AN ANALYSIS OF THE
COST OF CAPITAL HYPOTHESES

by

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ABSTRACT

The cost of capital has received much theoretical and empirical study in recent years. Two contradictory views have emerged concerning the effect of capital structure on the cost of capital. Writers such as Modigliani and Miller maintain that the cost of capital is independent of the relative proportion of liabilities to owners' equity and depends only on the risk associated with the type of business the firm is in. The opposite view is taken by those writers who support what is known as the traditional view. These writers maintain that judicious use of debt can reduce the firm's cost of capital.

The purpose of this paper is to determine which approach appears to be the more accurate in a real world situation. We first investigate the many difficulties associated with the empirical tests which have been applied to evaluate the two conflicting hypotheses. The problems associated with these tests lead us to reject them as a means of resolving the cost of capital controversy. Instead, we choose a theoretical approach. Based on suggestions made by Modigliani and Miller and by the traditional writers, we postulate the way in which debt and equity capitalization rates are expected to respond to increases in the amount of debt in the capital structure. The Modigliani and Miller hypothesis and the traditional hypothesis are studied in detail and computer models for each hypothesis are then developed. The hypothe-

tically determined capitalization rates are used as independent variables in the models to develop relationships between the dependent variables and the debt-equity ratio. The response of the dependent variables to changes in leverage is studied to see if it represents rational investor behavior.

Real world factors are introduced into the analysis. Effects of corporate income tax on both the Modigliani and Miller hypothesis and the traditional hypothesis are investigated. In addition, we study the way in which legal restrictions and limited personal liability may restrict the Modigliani and Miller arbitrage process.

The three computer models used in the analysis are described in detail in the appendices. These models have been developed so as to be as flexible as possible. All parameters are specified by the user, so the models may be adapted to a variety of situations. Program listings and the output used in our analysis are also included.

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CHAPTER I

INTRODUCTION

The cost of capital has received a great deal of both theoretical and empirical attention in recent years. However, the many studies, instead of providing a precise and workable definition, have resulted in much controversy. That this controversy warrants further study can be seen from the following definition, which shows the importance of the cost of capital in finance.[1.]

> "The cost of capital for a firm is a discount rate with the property that an investment with a rate of profit above (below) this rate will raise (lower) the value of the firm."

Although the cost of capital is easily defined, it is very difficult to determine.

In part this difficulty results from the fact that the finance function is composed of three closely interrelated problems:[2.]

> "1. How large should an enterprise be, and how fast should it grow?
>
> 2. In what form should it hold its assets?
>
> 3. What should be the composition of its liabilities?"

We observe that the cost of capital which determines the cut off point for future asset expenditures is itself influenced by the type and amount of assets purchased. As Gordon states, the cost of capital is not a constant but "a function of the level of the firm's investment with the parameters of the function depending on the firm's dividend rate, debt-equity ratio, rate of return on investment, and/or other variables."[3.]

A major difficulty in determining the cost of capital for use as an investment decision criteria results from the fact that it does not compensate for the uncertainty attached to the possible returns from the firm's assets. Authors such as Cord[4.] and Paine[5.] have studied ways in which risk may be introduced, and have found that in many cases, projects with expected returns which exceed the estimated average cost of capital should be rejected due to abnormal risk, while other projects with low or negative rates of return should be accepted because they reduce overall corporate risk. They have determined an asset protfolio which provides the optimum combination of return and risk.

Many authors have also studied the companion problem of the optimum composition of liabilities and owners' equity with which to finance the asset portfolio. It is in this area that much controversy has arisen. Writers such as Modigliani and Miller[6.] characterize a group which maintains that no matter what combination of debt and equity the firm has in its capital structure, the cost that the firm pays to acquire its capital is a constant. Many other traditional writers, in a group of which Solomon[7.] is representative, maintain that the judicious use of debt can reduce the firm's average cost of capital and therefore reduce the cutoff rate for investment project acceptance. It is the question of whether or not the average cost of capital can be reduced through the use of debt that we wish to study in this paper. We will study this problem in isolation, but before proceding, it should be noted that an optimal capital structure for the firm can only be determined through a consideration of its assets. As Johnson indicates, "there are certain instances where a major investment will change the entire cost of capital function for a firm. A major investment in jet aircraft apparently lowered the break-even point on the airlines, with a consequent reduction in their risk class, even though they remained in the same type of business."[8.]

If we are to disregard the asset side of the firm's balance sheet, our analysis must then be made by considering all investment to be in assets which are similar in risk to those already held by the firm.

Types of Risk

As Schwartz has noted, the individual firm faces two types of risk.[9.] The first of these is the external or business risk. This is determined from the stability of earnings of the firm, and the safety, liquidity and marketability of its assets. The firm has no control over this type of risk since it is dictated by the type of business the firm is in and is not subject to any control by the financial decision makers. Business risk can be excluded from our analysis by considering capital structures only for firms in what Modigliani and Miller have called "equivalent return" classes[10.] or in what Wippern has called equivalent risk classes.[11.]

There are many ways in which these equivalent return or risk classes can be defined. A standard statistical method would be to include all firms in the same risk class if they have similar coefficients of variation for income, where the coefficient of variation is measured by the standard deviation of income divided by the mean expected income. It should be noted that this method measures variation relative to size, so firms of greatly differing size can be included in the same risk class. Modigliani and Miller use an alternative but similar definition of risk class. They view the firm as yielding a stream of profits over time, but the elements of this stream are uncertain and extend indefinitely into the future. However, the mean value of the stream over time is finite and represents a random variable which can be described by a subjectively assigned probability distribution. Then, the average value over time of the stream of income accruing to the firm is the annual return to the firm, and the mathematical expectation of this average is the expected annual return.

The firm generates an income stream,

$$X(1),\ X(2),\ldots\ldots\ldots\ldots\ldots\ldots X(T),$$

whose elements are subject to the joint probability distribution,

$$\mathcal{X}_i\ \left[X(1),\ X(2),\ldots\ldots\ldots\ldots X(T)\right]\ .$$

Average annual return to the firm is:

$$X = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} X(t)$$

so the expected annual return is:

$$\bar{X} = E(X)$$

Then for firms in any given equivalent return class, the ratio of the annual return to the expected annual return will be a constant, ($\frac{X}{\bar{X}}$ = constant). All shares in any expected return class are equally desirable from the point of view of business risk.

The other type of risk for the firm is the internal or financial risk which depends on its capital structure. This risk depends on the proportion of fixed commitment liabilities to equity capital. In this paper, we will deal with firms in a given risk class, and see if there is some combination of debt and equity which will result in a maximum value for the firm, and, therefore, a minimum cost of capital.

Models of the Firm

There are two types of models which can be developed to study the effect of leverage on the cost of capital. In the first model, the company substitutes debt for equity. This is the type of model presented by Mao.[12.] In this case, the company is considered to have a fixed group of assets which yield earnings of a given risk class. Initially, the company has only equity in its capital structure. The owners issue debt and retain the proceeds, thereby keeping the earnings and assets constant. That is, the company issues debt and uses the proceeds to redeem outstanding stock held by the owners. The advantage of this model is that it keeps the assets constant, and so allows a direct comparison of the value of the firm at varying levels of debt.

The second type of model is the one used by Solomon.[13.] In this model, as debt is issued, it is used to acquire additional assets, so the company is allowed to expand. This model permits the ready identification of the marginal cost of each increment of debt. However, to derive useful results, we must assume that the new assets purchased produce earnings of the same yield and quality as the original assets.

For our work, we shall use a model of the first type. As has been previously noted, an expansion of assets can easily change the risk class of the firm, so ideally assets should be kept constant. Also, this model is the type used by Modigliani and Miller, whose hypothesis we want to investigate.

The models to be developed will be based on the following assumptions:

1. The company has a fixed group of assets on which it earns a constant rate of return of a given quality. Equivalently, the company has expected earnings of $\bar{X}$, which belong to a given risk class. This means that we are not considering business risk in our analysis. We determine how changes in leverage, and therefore financial risk, can influence the value of companies with earnings of a given business risk.

2. The company is considered to have only two sources

of funds: long term debt and equity.

3. The structure of market capitalization rates is given and does not change over time. The debt and equity markets establish capitalization rates to apply to firms in a given risk class.

In determining these capitalization rates, the markets will consider such factors as variability in earnings, length of time the firm has been in business, the nature of the business, their estimations for further success, and the ease with which securities can be traded on the market. Since all these factors have been considered in assigning capitalization rates to earnings of a given risk class, the capitalization rate within a particular risk class should be a function only of leverage. That is, by working within a given risk class, we have extracted all business risk from our analysis and are dealing only with financial risk. This financial risk is then the risk of default on debt. If the ratio of equity to debt is high, the interest coverage within a given risk class will also be high. Also, in the case of default, the asset value in relation to the claim from senior securities is large. Then the capitalization rates for debt and equity within any given risk class should be a function only of the relationship of debt to equity within the firm. In our studies, we shall use capitalization rates which increase as the absolute amount of debt in the capital structure increases. Functions of this type are similar to those in which the capitalization rate is a positive function of the debt-equity ratio or the ratio of debt to total value. In the models to be developed, debt is used to replace equity so any increase in debt results in a decrease in equity. Therefore, positive functions of the amount of debt are equivalent to negative functions of the amount of equity, the equity to debt ratio, or the ratio of equity to total value.

The Dividend Effect

It has been shown that the debt and equity capitalization rates should be functions only of the amount of debt in the capital structure for earnings of a given risk class. Then could two firms in the same risk class with the same capital structure have different equity capitalization rates due to a difference in dividend policy? The controversy surrounding the dependence of the equity capitalization rate on dividend policy is almost as great as that surrounding the effect of capital structure on the cost of capital. While it is not the purpose of this paper to resolve the dividend controversy, some indication will be given at this time as to why we think equity capitalization rates should be independent of dividends. That is, we propose that <u>expected</u> future earnings are the source of the value of a stock, so the cost of equity is measured by an <u>expected</u> earnings-price ratio.

Of those writers who regard dividends as being the determining factor of stock prices, the foremost is M. J. Gordon,[14.] who has provided both empirical and theoretical models to support his views. In his empirical findings, he uses regression analysis to show that share price is dependent on dividends, dividend growth rate, earnings variability, and corporate size. Unfortunately, empirical studies of this type reveal little about stock price dependence on dividends. It is _expected_ earnings and not present earnings that determine stock price. It seems likely that corporate dividend policy is more stable than present earnings and also reflects management's expectations of future earnings. Dividend policy is set by insiders who should have the best possible knowledge of the firm's expected future performance. Dividends therefore are a better surrogate for expected future earnings than are present earnings, which tend to fluctuate widely.

Gordon also provides a theoretical argument supporting his dividend hypothesis.[15] Since uncertainty increases with time, investors prefer a certain sum today to a larger, uncertain sum in the future. This leads Gordon to postulate that the rate at which dividends are discounted must increase with time. If this is true, then an increase in earnings retention will lead to an increase in the equity capitalization rate. Lintner follows an analysis somewhat similar to Gordon's.[16] He finds that, since uncertainty increases with time and since uncertainty is discounted by the investor, present dividend payouts should be increased. This results because "the relevant marginal cost of capital is not only greater than current earnings yields by amounts that increase with the size of the budget, but is necessarily rising at the optimum point."

Unfortunately, both Lintner and Gordon have combined the investment and financing decision. In their models, their only source of funds for further corporate expansion is from retained earnings. The company is prevented from issuing additional debt or equity, and so will be faced by a continually changing debt-equity ratio. In addition, the policies developed from their models would lead to eventual liquidation of all companies by establishing too high a cutoff rate for expenditure. What these authors have failed to do is to consider the opportunity cost of paid out dividends. If the investor shows a preference for dividends rather than earnings retention and growth, we must ask what he wants these dividends for. If he answers that he needs the funds for consumption, then he should not have bought the stock in the first place. If he wants dividends to purchase other stocks and diversify his portfolio, then he could probably accomplish this diversification more effectively by selling some stock. If the investor has an investment which yields him a higher utility than the stock, then instead of putting only his dividends in this investment, he should sell his stock also.

Many other authors have provided both empirical and theoretical work to show that any preference for dividends is irrational. Miller and Modigliani[17.] have shown that if we assume:

1. perfect capital markets in which all traders have equal and costless access to information about price and other data, and there are no brokerage costs, transfer fees

or tax incentives;

2. rational behavior in which investors prefer more wealth to less and are indifferent between cash payments and capital gains; and

3. perfect certainty in which every investor knows the future profit and investment programs for the corporation; then under these circumstances, we will get the equivalent value for a share by discounting cash flow, by using an investment opportunity approach, by discounting streams of dividends, or by discounting streams of earnings.

Moreover, they can extend these results to the case of uncertainty if every trader prefers more wealth to less wealth, regardless of the form this wealth may take, and believes that other traders behave this way also.

Lindsay and Sametz support the use of earnings in share evaluation.[18.] They state:

> "Earnings are the fundamental determinant of stock prices; the stockholder's preference for cash dividends versus capital appreciation depends primarily on his personal marginal income tax rate. But since stockholders tend to hold those stocks whose cash payout ratio fits their own desires, stock prices are influenced primarily by statistics on earnings, not dividends."

Their views support our contention that, if two firms with the same debt-equity ratio (same financial risk) and in the same risk class (same business risk) have different dividend payout policies, both stocks will sell at the same price, for each firm will be able to attract a group of stockholders that approves of its dividend policy.

Irwin Friend and Marshall Pũckett have presented both intuitive and empirical results to refute the dividend hypothesis.[19] They think that, since investors continue to buy shares at the prevailing market price, it is indicated that this price offers a rate of return at least as high as could be obtained from other investments of a comparable risk. Then, if investors are willing to buy and hold these shares, they should be indifferent if the present value of the additional future returns resulting from earnings retention equals the amount of dividends foregone. Also, there is a tax advantage favoring earnings retention as opposed to dividend payout.

They cite three behavioral assumptions necessary if retained earnings are to consistently receive a lower market valuation than dividends. These are:[20]

"1. The average holder of common stock possesses, at the margin of his portfolio, a very strong preference for current income over future income (a situation which could hardly be expected to persist over time.)

2. The expected increase in earnings arising from increased per share investment is viewed as involving a much higher degree of risk than that attaching to earnings on existing corporate assets.

3. The profitability of incremental corporate investment, as viewed by shareholders, is extremely low relative to the competitive yield prevailing in the stock market."

Since new stock, which implies the substitution of current for future income, can be issued at near market prices, this serves to refute the first two assumptions. Since marginal profit rates in most industries appear to be quite high, and in growth industries, incremental investment is highly profitable, the third assumption must also be false.

They note that in any statistical studies where price is regressed against both dividends and retained earnings, if the coefficients of these terms differ, then the payout position is not in equilibrium and stock price could be increased by increasing either dividends or earnings retention. Their empirical results show that, when other pertinent variables are included, the coefficients for price and retained earnings are nearly equal. In the models that follow, we will assume that the equity capitalization rate is independent of dividend payout, and depends only on leverage.

Limitations of Empirical Tests

To test the effect of leverage on the cost of capital, many empirical tests have been made. The first of these was made by Modigliani and Miller to support their contention that the cost of capital was independent of leverage. They defined the cost of capital as total earnings after tax divided by the market value of all securities, and found that this cost, when expressed as a function of the ratio of debt to total value, was independent of the debt in the capital structure. That is, if

$$\frac{X^t}{V} = a + b\frac{L}{V},$$

where: X^t is after-tax earnings,

V is total value of the firm,

L is the amount of debt in the capital structure, the value of b is not statistically significant.

One of the major problems in any statistical work of this type is to make certain that all data is for firms in the same risk class. In a recent paper, Wippern attempted to determine if "objective determinable risk classes exist? And do these classes correspond to industry groups?"[21.] As a measure of business risk, he used the variability of operating earnings per share for firms in eight industries, including oil, electric utilities, paper and rubber companies. Variability was measured by the antilog of the standard error around the logarithmic regression of annual earnings over a ten year period. His results showed that for the proxy variable chosen, there was as much variation within particular industry groups as there was among different groups. He could only conclude that "industry groups do not provide an adequate basis on which to insure homogeneity of basic business uncertainty."[22.]

The data used by Modigliani and Miller were for electric utilities and oil companies. As Fisher has noted, the electric utilities do not constitute a valid group on which to test the effect of financial risk on the cost of capital.[23.] These utilities are controlled by regulatory bodies which prevent them from maximizing profit. Then, if a decline in earnings were to occur, the regulatory bodies would relax their restrictions to allow earnings to return to a "fair" level. A public utility with the same apparent business risk or fluctuation in earnings as a manufacturing company would be much less likely to default on its debt. Also, as Barges points out, for the utility sample only 8 of the 43 utilities studied had debt to total market value ratios between 0% and 50%, and most of the observations were between 50% and 80%.[24.] Then, if the cost of capital curve was actually saucer-shaped, there would not be enough observations in the declining cost portion. The findings of Wippern and Barges indicate a fundamental problem in using empirical tests to determine the effect of leverage on the cost of capital.

If there is an optimal capital structure which results in a minimum cost of capital, we would expect all firms in the same risk class to have this capital structure. The fact that there is a wide range of capital structures for the samples from the oil industry probably indicates that these firms are not in the same risk class. Weston found that the oil companies in the risk class used by Modigliani and Miller included the following:[25.] "fully integrated oil companies, oil companies strong in refining, oil companies strong in distribution; some regional in their operations, some with heavy investments in troubled international regions; some with stable, assured or rising income from petrochemicals or uranium or other minerals." The oil companies, therefore, could not belong to the same risk class.

In the equation used by Modigliani and Miller in their empirical studies, V, which may be subject to random variation, appears in the denominator of both the dependent and independent variables. This tends to improve the correlation and bias the results against the traditional view.

If the firms in the sample used by Modigliani and Miller belonged to the same risk class, then we should only need to include some variable describing financial risk. Unfortunately, since at least those firms in the oil industry belonged to different risk classes, other variables such as growth and firm size should have been included. Weston found that when growth was included, the lack of any change in the cost of capital with changes in the capital structure was the result of the negative correlation between leverage and earnings growth.[26]

Finally, to refute Modigliani and Miller's empirical results, we should note that this data indicates that the "after-tax" cost of capital is independent of leverage. In a subsequent paper, they discover that due to corporate tax, the use of debt should actually result in a decreasing cost of capital, something their results do not show.

Modigliani and Miller undertook a second study for the purpose of determining the cost of capital in the electric utility industry.[27] They used two stage least-squares regression to express the value of the firm as a function of the tax deductability of the interest on debt, firm size, earnings and rate of growth. Their model allowed them to introduce debt as an additional explanatory variable and they found that the coefficient of the debt term was not statistically significant for any of the three years studied.

There are many reasons why the results of their tests are not useful in our analysis. As noted previously, the electric utility industry is not representative of industry in general. In fact, Wippern does not even consider it in his empirical investigation of the leverage effect.[28.] He states:

> "The electric utility industry, one that is most frequently chosen to test for capital structure effects, was excluded from this sample because it is believed to be an inappropriate group from which to draw conclusions regarding shareholder responses to financial risk. Interest charges are included among the expenses allowed by the commissions in determining electric utility rates. Further, the regulatory agencies appear to have a significant influence over the financial structure adopted by firms in this industry. It is, therefore, doubtful whether fixed commitment financing exposes the electric utility shareholder to financial risk in the same manner and/or to the same extent as the shareholder of a non-regulated firm."

Modigliani and Miller do not attempt to justify their choice of the electric utility industry as a test for their model. They note that:[29.]

> "corporate income taxes are deductible in computing the earnings allowed on the rate base. To the extent that tax is thus passed on, the ultimate value of the tax subsidy on interest is correspondingly reduced."

However, as Gordon observes, their model describes industries in which the before tax earnings is an exogenous random variable whereas the result of utility regulation is to make after tax earnings the exogenous variable.[30.]

Furthermore, as Modigliani and Miller admit, their objective is to estimate the cost of capital and not to test conflicting views about the effects of leverage on valuation. They do not want a precise estimate of the leverage effect, but only want to make certain that leverage will not significantly influence their results. From the results of the mathematical models of Chapter V, the effect of leverage on the cost of capital is not great. This is particularly so for relatively safe industries such as regulated utilities where the required debt yield should increase only slightly if at all to compensate for the increased financial risk of leverage.

We must reject the results of this test made by Modigliani and Miller. They have considered an industry which is far from being typical. Business and financial risk is almost non-existent due to regulation. Moreover, the capital structure is controlled by the regulatory agency so that the range of capital structures required to test the cost of capital hypothesis can not be found in the electric utility industry.

The empirical studies done by those espousing the traditional view are also of limited usefulness. Weston has analyzed Modigliani and Miller's data, and after including terms to compensate for growth and size, he found that these data actually supported the traditional view. However, as noted earlier, the electric utilities do not constitute a valid sample and the oil companies are not a homogeneous risk class, so the data cannot be considered as conclusive.

Barges[31] used railroads, department stores and cement companies to test the effects of leverage on the cost of capital. His data for railroads indicated that the cost of capital was a saucer-shaped curve. Unfortunately, most of his effort was devoted towards determining how the equity capitalization rate changed with leverage. Data from these tests can be of only limited usefulness in choosing between Modigliani and Miller's approach and the traditional approach to the effect of leverage on the cost of capital.

Ronald Wippern has tried to avoid the problem of risk class determination by including business and financial risk in a single variable. He suggests that the subjective probability distribution of future returns can best be determined from a study of past performance. He determines the logarithmic regression of income on time for a ten year period. Then, knowing the standard error for the regression line and the current income predicted from the regression line, he determines the minimum expected income within a given confidence interval. By comparing the fixed interest charges which are a function of leverage to the minimum expected income, he obtains a proxy uncertainty variable which includes both business and financial risk. The results of his study tend to suggest that effective use of leverage can reduce the cost of capital beyond the advantage provided by tax deductability of the interest charges. However, future business risk is a function of much more than past earnings fluctuations. Also, current fixed interest charges may differ greatly from future expected interest charges. Until all investigators can agree on exact measures of financial and business risks, any conclusions based on arbitrary risk parameters will remain suspect.

We do not think that empirical tests can be developed which will determine whether the average cost of capital is a function of leverage. We have noted that it is almost impossible to get data for a homogeneous risk class which is necessary if we are to exclude business risk and study only the influence of financial risk. The yield on equities will always be influenced by market imperfections. Institutional restrictions and imperfect knowledge on the part of investors will prevent unlisted or unknown stocks from selling at as high a price as listed stocks, even though they may belong to the same risk class. Since restricted lists exist for banks and other institutions, certain stocks will have their prices bid up. If the investor is irrational enough to be influenced by dividends, stocks in the same risk class could sell at different price-earnings multiples. In gathering data for empirical tests, measurement errors may result. We have no way of determining expected earnings for a growth stock, so any capitalization rate calculated from current earnings and price will be low. Due to the many problems involved in empirical testing, we think that the cost of capital controversy can be resolved only from a theoretical analysis of the behavioral characteristics suggested by the models developed by Modigliani and Miller and the traditionalists. In the following chapters we shall develop each model and examine the type of behavior required from investors if the models are to be correct.

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CHAPTER II

THE MODIGLIANI AND MILLER HYPOTHESIS

Assumptions

Modigliani and Miller[1.] propose that the average cost of capital to the firm is independent of the firm's capital structure. They maintain that the real cost of debt to the firm after taking into account the increase in the equity capitalization rate resulting from increased leverage, is such that the marginal cost of capital must be equal to the average cost of capital. This implies that the total market value of a firm is unaffected by the composition of its capital structure.

In their initial model, Modigliani and Miller make the following assumptions:

1. They first assume that all firms can be grouped in homogeneous risk classes. As shown in the introduction, this implies that investors regard income from all firms in a given class as equally risky. Although the absolute amount of income may differ (the firms may be of different sizes), the variation in income as compared to mean expected income is equal throughout the class, so a uniform capitalization rate, Ko,[2.] may be applied to the earnings of all firms in the risk class.

2. All investors have assigned the same subjective probability distribution to the returns of a particular firm

in a given risk class. This means that all present and future investors will have identical estimates of the expected average income, $\overline{X}$.

3. All stocks and bonds are traded in perfect capital markets. As defined by Modigliani and Miller,[3]

> "in perfect capital markets, no buyer or seller (or issuer) of securities is large enough for his transactions to have an appreciable impact on the then ruling price. All traders have equal and costless access to information about the ruling price and about all other relevant characteristics of shares. No brokerage fees, transfer taxes or other transactions costs are incurred when securities are bought, sold or issued, and there are no tax differentials either between distributed and undistributed profits or between dividends and capital gains."

Since markets are perfect, investors are assumed to be able to borrow unlimited amounts at the same borrowing rate faced by corporations.

4. In the first model developed by Modigliani and Miller, they do not consider corporate income tax. This means that any tax incentive resulting from debt financing is neglected.

Working within this framework, we will now develop the arguments put forth by Modigliani and Miller. We will find that their model is consistent with the assumptions made, but when real world characteristics such as corporate taxes, limited liability, and investor attitude toward risk are introduced, the model will lead to paradoxical results.

Throughout our analysis, the following symbols will be used. Since all models may be considered as describing different firms in the same risk class but with a different composition of debt and equity, or as describing the same firm with earnings characteristic of a given risk class but with a changed debt-equity ratio, no subscripts will be used to refer to different firms.

$\bar{X}$ = expected annual return on assets, where $\bar{X}$ is determined as in the introduction.

L = amount of debt in the capital structure (at market value.)

S = amount of equity in the capital structure (at market value.)

$V = L + S$ = total market value of the company.

K_i = debt capitalization rate. It is determined by the debt market and is assumed to remain constant over time. Since it is a value assigned to a particular risk class, it can only be a function of the capital structure of firms within the risk class to which it is assigned.

K_e = equity capitalization rate. It is assigned by the equity market to equity of firms within a particular risk class, so is a function only of capital structure. It is assumed to remain constant over time.

Y = return to investor from a given investment portfolio.

K_o = average cost of capital to firms within a particular risk class. (It replaces ρ in Modigliani and Miller's analysis.)

A complete list of symbols is given in Appendix I.

Proposition I

Consider any firm in a given risk class and having earnings $\bar{X}$. Then the value of this firm is given by:

$$V = L + S$$

$$= \frac{\bar{X}}{K_o} \quad (1)$$

Modigliani and Miller's Proposition I states that V, "the market value of any firm, is independent of its capital structure and is given by capitalizing its expected return at the rate, K_o, appropriate to its class."[4.]

Alternatively stated,

$$\frac{\bar{X}}{L + S} = \frac{\bar{X}}{V} = K_o, \tag{2}$$

so for firms in any given risk class, "the average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalization rate of a pure equity stream of its class."[5.]

Working within their assumptions, they prove Proposition I by showing that if two firms in the same risk class, but having different degrees of leverage in their capital structure, have different values, then through what they call arbitrage, shares will be bought and sold until, at equilibrium, the firms will have the same value.

Suppose both Company 1 and Company 2 are in the same risk class, and have equal expected earnings, $\bar{X}_1 = \bar{X}_2$. Now suppose Company 2 is levered and has a value higher than unlevered Company 1. An investor having an amount of shares, s_2, in Company 2 and therefore owning a fraction of the company, $\alpha = \frac{s_2}{S_2}$, will receive the following return from his portfolio:

$$Y_2 = \alpha(\bar{X} - K_i L_2) \tag{3}$$

If he wishes to sell his shares in Company 2, he would receive an amount, αS_2. Before he can invest in the unlevered company, if he wishes to keep his financial risk in Company 1 the same as it was in Company 2, he must acquire some personal debt. Now, by pledging his new holdings in Company 1 as collateral, he will borrow an amount, αL_2. This means that through his "homemade leverage" he has preserved the same debt-equity ratio he had in Company 2. That is, his new investment has the same financial risk as his old one. With the proceeds from his sale of stock plus his personal borrowing, he can purchase an amount $\alpha(S_2 + L_2)$ of the shares of the unlevered Company 1. He now owns a fraction:

$$\frac{s_1}{S_1} = \frac{\alpha(S2 + L2)}{S_1}$$

After he pays the interest on his personal debt, his total earnings from this new portfolio are:

$$Y_1 = \frac{\alpha(S_2 + L_2)}{S_1}\bar{X} - K_i \alpha L_2$$

$$(4) \qquad = \alpha \frac{V_2}{V_1}\bar{X} - K_i \alpha L_2$$

Comparing equations (3) and (4), this arbitrage process will remain profitable as long as $Y_1 > Y_2$. Arbitrage will only stop when $Y_1 = Y_2$, and at this time V_1 will equal V_2. Modigliani and Miller conclude that levered companies cannot command a premium over unlevered companies <u>because</u> investors can use personal leverage as a perfect substitute for corporate leverage.

Now consider the possibility that the levered Company 2 has a lower value than the unlevered Company 1. That is, $V_2 < V_1$. Now an investor will find it profitable to "switch" the shares of Company 2 for his shares of Company 1.

His return from his holdings in Company 1 is:

$$Y_1 = \frac{s_1}{S_1}\bar{X} = \alpha\bar{X}. \tag{5}$$

If his financial risk is to remain constant, he must purchase an amount of equity,

$$s_2 = \frac{S_2}{V_2} s_1 ,$$

and an amount of debt,

$$l_2 = \frac{L_2}{V_2} s_1 .$$

This will give him a claim to the same share of earnings in Company 2 that he had in Company 1. By acquiring this mixed portfolio, the investor "undoes" the leverage of the firm. Modigliani and Miller state that "it is this possibility of undoing leverage which prevents the value of levered firms from being greater than that of unlevered firms."[6.]

The return from his new holdings of equity and debt in Company 2 is now:

$$Y_2 = \frac{s_2}{S_2}(\bar{X} - K_i L_2) + K_i l_2$$

$$= \frac{s_1}{V_2}(\bar{X} - K_i L_2) + K_i \frac{L_2}{V_2} s_1$$

$$= \frac{s_1}{V_2}\bar{X} = \alpha\frac{S_1}{V_2}\bar{X} \tag{6}$$

Comparing equations (5) and (6), if V_2 is less than S_1, then it will pay the stockholder to sell his shares in Company 1 and replace them with the debt and equity of Company 2. Only when the value of the two firms are equal ($S_1 = V_2$, where firm 1 has only equity in its capital structure) will the arbitrage stop.

Proposition II

Proposition II states that "the expected yield of a share of stock is equal to the appropriate capitalization rate, K_o, for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between K_o and K_i."[7.] That is,

$$(7) \qquad K_e = K_o + (K_o - K_i) L/S$$

This is established as follows. The equity capitalization rate can be given by:

$$(8) \qquad K_e = \frac{\overline{X} - K_i L}{S}$$

But from Proposition I, equation (1), we know that

$$\overline{X} = K_o(S + L)$$

Making this substitution for $\overline{X}$ in equation (8), we get equation (7), which is Proposition II. Proposition II as determined in this fashion has no independent logic. It is derived from an expression for expected yield and from the average cost of capital hypothesis given in Proposition I.

Proposition II can also be developed from the definition of the average cost of capital.

$$\text{Let } w_1 = \frac{\text{debt}}{\text{total value}} = \frac{L}{V}$$

$$w_2 = \frac{\text{equity}}{\text{total value}} = \frac{S}{V}$$

Also, $V = L + S$. Then,

$$(9) \qquad K_o = w_1 K_i + w_2 K_e$$

$$K_e = \frac{K_o - w_1 K_i}{w_2}$$

Then substituting for w_1 and w_2,

$$K_e = \frac{K_o - \frac{L}{V} K_i}{\frac{S}{V}}$$

(7) $$= K_o + (K_o - K_i)\frac{L}{S}$$

Since Proposition I has not been used, the above relationship can be used to describe the behavior of K_e in the models of both Modigliani and Miller and the traditional writers. However, the model as used by Modigliani and Miller keeps K_o constant, so K_e is a linear function of leverage. The traditional writers treat K_o as a variable which depends on the capital structure and the values of K_e and K_i.

Effects of Corporate Tax

Modigliani and Miller then remove assumption 4 and consider corporate tax in their analysis. When corporate tax is included, the value of a firm within a risk class becomes a function of the tax rate and degree of leverage as well as the expected after-tax returns. This means that there is a tax advantage to using debt so that increased leverage will result in a higher value for the firm and, therefore, a lower cost of capital. Modigliani and Miller note that the valuation implied by their model now comes closer to that predicted by the traditional model. However, it is their view that this reduction in the cost of capital through the use of debt occurs only because of present tax laws and not because debt is inherently cheaper.

As before, the long run average earnings before interest and taxes can be denoted by X for the firm in a particular risk class. Then, if we let $Z = X / \overline{X}$, all firms in the same risk class will have the same value of Z. The average earnings, X, can be expressed as $\overline{X}Z$. Now, X^{τ}, the average earnings after tax but before interest, is given by

$$X^{\tau} = (1 - \tau)(X - K_iL) + K_iL$$

$$= (1 - \tau)X + \tau K_iL$$

$$= (1 - \tau)\overline{X}Z + \tau K_iL \tag{10}$$

where τ is the corporate tax rate, but

$$E(X^{\tau}) \equiv \overline{X}^{\tau} = (1 - \tau)\overline{X} + \tau K_iL.$$

So, replacing $(1 - \tau)\overline{X}$ in equation (10) by $\overline{X}^{\tau} - \tau K_iL$ gives

$$X^{\tau} = (\overline{X}^{\tau} - \tau K_iL)Z + \tau K_iL \tag{11}$$

$$= \overline{X}^{\tau}\left(1 - \frac{\tau K_iL}{\overline{X}^{\tau}}\right)Z + \tau K_iL$$

Now the distribution of X^{τ} is seen to depend on the tax rate and the degree of leverage (where $(K_iL)/\overline{X}^{\tau}$ is a measure of leverage) as well as on the value of Z for the risk class to which the company belongs.

From equation (10) we see that the long-run average stream of after-tax earnings is composed of two parts. The first is an uncertain stream, $(1 - \tau)\overline{X}Z$, where the uncertainty associated with this stream is dependent on the risk class to which the firm belongs. The second is a virtually certain stream resulting from the tax deductability of interest payments. This stream is certain to the extent that the tax deductability can be applied against current income or carried forward or back to offset income of other periods. Since the two streams have different degrees of uncertainty, they should be capitalized differently. The value of an unlevered company, V_u, can be determined by capitalizing after-tax earnings at a rate, K_o. Then,

$$V_u = \frac{(1 - \tau)\overline{X}}{K_o}$$

where K_o is now the appropriate cost of capital after tax.

The certain stream generated by the debt should be capitalized at a rate K_i. Then the value of a levered firm is determined by capitalizing the risky portion at a rate K_o and the certain portion at a rate K_i. Then,

$$V_L = \frac{(1 - \tau)\overline{X}}{K_o} + \frac{\tau K_i L}{K_i}$$

(12) $$= V_u + \tau L$$

Equation (12) implies that the value of a levered firm is equal to the value of an unlevered firm of the same risk class, plus an additional amount which increases with both the amount of debt in the capital structure and with the level of corporate taxes. We note that the additional term resulting from the tax saving has been capitalized at a more favorable rate than the uncertain stream.

Development of a Computer Model

To appraise the validity of Modigliani and Miller's hypothesis, we wish to study the shape of the function for K_e and the value of the firm after tax as the amount of debt in the capital structure changes. A computer program is developed based on the assumptions and equations of Modigliani and Miller to provide the required information. This program, and its output are given in Appendix II.

As noted in the introduction, the type of model used by Modigliani and Miller in their theoretical development assumes that the firm has a fixed group of assets and receives constant earnings from them, characteristic of the risk class that the firm belongs to. This means that in our analysis, $\bar{X}$ is treated as a constant. The firm is valued in the case when there is no corporate tax, by capitalizing its earnings at a rate characteristic of its risk class as given in equation (1).

(1) $$V = \frac{\bar{X}}{K_o}$$

When corporate tax is included in the analysis, the firm is valued by equation (12).

(12) $$V_L = \frac{(1 - \tau)\bar{X}}{K_o} + \tau L$$

The debt capitalization rate is first determined by using a hypothetical function to describe the behavior of the debt market. We have noted the many difficulties encountered when empirical data is analyzed to determine functions for K_e, K_i, etc. Then, instead of using empirical data, we postulate what we consider to be a reasonable function for the dependence of K_i on the level of debt in the capital structure, based on characteristics that both Modigliani and Miller and the traditional writers seem to agree upon. Both schools agree that K_i should be almost constant for the first small increments of debt. Then, as the amount of debt increases, the capitalization rate should increase at an increasing rate. The following functions have been developed to determine K_i in the Modigliani and Miller model and also in the traditional and net income models of the following chapter.

(13) $$K_i = a + bL$$

where: a = a constant arbitrarily chosen at 5%. It should be noted that the value of 'a' depends both on current economic conditions and on the risk class to which the firm belongs.

b = rate at which the capitalization rate increases for each successive unit of debt in the capital structure.

L = amount of debt in the capital structure.

Equation (13) shows that the required debt yield is a linear function of the amount of debt. This linear function does not represent rational behavior and can be shown to give paradoxical results when used in the traditional model.

(14) $$K_i = a + bL^2$$

This function shows an increasing aversion to debt as the level of debt increases. In this respect, it represents rational behavior.

(15) $$K_i = a + bL^3$$

This function shows an even greater investor aversion to increased levels of debt than did equation (14). Equation (14) may be characteristic of behavior of debt holders in industries which have a relatively stable level of earnings, such as regulated utilities, while equation (15) could be characteristic of the behavior of debt holders for an industry where earnings fluctuate more rapidly, such as the auto industry.

It has been suggested that the debt capitalization rate may remain constant as small amounts of debt are introduced into an all-equity capital structure. Then, when the amount of debt exceeds a certain level, the debt capitalization rate increases sharply with subsequent increments of debt. This suggests use of the following types of equations to describe the dependence of the debt capitalization rate on the amount of debt in the capital structure:

(16) $$K_i = a + b(L - \overline{A})$$

(17) $$K_i = a + b(L - \overline{A})^2$$

(18) $$K_i = a + b(L - \overline{A})^3$$

(19) $$K_i = a, \text{ for } L < \overline{A}$$

It should be noted that equations (13), (14), and (15) represent limiting cases of the above equations when $\overline{A} = 0$.

This program permits the user to specify the values for the parameters a, b, and $\overline{A}$ and also whether he wishes to use a first, second or third order equation to determine the cost of debt.

The user specifies the value of K_o to use in equations (1) and (12). In our analysis, we have used a value of 7% which corresponds to the after-tax equity capitalization rate used in the traditional model when there is no debt in the capital structure.

The Modigliani and Miller model first calculates the value of the firm assuming there is no corporate tax. Since it uses the after-tax capitalization rate to do this, the results are not realistic, but do not affect the basic shape of the curves, so the data is still useful for our analysis.

After the value of the firm is found, the value of the equity, S, is determined. The ratios of debt to total value and equity to total value are then determined. The equity capitalization rate before tax is then uniquely determined from equation (20).

(20) $$K_e = \frac{(K_o - w_1 K_i)}{w_2}$$

Using equation (12), the value of the firm after tax is then calculated. Knowing the value of the firm after tax and the amount of debt in the capital structure, the value of the equity is then determined. Then the equity capitalization rate is found by dividing the earnings accruing to the equity by the after-tax value of the equity. In the Modigliani and Miller model, the cost of equity is dependent on the cost of debt and the value of the firm as given by equation (1) in the before-tax case, or equation (12) in the after-tax case. In this model, K_e is the dependent variable and K_o and K_i are the independent variables.

This program uses an iterative procedure and operates as follows. The program uses the debt capitalization function, earnings capitalization rate K_o, earnings $\overline{X}$, tax rate τ , and increment to debt as specified by the user. Initially, the program assumes that there is no debt in the capital structure. In successive iterations, the program increases the debt by an increment specified by the user.

The output from the program then gives the following values for varying amounts of debt:

- the value of the firm before and after tax,
- the after-tax cost of capital,
- the before-tax and after-tax debt-equity ratio,
- the value of equity before and after tax,
- the average cost of debt,
- the average cost of equity before and after tax.

The program adds successive increments of debt and terminates when all the equity has been replaced by debt.

Footnotes

1. The analysis in this chapter is based primarily on their papers, "The Cost of Capital, Corporation Finance, and the Theory of Investment", pp. 261-97, and "Corporate Income Taxes and the Cost of Capital: A Correction", American Economic Review, Vol. LIII, No. 3, June, 1963, pp. 433-43.

2. In their analysis, Modigliani and Miller use ρ as the capitalization rate applied to earnings of a given risk class. We will use K_o instead of ρ so that the symbols used in the Modigliani and Miller analysis will be consistent with those used in the net income and traditional analysis.

3. Miller and Modigliani, op. cit., p. 412.

4. Modigliani and Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment", p. 268.

5. ibid., p. 268.

6. ibid., p. 272.

7. ibid., p. 270

CHAPTER III

THE TRADITIONAL HYPOTHESIS

The Net Operating Income Approach

The Modigliani and Miller hypothesis described in the previous chapter is what Durand[1] has called the net operating income approach to valuation of the firm. When this approach is used, the net operating income is capitalized using a rate characteristic of the risk class to which the firm belongs, to determine the total value of the firm. The value of the equity is then found by subtracting the amount of debt from the total value. Then, as long as the debt capitalization rate does not change, the equity capitalization rate is a linear function of the debt-equity ratio and is given by Modigliani and Miller's Proposition II. An example will make this clear. Suppose a company belongs to a risk class in which total earnings of $1,000 are capitalized at 10%. Suppose this company has $2,500 of debt at 4% in its capital structure. Then the value of the equity is given as follows:

Table I

VALUE OF THE CORPORATION

NET OPERATING INCOME MODEL

Net Operating Income	$1,000
Capitalized at 10%	x 10
Total Value	10,000
Less Total Debt	2,500
Value of Equity	$7,500

We should note that because of Proposition I, the total value of debt and equity must always be $10,000, regardless of their proportions.

The Net Income Approach

The alternative approach is what Durand calls the net income method. In this method, interest charges are first subtracted from the net operating income. The remaining net income is then capitalized at a rate applicable to that risk class to determine the value of the equity. If the equity capitalization rate is 10%, then for the firm described above, the value of the equity is now determined as follows:

Table II

VALUE OF THE CORPORATION

NET INCOME MODEL

Net Operating Income	$1,000
Less Interest	- 100
Net Income	900
Capitalized at 10%	x 10
	$9,000

Now the value of the equity is seen to be $9,000 instead of $7,500 as determined by the net operating income method. Furthermore, the value of the firm is now given by the total of debt and equity, so it is now $11,500. We note that the value of the firm increases as the amount of debt in the capital structure increases.

Table III

EFFECT OF CAPITAL STRUCTURE ON VALUE

NET INCOME MODEL

Amount of Debt	0	1,250	2,500
Value of Equity	10,000	9,500	9,000
Total Value	$10,000	$10,750	$11,500

The proponents of the net income method do not suggest that the procedure of replacing debt with equity and thereby increasing the value of the firm can continue indefinitely. The view of those supporting the net income approach to valuation is that for firms in a given risk class, the market value of the firm will first rise as the amount of debt in the capital structure is increased from zero to some point determined by the capital market's evaluation of the financial risk associated with varying degrees of leverage. Beyond this point, investors require a higher yield on equity to compensate for the increased financial risk resulting from leverage. This means that any "saving" from the use of debt is offset by the increase in "cost" of equity, so the value of the firm remains almost constant. At even higher amounts of debt, both debt and equity purchasers regard their holdings as very risky. The much higher yield that they demand results in a rapid decrease in the value of the firm.

Two Possible Computer Models

We now wish to describe two different models. We develop a "net income model" and a "traditional model". It should be noted that this terminology is similar to that used by Weston,[2.] but many writers tend to use the two terms interchangeably.

The net income model may be regarded as describing a possible extreme of investor behavior. It is used here since it was suggested by Modigliani and Miller as a theoretical alternative to their hypothesis. They state:[3.]

> "Without doing violence to this position (traditional hypothesis), we can bring out its implications more sharply by ignoring the qualification (that the earnings-price ratio or its reciprocal, the times-earnings multiplier, of a firm's equity will be only slightly affected by moderate amounts of debt in the capital structure) and treating the yield as a virtual constant over the relevant range."

This means that in what we have chosen to call the net income model, the equity holder is regarded as considering the moderate use of debt as not increasing financial risk sufficiently to warrant any increase in the equity capitalization rate. After a given level of debt is reached (depending on the risk class to which the firm belongs), the equity and debt capitalization rates increase rapidly.

In what we call the traditional model, we describe the investor as thinking that any use of debt results in increased financial risk. Even moderate use of debt results in an increase in the equity capitalization rate. The first derivative with respect to debt of the equity capitalization rate is a positive increasing function of the amount of debt. The equity holder regards increasing levels of debt as resulting in a more than proportionate increase in financial risk.

The net income model is developed as follows. The capitalization rates for both debt and equity are set by the market. As in the model developed to test the Modigliani and Miller hypothesis, a choice of three functions is available to determine the interest payments on debt. These functions take the same form as in the Modigliani and Miller model.

(16) $$K_i = a + b(L - \bar{A})$$

(17) $$K_i = a + b(L - \bar{A})^2$$

(18) $$K_i = a + b(L - \bar{A})^3$$

(19) $$K_i = a, \text{ for } L < \bar{A}$$

As in the previous model, $K_i = a$, in all cases where $L < \bar{A}$.

Also, since equity capitalization rates are now set by the market, we have a choice of three functions for K_e:

(21) $$K_e = c + d(L - \bar{A})$$

(22) $$K_e = c + d(L - \bar{A})^2$$

(23) $$K_e = c + d(L - \bar{A})^3$$

(24) $$K_e = c, \text{ for } L < \bar{A}$$

By using functions of this type, the equity yield remains constant until the amount of debt in the capital structure is increased to the level at which $L = \bar{A}$. This is in keeping with the objectives of the net income model. Once the amount of debt exceeds $\bar{A}$, the required yield increases at a rate which depends on the function selected. Equation (21) implies irrational behavior since aversion to financial risk is expressed as a linear function of the amount of debt. Equations (22) and (23) describe investors who require yields which increase rapidly when the level of debt exceeds $\bar{A}$.

In the analysis which follows, we assume that, initially, the debt holders require 5% interest on their holdings and that the yield on equity must be 7%. Corporate earnings must be sufficient to pay 5% to the debt holders and also return 7% after tax to the equity holders. These numbers are chosen because they are thought to be realistic for investors holding securities of a firm in a moderate risk class. It should be noted that the programs can be readily adapted to other interest rate functions. In addition, the level at which the capitalization rates first increase can be varied by changing $\bar{A}$ and the rate at which they increase can be changed by specifying b and d.

The total market value of the firm for the net income and traditional models is determined from an equation such as the one given by Lindsay and Sametz:[4.]

$$V = L + S$$

(25) $$= L + \frac{\bar{X} - K_i L}{K_e}$$

for the case when there are no corporate taxes, or, as given by Mao, when corporate taxes are included:[5.]

$$V = L + \frac{(\bar{X} - K_i L)(1 - t)}{K_e} \qquad (26)$$

where all variables have been previously defined.

In equation (26), the second term on the right hand side expresses the value of the equity as being the sum in perpetuity of the net after-tax earnings to equity. It should be noted that $\bar{X}$ is the expected average return on assets as specified in Modigliani and Miller's method of risk class determination, so all growth and fluctuation in earnings have been considered in determining $\bar{X}$. The value of the equity can then be accurately determined by applying a capitalization rate characteristic of its risk class.

The programs operate as follows. The user specifies the type of function he wants for K_e and K_i, including the level of debt, $\bar{A}$, up to which K_e and K_i are considered to remain constant. He specifies the expected average earnings on assets, $\bar{X}$, which in our analysis is considered to be $75. The tax rate and increment to debt are also specified. The programs then perform a series of calculations to determine the value of the firm, value of equity, debt and equity yields, average cost of capital and other parameters for various levels of debt. The equations used are equation (26) and equation (9), which expresses the average cost of capital as a function of the cost and amounts of debt and equity in the capital structure:

$$K_o = w_1 K_i + w_2 K_e \qquad (9)$$

The traditional model is identical to the net income model in all respects except for the functions describing K_e and K_i. The traditional writers suggest that equity holders are averse to even small increases in debt. Then, instead of keeping K_e constant until a debt level of $\bar{A}$ is reached, the capitalization rate should increase slightly as soon as any debt is introduced into the capital structure. Equations that can be used to describe the required debt yields are the same as equations (13), (14) and (15):

(13) $$K_i = a + bL$$

(14) $$K_i = a + bL^2$$

(15) $$K_i = a + bL^3$$

The equity capitalization rates can be given by the following equations:

(27) $$K_e = c + dL$$

(28) $$K_e = c + dL^2$$

(29) $$K_e = c + dL^3$$

A program listing, instructions for use, and output of the traditional model is given in Appendix III. The net income model is given in Appendix IV.

Marginal Cost of Debt

Also included in the programs for the traditional model and the net income model are procedures to calculate the marginal cost of debt for the type of model developed by Modigliani and Miller and for the model developed by the traditional writers. Although the marginal costs will not be analyzed in depth, their relationship to the average cost curve and the maximum value of the firm will be considered in the following chapters.

In the model developed by Modigliani and Miller, the total value of assets is kept constant and debt is used to reduce equity holdings ($dL = -dS$). In determining the marginal cost of debt, we need consider only the cost of the new increment of debt plus the way in which the new debt influences the return required on the previous debt. The marginal cost of debt can be determined from

$$\frac{d}{dL}(K_i L) = L\frac{dK_i}{dL} + K_i$$

If we determine the marginal cost of debt for the Modigliani and Miller model from equations (16), (17), and (18), we get:

$$\frac{d}{dL}(K_i L)_{16} = bL + a + b(L - \bar{A}) \tag{30}$$

$$\frac{d}{dL}(K_i L)_{17} = 2bL^2 - 2\bar{A}bL + a + b(L - \bar{A})^2 \tag{31}$$

$$\frac{d}{dL}(K_i L)_{18} = 3bL^3 - 6\bar{A}bL^2 + 3bL\bar{A}^2 + a + b(L - \bar{A})^3 \tag{32}$$

If we use equations (13), (14), and (15) to determine the required yield on debt, the marginal cost of debt is given by the following equations:

$$\frac{d}{dL}(K_i L)_{13} = bL + a + bL \tag{33}$$

$$\frac{d}{dL}(K_i L)_{14} = 2bL^2 + a + bL^2 \tag{34}$$

$$\frac{d}{dL}(K_i L)_{15} = 3bL^3 + a + bL^3 \tag{35}$$

In the model developed by the traditional writers, K_o is the dependent variable and K_e and K_i are the independent variables. We noted previously that in the Modigliani and Miller model, K_e was the dependent variable and K_o and K_i were the independent variables. This meant that a unique cross-relationship was assumed to exist between the debt and equity sectors of the market, which kept K_o and V constant. The traditional writers do not assume that this relationship exists. Since K_e and K_i are thought by them to be determined independently, the marginal cost of debt in the traditional and net income models must include a term to compensate for the change in the cost of equity resulting from increased use of debt. The marginal cost of debt is now composed of two components. The first component accounts for the increased cost of debt. In an iterative model, it may be given by the following expression:

(36) $$K_i' \text{ marg} = \frac{K_{i(L+\Delta)}(L+\Delta) - K_i L}{\Delta}$$

where Δ is the amount by which the debt has been increased and $K_{i(L+\Delta)}$ is the interest cost at the higher debt level.

The second component, giving the effect of increased debt on the value of equity, is given for small increments by:

(37) $$K_i'' \text{ marg} = \frac{(\bar{X} - K_i L)\ (1 - \tau)}{\Delta}\left[\frac{K_{e(L+\Delta)}}{K_{e(L)}} - 1\right]$$

where $K_{e(L+\Delta)}$ is the capitalization rate at the higher debt level, and $K_{e(L)}$ is the capitalization rate at the lower level.

The total marginal cost of debt can now be determined from the sum of the marginal cost of increases in debt charges and the marginal cost of the change in the value of the equity:

$$K_{i\,marg} = K_i{'}_{marg} + K_i{''}_{marg}$$

for small changes.

Footnotes

1. David Durand, "Cost of Debt and Equity Funds for Business: Trends and Problems of Measurement", Conference on Research on Business Finance, New York, National Bureau of Economic Research, 1952, pp. 215-247.
2. J. F. Weston, op. cit., pp. 105-112
3. Modigliani and Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment", p. 268.
4. Lindsay and Sametz, op. cit., p. 129
5. James C. T. Mao, op. cit., chapter 11.

CHAPTER IV

AN ANALYSIS OF THE MODIGLIANI AND MILLER HYPOTHESIS

In Chapter II, the Modigliani and Miller arbitrage argument has been used to show that if markets are perfect, the value of two firms with the same earnings and in the same risk class must be equal even if their capital structures are different. We now wish to examine some of the implications of the Modigliani and Miller hypothesis and to see how more realistic assumptions can influence their results.

Modigliani and Miller state:[1]

> "Our Propositions I and II, as noted earlier, do not depend for their validity on any assumption about individual risk preferences. Nor do they involve any assertion as to what is an adequate compensation to investors for assuming a given degree of risk. They rely merely on the fact that a given commodity cannot consistently sell at more than one price in the market; or more precisely, that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the price of the two components."

It is our contention that Modigliani and Miller's Proposition II *is* a description of equity holder behavior in reaction to increasing debt in the capital structure. Moreover, we hope to show that the behavior described by Modigliani and Miller is not realistic in a real world context where corporate income tax, limited personal liability and risk-averse investors exist. Also, we suggest that due to such factors as taxes, limited liability and institutional restrictions, the bundle of commodities may depend on the composition of its parts.

Arbitrage

Modigliani and Miller's Proposition I is based on their arbitrage idea. We will now examine some of the factors that may impede this arbitrage or switching. Modigliani and Miller maintain that whenever the value of two similar companies with equal earnings differs, it will pay investors to sell their shares in the higher-valued company and buy shares in the lower-valued company. There are several factors which impede this process. First, if the value of shares in the "overpriced" company has increased, the investor will be subject to capital gains tax. This may well wipe out all gains from purchase of the undervalued shares. Also, the investor must pay brokerage fees on his arbitrage transaction. The combination of these two costs may make it more profitable for him to retain his "overpriced" shares.

If arbitrage is to be effective, a great amount of money must be available for these transactions. Modigliani and Miller think that funds can be raised by purchasing equity on margin or by pledging shares of the undervalued company as security for personal loans. There are several factors which limit the effectiveness of margin buying. When Modigliani and Miller published their first paper, legal restrictions required the investor to own outright, 50% of the total equity. This has now been increased to 90%, so the amount of funds available on margin is very small. Even this small amount of margin buying is withheld from the institutions which account for a large fraction of equity purchases. Mutual funds, most personal trust funds, closed end trust, fire, casualty and life insurance companies are all prevented from buying stock on margin, either through direct restrictions imposed by their charter, or through the dictates of prudent behavior.

When we introduce the possibility of personal bank loans, secured by the equity purchases, many difficulties are encountered. If investors are to be able to undo corporate leverage through personal leverage, the interest rates paid by the corporation and by the individual must be identical, as can be seen from a study of equations (3) through (6). Modigliani and Miller "conjecture that the curve for bond yields as a function of leverage will turn out to be a non-linear one."[2.] They agree with traditional theorists that, because of the increased risk of default of interest payments associated with excessive amounts of leverage, debt yields must increase as the amount of debt in the capital structure increases. However, if "homemade" and corporate leverage are to be substitutes, the individual securing the personal loan and the corporation must face identical functions for Ki. This is not possible when we consider that corporate debt is backed only by the assets of the corporation while the personal loan is secured by unlimited personal liability. At extreme levels of corporate leverage, the individual can obtain debt funds at a lower cost than can the corporation. Only if the individual is identical in all respects to the corporation can corporate and homemade leverage be considered as equivalent.

However, Modigliani and Miller see another source of funds:[3]

> "Under normal conditions, moreover, a substantial part of the arbitrage process could be expected to take the form not of having arbitrage operators go into debt on personal account to put the required leverage into their portfolios, but simply of having them reduce the amount of corporate bonds they already hold when they acquire underpriced unlevered stock."

In a real life situation, any action such as this will most likely change the overall financial risk of the investment portfolio. Before purchasing the underpriced, unlevered stock, the investor has in his portfolio a combination of bonds and stocks which yield him earnings of a given mean and variance or of a given financial risk. Any attempt to sell some of the bonds in his existing portfolio to provide funds to create homemade leverage for the purchase of the undervalued security will probably alter the financial risk. Unless the investor holds the same proportion of bonds and stock in the overpriced security, the procedure described by Modigliani and Miller will result in a greater amount of funds being invested in a particular security, thereby changing the mean and variance of returns from the portfolio. If Modigliani and Miller allow financial risk to change, they introduce complications into their analysis which are impossible to resolve without first determining utility functions for investors.

Table IV shows the value of two companies with the same earnings and in the same risk class but with different capital structures. The equity of Company Y is capitalized at a higher rate to compensate for the financial risk from leverage.

Table IV

USE OF ARBITRAGE

	Company X	Company Y
Expected Earnings	$1,000	$1,000
Debt	-	3,000
Interest (4%)	-	120
Returns to Shareholders	1,000	880
Capitalization Rate	10%	11%
Value of Equity	10,000	8,000
Value of Firm	10,000	11,000
D/E Ratio	0	37.5

Through the Modigliani and Miller arbitrage argument, the investor, who is assumed to hold $1,000 of equity in Company Y, will sell these shares. To keep his financial risk constant, he will borrow $375 and invest this total of $1,375 in Company X.

In his old portfolio, he earned 11% on $1,000 or $110. In this new one, he earns: $1,375(.1) - $375(.04) = $122.50. However, if he uses the $375 he borrowed to purchase more shares in Company Y, he would have an income of $136.25, or an increase of $13.75 over what he could receive if he invested in Company X. The question of whether this added $13.75 is sufficient to offset the increased financial risk cannot be answered by any of the models developed to date.

Barges describes these circumstances which limit the usefulness of the arbitrage process in equating the value of similar firms with different capital structures.[4] Consider firms X and Y as described previously. Since the investor can receive a greater return at the same risk by switching his investment from Company Y to Company X, the implication is that this disparity in market value will be erased by arbitrage. Sale of stock of Company Y will deflate its price, driving its yield up, while the purchase of stock in Company X will increase its price, driving its yield down. Barges postulates that these changes in yields may induce the following reaction, which serves to counter the effectiveness of arbitrage. Since the yield of stock X has declined, some of the original shareholders of Company X may think that this new lower yield is insufficient to compensate them for their risk. Moreover, these investors note that the yield of stock Y has been driven up, while its risk has remained constant. Then some of the shareholders of Company X who do not like margin buying will now find stock Y more attractive. While the Modigliani and Miller arbitrage process is acting to cause investors to sell stock Y (depressing its price) and buy stock X (inflating its price), an induced reaction resulting from the arbitrage process is causing another group of investors to sell stock X (depressing its price) and buy stock Y (inflating its price). The actions of these two groups of investors may counterbalance, resulting in unchanged stock prices. We note that investors selling stock X and buying stock Y are increasing their financial risk, but they may well regard the increased yield as adequate compensation.

We note that if the induced reaction is to procede, we must have a group of investors who dislike margin buying. If this were not so, the original investors in Company X could improve their return by taking a margined position in Company X. We now examine the way in which limited liability influences the risks of margin buying.

Limited Liability

Modigliani and Miller assert that arbitrage is possible since homemade leverage is a substitute for corporate leverage. However, limited personal liability protects the equity holder in the levered company. The investor using personal leverage receives no such protection.

Suppose an investor holds $7,000 of equity in Company Y and that the remainder of the company's capital structure is composed of $3,000 worth of debt. If Company Y has a greater value than Company X, Modigliani and Miller suggest that the investor can improve his return by selling his shares of Company Y for $7,000. To keep the financial risk of his new portfolio equal to that of his old one, he must borrow $3,000 and invest the total of $10,000 in Company X. Modigliani and Miller maintain that return has been increased but financial risk remains constant. We submit that Modigliani and Miller are correct only if limited personal liability for equity holders does not exist. In our example, the shareholder of Company Y could lose a maximum of $7,000. After he has engaged in homemade leverage, his maximum possible loss has been increased to $10,000. As Durand notes,[5.] in practice the risk of losing the entire $10,000 is small since the bank or broker will sell out the stockholder to meet margin requirements, but the protection of maximum loss greatly increases the risk of smaller loss. In this context, we should note another difference between personal and corporate leverage. When a corporation borrows, interest on debt has first claim to the earnings of the corporation. When the stockholder engages in personal leverage, this is no longer so. Any dividend payment is now at the discretion of management. While the firm may have ample earnings, it may decide to retain them. In this case, the shareholder who has engaged in personal borrowing to create homemade leverage will be forced to liquidate some of his holdings to meet personal interest charges. This can result in losses if the liquidation occurs at an inopportune time.

In summary, we have noted that there are many real world restrictions which may prevent the arbitrage process of Modigliani and Miller from equating the value of two firms with equal earnings and in the same risk class but having different capital structures. Restrictions on margin buying limit its usefulness. Also, we have seen that homemade leverage is more risky than corporate leverage. If a group of investors exist who are averse to margin buying, the arbitrage process may induce results which cancel out its effectiveness.

Behavior of Debt and Equity Holders

Even though we have serious doubts about the effectiveness of the arbitrage process, we wish to examine some of the behavioral assumptions implied by Proposition II, given that Proposition I is correct. Also, we want to examine the effect of corporate taxes and the tax-deductability of interest payments on the Modigliani and Miller model.

We suggest that Proposition II is a description of investor reaction to the introduction of debt into the capital structure. Proposition II, developed in Chapter II and repeated below, gives the equity capitalization rate as a function of the debt-equity ratio:

(7) $$K_e = K_o + (K_o - K_i)L/S$$

and as we noted previously, it is based on the assumption that the weighted average cost of capital is a constant dependent on the risk class to which the firm belongs. Therefore, K_e, which is the dependent variable in the Modigliani and Miller model, becomes a function of the independent variables K_o, which is indicative of investor appraisal of the risk class, K_i, which is determined by the debt market, and L, the amount of debt in the capital structure. Both the traditional writers and Modigliani and Miller are agreed that the debt capitalization rate should increase as the financial risk, determined by the amount of debt in the capital structure, increases. What they are not agreed upon is the way in which the equity capitalization rate should increase. In order to better analyze the behavioral implications of Modigliani and Miller's model, let us determine some of the reasons why the debt and equity capitalization rates should increase with increased use of debt.

Both debt and equity holders are concerned with the risk of default. The debt holders are concerned with the fact that they may not receive their interest payments. The equity holders are concerned with the fact that if the debt holders do not receive their interest payments, the equity holders may lose control of the company. Both groups of investors are then interested in the amount by which the firm's assets can decline in value before they become less than its liabilities and the firm becomes insolvent. The likelihood of insolvency can be determined by the equity-debt ratio. For example, if the ratio is 19:1, the firm's assets may fall 95% in value before the debt is no longer covered. However, if the ratio is 1:4, default will occur if the assets lose only 20% of their value. For the first firm, the risk fo default is very small, for even if the earnings fluctuate greatly, the chances of not being able to cover the bond interest is remote. In the second case, even slight fluctuations in earnings may result in the firm's being unable to meet its large interest bill, and if the firm defaults, the assets would probably not be worth enough to pay the bonds off in full. Both bond and stock holders of the second company should require higher yields in order to compensate them for their risk.

Use of debt may influence the firm's future investment strategy. A highly levered firm may be forced to pass up investments which an unlevered firm in the same risk class can undertake. The highly levered firm may have insufficient funds to meet required interest payments and still undertake all the planned investments which have a positive present value. Since these profitable investments can be undertaken by the unlevered firm, its value will be increased relative to the levered firm. If the highly levered firm resorts to external debt or equity markets, it may find that any borrowing will result in creditors' imposing restrictions on the firm's financial policies. Equity holders will express a reluctance to purchase shares of these unsound corporations and can be enticed to do so only by offering substantial premiums in yield.

If excessive use of leverage does result in restrictions on the firm's investment policies, we can expect a change in the expected average income, $\bar{X}$, and the firm may be placed in a different risk class. The fact that use of debt increases the financial risk of the business and results in operating limitations which cause debt and equity holders to require higher yields tends to support Modigliani and Miller's contention that the real cost of debt is the same as the average cost of capital.

There are several other factors, however, which tend to make debt "cheaper" than equity. Since corporate income is taxed and interest on debt is a tax deduction, there is an advantage us using debt in both Modigliani and Miller and in the traditional model. Although use of debt increases the expected per share returns, it also increases the variability of these returns. However, due to the existence of limited liability, the gains possible through leverage are infinite, while the losses are limited, so the distribution of returns tends to be favorably skewed. Since debt contracts are made in monetary terms instead of real dollar terms, a bonus will accrue to equity holders if debt is contracted prior to an inflation. In a recession, debt is a disadvantage to the firm. Using our computer programs, we can now examine these implications for a hypothetical firm.

Analysis of Results From Computer Model

We use our computer model to determine the value of the firm as predicted by the Modigliani and Miller model. In the following chapter, we repeat the calculations using models based on the net income hypothesis and on the traditional hypothesis.

The net operating income hypothesis of Modigliani and Miller begins by positing a constant value for K_o. The necessary equilibrium value of K_e is then determined from Proposition II, using a debt yield, K_i, as set by the market. We wish to study the Modigliani and Miller model by using a hypothetical function to describe the dependence of K_i on L.

While this function has been chosen intuitively, it should be noted that its form is characteristic of rational investor behavior and is supported by many empirical studies. The results of our analysis are not dependent on the exact shape of the K_i function, but only on its form. As long as investors are risk-averse, ($\frac{d^2K_i}{dL^2}$ is positive), our findings will hold.

The function for K_i that we use is:

$$K_i = .05 + 5 \times 10^{-9} (L - 125)^3$$

This function describes a debt market in which bond purchasers do not require any additional compensation for risk until there is $125 of debt in the capital structure. This description of the behavior of debt holders is the same as that given by the net income model.

Figure I gives the value of the firm, average cost of debt, marginal cost of debt, average cost of equity, and average cost of capital for a firm earning $75, and for which there is no corporate income tax. In keeping with Modigliani and Miller's method of analysis, our data is plotted using the debt-equity ratio as a measure of leverage.

The data used in constructing this graph is given in Appendix II. From a study of the data and graph, the following implications of Modigliani and Miller's model are noted. As long as there is no corporate tax, the value of the firm and its weighted average cost of capital are constant. As given by Proposition II, as long as K_i remains constant, K_e is a linear increasing function of the debt-equity ratio. In

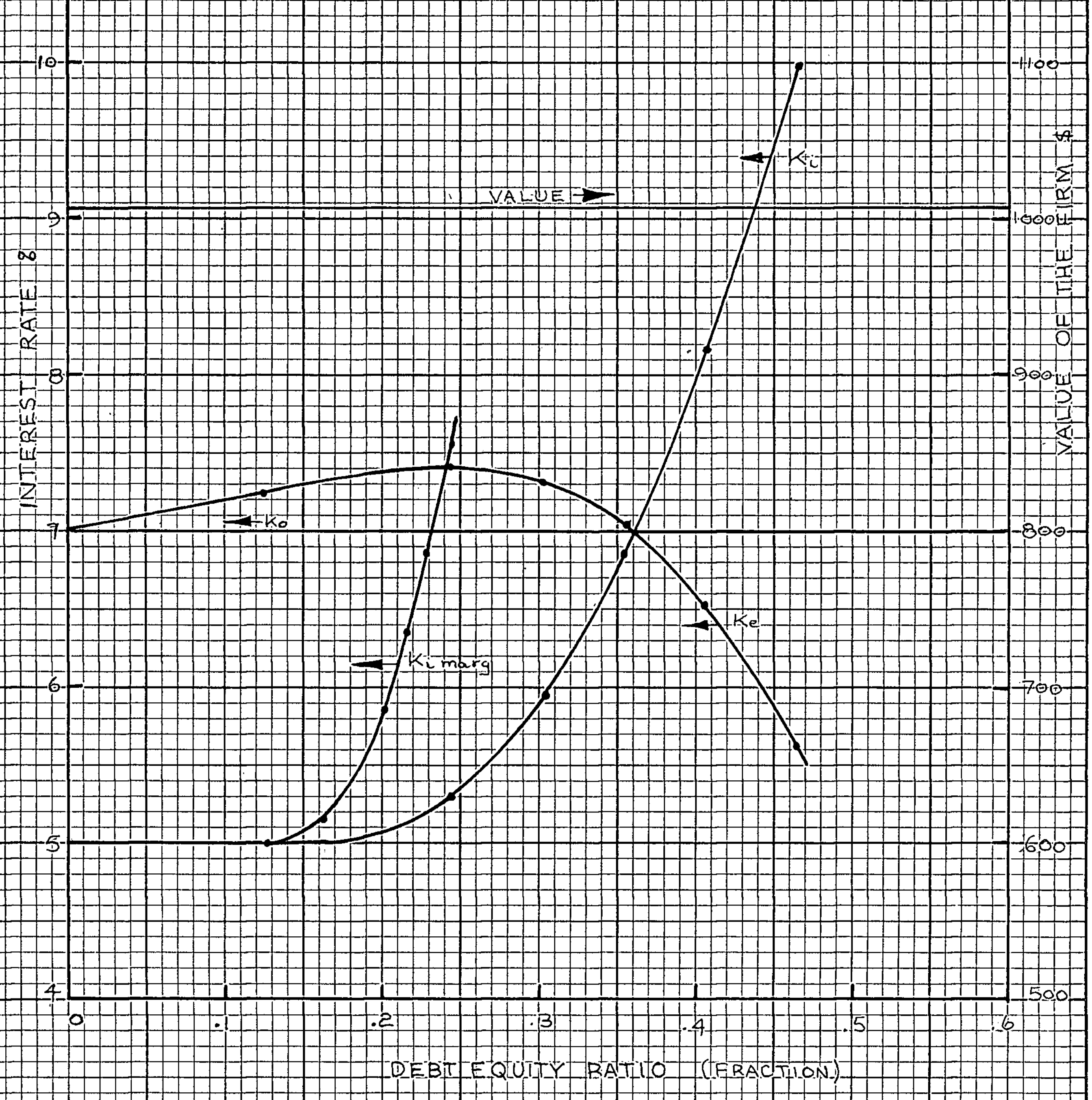
Figure I

MODIGLIANI AND MILLER HYPOTHESIS -

COST OF FUNDS AND VALUE OF THE FIRM AS A

FUNCTION OF THE DEBT-EQUITY RATIO

(No Corporate Income Tax)



this case, K_i remains constant until the amount of debt in the capital structure exceeds $\bar{A}$, where $\bar{A} = 125$. When L exceeds $\bar{A}$, which in this model occurs at a debt-equity ratio of approximately .125, K_i then begins to increase. However, if K_i is to increase and K_o remain constant, K_e must now increase at a decreasing rate. Moreover, following a proof given by Robichek and Myers, we can show that when the marginal cost of debt exceeds the average cost of equity, the average cost of equity must start to decline with further increases in the debt-equity ratio.[6]

The total interest paid by the corporation is K_iL, so the marginal rate is given by:

(38) $$M = \frac{d}{dL}(K_iL) = K_i + L\frac{dK_i}{dL}$$

Also, the average cost of capital is given by:

(9) $$K_o = w_1K_i + w_2K_e$$

$$K_o = \frac{L}{V}K_i + \frac{S}{V}K_e$$

(39) $$K_oV = LK_i + SK_e$$

Then, writing equation (39) in differential form:

(40) $$VdK_o + K_odV = K_idL + LdK_i + K_edS + SdK_e$$

But Modigliani and Miller maintain that V and K_o are both constant, so:

(41) $$0 = K_idL + LdK_i + K_edS + SdK_e$$

However, in the Modigliani and Miller model, debt is being substituted for equity, so:

$$dL = -dS$$

Then, making this substitution in equation (41) gives:

(42) $$0 = K_i dL + L dK_i + S dK_e - K_e dL$$

The point at which K_e starts to decline occurs when its first derivative $dK_e = 0$. Then, solving for this point from equation (42) gives:

(43) $$0 = K_i dL + L dK_i - K_e dL$$

Then:

(44) $$K_e = K_i + L \frac{dK_i}{dL}$$

$$= M$$

From Proposition I, when $K_i = K_o$, then K_e must also equal K_o.

(9) $$K_o = w_1 K_i + w_2 K_e$$

but if $K_i = K_o$

then $K_o = w_1 K_o + w_2 K_e$

(45) $$\left(\frac{1 - w_1}{w_2}\right) K_o = K_e$$

But $w_1 + w_2 = 1$,

so $w_2 = 1 - w_1$,

so $K_e = K_o$.

Since K_e is declining in this region, for debt-equity ratios beyond those at which $K_o = K_i$, a necessary condition is that $K_e < K_i$.

From the above analysis, we note that if K_o is to remain constant for all debt-equity ratios, the following behavior is required for K_e if K_i is to increase with increasing use of debt. K_e first increases at a constant rate as long as K_i is constant. Then, once K_i starts to increase, K_e increases, but at a decreasing rate. When the marginal cost

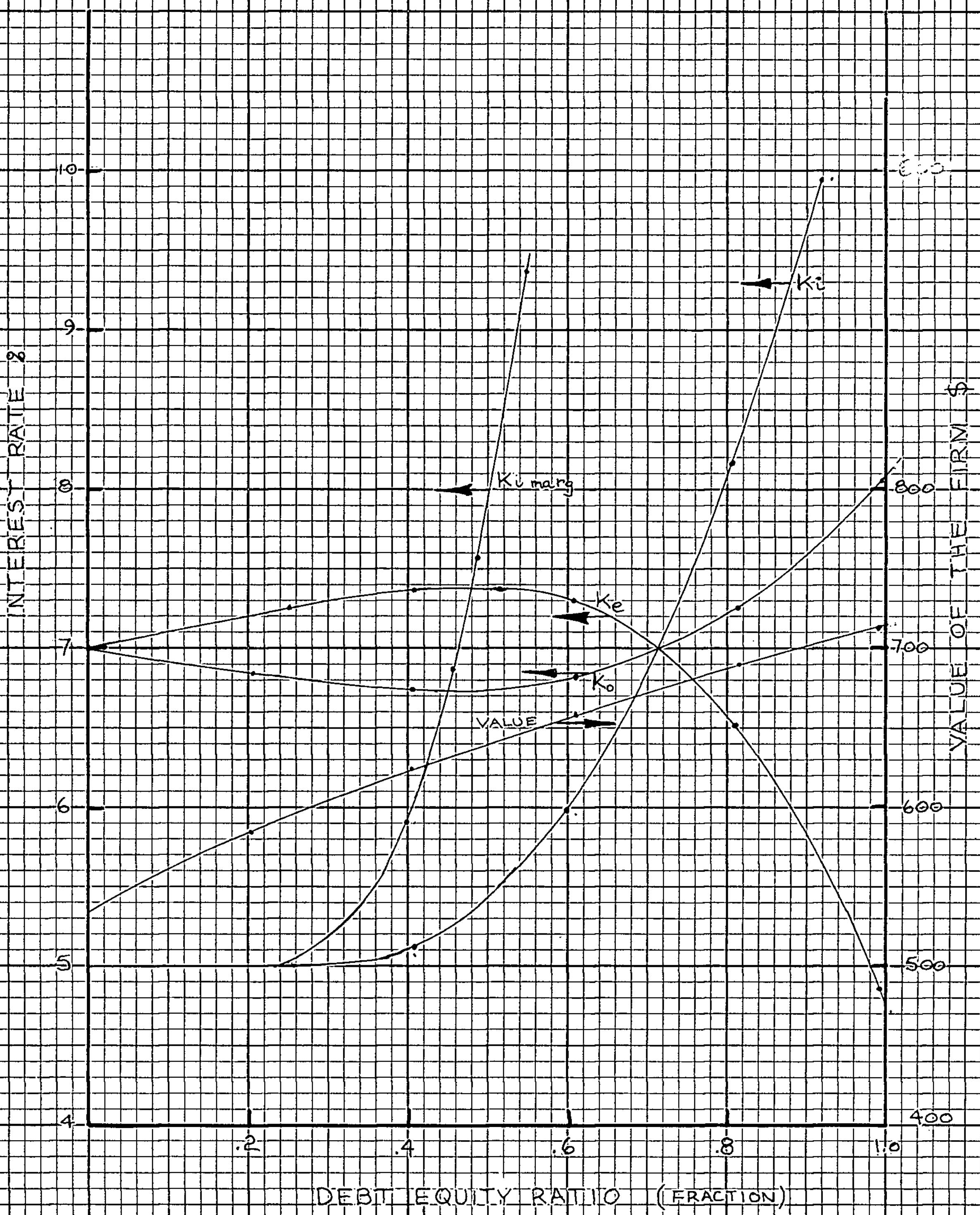
Figure II

MODIGLIANI AND MILLER HYPOTHESIS -

COST OF FUNDS AND VALUE OF THE FIRM AS A

FUNCTION OF THE DEBT-EQUITY RATIO

(50% Corporate Tax)



of debt exceeds the average cost of equity, K_e must start to decline. Furthermore, once K_i exceeds the average cost of capital, the average cost of debt is greater than the average cost of equity. This behavior implies that, as debt is introduced into the capital structure, thereby increasing financial risk, equity holders require a linear increase in their yield. As greater amounts of debt are used, the compensation for financial risk increases but at a decreasing rate. Then a point is reached at which the compensation required decreases as more debt is employed. Finally, the senior claim is capitalized at a higher rate than the subordinate claim. Modigliani and Miller state that the shares of an overlevered company would be purchased by risk lovers, but it is difficult to conceive of an investor who is so fond of risk that he prefers both a lower yield and more uncertainty to a higher yield and greater certainty. We maintain that if investors are averse to risk and are operating in perfect markets, the interest rate on debt cannot be greater than the yield on equity, for the debt holders are in a preferred position to the equity holders.

Robichek and Myers[7.] think that for risk-averse investors and perfect markets, the cost of debt cannot behave in such a way as to force the equity capitalization rate to decline. They state that if the company adds an increment of debt, ΔL, the interest rate on this additional debt cannot be greater than the expected return required by stockholders as long as the debt holders are in a preferred position.

to the stockholders with regards to all cash flows and assets of the company.

However, an analysis of their argument reveals that they define the marginal cost of the increment of debt, Δ L, as being the interest paid only on that increment of debt. They maintain that additional increments of debt do not increase the cost of existing debt. When we include a component of marginal cost to cover the increased cost of debt already held by the company, we note from Figure II that the average cost of debt is still much lower than the average cost of equity when the cost of equity starts to decline. Also, even if we accept Robichek and Myers' definition of marginal cost, we still cannot explain the behavior of equity holders. From Figure II, we note that as soon as K_i increases, K_e increases at a decreasing rate. Robichek and Myers provide no explanation for this paradoxical behavior of equity holders.

The illogical behavior required by the Modigliani and Miller model tends to support the traditional position which maintains that both debt and equity capitalization rates are established independently by the markets.

Figure II gives the cost of funds and the value for a firm with the same earnings and debt charges as used in Figure I, but paying a 50% corporate income tax. As in Figure I, the average cost of equity is observed to decrease once the marginal cost of debt exceeds it. Also, the average cost of debt, equity, and capital again intersect at one point. However, when corporate tax is introduced into the analysis, the value of the firm is seen to increase as debt is introduced into the capital structure. The average cost of capital is observed to decrease for small increments of debt, reach a minimum where the marginal cost of debt is equal to the average cost of capital, and then increase. The data for this figure are given in Appendix II, except for the marginal cost of debt which is determined from the net income model of Appendix IV.

Figure III shows the influence of tax rate on the value of the firm for different debt-equity ratios. The cost of debt is once again given by the function:

$$K_i = .05 + 5 \times 10^{-9} (L - \bar{A})^3$$

Data is plotted for a firm with expected earnings of $75, and paying tax at rates of 0%, 30%, 50% and 70%.

In all cases, the effect of corporate income tax is to increase the value of the firm as the amount of debt in the capital structure is increased. It is obvious that if we resort to an all-debt capital structure, the value of the firm will be equal to $1,071 (the value of the firm when there is no tax), regardless of the prevailing corporate tax rate. The implication of Modigliani and Miller's model is to suggest that the firm use the maximum amount of debt possible.

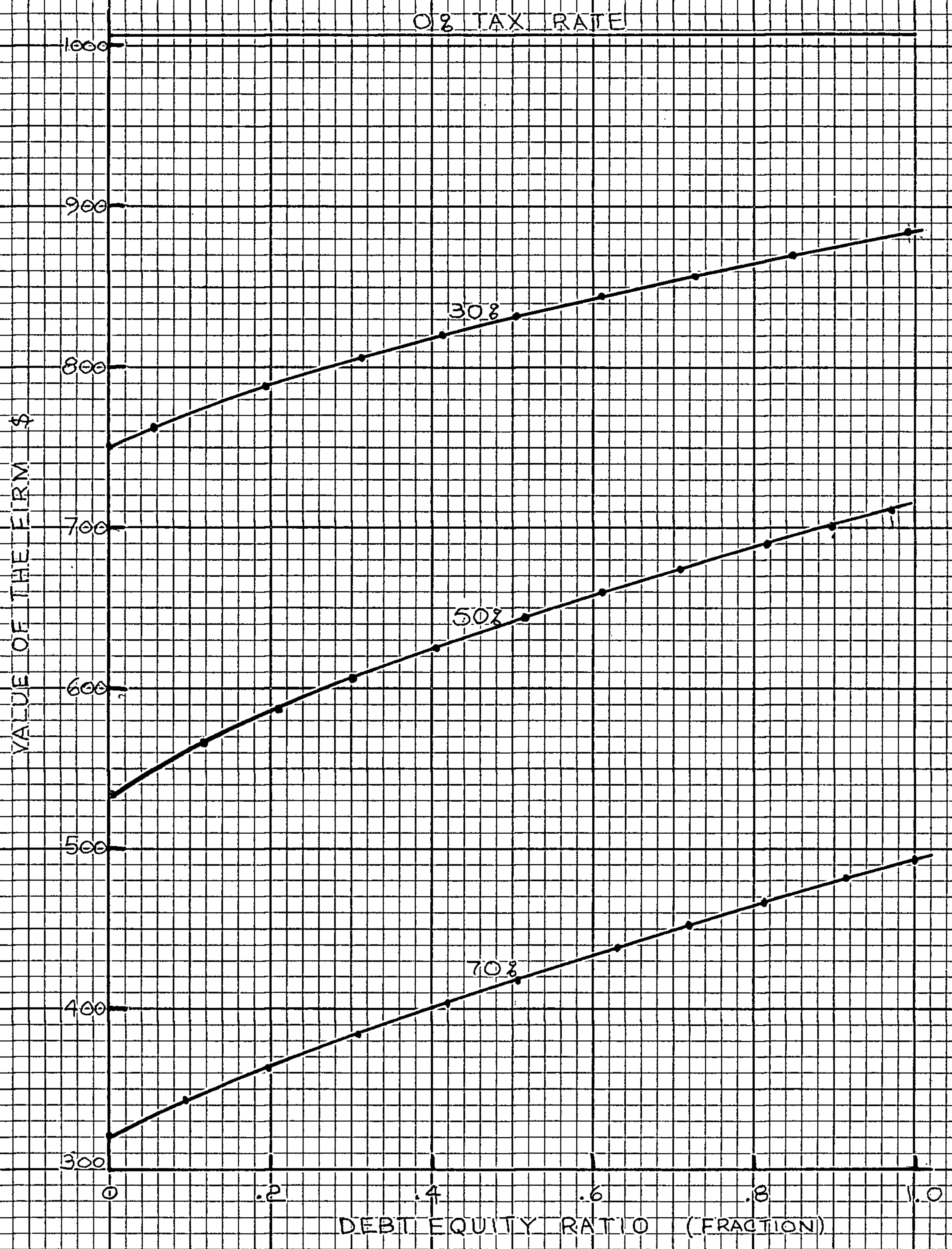
Also, from Figure III, we observe that the percentage

Figure III

MODIGLIANI AND MILLER HYPOTHESIS

EFFECT OF CORPORATE INCOME TAX RATE

ON THE VALUE OF THE FIRM



increase in value for any given increment of debt increases with increasing tax rates. If Modigliani and Miller's model provides a valid description of investor behavior, we would expect corporations to increase their use of debt as corporate income tax increases. In practice, the past 50 years have seen a great increase in corporate tax but a reduced use of debt financing. Data for Figure III is given in Appendix II.

Two other observations about Modigliani and Miller's hypothesis can be made from Figures II and III. First, since use of debt financing does increase the value of the firm, the results of the Modigliani and Miller hypothesis and the traditional hypothesis are similar. Therefore, it is very difficult to prove or disprove either hypothesis based on empirical evidence. As Modigliani and Miller admit, "the tax advantages of debt financing are somewhat greater than we originally suggested and, to this extent, the quantitative difference between the valuations implied by our position and by the traditional view is narrowed."[8]

Our second observation is that our theoretical work, based on Modigliani and Miller's model and assumptions, refutes their own empirical evidence. In their empirical work, in which they regressed cost of capital against the ratio of debt to total value, they found that the cost of capital was not significantly dependent on the debt-to-total-value ratio. Their definition of cost of capital was the ratio of total earnings <u>after taxes</u> to the market value of all securities.

If Modigliani and Miller did use after-tax data, our Figure II suggests that the cost of capital should decrease initially with increases in the debt-to-value ratio.

Analysis of Subjective Probability Distributions of Earnings

In the preceding analysis, we have considered $\bar{X}$ as the expected average value of the income to the firm. However, we know that although $\bar{X}$ is the most likely value, many other levels of income are possible. We now wish to consider a probability distribution for the income of firms in a particular risk class. Using ideas developed by Barges,[9.] we can determine how future expected economic conditions may favor the use of leverage and how the existence of limited corporate liability can influence the probability distribution in such a way that personal and corporate leverage are not perfect substitutes.

Let us consider two corporations,[10.] identical in all respects except that Company A is 100% equity financed while Company B has $200 worth of debt in its capital structure on which it pays interest at 5%, so that residual earnings to equity holders are reduced by $10. The hypothetical investor will be considered to arrive at the following subjective probability distribution to describe income from firms in this risk class.

Table V

SUBJECTIVE PROBABILITY DISTRIBUTION OF CORPORATE INCOME

Returns to Company	Subjective Probability Distribution
$25	10%
50	25%
75	30%
100	25%
125	10%

$$\bar{X} = \$75$$

In Company A, the distribution of shareholders' returns are the same as the distribution of returns to the Company. In Company B, the shareholders' returns are reduced by the interest payments of $10. The following table gives returns to shareholders in the two companies.

Table VI

DISTRIBUTION OF RETURNS TO SHAREHOLDERS FOR LEVERED & UNLEVERED FIRMS (SYMMETRICAL DISTRIBUTION)

Company A (unlevered)	Company B (levered)	Subjective Probability Distribution
$25	$15	10%
50	40	25%
75	65	30%
100	90	25%
125	115	10%

Company A - $\overline{X}$ = \$75 Company B - $\overline{X}$ = \$65

For the unlevered and levered company, the absolute range of the distributions are the same. However, the lower limit of \$25 for the unlevered company represents a potential loss of 67% $= \frac{75 - 25}{75}$, and the upper limit of \$125 represents a potential gain of 67% $= \frac{125 - 75}{75}$. For the levered company, the potential loss is now 77% $= \frac{65 - 15}{65}$, and the potential gain is now 77% $= \frac{115 - 65}{65}$. The potential gains and losses for Company B are greater than those of Company A, making the shares of Company B more "risky" than those of Company A. Before considering how much compensation equity holders should require for this added risk, that is, how much higher will K_e be for the levered firm than it is for the unlevered firm, we shall study how investors' expectations of future economic conditions can influence the desirability of leverage.

Suppose the investor thinks that the probability of inflation in the future is greater than the probability of price stability or deflation. If inflation occurs, debt interest and principal repayments remain fixed in monetary terms. However, it is generally assumed that the money earnings of corporations rise with an increase in general price level. This increase will partially or completely offset any decline in the real value of the dollar amount of the original earnings expectations of shareholders. The investor will now expect higher values of future income, so the probability distribution of earnings for all firms in this risk class will become skewed, and could take the following form:

Table VII

DISTRIBUTION OF RETURNS TO SHAREHOLDERS FOR LEVERED & UNLEVERED FIRMS (SKEWED DISTRIBUTION)

Company A (unlevered)	Company B (levered)	Subjective Probability Distribution
$25	$15	10%
50	40	20%
75	65	25%
100	90	20%
125	115	15%
150	140	10%

When possible shareholder gains and losses are calculated from the skewed distribution for Company A, the potential loss is $67\% = \frac{75 - 25}{75}$, and the potential gain is $100\% = \frac{150 - 75}{75}$. For Company B, the potential loss is $77\% = \frac{65 - 15}{65}$ and the potential gain is $115\% = \frac{140 - 65}{65}$. In both cases, the potential losses are the same as for the symmetrical distribution, but the potential percentage gain for Company B is greater than that for Company A. If Company B had employed an even higher level of leverage, the skewness of returns to shareholders of Company B would be further increased as would the potential gains of stock B compared to stock A. If a strong inflationary sentiment prevails in the minds of shareholders, they will find that leverage will increase their possible returns. It is obvious that, if investors expect future deflation, shares in unlevered companies will be more valuable to them.

In our previous examples for the levered company, there was no chance of default on interest payments whether the earnings distribution was symmetrical or skewed. Now, using a symmetrical distribution of expected earnings, we can show that for highly levered companies, the existence of limited personal liability for equity holders can result in a favorably skewed distribution of expected returns. Suppose Company B now has $1,000 of debt in its capital structure. Then the distribution of returns is as follows:

Table VIII

DISTRIBUTION OF RETURNS TO SHAREHOLDERS FOR LEVERED & UNLEVERED FIRMS EFFECTS OF LIMITED LIABILITY

Company A (unlevered)	Company B (levered)	Subjective Probability Distribution
$25	$-25	10%
50	0	25%
75	25	30%
100	50	25%
125	75	10%

For the shareholders in Company B, the maximum possible return is now \$75. If limited liability did not exist, the maximum possible loss would be \$-25 and the maximum possible gain and loss would be symmetrical about the mean. With the existence of limited personal liability for equity holders, the maximum possible loss is \$0. Thus, the negative region of the distribution of returns is truncated. The mean of the distribution must therefore be shifted upward, and the possible gain expressed as a percentage of this new mean is greater than the possible loss. The existence of limited liability limits the shareholder's possible loss, but does not limit his possible gain, resulting in a distribution which is favorably skewed. This skewed distribution does not exist when personal leverage is used as a substitute for corporate leverage, so the two forms of leverage cannot be regarded as being identical in risk.

We now examine how leverage influences the distribution of yields to equity holders. By doing this, we hope to determine the way in which the equity holders should react to the increased financial risk resulting from leverage.

In the preceding part of our analysis, we assumed that the interest required by the debt market was given by the function $K_i = .05 + 5 \times 10^{-9} (L - 125)^3$. If Proposition II were to hold, the equity capitalization rate was uniquely determined by the cost of debt and the risk class to which the firm belonged. In this part of our analysis, we want to determine if the equity capitalization rate derived from Modigliani and Miller's Proposition II is a realistic description of investor behavior. More specifically, should the equity capitalization rate increase at a constant rate as debt is first introduced into the capital structure and then increase at a decreasing rate, and finally decrease for successivley larger amounts of debt?

We use the following procedure. To provide a direct comparison with the Modigliani and Miller model, we assume that the debt market still requires an interest yield given by the function $K_i = .05 + 5 \times 10^{-9} (L - 125)^2$. The equity investor is assumed to arrive at the following conclusions concerning firms in this risk class. The investor expects an average annual income of $\bar{X}$ = \$75. However, now he considers the possible dispersion of this income. He concludes that there is a 10% chance that annual income will be no greater than \$25 per year. He also concludes that there is a 10% chance that annual income will exceed \$125. The exact form of the subjective probability distribution is not required in our analysis; however, we shall assume that the distribution is symmetrical. Since the equity investor is dealing with firms in the same risk class, all influence of business risk has been removed from our analysis and we concentrate only on financial risk. The equity holder is now concerned with the possibility that the firm will default on its interest payments. We suggest that some indication of the chance of default can be determined from the earnings accruing to equity holders after the interest on debt has been paid. As the amount of debt in the capital structure increases, the residual earnings to equity holders decreases. Also, the total interest bill increases, so the ratio of residual earnings to shareholders divided by the interest cost decreases at an ever increasing rate. We can make adjustments to our net income model and use it to determine the data required for our analysis. As we have seen, the Modigliani and Miller model forces a particular behavior upon the investor. Then, to see if this behavior is rational, we can not use the Modigliani and Miller model, but must use a model which will provide us with the data on which the investor bases his behavior. Since the net income model determines the value of equity from the function

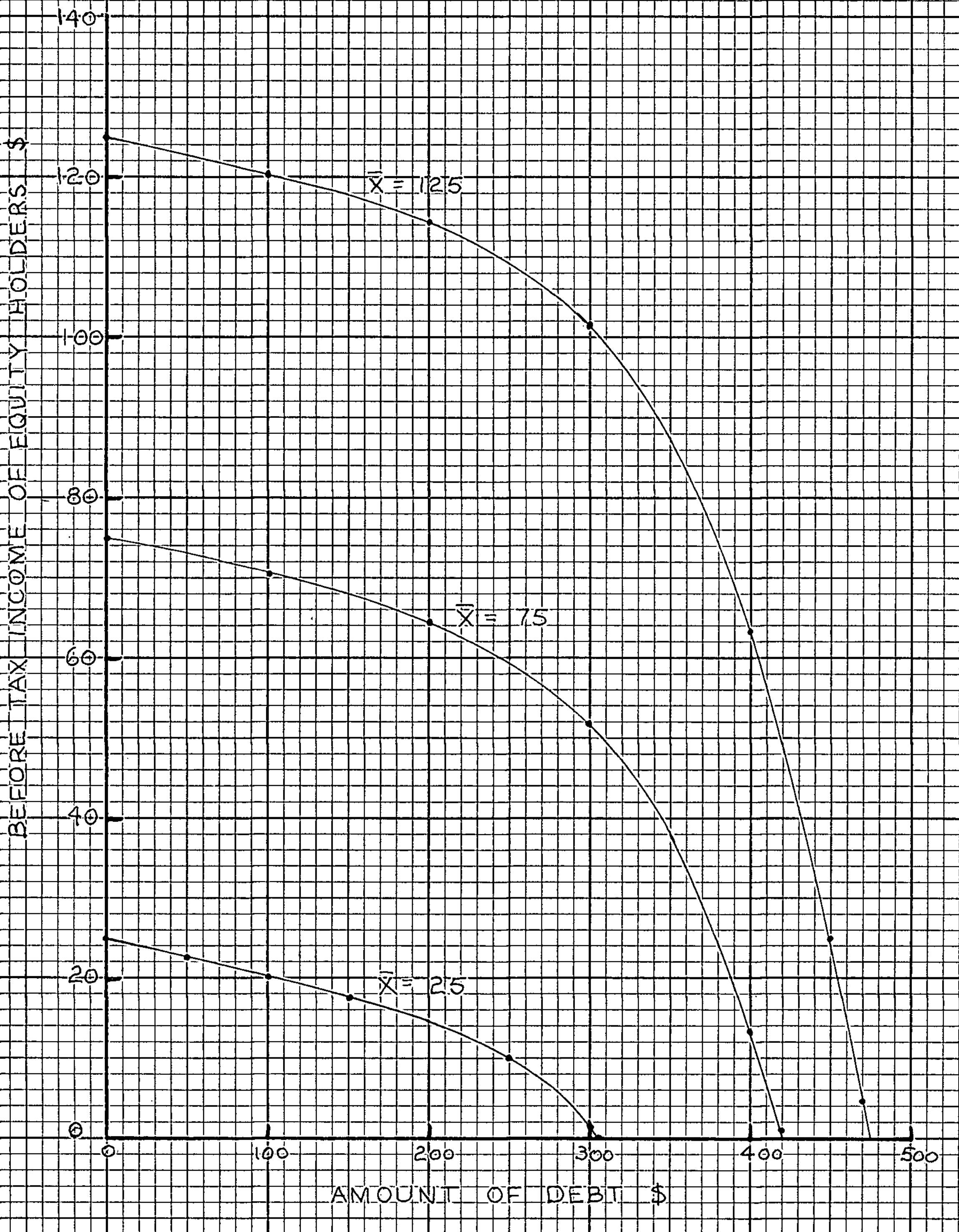
$$S = \frac{(\overline{X} - K_i L)}{K_e} (1 - \tau) \tag{40}$$

this model can be used to determine the residual earnings accruing to the equity holders if we set $K_e = 1$. Then in the data presented in Appendix IV, the column headed Value of Equity, on page 129 of this appendix gives the before-tax returns to shareholders instead of the value of the equity. Data are provided for cases in which expected corporate earnings are \$25, \$75 and \$125.

The before tax earnings of the shareholders is shown in Figure IV for various amounts of debt. From this figure, we observe that the residual earnings to equity first decrease at a linear rate and continue to do so as long as K_i is constant. As soon as the amount of debt exceeds $\overline{A}$ and K_i starts to increase, the residual earnings to equity dec-

Figure IV

BEFORE-TAX EARNINGS OF SHAREHOLDERS FOR VARIOUS LEVELS OF CORPORATE EARNINGS



rease at an increasing rate. Moreover, since the total interest bill is continually increasing, the ratio of residual earnings to interest payments is decreasing rapidly. The following table gives the ratio of residual before-tax earnings to equity holders divided by the total interest bill for expected earnings of $25, $75 and $125 and for various levels of debt in the capital structure.

Table IX

INTEREST COVERAGE OF RESIDUAL INCOME TO EQUITY HOLDERS

Amount of Debt	Ratio of Before-Tax Income to Equity Holders Divided by Interest Charges		
	X = 25	X = 75	X = 125
0	∞	∞	∞
50	9.0	29.0	49.0
100	4.0	14.0	24.0
150	2.3	9.0	15.6
200	1.4	6.2	11.0
250	.7	4.0	7.4
300	.09	2.3	4.4
350	-	1.0	2.3
400	-	.2	1.0
450	-	-	.3

From Table IX, when expected earnings are $25, with zero debt in the capital structure, the interest coverage is infinite. After $50 of debt has been introduced into the capital structure, there is still enough additional income to cover interest 9 times. The next $50 of debt introduced, making a total of $100 of debt in the capital structure, reduces the interest coverage to 4.0. This is still a fairly safe level of coverage and the equity holder faces little risk of default. However, consider his behavior as described by Modigliani and Miller. From Figure I, we observe that K_e is a linearly increasing function of the debt-equity ratio as long as the amount of debt is less than $125. This implies that the investor, who is attempting to compensate for financial risk by increasing his equity capitalization rate, associates as much risk with the first $50 of debt which reduces interest coverage from infinity to 9.0 as he does with the next $50 which reduces it from 9.0 to 4.0. We would expect that investors would find little risk associated with an interest coverage of 9.0 and probably would not require any additional compensation for bearing this risk. The next increment of debt further increases risk, but a coverage of 4.0 should not require much compensation in the form of higher equity yields. The next $50 of debt added increases total debt to $150 and reduces interest coverage to 2.3. Now there is a fair amount of financial risk associated with the capital structure. Since the amount of debt has exceeded $125, debt holders require increased compensation for their risk, so K_i starts to increase. We would expect a noticable increase in K_e, but Modigliani and Miller say this is not so. If K_e does increase, it increases at a decreasing rate, implying that equity holders now associate less risk with each successive increment of debt. As can be seen from Figure I, at even higher levels of leverage, when financial risk becomes extreme, K_e may turn downward and even be less than the yield required when there is no debt in the capital structure. Modigliani and Miller maintain that this may be caused by the risk-seeking investor who gambles on the firm's receiving the maximum possible earnings of $125 rather than the minimum earnings of $25. However, as Table IX shows, even if earnings are $125, at high levels of leverage there is still a good chance of default. We maintain that Modigliani and Miller's equity holder who views the first few fifty dollar increments to debt as being equally risky and views further increments as decreasing the financial risk is acting irrationally. We must agree with Hirshleifer who "feels very easy in asserting convexity of the indifference curves (diminishing marginal rate of substitution between certain income and expected uncertain income)."[11] We suggest an equity capitalization rate which remains constant or increases only slightly with the first few increments of debt. Then, as more debt is introduced, increasing the financial risk, the equity capitalization rate increases at an ever increasing rate to compensate for the investor's aversion to this increased risk. In the following chapter, we will study the position taken by the traditional writers who maintain that debt and equity markets independently determine the functions for K_e and K_i.

Footnotes:

1. Modigliani and Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment", p. 279

2. ibid.

3. ibid.

4. A. Barges, loc. cit.

5. D. Durand, "The Cost of Capital, Corporation Finance and the Theory of Investment: Comment", American Economic Review, Vol. XLIX, No. 4, Sept. 1959, pp. 639-55

6. Robichek and Myers, Optimal Financing Decisions, Prentice-Hall, Englewood Cliffs, N.J., 1965, chapter 5.

7. ibid., pp. 34-36

8. Modigliani and Miller, "Corporate Income Taxes and the Cost of Capital", American Economic Review, Vol. LIII, No. 3, June 1963, p. 434

9. A. Barges, op. cit., chapter 6

10. Throughout this analysis we will assume that the firm does not pay any corporate income tax.

11. J. Hirshleifer, "Risk, the Discount Rate and Investment Decisions", American Economic Review, Vol. LI, No. 2, May 1961, p. 115

CHAPTER V

AN ANALYSIS OF THE TRADITIONAL HYPOTHESIS

The Traditional Hypothesis

In this chapter we study the results obtained using the traditional and net income models. In these models, K_e and K_i are determined by the equity and debt markets respectively. While the Modigliani and Miller model treats K_e as the dependent variable, in these models, K_o is the dependent variable and is determined from the weighted average of the costs of debt and equity in the capital structure. The traditional model considers small increases in debt as causing slight increases in the cost of both debt and equity. As subsequent amounts of debt are added, the cost of both debt and equity increases at an increasing rate. Functions which meet these requirements are:

$$K_i = .05 + 1 \times 10^{-9} L^3$$

$$K_e = .07 + 1 \times 10^{-9} L^3$$

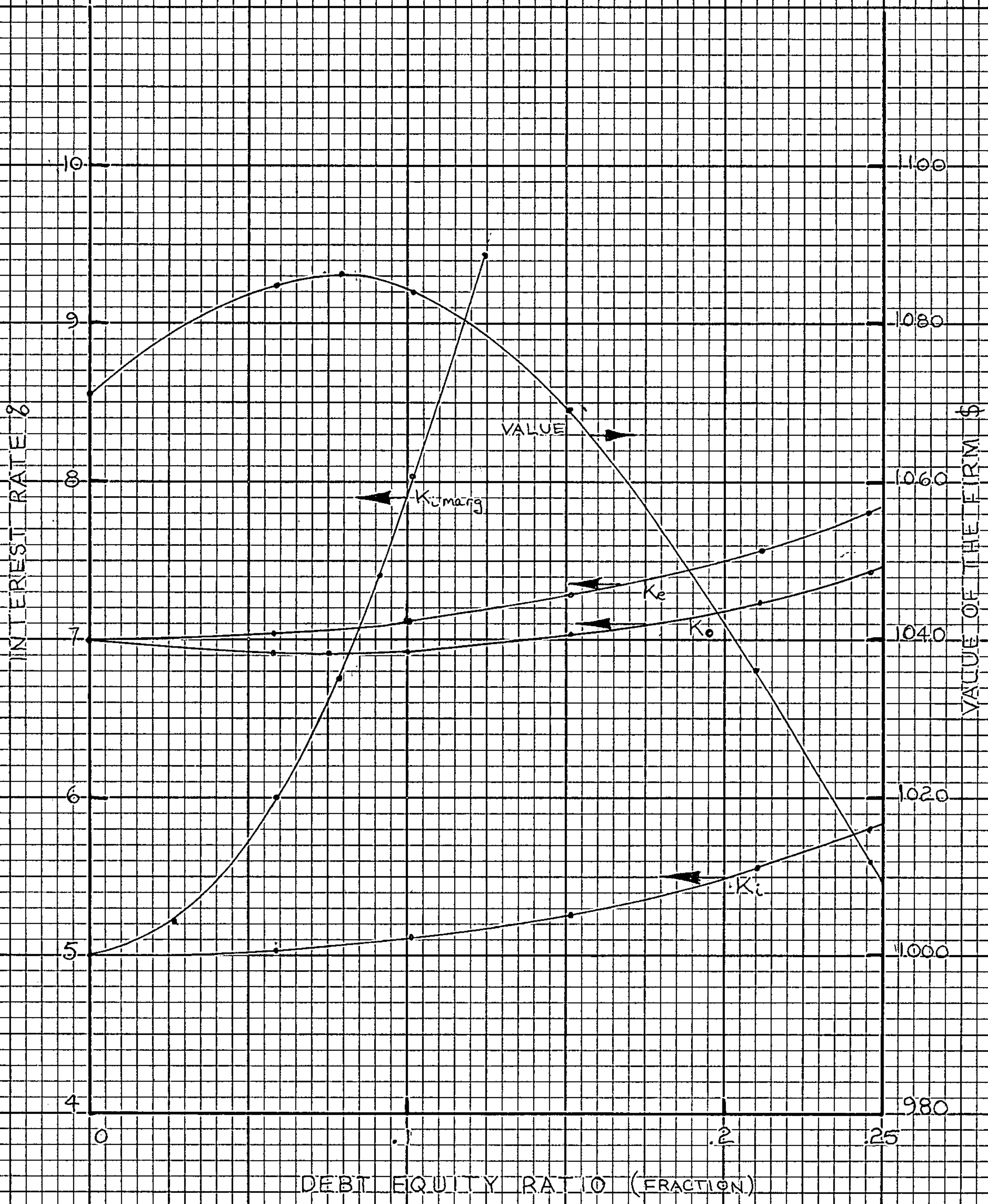
The above two functions were chosen as being representative of a moderately risky corporation which would have a maximum value at a debt-equity ratio of about 40% or at a debt-total value ratio of about 29%. The company is assumed to have an expected annual income of $75. The results of the model for cases in which the company pays no corporate income tax and in which it pays 50% corporate income tax are

Figure V

TRADITIONAL HYPOTHESIS

COST OF FUNDS AND VALUE OF THE FIRM AS A FUNCTION OF THE DEBT-EQUITY RATIO

(No Corporate Income Tax)



given in Appendix III.

We shall first analyze the case in which the firm does not pay any corporate tax. Figure V shows the value of the firm, the marginal and average cost of debt, the average cost of equity and the average cost of capital, all plotted against the debt-equity ratio. The results are not intended to be realistic, but are presented as an aid in determining the relationship between the marginal cost of debt and the average cost of capital.

We note several differences between Figure V and Figure I. In both cases, K_i is an increasing function of the amount of debt in the capital structure. In the traditional model, K_e is also an increasing function of the amount of debt. Instead of the value of the firm being constant, it now increases as debt is introduced, reaches a maximum at a debt-equity ratio of about .08, and then starts to decline. The weighted average cost of capital is no longer constant. It is now saucer-shaped. As debt is introduced, the weighted average cost of capital decreases slowly. When there is no corporate income tax, it reaches a minimum at the same debt-equity ratio at which the value of the firm is maximized. It then increases slowly as more debt is added. One other observation should be made about the average cost of capital curve. Although there is a distinct minimum, the curve is very flat. The model has been run using many other types of functions for K_e and K_i and in all cases the average cost curve was only slightly dished. If the functions we have used are at all realistic, this shallow curvature will make it almost impossible to use empirical data to test the validity of the Modigliani and Miller hypothesis or the traditional hypothesis. We also note that the minimum cost of capital occurs where the marginal cost of debt first exceeds the average cost of capital. Up to this point, any use of debt serves to reduce the weighted average cost of capital, but beyond this point, it will raise it. We should also note that the marginal cost used in this case is different from that used in the Modigliani and Miller analysis. The marginal cost of debt in this model includes a term to compensate for any change in the value of equity. The marginal costs of debt for both the Modigliani and Miller model and the traditional model are given in the data of Appendix III, where SO indicates the traditional calculation method and MM indicates the Modigliani and Miller calculation method.

Using the same functions for K_e and K_i, the data is plotted in Figure VI for a firm paying 50% corporate income tax. The firm now has a maximum value at a debt-equity ratio of 40%. However, when tax is introduced into the analysis, the minimum cost of capital occurs at a debt-equity ratio of 20%, long before the firm reaches its maximum value. We note that maximizing the value of the firm is not equivalent to minimizing the weighted average cost of capital when corporate taxes exist. However, the minimum average cost of capital is still that value at which the marginal cost of debt (including a cost for the changing value of equity) is equal

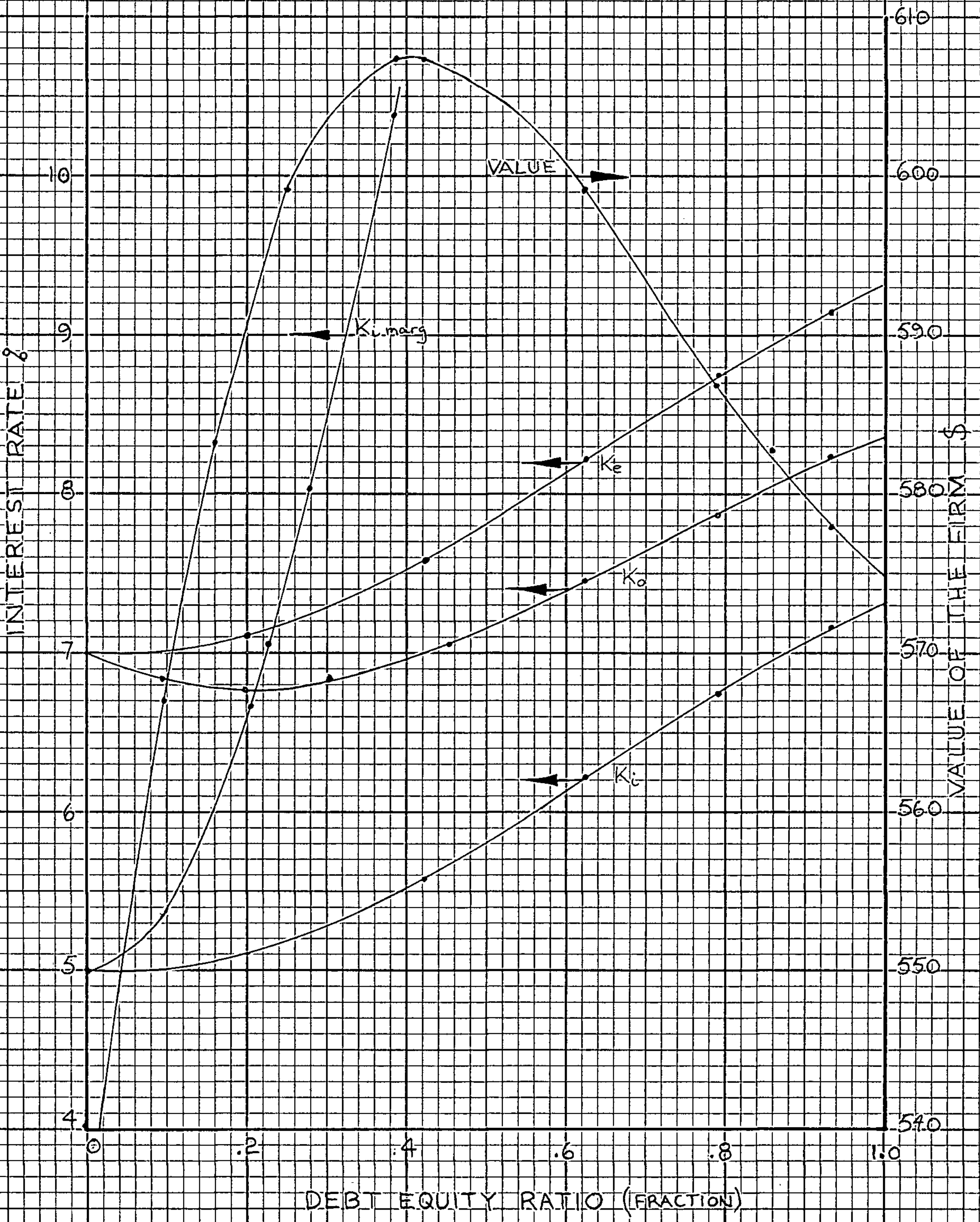
Figure VI

TRADITIONAL HYPOTHESIS

COST OF FUNDS AND VALUE OF THE FIRM AS A

FUNCTION OF THE DEBT-EQUITY RATIO

(50% Corporate Tax)



to the average cost of capital.

The Net Income Hypothesis

In the net income model, the costs of debt and equity remain constant until a certain amount of debt is introduced into the capital structure. Only when this level is exceeded is the financial risk great enough to warrant an increase in the capitalization rates. So that this analysis will be consistent with that of Chapter IV, we use the same function for K_i as was used in the Modigliani and Miller model.

$$K_i = .05 + 5 \times 10^{-9} (L - 125)^3$$

Also, $$K_e = .07 + 5 \times 10^{-9} (L - 125)^3$$

Again the firm is assumed to have expected annual earnings of $75 and pay corporate income tax at a rate of 50%. Data for the above case and for the case in which the firm does not pay any corporate tax are given in Appendix IV. From the data for the case in which the firm pays no corporate tax, the minimum cost of capital and maximum value of the firm occur at the same debt-equity ratio. The minimum cost of capital is equal to the marginal cost of debt as it was in the traditional model.

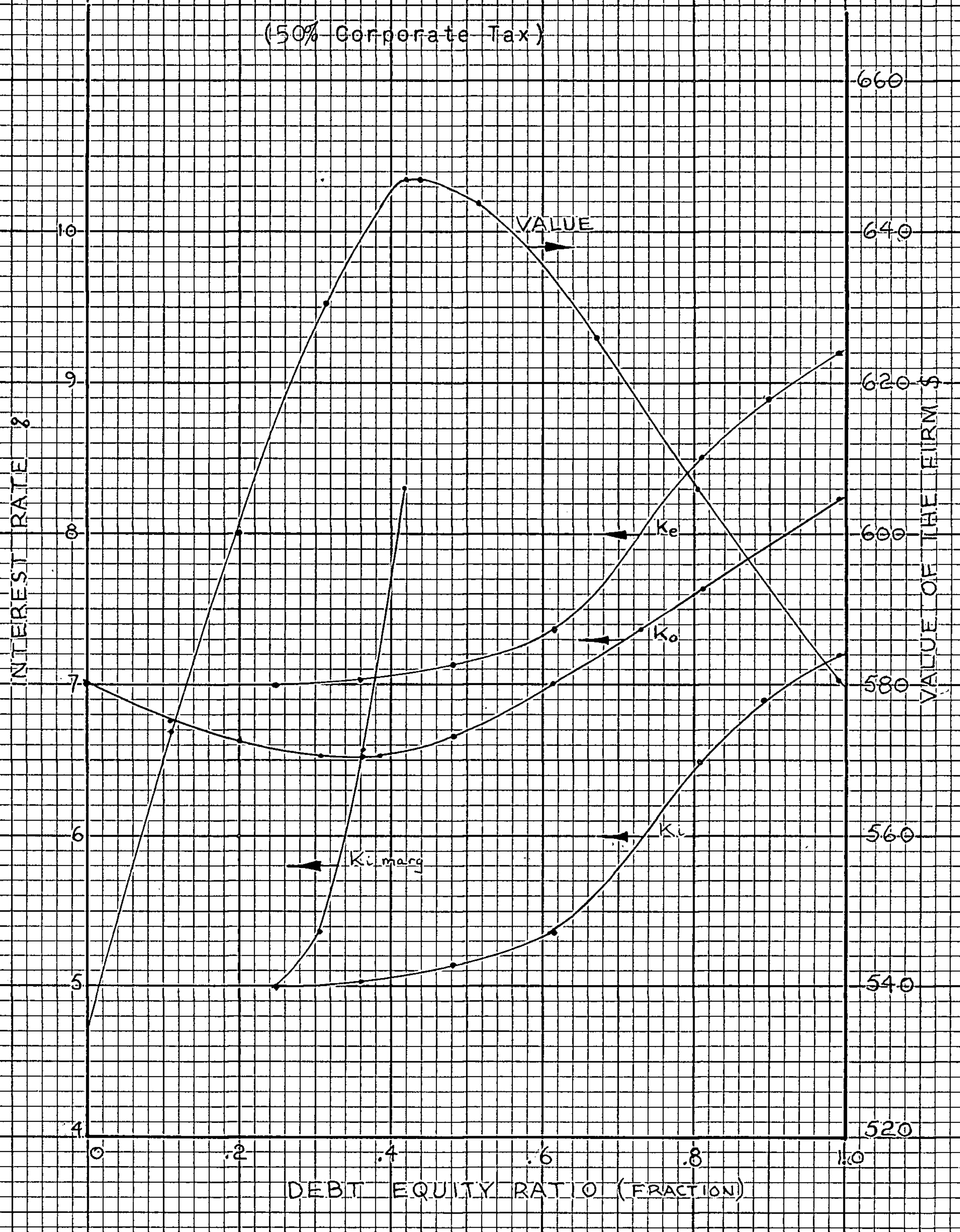
The data for the case in which the firm pays 50% corporate tax is given in Figure VII. As in the traditional model, the value of the firm increases as debt is added, reaches a maximum, and then decreases with the addition of still greater amounts of debt. In this case, the curve is more symmetrical than it was in the traditional model. The

Figure VII

NET INCOME HYPOTHESIS

COST OF FUNDS AND VALUE OF THE FIRM AS A FUNCTION OF THE DEBT-EQUITY RATIO

(50% Corporate Tax)



curves for K_e and K_i are constant until the amount of debt exceeds \$125 (a debt-equity ratio of about .23). The marginal cost curve coincides with the average cost of debt curve as long as the cost of debt remains constant. When the cost of debt starts to increase, the marginal cost of debt increases at a much greater rate than it did in the traditional model. The weighted average cost of capital is a minimum where it is equal to the marginal cost of debt. The cost of capital is not a minimum at the debt-equity ratio where the value of the firm is maximized. In this model, however, the debt-equity ratios at which the cost of capital is minimized and at which the value of the firm is maximized are much closer than they were in the traditional model. We also note that the net income model results in a more dished average cost curve with a more pronounced minimum.

CHAPTER VI

CONCLUSIONS

In this study, we have attempted to determine if the cost of capital of the firm should be unaffected by its capital structure, as suggested by Modigliani and Miller, or if the judicious use of debt can result in a decrease in the cost of capital as suggested by the traditional writers. We were forced to reject empirical methods as procedures for testing the two conflicting hypotheses. We found that it was difficult, if not impossible, to divide cpmpanies into groups in which business risk was constant. Moreover, even if these groups could be determined, the firms in a particular risk class would usually have nearly identical capital structures.

Instead, we studied the assumptions on which each model was based. Modigliani and Miller proposed that the average cost of capital, K_o, and the value of the firm, V, were independent of the financial structure. From this, they developed Proposition II, which expressed K_e as a function of the debt-equity ratio. They validated these propositions through their arbitrage argument.

There are several factors which can impede the arbitrage process. Legal restrictions prevent many large groups of investors from buying on margin. Furthermore, even those investors who can buy on margin can borrow only 10% of the total funds required. A further restriction to the arbitrage process occurs if investors view personal borrowing for margin purchases as being more risky than corporate borrowing. Modigliani and Miller maintain that arbitrage will occur, for personal or homemade leverage is a perfect substitute for corporate leverage. We suggest that this contention is not correct, for personal leverage results in unlimited personal liability, while corporate leverage results in limited personal liability. Also, Modigliani and Miller neglect the possibility that the new yield relationships brought about by the arbitrage process may not provide satisfactory risk compensation to the original shareholders. They do not consider the possibility that the original shareholders in the undervalued company, upon having their equity yield reduced by arbitrage operators, may find the shares of the overvalued company more desirable. Before a valid model can be developed along the lines pursued by Modigliani and Miller, we need to know the tradeoff the investor requires between higher yield and increased financial risk.

The Modigliani and Miller computer model is run using a hypothetical function to describe the yield required by the debt market in response to increases in leverage. It should be emphasized that the yield function is not based on any empirical evidence. However, the function does describe a risk-averse investor. Although the exact description of behavior may not be correct, it should be noted that any other function with a positive second derivative (a risk-averse investor) will give results similar to what we have obtained. From this analysis, we observe that the equity capitalization rate determined from Proposition II cannot represent rational behavior on the part of equity holders. As debt is introduced into the capital structure, the equity holders require linearly increasing compensation for risk as long as the cost of debt is constant. As soon as the cost of debt starts to increase, the yield required by the equity holders increases at a decreasing rate. When the marginal cost of debt exceeds the average cost of equity, any further increases in leverage result in a decrease in the required yield on equity. At still higher levels of debt, we have a paradox in which the senior claim is capitalized at a higher rate than the subordinate claim.

From an analysis of the Modigliani and Miller hypothesis, we note that there are many factors present in the real world which can impede the arbitrage process. However, if we admit that arbitrage is possible, then the behavior of equity holders as determined from Proposition II is irrational. Moreover, an introduction of corporate tax into the Modigliani and Miller model results in a continually increasing value of the firm for increasing use of debt, and suggests the optimal capital structure to be one composed 100% of debt.

The traditional writers think that the cost of debt as a function of leverage is determined by investors in the debt market. The cost of equity is determined in a similar fashion from the preferences of investors in the equity market. The traditional writers do not think that any cross relationship exists between the cost of debt and equity to keep the average cost of capital and value of the firm constant. In their approach, K_e and K_i are determined independently by the markets and the average cost of capital is determined by weighting the cost of debt and of equity by their proportions in the capital structure. Modigliani and Miller maintain that the risk associated with income in any risk class cannot be altered by changing the composition of the firm's liabilities. They state that:[1.]

> "a given commodity cannot consistently sell at more than one price in the market; or more precisely, that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components."

The traditional writers contend that due to market imperfections, the value of the whole will depend on the composition of its parts. Durand has shown how institutional restrictions result in the creation of a superpremium which is attached to high grade corporate bonds.[2.] Life insurance companies, pension trusts, etc., all face legal restrictions which compel them to purchase high grade corporate bonds. The demand created by these restrictions results in lower interest costs than would occur in purely competitive markets. The deductability of interest charges on corporate debt means that there is a tax advantage to using debt. Limited liability protects the equity holder in the levered company, while no such protection is offered the investor who creates personal leverage through buying on margin. The traditional writers think that these and other restrictions will hinder the arbitrage process and invalidate Proposition I and Proposition II which is based on it.

As Vickers notes, "the assumption of market perfection should not form the starting point of analysis or provide the paradigm."[3.] Unfortunately, Modigliani and Miller have studied the special case of perfect competition. The general case involves an imperfectly competitive environment where the firm can discriminate against its sources of capital, thereby influencing the elasticity of its interest cost function. In fact, Schwartz has developed a procedure by which the firm discriminates solely against the suppliers of debt funds to influence its cost of debt.[4.]

We think that due to the many differences that exist between real world markets and perfect markets, the traditional model is likely to give more realistic results than the Modigliani and Miller model. In addition, we suggest that the behavior of the equity market predicted by the Modigliani and Miller model is irrational. Our analysis does not allow us to choose between the traditional and net income hypotheses, but this is of minor importance since both models indicate that the firm can influence its cost of capital through a judicious choice of the amount of debt and equity in its capital structure.

Footnotes

1. Modigliani and Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment", p. 281

2. D. Durand, loc. cit.

3. Douglas Vickers, "Elasticity of Capital Supply, Monopsonistic Discrimination, and Optimum Capital Structure", Journal of Finance, Vol. XXII, No. 1, March 1967, pp. 1-9

4. Eli Schwartz, loc. cit.

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APPENDIX I

LIST OF SYMBOLS

K_e - equity capitalization rate

K_i - debt capitalization rate

K_o - weighted average cost of capital

L - amount of debt in the capital structure (at market value)

S - amount of equity in capital structure (at market value)

V - total market value of company

V_L - after-tax market value of a levered company

V_u - after-tax market value of an unlevered company

X - average annual before-tax earnings of corporation

$\bar{X}$ - expected average annual before-tax earnings of corporation

$\bar{X}^t$ - expected average annual after-tax earnings of corporation

Y - return to investor from a given portfolio of equity and debt

APPENDIX II

MODIGLIANI AND MILLER MODEL

Description of Program

The program user is given the choice of three debt capitalization functions by choosing a value for the variable SELECT.

If SELECT = 1, the function used is

$$K_i = a + b(L - \bar{A})$$

SELECT = 3,

$$K_i = a + b(L - \bar{A})^2$$

SELECT = 5,

$$K_i = a + b(L - \bar{A})^3$$

The user specifies values for a and b. The following data is also specified by the user:

XBAR = the earnings to be capitalized (expected net operating income before tax).

TAX = average corporate tax rate.

RHO = capitalization rate for earnings of firms in this risk class. In the before-tax case, it is the value of K_o applied to the particular risk class. In the after-tax case, it is the rate at which earnings of unlevered firms in the particular risk class are capitalized.

DELT = amount by which the level of debt in the capital structure should be increased for the next calculation.

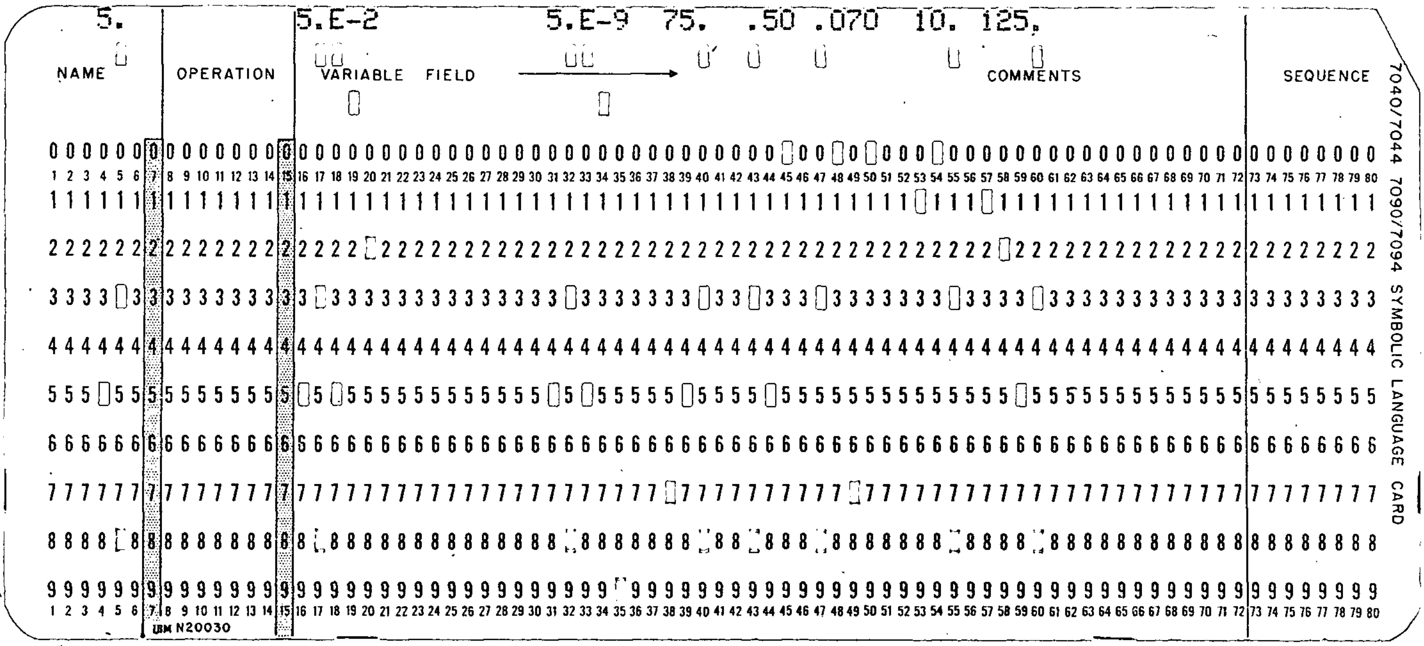
ABAR = $\bar{A}$ = the level of debt below which Ki remains constant. $\bar{A}$ may be set equal to zero to get functions similar to those described in the traditional hypothesis.

SENT = sentinel which will terminate program if not set equal to 0.

The parameters are read off one data card with the format and order as listed below.

Parameter	Format	Ending Column
SELECT	F5.0	5
A	E15.0	20
B	E15.0	35
XBAR	F5.0	40
TAX	F5.0	45
RHO	F5.0	50
DELT	F5.0	55
ABAR	F5.0	60
SENT	F5.0	65

A sample data card is given below.



MODIGLIANI AND MILLER HYPOTHESIS

```
C MODIGLIANI AND MILLER MODEL - EQUITY CAPITALIZATION RATE DETERMINATION
C  COST OF DEBT IS GIVEN BY THE MARKET -WE SPECIFY SHAPE,EARNINGS,TAXES
C
C DEFINE REAL VARIABLES
      REAL KOAT,L,KIBT,KEBT,KEAT
2     READ(5,100)SELECT,A,B,XBAR,TAX,RHO,DELT,ABAR,SENT
C SENTINAL WILL TERMINATE PROGRAM IF NON-ZERO
      IF(SENT.NE.0.)GO TO 3
      VBT=XBAR/RHO
      WRITE(6,200)XBAR,TAX,A,B,VBT,SELECT,RHO,DELT,ABAR,SENT
      WRITE(6,202)
C INITIALIZE ORIGINAL VALUES
      L=0.0
1     CONTINUE
C DETERMINE WHICH FUNCTION TO USE FOR KIBT
      IF(L-ABAR)10,10,11
10    KIBT=A
      GO TO 12
11    CONTINUE
      IF(SELECT.EQ.1.)KIBT=A+B*(L-ABAR)
      IF(SELECT.EQ.3.)KIBT=A+B*(L-ABAR)**2.
      IF(SELECT.EQ.5.)KIBT=A+B*(L-ABAR)**3.
12    CONTINUE
C THE FOLLOWING STATEMENTS DETERMINE KEBT (EQUITY BEFORE TAX)
      S=VBT-L
      W1=L/VBT
      W2=S/VBT
      W3=W1/W2
      KEBT=(RHO-W1*KIBT)/W2
C THE FOLLOWING STATEMENTS DETERMINE KEAT(EQUITY AFTER TAX)
      VAT=(XBAR*(1.-TAX))/RHO+TAX*L
      SAT=VAT-L
C   DETERMINE EARNINGS TO EQUITY (EQEARN)
      EQEARN=(1.-TAX)*(XBAR-KIBT*L)
C   KEAT=EARNINGS TO EQUITY/VALUE OF EQUITY
      KEAT=EQEARN/SAT
      W1AT=L/VAT
      W2AT=SAT/VAT
      W3AT=W1AT/W2AT
      KOAT=W1AT*KIBT+W2AT*KEAT
      WRITE(6,201)VAT,KOAT,W3,W3AT,L,S,SAT,KIBT,KEBT,KEAT
201   FORMAT(1X,1F9.3,1F10.6,2F10.6,3F10.3,3F10.6)
C INCREMENT DEBT BY AN AMOUNT DELT
      L=L+DELT
C CHECK TO SEE IF ENOUGH VALUES HAVE BEEN COMPUTED
      IF(W2-.0001)2,2,1
3     STOP
```

PROGRAM LISTING (cont'd)

MODIGLIANI AND MILLER HYPOTHESIS

```
  100     FORMAT(F5.0,2E15.0,6F5.0)
  200     FORMAT(1H1,37X,25HTHE VALUE OF THE FIRM FOR/
         1 39X,23HVARYING AMOUNTS OF DEBT//
         2 37X,27HMODIGLIANI AND MILLER MODEL//
         3 15X,40HTHE FIRM HAS DETERMINISTIC EARNINGS OF $,F5.0/
         4 15X,35HTHE FIRM HAS AN AVERAGE TAX RATE OF,F6.3/
         5 15X,41HTHE COST OF DEBT IS GIVEN BY THE FUNCTION,
         6 F16.9,1H+,F13.11,1HL/
         8 15X,37HTHE VALUE OF THE FIRM BEFORE TAX IS $,F10.3/
         9 20X,7HSELECT=,F5.0,10X,4HRHO=,F6.3,5X,
         7 5HDELT=,F5.0,5X,5HABAR=,F5.0,5HSENT=,F5.0///)
  202     FORMAT(4X,5HVALUE,4X,7HAT COST,3X,7HBT DEBT,3X,7HAT DEBT,
         1 3X,5HVALUE,5X,5HVALUE,5X,5HVALUE,5X,7HAVERAGE,3X,7HAVERAGE,
         2 2X,7HAVERAGE/4X,5HAFTER,6X,2HOF,6X,6HEQUITY,4X,6HEQUITY,
         3 6X,2HOF,6X,5HBT OF,5X,5HAT OF,6X,4HCOST,5X,7HCOST BT,2X,
         4 7HCOST AT/5X,3HTAX,5X,7HCAPITAL,
         74X,5HRATIO,5X,5HRATIO,5X,4HDEBT,
         5 5X,6HEQUITY,4X,6HEQUITY,4X,7HOF DEBT,2X,9HOF EQUITY,
         6 1X,9HOF EQUITY///)
          END
  $ENTRY
```

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

MODIGLIANI AND MILLER MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.500
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.000000005$(L-125)^3$
THE VALUE OF THE FIRM BEFORE TAX IS $ 1071.429
SELECT= 5. RHO= 0.070 DELT= 10. ABAR= 125.SENT= -0.

VALUE AFTER TAX	AT COST OF CAPITAL	BT DEBT EQUITY RATIO	AT DEBT EQUITY RATIO	VALUE OF DEBT	VALUE BT OF EQUITY	VALUE AT OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST BT OF EQUITY	AVERAGE COST AT OF EQUITY
535.714	0.070000	0.000000	0.000000	0.000	1071.429	535.714	0.050000	0.070000	0.070000
540.714	0.069815	0.009421	0.018843	10.000	1061.429	530.714	0.050000	0.070188	0.070188
545.714	0.069634	0.019022	0.038043	20.000	1051.429	525.714	0.050000	0.070380	0.070380
550.714	0.069455	0.028807	0.057613	30.000	1041.429	520.714	0.050000	0.070576	0.070576
555.714	0.069280	0.038781	0.077562	40.000	1031.429	515.714	0.050000	0.070776	0.070776
560.714	0.069108	0.048951	0.097902	50.000	1021.429	510.714	0.050000	0.070979	0.070979
565.714	0.068939	0.059322	0.118644	60.000	1011.429	505.714	0.050000	0.071186	0.071186
570.714	0.068773	0.069900	0.139800	70.000	1001.429	500.714	0.050000	0.071398	0.071398
575.714	0.068610	0.080692	0.161383	80.000	991.429	495.714	0.050000	0.071614	0.071614
580.714	0.068450	0.091703	0.183406	90.000	981.429	490.714	0.050000	0.071834	0.071834
585.714	0.068293	0.102941	0.205882	100.000	971.429	485.714	0.050000	0.072059	0.072059
590.714	0.068138	0.114413	0.228826	110.000	961.429	480.714	0.050000	0.072288	0.072288
595.714	0.067986	0.126126	0.252252	120.000	951.429	475.714	0.050000	0.072523	0.072523
600.714	0.067836	0.138088	0.276176	130.000	941.429	470.714	0.050001	0.072762	0.072762
605.714	0.067691	0.150307	0.300614	140.000	931.429	465.714	0.050017	0.073004	0.073004
610.714	0.067553	0.162791	0.325581	150.000	921.429	460.714	0.050078	0.073243	0.073243
615.714	0.067429	0.175549	0.351097	160.000	911.429	455.714	0.050214	0.073473	0.073473
620.714	0.067324	0.188590	0.377179	170.000	901.429	450.714	0.050456	0.073686	0.073686
625.714	0.067243	0.201923	0.403846	180.000	891.429	445.714	0.050832	0.073870	0.073870
630.714	0.067194	0.215559	0.431118	190.000	881.429	440.714	0.051373	0.074015	0.074015
635.714	0.067186	0.229508	0.459016	200.000	871.429	435.714	0.052109	0.074106	0.074106
640.714	0.067226	0.243781	0.487562	210.000	861.429	430.714	0.053071	0.074127	0.074127

645.714	0.067323	0.258389	0.516779	220.000	851.429	425.714	0.054287	0.074060	0.074060
650.714	0.067488	0.273345	0.546689	230.000	841.429	420.714	0.055788	0.073885	0.073885
655.714	0.067732	0.288660	0.577320	240.000	831.429	415.714	0.057604	0.073578	0.073578
660.714	0.068064	0.304348	0.608696	250.000	821.429	410.714	0.059766	0.073115	0.073115
665.714	0.068497	0.320423	0.640845	260.000	811.429	405.714	0.062302	0.072467	0.072467
670.714	0.069043	0.336898	0.673797	270.000	801.429	400.714	0.065243	0.071603	0.071603
675.714	0.069714	0.353791	0.707581	280.000	791.429	395.714	0.068619	0.070488	0.070488
680.714	0.070524	0.371115	0.742230	290.000	781.429	390.714	0.072461	0.069087	0.069087
685.714	0.071487	0.388889	0.777778	300.000	771.429	385.714	0.076797	0.067357	0.067357
690.714	0.072616	0.407129	0.814259	310.000	761.429	380.714	0.081658	0.065254	0.065254
695.714	0.073927	0.425856	0.851711	320.000	751.429	375.714	0.087074	0.062729	0.062729
700.714	0.075434	0.445087	0.890173	330.000	741.429	370.714	0.093076	0.059729	0.059729
705.714	0.077152	0.464844	0.929688	340.000	731.429	365.714	0.099692	0.056198	0.056198
710.714	0.079099	0.485149	0.970297	350.000	721.429	360.714	0.106953	0.052072	0.052072
715.714	0.081290	0.506024	1.012048	360.000	711.429	355.714	0.114889	0.047285	0.047285
720.714	0.083741	0.527495	1.054990	370.000	701.429	350.714	0.123531	0.041763	0.041763
725.714	0.086470	0.549587	1.099174	380.000	691.429	345.714	0.132907	0.035427	0.035427
730.714	0.089494	0.572327	1.144654	390.000	681.429	340.714	0.143048	0.028193	0.028193
735.714	0.092831	0.595745	1.191489	400.000	671.429	335.714	0.153984	0.019967	0.019967
740.714	0.096499	0.619870	1.239741	410.000	661.429	330.714	0.165746	0.010650	0.010650
745.714	0.100516	0.644737	1.289474	420.000	651.429	325.714	0.178362	0.000135	0.000135
750.714	0.104901	0.670379	1.340757	430.000	641.429	320.714	0.191863	-0.011694	-0.011694
755.714	0.109673	0.696833	1.393665	440.000	631.429	315.714	0.206279	-0.024964	-0.024964
760.714	0.114851	0.724138	1.448276	450.000	621.429	310.714	0.221641	-0.039809	-0.039809
765.714	0.120456	0.752336	1.504673	460.000	611.429	305.714	0.237977	-0.056375	-0.056375
770.714	0.126506	0.781473	1.562945	470.000	601.429	300.714	0.255318	-0.074821	-0.074821
775.714	0.133021	0.811594	1.623188	480.000	591.429	295.714	0.273694	-0.095317	-0.095317
780.714	0.140023	0.842752	1.685504	490.000	581.429	290.714	0.293136	-0.118048	-0.118048
785.714	0.147532	0.875000	1.750000	500.000	571.429	285.714	0.313672	-0.143213	-0.143213
790.714	0.155568	0.908397	1.816794	510.000	561.429	280.714	0.335333	-0.171028	-0.171028
795.714	0.164153	0.943005	1.886010	520.000	551.429	275.714	0.358149	-0.201726	-0.201726
800.714	0.173308	0.978892	1.957784	530.000	541.429	270.714	0.382151	-0.235562	-0.235562
805.714	0.183054	1.016129	2.032258	540.000	531.429	265.714	0.407367	-0.272808	-0.272808
810.714	0.193413	1.054795	2.109589	550.000	521.429	260.714	0.433828	-0.313764	-0.313764
815.714	0.204407	1.094972	2.189944	560.000	511.429	255.714	0.461564	-0.358752	-0.358752
820.714	0.216059	1.136752	2.273504	570.000	501.429	250.714	0.490606	-0.408124	-0.408124
825.714	0.228390	1.180233	2.360465	580.000	491.429	245.714	0.520982	-0.462264	-0.462264
830.714	0.241423	1.225519	2.451039	590.000	481.429	240.714	0.552723	-0.521586	-0.521586
835.714	0.255180	1.272727	2.545455	600.000	471.429	235.714	0.585859	-0.586548	-0.586548
840.714	0.269685	1.321981	2.643963	610.000	461.429	230.714	0.620421	-0.657646	-0.657646
845.714	0.284961	1.373418	2.746836	620.000	451.429	225.714	0.656437	-0.735423	-0.735423

THE VALUE OF THE FIRM FOR VARYING AMOUNTS OF DEBT

MODIGLIANI AND MILLER MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.500
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.00000001 L^3
THE VALUE OF THE FIRM BEFORE TAX IS $ 1071.429
SELECT= 5. RHO= 0.070 DELT= 10. ABAR= -0.SENT= -0.

VALUE AFTER TAX	AT COST OF CAPITAL	BT DEBT EQUITY RATIO	AT DEBT EQUITY RATIO	VALUE OF DEBT	VALUE BT OF EQUITY	VALUE AT OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST BT OF EQUITY	AVERAGE COST AT OF EQUITY
535.714	0.070000	0.000000	0.000000	0.000	1071.429	535.714	0.050000	0.070000	0.070000
540.714	0.069815	0.009421	0.018843	10.000	1061.429	530.714	0.050001	0.070188	0.070188
545.714	0.069634	0.019022	0.038043	20.000	1051.429	525.714	0.050008	0.070380	0.070380
550.714	0.069456	0.028807	0.057613	30.000	1041.429	520.714	0.050027	0.070575	0.070575
555.714	0.069283	0.038781	0.077562	40.000	1031.429	515.714	0.050064	0.070773	0.070773
560.714	0.069114	0.048951	0.097902	50.000	1021.429	510.714	0.050125	0.070973	0.070973
565.714	0.068951	0.059322	0.118644	60.000	1011.429	505.714	0.050216	0.071174	0.071174
570.714	0.068795	0.069900	0.139800	70.000	1001.429	500.714	0.050343	0.071374	0.071374
575.714	0.068646	0.080692	0.161383	80.000	991.429	495.714	0.050512	0.071573	0.071573
580.714	0.068507	0.091703	0.183406	90.000	981.429	490.714	0.050729	0.071767	0.071767
585.714	0.068378	0.102941	0.205882	100.000	971.429	485.714	0.051000	0.071956	0.071956
590.714	0.068262	0.114413	0.228826	110.000	961.429	480.714	0.051331	0.072136	0.072136
595.714	0.068160	0.126126	0.252252	120.000	951.429	475.714	0.051728	0.072305	0.072305
600.714	0.068074	0.138088	0.276176	130.000	941.429	470.714	0.052197	0.072458	0.072458
605.714	0.068006	0.150307	0.300614	140.000	931.429	465.714	0.052744	0.072594	0.072594
610.714	0.067958	0.162791	0.325581	150.000	921.429	460.714	0.053375	0.072706	0.072706
615.714	0.067934	0.175549	0.351097	160.000	911.429	455.714	0.054096	0.072792	0.072792
620.714	0.067934	0.188590	0.377179	170.000	901.429	450.714	0.054913	0.072845	0.072845
625.714	0.067962	0.201923	0.403846	180.000	891.429	445.714	0.055832	0.072861	0.072861
630.714	0.068021	0.215559	0.431118	190.000	881.429	440.714	0.056859	0.072833	0.072833
635.714	0.068112	0.229508	0.459016	200.000	871.429	435.714	0.058000	0.072754	0.072754
640.714	0.068240	0.243781	0.487562	210.000	861.429	430.714	0.059261	0.072618	0.072618

645.714	0.068407	0.258389	0.516779	220.000	851.429	425.714	0.060648	0.072416	0.072416
650.714	0.068616	0.273345	0.546689	230.000	841.429	420.714	0.062167	0.072141	0.072141
655.714	0.068870	0.288660	0.577320	240.000	831.429	415.714	0.063824	0.071783	0.071783
660.714	0.069172	0.304348	0.608696	250.000	821.429	410.714	0.065625	0.071332	0.071332
665.714	0.069527	0.320423	0.640845	260.000	811.429	405.714	0.067576	0.070777	0.070777
670.714	0.069936	0.336898	0.673797	270.000	801.429	400.714	0.069683	0.070107	0.070107
675.714	0.070404	0.353791	0.707581	280.000	791.429	395.714	0.071952	0.069309	0.069309
680.714	0.070935	0.371115	0.742230	290.000	781.429	390.714	0.074389	0.068371	0.068371
685.714	0.071531	0.388889	0.777778	300.000	771.429	385.714	0.077000	0.067278	0.067278
690.714	0.072197	0.407129	0.814259	310.000	761.429	380.714	0.079791	0.066014	0.066014
695.714	0.072936	0.425856	0.851711	320.000	751.429	375.714	0.082768	0.064563	0.064563
700.714	0.073753	0.445087	0.890173	330.000	741.429	370.714	0.085937	0.062907	0.062907
705.714	0.074650	0.464844	0.929688	340.000	731.429	365.714	0.089304	0.061027	0.061027
710.714	0.075633	0.485149	0.970297	350.000	721.429	360.714	0.092875	0.058902	0.058902
715.714	0.076704	0.506024	1.012048	360.000	711.429	355.714	0.096656	0.056511	0.056511
720.714	0.077868	0.527495	1.054990	370.000	701.429	350.714	0.100653	0.053831	0.053831
725.714	0.079130	0.549587	1.099174	380.000	691.429	345.714	0.104872	0.050835	0.050835
730.714	0.080493	0.572327	1.144654	390.000	681.429	340.714	0.109319	0.047497	0.047497
735.714	0.081961	0.595745	1.191489	400.000	671.429	335.714	0.114000	0.043787	0.043787
740.714	0.083539	0.619870	1.239741	410.000	661.429	330.714	0.118921	0.039675	0.039675
745.714	0.085232	0.644737	1.289474	420.000	651.429	325.714	0.124088	0.035127	0.035127
750.714	0.087042	0.670379	1.340757	430.000	641.429	320.714	0.129507	0.030108	0.030108
755.714	0.088976	0.696833	1.393665	440.000	631.429	315.714	0.135184	0.024578	0.024578
760.714	0.091037	0.724138	1.448276	450.000	621.429	310.714	0.141125	0.018496	0.018496
765.714	0.093230	0.752336	1.504673	460.000	611.429	305.714	0.147336	0.011817	0.011817
770.714	0.095559	0.781473	1.562945	470.000	601.429	300.714	0.153823	0.004495	0.004495
775.714	0.098028	0.811594	1.623188	480.000	591.429	295.714	0.160592	-0.003524	-0.003524
780.714	0.100644	0.842752	1.685504	490.000	581.429	290.714	0.167649	-0.012294	-0.012294
785.714	0.103409	0.875000	1.750000	500.000	571.429	285.714	0.175000	-0.021875	-0.021875
790.714	0.106329	0.908397	1.816794	510.000	561.429	280.714	0.182651	-0.032332	-0.032332
795.714	0.109409	0.943005	1.886010	520.000	551.429	275.714	0.190608	-0.043734	-0.043734
800.714	0.112652	0.978892	1.957784	530.000	541.429	270.714	0.198877	-0.056157	-0.056157
805.714	0.116065	1.016129	2.032258	540.000	531.429	265.714	0.207464	-0.069681	-0.069681
810.714	0.119651	1.054795	2.109589	550.000	521.429	260.714	0.216375	-0.084396	-0.084396
815.714	0.123416	1.094972	2.189944	560.000	511.429	255.714	0.225616	-0.100395	-0.100395
820.714	0.127365	1.136752	2.273504	570.000	501.429	250.714	0.235193	-0.117783	-0.117783
825.714	0.131501	1.180233	2.360465	580.000	491.429	245.714	0.245112	-0.136673	-0.136673
830.714	0.135831	1.225519	2.451039	590.000	481.429	240.714	0.255379	-0.157186	-0.157186
835.714	0.140359	1.272727	2.545455	600.000	471.429	235.714	0.266000	-0.179455	-0.179455
840.714	0.145090	1.321981	2.643963	610.000	461.429	230.714	0.276981	-0.203625	-0.203625
845.714	0.150029	1.373418	2.746836	620.000	451.429	225.714	0.288328	-0.229856	-0.229856

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

MODIGLIANI AND MILLER MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.300
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.000000005$(L-125)^3$
THE VALUE OF THE FIRM BEFORE TAX IS $ 1071.429
SELECT= 5. RHO= 0.070 DELT= 10. ABAR= 125.SENT= -0.

VALUE AFTER TAX	AT COST OF CAPITAL	BT DEBT EQUITY RATIO	AT DEBT EQUITY RATIO	VALUE OF DEBT	VALUE BT OF EQUITY	VALUE AT OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST BT OF EQUITY	AVERAGE COST AT OF EQUITY
750.000	0.070000	0.000000	0.000000	0.000	1071.429	750.000	0.050000	0.070000	0.070000
753.000	0.069920	0.009421	0.013459	10.000	1061.429	743.000	0.050000	0.070188	0.070188
756.000	0.069841	0.019022	0.027174	20.000	1051.429	736.000	0.050000	0.070380	0.070380
759.000	0.069763	0.028807	0.041152	30.000	1041.429	729.000	0.050000	0.070576	0.070576
762.000	0.069685	0.038781	0.055402	40.000	1031.429	722.000	0.050000	0.070776	0.070776
765.000	0.069608	0.048951	0.069930	50.000	1021.429	715.000	0.050000	0.070979	0.070979
768.000	0.069531	0.059322	0.084746	60.000	1011.429	708.000	0.050000	0.071186	0.071186
771.000	0.069455	0.069900	0.099857	70.000	1001.429	701.000	0.050000	0.071398	0.071398
774.000	0.069380	0.080692	0.115274	80.000	991.429	694.000	0.050000	0.071614	0.071614
777.000	0.069305	0.091703	0.131004	90.000	981.429	687.000	0.050000	0.071834	0.071834
780.000	0.069231	0.102941	0.147059	100.000	971.429	680.000	0.050000	0.072059	0.072059
783.000	0.069157	0.114413	0.163447	110.000	961.429	673.000	0.050000	0.072288	0.072288
786.000	0.069084	0.126126	0.180180	120.000	951.429	666.000	0.050000	0.072523	0.072523
789.000	0.069011	0.138088	0.197269	130.000	941.429	659.000	0.050001	0.072762	0.072762
792.000	0.068940	0.150307	0.214724	140.000	931.429	652.000	0.050017	0.073004	0.073004
795.000	0.068872	0.162791	0.232558	150.000	921.429	645.000	0.050078	0.073243	0.073243
798.000	0.068810	0.175549	0.250784	160.000	911.429	638.000	0.050214	0.073473	0.073473
801.000	0.068756	0.188590	0.269414	170.000	901.429	631.000	0.050456	0.073686	0.073686
804.000	0.068713	0.201923	0.288462	180.000	891.429	624.000	0.050832	0.073870	0.073870
807.000	0.068684	0.215559	0.307942	190.000	881.429	617.000	0.051373	0.074015	0.074015
810.000	0.068675	0.229508	0.327869	200.000	871.429	610.000	0.052109	0.074106	0.074106
813.000	0.068688	0.243781	0.348259	210.000	861.429	603.000	0.053071	0.074127	0.074127

816.000	0.068729	0.258389	0.369128	220.000	851.429	596.000	0.054287	0.074060	0.074060
819.000	0.068803	0.273345	0.390492	230.000	841.429	589.000	0.055788	0.073885	0.073885
822.000	0.068914	0.288660	0.412371	240.000	831.429	582.000	0.057604	0.073578	0.073578
825.000	0.069070	0.304348	0.434783	250.000	821.429	575.000	0.059766	0.073115	0.073115
828.000	0.069275	0.320423	0.457746	260.000	811.429	568.000	0.062302	0.072467	0.072467
831.000	0.069536	0.336898	0.481283	270.000	801.429	561.000	0.065243	0.071603	0.071603
834.000	0.069861	0.353791	0.505415	280.000	791.429	554.000	0.068619	0.070488	0.070488
837.000	0.070256	0.371115	0.530165	290.000	781.429	547.000	0.072461	0.069087	0.069087
840.000	0.070728	0.388889	0.555556	300.000	771.429	540.000	0.076797	0.067357	0.067357
843.000	0.071286	0.407129	0.581614	310.000	761.429	533.000	0.081658	0.065254	0.065254
846.000	0.071938	0.425856	0.608365	320.000	751.429	526.000	0.087074	0.062729	0.062729
849.000	0.072691	0.445087	0.635838	330.000	741.429	519.000	0.093076	0.059729	0.059729
852.000	0.073555	0.464844	0.664063	340.000	731.429	512.000	0.099692	0.056198	0.056198
855.000	0.074538	0.485149	0.693069	350.000	721.429	505.000	0.106953	0.052072	0.052072
858.000	0.075650	0.506024	0.722892	360.000	711.429	498.000	0.114889	0.047285	0.047285
861.000	0.076901	0.527495	0.753564	370.000	701.429	491.000	0.123531	0.041763	0.041763
864.000	0.078300	0.549587	0.785124	380.000	691.429	484.000	0.132907	0.035427	0.035427
867.000	0.079858	0.572327	0.817610	390.000	681.429	477.000	0.143048	0.028193	0.028193
870.000	0.081584	0.595745	0.851064	400.000	671.429	470.000	0.153984	0.019967	0.019967
873.000	0.083490	0.619870	0.885529	410.000	661.429	463.000	0.165746	0.010650	0.010650
876.000	0.085586	0.644737	0.921053	420.000	651.429	456.000	0.178362	0.000135	0.000135
879.000	0.087884	0.670379	0.957684	430.000	641.429	449.000	0.191863	-0.011694	-0.011694
882.000	0.090396	0.696833	0.995475	440.000	631.429	442.000	0.206279	-0.024964	-0.024964
885.000	0.093132	0.724138	1.034483	450.000	621.429	435.000	0.221641	-0.039809	-0.039809
888.000	0.096105	0.752336	1.074766	460.000	611.429	428.000	0.237977	-0.056375	-0.056375
891.000	0.099326	0.781473	1.116390	470.000	601.429	421.000	0.255318	-0.074821	-0.074821
894.000	0.102810	0.811594	1.159420	480.000	591.429	414.000	0.273694	-0.095317	-0.095317
897.000	0.106567	0.842752	1.203931	490.000	581.429	407.000	0.293136	-0.118048	-0.118048
900.000	0.110612	0.875000	1.250000	500.000	571.429	400.000	0.313672	-0.143213	-0.143213
903.000	0.114957	0.908397	1.297710	510.000	561.429	393.000	0.335333	-0.171028	-0.171028
906.000	0.119615	0.943005	1.347150	520.000	551.429	386.000	0.358149	-0.201726	-0.201726
909.000	0.124601	0.978892	1.398417	530.000	541.429	379.000	0.382151	-0.235562	-0.235562
912.000	0.129927	1.016129	1.451613	540.000	531.429	372.000	0.407367	-0.272808	-0.272808
915.000	0.135608	1.054795	1.506849	550.000	521.429	365.000	0.433828	-0.313764	-0.313764
918.000	0.141659	1.094972	1.564246	560.000	511.429	358.000	0.461564	-0.358752	-0.358752
921.000	0.148093	1.136752	1.623932	570.000	501.429	351.000	0.490606	-0.408124	-0.408124
924.000	0.154925	1.180233	1.686047	580.000	491.429	344.000	0.520982	-0.462264	-0.462264
927.000	0.162170	1.225519	1.750742	590.000	481.429	337.000	0.552723	-0.521586	-0.521586
930.000	0.169844	1.272727	1.818182	600.000	471.429	330.000	0.585859	-0.586548	-0.586548
933.000	0.177960	1.321981	1.888545	610.000	461.429	323.000	0.620421	-0.657646	-0.657646
936.000	0.186536	1.373418	1.962025	620.000	451.429	316.000	0.656437	-0.735423	-0.735423

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

MODIGLIANI AND MILLER MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.700
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.000000005$(L-125)^3$
THE VALUE OF THE FIRM BEFORE TAX IS $ 1071.429
SELECT= 5. RHO= 0.070 DELT= 10. ABAR= 125.SENT= -0.

VALUE AFTER TAX	AT COST OF CAPITAL	BT DEBT EQUITY RATIO	AT DEBT EQUITY RATIO	VALUE OF DEBT	VALUE BT OF EQUITY	VALUE AT OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST BT OF EQUITY	AVERAGE COST AT OF EQUITY
321.429	0.070000	0.000000	0.000000	0.000	1071.429	321.429	0.050000	0.070000	0.070000
328.429	0.069574	0.009421	0.031404	10.000	1061.429	318.429	0.050000	0.070188	0.070188
335.429	0.069165	0.019022	0.063406	20.000	1051.429	315.429	0.050000	0.070380	0.070380
342.429	0.068773	0.028807	0.096022	30.000	1041.429	312.429	0.050000	0.070576	0.070576
349.429	0.068397	0.038781	0.129271	40.000	1031.429	309.429	0.050000	0.070776	0.070776
356.429	0.068036	0.048951	0.163170	50.000	1021.429	306.429	0.050000	0.070979	0.070979
363.429	0.067689	0.059322	0.197740	60.000	1011.429	303.429	0.050000	0.071186	0.071186
370.429	0.067354	0.069900	0.233000	70.000	1001.429	300.429	0.050000	0.071398	0.071398
377.429	0.067033	0.080692	0.268972	80.000	991.429	297.429	0.050000	0.071614	0.071614
384.429	0.066722	0.091703	0.305677	90.000	981.429	294.429	0.050000	0.071834	0.071834
391.429	0.066423	0.102941	0.343137	100.000	971.429	291.429	0.050000	0.072059	0.072059
398.429	0.066135	0.114413	0.381377	110.000	961.429	288.429	0.050000	0.072288	0.072288
405.429	0.065856	0.126126	0.420420	120.000	951.429	285.429	0.050000	0.072523	0.072523
412.429	0.065587	0.138088	0.460293	130.000	941.429	282.429	0.050001	0.072762	0.072762
419.429	0.065331	0.150307	0.501022	140.000	931.429	279.429	0.050017	0.073004	0.073004
426.429	0.065095	0.162791	0.542636	150.000	921.429	276.429	0.050078	0.073243	0.073243
433.429	0.064887	0.175549	0.585162	160.000	911.429	273.429	0.050214	0.073473	0.073473
440.429	0.064719	0.188590	0.628632	170.000	901.429	270.429	0.050456	0.073686	0.073686
447.429	0.064602	0.201923	0.673077	180.000	891.429	267.429	0.050832	0.073870	0.073870
454.429	0.064548	0.215559	0.718531	190.000	881.429	264.429	0.051373	0.074015	0.074015
461.429	0.064572	0.229508	0.765027	200.000	871.429	261.429	0.052109	0.074106	0.074106
468.429	0.064687	0.243781	0.812604	210.000	861.429	258.429	0.053071	0.074127	0.074127

475.429	0.064910	0.258389	0.861298	220.000	851.429	255.429	0.054287	0.074060	0.074060
482.429	0.065257	0.273345	0.911149	230.000	841.429	252.429	0.055788	0.073885	0.073885
489.429	0.065745	0.288660	0.962199	240.000	831.429	249.429	0.057604	0.073578	0.073578
496.429	0.066392	0.304348	1.014493	250.000	821.429	246.429	0.059766	0.073115	0.073115
503.429	0.067217	0.320423	1.068075	260.000	811.429	243.429	0.062302	0.072467	0.072467
510.429	0.068239	0.336898	1.122995	270.000	801.429	240.429	0.065243	0.071603	0.071603
517.429	0.069477	0.353791	1.179302	280.000	791.429	237.429	0.068619	0.070488	0.070488
524.429	0.070952	0.371115	1.237051	290.000	781.429	234.429	0.072461	0.069087	0.069087
531.429	0.072686	0.388889	1.296296	300.000	771.429	231.429	0.076797	0.067357	0.067357
538.429	0.074699	0.407129	1.357098	310.000	761.429	228.429	0.081658	0.065254	0.065254
545.429	0.077012	0.425856	1.419518	320.000	751.429	225.429	0.087074	0.062729	0.062729
552.429	0.079649	0.445087	1.483622	330.000	741.429	222.429	0.093076	0.059729	0.059729
559.429	0.082632	0.464844	1.549479	340.000	731.429	219.429	0.099692	0.056198	0.056198
566.429	0.085984	0.485149	1.617162	350.000	721.429	216.429	0.106953	0.052072	0.052072
573.429	0.089727	0.506024	1.686747	360.000	711.429	213.429	0.114889	0.047285	0.047285
580.429	0.093887	0.527495	1.758316	370.000	701.429	210.429	0.123531	0.041763	0.041763
587.429	0.098486	0.549587	1.831956	380.000	691.429	207.429	0.132907	0.035427	0.035427
594.429	0.103548	0.572327	1.907757	390.000	681.429	204.429	0.143048	0.028193	0.028193
601.429	0.109100	0.595745	1.985816	400.000	671.429	201.429	0.153984	0.019967	0.019967
608.429	0.115164	0.619870	2.066235	410.000	661.429	198.429	0.165746	0.010650	0.010650
615.429	0.121766	0.644737	2.149123	420.000	651.429	195.429	0.178362	0.000135	0.000135
622.429	0.128932	0.670379	2.234595	430.000	641.429	192.429	0.191863	-0.011694	-0.011694
629.429	0.136686	0.696833	2.322775	440.000	631.429	189.429	0.206279	-0.024964	-0.024964
636.429	0.145054	0.724138	2.413793	450.000	621.429	186.429	0.221641	-0.039809	-0.039809
643.429	0.154063	0.752336	2.507788	460.000	611.429	183.429	0.237977	-0.056375	-0.056375
650.429	0.163738	0.781473	2.604909	470.000	601.429	180.429	0.255318	-0.074821	-0.074821
657.429	0.174105	0.811594	2.705314	480.000	591.429	177.429	0.273694	-0.095317	-0.095317
664.429	0.185190	0.842752	2.809173	490.000	581.429	174.429	0.293136	-0.118048	-0.118048
671.429	0.197020	0.875000	2.916667	500.000	571.429	171.429	0.313672	-0.143213	-0.143213
678.429	0.209623	0.908397	3.027990	510.000	561.429	168.429	0.335333	-0.171028	-0.171028
685.429	0.223023	0.943005	3.143351	520.000	551.429	165.429	0.358149	-0.201726	-0.201726
692.429	0.237249	0.978892	3.262973	530.000	541.429	162.429	0.382151	-0.235562	-0.235562
699.429	0.252327	1.016129	3.387097	540.000	531.429	159.429	0.407367	-0.272808	-0.272808
706.429	0.268284	1.054795	3.515982	550.000	521.429	156.429	0.433828	-0.313764	-0.313764
713.429	0.285149	1.094972	3.649907	560.000	511.429	153.429	0.461564	-0.358752	-0.358752
720.429	0.302947	1.136752	3.789174	570.000	501.429	150.429	0.490606	-0.408124	-0.408124
727.429	0.321707	1.180233	3.934109	580.000	491.429	147.429	0.520982	-0.462264	-0.462264
734.429	0.341455	1.225519	4.085064	590.000	481.429	144.429	0.552723	-0.521586	-0.521586
741.429	0.362221	1.272727	4.242424	600.000	471.429	141.429	0.585859	-0.586548	-0.586548
748.429	0.384031	1.321981	4.406605	610.000	461.429	138.429	0.620421	-0.657646	-0.657646
755.429	0.406913	1.373418	4.578059	620.000	451.429	135.429	0.656437	-0.735423	-0.735423

APPENDIX III

TRADITIONAL MODEL

Description of Program

The user is given the choice of three functions for required yield on debt and three functions for required yield on equity by choosing values for the variables SELECT and TELECT.

If SELECT = 1, the function used is

$K_i = a + bL$

SELECT = 3,

$K_i = a + bL^2$

SELECT = 5,

$K_i = a + bL^3$

If TELECT = 2,

$K_e = c + dL$

TELECT = 4,

$K_e = c + dL^2$

TELECT = 6,

$K_e = c + dL^3$

The user supplies the following additional parameters:

A = a in equation for debt function

B = b in equation for debt function

C = c in equation for equity function

D = d in equation for equity function

XBAR = the net operating income or expected average annual earnings of the firm before tax

TAX = average corporate tax rate

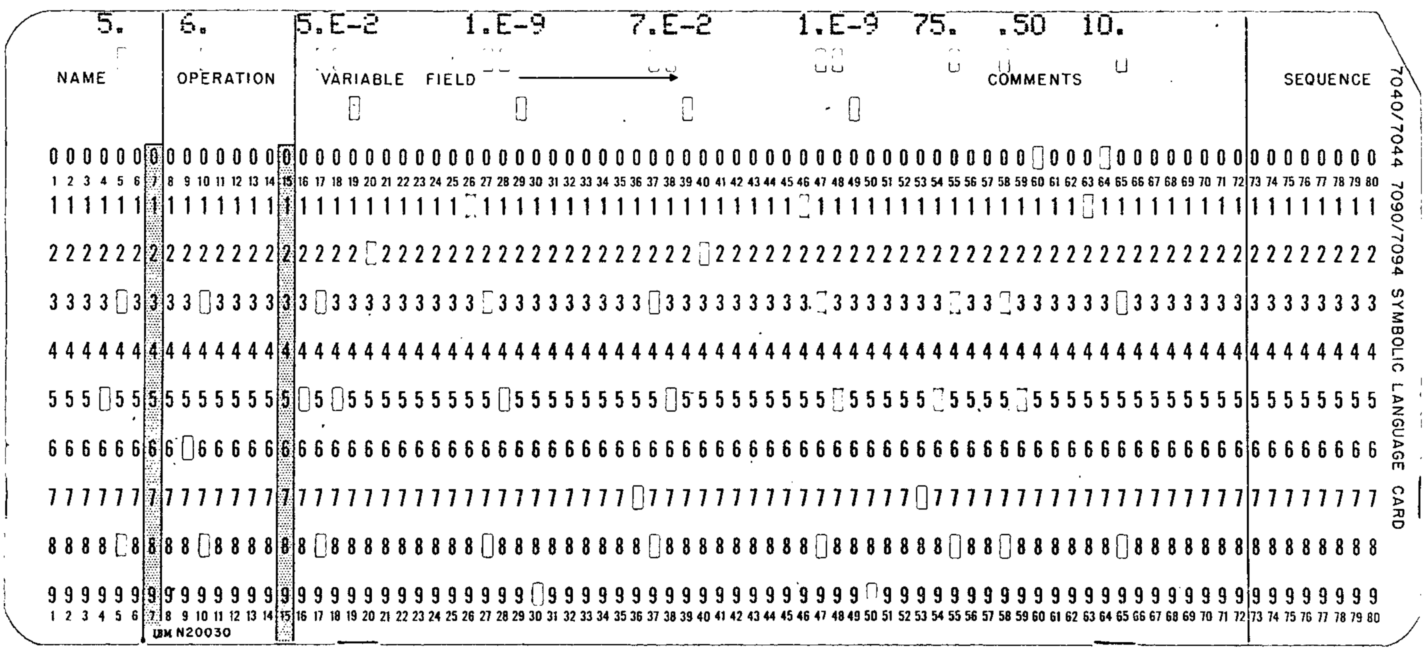
DELT = amount by which debt should be increased for each successive calculation

SENT = sentinel which will terminate program if not set equal to zero.

The parameters are read off one data card in the format and order listed below.

Parameter	Format	Ending Column
SELECT	F5.0	5
TELECT	F5.0	10
A	E10.0	20
B	E10.0	30
C	E10.0	40
D	E10.0	50
XBAR	F5.0	55
TAX	F5.0	60
DELT	F5.0	65
SENT	F5.0	70

A sample data card is given below.



PROGRAM LISTING

TRADITIONAL HYPOTHESIS

```
C VALUE OF THE FIRM - DETERMINISTIC MODEL
C TRADITIONAL MODEL
C MODEL ALLOWS CHOICE OF THREE TYPES OF FUNCTION FOR DEPENDENCE OF
C     DEBT AND EQUITY CAPITALIZATION RATES ON AMOUNT OF DEBT
C WE SPECIFY INTEREST FUNCTIONS,EARNINGS,TAX RATE,AND INCREMENT
C
C DEFINE REAL VARIABLES
      REAL L,KESTOR,ICSTOR,KIAVG,KEAVG,KIMARW,INTCOS,KOAVG,
     1KMARGL,KMARGS,KMARGR
2     READ(5,100)SELECT,TELECT,A,B,C,D,XBAR,TAX,DELT,SENT
100   FORMAT(2F5.0,4E10.0,4F5.0)
C SENTINAL WILL TERMINATE PROGRAM IF NON-ZERO
      IF(SENT.NE.0.)GO TO 3
      WRITE(6,200)XBAR,TAX,C,D,A,B,SELECT,TELECT,DELT,SENT
      WRITE(6,202)
C INITIALIZE ORIGINAL VALUES
      L=0.0
      KESTOR=C
      ICSTOR=0.0
1     CONTINUE
C DETERMINE WHICH FUNCTIONS TO USE FOR KIAVG AND KEAVG
      IF(SELECT.EQ.1.)KIAVG=A+B*L
      IF(SELECT.EQ.3.)KIAVG=A+B*L**2.
      IF(SELECT.EQ.5.)KIAVG=A+B*L**3.
      IF(TELECT.EQ.2.)KEAVG=C+D*L
      IF(TELECT.EQ.4.)KEAVG=C+D*L**2.
      IF(TELECT.EQ.6.)KEAVG=C+D*L**3.
C THE NEXT FUNCTIONS DETERMINE THE MARGINAL INTEREST RATE WHERE
C     KIMARW = D/DL OF (KIAVG*L)
C            = KIAVG + D/DL OF KIAVG MULTIPLIED BY L
      IF(SELECT.EQ.1.)KIMARW=B*L+A+B*L
      IF(SELECT.EQ.3.)KIMARW=2.*B*L**2.+A+B*L**2.
      IF(SELECT.EQ.5.)KIMARW=3.*B*L**3.+A+B*L**3.
C DETERMINE THE INTEREST COST OF DEBT
      INTCOS=KIAVG*L
C DETERMINE THE AFTER TAX EARNINGS FOR EQUITY
      EQEARN=(XBAR-INTCOS)*(1.-TAX)
C DETERMINE VALUE OF EQUITY
      S=EQEARN/KEAVG
C DETERMINE TOTAL VALUE OF FIRM
      V=L+S
C DETERMINE DEBT RATIO
      W1=L/V
C DETERMINE EQUITY RATIO
      W2=S/V
C DETERMINE THE DEBT EQUITY RATIO
      W3=W1/W2
```

PROGRAM LISTING (cont'd)

TRADITIONAL HYPOTHESIS

```
C DETERMINE AVERAGE COST OF CAPITAL
      KOAVG=W1*KIAVG+W2*KEAVG
C THE FOLLOWING FIVE STATEMENTS ARE USED TO DETERMINE THE MARGINAL COST
C  OF DEBT BY AN INCREMENTAL APPROACH. WE CONSIDER THE ADDED DEBT
C  CHARGES AND ALSO THE EFFECT OF ADDED DEBT ON THE COST OF EQUITY
      KMARGL=(INTCOS-ICSTOR)/DELT
      KMARGS=((EQEARN/KESTOR)*KEAVG-EQEARN)/DELT
      KMARGR=KMARGL+KMARGS
      KESTOR=KEAVG
      ICSTOR=INTCOS
      WRITE(6,201)V,L,S,KIAVG,KEAVG,W1,W3,KOAVG,KIMARW,KMARGR,
     1W2,KMARGL,KMARGS
201   FORMAT(1X,1F9.3,2F10.3,10F10.6)
C INCREMENT BY AN AMOUNT DELT
      L=L+DELT
C CHECK TO SEE IF ENOUGH VALUES HAVE BEEN COMPUTED
      IF(W2-.00001)2,2,1
3     STOP
200   FORMAT(1H1,37X,25HTHE VALUE OF THE FIRM FOR/
     1 39X,23HVARYING AMOUNTS OF DEBT//
     8 42X,17HTRADITIONAL MODEL//
     2 15X,40HTHE FIRM HAS DETERMINISTIC EARNINGS OF $,F5.0/
     3 15X,35HTHE FIRM HAS AN AVERAGE TAX RATE OF,F6.3/
     4 15X,43HTHE COST OF EQUITY IS GIVEN BY THE FUNCTION,F14.9,1H+,
     5 F13.11,1HL/15X,41HTHE COST OF DEBT IS GIVEN BY THE FUNCTION,
     6 F16.9,1H+,F13.11,1HL/20X,7HSELECT=,F5.0,8X,7HTELECT=,F5.0,5X,
     7 5HDELT=,F5.0,5X,5HSENT=,F5.0///)
202   FORMAT(4X,5HVALUE,5X,6HAMOUNT,4X,5HVALUE,
     1 4X,7HAVERAGE,3X,7HAVERAGE,3X,8HFRACTION,4X,4HDEBT,4X,
     2 7HAVERAGE,3X,8HMARGINAL,2X,8HMARGINAL,2X,8HFRACTION,2X,
     3 8HMARGINAL,2X,8HMARGINAL/6X,2HOF,8X,2HOF,8X,2HOF,6X,4HCOST,6X,
     4 4HCOST,8X,2HOF,6X,6HEQUITY,4X,4HCOST,5X,7HCOST MM,3X,7HCOST SO,
     5 6X,2HOF,4X,9HCOST DEBT,1X,10HCOST EQUTY/5X,4HFIRM,6X,4HDEBT,
     6 5X,6HEQUITY,3X,7HOF DEBT,2X,9HOF EQUITY,4X,4HDEBT,6X,5HRATIO,
     7 2X,10HOF CAPITAL,1X,7HOF DEBT,3X,7HOF DEBT,4X,6HEQUITY,3X,
     8 7HPORTION,3X,7HPORTION///)
      END
$ENTRY
```

THE VALUE OF THE FIRM FOR VARYING AMOUNTS OF DEBT

TRADITIONAL MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.000
THE COST OF EQUITY IS GIVEN BY THE FUNCTION 0.07000000+ 0.00000000 L^3
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.00000000 L^3
SELECT= 5. TELECT= 6. DELT= 10. SENT= -0.

VALUE OF FIRM	AMOUNT OF DEBT	VALUE OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST OF EQUITY	FRACTION OF DEBT	DEBT EQUITY RATIO	AVERAGE COST OF CAPITAL	MARGINAL COST MM OF DEBT	MARGINAL COST SO OF DEBT	FRACTION OF EQUITY
1071.429	0.000	1071.429	0.050000	0.070000	0.000000	0.000000	0.070000	0.050000	-0.000000	1.000000
1074.270	10.000	1064.270	0.050001	0.070001	0.009309	0.009396	0.069815	0.050004	0.050107	0.990691
1077.020	20.000	1057.020	0.050008	0.070008	0.018570	0.018921	0.069637	0.050032	0.050755	0.981430
1079.584	30.000	1049.584	0.050027	0.070027	0.027788	0.028583	0.069471	0.050108	0.052060	0.972212
1081.868	40.000	1041.868	0.050064	0.070064	0.036973	0.038393	0.069325	0.050256	0.054032	0.963027
1083.779	50.000	1033.779	0.050125	0.070125	0.046135	0.048366	0.069202	0.050500	0.056680	0.953865
1085.223	60.000	1025.223	0.050216	0.070216	0.055288	0.058524	0.069110	0.050864	0.060013	0.944712
1086.107	70.000	1016.107	0.050343	0.070343	0.064450	0.068890	0.069054	0.051372	0.064033	0.935550
1086.340	80.000	1006.340	0.050512	0.070512	0.073642	0.079496	0.069039	0.052048	0.068743	0.926358
1085.835	90.000	995.835	0.050729	0.070729	0.082886	0.090376	0.069071	0.052916	0.074141	0.917114
1084.507	100.000	984.507	0.051000	0.071000	0.092208	0.101574	0.069156	0.054000	0.080221	0.907792
1082.278	110.000	972.278	0.051331	0.071331	0.101637	0.113136	0.069298	0.055324	0.086973	0.898363
1079.077	120.000	959.077	0.051728	0.071728	0.111206	0.125120	0.069504	0.056912	0.094382	0.888794
1074.837	130.000	944.837	0.052197	0.072197	0.120949	0.137590	0.069778	0.058788	0.102428	0.879051
1069.504	140.000	929.504	0.052744	0.072744	0.130902	0.150618	0.070126	0.060976	0.111084	0.869098
1063.032	150.000	913.032	0.053375	0.073375	0.141106	0.164288	0.070553	0.063500	0.120321	0.858894
1055.388	160.000	895.388	0.054096	0.074096	0.151603	0.178694	0.071064	0.066384	0.130103	0.848397
1046.547	170.000	876.547	0.054913	0.074913	0.162439	0.193943	0.071664	0.069652	0.140389	0.837561
1036.502	180.000	856.502	0.055832	0.075832	0.173661	0.210157	0.072359	0.073328	0.151133	0.826339
1025.254	190.000	835.254	0.056859	0.076859	0.185320	0.227476	0.073153	0.077436	0.162287	0.814680
1012.821	200.000	812.821	0.058000	0.078000	0.197468	0.246057	0.074051	0.082000	0.173799	0.802532
999.230	210.000	789.230	0.059261	0.079261	0.210162	0.266082	0.075058	0.087044	0.185612	0.789838

984.525	220.000	764.525	0.060648	0.080648	0.223458	0.287760	0.076179	0.092592	0.197570	0.776542
968.759	230.000	738.759	0.062167	0.082167	0.237417	0.311333	0.077419	0.098668	0.209916	0.762583
951.995	240.000	711.995	0.063824	0.083824	0.252102	0.337081	0.078782	0.105296	0.222292	0.747898
934.307	250.000	684.307	0.065625	0.085625	0.267578	0.365333	0.080273	0.112500	0.234740	0.732422
915.776	260.000	655.776	0.067576	0.087576	0.283912	0.396477	0.081898	0.120304	0.247208	0.716088
896.491	270.000	626.491	0.069683	0.089683	0.301174	0.430972	0.083660	0.128732	0.259642	0.698826
876.544	280.000	596.544	0.071952	0.091952	0.319436	0.469370	0.085563	0.137808	0.271995	0.680564
856.032	290.000	566.032	0.074389	0.094389	0.338772	0.512339	0.087614	0.147556	0.284223	0.661228
835.052	300.000	535.052	0.077000	0.097000	0.359259	0.560694	0.089815	0.158000	0.296286	0.640741
813.701	310.000	503.701	0.079791	0.099791	0.380975	0.615445	0.092171	0.169164	0.308149	0.619025
792.075	320.000	472.075	0.082768	0.102768	0.404002	0.677858	0.094688	0.181072	0.319784	0.595998
770.269	330.000	440.269	0.085937	0.105937	0.428422	0.749541	0.097369	0.193748	0.331169	0.571578
748.371	340.000	408.371	0.089304	0.109304	0.454320	0.832575	0.100218	0.207216	0.342284	0.545680
726.467	350.000	376.467	0.092875	0.112875	0.481784	0.929695	0.103239	0.221500	0.353117	0.518216
704.636	360.000	344.636	0.096656	0.116656	0.510902	1.044531	0.106438	0.236624	0.363663	0.489098
682.950	370.000	312.950	0.100653	0.120653	0.541767	1.182296	0.109818	0.252612	0.373916	0.458233
661.477	380.000	281.477	0.104872	0.124872	0.574472	1.350020	0.113383	0.269488	0.383884	0.425528
640.277	390.000	250.277	0.109319	0.129319	0.609111	1.558272	0.117137	0.287276	0.393567	0.390889
619.403	400.000	219.403	0.114000	0.134000	0.645783	1.823129	0.121084	0.306000	0.402979	0.354217
598.902	410.000	188.902	0.118921	0.138921	0.684587	2.170443	0.125229	0.325684	0.412133	0.315413
578.813	420.000	158.813	0.124088	0.144088	0.725623	2.644620	0.129576	0.346352	0.421045	0.274377
559.171	430.000	129.171	0.129507	0.149507	0.768995	3.328916	0.134127	0.368028	0.429735	0.231005
540.004	440.000	100.004	0.135184	0.155184	0.814809	4.399816	0.138888	0.390736	0.438223	0.185191
521.334	450.000	71.334	0.141125	0.161125	0.863170	6.308315	0.143862	0.414500	0.446531	0.136830
503.179	460.000	43.179	0.147336	0.167336	0.914187	10.653260	0.149052	0.439344	0.454684	0.085813
485.551	470.000	15.551	0.153823	0.173823	0.967972	30.222219	0.154464	0.465292	0.462704	0.032028
468.459	480.000	-11.541	0.160592	0.180592	1.024635	-41.591989	0.160099	0.492368	0.470620	-0.024635

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

TRADITIONAL MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.500
THE COST OF EQUITY IS GIVEN BY THE FUNCTION 0.07000000+ 0.00000000IL^3
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.00000000IL^3
SELECT= 5. TELECT= 6. DELT= 10. SENT= -0.

VALUE OF FIRM	AMOUNT OF DEBT	VALUE OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST OF EQUITY	FRACTION OF DEBT	DEBT EQUITY RATIO	AVERAGE COST OF CAPITAL	MARGINAL COST MM OF DEBT	MARGINAL COST SO OF DEBT	FRACTION OF EQUITY
535.714	0.000	535.714	0.050000	0.070000	0.000000	0.000000	0.070000	0.050000	-0.000000	1.000000
542.135	10.000	532.135	0.050001	0.070001	0.018446	0.018792	0.069632	0.050004	0.050054	0.981554
548.510	20.000	528.510	0.050008	0.070008	0.036462	0.037842	0.069279	0.050032	0.050385	0.963538
554.792	30.000	524.792	0.050027	0.070027	0.054074	0.057166	0.068946	0.050108	0.051062	0.945926
560.934	40.000	520.934	0.050064	0.070064	0.071310	0.076785	0.068638	0.050256	0.052103	0.928690
566.889	50.000	516.889	0.050125	0.070125	0.088201	0.096732	0.068361	0.050500	0.053525	0.911799
572.611	60.000	512.611	0.050216	0.070216	0.104783	0.117048	0.068120	0.050864	0.055342	0.895217
578.053	70.000	508.053	0.050343	0.070343	0.121096	0.137781	0.067921	0.051372	0.057569	0.878904
583.170	80.000	503.170	0.050512	0.070512	0.137181	0.158992	0.067768	0.052048	0.060219	0.862819
587.917	90.000	497.917	0.050729	0.070729	0.153083	0.180753	0.067667	0.052916	0.063303	0.846917
592.254	100.000	492.254	0.051000	0.071000	0.168847	0.203147	0.067623	0.054000	0.066830	0.831153
596.139	110.000	486.139	0.051331	0.071331	0.184521	0.226273	0.067641	0.055324	0.070807	0.815479
599.538	120.000	479.538	0.051728	0.071728	0.200154	0.250241	0.067725	0.056912	0.075239	0.799846
602.418	130.000	472.418	0.052197	0.072197	0.215797	0.275180	0.067881	0.058788	0.080126	0.784203
604.752	140.000	464.752	0.052744	0.072744	0.231500	0.301236	0.068114	0.060976	0.085469	0.768500
606.516	150.000	456.516	0.053375	0.073375	0.247314	0.328575	0.068429	0.063500	0.091265	0.752686
607.694	160.000	447.694	0.054096	0.074096	0.263290	0.357387	0.068830	0.066384	0.097507	0.736710
608.274	170.000	438.274	0.054913	0.074913	0.279479	0.387885	0.069323	0.069652	0.104187	0.720521
608.251	180.000	428.251	0.055832	0.075832	0.295931	0.420314	0.069913	0.073328	0.111294	0.704069
607.627	190.000	417.627	0.056859	0.076859	0.312692	0.454951	0.070605	0.077436	0.118316	0.687308
606.410	200.000	406.410	0.058000	0.078000	0.329810	0.492114	0.071404	0.082000	0.126739	0.670190
604.615	210.000	394.615	0.059261	0.079261	0.347328	0.532164	0.072314	0.087044	0.135046	0.652672

602.263	220.000	382.263	0.060648	0.080648	0.365289	0.575520	0.073342	0.092592	0.143723	0.634711
599.379	230.000	369.379	0.062167	0.082167	0.383730	0.622666	0.074492	0.098668	0.152751	0.616270
595.997	240.000	355.997	0.063824	0.083824	0.402686	0.674162	0.075770	0.105296	0.162113	0.597314
592.153	250.000	342.153	0.065625	0.085625	0.422188	0.730667	0.077181	0.112500	0.171795	0.577812
587.888	260.000	327.888	0.067576	0.087576	0.442261	0.792954	0.078731	0.120304	0.181780	0.557739
583.246	270.000	313.246	0.069683	0.089683	0.462927	0.861944	0.080424	0.128732	0.192054	0.537073
578.272	280.000	298.272	0.071952	0.091952	0.484201	0.938740	0.082268	0.137808	0.202605	0.515799
573.016	290.000	283.016	0.074389	0.094389	0.506094	1.024677	0.084267	0.147556	0.213424	0.493906
567.526	300.000	267.526	0.077000	0.097000	0.528610	1.121387	0.086428	0.158000	0.224502	0.471390
561.850	310.000	251.850	0.079791	0.099791	0.551748	1.230890	0.088756	0.169164	0.235835	0.448252
556.038	320.000	236.038	0.082768	0.102768	0.575501	1.355716	0.091258	0.181072	0.247420	0.424499
550.135	330.000	220.135	0.085937	0.105937	0.599853	1.499083	0.093940	0.193748	0.259257	0.400147
544.186	340.000	204.186	0.089304	0.109304	0.624787	1.665150	0.096808	0.207216	0.271350	0.375213
538.234	350.000	188.234	0.092875	0.112875	0.650275	1.859391	0.099869	0.221500	0.283703	0.349725
532.318	360.000	172.318	0.096656	0.116656	0.676288	2.089162	0.103130	0.236624	0.296327	0.323712
526.475	370.000	156.475	0.100653	0.120653	0.702787	2.364592	0.106597	0.252612	0.309230	0.297213
520.739	380.000	140.739	0.104872	0.124872	0.729733	2.700039	0.110277	0.269488	0.322429	0.270267
515.139	390.000	125.139	0.109319	0.129319	0.757078	3.116545	0.114177	0.287276	0.335936	0.242922
509.701	400.000	109.701	0.114000	0.134000	0.784773	3.646258	0.118305	0.306000	0.349769	0.215227
504.451	410.000	94.451	0.118921	0.138921	0.812765	4.340886	0.122666	0.325684	0.363947	0.187235
499.406	420.000	79.406	0.124088	0.144088	0.840998	5.289240	0.127268	0.346352	0.378490	0.159002
494.586	430.000	64.586	0.129507	0.149507	0.869415	6.657831	0.132119	0.368028	0.393420	0.130585
490.002	440.000	50.002	0.135184	0.155184	0.897955	8.799632	0.137225	0.390736	0.408759	0.102045
485.667	450.000	35.667	0.141125	0.161125	0.926560	12.616629	0.142594	0.414500	0.424530	0.073440
481.590	460.000	21.590	0.147336	0.167336	0.955170	21.306521	0.148233	0.439344	0.440757	0.044830
477.776	470.000	7.776	0.153823	0.173823	0.983725	60.444437	0.154148	0.465292	0.457464	0.016275
474.230	480.000	-5.770	0.160592	0.180592	1.012168	-83.183978	0.160349	0.492368	0.474678	-0.012168

APPENDIX IV

NET INCOME MODEL

Description of Program

The user is given the choice of three functions for required yield on debt and three functions for required yield on equity by choosing values for the variables SELECT and TELECT.

If SELECT = 1, the function used is

$$K_i = a + b(L - \bar{A})$$

SELECT = 3

$$K_i = a + b(L - \bar{A})^2$$

SELECT = 5

$$K_i = a + b(L - \bar{A})^3$$

If TELECT = 2,

$$K_e = c + d(L - \bar{A})$$

TELECT = 4,

$$K_e = c + d(L - \bar{A})^2$$

TELECT = 6,

$$K_e = c + d(L - \bar{A})^3$$

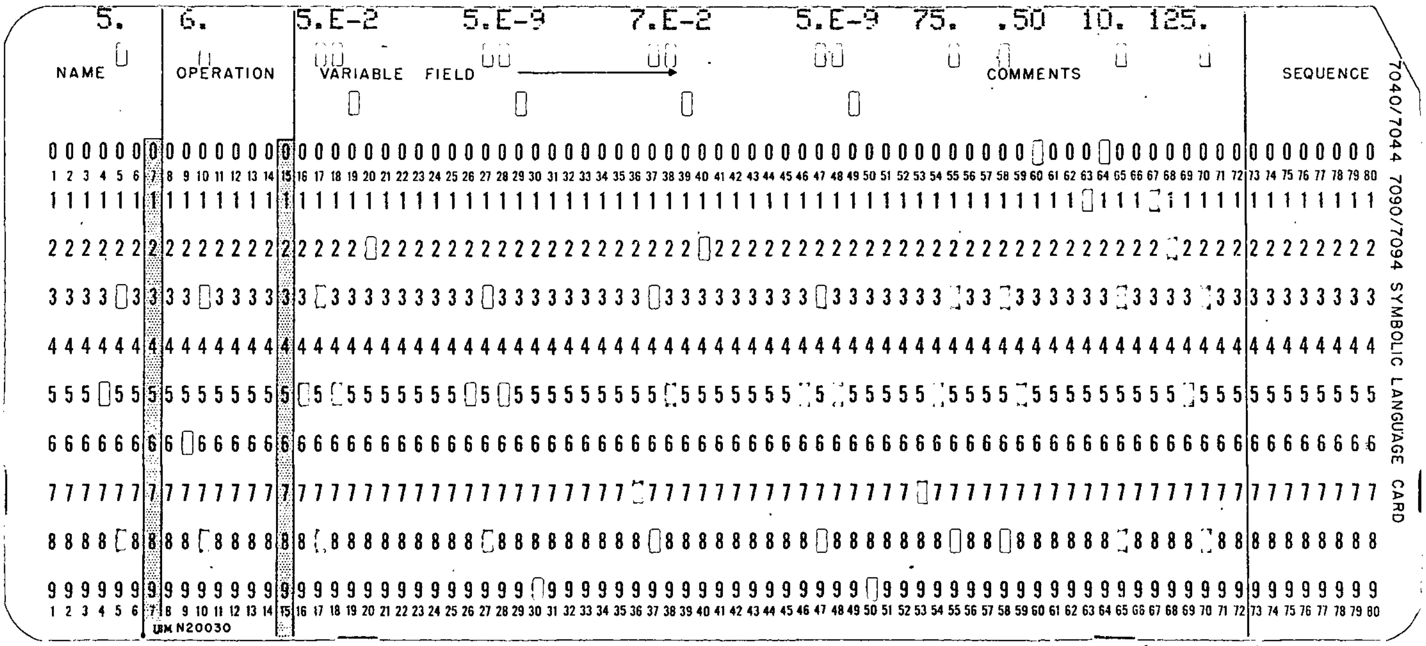
All the data specified by the user is identical to that used in the traditional model of Appendix III. In addition, the user must specify ABAR.

ABAR = $\bar{A}$ = the level of debt below which K_e and K_i remain constant.

The parameters are read off one data card in the format and order listed below.

Parameter	Format	Ending Column
SELECT	F5.0	5
TELECT	F5.0	10
A	E10.0	20
B	E10.0	30
C	E10.0	40
D	E10.0	50
XBAR	F5.0	55
TAX	F5.0	60
DELT	F5.0	65
ABAR	F5.0	70
SENT	F5.0	75

A sample data card is given below.



PROGRAM LISTING

NET INCOME MODEL

```
C VALUE OF THE FIRM - DETERMINISTIC MODEL
C NET INCOME MODEL
C MODEL ALLOWS CHOICE OF THREE TYPES OF FUNCTION FOR DEPENDENCE OF
C     DEBT AND EQUITY CAPITALIZATION RATES ON AMOUNT OF DEBT
C     IN THIS MODEL CAPITALIZATION RATES ARE CONSTANT UNTIL
C        L IS GREATER THAN ABAR
C WE SPECIFY INTEREST FUNCTIONS,EARNINGS,TAX RATE,AND INCREMENT
C
C DEFINE REAL VARIABLES
      REAL L,KESTOR,ICSTOR,KIAVG,KEAVG,KIMARW,INTCOS,KOAVG,
     1KMARGL,KMARGS,KMARGR
2     READ(5,100)SELECT,TELECT,A,B,C,D,XBAR,TAX,DELT,ABAR,SENT
100   FORMAT(2F5.0,4E10.0,5F5.0)
C SENTINAL WILL TERMINATE PROGRAM IF NON-ZERO
      IF(SENT.NE.0.)GO TO 3
      WRITE(6,200)XBAR,TAX,C,D,A,B,SELECT,TELECT,DELT,ABAR,SENT
      WRITE(6,202)
C INITIALIZE ORIGINAL VALUES
      L=0.0
      KESTOR=C
      ICSTOR=0.0
1     CONTINUE
C DETERMINE WHICH FUNCTIONS TO USE FOR KIAVG AND KEAVG
      IF(L-ABAR)10,10,11
10    KIAVG=A
      KEAVG=C
      KIMARW=A
      GO TO 12
11    CONTINUE
      IF(SELECT.EQ.1.)KIAVG=A+B*(L-ABAR)
      IF(SELECT.EQ.3.)KIAVG=A+B*(L-ABAR)**2.
      IF(SELECT.EQ.5.)KIAVG=A+B*(L-ABAR)**3.
      IF(TELECT.EQ.2.)KEAVG=C+D*(L-ABAR)
      IF(TELECT.EQ.4.)KEAVG=C+D*(L-ABAR)**2.
      IF(TELECT.EQ.6.)KEAVG=C+D*(L-ABAR)**3.
C THE NEXT FUNCTIONS DETERMINE THE MARGINAL INTEREST RATE WHERE
C     KIMARW = D/DL OF (KIAVG*L)
C            = KIAVG + D/DL OF KIAVG MULTIPLIED BY L
      IF(SELECT.EQ.1.)KIMARW=B*L+A+B*(L-ABAR)
      IF(SELECT.EQ.3.)KIMARW=2.*B*L**2.-2.*ABAR*B*L+A+B*(L-ABAR)**2.
      IF(SELECT.EQ.5.)KIMARW=3.*B*L**3.-6.*ABAR*B*L**2.+3.*B*L*ABAR**2.
     1  +A+B*(L-ABAR)**3.
12    CONTINUE
C DETERMINE THE INTEREST COST OF DEBT
      INTCOS=KIAVG*L
C DETERMINE THE AFTER TAX EARNINGS FOR EQUITY
      EQEARN=(XBAR-INTCOS)*(1.-TAX)
```

NET INCOME MODEL

```
C DETERMINE VALUE OF EQUITY
      S=EQEARN/KEAVG
C DETERMINE TOTAL VALUE OF FIRM
      V=L+S
C DETERMINE DEBT RATIO
      W1=L/V
C DETERMINE EQUITY RATIO
      W2=S/V
C DETERMINE THE DEBT EQUITY RATIO
      W3=W1/W2
C DETERMINE AVERAGE COST OF CAPITAL
      KOAVG=W1*KIAVG+W2*KEAVG
C THE FOLLOWING FIVE STATEMENTS ARE USED TO DETERMINE THE MARGINAL COST
C  OF DEBT BY AN INCREMENTAL APPROACH. WE CONSIDER THE ADDED DEBT
C  CHARGES AND ALSO THE EFFECT OF ADDED DEBT ON THE COST OF EQUITY
      KMARGL=(INTCOS-ICSTOR)/DELT
      KMARGS=((EQEARN/KESTOR)*KEAVG-EQEARN)/DELT
      KMARGR=KMARGL+KMARGS
      KESTOR=KEAVG
      ICSTOR=INTCOS
      WRITE(6,201)V,L,S,KIAVG,KEAVG,W1,W3,KOAVG,KIMARW,KMARGR,
     1W2,KMARGL,KMARGS
201   FORMAT(1X,1F9.3,2F10.3,10F10.6)
C INCREMENT BY AN AMOUNT DELT
      L=L+DELT
C CHECK TO SEE IF ENOUGH VALUES HAVE BEEN COMPUTED
      IF(W2-.00001)2,2,1
3     STOP
200   FORMAT(1H1,37X,25HTHE VALUE OF THE FIRM FOR/
     1 39X,23HVARYING AMOUNTS OF DEBT//
     8 42X,16HNET INCOME MODEL//
     2 15X,40HTHE FIRM HAS DETERMINISTIC EARNINGS OF $,F5.0/
     3 15X,35HTHE FIRM HAS AN AVERAGE TAX RATE OF,F6.3/
     4 15X,43HTHE COST OF EQUITY IS GIVEN BY THE FUNCTION,F14.9,1H+,
     5 F13.11,1HL/15X,41HTHE COST OF DEBT IS GIVEN BY THE FUNCTION,
     6 F16.9,1H+,F13.11,1HL/20X,7HSELECT=,F5.0,8X,7HTELECT=,F5.0,5X,
     7 5HDELT=,F5.0,5X,5HABAR=,F5.0,5HSENT=,F5.0///)
202   FORMAT(4X,5HVALUE,5X,6HAMOUNT,4X,5HVALUE,
     1 4X,7HAVERAGE,3X,7HAVERAGE,3X,8HFRACTION,4X,4HDEBT,4X,
     2 7HAVERAGE,3X,8HMARGINAL,2X,8HMARGINAL,2X,8HFRACTION,2X,
     3 8HMARGINAL,2X,8HMARGINAL/6X,2HOF,8X,2HOF,8X,2HOF,6X,4HCOST,6X,
     4 4HCOST,8X,2HOF,6X,6HEQUITY,4X,4HCOST,5X,7HCOST MM,3X,7HCOST SO,
     5 6X,2HOF,4X,9HCOST DEBT,1X,10HCOST EQUTY/5X,4HFIRM,6X,4HDEBT,
     6 5X,6HEQUITY,3X,7HOF DEBT,2X,9HOF EQUITY,4X,4HDEBT,6X,5HRATIO,
     7 2X,10HOF CAPITAL,1X,7HOF DEBT,3X,7HOF DEBT,4X,6HEQUITY,3X,
     8 7HPORTION,3X,7HPORTION///)
      END
```

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

NET INCOME MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.000
THE COST OF EQUITY IS GIVEN BY THE FUNCTION 0.07000000+ 0.000000005$(L-125)^3$
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.000000005$(L-125)^3$
SELECT= 5. TELECT= 6. DELT= 10. ABAR= 125.SENT= -0.

VALUE OF FIRM	AMOUNT OF DEBT	VALUE OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST OF EQUITY	FRACTION OF DEBT	DEBT EQUITY RATIO	AVERAGE COST OF CAPITAL	MARGINAL COST MM OF DEBT	MARGINAL COST SO OF DEBT	FRACTION OF EQUITY
1071.429	0.000	1071.429	0.050000	0.070000	0.000000	0.000000	0.070000	0.050000	-0.000000	1.000000
1074.286	10.000	1064.286	0.050000	0.070000	0.009309	0.009396	0.069814	0.050000	0.050000	0.990691
1077.143	20.000	1057.143	0.050000	0.070000	0.018568	0.018919	0.069629	0.050000	0.050000	0.981432
1080.000	30.000	1050.000	0.050000	0.070000	0.027778	0.028571	0.069444	0.050000	0.050000	0.972222
1082.857	40.000	1042.857	0.050000	0.070000	0.036939	0.038356	0.069261	0.050000	0.050000	0.963061
1085.714	50.000	1035.714	0.050000	0.070000	0.046053	0.048276	0.069079	0.050000	0.050000	0.953947
1088.571	60.000	1028.571	0.050000	0.070000	0.055118	0.058333	0.068898	0.050000	0.050000	0.944882
1091.429	70.000	1021.429	0.050000	0.070000	0.064136	0.068531	0.068717	0.050000	0.050000	0.935864
1094.286	80.000	1014.286	0.050000	0.070000	0.073107	0.078873	0.068538	0.050000	0.050000	0.926893
1097.143	90.000	1007.143	0.050000	0.070000	0.082031	0.089362	0.068359	0.050000	0.050000	0.917969
1100.000	100.000	1000.000	0.050000	0.070000	0.090909	0.100000	0.068182	0.050000	0.050000	0.909091
1102.857	110.000	992.857	0.050000	0.070000	0.099741	0.110791	0.068005	0.050000	0.050000	0.900259
1105.714	120.000	985.714	0.050000	0.070000	0.108527	0.121739	0.067829	0.050000	0.050000	0.891473
1108.562	130.000	978.562	0.050001	0.070001	0.117269	0.132848	0.067655	0.050049	0.050069	0.882731
1111.161	140.000	971.161	0.050017	0.070017	0.125994	0.144157	0.067497	0.050489	0.051807	0.874006
1113.043	150.000	963.043	0.050078	0.070078	0.134766	0.155756	0.067383	0.051484	0.056339	0.865234
1113.732	160.000	953.732	0.050214	0.070214	0.143661	0.167762	0.067341	0.053154	0.065278	0.856339
1112.757	170.000	942.757	0.050456	0.070456	0.152774	0.180322	0.067400	0.055619	0.077138	0.847226
1109.670	180.000	929.670	0.050832	0.070832	0.162210	0.193617	0.067588	0.058999	0.092394	0.837790
1104.057	190.000	914.057	0.051373	0.071373	0.172093	0.207864	0.067931	0.063414	0.110967	0.827907
1095.558	200.000	895.558	0.052109	0.072109	0.182555	0.223324	0.068458	0.068984	0.132714	0.817445
1083.883	210.000	873.883	0.053071	0.073071	0.193748	0.240307	0.069196	0.075829	0.157417	0.806252

1068.829	220.000	848.829	0.054287	0.074287	0.205833	0.259180	0.070170	0.084069	0.184785	0.794167
1050.296	230.000	820.296	0.055788	0.075788	0.218986	0.280386	0.071408	0.093824	0.214451	0.781014
1028.293	240.000	788.293	0.057604	0.077604	0.233397	0.304455	0.072936	0.105214	0.245983	0.766603
1002.938	250.000	752.938	0.059766	0.079766	0.249268	0.332033	0.074780	0.118359	0.278896	0.750732
974.461	260.000	714.461	0.062302	0.082302	0.266814	0.363910	0.076966	0.133379	0.312675	0.733186
943.185	270.000	673.185	0.065243	0.085243	0.286264	0.401079	0.079518	0.150394	0.346792	0.713736
909.508	280.000	629.508	0.068619	0.088619	0.307859	0.444792	0.082462	0.169524	0.380734	0.692141
873.886	290.000	583.886	0.072461	0.092461	0.331851	0.496673	0.085824	0.190889	0.414022	0.668149
836.804	300.000	536.804	0.076797	0.096797	0.358507	0.558863	0.089627	0.214609	0.446236	0.641493
798.756	310.000	488.756	0.081658	0.101658	0.388104	0.634264	0.093896	0.240804	0.477024	0.611896
760.219	320.000	440.219	0.087074	0.107074	0.420931	0.726911	0.098656	0.269594	0.506116	0.579069
721.641	330.000	391.641	0.093076	0.113076	0.457291	0.842609	0.103930	0.301099	0.533322	0.542709
683.422	340.000	343.422	0.099692	0.119692	0.497497	0.990037	0.109742	0.335439	0.558539	0.502503
645.908	350.000	295.908	0.106953	0.126953	0.541873	1.182801	0.116116	0.372734	0.581736	0.458127
609.388	360.000	249.383	0.114889	0.134889	0.590756	1.443532	0.123074	0.413104	0.602951	0.409244
574.094	370.000	204.094	0.123531	0.143531	0.644494	1.812894	0.130641	0.456669	0.622276	0.355506
540.198	380.000	160.198	0.132907	0.152907	0.703446	2.372063	0.138838	0.503549	0.639846	0.296554
507.826	390.000	117.826	0.143048	0.163048	0.767980	3.309978	0.147689	0.553864	0.655830	0.232020
477.054	400.000	77.054	0.153984	0.173984	0.838479	5.191140	0.157215	0.607734	0.670419	0.161521
447.924	410.000	37.924	0.165746	0.185746	0.915333	10.810977	0.167439	0.665279	0.683815	0.084667
420.444	420.000	0.444	0.178362	0.198362	0.998945	*********	0.178383	0.726619	0.696225	0.001055
394.594	430.000	-35.406	0.191863	0.211863	1.089726	-12.144985	0.190069	0.791874	0.707860	-0.089726

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

NET INCOME MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.500
THE COST OF EQUITY IS GIVEN BY THE FUNCTION 0.07000000+ $0.000000005(L-125)^3$
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ $0.000000005(L-125)^3$
SELECT= 5. TELECT= 6. DELT= 10. ABAR= 125.SENT= -0.

VALUE OF FIRM	AMOUNT OF DEBT	VALUE OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST OF EQUITY	FRACTION OF DEBT	DEBT EQUITY RATIO	AVERAGE COST OF CAPITAL	MARGINAL COST MM OF DEBT	MARGINAL COST SO OF DEBT	FRACTION OF EQUITY
535.714	0.000	535.714	0.050000	0.070000	0.000000	0.000000	0.070000	0.050000	-0.000000	1.000000
542.143	10.000	532.143	0.050000	0.070000	0.018445	0.018792	0.069631	0.050000	0.050000	0.981555
548.571	20.000	528.571	0.050000	0.070000	0.036458	0.037838	0.069271	0.050000	0.050000	0.963542
555.000	30.000	525.000	0.050000	0.070000	0.054054	0.057143	0.068919	0.050000	0.050000	0.945946
561.429	40.000	521.429	0.050000	0.070000	0.071247	0.076712	0.068575	0.050000	0.050000	0.928753
567.857	50.000	517.857	0.050000	0.070000	0.088050	0.096552	0.068239	0.050000	0.050000	0.911950
574.286	60.000	514.286	0.050000	0.070000	0.104478	0.116667	0.067910	0.050000	0.050000	0.895522
580.714	70.000	510.714	0.050000	0.070000	0.120541	0.137063	0.067589	0.050000	0.050000	0.879459
587.143	80.000	507.143	0.050000	0.070000	0.136253	0.157746	0.067275	0.050000	0.050000	0.863747
593.571	90.000	503.571	0.050000	0.070000	0.151625	0.178723	0.066968	0.050000	0.050000	0.848375
600.000	100.000	500.000	0.050000	0.070000	0.166667	0.200000	0.066667	0.050000	0.050000	0.833333
606.429	110.000	496.429	0.050000	0.070000	0.181390	0.221583	0.066372	0.050000	0.050000	0.818610
612.857	120.000	492.857	0.050000	0.070000	0.195804	0.243478	0.066084	0.050000	0.050000	0.804196
619.281	130.000	489.281	0.050001	0.070001	0.209921	0.265696	0.065802	0.050049	0.050039	0.790079
625.580	140.000	485.580	0.050017	0.070017	0.223792	0.288315	0.065541	0.050489	0.051017	0.776208
631.522	150.000	481.522	0.050078	0.070078	0.237522	0.311512	0.065328	0.051484	0.053887	0.762478
636.866	160.000	476.866	0.050214	0.070214	0.251230	0.335524	0.065190	0.053154	0.058768	0.748770
641.379	170.000	471.379	0.050456	0.070456	0.265054	0.360644	0.065155	0.055619	0.065727	0.734946
644.835	180.000	464.835	0.050832	0.070832	0.279141	0.387234	0.065249	0.058999	0.074811	0.720859
647.029	190.000	457.029	0.051373	0.071373	0.293650	0.415729	0.065500	0.063414	0.086041	0.706350
647.779	200.000	447.779	0.052109	0.072109	0.308747	0.446649	0.065934	0.068984	0.099406	0.691253
646.941	210.000	436.941	0.053071	0.073071	0.324604	0.480614	0.066579	0.075829	0.114856	0.675396

644.415	220.000	424.415	0.054287	0.074287	0.341395	0.518361	0.067459	0.084069	0.132307	0.658605
640.148	230.000	410.148	0.055788	0.075788	0.359292	0.560773	0.068602	0.093824	0.151633	0.640708
634.146	240.000	394.146	0.057604	0.077604	0.378462	0.608911	0.070035	0.105214	0.172680	0.621538
626.469	250.000	376.469	0.059766	0.079766	0.399062	0.664065	0.071784	0.118359	0.195266	0.600938
617.231	260.000	357.231	0.062302	0.082302	0.421236	0.727821	0.073877	0.133379	0.219192	0.578764
606.592	270.000	336.592	0.065243	0.085243	0.445110	0.802157	0.076341	0.150394	0.244254	0.554890
594.754	280.000	314.754	0.068619	0.088619	0.470783	0.889584	0.079204	0.169524	0.270256	0.529217
581.943	290.000	291.943	0.072461	0.092461	0.498331	0.993345	0.082494	0.190889	0.297019	0.501669
568.402	300.000	268.402	0.076797	0.096797	0.527796	1.117727	0.086241	0.214609	0.324392	0.472204
554.378	310.000	244.378	0.081658	0.101658	0.559185	1.268528	0.090474	0.240804	0.352260	0.440815
540.110	320.000	220.110	0.087074	0.107074	0.592472	1.453821	0.095225	0.269594	0.380547	0.407528
525.820	330.000	195.820	0.093076	0.113076	0.627591	1.685217	0.100524	0.301099	0.409219	0.372409
511.711	340.000	171.711	0.099692	0.119692	0.664438	1.980074	0.106403	0.335439	0.438283	0.335562
497.954	350.000	147.954	0.106953	0.126953	0.702876	2.365602	0.112896	0.372734	0.467786	0.297124
484.694	360.000	124.694	0.114889	0.134889	0.742736	2.887064	0.120035	0.413104	0.497805	0.257264
472.047	370.000	102.047	0.123531	0.143531	0.783821	3.625788	0.127854	0.456669	0.528446	0.216179
460.099	380.000	80.099	0.132907	0.152907	0.825909	4.744125	0.136389	0.503549	0.559837	0.174091
448.913	390.000	58.913	0.143048	0.163048	0.868766	6.619956	0.145673	0.553864	0.592123	0.131234
438.527	400.000	38.527	0.153984	0.173984	0.912144	10.382280	0.155741	0.607734	0.625458	0.087856
428.962	410.000	18.962	0.165746	0.185746	0.955795	21.621955	0.166630	0.665279	0.660006	0.044205
420.222	420.000	0.222	0.178362	0.198362	0.999472	*********	0.178372	0.726619	0.695926	0.000528
412.297	430.000	-17.703	0.191863	0.211863	1.042937	-24.289969	0.191004	0.791874	0.733387	-0.042937

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

NET INCOME MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 25.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.000
THE COST OF EQUITY IS GIVEN BY THE FUNCTION 1.00000000+ -0.00000000L
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.0000000C$(L-125)^3$
SELECT= 5. TELECT= 6. DELT= 10. ABAR= 125.SENT= -0.

VALUE OF FIRM	AMOUNT OF DEBT	VALUE OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST OF EQUITY	FRACTION OF DEBT	DEBT EQUITY RATIO	AVERAGE COST OF CAPITAL	MARGINAL COST MM OF DEBT	MARGINAL COST SO OF DEBT	FRACTION OF EQUITY
25.000	0.000	25.000	0.050000	1.000000	0.000000	0.000000	1.000000	0.050000	0.000000	1.000000
34.500	10.000	24.500	0.050000	1.000000	0.289855	0.408163	0.724638	0.050000	0.050000	0.710145
44.000	20.000	24.000	0.050000	1.000000	0.454545	0.833333	0.568182	0.050000	0.050000	0.545455
53.500	30.000	23.500	0.050000	1.000000	0.560748	1.276596	0.467290	0.050000	0.050000	0.439252
63.000	40.000	23.000	0.050000	1.000000	0.634921	1.739130	0.396825	0.050000	0.050000	0.365079
72.500	50.000	22.500	0.050000	1.000000	0.689655	2.222222	0.344828	0.050000	0.050000	0.310345
82.000	60.000	22.000	0.050000	1.000000	0.731707	2.727273	0.304878	0.050000	0.050000	0.268293
91.500	70.000	21.500	0.050000	1.000000	0.765027	3.255814	0.273224	0.050000	0.050000	0.234973
101.000	80.000	21.000	0.050000	1.000000	0.792079	3.809524	0.247525	0.050000	0.050000	0.207921
110.500	90.000	20.500	0.050000	1.000000	0.814480	4.390244	0.226244	0.050000	0.050000	0.185520
120.000	100.000	20.000	0.050000	1.000000	0.833333	5.000000	0.208333	0.050000	0.050000	0.166667
129.500	110.000	19.500	0.050000	1.000000	0.849421	5.641026	0.193050	0.050000	0.050000	0.150579
139.000	120.000	19.000	0.050000	1.000000	0.863309	6.315789	0.179856	0.050000	0.050000	0.136691
148.500	130.000	18.500	0.050001	1.000000	0.875421	7.027058	0.168350	0.050049	0.050008	0.124579
157.998	140.000	17.998	0.050017	1.000000	0.886089	7.778799	0.158230	0.050489	0.050228	0.113911
167.488	150.000	17.488	0.050078	1.000000	0.895585	8.577172	0.149264	0.051484	0.050936	0.104415
176.966	160.000	16.966	0.050214	1.000000	0.904130	9.430793	0.141270	0.053154	0.052258	0.095870
186.423	170.000	16.423	0.050456	1.000000	0.911907	10.351624	0.134104	0.055619	0.054316	0.088093
195.850	180.000	15.850	0.050832	1.000000	0.919069	11.356279	0.127649	0.058999	0.057228	0.080931
205.239	190.000	15.239	0.051373	1.000000	0.925750	12.467923	0.121809	0.063414	0.061116	0.074251
214.578	200.000	14.578	0.052109	1.000000	0.932061	13.719185	0.116508	0.068984	0.066098	0.067939
223.855	210.000	13.855	0.053071	1.000000	0.938107	15.156798	0.111679	0.075829	0.072296	0.061893

233.057	220.000	13.057	0.054287	1.000000	0.943976	16.849344	0.107270	0.084069	0.079828	0.056024
242.169	230.000	12.169	0.055788	1.000000	0.949751	18.900902	0.103234	0.093824	0.088816	0.050249
251.175	240.000	11.175	0.057604	1.000000	0.955509	21.476605	0.099532	0.105214	0.099378	0.044491
260.059	250.000	10.059	0.059766	1.000000	0.961322	24.854368	0.096132	0.118359	0.111636	0.038678
268.802	260.000	8.802	0.062302	1.000000	0.967256	29.540376	0.093005	0.133379	0.125708	0.032744
277.384	270.000	7.384	0.065243	1.000000	0.973379	36.563780	0.090128	0.150394	0.141716	0.026621
285.787	280.000	5.787	0.068619	1.000000	0.979752	48.387862	0.087478	0.169524	0.159778	0.020248
293.986	290.000	3.986	0.072461	1.000000	0.986440	72.746994	0.085038	0.190889	0.180016	0.013560
301.961	300.000	1.961	0.076797	1.000000	0.993506	**********	0.082792	0.214609	0.202548	0.006494
309.686	310.000	-0.314	0.081658	1.000000	1.001014	**********	0.080727	0.240804	0.227496	-0.001014

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

NET INCOME MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 75.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.000
THE COST OF EQUITY IS GIVEN BY THE FUNCTION 1.00000000+ -0.00000000L
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ 0.000000005$(L-125)^3$
SELECT= 5. TELECT= 6. DELT= 10. ABAR= 125.SENT= -0.

VALUE OF FIRM	AMOUNT OF DEBT	VALUE OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST OF EQUITY	FRACTION OF DEBT	DEBT EQUITY RATIO	AVERAGE COST OF CAPITAL	MARGINAL COST MM OF DEBT	MARGINAL COST SO OF DEBT	FRACTION OF EQUITY
75.000	0.000	75.000	0.050000	1.000000	0.000000	0.000000	1.000000	0.050000	0.000000	1.000000
84.500	10.000	74.500	0.050000	1.000000	0.118343	0.134228	0.887574	0.050000	0.050000	0.881657
94.000	20.000	74.000	0.050000	1.000000	0.212766	0.270270	0.797872	0.050000	0.050000	0.787234
103.500	30.000	73.500	0.050000	1.000000	0.289855	0.408163	0.724638	0.050000	0.050000	0.710145
113.000	40.000	73.000	0.050000	1.000000	0.353982	0.547945	0.663717	0.050000	0.050000	0.646018
122.500	50.000	72.500	0.050000	1.000000	0.408163	0.689655	0.612245	0.050000	0.050000	0.591837
132.000	60.000	72.000	0.050000	1.000000	0.454545	0.833333	0.568182	0.050000	0.050000	0.545455
141.500	70.000	71.500	0.050000	1.000000	0.494700	0.979021	0.530035	0.050000	0.050000	0.505300
151.000	80.000	71.000	0.050000	1.000000	0.529801	1.126761	0.496689	0.050000	0.050000	0.470199
160.500	90.000	70.500	0.050000	1.000000	0.560748	1.276596	0.467290	0.050000	0.050000	0.439252
170.000	100.000	70.000	0.050000	1.000000	0.588235	1.428571	0.441176	0.050000	0.050000	0.411765
179.500	110.000	69.500	0.050000	1.000000	0.612813	1.582734	0.417827	0.050000	0.050000	0.387187
189.000	120.000	69.000	0.050000	1.000000	0.634921	1.739130	0.396825	0.050000	0.050000	0.365079
198.500	130.000	68.500	0.050001	1.000000	0.654912	1.897812	0.377834	0.050049	0.050008	0.345088
207.998	140.000	67.998	0.050017	1.000000	0.673085	2.058895	0.360581	0.050489	0.050228	0.326915
217.488	150.000	67.488	0.050078	1.000000	0.689692	2.222608	0.344846	0.051484	0.050936	0.310308
226.966	160.000	66.966	0.050214	1.000000	0.704952	2.389283	0.330446	0.053154	0.052258	0.295048
236.423	170.000	66.423	0.050456	1.000000	0.719052	2.559372	0.317229	0.055619	0.054316	0.280948
245.850	180.000	65.850	0.050832	1.000000	0.732153	2.733474	0.305064	0.058999	0.057228	0.267847
255.239	190.000	65.239	0.051373	1.000000	0.744400	2.912364	0.293842	0.063414	0.061116	0.255600
264.578	200.000	64.578	0.052109	1.000000	0.755920	3.097024	0.283470	0.068984	0.066098	0.244080
273.855	210.000	63.855	0.053071	1.000000	0.766829	3.288692	0.273867	0.075829	0.072296	0.233171

283.057	220.000	63.057	0.054287	1.000000	0.777229	3.488913	0.264964	0.084069	0.079828	0.222771
292.169	230.000	62.169	0.055788	1.000000	0.787216	3.699609	0.256701	0.093824	0.088816	0.212784
301.175	240.000	61.175	0.057604	1.000000	0.796879	3.923174	0.249025	0.105214	0.099378	0.203121
310.059	250.000	60.059	0.059766	1.000000	0.806299	4.162602	0.241890	0.118359	0.111636	0.193701
318.802	260.000	58.802	0.062302	1.000000	0.815554	4.421655	0.235256	0.133379	0.125708	0.184446
327.384	270.000	57.384	0.065243	1.000000	0.824719	4.705115	0.229089	0.150394	0.141716	0.175281
335.787	280.000	55.787	0.068619	1.000000	0.833863	5.019129	0.223356	0.169524	0.159778	0.166137
343.986	290.000	53.986	0.072461	1.000000	0.843057	5.371721	0.218032	0.190889	0.180016	0.156943
351.961	300.000	51.961	0.076797	1.000000	0.852367	5.773568	0.213092	0.214609	0.202548	0.147633
359.686	310.000	49.686	0.081658	1.000000	0.861863	6.239184	0.208515	0.240804	0.227496	0.138137
367.136	320.000	47.136	0.087074	1.000000	0.871611	6.788837	0.204284	0.269594	0.254978	0.128389
374.285	330.000	44.285	0.093076	1.000000	0.881681	7.451726	0.200382	0.301099	0.285116	0.118319
381.105	340.000	41.105	0.099692	1.000000	0.892143	8.271547	0.196796	0.335439	0.318028	0.107857
387.566	350.000	37.566	0.106953	1.000000	0.903071	9.316834	0.193515	0.372734	0.353836	0.096929
393.640	360.000	33.640	0.114889	1.000000	0.914542	10.701599	0.190529	0.413104	0.392658	0.085458
399.294	370.000	29.294	0.123531	1.000000	0.926636	12.630713	0.187832	0.456669	0.434616	0.073364
404.495	380.000	24.495	0.132907	1.000000	0.939442	15.513122	0.185416	0.503549	0.479828	0.060558
409.211	390.000	19.211	0.143048	1.000000	0.953053	20.300620	0.183279	0.553864	0.528416	0.046947
413.406	400.000	13.406	0.153984	1.000000	0.967571	29.836818	0.181420	0.607734	0.580498	0.032429
417.044	410.000	7.044	0.165746	1.000000	0.983109	58.203133	0.179837	0.665279	0.636196	0.016891
420.088	420.000	0.088	0.178362	1.000000	0.999790	**********	0.178534	0.726619	0.695627	0.000210
422.499	430.000	-7.501	0.191863	1.000000	1.017754	-57.324680	0.177515	0.791874	0.758915	-0.017754

THE VALUE OF THE FIRM FOR
VARYING AMOUNTS OF DEBT

NET INCOME MODEL

THE FIRM HAS DETERMINISTIC EARNINGS OF $ 125.
THE FIRM HAS AN AVERAGE TAX RATE OF 0.000
THE COST OF EQUITY IS GIVEN BY THE FUNCTION 1.00000000+ -0.00000000L
THE COST OF DEBT IS GIVEN BY THE FUNCTION 0.05000000+ $0.000000005(L-125)^3$
SELECT= 5. TELECT= 6. DELT= 10. ABAR= 125.SENT= -0.

VALUE OF FIRM	AMOUNT OF DEBT	VALUE OF EQUITY	AVERAGE COST OF DEBT	AVERAGE COST OF EQUITY	FRACTION OF DEBT	DEBT EQUITY RATIO	AVERAGE COST OF CAPITAL	MARGINAL COST MM OF DEBT	MARGINAL COST SO OF DEBT	FRACTION OF EQUITY
125.000	0.000	125.000	0.050000	1.000000	0.000000	0.000000	1.000000	0.050000	0.000000	1.000000
134.500	10.000	124.500	0.050000	1.000000	0.074349	0.080321	0.929368	0.050000	0.050000	0.925651
144.000	20.000	124.000	0.050000	1.000000	0.138889	0.161290	0.868056	0.050000	0.050000	0.861111
153.500	30.000	123.500	0.050000	1.000000	0.195440	0.242915	0.814332	0.050000	0.050000	0.804560
163.000	40.000	123.000	0.050000	1.000000	0.245399	0.325203	0.766871	0.050000	0.050000	0.754601
172.500	50.000	122.500	0.050000	1.000000	0.289855	0.408163	0.724638	0.050000	0.050000	0.710145
182.000	60.000	122.000	0.050000	1.000000	0.329670	0.491803	0.686813	0.050000	0.050000	0.670330
191.500	70.000	121.500	0.050000	1.000000	0.365535	0.576132	0.652742	0.050000	0.050000	0.634465
201.000	80.000	121.000	0.050000	1.000000	0.398010	0.661157	0.621891	0.050000	0.050000	0.601990
210.500	90.000	120.500	0.050000	1.000000	0.427553	0.746888	0.593824	0.050000	0.050000	0.572447
220.000	100.000	120.000	0.050000	1.000000	0.454545	0.833333	0.568182	0.050000	0.050000	0.545455
229.500	110.000	119.500	0.050000	1.000000	0.479303	0.920502	0.544662	0.050000	0.050000	0.520697
239.000	120.000	119.000	0.050000	1.000000	0.502092	1.008403	0.523013	0.050000	0.050000	0.497908
248.500	130.000	118.500	0.050001	1.000000	0.523139	1.097047	0.503018	0.050049	0.050008	0.476861
257.998	140.000	117.998	0.050017	1.000000	0.542641	1.186464	0.484501	0.050489	0.050228	0.457359
267.488	150.000	117.488	0.050078	1.000000	0.560772	1.276723	0.467310	0.051484	0.050936	0.439228
276.966	160.000	116.966	0.050214	1.000000	0.577689	1.367922	0.451319	0.053154	0.052258	0.422311
286.423	170.000	116.423	0.050456	1.000000	0.593529	1.460198	0.436418	0.055619	0.054316	0.406471
295.850	180.000	115.850	0.050832	1.000000	0.608416	1.553730	0.422511	0.058999	0.057228	0.391584
305.239	190.000	115.239	0.051373	1.000000	0.622463	1.648746	0.409515	0.063414	0.061116	0.377537
314.578	200.000	114.578	0.052109	1.000000	0.635772	1.745534	0.397358	0.068984	0.066098	0.364228
323.855	210.000	113.855	0.053071	1.000000	0.648438	1.844449	0.385975	0.075829	0.072296	0.351562

333.057	220.000	113.057	0.054287	1.000000	0.660548	1.945923	0.375311	0.084069	0.079828	0.339452
342.169	230.000	112.169	0.055788	1.000000	0.672183	2.050482	0.365317	0.093824	0.088816	0.327817
351.175	240.000	111.175	0.057604	1.000000	0.683420	2.158760	0.355948	0.105214	0.099378	0.316580
360.059	250.000	110.059	0.059766	1.000000	0.694331	2.271517	0.347166	0.118359	0.111636	0.305669
368.802	260.000	108.802	0.062302	1.000000	0.704986	2.389673	0.338936	0.133379	0.125708	0.295014
377.384	270.000	107.384	0.065243	1.000000	0.715451	2.514333	0.331227	0.150394	0.141716	0.284549
385.787	280.000	105.787	0.068619	1.000000	0.725790	2.646839	0.324013	0.169524	0.159778	0.274210
393.986	290.000	103.986	0.072461	1.000000	0.736066	2.788826	0.317270	0.190889	0.180016	0.263934
401.961	300.000	101.961	0.076797	1.000000	0.746341	2.942303	0.310975	0.214609	0.202548	0.253659
409.686	310.000	99.686	0.081658	1.000000	0.756677	3.109765	0.305112	0.240804	0.227496	0.243323
417.136	320.000	97.136	0.087074	1.000000	0.767136	3.294343	0.299662	0.269594	0.254978	0.232864
424.285	330.000	94.285	0.093076	1.000000	0.777779	3.500025	0.294613	0.301099	0.285116	0.222221
431.105	340.000	91.105	0.099692	1.000000	0.788671	3.731967	0.289953	0.335439	0.318028	0.211329
437.566	350.000	87.566	0.106953	1.000000	0.799879	3.996966	0.285671	0.372734	0.353836	0.200121
443.640	360.000	83.640	0.114889	1.000000	0.811469	4.304169	0.281760	0.413104	0.392658	0.188531
449.294	370.000	79.294	0.123531	1.000000	0.823515	4.666198	0.278214	0.456669	0.434616	0.176485
454.495	380.000	74.495	0.132907	1.000000	0.836092	5.100987	0.275030	0.503549	0.479828	0.163908
459.211	390.000	69.211	0.143048	1.000000	0.849282	5.634923	0.272206	0.553864	0.528416	0.150718
463.406	400.000	63.406	0.153984	1.000000	0.863173	6.308526	0.269742	0.607734	0.580498	0.136826
467.044	410.000	57.044	0.165746	1.000000	0.877861	7.187397	0.267641	0.665279	0.636196	0.122139
470.088	420.000	50.088	0.178362	1.000000	0.893450	8.385239	0.265908	0.726619	0.695627	0.106550
472.499	430.000	42.499	0.191863	1.000000	0.910055	10.117916	0.264551	0.791874	0.758915	0.089945
474.237	440.000	34.237	0.206279	1.000000	0.927806	12.851563	0.263581	0.861164	0.826179	0.072194
475.262	450.000	25.262	0.221641	1.000000	0.946847	17.813510	0.263013	0.934609	0.897536	0.053153
475.531	460.000	15.531	0.237977	1.000000	0.967340	29.618880	0.262864	1.012329	0.973109	0.032660
475.000	470.000	5.000	0.255318	1.000000	0.989473	93.990874	0.263158	1.094444	1.053015	0.010527
473.627	480.000	-6.373	0.273694	1.000000	1.013456	-75.314352	0.263921	1.181074	1.137377	-0.013456