a matched load at all times. This is a significant feature of the machine because a mismatched load would cause the magnetron to operate in a highly unstable fashion.

The design of the main accelerator is based on the results of work carried out by West\(^{(3)}\). From the theoretical investigation of the dielectric disc loaded cavity, a design for the main accelerator section was selected which had the following parameters:

- tube diameter: 7.669 cm
- dielectric constant: 93
- disc thickness: 0.577 cm \(\text{(2-27)}\)
- diameter of disc hole: 2.0 cm
- length of each accelerating cavity: 0.5 M
In order that each disc and cavity section be demountable, the method of construction shown in Figure 7 was chosen. Copper rings are first shrunk on to each ceramic disc and then the rings holding the discs are fitted into the recesses at the ends of the copper tube sections which also act as spacers for the discs. The tube sections are aligned and held in position by inserting them into a cylindrical vacuum envelope which has end plates secured by bolts.

2.2.2 Measured Q

The $Q$ of the main cavity was measured by the same method as used for the pre-accelerator. The values of the loaded and unloaded $Q$ were found to be

$$Q_u = 8820 \quad \ldots (2-28)$$
$$Q_1 = 3275 \quad \ldots (2-29)$$

These values are considerably affected by a coating of lead borate glaze which was applied to the exposed surfaces of the discs to inhibit vacuum breakdown\(^9\). The properties of the glaze have been investigated by Luthra\(^{10}\) who found that the microwave power absorption in a coating thickness of 0.004 inches was as great as the absorption in the titania disc. In the current version of the main accelerator cavity no attempt was made to achieve a minimum thickness in the glaze coating and consequently a considerably higher cavity $Q$ could be obtained.
Fig. 7 Construction of the Main Cavity
2.2.3 The Field Pattern

The field pattern was determined theoretically, neglecting the center hole in the discs, by postulating a forward and a backward wave of a given mode type in each air and each dielectric region. The relative magnitudes of these component waves can be determined by the use of Floquet's theorem and the application of the requisite boundary conditions at the air-dielectric interfaces. The cavity has been designed so that, at the operating frequency, the phase change between discs is $\pi$, and also, the impedance of the air region is equal to the impedance of the dielectric region. Thus, there are two possible standing wave patterns, corresponding to two $\pi$-modes, which have nodal planes differing in position from one another by one-quarter of a wavelength.

The field pattern was checked experimentally to ensure that the proper mode was being excited in the cavity, that no distortion of the field pattern was caused by the coupling hole between the waveguide and the cavity, and to determine the value of the electric field strength in the region of the central hole of the disc. The measured values of electric field strength were used, also, to determine the voltage gain of the cavity.

The measurement of electric field strength in the cavity was carried out by means of a perturbation technique which consisted of measuring the resonant frequency of the cavity as a function of the position of a small perturbing body in the cavity.

The technique is based on a proposition, which can
be derived from Maxwell's equations, that for a closed, oscillating, loss free system, the ratio of the stored energy to the resonant frequency of the system does not vary with adiabatic changes of the system. In the case of a cavity which is being excited from an external source, the proposition holds true provided the losses in the cavity are small. It is possible to write, therefore, that

\[
\frac{df}{f} = \frac{dU}{U} \quad \ldots (2-30)
\]

where \( f \) is the unperturbed resonant frequency, \( df \) is the change in resonant frequency, \( U \) is the total energy stored in the cavity, \( dU \) is the change in stored energy due to the introduction of the perturbing body into the electromagnetic field of the cavity.

The perturbing body was selected to be a dielectric material with a relative permeability of unity. The change in stored energy due to the introduction of the perturbing body may be calculated in terms of the electric field strength only. If the dielectric is spherically shaped, this change in stored energy can be shown to be

\[
dU = \pi a^3 \left( \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1 + 2\varepsilon_0} \right) \varepsilon_0 E_0^2 \quad \ldots (2-31)
\]

where

- \( a \) is the radius of the dielectric sphere
- \( \varepsilon_0 \) is the permittivity of free space
$\varepsilon_1$ is the dielectric constant of the sphere

$E_0$ is the electric field strength in the cavity before the introduction of the sphere.

It is assumed that the perturbing body is sufficiently small that the electric field in the region of the sphere can be considered to be uniform. The change in stored energy given by equation (2-31) may be written as

$$dU = K_p \varepsilon_0 E_0^2$$ \hspace{1cm} (2-32)

where

$$K_p = \pi a^3 \left( \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1 + 2\varepsilon_0} \right)$$

$K_p$ will be referred to as the perturbation constant of the sphere. In order to make the perturbing body as small as possible and still have $K_p$ large, a material with a high dielectric constant is required. Some difficulty was experienced in obtaining a material with a sufficiently high dielectric constant which could easily be formed into the required shape. Since titanium dioxide powder was available, a small sphere was manufactured by mixing some of the powder with a binder of amyl acetate and moulding the mixture into a spherical shape of approximately 0.7 cm diameter. The perturbation constant of the sphere was then determined experimentally as described below.
The experiment consisted of measuring the resonant frequency of an empty cylindrical cavity with the dimensions shown in Figure 8 and then noting the change in resonant frequency when the perturbing body was suspended by a thin nylon thread at the mid point of the axis of the cavity. The ratio of the change in resonant frequency to the unperturbed resonant frequency was found to be

\[ \frac{df}{f} = 2.29 \times 10^{-3} \quad \text{...(2-33)} \]

The cavity was excited in the \( \text{TM}_{010} \) mode for which the stored energy is given by

\[ U = \frac{\pi \varepsilon_0 d E_0^2 b^2}{2} J_1^2(kr) \quad \text{...(2-34)} \]

where

- \( b \) is the radius of the cavity
- \( d \) is the length of the cavity
- \( E_0 \) is the peak electric field intensity
- \( J_1(kr) \) is the Bessel function of order one and first kind
- \( k = \frac{2.405}{b} \)

From the dimensions given in Figure 8,

\[ U = \varepsilon_0 E_0^2 \times 0.460 \times 10^{-4} \text{ Joules} \quad \text{...(2-35)} \]
Fig. 8  Cavity for Determining the Perturbation Constant
On combination of equation (2-31) and equation (2-34) with equation (2-30)

\[
\frac{df}{f} = \frac{K_p E_0^2}{E_0^2 \times 0.460 \times 10^{-4}} \tag{2-36}
\]

On substitution of the value for \( \frac{df}{f} \) from equation (2-33)

\[
K_p = 33.4 \times 10^{-9} \tag{2-37}
\]

The result of the perturbation of one of the main cavities is shown in Figure 9 where the change in resonant frequency due to the perturbation has been plotted as a function of the distance of the perturbing body from one end of the cavity. Thus, this figure represents the square of the electric field strength on the axis of the cavity. The figure shows that the proper mode is being excited, that is, the accelerating field has its maximum value at the mid point of the air region. The fields in each section are remarkably similar to one another with the exception of the section between disc number nine and disc number ten. In this section the field is slightly greater in magnitude than in the other sections, but the increase is not sufficient to be of great significance. In the region of the coupling hole, between disc five and disc six, no distortion of the accelerating field was observed. The electric field intensity in the region of the central hole in the disc was found to be very weak and within the hole no field could be detected beyond a distance of about 0.2 cm. from the edge of the disc. The measured value of field in this region might be expected to be inaccurate due to the proximity effect of the disc on the perturbing body, but since the field is weak the inaccuracy is not important.
Fig. 9 Perturbation of the Electric Field in the Main Accelerator.
2.2.4 Voltage Gain

In previous work\(^3\), the electric field strength along the axis of the cavity has been approximated by assuming that the field strength was zero in the disc region and that it was given in the air region by the following expression.

\[
E_z = E_0 \cos \beta_a (z - \frac{P}{2}) \tag{2-38}
\]

where

- \(z\) is the distance measured from a disc
- \(E_0\) is the peak value of the electric field
- \(P\) is the distance between discs
- \(\beta_a\) is the propagation constant in the air region and is equal to 6.689

A plot of this expression is shown in Figure 10.

The measured value of electric field strength on the axis of the cavity in an air region can be obtained from any one of the section of Figure 9 by taking the square root of each of the values plotted. In order to minimize errors due to inaccuracies in measurement of the field strength, an average section was plotted from the first eight sections shown in Figure 9. This average section is also plotted in Figure 10. A comparison of the voltage gain based on the theoretical approximation of electric field strength and the voltage gain based on the measured field strength will now be made.
Fig. 10 Average Electric Field Strength in the Main Accelerator.
The accelerating field in the main cavity varies sinusoidally with respect to time. The value of the field at any point along the axis of the cavity can be expressed at any instant by

$$E = E' \sin \omega t$$

where $E'$ is the maximum value which the field attains at the point under consideration and, hence, is a function of $z$. Since the phase velocity of the accelerating wave is made equal to the velocity of light, an electron, travelling for all practical purposes at the same velocity, will remain in phase with the accelerating wave. If the electron is at the mid point of an air region at the instant when the accelerating field is a maximum, the field which it experiences may be written in terms of the distance from a disc as

$$E = E' \sin \frac{\omega}{c} z$$

where $z$ is the distance from a disc. The voltage gain of an electron which travels the length of one air region is therefore given by

$$V = \int_{-q/2}^{p + q/2} E' \sin \frac{\omega}{c} z \, dz \quad \ldots (2-39)$$

where $p$ is the length of an air region and $q$ is the disk
thickness. In order to determine the voltage gain from the results of Section 2.2.3, the above integral must be evaluated with \( E' \) replaced by the measured value of the electric field strength on the axis, \( E_m(z) \). The integral is not evaluated directly. Instead, a comparison is made between the value of this integral and the value obtained from the theoretical approximation. The two integrals to be compared are the following:

\[
\int_{-q/2}^{p + q/2} E_m(z) \left( \sin \frac{\omega}{c} z \right) \, dz
\]

\[
\int_{0}^{p} E \cos \left( \beta_a z - \frac{\beta_g \beta_a}{2} \right) \left( \sin \frac{\omega}{c} z \right) \, dz \quad \ldots (2-40)
\]

From Figure 10 a plot is made of each of the above integrands. The plots are shown in Figure 11 and from them the integration is carried out graphically. From Figure 11 it was found that the measured value of the voltage gain agrees with the value obtained from the theoretical approximation to within two percent. On integration of the second of equations (2–40) the theoretical value of voltage gain over one air section is 0.0214 \( E \).

2.3 The Test Bench

The characteristics of both the pre-accelerator cavity and
Fig. 11 Numerical Integration for Determining Voltage Gain.
the main cavity were determined from measurements carried out on a low power S-band bench. A block diagram of the equipment is shown in Figure 12.

The RF supply was a reflex klystron capable of being tuned over a frequency range of approximately 100 Mc/s by means of tuning probes. Modulation of the RF output signal was accomplished by a 3 Kc/s square wave applied to the grid of the klystron. The square wave had a sufficiently negative going voltage to cut off the tube so that the RF output consisted of pulses of RF signal, each pulse being 0.167 ms long. In order to increase the frequency stability of the klystron, a ferrite isolator and an attenuator were placed in the waveguide run between the klystron and the cavity under test.

For monitoring purposes a small fraction of the RF power was coupled out of the main waveguide. Part of this signal was fed through a crystal detector and a transistor amplifier to an oscilloscope, where the modulation waveform could be observed. Another portion of the signal was fed to a cavity wavemeter in order to measure the frequency. The wavemeter was connected to an indicating unit consisting of a high gain, tuned amplifier with a narrow bandwidth. The amplifier was tuned to the modulation frequency, the narrow bandwidth being important in reducing the detected noise.

A calibrated attenuator was included in the waveguide run and was used in conjunction with a slotted line for accurately measuring the VSWR in the waveguide. The shielded probe in the slotted line was connected to a second tuned amplifier indicator
Fig. 12 Low Power Test Bench.
When it was necessary to find the resonant frequency of a cavity, a probe antenna was inserted into the cavity and the output of the probe was connected to one of the indicator units. The frequency was then adjusted for maximum output from the probe, and this frequency was determined in the usual manner. To ensure that the unloaded resonant frequency was being measured, the probe was moved until no change in resonant frequency could be observed with a change in probe position while maintaining an output signal from the probe which was adequate to drive the tuned amplifier indicator unit. This procedure ensured a loose coupling between the probe and the electromagnetic fields in the cavity.
3. HIGH POWER MEASUREMENTS

3.1 High Power Apparatus

3.1.1 RF Source

The high power RF source consisted of a two megawatt S-band magnetron and modulator unit. Power was provided in pulses of two microseconds duration at a repetition rate of 60 pulses per second. The output power of the magnetron was continuously variable from zero to a maximum of two megawatts peak power. The frequency could be varied over a range of two megacycles by means of a phase shifter mismatch unit and by adjustment of the magnetron magnet current.

A ferrite isolator was used between the magnetron and the cavity under test to improve the frequency stability of the magnetron. To avoid sparking and to prevent excessive heating of the ferrite strips, the isolator was operated in a pressurised atmosphere. It was necessary, therefore, to divide the waveguide run into two sections. This was accomplished by a commercial high power window. The waveguide from the magnetron through the isolator to the window was pressurised to 15 psi with nitrogen and the remainder evacuated to a pressure below $10^{-5}$ mm Hg.

The RF power was measured by extracting a small fraction of the output of the magnetron by a directional coupler which fed into a thermistor unit. The thermistor mount was of conventional design.

The thermistor formed part of the bridge circuit shown in
Figure 13 and the power absorbed was deduced from the change in DC current necessary to balance the bridge.

Figure 13. Thermistor Bridge Circuit For Measuring Microwave Power.

The thermistor and its associated bridge circuit were calibrated by means of a water load consisting of a glass tube filled with water and inserted in a waveguide section. The tube was slanted in the guide, hence, it was possible to obtain a good impedance match and a uniform distribution of power dissipation along the length of the water column. A constant water pressure in the tube was ensured by using a simple gravity feed system.
3.1.2 Vacuum System

The vacuum pumping system consisted of two units, each containing a mercury diffusion pump of two litre/sec capacity and a mechanical rotary vane backing pump of 57 l/minute capacity. One pumping unit was connected directly to the electron gun so that the pressure in the gun region could always be maintained at a value less than $5 \times 10^{-6}$ mm Hg. The second unit was connected to the pre-accelerator section through the feed waveguide and the pressure in this region was maintained at about $10^{-5}$ mm Hg. Both pumping units were connected through an alarm system so that in the event of vacuum leak or failure of the cooling water supply they would automatically shut off. Pressures were measured with both pirani and ion gauges.

3.1.3 The Electron Gun

Initial tests were carried out using a gun with a tungsten filament-cathode and four plane focusing electrodes. Details of this gun are given by Goud.\(^{(12)}\) Characteristics obtained in bench tests could not be repeated when the gun was located in its operating site and despite many modifications a satisfactory performance was not obtained. The gun was originally designed for DC operation and this created difficulties in measuring the output of high energy electrons from the pre-accelerator. As mentioned above, the magnetron operated in two microsecond pulses at a 60 c/s repetition rate and current passed during the magnetron off-period tended to mask the accelerated beam.

For these reasons it was eventually decided to abandon this particular gun. Since a current of the order one milliampere
was sufficient for test purposes, it was found that a conventional gun of the type used in commercial television tubes was adequate.

This gun had an oxide cathode and was, therefore, likely to be damaged by water vapour if exposed to the atmosphere. It was found that if the heater of the gun was operated at a reduced voltage while air was being admitted to the gun envelope very little damage resulted to the cathode. If this procedure was followed, the gun could be re-used after evacuation of the air by performing the cathode activating cycle.

In order to eliminate the current passed during the magnetron off-period the gun was pulsed. A circuit was designed and built which provided a positive going pulse to the grid of the gun. The pulse amplitude was variable up to 800 volts and the pulse length was variable from one half to five microseconds. A rise and fall time of approximately 0.03 microsecond each was obtained. The pulse circuit was triggered from the magnetron trigger circuit so that electrons entered the pre-accelerator only when RF power was flowing.

3.1.4 The Magnetic Analyser

The energy of the accelerated electrons was measured by means of a magnetic analyser. The magnetic field was provided by an electromagnet with two flat pole pieces as shown in Figure 14. The magnetic field strength could be varied by changing the current in the two coils.

The electrons, on emerging from the pre-accelerator, entered a three-quarter inch diameter glass tube. The end portions of this tube, each about two inches long, were straight
while the center portion was curved in the form of a circle of radius 15 cm. The magnetic field was applied over the curved portion of the tube, the magnet pole pieces being shaped for this purpose as shown in Figure 15.

The magnetic field was adjusted until the electrons followed a circular path down the center of the glass tube and were collected by a target electrode at the end of the tube. The magnetic flux density was measured with a Hall probe, and the electron energy was calculated in the following manner. The motion of the fast electron in a uniform magnetic field is
Figure 15. Magnetic Analyser Pole Pieces.

given by (13)

\[
\frac{mv}{\sqrt{1 - \beta^2}} = ReB
\]

...(3-1).

where

- \( m \) is the rest mass of the electron
- \( v \) is the velocity of the electron
- \( \beta = \frac{v}{c} \), where \( c \) is the speed of light
- \( R \) is the radius of the electron path
- \( e \) is the electron charge
- \( B \) is the magnetic flux density
The left hand side of equation (3-1) is the momentum, \( P \), of an electron. The energy in electron volts, \( T \), is given in terms of the momentum from the following expression.

\[
T = \frac{1}{1.60 \times 10^{-19}} \sqrt{P^2c^2 + m^2c^4 - mc^2} \quad \ldots (3-2)
\]

The main difficulty in the use of the magnetic analyser was due to the magnetic fringing field. For electron energies which would normally be expected from the pre-accelerator cavity, that is 70 KEV or more, the fringing field is of no consequence since it is not sufficiently strong to deflect a high energy electron by an appreciable amount. It was found, however, that the fringing field extended a distance of the order of a foot from the edge of the pole pieces and, even at this distance, was strong enough to deflect the low energy electrons in the region of the electron gun. A field strength of the order of 10 gauss was sufficient to deflect electrons leaving the cathode so that they could not enter the pre-accelerator. Attempts to reduce the field below this value by mumetal shields were not successful.

The undesirable effect of the fringing field was partly overcome by placing a small permanent magnet in such a position that the fringing field was cancelled in the cathode region. In the operation of the magnetic analyser, it was necessary to adjust the magnetic field to enable the electrons to pass through the full length of the glass tube. Each time the magnetic field was changed, the beam would disappear entirely until a new position was found for the permanent magnet so that the fringing
field was neutralized. The new position of the permanent magnet was not always easy to find and, in fact, it was often impossible to find such a position.

Later, a slightly more elaborate system of neutralizing the fringing field was used. The permanent magnet was replaced by a pair of coils of a few turns each with a current flowing through them which could be easily adjusted to neutralize the fringing field. In addition to the two coils, two pairs of electrostatic deflection plates were added near the entrance to the pre-accelerator. The combination of electrostatic and magnetic deflection was sufficient to enable the fringing field to be neutralized in nearly all cases, the exception being the case when a very high magnetic field was required. In this case a slight adjustment in the position of the coils was necessary for satisfactory operation.

3.2 High Power Test of the Pre-accelerator

The high power test of the pre-accelerator was carried out using the apparatus shown in the block diagram of Figure 16. To obtain different power levels in the cavity, it was found more convenient to use a variable water attenuator than to change the power output of the magnetron since the magnetron was more stable when operated at a high power level.

After the electron gun voltages were adjusted for maximum beam current into the pre-accelerator, the RF power was turned on and adjusted to some convenient level. The energy of the accelerated electrons was then measured by means of the magnetic analyser. The power flowing into the cavity was then changed
Fig. 16 High Power Test Equipment.
by means of the variable water attenuator, and the energy was measured again.

The measured values of energy are shown in Figure 17. The theoretical values of maximum energy given by Figure 6 are reproduced in Figure 17 to enable a comparison to be made. In the range of energies of interest, say those above 70 KEV, the measured values of energy agree very well with the theoretical values.
Fig. 17 Measured Electron Energy from Pre-accelerator.
4. CONCLUSIONS

The energy stored and the power dissipated in the pre-accelerator cavity have been calculated by assuming that the fields in the cavity are those of a radial transmission line loaded by the capacitance of the gap. The unloaded $Q$ of the cavity was calculated in this way, using theoretical values for the surface conductivity. The $Q$ was checked by experimental measurement and although a lower value was obtained the discrepancy was consistent with the effects of surface roughness and losses in the feeding mechanism. As a result it was possible to predict the accelerating field intensity in the cavity as a function of microwave power input.

The cavity was evacuated and an electron beam, produced by a 2 KV gun, was injected. The output electron energy was measured by means of a magnetic analyser, the experiment being repeated for a range of energies up to 340 KEV corresponding to microwave input powers of up to 190 KW. Good agreement was obtained between measured values and the values predicted by calculation.

The field pattern in the main accelerator was checked by using a perturbation technique. In previous theoretical work the field pattern had been calculated by assuming solid discs so an experimental measurement of the field in the central hole of the disc was made. It was found that the electric field strength in the central hole was very weak, hence, the error in voltage gain caused by assuming the discs to be solid is negligible. The voltage gain which was found from the results of the
perturbation of the cavity agreed very well with the theoretical value.

As a result of this investigation it is concluded that the performance of the pre-accelerator and main accelerator cavities is in accordance with design expectation and that the final assembly of the accelerator can now be completed.
APPENDIX A

THE FIELDS IN THE PRE-ACCELERATOR CAVITY

An expression for the fields in the cavity which everywhere satisfies the boundary conditions may be found in the form of an infinite series. The cavity is divided into two regions, A and B, as shown in Figure A-1.

Fig. A-1 Cavity with Circular Symmetry
In region A the fields are written

\[ E_{za} = \sum_{n=0}^{\infty} A_n \frac{J_0(a_n r) + C_n N_0(a_n r)}{J_0(a_n r_1) + C_n N_0(a_n r_1)} \cos \frac{\pi n}{2} (Z + \ell) \]  \hspace{1cm} \ldots (A-1)

\[ E_{ra} = \sum_{n=0}^{\infty} - \frac{A_n}{a_n} \frac{J_1(a_n r) + C_n N_1(a_n r)}{J_0(a_n r_1) + C_n N_0(a_n r_1)} \frac{\pi n}{2} \sin \frac{\pi n}{2}(Z + \ell) \]  \hspace{1cm} \ldots (A-2)

\[ H_{\rho a} = - j \omega \varepsilon \sum_{n=0}^{\infty} \frac{A_n}{a_n} \frac{J_1(a_n r) + C_n N_1(a_n r)}{J_0(a_n r_1) + C_n N_0(a_n r_1)} \cos \frac{\pi n}{2} (Z + \ell) \]  \hspace{1cm} \ldots (A-3)

where \( a_n^2 = \omega^2 \mu \varepsilon - \left(\frac{\pi n}{2} \right)^2 \).

At radius \( r = r_2 \) \( E_{za} \) must be zero, hence,

\[ C_n = - \frac{J_0(a_n r_2)}{N_0(a_n r_2)} \]  \hspace{1cm} \ldots (A-4)

In region B the fields are written

\[ E_{zb} = \sum_{m=0}^{\infty} B_m \frac{J_0(\tilde{a}_m r) \cos \frac{\pi n}{2d}(Z + \ell)}{J_0(\tilde{a}_m r_1)} \]  \hspace{1cm} \ldots (A-5)

\[ E_{rb} = \sum_{m=0}^{\infty} - \frac{B_m}{\tilde{a}_m} \frac{J_1(\tilde{a}_m r) \sin \frac{\pi n}{2d}(Z + \ell)}{J_0(\tilde{a}_m r_1)} \]  \hspace{1cm} \ldots (A-6)

\[ H_{\rho b} = - j \omega \varepsilon \sum_{m=0}^{\infty} \frac{B_m}{\tilde{a}_m} \frac{J_1(\tilde{a}_m r) \cos \frac{\pi n}{2d}(Z + \ell)}{J_0(\tilde{a}_m r_1)} \]  \hspace{1cm} \ldots (A-7)

where \( \tilde{a}_m^2 = \omega^2 \mu \varepsilon - \left(\frac{\pi n}{2d} \right)^2 \).
The $A_n$ and the $B_m$ of the above expressions are found by matching the fields in the two regions at their common surface, that is, along $r = r_1$. On equating $E_{za}$ to $E_{zb}$ at $r = r_1$, the following expression is obtained:

$$A_o + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi}{2\ell} (Z + \ell) = B_o + \sum_{n=1}^{\infty} B_m \cos \frac{m\pi}{2d} (Z + d) \quad \ldots (A-8)$$

Similarly, by equating $H_{\rho a}$ to $H_{\rho b}$ at $r = r_1$ it is found that

$$\frac{A_o}{a_o} R_{ao} + \sum_{n=1}^{\infty} \frac{A_n}{a_n} R_{an} \cos \frac{n\pi}{2\ell} (Z + \ell)$$

$$= \frac{B_o}{a_o} R_{bo} + \sum_{m=1}^{\infty} \frac{B_m}{a_o} R_{bm} \cos \frac{m\pi}{2d} (Z + d) \quad \ldots (A-9)$$

where

$$R_{an} = J_1(a_n r_1) + C_n N_1(a_n r_1)$$

$$J_0(a_n r_1) + C_n N_0(a_n r_1)$$

and

$$R_{bm} = \frac{J_1(b_m r_1)}{J_0(b_m r_1)}.$$  

Expressions for the $A_n$ and the $B_m$ are obtained by expanding the right hand side of equation (A-8) and the left hand side of equation (A-9) in Fourier series as follows:
\[ A_0 = \frac{1}{2\ell} \int_{-d}^{d} \left[ B_0 + \sum_{m=1}^{\infty} B_m \cos \frac{m\pi}{2d}(Z + d) \right] dZ \] 

...(A-10)

\[ A_n = \frac{1}{\ell} \int_{-d}^{d} \left[ B_0 + \sum_{m=1}^{\infty} B_m \cos \frac{m\pi}{2d}(Z + d) \right] \cos \frac{n\pi}{2\ell}(Z + \lambda) dZ \] 

...(A-11)

\[ \frac{B_0}{a_0} R_{bo} = \frac{1}{2d} \int_{-d}^{d} \left[ \frac{A_0}{a_0} R_{ao} + \sum_{n=1}^{\infty} \frac{A_n}{a_n} R_{an} \cos \frac{n\pi}{2\ell}(Z + \lambda) \right] dZ \] 

...(A-12)

\[ \frac{B_{\beta}}{a_{\beta}} R_{bp} = \frac{1}{d} \int_{-d}^{d} \left[ \frac{A_0}{a_0} R_{ao} + \sum_{n=1}^{\infty} R_{an} \cos \frac{n\pi}{2\ell}(Z + \lambda) \right] \cos \frac{n\pi}{2d}(Z + d) dZ \] 

...(A-13)

Integration and evaluation of the above expressions yields

\[ A_0 = B_0 \frac{d}{\ell} \] 

...(A-14)

\[ A_n = \frac{4B_0}{n\pi} \sin \frac{n\pi d}{2\ell} \cos \frac{n\pi}{2} + \frac{2}{\ell} \sum_{m=1}^{\infty} n B_m X_{mn} \] 

...(A-15)

where

\[ X_{mn} = \frac{\sin\left(\frac{n\pi}{2} - \frac{n\pi d}{2\ell}\right) - \cos\pi \sin\left(\frac{n\pi d}{2\ell} + \frac{n\pi}{2}\right)}{n! \left[ \left( \frac{m}{d} \right)^2 - \left( \frac{n}{\ell} \right)^2 \right]} \]
\[
\frac{R_o R_{bo}}{a_o} = A_o \frac{Rao}{a_o} + \frac{1}{\pi d} \sum_{n = 1}^{\infty} \frac{Ran}{a_n} \sin \left(\frac{n\pi d}{2k}\right) \cos \frac{n\pi}{2} 
\]

...(A-16)

\[
\frac{B_p R_{bp}}{B_o a_p} = \frac{1}{d} \sum_{n = 1}^{\infty} 2n \frac{A_n}{a_n} \frac{Ran X_{pn}}{\sin \left(\frac{n\pi d}{2k}\right) \cos \frac{n\pi}{2}} 
\]

...(A-17)

where \(X_{pn}\) is the same as \(X_{mn}\) except that \(m\) is replaced by \(p\).

By substituting equations (A-14) and (A-15) into equations (A-16) and (A-17) the following equations can be obtained:

\[
\frac{R_{bo}}{a_o} = \frac{Rao}{a_o} \frac{d}{\pi d} + \frac{4k}{\pi^2 d} \sum_{n = 1}^{\infty} \frac{Ran}{a_n^2} \left(\sin \frac{n\pi d}{2k} \cos \frac{n\pi}{2}\right)^2 
\]

\[
+ \frac{2}{\pi d} \sum_{n = 1}^{\infty} \sum_{m = 1}^{\infty} \frac{Ran}{a_n} \frac{B_m}{B_o} \left(\sin \frac{n\pi d}{2k} \cos \frac{n\pi}{2}\right) X_{mn} 
\]

...(A-18)

\[
\frac{B_p R_{bp}}{B_o a_p} = \frac{8}{\pi d} \sum_{n = 1}^{\infty} \frac{Ran}{a_n} X_{pn} \sin \frac{n\pi d}{2k} \cos \frac{n\pi}{2} 
\]

\[
+ \frac{4}{d} \sum_{n = 1}^{\infty} \sum_{m = 1}^{\infty} n^2 \frac{Ran}{a_n} \frac{B_m}{B_o} X_{pn} X_{mn} 
\]

...(A-19)
The resonant frequency of the cavity may be found by substituting values of frequency into equation (A-18) until a value is found which satisfies the equation. It was found that the last term of equation (A-18) could be neglected when calculating the resonant frequency. In the case of the pre-accelerator cavity, the resonant frequency was calculated to be 3100 Mc/s.

Evaluation of equations (A-15) and (A-17) provide the following values for the coefficients $A_n$ and $B_m$:

\[
\begin{align*}
A_1 &= 0 \\
B_1 &= 0 \\
\frac{A_2}{A_0} &= -1.617 \\
\frac{B_2}{B_0} &= 0.369 \\
A_3 &= 0 \\
B_3 &= 0 \\
\frac{A_4}{A_0} &= -0.254 \\
\frac{B_4}{B_0} &= 0.364
\end{align*}
\]

Hence, the magnetic fields in the two regions can be written as

\[
H_{na} = -j\omega A_0 \left[ G_0 J_1(a_0 r) + C_0 N_1(a_0 r) \\
- 1.617 G_2 J_1(a_2 r) + C_2 N_1(a_2 r) \cos \frac{\pi}{\lambda} (Z + \ell) \\
- 0.254 G_4 J_1(a_4 r) + C_4 N_1(a_4 r) \cos \frac{2\pi}{\lambda} (Z + \ell) \right]
\]

where

\[
G_n = \frac{a_n}{a_n J_0(a_n r_1) + C_n N_0(a_n r_1)}
\]

\[
C_n = -\frac{J_0(a_n r_2)}{N_0(a_n r_2)}
\]
\[ H_{\phi b} = - j \omega \epsilon B_0 \left[ F_0 J_1 (\tilde{a}_0 r) + 364 F_2 J_1 (\tilde{a}_2 r) \cos \frac{\pi}{d} (Z + d) \right. \\
\left. + 364 F_4 J_1 (\tilde{a}_4 r) \cos \frac{2\pi}{d} (Z + d) \right] \quad \text{(A-22)} \]

where

\[ F_n = \frac{1}{a_m J_0 (a_m r_1)} \]
REFERENCES


