THE RELEVANCE OF GAME THEORY IN ITS APPLICATION TO DECISION MAKING IN COMPETITIVE BUSINESS SITUATIONS

by

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B. E. (Hons.) University of Malaya, 1965.

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENT FOR THE DEGREE OF
MASTER OF BUSINESS ADMINISTRATION

In the Faculty
of
Commerce and Business Administration

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

July, 1968.

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ABSTRACT

It is known that in situations where we have to make decisions, and in which the outcomes depend on opponents whose actions we have no control, current quantitative techniques employed are inadequate. These are depicted in the complex competitive environment of to-day's business world.

The technique that proposes to overcome this problem is the methodology of game theory. Game theory is not a new invention but has been introduced to economists and mathematicians with the publication of von Neumann and Morgenstern's book "Theory of Games and Economic Behavior". Since then it has remained relatively obscured as a tool for aiding management decision making partly because it has become more mathematical in nature, and, as a result of the vast amounts of computation involved for any reasonable size problem.

However, with the advent of the high speed electronic computer the possibility of practical application to large scale business problems is within sight. Efforts are therefore seen to undergo reorientation. The recent development of a competitive decision model at Shell Development Company in California is a step forward in this direction.

This thesis examines various aspects of game theory that would appropriately lead to areas of fruitful practical applications. A series of examples were discussed to demon-

strate the analytical process of game theory. The features of the model that the Shell research group set up were analysed and discussed for possible applications to industrial situations. Our general conclusion is that, although the Shell model does indicate to us the sort of results that a game theoretic model could provide and to leave us with a clearer understanding of the problem, it has not answered the question of how we should proceed to use these results.

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ACKNOWLEDGMENT

Much of the ideas generated in the thesis were largely due to the many sessions of stimulating discussions with Professor J. Swirles. It was through him that the subject of the thesis was brought to my attention.

I wish to offer my most sincere thanks to him for the interest and encouraging words he had shown in my work as my thesis supervisor.

CHAPTER I

INTRODUCTION

Trends in Business Research

The expansion of business organizations to multi-national, multi-product and multi-level dimensions in an environment of ever-increasing complexities calls for more scientifically based rather than "rules of thumb" or "intuitive" decisions. This trend to systematic methodologies has sparked off the growth and development of quantitative techniques of operations research and management science, the tool-kit of tomorrow's manager. Further impetus has been given to this advancement with the advent of the high-speed electronic computer. The impact of such developments and the successful application to business activity is summed up by Hillier and Lieberman:

> "... some of the problems ..... have been solved by particular techniques of operations research. Linear programming has been used successfully in the solution of problems concerned with assignment of personnel, blending of materials, distribution and transportation, and investment portfolios. Dynamic programming has been successfully applied to such areas as planning advertising expenditures, distributing sales effort, and production scheduling. Queueing theory has had applications in solving problems concerned with traffic conjestion, servicing machines subjected to breakdown, determining the level of a service force, air traffic scheduling,

> design of dams, production scheduling, and hospital operation. Other techniques of operations research, such as inventory theory, game theory, and simulation, also have been successfully applied in a variety of context."[1]

Yet, in the midst of all this progress we can still find resistance to change. Change is something that cannot be achieved overnight, but is a gradual process. However, we cannot attribute this resistance to inadequacies in coping with such development. This is perhaps due to apathy and to the attitude of some of the managers belonging to the school which subscribes to the belief that management is an art, not a science.

In the past and currently, the emphasis on management science development has been in the areas of optimization techniques. The logic of the "optimum" solution and the elaborate mathematical procedures for solving it are indeed beyond dispute. However, in situations where stochastic simulation and the generation of statistical distribution functions are no longer available and we can no longer describe the variables as random, we have to look for other solution techniques.

The situation we have in mind is that where decisions have to be made in a complex competitive environment. This is one area where the stochastic techniques of decision theory,

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1. Hillier, F.S., Liebernman, G.J., "Introduction to Operations Research," Holden Day Inc., San Francisco 1967 P. 7.

simulation in industrial dynamics, risk and venture analysis, both linear and non-linear optimization techniques and optimization of stochastic situations are inadequate and incapable of giving satisfactory solution To illustrate this in a very elementary way, let us consider the following situation:

Three oil companies, A, B and C marketing gasoline in a given area have respectively 500, 300 and 200 service stations to serve as outlets for their products. Through research and forecasting facilities it is known to each that the volume of the gasoline market will increase by approximately 10 % each year for the next ten years. Besides, the rate of deterioration of service stations measured as a percentage of the existing total can be calculated from historical records or operations research studies.

Faced with this situation what would for example be the appropriate size of the marketing investment X_B of company B viewed over the next ten years? A first reaction could be to maintain the status quo, that is, to work for the same share in the expanding market by investing in a proportion of 5: 3: 2. Alternatively company B would like to know what would happen if it makes investment X_B such as to include a sizable amount, say 100 stations which include both the new market opportunities and deterioration of the older stations. Would company A and C let such a move go by unchallenged and allow B to bite into their share of the new market?

Retaliation followed by retaliation could ensue with nobody gaining anything. The answer to seek is therefore an optimum X_B in view of the competitive environment in which the firm is operating.

Game theory may overcome this decision-making problem. In essence, game theory is, as Shubik describes:

> "...a method for the study of decision-making in situations of conflict. It deals with problems in which the individual decision-maker is not in complete control of the factors influencing the outcome....
>
> The essence of a game problem is that it involves individuals with different goals or objectives whose fates are interlocked....
>
> The problem of game theory is more difficult than that of simple maximization. The individual has to work out how to achieve as much as possible, taking into account that there are others whose goals are different and whose actions have an effect on all. A decision-maker in a game faces a cross-purposes maximization problem. He must plan for an optimal return, taking into account the possible actions of his opponents." [2]

Much of the research done in game theory has been mainly concerned with the mathematical aspects of formulation and solution since the inception of the monumental work of von

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2. Shubik, Martin, "The Uses of Game Theory in Management Science," Management Science, Volume 2, No.1, October 1955. p. 40.

Neumann and Morgenstern,[3] but relatively less on its applicability to practical decision-making situations. One of the most outstanding is the research done by Shell in California.[4] The offspring of this research is a body of belief that this technique will soon be a significant aid to business decision making instead of remaining a mathematician's delightful pastime.

To see where and when the methodological tools of game theory as provided by gaming and simulation, with the advancement of computer technology, assist in the effective analysis of business and other economic situations, we turn to some assessments of Professor Martin Shubik.

> "The methodologies we have discussed are new ..... The bringing in of radical ly new approaches and techniques usually take a sizable fraction of a generation..... .... It is not unreasonable however, to expect that within twenty years many large firms and sections of the government will have detailed simulations of different aspects of the envi-

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3. von Neumann, J., and Oskar Morgenstern. "Theory of Games and Economic Behavior." Princeton University Press, Princeton (1947). Mathematical research into the theory of games includes: Kuhn, H., and Tucker, A.W., (eds.) "Contributions to the Theory of Games." Vol. I and II: Annals of Mathematical Studies 24 and 28. Princeton University Press, Princeton 1950, 1953. Various journal articles of the Society of Industrial and Applied Mathematics and publications of Rand Corporation.

4. Hughes, R.R., and Ornea, J.C. "Decision-Making in Competitive Situations." Proceedings World Petroleum Congress, Mexico City, mimeograph copy.

> ronment in which they operate. If they do so they will be in a position to explore policy alternatives and, in some cases, will also find it worthwhile to use their simulations to provide the environment for operational games in the same way as military operational games are used today. There is no royal road to this state of affairs, nor will it necessarily herald the millenium. A new methodology is becoming available at the appropriate time, when the increase in speed of technological change, joined with the size of population and the complexity of modern society, makes it imperative for us to be in a position to examine and integrate models of economic fine structure..... The methodological tools provided by gaming and simulation are making it feasible to uncover and examine in an organized manner much of the important fine structure of the firms and markets in which firms operate. Methodology and techniques alone do not provide answers and cure-alls. However, the progress to date and the costs of the progress in gaming and simulation indicate that they can be of immediate and direct use in business operations, that they are beginning to supply data and enlarge the knowledge of both the empiricist and economic theorists, and that they are beginning to provide a base for the construction of aids to guide in the framing of policy."[5]

It is recognizable that in the next two or three decades, the revolution in business thinking and the environment in which business operates will have changed so much, the survival of a firm in a competitive world depends

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5. Shubik, Martin, "Simulation and Gaming: Their Value to the Study of Pricing and Other Market Variables," in "Model of Markets," edited by Alfred R. Oxenfeldt, Columbia University Press, New York, 1963, pp. 307-338.

entirely on the strategic decisions which its executives make. In the process, one of the most challenging intellectual frontiers of business research lies in the dynamics of business organization and decision making in a highly competitive environment. It would thus appear timely for management to investigate in the strategic area of decision making using game theory once computational barriers are overcome and the problematic areas of the theory are resolved. Of these areas, the most important are those of measurement and intercomparability of utilities.

Examples of Application of Game Theory

The Canadian pulp and paper industry, and, for that matter the British Columbia pulp and paper industry as we were told in the editorial of "The Province", "has been seriously weakened by massive increase of over-capacity". These were the remarks of Mr. R. M. Fowler, President of the Canadian Pulp and Paper Association.[6] Mr. Fowler had correctly indicated this to the "spirit of competition". However, he went on to point out that these competitive forces were "vigorously aided and abetted by interprovincial competition which finds its roots in federal policies to support provin-

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6. "The Province", editorial "Competition and the Dinosaurs," February 6, 1968.

cial effort to expand."[7] In a statement to the editor of "The Province" Dr. J.W. Sutherland has this to say:

> "...Under the free enterprise system, capacity increases in anticipation of, or in response to, increase in demand. If there has been an increase in capacity in the pulp and paper industry we hope it is because at some point producers felt that there would be an increase in future demand. Current over-capacity merely indicates that the forecast of producers were in error.....
>
> ..., one cannot presume that the government's aims and objectives are identical with those of industry. Industry must compete in the market for economic advantage. Government on the other hand, are primarily concerned with political advantage. They invest in the benevolent basis of realizing advantages for the industrial sector."[8]

Bearing what has been said, we can now take a closer look at the British Columbia pulp and paper industry. The situation as it is, spells an urgent need for a realignment of market and production strategies. What ails the companies in the competition as shown by their past performance of a steady decline in profits over the years 1965-1967 despite increased sales[9] calls for special diagnosis. It is here that the new methodology of game theory would seem appropriate, though not as a panacea for the strategic ills, but could

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7. Ibid.

8. Sutherland, J.W., "Exports Suffer for Lack of Free Enterprise" in a letter to the editor of "The Province", February 12, 1968.

9. "Pulp and Paper Magazine of Canada." (August 1967). Most companies reported losses. Notably all the companies have attributed losses to price immobility and rise in wages. Other sources: Company reports 1965-1967.

improve a company's overall position.

Granted that there may be a short world market demand, and that at the moment the pulp and paper industry is in the doldrums, a company as a party in the competition must not overlook certain aspects of the markets. Competitive pressures have been building up with the entry of new firms and with extensive increases in existing capacities. In the industrial survey of British Columbia pulp and paper industry, published by B. C. Hydro[10] there were 17 mills operated by 10 companies. To this date we have 19 mills, and still more mills are to be constructed or are under construction.[11] Have these investments been undertaken with total disregard for competitive pressures and for the over-supply prevailing in the pulp and paper markets?

There is considerable similarity of this industry, regional as well as national, with the oft-quoted and familiar examples of oligopolistic industries in steel, automobiles, oils and others. We can thus portray the British Columbia pulp and paper market as a problem in oligopolistic competition.

The leading competitor is MacMillan Bloedel Limited

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10. "The Pulp and Paper Industry of British Columbia." Industrial Development Department, B.C. Hydro and Power Authority. October 1966. 2nd. Edition.

11. "Province of British Columbia Budget Speech" by W.A.C. Bennett, Premier and Minister of Finance, February 1968, p. 25.

with a number of other competitors who are either subsidiaries or associates of major companies in the U.S., Britain and Sweden or formed from groups of local independent companies. Here the entrepreneur of a firm can no longer act according to well-known laws of maximization of economic theory. He becomes a competitor whose success is determined not only by his own decision but by other entrepreneurs who he is operating against, in a situation that affords them the opportunity to exercise their free will in a manner that will vitally influence him. Joint ventures are typical in any new investment because of their immense size. Aside from telling where the investment should stop, the techniques of capital budgeting theory are totally lacking in all these considerations. Each firm tries to outdo the others with larger mills. Although this is in line with economies of large-scale, there is a point where less favorable operations may result upon bringing in the effects of competition, threats, counter-threats, entry and the like. In planning its moves, a firm should consider how its competitors will react to them. This, in effect is the crux of the problem.

For some time lately a series of shutdowns were instituted by a number of firms,[12] and yet the construction of mills is still going on. It would be interesting to know what

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12. "Pulp and Paper Magazine of Canada" (June and July 1967).

the bases for such strategies were. Nevertheless, in the midst of all this competition and outcry some firms were able to improve their share of the markets and increase earnings. What possible explanation can we offer? In the face of inflated labour costs and competition, regional, national and international, what strategic choice should a firm whose investment runs to millions make?

To advocate the use of game theoretic methods we must also note the limitations of the mechanics of the solution. Investment decisions will involve both buyers and sellers market and a firm making a decision will have to consider both oligopolistic and oligopsonistic competition.[13] All these factors can and will make a complete game theoretic model extremely unwieldy but if we accept the fact that computational problems can be solved in the near future by large-scale computing machine technology we can foresee immense applicability of game theory.

We have thus far encountered a problem in industry where game theory could be operative. As far as the literature on the subject goes, the areas in which game theory is applicable have, over the years, been considerably extended.

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13. For a firm the variables in an oligopolistic market may be price, differentation of product, etc.. In an oligopsonistic market we have interest on loans, labor, wages, transportation and material costs, etc..

Shubik,[14] Friedman[15] and Charnes and Cooper,[16] have shown how game theory can be used in the allocation of advertising expenditures. Game theoretic models for competition between two refineries with fixed demand and other problems including capital budgeting in the oil industry were discussed by G.H. Symonds[17,18] and E.G. Bennion.[19]

Within an organisational system there are competing units. Successful decentralization with respect to a set of decisions within a system, means that the independent actions of the individuals in control of the sub-systems

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14. Op . cit. p. 47-48

15. Friedman, Lawerence, "Game-Theory Models in the Allocation of Advertising Expenditures," Operations Research Volume VI, No. 5, September-October 1958, pp. 699-709. Also located in "Mathematical Models and Methods in Marketing" by F.M. Bass, R.D. Buzzell and others, Richard D. Irwin, Inc., 1961, pp.220-244.

16. Charnes, A. and Cooper, W.W., "A Constrained Game Formulation of Advertising Strategies," Econometrica, 22, October 1954.

17. Symonds, G.H., "Applications to Industrial Problems, Including Scheduling and Technological Research," Econometrica, 22, October 1954, p. 526.

18. _____, "Linear Programming: The Solution of Refinery Problems," Esso Standard Oil Company, New York, 1955 Chapter 5.

19. Bennion, E.G., "Capital Budgeting and Game Theory", Harvard Business Review, Volume 34, No. 6 (November-December) 1956, pp. 115-123.

achieve the same outcomes as a single decision maker making all the decisions. The postulation of a method of resolving this conflict of interest for easier decentralized decision-making as a solution of an n-person game was examined by Shubik.[20]

Another area where game theory has drawn attention is in the dynamics of labour-management bargaining.[21] The interactions of the parties concerned require an analysis of the process of concession and the determination of the point where agreement would be reached on the basis of some arbitration schemes. Shubik[22] has shown how game theoretic methods could be used in accounting in the assignment of joint costs and the construction of an incentive system for decentralized control. Such attempts to apply the technique to various

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20. Shubik, Martin, "Games, Decisions and Industrial Organization," Management Science, Volume 6, July 1960, pp. 455-474. The paper discussed the application and influence of game theory from 2-person to n-person and especially oriented toward possible applications in industry. In this connection it is worthwhile to mention that there are many solution concepts to n-person games according to Luce and Raiffa (Ref. Footnote 8, Chapter II). However in a recent mathematical exposition J.B. Rosen (Ref. Footnote 7, Chapter III) has shown the existence of a unique equilibrium point under certain constrained conditions.

21. Cross, J.C. "A Theory of the Bargaining Process," American Economic Review, Volume 55, 1965, pp. 67-94.

22. Shubik, Martin, "Incentives, Decentralized Control; The Assignment of Joint Costs and Internal Pricing," Management Science, April 1962, pp. 325-343.

management disciplines go to show the wide scope and versatility of its applications.

Game theory has also made inroads into other disciplinary studies, namely national and international politics and psychology.[23]

Purpose and Scope of Thesis

It is becoming more evident that the ability to deal effectively with various facets of competition, within and without the industry, ultimately determines whether an enterprise stands or falls. A major objective of the thesis is to draw attention to a body of theory, namely game theory, which has drawn considerable interest in recent years, and which in the course of time may become an indispensable tool which management can rely on in seeking optimum decisions in competitive situations. For this reason, an attempt is made at bringing together the main body of game theory in an application oriented context,[24] rather than the rigorous mathematical treatment that the subject is currently receiving since the von Neumann and Morgenstern classic.[25]

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23. A book which gives the general disciplinary areas where game theory has made progress is "Game Theory and Related Approaches to Social Behavior", John Wiley & Sons, Inc., New York, 1964, by Martin Shubik. The Journal of Conflict Resolution contains a number of articles which deals with various aspects of game theory.

24. This is especially done in "Strategy and Market Structure", John Wiley (2nd Edition), 1960, by Martin Shubik.

25. op. cit.

Some areas of management where the theory has found use and could potentially be used are observed. On the assumption that advances in computer technology are such that vast amounts of data and computation can eventually be handled we then propose to see how game theory can elucidate and contribute to strategic decision making in the oil industry. The mathematical model that we examine in this connection is the Shell model. Attention is drawn to the applicability of the model from the standpoint of the results that could be produced, and the possible use that could be made of such results so that more light will be shed on decision making in competitive situations.

Having outlined the aim of the study, Chapter II is designed to bring out the fundamental aspects of game theory. We start off with simple 2-person constant sum games. The more difficult theories of solution of nonconstant sum games are introduced. Wherever applications of these theories have made important contributions, mention is made of them. Implications of game theoretic results to our discussion, especially those from Hughes and Ornea's paper[26] are highlighted. Extensive games, a framework for moving to dynamics, form the last section of Chapter II.

In Chapter III the focus is on continuous variable and

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26. op. cit.

payoff spaces. This forms the basis for examining continuous games. The direction pursued is, as the title suggests, the dynamics of game situations.

To tie up all that has been said on the essence of game theory we discuss the structural features of the Shell model in Chapter IV. An attempt is made at considering the applicability of such a model to an industrial situation characterized by the British Columbia oil industry.

The Shell model has been successful in providing results of problems arising from competitive situations. In view of this, Chapter V sums up the survey with a note on the relevance of game theory.

CHAPTER II

PERSPECTIVES IN THE THEORY OF GAMES

1. Introduction

The virtues of mathematics are indeed many and varied. Its use as a powerful device to make precise concepts in social and economic sciences has led to the formulation of various theories of social and economic behavior. For,

> "Mathematical analysis is as extensive as nature itself, it defines all perceptible relations, measures time, spaces, forces, temperatures..... Its chief attribute is clearness, it has no marks to confused notions. It brings together phenomena the most diverse, and discovers the hidden analogies which unite them.... It seems to be a faculty of the human mind destined to supplement the shortness of life and the imperfection of the senses."
>
> --Fourier, "Analytical Theory of Heat"[1]

However, we are ever in danger of treating mathematics in a business research paper for the sake of mathematics itself. This has caused us to lose sight of what we were initially after, namely to make concepts precise, clearer and useful. Mathematics, as a language in social and economic sciences of which business is a part, should be used on the

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1. Quoted in Herbert A. Simon, "Models of Man", John Wiley and Sons, Inc, New York, 1961, (2nd. Edition)p.1.

basis of utilitarian rather than aesthetic grounds. The large number of mathematical essays in management science dealing with game theory look mainly at interesting mathematical questions. As decision makers, our interest in mathematical techniques is not the elaborate proofs, but rather their applicability to practical situations.

On the other hand opponents of the role of mathematical models in business have claimed that these are too sterile and restricted, and, that in the oversimplification and abstraction of the representation of the phenomena, the subtleties and vital content of the subject matter are often destroyed. However, even with highly simplified assumptions, frequently the problems posed are mathematically unmanageable. True, the language of mathematics is poor in adjectives. But, inspite of this drawback, a good mathematical model can display analysis of the type which enables us to follow through a chain of reasoning not possible in a verbal description of business phenomena furnished with rich adjectives. We have to admit that mathematics does serve us as a vehicle for the study of the properties of the system. To see this implication we turn to Shubik.

> "We need insights and breadth of view. It is necessary to couple this with logical clarity and analytical ability. The type of mathematical thinking exemplified by the theory of games provides a useful methodology such that with care at least some aspects of the

essential features of important problems can be examined."[2]

At the micro-economic level of the firm and market, institutional writings have been far too broadly descriptive and of use only in conveying a general picture of a firm or industry, not as an aid to decision-making for the operator of a firm. In the study of the competitive environment, we are told that,

> "The theory of oligopoly has long been one of the most unsatisfactory areas of economic theory. On the one hand, there have been a series of mathematical models of markets which have depended on simplifications so drastic as to render them of highly limited value for any purpose. On the other hand, many of the verbal descriptions have tended to be historical anecdotal in nature, providing little possibility for generalization. ...., the oligopolistic form is dominant. The elegant and simple theories of pure competition and of monopoly do not provide sufficient insight into the major market mechanisms of market structure..... ..... Among the most glaring omissions from economic analysis has been the lack of explicit attention to important distinguishing features between different competitive situations, such as type of distribution and retailing system within which a manufacturing firm operates..."[3]

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2. Shubik, Martin, "Game Theory and Related Approaches to Social Behavior", op . cit., p.4.

3. Shubik, Martin, "Simulation and the Theory of the Firm", in "Contribution to Scientific Research in Management", The proceedings of the Scientific Program Following the Dedication of the Western Data Processing Center, Graduate School of Business Administration, University of California, Los Angeles, Jan. 29-30, 1959, p. 70-71.

What we need thus is a methodology which would prescribe guidelines for the positive treatment of different market forms. This approach would of necessity be different from the patchwork theory that has surrounded the economics of oligopoly. It is generally believed here that the methods of game theory could help us find answers to problems arising from competitive situations, thereby improving our decision making capabilities. Hopefully, new insights and further light would be shed on the nature of cross-purposes maximization problems concerning competition and collusion as we probe deeper into our analysis.

Before we begin to discuss the various theories of solution of games it might be appropriate to introduce the basic concepts that are used to characterize a game situation. Some of the important assumptions that are implicit or explicit in the formulation process will be outlined.

We describe a competitive situation as a <u>game</u> involving <u>players</u> or individual autonomous decision-making units, each of which has a set of <u>strategies</u> which specifies courses of action for every possible set of <u>moves</u> of his opponents. The <u>rules</u> of the game specify the variables that each player controls, the <u>information conditions</u>, and other relevant aspects of the environment prior to the <u>choice</u> of an action from a set of alternatives. The payoffs are the values assigned to the outcome of the <u>play</u> of the game. These elements form the building blocks of game theory.

A highly complicated model of reality will mean the examination of a multitude of details, and, it is very likely that the purpose of the investigation would be lost in the maze. By making an abstraction with simplifying assumptions we can discover some of the non-obvious properties of the real world. A game theory model of a real world conflict situation has these objectives in mind when we attempt to put the latter down on paper. It is not realistic at the outset to enumerate all the assumptions that are associated with game theory as details and problems will arise as we proceed with our analysis. What we propose to do is to recognize some of the more important ones and point out those that we will encounter as we move along. True, some of the assumptions rest on shaky grounds, but we will have something to say about them in a later chapter.

The rules of behavior in game theory pertain to those of the general decision theory postulates of which one of the most comprehensive is found in the Savage Axioms.[4] By rational behavior of the decision maker, the assumption

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4. Savage, Leonard, J., "The Foundations of Statistics", John Wiley and Sons, Inc., New York, 1964. The book contains new concepts in the modern subjectivistic theory of decision making. In the main, the postulates of decision theory and that of game theory have very much in common e.g. a preference ordering of outcomes, inadmissibility of dominated actions, maximization of expected utility, etc., except that in game theory the assignment of probabilities is conditional on the results.

does not imply value judgement, but, that given his desires as known, his choice of actions should be consistent with his desires. However, we know that as human beings we are not infallible and thus imperfections are usually present. This assumption of individual behavior is crucial to the whole superstructure of game theory and it must be stressed that there should be no confusion about this. The term rational is, unfortunately far from being precise and while it assumes individuals maximizing something, it in a way pictures the individual as a conservative, pessimistic being. The opponent is assumed to be going all out to render him as much damage as possible and is intelligent and fully understands how to take advantage of any situation.

To supplement our rationality postulate we turn to the concept of a utility function which describes the decision maker's preferences, and the problem in game theory becomes one of maximizing expected utility. The measurability of utility is an age old controversy and we shall not dwell on it here.[5] To be able to assign a numerical

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5. Some experiments have been performed on the measurability of utility as an aid to decision-making. While there are many the ones that stand out are:
Grayson, C. Jackson, "Decisions Under Uncertainty, Drilling Decisions by Oil and Gas Operators", Boston, Harvard University, Division of Research, Graduate School of Business Administration, 1960.
Qwalm, R.C. "Utility Theory-Insights into Risk Taking", Harvard Business Review, November-December, 1966, pp.123-126. Green, P.E. "Risk Attitudes and Chemical Investment Decisions", Chemical Engineering Progress, January 1963, pp. 35-40.

value to the element in the game matrix, to be described in the next section, we have to assume that the payoffs from an action under all possible circumstances are numerically measurable.

Game theory which von Neumann and Morgenstern[6] developed is a normative theory. The concept of solution is plausibly a set of rules for each participant which tells them how to behave in every situation that may conceivably arise. Further, as mentioned above, it is assumed we are playing an adversary who in all respects is rational too, and, whose interests are diametrically opposed to ours. The question we might ask is, if our opponents display irrationality does the solution still hold . If we assume the players are similarly motivated the problem does not arise. However, in the event of facing an irrational opponent under complete information the normative theory would not necessarily be the best to follow as we could exploit our opponent's weakness. In realistic situations the assumption of complete information must give way to ignorance, uncertainty and indeterminate economic situations requiring more complex game analysis.[7]

To be able to construct the payoff matrix in the discrete choice games and payoff space in continuous variable

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6. op . cit. "Theory of Games and Economic Behavior."

7. Shubik, Martin, "Information, Risk, Ignorance and Indeterminacy", Quarterly Journal of Economics, Volume 68. (1954), pp. 629-640.

space games we must have all the necessary information. In order to characterize a game it must be assumed that we can specify all the variables which control the possible outcomes and all the values they may assume under different circumstances.

Leaving the controversial issues for discussion later we shall in the following sections turn our attention to the formulation of games.

2. Theory of Solution of 2-person Constant Sum Games.

This class of 2-person games is by far the most extensively developed and familiar part of game theory.[8] In general they fall into two categories, namely games where both players have finite strategies and those where at least one of the players has an infinite set of pure strategies. The first of these are known as matrix games or games in normalized form while the second type includes games of search and duels. The solution concept of the latter class of games, having value in military work and weapon defence system, is the same as the former but the mathematical difficulties

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8. 2-person games have played a central role in the theory of games mainly because economic theories of linear programming and activity analysis receive much of their impetus from the interrelationship with 2-person games. For an elaborate enumeration of the historical and mathematical prominence of 2-person theory, one is referred to Luce, R.D. and Raiffa, H., "Games and Decisions", John Wiley and Sons, Inc., New York, (1957) pp. 56-57.

are greater. In the discussion that follows we will be concerned with matrix games. A further type of 2-person games called <u>extensive games</u> will be introduced in a later section as we move away from the primarily static nature of normalized games to the dynamic features inherent in extensive games formulation.

If S_1 and S_2 are the set of strategies for player 1 (henceforth called P_1) and player 2 (henceforth called P_2) respectively, every pair of strategies $s_i \in S_1$ and $s_j \in S_2$ will have two payoff functions $R_1(s_i,s_j)$ and $R_2(s_i,s_j)$ associated with it. These represent amounts going respectively to P_1 and P_2 when they choose the strategies s_i and s_j.

By definition in a constant sum game the algebraic sum of the payoffs equals some constant.

(2-1) $$R_1(s_i,s_j) + R_2(s_i,s_j) = C$$

In the limit we have a zero sum game where C=0. In other words we have a strictly competitive game, meaning what is gained by one of the players is lost by the other. There is thus no motivation for collusion in 2-person zero sum games.

(2-2) $$R_1(s_i,s_j) = -R_2(s_i,s_j)$$

As the name implies we can represent the payoffs to P_1 in the form of a matrix as shown in table (2-1).

The behavioristic assumptions concerning the method of play of zero-sum games is given by the minimax and maximin principles. P_1 in the above game, if he assumes the postulates outlined earlier, will achieve his optimal strategy by choosing i such that

$$\max_i \min_j R_1(s_i, s_j)$$

Similarly P_2's choice of j must be such as to provide him at least,

$$\text{(2-3)} \quad \max_j \min_i R_2(s_i, s_j) = \max_j \min_i -R_1(s_i, s_j)$$

or

$$\text{(2-4)} \quad \max_i \min_j R_1(s_i, s_j) = \min_j \max_i R_1(s_i, s_j)$$

The value of the game is thus determined and since a solution exists we have an equilibrium pair of strategies.[9] The game is said to possess a <u>saddle-point</u> and is called a <u>strictly determined game</u>. A mixed strategy must be employed in cases where pure strategies do not yield saddle-point solutions. By a mixed strategy we mean for P_1 an

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9. Further discussion of the mathematical concepts and proofs of solutions of zero-sum 2-person games can be found in McKinsey, J. "<u>Introduction to the Theory of Games</u>". The Rand Series, McGraw-Hill Book Co., Inc., New York, 1952. Chapter 1 and 2.

P_2's Pure Strategies

P_1's Pure Strategies	s_1	s_2	.....	s_j	.....	s_n
s_1	$R_1(s_1,s_1)$	$R_1(s_1,s_2)$	.....	$R_1(s_1,s_j)$	.....	$R_1(s_1,s_n)$
s_2	$R_1(s_2,s_1)$	$R_1(s_2,s_2)$	.....	$R_1(s_2,s_j)$	.....	$R_1(s_2,s_n)$
.....	.....			.....		
s_i	$R_1(s_i,s_1)$	$R_1(s_i,s_2)$	.....	$R_1(s_i,s_j)$	.....	$R_1(s_i,s_n)$
.....	.....			.....		
s_m	$R_1(s_m,s_1)$	$R_1(s_m,s_2)$	.....	$R_1(s_m,s_j)$	.....	$R_1(s_m,s_n)$

Table (2-1): Payoff Matrix for 2-Person Constant Sum Game

ordered M-tuple $[x_1, x_2, \ldots\ldots, x_m]$ satisfying the conditions

(2-5) $\quad x_i \geqslant 0 \quad (i=1,2,\ldots\ldots,m)$

$$\sum_{i=1}^{m} x_i = 1 \ ,$$

that is, the probability with which P_1 chooses strategies i to m. Similarly for P_2 we have an ordered n-tuple $[y_1, y_2, \ldots\ldots, y_n]$ satisfying

(2-6) $\quad y_j \geqslant 0 \quad (j=1,2,\ldots\ldots,n)$

$$\sum_{j=1}^{n} y_j = 1$$

If P_1 employs mixed strategy $X = [x_1, x_2, \ldots\ldots, x_m]$ and P_2 employs mixed strategy $Y = [y_1, y_2, \ldots\ldots, y_n]$ then the mathematical expectation of P_1 is

(2-7) $$E(X,Y) = \sum_{j=1}^{n} \sum_{i=1}^{m} R_1(s_i, s_j)\, x_i\, y_j$$

If it happens that for some X^* in S_1 and some Y^* in S_2 we have,

(2-8) $$E(X,Y^*) \leqslant E(X^*,Y^*) \leqslant E(X^*,Y)$$

for all X in S_1 and all Y in S_2 then X^* and Y^* are the optimal mixed strategies for P_1 and P_2 respectively. $E(X^*,Y^*)$ is called the value of the game to P_1, X^* and Y^* being the

solution to the game or as McKinsey calls, a <u>strategic saddle-point</u>.[10] Thus, for games without pure equilibrium strategies we are led to a mixed strategy equilibrium. As a matter of fact, the pure strategy equilibrium pair is a special case of the general mixed strategy equilibrium solution with the probability of one of selecting the maximin strategy (for P_1). In this sense the theory of 2-person strictly competitive game should be adequately described by the theory of mixed strategy equilibrium pairs.

To pursue the notion of mixed strategy further we note that mathematically the mixed strategy solution is irreproachable, but the problem is in interpreting it. In practice we would question selecting a strategy on some randomized procedure. It might be argued that mixed strategies are desirable in order to withhold from our opponents knowledge of the pure strategy we will use to avoid them exploiting us. Whenever we refer to chance device in assigning probabilities to various choices, we are intuitively having in our minds games which will be played many times in succession. Business decisions are usually of "one-shot" nature. Once we have committed ourselves to a capital expenditure the flexibility of change

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10. Ibid. Chapter 2. The chapter includes mathematical proofs of the expression (2-7) and the minimax theorem.

on the whole is minimal. There are, however, cases where mixed strategies are operative, and, in fact, quite useful as in open market operations and currency devaluation where it is used to minimize the chance of speculators taking advantage of information leaks. Nevertheless we are drawn to this striking statement by Luce and Raiffa.

> "A strategy which is good in the total context of the conflict of interest may appear to be poor in a limited context. In evaluating strategies this distinction between context is, of course, important, but it is often difficult to maintain when considering particular cases.....
>
> ....Unfortunately, the strategist is often evaluated in terms of the outcome of the adopted choice rather than in terms of its strategic desirability in the whole risky situation."11

It must be pointed out that in the discussion so far, the formulation of 2-person zero-sum or strictly competitive games glides over the method of solution. We have not given it any rigorous mathematical treatment. Indeed several articles and books have been written that are concerned with the mathematical details of strictly competitive games and we have given little consideration to this aspect. We have for reasons explained in Chapter I that the purpose here is to determine applications for game theory, not

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11. op. cit. p. 76.

getting involved in mathematical intricacies. However, a number of methods are employed in the solution of 2-person zero-sum games. These include trial and error, simplex method and the like.[12]

As far as applications go, there are few situations or organizations that can be characterized successfully by means of 2-person constant sum games. One may think of a situation where two firms are involved in an advertizing competition for a market that is already saturated so that what is lost by one is gained by the other.[13] As far as direct application is concerned there appears according to Shubik[14] very few in the areas of economics, social or industrial organization but have been applied in military problems.

3. Theories of Solution of n-Person Games

The moment we move away from constant sum 2-person games the analysis becomes involved. Extra game theoretic considerations such as "bargaining psychologies of the players" and "interpersonal comparison of utility" will add to the complexities and modify the solution. By n-person games we

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12. For a detail of the methods, see Appendix 6 in Luce and Raiffa, Ibid.

13. See Charnes and Cooper, op . cit.

14. Op . cit. "Games, Decisions and Industrial Organization."

include variable sum games for $n \geq 2$ and constant sum games for $n > 2$. As the title suggests there is no single theory of n-person games but a number of them.[15] We shall however be concerned with von Neumann-Morgenstern and Nash[16] theories. Broadly, the theories fall within two main distinctions, namely cooperative and non-cooperative theories, with realistic cases falling in between these polar extremes. In cooperative theory we are dealing with cases where there is a tendency to joint maximization through cooperation or collusion, open preplay communication and side payments. The latter type examines situations where there is no or limited communication, no side payment and independent individual action.

Before proceeding to give the definition of a non-cooperative 2-person nonconstant sum game it is proposed to present a classical example of this class of games. The purpose is to bring out some of the interrelationships that exist between the strategies and objectives of several decision makers. In particular, the concept of equilibrium would be introduced.

Let us turn to the famed "Prisoners' Dilemma Game" attributed to A.W. Tucker. Two suspects taken separately

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15. Luce and Raiffa op . cit. Chapter 6 gives a critical analysis of the important theories.

16. Nash, J.F., "The Bargaining Problem", Econometrica, Volume 18, (April 1950), pp. 155-162 and "Two-Person Cooperative Games", Econometrica, Volume 21, (January 1953) pp. 128-140.

into custody are confronted with the problem of either confessing and turning state evidence or not confessing. Both know that if they do not confess they will escape with relatively light sentences. However, if one confesses and the other does not, the non-confessor will be faced with heavy punishment. Finally if both confess they will receive charges that are heavy but not as heavy as the most severe one. The game matrix is drawn with figures respresenting the prisoners' payoffs in table (2-2)

		Prisoner 2	
		Not Confess b_1	Confess b_2
Prisoner 1	Not Confess a_1	(-1,-1)	(-10,0)
	Confess a_2	(0,-10)	(-8,-8)

Table (2-2):[17] Prisoners' Dilemma Game

Since the strategy a_2 dominates a_1 and strategy b_2

17. Note that the first numeral within the brackets according with convention refers to payoff going to row player, i.e. Prisoner 1, while the second numeral refers to payoff to columnplayer, i.e. Prisoner 2.

dominates b_1, it will be observed that by applying the criterion of individual rationality both prisoners will be motivated to adopt the position of confessing. The strategies a_2 and b_2 are maximin strategies and the outcome (-8,-8) is said to be an equilibrium outcome. The outcome (-1,-1) involving strategies a_1 and b_1 is termed by Luce and Raiffa[18] as a quasi-equilibrium outcome and occurs only in a repeated game which eventually settles at (-8,-8), caused by one of the players "defecting" or "double-crossing". One point of note is that although the payoff (-8,-8) is in equilibrium, it is not a jointly desirable one.

Many 2x2 games of this nature have been developed and a complete classification of 2x2 games with outcomes on an ordinal scale can be found in an article by Rapoport and Guyer.[19] It is not proposed to go into a discussion of these, as, although recognizing the value of such games in bringing out the essential nature of conflict and cooperation and enabling anyone interested in thinking logically about competitive situations, they are too restrictive for most analysis of industrial situations. It must be pointed out that even

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18. op . cit. p. 98.

19. Rapoport, Anatol and Guyer, Melvin, "A Taxonomy of 2x2 Games", General Systems: Yearbook for the Society of General Systems Research, Volume XI (1966). pp. 203-214.

in a 2x2 game we do not necessarily have unique stable equilibrium as the above case, but conditions such as strongly stable or Pareto equilibrium, stable and unstable equilibrium and other properties such as non-conflict, threat-vulnerable, essential, inessential and the like.[20]

Although we began to define the concept of solution of non-cooperative nonconstant sum games for 2 players it can be generalized for any number of players using mixed strategies.[21] Using the notations set out in formulating 2-person zero sum games we have for a pair of strategies $(\bar{s}_1, \bar{s}_2)$ to be in equilibrium the conditions that

(2-9) $$R_1(\bar{s}_1, \bar{s}_2) \geqslant R_1(s_1, \bar{s}_2) \quad \text{for } s_1 \epsilon S_1$$

$$R_2(\bar{s}_1, \bar{s}_2) \geqslant R_2(\bar{s}_1, s_2) \quad \text{for } s_2 \epsilon S_2$$

This amounts to saying that given that P_2 selects his strategy $\bar{s}_2$, P_1 will select his strategy s_1 which maximizes his payoff against P_2's choice. The same applies to P_2. Thus for equilibrium the strategy pair $(\bar{s}_1, \bar{s}_2)$ must be jointly admissible. In general if ζ_i is mixed strategy to player i (denoted by P_i) then the payoff function to P_i is defined by

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20. Ibid. We shall have more to say about Pareto equilibrium later in the section.

21. Nash, J.F., has shown in his paper "Non-cooperative Games" Annals of Mathematics, 54, (September) 1951. pp.286-296, that a mixed strategy equilibrium solution exists in non-cooperative games with finite sets of pure strategies.

$R_i(\zeta_1, \zeta_2, \ldots\ldots, \zeta_n)$. If the n-tuple of mixed strategies $(\zeta_1, \zeta_2, \ldots\ldots, \zeta_n)$ is denoted by Φ we can let the expression (ϕ, η_i) to stand for $(\zeta_1, \zeta_2, \ldots\ldots, \zeta_{i-1}, \eta_i, \zeta_{i+1}, \ldots\ldots, \zeta_n)$. For Φ to be in equilibrium we must satisfy

$$(2\text{-}10) \qquad R_i(\Phi) \quad \max_{\text{all } \eta_i} \; R_i(\phi, \eta_i)$$

meaning that each player maximizes his expected payoff if the strategies of the others are held fixed. This definition of equilibrium is essentially static and does not reflect the psychological factors of non-cooperative games. We will consider the dynamics of action and reaction, interplay and motion in the next chapter.

A decision to cooperate arises because in general there is much more to gain by cooperating than not cooperating. In spite of gains from joint action, interests are usually opposed when we come to dividing the profits. It is here the cooperative theories play their roles in deciding what is a "fair" division.

In von Neumann and Morgenstern theory we are introduced to the notion of the characteristic function of a game, a function which assigns a value to every subset of players. If we have n-players the characteristic function has 2^n values. For example, if we have a situation involving 3 players, we have

$$\vartheta(\phi) = 0$$

where ϕ is the empty set involving no player

$$\vartheta(1) = z_1 \qquad \vartheta(2) = z_2 \qquad \vartheta(3) = z_3$$
$$\vartheta(1,2) = z_4 \qquad \vartheta(1,3) = z_5 \qquad \vartheta(2,3) = z_6$$
$$\vartheta(1,2,3) = z_7$$

where 1,2,3 represent the players and z_i for $i = 1,2,\ldots\ldots,7$ are the payoffs to the various coalitions.

The characteristic function has a further property that given a partition of the players into two subsets S and T, then

(2-11) $\quad \vartheta(S \cup T) \geqslant \vartheta(S) + \vartheta(T)$

when $S \cap T = \phi$

The condition given in (2-11) is fundamental in expressing von Neumann and Morgenstern solution concept of the set of imputations. By imputation it is meant that if $\alpha = (\alpha_1, \alpha_2)$ represents a set of values in a 2-person situation such that

(2-12) $$\alpha_1 + \alpha_2 = \vartheta(1,2)$$
$$\alpha_1 \geqslant \vartheta(1)$$
$$\alpha_2 \geqslant \vartheta(2)$$

then α is an imputation.

The solution to this game consists of a set of imputations which dominates all others but do not dominate each other. This is equivalent to saying that the solution lies

on the Pareto surface.[22] Beyond that the solution does not narrow the field of choice as much as we would like.

Pictorially the outcomes of the game can be represented by the convex region R shown in figure (2-1). The undominated set or the Pareto optimal set lies on the section formed by the extreme points a,b,c and d. If (u,v) represents the maximin values of the game then the boundary of R between the dashed vertical and horizontal through (u,v) as represented by lbcm, is the negotiation set of the game. This is what von Neumann and Morgenstern terms the cooperative solution.

The Nash solution[23] for 2-person cooperative games is based on four basic assumptions. They are; (1) there is no interpersonal comparison of utilities or in other words we can pick a natural position of status quo, (2) the solutions are Pareto optimal, (3) the solution is independent of irrelevant alternatives and (4) the symmetry nature of "fairness" is specified. We will not dwell on their limitations and question their validity as the opportunity will arise again when we discuss the dynamics of the bargaining process. We should however recognize that the formulation has advanced

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22. A division of proceeds is said to be Pareto optimal if no individual can increase his welfare by departing from it without at least one other individual suffering a decrease in his welfare. This icea is similar to the contact curve of Edgeworth indifference solution in 2-dimensional economics.

23. Nash, J.F., "The Bargaining Problem" and "Two-person Cooperative Games", op . cit.

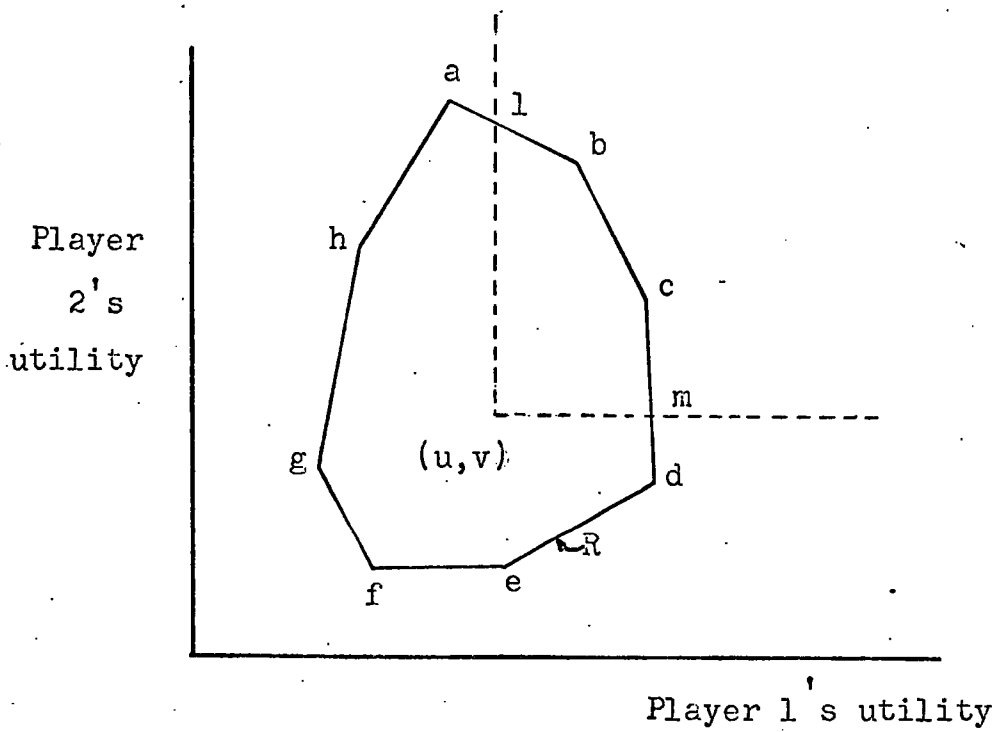

Figure (2-1). Pareto Optimal and Negotiation Sets

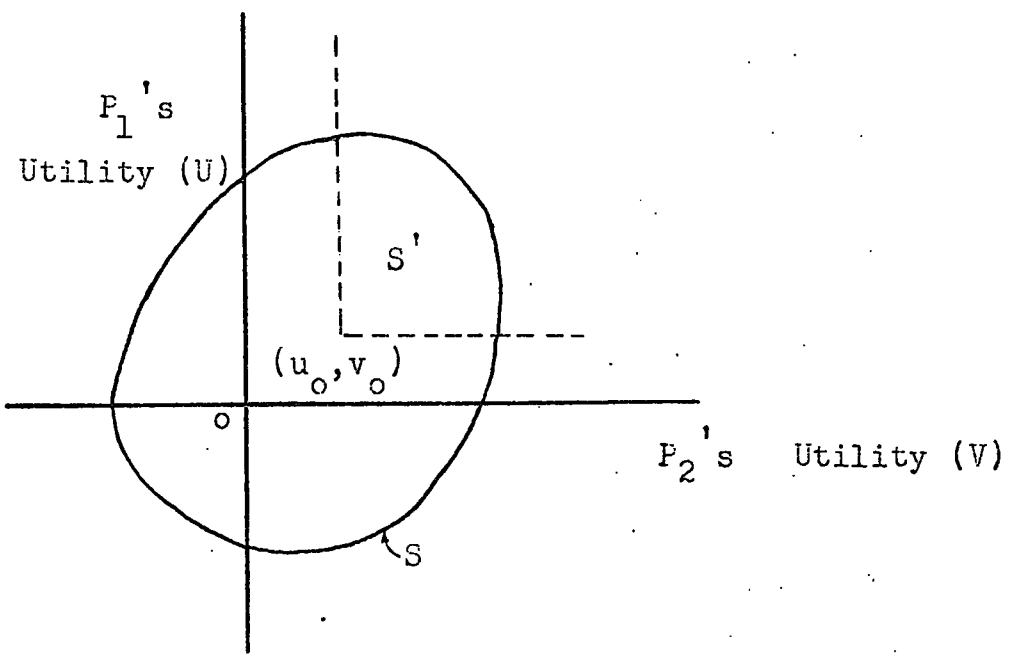

Figure (2-2). Linear Transformation of Convex Utility Function

our understanding of conflict situations by introducing the role of threats. Thus if we accept the Nash assumptions the analysis proceeds as follows:

Given two utility functions U and V of two players P_1 and P_2 we can describe a convex region[24] representing all possible prospects of payoffs. This is shown in figure (2-2). Amongst these there is one situation in which no trade ensues or the status quo position where the players act non-cooperatively. We denote this by (u_o, v_o). By a linear transformation of utility we can always reduce this to the (0,0) point.

The Nash solution is the determination of a unique point[25] based on his assumptions of the game represented by the region S and the status quo point (u_o, v_o). We can denote such a game by $[S, (u_o, v_o)]$. This point must of course be on the Pareto surface because no points will be considered if both players can jointly improve their position by choosing another prospect. Hence if we transform (u_o, v_o) into (0,0) the resultant game described by the region above and to the right of status quo point can be considered to be independent of the large game S (assumption 3). Nash solution is then the maximum of the product of all (u', v') of the new game $[S', (0,0)]$, i.e.

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24. The region is convex because for any two prospects with different utility pairs there will always be other prospects whose utility pair lies on the straight line connecting them. If the region represents finite set of trades, i.e. commodities are not infinitely divisible we have a convex polygon.

25. Convexity of S or S' ensures one maximal element.

Find $u'_* v'_* \geqslant u' v'$ for all (u', v') belonging to S'

Geometrically the solution is given by the point of tangency of the Pareto surface and the equilateral hyperbola UV=constant, shown in figure (2-3). One is able to compare Nash's criterion of "fair division" with one in which the utilities U and V are made equal and the other which is the maximization of the sum of utilities U and V.[26] The points, a, b and c in figure (2-3) represent these respectively.

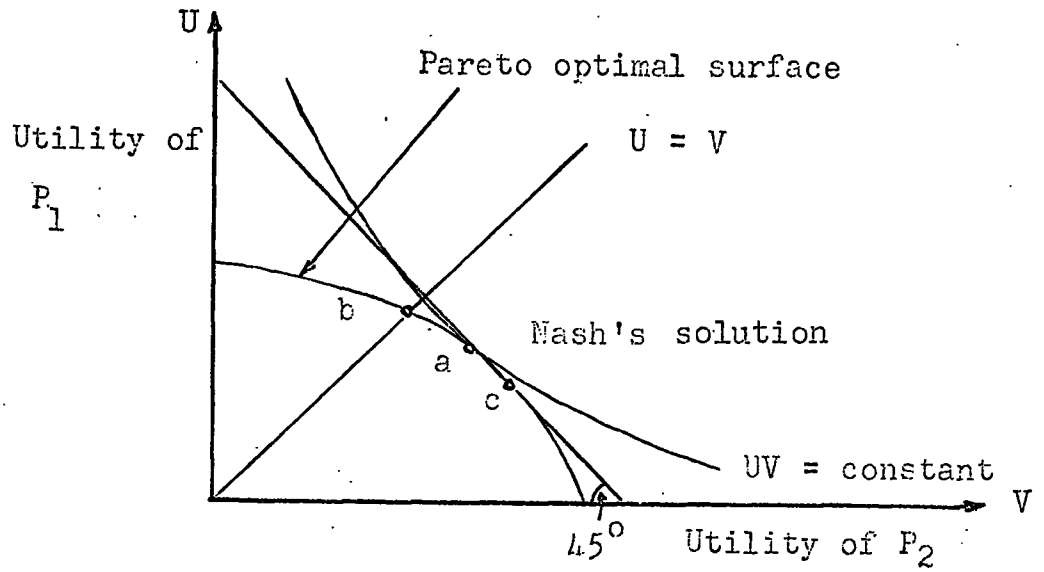

Figure (2-3): Comparison of Nash's and Other "Fair Division" Criteria

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26. $U=f(V)$
$U+V=f(V)+V$
$\frac{d(U+V)}{dV}=f'(V)+1=0$
$\therefore f'(V)=-1$

i.e. the point of joint maximun on the Pareto surface has a slope of -1.

Nash's axioms can be generalized for n-players. The problem becomes,

(2-13) $\max \prod U_i$ for $i=1,2,\ldots\ldots,n$

The problem however is in finding the status quo position because unlike 2-person games there is a possibility of forming coalition structures.

We shall deal briefly here with the concept of optimal threat strategies with and without side payments. A "fair division" of profits as a measure of payoff can only be determined after analysing the optimal threat strategies equivalent to the status quo point outlined above.

Two players in a cooperative game, having the object of making the outcome to themselves as favourable as possible, will pick threat strategies that guarantee the largest amount of damage to their opponent. In doing so it is inevitable that the threatener will have to bear a certain cost. If we assume as Nash does that both players have the same evaluation for outcomes then the optimal threat strategies will be for the players to

(2-14) $\min_{1} \max_{2} (R_1-R_2) = \max_{2} \min_{1} (R_1-R_2)$

where R_1 and R_2 are the payoffs to P_1 and P_2. What this means is for P_1 to maximize (R_1-R_2) and P_2 to minimize this quantity.

It is for this reason that the threat curve is sometimes known as the maximin surface (also called Pareto minimal surface). The pair of threat strategies for games where players collude and side payments are permitted will be different from that in which no side payment is permitted. When the players have identical cost structure they coincide. Where side payment is permitted the point of "fair division" can be determined geometrically by a 45° line drawn from the threat point to the joint maximun line, the construction of which was demonstrated in figure (2-3). The determination of no side payment optimal threat strategies and the Nash solution is slightly more complicated.[27] We will however proceed with the general approach of obtaining the Pareto optimal and optimal threat surface which is fundamentally more important and forms the basis for calculating optimal threat and Nash equilibrium points.

A rational player 1 (P_1) will attempt to

$$\max_{s_i} \min_{s_j} R_1(s_i, s_j)$$

Player 2 (P_2) will make every effort to keep this minimal. This condition alone is insufficient to define an outcome. If we assume that P_2 is restricted to making a fixed profit

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27. Shubik, "Strategy and Market Structure", op . cit. Appendix C, pp. 360-361.

$R_2(s_i,s_j)=C$, then for the profit to be Pareto optimal,

$$(2\text{-}15)\quad dR_1(s_i,s_j)=\frac{\partial R_1(s_i,s_j)}{\partial s_i}\,ds_i+\frac{\partial R_1(s_i,s_j)}{\partial s_j}\,ds_j=0$$

Similarly,

$$(2\text{-}16)\quad \frac{\partial R_2(s_i,s_j)}{\partial s_i}\,ds_i+\frac{\partial R_2(s_i,s_j)}{\partial s_j}\,ds_j=0$$

The above two equations are summarized in the Jacobian

$$(2\text{-}17)\quad \begin{vmatrix} \dfrac{\partial R_1(s_i,s_j)}{\partial s_i} & \dfrac{\partial R_2(s_i,s_j)}{\partial s_i} \\[2ex] \dfrac{\partial R_1(s_i,s_j)}{\partial s_j} & \dfrac{\partial R_2(s_i,s_j)}{\partial s_j} \end{vmatrix} = 0$$

The expression (2-17) is satisfied by 2 curves, the Pareto optimal and the optimal threat curves. The typical shape of the curves and the location of the game theoretic points for the 2-person case discussed above is shown in figure (2-4).

We have shown in general how Nash theory could be generalized for n-person non-cooperative games [equation (2-10)] and cooperative games [equation (2-13)]. It was noted then that we have to explicitly incorporate coalition structures in our analysis, given that cooperation is allowed. If τ denotes the partition of the players into coalition, the Nash equilibrium is rightfully described by,

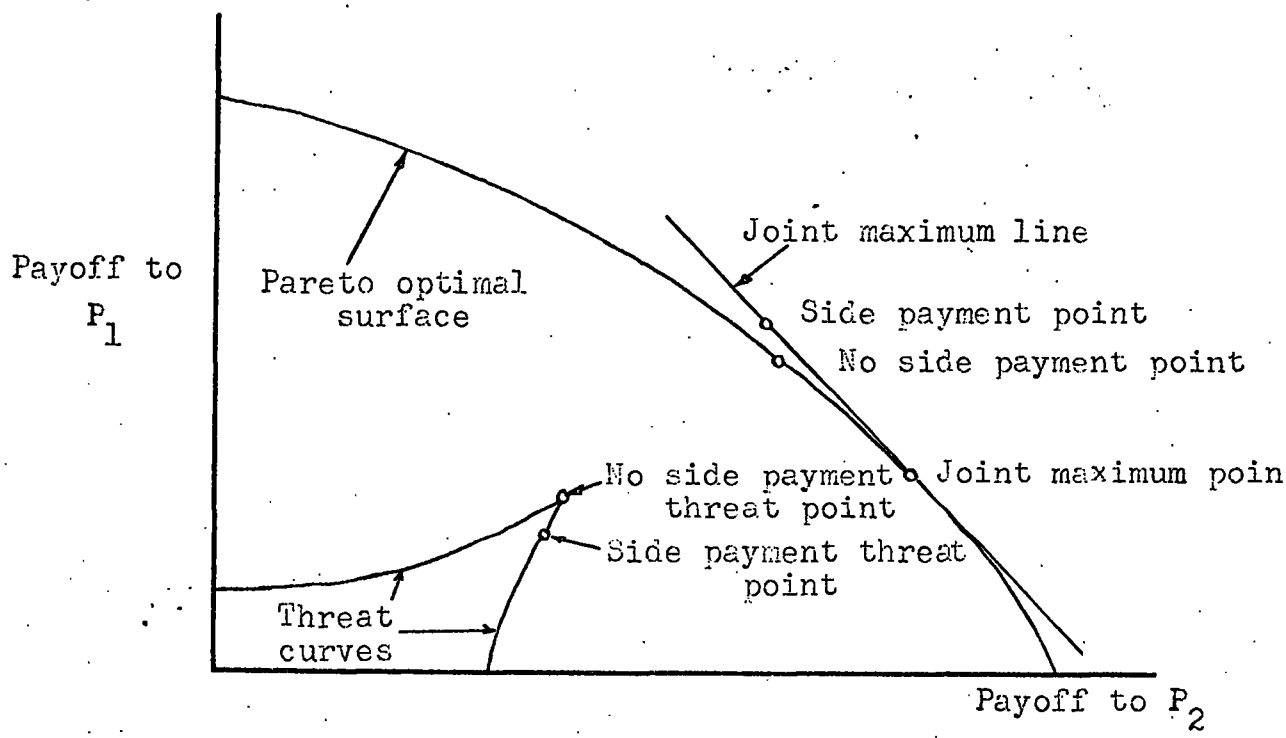

Figure (2-4):[28] Game Theoretic Points on Payoff Space

$$[(\zeta_1, \zeta_2, \ldots\ldots, \zeta_n), \tau]$$

where $(\zeta_1, \zeta_2, \ldots\ldots, \zeta_n)$ constitute, as defined earlier the n-tuple of mixed strategies in equilibrium. If

$$\tau = [\{1\}, \{2\}, \ldots\ldots, \{n\}]$$

the coalition structure is said to have no non-trival coalitions and thus we have the condition of non-cooperation. In cooperative games the specification of τ is important and

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28. Source: Shubik, "Strategy and Market Structure". Chapter 5.

the equilibrium condition will depend on it. Between the cases where we have perfect cooperation and non-cooperation there is a host of possibilities. It is observed in practice that the coalition structure has certain limitations placed on its contemplated changes. This is specified in a rule of admissible coalition changes summarized in the function $\psi(\tau)$.[29] These rules may be thought of as sociological, costs or other restriction,[30] likened to the boundary conditions of classical mathematical physics problems.

In general one is led in a cooperative game to search for a pair $[\{\zeta(T_1), \zeta(T_2), \ldots\ldots, \zeta(T_n)\}, \tau]$, where $\tau=(T_1, T_2, \ldots\ldots, T_t)$ and $\zeta(T_i)$ denotes a typical correlated mixed strategy jointly chosen by the players in the coalition structure T_i. If it is in equilibrium then for at least one player j in $M \in \psi(\tau)$ i.e. M is a possible coalition change in τ,

$$R_j[\zeta(T_1), \zeta(T_2), \ldots, \zeta(T_n)] > R_j[\zeta(M), \zeta(\bar{M})].$$

$\zeta(\bar{M})$ is the mixed strategy of the coalition formed by the remaining players not in M, chosen such as to minimize the payoff to j. This, of course, is a conservative definition of equilibrium of an n-person cooperative game.

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29. A discussion of the function $\psi(\tau)$ is given in Luce and Raiffa, op . cit. Section 7.6 of Chapter 7 and Chapter 10.

30. For example, in the coalition structure $\tau = [\{1\}, \{2\}, \{3\}]$ formed by players 1,2,3 we might impose the condition that the admissible coalitions are $\psi(\tau)=[\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}]$.

Although there are few direct applications of the theories of n-person nonconstant sum games to operating problems in industry, the potentialities of this part of game theory is great. Most of the difficulties are computational in nature, but, with the rapidity at which computer technology is advancing this will eventually be overcome. We have already mentioned one breakthrough in the use of these theories in the oil industry.[31] Charnes and Cooper[32] had made a study using an n-person game model with Nash equilibria in association with the Chicago Area Transportation Study to simulate patterns of traffic flow when the huge size of the problem made ordinary cut-and-tried simulations prohibitive for electronic computer runs. In other aspects Shubik[33] has written about the possible use of game theory in joint costs allocation for decentralized decision making.

4. Game Theoretic Points: Their Implications to Decision Making.

The above discussion has mainly centred around the

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31. Hughes and Ornea, op . cit.

32. Charnes, A. and Cooper, W.W. "Extremal Principles for Simulating Traffic Flows Over a Network of City Streets" Proceedings of the National Academy of Sciences, Volume 44:2 February 1958, pp. 201-204.

33. Shubik, Martin, "Incentives, Decentralized Control, The Assignment of Joint Costs and Internal Pricing," Loc.cit.

concept of equilibrium and threat strategies under various context in a static setting. It is thus appropriate here to examine qualitatively in greater details what implications, basically these concepts have in decision making in competitive situations.

The word equilibrium in a sense conveys a feeling of a state of balance or immobility. In arriving at the equilibrium (in the non-cooperative sense), we stated that any unilateral movement by a player away from it will make him less well off than he was at the equilibrium. It is reasonable to expect that none of the players will be motivated to shift. But, it is certainly possible for anyone player to move away from the equilibrium (against his immediate interest) in order to motivate another to shift (in pursuit of immediate gains or taking any consequence of the shift) so as to obtain for the initiator of the shift a larger payoff eventually than he gets at the equilibrium. An example would be the price cutting move by a financially larger firm to drive out the weaker ones so that it will be left with larger shares at the end. One could induce another player to shift strategy if the other player sees that it is to his advantage to shift rather than to suffer the consequences of the first player shifting. In another situation, the shifting of the strategy of a player may force a second player to shift also because of the

advantage of the latter in doing so.

In a competitive decision situation we can broadly identify four general strategies -equilibrium, cooperative, threat and defence. We can, as was done in the sections before this, determine these points and plot them out graphically in payoff space. If it is a one decision variable problem involving two players, which is very unlikely in practical situations we could also draw out the conflict situation on a 2-dimensional variable plane. Most game theory analysis in text and papers end here leaving the decision maker who would eventually use the technique to decide for himself a host of unanswered questions.[34]

Let us picture a situation in which two competing firms are faced with manufacturing and marketing investment decisions. We could on the assumption of perfect information construct the payoff function profiles representing the conflict situation in figure (2-5). One question that comes into the fore is the changes that are likely to occur on the position of the equilibrium point (E/E) given changes in the restrictions that may be placed on the actions of the decision maker. For example, an obvious one would be that investment cannot be infinite, given limitations of financing, shareholders sanctions, etc.

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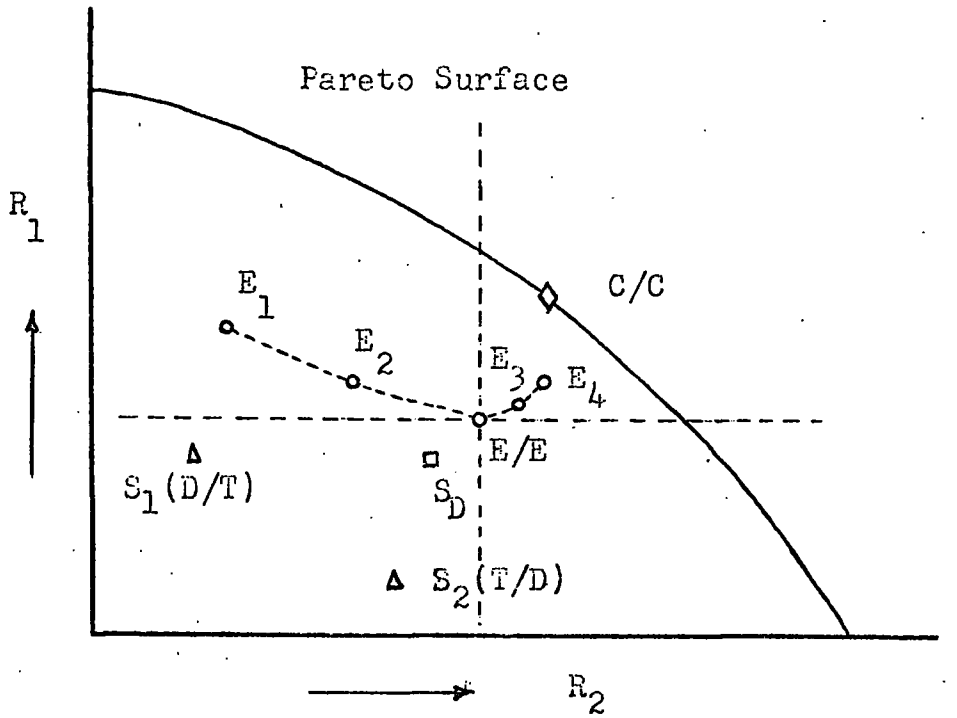
34. An exception is the paper by Hughes and Ornea, op. cit.



E_1, E_2, E_3, E_4, are the equilibrium points resulting from various restrictions placed on the firms.

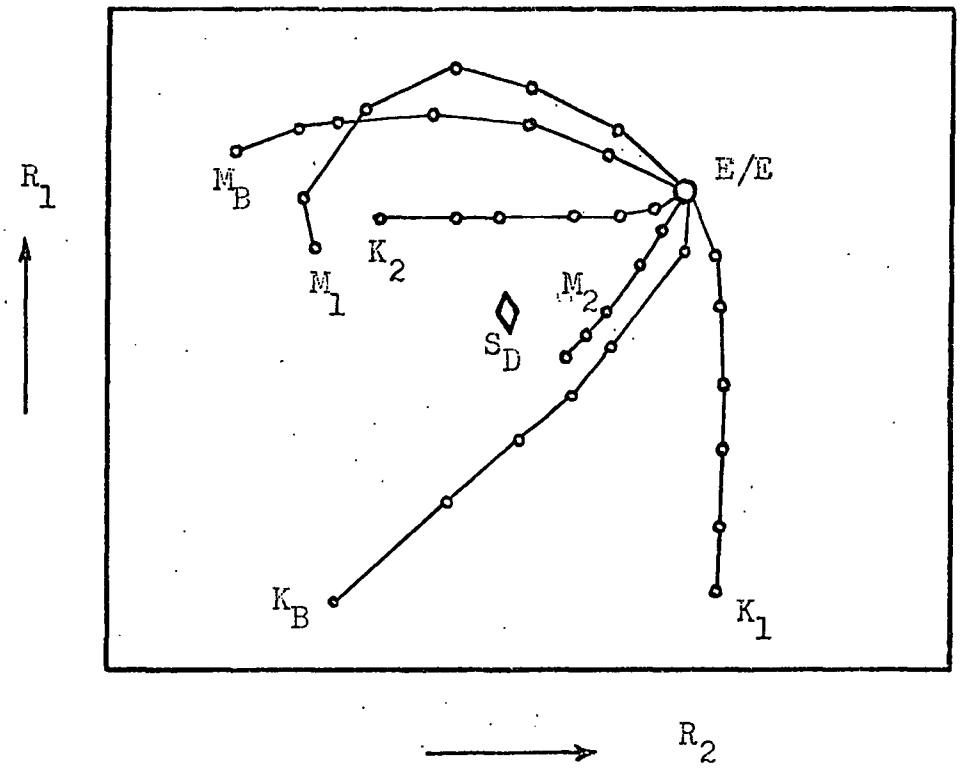
Figure (2-5):[35] Effect of Constraints on Position of Equilibrium Points

In the figure (2-5) any action that would cause the movement of the equilibrium point to the left of the vertical through E/E towards the Pareto surface would mean gains to player 2. This applies to any movement of the equilibrium point above the horizontal through E/E for player 1. A movement in the south-west direction from E/E is unwel-

35. Adapted from results in Hughes and Ornea's Model., Ibid.

comed to both as this means that both will have to throw in large investment with relatively little gains. An uncoordinated action would lead to this mess. The opposite traverse of the equilibrium point will only come about through cooperation, and, as we have pointed out earlier, the ultimate point of division will depend on the players.

Just as much as we are interested in post optimal analysis in linear programming and other mathematical techniques we must not limit our interest in game theory at the equilibrium solutions. As decision makers we would like to know how sensitive the results of the analysis are to small changes in the variables of the problem. These may be due to inaccuracies in the forecasts and other contingencies and, that for all we know we might in fact not be operating at where we thought we are or ought to be. Further it would be interesting to investigate the movement of the equilibrium positions when the firms as competitors either singly or both simultaneously undertake small changes in various forms of investment. It might be possible that we find it advantageous in making a certain kind of investment while in others we would only sow the seeds of our financial ruin. Figure (2-6) is a typical example of such an analysis and it will be noted that there are certain investments, in this particular case marketing investments which are generally costlier to firm 2 while manufacturing investments are



M = Increase in marketing investment
K = Increase in manufacturing investment
1,2,B = Player 1,2 or both respectively

Figure (2-6):[36] Effect of Small Changes in Investment on Position of Equilibrium Points

costlier to firm 1. It also appears that firm 1 could well improve its market share by adopting an aggressive marketing policy. Such analysis would be an invaluable aid to management decision making as they form guidelines to strategy

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36. Source: Hughes and Ornea, Ibid.

selection and policy formulation. This would be a far cry from just offering management an equilibrium point and expecting rational maximin behavior to respond without investigating any underlying flexibility in the results offered.

Turning to threat and defence strategies we note that they are reciprocal in nature, meaning that when one of the players threatens the other is actually employing a defensive strategy and vice versa. To be sure, a threat once carried out is no more a threat. It is only effective at the bargaining or negotiating table if its use is kept at bay, to be unleashed anytime the outcomes are not favourable to the party concerned. We will have the occasion to view the dynamics of the process of bargaining in the next chapter where we will also introduce the concept of concession.

The threat point S_1 in figure (2-5) is the maximun of the payoff R_1 with respect to firm 1's strategy and minimum to firm 2's strategy. In effect firm 1 is defending against any threat from firm 2. Similarly S_2 is the maximin point of payoff function R_2 for firm 2. There is, of course, a limit at which firm 1 and 2 can exercise their threat strategy. This is tied in with the restriction placed on equilibrium point calculations in the earlier part of the section. Finally point S_D describes the threat-defence strategies discussed in section 3 equation (2-14), where firm 1 tries to maximize R_1-R_2 and firm 2 attempts to minimize this quantity.

In going through the theoretical exercise of game theory we have implicitly assumed a natural tendency for the players to coordinate their choice of strategies. The equilibrium point was arrived at with both players choosing their equilibrium strategy. Likewise, the Pareto solution results when both players are drawn by various reasons, including the prospect of joint maximal payoff to cooperate. It seems thus that consideration have not been made for situations where it is quite possible for independent uncoordinated actions on the part of the players. What would then be the outcome is a question that brings us to the interesting area of cross-strategies. This was done in Hughes and Ornea's paper[37] and figure (2-7) and figure (2-8) are some of the results. It appears that with cooperation both players have payoffs on the Pareto surface. If one of the players had initially chosen a cooperative strategy, a defence or equilibrium strategy taken by the other will improve the latter's payoff. With a threat strategy both firms suffer reduced payoffs. This is shown in figure (2-7). Figure (2-8) is the case where the reference strategy is the equilibrium strategy. In this case, too, increased payoff goes to the player selecting a cooperative strategy while a defence strategy makes little difference to the results. Again

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37. Ibid.

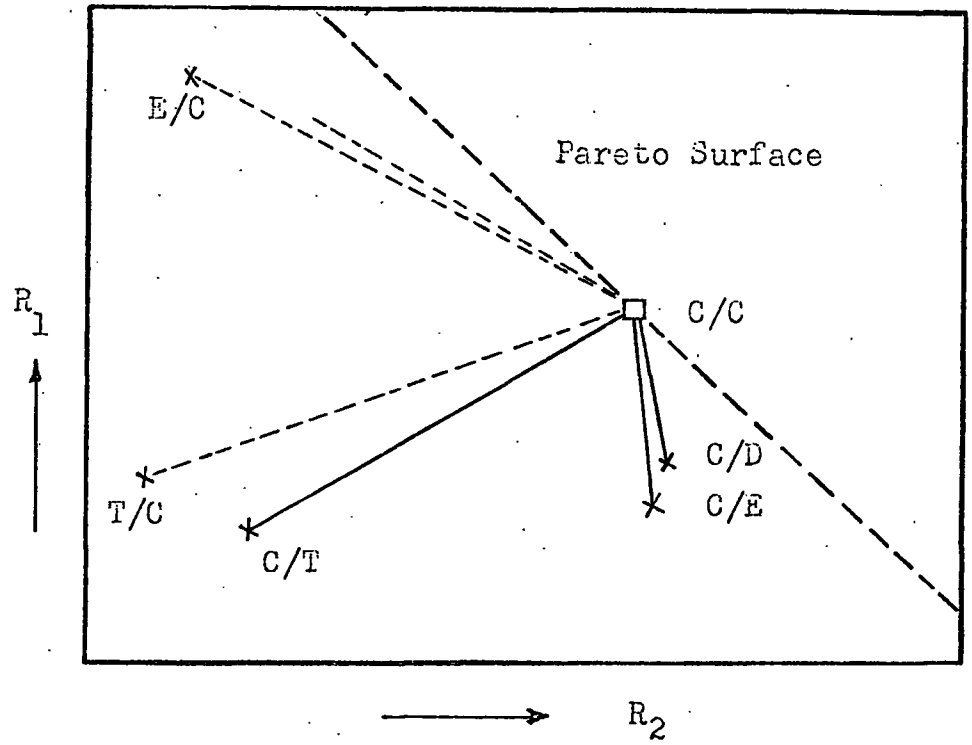

Figure (2-7): Cross Strategies When One Firm Uses Cooperative Strategy

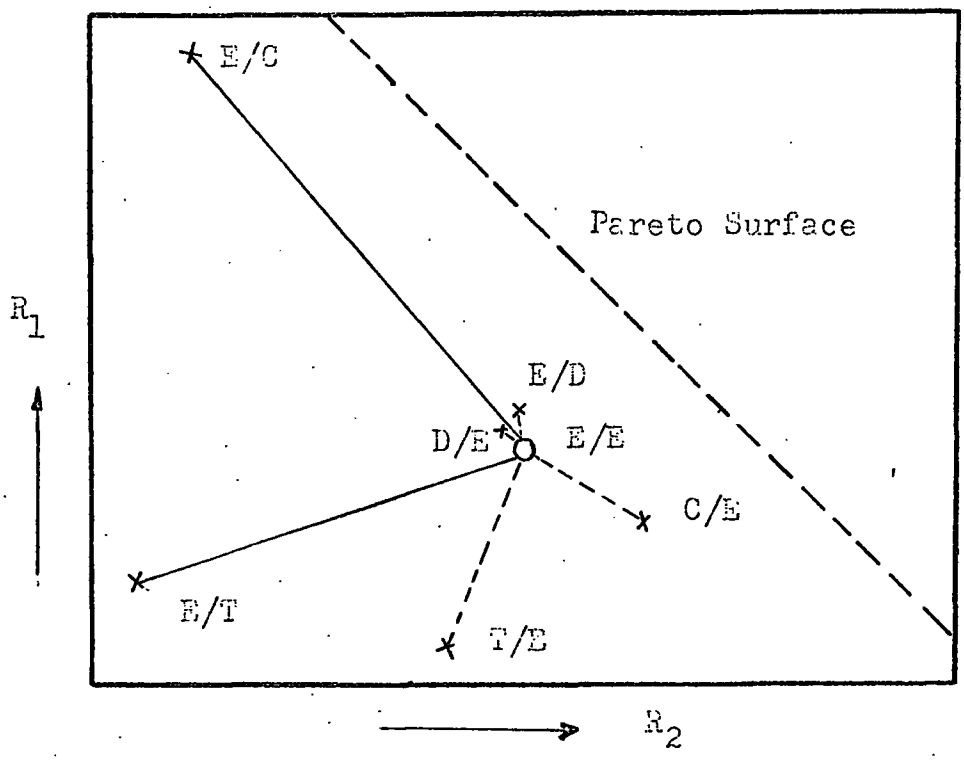

Figure (2-8): Cross Strategies When One Firm Uses Equilibrium Strategy

threat strategies are seen to run against mutual interest.

If the above were a "one-shot" game then we would expect the results expounded. In the case where the game is repeatable we would not expect the result to remain to the advantage of one player for long, as, by then the good Samaritan will have learnt that his good wills are being abused and we can expect the game to degenerate to a vicious end or to some more equitable results.[38]

5. Extensive Games

In our cursory survey thus far we were drawn on various occasions to gloss over the dynamic aspects of game theory. Partly to set the foundations for the material in the next chapter which deals with this area of game theory, and, as an alternative formulation of a game in normalized form, the extensive form of a game will be described here.

The matrix game in table (2-2) can be represented by a game tree shown in figure (2-9). Player P_1 starts at P_1, the vertex with the name of the player, making his choice from two alternatives. Since we are considering a case where P_2 has no knowledge of P_1's choice, similar to

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38. The tendencies for players to behave in such a manner were found in experiments by Russell L. Ackoff, David W. Conrath and Nigel Howard in their work, "A Model Study of the Escalation and De-escalation of Conflict", Management Science Center -University of Pennsylvania (1967).

the case where they make a single move simultaneously, the vertices P_2 are enclosed by a single curve, indicating that the group of vertices belong to the same information set. In the case where P_2 is allowed to make his move after P_1 has done and revealed his choice, the vertices P_2 are circled separately producing two information sets, indicating that P_2 can distinguish P_1's choice. We thus have the condition of perfect information in respect of P_1's choice for P_2. This is shown in figure (2-10) and the equivalent normalized game is shown in table (2-3). When both players have made their choice, the game leads to one of the four payoffs through a path starting from P_1 and ending at the payoff, giving a complete description of the play of the game. Often during the play of a game chance moves as distinguished from personal moves of the players are involved. To this which we assign P_0 we must associate a probability distribution on the alternatives. Notice that the vertices in the same information set contain the same number of alternatives (otherwise the vertices can be distinguished). Proceeding thus the anatomy of any game can be described, no matter how complicated the moves or information conditions may be.

More generally we have an n-person game in extensive form, described mathematically as follows:

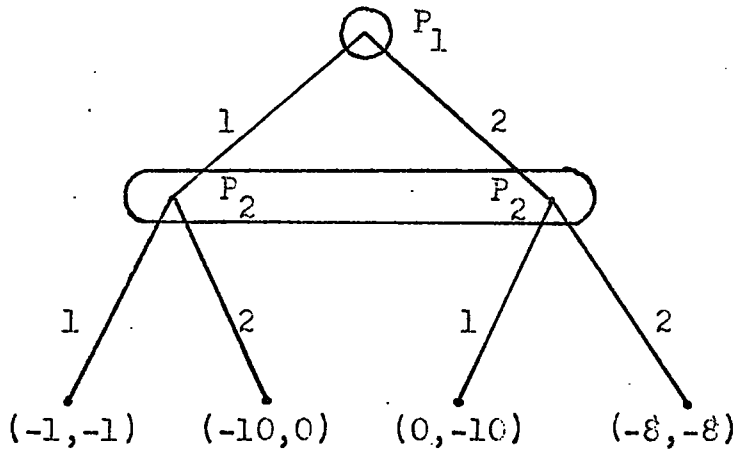

Figure (2-9): Game Tree Representation of Table (2-2)

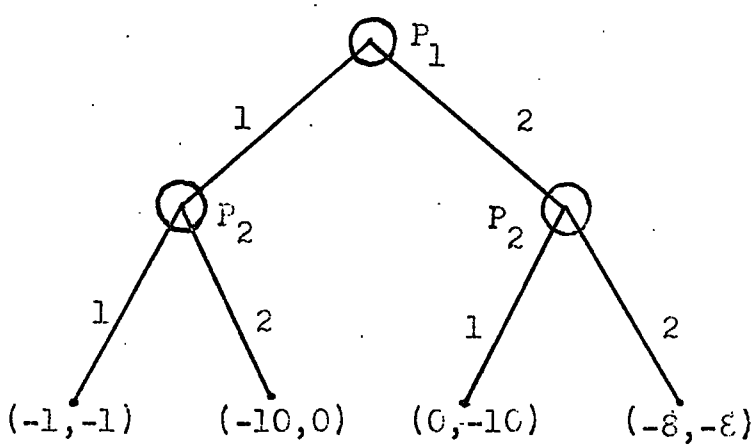

Figure (2-10): Game Tree Representation of Table (2-3)

(1,1;2,1)	(1,1;2,2)	(1,2;2,1)	(1,2;2,2)
(-1,-1)	(-1,-1)	(-10,0)	(-10,0)
(0,-10)	(-8,-8)	(0,-10)	(-8,-8)

Table (2-3); Matrix Game: P_1's Choice Known to P_2

1) A partition of the moves into n+1 index set $P_0, P_1 \ldots$ $\ldots, P_n$ consisting of chance and personal moves. This is known as the <u>player partition</u>.

2) For each chance move with j alternatives, a probability distribution, which assigns a positive probability to each alternatives.

3) For each play W, a payoff function $R(W) = [R_1(W) \ldots\ldots, R_n(W)]$.

4) A partition of moves into sets U so that each U is contained in some P_i and A_j for some i and j, and no U contains two moves belonging to the same play. This partition is called the <u>information partition</u>.[39]

The first three of these specifications are self-explanatory. In (4), what is meant is that the set of all moves (a move is the selection of one among a set of alternatives at a choice point in the game) are partitioned into sets $A_1, A_2, \ldots\ldots, A_j, \ldots\ldots$ where A_j contains those moves with j alternatives. Thus each U must belong to some P_i and A_j for some i and j. The game proceeds as a continuous path with no branching, or, in other words one cannot distinguish the vertices in the same information set and makes a choice as if faced with one move.

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39. Shubik, "<u>Strategy and Market Structure</u>", op.cit p.190

We can think of a pure strategy in an extensive game of player i with q different information sets as the selection of a q-tuple of integers $(y_1, y_2, \ldots, y_k, \ldots, y_q)$ such that the number y_k represents the branch chosen in the kth information set, where y_k is positive integer $\leq j$, the number of alternatives in the kth information set. The mixed strategy could be regarded as a "random" selection from a number of fixed plans, the pure strategies, by the strategist or the corporation directors or officers.

Inspite of the dynamic overtones of the extensive game model presented above we have not actually achieved a dynamic model of a market. The above game is limited to a finite length with payoffs at the end. In real life economic situations payments are made continuously and the game has no definite termination point. Unlike the one period game, in an infinite game at any play during a subgame, the knowledge that the supergame is expected to last for subsequent periods, no matter how small the probability, alone makes it possible to carry out reprisals in the event of defection or double-crossing. The whole course of strategies may have to be changed under such circumstances where the degree of fixed investment, flexibility in advertising, research or pricing policy all of which affects the maneuverability of the firms and

their ability to enforce stability in the market.

For a mathematical description of an n-person infinite game Γ, we have, with the first and second conditions similar to that for the finite game the remaining as follows:

3) A partition $\mathcal{U}$ of the moves into information sets U

4 For each path of play up to the t.th move $h_t \in H_t$ (the set of all plays up to the t.th move), a payoff function

$$R_t(h_t) = [R_{1t}(h_t), R_{2t}(h_t), \ldots\ldots, R_{nt}(h_t)]$$

5) For any player i, the sum $\sum_{t=o}^{\infty} R_{it}(h_t)$ is bounded.[40]

Figure (2-11) represents for simplification a duopoly dynamic game. During the first period players P_1 and P_2 make simultaneous decisions, at which point payoffs as a result of these decisions and described by the function $R_1(h_1)$ are made. The game proceeds through subsequent periods during which further payoffs are made. One of the criteria for a multi-period decision is the maximization of a present value function. Through a discount rate which could be a function of time, the discounted future income stream sets a bound to the sum of payoffs, satisfying condition (5). In the event of one of the players dropping out

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40. Ibid. p. 198

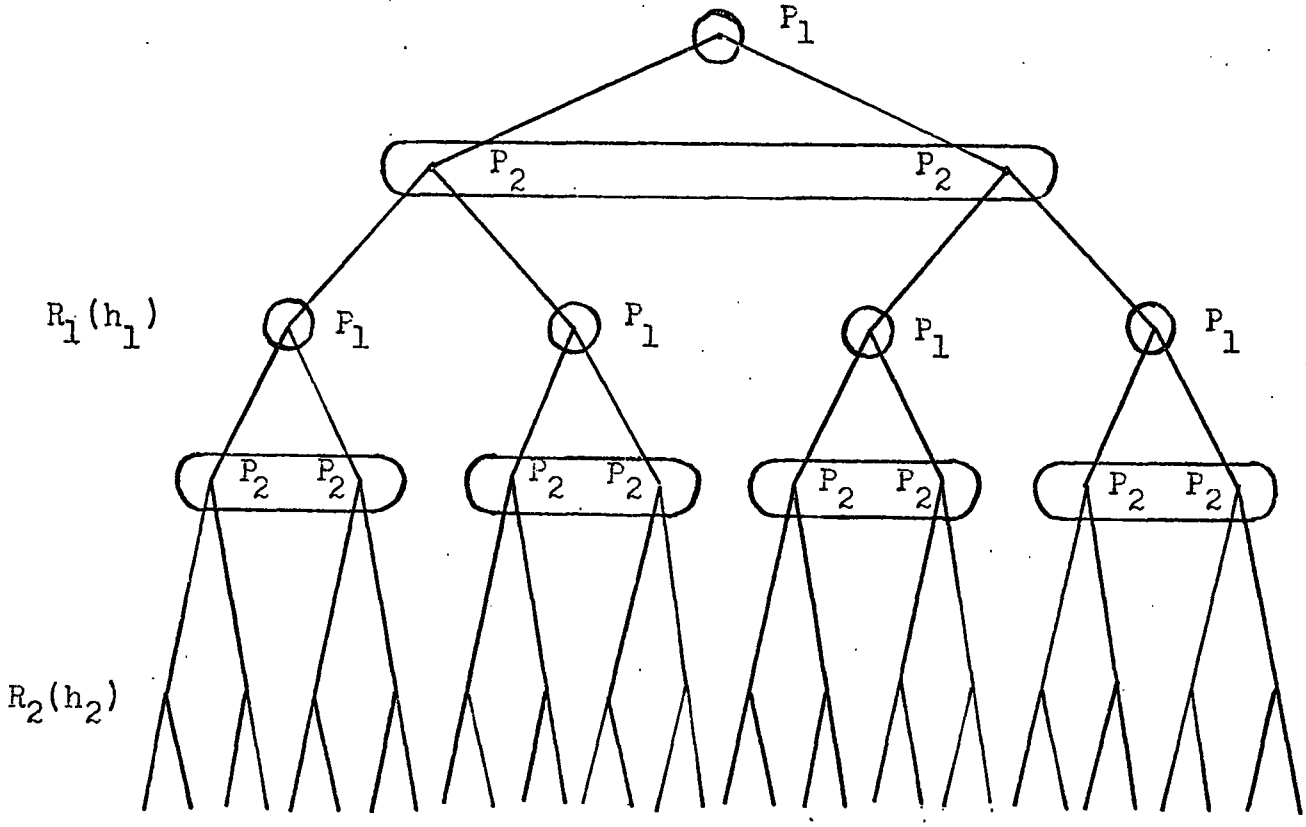

Figure (2-11): A simple Dynamic Duopoly Game

the remaining player is left to exercise monopolistic control from then on.

All the above formalization of the infinite game tree gives only a complete description of what can possibly happen. The game tree per se does not prescribe any pattern of behavior but represents a method whereby the dynamic features of an interactional competitive environment can be examined in toto. We shall leave the discussion of solving dynamic games to the next chapter.

CHAPTER III

GAMES AS DECISION TOOLS IN DYNAMIC SETTINGS

1. Introduction

There has been a tendency in the development of techniques in management science and operations research to build from simple static models to those involving complex dynamic analysis. Theories and models proceeding from elementary expository forms of single-period situations are extended to cases with a touch of pseudo-dynamic flavor, leading eventually to all-encompassing dynamic models where the time element plays a major role. The path which game theory has traced follows a very similar course with one exception, that is, the emphasis and swing to the consideration of the dynamic aspect of the theory still leaves many gaps to be filled. In this chapter, we shall examine how different formulations can be made to construct models of dynamic games situations. Notably the work of Shubik[1] and Hughes and Ornea[2] which we have consistently emphasized at the outset will be involved.

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1. op . cit. "Strategy and Market Structure".
2. op . cit.

As we have seen in our encounter with the essence of game theory in the previous chapter, formulation becomes increasingly complicated when we leave 2-person analysis to the general case of multiperson situations. We could therefore expect the mathematics of dynamic game situations for the general n-person case to be even more complex. In many of the situations we will be discussing in the following pages, we are of necessity drawn to make some simplifying assumptions to bypass the difficulties of mathematical formulations, rather than to be bogged down in our effort to deal effectively with real world situations. On the other hand, we should not be necessarily satisfied with the tools presently available.

The seemingly dynamic overtones of a game in its normalized form played many times is inadequate representation of actual situations. When we conjecture a competitive situation as a one period model we have evidently left aside such considerations as the learning processes and other psychological attitudes as well as adaptive characteristics of the players. The combined interplay of these forces resulting from internal (endogenous to the players) and external (environment) sources will have bearing on the payoff functions. Hence, it would be reasonable to assert that the payoff functions to the players could not remain unchanged as the game progresses over many plays.

In the main, the expectations of the players are likely to exhibit ephemeral behavior. As a consequence, in order to make game theory truely a normative theory we are besieged by a need to formulate suitable criteria for achieving optimality in our decisions. The answer is in some way provided by the processes dealt with in dynamic programming.[3]

Leaving the single-stage or -period game we are invariably drawn into a consideration of the way in which the quantity of resources change in a multi-stage process. It is obvious that as time goes on the amount of resources credited to each player through which they are further able to obtain or develop will be functionally related to one another and to time. What this means to the strategic alternatives available to the players is that they will not be limited to any same number of alternatives in any one subgame period. It is thus appropriate to view the alternatives available as a continuous function rather than discrete value variables. The dynamics are not restricted to controllable variables intrinsic to the players but to environmental forces, e.g. changes in taste over time, adding to the complexity of the choice situation. All these, as was remarked much earlier in the thesis, can make the model extremely un-

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3. Bellman, Richard, "Dynamic Programming" Princeton University Press, Princeton, 1957.

wieldy and, it would be wise to pause and examine what orders of magnitude in the features of the model we are expecting before we pass the point of diminishing returns. Imagine the immensity of the computer program if we have to enumerate all the alternatives pertaining to the various stages, what-more, the combinations of such! This is clearly an impossible task and such exhaustive examination is out of the question. Fortunately most of these are irrelevant and do not warrant attention and work in heuristic programming and artificial intelligence[4] have taken a step in the direction of formalizing various rules of thumb in attempting to portray the search processes in decision making.

In the context of dynamic games we are naturally interested in the applicability of the concepts of equilibrium and other game theoretic points to its solution. Their importance cannot be underplayed even as a result of the hazy picture we have presented above, for, they form the basis from which the normative structure of game theory is built. How would we then view the single period equilibrium solution which was so emphatically displayed in the preceding chapter? In the words of Bellman,

> "These multi-stage game may be considered not only to constitute an extention of the single-stage theory, but in many ways they

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4. Minsky, M. "Steps Toward Artificial Intelligence", Proceedings of the IRE, January 1961, pp. 8-30.

> may be considered to be more fundamental. The single-stage game may be conceived of as a steady-stage version of an original dynamic process, namely the multi-stage process."[5]

We might also make some remarks in this regard about the type of equilibrium solution that can possibly arise in dynamic games. However, it must be realized first that no matter what the goals a corporation or individual professes they are in some ways related to the problem of survival, for unless a firm can survive, it is pointless to think of maximizing profits and growth. With this view it appears that when we apply the non-cooperative concept to a game of economic survival, almost any state yielding every imputation of wealth can be enforced as an equilibrium state by carrying a sufficiently violent threat. For, in a game that is played only once, as in the case of the prisoners' dilemma game we would expect the Nash non-cooperative equilibrium to result, with the players selecting their second strategy. Even if the game was to be played many times over a finite length of time, by a process of "backward induction" this equilibrium would persist because of the fear hovering in the game before the last of the opponent defecting from the joint maximal solution. With the game over, reprisals cannot be carried out. No such lack of opportunity exists in an infinite game.

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5. op . cit. p. 283.

The temporary gains of the double-crosser may cost him more if his opponent carries out a stinging reprisal. Further once a defection is made, in a world where suspicion finds permanent roots in the minds of people, much effort is required to erase this and to rebuild the confidence that was shattered.

Thus, it may even be possible as the above consideration shows, to have a joint maximum enforced non-cooperatively if we consider economic survival as a mainstream force in the goals of a firm. It is therefore difficult from this standpoint to draw a clear cut distinction between cooperative and non-cooperative behavior in a dynamic situation. In this regard the strict dichotomy between these two forms of behavior must give way to various measures of cooperation. Of this, we have mentioned at the start of our preamble into the different theories of solution of games in chapter II. However, Shubik believes that,

> "The key to the examination of the possible equilibrium states rests with our ability to develop a calculus of plausibility for threats."[6]

With this note we shall in the following sections determine how the processes of game theory are employed in

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6. op. cit. "Game Theory and Related Approaches to Social Behavior." p. 69.

decision making in dynamic settings. When, in fact analytical formulation of descriptive or intricated quantifiable knowledge becomes exceedingly difficult we shall have to resort to simulation techniques. Simulation models with the aid of electronic computers have considerably extended our ability to deal with a wide range of the still largely descriptive and nebulous areas of management decision making.

2. Continuous Variable Strategy and Payoff Space

One of the prime purposes of constructing payoff matrices in 2-person games is to show the essential nature of conflict and why it arises in situations where interests are opposed. In setting up matrices involving a few strategies the object is to demonstrate certain basic concepts on which game theory is built. The real world could be very simple to deal with if in fact such are what we encounter in the business environment, which we know to be highly complex. Not discounting that we can have discrete choice situations but surely containing much more strategies than what we have shown, it is interesting to note that in industrial life we will have to deal with choice of strategic variables described by continuous functions. This fact is for example observed in investment in marketing or other facilities, where over a period of time the size of such investment can be varied continuously.

To facilitate our discussion of the concept of continuous variable and payoff functions we shall consider a simple example of a 2-person situation where each player is concerned with one decision variable. By increasing the number of variables or players or both we will only complicate our analysis with an increase in the dimensionality of the problem, making it difficult to visualize on a 2-dimensional plane. Constructing the graphical relationship is a first step in attempting to show how the path to an equilibrium is followed on the continuum of strategic choice. Figure (3-1) is such a representation. The quantities with subscript 1 and 2 are those associated with players 1 and 2 respectively. X is the decision variable and Rs the payoffs to the players. The dotted and light contours are iso-profit or payoff lines such that $R_i' > R_i'' > R_i''' > R_i''''$ for i = 1 and 2. This graphical representation is equivalent to the Edgeworth indifference analysis for a duopoly market in economic text. The loci of the players maxima are shown as dark unbroken and broken lines with M_1 and M_2 the monopoly profits. At their intersection we have the non-cooperative equilibrium, the point denoted by E/E. In a cooperative game the players would attempt to achieve joint maximization on the Pareto surface. Naturally the set of solutions, the negotiation set in the sense of von Neumann and Morgenstern, on this surface must be such that each player individually must

obtain more than they each alone could have obtained at the equilibrium point E/E. The significance of the D/T and T/D, the defence and threat points were mentioned in Section 4 of the preceding chapter.

For a 2-person case, as the number of variables increases, the increase in dimensionality of the problem makes it impossible to represent this graphically on a variable space. Representation on payoff space could overcome this. The equivalent of figure (3-1) plotted in payoff space is shown in figure (3-2). In general the payoff to a player i in an n-person situation is related by a continuous function R_i having a finite range over his decision variables x_i, y_i, z_i, and so on. The function for player i is given by $R_i[(x_1, y_1, z_1, \ldots), \ldots\ldots, (x_i, y_i, z_i, \ldots), \ldots\ldots (x_n, y_n, z_n, \ldots)]$ and limited to a finite range because in practice we would not expect the players to have infinite resources. Other restrictions out of practical considerations that would place constraints on the actions of the players include cases where one or more players control access to a fixed amount of physical resources, legal matters concerning the operation of the business enterprises and others. The existence and uniqueness of solution of such a game where the constraints and payoff functions to each player depends on the strategy of every player, requires complex mathematical analysis.

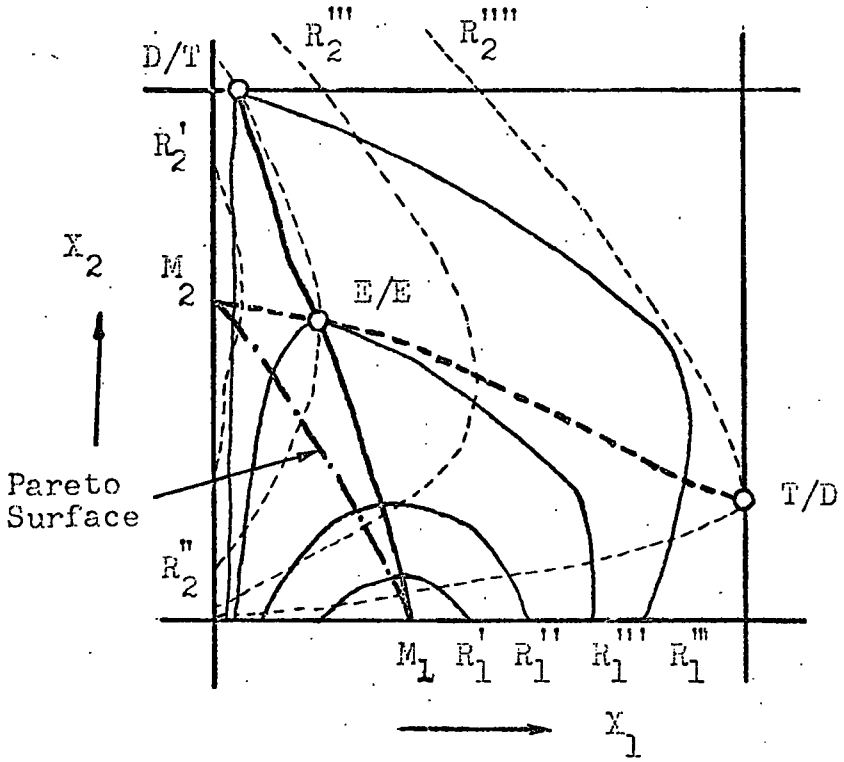

Figure (3-1): 2-Person Game in Variable Space

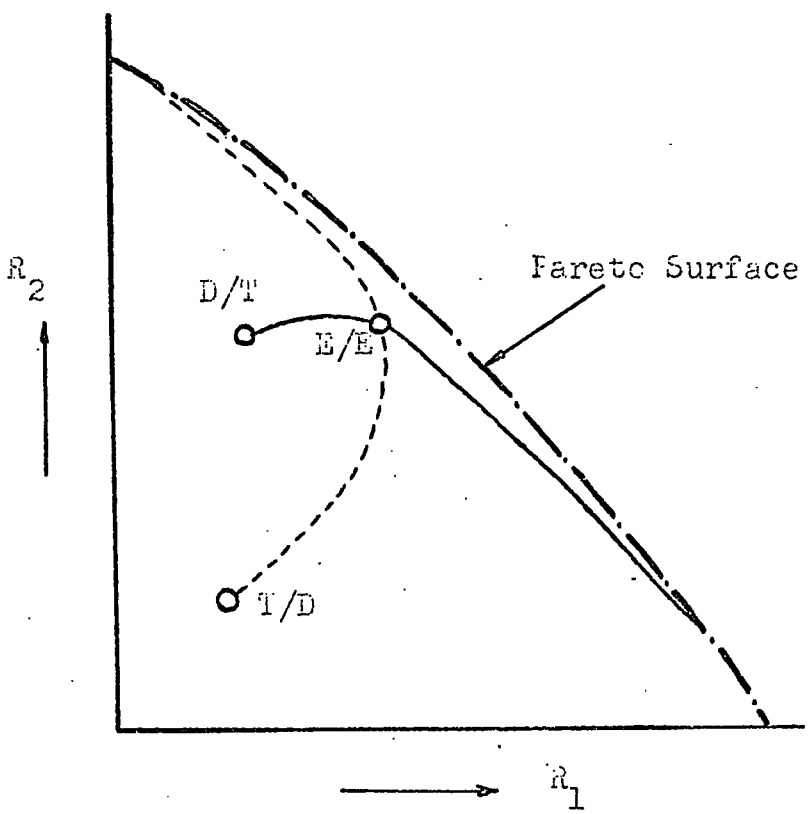

Figure (3-2): 2-Person Game in Payoff Space

However, J.B. Rosen[7] has shown the condition to hold under requirements of appropriate concavity in the payoff functions. For a simple exposition of the type of formulation involving continuous functions of dynamic games we shall outline the approach using the technique of dynamic programming.[8] To relieve us of undue mathematical and conceptual difficulties, 2-person games will be examined.

In the case where the game is zero-sum the expectation of player 1 (P_1) when he chooses his strategies from a continuum rather than a discrete set of moves is defined by E(F,G), where F is the distribution function[9] from which P_1 chooses his strategic variable x defined in the interval [0,1]. G is the distribution function from which player 2 (P_2) chooses his strategic variable y defined in the interval [0,1]. Then E(F,G) is given by the Stieltjes integral[10]

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7. Rosen, J.B., "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games." Econometrica, Volume 33, No. 3, July 1965, pp. 520-534.

8. Bellman, op . cit., Dreyfus, Stewart, "Dynamic Programming", in "Progress in Operations Research", Volume I. edited by Russel L. Ackoff., John Wiley and Sons, New York 1961. pp. 211-242.

9. The discussion of distribution functions as applied to game theory is found in Chapter 8 of McKinsey "Introduction to the Theory of Games", op . cit. In this case F. is the cumulative distribution function governing the frequency with which player 1 chooses x.

10. This is a continuous equivalent of the discrete formulation given in equation (2-7) of Chapter II.

$$(3\text{-}1) \qquad E(F,G) \equiv \int_0^1 \int_0^0 R(x,y)dF(x)dG(y)$$

If $R(x,y)$ the payoff function is jointly continuous in x and y in the closed unit square then it can be shown[11] that

$$(3\text{-}2) \qquad \int_0^1 \int_0^1 R(x,y)dF(x)dG(y) = \int_0^1 \int_0^1 R(x,y)dG(y)dF(x)$$

and hence the condition

$$(3\text{-}3) \qquad \max_F \min_G E(F,G) = \min_G \max_F E(F,G)$$

Taking the analysis one step further we put restrictions on the resources of the players represented at some stage of the dynamic game by m-dimensional vectors x and y. For a zero-sum case where the survivor of the game benefits with the ruin of his opponent, we call this a <u>game of survival.</u> In the more general case we have a non-zero sum dynamic game, <u>a game of economic survival</u> which we shall leave to a later section. Let us for the moment side step from our object of formalizing an infinite process because of conceptual difficulties and consider an N-state process. P_1 makes an allocation of his resources at the beginning of each stage by a vector u and P_2 by a vector v such that $0 \leq u \leq x, 0 \leq v \leq y$, where the inequalities hold component

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11. McKinsey, op. cit. pp. 186-193.

wise. This is equivalent to a strategy in the subgame under consideration. As a consequence of this action P_1 receives payoff of $R(u,v;x,y)$, a scalar function and P_2 by definition the negative of this quantity. With these payoffs the resources of P_1 is transformed from x into $T(x,y;u,v)$ and P_2 from y into $S(x,y;u,v)$. After N periods the total return to P_1[12] is

$$(3\text{-}4) \qquad R_N = R_N(u,u_1,u_2,\ldots,u_{N-1},v,v_1,v_2,\ldots,v_{N-1};x,y)$$

$$= R(u,v) + R(u_1,v_1) + \ldots + R(u_{N-1},v_{N-1})$$

If $R(u,v;x,y)$ is a continuous function in u and v for all finite values of u,v,x and y and $T(x,y;u,v)$ and $S(x,y;u,v)$ are also continuous over finite values of the vector variables then the value of the N-stage game is given by

$$(3\text{-}5) \qquad V_N = E(F,G) = \max_F \min_G \left[\iint R_N \, dF(u,u_1,u_2,\ldots,u_{N-1}) \, dG(v,v_1,v_2,\ldots,v_{N-1}) \right]$$

$$= \max_G \min_F \left[\qquad \right]$$

where F and G are distribution functions over regions of quite complicated forms defined by the inequalities

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12. These values should rightly be adjusted to its present value through a discount rate function.

$$(3\text{-}6) \qquad \begin{array}{ll} 0 \leq u \leq x, & 0 \leq v \leq y, \\ 0 \leq u_1 \leq T, & 0 \leq v \leq S, \\ \vdots & \\ 0 \leq u_{N-1} \leq T_{N-1}, & 0 \leq v \leq S_{N-1} \end{array}$$

The quantities T and S depend upon x,y,u and v; T_1 and S_1 depends upon x,y,u,v,u_1 and v_1 , and so on.

On applying the principle of optimality and functional equations of dynamic programming[13] where the sequence of functions is defined as

$$(3\text{-}7) \qquad f_N(x,y)=V_N \quad \text{for } N=1,2\ldots\ldots\ldots$$

we obtain the following recurrence relations[14]

$$(3\text{-}8) \qquad f_1(x,y)= \max_F \min_G \left[\int_{\substack{0 \leq u \leq x \\ 0 \leq v \leq y}} \int R(u,v;x,y)dF(u)dG(v) \right]$$

$$= \min_G \max_F \left[\qquad \right]$$

$$f_{N+1}(x,y)= \max_F \min_G \left[\int_{\substack{0 \leq u \leq x \\ 0 \leq v \leq y}} \int \left[R(u,v;x,y)\ f_N(T,S) \right] dF(u)dG(v) \right]$$

$$= \min \max \left[\qquad \right]$$

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13. For proof of applicability of this principle to game processes see Bellman, op . cit. pp. 291-292.

14. The existence and uniqueness of solution of this approach is found in Bellman, Ibid.

With this background we turn to the general case of an infinite game in which we consider the function $f(x,y;k)$, the value to P_1 of the infinite game beginning at the kth stage or subgame where P_1 and P_2 possess resources x and y at this stage and both employ optimal strategies. The functional equations describing the subsequent periods are given in the recurrence relation.

$$(3\text{-}9) \qquad f(x,y;k)$$

$$= \max_{F} \min_{G} \left[\int_{0 \le u \le x} \int_{0 \le v \le y} \left[R(u,v;x,y;k) \quad f(T_k,S_k;k+1) \right] dF(u)dG(v) \right]$$

$$= \min_{G} \max_{F} \left[\qquad \right]$$

Further complications could be added to the model but this would take us away from the purpose of this section which is to show the change in formulation when we consider continuous rather than discrete functions.

3. Dynamics of Game Situations

With the background of game theoretic reasoning developed thus far we shall investigate some situations exhibiting dynamic game characteristics. There are basically two approaches in the consideration of dynamic operational systems, namely continuous or discrete time sequences.[15] We usually

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15. Morse, Philip M., "Dynamics of Operational Systems", in Ackoff's "Progress in Operations Research", op . cit. Chapter 6.

think of the first category as those where the states of the system changes as a continuous function of time. Decision criteria could be based on a comparison of the times for the systems to reach a predetermined state or other measures that involve consideration of time as a continuous variable. The case of discrete time systems are exemplified by the period by period games we observed earlier and which we will discuss as games of economic survival in the remaining section of the chapter. For the present we will be concerned with continuous time systems.

The first situation we shall analyse is the decision to penetrate markets exhibiting various growth patterns.[16] A number of complex factors are involved, of which the capability of the company, objectives, time, markets, competition, costs, investments and experience are a few. At best we shall consider some of these because the interrelationship of these factors can be exceedingly complicated. The environment pictured is a market composed of many products in which companies are competing for a share. To simplify our exposition we shall consider the company playing against a unified

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16. The analysis is based on the ideas of the model in R.W. Allen's "Factors Influencing Market Penetration", Management Science, Series A, Volume 13, September 1966, pp. 22-43, but modified for the inclusion of explicit game theoretic reasoning and method of formulation.

opponent, namely "competition" in the same product market. Thus if, [17]

S = market share of company at time t.

C = market share of competition at time t.

S' = rate of change of market share of company at time t.

C' = rate of change in market share of competition at time t.

α = a payoff factor ≤ 1 to the company which is a function of the market investment strategies of the company and competition. The payoff expectations changes with time through experience gained (following the exponential learning curve theory) in the process. That is $\alpha = f(x,y;t)$ where x and y are the strategies of the company and competition respectively.

β = a payoff factor ≤ 1 to competition similarly defined as for α. Thus $\beta = g(x,y;t)$[18]

γ = a loss rate factor on the company's shares (business losses in the form of losing contracts, etc..)

δ = as defined for γ but on the competition's shares.

M_a = the available market at time t. $M_a = M - C - S$

A pictorial representation of the market interaction is shown in figure (3-3). We have the rate of change of market shares

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17. All the variables discussed can be reduced to the common yardstick of dollar units.

18. For a constant or fixed available market $\alpha + \beta = 1$ at the steady state equilibrium, i.e. we have a constant-sum game.

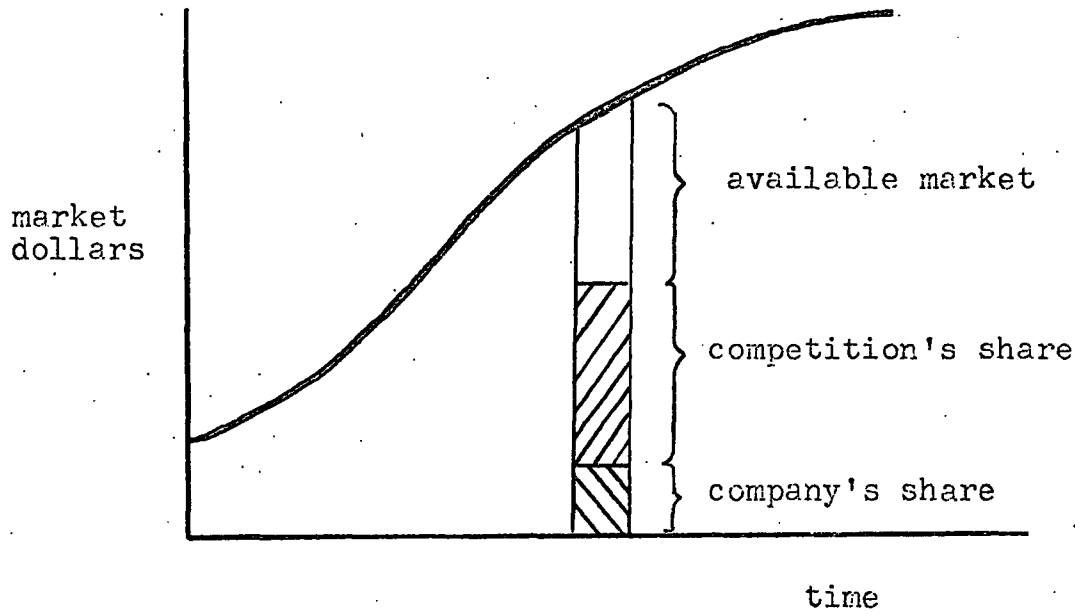

Figure (3-3): Market Trend With Share Distribution Relationship

of a company and competition given by,

(3-9) $$\frac{dS}{dt} = \alpha M_a - \gamma S$$

(3-10) $$\frac{dC}{dt} = \beta M_a - \delta C$$

Substituting for M_a in equation (3-9) and (3-10) we have

(3-11) $$S' = \alpha(M-S-C) - \gamma S$$

(3-12) $$C' = \beta(M-S-C) - \delta C$$

Differentiating (3-11) with respect to time we obtain

(3-13) $$S'' = \alpha(M' - S' - C') - \gamma S'$$

Replacing for C' and C and rearranging we are left with the 2nd order linear differential equation

(3-14) $$S'' + (\alpha + \beta + \gamma + \delta)S' + (\alpha\delta + \beta\gamma + \gamma\delta)S = \alpha(M' + \delta M)$$

The equation can be solved for various market conditions (for example, constant, linear or transcendental) and the competitive behavior of the participants as described by their interlocking strategy spaces.

We are left with finding a criterion for the decision to penetrate the market. Two measures could be thought of, (1) the time to reach a positive gain G given by

(3-15) $$G = S - K$$

Where K is the total cost involving the initial investment, recurring investment and operating cost, all the costs and gains being reduced to present values, and (2) the gain over investment ratios. These two criteria are likened to the net present values and benefit cost ratios of capital budgeting theory. The results should indicate to us what investment plans or strategies to follow in order to reap most from a market penetration. We do recognize that subjective factors will influence the results but until we are able to quantify them, they rest mainly in individual judgement and can be

accounted for in the analysis by putting more or less severe constraints on the problem or changing the values of the constant terms.

Let us now turn to another process, the bargaining process, where economists have traditionally left to psychologists the problem of providing a specific solution rather than defining a delimited area as in the case of von Neumann and Morgenstern negotiation set. This is thought to depend on the "bargaining abilities" of the parties concerned. We have seen how the Nash Axiomatic approach isolates for us a point on the Pareto optimal surface that is satisfied by maximizing the product of the players utilities measured from the point of total disagreement. J.G. Cross had pointed out two short-comings of Nash's theory in that,

> "...First, it offers no analysis of the dynamic process of disagreement-concession-agreement that constitutes the very essence of the bargaining process. We are given only a solution criterion with no insight into its raison d'etre..... Second, acceptance of the descriptive interpretation of the Nash model would imply acceptance of the conclusion that all the information which is necessary for the analysis is contained in the set of possible utility-payoff combination."[19]

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19. Cross, John G, "A Theory of the Bargaining Process" op . cit. p. 69.

In what follows we will incorporate the above observations in a labour-management bargaining model. Cross[20] makes two distinctive cases in the bargaining process, the pure bluffing and the pure intransigent cases, but recognizes that both are present in varying magnitudes in actual situations. The analysis of bargaining and negotiations involving bluffing is still in its qualitative stages and remains to be formulated mathematically.[21] Our attention however, will be directed at the intransigent case where genuine disagreement arises.

For a 2-person situation disagreement results because

(3-16) $$q_1 + q_2 > M$$

where q_1 and q_2 are the quantities of a commodity demanded by player 1 (P_1) and player 2 (P_2), having an available

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20. Ibid. pp. 70-71.

21. An enlightening article bringing in misrepresentation of preference (bluffing) in a bargaining and negotiation interaction and the possible change in utility evaluation as the negotiation proceeds is "Negotiation: A Device for Modifying Utilities" by F.C. Ikle in collaboration with N. Leites, Conflict Resolution, Volume VI, No. 1, pp. 19-28. Also found in Shubik's "Game Theory and Related Approaches to Social Behavior", op . cit. pp. 243-257.

amount M.[22] In business, time has a monetary cost brought out in the discounting function and utility in that our disposition and value of an agreement changes through time.[23] If initially P_1 expects P_2 to concede at some rate r_2 and P_2 expects a concession rate r_1 from P_1 then the expected time for P_1 to receive q_1 is $(q_1 + q_2 - M)/r_2 = \tau$. The value of the utility of agreement to P_1 is the difference between the present value at discount rate a of the utility of q_1 and the costs C_1 which is assumed to be fixed and recurring through time. Hence,

$$(3\text{-}17) \qquad U_1 = f(q_1)e^{-a\tau} - \int_0^{\tau} C_1 e^{-at} dt$$

$$= f(q_1)e^{-a\tau} + \frac{C_1}{a}e^{-a\tau} - \frac{C_1}{a}$$

To determine the value of q_1 which maximizes U_1 we derive the first and second derivatives of U_1 with respect to q_1 and arrive at two conditions:[24]

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22. The quantity M need not be a fixed quantity. If we are considering labour-management wage negotiation, disagreement arises whenever $Q_1 > Q_2$ where Q_1 is the union wage demand. Unlike the case where the two players individually strives for the greatest q_i player 2 (management) in the labour-management negotiation prefers a low Q_2. Hence to avoid sign confusion in the formulation we will retain the definition $q_1 + q_2 > M$ for disagreement.

23. Ikle and Leites, op. cit.

24. This is done in the Appendix

(3-18) $$\left[f(q_1) + \frac{C_1}{a} \right] \frac{a}{r_2} = f'(q_1)$$

(3-19) $$\left[\frac{f''(q_1)}{f'(q_1)} r_2 - a \right] < 0$$

As the play progresses each party acquires information and modifies their expectations. We can characterize the learning process by the following conditions:

(3-20) $$\begin{aligned} \frac{dr_2}{dt} &> 0 \quad \text{if} \quad -\frac{dq_2}{dt} > r_2 \\ \frac{dr_2}{dt} &= 0 \quad \text{if} \quad -\frac{dq_2}{dt} = r_2 \\ \frac{dr_2}{dt} &< 0 \quad \text{if} \quad -\frac{dq_2}{dt} < r_2 \end{aligned}$$

meaning that if P_2 concedes faster than is expected P_1 will increase his estimate of P_2's concession rate: if P_2 concedes just as rapidly as is expected P_1 will retain his original estimate r_2 and so on. In fact P_1's expectations change directly in proportion with the discrepancy between his expectations and P_2's concession rate, that is,

(3-21) $$\frac{d\dot{r}_2}{d(-r_2-\dot{q}_2)} > 0 \qquad \text{where } \dot{r}_2 = \frac{dr_2}{dt}, \quad \dot{q}_2 = \frac{dq_2}{dt}$$

To determine $-\dot{q}_1$, the concession rate of P_1 we differentiate

equation (3-18) with respect to time.[25] We get,

$$(3\text{-}22) \qquad \dot{q}_1 = - \frac{1}{\frac{f''(q_1)}{f'(q_1)} r_2 - a} \frac{dr_2}{dt}$$

From equation (3-19) and (3-22) we see that q_1 varies in the same direction as r_2. What this means is that if P_1 discovers that P_2 is conceding at a rate higher than his expectations, he will increase his demand q_1, in line with the learning theory conditions above. Similar expressions can be arrived for P_2, the subscripts being correspondingly replaced and a different discount rate, b put in place of a.

It can be shown [26] that the Nash solution under symmetric conditions, with appropriate transformation of the origin of the utility functions is a particular case of the general model. By assuming suitable utility and learning functions and applying the general model we can determine equations describing the players demand as a function of time and the time taken to reach agreement. This requires the solution of complicated differential equations which are best left to research oriented to computional methods and numerical solutions.

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25. Refer to Appendix

26. Cross, John G, op.. cit. pp. 81-84.

4. Games of Economic Survival

We shall portray in this section the relationship between the market and financial aspects of a firm through the formulation of a game of economic survival as developed by Shubik.[27] It was mentioned in the introduction to this chapter that survival per se could be a goal as could the maximization of the discounted value of the dividend payment. To characterize the dynamic play we distinguish the fortunes of the firm as divided into a corporate and a withdrawal account. The gains and losses of the firm are reflected in the former while the decision to make payments into his withdrawal account rests with the decision maker and his assessment of his chance of surviving. One restriction, however, is that once payments are made to the withdrawal account they can no longer be available for use in the corporate account. To some extent this is unrealistic because capital is normally available in the sense that owners can shell out reserves for reinvestment or loans could be floated. But, to reduce the complexity of the problem and to bring out essential relationships we have to forgo this added complication.

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27. Shubik, "Strategy and Market Structure", op. cit., and Shubik and Thomson, G.L., "Game of Economic Survival" Naval Research Logistics Quarterly, Volume 6, 1959, pp. III-123.

Before we venture to formalize the case for n-person let us make a brief excursion into 2-person games of economic survival. In each period prior to the ruin of one of them we can identify for each player a market and a financial move. The market action determines the payoff while the financial move decides the payment to the withdrawal account. A 2-person game of economic survival can thus be characterized by:

1) a payoff function $R_{i,t}$ in continuous variable space at time t for player i=1,2.

2) a discount rate $\rho_{i,t}$ where the subscripts are similarly defined.

3) A_1 and A_2 are ruin payments, B_1 and B_2 the ruin conditions such that $A_1 < B_1$; $A_2 < B_2$.

4) $C_{1,t}$;$C_{2,t}$ the corporate accounts and $W_{1,t}$; $W_{2,t}$ the withdrawal accounts to the players denoted by the subscripts 1 and 2 at time t.

5) a firm will be ruined in time τ if its assets which we have condensed into the functions we termed corporate accounts fall below the ruin conditions. That is, $C_{i,\tau} < B_i$.

6) $R^*_{i,t}$ the value of the income to the survivor i as soon as his opponent is ruined.

In any period t the corporate account to the ith player is given by,

$$C_{i,t} = C_{i,t-1} + R_{i,t} - W_{i,t}. \tag{3-23}$$

If the firm i never liquidates the long-run discounted payoff

$$(3\text{-}24) \qquad D_i = \sum_{t=0}^{\infty} W_{i,t}\, \rho_i^{\,t}$$

and if it liquidates at the end of τ periods.

$$(3\text{-}25) \qquad D_i = \sum_{t=0}^{\tau-1} W_{i,t}\, \rho_i^{\,t} + A_i \rho^{\tau}$$

In a game of economic survival and in general, games having infinite duration, the threat of reprisals induces equilibria which do not exist in single period games. A player can employ a violent threat in an effort to maintain certain status quo. It is thus possible to enforce non-cooperative equilibrium at the joint maximum. Much of the ability to carry out effective threats lies in the flexibility of the firms, a factor founded on their costs and financial structures. Assuming that each player adopts a <u>stationary strategy</u>, one in which the market and financial moves are repeated through time, we can define a <u>stationary state solution</u> as the set of stationary outcomes, which the players can enforce by a pair of threat strategies. We denote a stationary strategy for a player i by $\bar{s}_{i,t} \in S_{i,t}$, his set of particular strategies. There is a note of unrealism in our assumption, but it brings out the assertion that in infinite games our notion of equilibrium becomes nebulous, and the more so the more paranoid are the players especially when we consider their suicidal attempts to control their opponents.

Asymmetry in the assets of the players are important considerations in a game of economic survival. Legal and other ethical questions set up constraints on the actions of the players. Leaving these aside a financially stronger player will not accept a long-run stationary equilibrium if it is possible to drive out the opponent and recoup more than enough to equal the stationary state income. The equivalent steady state payoff M_i at which he is indifferent is determined by solving the following equation:

$$(3\text{-}26) \quad D_i \equiv \sum_{t=0}^{\infty} M_i \rho^t = \sum_{t=0}^{\tau} R_{i,t} \rho^t + \sum_{t=\tau-1}^{\infty} R^*_{i,t} \rho^t$$

Clearly we cannot discount the fact that the opponent $j \neq i$ can retaliate other than adopting a defensive strategy. Therefore D_i is better defined as:

$$\min_{s_{j,t}} \max_{s_{i,t}} D_i$$

As we noted, legal and social restraints will modify somewhat the goals of the financially stronger firms.

The foregoing argument is brought out more clearly in a graphical analysis in a one decision variable problem. The α lines in figure (3-4) represent the equivalent steady state payoff M_i that player 1 can enforce as outlined in equation (3-26). αs with increasing number of (') represent lower initial assets of player 1. L_1 and L_2 lines are the

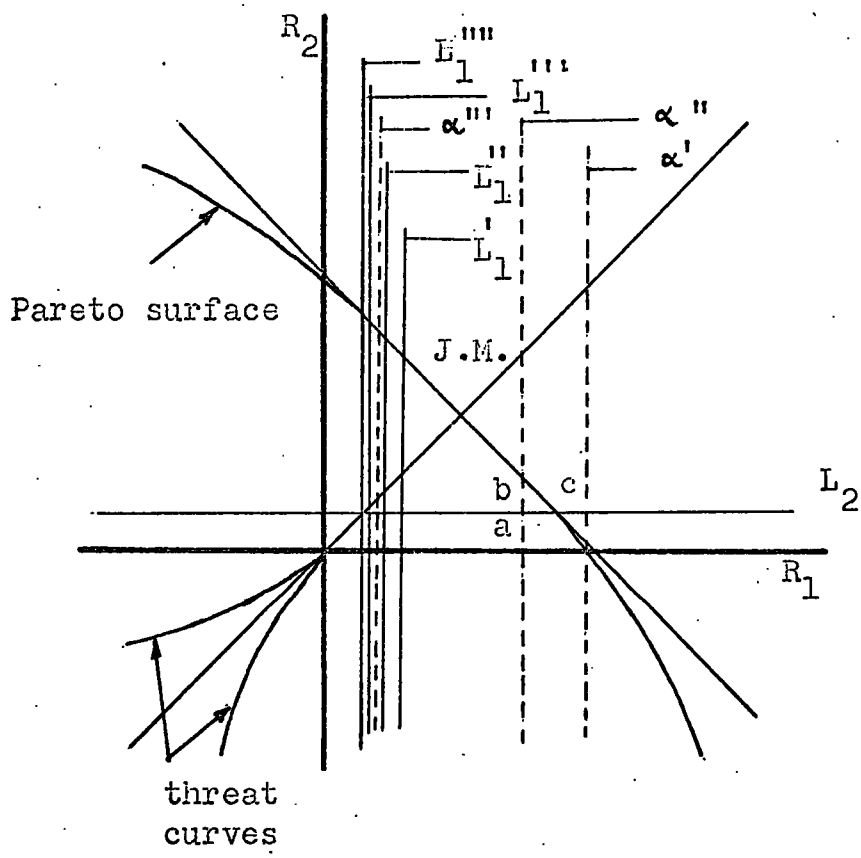

Figure (3-4):[28] Graphical Representation of a Game of Economic Survival

liquidation values, the returns from investment at long run rate of return of players 1 and 2 respectively (for player 1 the number of (') in the Ls correspond to the asset levels in the αs). At α' we note that no possible equilibrium is attractive to player 1 except the ruin of his opponent. If player 1 starts out with assets in the α'' position then the possible steady state equilibria is limited to the region

28. Adapted from Shubik, "Strategy and Market Structure", op . cit. p. 239.

on the Pareto curve marked off by the lines α'' and L_2. Similar explanation can be made for the α''' and L_2 lines, only that in this case the assets of player 1 are such that he can barely ruin player 2. We observe that as the assets of the players become more symmetrical the set of possible equilibria becomes increasingly larger.

A factor which makes the equilibrium concept in a dynamic game more confounding is the role played by chance. An otherwise stable stationary state will unexpectedly be destroyed as chance fluctuations in the payoffs cause misfortunes to run against one of the players. We might think of such a situation as a failure in an advertising campaign or an even more drastic event. The values assigned to a payoff matrix element is an expected value over some probability distribution. As a result of certain market and financial policies, there is a probability that the firm will be ruined. This has been considered by Shubik and Thompson as a random walk problem.[29]

Other factors that contribute to the shaping of competition are the structures of the markets distinguished by their manufacturing, distribution and retailing systems and the type of products traded. The combined action involving the multitude of decision variables, for example,

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29. Shubik, Ibid. pp. 247-249 and Shubik and Thompson, op . cit.

price, advertising, production and inventory scheduling, etc., will expand considerably the formal apparatus we have erected above. Computational difficulties will call for experimentation by simulation with high speed computers.

When we expand our analysis to situations involving more than two players the notion of threat becomes even more awkward to handle. Measures similar to the ψ function for coalition structures could be employed to describe the interaction. The assumption is made that we can divide an n-player interaction into 3 sets containing k, r and n-k-r players. The (k_i) - (r_j) stable market equilibrium[30] specifies that the joint action by the r players in the r set cannot yield them more than maintaining the steady state on the assumption that each of the k players in the k set is committed to a threat strategy while those of the players in the n-k-r use their steady state strategy. The problem is in defining these different sets and just as in the ψ stability function economic and extra-economic information will be required to limit the combinatorial possibilities. In the case of a market dominated by a single firm, it can be described by (1_i) - (r) stability. (1_i) stands for the dominant firm. (r) stands for any set of r players not including the dominant firm, and, since we do not need to distinguish between

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30. Shubik, Ibid. Chapter 11.

different sets of r players we do not use a subscript with (r). For complete domination we have (1_i) - (n-1) stability. The condition for the general case of (k_i) - (r_j) stability can be expressed as follows:

$$(3\text{-}27) \qquad \max_{V_j} R_i\,(\bar{S},\bar{T}^{k_i},V^j) = R\,(\bar{S},\bar{T}^{k_i}) \text{ for } j \neq i$$

which means that the player j in set r is maximizing his payoff using his optimal strategy V^j against the threat strategies of all the players in the k set described by the vector $\bar{T}^{k_i}$ while the rest n-k-r players use their stationary strategies represented by the vector $\bar{S}$. The reasoning can be extended to other players in r. For equilibrium the index of (r_j) - (k_i) stability must be defined with the roles in the expression (3-27) reversed.

The stability criteria that we examined in the general case is helpful only in showing us what we can infer from an equilibrium state. No explicit consideration of the dynamic path to equilibrium or from one equilibrium to another is given except in the 2-person cases we have given in section 3 and the earlier part of this section.[31] As was suggested at the beginning of the chapter we note that our concept of solution depends on the understanding of threat strategies.

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31. The path to equilibrium for a 2-person case is discussed by means of graphical analysis by Shubik and Thompson, op . cit. pp. 120-123

Indeed the threat components of a threat strategy can be almost any function and can involve very complex behavior. It is hoped that some light would be thrown on the nature of these complexities through gaming and simulation experiments.

CHAPTER IV

STRATEGY AND STRUCTURE IN THE OIL AND GAS INDUSTRY: AN APPLICATION OF THE SHELL MODEL.[1]

1. Overview of the Industry:

A major goal of this study is related to the problem of how game theory methods can be used to elucidate and contribute to strategic decision making. Factors affecting gasoline sales in the medium range future of the North American oil and gas industry are areas offering opportunities for such investigation. The economic background of the industry had given rise to strongly oriented oligopolistic competition requiring the use of sophisticated decision tools.

"Fortune"[2] provides an informative listing of oil companies. The vast number of competitors in the business reaching for shares of the market makes it of considerable urgency for a competing firm to search for policies aimed not only for survival or status quo position but for expanding profitable ventures.

By far and large we can perceive the petroleum industry as comprising three stages in the vertical structure, namely crude producing; refining and marketing, exploration and

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1. Hughes and Ornea, op . cit.

2. "Fortune", June 1967, pp. 196-213. For the Canadian scene, refer to "survey of Oils 1968", publication of the Financial Post.

drilling being ordinarily conceived within crude production functions.[3] Interspaced between these divisions and the consuming public at one end are competitive markets. If there were no integration the crude and products would pass through the successive markets to the consumer market. Conversely if every firm were fully integrated and completely balanced there would be no market beyond those over which the final products pass into the hands of the ultimate consumer. However, although there is full integration, no major[4] company is completely balanced and most independents are non-integrated or partially integrated.[5] For the fully integrated concerns, sometimes more crude may be produced for them to refine while at other times they may refine more crude than they produce. The intermediate markets provide buffers for such imbalance. In the product market it is possible that they may market more than they refine or vice versa. The unpredictable occurrence and exhaustible nature of crude deposits have made it often times economical for a company to sell the crude it produces for other refineries,

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3. Hamilton, D.C. "Competition in Oil", Harvard University Press. Cambridge (1958), p. 4.

4. A distinguishing factor between a major and an independent marketer is the customary 2 cent price differential allowed to the latter on the market front.

5. Hamilton, D.C., op . cit.

while it may find more to its advantage to buy crude from sources located near its refineries.[6] The interaction of these forces as determined by the policies of the various companies gives rise to competition or collusion which we can analyse by examining the interlocking strategy spaces of the parties to conflict or co-operation.

2. Strategic Decisions in the Oil Industry

The foregoing discussion of the general structure of the oil industry provides a basis for examining the types of decisions that an oil company must consider. Broadly, in line with the features of the industry, three areas of strategic decisions can be identified. These are concerned with marketing, refining, and exploration, a fact noted in our dissection of the industry into different levels. In the model to be discussed we are mainly interested in marketing decisions, largely because it is here that competition is keenest. Importance of optimum decision making in the other levels of the industry are not to be underplayed. In fact the proper approach is to adopt a systems viewpoint in order to strive for overall optimality.

Being an area where there is tremendous growth, continual change in character, location and operation of facilities the consummer market is closely watched by rival firms. In

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6. Ibid. p. 7.

no other sphere is it easier for operators to enter or exit. Excess capacity is recognized to predominate among retail service stations and is generally the cause of depressed earnings. Marketing investment decisions thus determine to a large extent the success of a firm. These investments both for new opportunities and to develop, modernize and expand existing investments in service stations and other facilities run to substantial proportions of the overall investment figures of oil firms. To quote some comparative figures, in 1965 Imperial Oil invested 21.6 million dollars in marketing, which is 25.2 percent of total capital expenditure.[7] For the same period Shell invested 12.1 million dollars, which is 28.6 percent of capital expenditure.[8]

Price stabilization in the last few years has produced a general shift in marketing strategies to non-price and other forms of competition. This has opened a new arena for implementing marketing investment decisions. Also it is predicted, though not an immediate issue in our present decision problem, that trends in shopping habits will greatly change the manner of marketing gasoline. Service stations will gradually become part of retail complexes as powerful

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7. Imperial Oil Limited, Annual Report 1965.

8. Shell Canada Limited, Annual Report 1965.

competition from giant retail stores and general merchandise chains become an important factor in gasoline marketing.[9]

Refining has been the most fully developed branch of the industry and where independent entry finds staunchest barriers largely because of higher investment outlays. Refining processes are essentially the same and only with great effort through reductions in costs can a firm expand its margin by fractions of a cent. Thus the areas of strategic concern to a refiner are price and availability of crude, price and availability of pipeline and other means of transportation, problem of access to markets and technological development.[10]

In exploration and drilling independents have until recently played prominent roles. The majors with ability at marshalling the ever increasing capital required for expenditure in such activities have taken over large proportions of these activities.

Competition in the oil industry is linked to the erratic fluctuations and annual additions to capacity and new oil field discoveries. Competitive strategies are ne-

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9. "Canadian Petroleum", October 1967, p. 17.

10. de Chazeau, M.G., and Kahn, A.E., "Integration and Competition in Petroleum Industry", Yale University Press, New Haven (1959).

cessary in the face of uncertainty arising from lack of control of raw materials, transportation, processing and marketing. In the pursuit of security it is observed that oil firms have over the years proceeded on an extensive horizontal and vertical program of integration. For, as de Chazeau and Kahn remarked,

> "Competitive strategies have many facets, especially in an industry as dynamic as this one. Integration itself has been one of these facets. Investment, innovation and non-price competition are others". 11

At the present time, this is however the subject of much controversy especially in view of public policies directed at the organization of the oil industry and its tendency to monopolistic control.

Finally, though it may seem that the direction in which the gasoline market is heading signals for new strategic moves,

> "Competition for gasoline sales in the last 2 years has strangely brought very few radical changes in marketing patterns across Canada and the U.S... Markets are growing but major companies appear to hold tenaciously onto their shares of the business... The regional character of the market prevails (in the U.S.) despite efforts of some majors to expand nationally." 12

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11. Ibid. p. 375.

12. "The Oil and Gas Journal", February 1967, Volume 65, No. 6, p. 56.

Oil companies are faced with other problems of a competitive nature outside the confines of the immediate industry. Trends in inter-fuel competition have loomed large of recent years.[13] Furthermore, serious air pollution problems have taken changes in public policies and control and accelerated the advent of other forms of energy. The output composition of the industry will then have to take drastic changes to meet with times.[14]

3. The Shell Model [15]

Although we may have kindled an interest in the model developed at Shell by frequently referring to it at various stages in the preceding chapters, we have not made any significant examination of the composition of its basic structure. In this section we shall attempt to outline its main

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13. Ibid. p. 52.

14. Success by British Petroleum to produce edible proteins at their French refinery goes to show the gradual change in typical refinery product balances, "Canadian Petroleum News," June, 1957, p. 12.

15. Part of this section, especially the analytical details is the result of private communication with Professor J. Swirles, a member of the team responsible for setting up the model.

features, paving a way for possible application to an industrial situation characterized by the British Columbia oil and gas industry. But, it must be asserted that we shall not in any way resort to detailed computation for obtaining game theoretic points nor to concern ourselves solely with the validity of the mode of characterization. The Shell model is an attempt at integrating what we have discussed in the foregoing chapters. It represents one of the few efforts at endeavouring to construct a model of a complex competitive situation in order to provide meaningful results to aid competent management in making decisions. Hence, taken that the model does provide us with a method of calculating these points our purpose is to raise issues as to the manner we proceed to utilize these results, a question that has not been resolved fully.

The Shell model code named STRATCOM was intended to look at a multiperson non-zero sum game in continuous variable space. In their paper Hughes and Ornea[16] considered the case of a duopoly market situation only. Figure (4-1) in general shows the structure of the model from the viewpoint of a single company. A basic consideration in the model is the exclusion of exploration and production activities. The

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16. op . cit.

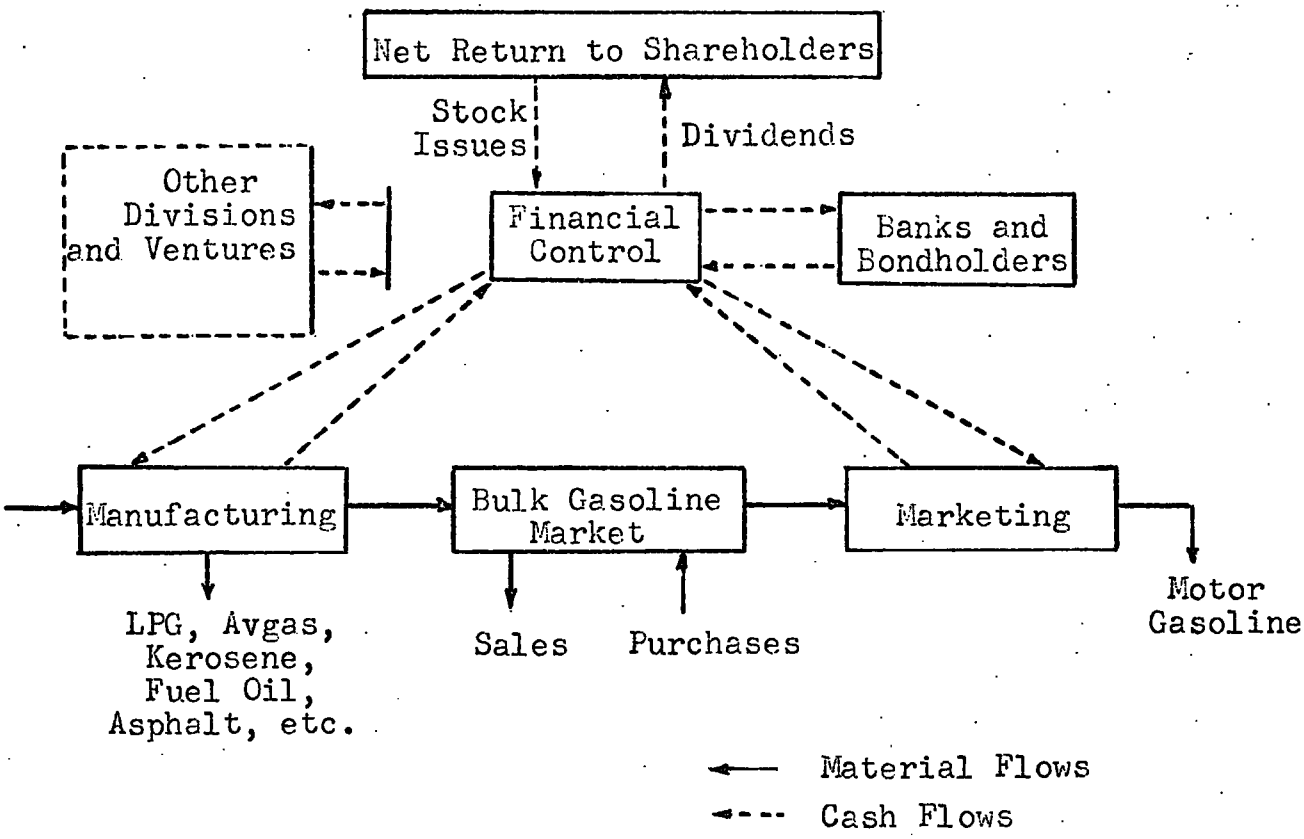

Figure (4-1): STRATCOM; Structure of Model

price of crude oil was fixed for this reason and fluctuations in its availability were not considered. Other ventures, e.g. chemical companies which contribute to the financial success of an oil company were not considered as part of the model.[17] Gasoline was the prime product with regard to decision making. Other refinery products were assumed to be sold at fixed prices irrespective of the production level of the refinery.

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17. Hughes and Ornea.

A set of competitors which may compose of single or groups of companies are chosen. For the initial states, the current total debt, investment in manufacturing facilities (including facilities for marketing byproducts and handling the bulk gasoline market) and investment in retail gasoline marketing facilities of each competitor are taken. The performance of the competitors over a number of time periods are calculated. Altogether three time periods in a ten year horizon are considered. Decisions in the first period apply to the first and second years, those in the second period run from the third to the fifth year, while the third period concern the remainder of the ten years. In each time period the set of decisions involve investments in refineries and marketing, debt, retail price of gasoline and the buying and selling of bulk gasoline. To enumerate they are:

1) Increase in debt (ΔB)
2) New investment in manufacturing (ΔK)
3) New investment in marketing (ΔM)
4) Tankwagon price for retail sales (P)
5) Price margin for selling gasoline in bulk market (Q^+)
6) Price margin for buying gasoline in bulk market (Q^-)

The effect of the first three decisions is felt throughout the subsequent periods. The latter three affect the current time period only and these prices are assumed constant over the entire period concerned, not too realistic for implementation at first sight. This can be taken to be

an average over the entire period and is reasonable approximation to an overall strategy. In actual practice day-to-day consideration of the balance of refinery runs, inventory position and individual bulk sales and purchases govern the bulk price margin.

A market initially having a sales volume of $V(0)$ and predicted to grow at a uniform rate such that at time t

(4-1) $$V(t) = V(0)(1+\alpha t)$$

where α is a constant, is taken as the starting point. If,

$M_i(t)$ = the marketing investment at time t of player i

$K_i(t)$ = the manufacturing investment at time t of player i

$B_i(t)$ = the debt position at time t of player i

t = 0 for the variables above represents initial condition

then total marketing investment at time t is

(4-2) $$M(t) = \sum_i M_i(t)$$

and similarly for manufacturing investment

(4-3) $$K(t) = \sum_i K_i(t)$$

$M_i(t)$ and $K_i(t)$ the marketing and manufacturing investment of player i at time t is given by

(4-4) $$M_i(t) = aM(t-1) + \Delta M(t-1)$$

(4-5) $K_i(t) = bK(t-1) + \Delta K(t-1)$

where a and b are parameters, their values being determined by the deterioration of investment functions. If it is assumed that the deterioration is 10 percent for manufacturing and marketing facilities then $a = b = 0.9$.
With these investment levels the supply at time t, $V_i(t)$ of competitor i to meet the market demand is determined by the following equation.[18]

(4-6) $$\frac{V_i(t)}{V(t)} = \frac{M_i(t)}{M(t)} f(P_i(t), \bar{P}(t))$$

where $P_i(t)$ is the tankwagon price for retail sales of competitor i and $\bar{P}(t)$ is the weighted average price given by,

(4-7) $$P(t) = \frac{\sum_i M_i(t)P_i(t)}{M(t)}$$

The bulk prices are determined by an algorithm deviced for the model. A simple interpretation of the Bulk Algorithm for a 2-person case is shown graphically in figure (4-2). The horizontal scale represents the volume of sales at time t. This is divided between the players, V_1 going to player 1 and V_2 to player 2, these values having been calculated from the variables of foregoing equation (4-6). At prices Q_1^+ on the upper broken curve player 1 sells bulk gasoline and buys at

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18. The relationship is based on an empirical study done at Shell.

prices Q_1^- on the lower dashed curve. The same goes for player 2, subscript 1 being replaced with 2. In the given representation player 1 buys gasoline to fulfill his market demands.

It must be realized that the actual program is much more complicated than these when we consider more than 2 players. The above have been a simplified abstraction to highlight the important features. Proceeding with given values for decisions and initial states, the STRATCOM program computes year by year for each player (1) the investment levels ($\delta M_i(t+1) = M_i(t+1) - M_i(t)$ and $\delta K_i(t+1) = K_i(t+1) - K_i(t)$), (2) sales ($V_i(t)$) and production levels (C_i), and (3) the financial balance including new loans ($\delta B_i(t)$).

Limits are imposed on the model for economic reasons and calculational considerations in order to achieve realism. The retail gasoline price is limited to

$$P_i(t) \leq \text{Constant}$$

for otherwise a cooperative strategy by all the players will place no bounds on the price. In reality consumers' reaction, legal and ethical restrictions, threat of entry set the limits. For a similar reason bulk gasoline prices are kept at levels comparable to actual situations. The interest rate is based on the policy of setting a debt capacity of

the company. A linear relationship between debt and total capital investment is assumed. For the model,

(4-8) $B_i(t) \leq k(K_i(t) + M_i(t))$ where k is a constant.

Finally, the bond interest rate r_i is taken to be a function of the debt - capital ratio. Thus,

(4-9) $$r_i = f\left(\frac{B_i(t)}{K_i(t) + M_i(t)}\right)$$

For each set of decisions the resulting financial balances can be compared, the criterion being to maximize the

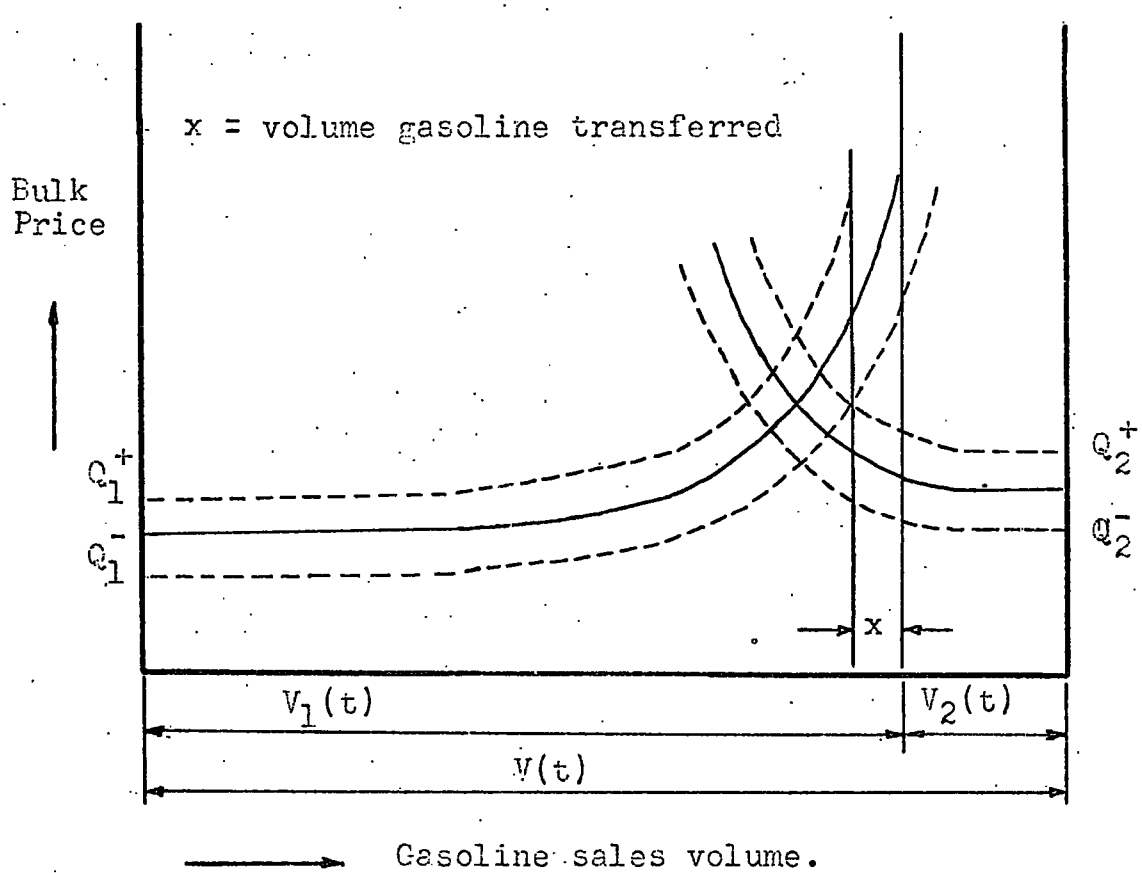

Figure (4-2): Simplified Graphical Representation of Bulk Algorithm

total discounted cash flows over the specified time horizon (in this case 10 years). In each year the cash flow equation for ith player is given by,

$$(4\text{-}10)\quad Z_t = P_i(t)V_i(t) + \Delta B_i(t) - \Delta M_i(t+1) - \Delta K_i(t+1) - r_i B_i(t) - \gamma C_i(t) \pm xQ^{\pm} + (\text{profit from sale of by products})$$

where

$C_i(t)$ = the crude production

γ = crude oil cost

Q_i^+ = price margin for selling bulk

Q_i^- = price margin for buying bulk

x = the bulk bought or sold as determined by the Bulk Algorithm

From the yearly cash flow the total discounted cash flow (R) is determined, a discount rate ρ related to the cost of capital being used. Thus with the initial parameters known, the values of the decision variables chosen, for the ith player the total discounted cash flow can be represented for an n-player situation by a function,

$$(4\text{-}11)\quad R_i[M_1(0),K_1(0),B_1(0),\ldots,M_i(0),K_i(0),B_i(0),\ldots,M_n(0),K_n(0),B_n(0),\ldots,\Delta M_1,\Delta K_1,Q_1^+,Q_1^-,\Delta B_1,P_1,\ldots,\Delta M_i,\Delta K_i,Q_i^+,Q_i^-,\Delta B_i,P_i,\ldots,\Delta M_n,\Delta K_n,Q_n^+,Q_n^-,\Delta B_n,P_n]$$

If for each player we can summarize the initial states and decision variables by a vector X as was done in chapter III, section 2, then (4-11) becomes,

$$R_i(X_1, X_2, \ldots, X_i, \ldots, X_n)$$

For a 2 player situation

Player 1's payoff becomes $R_1(X_1, X_2)$

Player 2's payoff becomes $R_2(X_1, X_2)$

The Alternate Play Algorithm[19] in the program searches for an equilibrium point following the steps outlined in equation (2-9) of chapter II.

Having obtained the game theoretic points it would be interesting to analyse their significance and usefulness in decision making. Before entering into such a discussion the B.C. gasoline market will be examined so as to provide the necessary background.

4. The B.C. Gasoline Market Structure

The petroleum market in British Columbia is dominated by seven major firms, two of which are really the marketing

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19. For details of this, one is referred to the paper "A Heuristic Algorithm for Calculating Equilibrium Points in Competitive Situations", by Professor J. Swirles presented at the Canadian Operations Research Society Annual Meeting in Toronto, May 10, 1968.

subsidiaries of two of the majors. In effect the number of separate entities engaged in the market is five. Minor firms in B.C. have recently taken on a primary importance in the major metropolitan areas.[20] B.C. is attributed with a larger number and variety (both in size and location) of refining facilities than would be expected in a market of its size. As a result it has refining capacity in excess of its immediate needs. In addition refinery outputs from Alberta and the U.S. have also entered the market.[21] Six refineries with varying capacities serve the market. Despite the advantage of economies of large scale, high transportation costs for both crude petroleum and refined products and limited size of particular markets make it favorable to operate smaller refineries with higher operating costs. Considerations of these kinds can be optimized by an analysis using linear programming techniques. The Shell model described provides only the size of the manufacturing investment to be undertaken.

Since 1955 the shares of major brands in the retail market have seen a decline.[22] These changes have taken

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20. "An Analysis of Competition and Price Behavior in the British Columbia Petroleum Industry," prepared for Imperial Oil Limited, Toronto, Ontario, Canada, by Stanford Research Institute, Menlo Park, California, May 1964, p. VII-35. Most of the observations in this section are derived from this study.

21. Ibid - p. VII-34.

22. Ibid. p. VII-14.

place amidst a period of general expansion. Influx of minor and independent brands have accounted largely for this. During this period the independents have invested in 18.6% of the total number of service stations built.[23]

The entire province of B. C. may not be representative for the whole Canadian or U. S. markets, its regional character is sufficiently different to require separate analysis.

Effective gasoline retailing to combat effective competition is most accentuated around the highly populated areas of Vancouver and Vancouver Island. Studies and data have shown that increase in outlets in these high volume markets have not matched the general pattern of increasing sales. This time lag could be regarded as an area worth exploring and exploiting by oil companies looking for profitable ventures and by researchers seeking problems for application of operations research techniques.

The variations in market shares among competing firms between regions is an observation worth noting. Imperial Oil Limited with the highest province wide average market share does not command the same percentage in the major populated areas as it does in the Northern and Southern Interior of B. C. Major companies have experienced substantial de-

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23. Ibid. p. VII-14.

clines in the city areas.[24] By contrast minor brands with a lower province wide average have gained considerable prominence in the metropolitan markets around Vancouver and Vancouver Island. This diversity and instability in ranking of various brands among regions over time is further evidence of the need to cope with market variety and to construct effective analysis of the impact of structural changes on company policies.

Prior to 1955 the majors accounted for all the outlets in B. C. By 1963 marketing outlets for minor and independent brands have become significant. The threat of entry of these independent brands and their impact on the gasoline retailing strategies of the majors are areas of worthwhile investigation and research for students of game theory.

In the Stanford Research Institute Study[25] the small firms are found to experience rapid increases in market shares against the declines of their major counterparts. This is apparently in agreement with the situation Hughes and Ornea[26] constructed in their paper to demonstrate game theoretic calculations. Here extensive marketing investment is not attractive to large size firms.

The structure of the petroleum industry which we have

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24. Ibid. p. VII-20

25. Ibid. p. VII-34

26. op. cit.

just outlined could provide a suitable framework for an analysis of competitive choice and the development of effective marketing policies of a firm.

4. Applicability of the Shell Model

The model, based on the framework discussed in the previous section we like to see run on the B.C. gasoline market and possibly other industrial situations should be able to handle cases involving multiperson situations. The five majors including the two marketing subsidiaries and the group of independents could be characterized as a 6-player game. The extent of integration and retail price of gasoline have been cited as distinguishing factors between majors and independents.

Assuming that the model constructed is feasible in terms of producing calculable results, and that for the output we are given a set of equilibrium strategies for each player, the purpose then as keynoted in the thesis is to draw attention to what possible use can be made of these results. To various extents we have illustrated the type of analysis that was done in Hughes and Ornea's paper. At this point we would like to reiterate our belief that in the near future the approach provided by game theoretic analysis to elaborate practical problems concerning competition will surpass sophisticated techniques currently in use. However, the answers to the questions of when and how we

should "take-off" after results provided by a model similar to that done at Shell have been obtained remain to be determined.

Let us return to the simple example in Chapter I and stretch it to bring out this point. Assume the gasoline market share distribution that prevails in B.C. is such that the leading major (Imperial Oil Limited) holds 30 percent of the total, the rest of the majors hold 15 percent each while the independent group holds the remaining 10 percent. Our concern then becomes one of viewing the market from the point of a firm in the major class but excluding that of the leading major. Given that the relevant data have been obtained, a computer simulation on a model similar to the Shell model but modified to suit conditions and explicit policies of the company is conducted. If feasible results from the computer run are produced, should we then adopt them as policies or strategic guidelines, or should we employ strategies other than the recommendations of the equilibrium strategies? In short how do we use these results. We have to some length discussed the implications of game theoretic points to decision making in Section 4, Chapter II. Some of these concepts could be the basis for formulating our initial take-off, but they are by no means the all-inclusive answers. Even then fundamentally we are still faced with a series of unresolved questions as to what actions we should ultimately choose in order to be

most well off in the medium and long run future. Of course we could explore possible immediate gains.

Coming back to the example, suppose that the market has at present 1000 service stations and is predicted that the volume of gasoline consumption is likely to grow in the next 2 years by 20 percent. It is noted that each year the existing stations suffer a 10 percent deterioration rate. This means that current investment would not only have to take into consideration new market opportunities but also the rebuilding of older investments. The estimated number of service stations required to fill this new volume is taken to respond proportionally to the increase, that is to 1200 stations. Under these circumstances, the question any management would like to see answered is:"Given that a computer run on the relevant data recommends an investment of 30 stations, should management accept these results or should they take alternative actions based on the picture of the resulting situation which the output of the computer program provides?"

Some idea of the courses of action to look for could be approached by examining the influence of threat points and Pareto surface on the decision. To achieve outcomes on the Pareto surface we observed in Chapter II that some form of cooperation or collusion may have to be obtained. Threat strategies may in all eventuality keep everyone on the status

quo or if carried out will result in losses or less gains on the part of the competitors. There may be some opportunities for a little edging in on the shares of other competitors. It is here that we hope to discover, through the methodology of game theory, the best line of penetration or defence.

In a simple one decision variable of a one period constant sum model the normative rule of game theory prescribes the maximin solution. As the model becomes non-constant sum and more complex with additional variables and extending over many periods this rule does not apply and will not necessarily yield equilibrium solutions. Thus it appears that the question of how we can use the results provided by a model such as that done at Shell remains to be resolved. The thesis, in its effort to find applicability for the use of game theory in the development of competitive models, has reached such an impasse. It is therefore hoped that further work in this direction would be carried out. Such contribution would be most welcomed by pragmatists. Only thus can the technique of game theory methods find a place in management decision aids as others such as linear programming have currently found.

A few general remarks could be made about the Shell model. Indeed it can be attributed as a significant contribution in the field of operations research and management science. Much have been said and views have been made regularly in learned journals regarding possible use of game theory to practical

business situations but very little have actually been done. The main areas seemed to be those in psychology, where simple matrix games have been designed to test human behavior. The Shell model could be regarded as one of the first attempts at modeling competitive business situations. It represents a cornerstone from which more sophisticated and more realistic models can be constructed. However, as it now stands, it is not without shortcomings. Most of these have been recognized by Hughes and Ornea. These are associated with the deterministic nature of STRATCOM. More realistic models should take into account uncertainties which could as a first approximation be taken care of in an elaborate sensitivity analysis. Clearly the model has not reached a stage of refinement for full-proof application to real situations. The inadequacies as demonstrated earlier and its simplicity and deterministic character are not really limits to its eventual fruitful use in dynamic competitive situations because the model can be enlarged to take into account such factors. What it leaves us with, is the sort of results and comparisons which might be made with a more accurate model.

CHAPTER V

SUMMARY AND CONCLUSIONS

While it is recognized that in the past progress in the use of game theory for business applications has been relatively insignificant, its position as a management tool for the effective analysis of competitive business situations in the future will no doubt improve. This has been largely due to advances in high speed computer technology. The increasing complexity of the business environment is demanding a totally new way of examining the forces at play within and without the firm. Efforts have been expended in showing the nature of such situations in Chapter I where the general conclusion reached is that there is a need for a new and effective tool to solve the type of strategic problems which modern management faces.

In Chapter II the direction pursued is one mainly of exposition. Though many books and papers have dealt in some ways with various aspects of game theory, they are either too rigorously mathematical in nature or they lack of sufficient depths to be useful for a person seeking operational principles. An exception, however, is the book "Strategy and Market Structure" by Martin Shubik [1]. This chapter in effect

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1. Op. cit.

attempts at giving a bird's eye view of the various aspects and concepts of game theory and especially to highlight important relationships. It is oriented from a practical point of view and examples from the next two chapters draw considerably from the principles enunciated. In particular, we saw how game theoretic points influenced strategic decision making. To extend an understanding of game theory beyond simple concepts, and in view of a desire to find applications, we have demonstrated the kinds of results that could be obtained from an experiment of the type done at Shell [2].

That there is a complete and correct 2-person zero or constant sum game theory is the general concensus. Indeed this theory is so solidly founded that it is difficult to conceive that it could be incorrect. On the other hand as we have shown this is not the case with non-zero sum and n-person games. This, we have seen is related to the problem of coalition formation. But, with the increasing capabilities of computers such combinatorial problems could eventually be overcome. Furthermore our answer to a problem is only as accurate as the data themselves, so much so we find that it is often necessary in dealing with the subjective environment to make simplifying assumptions.

A number of critical comments have been directed at

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2. Op. cit.

the relevance of game theory. David W. Miller [3] had attributed the shortcomings of the theory to basic methodological considerations relating to the measurement of utilities, and some paradoxical difficulties with the concept of rationality. However, he did recognize that,

> ".... it might be assumed that for many kinds of competitive situations, dollar amounts, in one form or another, represents satisfactory measures of payoff." [4]

Besides, Hughes and Ornea[5] have pointed out that game theory per se assumes nothing. The rationality issue thus does not affect our results in any appreciable way. What game theoretic methods go out to find is the <u>best action for us regardless of what our opponents do</u>.

The object of going through the analysis of dynamic games in Chapter III is primarily to bring out conceptual difficulties in proceeding from single period to continuous games. The application of the processes of dynamic programming methods to a game situation was discussed. Discrete variable functions give way to continuous variable functions as we probe deeper into the dynamics of game situations. Two examples of industrial situations were taken to show the use of games as decision

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3. Miller, David W. "<u>The Relevance of Game Theory</u>", in "Models of Markets" edited by Alfred Oxenfeldt, Columbia University Press, New York, 1963, pp. 265-306.

4. Ibid.

5. Loc. cit.

tools in dynamic settings. In general it was found that the computation can be substantial and involving. The last section of the chapter shows how an on-going market situation can be characterized as a game of economic survival. Leaving aside social and other restraints, financial strength, as expected, is a major factor in determining who the survivors would be. The analysis also brought out the effects of threat strategies on equilibrium point solutions, which cover an increasingly larger region of the Pareto surface as the financial assets of the players or competitors become more symmetrical.

Our interest subsequently turned to the applicability of game theory to the oil industry with special implications to the situation in British Columbia. The overall structure of the industry and the kinds of strategic decision problems that arise at the various levels of the industry were examined. One of the most crucial we noted is in the area of marketing investment decisions. It thus appears that the situation calls for the sort of analysis provided by game theory. In this connection, the basic approach that could be applied is STRATCOM, the competitive decision model developed at Shell[6].

Some of the essential features of the model were discussed so as to indicate its relevance in such a practical situ-

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6. Op. cit.

ation. As to its applicability and potentiality for other industrial situations we noted that further modifications are required before any idea of implementation could successfully materialize. We are especially concerned here with the question of how the results provided by a model such as this should be used. Nevertheless, it is generally agreed that the Shell model represents one of the first few concrete attempts at providing the sort of results that are meaningful to management contemplating game theory for complex competitive decision problems. It must be said that even the ardent critic of game theory, D. W. Miller would agree to this by his statement that,

> "... A theory may very well deepen understanding by raising questions even if the answer offered by the theory, are for one or another reason, unsatisfactory." 7

Finally, it must be mentioned that while the thesis has not contributed to basic research other than the comments on some of the foremost literature dealing with practical applications of game theory, it is hoped, it has at least aroused an interest and opened up new efforts in seeking use for game theoretic methods.

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7. Op. cit. p. 267

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APPENDIX

1) $U_1 = f(q_1)e^{-a\tau} + \frac{C_1}{a}e^{-a\tau} - \frac{C_1}{a}$

$$\frac{\partial U_1}{\partial q_1} = e^{-a\tau} f'(q_1) + f(q_1)(-a)e^{-a\tau}\frac{\partial \tau}{\partial q_1} + \frac{C_1}{a}(-a)e^{-a\tau}\frac{\partial \tau}{\partial q_1}$$

Given that $(q_1 + q_2 - M)/r_2 = \tau$

$$\frac{\partial \tau}{\partial q_1} = \frac{1}{r_2} \qquad \frac{\partial^2 \tau}{\partial q_1^2} = 0$$

For $\frac{\partial U_1}{\partial q_1} = 0$

$$f'(q_1) - f(q_1)\frac{a}{r_2} - \frac{C_1}{r_2} = 0$$

or $\left[f(q_1) + \frac{C_1}{a} \right] \frac{a}{r_2} = f'(q_1)$

To satisfy maximum conditions $\frac{\partial^2 U_1}{\partial q_1^2} < 0$

$$\frac{\partial^2 U_1}{\partial q_1^2} = e^{-a\tau} f''(q_1) + f'(q_1)(-a)e^{-a\tau}\frac{\partial \tau}{\partial q_1} - af'(q_1)e^{-a\tau}\frac{\partial \tau}{\partial q_1}$$

$$-af(q_1)(-a)e^{-a\tau}\left(\frac{\partial \tau}{\partial q_1}\right)^2 - af(q_1)e^{-a\tau}\frac{\partial^2 \tau}{\partial q_1^2}$$

$$C_1 a e^{-a\tau}\left(\frac{\partial \tau}{\partial q_1}\right)^2 - C_1 e^{-a\tau}\frac{\partial^2 \tau}{\partial q_1^2}$$

This reduces to

$$f''(q_1) - \frac{a}{r_2} f'(q_1) < 0$$

$$\frac{f''(q_1)}{f'(q_1)} r_2 - a < 0$$

2) $$f'(q_1) - \frac{a}{r_2} \left[f(q_1) + \frac{C_1}{a} \right]$$

Rearranging,

$$r_2 f'(q_1) = af(q_1) + C_1$$

Differentiating with respect to time t

$$r_2 f''(q_1) \frac{dq_1}{dt} + \frac{dr_2}{dt} f'(q_1) = af'(q_1) \frac{dq_1}{dt}$$

Writing $\frac{dq_1}{dt} = \dot{q}_1$ and $\frac{dr_2}{dt} = \dot{r}_2$

This reduces to after rearranging to,

$$\dot{q}_1 = - \frac{\dot{r}_2}{\frac{f''(q_1)}{f'(q_1)} r_a - a}$$