AN INVESTIGATION OF GORDON'S COMMON
STOCK VALUATION MODEL

by

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We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA

September, 1968
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Date SEPTEMBER 13, 1968
ABSTRACT

The purpose of this study was to determine whether a model based upon the dividend formulation of Myron J. Gordon provides an adequate explanation of the variation in common stock prices. In particular, the hypothesis tested is that investors mainly consider dividends, the rate of growth in dividends and the risk characteristics of the firms in valuing shares of common stock.

The model was tested by using multiple regression analysis on a cross-section of U.S. companies in the machinery industry in each of the years 1956 to 1965. Alternative measures or proxies for "normalized" earnings, growth, business risk and financial risk were used in testing the model.

Empirical results supported the proposition that Gordon's model provides an adequate explanation of the variation in stock prices. On average, between 76% and 81% of the total variation was explained by the model over the ten-year period. The $R^2$ values for each year ranged between .68 and .90. The coefficients of the dividend, growth and size variables were significant at the 5% level or better in almost all the years tested and except for growth in two years, these coefficients had the sign indicated by the theory. Business risk and financial risk performed poorly as explanatory variables. The coefficients for both variables were not statistically significant at the 5% level in most years and the business risk coefficient frequently did not have the expected sign.
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CHAPTER I

INTRODUCTION TO THEORETICAL COMMON
STOCK VALUATION MODELS

A. Purpose of the Study

The purpose of this study is to determine whether a model based upon the dividend formulation of Myron J. Gordon provides an adequate explanation for the variation in common stock prices. The Gordon model, which is commonly labeled the "Gordon and Shapiro model,"\(^1\) equates the value of a share of common stock to the present value of all its future dividends.

It will be shown that under certain assumptions the price of a share equals the current dividend divided by the difference between the rate of return required by investors and the rate of growth in the dividend. The investors' required rate of return also is the rate at which the dividend expectation is discounted. This rate of profit depends upon the growth rate of the dividend and the risk characteristics of the firm. The hypothesis to be tested, then, is that investors, in valuing a share of common stock, mainly consider the dividend, the growth rate of the dividend and the risk characteristics of the company.

\(^1\)The initial article on this model appeared in Myron J. Gordon and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," Management Science, III (October, 1956), 102-110.
B. Importance of the Study

By testing a stock valuation model incorporating a hypothesis about investor behavior we hope to gain information on the determinants of share prices and the relative weights attached to these determinants by the stock market. This information would be useful not only to investors seeking to maximize their wealth but also to corporate managers seeking to maximize the market price of the company's stock.

C. Methodology

The model was tested by using multiple regression analysis on a cross-section of firms in the machinery industry in the United States in each of the years 1956 to 1965. The method of least-squares was used to estimate the parameters of the regression equations. Historical data was provided by Standard and Poor's Compustat service.

D. Outline of Presentation

The rest of this chapter will introduce the present value concept of valuation, discuss several growth models of stock valuation, describe the alternative formulations of constant growth and conclude by stating the reasons for adopting Gordon's model for testing purposes. Chapter II describes the theoretical basis of the Gordon model, including its assumptions, components and interpretation. Chapter III reviews the statistical results of Gordon's investigations. Chapter IV describes the formulation and testing of the model used in this study, including the measurement
problems encountered. Chapter V presents the results and conclusions of the study.

E. Present Value Approach to Valuation

Although there is evidence that present value approaches to stock valuation date back at least to 1869, J.B. Williams is generally credited as the originator of stock valuation models. In a book published in 1938, Williams defined the investment value of a stock as the present value of its future infinite stream of dividends:\(^3\)

\[
Vo = \sum_{t=1}^{\infty} \pi_t v^t = \pi_1 v + \pi_2 v^2 + \pi_3 v^3 + \ldots
\]  
(1-1)

where Vo = investment value at start

\[\pi_t = \text{dividend in year } t\]

\[v = \frac{1}{1 + i}, \text{ by definition}\]

\[i = \text{interest rate sought by the investor.}\]

By replacing Vo with Po, v with \(\frac{1}{1 + i}\), Equation (1) can be written in a more familiar form as

\[
Po = \sum_{t=1}^{\infty} \frac{D_t}{(1 + i)^t} = \frac{D_1}{1 + i} + \frac{D_2}{(1 + i)^2} + \frac{D_3}{(1 + i)^3} + \ldots
\]  
(1-2)

If the dividend stream remains constant, as is the case with preferred stock, the equation reduces to

\[
Po = \frac{D}{1}
\]  
(1-3)

---


F. Growth Models of Valuation

An investor who purchases a share of common stock for the future dividends realistically will expect a growing stream of dividends. Although the growth rate will vary over time, shareholders can be considered as forming a subjective probability distribution of future growth rates at any particular period of time. The mean of this distribution can be considered as the constant normal growth rate.

This section describes and illustrates various types of growth encountered in stock valuation models.

1. "No Growth"

A static company can be defined as one which expects to receive a constant annual net income, \( Y_t \), from its existing assets and has no future investment opportunities that will provide a rate of return greater than the return expected by shareholders. Hence, there is no growth possible for earnings or dividends and the price of the company's share is the sum of an annuity discounted at the shareholder's required rate of return, \( k \),

\[
P = \sum_{t=1}^{\infty} \frac{Y_t}{(1 + k)^t} = \frac{Y}{k}
\]  

(1-4)

and \( k = \frac{Y}{P} \)  

(1-5)

being the reciprocal of the price-earnings ratio.

2. Simple Growth

To illustrate simple growth it is useful to adopt the model of Ezra Solomon, which is based upon the investment opportunities approach.
to valuation. The price of a common share is equated to the sum of the present worth of the constant annual net earnings from existing assets and the premium resulting from the corporation's ability to invest a constant amount at rates of return greater than the shareholders required rate of return.

Let \( P = \text{market price of all-equity company} \)

\( Y = \text{constant annual net earnings from existing assets} \)

\( b = \text{fraction of earnings retained for investment} \)

\( r = \text{constant rate of return on future investment opportunities} \)

\( k = \text{shareholder's required rate of return} \)

\( r > k \)

Then an investment of \( bY \) in each year starting in period 0 (the present) will generate additional perpetual income streams of \( (bY)\overset{r}{(1 + k)} \), each with a present value at the time the investment is made equal to

\[
\sum_{t=1}^{\infty} \frac{(bY)r}{(1 + k)^t} = \frac{bYr}{k}
\]

which is the value of the first payment. The present worth today of this stream of additional payments \( bYr/k \) is

\[
\sum_{t=1}^{\infty} \frac{bYr/k}{(1 + k)^t} = \frac{bYr}{k^2}
\]

To produce this additional income, it was necessary each year to invest \( bY \) which has a present value of \( bY/k \). The net present

value of the additional income from investment opportunities, therefore, is
\[
\frac{bYr}{k^2} - \frac{bY}{k}
\]  
which simplifies to
\[
\frac{bY}{k} \left\{ \frac{r-k}{k} \right\}
\]  
Then the price of a common share is
\[
P = \frac{Y}{k} + \frac{bY}{k} \left\{ \frac{r-k}{k} \right\}
\]  
The second term is the investment opportunities growth factor. It increases proportionately with the profitability of investment, being \( r-k \). If \( r=k \), the growth factor equals zero, while if \( r<k \), the price is decreased.

3. Constant Growth

While the simple growth model assumed a constant amount of investment each year, the dynamic growth model assumes increasing investment at a constant rate.

Myron J. Gordon has developed a dividend model which is frequently used\(^5\) for growth models of stock valuation.\(^6\) The model is based upon the following assumptions:

(a) the corporation engages in no outside equity financing;
(b) it does not use debt financing;
(c) it will earn a return, \( r \), on investment in every future period;

---

\(^5\)For example, see Solomon, Theory of Financial Management, pp. 62-67.

(d) it will retain the fraction, b, of its income in every future period.

Since there is no outside financing, the expected dividend is a function of the corporation's current income, the investment or retention rate (which are the same) and its rate of return on investment.

The dividend in any future period is certain

\[ D_t = (1 - b) Y_t \]  \hspace{1cm} (1-11)

If the net income when \( t = 0 \) was \( Y_0 \) then

\[ Y_1 = Y_0 + rbY_0 = Y_0 (1 + rb) \]  \hspace{1cm} (1-12)

since during period \( t = 0 \), retained earnings of \( bY_0 \) have been invested at rate \( r \). Income in \( t=1 \) therefore equals the sum of \( Y_0 \) and \( rbY_0 \). Equation (1 - 12) is a compound interest expression which, if \( Y_t \) grows continuously at the rate \( g = br \), can be written.

\[ Y_t = Y_0 (1 + rb)^t \]

\[ = Y_0 e^{rbt} \]  \hspace{1cm} (1-13)

The product \( rb \) is also necessarily the rate at which earnings and reinvestment as well as dividends will grow.

If \( k \) is the stockholder's required return on the dividend \( D_t \), \( t \to \infty \) the share value at the end of \( t = 0 \) is

\[ P_0 = \int_0^\infty D_t e^{-kt} dt \]  \hspace{1cm} (1-14)

which can be rewritten using equations (1-11) and (1-13) as

\[ P_0 = \int_0^\infty (1 - b) Y_0 e^{rbte^{-kt}} dt \]  \hspace{1cm} (1-15)

\[ P_0 = \frac{(1 - b)Y_0}{k - rb}, \hspace{0.5cm} (k > rb) \]  \hspace{1cm} (1-16)
Equation (1-16) can be further simplified to

\[
Po = \frac{Do}{k-g}, \quad (k > g)
\]

(1-17)

To illustrate: A corporation pays a dividend of $1.00 and has a dividend growth of 5% per annum while investors require a return of 10% per annum.

\[
Po = \frac{\$1.00}{.10 -.05} = \$20.00
\]

A stock bought at this price would provide the investor with a dividend yield of \( \frac{\$1.00}{\$20.00} = 5\% \) plus a capital gain of 5% per annum on the increase in stock price. This relationship readily can be seen by solving Equation (1-17) for \( k \)

\[
k = \frac{Do}{Po} + g
\]

(1-18)

In the case of a constant dividend stream previously referred to in equation (1-3), \( g = 0 \). The particular merit of this model for testing purposes is that stock price is given in terms of current values instead of expected values of \( b, Y \) and \( r \).

\[\text{The same result can be obtained using algebra:}\]

\[
Po = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \ldots
\]

\[
= \frac{Do (1+g)}{1+k} + \frac{(Do (1+g)^2}{(1+k)^2} + \frac{Do (1+g)^3}{(1+k)^3} + \ldots
\]

\[
= \sum_{t=1}^{\infty} \frac{Do (1+g)^t}{(1+k)^t} = \frac{Do}{k-g}
\]
G. Alternative Formulations of Constant Growth:

Several hypotheses, consistent with constant rates of growth, have been advanced to explain common stock values. The most important of these adopt the dividends, investment opportunities, stream of earnings or discounted cash flow approach.

1. Dividend Approach

It has been popular since the time of J.B. Williams to equate the price of a common share to the present value of its future perpetual stream of dividends. The Gordon-Shapiro model discussed above illustrates the dividend approach to valuation.

The tendency of writers of the "Dividend" school is to consider the division of earnings between dividends and retentions critical to the variation in price of a common share. In his 1953 study, O. Harkavy concluded that while dividends affect price more in the short-run than in the long-run, "...given two stocks similar in all respects but dividend payout, a higher price will be paid for the stock of the company distributing a greater proportion of its earnings in dividends."

---


9 O. Harkavy, Ibid., 297.
2. **Investment Opportunities Approach**

To best illustrate that the investment opportunities approach is equivalent to the dividend approach, the model of Modigliani and Miller will be described.\(^\text{10}\) While Solomon adopted an investment opportunities approach in his "simple growth" model, he used the Gordon-Shapiro model as the basis for his "dynamic growth" model which assumes a constant rate of reinvestment rather than a constant amount of reinvestment.\(^\text{11}\)

The investment opportunities approach states that the price of a common share is determined by the earning power of the corporation's existing assets and the premium arising from the ability of the firm to make investments yielding a rate of return greater than that required by the shareholders. To illustrate this approach,

\[
\begin{align*}
Y &= \text{constant annual income from existing assets;} \\
b &= \text{fraction of income retained and reinvested;} \\
r &= \text{perpetual rate of return on new investment} \\
k &= \text{shareholders' required rate of return;} \\
r &> k \\
\end{align*}
\]

At the end of year 1, the corporation invests \(bY\), which returns an income at the end of year 2 of \((bY)r\). At this time, a second investment is made consisting of \(bY + b(bYr)\) or \(bY(1 + br)\). The firm continues to invest as shown in Table 1-1 so that both income and reinvestment increase over time at the rate \(br\).


**TABLE 1-1**

Investment Opportunities Approach to Constant Growth

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Constant Income</th>
<th>Δ Income</th>
<th>Total Income</th>
<th>Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Y</td>
<td>-</td>
<td>Y</td>
<td>bY</td>
</tr>
<tr>
<td>2</td>
<td>Y</td>
<td>(bY) r</td>
<td>Y(1 + br)</td>
<td>bY(1 + br)</td>
</tr>
<tr>
<td>3</td>
<td>Y</td>
<td>bY(1 + br) r</td>
<td>Y(1 + br)²</td>
<td>bY(1 + br)²</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

\[ I_t \]

The annual investment \( I_t \) generates an annual income \( I_t r \) with a present value at the time the investment is made equal to

\[
\sum_{t=1}^{\infty} \frac{I_tr}{(1 + k)^t} = \frac{I_tr}{k}
\]

The net benefit in year \( t \), therefore, equals

\[
\frac{I_tr}{k} - I_t = I_t \left( \frac{r - k}{k} \right)
\]

Since the investment in year \( t \) equals \( bY (1 + br) t^{-1} \), the present value of the total net benefits equal

\[
\sum_{t=1}^{\infty} \frac{bY(1 + br)t^{-1} (r - k)/k}{(1 + k)^t}
\]

\[
= \frac{r - k}{k} bY \sum_{t=1}^{\infty} \frac{(1 + br)^{t-1}}{(1 + k)^t}
\]

\[
= \frac{bY}{k - br} \left( \frac{r - k}{k} \right)
\]

The price of a share under the investment opportunities approach equals the sum of the present values of the constant income and the
income from investment opportunities, that is,

\[ P = \sum_{t=1}^{\infty} \frac{Y_t}{(1+k)^t} + \left( \frac{bY}{k - br} \right) \left( \frac{r - k}{k} \right) \]

\[ = \frac{Y}{k} + \left( \frac{bY}{k - br} \right) \left( \frac{r - k}{k} \right) \]  

(1-19)

For growth to occur, it is necessary that \( r > k \), for if \( r = k \), the second term equals zero and if \( r < k \), \( P \) will decline.

Equation (1-19) can be simplified further by taking out the common factor \( Y/k \),

\[ P = \frac{Y}{k} \left( \frac{1 + b(r - k)}{k - br} \right) \]

\[ = \frac{Y(1 - b)}{k - br} \]  

(1-20)

Since \( D_t = (1 - b)Y_t \)  

(1-11)

and \( br = g \), equation (1-20) is equivalent to (1-17).

3. Stream of Earnings Approach

The value of a share also can be defined as the sum of the present value of future earnings less the present value of the investments in the respective year of the earnings; that is,

\[ P = \sum_{t=1}^{\infty} \frac{(Y_t - I_t)}{(1+k)^t} \]  

(1-21)

If we assume that the corporation finances its investments solely through retained earnings,\(^{12}\) the investment in periods 1, 2, 3, ...

\(^{12}\)In the M&M formulation, outside financing can be taken into account without altering the result.
\(\infty\) is \(bYo, bYo(1 + br), bYo(1 + br)^2, \ldots, bYo(1 + br)^{t-1}\). The present value of the total investment is

\[
\sum_{t=1}^{\infty} \frac{bYo(1 + br)^{t-1}}{(1 + k)^t}
\]

while the present worth of the total net income is

\[
\sum_{t=1}^{\infty} \frac{Yo(1 + br)^{t-1}}{(1 + k)^t}
\]

Hence,

\[
Po = \sum_{t=1}^{\infty} \frac{Yo(1 + br)^{t-1} - bYo(1 + br)^{t-1}}{(1 + k)^t}
\]

\[
= \sum_{t=1}^{\infty} \frac{Yo(1 - b) (1 + br)^{t-1}}{(1 + k)^t}
\]

\[
= \frac{Yo(1 - b)}{k - br}
\]

which is equivalent to (1-20).

4. Discounted cash flow approach

This approach equates the price of a common share to the present value of all future net cash flows between the corporation and its shareholders. The sources of funds to the corporation are the net income \(Y_t\) and in a debt-free corporation, from the sale of additional stock, \(\bar{z}_t\). The uses of funds are for investments, \(I_t\), and dividends, \(D_t\). Equating the sources and uses of funds

\[
Y_t + \bar{z}_t = I_t + D_t
\]

or

\[
D_t = Y_t + \bar{z}_t - I_t
\]

(1-23)

in other words, the shareholders expect to receive a dividend flow equal to the total cash flow of the firm less the investment costs;
hence price
\[ P = \sum_{t=1}^{\infty} \frac{(Y_t - I_t + \delta_t)}{(1 + k)^t} \]  
(1-24)

Assuming, as in the previous formulations that \( \delta_t = 0 \)
\[ P = \sum_{t=1}^{\infty} \frac{(Y_t - I_t)}{(1 + k)^t} \]

which is the same as the stream of earnings approach (1-21).

H. Reasons for Adopting Gordon's Model

It might well be asked since the alternative formulations are equivalent, what are the advantages in adopting Gordon's dividend formulation. First, it is useful in testing because in a simple form it incorporates current values of \( r, \delta, \) and \( Y \) which are more readily ascertained than expected values. Hypotheses dealing with expectations will be discussed in Chapter III which expands the simple model into an empirical model.

Secondly, we have assumed implicitly in the previous section an ideal world of certainty. Modigliani and Miller correctly pointed out that once the investment plan is given, dividend policy becomes irrelevant.\(^{13}\) The financing of investment can be through retained earnings or by sale additional common shares without affecting the price of shares. Investors wishing the cash equivalent of greater dividends could accomplish the same by selling a portion of their shares or by pledging them as collateral for a loan.

If market imperfections exist, however, investors may not wish to postpone present consumption of dividends. Assuming that investors are risk averse, any shifting of the time pattern of dividends into

\(^{13}\) Miller and Modigliani, "Dividend Policy," 411-433.
the future, may cause them to attach greater risk to future dividends. Accordingly, the required rate of return $k$ will be increased. The effect of dividend policy on the required rate of return of shareholders will be discussed more fully in the next chapter which presents Gordon's empirical model.
CHAPTER II

GORDON'S STOCK VALUATION MODEL

Gordon's stock valuation model states that the price of a share is equal to the current dividend divided by the difference between the shareholders' required rate of return and the rate of growth of the dividend; that is,

\[ P_0 = \frac{D_0}{k - rb} \]  

or \[ P_0 = \frac{(1 - b)Y_0}{k - rb} \]

A. Assumptions

The model makes the following assumptions regarding the corporation:

1. that it issues no new shares;
2. that it maintains a constant debt-equity ratio;
3. that it will earn a rate of return, \( r \), on its investment and
4. that it will retain a constant fraction, \( b \), of its income in every future period.

Accordingly, the dividend will grow at the rate \( br \).

There is ample evidence that corporations, "particularly those engaged in manufacturing, undertake relatively little outside equity
Furthermore, the work of Gordon gives evidence that corporations attempt to maintain a constant debt-equity ratio. Accordingly, "... it is quite reasonable to assume that in estimating a corporation's future dividends investors do not consider the possible future stock sales by a corporation as being material and that they expect the corporation to maintain its existing debt equity ratio."³

The remaining two assumptions perhaps are more objectionable. While a corporation is unlikely to retain a fraction b of its income in every future period, this is not important. What is relevant is what investors expect the corporation to do. The rational investor will estimate b and r for future periods. The findings of Lintner provide evidence that firms follow a policy of paying a stable fraction of their normal earnings as dividends.⁴

Given b, the retention and hence the investment rate, a corporation is expected to earn a return of r on its investment in every future period. This assumption does not exclude the possibility that r will vary if b varies. It does exclude the possibility of r taking on different values when b is constant. The behavioral hypothesis behind this assumption is that"... if investors were polled on the change they expect in the rate of profit a corporation

---


will earn, the typical result will be a frequency distribution with mean zero and a small standard deviation. The quality of the empirical results will turn in large measure on the accuracy of this speculation."

B. Interpretation

The question whether the price of a share is independent of dividend policy will depend upon the assumption made with respect to the behavior of $r$ and $k$ as $b$ is varied.

**Case 1:** If $r$ and $k$ are independent of $b$, then

$$\frac{\partial P}{\partial b} = (r - k) \frac{Yo}{(k - rb)^2} \quad (2-3)$$

The condition when the share's price is maximized can be found by setting the partial derivative of $P$ with respect to $b$ equal to zero, in which case $r$ equals $k$. At this point, a share's price is independent of the retention rate $b$. If $r > k$, $P$ will rise with $b$ and conversely if $r < k$, $P$ will fall with $b$. The fact that $P$ will rise or fall with $b$ when $r \geq k$ is not due to financing the investment through retention but to the profitability of investment. If funds were raised through sale while $b$ was kept constant, the price would still change.

**Case 2:** If $r$ and $k$ are not independent of $b$, $P$ will not rise or fall indefinitely with $b$, however, because the rate of return $r$, is not a constant. Provided there are no indivisibilities in

---

the firm's investment opportunities \( r \) will fall as \( b \) increases. In this event,

\[
\frac{\partial P}{\partial b} = \left[ r - k - b(1 - b) \frac{\partial r}{\partial b} \right] \frac{Yo}{(k - rb)^2} \tag{2-4}
\]

By setting \( \frac{\partial P}{\partial b} = 0 \), we can also set the expression

\[
(r - k - b(1 - b) \frac{\partial r}{\partial b}) = 0.
\]

When \( b = 0 \), price is maximized when \( r = k \). If \( r > k \) at \( b = 0 \), \( P \) is maximized at a positive value of \( b \) since as \( b \) increases, \( r \) falls and \( b(1 - b) \frac{\partial r}{\partial b} \) which is negative because \( \frac{\partial r}{\partial b} \) is negative, increases in absolute amount.

At \( b = 1 \), \( P = 0 \) regardless of the value of \( r \) and \( k \). Despite the profitability of investment, a corporation's shares will have zero value if it is expected never to pay a dividend. Gordon admits, however, that his model is not the best means of dealing with the situation of a non-dividend paying corporation.\(^6\)

In order to use Equation (2-2) to solve for the optimum price we must be able to observe the variables. While \( Yo, r \) and \( b \) can be obtained from historical data, the shareholders' required return must be derived. Equation (2-2) can be rewritten as

\[
\frac{(1 - b)Yo}{Po} = k - br
\]

so that the left side is equal to \( d \), a corporation's dividend yield based on its current dividend. Then a sample of similar corporations of equivalent risk can be taken and used to estimate the parameters of

\[
d = a_o - a_1 br
\]

where \( a_o \) is an estimate of \( k \) and \( a_1 \) should equal 1.

\(^6\)Ibid., 40 fn.
Gordon, however, found that \( a_1 \) was significantly less than 1.\(^7\) But if \( k \) is a constant, independent of \( br \), \( d \) could become negative for a sufficiently high growth rate and the share's price would pass through infinity. It is unlikely, however, that \( br \) is greater than \( k \) as \( r \) would have to be greater than \( \frac{1}{b} k \) or at minimum \( r > \frac{1}{b} k \) with \( 0 \leq b \leq 1 \). Yet according to our assumption, investors would not likely expect this rate of return on investment in every future period. Moreover, a corporation whose return on investment exceeded its cost of capital would doubtless engage in outside financing until \( r = k \). Nevertheless, there are some firms which are expected to earn a high rate of return on investment for a long time and which do not engage in outside financing whose shares still sell at finite prices.

An alternative explanation is that \( k \) is not independent of \( br \) but is an increasing function of \( br \). Gordon suggests that this requirement would be satisfied by the expression

\[
d = a_0 (1 + br)^{-a_1}
\]  

(2-7)

Then as \( br \) increases, the required dividend yield, \( d \), would fall though not as rapidly and would asymptotically approach zero.

These alternative functions for \( d \) are illustrated below:

---

Since \( d = k - br \), the expression in (2-7) can be substituted in the original model \( P_0 = \frac{Y_0 (1 - b)}{k - br} \) to give

\[
P_0 = Y_0 (1 - b) \frac{1}{a_o} (1 + br)^{a_1} \tag{2-8}
\]

When \( b = 0 \), \( P = Y_0 (1 - b) \frac{1}{a_o} \). As \( b \) rises, the multiplier increases to \( \frac{1}{a_o} (1 + br)^{a_1} \). Taking the current dividend \( Y_0(1 - b) \) as given, the larger the expected growth of the dividend \( br \), the higher the price investors will pay for the share, provided the parameters of (2-8) are positive.

For this formulation to be theoretically sound, the firm's cost of capital \( k \) must be assumed to be an increasing function of \( br \), the expected rate of growth in its dividend.

\[
k - br = a_o (1 + br)^{-a_1} \tag{2-7}
\]

implies that

\[
k = a_o (1 + br)^{-a_1} + br \tag{2-9}
\]

and

\[
\frac{\partial k}{\partial br} = -a_1 a_o (1 + br)^{-a_1} = 1 - \frac{a_1 a_o}{(1 + br)^{a_1}} \tag{2-10}
\]

The product \( a_o a_1 \) should equal approximately 1. Assuming \( a_o a_1 > 1 \), \( k \) will fall as \( b \) increases from 0. As \( b \) and hence \( br \) rise, \( \frac{\partial k}{\partial br} \) will become permanently positive.

In (2-9) when \( b = 0 \), \( a_o \) is a firm's cost of capital. As \( br \) rises \( a_o (1 + br)^{-a_1} \) falls thus moderating the increase of \( k \) with \( br \).

It follows that \( a_1 \) can be considered "as the price investors are willing to pay for growth in the dividend."\(^8\)

---

The greater the value of $a_1$, the larger the price investors are willing to pay for dividend growth.

Formerly, under the assumption that $k$ was a constant, price was independent of $b$ when $r = k = a_0$. In this case, $P$ depends on the dividend and investment rates. When $br = o$, $k = a_0$. If $r = a_0$, $P$ will change with $b$ because $k$ changes. Not only does the value of a share change due to the return on investment but also because of the fraction of earnings retained or paid out in dividends.

If the assumption that $\frac{\partial k}{\partial br} > 0$ is correct, it follows that there is an optimum price associated with a finite dividend rate when the rate of return is a constant. The optimum dividend or retention rate can be found by differentiating (2-8) with respect to $b$,

$$\frac{\partial P}{\partial b} = \frac{Y_0 (1 + br)^{a_1}}{a_0} \left[ \frac{ra_1 (1 - b) - 1}{(1 + br)} \right]$$

(2-11)

When $\frac{\partial P}{\partial b} = 0$, the expression $\left[ \frac{ra_1 (1 - b) - 1}{(1 + br)} \right] = 0$. Solving for $b$,

$$b = \frac{a_1}{a_1 + 1} - \frac{1}{r(a_1 + 1)}$$

(2-12)

Then $P_o$ is maximized by this function of $b$ if $r$ is independent of $b$. Moreover, the optimum retention rate is an increasing function of the rate of return, $r$, and of $a_1$, the price investors are willing to pay for growth in the dividend.

Gordon notes that the proposition that $k$ is an increasing function of $br$ does not imply that $P_o$ is a decreasing function of $br$. From (2-8)

$$P_o = Y_0 \left(1 - b\right) \frac{1}{a_0} \left(1 + br\right)^{a_1}$$
it can be seen that for a given $b$, share price will always increase through a rise in $r$ and for a given $r$ and $a_1$, $P_0$ will rise and then fall with an increase in $b$.

C. Assumptions under which $k$ is an Increasing function of $br$:

The basic premise of Gordon's model is that the value of a share is equal to the present value of the future stream of dividends arising from the share. The present value may be arrived at by either of two methods: by discounting the expected value of the dividends at an interest rate which reflects risk or by discounting the certainty equivalent of the future dividends at the pure rate of interest.

The second assumption is that the uncertainty of dividend increases with its time in the future. In terms of the standard deviation, $\sigma$, this means that at the end of year $n$, $\sigma_n > \sigma_{n-1}$. Similarly, $\sigma_{n+1} > \sigma_n$.

The effect of increasing uncertainty on $k$ is indeterminate; that is, $k_n$ may be less than, equal to, or greater than $k_{n+1}$. If however, $k_t$ is an increasing function of $t$ then a corporation's cost of capital $k$ being a weighted average of the $k_t$'s is an increasing function of the rate of growth in the dividend.\(^9\)

\(^9\)A mathematical proof of this proposition is provided by Ramesh Gangolli in Gordon, *Ibid.*, 49-50. Also see H. Chen, "Valuation under Uncertainty," *Journal of Financial and Quantitative Analysis*, II (September, 1967), 313-325. Chen points out that while the risk of a future dividend increases with time it is more important that we know the rate at which this risk increases. He shows that if risk (1) increases at a constant rate over time $k_t = k_t + 1$, for all $t=1, 2, 3, \ldots$; (2) increases at a decreasing rate over time, $k_1 > k_2 > k_3 > \ldots > k_t > k_t + 1$; or (3) increases at an increasing rate over
Deductive argument cannot solve the problem of how the $k_t$'s behave and it should be regarded as a question of fact. Substantiation for this proposition must be based on a model incorporating this proposition in explaining common share prices.

time, $k_1 < k_2 < k_3 < \ldots < k_L < k_{L+1}$. Since Gordon's proposition that $k$ is an increasing function of $b$ is based on the assumption that the $k_t$'s increase with $t$, either it must be assumed that the risk of future dividends must increase at an increasing rate or it must be shown that $k$ increases with $b$ regardless of whether the $k_t$'s increase with $t$, to substantiate the theory.
CHAPTER III

REVIEW OF GORDON'S EMPIRICAL INVESTIGATIONS

A. Linear Models

Gordon's earliest empirical work was reported in his paper "Dividends, Earnings and Stock Prices." It attempted to establish whether investors used earnings, dividends or a combination of the two in arriving at the valuation of a share. The following linear function was used:

\[ P = a_0 + a_1 D + a_2 Y \]  \hspace{1cm} (3-1)

where

- \( P \) = year-end price
- \( D \) = dividends paid during year
- \( Y \) = earnings paid during year

Eight samples consisting of four industries for the years 1951 and 1954 were used to estimate the parameters of (3-1). The industries groups used for testing were Chemicals (32), Foods (52), Steel (34) and Machine Tools (46). In order to obtain samples of sufficient size some fringe classifications were included such as pharmaceutical manufacturers under Chemicals and forging manufacturers and other steel fabricators with the basic producers under Steel.²

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²Ibid., 100.
In addition, considerable variation existed among the firms with respect to such attributes as size, profitability, market structures for supply and demand and type of shareholder. Table 3-1 below presents the least-squares estimates of the parameters, their standard errors (the standard deviation of the sample observations about the regression or estimating line), the multiple correlation coefficients ($R$) and the coefficients of determination ($R^2$).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Constant Term</th>
<th>Coefficient of standard error of</th>
<th>Multiple Correlation</th>
<th>Coefficient of determination $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$D$</td>
<td>$Y$</td>
<td></td>
</tr>
<tr>
<td>1951</td>
<td>Chemicals</td>
<td>-7.0</td>
<td>-.8</td>
<td>16.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Foods</td>
<td>.1</td>
<td>7.0</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Steels</td>
<td>5.5</td>
<td>6.6</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Machine Tools</td>
<td>2.4</td>
<td>12.0</td>
<td>.8</td>
</tr>
<tr>
<td>1954</td>
<td>Chemicals</td>
<td>-3.0</td>
<td>25.7</td>
<td>.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Foods</td>
<td>-.4</td>
<td>10.4</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Steels</td>
<td>8.7</td>
<td>8.4</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Machine Tools</td>
<td>6.3</td>
<td>5.5</td>
<td>4.1</td>
</tr>
</tbody>
</table>
Gordon expected that if the dividend coefficient is considered an estimate of the rate of the profit the regression statistics could be evaluated as follows:

a. Since the dividend yields on good preferred stocks during these years was 4 - 5% and companies acquired through mergers had been selling at five times earnings before income tax, it would be reasonable to expect the dividend coefficient being the reciprocal of the dividend yield, to range between 10 and 25;

b. the rate of profit (and hence the dividend coefficient) should be ranked according to industry characteristics in the following order:

(1) Chemicals were characterized by size, growth and stability;
(2) foods represented a stable industry;
(3) steels included large companies and was vulnerable to cyclical trends; and
(4) machine tools consisted of comparatively small corporations which were cyclically vulnerable;

and

c. Since 1951 and 1954 differed in terms of business expectations, the coefficients between years should reflect the difference.

The results for the model incorporating both earnings and dividends were hardly encouraging. The fact that the dividend coefficient is large and statistically significant in seven out of eight samples would appear to discount the earnings hypothesis. Also, the income coefficients except for Chemicals in 1951 were very low as a measure
of the price investors placed on earnings. The dividend coefficient for Chemicals varies widely from -.8 in 1951 to 25.7 in 1954. Because there is high intercorrelation between the independent variables making the coefficients highly unstable there is little predictive value in this model.

If investors purchase a share of stock for its expected future dividends this hypothesis might be represented by the following equation:

\[ P = a_0 + a_1D + a_2(Y - D) \] (3-2)

where retained earnings \((Y - D)\) is used as a proxy measure of \(br\), the growth in the dividend. If \(B\) represents the book or asset value per share of common then

\[ br = \frac{(Y - D)}{(Y)} \cdot \frac{(Y)}{(B)} = \frac{Y - D}{B} \] (3-3)

Gordon preferred to use the absolute measure of growth \((Y - D)\) rather than to deflate retained earnings by book value because price and dividend are absolute quantities in the model.\(^3\) A weakness of this approach however is that two companies may have the same retained earnings but have vastly different rates of return on investment. Investors, it is assumed, will find a lower rate of growth starting from a high book value preferable to a higher rate of return based on a low asset value which will be discounted more heavily in the distant future.

The dividend coefficients proved more reassuring than under the first model. Only the 1951 coefficient for machine tools appeared too large. The range of variation was much smaller than previously

\(^3\)Ibid., 101.
tested although still quite large. Steels - 1951 and machine tools - 1954 were low and chemicals - 1954 was high. The standard error was much reduced.

Since the retained earnings coefficient represented the price investors were willing to pay for dividend growth, it was predictable that the coefficients should be positive. Otherwise, the results were generally poor, particularly as the coefficients are low.

The second model did provide a superior means of estimating price variation with respect to the dividend given the firm's retained earnings as compared to earnings.

It should be noted that the dividend coefficient in Table 3-2 is the sum of the dividend and earnings coefficients in Table 3-1. The increase in dividend in (3-1) causes an equivalent reduction in retained earnings while under (3-2) the retained earnings are held constant. In addition, the multiple correlation and R² statistics are the same for each year in the two tables, since price is a linear function of the same variables. Finally, the earnings and retained earnings coefficients and their standard errors are the same since a change in earnings is the same as a change in retained earnings with the dividend held constant. Not only are the coefficients low but in the case of machine tools - 1951 and chemicals - 1954, the coefficients are not statistically significant at the five per cent level.

The retained earnings coefficients are more encouraging when viewed as the price investors are willing to pay for growth in the dividend than as earnings coefficients. If the growth coefficient is low, an increase in dividends with a corresponding decrease in
TABLE 3-2
Regression of Price on Dividend and Retained Earnings

<table>
<thead>
<tr>
<th>Sample</th>
<th>Constant Term</th>
<th>Coefficient and standard error of D</th>
<th>Multiple Correlation</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1951</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>-7.0</td>
<td>15.9</td>
<td>16.7</td>
<td>.93</td>
</tr>
<tr>
<td></td>
<td>(2.7)</td>
<td>(3.1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foods</td>
<td>.1</td>
<td>12.5</td>
<td>5.5</td>
<td>.90</td>
</tr>
<tr>
<td></td>
<td>(1.1)</td>
<td>(.9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steels</td>
<td>5.5</td>
<td>8.6</td>
<td>2.0</td>
<td>.86</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(.6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine Tools</td>
<td>2.4</td>
<td>12.8</td>
<td>.8</td>
<td>.90</td>
</tr>
<tr>
<td></td>
<td>(1.0)</td>
<td>(.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>-3.0</td>
<td>30.0</td>
<td>.3</td>
<td>.92</td>
</tr>
<tr>
<td></td>
<td>(2.6)</td>
<td>(3.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foods</td>
<td>-.4</td>
<td>15.9</td>
<td>5.6</td>
<td>.91</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(1.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steels</td>
<td>8.7</td>
<td>10.4</td>
<td>2.0</td>
<td>.94</td>
</tr>
<tr>
<td></td>
<td>(1.4)</td>
<td>(.8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine Tools</td>
<td>6.3</td>
<td>9.6</td>
<td>4.1</td>
<td>.89</td>
</tr>
<tr>
<td></td>
<td>(1.2)</td>
<td>(.6)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

retained earnings will increase the price of a share more than when the growth coefficient is higher.

The dividend hypothesis in Model 2 is superior in predicting how price will change with the dividend given the retained earnings than when given the firm's earnings. The dividend coefficients are greater than for Model 1 and the standard errors are less.

There are certain limitations, however, in this kind of model. A scale factor, part of which is reflected by the presence of both
high-priced and low-priced stocks in the samples help cause correlation between variables and variation in the coefficients among industries. The influence of scale on the coefficients might be reduced by deflating the variables, by book value for instance, or by the use of logs, or by a combination of the two.

Secondly, the current values of dividends and earnings in any one year may vary from the normal values resulting from the average over some prior period. Hence a combination of current values and past averages for dividends and retained earnings might provide a better model.

The absence of other variables such as corporation size, debt-equity ratio and variability of earnings which the investor may take into account in valuing a share, would bias the retained earnings and dividend coefficients.

Lastly, a model using the actual growth rate rather than an index of growth represented by retained earnings might be an improvement. Empirically, there are problems, however. The use of book value as measure of return on investment is questionable because of variation in accounting practice. The use of past values to predict future earnings might be questioned especially if other variables are used by investors to predict future earnings. Retained earnings are quite unstable and each industry may show a different pattern of variation over time.
B. Refinement

The refined model next tested by Gordon was:

\[ \frac{P}{W} = a_0 + a_1 \frac{D}{W} + a_2 \left[ \frac{D}{W} - \frac{\bar{D}}{W} \right] + a_3 \bar{g} + a_4 (g - \bar{g}) \] (3-4)

where \( \frac{P}{W} \) = year-end price divided by book value in current year,

\( \frac{\bar{D}}{W} \) = average dividend for the prior five years divided by the book value,

\( \frac{D}{W} \) = current year's dividend divided by book value

\( \bar{g} \) = average retained earnings for the prior five years divided by book value,

\( g \) = current year's retained earnings divided by book value.

The assumption is made that in arriving at the current dividend and its growth, investors look at the average over the prior five years and the amount by which the current years values depart from these averages. Deflation by book value was undertaken to eliminate the scale effect although its use is debatable.\(^4\)

In interpreting the coefficients, if \( a_1 = a_2 \) or \( a_3 = a_4 \) implies that investors ignore the five-year average dividend or growth in dividend in favor of the current values; if \( a_2 = 0 \) this implies that the current dividend is ignored; if \( a_1 > a_2 \) this "implies that investors adjust to a change in the dividend with a log, i.e., the elasticity of expectations is less than one."\(^5\) When \( a_1 < a_2 \), of course, the reverse is true.

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\(^4\)Ibid., 104 fn.

\(^5\)Ibid., 105.
### TABLE 3-3

Regression of Price on Dividend, Retained Earnings, Change in Dividend, Change in Retained Earnings, All Deflated by Book Value.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Constant Term</th>
<th>Coefficient &amp; standard error of</th>
<th>Multiple Correlation</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>D/W</td>
<td>D/W-D/W</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>g</td>
<td>g-Ｂ</td>
<td></td>
</tr>
<tr>
<td>1951</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>-.23</td>
<td>12.42 (2.63)</td>
<td>9.79 (5.98)</td>
<td>18.74 (5.96)</td>
</tr>
<tr>
<td>Foods</td>
<td>.04</td>
<td>14.04 (1.04)</td>
<td>8.06 (2.49)</td>
<td>3.16 (1.39)</td>
</tr>
<tr>
<td>Steels</td>
<td>.15</td>
<td>9.88 (1.05)</td>
<td>6.38 (1.87)</td>
<td>1.45 (1.09)</td>
</tr>
<tr>
<td>Machine Tools</td>
<td>.12</td>
<td>12.62 (1.17)</td>
<td>5.93 (2.75)</td>
<td>.12 (.99)</td>
</tr>
<tr>
<td>1954</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>.54</td>
<td>17.38 (2.92)</td>
<td>12.71 (8.93)</td>
<td>.12 (6.39)</td>
</tr>
<tr>
<td>Foods</td>
<td>-.03</td>
<td>15.51 (1.04)</td>
<td>8.74 (2.82)</td>
<td>5.15 (1.66)</td>
</tr>
<tr>
<td>Steels</td>
<td>.18</td>
<td>9.69 (.99)</td>
<td>3.85 (1.13)</td>
<td>2.02 (.68)</td>
</tr>
<tr>
<td>Machine Tools</td>
<td>.05</td>
<td>11.65 (1.16)</td>
<td>6.06 (1.74)</td>
<td>3.70 (1.12)</td>
</tr>
</tbody>
</table>

In comparison to Table 3-2 there is a slight improvement in the dividend coefficients whose range of fluctuation among samples has been reduced. Moreover, all but the chemicals coefficients are significant at the five per cent level. Since in every sample, $a_1 > a_2$ it can be inferred that investors will not take an increase in dividends into account until the average of dividends has increased.
The growth coefficients are disappointing in that the coefficients for $g$ are poorer than for $Y - D$ in Table 3-2 and three of these are not statistically significant. Furthermore, in five samples, the change in growth coefficients exceeded those for average growth showing that in most cases, investors preferred a current change in retained earnings over past performance. The averages used do not yield reliable estimates of what investors are willing to pay for retained earnings or growth.

While this model is a great improvement over the first model there were several ways, Gordon felt it could be improved. The scale factor might be treated differently through another relationship among the variables. Five of the $R^2$ values are lower than in Table 3-2 due to the deflation by book value. The representation of growth might also be improved.

C. Nonlinear Model

The results of testing a nonlinear model were reported in Gordon's 1960 paper "The Optimum Dividend Rate."\(^6\) The new estimating equation including risk variables was

$$\ln(P/B)_t = \ln a_0 + g_t \ln a_1 + a_2 \ln (D/B)_t$$
$$+ a_3 \ln S_t + a_4 \ln U$$

(3-5)

where $P_t = \text{average of high and low prices for the months September, October and November of year } t,$

$B_t = \text{book value per share at end of year } t,$

\[ S_t = \text{product of current retention rate} \]
\[ b_t = \frac{(Y_t - D_t)}{Y_t} \text{ and the current earnings rate, } r_t = \frac{Y_t}{B_t}, \]
\[ D_t = \text{dividend paid during year } t, \]
\[ S_t = \text{size of the corporation measured by total book value of the common equity at the end of year } t \text{ and} \]
\[ U = \text{instability of past earnings measured by the standard deviation of the return, } r, \text{ on common equity over the period } 1934-1954. \]

An additional risk variable consisting of the ratio of net worth plus long-term debt to net worth was included originally but was deleted because the coefficient values were unsatisfactory.\(^7\)

Deflation by book value was undertaken to avoid correlation between price and dividends due to low-priced and high-priced shares being included in the samples.

While the eight samples consisted of the same industries and years as used previously, the criteria used for a firm's inclusion differed somewhat, reducing the sample sizes:

1. the firm must be listed on a security exchange;
2. data on income and net worth must be published continuously since 1934; and
3. the dividend in the sample year must be at least 2% of the book value per share.

\(^7\)Ibid., 101. Also see M.J. Gordon, The Investment, Financing and Valuation of the Corporation (Homewood, Illinois: Richard D. Irwin, Inc., 1962), 146-147. The eight samples were split evenly between positive and negative correlation, only three were statistically significant at the five per cent level and two had the wrong sign.
TABLE 3-4
Regression of Price on Dividend, Growth Rate, and Uncertainty Variables.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Dividend</th>
<th>Growth Rate</th>
<th>Size</th>
<th>Uncertainty</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lnD/B</td>
<td>g</td>
<td>lnS</td>
<td>lnU</td>
<td></td>
</tr>
<tr>
<td>1951</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>.825</td>
<td>4.56</td>
<td>.077</td>
<td>-.081</td>
<td>.757</td>
</tr>
<tr>
<td>(32)</td>
<td>(.132)</td>
<td>(2.29)</td>
<td>(.038)</td>
<td>(.104)</td>
<td></td>
</tr>
<tr>
<td>Foods</td>
<td>1.023</td>
<td>5.51</td>
<td>.024</td>
<td>-.283</td>
<td>.865</td>
</tr>
<tr>
<td>(49)</td>
<td>(.063)</td>
<td>(1.07)</td>
<td>(.026)</td>
<td>(.064)</td>
<td></td>
</tr>
<tr>
<td>Steels</td>
<td>.733</td>
<td>2.10</td>
<td>.002</td>
<td>-.002</td>
<td>.740</td>
</tr>
<tr>
<td>(34)</td>
<td>(.095)</td>
<td>(.90)</td>
<td>(.029)</td>
<td>(.132)</td>
<td></td>
</tr>
<tr>
<td>Machine</td>
<td>.874</td>
<td>1.54</td>
<td>.041</td>
<td>-.085</td>
<td>.757</td>
</tr>
<tr>
<td>Tools</td>
<td>(.078)</td>
<td>(.82)</td>
<td>(.042)</td>
<td>(.082)</td>
<td></td>
</tr>
<tr>
<td>(46)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>.822</td>
<td>-.11</td>
<td>.088</td>
<td>-.018</td>
<td>.792</td>
</tr>
<tr>
<td>(31)</td>
<td>(.130)</td>
<td>(1.92)</td>
<td>(.039)</td>
<td>(.100)</td>
<td></td>
</tr>
<tr>
<td>Foods</td>
<td>.953</td>
<td>3.87</td>
<td>.084</td>
<td>-.165</td>
<td>.903</td>
</tr>
<tr>
<td>(45)</td>
<td>(.055)</td>
<td>(1.08)</td>
<td>(.024)</td>
<td>(.057)</td>
<td></td>
</tr>
<tr>
<td>Steels</td>
<td>.835</td>
<td>3.71</td>
<td>.022</td>
<td>.136</td>
<td>.846</td>
</tr>
<tr>
<td>(29)</td>
<td>(.091)</td>
<td>(.85)</td>
<td>(.021)</td>
<td>(.098)</td>
<td></td>
</tr>
<tr>
<td>Machine</td>
<td>.694</td>
<td>3.24</td>
<td>.028</td>
<td>-.131</td>
<td>.774</td>
</tr>
<tr>
<td>Tools</td>
<td>(.061)</td>
<td>(.87)</td>
<td>(.044)</td>
<td>.089</td>
<td></td>
</tr>
<tr>
<td>(43)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The addition of size and uncertainty variables were an improvement. All the size coefficients had the correct sign and three were statistically significant. For the uncertainty coefficient, only one had the wrong sign while two were statistically significant. Gordon regarded the data as supporting the hypotheses that price is directly related to size and inversely related to the instability of past earnings.  

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8Gordon, "Optimum Dividend Rate," 102.
While the dividend coefficients $a_2$ are all highly significant, all but foods - 1951 are less than one. The theory predicts that $a_2 = 1$. The growth coefficients with one exception, chemicals - 1954, have the right sign and are significant. The dividend and growth coefficients are an improvement over the results of the linear models but could be improved further.

D. Nolev Model

The next empirical model employed by Gordon was reported in his paper "The Savings, Investment, and Valuation of a Corporation,"9 the basis of which are used in developing Chapter II. Equation (2-8),

$$\text{Po} = \frac{1}{a_0} \left(1 + br\right)^{a_1}$$

was expanded to take into account corporation size and instability of earnings:

$$\text{Po} = a_S \frac{Yo}{Bo} \left(1 - \frac{Do}{Yo}\right)(1 + rb)^{a_1} (1 + u)^{a_2} S^{a_3}$$  \hspace{1cm} (3-6)

where

- $a_S = \frac{1}{a_0}$
- $Yo(1-b) = Do$
- $r = \frac{Yo}{Bo}$
- $b = 1 - \frac{Do}{Yo}$
- $rb = \frac{Yo - Do}{B}$
- $S = \text{index of size} = \text{sum of net plant account and working capital}$
- $u = \text{index of instability of earnings}$

To arrive at $Do$, the normalized current dividend, it is hypothesized that investors use an exponentially weighted average of a corporation's

---

past dividends. Similarly, investors take an exponentially weighted average of past earnings to arrive at the normalized current earnings $Y_0$. The return on investment $r$ is assumed to be the normalized return on existing assets, where $B_0$ is the actual net worth or book value per share of stock at the end of $t = 0$. The instability index, $u$, is defined as the absolute (i.e., without regard to sign) average of the year-to-year change in rate of return, $Y_t/B_t$, over the period 1947 to the sample year.

Sample data was derived from 48 food and 48 machinery corporations for the years 1954 to 1957. The firms were selected from the industrial groups of the Value Line Investment Survey at the end of 1958. The criteria employed excluded firms under the following conditions:

(1) if data on earnings and dividends were not available since 1947;

(2) if abnormal market interest in the shares occurred during the sample years;

(3) if the firm's dividend fell below 2% of its book value in two or more years during the period 1951 to 1958.

The normalized earnings and dividends variables are defined as follows:

$$D'_t = \beta D_t + (1 - \beta)D'_{t-1}; \text{ and}$$

$$Y'_t = \lambda Y_t + (1 - \lambda)Y'_{t-1}.$$

If the smoothing constants $\beta$ and $\lambda$ equal one then the investor is implied to consider only the most recent values of dividends and
earnings, whereas if \( \beta \) and \( \lambda \) equal zero, only the normalized earnings and dividends in the prior period is considered. The following combinations of \( \beta \) and \( \lambda \) were tested on the sample data:

1. \( \beta = 1.0; \lambda = 1.0 \)
2. \( \beta = 0.8; \lambda = 0.7 \)
3. \( \beta = 0.8; \lambda = 0.5 \)
4. \( \beta = 0.8; \lambda = 0.3 \)
5. \( \beta = 0.6; \lambda = 0.5 \)
6. \( \beta = 0.6; \lambda = 0.3 \)

Values for \( \delta^t \) and \( r_b \) were calculated. The set \( \beta = 0.6 \) and \( \lambda = 0.5 \) was chosen because in every sample it provided the highest multiple correlation.\(^{10}\)

Initial values of 0.05 and 0.08 times the year-end book value were assigned arbitrarily to \( \delta^t \) and \( Y^t \) respectively for 1957. Each subsequent year's values were determined by the equations for \( \delta^t \) and \( Y^t \) except that \( \delta^t \) was always taken as the greater of the actual dividend or two per cent of \( B^t \).

The log form of (3-6) was used to yield a linear expression,

\[
\ln P = \ln a_5 + a_4 \ln D^t + a_1 \ln (1 + br) + \\
+ a_2 \ln (1 + u) + a_3 \ln S
\]

The dividend coefficient \( a_4 \) is included in order that this parameter can be estimated also and is not forced to equal one. The results are presented in Table 3-5.

The dividend coefficient is significantly less than one, ranging from 0.82 to 0.93 with the standard error in each sample being very small. The significance is that heretofore it has been assumed that the coefficient equals one, meaning that a doubling of the dividend, other things being equal, would double the price. Gordon suggests several possible explanations for this. Investors

\(^{10}\)Ibid., Appendix B, 50-51.
\textbf{TABLE 3-5}

Regression of Price on Exponentially Averaged Dividend and Growth Rate and on Risk Variables *

<table>
<thead>
<tr>
<th></th>
<th>\textbf{ln P}</th>
<th>\textbf{ln a5}</th>
<th>\textbf{ln D'}</th>
<th>\textbf{ln (1 + br)}</th>
<th>\textbf{ln (1 + u)}</th>
<th>\textbf{ln S}</th>
<th>\textbf{R^2}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Coeff.</td>
<td>Mean</td>
<td>Coeff.</td>
<td>Mean</td>
<td>Coeff.</td>
<td>Mean</td>
</tr>
<tr>
<td>FOOD SAMPLE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td>3.70</td>
<td>2.55</td>
<td>.671</td>
<td>.83</td>
<td>.039</td>
<td>11.80</td>
<td>.031</td>
</tr>
<tr>
<td></td>
<td>(.57)</td>
<td>(.15)</td>
<td>(.494)</td>
<td>(.06)</td>
<td>(.021)</td>
<td>(1.49)</td>
<td>(.024)</td>
</tr>
<tr>
<td>1955</td>
<td>3.69</td>
<td>2.56</td>
<td>.672</td>
<td>.93</td>
<td>.040</td>
<td>9.87</td>
<td>.029</td>
</tr>
<tr>
<td></td>
<td>(.59)</td>
<td>(.13)</td>
<td>(.503)</td>
<td>(.05)</td>
<td>(.021)</td>
<td>(1.29)</td>
<td>(.022)</td>
</tr>
<tr>
<td>1956</td>
<td>3.59</td>
<td>2.62</td>
<td>.654</td>
<td>.92</td>
<td>.042</td>
<td>8.76</td>
<td>.027</td>
</tr>
<tr>
<td></td>
<td>(.54)</td>
<td>(.14)</td>
<td>(.500)</td>
<td>(.05)</td>
<td>(.020)</td>
<td>(1.34)</td>
<td>(.020)</td>
</tr>
<tr>
<td>1957</td>
<td>3.54</td>
<td>2.49</td>
<td>.600</td>
<td>.83</td>
<td>.042</td>
<td>9.87</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td>(.52)</td>
<td>(.12)</td>
<td>(.488)</td>
<td>(.05)</td>
<td>(.018)</td>
<td>(1.25)</td>
<td>(.010)</td>
</tr>
<tr>
<td>MACHINERY SAMPLE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td>3.53</td>
<td>2.42</td>
<td>.609</td>
<td>.88</td>
<td>.047</td>
<td>4.16</td>
<td>.035</td>
</tr>
<tr>
<td></td>
<td>(.47)</td>
<td>(.22)</td>
<td>(.412)</td>
<td>(.09)</td>
<td>(.025)</td>
<td>(1.32)</td>
<td>(.023)</td>
</tr>
<tr>
<td>1955</td>
<td>3.62</td>
<td>2.57</td>
<td>.619</td>
<td>.83</td>
<td>.045</td>
<td>6.07</td>
<td>.035</td>
</tr>
<tr>
<td></td>
<td>(.51)</td>
<td>(.19)</td>
<td>(.465)</td>
<td>(.07)</td>
<td>(.022)</td>
<td>(1.39)</td>
<td>(.020)</td>
</tr>
<tr>
<td>1956</td>
<td>3.67</td>
<td>2.41</td>
<td>.553</td>
<td>.82</td>
<td>.054</td>
<td>7.68</td>
<td>.035</td>
</tr>
<tr>
<td></td>
<td>(.55)</td>
<td>(.17)</td>
<td>(.518)</td>
<td>(.05)</td>
<td>(.024)</td>
<td>(1.12)</td>
<td>(.019)</td>
</tr>
<tr>
<td>1957</td>
<td>3.36</td>
<td>2.23</td>
<td>.557</td>
<td>.85</td>
<td>.055</td>
<td>3.91</td>
<td>.033</td>
</tr>
<tr>
<td></td>
<td>(.52)</td>
<td>(.17)</td>
<td>(.518)</td>
<td>(.05)</td>
<td>(.021)</td>
<td>(1.25)</td>
<td>(.017)</td>
</tr>
</tbody>
</table>

* The standard deviations and standard errors of the means and coefficients respectively are shown in parentheses below the mean and coefficient.
may place a lower price relative to dividend on high priced shares than on low priced shares. Alternatively, there may be some normal level for the dividend and investors expect low dividends will more likely rise than will high dividends. Lastly, errors in measurement may bias the coefficient downwards.\textsuperscript{11}

The results do show an improvement in statistical significance and in the narrower range of fluctuation for the dividend and growth coefficients. The fact that the machinery results are poorer than those for the food sample could be attributed to investors being less confident that the expected rate of growth will be achieved and accordingly placing a lower price on the stock. Alternatively, an inadequate measurement for the expected rate of growth for the machinery sample may have been used.\textsuperscript{12}

The instability of earnings coefficient has a negative sign and exceeds its standard error for every sample, supporting the hypothesis that price varies inversely with earnings instability. Similarly, the size coefficient has the correct sign in every sample and is significant at the five per cent level in six of eight years.

To further validate the results, Gordon examined the food sample and determined that there was no significant bias imparted to the coefficient estimates through the existence of correlation between the squares of the residuals and the growth rate and dividend variables.\textsuperscript{13} Although the dependent variable, price, was not

\textsuperscript{11}\textsuperscript{Ibid.\textsuperscript{,} 46.}

\textsuperscript{12}\textsuperscript{Ibid.}

deflated this problem of heteroscedasticity apparently was avoided through the use of a linear equation in logs.

A second statistical problem is that of "firm effects" which arises when the same sample is used in successive cross-sections over time. If the average of the residuals for a firm departs significantly from zero it is not due to random disturbances but to the existence of one or more independent variables excluded from the model and peculiar to each firm. These firm effects increase the variance of the estimate of the dependent variable and possibly bias the parameter estimates of the independent variables.\(^{14}\)

The use of Kuh's analysis of variance test on the food samples determined that the firm effects were highly significant. In order to eliminate the problem and to secure a closer correspondence between the actual and estimated value of the dependent variable and also possibly revised parameter estimates additional variables might have to be included in the model or different measurement rules for the independent variables used.\(^{15}\)

This model has shown great promise as an attempt to explain stock price variation. The coefficients for growth and dividends are highly significant in terms of their size in relation to their standard errors and show a fairly small range of variation among samples. The risk variables used have improved the results but there is still room for improvement in the treatment of risk and the inclusion of leverage

---


could be undertaken. Finally, more accurate measurement of the variables is possible.

Using the same sample data, Gordon tested two additional models, which he labeled the "Simlev" and "Adlev" models. The results of these models were reported together with the "Nolev" model of equation (3-6) in his book, "The Investment, Financing, and Valuation of the Corporation."

E. Simlev Model

The logarithmic form used to incorporate further risk variables was

\[ \ln P = \ln a_0 + a_1 \ln D + a_2 \ln (1+br) + a_3 \ln (1 + \sigma/W) + \]
\[ + \quad a_4 \ln (1+h) + a_5 \ln \pi + a_6 \ln \mu + a_7 \ln S \quad (3-8) \]

where
\[ P = \text{Price of a common share} \]
\[ D = \text{Current dividend} \]
\[ br = \text{Growth rate of dividend} = \{\frac{\bar{Y} - D}{\bar{Y}}\} = \frac{\bar{Y} - D}{W} \]
\[ \bar{Y} = \text{Earnings normalized by exponential smoothing} \]
\[ (\sigma/W)_t = \text{Earnings instability index (leverage free)} = \]
\[ (\sigma/W)_t \frac{W_t}{W_t + L_t} \]

where \( W_t + L_t = \text{net worth + net debt per share = operating assets per share} \)
\[ h = \text{Debt-equity ratio} = L/W \]
\[ \pi = \text{Operating asset liquidity index} = \frac{7 \text{ INV} + 5 \text{ OOA} + 3 \text{ PE}}{5 \text{ (INV + OOA + PE)}} \]

where \( \text{INV} \) = normal inventory, \( \text{OOA} \) = other operating assets such as deferred charges and investments in other companies, and \( \text{PE} \) = plant and equipment net of depreciation all of which are arbitrarily weighted according degree of liquidity.\(^{17}\)

\[
\mu = \text{Debt maturity index} = \frac{1 + L/W}{1 + L'/W}
\]

where \( L = (\text{current liabilities}) + (\text{intermediate-term debt}) + (\text{long-term debt}) + (\text{liability reserves such as pension liabilities}) + (\text{preferred stock}) - (\text{cash and government bonds}) - (\text{accounts receivable}) \)

and \( L' = \) the sum of the items comprising \( L \), weighted according to their maturity so that \( \mu \) increases with maturity.\(^{18}\)

\[ S = \text{Size} = \text{assets} - \text{current liabilities}. \]

The dividend coefficients were only slightly lower for the Simlev model compared to the Nolev model. The use of the current dividend versus an exponentially smoothed average of the dividend would seem preferable because of the ease of computation. The growth coefficients, similarly, are highly significant performing again better for the food sample. The measurement of growth appears slightly better than for the Nolev model. The problems of measuring return on investment in the machinery industry remain unsolved.

\(^{17}\text{Ibid.}, 74-75, 162.\)

\(^{18}\text{Ibid.}, 80, 163.\)
<table>
<thead>
<tr>
<th></th>
<th>1954</th>
<th>1955</th>
<th>1956</th>
<th>1957</th>
<th>1958</th>
<th>Average</th>
</tr>
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<tbody>
<tr>
<td><strong>FOOD SAMPLE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Const. term, ln$a_0$</td>
<td>2.488</td>
<td>2.458</td>
<td>2.584</td>
<td>2.553</td>
<td>2.668</td>
<td>2.550</td>
</tr>
<tr>
<td>S.E.</td>
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<td>.146</td>
<td>.169</td>
<td>.134</td>
<td>.158</td>
<td>.155</td>
</tr>
<tr>
<td>Dividend, $a_1$</td>
<td>.784</td>
<td>.859</td>
<td>.885</td>
<td>.789</td>
<td>.711</td>
<td>.804</td>
</tr>
<tr>
<td>S.E.</td>
<td>.066</td>
<td>.058</td>
<td>.064</td>
<td>.049</td>
<td>.056</td>
<td>.059</td>
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<tr>
<td>S.E.</td>
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<td>1.508</td>
<td>1.789</td>
<td>1.849</td>
<td>1.930</td>
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<tr>
<td>Earn. inst., $a_3$</td>
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<td>-4.442</td>
<td>-3.885</td>
<td>-8.562</td>
<td>-3.661</td>
<td>-5.234</td>
</tr>
<tr>
<td>S.E.</td>
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<td>1.812</td>
<td>2.432</td>
<td>2.201</td>
<td>2.440</td>
<td>2.202</td>
</tr>
<tr>
<td>Leverage, $a_4$</td>
<td>-.522</td>
<td>-.416</td>
<td>-.255</td>
<td>-.343</td>
<td>-.175</td>
<td>-.342</td>
</tr>
<tr>
<td>S.E.</td>
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<td>.151</td>
<td>.163</td>
<td>.120</td>
<td>.47</td>
<td>.148</td>
</tr>
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<td>Asset liq., $a_5$</td>
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<td>.343</td>
<td>.175</td>
<td>.285</td>
<td>.164</td>
<td>.258</td>
</tr>
<tr>
<td>S.E.</td>
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<td>.181</td>
<td>.202</td>
<td>.136</td>
<td>.186</td>
<td>.184</td>
</tr>
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<td>Debt mat., $a_6$</td>
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<td>-.019</td>
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TABLE 3-6 CONTINUED

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<td>Dividend, ( a_1 )</td>
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<td>.321</td>
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<td>.331</td>
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<td>.161</td>
<td>-.628</td>
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<td>.663</td>
<td>.495</td>
<td>.574</td>
<td>.627</td>
<td>.563</td>
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<td>Coeff. of det., ( R^2 )</td>
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<td>.880</td>
<td>.913</td>
<td>.890</td>
<td>.881</td>
<td>.883</td>
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The risk variables produced mixed results. The earnings instability coefficients while having the right sign are not statistically significant for two food samples and three machinery samples. In two of the machinery samples the coefficient is exceeded by its standard error. In addition, the coefficients fluctuate widely. The coefficient is higher but less significant than for the Nolev Model. Nevertheless, Gordon believed that without the use of a leverage-free earnings instability measurement versus a levered measurement the results would have been poorer. 19

The leverage coefficient performed fairly well for the food sample but poorly for the machinery sample. Four of five machinery samples had the wrong sign. The lack of inverse correlation may be attributed to the correlation of leverage with earnings instability directly, and indirectly through earnings instability based on levered earnings. More plausibly perhaps, the leverage variable may measure profitability and security rather than leverage since secure profitable firms are more likely to use debt financing. 20 A better measurement of return than return on book value of equity may be necessary.

The operating asset liquidity and debt maturity variables performed poorly and Gordon concludes that they contribute little or nothing to the model. 21

Only the size variable performed well as a risk variable. All the samples had the right sign and eight of ten samples were

19 Ibid., 165.
20 Ibid., 166.
21 Ibid., 166-167.
statistically significant. This was an improvement on the Nolev model for the food sample while the size coefficients for the machinery sample were the same.

F. **Adlev Model**

\[
\ln P_0 = \ln a_0 + a_1 \ln D + a_2 \ln (1+br) + a_3 \ln (1+\sigma/w) + \\
+ a_4 \ln (1+h - ih/k) + a_5 \ln \pi + a_7 \ln S \quad (3-9)
\]

A new leverage variable \((1+h - ih/k)\) was substituted for \((1+h)\) where \(i = \text{interest on debt} \) and \(k = \text{rate of return required by investors on a share if a corporation's leverage and retention rates are both equal to zero. Also, the risk variable \(\mu\) was omitted because of the poor results in the previous model.}

The leverage coefficient in the food samples performed exceptionally well. Not only did \(a_4\) increase in size over the five year period but also the range of variation was reduced while the ratio of size to standard error increased from approximately two in the Simlev model to from three to five. The machinery sample values, however, failed to show improvement.

G. **Eko Model**

\[
\ln P = \ln a_0 + a_1 \ln D + a_2 \ln (1+br+vq) \\
+ a_3 \ln (1+\sigma/w) + a_4 \ln (1+h - ih/k) + \\
a_5 \ln \pi + a_7 \ln S + a_8 \ln (1 + q) \quad (3-10)
\]

The Eko model is an expansion of the Adlev model to include outside equity financing. The rate of growth in the dividend, \(br\),
<table>
<thead>
<tr>
<th></th>
<th>1954</th>
<th>1955</th>
<th>1956</th>
<th>1957</th>
<th>1958</th>
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<td>.787</td>
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<td>.063</td>
<td>.044</td>
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<td>8.959</td>
<td>10.442</td>
<td>10.038</td>
<td>10.690</td>
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<td>1.535</td>
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<td>.181</td>
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<td>1.542</td>
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<td>.666</td>
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<td>.283</td>
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<td>.879</td>
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is increased by the equity accretion rate \( v_q \) and the stock financing variable \( q \) is added where

\[
v = \text{fraction of funds invested by new shareholders during a period that accrues to the equity of existing shareholders at the start of the period}
\]

\[
q = \text{funds raised through outside equity financing as a fraction of the net worth per share.}
\]

The outside financing rate was measured by

\[
q_t = \frac{n_t}{N_{t-1}} \frac{P_t}{W_{t-1}} \quad (3-11)
\]

where

\[
N = \text{number of shares outstanding}
\]

\[
W = \text{book value or net worth per share.}
\]

For the food sample, the outside equity financing rate coefficient \( a_q \) has the right sign in every year but is not statistically significant in any year and fluctuates widely. Moreover, the accuracy of growth variable coefficient \( a_2 \) has been reduced by the addition of \( v_q \) resulting from correlation between \( v_q \) and \( q \). This correlation, in turn, has created correlation between the outside financing rate and share price.\(^{22}\)

The machinery sample again proved to be even worse. The outside equity financing coefficient has the wrong sign in two years and did not exceed its standard error. The growth coefficient was reduced in every year and the ratio to its standard error was reduced in some years.

\(^{22}\)Ibid., 172.
### TABLE 3-8

Regression Statistics for the Eko Model

<table>
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<tr>
<th></th>
<th>1954</th>
<th>1955</th>
<th>1956</th>
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<th>1958</th>
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<td>.300</td>
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<td>1.212</td>
<td>1.216</td>
<td>1.542</td>
<td>1.322</td>
</tr>
<tr>
<td>Earn. inst., ( a_3 )</td>
<td>-2.352</td>
<td>-2.858</td>
<td>-.186</td>
<td>-4.566</td>
<td>-.588</td>
<td>-2.110</td>
</tr>
<tr>
<td>S.E.</td>
<td>1.685</td>
<td>1.600</td>
<td>1.576</td>
<td>1.941</td>
<td>2.016</td>
<td>1.764</td>
</tr>
<tr>
<td>Leverage, ( a_4 )</td>
<td>.277</td>
<td>-.274</td>
<td>-.038</td>
<td>-.142</td>
<td>.322</td>
<td>.029</td>
</tr>
<tr>
<td>S.E.</td>
<td>.281</td>
<td>.297</td>
<td>.292</td>
<td>.318</td>
<td>.408</td>
<td>.319</td>
</tr>
<tr>
<td>Asset liq., ( a_5 )</td>
<td>-.218</td>
<td>-.169</td>
<td>.069</td>
<td>.029</td>
<td>.646</td>
<td>.051</td>
</tr>
<tr>
<td>S.E.</td>
<td>.358</td>
<td>.324</td>
<td>.274</td>
<td>.322</td>
<td>.341</td>
<td>.324</td>
</tr>
<tr>
<td>Size, ( a_7 )</td>
<td>.064</td>
<td>.111</td>
<td>.135</td>
<td>.116</td>
<td>.062</td>
<td>.098</td>
</tr>
<tr>
<td>S.E.</td>
<td>.050</td>
<td>.046</td>
<td>.037</td>
<td>.046</td>
<td>.048</td>
<td>.045</td>
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<tr>
<td>Stock issue, ( a_8 )</td>
<td>.382</td>
<td>-.709</td>
<td>-1.539</td>
<td>-.814</td>
<td>.891</td>
<td>-.358</td>
</tr>
<tr>
<td>S.E.</td>
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<td>.768</td>
<td>.716</td>
<td>.937</td>
<td>1.103</td>
<td>.901</td>
</tr>
<tr>
<td>Coeff. of det., ( R^2 )</td>
<td>.830</td>
<td>.872</td>
<td>.920</td>
<td>.892</td>
<td>.879</td>
<td>.879</td>
</tr>
</tbody>
</table>
H. Conclusions from Empirical Findings

Of the three models tested, the Adlev model performed best particularly for the food sample. While theoretically outside equity financing should be recognized, the Eko model did not perform as well possibly because of the difficulty is measuring that variable.

The food sample results for the Adlev model show that the dividend, growth and leverage variables are highly significant and have a small range of fluctuation. The remaining variables have the right sign and exceed their standard error in every year. Moreover, in terms of statistical significance at the five per cent level, the size coefficient is significant in four years, earnings instability in three years and operating asset liquidity in two years. With better measurement, the earnings instability coefficient could be expected to be stronger.

The machinery sample under the Adlev model performed only modestly well. Only the dividend coefficient is highly significant in every year and stable over the five year period. The growth rate coefficient performs well in the first three years, then drops until it is not statistically significant in the last year. While the earnings instability and size coefficients have the right sign in every year they each are statistically significant in only three years. The leverage and asset liquidity coefficients have the wrong sign in two years. Gordon concludes that the coefficients "do not provide reliable estimates of the parameters."23

23Ibid., 175.
CHAPTER IV

FORMULATION AND TESTING OF THE MODEL

This chapter presents the model used for testing and specifies the regression variables and the alternative measures used for some of the variables. The data and methodology also will be described.

A. Selection of Variables

1. Basic Model

The basic empirical model of equation (2-8) expresses the dependent variable, price, in the following functional relationship to the independent variables, dividends and the growth rate:

\[ P_o = D_o \left( \frac{1}{a_o} \right) (1 + br)^{a_1} \]

By replacing \( \frac{1}{a_o} \) with \( a_0 \) and \( a_1 \) with \( a_2 \) and by including a dividend coefficient \( a_1 \) which is allowed to vary instead of being constrained to equal one, the above equation can be modified to

\[ P_o = a_0 D_o a_1 (1 + br)^{a_2} \]  \hspace{1cm} (4-1)

For testing purposes, the logarithmic form is used:

\[ \ln P = \ln a_0 + a_1 \ln D + a_2 \ln (1 + br) \]  \hspace{1cm} (4-2)

2. Risk Variables

Risk is defined in this study as the variability of the corporation's expected returns from investment, due to unforeseen circumstances. These future returns are random variables with a joint probability
distribution. The investor is assumed to be able to determine subjectively the probability distribution's variance or dispersion about its expected value.¹

If the standard deviation of expected returns is considered an index of risk, the price of a share over time other things being equal, will be a decreasing function of the probability distribution's dispersion.

a. **Business Risk**: Business risk can be considered as the variability of a firm's operating income. It arises from the nature of the firm's existing assets and its future investments. Different investment projects will have different probability distributions of return. The risk of a decline in operating income will endanger the ability of the corporation to pay interest charges on its debt and dividends to shareholders.

b. **Financial Risk**: Financial risk arises from the method of financing which causes volatility or additional fluctuations in the shareholder's earnings. The use of debt financing may increase the rate of return to shareholders but it will also increase the variability of the expected return. The variability of expected earnings to the shareholders it has, and will be argued, is related to the financial structure of the corporation besides the nature of its investments.

c. **Size**: An inverse relationship is postulated between the size of a firm and the stability of its operating income. While size should be reflected in the measurement of business risk by

reducing the variability of expected earnings, a size variable is included nevertheless on the proposition that investors will consider, independently, the variability of income and size in arriving at their measure of risk. Two companies with historically identical standard deviations of operating income, for example, could, it is proposed, be in different risk classes if they differed in size, other things remaining the same. It is conceivable that the shares of large corporations may command a premium because of greater investor interest and greater marketability of the shares.

3. Complete Model

If we let

\[ u = \text{index of business risk} \]
\[ h = \text{index of financial risk} \]
\[ S = \text{index of size} \]

then our complete empirical model\(^2\) is

\[
\ln P = \ln \alpha_0 + \alpha_1 \ln D + \alpha_2 \ln (1 + br) + \alpha_3 \ln (1 + u) + \alpha_4 \ln (1 + h) + \alpha_5 \ln S
\]  

(4-3)

B. Data

The historical data used in this study is provided by Standard and Poor's Compustat service. The Compustat tape which was used here contains annual financial information for approximately nine hundred industrial companies for the period 1946 to 1965.

\(^2\)It should be noted that since \( \ln 1 = 0 \), the use of \( \ln (1+u) \) and \( \ln (1+h) \) allows the risk variables to remain positive provided \( u \) and \( h \) are positive.
This information was gathered from company records, from the Securities Exchange Commission and from outside sources. Some of the data has been adjusted by Standard and Poor's for greater comparability among firms. For the study at hand, data were taken for seventy-seven companies in the machinery industry in the United States. More will be said on this later.

C. Measurement of the Variables

1. Dependent Variable - Price (P)

The Compustat tape provides annual absolute high, low and close prices for the companies listed on national stock exchanges and the bid prices for over-the-counter issues. These prices are given on a calendar-year basis, regardless of the fiscal year-end and have been adjusted by Standard and Poor's for stock splits and stock dividends except when these occur between a fiscal year-end and the calendar year-end.

For the purpose of this study, the year-end closing price has been chosen. Arguments can be and have been offered for other measures. Yearly averages of daily average prices probably would receive a lot of support but these are not readily available. There is also support for using the mean of the high and low prices during the year. This is readily available on the Compustat tape. Our model is an attempt to describe those phenomena which determined individual share prices at a specific point in time rather than average prices over

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the year.

2. **Independent Variables**

   a. **Dividends (D)**. The current per-share dividend in the year of the cross-section is used. The data is adjusted for stock splits and stock dividends on the Compustat tape and excludes preferred stock payments in lieu of cash and also excludes spin-offs.

   The current dividend is chosen as the best measure of normal dividends on the ground that while it is true that fluctuations in dividends occur, there is evidence that corporations seek a fair degree of stability in dividends in the face of rising or falling earnings. Thus, "normalized" dividends (some average of past dividends) would not be appropriate.

   b. **Growth Rate (br)**. Growth rates can be calculated using various measures including dollar sales, net profit on total capital invested, per-share earnings, total assets, fixed assets, total net income or retained earnings. Increase in price or changes in the ratio of retained earnings to per-share earnings are also possible "growth" variables. The time period from which results might be taken for the purposes of prediction might be the previous year or an average of several previous years.

   Insofar as Gordon's model is concerned with its attention to the growth in dividends, we will define growth as the product of

---


the rate of earnings retention, \( b \) and the profitability of investment, \( r \); that is, growth,

\[ g = br \]

The retention rate is measured by the fraction of "normal earnings," \( \bar{Y} \), retained in each period:

\[ b = \frac{\bar{Y} - D}{Y} \]

Equivalently, the retention rate, \( b = 1 - \frac{D}{Y} \) where \( \frac{D}{Y} \) = the payout rate.

The profitability of investment, \( r \), can be measured in such ways as

(1) return on common equity, \( E \):
\[ r = \frac{\bar{Y}}{E} \]

(2) return on total shareholders' funds, \( W \):
\[ r = \frac{\bar{Y}}{E + PS} = \frac{\bar{Y}}{W} \]
where \( PS \) = preferred stock;

(3) return on total capital, \( C \):
\[ r = \frac{\bar{Y}}{E + PS + D} = \frac{\bar{Y}}{C} \]
where \( D \) = noncurrent debt; and

(4) return on total assets, \( A \),
\[ r = \frac{\bar{Y}}{A} \]

The return on common equity or net worth measures have a strong appeal from the viewpoint of the shareholders, who as owners are interested in the return on the money they have invested in the past.
It is a measure that can be used to compare different investments. Alternatively, the return on total capital or total assets measures receive strong support from investment analysts.\(^6\)

The after-tax earnings should be adjusted to include interest expense, in order that the effect of changes in capital structure is minimized. Hence these measures provide a consistent long-term measure of corporation performance and a comparison of the earning power of firms with different capital structures.

Superior to any of these average rates of return is the incremental or marginal rate of return. Without access to corporation records, however, it would be extremely difficult to even approximate this measure. Our tacit assumption is that investors use an average rate of return based upon the historic cost or book value of assets.

For testing purposes both the return on total assets and net worth were chosen. The first was used in conjunction to operating income and the second in conjunction with net income before dividends. Hence, the measurements of growth are:

\[
br = \frac{\tilde{Y} - D}{\tilde{Y}} \frac{\tilde{Y}}{A} = \frac{\tilde{Y} - D}{A}
\]

and

\[
br = \frac{\tilde{Y} - D}{W}
\]

respectively.

Since per-share earnings are subject to, in some instances, substantial fluctuations it was felt necessary to normalize earnings. As a first approximation, earnings per share for each corporation were regressed on time for the period 1956 to 1965 using the linear

function,

\[ \hat{Y}_t = a + bT \]

The normalized earnings, \( \hat{Y} \), was defined as the predicted value in each year obtained from the trend line.

The second step was to eliminate any bias arising from the assumption of a linear trend. In addition, two logarithmic equations also were fitted for each company:

\[ \ln \hat{Y}_t = a + bT \]
\[ \hat{Y}_t = a + b (\ln T) \]

The computed trend value for each firm's normalized earnings was taken from whichever equation of the three provided the best fit, chosen on the basis of minimizing \( \sum (Y - \hat{Y})^2 \); that is, the sum of the squared deviations between actual earnings and estimated earnings.\(^7\)

Besides eliminating a linear bias, the use of these two logarithmic functions is supported by the nature of earnings growth. The arithmetic straight line represents a constant amount of growth per year, whereas the logarithmic straight lines represent constant percentage rate of growth.

The starting point of the earnings trend is also important. Our initial assumption was that investors use historical data to determine future expected values of dividends, growth and risk. Accordingly, the trend equation was changed to cover a period up

\(^7\)Properly, goodness of fit is based upon the trend line with the lowest standard error of estimate. Since \( S.E. = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n - k}} \) and \( (n - k) \) was the same for each trend line the use of \( \sum (Y - \hat{Y})^2 \) is equivalent. This topic is treated at length in Mordecai Ezekiel and Karl A. Fox, *Methods of Correlation and Regression Analysis*, 3rd ed. (New York: John Wiley & Sons, Inc., 1959), Ch. VII.
to and including the cross-section year.

The actual period covered should be long enough to encompass the average business cycle but short enough so that lack of financial information on the Compustat tape would not seriously deplete the sample. A ten year period was chosen for each year's trend equation; that is, from $t - 9$ to $t$ where $t$ was the cross-section year. The machinery sample, originally consisting of seventy-seven firms, had been reduced to fifty-six companies when the 1956-1965 trend equations were used. By taking the period back ten years, an additional thirteen companies were deleted leaving a sample of forty-three firms.

Both operating income (income before taxes and fixed charges) and net income or earnings before preferred and common dividends were tested for their explanatory powers.

c. Business Risk ($u$):

The selection of a sample consisting of one industry or more accurately, a group of closely related industries is an attempt to control for the variation in share price due to business risk. A pure "homogeneous risk class" is however, unlikely to exist where the number of companies in any class exceeds one. Companies comprising an industry sample, while manufacturing similar products with like technology retain differences as a result among other things, of serving different geographical areas or market segments, maintaining varying degrees of product diversification or quality, or having different abilities of management.

Business or operating risk relates to the variability of operating income or earnings before interest and taxes. A useful
measure of this risk, \( u \), is the standard deviation\(^8\) of income, \( Y \); that is,
\[
u = f (\sigma_Y)
\]
As \( u \) increases, the desirability to most investors of the share of common stock to which \( u \) attaches declines and the price, other things remaining unchanged, will be reduced.

Two measures of business risk were tested:

(1) \( U = \frac{\sigma_Y}{\bar{Y}} \)

and (2) \( U = \frac{\sigma_Y}{W} \)

where \( \bar{Y} = \) average income over a ten-year period
\( W = \) net worth.

The first measure, called the "coefficient of variation," has been used in tests by Barges\(^9\) and Benishay\(^10\) and frequently is recommended as a measure of relative risk by financial writers.\(^11\) The second measure was used by Gordon (see Chapter III). Both are designed to

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\(^8\)Alternative measures of dispersion such as skewness might also have been used but are beyond the scope of this study. Cf. Fred D. Arditti, "Risk and the Required Return on Equity," Journal of Finance, XXII (March, 1967), 19-36.


account for the influence of firm size on the measurement of business risk. In the homogeneous risk class referred to above, the business risk of each firm would be the same.

d. Financial Risk: (h)

As a proxy for financial risk, we chose two measures reflecting the long-term debt paying ability of a firm; that is, the capacity to hold debt in its capital structure:

(1) \[ \text{times} - \text{fixed charges} - \text{earned} = \frac{\text{operating income}}{\text{fixed charges}} \]
and (2) debt-equity ratio.

Unfortunately the data on fixed charges was unavailable on the Compustat tape for most firms in the machinery sample and depleted the sample so greatly that the first measurement was abandoned.

With respect to the debt-equity ratio, long-term debt was chosen in the numerator. Previous tests by Modigliani and Miller included preferred stock since both preferred shareholders and bondholders receive payment before residual earnings may be distributed to common shareholders. A separate test was conducted to include current liabilities, long-term debt and preferred shares in the numerator of the leverage variable.

e. Size (S):

Numerous ways exist to measure size, such as total book value of common equity, total of net plant account and working capital, assets less current liabilities, book value of invested capital and market value of equity. This study used total assets in millions of dollars. The choice was based on expediency - it is simple to calculate. There are no apparent theoretical grounds to favor one measure over another.
D. Methodology

The machinery industry for the period 1956 to 1965 was selected for testing purposes. Thus this study extends Gordon's work which covered the period 1954 to 1958. Unlike the electrical utility industry and others, relatively little attention has been devoted to the machinery industry. A ten year period was chosen in the hope that additional information would result. There was a sufficiently large number of companies, totalling seventy-seven, whose financial data was available on the Compustat tape. The firms are classified by Standard and Poor's as follows: machine tools (8), agricultural (6), construction and materials handling (8), industrial (11), metal fabricating (13), oil well (6), specialty (16), steam generating (4) and general industrial (5).

It was necessary to set certain criteria for a firm's inclusion in the sample. To afford comparability between years, financial information on a company had to be available in each year of the ten year period otherwise the firm was deleted from the sample. Moreover, since logarithms were used, the variables for each firm were required to have values greater than zero.

The multiple regression analysis was performed by the U.B.C. TRIP program at the Computing Centre. A separate routine was required for the least-squares fitting of the earnings trend regression.
CHAPTER V

RESULTS AND CONCLUSIONS

In the initial tests, the most important objectives were to select the appropriate trend period and to choose the curve of best fit for normalized earnings. For this purpose, the variables were defined as follows:

\[ P = \text{year-end price in cross-section year} \]
\[ D = \text{current dividend in cross-section year} \]
\[ br = \text{growth rate of dividend} \]
\[ = \frac{\bar{Y} - D}{A} \]
\[ \bar{Y} = \text{normalized earnings in cross-section year} \]
\[ A = \text{total assets at book value} \]
\[ u = \text{index of business risk} \]
\[ = \frac{\sigma_Y}{\bar{Y}} \text{(coefficient of variation)} \]
\[ \sigma_Y = \text{standard deviation of net income about trend line} \]
\[ \bar{Y} = \text{average of net income from t-9 to t where t is cross-section year} \]
\[ h = \text{index of financial risk} \]
\[ = \frac{L}{E} \]
\[ L = \text{long-term debt} \]
\[ E = \text{book value of common equity} \]
\[ S = \text{size of the firm in millions of dollars of total assets}. \]
A. Regression Results

The sample estimates of the regression statistics are shown in Table 5-1. The coefficient values in each year represent the influence on price that investors attribute to each variable. The dividend coefficient ranges in magnitude from .5671 in 1965 to .8463 in 1956 and averages .7088. This means that for a cross-section of firms in 1956, for example, a 100% change in the dividend of a stock is associated with an 85% change in the price of that stock. The coefficients of the other variables can be similarly interpreted although the particular logarithmic form of the growth, business risk and financial risk variables complicates the analysis.

The dividend, growth and size coefficients statistically performed well. The dividend coefficient was "highly significant," that is, statistically significant at the 1% level and had the expected sign indicated by the theory in every year.

The growth coefficient has the expected sign in every year and is highly significant in eight of the ten years while the size coefficient is significant at the 5% level in eight of ten years including six years where it is significant at the 1% level and has the correct sign in all years. In one year, size is significant at the 10% level. The growth coefficient fluctuates considerably, however, from a high of 10.2215 in 1956 to a low of 1.9181 in 1965. The drastic decline after 1962 both in magnitude and ratio of coefficient to its standard error may indicate a disenchantment on the part of investors with "growth" in the stock market.
TABLE 5-1

Regression Statistics for Ln P = lna₀ + a₁ ln D + a₂ ln (1 + br) + a₃ ln (1 + u) + a₄ ln (1+h) + a₅ lnS

<table>
<thead>
<tr>
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<td></td>
<td>MACHINERY SAMPLE, n=43</td>
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</tr>
<tr>
<td>Constant, ln a₀</td>
<td>2.3755</td>
<td>1.8561</td>
<td>2.2242</td>
<td>2.3696</td>
<td>2.0085</td>
<td>2.6198</td>
<td>2.4222</td>
<td>2.4886</td>
<td>2.5068</td>
<td>2.6916</td>
<td>2.3561</td>
</tr>
<tr>
<td>Dividend, a₁</td>
<td>.8463ᵃ</td>
<td>.8137ᵃ</td>
<td>.8051ᵃ</td>
<td>.7065ᵃ</td>
<td>.5831ᵃ</td>
<td>.6235ᵃ</td>
<td>.6706ᵃ</td>
<td>.8240ᵃ</td>
<td>.6480ᵃ</td>
<td>.5671ᵃ</td>
<td>.7088</td>
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<td>S.E.</td>
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<td>.0877</td>
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<td>.0780</td>
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<td>S.E.</td>
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<td>2.0462</td>
<td>2.1529</td>
<td>2.1055</td>
<td>2.0838</td>
<td>1.3872</td>
<td>2.1234</td>
<td>2.3181</td>
<td>1.9993</td>
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<tr>
<td>Business Risk, a₃</td>
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<td>.1159</td>
<td>.3525</td>
<td>.4788ᵃ</td>
<td>.3507ᶜ</td>
<td>.1660</td>
<td>.0420</td>
<td>.4536ᵃ</td>
<td>.3053ᵇ</td>
<td>.0346</td>
<td>.2378</td>
</tr>
<tr>
<td>S.E.</td>
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<td>.1456</td>
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<td>.1470</td>
<td>.1861</td>
<td>.1773</td>
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<tr>
<td>Financial Risk, a₄</td>
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<td>.2648</td>
<td>.5364ᵇ</td>
<td>-.0206</td>
<td>-.0360</td>
<td>.1176</td>
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<td>-.3103</td>
<td>-.2554</td>
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<td>.3063</td>
<td>.2845</td>
<td>.1678</td>
<td>.1909</td>
<td>.2441</td>
<td>.2573</td>
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<tr>
<td>Size, a₅</td>
<td>.0590</td>
<td>.1165ᵃ</td>
<td>.1163ᵃ</td>
<td>.1098ᵃ</td>
<td>.1525ᵃ</td>
<td>.0768ᶜ</td>
<td>.0976ᵇ</td>
<td>.1096ᵇ</td>
<td>.1381ᵃ</td>
<td>.1562ᵃ</td>
<td>.1132</td>
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<td>.7995</td>
<td>.7027</td>
<td>.8037</td>
</tr>
</tbody>
</table>

ᵃ significant at the .01 level
ᵇ significant at the .05 level
ᶜ significant at the .10 level
The business and financial risk coefficients performed poorly. Neither coefficient was significant at the 5% level and both, at times, carried unexpected signs. The coefficient for business risk was positive in every year while the coefficient for financial risk was negative in six years.

The coefficient of determination, $R^2$, which is the ratio of explained variance to total variance of the dependent variable is considered high for cross-sectional regression. It averages .8037 over the ten-year period, with a range of .7027 to .8957.

In the previous test net income before preferred and common dividends was used to normalize earnings and normal earnings in turn was used in the measurement of growth. Since net income should be associated with the rate of return on net worth, $W$, rather than on total assets, the growth variable was changed to

$$ \text{br} = \left( \frac{\bar{Y} - D}{\bar{X}} \right) \quad \left( \frac{\bar{X}}{W} \right) = \frac{\bar{Y} - D}{W}, $$

where $W =$ book value of common equity and preferred stock.

Table 5-2 presents the regression statistics.

There is little change in the dividend, growth and size coefficients. They have the expected sign in every year and the dividend coefficient is highly significant in every year. The growth coefficient is highly significant in seven years out of ten years and significant at the 5% level in another, while assets are highly significant in eight years and significant at the 10% level in another year. This is a slight improvement in significance. One notable change is the decrease in the fluctuation of the growth coefficient which ranges from 1.5636 to 7.7929.
| TABLE 5-2 Regression Statistics for \( \ln P = \ln a_0 + a_1 \ln D + a_2 \ln (1+br) + a_3 \ln(l+u) + a_4 \ln(l+h) + a_5 \ln S \) After Change in Growth Variable from \( br = \frac{\bar{Y} - D}{A} \) to \( \frac{\bar{Y} - D}{W} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Constant, \( \ln a_0 \) | 2.4290         | 1.8820         | 2.2079         | 2.3943         | 2.0530         | 2.6452         | 2.4094         | 2.5101         | 2.5284         | 2.7096         | 2.3769        |
| Dividend, \( a_1 \) | .8538<sup>a</sup> | .8180<sup>a</sup> | .8073<sup>a</sup> | .7165<sup>a</sup> | .5946<sup>a</sup> | .6222<sup>a</sup> | .6757<sup>a</sup> | .8211<sup>a</sup> | .6463<sup>a</sup> | .5686<sup>a</sup> | .7124         |
| S.E.           | .0818          | .0709          | .0557          | .0742          | .0741          | .0853          | .0882          | .0593          | .0756          | .0926          | .0758         |
| Growth, \( a_2 \) | 6.9011<sup>a</sup> | 5.6806<sup>a</sup> | 5.8653<sup>a</sup> | 6.1985<sup>a</sup> | 7.7929<sup>a</sup> | 7.9456<sup>a</sup> | 7.0063<sup>a</sup> | 2.2474<sup>b</sup> | 1.5636         | 1.6957         | 5.0897        |
| S.E.           | 1.3302         | 1.2334         | 1.1892         | 1.3944         | 1.4942         | 1.5479         | 1.5279         | 1.0485         | 1.4808         | 1.5898         | 1.3836        |
| Business Risk, \( a_3 \) | .0650          | .1110          | .4156          | .4569<sup>a</sup> | .3287<sup>c</sup> | .1159          | .0601          | .4485<sup>a</sup> | .3082<sup>b</sup> | .0157          | .2326         |
| S.E.           | .1417          | .1349          | .2519          | .1681          | .1697          | .1434          | .2656          | .1179          | .1491          | .1864          | .1729         |
| Financial Risk, \( a_4 \) | .4586          | -.0127         | .2603          | -.2904         | -.3473         | -.1090         | -.6319<sup>b</sup> | -.3230<sup>c</sup> | -.3678<sup>c</sup> | -.2938         | -.1657        |
| S.E.           | .2770          | .2346          | .2251          | .2822          | .2721          | .3034          | .2805          | .1738          | .1926          | .2322          | .2474         |
| Size, \( a_5 \) | .0556          | .1157<sup>a</sup> | .1179<sup>a</sup> | .1085<sup>a</sup> | .1441<sup>a</sup> | .0739<sup>c</sup> | .0988<sup>a</sup> | .1078<sup>a</sup> | .1362<sup>a</sup> | .1517<sup>a</sup> | .1110         |
| S.E.           | .0403          | .0350          | .0339          | .0368          | .0375          | .0404          | .0369          | .0216          | .0280          | .0357          | .0346         |
| \( R^2 \)      | .8318          | .8701          | .9022          | .8204          | .8021          | .7429          | .7716          | .8954          | .7978          | .7062          | .8140         |

<sup>a</sup> significant at the .01 level  
<sup>b</sup> significant at the .05 level  
<sup>c</sup> significant at the .10 level
The business risk coefficients overall, are slightly lower in magnitude with the same level of significance. There is no improvement in sign. Financial risk, however, has the sign predicted by the theory in eight out of ten years but is significant at the 5% level in only one of those years and significant at the 10% level in two others. The coefficient of determination showed slight improvement to .8140 over ten years.

An alternative measure of business risk next was tested by changing

\[ u = \frac{\sigma_Y}{Y} \]

to

\[ u = \frac{\sigma_Y}{W}. \]

The results are presented in Table 5-3.

There was no significant change in the dividend, growth, financial risk and size coefficients. The \( R^2 \) values also were very close to previous results, averaging .8129 over the testing period. Business risk as an explanatory variable appeared to have less influence than the little it had before. Although one year had the expected sign, the fluctuation of the coefficient over the years studied increased considerably, with a range of -2.715 to 2.7266.

The use of net income in deriving normalized earnings also affects the measurement of business risk. The theory indicates that the variability of earnings applies to net operating income; that is, net income before income taxes, fixed charges and depreciation. Accordingly, in the next test, operating income was used in conjunction with the risk and growth proxies

\[ u = \frac{\sigma_Y}{W} \]

and

\[ br = \frac{\tilde{Y} - D}{A} \]
| TABLE 5-3 |
| Regression Statistics for $\ln P = \ln \alpha_0 + \alpha_1 \ln D + \alpha_2 \ln (1+br) + \alpha_3 \ln (1+u) + \alpha_4 \ln (1+h) + \alpha_5 \ln S$ After Change in Business Risk Variable from $u = \frac{\sigma_U}{Y}$ to $\frac{\sigma_U}{W}$ |

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<sup>a</sup> significant at the .01 level  
<sup>b</sup> significant at the .05 level  
<sup>c</sup> significant at the .10 level
The growth proxy includes the rate of return on total assets while the other variables were left unchanged. The regression statistics are presented in Table 5-4.

Over the ten-year period, the dividend coefficient ranged slightly lower, averaging .6706, but remained highly significant in every year. The use of operating income in the growth variable considerably reduced the magnitude of the growth coefficient. The generally lower absolute levels were evidenced particularly in 1964 and 1965, when the coefficient became negative. In addition, the coefficient was significant at the 1% level in only five years and significant at the 5% in only two other years. The size coefficient ranged slightly higher in most years but remained significant at the 5% level in eight out of the ten years.

The risk variables did not show improvement either. The coefficient for financial risk was about the same as in previous tests, having the expected sign in six years but not being statistically at the 5% level in any of the years. Business risk had the expected sign in five years but was significant in none of these years. The coefficient for business risk was significant at the 5% level in only three years.

To determine whether the poor results for risk were caused by the use of \( u = \sigma_Y/W \), the same model was tested except that the coefficient of variation was used as the proxy for business risk, that is,

\[
u = \frac{\sigma_Y}{\bar{Y}}\]
TABLE 5-4

Regression Statistics for \( \ln P = \ln a_0 + a_1 \ln D + a_2 \ln (1+br) + a_3 \ln (1+u) + a_4 \ln (1+h) + a_5 \ln S \) After Change to Operating Income with \( u = \frac{\sigma_Y}{W} \) and \( br = \frac{\bar{Y} - D}{A} \)

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\(^a\) significant at the .01 level  
\(^b\) significant at the .05 level  
\(^c\) significant at the .10 level
As Table 5-5 shows, the coefficient for business risk performed poorly. Only three years had the sign indicated by the theory compared to five years in Table 5-4 and only one year was significant compared to three years previously. In neither case did the business risk coefficient have the correct sign in the year in which it was significant. There was, however, a considerable reduction in the range of fluctuation over the years.

More importantly, the use of the coefficient of variation allowed the growth coefficient to "improve." In all ten years, compared to eight years previously, growth has the expected sign. In addition, eight of the years are significant, being one more than before. The range of fluctuation is slightly reduced as well. Comparison of Table 5-2 with 5-3 and Table 5-4 with 5-5 indicates that there is a slight advantage in using \( \sigma_y / \bar{Y} \) instead of \( \sigma_y / W \) as a business risk proxy.

Using the same model reported in Table 5-4, the financial risk proxy was changed from \( h = L/E \) to

\[
h = \frac{L + P.S. + C.L.}{E}
\]

where

- \( P.S. = \) book value of preferred stock
- \( C.L. = \) current liabilities.

The results tabulated in Table 5-6 show no appreciable improvement in the performance of this risk coefficient compared to previous tests nor in the effects on any other variable. Although one more year has the expected sign, none of the years are significant at the 5% level. The range, however, was slightly reduced.
TABLE 5-5

Regression Statistics for $\ln P = \ln \alpha_0 + \alpha_1 \ln D + \alpha_2 \ln (1+br) + \alpha_3 \ln (1+u) + \alpha_4 \ln (1+h) + \alpha_5 \ln S$ After Change to Operating Income with $u = \frac{\sigma_Y}{\overline{Y}}$

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a significant at the .01 level
b significant at the .05 level
c significant at the .10 level
### Table 5-6

Regression Statistics for $\ln P = \ln a_0 + a_1 \ln D + a_2 \ln (1+br) + a_3 \ln (1+u) + a_4 \ln (1+h) + a_5 \ln S$ After Change to Operating Income with $u = \frac{oY}{W}$ and $h = \frac{D+P.S+C.L.}{E}$

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\(^a\) significant at the .01 level  
\(^b\) significant at the .05 level  
\(^c\) significant at the .10 level
B. Conclusions

The purpose of this thesis was to determine whether a stock valuation model based upon the dividend formulation of Myron J. Gordon provides an adequate explanation of the variation in stock prices over an extended period of time. It can be concluded that the empirical results support the theory that investors mainly consider the current dividend, its expected rate of growth and its risk in determining the price of a share. Tests conducted on a sample taken from the machinery industry, showed that on average over a ten-year period, between 76% and 81% of the total variance in share price was explained by the model. The lowest $R^2$ value in any year was .6781, while the highest value was .9022.

The coefficients of dividends, growth and size were statistically significant at the 5% level or better in most of the years tested. The dividend coefficient, in particular, was highly significant in every year. Furthermore, except for the growth coefficient in two years, these coefficients had the expected sign indicated by the theory. In terms of stability, only the growth coefficient fluctuated widely. The change in absolute level and stability of growth can be partly explained by the measurements used for corporate earnings or return on investment. The most important reason, however, may be changing investor valuation of growth in the stock market.

Except for a short period in early 1962, the stock market has been an expansionary phase since 1960. The tests show that after 1962, however, there was a drastic decline in the growth coefficient which corresponds to the market's reassessment of "growth stocks."
That this decline continued until 1965 probably relates to the industry’s prospects rather than to growth expectations for the overall market.

The good performance of the size variable is a mixed blessing. The fact that size appeared to be highly correlated with both financial and business risk may have contributed to the failure of the other risk variables. More importantly, the lack of statistical significance in most years would indicate that the measurement of risk was at fault. In particular, measures of business risk other than the standard deviation of income might be more successful. There is also a strong possibility that in assessing risk, investors consider such factors as quality of management, which is not derived from the balance sheet or income statement of the corporation. Lastly, it is possible that the machinery industry, itself, is not broadly representative of the stock market.

Although the model proved generally successful in predicting share price variation, its usefulness to investors for security selection is limited. Cross-section averages for an industry may ignore conditions peculiar to certain companies within the industry. Omission of other important determinants of stock prices common to an industry also is possible if less likely. A more common problem is that of accurate measurement of the variables. Assuming the problem of measurement can be overcome it is probable that the nature of the relationship between variables will change from year to year and this study indicates this is frequently the case. The weight attached to growth in dividends and earnings was found to be particularly vulnerable to change over time. Lastly,
the mechanistic use of past or current earnings - even if normalized -
to the extent that its values deviate from investors expectations
limit the model's usefulness.

The main value of Gordon's model is in specifying the
variables important to stock valuation and in clarifying the nature
of the relationships. To this extent, this model is worthy of
study by investor and corporation officer alike. To the academic
community remains the problems of empirical testing of hypotheses,
especially in the field of risk, and the extension of theory on
the nature and formulation of investor's expectations.
BIBLIOGRAPHY

Books


**Articles**


