NOISE FIGURE
OF
THE TRANSISTOR AMPLIFIER

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Abstract

The noise figure of the transistor amplifier is of special interest since the large quantity of excess noise present limits the usefulness of the transistor as a low-noise amplifier.

While the equivalent noise voltages generated in the emitter and collector leads vary inversely with frequency, the noise figure does not. This variation from the inverse frequency characteristic is produced by the decrease of the effective gain at higher frequencies. The subject of this thesis is the investigation of this variation.

The variation of the noise figure with frequency has been measured for several different transistors, with varying emitter and collector currents.

Equivalent diagrams have been suggested which explain the variation of the noise figure with frequency if transit time dispersion is negligible. For low frequencies, the noise figure is given by,

\[ F = \frac{N_e^2}{4KTR_B R_g} \left( \frac{R_g + R_C + R_b}{R_b + R_m} \right)^2 \]

Variation of the noise figure due to the stray capacities is given approximately by

\[ F = \frac{N_e^2}{4KTR_B R_g} \left[ \frac{(R_g + R_C + R_b)^2 + R_g^2 (R_e + R_b)^2 + 2R_g (R_e + R_b)(C_1 C_2) \omega^2 \times 10^{-12}}{(R_b + R_m)^2} \right] \]
The variation, due to transit time effects, ignoring transit time dispersion, is

\[ F = \frac{N_0^2 (T_e + T_r + T_0)^2}{4kT R q} \left\{ \frac{(T_e/T_r)^2 + \omega^2 T_e}{1 + e^{-\omega^2 T_e/T_r} - 2e^{-\omega T_e/T_r} \cos \omega T_e} \right\} \]

The operating conditions for minimum noise figure of the transistor amplifier are dependent on the frequency. At low frequencies, a low value of emitter current should be used, and the lowest possible value of collector voltage which is compatible with the desired gain.

At higher frequencies a low value of emitter current should also be used but the value of the collector voltage is determined by the frequency. The higher the frequency, the higher will be the required collector voltage.

The best noise figures at higher frequencies will be obtained with transistors with the following characteristics:

(i) \( r_m \) large
(ii) close spacing of point contacts
(iii) high value of resistivity
(iv) small transit time dispersion

Equivalent diagrams have been suggested which partially account for the variation of the noise figure with frequency. It has been suggested that transit time dispersion would explain the remaining deviation of the noise figure from the inverse frequency characteristic.
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NOISE FIGURE
OF
THE TRANSISTOR AMPLIFIER

I Introduction

The noise figure of an amplifier is of the utmost importance, for example the limit of sensitivity of an amplifier is determined by this factor. In the transistor or crystal triode amplifier, due to the large quantity of excess noise present, this rating of an amplifier is of special importance. Although sufficient gain is present in an amplifier, unless the signal power output is large with respect to the noise output, the signal may be masked completely or marred sufficiently to destroy its usefulness as a means of communicating information.

The type-A transistor developed at the Bell System Telephone Laboratories by Drs. Bardeen and Brattain under the direction of Dr. Shockley* consists of two small cats' whiskers in contact with a small block of germanium. One of the probes is biased in the positive direction and is called the emitter. The second probe is biased in the negative direction and is designated the

# All numbered references given in "Literature Cited"
collector. Variation of current in the emitter circuit produces a variation of from two to five times that in the collector circuit. Therefore the transistor acts as a current amplifier. Also, because the input circuit is a low impedance and the output circuit is a high impedance, a voltage gain is obtained. Power gains of the order of 20 db or 100 times and voltage gains of 30 db or 35 times are easily obtained with the type-A transistor.

Although the gain of the crystal triode compares well with that of a vacuum triode, the noise figure is much worse. The noise figure of a low-noise vacuum-tube amplifier is of the order of 10 db in a range of 100 cycles up to 30 megacycles. By comparison the transistor amplifier has a noise figure of about 70 db at 100 cycles and of about 30 db at 10 megacycles, with wide variation in individual transistors. In the lower frequency range the noise figure decreases almost inversely with frequency. At higher frequencies of the order of 100 kilocycles to 10 megacycles, variation of current amplification increases the noise figure to values higher than that to be expected from the inverse frequency variation. This variation can be explained by the effects of the actual physical capacitance of the transistor circuit and of the transit time effects in the germanium triode.

The magnitude of the noise generated in the transistor is nearly proportional to the collector voltage, but the greater the collector voltage, the higher the
frequency before the amplifier deviates from the noise spectrum that is inversely proportional to frequency.

The equivalent noise voltage generated in the emitter and the collector leads have been measured by H. C. Montgomery of the Bell Laboratories. These noise voltages squared vary nearly inversely with frequency. The noise figure will also vary inversely with frequency at low frequencies. However at higher frequencies the noise figure will no longer vary in this manner.

The subject of this thesis is the investigation of this variation.

II General Theory

A Noise Measurement

1. Noise Figure

The noise figure $F$ of a four-terminal network has been defined by Friis as the ratio of the available signal-to-noise ratio at the signal generator terminals to the available signal-to-noise ratio at its output terminals.

![Four-Terminal Network Diagram](Fig. 1 Four-Terminal Network)
Several definitions will be given to make possible a more precise mathematical definition of this quantity.

2. Available Signal Power

The available signal power $S_j$ is the maximum that a generator of impedance $R_0$ ohms and electromotive force $E$ can deliver. This is equal to

$$ S_j = \frac{E^2}{4R_0} \quad (1) $$

3. Gain

The gain of a network is the ratio of the available signal power at the output terminals of the network to the available signal power at the output terminals of the signal generator. If $S$ equals the available output power at the output terminals, the gain

$$ G = \frac{S}{S_j} \quad (2) $$

4. Available Noise Power

The available noise power at the input of the network is that generated by the thermal agitation in the equivalent internal-resistance of the signal generator. This available thermal noise of Johnson-noise power is equal to $KTB$, where $K$ is Boltzman's constant, $T$ is the absolute temperature, and $B$ is the bandwidth.

The available noise power $N$ at the output of the network is due to all the noise sources in the network and the signal generator thermal noise, i.e.,

$$ N = FGKTB \quad (3) $$
5. Effective Bandwidth

The effective bandwidth $B$ of the network is the bandwidth of an ideal band-pass network with gain $G$ which would produce an output of

$$G K TB = \int G_f K T d f$$

(4)

where $G_f$ is the gain at frequency $f$. Therefore,

$$B = \frac{1}{G} \int G_f d f.$$ 

(5)

6. Noise Figure

The noise figure will now be defined in terms of $S_q, S, K TB$, and $N$. Repeating, the noise figure $F$ is the ratio of the available signal-to-noise ratio at the input terminals to the available signal-to-noise ratio at the output terminals. Therefore,

$$F = \frac{S_q / K TB}{N/S}$$

(6)

7. Noise Figure Measured with Signal Generator

If the signal generator output is increased until the total power output is double that due only to the internal noise of the network and the Johnson-noise in the generator impedance, then the noise figure

$$F = \frac{S_g}{K TB} = \frac{V^2}{4 R K TB}$$

(7)

If $K = 1.38 \times 10^{-23}$ joules/°C, $T = 290$° Kelvin, this
becomes

\[ F = \frac{V^2}{4R B} \]  

(8)

The noise figure can now be determined by calculating the effective bandwidth by manually integrating the gain-versus-frequency curve.

8. Noise Figure Measured with Noise Diode

The mean-square noise current of a temperature limited diode is given by the Schottky formula as

\[ \overline{i^2} = 2e I_D B \]  

(9)

If the diode load is \( R \) the mean-square-noise voltage due to the diode current and from Johnson-noise is

\[ \overline{e^2} = 4kT RB \left( 1 + \frac{eI_D R}{2kT} \right) \]  

(10)

Fig. 2 Diode Noise Generator

With the noise generator off, the output noise power is

\[ N = FGKTB \]

With the noise generator on, the total output power is
\[ N_0 = kTB \left( F + \frac{eI_0R}{2kT} \right) \] (11)

Solving for \( F \) gives
\[ F = \frac{eI_0R}{2kT} \left( \frac{1}{N_0/N - 1} \right) \] (12)

for \( T = 290^\circ K \),
\[ F = \frac{20I_DR}{(N_0/N - 1)} \] (13)

For the special case where the noise output is doubled
\[ F = 20I_DR \] (14)

9. Comments

The noise diode method of measuring noise figure is far the easiest both from a point of time and equipment. The noise figure can be obtained by a single measurement and an easy calculation. On the other hand, for the signal generator method, the bandwidth has to be laboriously plotted and many calculations made, although to a fair approximation \( B \) can be taken as the distance between half-power points.

The advantages of the diode noise source are limited however. At high frequencies the impedance of the generator is difficult to determine and it is necessary to minimize reactive components to prevent variation over the bandwidth. At low frequencies the Schottky formula does not apply due to flicker effect.4
Transistor Theory

The type-A transistor consists of a small block of germanium against which two point contacts are made. The germanium used is similar to that in high-back-voltage rectifiers such as described by Torrey and Whitmer. The surface is ground flat and then etched and welded on a brass plug. This plug is forced into the metal barrel of the transistor as shown in Fig. 3. The point contacts are made of phosphor-bronze wire .002" to .005" in diameter and cut in wedge shape to facilitate close spacing. These are welded on to supporting pins on an insulating plug. After the points are manually adjusted until the separation is approximately .002", the insulating plug is pushed into the metal
cartridge until contact is made with the germanium slug.

One of the points is biased in the positive direction and is called the emitter. The second point, the collector, is biased in the negative direction. The brass base makes an ohmic contact with the germanium crystal. The emitter and the collector can be considered separately as two diodes. The emitter is biased in the forward direction and the collector in the backwards direction. Therefore, the emitter exhibits a low resistance and the collector exhibits a high resistance.

When the emitter is placed sufficiently close to the collector, variation of the emitter current will influence the collector current. The variation of the collector current is actually greater than that of the emitter current, so current amplification takes place. Also, since the emitter circuit is of low impedance while the collector circuit is of high impedance, a voltage gain can be obtained. This current amplification can be explained by the theory of solid state. Only a brief summary will be given of one proposed by Bardeen and Brattain. 6

The flow of electricity in semiconductors consists of negative electrons and positive holes. The negative carriers are excess electrons which are free to move or have energies in the conduction band of the crystal. The positive carriers are defect electrons or unoccupied energy states in the filled band of the crystal. The
germanium used in the transistor is n-type which has an excess of negative carriers. The current from the emitter, which is biased in the positive direction, consists largely of holes. These holes are attracted towards the negatively biased collector. The largest part of this hole current from the emitter flows into collector circuit since the collector contact offers only a low resistance to these holes. The hole current flowing into the collector circuit also alters the space charge in the barrier layer at the collector. This change in the barrier layer charge reduces the resistance to the flow of electrons from the collector. In this fashion a current change in the emitter circuit can produce a change of the collector current of as much as ten times. Normally this change is of the order of two.

Fortunately for small signal consideration, linear signal theory can be used and the operation of the transistor can be represented by an equivalent circuit. The circuit used most often is the equivalent T shown in Fig. 4.

![Circuit Representation of Transistor](image)

**Fig. 4 Circuit Representation of Transistor**

Fortunately this equivalent diagram has a significant parallel with the actual physical situation. The base
resistance $r_b$ is actually the resistance to the flow of current from the germanium to the base. The emitter resistance $r_e$ is the actual resistance to the flow of current from the emitter into the crystal. The collector resistance $r_c$ is the actual resistance to the flow of current from the collector into the crystal. The mutual transfer resistance $r_m$ represents the active property of the network produced by the alteration of the impedance to the flow of current in the collector circuit by the emitter current.

This representation of the transistor is only applicable at low frequencies. The contact impedances $r_b$, $r_e$, and $r_c$ are pure resistances in the useful range of the transistor. This is as would be expected from previous measurements on crystal diodes. However the mutual transfer-impedance $r_m$ will vary at higher frequencies and should be represented by $Z_m$ which is a function of frequency.

The equivalent resistances can be determined directly from the static characteristics of the transistor. Since the transistor is essentially a current amplifying device, and because the voltage is a single-valued variable of the current, the emitter current and the collector current are selected as independent variables. The static characteristics of the transistor are shown in Fig. 5 where $E_e$ and $E_c$ are plotted against $I_e$ and $I_c$. Slopes of the different curves yield the values of the equivalent circuit components. Since the currents are the independent
Fig. 5 Static Characteristics of Transistor
variables,
\[ V_e = f_1(I_e, I_c) \]  \hspace{1cm} (15)
\[ V_c = f_2(I_e, I_c) \]  \hspace{1cm} (16)

Expanding these functions in a Taylor series and considering only first order terms,
\[ \Delta V_e = \frac{\partial f_1}{\partial I_e} \Delta I_e + \frac{\partial f_1}{\partial I_c} \Delta I_c \]  \hspace{1cm} (17)
\[ \Delta V_c = \frac{\partial f_2}{\partial I_e} \Delta I_e + \frac{\partial f_2}{\partial I_c} \Delta I_c \]  \hspace{1cm} (18)

Therefore the general circuit impedances are as follows:

The input impedance with the output open-circuited for AC is,
\[ R_{11} = \frac{\partial V_e}{\partial I_e} \]  \hspace{1cm} (19)

The output impedance with the input open-circuited for AC is,
\[ R_{22} = \frac{\partial V_c}{\partial I_c} \]  \hspace{1cm} (20)

The backward transfer impedance with the input open-circuited for AC is,
\[ R_{12} = \frac{\partial V_e}{\partial I_c} \]  \hspace{1cm} (21)

The forward transfer impedance with the output open-circuited for AC is,
\[ R_{21} = \frac{\partial V_e}{\partial I_e} \]  \hspace{1cm} (22)

Therefore in the static characteristics
\[ R_{11} = \text{slope of curve 5 (a)} \]  \hspace{1cm} (23)
\[ R_{12} = \text{slope of curve 5 (b)} \]  \hspace{1cm} (24)
\[ R_{21} = \text{slope of curve 5 (c)} \]  \hspace{1cm} (25)
\[ R_{22} = \text{slope of curve 5 (d)} \]  \hspace{1cm} (26)
From these the circuit parameters are ascertained,

\[ r_e = R_{11} - R_{12} \quad \text{and} \quad r_c = R_{22} - R_{12} \]

\[ r_b = R_{12} \quad \text{and} \quad r_m = R_{21} - R_{12} \]

The equivalent diagram so far given does not include any noise sources that are actually present in the transistor. Peterson has shown that the noise in a 4-terminal network can be considered as originating in two noise generators included in the arms of the network as shown in Fig. 6, where

\[ N_e = \text{Equivalent noise voltage in emitter lead} \]

\[ N_c = \text{Equivalent noise voltage in collector lead} \]

Fig. 6 Equivalent Circuit of Transistor

The noise voltage per unit bandwidth has been measured in some detail by Montgomery. It has been shown that for the crystal diode, the noise voltage squared per unit bandwidth varies nearly inversely with frequency. The
The slope of the noise spectrum varies between 0.9 and 1.2. The magnitude of $N_e$ is of the order of one-hundredth of the value of $N_c$. Also, some correlation between the two noise sources has been demonstrated, but it is of a highly unpredictable magnitude.

No complete theory of the sources of the excess noise in semiconductors has been presented. A. Van der Ziel has shown that if a sufficiently wide distribution of relaxation times is considered the inverse variation of noise power with frequency can be explained. He suggests that this wide distribution function could be produced by five different sources of noise. These are thermal noise, shot noise, flicker effect, local fluctuations in conductivity due to diffusion of foreign atoms or lattice distortions, and spontaneous fluctuations in temperature.

Although the cause of the excess noise in semiconductors is not fully understood, considerable improvement has been made with the noise level in crystal diodes, and it is hoped that similar improvements will be made in the transistor.

### III Description of Equipment

The equipment was designed to measure the noise figure of the transistor amplifier in the frequency range from 100 cycles per second to 10 megacycles per second. Where possible the noise-diode generator method of measuring the noise figure was used because of this greater ease of
Measurement and greater accuracy. This was done in the range 75 kilocycles to 10 megacycles where the most extensive measurements were made. In the audio frequency range however, measurement was made by the signal generator method. Because of this the equipment divided itself into two test sets, that used at audio frequencies and that used at radio frequencies.

A Low Frequency Equipment

The low frequency test set, a block diagram of which is shown in Fig. 20, consisted of an audio signal generator with a range of from 10 cycles to 100 kilocycles, a vacuum tube voltmeter for measuring the input signal, an attenuator, the transistor amplifier, a high-gain low-noise preamplifier, a selective amplifier, and a square law output meter.

The complete circuit diagram is shown in Fig. 21. The high level of the noise in the transistor at the lower audio frequencies made this method of measurement practical. The necessary input needed to double the power output of the circuit was obtained with a reasonable attenuation from an easily measurable voltage. This fact minimized stray field consideration which makes this method of measurement difficult at low signal levels. The high gain preamplifier consisted of two 6AK5 pentodes with ample decoupling to ensure stability and a DC filament supply to reduce hum. The selective amplifier employed a twin-T feedback circuit with coupling through a cathode follower and a series
connected amplifier. The output of the selective amplifier was fed to a square-law detector suggested by Miller.\(^\text{13}\)

The frequency was changed by plug-in resistors in the twin-T feed-back circuit. The difficulty with this circuit was the necessity of realigning it for each frequency and the replotted of the selectivity curve. It was found necessary to use a long time constant of approximately thirty seconds in the output circuit to smooth out periodic fluctuations in the noise output.\(^\text{14}\) Unfortunately this increased the possibility of zero drift during measurement. The whole test set, with the exception of the signal generator and vacuum tube voltmeter, was built on a single 17" x 13" x 4" chassis, with compartment shielding, and mounted on sponge rubber to minimize instability.

\textbf{B \ High Frequency Equipment}

The high frequency test set, a block diagram of which is shown in Fig. 20, consisted of a diode noise generator, a preamplifier, the transistor amplifier, a cathode follower, an AR88 RCA receiver, a square law detector similar to that used in the low frequency range, and a vacuum tube voltmeter output meter.

The complete circuit diagram is shown on Fig. 23. The noise diode used was a Sylvania type 5722 which has a maximum current rating of 30 ma. With a load of 1000 ohms, which is approximately the input resistance for the best noise figure of the transistor amplifier, noise figures up to 600 may be measured for double output power. With an
increase of only one third output power, which is about the minimum for reliable measurement, noise figures of up to 1800 can be measured. For noise figures above this a 6AC7 preamplifier which had a power gain of about seventeen was used. This made possible the measurement of noise figures up to 30,000. It is well to point out at this time that all calculations depend on the fact that the noise figure of all circuits except the transistor amplifier can be ignored. In cascaded amplifiers the principle source of noise is in the early stages. The total noise figure is given by

\[ F_{1+2} = F_1 + \frac{F_2 - 1}{G_1} \]  

(27)

where \( F_1 \) is the noise figure of the first stage, \( F_2 \) is the noise figure of the second stage, and \( G_1 \) is the gain of the first amplifier. As the minimum noise figure of the transistor is 300 with a gain of 10 to 100, and the noise figure of following amplifiers is 10 or less, all noise following the transistor amplifier can be ignored. When the amplifier is used preceding the transistor, the noise figure is above 1800 so that the noise of the preamplifier with a gain of 17 and a noise figure of 10 need not be considered.

The AR88 receiver was especially suitable because of its low frequency range. The receiver was linearized by operating with the beat frequency oscillator on.

The operation of the set was very simple, allowing rapid compilation of data in the frequency range most desired. The only difficulty of design was in minimizing the input capacity to the transistor amplifier. Since the
5722 diode has an internal capacity of $3 \text{mF}$, this input capacity could not be reduced much below $10 \text{mF}$. Rather than tune this capacity out, it was taken into account in the theoretical calculation to follow.

The high frequency test set, except for the radio receiver, the vacuum tube voltmeter and the power supplies, was also mounted on a 17" x 13" x 4" chassis as a complete unit.

Both sets have built-in controls to vary and measure the emitter and the collector current. From these two readings the operating point on the characteristic curves may be determined since the emitter and collector voltages are both dependent variables of the emitter and collector currents.

IV Theory Applied to Investigations

An analysis of the transistor will now be made from the consideration of the theoretical equivalent circuit. A first analysis will be made for low frequencies where capacitive and transit time effects can be ignored. This will then be extended to include capacities which may

![Equivalent Circuit of Transistor](image)

*Fig. 7 Equivalent Circuit of Transistor*
be ignored, except at the highest frequencies. Then variation of the noise figure due to $j_m$ alone will be considered.

A Low Frequency Case

The noise figure has been defined as the available signal-to-noise ratio at the signal generator to the available signal-to-noise ratio at the output terminals. Consider the equivalent circuit diagram of the transistor shown in Fig. 8.

![Fig. 8 Low Frequency Equivalent Diagram]

The available signal power $S$ at the input terminals is $\frac{E^2}{4R_g}$. The available noise power at the input terminals is $KTB$. Therefore, the available signal-to-noise ratio at the input terminals is

$$\frac{E^2}{4R_g KTB}$$

The available signal power at the output terminals will now be calculated.

The current flowing in circuit two due to the voltage $E$ is one

$$I_2 = \frac{-M_{12}}{D} E$$

(28)
where $D$ is the circuit determinate

$$D = \begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix}$$

(29)

and $M_{12}$ is the minor obtained by cancelling row 1 and column 2. Therefore, the signal output power is

$$P_s = \left( \frac{M_{12}}{D} \right)^2 R_L E^2$$

(30)

The output noise power due to $N$, the thermal noise in the generator, is similarly

$$P_N = \left( \frac{M_{12}}{D} \right)^2 R_L 4KTB$$

(31)

The output noise power due to the noise generator $N_{e}$ in the emitter lead is

$$P_{N_e} = \left( \frac{M_{12}}{D} \right)^2 R_L N_e^2$$

(32)

The output power due to the noise generator $N_c$ in the collector lead is

$$P_{N_c} = \left( \frac{M_{22}}{D} \right)^2 R_L N_c^2$$

(33)

Since the noise voltages are statistically independent events, their output powers may be added arithmetically. This gives a total noise output power

$$P = \left( \frac{M_{12}}{D} \right)^2 R_L \left( 4KTB + \overline{N_e}^2 \right) + \left( \frac{M_{22}}{D} \right)^2 R_L \overline{N_c}^2$$

(34)

A possibility exists of $N_e$ and $N_c$ not being completely independent but the correlation has not been determined as yet, and therefore will not be considered.
\[ F = \frac{\text{available signal at input}}{\text{available noise at input}} \times \frac{\text{noise at output}}{\text{signal at output}} \]

\[ = E^2 \left\{ \frac{(\frac{M_{12}}{D})^2 R_L (4KT \, T + \overline{N_e}^2) + (\frac{M_{22}}{D})^2 R_L \overline{N_c}^2}{4 R_g K T B} \right\} \]

\[ = \frac{1}{4KT B R_g} \left\{ \overline{N_e}^2 + \left( \frac{M_{22}}{M_{12}} \right)^2 \overline{N_c}^2 \right\} \]

\[ = \frac{1}{4KT B R_g} \left\{ \overline{N_e}^2 + \left( \frac{R_{11}}{R_{21}} \right)^2 \overline{N_c}^2 \right\} \]

\[ = \frac{1}{4KT B R_g} \left\{ \overline{N_e}^2 + \left( \frac{R_g + r_e + r_b}{r_m + r_b} \right)^2 \overline{N_c}^2 \right\} \]

(35)

Actually any correlation between \(N_e\) and \(N_c\) should be taken into account, but for simplifying purposes \(N_e\) will be ignored, since \(N_c\) is approximately 50 db. below \(N_c^{\text{17}}\), and \(\frac{R_{21}}{R_{11}}\) is never larger than about 35, \(N_c\) will only slightly affect the noise figure. Therefore, since

\[ F = \frac{1}{4KT B R_g} \left\{ \frac{R_g + r_e + r_b}{r_m + r_b} \right\}^2 \overline{N_c}^2 \]

(36)

If \(N_c\) varies inversely proportional to frequency

\[ F \propto \frac{1}{F} \left\{ \frac{R_{11}}{R_{21}} \right\}^2 \]

(37)

This variation of noise figure is shown on Fig. 9.
Fig. 9 Variation of Noise Figure at Low Frequencies
B High Frequency Case

1. Neglecting Transit Time

For higher frequencies the equivalent circuit diagram of the transistor amplifier is as shown in Fig. 10.

Fig.10 High Frequency Equivalent Diagram Neglecting Transit Time

$C_1$ is the input capacity, $C_2$ is the capacity between the emitter and the collector, and $C_3$ is the barrier layer capacity. This model was suggested by Bradner-Brown.

As in the low frequency case, the available signal-to-noise ratio at the input terminals is

$$\frac{E}{4kT B R_g}$$

To obtain the output powers, first write the general mesh equations for the active network, ignoring $N_e$. 
\[ Z_{11} i_1 + Z_{12} i_2 + Z_{13} i_3 + Z_{14} i_4 + Z_{15} i_5 = E \]
\[ Z_{21} i_1 + Z_{22} i_2 + Z_{23} i_3 + Z_{24} i_4 + Z_{25} i_5 = 0 \]
\[ Z_{31} i_1 + Z_{32} i_2 + Z_{33} i_3 + Z_{34} i_4 + Z_{35} i_5 = -N_c + r_{m} (i_2 - i_4) \]
\[ Z_{41} i_1 + Z_{42} i_2 + Z_{43} i_3 + Z_{44} i_4 + Z_{45} i_5 = -N_c - r_{m} (i_2 - i_4) \]
\[ Z_{51} i_1 + Z_{52} i_2 + Z_{53} i_3 + Z_{54} i_4 + Z_{55} i_5 = 0 \]

This can be rewritten in the passive form as,

\[ Z_{11} i_1 + Z_{12} i_2 + Z_{13} i_3 + Z_{14} i_4 + Z_{15} i_5 = E \]
\[ Z_{21} i_1 + Z_{22} i_2 + Z_{23} i_3 + Z_{24} i_4 + Z_{25} i_5 = 0 \]
\[ Z_{31} i_1 + Z_{32} i_2 + Z_{33} i_3 + Z_{34} i_4 + Z_{35} i_5 = N_c \]
\[ Z_{41} i_1 + Z_{42} i_2 + Z_{43} i_3 + Z_{44} i_4 + Z_{45} i_5 = -N_c \]
\[ Z_{51} i_1 + Z_{52} i_2 + Z_{53} i_3 + Z_{54} i_4 + Z_{55} i_5 = 0 \]

where
\[ Z_{32}' = Z_{32} - r_{m} \quad Z_{35}' = Z_{35} + r_{m} \]
\[ Z_{42}' = Z_{42} + r_{m} \quad Z_{45}' = Z_{45} - r_{m} \]

If the circuit determinant
\[ D = \begin{vmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ Z_{31} & Z_{32}' & Z_{33} & Z_{34} & Z_{35}' \\ Z_{41} & Z_{42}' & Z_{43} & Z_{44} & Z_{45}' \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} \end{vmatrix} \]

the output signal equals
\[ \left( \frac{M_{13}}{D} \right)^2 E^2 R_L \]

and the output noise power equals
\[ \left( \frac{M_{33} + M_{43}}{D^2} \right)^2 N_c^2 R_L \]
Therefore, the circuit noise figure

\[ F = \frac{1}{4kT R_g} \left( \frac{M_{33} + M_{44}}{M_{03}} \right)^2 N_c^2 \]  

(43)

where,

\[ Z_{11} = R_g - jX_1 \quad Z_{25} = -r_e \quad Z_{44} = r_c - jX_3 \]
\[ Z_{12} = jX_1 \quad Z_{34} = 0 \quad Z_{45} = jX_3 - r_m \]
\[ Z_{13} = 0 \quad Z_{32} = -r_b - r_m \quad Z_{51} = 0 \]
\[ Z_{14} = 0 \quad Z_{33} = r_b + r_c + R_L \quad Z_{52} = -r_e \]
\[ Z_{15} = 0 \quad Z_{34} = -r_c \quad Z_{53} = 0 \]
\[ Z_{21} = jX_1 \quad Z_{35} = r_m \quad Z_{54} = jX_3 \]
\[ Z_{22} = r_e + r_b - jX_1 \quad Z_{41} = 0 \quad Z_{55} = r_e - j(X_2 + X_3) \]
\[ Z_{23} = -r_b \quad Z_{42} = r_m \]
\[ Z_{24} = -r_e \quad Z_{43} = -r_c \]

This formula becomes, after considerable manipulation,

\[ F = \frac{\left[ r_e^2 R_g^2 \omega^2 C_2^2 10^{-12} + R_g^2 (r_e + r_b)^2 10^{-12} \omega^2 C_2^2 \right]}{4kT R_g \left[ (r_e + r_c + r_b - r_b r_m)^2 \omega^2 C_2^2 10^{-12} + r_e^2 r_b^2 \omega^2 C_3^2 10^{-24} + 2 r_e r_b (r_e + r_c + r_b - r_b r_m) \omega^2 C_2 C_3 10^{-12} + r_e^2 r_b^2 \omega^2 C_2^2 C_3^2 10^{-24} + (r_b + r_m)^2 \right]} \]  

(44)

where \( \omega = 2\pi f \) and \( f \) is in megacycles per second and \( C_1, C_2, + C_3 \) are in \( \mu F ds \). A typical curve for variation of \( F \) is shown in Fig. 11 for \( C_1 = 10, C_2 = 2, C_3 = 1 \).

\( r_e = 700 \mu \Omega, r_b = 300 \mu \Omega, r_c = 15,000 \mu \Omega, r_m = 65,000 \mu \Omega \)
\( R_g = 1000 \mu \Omega \)
Fig. 11 Variation of Noise Figure Neglecting Transit Time
At all but very high frequencies

\[ F = \frac{N_c}{4\pi^2 R_g} \left\{ \frac{(R_g + r_o + r_b)^2 + \left[(r_o + r_b)C_1 + 2r_c(r_o + r_b)C_2\right] R_g^2 \omega_{10}^{-2}}{(r_o + r_m)^2} \right\} \tag{45} \]

It is seen that stray capacities except for frequencies above 10 megacycles, if kept to the smallest possible values, have little effect on the variation of the noise figure. Therefore, if

\[ N_c^2 \ll \frac{1}{F} \]

\[ F \ll \frac{1}{f} \left[ \left( \frac{R_{11}}{R_{22}} \right)^2 + \frac{R_g^2 (r_o + r_b)^2 C_1^2 + 2r_c (r_o + r_b) C_2}{R_{22}^2} \omega_{10}^{-2} \right] \tag{46} \]

2. Neglecting Stray Capacities

In the previous section, it has been shown that stray capacities have very little effect on the noise figure, for below 10 megacycles. The equivalent diagram may therefore be shown in Fig. 12.

![Fig. 12 High Frequency Equivalent Diagram Neglecting Stray Capacities](image-url)
The noise figure is then

\[ F = \frac{N_c^2}{4kT R_g} \left( \frac{R_g + r_e + r_h}{j_m + r_b} \right)^2 \]  

(47)

and since \( r_b \) is very small with respect to \( j_m \)

\[ F = \frac{N_c^2}{4kT R_g} \left( \frac{R_g + r_e + r_h}{j_m} \right)^2 \]  

(48)

where \( j_m \) is some function of frequency.

In consideration of the filamentary transistor, Shockley, Pearson and Haynes have developed a formula which gives the variation of \( \alpha_e \) with frequency. This expression takes into account the recombination of holes with electrons and the time delay of holes passing to the collector. It can be shown that \( \alpha_e \) is equivalent to \( \frac{j_m}{r_e} \), and therefore any variation of \( \alpha_e \) with \( \omega \) will be duplicated in \( j_m \).

They have given

\[ \alpha_e = \gamma (1 + b) \beta \]  

(49)

where

\[ \beta = \frac{1 - \exp[-i\omega T_e - (T_c/T_p)]}{i \omega T_e + (T_c/T_p)} \]  

(50)

\( T_e \) is the transit time, \( T_p \) is the hole lifetime, \( \gamma \) is the fraction of the emitter current carried by holes, and \( b \) is the ratio of the mobility of electrons to the mobility of holes.

Hence, for the filamentary transistor,

\[ j_m \alpha = \frac{1 - \exp[-i\omega T_e - (T_c/T_p)]}{i \omega T_e + (T_c/T_p)} \]  

(51)

or

\[ j_m = \gamma_m \left[ \frac{1 - \exp[-i\omega T_e - (T_c/T_p)]}{i \omega T_e + (T_c/T_p)} \right] \]  

(52)
Bardeen has given the hole lifetime as $2 \times 10^{-7}$ seconds and the transit time of the holes as $.5 \times 10^{-7}$ seconds.

Then

$$F = \frac{Nc^2(\beta_g + \beta_e + \beta_m)}{4KB_q \beta_g \beta_m} \left\{ \frac{(\tau_e/\tau_p)^2 + \omega^2 \tau_e^2}{1 + e^{-2\tau_e/\tau_p} - 2 e^{-\tau_e/\tau_p} \cos \omega \tau_e} \right\}$$  \hspace{1cm} (53)

and, as a function of frequency,

$$F \propto \frac{1}{f} \left\{ \frac{(\tau_e/\tau_p)^2 + \omega^2 \tau_e^2}{1 + e^{-2\tau_e/\tau_p} - 2 e^{-\tau_e/\tau_p} \cos \omega \tau_e} \right\}$$  \hspace{1cm} (54)

The variation of $F$ with frequency for the particular case having $\tau_p = 2 \times 10^{-7}$ seconds and $\tau_e = .5 \times 10^{-7}$ seconds is shown in Fig. 13.

This variation does not include the effects due to transit time dispersion. In the type-A transistor, the path that all the holes follow is not the same. The holes on leaving the emitter tend to follow the potential gradient. The shape of this field caused by two non-point sources in contact with a relatively large body having different potentials, is extremely complex. It is further complicated by the presence of space charges and a rectifying barrier. Even if the shape of the field could be determined, the problem is still further complicated by the existence of two different types of current carriers which must be considered statistically. Therefore, no attempt has been made to predict the variation of $F$ due to transit time dispersion.
V Results

The variation of the noise figure of the transistor amplifier with emitter current, collector current, and frequency was measured.

In Fig. 14 is shown the variation of the noise figure for four different transistors in the frequency range 100 kilocycles to 10 megacycles. It is easily seen that there is very little uniformity in the results obtained from various transistors. At one megacycle, the noise figures vary from 600 to 32,000, a range of 17 db. Variation of the magnitude of $\nu_m$ under similar operating conditions will result in wide variation of the noise figure, since $F$ is inversely proportional to $\nu_m$ squared. This alone would not account for the wide variation of the noise figure, but probably the noise voltages generated vary considerably in different units.

In Fig. 15 the variation of noise figure with $V_C$ is shown for a single unit at a frequency of 500 kilocycles. The deviation of the curve from the general pattern at the lower voltages is due to the increase of transit time effects. It is seen that the noise figure is very nearly proportional to the collector voltage.

In Fig. 16 the variation of noise figure for a transistor amplifier in the range from 100 cycles to 10 megacycles is shown for two different operating points. The variation of the noise figure in the frequency range from 100 kilocycles to 10 megacycles is shown in Fig. 17 for different values of collector current. The noise figure
Fig. 13 Variation of Noise Figure Neglecting Stray Capacities
Fig. 14 Variation of Noise Figure between Transistors
Fig. 15 Variation of Noise Figure with Collector Voltage
Fig. 16 Variation of Noise Figure with Operating Point
Fig. 17 Variation of Noise Figure for Different Collector Currents
variation in the frequency range from 100 kilocycles to 10 megacycles is plotted in Fig. 18 for different values of emitter current.

These curves indicate that the noise figure decreases at some rate less than inversely proportional to frequency at radio frequencies. In fact, the noise figure increases above a certain frequency. The lower the voltage on the collector, the lower is the frequency at which these effects become important. The variation in both the curves of noise figure for different collector and emitter currents can be attributed to the changes in collector voltage. However, the lowest noise figure at a particular frequency is not obtained with the highest collector voltage. Fig. 19 shows the optimum values of $I_c$ for varying emitter currents.

<table>
<thead>
<tr>
<th>Freq</th>
<th>$I_e$</th>
<th>$I_c$</th>
<th>$F$</th>
<th>$I_e$</th>
<th>$I_c$</th>
<th>$F$</th>
<th>$I_e$</th>
<th>$I_c$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mc</td>
<td>0.2 m</td>
<td>1.0 m</td>
<td>720</td>
<td>0.4 m</td>
<td>2.0 m</td>
<td>10.0 m</td>
<td>0.8 m</td>
<td>2.0 m</td>
<td>1150</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>350</td>
<td>2.0</td>
<td>390</td>
<td>2.0</td>
<td>370</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>270</td>
<td>3.5</td>
<td>370</td>
<td>3.5</td>
<td>340</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3.0</td>
<td>270</td>
<td>3.5</td>
<td>370</td>
<td>3.5</td>
<td>490</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 19 Optimum Operating Points at Different Frequencies

The above tabulation shows that the best noise figure is obtained with a low emitter current and that the best collector current depends on the frequency.
Fig. 18 Variation of Noise Figure for Different Emitter Currents
VI Conclusions

(a) The noise figure of the transistor amplifier is very nearly inversely proportional to frequency at frequencies where the transit time is not important. This agrees with the low frequency equivalent diagram for which

$$F = \frac{N_c^2}{4KTBR_q} \left\{ \frac{R_g + r_e + r_b}{r_b + r_m} \right\}^2$$  \hspace{1cm} (55)

This agreement depends on the inverse frequency characteristic of $N_c^2$.

(b) The variation of the noise figure with frequency will be further changed by the stray capacities as shown in Fig. 11 and

$$F = \frac{N_c^2}{4KTBR_q} \left\{ \frac{(R_g + r_e + r_b)^2 + 1}{r_b + r_m} \right\}$$  \hspace{1cm} (56)

This only becomes important above 7 or 8 megacycles, and is usually overshadowed by other effects.

(c) The variation of noise figure will be modified by change in $j_m$ with frequency. This variation is produced by recombination of holes with electrons during the time taken for the holes to pass from the emitter to the collector and the reduction due to the time of transit being comparable with the period of the signal. The noise figure, as plotted in Fig. 13 is given by

$$F = \frac{N_c^2}{4KTBR_g r_m^2} \left\{ \frac{(T_e/T_B)^2 + \omega^2 T_e^2}{1 + e^{-2\pi T_e/T_B} \cos \omega T_e} \right\}$$  \hspace{1cm} (57)
Only at high collector voltage where the transit time is high do the above variations explain the observed changes of the noise figure with frequency. The noise figure at higher frequencies is considerably above that to be expected. This increase probably can best be explained by transit time dispersions. That is, some holes in passing from the emitter to the collector, do not follow the direct path but follow a round-about path. Therefore, not all the holes will have the same transit time, and as a result at the higher frequencies decreases the effective value of the transfer impedance.

(d) Where the noise level is of primary importance, some general rules for the operation of the transistor amplifier can be given.

At low frequencies, a low value of emitter current should be used. Also, the lowest possible value of collector voltage should be used that is compatible with the desired gain.

At higher frequencies a low value of emitter current should be used but the value of the collector voltage is determined by the frequency. Fig. 19 indicates the variation of the best operating collector voltages at several radio frequencies.

(c) Some general criteria can be established as to the characteristics that are most desired for operating with the lowest possible noise figure.
The larger the value of $r_m$ the lower the noise figure will be since $F$ is very nearly inversely proportional to $r_m^2$. This applies at both high and low frequencies. The high frequency noise figure can be improved by reducing the transit time effects. The transit time

$$T_c = \frac{2\pi S}{3\mu_h \rho \bar{I}_c}$$

(58)

and therefore the closer the spacing, the smaller the transit time. The value of $\bar{I}_c$ is determined by other considerations and therefore can not be greatly altered. Also, the higher the value of the resistivity, $\rho$, the lower the transit time. Bradner-Brown has shown variations of $\alpha_e$ with frequency, with spacing, collector current, and bulk resistivity.

The largest reduction of $\beta_m$ with frequency is caused by transit time dispersion. This has been reduced by using magnetic biasing and by making the transistor in the form of a filament. In the filamentary transistor, very little dispersion can take place, and

$$F = \frac{N e^2 (R_e + R_t + R_b)}{4KT B R_g r_m^2} \left\{ \left( T_c / T_p \right)^4 + \frac{\omega^2 T_c}{1 + e^{-2T_c/T_p} - 2e^{-T_c/T_p}\cos\omega T_e} \right\}$$

(59)

would very nearly apply. With magnetic biasing as used by Bradner-Brown, and Shockley and Suhl, the magnetic field tends to make the holes travel from the emitter to the collector within narrower time limits, thus reducing transit time dispersion.

Variation with frequency of the noise figure of the transistor amplifier has been plotted and this variation has been partly explained due to capacitance and transit time.
effects. It has been suggested that the observed variations from the theory could be caused by transit time dispersion. Some general rules for low noise operation of the transistor have been given and a list of desirable characteristics have been given for low noise operation.
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W.L. Hatton
VTVM

OSC.  

Transistor Amp.  

Pre-Amp.  

Selective Amp.  

Square Law Det.  

Output Meter  

Low-Frequency Block Diagram

Diode Noise Gen.  

Pre-Amp.  

Transistor Amp.  

Cath. Follower  

AR88 Receiver  

Square Law Output Meter  

High-Frequency Block Diagram

Fig. 20 Block Diagrams of Test Equipment
Fig. 21 Low Frequency Equipment Circuit Diagram
Fig. 22  Low Frequency Test Set
Fig. 23  High Frequency Equipment Circuit Diagram
Fig. 24 High Frequency Test Set