COHERENT LOCAL OSCILLATORS FOR A 21 C.M. SUPERSYNTHESIS EXPERIMENT

by

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ABSTRACT

A method based on phaselock techniques, for synchronizing the local oscillator signals in a proposed two-dish supersynthesis at 1420 MHz is presented. To demonstrate the feasibility of this method, the design and construction of a working system that provides phase-coherent, 1390 MHz signals at two sites, separated by a time-varying path length, is described.

The phase accuracy of this system is $\pm 5^\circ$. A provision for introducing a known phase difference between the two signals, in a manner that is suitable for interfacing with a digital computer, is included. Also, operation of the system over a frequency range greater than the expected range of doppler shift is possible, without the risk of locking to a wrong sideband.

Transistor microwave oscillators at 1.4 GHz are used as voltage-controlled oscillators in this system. The performance of these devices is compared with that of the conventional voltage-controlled crystal oscillator/multiplier chain.

Test results are given, which indicate that the system is suitable for use in an operational environment.
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1. INTRODUCTION

1.1 Supersynthesis

The requirements for greater resolution and sensitivity in radio astronomy experiments can only be met by radio telescope antennas with apertures larger than those presently available. The high cost and the difficulties involved in the construction of large conventional antennas, has led to the development of aperture synthesis techniques.

The signal delivered to the receiver of a radio telescope can be considered as that resulting from the vector addition of the currents that are induced at each point across the aperture. For a parabolic dish, this vector addition is accomplished automatically at the focus. In the case of a multi-element array, the addition can be performed by connecting each of the elements to the receiver through equal length cables.

For an N element array, the current at the receiver input terminals that results from an incident plane wave is \( \sum_{x} I_{x} e^{j\phi_{x}} \), where \( \phi_{x} \) is the phase of the wave at the xth element. The power delivered to the receiver would then be given by

\[
P = \alpha \left( \sum_{x} I_{x} e^{j\phi_{x}} \sum_{y} I_{y} e^{-j\phi_{y}} + \sum_{x \neq y} I_{x} I_{y} \cos (\phi_{x} - \phi_{y}) \right) (1-1)
\]

The first term in (1-1), is simply N times the power received by a single element. The terms \( I_{x} I_{y} \cos (\phi_{x} - \phi_{y}) \) correspond to the output of a correlation receiver connected between the xth and the yth elements. If, then, it were possible to measure separately the currents at a pair of elements in the array, then the results ob-
obtained from all such element pairs could be combined to provide the same information as would be obtained from the physical array. This, indeed, is the basis of the aperture synthesis method. The spacing and orientation of two small receiving elements is varied over an area equivalent to that of the physical aperture that is to be synthesized.

In the method of supersynthesis, developed by Ryle,\(^{(1)}\) two dish antennas are constrained to move along an east-west baseline. Movement of the dishes in the north-south direction is accomplished by making use of the rotation of the earth. This can be seen with the aid of Figure 1-1.

![Figure 1-1](image)

**Figure 1-1** Supersynthesis, showing the relationship between the dish positions and the synthesized aperture.
During each 24 hour rotation of the earth, the positions of the two dishes A and B appear as in Figure 1-la, to an observer located at the source. The locus of B relative to A traces out a strip in the elliptical area shown in Figure 1-lb. Thus, by changing the separation between the dishes in increments of less than a dish diameter, it is possible to synthesize a complete elliptical aperture. The major axis of this ellipse is equal to 2D, where D is the maximum dish separation. The minor axis is then equal to 2D \sin \phi, where \phi is the declination angle of the source. It can also be seen from Figure 1-l, that the observation period for each dish separation d need only be 12 hours, because of the symmetry.

Once the aperture has been synthesized, an appreciable area of sky can be scanned without having to make additional observations. This is accomplished by introducing a progressive phase shift in the computations, to "steer" the synthesized beam. On this basis, it can be shown\(^{(2)}\) that the total observing time required to map a portion of the sky with a given resolution is of the same order as that required for a conventional instrument with the same aperture.

In order to use the supersynthesis technique successfully, two conditions must be met. The first is that the source be constant during the entire observing period. This is generally the case for radio sources. The second condition is that accurate phase relationships be maintained during each twelve hour measurement at a given dish separation. This latter condition becomes increasingly difficult to meet at the shorter wavelengths.

1.2 The Proposed Penticton Supersynthesis Experiment

A new radio telescope employing the supersynthesis prin-
principle is now under construction at the Dominion Radioastrophysical Observatory at Penticton, B.C. One of the experiments that will be conducted with this instrument is a radio survey of the northern sky. The frequency range of interest here is the hydrogen line radio spectrum, which occupies the band $1420 \pm 1\text{MHz}$.

The proposed radio telescope can be described with the aid of Figure 1-2.

![Diagram of Proposed Supersynthesis Radio Telescope](image)

**Figure 1-2** Proposed Supersynthesis Radio Telescope
The dish antennas \( D_1 \) and \( D_2 \) of Figure 1-2 are each mounted on a trolley. The trolleys are movable along a railway track that is coincident with an east-west baseline, so that the distance \( d \), between the dishes, can be varied from 30 ft. to 2000 ft. At the focus of each dish, the received signals are amplified by a low noise microwave preamplifier and heterodyned with a 1390 MHz local oscillator (L.O.) signal. The resulting 30 MHz I.F. signals are then fed through equal length cables to a central location for signal processing.

1.3 Phase Coherence Requirements

The resolution that can be obtained by supersynthesis is critically dependent upon the degree to which any random differential phase shifts between the two signals applied to the central processor are eliminated. The allowable phase error here has been specified as \( \pm 15^\circ \) maximum. This corresponds to a resolution of about 3 seconds of arc, for a 2000 ft. baseline interferometer operating at 1420 MHz.

There are a number of contributing factors which give rise to this random phase error. There is, for example, a limit to the accuracy with which the value of \( d \) can be specified and maintained, using current civil engineering techniques. Drifts in the relative amounts of phase shift that are introduced by the microwave preamplifiers are another source of error. The most serious factor, however, is the variation in the length of the cables that deliver the L.O. signals to the dishes.

If, for example, the thermal expansion coefficient of copper and a nominal cable length of 305 meters (1000 ft.) are
assumed, then the change in length that would result from, say, a 2\(0^\circ\) change in the average temperature of the cable, is about 10 cm. This corresponds to a phase change of almost 180\(^\circ\) for a 1390 MHz L.O. signal. It is obvious then, that the requirements for phase coherence cannot possibly be met, unless steps are taken either to reduce this error, or to compensate for it.

Up to the present time, attempts to control the phase error due to unequal cable lengths have not gone beyond the expedient of simply cutting the high frequency cables to the same length and burying them a few feet below ground level. Calibration tests are run to determine the differential phase shift through the cables for different times of the day and year. The phase corrections obtained in this manner are then applied to the observation results. This procedure has been satisfactory at the longer wavelengths. However, at 21 cm, the differential phase shift for buried cables becomes large and unpredictable to such a degree, that the validity of applying corrections based on an earlier measurement is open to question. In one published result,\(^3\) describing a supersynthesis at 21 cm, the differential phase shift between two buried cables 400 ft. long varied by as much as 18\(^\circ\) over a three hour time interval.

What is required then, is a system that will provide L.O. signals at 1390 MHz to the two dishes, phase coherent to within, say, \(\pm 5^\circ\), and independent of the cable lengths.

1.4 Introduction of a Programmed Phase Difference

The main purpose of the coherent L.O. system proposed in the previous section would be to eliminate the phase error due to the L.O. cable runs. It would be very advantageous though, if this
A single, programmable, continuously variable phase difference between the L.O. signals at the two dishes. The other is the introduction of a programmable change in the frequency of the L.O. signal. The first requirement arises in the following manner.

In order to synthesize an aperture whose plane is normal to the incoming wave front, the effect of the path length difference $h_1$, in Figure 1-2 must be removed. This can be achieved for all frequencies in the band $\omega_1 \pm \Delta \omega_1$ by introducing a phase difference $\alpha$ and a path length difference $h_3$, as shown in Figure 1-2. The phase difference between the signals at A and B is then

$$\phi(\omega_1 + \omega_1) = -\alpha + \frac{\omega_1 h_1}{c} - \frac{\omega_2 h_3}{c} + \frac{(\Delta \omega_1)(h_1 - h_3)}{c} \quad (1-2)$$

where

$$\omega_1 = 2\pi \times 1420 \times 10^6 \text{ rad/s}$$
$$\omega_2 = 2\pi \times 1390 \times 10^6 \text{ rad/s}$$
$$\omega_3 = \omega_1 - \omega_2 = 2\pi \times 30 \times 10^6 \text{ rad/s}$$
$$c = 3 \times 10^8 \text{ meters/s}$$

The value of $\alpha$ can be programmed to make the first three terms of (1-2) equal to zero. Then, by maintaining $h_1 = h_3$, the phase difference $\phi$ can be made to vanish for all frequencies in the band $\omega_1 \pm \Delta \omega_1$.

If a finite error $\phi$ is accepted for the edges of the band, then the value of $h_3$ can be changed in discrete steps, by switching in cables whose lengths are multiples of a common factor. For
example, if the maximum allowable error $\theta$ at the band edges is $5^\circ$, then from the last term in (1-2), the cable length $h_3$ can be switched in increments of 4.17 meters. For such an increment only 13 separate cables are required. Any path difference up to a maximum of 2000 ft. can then be tracked to within 4.17 meters, by a suitable combination of these cables.

The maximum rate of change of the path difference $h_\perp$ is given by

$$h_\perp \max = W_e d$$

(1-3)

where $W_e$ is angular rotation rate of the earth about its axis. For $d = 610 \text{ meters (2000 ft.)}$

$$h_\perp \max = 4.44 \text{ cm/s}$$

If the cable delay is switched in increments of 4.17 meters, then the shortest time interval between cable switchings is

$$t_{\min} = \frac{\Delta h}{h_\perp \max} = 94 \text{ seconds}$$

When $h_\perp$ is changing at this maximum rate, the required rate of change of the programmed phase difference $\alpha$ is

$$\alpha_{\max} = 75.6^\circ/s$$

From equation (1-2), it can be seen that at each cable switching, the value of $\alpha$ must change by \(\frac{\omega_2 (\Delta h_3)}{c}\). For $\Delta h_3 = 4.17 \text{ meters}$, the change is about $150^\circ$. Hence, the programmed phase difference $\alpha$ must vary smoothly between cable switchings and undergo a step change when the cable is switched. Because the value of $\alpha$ must go through $2\pi$ radians
many times during the observing period and the time interval between cable switchings is relatively short, a phase shifter that is modulo $2\pi$ is desirable. The rotary phase shifter, (resolver) which produces an electrical phase shift proportional to a mechanical angular displacement is such a device.

1.5 Introduction of a Programmed Frequency Offset

The signals received at the antennas have superimposed upon them a doppler shift. This doppler shift normally has two components. These are $f_e$, the doppler shift due to the earth's rotation about its axis and $f_s$, due to the motion of the earth about the sun. The range of these shifts are as follows:

$$f_e = \pm 1.4 \text{ kHz}$$
$$f_s = \pm 140 \text{ kHz}$$

At the central processor, the I.F. signal from each dish is to be resolved in frequency by a 100 channel spectrometer. It is thus essential that all doppler shift components be removed prior to this operation. The L.O. frequency therefore, must be programmable over a frequency interval that is at least as great as the expected range of doppler shifts.

When external galaxies are observed, an additional doppler shift component due to the motion of the source is often encountered. This component may be as large as 8 MHz. However, the day to day variation is so small, that during the time of observation of the galaxy, the doppler component associated with the motion of the latter can be regarded as a constant frequency offset. The L.O.
frequency should, therefore, be manually tunable over a range of \( \pm 8 \text{ MHz} \) about 1390 MHz.

1.6 Summary of Requirements for the Coherent L.O. System

It was established in the preceding sections, that in order to perform a high resolution supersynthesis at 21 cm, a system that would maintain the phase coherence of the local oscillator signals was necessary. The requirements that such a system must satisfy can be summarized as follows:

(a) The L.O. signals must be delivered to the dishes phase coherent to within \( \pm 5^\circ \), independent of the L.O. cable lengths.

(b) The L.O. power level at each dish must be sufficient to satisfy the requirements of the low-noise mixers - about 10 milliwatts are required.

(c) There must be a provision for introducing a predetermined phase difference between the L.O. signals at the dishes. This phase difference must be introduced in such a manner that it can be controlled by a digital computer.

(d) The frequency of the L.O. signal must be variable over a \( \pm 300 \text{ kHz} \) range during any 12 hour observing period. This frequency change must be controllable by a digital computer.

(e) The L.O. frequency must be mechanically tunable over the range 1390 \( \pm 8 \text{ MHz} \).

The design of a system that satisfies the above requirements is the problem to which this thesis is addressed.
2. A REPRESENTATIVE COHERENT L.O. SYSTEM

2.1 Introduction

The coherent L.O. system that was chosen for development is described in the following chapter. However, before proceeding to this system, one of the other systems that was considered is discussed. The reasons for this are twofold:

(1) The system described in Chapter 3 may at first glance appear to be an unduly sophisticated solution to what is apparently a straightforward problem. A discussion of the shortcomings of one of the "simpler" solutions should justify the need for a more complex system.

(2) The system to be described here is not meant to be presented as a viable alternative to the system described in Chapter 3. It has, however, certain features that are necessary to all coherent L.O. systems. A consideration of these essential features provides some insight into the nature of the design problem.

2.2 The Representative System

The simple system previously referred to is illustrated in Figure 2-1.

A subharmonic of the desired L.O. frequency is transmitted from the central station to each dish. (This is done to avoid the cable loss encountered at 1390 MHz. The attenuation of 1000 ft. of 0.5 inch Heliax cable for example, is about 40 dB. On the other hand, the attenuation of the same cable at the ninth subharmonic frequency is only 10 dB.) At each dish, part of the signal is multiplied up
to the L.O. frequency. The remainder is amplitude modulated by a low frequency signal and returned to the central station. Here the modulated return signals from the dishes are fed into a balanced mixer whose output is tuned to the modulation frequency $\omega_m$. The mixer output signal $V_m$ is given by

$$V_m = kE_1E_2 \sin \left( \frac{2\omega}{c} (R_a - R_b) \right) \cos (\omega_m T)$$

Figure 2-1 Representative Coherent L.O. System

where $E_1$ and $E_2$ are the carrier amplitudes at the inputs to the balanced mixer. The phase difference between the input signals to the multipliers is then

$$\phi_d = \sqrt{S_a} - \sqrt{S_b} = \frac{\omega R_b}{c} - \frac{\omega R_a}{c} \quad (2-2)$$
The detected envelope of (2-1) can then be used as an error signal to adjust the length of one of the lines to make \( \phi_d = 0 \).

2.3 **Disadvantages with the System of Figure 2-1**

The method outlined above was not implemented for the following reasons:

(a) In order to maintain the phase accuracy at 1390 MHz to within \( \pm 5^\circ \), it is evident that the differential phase error of the cables at the ninth subharmonic must be made zero to within \( \pm 5/9^\circ \). Thus, while the attenuation problem has been eased somewhat, the phase accuracy requirement has become almost un-realizable.

(b) For a 10 dB cable loss at 154.4 MHz, about 1 watt of power would have to be transmitted in order to drive the multiplier chains at a level of, say 100 mW. This would present considerable isolation problems at the inputs to the balanced mixer.

(c) In Equation (2-1), the sine function is zero for two values of the argument, these being 0° and 180°. This results in an ambiguity in the value of \( \phi_d \) of 90°. Additional circuitry would be necessary at the control station, to overcome this difficulty.

(d) The adjustable line of Figure 2-1 is not suitable for introducing a modulo 2\( \pi \) phase difference between the two output signals. On the other hand, rotary phase shifters do not operate at 1390 MHz.

(e) If the frequency \( \omega_1 \) is changed, the phases of the 1390 MHz signals will shift by different amounts, unless the multiplier cavities have identical frequency-to-phase relationships.
2.4 Design Guidelines for the Coherent L.O. System

From Sections 2.2 and 2.3, certain design guidelines can be drawn:

(a) The phase shift introduced by each cable must be measured over a forward and return path. One-half of the differential phase shift must then be applied to the forward path of one of the cables, in order to bring the signals at the end of the cables into phase synchronization.

(b) In order to avoid errors due to reflections of the transmitted signal from some point along the cable, the transmitted and return signals must be distinguishable. In the system of Figure 2-1, this was done by modulating the return signal.

(c) Use of a subharmonic frequency to equalize the cables is to be avoided. This immediately introduces the problem of cable attenuation.

(d) Because of the unsuitability of adjustable lines for the purpose, a method of introducing the programmed phase difference at a lower frequency must be devised.
3. A PHASE-LOCKED LINK

3.1 Introduction

One of the more obvious methods of obtaining phase coherence at the two antennas was presented in Chapter 2. From an evaluation of this method, with its shortcomings, it was established that the design of an acceptable coherent L.O. system would have to proceed along certain guidelines. The following system, based on phaselock techniques, was devised using these guidelines.

3.2 The Phase-Locked Link - Principles of Operation

The operation of the link can be described with the aid of Figure 3-1.

(a) A control signal at the desired local oscillator frequency (typically 1390 MHz) is obtained from a frequency synthesizer. The frequency and phase of this signal are represented by (1). The signal (1) and a low frequency reference signal (2) are applied to the upper-sideband generator, to produce (3). The frequency of the signal (2) was chosen to be 2 MHz, for reasons that will be developed later.

(b) The signal (3) at 1392 MHz, is transmitted down the cable, along with the signal (4), which has the same frequency as (2), but which has for the moment, an unrelated phase.

(c) The signals (3) and (4) arrive at the satellite station delayed in time by \( \frac{R}{c} \) and are represented by (5) and (6) respectively.

(d) At the satellite, the signals (5) and (6) are applied to the lower-sideband generator, to obtain the output (7) at a frequency of 1390 MHz. This is the signal that must be made phase
Figure 3-1

Phase-Locked Link - Theory of Operation
coherent with the control signal (1). Part of the signal (7) is transmitted down the cable to the control station.

(e) At the control station, the signal (7), delayed in time by $\frac{R}{c}$, is now given by (8). The signals (3) and (8) are heterodyned to produce the I.F. signal (9), at 2 MHz.

(f) The signal (9) is heterodyned with (4), to obtain (10), at the sum frequency of 4 MHz.

(g) The signal (10) is applied to a divide-by-two circuit, to give (11), at 2 MHz.

(h) The signal (11) is given a programmed phase shift $\alpha$ and is used as the low frequency input signal to the upper-sideband generator. So we have

$$\Theta_2 = \Theta_3 + \frac{\omega_1 R}{c} + \alpha \quad (3-1)$$

Substituting for $\Theta_2$ in (7), we have for (7)

$$\frac{\omega_1}{\Theta_1} + \alpha$$

The phase difference between the control signal and the output signal at the satellite station, is thus independent of the cable length $R$ and can be made to take on any desired value by an appropriate choice of $\alpha$.

The control station could be placed at one of the antennas and the satellite station at the other. In this case there would be a single cable run of 2000 ft. Alternatively, the system shown in Figure 3-1 could be duplicated. The satellite stations would then be located at the dishes and the dual control station at a central point. The cable runs from the dual control station to each dish
would then be 1000 ft. long. This set-up would increase the complexity of the system, but would permit the use of lossier and cheaper cable.
3.3 The Translation Loop as a Sideband Generator

3.3.1 Phase and Frequency Relationships

The upper and lower sidebands are generated by phase-locked translation loops, as shown in Figure 3-2. The input signal to the loop at frequency \( \omega_i \) is heterodyned with the output of the voltage-controlled oscillator whose frequency is \( \omega_0 \). The Intermediate Frequency signal and a signal at a reference frequency \( \omega_r \), are compared in a phase detector. The filtered output of the latter is then used to control the frequency of the oscillator. We have:

\[
\begin{align*}
\omega_0 & > \omega_i \\
\omega_0 &= \omega_i + \omega_r \\
\theta_0 &= \theta_i + \theta_r
\end{align*}
\]

\[
\begin{align*}
\omega_0 & < \omega_i \\
\omega_0 &= \omega_i - \omega_r \\
\theta_0 &= \theta_i - \theta_r
\end{align*}
\]

3.3.2 Advantages of the Translation Loop

One might consider, on the basis of Equations (3-2) to (3-5), the possibility of generating the upper and lower sidebands by a scheme of direct mixing, amplification and filtering, as in Figure 3-3.

In the case of the upper-sideband generator, sufficient power must be provided at the translated frequency to drive the L.O. port of Mixer \( M_1 \) and to transmit down the cable. The power requirements at the output of the lower-sideband generator are similar. Several milliwatts at least are involved and the scheme of Figure
Figure 3-2
Basic Translation Loop

Figure 3-3
Sideband Generation by a Direct Method
3-3 would require microwave amplifiers at 1392 mhz and 1390 mhz. Transistor amplifiers giving 30 db gain at L-band are available, but they are very expensive and their long-term phase stability is at best $\pm 4^\circ$.

The upper and lower sidebands generated by the method of Figure 3-3 are separated by $2\omega_r$. If $f_r$ is required by other considerations to be low, i.e., 2 MHz, then a bandpass filter of the necessary loaded Q becomes unrealizable. Regardless of $\omega_r$, a narrow bandwidth at the microwave frequency would be necessary from noise considerations. This would require a multipole filter, having a large group delay, and the phase shift through the filter would be intolerably sensitive to frequency changes, or to a drift in the center frequency of the filter.

The Phase-Locked Translation Loop circumvents these difficulties. The sideband frequency is obtained at the output of an oscillator, and not a mixer; consequently a large amount of power is available. Only one sideband appears at the output when the loop is locked, and the loop can be designed to track a change in input frequency with a very small phase error.

3.4 The Problem of Wrong-Sideband Lock and its Solution

The simple translation loop of Figure 3-2 can lock to either sideband of the input frequency $\omega_1$, these sidebands being separated by $2\omega_r$. Normally a wrong-sideband lock is avoided by restricting the frequency range of the VCO to less than $2\omega_r$. In the event, however, that the loop is required to track an input whose frequency must be changed over a range greater than this, the possibility of a wrong-sideband lock is an immediate problem. From
Translation Loop with Sideband Discrimination
Figure 3-1, the result of one of the loops being locked to a wrong sideband is a frequency difference between the control and output signals of $2\omega_r$.

A method of avoiding a wrong-sideband lock has been devised and the principles can be explained with the aid of Figure 3-4.
The loop is locked through Phase Detector (1) and the Loop Filter.
A sweep signal $V_s$, which is switched on by the output of Phase Detector (2) is made available at the input of the VCO. For the Phase Detectors (1) and (2), we have:

$$V_{d1} = K_{d1} \cos (\theta_{if} - (\theta_r - 90^\circ)) = K_{d1} \cos X \quad (3-6)$$

$$V_{d2} = K_{d2} \cos (\theta_{if} - \theta_r) = K_{d2} \cos (X - 90^\circ) \quad (3-7)$$

where

$$X = \theta_{if} - \theta_r + 90^\circ \quad (3-8)$$

Also

$$\theta = K_0 V_f \quad (3-9)$$

where

$$V_i = K_{d1} F_1(0)V_{dl} \quad (3-10)$$

and $F(s)$ is the transfer function of the Loop Filter.

The Phase Detector Characteristic is shown in Figure 3-5.
When the loop is locked the output of Phase Detector (1) is at a null, i.e., $V_{d1} = 0$. From Figure 3-5 two such nulls exist, but only one of these nulls corresponds to a phase-locked condition, and this can be shown as follows:

Suppose that the loop is locked to a lower sideband and that $V_{d1}$ is nulled at (b), where $X = (90^\circ)$. 
Then \( \omega_i < \omega_o \)

and \( X = (\theta_i - \theta_o + 90^\circ) \) \( (3-11) \)

Let \( X \) be perturbed to a slightly lower value. \( V_{dl} \) becomes slightly positive. This signal is then integrated by the Loop Filter according to \( (3-10) \), and this results in an increase in \( V_f \). \( \dot{\theta} \) then increases according to \( (3-9) \), i.e., the oscillator runs faster, and from \( (3-11) \), \( X \) continues to decrease. Similarly, if the initial perturbation of \( X \) were in the direction of increasing \( X \), then \( X \) would continue to increase. The null at \( (b) \) then, is unstable for a lower-sideband lock. By a similar argument, the null at \( (a) \) is stable.

For an upper-sideband lock the situation is reversed; the
null at (a) is unstable and the null at (b) is stable.
So we have;

For Lower-Sideband Lock \hspace{1cm} \text{For Upper-Sideband Lock} \\
X = -90^\circ \hspace{1cm} X = +90^\circ \\
V_{d2} = K_{d2} \cos(180^\circ) = -K_{d2} \hspace{1cm} V_{d2} = K_{d2} \cos(0^\circ) = +K_{d2} \\

Thus the magnitude and polarity of $V_{d2}$ can be used to indicate either a loss of lock, or a wrong-sideband lock.

If the threshold voltage $V_t$ is set to

- $-K_{d2} < V_t < 0$ for lower-sideband lock
- $0 < V_t < K_{d2}$ for upper-sideband lock

then the VCO will begin to sweep when lock is lost and continue sweeping until lock is regained. The loop can be kicked out of a wrong-sideband lock, by employing, for example, a sawtooth sweep waveform.

3.5 The Phase-Locked Link as a Coherent L.O. System

As a coherent L.O. system, the phase-locked link of Figure 3-1 has several desirable features.

The compensation for the changing cable length is accomplished without the use of line stretchers, servomotors or other mechanical devices. This compensation is done at the microwave frequency; accordingly, varactor multipliers, which require a high drive level for efficient operation and which multiply phase errors, are not used. The transmitted and return frequencies are different, so that the problem of discriminating between a return signal originating at the satellite station, and one that has been reflected
from some point along the cable, is avoided. Use of the phase-locked loops for the sideband generators makes amplification at the microwave frequency unnecessary. Each loop is, in fact, a tracking receiver and transmitter. The input noise bandwidth to the loop is moved to a narrow baseband by the loop filter and a considerable improvement in signal-to-noise ratio can be achieved. The loop is thus capable of locking to a weak signal and re-transmitting a relatively high level signal at the sideband frequency. Finally, the desired programmable phase difference between the control and output signal can be introduced at the reference frequency $\omega_r$. Since $\omega_r$ is fixed, a phase shifter of only narrow-band capability is required.

This system does have one shortcoming that is shared by all other coherent L.O. systems that are based on the general principles of Section 2.4. Referring to Figure 3-1, the phase coherence is maintained by determining the phase shift due to the cable over a two-way path and then applying one-half of this phase shift to the forward path. The difficulty arises as follows.

The phase shift over the two-way path can only be resolved to within a multiple of $2\pi$ radians. When this phase shift is divided by two, there are two possible results. One of these is the signal represented by (11) in Figure 3-1. The other, is this same signal, shifted in phase by $180^\circ$. In the latter case, this $180^\circ$ phase shift appears at (7). Thus, if the system should lose lock, there is the possibility that the signal (7), may be $180^\circ$ out of phase when lock is regained.

For the duplicate system, described at the end of Section 3.2, this problem could be overcome as follows. The phase
difference between the signals (\(ll\)) from each half of the duplicate system could be monitored. If this phase difference changes suddenly be 180°, the programmed value of \(\alpha\) could be changed by this amount, thus maintaining the correct phase relationship between the signals (\(7\)) at the satellites.

Another alternative would be to operate the duplicate system of Section 3.2, at the second sub-harmonic of the desired L.O. frequency. The signals (\(7\)) would drive frequency doublers at each dish. Any phase reversals at the sub-harmonic frequency, then, would not affect the phase of L.O. signals. This alternative, however, requires frequency multiplication — an operation that should be avoided, for the reasons given in Section 2.3.

3.6 An Alternative Phaselock Method

A phase-locked link that has certain similarities to the system described in Section 3.2 has been developed independently by Airborne Instruments Laboratories(4). This system, hereafter referred to as the A.I.L. system, is shown in Figure 3-6.

The control signal \(\omega_1/\theta_1\), is obtained from a frequency synthesizer and applied to the control station. The low frequency reference signal \(\omega_2/\theta_2\), is generated at the satellite station. If in Figure 3-6, the output of the divide-by-two circuit is assumed to be \(\omega_2/\theta_3\), then it can readily be shown that the output signal at the satellite station is \(\omega_1/\theta_1\). As in the system of Figure 3-1, a phase difference \(\alpha\), between the control signal and the output signal can be introduced by inserting a phase shift \(\alpha\) between the divide-by-two and the upper-sideband generator.
Figure 3-6

An Alternative Phase-Locked Link
The phase-locked link of Figure 3-1, which we shall refer to as the Penticton system, has several advantages over the A.I.L. system.

The Penticton system requires two phase-locked sideband generators; the A.I.L. system, three. For the duplicate arrangement discussed in Section 3.2, the relative complexity would be four sideband generators for the Penticton system, as opposed to six for all the A.I.L. system. Referring to Figure 3-6, the instrumentation of the satellite station, for the A.I.L. system consists of no less than two sideband generators, a mixer and a signal generator at frequency \( \omega_2 \). For the Penticton system, only a lower-sideband generator is required at the satellite. Because the space available at each dish for the coherent L.O. system is limited, this difference in the amount of instrumentation required at the dishes is significant.

In the A.I.L. system, the reference signal is transmitted from the satellite station to the control station, then back to the satellite station, at the same frequency \( \omega_2 \). This system is, thus, vulnerable to errors arising from reflections of the reference signal. To avoid such errors, two separate cable paths of length \( R \) would be necessary. For a duplicate system, four additional cables would be required. This would raise the cost of the system by several thousand dollars.

The A.I.L. system permits phase synchronization over a frequency range of 500 Hz\(^4\). This range is not adequate for the doppler tracking requirements of Section 1-6. On the other hand, the Penticton system can easily track over the expected range of doppler shift, without locking to the wrong sideband.
4. ANALYSIS AND DESIGN

4.1 The Linear Loop Equations

The basic translation loop is shown in Figure 4-1, where:

\[ \theta_i = \text{phase of the input signal} \]
\[ \theta_o = \text{phase of the output signal} \]
\[ \theta_r = \text{phase of the reference signal} \]
\[ \theta_{nl} = \text{contaminating phase disturbance at the Intermediate Frequency (I.F.)} \]
\[ \theta_{n2} = \text{contaminating phase disturbance at the output of the voltage-controlled oscillator (VCO).} \]

When the loop is locked, the phase detector operates about a null and for small variations of phase about the locked condition the output voltage \( V_d \) is approximately a linear function of the dif-
ferential phase input.

\[ V_d(s) = K_d(\theta_i(s) - \theta_o(s) + \theta_n(s) - \theta_r(s)) \quad (4-1) \]

also;

\[ \dot{\theta}_o(s) = \dot{\theta}_n(s) + K_o F(s)V_d(s) \quad (4-2) \]

where in (4-2), \( K_o \) is the voltage-to-frequency gain constant of the VCO and \( F(s) \) is the response function of the Loop Filter.

The response of the loop for the excitations \( \theta_i(s), \theta_n(s) \) and \( \theta_n(s) \) is then;

\[ \theta_o(s) = H(s)\theta_i(s) + H(s)\theta_n(s) - (1 - H(s))\theta_n(s) \quad (4-3) \]

where;

\[ H(s) = \frac{K_o K_d F(s)}{s + K_o K_d F(s)} \quad (4-4) \]

\[ 1 - H(s) = \frac{s}{s + K_o K_d F(s)} \quad (4-5) \]

The order of the closed loop response \( H(s) \) is determined by the choice of the loop filter function \( F(s) \). In order to track a ramp input phase (a frequency offset), with a finite steady-state phase error, a second order system is required. Such a system, in which the loop gain, damping and bandwidth are independently variable can be realized in \( F(s) \) has the form;

\[ F(s) = \frac{A_o(1 + t_2 s)}{(1 + A_o t_1 s)} \quad (4-6) \]
\[ F(s) = \frac{V_2}{V_1} = A_0 \left( 1 + t_1 s \right) \frac{1}{1 + A_0 t_2 s} \]

\[ A_0 = \text{open loop gain of the Operational Amplifier} \]

\[ t_1 = R_1 C \] \hspace{1cm} (4-7)

\[ t_2 = (R_1 + R_2) C \] \hspace{1cm} (4-8)

**Figure 4-2**
Realization of \( F(s) \) by an Operational Amplifier

**Figure 4-3**
Frequency Response of \( F(s) \)
Figure (4-2) illustrates how $F(s)$ is realized by using an operational amplifier. The frequency response of $F(s)$ is shown in Figure (4-3). When $F(s)$ is given by (4-6), then $H(s)$ and $1 - H(s)$ become:

$$H(s) = \frac{\omega_n^2 + 2z\omega_n s}{s^2 + 2z\omega_n s + \omega_n^2}$$  \hspace{1cm} (4-9)

$$1 - H(s) = \frac{s^2}{s^2 + 2z\omega_n s + \omega_n^2}$$  \hspace{1cm} (4-10)

where:

$$\omega_n = \left( \frac{K_d K_o}{t_1} \right)^{\frac{1}{2}}$$  \hspace{1cm} (4-11)

$$z = \frac{t_2}{2} \left( \frac{K_o K_d}{t_1} \right)^{\frac{1}{2}}$$  \hspace{1cm} (4-12)

and $\frac{1}{A_0 t_1} \ll 2z\omega_n$

(This last condition is true for all but very narrow band phase locked loops.)

4.2 Phase Errors Due to the Translation Loop

The purpose of the translation loop is to translate the frequency and phase of the input signal according to the relationships given in Equations (3-2) to (3-5). Any deviation of the output phase from these relationships contributes to the over-all system phase error. The design of the loop, then, involves the choice of loop parameters and components that will tend to minimize this
The phase errors introduced by the loop fall into three categories; these being drift errors, tracking errors and phase jitter due to noise.

The drift errors can be attributed to the change, over a period of time, of the phase shifts introduced by the various elements in the loop. This, in fact, is the sort of phase error that the system was supposed to eliminate. Since the main culprit here is the I.F. amplifier, a judicious choice of amplifier configuration is required. This, in turn, has a bearing on the choice of the I.F. frequency. More will be said about this point in a later section.

The second type of phase error occurs when there is a difference between the frequency being tracked and the free-running frequency of the VCO, i.e., the frequency that the VCO generates when the output of the phase detector is zero. This frequency difference can change in either of two ways:

1. There is a programmed change in the desired L.O. frequency, as would occur while a doppler shift was being tracked out.
2. There is a change in the characteristics of the VCO or its control circuit; for example, expansion of the oscillator cavity, or a DC offset in the control circuit.

The anticipated range of the frequency offset can be estimated as follows:

**Programmed Offset for Doppler Tracking Requirements**
*(Based on the anticipated range of $f_e$ in $f_s$ in Section 1.5.)*

142 kHz
Offset Due to Expansion of Oscillator Cavity (Brass) (If the expansion coefficient of Brass and a temperature range of $10^0\text{C}$ are assumed, then the resonant frequency of the cavity will vary by this amount) 200 kHz

Offset Due to Drift in DC Level of VCO Control Circuit (estimate based on measured DC drifts) 1.5 MHz

Total 1.84 MHz

The frequency offset can thus be expected to vary over a range of, say, 2 MHz. Accordingly, the design was initiated by specifying an allowable error rate of $1^0/\text{MHz}$. For a second-order loop, the steady-state phase error per unit frequency offset is just the inverse of the DC loop gain. Hence the required loop gain is

$$K_{d} = 3.6 \times 10^8$$

(4-13)

From Figure 3-1, it can be seen that if both loops develop the same error $e$, due to a frequency offset, the resultant phase error between the control signal and the output signal at the satellite station is $\frac{3e}{2}$. For the chosen value of DC loop gain, this amounts to $3^0$. Higher values of loop gain could, of course, reduce this error, provided loop stability can be maintained. Also, for the duplicate system discussed in Section 3-2, the output signal at each satellite station will be affected in roughly the same manner, so that the differential error will be less than the above figure.

4.3 Phase Jitter Due to Noise

4.3.1 Introduction

Apart from the slowly varying phase errors discussed in Section 4-2, there is always present the phase jitter resulting
from noise in the phase-locked loop. This phase jitter can be considered as the response of the loop to random inputs $\theta_{n1}$ and $\theta_{n2}$ which are introduced at the I.F. input of the phase detector and at the output of the VCO respectively. (See Figure 4-1)

4.3.2. I.F. Noise

The phase-locked loop responds to the random phase signal $\theta_{n1}$ as a linear system with a response function $H(s)$. If it is assumed that the I.F. noise that produces the phase jitter $\theta_{n1}$ can be described by Rice's representation of narrow-band noise\(^1\), i.e.,

\[
n_{if}(t) = n_c(t) \cos \omega_{if} t + n_s(t) \sin \omega_{if} t
\]

(4-14)

where \(n_c n_s = n_c \cdot n_s = 0\)

and \(n_c^2 = n_s^2 = n_{if}^2\)

with $n_c$ and $n_s$ having a spectrum from DC to $B_{if}/2$, $B_{if}$ being the I.F. bandwidth, then the phase jitter at the output of the loop, due to $n_{if}(t)$ is\(^2\)

\[
U_{ol}^2 = \frac{1}{2(SNR)_{if}} \frac{(BL)}{(B_{if})}
\]

(4-15)

where $B_L$ is the loop bandwidth, given by

\[
B_L = \int_0^\infty |H(w)|^2 \ dw = \pi \omega_n (z + \frac{1}{4z}) \ rad/s
\]

(4-16)

An estimate of $U_{ol}^2$ can be made as follows:

The amount of power provided by the VCO at the control station is
conservatively, 40 milliwatts, or +16 dBm. If, as proposed, 1000 ft. of 0.5 inch Heliax cable is used between the satellite and the control stations, the attenuation is about 40 dB. With the instrumentation scheme of Figures 5-1 and 5-2, an additional 12 dB loss at each end is incurred in coupling the signal into and out of the cable. The signal level at the input of the mixer of the satellite loop is then -48 dBm. If at the satellite loop, the noise figure of the mixer and I.F. amplifier combination is to say 10 dB, the I.F. bandwidth about 3 MHz and the temperature 290° Kelvin, then the noise level referred to the mixer input is -100 dBm. The signal-to-noise ratio at the input of the phase detector is then about 50 dB. If we disregard for the moment, the ratio $\frac{B_L}{B_{if}}$ in (4-15), the output phase jitter is then

$$U_{ol}^{(rms)} = 0.14^\circ$$

Because the signal-to-noise ratio at the input of the loop is high, the ratio of $B_L$ to $B_{if}$ need not be very much less than one. If the phase-locked loop were required to track a signal that is buried deeply in the noise, i.e., $(SNR)_{if}$ is very small, a loop bandwidth of a few Hz would be necessary. In this case, however, $B_L$ or equivalently, $\omega_n$, can be several orders of magnitude larger, without seriously increasing the phase jitter due to noise at the I.F. This result will become significant, when the phase jitter due to oscillator noise is considered in the next section.

4.3.3 Oscillator Noise

The contaminating signal $\theta_{n2}$ in Figure (4-1) can be identified with the phase jitter of the VCO due to a noisy oscillator
signal. The loop responds to $\theta_n^2$ as a linear system with a transfer function $1 - H(s)$.

Figure 4-4 Noise Processes in the VCO

A simple model of the oscillator is shown in Figure 4-4. The oscillator signal is generated by a feedback loop of bandwidth $B_a$ and reaches the output terminals of the device through an inter-stage of bandwidth $B_b$. The phase jitter $\theta_n^2$ is the result of two independent noise processes associated with the oscillator. These are shown in Figure 4-4 as $n_{2a}$ and $n_{2b}$. Both $n_{2a}$ and $n_{2b}$ are assumed to be narrow-band Gaussian, centered about the oscillator frequency $\omega_0$ and representable by the general form of (4-14) with bandwidths $B_a$ and $B_b$ respectively.

4.3.4 "Elastic" Noise

The disturbance $n_{2b}$ in Figure 4-4 represents the noise
that is added by the output stage after the oscillator clock frequency has been generated by the feedback loop. This noise could, for example, be attributed to amplifier stages and/or multiplier chains at the output of a master oscillator. We can regard \( n_{2b} \) as a sequence of random pulses, each of which perturbs the phase of the oscillator signal. These perturbations are "elastic" in the sense that the oscillator phase is momentarily disturbed when a pulse arrives, but returns to its original value after the pulse disappears. Under such conditions and with the assumption that the signal-to-noise ratio is high, (greater than 15 dB) the mean-square phase jitter due to \( n_{2b} \) is

\[
\overline{\Theta^2_{n2b}} = \frac{1}{2(SNR)_b} \tag{4-17}
\]

where \((SNR)_b\) is the signal-to-noise ratio at the oscillator output.

If it is assumed that the spectrum of \( \Theta_{n2b} \) extends from \( \omega = 0 \) to \( \omega = \frac{B_b}{2} \), then an equivalent uniform spectral density is

\[
\phi_b(\omega) = \frac{\overline{\Theta^2_{n2b}}}{(\frac{B_b}{2})} = \frac{2\Theta^2_{n2b}}{B_b} \tag{4-18}
\]

The phase jitter at the output of the translation loop is then given by

\[
\overline{U^2_{b}} = \frac{B_b}{2} \int_{0}^{\frac{B_b}{2}} \phi_b(\omega) \left| 1 - H(\omega) \right|^2 \, d\omega \tag{4-19}
\]
For the second order loop (4-19) can be evaluated approximately by integrating over simple geometric areas to give

\[
\overline{U_b^2} = \frac{1}{2 \text{SNR}_b} (1 - \frac{\omega_n}{B_b}) \text{ for } \omega_n < B_b
\]

= 0 \text{ for } \omega_n > B_b \tag{4-20}

From equation (4-20) it is apparent that in order to track out the phase jitter due to \( n_{2b} \), a large \( \omega_n \) or large loop bandwidth is required. When the oscillator output bandwidth is much greater than \( \omega_n \), as is usually the case for microwave oscillators, a small phase jitter \( \overline{U_b^2} \) can be achieved only by maintaining a high oscillator output signal-to-noise ratio.

4.3.5 "Non-Elastic" Noise

The noise process \( n_{2a} \) in Figure 4-4 produces a phase disturbance that is markedly different from that produced by \( n_{2b} \). If as in the case of \( n_{2b} \), the process \( n_{2a} \) is considered as a sequence of random pulses, then as before, each pulse shifts the oscillator phase by a minute amount. These phase shifts, however, occur within the feedback loop that generates the clock frequency. Each incremental phase change becomes a permanent part of the oscillator phase, with the result that the latter executes a random walk. The error accumulated over an observing time \( t \), has equal probability of being positive or negative and so has zero mean value. However, the mean-square error taken over an infinite time interval is not bounded. Van Blerkom has obtained the following result for the mean-square oscillator phase error due to this "non-elastic" noise.\(^7\)
\[ \frac{\theta_{n2a}^2}{\theta_{n2a}^2} = \frac{B_{at}}{2(SNR)_a} \] (4-21)

where

\[ B_{a} = \text{bandwidth of the oscillator feedback loop} \]
\[ t = \text{observing time} \]
\[ (SNR)_a = \text{signal-to-noise ratio in the oscillator feedback loop} \]

In order to determine the effect of the oscillator noise \( n_{2a} \) on the phase-locked loop, an expression for the mean-square phase error at the output of the loop is required. An approximate result can be obtained as follows.

The unbounded mean-square phase error of the oscillator signal due to \( n_{2a} \) arises as the result of the algebraic addition of the incremental phase errors introduced by the arrival of each new pulse of \( n_{2a} \). In order to make this mean-square error finite and small, there must be introduced a "mechanism" that tends to return the phase error to zero after each such incremental phase step. This "reset" must be made within a time that is comparable to the interval between noise pulses and must be made with a minimum of overshoot consistent with a fast response. This "mechanism" of course, is the phase-locked loop. One would expect, therefore, that the mean-square phase error at the output of the loop would be comparable to that generated by the VCO in a time \( t \), where \( t \) is the response time of the loop. Using the value \( t = \frac{1}{\omega_n} \) for a second-order loop, the approximate result is then

\[ \frac{U_a^2}{\theta_{n2a}^2} = \frac{1}{2(SNR)_a \omega_n} \] (4-22)
This result, when combined with (4-20), gives for the mean-square phase error at the output of the loop, due to oscillator noise

\[ U_{o2}^2 = U_a^2 + U_b^2 = \frac{1}{2 \text{SNR}} \frac{B_a}{\omega_n} + \frac{1}{2 \text{SNR}} \frac{(1 - \omega_n B_b)}{B_b} \quad (4-23) \]

The mean-square phase error \( U_{o1}^2 \), due to the I.F. noise, has not been included. The results of Section 4.3.2, however, show that this component of the output noise is sufficiently small, that it can be neglected.

4.4 Choice of a Voltage-Controlled Oscillator (VCO)

4.4.1 Introduction

In order to realize the sideband generators of Figure 3-1 using phase-locked translation loops, voltage-controlled oscillators with free-running frequencies around 1.4 GHz are required. In this Section, the performance of two types of oscillators is discussed and the reasons for the choice of one over the other are given.

4.4.2 The Voltage-Controlled Crystal Oscillator (VCXO) and Multiplier

A commonly used method of obtaining a pure signal at a microwave frequency is to varactor multiply the signal from a crystal oscillator. Crystals with resonant frequencies in the range 1-10 MHz are usually used and voltage control of the frequency is obtained by a voltage-variable capacitor in the crystal oscillator unit. In terms of the oscillator model of Figure 4-4, the crystal oscillator and multiplier chain would correspond to the feedback loop and the output stage respectively. In such a case, the first term of (4-23) can be made very small, owing to the very small value of \( B_a \) which is
determined by the Q of the oscillator crystal. For frequencies up to 100 MHz or so, the second term in (4-23) is also small, since \((SNR)_b\) is large. The VCXO-Multiplier then, is a logical choice for phase-locked loops operating at these lower frequencies.

When, however, the required frequency is in the GHz range, the second term of (4-23) dominates. It has been shown that varactor multiplication by a factor \(N\) results in a degradation of the signal-to-noise ratio of \(N^2\). If, for example, a 5 MHz crystal oscillator were used, the degradation of \((SNR)_b\) at 1400 MHz would be about 50 dB. In addition, varactor multipliers, for their efficient operation must be driven at substantial power levels — several hundred milliwatts at least. This means that a power amplifier is required between the VCXO and the multiplier chain, which introduces additional noise. This noise occupies the bandwidth \(B_b\), determined by the Q of the multiplier cavities, and at 1400 MHz is at least a few MHz wide. The result, in practical terms, is that the best frequency synthesizers which generate frequencies by such a method, have signal-to-noise ratios of no better than 40 dB. Accordingly, the phase jitter at the output of a translation loop using a VCXO-Multiplier Chain would be

\[
U_{o2}^2 = \frac{1}{2(SNR)_b} = \frac{1}{2 \times 10^4} \text{ radians}^2
\]

which corresponds to an rms phase jitter of

\[
U_{o2}^{\text{rms}} = 0.5^\circ
\]

Crystal oscillators are by nature of their high Q, very narrow band devices and their tuning range, whether by electrical or
mechanical means, is usually restricted to about 0.2%. The required frequency range of \pm 8 MHz at the microwave frequency, would then make necessary the use of more than one crystal oscillator unit, or of a tapped multiplier chain.

The complexity of the VCO-Multiplier Chain and the unsuitability of the latter for wide-band use, led to the consideration of an alternative - the Transistor Microwave Oscillator (TMO), which is discussed in the following section.

4.4.3 The Transistor Microwave Oscillator (TMO)

Any transistor is potentially unstable at a frequency that is high enough such that its gain as an amplifier approaches unity, this potential instability extending over a range of bias conditions. In the case of oscillations at 1400 MHz, this unstable operation is possible with a UHF power transistor operating in the common base configuration. The oscillator can then be realized, by coupling this unstable device to a cavity that is resonant at the desired frequency of operation. The frequency of oscillation can then be controlled by tuning the cavity with a voltage-variable capacitor, or by varying the bias conditions of the transistor, the latter having the effect of altering the reactive load that the transistor presents to the cavity. Such a TMO was designed and built using a 2N 3866 UHF power transistor coupled into a quarter-wavelength cavity at 1390 MHz. Output power into a 50 ohm load was 60 milliwatts and frequency control over a range of 8 MHz was achieved by varying the emitter current. The details of the design and construction are given in a later section.

The TMO, considered in the light of Equation (4-23) and
Figure 4-4, has several interesting features.

The output power is high — 50 milliwatts at 1.4 GHz being typical. In terms of the oscillator model of Figure 4-4, the bandwidth of the feedback loop $B_a$ and that of the output stage $B_b$ are equal and given by $\omega_0$, where $\omega_0$ is the oscillator frequency, and $Q_L$ is the loaded $Q$ of the oscillator cavity. $Q_L$, however, is considerably less than the unloaded cavity $Q$, as a relatively large amount of power is coupled out. Because the output signal level is high and delivered at the microwave frequency, without undergoing a multiplication process, the signal-to-noise ratio $(\text{SNR})_b$ in Figure 4-4 is sufficiently large that phase jitter $\overline{U^2}_b$ in (4-23) can be neglected. This is true in spite of the large output bandwidth $B_b$ resulting from the low value of $Q_L$. It is also true, that the phase jitter $\overline{U^2}_a$ due to the "non-elastic" noise, can be made very small. The reason for this is as follows.

At the frequency of oscillation, the gain of the transistor in the TMO is only marginally greater than unity. It follows then, that the signal level at any point within the feedback loop of Figure 4-4 must be nearly equal to the signal level at the oscillator output, so that $(\text{SNR})_a$ is large. We have for the TMO

$$\overline{U^2}_{o2} = \overline{U^2}_a \leq \frac{1}{2(\text{SNR})_a} \frac{B_a}{\omega_n}$$

where a value of $z = 1$ has been assumed for convenience.

If an oscillator bandwidth of 140 MHz (corresponding to a loaded $Q$ of 10 at 1.4 GHz) and a conservative estimate of $(\text{SNR})_a$ of 80 dB are assumed, then the value of $f_n = \omega_n/2\pi$ required to maintain
a 1° rms phase jitter is

\[ f_n = \frac{1 \times 10^8}{2 \times 10^8 \times \frac{1}{3600}} = 1.8\text{KHz} \]

A value of \( f_n = 50 \text{kHz} \), which improves the rms phase jitter by a factor of about 6.5, was chosen, i.e., for

\[ \omega_n = 3.14 \times 10^5 \]

\[ U_{o2 \text{ rms}} = 0.15° \]

4.4.4 Comparing the VCXO-Multiplier and the TMO

The results of Section 4.4.2 indicate that the best that one could expect in terms of phase jitter, from a VCXO-Multiplier is about 0.5° rms. The result (4-26) indicates that if the TMO were used, the rms phase jitter at the output of either translation loop could be maintained, for all practical purposes, equal to that of the control signal.

The TMC consists of a UHF transistor and a Cavity. (The 2N 3866 sells for about $4.00.) The VCXO-Multiplier, on the other hand, is a complex and expensive device.

Emitter-current control of the frequency of the TMO over the required 300 kHz range and mechanical tuning of the cavity over ±8 MHz are easily accomplished. The VCXO, however, is not suitable for wideband use.

The VCXO has the advantage of long-term frequency stability. This is of secondary importance here, as the loop oscillators are locked to a control signal that provides this stability.

For these reasons, the TMO was chosen for use as the VCO in the phase-locked link.
4.5 Intermediate Frequency and I.F. Bandwidth Requirements

4.5.1 Effect of I.F. Filter

In the translation loop of Figure 4-1 there is, in general, an Intermediate Frequency amplifier which can be characterized by an impulse response \( h_B(t) \). Introducing this I.F. filter is equivalent to cascading with the loop filter, a low-pass filter which has an impulse response \( h_L(t) \), given by

\[
h_B(t) = h_L(t) \cos \omega_{if} t \tag{4-27}
\]

In the frequency domain, this means that the equivalent low-pass filter poles are obtained by shifting the pole cluster at \( +j\omega_{if} \) of the I.F. filter down to the origin.

Suppose, for example, that the I.F. filter response is maximally flat, with symmetrical 40 dB/decade skirts. The equivalent low-pass filter function is then

\[
M(s) = \frac{\omega_{nl}^2}{s^2 + 2z\omega_{nl}s + \omega_{nl}^2} \tag{4-28}
\]

where \( \omega_{nl} = \frac{B_{if}}{2} \) \( z = 0.707 \)

\( B_{if} = \) I.F. bandwidth

The effect of \( M(s) \) on the performance of the loop can be determined by considering the open loop response

\[
G(s) = \frac{H}{1 - H} = \frac{K_0 K_d F(s) M(s)}{s}
\]

With \( F(s) \) given by (4-6) and \( M(s) \) by (4-28), the open loop gain function becomes
\[ G(s) = \frac{K_o K_d A_o (s^2 + 1) \omega_{nl}^2}{s(sA_o t_1 + 1)(s^2 + 2\omega_{nl} s + \omega_{nl}^2)} \]

A Bode Plot of (4-29) is shown in Figure 4-5.

From Figure 4-5, it can be seen that if the roll-off term at \( \omega_{nl} \) is not present, the loop is stable, with a phase margin \( L \), given by \( L = \pi/2 - \text{phase at } \omega_c = \pi/2 - \frac{1}{4\omega_{nl}^2} \) radians.
For a damping ratio of $z = 1$, the phase margin is $80^\circ$.

The I.F. filter of bandwidth $B_{if}$ produces an additional roll-off at $\omega_{nl} = B_{if}/2$ radians/sec. In order to prevent the loop performance from being unduly influenced by the presence of the IF filter, it is necessary to keep the ratio of $\omega_{nl}$ to $\omega_c$ large.

4.5.2 Choice of I.F. Bandwidth

The additional phase lag introduced at the crossover frequency $\omega_c$ in Figure 4-5, is a function of the ratio $\omega_{nl}/\omega_c$, and of the damping ratio $z_l$. This function is tabulated in most references on servomechanisms. If we assume a maximally flat response for which $z_l = 0.707$, then, in order to keep this lag contribution at the crossover frequency less than, say, $10^\circ$, we must have $\omega_{nl}/\omega_c > 8$.

Now since $\omega_{nl} = B_i/2$, and $\omega_c = 2\omega_n$ we must have

$$\frac{B_{if}/\omega_n}{\omega_n} > 32z$$

For typical loop damping ratios, then, the I.F. bandwidth should be at least 30 times greater than $\omega_n$ for the loop. For the value of $f_n = 50$ KHz specified in Section 4.4.2, the I.F. bandwidth should be greater than $1.5$ MHz.

4.5.3 Choice of the Intermediate Frequency for the Translation Loop

For the required I.F. bandwidth of $1.5$ MHz, any I.F. greater than say, $1$ MHz would suffice. The lowest possible frequency is desirable from the point of view of minimizing the effect of time delay drifts at the I.F. Furthermore, it is relatively easy to obtain high I.F. gains with phase stability if a low I.F. is used.

For these reasons, an I.F. of $2$ MHz was chosen. The
required minimum I.F. bandwidth of 3 MHz was obtained by rolling off the response of a Fairchild uA 702C operational amplifier.

4.5.4 Derivation of the Other Loop Parameters

Two of the loop parameters have already been chosen. These are:

\[ K_0 K_d A_o = 3.6 \times 10^8 \quad (4-13) \]
\[ \omega_n = 3.14 \times 10^5 \text{ rad/s} \quad (4-25) \]

The open loop gain of the uA 702C is 3000. (This figure will vary from unit to unit and this variation is compensated for by a gain adjustment in the circuit design.)

Using \( A_o = 3000 \) in (4-13), we have

\[ K_o K_d = 1.2 \times 10^5 \quad (4-30) \]

Using a damping ratio of say, \( z = 1 \) in Equations (4-11) and (4-12), the loop filter time constants \( t_1 \) and \( t_2 \) can be evaluated as follows:

\[ t_1 = \frac{K_o K_d}{\omega_n^2} = 1.215 \times 10^{-6} \text{ sec.} \quad (4-31) \]
\[ t_2 = \frac{2z}{\omega_n} = 6.36 \times 10^{-6} \text{ sec.} \quad (4-32) \]

The time constants are related to the element values of the active filter by (4-7) and (4-8), i.e.,

\[ t_1 = R_1 C \quad (4-7) \]
\[ t_2 = (R_1 + R_2)C \quad (4-8) \]

so that
\[ \frac{R_2}{R_1} = \frac{t_2}{t_1} - 1 = 4.24 \]

Choosing

\[ R_2 = 10K \quad (4-33) \]

\[ R_1 = \frac{10K}{4.24} = 2.36K \quad (4-34) \]

then, from (4-7),

\[ C = \frac{t_1}{R_1} = 515 \text{ p.f.} \quad (4-35) \]

Choosing a standard value of \( R_1 = 2.2K \), \( t_2 \) becomes

\[ t_2 = 6.28 \times 10^{-6} \text{ sec.} \quad (4-36) \]

and \( z \) is slightly modified to become

\[ z = \omega_n t_2^2 / 2 = 0.988 \quad (4-37) \]

The numerical values obtained using these design parameters have been included in Figure 4-3 and Figure 4-5.

4.6 Hold-In Range

The maximum frequency range over which the loop will track without unlocking can be determined from the steady-state error equation of the second order loop

\[ E_{ss} = \frac{\Delta \omega}{K_o K_d A_o} \quad (4-38) \]

where \( \Delta \omega \) is the frequency offset.

Now since the phase detector has a dynamic range of \( \pm \pi/2 \) radians, the loop drops out of lock when the error exceeds these bounds.

The Hold-In Range is then
\[ \Delta \omega = K_0 K_d A_0 \frac{\pi}{2} = 3.6 \times 10 \times 1.07 \text{ rad/s}, \text{ which corresponds to } 90 \text{ MHz.} \]

Such a Hold-In Range is not possible in practice, as one of the loop elements will saturate first. An experimental verification that \( \Delta \omega_H \) meets the tracking requirements is necessary and this is provided in Chapter 6.

### 4.7 Lock-In Range

If the loop is initially unlocked, and the input signal (of sufficient signal-to-noise ratio) is introduced, then there is a finite frequency range of the input signal for which the loop will immediately achieve lock. This range is defined as the Lock-In Range, and for the second order loop it has been shown that the latter is equal to the high frequency loop gain.

\[ \Delta \omega_L = K_0 K_d \frac{t_2}{t_1} = 2z\omega_n \]  \hspace{1cm} (4-39)

For \( z = 1 \) and \( \omega_n = 2\pi \times 50 \times 10^5 \),

\[ \Delta f_L = \Delta \omega_L / 2\pi = 100 \text{ kHz} \]  \hspace{1cm} (4-40)

In Section 4.4.3, a value of \( f_n = 50 \text{ kHz} \) was chosen in order to suppress the rms phase jitter at the output of the loop. A smaller value of \( f_n \) could certainly have been specified without increasing the phase jitter to an unacceptable value. One of the effects of lowering the value of \( f_n \), i.e., \( \omega_n \), would be to decrease the loop bandwidth and provide additional filtering of the input noise to the loop. The results of Section 4.3.1, however, showed that the phase jitter due to the I.F. noise was already negligible. Consequently there is little to be gained by such a change. On the
other hand, Equation 4-39 indicates that in order to acquire lock quickly, the largest possible value of $\omega_n$ should be used. The design value of $f_n = 50$ kHz then, represents a compromise between a high value of $\omega_n$ and the required large ratio of $B_{\text{if}}/\omega_n$ for loop stability.

4.8 Sweep Waveform

The maximum VCO sweep rate that can be used to acquire lock is $\omega_n^2$. For, in order to acquire lock, the VCO must be within a range of frequencies that is equal to the Lock-In Range, which is equal to $2\pi \omega_n$. The time taken by the loop to lock up is in the order of $\pi \omega_n$, which establishes the maximum sweep rate at about $\omega_n^2$.

If the sawtooth sweep waveform proposed in Section 3.3.3 is to be employed, then the rise time of the leading edge of the sawtooth must be fast enough to ensure that the loop will be kicked out of lock, in the event that a wrong-sideband lock is encountered. For a 5 MHz sweep width, the required rise time is about 300 usec.
5. INSTRUMENTATION

5.1 Components

The instrumentation of the phase-locked link discussed in Section 3.2 is shown in Figures 5-1 and 5-2.

The voltage-controlled oscillators and all of the circuitry of Figures 5-3 to 5-5 were designed and built for the project; however the following items were purchased from outside suppliers:

Mixers M1, M2, M5
These are Sage Laboratories Inc. balanced mixers:
input bandwidth 1.0 - 2.0 GHz
noise figure (with a 1.2 dB N.F. amplifier) 7.0 dB
isolation between L.O. and signal ports 25 dB
conversion loss (2 MHz I.F.) 6.0 dB

Mixer M3
Relcom Corp. double-balanced mixer. Input bandwidth 0.2 - 500 MHz.

Directional Couplers DC1, DC2, DC3, DC4
Microlab/FXR Model CA-45N dB directional couplers. Isolation is 23 dB.

Resistive Power Splitters/Combiners RC1, RC2, RC3
Microlab/FXR Model DA-4FN
loss between any two ports 6.0 dB
bandwidth DC - 2.0 GHz

High-Pass Filters F1, F2
Microphase Corp. Model HT-1000 AB
5.2 Control Station

A block diagram of the control station is shown in Figure 5-1. The high-level transmitted signal at 1392 MHz and the low-level level received signal at 1390 MHz are dupplexed to the cable by the directional couplers DC1 and DC2. This dupplexing is necessary to prevent the signal at 1392 MHz from overloading the mixer M2. The 2 MHz signal which is propagated down the cable must be prevented from entering the 2 MHz I.F. stage of the upper-sideband generator or the 2MHz I.F. stage that follows mixer M2. This is accomplished by the high-pass filter Fl, which has a 60 dB insertion loss at 2 MHz. The resistive power splitter/combiner RC2 is used to combine the 1392 MHz and the 2 MHz signals for propagation down the cable.

The mixer M3 is a double-balanced or ring-modulator type. Such a device produces an output at the I.F. port only if both the L.O. and input signal are present. This particular choice for M3 was necessary in order to suppress the second harmonic components of the signal and L.O. frequencies, which appear at 4 MHz. These 4 MHz components add vectorially to the 4 MHz output signal of M3 obtained by the mixing of a 2 MHz signal and a 2 MHz L.O. The addition of such a spurious phase term at 4 MHz results in a phase error between the control signal and the output signal at the satellite, that varies periodically with changing line length.

Division by two is accomplished by incorporating the mixer M4 in a regenerative loop, tuned to the desired output frequency of
Figure 5-1
Instrumentation - Control Station
Figure 5-2

Instrumentation - Satellite Station
2 MHz. For this application, such a method has a distinct advantage over, say, a binary divider. The latter would provide no output at 2 MHz until a 4 MHz signal was applied to the input of the device. When one considers the method by which this 4 MHz signal is derived, it can be seen that considerable difficulty in getting the system to lock up could be anticipated, if such a divider were used. In the regenerative mixer of Figure 5-1, the feedback loop sustains an oscillation at 2 MHz. This 2 MHz signal provides the local oscillator signal for mixer M4 and also serves as the reference signal for the upper-sideband generator. The provision of this reference signal and a control signal enables the upper-sideband generator to lock. A 4 MHz signal then appears at the input of mixer M4, which synchronizes the free-running frequency of the regenerative mixer and locks up the system.

5.3 Satellite Station

A block diagram of the satellite station is shown in Figure 5-2. The instrumentation is similar to that used for the upper-sideband generator at the control station. Here, however, the trigger circuit for the sweep has been changed to provide a lower-sideband lock and the sweep waveform has been inverted so that the VCO is initially driven to a lower frequency and is then swept toward a higher frequency.

5.4 Circuitry

Figure 5-3

The I.F. amplifier, A1, is a Fairchild uA 702C operational amplifier. The resistors R1 and R4 and the compensating capacitor
All T's are 20 turns 24 Formex bifilar wound on Philips 2F6-583-2Z2 ferrite core.
All RFC's are Philips 1422-585-0015B.

All transistors are 2N 124.
All diodes are 1N 4154.
All by-pass C's are 0.1 µf.

Figure 5-3 Translation Loop
I.F. Channel, Reference Channel, Loop Phase Detector
Figure 5-4: Translation Loop (shown for lower-sideband lock)
Lock Indication Circuits, Sweep Circuit, Loop Filter.
C2 were chosen to provide a closed-loop gain of 54 dB and a video bandwidth of 4.5 MHz. The amplifier response was rolled off at the lower end at 50 kHz by choosing $C_1 = 0.022 \mu F$, to provide an I.F. bandwidth of about 4 MHz with a center frequency of 2 MHz. This ensures that the I.F. bandwidth requirements of Section 4.5.3 are met.

In Section 4.5.1, the need to keep the I.F. filtering to a minimum was emphasized. Having established the I.F. bandwidth by shaping the frequency response of $A_1$, the other I.F. stages were deliberately made broad-band.

The limiter stage $A_2$, is a Fairchild uA 703C emitter-coupled RF amplifier, with a DC to 150 MHz capability. Hard limiting without saturation is achieved by loading the output of the uA 703C with $R_5$.

Wide-band interstage coupling is realized by using bifilar-wound distributed-line transformers, of the type described by Ruthroff. The transformers used in the circuits of Figures 5-3 to 5-5 exhibit a flat response from 20 kHz to 15 MHz.

These bifilar-wound transformers are also used in the unbalanced-to-balanced hybrids which drive the loop phase detector, quadrature phase detector, phase shifter and mixer $M_4$. Under the assumption that the transformers are ideal over the frequency range of interest, it can readily be found that:

1. The balanced arms are isolated from each other, with the impedance looking into either unbalanced port equal to one-half the impedance across the balanced arms.
2. When the unbalanced arms are terminated to ground by equal
impedances $R$, the output impedance at either balanced arm to
ground is $2R$. The balanced arms are isolated from each other.

(3) If a signal is applied to each unbalanced port, then the sum of
these signals appears at one of the balanced ports and their
difference at the other balanced port.

The loop phase detector is of the 4-diode type, driven from
two unbalanced-to-balanced hybrids. It can be seen from Figure 5-3
that the device consists of a pair of conventional 2-diode phase
detectors placed back-to-back. The advantage of the 4-diode circuit
is that when the I.F. and reference signals are in quadrature, the
resulting null is insensitive to unbalances in either or both of
these signals.

The $90^\circ$ phase shift required for the lock indication scheme
of Section 3.3.3 is introduced into the reference channel of the
loop phase detector by $R31$ and $C6$ which are connected across the
balanced arms of the hybrid made from transformers $T5$ and $T6$.

Figure 5-4

The loop filter, which uses a uA 702C operational ampli-
fier, has already been described in Section 4.1. The output of the
loop filter drives transistor $Q10$, which in turn controls the collec-
tor current of $Q11$. In the absence of a sweep signal, transistor $Q11$
acts as a current sink that determines the collector current of the
transistor oscillator and the frequency of oscillation of the latter.

The reference signal at $C$, which is $90^\circ$ out of phase with
the reference signal applied to the loop phase detector, is coupled
to the hard limiter $A5$ through transformer $T11$. The output of $A5$
and the I.F. signal are applied to the unbalanced arms of the hybrid T12, T13. The sum and difference of these signals appear across CR5 and CR6 respectively. The detected output currents are summed by the resistive network, to give a signal that is proportional to the phase difference. For the case of a lower-sideband lock, this signal is applied to the non-inverting input of comparator A6, as shown in Figure 5-4.

The threshold of the comparator is adjusted by potentiometer A6. When the loop is locked to the lower sideband, the voltage at pin 3 of A6 is negative and less than \( V_t \), where \( V_t \) is the threshold level. If the loop loses lock or locks to the upper sideband, the voltage at pin 3 becomes greater than \( V_t \). A6 then goes to the ON state. The loss of lock is indicated by a lamp which is driven by transistor Q9.

Transistors Q8, Q17, and Q16 form a trigger circuit that turns on the astable circuit consisting of Q14 and Q15.

The output of the astable circuit is differentiated and DC restored by C13, R75 and CR9, to produce a sawtooth waveform. The sweep waveform drives the current sources Q12 and Q13. The latter are connected in push-pull, and sweep the collector current and consequently, the frequency of the transistor oscillator. The clock rate of the astable circuit is approximately 2 Hz, which results in a sweep rate for the VCO of about 4 MHz/sec.

For the upper-sideband generator, resistor R50 is connected to pin 2, i.e., the inverting input of comparator A6. Also, the sweep waveform is inverted, by reversing CR9 and connecting R75 to the +12 volt supply.
Figure 5-5

The I.F. signal from mixer M2 is applied to pin E in Figure 5-5. The I.F. stage for mixer M2 consists of an amplifier A8, followed by a bandpass filter at 2 MHz of 250 kHz bandwidth. The 2 MHz I.F. signal at pin G is then fed to the signal port of the double-balanced mixer M3.

The output of the mixer M3 at pin H is amplified by A9, hard limited by A10 and filtered by a 4 MHz bandpass filter consisting of L2, C21 and R103. The bandwidth of the latter is 750 kHz.

The limited signal at 4 MHz is then fed to the signal port of the balanced mixer M4. The I.F. output of this mixer is amplified and limited by A1, filtered at 2 MHz and fed back from the emitter of Q23 to the L.O. port of M4. Since the L.O. signal appears in phase at both balanced output ports of the hybrid T17, T18, the diodes CR13 and CR14 are switched alternately at the 2 MHz L.O. frequency. In the absence of a signal at the input of mixer M4, the feedback loop sustains a free-running oscillation at 2 MHz.

The 4 MHz signal from M3 then synchronizes this free-running oscillation to perform the division by two of the 4 MHz frequency. The divider output is taken from the emitter of Q24, attenuated 40 dB by R114, R115, R116, R117 and then fed to the reference input of the upper-sideband generator. This attenuation can be reduced as necessary, when the rotary phase shifter is placed between the divider and the upper-sideband generator.

5.5 The Design of the Transistor Microwave Oscillator

The design of the oscillator involved two steps. These were:
(a) The selection of an active device that was potentially unstable at the frequency of interest and that was capable of providing a sufficient amount of power at this frequency.

(b) The choice of a resonant structure that would utilize the instability of (a) and allow the power generated by the active device to be delivered to an external load at the desired frequency.

Several UHF power transistors were evaluated for their suitability for use as oscillators at 1390 MHz. For each transistor, common base $y$-parameters at 1390 MHz were measured for a range of collector-to-base voltage and collector current values. The stability of the transistors for each set of bias conditions was studied, using a modification of a method developed by Linvill,\(^{(10)}\) as follows:

The transistor, on a linear basis, can be represented by a two part network that is driven by a source of admittance $Y_s$ and terminated by a load of admittance $Y_L$. For some specified source and load, the voltages $V_1$ and $V_2$ will appear at the input and output terminals respectively. Furthermore, because the system is linear, the source can be scaled so that $V_1$ takes on a value of unit magnitude with zero phase. This scaling in no way affects the input and output admittances or the power gain of the transistor. The substitution theorem can then be invoked, to replace the source with a voltage generator $V_\perp$ (unit magnitude and zero phase) and the load with a voltage source $V_2$ that applies to the output terminals the same voltage as that taken by the load. This representation is shown in
Figure 5-6. Thus any excitation and load applied to the transistor can be simulated by a suitable choice of the quantities $L$ and $M$, in the complex voltage generator $V_2$.

$$V_1 = 1/0^\circ \quad (5-1)$$

$$V_2 = \frac{-y_{21}}{2} (L + jM) \quad (5-2)$$

Figure 5-6 Representation of the Transistor as a Linear Two-Port

The input power $P_i$ is then

$$P_i = \text{Re}(V_1 I_1^*) = g_{11} - \frac{\text{Re}(y_{12}y_{21})L}{2 g_{22}} + \frac{\text{Im}(y_{12}y_{21})M}{2 g_{22}} \quad (5-3)$$

and the output power is

$$P_o = -\text{Re}(V_2 I_2^*) \quad (5-4)$$

which results in

$$(L - 1)^2 + M^2 = 1 - 4 \frac{g_{22} P_o}{|y_{21}|^2} \quad (5-5)$$

In the three-dimensional space of $L$, $M$ and $P$, where $P$ represents power, (5-3) is the equation of a plane and (5-5) the equation of a
paraboloid. The curve $P_o = 0$ is a circle of unit radius, centered at $L = 1, M = 0$ lying in the $L$-$M$ plane.

The projection of $\text{grad } P_i$ on the $L$-$M$ plane and the line of intersection of the $P_i$ and $L$-$M$ planes can be easily determined from (5-3). Referring to Figure 5-7, we have

$$\phi = \tan^{-1} \frac{\text{Im}(y_{12} y_{21})}{-\text{Re}(y_{12} y_{21})}$$

(5-6)

The line of intersection, i.e., $P_i = 0$, is perpendicular to the gradient direction and has an $M$-intercept given by

$$b_M = \frac{-2 \varepsilon_{22}\varepsilon_{11}}{\text{Im}(y_{12} y_{21})}$$

(5-7)

![Diagram of paraboloid and planes](image)

**Figure 5-7** Geometrical Interpretation of $P_i$ and $P_o$
If the load admittance is given by

\[ Y_L = g_L + j b_L \]  \hspace{1cm} (5-8)

the above can be mapped into the L-M plane by the relationship

\[ Y_L = -\frac{I_2}{V_2} \]  \hspace{1cm} (5-9)

Equation (5-9) maps the admittance plane into the \( P_o = 0 \) circle in the L-M plane. This map, the so-called Linvill Chart, can be obtained from the Smith Chart by translating the latter one unit to the right and a rotating through 180°. The load conductance \( g_L \) and the load susceptance \( b_L \), are then related to the quantities \( r \) and \( x \) of the Smith Chart, by the following:

\[ g_L = g_{22} r \]  \hspace{1cm} (5-10)

\[ b_L = g_{22} x - b_{22} \]  \hspace{1cm} (5-11)

The \( y \)-parameters for the 2N 3866 at \( f = 1390 \) MHz, \( V_{cb} = 18 \) volts, \( I_c = 50 \) mA were as follows:

\[ y_{11} = (10.8 - j 7.6) \] mmhos.

\[ y_{12} = (-31.5 - j 2.1) \] mmhos.

\[ y_{21} = (0 - j 40.0) \] mmhos.

\[ y_{22} = (35 + j 70) \] mmhos.

The position of the input power plane relative to the L-M plane, for the above data, is shown in Figure 5-8.
Figure 5-8

Linvill Chart for 2N3366 at 1390 MHz

V_{cb} = 18 \text{ volts}
I_c = 50 \text{ mA}
It is of interest that in Figure 5-8, the input power plane intersects the L-M plane within the area of the unit circle. Over the crosshatched region in Figure 5-8, $P_1$ is negative for positive values of $P_0$. The transistor will then oscillate when terminated in any load value that corresponds to the values of $L$ and $M$ falling within this region. On the basis of Equations (5-10) and (5-11) and the measured data, it can be seen that this unstable region corresponds low values of load conductance and a negative load susceptance. Just such an admittance is that presented by an anti-resonant circuit that is slightly below anti-resonance at 1390 MHz.

![Figure 5-9 Equivalent Circuit of the Transistor and Load](image-url)
Frequency and Power vs Collector Current

Transistor Microwave Oscillator

Figure 5-10
The required anti-resonant structure was realized by the two transmission lines $T_1$ and $T_2$ in parallel, as shown in Figure 5-9. The line $T_1$ is terminated in a short circuit and $T_2$ is terminated by a tuning stub, represented by the variable capacitance $C_s$. The transistor is probe-coupled to the junction of the two lines at P. The probe coupling capacitance is represented by $C_p$. The length $l_1 + l_2$ is about 15% shorter than one-quarter wavelength at 1390 MHz and the capacitance $C_s$ can be adjusted so that the admittance $Y_L$, seen by the collector terminals of the transistor, falls within the unstable region of Figure 5-9. The oscillator can then be mechanically tuned with $C_s$ and the frequency of oscillation pulled by varying the collector current. The latter changes the value of the transistor output capacitance $C_o$. The details of the construction of the oscillator are shown in the Appendix. The design and construction of the VCO centered at 1392 MHz is similar.

The variation of the frequency and the power output of the TMO as a function of the collector current is shown in Figure 5-10.

The frequency is linear with collector current for only a limited range of the latter. Over the interval of ±1 MHz about 1390 MHz the oscillator gain constant is approximately 330 kHz/mA.

The output power in Figure 5-10 exhibits an almost linear variation with frequency. About one-tenth of the output power would be fed to the L.O. port of the 1420 MHz-to-30 MHz mixer at the satellite antenna. The resulting change in L.O. level would then be about 1 mW, over a 2 MHz tracking range. More precise control of the power level could be obtained with the use of a PIN diode leveller.
6. PERFORMANCE OF THE SYSTEM

6.1 Testing the Phase-Locked Link

The performance of the phase-locked link was checked, using the test configuration of Figure 6-1.

The control signal and the 2 MHz reference signal were obtained from a Hewlett-Packard 614A L-band signal generator and a Tektronix Model 191 signal generator respectively. The frequency at the output of the satellite station was monitored with a Beckman Model 6146 counter fitted with a Model 609 Heterodyne Unit. Phase comparisons were made directly at the microwave frequency, by applying the control and output signals to the A and B channels respectively, of the HP 140A sampling oscilloscope. The attenuation of the cable and its time varying path length were simulated by a
variable attenuator and a GR Model LK-20L adjustable line.

The system steady state phase error for a frequency offset was measured by first locking the system and then changing the frequency of the control signal. With the sampling oscilloscope triggered on channel B, the resulting phase error was measured from the displacement of the trace on channel A relative to that of channel B. The Hold-In Range was determined by observing with the counter, the range of the control signal frequency over which the system remained in lock.

The Lock-In Range was determined by unlocking the loop, then slowly changing the control frequency until lock was attained.

Finally, the performance of the system with regard to its ability to track out errors due to changes in the cable length was checked. A 20 C.M. change in line length was introduced by means of the adjustable line, while the control and the output signals were observed on the sampling oscilloscope.

6.2 Results

The steady-state phase error rate for each translation loop was set to a value of 1°/MHz, by adjusting the values of resistor R22 and potentiometer R21. Since no stability problems were evident for this value of loop gain, the value of the latter was then increased by a factor of three, in order to decrease the steady-state phase error rate to 0.3°/MHz. This corresponded to increasing the values of $\omega_n$ and $z$ for each translation loop by a factor of 1.6, since both these loop parameters are proportional to the square root of the DC loop gain. This increase in loop gain did not introduce any apparent difficulties with regard to lock acquisition. The
steady-state phase error rate between the control signal and the output signal at the satellite loop was then measured and found to be 0.5°/MHz. This is about 1.5 times the error generated by one of the translation loops as predicted in Section 4.2. In Section 4.2, the total frequency offset that could be introduced into the loop was estimated to be 2 MHz. On the basis of this estimate, the increased value of loop gain would then result in a system phase error of less than 1.5° maximum.

The Hold-In Range which was limited by the dynamic range of the loop filter A7, was measured and found to be ±8 MHz about 1390 MHz.

The Lock-In Range was measured and found to be ±200 kHz about 1390 MHz.

In the final test, the sampling oscilloscope observations indicated that the change in cable length of one wavelength at 1390 MHz was tracked out by the system to within a small error. This error appeared as a periodic displacement of one of the traces relative to the other, with a maximum deviation of 1.5°. After investigation, this phenomenon was attributed to the generation of spurious second harmonic components of the L.O. signal at the output of mixer M3. This error could be further reduced by replacing mixer M3 with a unit that has a lower harmonic content.
7. CONCLUSIONS

A new method of obtaining phase-coherent local oscillator signals for a two-dish supersynthesis at 1420 MHz has been devised. The feasibility of this method has been demonstrated by the design and construction of a system that provides phase-coherent, 1390 MHz signals at two sites that are separated by a time-varying path length.

Tests performed indicate that in a laboratory environment the phase error of the system is well within the allowable limits of $15^\circ$ maximum. Field tests, which would have brought out the phase errors due to temperature effects on the system components, were not possible at this time. However, these temperature effects are anticipated to be small, as both the control station and the satellite station will be operated in a temperature controlled environment during the actual experiment.

The periodic phase error described in Section 6-2, although within the design limits, could be further reduced by using a double-balanced mixer that better suppresses the second harmonic components of the input signals. Such mixers are commercially available.

During the laboratory tests the maximum attenuation that was introduced between the control and satellite stations was 30 dB. With more attenuation, the loop at the satellite station no longer hard-limits the incoming signal. If, as proposed, the more economical 0.5 inch Heliax cable is to be used, the attenuation for a 1000 ft. run would be 40 dB. The additional 10 dB loss could be made up by replacing the resistive power splitters/combiners RC2 and RC3 by a reactive coupling scheme. For example, the 2 MHz signal could be coupled to the cable through a shunt arm that is short-circuited at
1390 MHz by a suitable capacitor placed one-quarter wavelength along the shunt arm.

Alternatively, the I.F. gain of the satellite loop could be increased by 10 dB. It would be advisable to add this gain as a wide-band stage in order not to introduce more I.F. filtering. This wide-band gain could be achieved by cascading with the limiter stage A2, another uA 702C limiter stage.

The duplicate system, which calls for two satellite stations and a dual control station driven from a single control signal at 1390 MHz, is presently being considered for implementation. The reasons for this choice are as follows.

The instrumentation for the satellite station is simpler than that required for the control station. The duplicate system permits the use of a satellite station at each dish, where space is at a premium. Also, for a 2000 ft. baseline, the cable runs need only be 1000 ft. long.

For the duplicate system the steady-state phase error due to a frequency offset of the control signal, will tend to be the same for each dish, if the loop gains are equal. The differential tracking error, then, can be made much less than the tracking error for a single system.

The differential phase shift due to the cables can be conveniently monitored in the duplicate system. In Section 3.5, it was pointed out that this information could be used to detect and correct for the reversal in phase of the L.O. signals that may occur when the system regains lock after a momentary loss of lock.

A duplicate system, of course, requires the construction
of another system similar to the one that has already been built. Also, the frequency synthesizer that generates the control signal and the rotary phase shifter have yet to be interfaced with the PDP-9 computer. These tasks are not considered to be extremely difficult.

This system was developed for a specific application in radio astronomy. However, the technique is quite general and could be applied to many situations requiring the phase synchronization of signals at two sites that are separated by a time-varying path length. This method, with low-noise circuit components and a narrow loop bandwidth to suppress the I.F. noise, could be used with suitable antennas, over a broadcast path of many miles. For operation of the link at higher microwave frequencies, the requirements for the VCO could possibly be met by a Gunn Oscillator.

The system described in this thesis represents a considerable improvement over existing methods of obtaining phase coherence of L.O. signals in radio astronomy interferometer and supersynthesis experiments. With the continuing interest in supersynthesis at short wavelengths, the technique should prove useful in future experiments.
REFERENCES


Figure A-1  Details of the Transistor Microwave Oscillator