AN EMPIRICAL TEST OF A PROBABILISTIC MODEL
OF CONSUMER SPATIAL BEHAVIOUR

by

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We accept this thesis as conforming
to the required standard:

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Department of Commerce and Business Administration

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This thesis is concerned with making an empirical test of a probabilistic model of intra-urban retail trade interactions. The model, termed a probabilistic model of consumer spatial behaviour, is related to a series of models in social science known as gravimetric models. The particular model considered used store area and distance in time units as its major variables. It also includes an exponential parameter, the value of which must be estimated from empirical data. The major hypothesis on which the model is tested is based on the behaviour of this parameter. The hypothesis states that values of this parameter vary significantly, depending upon the type of shopping trip being considered. The type of shopping trip is determined by the particular type of shopping goods apparently sought by the consumer.

The hypothesis is tested by means of empirical data on consumer purchasing patterns gathered in the Vancouver Metropolitan area through the use of an interview survey conducted randomly by census tracts. The data are analyzed in an especially written, iterative computer programme. Statistical tests usually applied to such data are found to be inadequate to the analysis of the results. A special test which is intended to show the sensitivity of the model to the parameter is presented and applied. An independent test of the
representativeness of the data is presented.

The data are found to be representative, but the model is found to be insensitive to the behaviour of the parameter. Further, measures of variation in observed behaviour explained by the model are generally low. It is concluded that the model in its present form does not apply to Vancouver. The thesis is unable to conclude whether changes are required in the factors of the model or in the relationship specified, though there is evidence which shows that both may require attention.
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CHAPTER I

INTRODUCTION

Statement of the Problem

The problem to which this thesis is addressed is that of making an empirical test of a probabilistic model of intra-urban retail trade interactions. This model is the most recent of a long series of models in social science which are classed as gravimetric models. In particular, the model to be examined is that put forward by Huff and termed, by him, a probabilistic model of consumer spatial behaviour.¹

The practice of marketing makes many assumptions about the factors which influence consumer spatial behaviour. Gravimetric models are intended to make explicit some of these factors and their relationships. Such models have the general form:

\[ P = \frac{f(A)}{f(B)} \]

where \( P \) = a measure of potential; \( A \) = a measure of the attractive force of an area or point; \( B \) = a measure of the cost involved in responding to the attractive force. The units used for these measures vary with the nature of the problem. The mathematical form of the functions used frequently contain

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¹ David L. Huff, Determination of Intra-Urban Retail Trade Areas (Los Angeles: University of California, Graduate School of Business Administration, Division of Research, 1962) pp. 4-5.
an exponent or exponential parameter.

Purpose of the Study

The purpose of this study is to determine whether empirical data gathered in a particular geographical area, namely Metropolitan Vancouver, will tend to support a model which was formulated and tested in a pilot study elsewhere (the Los Angeles Metropolitan Area). If the data tend to support the model then the weight of this additional evidence would tend to show that the model might be generally applicable to the prediction of retail trade interaction in urban areas.

On the other hand, if the data do not support the model, an attempt will be made to determine whether the deficiency lies in the data, or in the model. If the data are deficient, then nothing more can be done with them directly. The task becomes one of determining the manner in which the data are deficient and of producing a new research design without such limitations. If, however, the model appears to be deficient, this may be the result of incorrect factors or variables, or incorrect relationships among factors, or both. Here, the data, the results of the study, and other sources of information must be considered together to attempt to detect the deficiencies. These may be such as to require a redefinition of factors included or a reconceptualization of the relationships used, or both. In order to prepare for this contingency, the development of the present model was examined in detail, both for internal consistency and against a background of
previous research on gravimetric models reported in diverse literature.

Methodology of the Study

The methodology of a scientific study must follow naturally from the hypotheses to be tested. Because this study seeks to do the same thing as Huff’s pilot empirical study, the basic hypothesis to be tested must also be the same. Huff’s model contains an exponential parameter. The hypothesis is that the empirically estimated value of this parameter is a function of the type of shopping trip for which it is estimated. By this Huff means that the value determined for consumers’ excursions to buy one type of shopping good will be significantly different from that obtained for other shopping goods.

In order to test his hypothesis, Huff used a saturation sampling technique on a few neighbourhoods selected to be homogeneous in terms of income. The individuals thus sampled were asked to report where they last bought a range of shopping goods items, and where they normally bought this same range of items. The empirical test was made on some of the information obtained in answer to the first question.

Since the present study was to be on a broader basis, it could not use the same sampling technique. Neither time nor money permitted this. Thus, it was necessary to use a random sampling technique over a large number of neighbourhoods. Further, because the data were gathered in a survey
which sought to develop other information regarding the shopping
trip as an entity, the particular questions which were intended
to elicit the empirical data had to be framed differently. The
present survey focussed on the last shopping trip, not on the
last purchase. While Huff's questions normally produced a
response for every item in the range, the survey used as a basis
for this study did not obtain a response in many cases since
many shopping goods are infrequently purchased. This reduced
the number of items in the sample, but it did hold other fac-
tors constant.

Given the model and the set of empirical observations,
the next task is to devise a means of determining whether the
theory (model) fitted the facts (set of empirical data). This
is the point at which the specific hypothesis put forward by
Huff -- that the exponential parameter of the model is a
function of the type of shopping trip -- is of central concern.

Given a suitable hypothesis, the technical detail of the
test must be determined. Huff chose to have the model predict
what consumer behaviour would be for a sequence of values of
the exponential parameter. These predictions were compared,
one by one, with the observations. The goodness-of-fit of
predictions with observations was measured by a test parameter
defined in such a way that it resembled the coefficient of
determination used in regression analysis. The objective was
to determine the value of the exponential parameter which
maximized the value of the test parameter.

To determine if the results of this procedure would
support the hypothesis, Huff performed some simple statistical tests of significance. He concluded that his data supported the hypothesis. This study re-examines Huff's data using a technique designed to show the sensitivity of the model to the exponential parameter. This technique examines the behaviour of the test parameter for a range of values of the exponent. When applied to the Vancouver data, the test shows that the model needs other factors and relationships in order to account for consumer behaviour in Vancouver.

The study then turns to examining the ways of approaching the task of defining other factors and relationships. Other relevant work is considered for its impact on this task, and for the aid which it does or can provide.

Throughout this study, three basic inter-related ideas are interwoven. First, there is concern for the properties and purposes of abstract models. Second, considerable attention is given to the empirical tests of gravimetric models used by various researchers at various times and places. This is of primary interest in determining the appropriate research design for this study. The third idea is concerned with suitable bases on which to judge the results of such research. Too frequently simple statistical tests mask, or overlook, information of fundamental value in this regard. More discriminating tests are needed.
Limitations of the Study

This study has limitations along three dimensions. First, the results are valid only for the Metropolitan Vancouver area wherein the sample was obtained. Second, because the sample was taken cross-sectionally, the results are only valid for that one point in time. Third, the study deals only with certain classes of shopping goods (clothing, home furnishings, and hardware), and only with certain kinds of stores (major department stores and some planned shopping centres). It would be incorrect to apply the results of this study beyond these bounds.

Definition of Terms

Several terms used throughout this study are basic to the understanding of it. The manner of their usage should be examined explicitly.

1. Shopping trip -- It is assumed that when a consumer makes an excursion from his travel base point (assumed to be his home) in quest of shopping goods he also has in mind the particular type of shopping goods he is seeking. That is, it is assumed that the shopping trip is purposive. It may, however, either be successful or unsuccessful in terms of obtaining or not obtaining by purchase the desired shopping goods. If the trip is unsuccessful, it is more difficult to objectively determine the purpose of the trip. Thus, for this
study, shopping trips are defined to be only those where shopping goods were purchased. While shopping goods may be obtained in a wide variety of stores, the definition of a shopping trip, in this study, is restricted to include only excursions to major department stores and planned shopping centres.

2. Shopping goods -- These are the goods for which the probable gain from making careful comparisons is thought to be large relative to the time and effort needed to shop effectively. Such goods may be classed as homogeneous, which consumers view as essentially similar, or heterogeneous, which are non-standardized and may contain a significant element of style or fashion. This study does not explicitly use this classification, but does attempt to group items into composite commodities which are consistent with it.

3. Major department stores -- These are the ones which fit the pattern of handling a wide variety of shopping goods, divided into major departments. With one exception, they are branches of regional or national corporate chains.

4. Planned shopping centres -- These are geographically contained agglomerations of complementary and competing retail firms which have been planned and developed as a self-contained unit on a tract of land devoted entirely to that complex of activities. Such centres may be characterized as neighbourhood, community, or regional centres. This study is concerned only with the latter two types which are large enough to have a
branch of one or more of the regional or national department store chains as a central focus.

Organization of the Written Report

This report is divided into chapters, each of which examines a specific aspect of the study. Chapter II is divided into two parts. The first part briefly states some basic notions and issues about abstract models and measurement which are relevant to the discussion of the development of the model and the task of reconceptualization. The second part is devoted to a survey of the literature of gravimetric models in social science. Particular attention is paid to the formulation of these models, but some consideration is also given to empirical tests which have been performed.

Chapter III deals with the specific incidence of gravimetric models in marketing. Previous work is examined for the nature of the processes of conceptualization and formulation. Of particular interest, however, is the nature of empirical research which has been held by some to support, and by others to reject, these models. This aids in the consideration of the bases on which evidence can be used to support or reject such models.

Chapter IV is devoted to the particular gravimetric model which is the main object of this study. In addition to Huff's original development, two additional approaches are presented. The relative strengths and weaknesses of each are
considered in view of some of the basic issues of abstract models. The chapter concludes that regardless of these strengths and weaknesses, it is still worthwhile to conduct an empirical test of the model.

Chapter V presents the empirical work of Huff, and of this study. The technique of evaluating the sensitivity of the model to the exponential parameter is presented.

Chapter VI is devoted entirely to the task of reconceptualizing and reformulating the model. The evidence developed in Chapter V serves as the indicator of how this task might be approached. Some very basic issues, and their probable impact, in dealing with the type of model considered in this study are examined. The need for better methods of judging the results of studies of this kind is also considered.
CHAPTER II
MODELS IN SOCIAL SCIENCE

Part I: Some Fundamental Notions and Limitations of Models

The term model is being encountered with increasing frequency in the literature of social science. It is used by some interchangeably with the term theory, although the term model connotes only the more immediate aspects of conceptualization and summarization.

In a general sense, a model is a representation of reality. In this same general sense it may be argued that all human perceptions of reality, and ways of thinking about it, are models. For example, the number system is a model by which physical objects can be represented by abstracting a particular common property. There are two basic steps involved in the use of models: (1) the abstraction of real objects and events into a model; and, (2) the application of results derived from the use of a model.

The use of any model may produce a degree of approximation in the results obtained. Consider an example of the use of the number system wherein a housewife has a box of mixed apples with which she wishes to bake pies. Using the number system she counts the number of apples but this would not tell her exactly how many pie shells to make -- the apples are not
homogeneous in size or quality. The number system model requires the assumption of complete homogeneity, indeed complete identity, of the objects represented to be completely accurate. Where this assumption is not fulfilled in the processes of abstraction and simplification of the real (empirical) situation, the results of using the model will be approximate.

It is important to note in the example that the counting model was applied for a specific purpose. Models may be developed for a number of purposes, but these are generally acknowledged to include understanding, prediction, and control of real world events. This implies that the type of model that should be used in a particular instance depends on the specific purposes involved. Further, the use of models to accomplish specific purposes has only two bases: (1) determination of results is faster, less expensive, or more accurate with the aid of the model; or (2) operation on the physical objects and events themselves (experimentation) is not possible.

Types of Models

Models are representations of reality, which implies a form of correspondence with the real world. Depending upon the nature of the correspondence, models may be classified as iconic, analogue, or symbolic.¹ Iconic models look like

reality; analogue models substitute one property for another; and symbolic models represent objects, events, and processes by symbols of a logical and/or mathematical character.

This study is concerned with symbolic models, especially mathematical models, which are the kind of models being actively sought and applied in all branches of social science, including marketing. However, there is more involved in the use of mathematical models than just symbolism. Mathematics consists of a set of axioms (basic definitions and assumptions) and a set of theorems derived from the axioms by legitimate processes of logical deduction. The truth of a mathematical theorem has nothing at all to do with whether or not it corresponds in any way with any real objects or events. A mathematical theorem is a purely abstract construct. Because of this basic quality of mathematics, it is extremely important to note that the use of a mathematical model to represent real objects and events rests squarely on the very broad and general assumption that the axioms and theorems of one or more branches of mathematics are legitimate statements when applied to some aspect of the real world process which the model purports to represent. If this assumption is not correct, then the model and the inferences drawn from it are not legitimate. Presumably, this would be discovered in the course of experimental or empirical research when it is determined that the model does not fit the facts.²

Therefore, it would be better to deduce this beforehand to avoid wasteful empirical research.

Variables and Relationships

Mathematical models are composed of variables or factors, and the relationships among them. A variable is any property found to have different values at different times, places, or circumstances. The nexus of model-building is the specification of the relationships among variables, and these may be of several forms. Three forms may be distinguished as: (1) deterministic causality; (2) probabilistic causality; and (3) correlation.

For much of the recorded history of scientific inquiry, the relationship between variables has been regarded as that of deterministic causality, also known as cause and effect. This is the classical viewpoint of physical necessity which dominated scientific thinking up to the nineteenth century. More contemporary research in mathematical physics has shown that the classical, deterministic concept is an artificial one even as applied to the simplest kinds of physical phenomena. The laws of nature are conditional. That is, they are true only if one abstracts from contingencies, and only to the extent that these contingencies may be neglected.

Probabilistic causality has been interpreted as a relationship in which one thing is necessary for the occurrence of some other thing, but not sufficient because other factors
are also involved.\(^3\) The study of such relationships has been
developed in mathematical statistics, and in the study of the
nature of stochastic processes. Many of the processes studied
in social science are probably stochastical and there appears
to be a sort of multiple probabilistic causality operating.
Any one factor, or several of many different factors, may
cause changes in a variable. No one causal factor is necessary,
nor is any of them sufficient for occurrence of the effect
under investigation, since still other unknown factors may
also contribute to the effect. This tends to make stochastic
process model-building rather intractable.

Correlation is the weakest form of relationship among
variables. Many types of nonsense relationships where correla-
tions are very high have been reported. Such relationships are
frequently termed spurious correlation.\(^4\) Purely statistical
associations of this kind are far less dependable than even
partial knowledge of actual causes. On the other hand, it
may be preferable to base predictions on a correlation between
variables than on pure guesswork, providing the inherent dangers

\(^3\) R.L. Ackoff, Scientific Method: Optimizing Applied Research

\(^4\) The term spurious correlation is used here with a common
sense meaning. There is also a precise meaning in mathematical
statistics referring to the case where apparent correlation is
higher than true correlation because of the presence of some
third variable (explicit or implicit) in common in the relation-
ship being studied. The earliest substantial investigation of
this seems to be G. Yule, "Why Do We Sometimes Get Nonsense
Correlations Between Time Series," Journal of the Royal
Statistical Society, LXXXIX (1926), pp. 1-64.
are recognized.\(^5\)

The Problem of Measurement

One of the key issues in the development and use of mathematical models in social science is that of measurement. Measurement may be of several forms but in common usage it implies some way of assigning numbers to properties of objects or events. Ackoff points out, however, that symbols other than numbers can also be used, as long as the symbols "are related to each other in the same way that the observed objects, events, or properties are or can be."\(^6\) Numbers are usually desired, however, because the models in which they are used can be manipulated mathematically.

Even if measurement is restricted to the use of numerical symbols, there are several types of measurement possible, depending upon the particular type of scale chosen: nominal, ordinal, or cardinal.\(^7\) A nominal scale is simply a measure which uses a number to name an entity and/or property of an

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\(^5\) A useful discussion of the use of statistics, particularly correlation, for inference of causation is found in Hubert M. Blalock, Jr., *Causal Inferences in Nonexperimental Research* (Chapel Hill: University of North Carolina Press, 1964.)


entity. It makes no sense to perform mathematical operations on these numbers. Very little information is conveyed by the use of these numbers other than as a means of identification.

An ordinal scale is a ranking measure which tells which of any two items is higher, lower, or the same on the scale. While a wide choice of numbers is available with which to do this, care must be taken that the numbers preserve the ordering relationship. Care must also be taken that nothing is inferred about the numerical difference between rank numbers -- assigning numerical rankings implies nothing about how far apart the items ranked are, or how much greater a property is in one as compared with another.

Cardinal scales are composed of two subclasses -- interval scales and ratio scales. Interval scales have an arbitrary zero point and a constant unit of measure. They are usually defined in an operational way with respect to fixed reference points. A ratio scale has both an absolute zero point and a constant unit of measurement. The Celsius and Fahrenheit temperature scales are examples of interval measures, while the Kelvin and Rankine scales are their ratio (absolute) counterparts. Such measures as inches, pounds, and dollars are also examples of ratio scales.

In the case of interval scales, differences between scale values can be expressed as multiples of each other, but individual values cannot. This notion is formalized by saying that interval scales are unique up to a positive linear trans-
formation (such as that used to convert Celsius to Fahrenheit measure). On the other hand, in the case of ratio scales, it does make sense to talk about values on one scale being multiples of values on another. Scales of this type, having an absolute zero, are unique up to a proportionate transformation. All the mathematical operations typically associated with numbers can be performed.

It appears that critics of mathematical models in social science, who claim that major factors cannot be measured, implicitly mean that they cannot be measured on ratio scales. Since they believe that these scales are the only ones that can be manipulated mathematically, they conclude that mathematical model-building is hopeless. Such critics would do well to reconsider the astounding progress made in many branches of physical science using only interval scales for measurement. Differences can be compared relatively using interval scales since the unit of measurement is constant and it was relative measures, not absolute measures, which were sought by the physical scientists. In social science, a properly constructed index number may be considered an appropriate interval scale.

It will be seen that social science has paid considerable attention to the results of physical science in terms of models or theories. Insufficient attention has been paid to the methodology and technique of physical science, probably because it is largely experimental, and social science has
been mostly non-experimental. However, the ingenuity and creativity of early physical scientists in overcoming the problems associated with obtaining satisfactory measurements should provide both example and hope to social scientists. This point cannot be over-emphasized, since it appears to contain a very important lesson for research in social sciences.

Part II: Gravimetric Models of Human Interaction

In recent years increasing attention has been given by social scientists to the so-called gravity and potential concepts of human interaction. This is a case where, either implicitly or explicitly, a model has been developed by the scientist based on Newton's Universal Law of Gravitation.  

In general terms, the gravity concept of human interaction postulates that an attracting force of interaction between two areas of human activity is created by the population masses of the two areas, and a friction against interaction is caused by the intervening space over which the interaction must take place. That is, interaction between the two centres of popu-

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8 This law may be expressed as follows:

\[ F = \frac{GMm}{d^2} \]

where \( M \) is a particle of mass at point \( A \), at a distance \( d \) from a second particle of mass \( m \) at point \( a \); a force acts on each mass, attracting them together along the line joining them; \( G \) is a universal constant, in this case the gravitational constant.
lation concentration varies directly with some function of the population size of the two centres and inversely with some function the distance between them. Mathematically the relationship may be expressed as follows:

\[ I_{ij} = \frac{f(P_i, P_j)}{f(D_{ij})} \quad (\text{II.1}) \]

where \( I_{ij} \) = the interaction between centre \( i \) and centre \( j \); \( P_i, P_j \) = the population of centres \( i \) and \( j \), respectively; and \( D_{ij} \) = the distance between centre \( i \) and centre \( j \).

Stated in this way, the gravimetric hypothesis is based on the postulates that: (1) to produce interaction, individuals must be in communication, directly or indirectly, with one another; (2) any particular individual, as a unit of a large group, may be considered to generate the same influence of interaction as any other individual; (3) the probable frequency of interaction generated by an individual at a given location is inversely proportional to the difficulty of reaching, or communicating with, that location; and (4) the friction against this transportation or communication is directly proportional to the intervening physical distance between the individual and the given location.

Earliest Incidences

Students of marketing are aware that gravimetric models have been proposed for some time, beginning with
Reilly's Law, which in terms of marketing thought is relatively old. However, gravimetric models are apparently much older, as the earliest known explicit formulation of the gravity concept of human interaction was made by Carey during the first half of the nineteenth century. Carey reasoned that social and physical phenomena are based on the same fundamental law, in a manner not unfamiliar today. He formulated his concept as follows:

Man, the molecule of society, is the subject of Social Science ... The great law of Molecular Gravitation is the indispensable condition of the existence of the being known as man ... The greater the number collected in a given space, the greater is the attractive force that is there exerted ... Gravitation is here, as everywhere, in the direct ratio of the mass, and the inverse one of distance.

After Carey's original statement of the concept, it was largely neglected. It appeared only partly expressed until quite recently. In 1885, Ravenstein produced an hypothesis regarding migration which was implicitly based on


the gravimetric concept. 12 Ravenstein was concerned with explaining migration and presented empirical evidence suggesting that migratory movement tends to be toward cities of large population and that the volume of movement decreased with distance between the source of migration and the centre of absorption. The relationship may be expressed mathematically:

\[ M_{ij} = \frac{f(P_i)}{D_{ij}} \]  

(II.2)

where \( M_{ij} \) = migration from source \( j \) to centre of absorption \( i \); \( f(P_i) \) = some function of the population of \( i \); and \( D_{ij} \) = the distance between source \( j \) and centre \( i \).

Subsequently, the gravimetric concept does not appear in the literature again for some thirty years, until the late 1920's when Young made a somewhat similar attempt to measure migration. 13 Young hypothesized that the relative volume of migration to a given destination from each of several source areas varies directly with the force of attraction of the destination and inversely with the square of the distance between the source and the destination. That is:

\[ M_{ij} = \frac{kZ_i}{D_{ij}^2} \]  

(II.3)

12 Ibid., p. 95. The original source is E.G. Ravenstein, "The Laws of Migration," Journal of the Royal Statistical Society, No. 6 (1885), 167-235 and No. 6 (1889), 241-305.
13 Ibid. The original source is E. C. Young, The Movement Of Farm Population (Ithaca: Cornell Agricultural Experiment Station, Bulletin 426, 1924).
where $Z_i = \text{the force of attraction of destination } i$; and $k = \text{a constant of proportionality}.$

About the same time, Reilly postulated his "Law of Retail Gravitation"\textsuperscript{14} which approaches the gravity concept somewhat differently. While this concept, its empirical support, and the work of subsequent investigators will be considered in some detail in the next chapter, the concept is included briefly here for completeness of the historical survey. Reilly postulated that a city will attract retail trade from an individual in its surrounding territory in direct proportion to the population size of the retail centre and in inverse proportion to the square of his distance away from the centre. Thus, for any two cities competing for retail trade, the point of equilibrium on the line joining them, where competitive influence is equal (the breaking-point) is described by the equation:

$$\frac{P_i}{d_{xi}^2} = \frac{P_j}{d_{xj}^2} \quad \text{(II.4)}$$

where $P_i, P_j = \text{population of cities } i \text{ and } j$, respectively; $x = \text{the point of equilibrium on the line joining } i \text{ and } j$; $d_{xi} = \text{distance from city } i \text{ to point } x$; $d_{xj} = \text{distance from}$

\textsuperscript{14} In addition to the work previously cited, this concept was earlier mentioned in W. J. Reilly, \textit{Methods for the Study of Retail Relationships} (University of Texas, Bureau of Business Research, Research Monograph No. 4, University of Texas Bulletin No. 2994, November, 1929).
city \( j \) to point \( x \); and \( D_{kj} = a_{xi} + a_{xj} \).

In the 1930's, an early sociological application of the concept was made by Bossard who examined the function of distance as a factor in marriage selection. Bossard found,

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15 Actually, Reilly expressed his concept somewhat differently. In Reilly, *The Law of Retail Gravitation*, p. 9, is found:

Two cities attract retail trade from any intermediate city or town in the vicinity of the breaking point approximately in direct proportion to the population of the two cities and in inverse proportion to the square of the distances from these two cities to the intermediate town.

Mathematically:

\[
\frac{B_a}{B_b} = \frac{P_a}{P_b} \times \frac{D_b^2}{D_a^2}
\]

where \( B_a, B_b \) = proportions of retail trade from the intermediate town attracted by cities \( A \) and \( B \), respectively; \( P_a, P_b \) = populations of cities \( A \) and \( B \), respectively; \( D_a, D_b \) = distances from the intermediate town to cities \( A \) and \( B \), respectively.

Equation (II.4) above can be derived from Reilly's formula, since at the breaking point, where competitive influences are equal, \( B_a = B_b \), or the intermediate community divides its trade equally between the two cities. Thus, \( B_a/B_b = 1 \), and:

\[
1 = \frac{P_a}{P_b} \times \frac{D_b^2}{D_a^2}
\]

or,

\[
\frac{P_a}{D_a^2} = \frac{P_b}{D_b^2}
\]

which is Equation (II.4).

16 James H.S. Bossard, "Residential Propinquity as a Factor in Marriage Selection," *American Journal of Sociology*, XXXVIII (September, 1932), 219-244.
from empirical tests in Philadelphia, that the number of marriages decreased as the distance between the premarriage residences of the principals increased.

Formalization: Social Physics

Beginning in the early 1940's, the gravity concept of human interaction was generalized by Stewart and Zipf. By referring to the relevant physical laws (models), equations


describing demographic force, energy and potential have been
developed. In fact, Stewart, an astrophysicist, claims that
this generalization is the basis of a new science, which he
calls "Social Physics". According to Stewart:

In its application, social physics relates directly
to the economic theory of location and to general
marketing theory -- for example, to the construction of
maps of trading centers ... The general possibilities
of its social physics usefulness extend over the
entire field of social statistics wherever averages
rather than individual human characteristics are
studied. Emphasis added. 18

Social physics is defined by Stewart as including all
studies of a mathematical type relating to human behaviour.
Social physics, he states:

... analyses demographic, economic, political and
sociological situations in terms of the purely physical
factors of time, distance, mass of material, and
number of people with recourse also to social factors
which can be shown to operate in a similar way to two
other physical agents, namely temperature and electrical
charge. 19

He writes that temperature as a social factor refers to the
level of activity of people as shown in bank deposits per
capita, mileage of railways and highways per square mile of
area, and percentage of workers in manufacturing industries,
for example. 20 The social quantity that Stewart substitutes
for electric charge (positive and negative) is desire. He

18 Stewart, Theory in Marketing, p. 19.
19 Stewart, Impact of Science on Society, III, p. 110.
20 Ibid., pp. 120-121.
writes:

Since desire in the sense of simple hunger is neutralized by its appropriate satisfaction, it is in this respect like a negative electric charge, while the desirable object possesses the neutralizing positive charge.21

This kind of framework is termed dimensional analysis by Stewart. As applied to social physics, a principal objective of dimensional analysis is to designate the leading factors by reference to which social physics as a whole can be summarized.

The immediate objective of the social physicist, according to Stewart, is to discover uniformities in social behaviour which can be expressed in mathematical form more or less corresponding to the known patterns of physical science. He is of the opinion that enough such regularities exist to justify the conclusion that certain types of human relations, on the average and only on the average, conform to mathematical formulas resembling the primitive laws of physics.

At the hands of Stewart and Zipf, a return was made to the original formulation in terms of Newtonian physics, as first set forth by Carey, namely that the force of interaction between two concentrations of population, acting along a line joining their centres, is directly proportional to the product of the populations of the two centres and inversely proportional

21 Ibid., p. 123.
to the square of the distance between them. That is, mathematically:

$$F_{ij} = \frac{P_i P_j}{D_{ij}^2}$$  \hspace{1cm} (II.5)$$

where $F_{ij} =$ the force of interaction between concentrations $i$ and $j$.

Following the analogy from physics, the energy of interaction between the two centres which results from this force would be:

$$E_{ij} = k \frac{P_i P_j}{D_{ij}}$$  \hspace{1cm} (II.6)$$

where $E_{ij} =$ energy of interaction between $i$ and $j$; and $k =$ a constant of proportionality equivalent to the gravitational constant of physics.\(^{22}\) Thus, the energy of interaction between any two centres of population increases as the product of the two populations increases, and falls off as the distance between the two centres increases. The total

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\(^{22}\) The concept of demographic energy is based on Newton's equation for the mutual energy of two masses in the gravitational field:

$$E = \frac{G M m}{\alpha}$$

where $G =$ the gravitational constant; $M =$ mass at a point; $m =$ mass at another point; and $\alpha =$ distance between the two points. Mutual energy refers to the force each mass exerts on the other, operating to bring the two masses together.
energy of interaction of a given region \( i \) would be the sum of the energy of interaction of \( i \) with each of the \( r > 1 \) other regions into which a given universe may be divided. Formally:

\[
E_i = k \sum_{j=1}^{n} \frac{P_i P_j}{D_{ij}}
\]

where \( E_i \) = total energy of interaction of region \( i \). Zipf, Stewart, and others\(^{23}\) have tested and applied this formulation of the gravity concept empirically, measuring the energy of interaction between pairs of cities by a variety of characteristics such as telephone calls, bus passenger movements, newspaper circulation, airline trips, and similar acts of communication.

Stewart has extended the physical analogy to include the concept of potential of population, which may be thought of as a measure indicating the intensity of the possibility of interaction. At a given location \( i \), the potential influence, or possibility of interaction, with respect to an

individual at \( i \), which is generated by the population of any given area \( j \), will be greater as the population of \( j \) is larger and will be less as the distance between \( i \) and \( j \) increases. Mathematically:

\[
V_{ij} = k \frac{P_j}{D_{ij}} \tag{II.8}
\]

where \( V_{ij} \) = potential at \( i \) of the population of area \( j \).

The total possibility of interaction between an individual at \( i \) and the population of all other areas in the particular universe under consideration, that is, the total population potential at \( i \), would be:

\[
V_i = k \sum_{j=1}^{n} \frac{P_j}{D_{ij}} \tag{II.9}
\]

where \( V_i \) = total population potential at \( i \).

Although Stewart did not carry out this next step, it could be argued that if \( V_{ij} \) represents the possibility of interaction from \( i \) with a particular area \( j \), and \( V_i \) represents the total possibility of interaction from \( i \) with all relevant \( j \), then in a relative frequency sense the probability of interaction from \( i \) with a particular \( j \) is:

\[
P_{ij} = \frac{V_{ij}}{V_i} = k \frac{P_j}{D_{ij}} \left/ \frac{k \sum_{j=1}^{n} \frac{P_j}{D_{ij}}} \right. \tag{II.10}
\]

An equation of this form will be encountered later.

The interpretation to be given to the concept of demographic potential is not entirely clear. Stewart writes of
population potential of a point as a measure of the proximity of people to that point, as a measure of aggregate accessibility, and more simply as a measure of influence of people at a distance. To point up the significance of this concept, Stewart and his associates have conducted a number of empirical studies. He reports high correlation within the United States of the spatial variation of population potential with spatial variation in a wide variety of sociological phenomena. Among these are rural population density, rural nonfarm population density, rural nonfarm rents, farmland values, miles of railroad track per square mile, miles of rural free delivery routes per square mile, density of rural wage earners in manufacturing, and rural death rates.  

In a more recent article, Stewart and Warntz compare cities in the United States and Great Britain equal in population but located in areas of varying population potential. They find that cities in areas of low potential tend to have larger areas, lower taxes, and a greater excess of births over deaths than cities of equal population in areas of high potential.  

In the historical development of the gravity model, the works of Zipf have also been influential. Zipf writes variously of the $\frac{P_i}{D}$ and $P_i P_j / D$ factors which, using the same


symbols as above, are the $P_i/D_{ij}$ and $P_iP_j/D_{ij}$ factors, respectively. These factors closely resemble Stewart's concepts of potential and demographic energy and have been interpreted as identical with these concepts on a number of occasions. However, in empirical analysis, Zipf's use of his relationships or factors differs from Stewart's use of his (Stewart's) demographic concepts. Essentially, Zipf examines, for pairs of cities, interaction phenomena and the $P_iP_j/D_{ij}$ factor where the entire factor is raised to some exponent. For example, for each pair selected from a set of cities for which materials are available he plots a log-log graph of the frequency of a particular phenomena and their $P_iP_j/D_{ij}$ factor. When the data for all the possible pairs of cities are plotted, Zipf finds a straight-line relationship.

Zipf finds straight-line relationships between the $P_iP_j/D_{ij}$ factor and many interaction phenomena between pairs of cities, such as bus passenger trips, railway passenger trips, airline passenger trips, telephone calls, and tonnage of railway express shipments. However, it must be remembered that in each case the slope of the straight line is the exponent of the entire $P_iP_j/D_{ij}$ factor, and not the exponent of $D_{ij}$ alone as in Stewart's concepts. Thus, Zipf's findings are not a direct test of the validity of Stewart's concepts.  

In application, the gravity and potential models lend

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26 However, in the non-typical case, when the slope of Zipf's straight line is unity, Zipf's use of his $P_iP_j/D$ relationship becomes identical with Stewart's use of demographic energy.
themselves to being mapped "topographically" to show contours of equal potential. From such maps, areas of different potential are readily discernible, and interrelations between areas are easily visualized. Stewart has plotted such maps for the United States.27

Carroll describes a technique by which a somewhat different aspect of the concepts is mapped.28 Carroll was concerned with describing the area over which urban centres have influence. As measures of influence, he used numbers of telephone calls and volume of highway traffic. Starting with assumptions of (1) a flat terrain, (2) urban influence proportional to city size, and (3) constant rate of change in the decline of influence with distance, Carroll arbitrarily assigned the population size as the measure of maximum influence of each centre. Since influence falls away in all directions, he generated a surface (or rather contours on this surface below the maximum point) from the equation:

\[ U_{ij} = k \frac{P_i}{D_{ij}^a} \]  

(II.11)

where \( U_{ij} \) = urban influence of centre \( i \) upon any point \( j \); and \( a \) = a constant exponent. The lines of intersection of the surfaces for the various adjacent centres constitute a

27 Stewart included such maps in his articles in: Geographical Review, XXXVII, 461-485; American Journal of Physics, XVIII, 239-253; and Theory in Marketing, 19-40.

map of the areas of influence of the centres. Carroll carried out this process for 21 major cities in southern Michigan, and determined a value of 2.8 for the exponent of distance from empirical tests of telephone messages and inter-city travel.

Extension of the Concepts

Many researchers have collected empirical evidence in an effort to test hypotheses developed from the general permutations of the concepts of demographic force, energy, and potential discussed above. This has led some to concentrate their attention on the distance factor, and others on the population factor. Such results, and the modifications to the models developed therefrom are of relevance in considering the factors involved in the model which is the main object of this thesis. The work is reviewed under the two factors — distance and population.

A. The Distance Factor

Empirical evidence developed by Price and Ikle among others, as well as Carroll, suggests that the impact of distance is not uniform, and that its relationship in the basic Equations (II.6) and (II.8) is not a simple inverse one, but one in which distance is raised to some power other than

one or two. Various exponents have been used as a result of empirical testing, ranging from one-half to over three. Isard and Peck have derived a value of 1.7 empirically.\textsuperscript{30} Carroll reports determined exponents of over three.\textsuperscript{31}

Anderson has suggested that the exponent itself is a variable, inversely related to the size of the population.\textsuperscript{32} This implies, taking the potential relationship for example, that:

\[ V_{ij} = k \frac{P_j}{D_{ij}^x} \quad \text{(II.12)} \]

where \( x = f(1/P_j) \)

On the other hand, Carrothers argues that the evidence may be interpreted differently; namely, that the exponent may be a variable function related inversely to distance itself, rather than to population.\textsuperscript{33} In this case, the interpretation would be that friction per unit of distance against interaction caused by short distances is disproportionately greater than friction per unit of distance caused by longer distances. For instance, friction against movement within an urban area is generally greater than that caused by an equal distance in the less densely developed space between two such areas. Or, again, an extra unit of distance added to a long movement is of


\textsuperscript{31} Carroll, \textit{op. cit.}

\textsuperscript{32} Anderson, \textit{op. cit.}, p. 287.

\textsuperscript{33} Carrothers, \textit{op. cit.}, p. 97.
less importance than an extra unit added to a short movement.—it is obvious that the relative effect is less. It is interesting to note that many freight and passenger transportation rate structures also reflect this point. In any event, the exponent of distance in Equation (II.12) becomes $x = f(1/D_{ij})$

If, however, distance were measured in terms of travel time, and increments in relative (say, percentage) terms, rather than absolute terms, this argument of population versus distance in the exponent function tends to vanish. This is a case where the type of measure (scale) used may be hiding the real issue.

In reporting investigations of the relation of distance to migration, Price reports another complication. The empirical evidence suggests that the function of distance is affected by the particular direction from the destination in which the measures are made, and that the distance factor of Equation (II.8) varies accordingly. This implies that distance should be treated as a vector, and not as a scalar.

B. The Population Factor

In calculating population potentials and energy of interaction for various countries, and for various kinds of activity within a country, Stewart found that frequently an area would have a pull, or influence, either greater or less

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34 Price, op. cit.
than would be expected from the simple formulations of the concepts. He concluded that population under one set of circumstances is not necessarily of the same importance as under other circumstances, that is, the populations are not homogeneous. This is contrary to the assumptions implied in his original formulation where a value of unity was implicitly assigned as a weight to the population element. By assigning values other than one to these weights, he sought to account for differences in degree of influence which would result from different characteristics of the populations. When this is done, the basic energy equation becomes:

\[ E_{ij} = \frac{k \cdot m_i \cdot P_i \cdot m_j \cdot P_j}{D_{ij}} \]  (II.13)

where \( m_i, m_j \) = molecular weights of an individual in \( i \), and an individual in \( j \), respectively. Stewart interprets molecular weight in this context as a measure of the individual's capacity for sociological interaction. Having admitted these molecular weights, however, it may be argued that adjustment has been made for cultural determinants of human behaviour, and that the relationships are no longer the purely objective, demographic species upon which Stewart founded his whole concept of social physics.

35 Stewart, Theory in Marketing, pp. 32-36.
36 This weight is equivalent to molecular weight in the physical analogy.
This formulation is similar to Dodd's interactance hypothesis in which he introduces variables other than those of population numbers and distance into the original formulation, by making them multipliers of the basic variables, in order to account for differentials in sex, income, education, and other (demographic) characteristics. The basic energy equation thus becomes:

\[ E_{ij} = k \frac{[\sum \phi_i P_i] [\sum \phi_j P_j]}{D_{ij}} \]  

(II.14)

where \( \sum \phi_i \) and \( \sum \phi_j \) = weighting factors for population \( P_i \) and \( P_j \), respectively.

But the application of simple indexes to the population factor in the equation may not be enough to account for observed differences in the influence of population in different circumstances. For instance, a larger population in one area than in another may of itself result in an influence for the first area larger proportionately than can be accounted for by the modification of population size by a simple multiplier. This may be the case where agglomeration economies are present, for example. Anderson suggests the possibility of raising the numerator of the basic equation to some power other than unity (implicit above). But this still implies that populations of


different kinds have equal influence. Therefore, it would seem more logical to raise the individual population elements to some power other than unity, and not necessarily the same power (nor even necessarily a constant power).

When the various modifications of the distance and population factors that have been discussed are added to the basic energy equation, it becomes:

$$E_{ij} = \frac{k \left( \sum \phi \right)^b \cdot \left( \sum \phi \right)^c}{D_{ij}^a}$$

Clearly, the task of evaluating this model against a particular set of empirical data seems very large indeed. For every pair of possible weighting factors, the three exponents must be evaluated simultaneously. Experience with quadratic programming problems would indicate that this is likely to be very time-consuming, even on a high-speed digital computer. Further, the results of such an evaluation for a very much simpler model, to be reported later, raise the distinct possibility that such an evaluation may prove to be inconclusive. It may then be proper to question the probable return to the effort expended in such activity. More careful analysis of the factors, and the measures of the factors, seems to offer a greater potential return.

The adjustments of the basic concepts of gravity and potential models which have been variously proposed, and described above, open the way for introducing different kinds
of key variables in place of population and distance. Different key variables also imply new interpretations to be given to the gravity and potential relationships developed.

Other Modifications and Adaptations

Stouffer has suggested a modification, of interest in connection with measuring population mobility. He suggests that there is no necessary relationship between distance and mobility, but that the number of persons going a given distance is directly proportional to the number of opportunities at that distance and inversely proportional to the number of intervening opportunities. That is, the function of distance is not necessarily continuous in the formulation. Mathematically, the relation may be expressed:

\[
\frac{\Delta y}{\Delta s} = \frac{a \cdot \Delta x}{x \cdot \Delta s}
\]

where \(\Delta y\) = number of persons moving from a point of origin to a circular band of width \(\Delta s\); \(x\) = cumulated number of opportunities between the origin and destination \(s\); \(\Delta x\) = number of opportunities within the band of width \(\Delta s\); and \(a\) = a constant. In testing the hypothesis, Stouffer encountered

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difficult problems of measuring opportunities. One approach used by him was to consider total in-migrants as such a measure, but, as Anderson has pointed out, there is an element of circularity involved. Other tests of Stouffer's hypothesis have been carried out by Bright and Thomas, Folger, Isbell, and Strodtbeck. All have experienced similar difficulty with the measure of opportunities.

For purposes of developing models for projecting national and regional product, Isard and Freutel consider aggregate income to be a critical variable and suggest the use of an income potential measure, in which regional (or national) income is substituted for the population factor. They also suggest that the friction against interaction caused by distance is not so much a function of the intervening physical space, but rather a function of the cost of traversing this space. They therefore utilize a measure of effective or economic distance, in which physical distance is modified by

40 Anderson, American Sociological Review, XX, 289.


transport cost. Thus, under this hypothesis, the basic potential equation becomes:

$$W_i = \sum_{j=1}^{a} K_{ij} \frac{Y_j}{D_{ij}^{a}}$$

(II.17)

where $W_i$ = income potential at $i$; $Y_j$ = income of region $j$; $a$ = a constant exponent; and $K_{ij}$ = a parameter which differs from one pair of regions to another and which is some function of transport cost between each pair of regions, but note that this seems also to be ultimately dependent on distance.

Carroll and Anderson have suggested that a time-cost measure of distance would be appropriate in many circumstances. For instance, it is argued that this might be particularly pertinent with respect to intra-metropolitan interaction when the time involved in communicative activities appears to be a critical factor.

In applying the gravitation principle to traffic analysis, Voorhees, Sharpe, and Stegmaier use time of travel as a measure of distance, classifying movement by mode of travel to account for differences in rates. At the same time, they suggest classifying trips by the nature of the destination, such as a shopping area, and measuring the size

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of the attracting influence of the destination in terms such as floor area devoted to the sale of a particular commodity of interest.

Harris has extended the potential concept to include a measure of market potential. He measures the markets in terms of retail sales and distance in terms of transport costs, then sums over all markets. Thus, the market potential is:

\[ R_i = \sum_{j=1}^{n} \frac{S_j}{C_{ij}} \]  

where \( R_i \) = market potential at \( i \); \( S_j \) = volume of retail sales in region \( j \); and \( C_{ij} \) = transport cost from \( i \) to \( j \). Note that this relationship implicitly constitutes raising the distance factor to a variable exponent since transport costs are a product of distance and rate, and rate is a function of distance.

Anderson has generalized the potential concept in such a way that the numerator may be identified as any given resource (including population) whose distribution may be

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46 The term function is used here in a sense slightly different from mathematics, in that while distance is a factor, many transport rates are determined in a somewhat arbitrary fashion with a view to competition, subsidies, and similar influences.
useful in describing the variations in intensity of potential of interaction which may be found among areas. In the exponent to which he raises the distance factor is included a measure of the impact of technological innovation which will, in general, tend to decrease the energy consumed in traversing distance. This generalized formulation may be expressed as:

\[ H_i = k \sum_{j=1}^{n} \frac{X_j}{D_{ij}^a} \]  

(II.19)

where \( H_i \) = potential at \( i \) created by resource \( X \) in region \( j \); \( X_j \) = the measure of a given resource in region \( j \); and \( a \) = a constant exponent.

Summary

As has been pointed out, the gravity and potential concepts of human interaction were developed originally from analogy to Newtonian physics of matter. The behaviour of individual molecules of matter is not normally predictable, but in large numbers molecular behaviour is predictable on the basis of mathematical probability. Similarly, while it is not yet possible to describe the actions and reactions of the individual human in mathematical terms, it is the implicit hypothesis of all the above-mentioned authors -- as well as


48 It is interesting to note, parenthetically, that there have been no reported attempts to analyze human interaction using the related concepts developed by Einstein in his various relativity theories.
econometricians, and those involved in sociometry and psychrometrics that interactions of groups of people may be described mathematically. Note, however, that none of the authors cited used a probabilistic approach, and where probability was mentioned, it was added to the discussion found in the literature. The possibility that interaction may be described mathematically is suggested by the phenomenon observed in all social sciences that people do, in fact, behave differently in groups than they do as individuals. A group, however, is not the same thing as an aggregation of individuals acting independently, and perhaps also without knowledge of the action of others.  

It seems important to keep in mind, while evaluating the various works discussed above, that although the use of analogy in developing a concept may be attractive, flexibility must be maintained. Otherwise, science is replaced by dogma. There may be a point at which referring back to the original analogy can be dysfunctional in that it may retard forward progress. At this point, the original analogy, and perhaps even much of the results generated from it, should be abandoned in favour of some other, more fruitful approach.

In the case of gravity and potential models, a fundamental difficulty arises from the different nature of the

two basic entities involved in the original physical model and the human interaction models: the individual human being can make decisions with respect to his actions, while the individual molecule (presumably) cannot. This does not necessarily imply that the interaction of humans in large numbers cannot be described mathematically, but it does cast some doubt regarding the validity of aggregative procedures used in empirically testing the mathematical relationships evolved. It does appear that there exists some form of threshold where the power of individual decision-making critically affects the results obtained. This threshold may vary among interaction phenomena along many different dimensions. It would seem desirable to investigate somehow this notion of thresholds before concepts based on aggregation are broadly applied. This is tantamount to investigating whether the basic mathematical operations are legitimate procedures in models of human behaviour.
CHAPTER III

SPECIFIC HISTORY OF GRAVIMETRIC MODELS IN MARKETING

All the so-called laws of retail gravitation now appearing in the literature of marketing, except the one to be discussed in Chapter IV of this paper, are based on the analysis of retail trade begun by Reilly in 1927.¹ This work was previously mentioned in Chapter II in order to place it in its proper historical perspective with respect to gravimetric models in other branches of social science.

Reilly's studies were carried out over a period of more than three years, and had as their objective the discovery of some method for measuring the retail trade influence of a city.² Converse has performed considerable work in extending and refining the results obtained by Reilly.³

According to Schwartz, the net result of the combined efforts of Reilly and Converse can be summarized by six

² Reilly defines "retail trade influence of a city" as "the amount of trade a city draws from its surrounding area." (Ibid., p. 56).
equations which are the laws of retail gravitation. It is the purpose of this chapter to survey these equations in the order in which they were developed, to discuss briefly some aspects of the methodology by which empirical support was drawn, and to look at some more recent work pertaining to them.

The Laws of Retail Gravitation

The original law, with which it all began, was stated by Reilly:

Two cities attract retail trade from any intermediate city or town in the vicinity of the breaking point approximately in direct proportion to the population of the two cities and in inverse proportion to the square of the distances from these two cities to the intermediate town.

Mathematically, this statement has the following form:

\[ \frac{B_a}{B_b} = \frac{P_a}{P_b} \times \frac{D_b^2}{D_a^2} \] (III.1)

---


5 The term "retail trade" in Reilly's work is apparently meant to include purchases of shopping goods, specialty goods, and convenience goods (Reilly, op. cit.). On the other hand, Converse refers to purchases of shopping goods, especially fashion goods (Converse, Retail Trade Areas in Illinois).

6 The term "breaking point" is defined by Reilly to be "the point up to which one city shows domination of retail trade, and beyond which the other city dominates" (Reilly, op. cit. p. 64).

7 Ibid., p. 9.
where $B_a, B_b = \text{the proportions of retail trade from the intermediate town attracted by cities } A \text{ and } B$, respectively; 
$P_a, P_b = \text{the populations of cities } A \text{ and } B$, respectively; 
and $D_a, D_b = \text{the distances from the intermediate town to cities } A \text{ and } B$, respectively.

Converse and his associates developed a second equation, known as the breaking point formula, used to measure the movement of shopping goods trade, and written:

$$D_b = \frac{D_a + D_b}{1 + \sqrt{\frac{P_a}{P_b}}} \quad \text{(III.2)}$$

In this equation, the breaking point between any two cities is the intermediate community which divides its shopping goods trade equally between the two cities. According to Converse, Equation (III.2) can be used to determine the boundaries of a town's normal trading area without performing any field-work, providing the differences in populations of the towns and cities being studied are relatively small. Distances used are those measured along improved automobile highways.

Another law of retail gravitation developed by Converse states:

A trading center and a town in or near its trade area divide the trade of the town approximately in direct proportion to the populations of the two towns and inversely as the squares of the distance factors, using 4 as the distance factor of the home town.$^8$

---

Mathematically, this statement has the equation:

\[
\frac{B_a}{B_b} = \frac{P_o}{H_b} \times \frac{4}{L^2}
\]  \hspace{2cm} (III.3)

where \( B_a \) = the proportion of trade going to the outside town; 
\( B_b \) = the proportion of trade retained by the home town; 
\( P_o \) = the population of the outside town; 
\( H_b \) = the population of the home town; 
\( L \) = the distance to the outside town; and 
\( 4 \) = the inertia factor. The inertia-distance factor, according to Converse, reflects the effort required to overcome the inertia of travelling to a store close at hand. Converse speculates that the purchases not made in the nearby large city because of this inertia factor are made either in the home town or in shopping centres in the suburbs of the large city.\(^9\)

Equation (III.3) deals only with one small town and one large town, which is not always the actual arrangement. If a small town loses a considerable amount of trade to two or more larger towns, then, according to Converse, the proportion lost to these towns is determined by using multiples of 4 to obtain a total inertia factor.\(^10\) Converse states that he has experimented with this method, and he has concluded that it seems to work satisfactorily.\(^11\)

Two additional laws of retail gravitation developed by

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\(^9\) Ibid.

\(^10\) If home town B loses trade to two larger towns, inertia factor = 2 x 4 or 8, and so forth.

\(^11\) Ibid.
Converse are basically Equations (III.1) and (III.2), but modified to increase predictive accuracy in those cases where a trading centre is more than twenty times the size of the intermediate town. The revised equations are:

\[
\frac{B_a}{B_b} = \frac{P_a}{P_b} \times \frac{D_b^3}{D_a^3} \tag{III.4}
\]

\[
D_b = \frac{D_a + D_b}{1 + \sqrt{\frac{P_a}{P_b}}} \tag{III.5}
\]

Converse regards Equations (III.4) and (III.5) as tentative, because "we do not have enough data as yet to measure the accuracy of this adjustment."\(^{12}\)

The final equation developed by Converse is a modification of Equation (III.3), and was tested only in Chicago and vicinity. Because of urban congestion, neighbouring small towns were found to retain a larger proportion of their fashion goods trade than Equation (III.3) predicts. Using suitable data, it was determined that the inertia-distance factor should be 1.5 for towns in the vicinity of Chicago, and thus the equation is:

\[
\frac{B_a}{B_b} = \frac{P_a}{P_b} \times \frac{1.5^2}{d^2} \tag{III.6}
\]

\(^{12}\) Ibid., p. 383.
Empirical Development of the Laws

Schwartz states that "the method used to develop Reilly's law is an example of the use of regression analysis to discover a scientific law." This statement may not be an entirely accurate characterization of what Reilly actually did, but this will become more apparent from the following discussion.

Reilly chose to investigate the phenomenon that he called "the division of retail trade between two cities" to determine what factors would explain this phenomenon (in a causal sense), and what relationship the factors bore to each other and the phenomenon. Primarily, it appears, by using introspection (and perhaps also implicit reference to Newton's Law of Gravity) Reilly ascertained that the specific factors of population of the cities and distance from the cities must be the determining factors. Reilly explicitly states:

It is so readily acceptable that the amount of outside trade which a city enjoys in any surrounding town is a direct function of the population of that city and an inverse function of the distance of the city from that town, that the general law needs no support.

Reilly uses distance in an inverse relationship because he implicitly seems to assume the image of an economic man, and

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13 Schwartz, op. cit., p. 16.
14 Reilly, op. cit., p. 71. The "general law" is given below as Equation (III.7).
reasons that is a priori inconvenient and costly for people to travel to shop. On the other hand, Reilly is not nearly so certain in his choice of population in a direct relationship. He asserts that it is not merely the fact of a large cluster of people alone which causes other people to travel to a large city to shop, but rather it is the existence of such attractions as large retail stores with a wide variety of goods, social and amusement attractions, and advertising media in the large city which cause people to buy there. But Reilly argues, deductively, that the existence of the attractions would not be economically feasible in the absence of larger population, ergo there must be a direct relationship between the factors which induce people to travel to a city to shop and the size of that city's population. Reilly concludes that, in view of this relationship, it is reasonable to use population as a "proxy" variable for these other factors. It is interesting to note that the implicit model of the shopper which Reilly

---

15 Reilly, op. cit., pp. 73-75, ventures a very long list of factors which may influence division of retail trade, and population and distance are only two among the many.

16 Subsequent studies have used other measures. In Frank Strohkark and Katherine Phelps, "The Mechanics of Constructing a Market Area Map." Journal of Marketing, XII (April, 1948), 495-496, it is concluded that substitution of "shopping line sales" in place of population yielded more accurate results. There is a serious question here as to whether the resulting statement is largely tautological, since in a causal sense it would state that shopping line sales cause shopping line sales.

17 Reilly, op. cit., pp. 30-32.
seems to use in reaching this conclusion is rather different from the economic man model of the shopper who is influenced by distance. How these different images of the shopper affect the operation of the model is not evident.

Reilly has summed up the basis for his use of population and distance as variables in his law as follows:

... Evidence secured ... shows that the population of a city and the distance from that city to another comparable city are the primary factors that condition the retail trade influence of that city; that population and distance are reliable indexes of the behavior of other factors; that other factors are either so closely related to, or so directly dependent upon, these two primary factors that the effects of the dependent factors tend to balance out when cities are compared on the basis of population and distance.\(^{18}\)

Having deduced and supported his position that these are the factors and the direction of their influence, Reilly puts the relationship in general form:

$$\frac{B_a}{B_b} = \frac{P_a^\nu}{P_b^\nu} \times \frac{D_b^n}{D_a^n}$$ (III.7)

The empirical phase of his research then became the problem of evaluating the exponents \(\nu\) and \(n\).

Reilly determined that the value of \(\nu\) should be one on the basis of studies which he does not present. He writes:

\(^{18}\) Ibid., pp. 31-32. The "evidence secured" is not presented.
... By personal investigation we have approximated the rate at which outside trade drawn by a city increases with the population of that city, and we have evidence to support the use of the first power of the population variable. \footnote{Ibid., pp. 71-72.}

Reilly then solved his general equation for \( \gamma \), and substituted appropriate values for the other variables. Population data were relatively easily obtained. Reilly chose to evaluate \( \gamma \) only for those intermediate towns which were breaking points between any two pair of trading centres. Thus, \( B_a / B_b = 1 \), and Reilly needed only to determine which towns were breaking points. This would also give him the necessary distances. Reilly's discussion of how this was handled is interesting:

We drove along the main automobile highways connecting larger cities and called upon the secretary of the Retail Credit Men's Association in intermediate cities and towns. On the basis of the records of credit inquiries kept in these offices, we were able to find that point at which the preponderance of retail trade ceased to flow in the direction of the city we had left and began to flow in the direction of the city we were approaching.\footnote{Ibid., p. 64.}

When the breaking point town was determined in this manner, Reilly could ascertain the distances by consulting a highway map. In this manner, \( \gamma \) was evaluated in 225 cases, and the frequency distribution of the results is presented in Table I and Figure 1. From this distribution Reilly concluded:
<table>
<thead>
<tr>
<th>Value of</th>
<th>No. of Cases</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 - 1.5</td>
<td>45</td>
<td>20.0</td>
</tr>
<tr>
<td>1.51 - 2.5</td>
<td>87</td>
<td>38.8</td>
</tr>
<tr>
<td>2.51 - 3.5</td>
<td>35</td>
<td>15.6</td>
</tr>
<tr>
<td>4.51 - 5.5</td>
<td>15</td>
<td>6.7</td>
</tr>
<tr>
<td>5.51 - 6.5</td>
<td>14</td>
<td>6.2</td>
</tr>
<tr>
<td>6.51 - 7.5</td>
<td>6</td>
<td>2.7</td>
</tr>
<tr>
<td>7.51 - 8.5</td>
<td>5</td>
<td>2.2</td>
</tr>
<tr>
<td>8.51 - 9.5</td>
<td>12</td>
<td>5.3</td>
</tr>
<tr>
<td>9.51 - 10.5</td>
<td>5</td>
<td>2.2</td>
</tr>
<tr>
<td>10.51 - 11.5</td>
<td>3</td>
<td>1.3</td>
</tr>
<tr>
<td>11.51 - 12.5</td>
<td>4</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Total 225

a - These data computed for plotting purposes: See Figure 1.

Figure 1

Reilly's Determination of the Distance Exponent in Reilly's Law

Relative Frequency of Values of the Exponent Empirically Determined
... a clear mode occurs in the range of 1.51 - 2.50 which shows that the exponent of distance is nearer to the second power than to any other even power.\footnote{21}

Over the intervening years the fact that there was a distribution in the values of the exponent, and that the value chosen was a modal or maximum likelihood estimate, has been lost from sight. The result is that students who encounter Reilly's law -- often in a first course in marketing -- gain the impression that it is much more precise and rigorous than has been shown. This is unfortunate, since many students then cannot understand why other similarly well-defined relationships have not been uncovered in marketing.

Tests of the Law of Retail Gravitation

Reilly undertook to test his law by computing the location of the breaking point between each pair of cities in a set of thirty pairs of trading centres by means of his equation. He then conducted field studies to determine the actual location of the breaking point towns, and compared the results. The comparison will not be presented here because of its length, but Reilly concluded that his law was "startlingly" accurate.\footnote{22} There is no doubt in examining the comparison that the agreement between the predicted and actual results is very close. It would have been interesting

\footnote{21}{William J. Reilly, Methods for the Study of Retail Relationships (Austin: University of Texas Press, 1929), p. 50.}

\footnote{22}{Reilly, The Law of Retail Gravitation, pp. 25-29.}
if Reilly had also worked backward from his empirical results and calculated the value of $n$ for these thirty pairs of trading centres. Undoubtedly the values would be close to two.

In 1943, Converse published a study in which he reported the results of his efforts to check the accuracy of Reilly's law. His study compared the predictions of Reilly's law with the movement of retail trade ascertained through consumer surveys wherein he determined how many families in the towns studied made their shopping goods purchases in nearby primary trading centres. Using Reilly's law in its original form, Converse predicted the relative division of retail trade for thirteen intermediate towns lying between Champaign-Urbana and five other primary trading centres. From the survey data he computed the percentages of consumers who bought shopping goods in Champaign-Urbana and the competing trading centres. He then calculated the coefficient of correlation between the predictions and the empirical results (least squares, $r = 0.88$, $r^2 = 0.77$), and concluded that, "on the whole, it [Reilly's law] works with a relatively high degree of accuracy." Converse later reported another study in the same geographical area involving the division of fashion goods purchases. In this study, the dollar amounts spent on fashion goods by the inhabitants of each town in the nearby trading

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24 Ibid., p. 48.
centres were estimated. From these, the relative division was calculated and compared with predictions based on Reilly’s law. In this study, the agreement was closer than before (least squares, $r = 0.93$, $r^2 = 0.86$). Even so, Converse felt it necessary to qualify and explain the high degree of agreement in a note of caution:

It should not, however, be concluded that the law will measure the movement of trade with such accuracy in all territories. The towns included in the computations used here are in a territory in which the primary trading centers are considerably larger than the intermediate towns. In areas where there is less difference in size between the primary and secondary trading centers or between the trading centers and the towns from which they draw trade, the law of retail gravitation may perhaps not predict the movement of trade with the accuracy found in the territory here studied.

More recently, another test of the law of retail gravitation was conducted by Reynolds. Reynolds used the same equation as Reilly did to determine the value of $n$, the exponent of the distance factor. While Reilly had concluded that this exponent should be equal to two on the basis of his empirical data, Reynolds sought to find out if other empirical data would support Reilly’s conclusion. Reynolds used data collected in 1935 for ninety-one Iowa counties, and data

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26 Ibid., p. 18.
collected in 1949 for southwest Iowa. From these, he computed regression equations for groceries, eggs and poultry, movies, farm machinery, lumber and cement, physicians' services, women's coats and dresses, men's good shoes, and men's suits.

The data which Reynolds used described the trading areas of specific trading centres for each of the products mentioned. The study itself is described as follows:

From ... [the] ... points, where trading area boundaries crossed roads, highway distances to the nearest mile ... were measured to the two trading centers in question and the populations of the centers ... were recorded to the nearest hundred. Two values each, $D(D_1/D_2$ and $D_2/D_1)$ and $P(P_1/P_2$ and $P_2/P_1$) then were computed for every breaking point.

... The problem resolved itself to finding the value of the constant, $b$, in the following linear equation, ...

\[ \log D = b \log P. \]

This $b$ is the exponent of $P$ in Converse's formula, where it is .50, or the square root.

In addition to calculating a $b$ value for each of the commodities he studied, Reynolds also computed coefficients of determination for these products. The results are shown in Table II, which includes also the conversion of $b$ values back to their equivalent $\eta$ values. These $\eta$ values are cumulated in Table III, and the relative frequency is plotted in Figure 2. These can be compared with Table III and Figure 3. It is granted that the number in the sample is very small,

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28 Note that only some of these commodities are shopping goods; namely, women's coats and dresses, men's good shoes, and men's suits.

### TABLE II

Reynolds' Test of Reilly's Law

<table>
<thead>
<tr>
<th>Product</th>
<th>1935</th>
<th>1949&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b$</td>
<td>$n = \frac{1}{6}$</td>
</tr>
<tr>
<td>Groceries</td>
<td>.31</td>
<td>3.24</td>
</tr>
<tr>
<td>Eggs and Poultry, Marketing of</td>
<td>.32</td>
<td>3.12</td>
</tr>
<tr>
<td>Movies</td>
<td>.31</td>
<td>3.24</td>
</tr>
<tr>
<td>Lumber and Cement</td>
<td>.22</td>
<td>4.54</td>
</tr>
<tr>
<td>Farm Machinery</td>
<td>n.s.</td>
<td>-</td>
</tr>
<tr>
<td>Physician</td>
<td>.22</td>
<td>4.54</td>
</tr>
<tr>
<td>Women's Coats and Dresses</td>
<td>.44</td>
<td>2.27</td>
</tr>
<tr>
<td>Men's Suits</td>
<td>.45</td>
<td>2.22</td>
</tr>
<tr>
<td>Men's Good Shoes</td>
<td>n.s.</td>
<td>-</td>
</tr>
</tbody>
</table>

<sup>a</sup> - Southwest Iowa only  
<sup>b</sup> - Eggs only  
<sup>c</sup> - Women's Coats only  
n.s. - not studied

TABLE III

Reynolds' Test of Reilly's Law - Evaluation of Distance Exponent

<table>
<thead>
<tr>
<th>Value of</th>
<th>No. of Cases</th>
<th>% of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 - 1.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.51 - 2.5</td>
<td>6</td>
<td>42.8</td>
</tr>
<tr>
<td>2.51 - 3.5</td>
<td>4</td>
<td>28.6</td>
</tr>
<tr>
<td>3.51 - 4.5</td>
<td>1</td>
<td>7.2</td>
</tr>
<tr>
<td>4.51 - 5.5</td>
<td>2</td>
<td>14.3</td>
</tr>
<tr>
<td>5.51 - 6.5</td>
<td>1</td>
<td>7.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>14</strong></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.

Reynolds' Test of Reilly's Law

Relative Frequency of Values of the Exponent Empirically Determined
but even so, there is a distinct resemblance to Reilly's data shown in this way. Shown these results, Reilly would undoubtedly suggest that they support his original contention.

Reynolds treated the results differently, and tested the significance of the difference between each $\phi$ value and 0.50. He found that for 1935 the differences were significant and concluded that these results were evidence warranting rejection of the breaking point equation (Equation III.2). Similar tests performed for the 1949 results produced significant differences for the products studied, except for the shopping goods items. Since the model was originally formulated for shopping goods, it would not seem that Reynolds' investigation produced any evidence on which it could be decided that the model should be rejected.

Reynolds carried his investigation a little further, and computed coefficients of determination in order to find the extent to which the variation in the location of the breaking point towns between trading centres was explained by the model. These coefficients are shown in Table II. For the shopping goods studied, these coefficients fall within the range 0.75 to 0.90. However, over the whole sample the range is considerably wider, and there seemed to be some relationship between the exponent $n$ and the coefficient $r^2$. For this reason, a scatter diagram has been plotted, and is given as Figure 3. While no attempt has been made to fit a curve to these data, it can be seen by inspection that there is a definite tendency to rapidly declining $r^2$ values with increas-
ing values. In words, this means that the power of account of the model -- measured by the coefficient of determination -- decreases with increasing values of the exponent. Whether this is significant, or largely tautological, is difficult to determine. Further, since Reynolds' data are not directly comparable to Reilly's or Converse's, it seems that the results, at best, are inconclusive in testing the breaking point relationship.

After publication of Reynolds' study, there was an exchange of views with Converse which unfortunately generated much heat and little light.\(^{30}\) Thus, the situation remained unchanged.

Relatively recently, Jung presented some data which, he maintains, contradict Reilly's law.\(^{31}\) Jung wrote that a study of buying habits of the residents of Columbia, Missouri showed no preference for buying at stores in St. Louis rather than Kansas City. According to Reilly's law, however, it would be expected that Columbia residents would have made a substantially larger proportion of their purchases in St. Louis.

Reilly has countered this result by arguing that Jung has not invalidated his general law. The most that has been done, according to Reilly, is to dispute the judgment that


N = 1 and n = 2. This immediately raises the question of what exactly is Reilly's law? Common usage, and decreasing frequency of reference to the original work, has produced the belief that the law is the particular equation in which \( N = 1 \) and \( n = 2 \). It would appear that Jung might not have consulted the original empirical work of Reilly, nor noted the qualifications which Converse applied, both of which have been discussed above.

**Summary**

The specific history of gravimetric models in marketing evolves from the basic law of retail gravitation stated by Reilly. Although Reilly, himself, made no mention of Newton's Law of Gravitation, nevertheless the similarity exists. As was mentioned earlier (in Chapter II), Stewart has written that Reilly's law can be derived from Newton's Law, and further that it represents the first recognition of demographic gravitation.\(^{33}\)

In his several studies, Reilly attempted to find methods for measuring the retail trade influence of a city. Proceeding in a deductively analytical manner, he concluded that distance and population (as a proxy variable for a range of attractive

\(^{32}\) Schwartz, *op. cit.*, p. 28, reports that Reilly stated this position at the Illinois Symposium in October, 1959.

\(^{33}\) John Q. Stewart, "Demographic Gravitation: Evidence and Application," *Sociometry*, XI (February-May, 1948), 35. The survey given in Chapter II shows that Stewart was not strictly correct in his statement.
influences) were the major determinants. He then, in effect, hypothesized the general form of the relationship involving the variables, and proceeded to evaluate his parameters empirically. On this basis, he concluded it should have the form of Equation (III.1) above. Reilly then tested the resulting form against another, smaller sample and found remarkably good agreement.\footnote{Schwartz, op. cit., p. 31, has computed the coefficient of correlation between predicted and actual distances in this sample to be +0.98, but points out a very slight upward bias.} As has been discussed, others have developed evidence which does not agree so closely, but in view of the original method of developing the law, this cannot be taken as sufficient evidence warranting rejection. Converse's reformulations are another issue, as the law was reworked for application in rather different circumstances.
A PROBABILISTIC MODEL OF INTRA-URBAN TRADE INTERACTIONS

The gravimetric models discussed in the preceding chapters have revealed one important characteristic in common; they are all macro-models. The human interactions studied are treated in a highly aggregated basis, at the national, regional, or inter-urban level.

Marketing, however, also operates at the micro level, the level of the firms and consumers where decisions are made. Since the time of the introduction of Reilly's law into the marketing literature, there have been some important changes in the institutional structure of marketing which affect both groups of decision makers and their relationship to each other. In many large cities, the notion of a single downtown shopping core, as considered by central place theory, has been superseded by a notion of many alternate centres of retailing activity with the rise of the planned shopping centre as a new form of retail institution. Two of the most important questions which may be asked by those investing large sums of money in promoting and developing such centres would be in regard to where the centre should be located, and how large the physical plant should be.

In attempting to answer such questions, Reilly's law has been applied, with modifications, to estimate trading areas of proposed shopping centres within cities (intra-urban). For
example, given a proposed site for a shopping centre, the number, proximity, and size of competing centres can be determined. Using the modified gravity model, the breaking points from the proposed centre to each of the existing centres may be calculated, and the results mapped to delineate the potential trading area of the proposed unit. Such a delineation could be imposed over a census tract map to determine the number of persons within the trading area. This information may then be used as a basis for determining the purchasing power potential of the trading area, and thus the potential sales and profitability of the proposed shopping centre. The same determination may be made for other alternative locations, and a certain amount of information may be developed for comparing all the alternatives.

The modifications made to Reilly's law for intra-urban applications involve changes to the factors chosen as measures of attractive and detractive forces. In one modification, developed by the Curtis Publishing Company, the factors chosen have been the square footage of each retail centre in place of population, and travel time between retail centres in place of physical distance.¹

Deductive reasoning may be used to support the choice of these factors in the same manner as Reilly argued for his original selection of population for his model. As in Reilly's

model, the arguments, in reference to shopping goods only, are developed from introspection and from the results of empirical research. With regard to the selection of shopping centre area (square footage) as a measure of the attractive power, it may be argued that it really is the number of items of the general class which the consumer may desire that are carried by the various shopping centres that is attractive. That is, it is the breadth and depth of the product assortment of a particular shopping good at the particular shopping centre, measured in a suitable way, which is the attraction. The consumer does not known in advance, generally, whether he (or she) may be able to obtain a particular shopping goods item at a particular centre, but may have formed some sort of a priori opinion regarding the relative probability that success in obtaining the desired item may be achieved in one or more places. Presumably, the greater the number of items carried by a particular shopping centre, the greater is the consumer's expectation that a shopping expedition to that centre will be successful. It is thus hypothesized that the degree of expectation, or relative subjective probability is directly proportional to the number of items. However, it is not easy to measure the number of items, particularly in a way which represents the consumer's view of it. Thus, it is further hypothesized that this is

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2 The notion of probability is highly subjective in this case.

measured (or approximated at least) by using the square footage of selling space, again in direct proportion. ⁴

Similarly, it may be argued that the consumer's perception of the effort and expense involved in travelling to various shopping centres modifies the situation. The costs of transportation, the effort in making the trip (a form of inertia), and other opportunities foregone, all tend to detract from the consumer's perception of a shopping centre.

It is not really possible to speak of quite the same probability or expectation as above after distance and cost are introduced. Rather, the notion of distance and cost requires the probability to be regarded as conditional, or better still to be regarded as being modified by a measure of likelihood related to the distance involved (as in Bayesian statistics). It is implicitly hypothesized that the likelihood measure should be multiplicative with the original probability. The result is a measure of the probability that the consumer would go from his particular starting point to the store in question.

It can be argued that when a store is close by, the effect of distance on the consumer's expectation is very small. As distance increases, however, it is hypothesized that the net probability will decrease rapidly, and tend to approach zero, but never quite reach it. This type of relationship is des-

⁴ Square footage may be some sort of total, or it may be further refined into that devoted to the sale of particular shopping goods items.
cribed by a declining exponential function, and the inverse distance-factor in Reilly's law may be assumed to be a suitable first approximation. On the other hand, it is not the physical distance which is relevant in the contemporary scene, but rather distance as measured in units of travel time by a direct route. This follows current common usage, and is probably a good measure of the consumer's perception of distance. 

It seems to be reasonably logical, in a deductive sense, to substitute store area and travel time into the equation of Reilly's law for use in the intra-urban situation. The breaking point is then the point at which the net probabilities of consumers going to one or the other of a pair of centres are equal. Knowing the store areas, and the travel time from one centre to the other, the breaking point can be determined as before, and the trading area progressively delineated.

One drawback of this approach seems to be that it considers only the behaviour of persons on or near the traffic artery joining two centres. In the usual case, the breaking points are determined separately and then joined on a map by straight lines. It may not be unreasonable to use main arteries when these are inter-urban highways passing through intermediate towns, but it does not seem to make good sense to ignore in effect, a very large mass of people who happen to

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5 This does not distinguish between what the consumer believes the distance to be, and what the distance actually is when objectively measured. This distinction is quite another issue.
live or work between these points. Further, this model seems to assume that consumers only consider, as shopping alternatives, the two centres immediately adjacent to their starting point, and no others. However, consumers do have a much wider latitude of choice, and they frequently exercise it. Finally, by terminating a store's retail trading area at the breaking point, this application excludes from consideration those persons whose probability of selecting the store is less than 0.5. On the other hand, even if a large number of consumers only chose a particular shopping centre for one quarter of their shopping trips, this would still generate an important volume of traffic to the centre. It seems unduly artificial to delimit the trading area at the breaking point.

Huff has commented on these, and other limitations of gravimetric models and their application to retail trade area analysis.6 In particular, he has questioned the assumption that the exponent of the distance factor which Reilly had originally estimated as 2 for inter-urban trade movements would be the same for urban areas and urban trade movements. Huff argues that this is particularly questionable when other studies (discussed in Chapter II) have shown that the exponent has ranged from 1.5 to over 3, depending on the type of trip, as well as the geographical setting being analyzed.

---
Huff argues also that the gravity model does not have a sound theoretical basis; that it does not reveal why the observed regularities occur as they do. In view of the discussion of Reilly's work in Chapter III, this argument may not be as valid as a cursory glance might suggest. In terms of the accepted methodology of social science, Reilly's law is on fairly firm ground. This is not to deny that it could be improved upon, however.

Huff's Approach to Consumer Spatial Behaviour

In view of the many limitations attached to the current fund of knowledge, Huff has developed and started to test a model which he believes overcomes some of these limitations. In terms of the accepted methodology of social science, Reilly's law is on fairly firm ground. This is not to deny that it could be improved upon, however.

Huff's Approach to Consumer Spatial Behaviour

In view of the many limitations attached to the current fund of knowledge, Huff has developed and started to test a model which he believes overcomes some of these limitations. In terms of the accepted methodology of social science, Reilly's law is on fairly firm ground. This is not to deny that it could be improved upon, however.

With regard to the application of his model, Huff writes:

The model provides a tentative operational basis for understanding and determining the retail trade area of a shopping center. The retail trade area of an existing or proposed shopping center can be ascertained by: (1) dividing the surrounding area into small statistical units; (2) calculating the probability of consumers from each of these units going to a particular shopping center; and (3) drawing lines connecting all statistical units having like probabilities. A retail trade area is thus not a fixed line circumscribing a shopping center, but rather a series of zonal probability contours.

---


8 Huff, Determination of Intra-Urban Retail Trade Areas, p. 5.
In order to determine these probabilities, Huff sets out to develop a model which, while it uses some of the conceptual properties of the gravity model, focuses on the consumer rather than on the retail firm per se. Huff writes:

Since the consumer is really the primary object of any trade area analysis, an explicit understanding is needed not only of the factors affecting his choice of a shopping center, but also of the choice process itself which gives rise to observable spatial behavior.  

Thus Huff concludes that the specific objectives of his study must be as follows:

1. A mathematical model will be formulated which represents a theoretical abstraction of consumer spatial behavior. Mathematical conclusions will be deduced from the model which, in turn, will be interpreted in terms of their behavioral implications.

2. The behavioral implications stemming from the model will serve as a frame of reference for designing an empirical study to test their validity. Actual consumer spatial behavior will be compared statistically to the mathematically derived behavior, i.e. expected behavior, in order to appraise the suitability of the model as a predictive and explanatory theory of consumer spatial behavior.

3. A method will be developed and exemplified by which the basic model of consumer spatial behavior can be used to delimit intra-urban retail trade areas.

In short, Huff, proposes to develop his model, test it empirically, and then apply it. This thesis is concerned only

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9 Ibid., p.12. Huff defines "spatial behavior" as referring to the observable courses of action that consumers take with respect to their choice of a shopping centre.

10 Ibid., p. 13.
with the first two of his three steps.

Huff's Concept of Utility and The Model

Because he is critical of the theoretical basis of Reilly's law, Huff seems to feel he would be on stronger ground to base his model on the theory of utility analysis as applied to individual choice behaviour. In order to do this, however, the definition of utility is modified from the ordinal concept of economic theory to a view which regards utility as being measured, in effect, on a ratio scale. This is perhaps best explained in Huff's own words:

... It is assumed that the ratio between the probabilities of a consumer's choosing any one of two particular shopping centers does not depend on the existence of other centers. This ratio is called the ratio of utilities of the two centers to a consumer.

Such postulated behavior differs markedly from traditional economic theory. The latter maintains that a consumer either always chooses one particular (the "most desirable") alternative with probability 1, or is indifferent between several "most desirable" alternatives (presumably chosen with equal probabilities). As a consequence, all other alternatives possess zero probabilities.

This difference in postulates is reflected in the definition of the word "utility". In the classical theory, utility is identical with the rank of an alternative and that alternative with the highest rank is always chosen. The definition being advanced here, however, asserts that utility is proportional to the probability of being chosen.  

---

11 This is because, while classical utility theory deals in ranked alternatives, Huff uses the ratio of two utilities. The discussion of the various kinds of scales in Chapter II shows that this is only meaningful when using ratio scales, since he is making a proportionate transformation.

12 Ibid., p. 4.
This approach seems to introduce unnecessary confusion, circularity, and inconsistency into the development of the model. After tracing Huff's reasoning in arriving at his model, it will also be shown that the same model can be devised without reference to the utility notion, and the conceptual confusion which it introduces.

Huff sets down some terms and definitions of elements on which his model is based, as follows:

1. A set of alternative shopping center choices which is represented as set J;

2. A subset of alternative shopping center choices which is represented as $J_0$. The subset $J_0$ of alternatives represents available alternatives which are in accord with a consumer's tastes and preferences. Any given alternative within the subset $J_0$ is represented as $j$ (where $j = 1, ..., n$); and

3. A positive "pay off" function $u$ is associated with each alternative shopping center indicating its "utility" to a consumer.\(^1\)

From these basic terms and definitions, Huff develops a set of basic propositions, which are summarized below:\(^2\)

1. The probability $P_j$ of a given alternative $j$ being chosen from among all alternatives in the subset $J_0$ is proportional to $u_j$. That is:

$$P_j = \frac{u_j}{\sum_{j=1}^{n} u_j} \quad (IV.1)$$

\(^1\) Ibid., p. 14.

\(^2\) Adapted from Ibid., pp. 14-16.
such that \( \sum_{j=1}^{n} P_{j} = 1 \); and \( 0 < P_{j} < 1 \). It seems here that what Huf is implicitly saying is that:

\[ P_{j} = k \ u_{j} \]

and

\[ k = \frac{i}{\sum_{j=1}^{n} u_{j}} \]

2. The ratio between the probabilities of a consumer's choosing any one of two particular shopping centres does not depend on the existence of other centres. This ratio is called the ratio of utilities of the two centres to a consumer, and thus:

\[ \frac{P_{1}}{P_{2}} = \frac{u_{1}}{u_{2}} \]

But, this logic is unnecessary, because, by Equation (IV.1):

\[
\frac{P_{1}}{P_{2}} = \frac{u_{1}/\sum_{j} u_{j}}{u_{2}/\sum_{j} u_{j}} = \frac{u_{1}}{u_{2}}
\]

and this is true for \( j = 1, \ldots, n \), regardless of the value of \( n \).

3. The properties of the pair \( (P_{1}, P_{2}) \) that determine the utility in \( (u_{1}, u_{2}) \) are: (a) the size \( S_{j} \) of a given shopping centre; and (b) the distance \( T_{ij} \) in time units, from a consumer's travel base \( i \) to a shopping centre \( j \). There is no argument with his choice of factors, space and time, since the discussion regarding other intra-urban studies has shown them to be logically acceptable. What is disturbing at this
point is the sudden switch from defining probabilities in terms of utilities and ratios of utilities to defining utilities in terms of probabilities. What Huff means is that a relative utility can be defined in terms of the relative frequency with which certain items (shopping centres) are chosen in a series of trials.\textsuperscript{15} The fact that probabilities are also defined as relative frequencies is immaterial to the discussion. It is inconsistent, circular, and unnecessary to infer that probability underlies utility when it is clearly the other way around. It may be the consumer's subjective probability estimate of success which defines or determines the utility, but this is quite different from the relative frequency notion which Huff seems to be discussing.

4. The next proposition is that the utility of a shopping centre is directly proportional to the area factor $S_j$, and inversely proportional to the time factor $T_{ij}$ raised to an exponent $\lambda$, where $\lambda$ is a parameter. Thus:

$$u_{ij} = k \frac{S_j}{T_{ij}^\lambda}$$

(IV.3)

where $u_{ij}$ = the utility of a shopping centre $j$ to a consumer at $i$. The total utility of all shopping centres relevant to the consumer at $i$ would be:

$$\sum_{j=1}^{\alpha} u_{ij} = k \frac{S_1}{T_{i1}^\lambda} + k \frac{S_2}{T_{i2}^\lambda} + \ldots + k \frac{S_n}{T_{in}^\lambda}$$

$$= k \sum_{j=1}^{\alpha} \frac{S_j}{T_{ij}^\lambda}$$

(IV.4)

\textsuperscript{15} There can be no doubt regarding what Huff intends, as he discusses it in some detail, \textit{Ibid.}, p. 16.
Thus, by Equations (IV.3) and (IV.4), the relative frequency probability that a consumer at \( i \) will go to a particular store \( j \) is defined as:

\[
Pr_{ij} = \frac{u_{ij}}{\sum_j u_{ij}} = \frac{S_j/T_{ij}^\lambda}{\sum_j S_j/T_{ij}^\lambda} \tag{IV.5}
\]

where the constant of proportionality \( \lambda \) in both Equations (IV.3) and (IV.4) is assumed to be the same, and thus vanishes from the probability definition. Equation (IV.5) is Huff's model. The parameter \( \lambda \) is to be estimated empirically to reflect the effect of travel time on various kinds of shopping trips, as it will be recalled that Huff does not believe that this should be equal to 2 in every case.

5. One further proposition is necessary in order to apply the model, and also for the computational procedure used to estimate \( \lambda \). This proposition states that the expected number of consumers at a given place \( i \), choosing to shop at \( j \), is proportional to the number of consumers at \( i \) and to the probability that a consumer at \( i \) will select \( j \) for shopping. Thus:

\[
E(C_i)j = Pr_{ij} \times C_i \tag{IV.6}
\]

where \( E(C_i)j \) = the expected number of consumers at \( i \) choosing shopping centre \( j \); and \( C_i \) = the number of consumers at \( i \). The definition of \( C_i \) must change depending on whether the model is being used to delineate trade areas, or to estimate the parameter. In the first case, it is the total number of
potential consumers in the area designated as \( i \). In the second case, it is the total number of consumers sampled from \( i \).

It can now be demonstrated how this same model, Equation (IV.5), can be developed without using the detour through a modified utility theory. It will be recalled that in discussing the basis for selecting the factors of store area and time for use in the intra-urban application of Reilly's law, it was argued that the consumer forms an opinion, a subjective probability, regarding the success of a shopping trip to a particular shopping centre. It was explained that this is thought, or assumed, to be on the basis of the breadth and depth of the selection of items available, and this is assumed to be adequately represented by store area. Thus, the subjective probability is taken to be proportional to store area:

\[
SP_{rj} = k_i S_j
\]  

(IV.7)

where \( SP_{rj} \) = the subjective probability; and \( k_i \) = a constant of proportionality. It is assumed that \( k_i \) is the same for all \( j \), otherwise there is no hope for a model where individuals form opinions inconsistently.

It was then assumed that a measure of likelihood is attached to this subjective probability to account for the fact that all shopping centres are not equally distant from consumers who do, under some circumstances, behave as economic men. This measure of likelihood is assumed to be adequately
represented by the inverse of $T_{ij}$ raised to an exponent. Accepting the evidence presented in Chapter II that this exponent differs from 2, depending on the trip type, it can be represented as $x$. It is also assumed that the measure of likelihood multiplies the subjective probability. Thus:

$$SL_j = S_P_j \cdot \frac{1}{T_{ij}^x}$$  \hspace{1cm} (IV.8)

where $S_L_j$ = the subjective likelihood that a consumer would go to shopping centre $j$.

There is no necessary reason for all these subjective measures to total 1 as in the relative frequency case. Therefore, on the assumption that all opinions are formed consistently, each subjective measure is scaled by a factor $k_2$ so that they do total 1. Thus

$$k_2 \sum_{j=1}^{n} S_L_j = k_2 S_L_1 + k_2 S_L_2 + \cdots + k_2 S_L_n$$

and,

$$k_2 S_L_j = k_2 k_2 \frac{S_j}{T_{ij}^x}$$

and thus,

$$k_2 \sum_{j=1}^{n} S_L_j = k_2 k_2 \sum_{j=1}^{n} \frac{S_j}{T_{ij}^x}$$  \hspace{1cm} (IV.9)

It is then possible to define a relative frequency measure of the probability that a consumer will go from $i$ to $j$, in comparison to all relevant alternatives ($j = 1, \ldots, n$):
where $FP_{ij} = \text{the relative frequency probability measure.}$

Equation (IV.10) is identical to Equation (IV.5), and was developed without reference to the notion of utility.

It was mentioned in Chapter II that models may be developed for purposes of explanation, prediction, and/or control. Both of the preceding developments contain the first two elements of models. However, even if the need for explanation is dropped, the model can still be developed. It is noted on the basis of empirical evidence that the number of people at $i$, going to $j$, is directly proportional (more or less) to store area, and inversely proportional (more or less) to the distance, measured in time units, from $i$ to $j$. Thus:

$$N_{ij} = \kappa \frac{S_j}{T_{ij}^x} \quad \text{(IV.11)}$$

where $N_{ij} = \text{the number at } i \text{ going to a particular } j$; and $\kappa = \text{a constant of proportionality}$. The same is true for all $j ( j = 1, \ldots, n)$, and therefore the total number of people
going to any $j$ is:

$$\sum_{j=1}^{n} N_{ij} = k \sum_{j=1}^{n} \frac{S_j}{T_{ij}}$$

(IV.12)

And the relative frequency of people at $i$ going to a particular $j$ out of all possible $j$ is:

$$F_{ij} = \frac{N_{ij}}{\sum_{j} N_{ij}} = \frac{S_j}{\sum_{j} T_{ij}}$$

(IV.13)

where $F_{ij}$ = the relative frequency. Equation (IV.13) is identical to Equations (IV.10) and (IV.5). Whether $F_{ij}$ is interpreted as a probability measure is immaterial. The model predicts the relative frequency, and presumably could be tested as to whether or not it agreed with a particular set of facts. Whether or not any explanation is involved is a matter of opinion, and interpretation.

Huff's Empirical Test and Evaluation of the Parameter

Huff chose to test his model in a suburban community within the Los Angeles Metropolitan Area. A judgment sample was used to determine which consumers would be asked to supply data. As Huff explains:

Within this community three rather distinct neighborhoods, i.e., clusters of households geographically delineated, were selected as consumer points of origin -- that is, the i's as depicted in the model ... The criteria used to select these neighborhoods were as follows:

1. Each neighborhood should be approximately equal in terms of population density; and,
2. Each of the three neighborhoods should be fairly homogeneous in terms of income. Because of the difficulty of obtaining income data, the value of dwelling units was substituted as a close approximation.16

After selecting the three neighborhoods on this basis, a mail survey was conducted by questionnaire sent to all households known to be in each area. The questionnaire used is reproduced as Appendix I. The number of questionnaires sent and returned is summarized in Table IV.

Additional data were obtained for 14 planned shopping centres which were to be the $j$'s in the model. These shopping centres were all within an arbitrarily defined 20 mile radius from each of the sample neighbourhoods. For each shopping centre information was obtained regarding the size of the centre, and its distance from each of the sample neighbourhoods.

Having meanwhile tabulated the numbers of consumers in each neighbourhood to determine, by neighbourhoods, how many had purchased what products at which stores, it was now possible for Huff to make empirical estimates of the exponential parameter. This could have been done by hand, but would have been very time-consuming. Therefore, an iterative computer programme for successive approximation was devised. According to Huff, the sequential steps involved in this programme were:

16 Huff, Determination of Intra-Urban Retail Trade Areas, p.22.
TABLE IV

Huff's Sample Data Pertaining to the Neighbourhood Studied

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960 Census: population</td>
<td>3812</td>
<td>4844</td>
<td>4559</td>
</tr>
<tr>
<td>1960 Census: household</td>
<td>885</td>
<td>974</td>
<td>1199</td>
</tr>
<tr>
<td>Mailing size (no. of household)</td>
<td>670&lt;sup&gt;a&lt;/sup&gt;</td>
<td>958&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1022&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Number of replies</td>
<td>123</td>
<td>331</td>
<td>312</td>
</tr>
<tr>
<td>Reply percentage</td>
<td>18.3</td>
<td>34.5</td>
<td>30.5</td>
</tr>
</tbody>
</table>

<sup>a</sup> - The difference from 1960 Census reflects the fact that some families had moved since the time of the Census.

Source: Huff, Determination of Intra-Urban Trade Areas, p. 23.
1. Assume a particular value of $\lambda$ which is greater than unity. Correspondingly, substitute the appropriate values for each of the appropriate alphabetic characters noted in the model and calculate the expected probabilities.

2. Compare the expected probabilities with the actual relative frequencies obtained from the survey data and calculate a correlation coefficient.

3. Continue to substitute incremental values for $\lambda$ until the highest correlation coefficient is obtained which will represent the optimum value of the parameter.

Two additional rules were applied by Huff in order to obtain necessary data and to carry out the computations:

1. The expected number of trips (sample size times the estimated probability) to any given shopping center had to be equal to or greater than one-half a trip. Fractional trips that were less than this minimum criterion were not included in calculating an estimate of the parameter.

---

17 The significance of this assumption, and interpretation of values less than unity, and even negative values, will be discussed in Chapter V.

18 It is obvious here that if these were called expected relative frequencies, in harmony with the non-explanatory development of the model, it would not affect the value of the empirical test.

19 This is not the same correlation coefficient as is usually considered in regression analysis, and probably should not be so called. The definition of this coefficient is explained below. That it is not the same became very clear during the study discussed in Chapter V when values of $R^2$ turned out to be negative.

20 Ibid., p. 23.

21 Why this criterion was established is not explained by Huff. Apparently, however, it was thought that in samples as large as his, values less than one-half were not significantly different from zero. It will be shown in Chapter V that this rule does change the value of the parameter.
2. The parameter was estimated for each neighborhood separately. Therefore, the distance measure, i.e. travel time, to each of the fourteen shopping centers was unique for each neighborhood. The center of each neighborhood was used as the point of origination in determining the travel time involved in getting to each of the selected shopping centers.\textsuperscript{22}

The correlation coefficient used by Huff to determine the optimum value of his exponential parameter is of considerable interest. This coefficient is:

\[
R = \sqrt{1 - \frac{\sum_{j=1}^{n} [C_{ij} - E(C_{ij})]^2}{\sum_{j=1}^{n} [C_{ij} - \bar{C}_i]}}
\] (IV.14)

Considering first the denominator of the fraction, which is being used as a standard of comparison in the coefficient, it seems to be based on the LaPlace Criterion (sometimes called the Principle of Insufficient Reason). What it argues is that if there were no factors operating to make some consumers prefer one shopping centre to another, then consumers' choices would be random, and it might be expected that they would be evenly distributed among all available alternatives. This assumption is compared to the observed number of persons in the sample neighbourhood by subtraction, and the results are squared and summed over all stores. Similarly, in the numerator, the predictions of the model are compared to the observations by subtracting, squaring, and summing. If the model provides no improvement over the random behaviour assumption, then the values of the fraction will be nearly one, and the coefficient of determination ($R^2$), which measures the power of account of

\textsuperscript{22} Ibid., p. 25.
the model proposed, will be hardly different from zero. On the other hand, if the model predicts the empirical data exactly, the numerator of the fraction will be zero, and $R^2$ will be 1. The correlation coefficient $R$ is merely the square root of the coefficient of determination and has no significance of its own. It merely extends the range of numerical results, and tends to spread the numbers out somewhat. As mentioned above, the programme calculates a value for $R$ in each iteration, and the optimum value of the exponential parameter is the one which maximizes $R$.

Results of the Empirical Test

Huff makes some useful comments on the nature of his empirical test:

... The empirical test was designed merely as a small-scale, experimental pilot study. An effort of this type cannot validate or invalidate the theoretical model. A larger number of more comprehensive investigations will be required to do so. It was expected, however, that the small-scale test would provide valuable experience for the design of more comprehensive empirical surveys and suggest desirable modifications of the model itself. The emphasis here is on methodology rather than verification.24

The results of the empirical test conducted on this particular sample are given in Table V for clothing, Table VI for furniture, and Table VII which summarizes the other two.

23 The explanation of how comes to be negative will be given in Chapter V when the design of the computer programme is discussed.


TABLE V

Results of Huff's Empirical Test for Commodity "Clothing"

<table>
<thead>
<tr>
<th>Shopping Centre</th>
<th>Neighbourhood 1</th>
<th>Neighbourhood 2</th>
<th>Neighbourhood 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Expected</td>
<td>Observed</td>
</tr>
<tr>
<td>1</td>
<td>71</td>
<td>70.76</td>
<td>148</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.27</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.04</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2.60</td>
<td>38</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.77</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.00</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0.00</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>0.99</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>0.78</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0.79</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ R = 0.99 \quad R = 0.98 \quad R = 0.90 \]

\[ \Lambda = 2.889 \quad \Lambda = 2.655 \quad \Lambda = 3.690 \]

### TABLE VI

**Results of Huff's Empirical Test for Commodity "Furniture"**

<table>
<thead>
<tr>
<th>Shopping Centre</th>
<th>Neighbourhood 1</th>
<th>Neighbourhood 2</th>
<th>Neighbourhood 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Expected</td>
<td>Observed</td>
</tr>
<tr>
<td>1</td>
<td>51</td>
<td>51.66</td>
<td>68</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.50</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.30</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3.43</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.98</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.00</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.00</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>1.31^a</td>
<td>16</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.00^a</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0.91</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0.91</td>
<td>6</td>
</tr>
</tbody>
</table>

\[ R = 0.99 \quad R = 0.94 \quad R = 0.96 \]

\[ \text{Lambda} = 2.542 \quad \text{Lambda} = 2.115 \quad \text{Lambda} = 3.247 \]

^a - These two values seem to be incorrectly interchanged in the published results, as was disclosed by recalculation (see Chapter V).

^b - This value was incorrectly published as 15.0, but was corrected in Huff. "Probabilistic Analysis of Consumer Spatial Behavior," p. 455.

Source: Huff, Determination of Intra-Urban Retail Trade Areas, p. 27.
### Table VII

**Composite Results of Huff's Empirical Test**

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Neighbourhood</th>
<th>Estimated Lambda</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clothing</td>
<td>1</td>
<td>2.812</td>
<td>0.99</td>
</tr>
<tr>
<td>Clothing</td>
<td>2</td>
<td>2.604</td>
<td>0.98</td>
</tr>
<tr>
<td>Clothing</td>
<td>3</td>
<td>3.779</td>
<td>0.96</td>
</tr>
<tr>
<td>Furniture</td>
<td>1</td>
<td>2.523</td>
<td>0.99</td>
</tr>
<tr>
<td>Furniture</td>
<td>2</td>
<td>2.115</td>
<td>0.94</td>
</tr>
<tr>
<td>Furniture</td>
<td>3</td>
<td>3.331</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Source: Huff, *Determination of Intra-Urban Retail Trade Areas*, p. 27.
Huff makes some further comments regarding the results of his test:

The expected behavior derived from the model corresponds quite closely to the actual behavior observed from the survey findings. However, contrary to what was expected, the estimates of lambda varied from neighborhood to neighborhood ... ... the investigation was designed ... to test the hypothesis that lambda was primarily a function of the type of shopping trip. Each of the three sample neighborhoods were selected on the basis of their homogeneity [in terms of income], and it was therefore expected that lambda would be approximately the same for each neighborhood with respect to a given type of shopping trip ...

It would be unwarranted to discard the extreme ... estimates ... unless it could be shown that these values were so far removed from the other estimates as to indicate the presence of factors other than sample variation. Therefore, a statistical criterion was constructed to test the hypothesis that the extreme values ... were not due to factors other than sample variation.

The particular test used was a simple statistic based on computing the ratio between the difference between the largest and second largest observations and the range. On the basis of this test, the hypothesis was accepted (at the one percent level) that the extreme values were due to sample variation. It is doubtful, however, whether such a simple test is meaningful. The numbers being considered here are exponents, not arithmetic measures of properties. Statistical tests of


the type used were originally designed for use on this latter type of measure. It is difficult to conceive how a variation of one unit in the exponent, which is a ten-fold change in the underlying property (or properties), could be the result of sample variation rather than other factors. 27

Satisfied that all values were accounted for by sample variation, Huff calculated a mean value of lambda for each of clothing and furniture, 3.191 and 2.723 respectively. It was not possible to test for significance of the difference of these two means since the numbers in the sample are so small. Huff concludes:

Despite the lack of conclusive statistical evidence that the mean ... estimates are valid representations of the clothing and furniture parameters, they do indicate that a consumer's spatial behavior is a function of the type of shopping trip. For example, the mean estimate for shopping trips involving clothing purchases is higher than the mean value estimated for furniture purchases, which confirms the observation that consumers are not willing to travel as far for clothing as they are for furniture purchases. 28

27 Out of curiosity, the exponents were converted to corresponding arithmetic measure by viewing them as logarithms, and thus looking up the corresponding anti-logarithms in tables. The same test statistics were computed using these arithmetic values. Though the statistics were higher in value, the hypothesis that extreme values were caused by sample variation still could not be rejected at the 1% significance level. 28

Ibid., p. 31.
Summary

The discussion of this particular probabilistic model has been in two parts: the development of the model; and the empirical testing of the model. Considering first the development of the model, it was shown that there seemed to be a certain amount of inconsistency in the logic applied by Huff. The model was derived also from a sort of Bayesian approach using subjective probabilities and likelihood measures. Since it may be thought that this development is similarly not acceptable, it was also shown that the same model could be devised in a manner which is free of any attempt at explanation of behaviour, but is aimed solely at ability to predict.

Since it was established that the model was not necessarily based on any particular explanation, it was possible to consider, separately and independently, the results of the empirical test. The major contribution of this work lies in the design and use of a programme for making successive approximations to the value of a parameter, using minimum squared deviation from observed values as a criterion, by means of a high-speed digital computer. This type of estimating procedure would not be practical by any other, lesser means of computation.

While the actual results of the empirical work appear encouraging, they must nevertheless be viewed with considerable caution. Huff emphasizes that his work is but a pilot study.
Even so, he seems to place a considerable amount of faith in the results.\textsuperscript{29} The usual type of statistical significance tests do not seem appropriate in this setting because of the fact that the parameters are exponents, rather than arithmetic measures. This raises another question regarding the design of an appropriate technique for objectively evaluating the results of this type of empirical work. An attempt at answering this question is advanced in Chapter V of this thesis.

\textsuperscript{29} While Huff mentions that a large-scale study was being initiated, \textit{(ibid., p.5)}, no further empirical work has been published. On the other hand, Huff has recently advocated that this same model be applied in another context: "The Use of Gravity Models in Social Research," \textit{Mathematical Explorations in Behavioral Science}, eds. Fred Massarik and Philburn Ratoosh (Homewood, Ill.: Richard D. Irwin, Inc., and The Dorsey Press, 1965), pp. 317-321. This raises a question with regard to the theoretical, or explanatory, content which Huff imputes to the model in its original setting.
CHAPTER V

EMPIRICAL TEST OF THE PROBABILISTIC MODEL
IN METROPOLITAN VANCOURVER

The empirical test of the probabilistic model of consumer spatial behaviour, discussed in Chapter IV, was designed and intended to be only a pilot study. Although Huff had stated that he was initiating a major empirical study, there are, as yet, no published results. Further, it was thought that this major study would again be conducted in the Los Angeles Metropolitan area, and therefore, that it would be useful to conduct a major study in a different area, namely Metropolitan Vancouver, both for its own sake, and for purposes of comparison should Huff publish his results in the interim.

It will be recalled from previous discussion that Huff had designed his pilot study both to check on methodology, and to test the hypothesis that the exponential parameter was a function of the type of shopping trip. The results of the pilot study appeared to indicate that the hypothesis should be accepted tentatively. It appeared to be desirable to obtain data which would allow examination of this tentative hypothesis on a much broader basis. Since others were desirous of conducting a wide-ranging survey from which to examine other aspects of shopping trips, it was determined that it would not be difficult to include items in the survey questionnaire from
which data could be generated in the form required for use in the model. It would also be necessary to obtain measures of the store areas involved, as well as travel times from the various neighbourhoods defined to each of the respective stores. This, also, was not thought to be a difficult problem.

The Sample and the Survey

To begin the study, it was necessary to determine the geographical groupings which are referred to as neighbourhoods. It was decided to use Census Tracts for this purpose for two reasons: (1) they are already defined; and, (2) a number of surveys had previously been conducted on this basis, and it was felt that they would probably yield data for future extension of this study which would thus be on a comparable basis. The survey was conducted in the Metropolitan Area of Greater Vancouver, and a map of this area showing the Census Tract boundaries is included here as Figure 4. The particular Census Tracts included in the sample are shown shaded in this figure. Since nearly fifty Census Tracts were covered -- though not all yielded useful results -- it is immediately obvious that Huff's criterion of homogeneity of income among neighbourhoods is not satisfied. The technical details of how the sample was taken are included as Appendix II.

After the method of sampling was determined, the data were obtained by personal interviews using a questionnaire especially designed for this survey. In fact, two different
questionnaires were used, but they differed only in the parts being used for other purposes. Thus, all the questionnaires (correctly) completed were potential data for this study. The particular questions of interest are reproduced as Appendix III. The responses to these, and other questions, were suitably coded and the whole survey was thus reduced to a deck of punched computer data cards, ready for subsequent analysis.

It may be noted, on comparing the questions asked in this survey with the questions asked by Huff, that there are marked differences. Huff basically elicited two separate responses: (1) place of last purchase of items A, B, C, etc.; and, (2) place of majority of purchases of items A, B, C, etc. On the other hand, the present survey was oriented toward examining several aspects of the shopping trip as an entity. Thus, the pertinent questions were framed in such a way that they determined what items were purchased at which stores on the last shopping trip. Since this study is primarily concerned with shopping goods, which are by nature infrequently purchased, many interviews did not generate useful data in response to these questions. Unfortunately, not all shopping trips are taken in quest of shopping goods, and those that are, are not always successful in terms of a purchase of an item. This had the net effect of reducing the size of the sample, which was already small relative to the area covered.

It may also be noted that whereas Huff's study dealt only with planned shopping centres, this study included the
major department stores centrally located in downtown Vancouver. There are at least two, related reasons for this. First, Vancouver is only a fraction of the size of an urban area such as Los Angeles and Vancouver's central district is readily accessible to all suburban areas considered. It was felt that it would be impossible to find a suburban area in Vancouver that would not be subject to considerable influence by these major stores. Second, this influence is reflected in the survey results which showed that many shopping goods purchases were made in these stores. Exclusion of these stores would have reduced the portion of the original sample used in the analysis to a small amount, and, indeed, would have made the balance of the analysis a questionable exercise. Altogether, ten stores and/or shopping centres were used in the study, and their locations are shown on the map, Figure 4.

The next step required in putting the raw data into a form which could be used in evaluating the model involved aggregation, by neighbourhoods, of the particular shopping goods purchased at particular stores. The questionnaire included space for up to three stores on one trip, and the whole range of items for each store indicated. The data were also coded in this manner such that stores were identified on three card columns, and associated items in three groups of columns. It was thus a relatively simple matter to sort the cards by neighbourhoods and apply the MVTAB programme from the Computing Centre Library to generate bi-variate frequency tables of items.
by stores for each neighbourhood. The contents of these tables were manually aggregated to produce the $C_{ij}$ data required to test the model.

This was not quite the end of this step, however. The numerical values obtained were quite small, and there was a serious question concerning whether they would be suitable for evaluation of the model. Thus, it was decided that further aggregation would be necessary, and so certain related items were combined to produce three composite commodities, A, B, and C. The items which define these are identified in Table VIII. This procedure produced numerical results for $C_{ij}$ which, while still small relative to Huff's virtually complete sample of a neighbourhood, were considered adequate to proceed.

The next item of information required was the appropriate area measurements for the stores. These were obtained by personal contact with suitable persons employed in the stores. The measurement used was based on the area devoted to retail selling, plus adjacent storage area. This latter was included because it was thought to be material in determining breadth and depth of selection of merchandise. The measurements are shown in Table IX.

---

1 The exercise was simple but time-consuming. MVTAB had to be separately set up for each neighbourhood, and there were approximately fifty of these. The set-up time greatly exceeded the computing time, which was generally less than one minute for each neighbourhood on the Computing Centre's IBM 7040 machine.
**TABLE VIII**

**Definition of Composite Commodities**

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Clothing for self</td>
</tr>
<tr>
<td></td>
<td>Children's Clothing</td>
</tr>
<tr>
<td>B</td>
<td>Furniture</td>
</tr>
<tr>
<td></td>
<td>House furnishings</td>
</tr>
<tr>
<td></td>
<td>China, glassware</td>
</tr>
<tr>
<td></td>
<td>Appliances</td>
</tr>
<tr>
<td>C</td>
<td>Sporting Goods</td>
</tr>
<tr>
<td></td>
<td>Toys</td>
</tr>
<tr>
<td></td>
<td>Hardware (including paint)</td>
</tr>
</tbody>
</table>
### TABLE IX

Areas of Stores in Metropolitan Vancouver

<table>
<thead>
<tr>
<th>Store Number</th>
<th>Store</th>
<th>Location</th>
<th>Area (thousand square feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hudson's Bay</td>
<td>Downtown</td>
<td>617</td>
</tr>
<tr>
<td>2</td>
<td>Woodward's</td>
<td>Downtown</td>
<td>573</td>
</tr>
<tr>
<td>3</td>
<td>Woodward's</td>
<td>Oakridge</td>
<td>225</td>
</tr>
<tr>
<td>4</td>
<td>Woodward's</td>
<td>Park Royal</td>
<td>122</td>
</tr>
<tr>
<td>5</td>
<td>Eaton's</td>
<td>Downtown</td>
<td>550</td>
</tr>
<tr>
<td>6</td>
<td>Eaton's</td>
<td>Brentwood</td>
<td>210</td>
</tr>
<tr>
<td>7</td>
<td>Eaton's</td>
<td>Park Royal</td>
<td>125</td>
</tr>
<tr>
<td>8</td>
<td>Simpson-Sears</td>
<td>Burnaby</td>
<td>240</td>
</tr>
<tr>
<td>9</td>
<td>Simpson-Sears</td>
<td>Richmond</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>Army and Navy</td>
<td>Downtown</td>
<td>42</td>
</tr>
</tbody>
</table>

a - These figures include only main store areas. They do not include food floors, or specialty shops in the case of shopping centres.
The final measurement required was that of the distances, in time units, from each neighbourhood to each store in the sample. To accomplish this measurement, the area was laid out on a grid system along major traffic arteries. The length of each part of the grid was determined by driving along it, at normal speed, and measuring the time required with a stop watch. All measurements were taken in the early evening or on weekends. While the traffic is lighter at these times than during normal business hours, it is consistently so over the whole sample. Since it is the relative effect of distance which is sought, the fact that these times are somewhat shorter than those which consumers actually experience was considered not too damaging to the investigation. The distance measures thus obtained were marked on the grid on a map. The most likely route from each neighbourhood to each store was traced on the map, and the distance obtained by adding up the appropriate grid elements. Again, as in Huff's study, the centre point of each neighbourhood was assumed to represent the point of origin for all consumers in that neighbourhood. The resulting measurements were the $\hat{\lambda}_j$ values required by the model.

Programme XLAMC for Evaluating the Exponential Parameter

In order to make use of the high-speed computing ability of the University of British Columbia Computing Centre's IBM 7040 machine, it was necessary to prepare a programme in a suitable programming language. The most useful language for this study was FORTRAN IV. A flow chart was prepared to show
the steps necessary in efficiently reaching the desired result, and is presented as Appendix IV. The equivalent FORTRAN programme is also presented as Appendix V.

There are two significant points to be noted about this programme. First, is the inclusion of the CALL PLOTS routine. Second is the inclusion of SUBROUTINE SRNEG. The CALL PLOTS routine is part of the Computing Centre's IBM 7040 library, and is used when it is more convenient to display the results of a computational routine in a X-Y plot than in tabular form.

In the particular calculation of empirical results of interest in this study, it was thought that it would be more informative to have the paired values of $R$ and $A$ displayed in an X-Y plot. This is particularly so when the usual statistical tests on the value of $A$ for each maximum value of $R$ are of doubtful merit. On the other hand, a graphical presentation of $R$ and $A$ shows the manner in which $R$ varies with $A$. By inspection, it can be seen how rapidly $R$ approaches its maximum value with either increasing or decreasing values of $A$. On this basis, the researcher can form an opinion about what may be termed the sensitivity of the model, or what amounts to the same thing, the uniqueness of the value of $A$ which determines the maximum value of $R$. For example, if the curve of $R$ versus

---

2 This flow chart was adapted from David L. Huff, Determination of Intra-Urban Retail Trade Areas (Los Angeles: University of California, Graduate School of Business Administration, Division of Research, 1962), Figure 5, p. 25.
\( \lambda \) has a definite and decided peak, the researcher might be led to conclude that the maximum \( \mathcal{R} \) and \( \lambda \) pair are quite uniquely determined, and that the model is quite sensitive to the value of the exponential parameter. The converse would also be true. If all, or most, of the \( \lambda \) values seemed thus uniquely determined, the researcher might consider himself on firmer ground in referring to a distribution of \( \lambda \) values. Given such a distribution, it might then be acceptable to perform statistical tests of significance on extreme values. If the \( \lambda \) values are not so uniquely determined, the researcher is only able to say that the model is insensitive to \( \lambda \); that it is meaningless to speak of a particular value of \( \lambda \) as being appropriate; and that the model will probably not provide useful predictions.

The need for SUBROUTING SRNEG is related to the discussion of the sensitivity of the model to values of \( \lambda \), but has a different interpretation. Those who are accustomed to dealing with coefficients of determination as used in regression analysis will know that \( \mathcal{R}^2 \) is never negative. It thus took considerable analysis of the programme routine and the underlying mathematics to determine why the computer reported negative values for \( \mathcal{R}^2 \) when processing empirical data for Vancouver. Referring back to Equation (IV.14) in Chapter IV, it is reproduced here for \( \mathcal{R}^2 \) rather than \( \mathcal{R} \), as follows:

\[
\mathcal{R}^2 = 1 - \frac{\sum [C_{ij} - \mathcal{E}(C_{ij})]^2}{\sum [C_{ij} - \sum_n C_{ij}]} 
\] (V.1)
It will be recalled that the denominator of the fraction in this definition was intended to describe purely random behaviour based on the Principle of Insufficient Reason. The numerator, on the other hand, measures the improvement of fit of the values estimated from the model. Thus, it will be smaller than the denominator and the fraction is less than one. If the model predicts perfectly, the numerator is zero, and $R^2$ is one. But what happens if, for some reason, the model does not fit the data as well as purely random behaviour? Exactly this: the numerator is greater than the denominator since the sum of squared deviations of estimated behaviour from the actual behaviour is greater than the sum of squared deviations of random behaviour from actual behaviour. In this event, the fraction is greater than one, and $R^2$ is negative. SUBROUTINE SRNEG merely tells the computer to report this result, take a large step forward, and try again.

But how can the model predict behaviour whose relationship to actual behaviour is worse than a purely random pattern? On logical grounds, this could only be the case when the model misrepresents the factors which actually determine behaviour. In other words, the model must be incorrect in some very fundamental way. Some suggestions as to how this might be will be made in Chapter VI, along with some proposed modifications to the basic model. At this time, however, it is merely stated that the occurrence of negative values of $R^2$ over the whole range of values of $\lambda$ being considered would appear to be very
strong evidence that the model should be rejected, at least in its present form.

One further comment needs to be made regarding programme XLAMC. In this routine, all values of estimates of shopping trips are included in calculating the parameter regardless of the size of the estimate. It will be recalled from the discussion of Huff's study in Chapter IV that all estimates of shopping trips less than one were equated to zero and disregarded in calculating the parameter. Huff does not state why this was done, nor does he include any comment regarding the effect that this might have on the estimate of the parameter. This step might possibly have a very small effect in his case where the sample for each neighbourhood contains a hundred or more consumers. For the Vancouver sample, however, it was thought that this would produce very misleading results, as the samples are of the order of one-tenth of the size used by Huff for each neighbourhood.

Application of Programme XLAMC to Huff's Data

It is an accepted part of scientific procedure when a piece of research is intended to extend the work of others, that the new research design be able to reproduce the preceding results. For this reason, it was decided that this should apply also to programme XLAMC. The data used were those published by Huff. The results of these trials are presented in Table X

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3 Huff, op. cit.; Table III, p.24; Table IV, p.25; Table V, p. 26, and Table VI, p. 27.
for clothing, Table XI for furniture, and are summarized in Table XII.

The best check on the fit between the two sets of expected values is given by the results for neighbourhood 2 for either commodity. In this case, Huff's programme was not required to equate any estimates to zero. It can be seen that the estimates of the number of consumers going to each store differ generally only in the second decimal place. This is considered to be an insignificant amount. More important are the slight differences noted in the resulting values of $\lambda$ and $\rho$. This can only be attributed to the difference in the way significant digits were saved in the two computational routines. Although Huff makes no explicit statement in this regard, it seems from the way in which the results are reported that he used only two significant figures for $\lambda$ and four significant figures (three places of decimals) for $\rho$. On the other hand, programme XLAMC used five significant figures for $\rho$ and three for $\lambda$. How this makes a difference is explained below.

Comparing the results obtained by each programme for neighbourhoods 1 and 3 shows that Huff's decision to equate estimates less than one-half to zero does make a difference to the value obtained for $\lambda$. Some might argue that the difference is significant, thus Huff should justify his decision. Others might argue that, in view of the following discussion, there are more fundamental issues to investigate.

One of the benefits obtained by processing Huff's data
<table>
<thead>
<tr>
<th>Shopping Centre (J)</th>
<th>Neighbourhood 1</th>
<th>Neighbourhood 2</th>
<th>Neighbourhood 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71</td>
<td>70.76</td>
<td>70.53</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.27</td>
<td>1.15</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.04</td>
<td>0.93</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.00</td>
<td>0.39</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2.60</td>
<td>2.89</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.77</td>
<td>0.69</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.00</td>
<td>0.27</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.00</td>
<td>0.22</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.00</td>
<td>0.11</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0.00</td>
<td>0.19</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>0.99</td>
<td>0.87</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>0.78</td>
<td>0.71</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0.79</td>
<td>0.72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>R = 0.99</th>
<th>0.99885</th>
<th>R = 0.98</th>
<th>0.98006</th>
<th>R = 0.96</th>
<th>0.96312</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lambda = 2.889</td>
<td>2.980</td>
<td>Lambda = 2.655</td>
<td>2.660</td>
<td>Lambda = 3.690</td>
<td>3.840</td>
</tr>
</tbody>
</table>
TABLE XI
Comparison of Huff's Evaluation of Parameter with Programme XLAMC Evaluation (Furniture)

<table>
<thead>
<tr>
<th>Shopping Centre (J)</th>
<th>Neighbourhood 1</th>
<th>Neighbourhood 2</th>
<th>Neighbourhood 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51</td>
<td>51.66</td>
<td>51.04</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.50</td>
<td>1.37</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1.30</td>
<td>1.17</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3.43</td>
<td>3.07</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.98</td>
<td>0.88</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.00</td>
<td>0.38</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.00</td>
<td>0.33</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.00</td>
<td>0.18</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>0.00</td>
<td>0.27</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>1.13</td>
<td>1.17</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.00</td>
<td>0.23</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0.91</td>
<td>0.83</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0.91</td>
<td>0.83</td>
</tr>
</tbody>
</table>

R = 0.99  0.99249  R = 0.94  0.93387  R = 0.96  0.95999
Lambda = 2.542  2.620  Lambda = 2.115  2.120  Lambda = 3.247  3.370
### TABLE XII

Comparison of Huff and XLAMC Parameter Estimates

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Neighbourhood</th>
<th>Huff Lambda</th>
<th>Huff R</th>
<th>XLAMC Lambda</th>
<th>XLAMC R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clothing</td>
<td>1</td>
<td>2.889</td>
<td>0.99</td>
<td>2.980</td>
<td>0.99885</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.655</td>
<td>0.99</td>
<td>2.660</td>
<td>0.98006</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.690</td>
<td>0.96</td>
<td>3.840</td>
<td>0.96312</td>
</tr>
<tr>
<td>Furniture</td>
<td>1</td>
<td>2.542</td>
<td>0.99</td>
<td>2.620</td>
<td>0.99249</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.115</td>
<td>0.94</td>
<td>2.120</td>
<td>0.93387</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.247</td>
<td>0.96</td>
<td>3.370</td>
<td>0.95999</td>
</tr>
</tbody>
</table>
with programme XLAMC was that the CALL PLOTS routine produced graphs showing the behaviour of $R$ with changes in $\lambda$ over the range for $\lambda$ of 0.5 to 4.5 in increments of 0.01. While the original plots more than filled an $8\frac{1}{2}'' \times 11''$ page, the interesting parts were those which show the curve at or near its peak. These curves are reproduced here as Figure 5 for clothing, and Figure 6 for furniture, traced exactly from the originals. On examination, the only curve that looks at all like one might expect is that for furniture for neighbourhood 2, but the value at the peak in this case is notably less than in every other curve. In every other case, the model has produced consistently high values of $R$ over a very wide range of values for $\lambda$.

This is indeed a very surprising result when first encountered. However, several conclusions may be drawn from it. First, one can raise the question whether Huff's tests for significance of particular $\lambda$ values are meaningful. It would seem not. Second, the model is insensitive to values of $\lambda$ over a very wide range. How wide this range might be was not determined, since the programme was instructed to stop at 4.5. Regardless of this, however, this result means that in this model any value of $\lambda$ over this wide range will predict about as well as any other value. This raises a very serious question about the value of the model.

When this result is contemplated further, it becomes less surprising. Recall from Chapter II that in applying this type
Figure 5
Huff’s Data for Clothing
Figure 6

Huff's Data for Furniture

![Graph showing Huff's Data for Furniture with labeled axes and points labeled N1, N2, and N3.]
of gravimetric model elsewhere, various researchers reported values of the exponential parameter varying from 1.5 to 3.0 and higher. This raises a serious question about gravimetric models in social science in general. Is the exponential parameter really a structural or behavioural parameter as is postulated, or is it a function of research design and the method of processing data? Before dealing any further in such general issues, however, it may be well to review the results of using programme XLAMC to handle data obtained in the survey of Vancouver.

Application of Programme XLAMC to Vancouver Data

As pointed out above, the data for Vancouver were grouped to produce three composite commodities: A, B, and C. On this basis, sufficient data in final form were obtained for further processing for 42 neighbourhoods for commodity A, 26 neighbourhoods for commodity B, and 40 neighbourhoods for commodity C. Because of the large number of neighbourhoods involved, the results of processing all this amount of data with programme XLAMC will not be presented in the same detail as above, but will be given only in summary form.

Before this is done, one additional comment should be made. When the curves of \( K \) versus \( \lambda \) were examined for all these computations, it was found that few of them resembled the ones previously obtained in processing Huff's data. In fact, they could be classified into three distinct types as
shown in Figure 7. Both Type I and Type II curves had distinct maxima in the range of \( \lambda \) considered. The Type I curve was the one which would normally have been expected to occur. Some Type I curves were sharply peaked; others were relatively broad. With Type III curves, there were two variants. The first, designated III+, had a steadily increasing value of \( \lambda \) with increasing \( \lambda \), but did not reach a maximum in the range considered. Conversely, the second, designated III−, had a steadily increasing value of \( \lambda \) with decreasing \( \lambda \). In most cases of this type of curve, it appeared that if a maximum occurs, it would be with negative values of \( \lambda \). This requires some interpretation on logical grounds, but this will be included later in discussion of the implication of all \( \lambda \) values less than 1.0. In addition to these three types of curves, there were those cases where no curve was obtained for any values of \( \lambda \) in the range considered. This is designated as undefined as it occurred when all values of \( \lambda^2 \) were negative (as discussed above) and thus \( \lambda \) was undefined in the mathematical sense.

The summary results of processing the Vancouver survey data are presented in Table XIII for Commodity A, Table XIV for Commodity B, and Table XV for Commodity C. Each table gives: (1) the Census Tract number; (2) the maximum value of \( \lambda \) obtained and its associated value of \( \lambda \) (each to three significant figures) when these values were obtained; (3) the type of curve obtained, which shows whether \( \lambda \) would be
Figure 1

Typical Curves Produced By Programme XLAMC
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### TABLE XIV

**Results of Application of Programme XLAMC for Commodity B**

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TABLE XV

Results of Application of Programme XLAMC for Commodity C

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above or below the range considered in the case of Type III curves, or may not exist in the case of undefined curves; and (4) the total number of consumers whose responses provided data in each Census Tract.

On examining the results presented in the tables as shown, it seemed that there might be several tendencies in the results. First, there appeared to be some relationship involving increasing values of the maximum $\lambda$ with increasing values of $\lambda$. Second, it appeared that there might be some significance in the difference of the $\lambda$ values obtained from Type I and Type II curves, the latter having higher values. Third, in a negative way, there did not seem to be any relationship whatsoever between the number of persons in the sample from each neighbourhood and the type of curve obtained. This includes also whether or not a maximum $\lambda$ was found in the range of $\lambda$ considered. To examine the first two tendencies more carefully, scatter diagrams were drawn of the maximum $\lambda$ and $\lambda$ pairs. These are presented here as Figure 8 for Commodity A, Figure 9 for Commodity B, and Figure 10 for Commodity C. It

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4 One additional classification occurs in Table XV which has not been mentioned. This is the category called invalid, which is intended to describe a peculiar result. In this situation, for several Census Tracts located near Oakridge, $\lambda$ became 1.0 (meaning certainty) for all values of $\lambda$ slightly greater than 2.0. Here again, one could hardly speak of a meaningful $\lambda$-pair.
Figure 8

Commodity A

$R_{\text{max}}$ versus $\Lambda$

- Type I Curve Values
- Type II Curve Values
Figure 9

Commodity B

$R_{max}$ versus $\Lambda$

- Type I Curve Values
- Type II Curve Values

$\Lambda$
Figure 10

Commodity C

$R_{max}$ versus $\Lambda$

- Type I Curve Values
- Type II Curve Values

$\Lambda$

$R_{max}$
was observed that the scatter was so great that the tendencies noted remain merely as tendencies, and no further action was taken on these diagrams. While some might argue that it would be possible to fit a straight line or a curve to these points, or perhaps perform a rank correlation, the interpretation of the relationship deserves some thought. The interpretation would be that greater values of \( \lambda \) produce greater powers of account (explained variation) in the model. The evidence does not seem strong enough for such a statement to be made with the degree of conviction that a regression curve would imply.

In addition to the above tables and figures, it seemed useful to bring all the results together in one place. Thus, Table XVI showing the maximum \( \omega \) and \( \lambda \) pairs for each Census Tract, for each commodity, was prepared. Again, on examining the results presented in this way, it is virtually impossible to detect any trends, not even that of central tendency. For this reason, as well as the reasons advanced above in discussing curve types, there does not seem to be any justification for conducting any further tests of statistical significance based on the distribution of \( \lambda \) values.

The results of this survey, and the data processed from it, must be examined with respect to Huff's hypothesis that the value of the exponential parameter is a function of the type of shopping trip being considered. This was also the hypothesis being tested by the Vancouver survey, since the
**TABLE XVI**

Table of Composite Results

<p>| Census Tract | Commodity A | | Commodity B | | Commodity C | |
|--------------|-------------|-----------------|-----------------|-----------------|-----------------|
|               | R max       | Lambda          | R max           | Lambda          | R max           | Lambda          |
| 00            | 0.735       | 4.32            | III-            |                | III+            |                |
| 07            | 0.675       | 1.56            | III+            |                | III+            |                |
| 08            | 0.910       | 1.94            | undefined       |                | III+            |                |
| 09            | III+        |                | ni*             |                | III+            |                |
| 10            | 0.960       | 1.51            | ni              |                | III+            |                |
| 11            | 0.318       | 1.11            | undefined       |                | ni              |                |
| 12            | undefined   |                | 0.610           | 2.31           | 0.473           | 1.34           |
| 14            | III-        |                | ni              |                | 0.646           | 1.22           |
| 15            | 0.954       | 2.45            | III+            |                | 0.356           | 2.81           |
| 16            | undefined   |                | III+            |                | III-            |                |
| 17            | 0.923       | 1.59            | III+            |                | III-            |                |
| 19            | 0.795       | 2.93            | III-            |                | III-            |                |
| 21            | 0.895       | 3.71            | 0.917           | 3.60           | 0.337           | 0.88           |
| 23            | undefined   |                | III-            |                | 0.337           | 0.88           |
| 24            | III-        |                | III-            |                | 0.337           | 0.88           |
| 25            | 0.245       | 2.14            | 0.337           | 1.80           | 0.337           | 0.88           |
| 26            | III-        |                | 0.416           | 2.00           | 0.337           | 0.88           |
| 27            | III-        |                | III-            |                | 0.337           | 0.88           |
| 28            | III-        |                | III-            |                | 0.337           | 0.88           |
| 29            | III-        |                | III-            |                | 0.337           | 0.88           |
| 31            | 0.841       | 2.09            | III-            |                | III-            |                |
| 32            | 0.885       | 3.54            | III-            |                | III-            |                |
| 33            | 0.582       | 0.57            | III-            |                | III-            |                |
| 34            | 0.902       | 1.40            | III-            |                | III-            |                |
| 35            | 0.639       | 1.40            | 0.792           | 1.83           | 0.959           | 2.17           |
| 36            | 0.887       | 2.39            | 0.848           | 1.30           | invalid         |                |
| 39            | III+        |                | ni              |                | invalid         |                |
| 40            | 0.895       | 1.75            | 0.475           | 1.71           | 0.926           | 1.28           |
| 41            | 0.732       | 0.94            | 0.776           | 1.54           | 0.552           | 1.12           |
| 43            | 0.717       | 1.29            | III-            |                | invalid         |                |
| 44            | 0.773       | 0.98            | ni              |                | invalid         |                |</p>
<table>
<thead>
<tr>
<th>Census Tract</th>
<th>Commodity A</th>
<th></th>
<th>Commodity B</th>
<th></th>
<th>Commodity C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R max</td>
<td>Lambda</td>
<td>R max</td>
<td>Lambda</td>
<td>R max</td>
</tr>
<tr>
<td>45</td>
<td>0.820</td>
<td>0.76</td>
<td>ni</td>
<td></td>
<td>invalid</td>
</tr>
<tr>
<td>46</td>
<td>0.964</td>
<td>1.97</td>
<td>0.982</td>
<td>2.98</td>
<td>invalid</td>
</tr>
<tr>
<td>47</td>
<td>0.823</td>
<td>1.45</td>
<td>0.258</td>
<td>1.04</td>
<td>0.852</td>
</tr>
<tr>
<td>48</td>
<td>0.426</td>
<td>1.09</td>
<td>0.501</td>
<td>1.79</td>
<td>undefined</td>
</tr>
<tr>
<td>49</td>
<td>III-</td>
<td></td>
<td></td>
<td></td>
<td>III-</td>
</tr>
<tr>
<td>51</td>
<td>0.531</td>
<td>2.92</td>
<td>ni</td>
<td></td>
<td>0.574</td>
</tr>
<tr>
<td>52</td>
<td>III-</td>
<td></td>
<td>ni</td>
<td></td>
<td>undefined</td>
</tr>
<tr>
<td>53</td>
<td>0.709</td>
<td>1.39</td>
<td>ni</td>
<td></td>
<td>0.654</td>
</tr>
<tr>
<td>54</td>
<td>0.896</td>
<td>1.45</td>
<td>ni</td>
<td></td>
<td>0.752</td>
</tr>
<tr>
<td>55</td>
<td>0.901</td>
<td>2.27</td>
<td>III-</td>
<td></td>
<td>III+</td>
</tr>
<tr>
<td>56</td>
<td>0.929</td>
<td>2.50</td>
<td>0.701</td>
<td>3.19</td>
<td>0.823</td>
</tr>
</tbody>
</table>

*The designation ni means this Census Tract was not included for the particular commodity.*
purpose was to perform a broader test on Huff's model. It can only be concluded that the results are so diffuse as to not lend any support to the hypothesis.

It might be argued that the samples in each Census Tract were too small to produce any amount of central tendency. Typically in statistical hypothesis testing, samples of less than 30 items are classed as small sample cases, and are treated with a modified body of theory and technique. Certainly, it is very tempting to conclude that the samples were too small compared to Huff's neighbourhood saturation technique. But, if the model requires this type of handling in every case, it is much too expensive to be generally useful. Further, before reaching any conclusions regarding the validity of the results of the Vancouver survey, it may be well to consider the results of treating the data somewhat differently.

Application of Programme XLAMS to Vancouver Data

One of the original hypotheses underlying this thesis -- not stated previously -- was that there is a significant difference in consumers' attitudes regarding stores located downtown versus stores located in shopping centres. This is derived from the study by Jonassen which sought to compare attitudes of consumers towards the two types of locations.5

5 C.T. Jonassen, The Shopping Center versus Downtown (Columbus, Ohio: Bureau of Business Research, College of Commerce and Administration, The Ohio State University, 1955), pp. 89-100.
If this is so, there should be a significant difference in the exponential parameter of a model formally the same as Huff's, but applied to stores rather than neighbourhoods.

The hypothesis was tested by, in effect, turning the computational routine used previously on its side to produce a programme named XLAMS. The programme is the same as XLAMC except that stores and neighbourhoods are interchanged. Whereas XLAMC computed by neighbourhoods across all stores, XLAMS computed by stores across all neighbourhoods.

The results of applying programme XLAMS to the same empirical data for Vancouver as before are summarized in Table XVII. Note that the parameter is now called by a different name, \( \eta \), to distinguish it from the previous parameter. This parameter \( \eta \) is a characteristic of stores, whereas the previous parameter \( \lambda \) was a characteristic of neighbourhoods.

The results of this treatment are relatively conclusive. While graphs were not obtained for every store and every commodity -- since the sample was heavier in some areas than others due to the arrangement of Census Tracts -- there are at least two things to be noted. First, the value of the parameter \( \eta \) lies within the range 0.51 to 0.80 for major downtown stores. For suburban stores, this parameter lies in the range 0.49 to 2.49. Second, the power of account (measure of explained variation), \( R^2 \), is also significantly different between downtown and suburban stores. For downtown stores \( R^2 \) is less than 10\% (even less than 5\%), whereas for suburban stores it is as much
TABLE XVII

Results of Application of Programme XLAMS to Vancouver Data

<table>
<thead>
<tr>
<th>Store</th>
<th>Location</th>
<th>Commodity A R</th>
<th>Commodity B R</th>
<th>Commodity C R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>central</td>
<td>0.232 0.51</td>
<td>0.137 0.79</td>
<td>0.209 0.80</td>
</tr>
<tr>
<td>2</td>
<td>central</td>
<td>0.283 0.61</td>
<td>undefined</td>
<td>0.248 0.68</td>
</tr>
<tr>
<td>3</td>
<td>suburb</td>
<td>est. 0.39 at 0.49</td>
<td>0.415 1.12</td>
<td>0.42 at 0.51</td>
</tr>
<tr>
<td>4</td>
<td>suburb</td>
<td>undefined</td>
<td>undefined</td>
<td>0.115 2.49</td>
</tr>
<tr>
<td>5</td>
<td>central</td>
<td>undefined</td>
<td>undefined</td>
<td>0.090 0.610</td>
</tr>
<tr>
<td>6</td>
<td>suburb</td>
<td>0.694 1.72</td>
<td>Note (c)</td>
<td>0.68327 2.24</td>
</tr>
<tr>
<td>7</td>
<td>suburb</td>
<td>undefined</td>
<td>undefined</td>
<td>undefined</td>
</tr>
<tr>
<td>8</td>
<td>Note (a)</td>
<td>0.324 0.99</td>
<td>undefined</td>
<td>0.112 0.64</td>
</tr>
<tr>
<td>9</td>
<td>suburb</td>
<td>undefined</td>
<td>undefined</td>
<td>undefined</td>
</tr>
<tr>
<td>10</td>
<td>Note (b)</td>
<td>0.227 1.04</td>
<td>0.643 2.83</td>
<td>undefined</td>
</tr>
</tbody>
</table>

(a) This store is suburban in terms of Vancouver, but central to the outlying Municipality of Burnaby (north).

(b) While this store is centrally located, it is less than one-tenth the size of other central stores. Further, it is noted for its price appeals in low-priced lines of merchandise such as shoes, clothing and other high volume items.

(c) This commodity has a curve of Type III*, and maximum would be at a value greater than 4.5.
as 40%. In any case, the results tend to support Jonassen's conclusions, which were reached by quite another route.

Further, this agreement tends to support the validity of the empirical data for Vancouver, in spite of the relatively small sample sizes. The parameter $\gamma$ includes, implicitly, many of the factors which Jonassen identifies and examines explicitly. However, the rather low power of account of the Huff-type model in this context tends to indicate that it omits a great deal.

Examination of some individual entries in Table XVII shows some anomalies, specifically the values of $\gamma$ determined for Stores 3, 8, and 10. These will each be considered separately. First, looking at Store 8 (Simpson-Sears in Burnaby), it was noted in the table that this store is suburban with respect to Vancouver's central core but is central in certain respects within the municipality of Burnaby. Further, it is relatively large compared to other stores in shopping centres, being the sole occupant of its location and the largest outlet which this chain has in British Columbia. Thus, it cannot be classified with either of the neatly dichotomized types of central department stores and suburban shopping centres. The results of programme XLAMS are similarly ambivalent, though the tendencies are in agreement, and this is not unexpected.

Second, on considering Store 10 (Army and Navy), located in the central core, the results are not clear-cut. This store is about one-tenth of the size of other major department stores.
and has a merchandising policy which is quite different from these other retail outlets. This policy may be described as aiming at high-volume using price appeal on low price-range lines of merchandise. This policy is implemented by buying the broken or discontinued lines of wholesalers and other retailers at or after the peak of the regular season, at liquidation prices. Thus, the product assortment carried in the store is frequently opportunistically determined. Consumers may be somewhat more dubious of their success in shopping at this store. As will be discussed extensively below, this may be reflected in a somewhat higher value for $\gamma$ for Commodity A (clothing) than for other central stores. Similarly, this store was not noted as a furniture emporium, and this seems to be reflected in the high value of $\gamma$ for Commodity B (furniture).

Finally, considering Store 3 (Woodward's at Oakridge), the results are somewhat conflicting. The $\gamma$ values are as low as for central stores but the $\mathcal{R}$ values are moderately higher. Perhaps this anomaly can be accounted for by re-introducing certain ideas developed in Chapter IV. It may be recalled that the basic model was developed by postulating that consumers form a subjective probability estimate of success regarding a particular store for particular items, and that this could be represented by store area. However, this would be modified by distance to produce a likelihood of shopping. But, if the subjective probability estimates were to be based on firmer knowledge, and thus were more certain, the distance factor
would have much less influence on likelihood. The model may fit this store better than downtown department stores but the influence of distance is less than for other suburban shopping centres.

This explanation was suggested by examining the sample of consumers who had patronized Oakridge. It was found that they tended to reside in fairly well-defined neighbourhoods known to be of moderately high and uniform status in a socio-economic sense. Many of these people have a pattern of shopping at Oakridge, and thus are probably more knowledgeable regarding the available product mix. Further, the Oakridge centre, as a centre, aims a considerable amount of advertising at the particular social strata which are well represented among the neighbourhoods having a notable tendency toward patronizing Oakridge. This, combined with the effects of status congruency, may account for the conflicting results obtained for this particular shopping centre. Many of these factors, which are merely mentioned here, are examined in more detail in Chapter VI.

It is emphasized again that while the models applied to stores and applied to neighbourhoods are the same in a formal mathematical sense, they are quite different in their interpretations when applied in different contexts. These differences are, unfortunately, hidden in the many factors which are implicit in the exponential parameter. However, the logical argument being made is that the results of using the empirical data for Vancouver in programme XLAMS are consistent with previous,
independently conducted research as well as with what is believed to be true about Vancouver consumers and retail institutions. Therefore, the results of using these same data in programme XLAMC are held to be valid and representative, despite their inconclusive nature. Thus, the diffuse nature of these results constitutes a test of Huff's hypothesis in the appropriate context but does not support the hypothesis, nor the predictive ability of the model when applied to neighbourhoods in Vancouver.

Concluding Remarks

It was mentioned above that Huff's computational routine included only estimates of the exponential parameter greater than one. While Huff does not offer any explanation for including only values greater than one, it will be seen below that the assumption that all values must be greater than one avoids some difficult logical explanations.

Since it had not been demonstrated that the parameter could not have values less than one, the programmes XLAMC and XLAMS began their iterations by assuming a value for the parameter of 0.5. As it turned out, the results of the computations produced a significant number of estimates of the parameter less than one for both programmes. In addition, programme XLAMC produced some Type III- curves which appeared as if the maximum value of $R$ -- if indeed there is a maximum -- would occur with negative values of the parameter. Each of these
situations merits individual interpretation.

Considering first the case of programme XLAMC where estimates of the parameter were less than one, but greater than zero, it would seem that the model was behaving as if distance (time) were a relatively minor factor. This follows from the fact that any positive exponent less than one reduces the value of the number with which it is associated. Since this distance measure appears in the denominator of a fraction, such an exponent would tend to increase the effect of factors which are included in the numerator -- in this case, store area. It will be recalled also that the power of account, $R^2$, showed a tendency to be less with lower values of the parameter. This is consistent with observation that the model is de-emphasizing distance as a factor, and weighing more heavily other factors which are only partially included in the numerator of the model as formulated.

The second situation involves programme XLAMC in those cases where the parameter would appear to be negative. Mathematically, a negative exponent occurring in the denominator becomes positive when the whole expression is moved to the numerator. This means that the distance factor then multiplies the store area factor. Two interpretations may be put on this. First, in terms of behaviour, this would suggest that the further away a store was from the consumer, the more attractive it would become. While some individuals undoubtedly enjoy journeying to shop for its own sake, the situation appeared as
if it would occur too frequently for this explanation to be logically acceptable. A more tenable explanation, though a rather mechanical one, is as follows. The programme XLAMC was written in such a way as to determine only a group of numbers, the $R$'s, which are intended to measure the power of account of the model with respect to certain empirical observations supplied to it. It may be that if the programme had been allowed to range downward over values of the parameter, through zero, and into negative values of increasing absolute magnitude, it would have produced two identical peaks. The first peak might have occurred with a small positive value of the parameter and the second with a negative value of more substantial absolute magnitude. In either case, the parameters would be the same in their effect on the numerator of the fraction; one by division by a small number, the other by multiplication by a large number. The interpretation of this mechanical explanation would be the same as for other values of the parameter less than one; namely, more weight would be shifted to other factors besides distance.

Finally, considering the results of programme XLAMS, it seems to be logical that distance is much less important in determining the attitudes that consumers hold toward major downtown stores than in the case of shopping centres. It is suspected that Vancouver residents are not very different in this respect than residents of other cities which have a well-defined core area for shopping. Jonassen's study tends to
support this view, as he states that "the effect of [distance] was minimized or overcome under certain situations and conditions by the presence of other variables".  

Why then might Huff have obtained such encouraging results with his model in Los Angeles, dealing only in suburban shopping centres? Perhaps it is because the area is so different in its structure that the model works reasonably well in it. Los Angeles is known to be a city made up of a conglomeration of suburbias. It does not have a well-defined and established central core, like Vancouver and many other cities. In this way, it is much more analogous as an urban setting to the inter-urban situation with fairly uniform trading centres as originally contemplated by Reilly. On the other hand, Converse's unbalanced model, having one or several relatively large trading centres, is more closely analogous to cities having well-developed cores. It seems that a well-developed urban shopping core distorts the situation such that the model requires other factors beyond space and distance to account for observed behaviour. Some of these factors were mentioned briefly above and will be discussed more fully in Chapter VI.

6 Jonassen, op. cit., p. 91.
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Restatement of the Problem

As stated in Chapter I, the problem to which this thesis was addressed was that of making an empirical test of a probabilistic model of intra-urban retail trade interactions. Specifically, the model which was examined was that put forward by Huff and termed, by him, a probabilistic model of consumer spatial behaviour.

Recapitulation of Purposes in View of Results Obtained

The purpose of this study was to discover whether empirical data gathered in Vancouver would tend to support a model formulated and tested in a pilot study elsewhere. If the data supported the model, or more specifically, an hypothesis about the model, then the purpose would have been accomplished. On the other hand, if the data did not support the model, the purpose would encompass the determination of whether the data or the model appeared to be deficient. If the data were deficient, the manner of its collection would have to be reviewed, and a new research design developed. If the model appeared to be deficient, then attention would have to be turned to the model itself.
The results of the empirical research, reported in Chapter V, showed that the data did not support the model, but that the data seemed to be representative since they tended to support another independent hypothesis— that consumers have different attitudes toward downtown versus suburban stores. A test of the sensitivity of the model was applied to the data of the original pilot study. The model was shown to be quite insensitive. Therefore, in line with the purposes stated, attention must be turned to the model, and consideration given to both factors and relationships. This chapter is devoted to this final element in the stated purpose.

The Need for a New Model

While some successes have been achieved in the past with the gravimetric model in social science in general, and marketing in particular, the results of this study tend to show that it may not be entirely suited to the intended purposes in the new contexts to which it has been applied. While the gravimetric model has a priori appeal, it will not necessarily legitimate the results of social science research by finding that the same models apply as in physical science. Social science must make its own abstractions, in terms of factors and relationships, based on evidence available to it just as physical science did in the first instance.

The evidence available to social science in general, or any particular social science, may admit of an infinitude
of possible models. Some method of determining criteria for choosing among models must be established. These must be based on the capabilities of the models themselves, as well as the purposes for which the research is undertaken. One basis for distinguishing among models is to determine whether the model is intended for purposes of explanation (or description), prediction, or control, as was stated earlier. Not all models are suited to all these purposes. Other bases may be the distinction between microscopic and macroscopic models, or between static and dynamic models.

Ideally, the model pertaining to a particular phenomenon should be macroscopic in the sense of applying to the whole of the relevant world, and dynamic in the sense of exhibiting the correct behaviour with the passage of time. It should satisfy all the purposes of explanation, prediction and control. Because such a model would include all relevant variables, all their necessary inter-relationships, and all appropriate time-dependencies, it would likely have to be viewed as a set of inter-related microscopic models containing the necessary aspects. An economic model of this kind, using a "building-block" approach, has recently been developed, and is currently being tested and extended.\(^1\) Such a model is an ultimate goal, however, and considerable research must be performed to specify the basic building blocks.

The gravimetric model, such as used in this study, can be viewed as an effort of basic research intended to specify factors and relationships which will describe or explain the spatial aspects of consumer behaviour. In the pilot study conducted by Huff, the model had the appearance of being quite descriptive in the sense of the amount of variation in data "explained" by the model was high. The explanatory nature of the model may be quite illusory, since the model was shown to be insensitive to the parameter which was considered to be so indicative of behaviour, and upon which the major hypothesis was based.

The basic model of the gravimetric type contains only two variables -- store area as a proxy measure of product assortment, and distance measured in time units. The empirical data presented in Chapter V showed that the model did not work well in Vancouver. The very low measures of explained variation generally obtained suggest that at least one other factor needs to be explicitly included.

However, the fundamental problem may be such that it cannot be remedied by merely including other factors. It may be recalled that in earlier discussions the consumer was viewed as being in a situation of decision-making under conditions of uncertainty. In this situation, the consumer was seen as making a subjective estimate of the probability of his success in shopping at various stores, but his information was limited. It seems acceptable to suggest that any source which provides
the consumer with additional information will influence the subjective probability estimates formed. While there are many such possible sources, the consumer's own experience or learning, inter-personal communications with family or social contacts, and the promotional activities of the vendor or store are perhaps among the more important. It may be that the latter two sources have their major impact on consumer behaviour by acting on the first source. Even if this were not so, each of these sources has the common property of being an on-going process. The gravimetric model is, however, a static framework of analysis. There is a significant difference in kind between the dynamic equilibrium, or steady state, of an on-going process, and the unmoving, cross-sectional aspect of a static analysis.

It was suggested above that all the various sources of information may exert their impact on the consumer through a learning process, and thus the subjective probability estimates which the consumer forms at any particular time, are the result of some combination of all the factors relevant to the particular consumer. What these factors might be, and how they are inter-related, becomes the proper subject for study in examining consumer behaviour, spatial or otherwise. The main point seems to be that it is unwarranted to treat the choice of a store for shopping as separate and distinct from any other aspect of consumer decision-making behaviour.

Learning is an individual process. It is never the same for any two persons. This fact raises a serious question
regarding the validity of analyses of behaviour using aggregative procedures, such as has been done in all studies of gravimetric models, including the present study and its precursors. It does not seem logical to ignore the individualistic nature of the underlying processes in order to devise a more manageable research methodology. Aggregative models imply that the fundamental mathematical axiom of addition is applicable, but it is not demonstrated, and highly doubtful, that the fundamental assumption of homogeneity among elements is satisfied.

Learning is also a time-dependent process, and this fact also has a significant impact on the methodology of research on such processes. Cross-sectional analyses, wherein all data are gathered at one point in time, do not seem appropriate either, but are the natural consequence of using static, aggregative models.

The literature on consumer behaviour in general is extensive. So, also, is the literature on both learning and choice processes in particular. This literature cannot be reviewed here, as this is more properly the subject of another study. What might be done can be suggested in general terms, however. Considering an individual, it may be possible to enumerate the various factors which might influence his choice of stores for shopping. These various factors may be combined into some sort of weighted function which can predict the probability of shopping at a particular store. Many other
individuals may be similarly considered, and the individual functions examined to determine if a pattern, which might be termed collectivities, emerge. Many possible statistical tools might be used in such analyses, but the multiple discriminant analysis procedure seems to hold considerable promise.\(^2\) While it is not intended to discuss either the theory or technique of multiple discriminant analysis in this study, it may be pointed out that the weightings of various factors in the discriminant function will tend to show which factors have the greatest effect on changing the classification of individuals from one collectivity to another.

The discriminant function is descriptive in the sense that it specifies important factors and the magnitude of their effects on the phenomenon of concern. It is but a first step -- a macroscopic generalization -- toward specifying the necessary microscopic mechanisms which actually determine behaviour. It does treat the individual, however, and it does not assume homogeneity among members of a particular collectivity, but rather seeks to measure it. Because of its general form, it may be possible to include learning processes by including factors which would measure the transient state of the consumer, such as lagged variables, for example. Altogether,

\(^2\) There are several sources which touch on this and related areas, however, for practical purposes, the best seems to be William W. Cooley and Paul R. Lohnes, *Multivariate Procedure for the Behavioral Sciences* (New York: John Wiley and Sons, Inc., 1962).
it seems to offer considerable hope for avoiding the many shortcomings, in terms of validity, imposed by the gravimetric model on the data gathered for it and the results obtained from it.

This study has shown that gravimetric models have severe shortcomings in terms of general validity, and has sought to suggest the reasons for these. Suggestions were also made for new approaches to the analysis of the phenomenon of consumer spatial behaviour, primarily by viewing it as being closely related to other kinds of decision-making and choice behaviour. Statistical tools, such as multiple discriminant analysis, offer considerable hope for useful quantitative models. Research in marketing directed along these lines would seem to offer much greater opportunity for success than has been achieved with gravimetric models.
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APPENDIX I

HUFF'S CONSUMER BEHAVIOR QUESTIONNAIRE

1. At what SHOPPING CENTER did you LAST make a purchase of the following items?

   Clothing............................... Household
   Furnishing...............................
   Food...................................... Hardware
   Cosmetics or
   Drugs.................................

2. How many members of your household normally work in excess of four hours per day? .................

3. What type of WORK does the major income earner of your household do?

   .......................................Self Employed? YES ( ) NO ( )

4. What are the APPROXIMATE AGES of those persons living within your household?

   MALE (...) (...) (...) (...) FEMALE (...) (...) (...) (...)

5. How many CARS in the family? ......................

6. At what shopping center do you NORMALLY MAKE THE MAJORITY of the following family purchases?

   Clothing............................... Household
   Furnishings...............................
   Food...................................... Hardware
   Cosmetics or
   Drugs.................................

7. What bracket as indicated below most closely fits your total FAMILY INCOME?

   $  0-1999..................  $5000-5999..................
   2000-3999..................  6000-7999..................
   4000-4999..................  8000-9999..................
   $10,000 & over..............

Source: Huff, Determination of Intra-urban Retail Trade Areas. p. 39.
APPENDIX II

TECHNICAL DETAIL OF VANCOUVER SAMPLE

Sampling

The group which conducted the survey was syndicated, and each syndicate was supplied with a listing of sample blocks. The interviewing plan involved a systematic sample with an interval of seven, therefore it was necessary to approach every seventh dwelling in the blocks assigned. Interviewing was begun with a random start, counting in a clockwise direction commencing at the Northwest corner of a block. On completion of a block, counting was continued without interruption at the next block assigned, each time beginning at the Northwest corner and counting in a clockwise direction. Throughout the list of blocks, every sixth dwelling was approached. No one contacted in an institution of any kind was interviewed. If an apartment block was encountered, each suite was treated as a dwelling, and counting was continued systematically. Interviewers were instructed to contact an adult female, preferably the female head of the household. If there was no woman in the household, an adjacent dwelling was substituted. One call-back was required. If the second call was unsuccessful in making contact, an adjacent dwelling was substituted.

In view of the block saturation sampling technique used by the Dominion Bureau of Statistics, it was desired not to use
a block already covered by them. An adjacent block was substituted in this event.

Definitions

The following definitions of terms were provided for guidance of the interviewers:

Dwelling: a dwelling is a structurally separate set of living premises, with a private entrance from outside the building, or from a common hallway or stairway inside. Entry is not to be through anyone else's living quarters.

Household: a person or group of persons occupying one dwelling is defined as a household. A person is a member of the household in whose dwelling he normally and regularly sleeps.

Questionnaire

Only one multi-part question in the survey was relevant to this study (see Appendix III). This question was a screening question, thus some answer would usually be obtained. The question was initially concerned with a visit to a local department store, whether or not a purchase was made. If such a visit occurred within the previous three months, interviewers were required to determine:

(a) which stores were visited, whether or not a purchase was made;
(b) for purchases made, the names of the department stores visited were to be recorded, together with the types of purchases in terms of predefined categories (normally departments).

The empirical data were developed from responses to this question.
APPENDIX III

QUESTIONS FROM VANCOUVER SURVEY

The following questions were used to develop the data for testing the model considered in this thesis:

When was the last time you went to any one of the local department stores? _______________________

a. What stores did you visit on that trip?

Hudson's Bay ....

Woodwards - Oakridge ....
Downtown ....
Park Royal ....

Eatons - Brentwood ....
Downtown ....
Park Royal ....

Simpson-Sears - Burnaby ....
Richmond ....

Army & Navy ....

Other (specify) ....

b. What did you buy at each department store you visited? (Write in store and check merchandise purchased)

........ ........ ........

Clothing (including hats and shoes) for yourself .... .... ....
Children's clothing .... .... ....
Furniture .... .... ....
House furnishings, piece goods, floor coverings or housewares .... .... ....
<table>
<thead>
<tr>
<th>Category</th>
<th>Country</th>
<th>Quality</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jewelry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosmetics or drug products</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China or glassware</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sporting goods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appliances *(including T.V. and radio)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Groceries</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardware (including paint)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nothing</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX IV

PROGRAMME XLAMC

FLOW CHART FOR SUCCESSIVE APPROXIMATION OF LAMBDA

1. Read $S_j, C_{ij}, T_{ij}, K, N$
   - initial value of $\lambda$
   - increment of $\lambda$
   - upper limit of $\lambda$

2. Set $i = 1$
   - where $i = 1, K$

3. Print $i$

4. Set $\sum_{j=1}^{n} C_{ij} = 0$

5. Set $\sum_{j=1}^{n} S_j T_{ij}^\lambda = 0$

6. Set $\sum_{j=1}^{n} S_j T_{ij}^\lambda$

7. Set $j = 1$
   - where $j = 1, N$

8. Start
APPENDIX IV (cont'd)

PROGRAMME XLAMC

FLOW CHART FOR SUCCESSIVE APPROXIMATION OF LAMBDA

2

Set

\[ \sum_{j=1}^{n} \left[ C_{ij} - E(C_{ij}) \right]^2 = 0 \]

5

Set

\[ \sum_{i=1}^{n} \left[ C_{ij} - \frac{1}{N} \left( \sum_{j=1}^{n} C_{ij} \right)^2 \right] = 0 \]

\[ E(C_{ij}) = \frac{S_j}{T_{ij}^\lambda} \times \sum_{j=1}^{n} C_{ij} \]

3

\[ \sum_{j=1}^{n} \left[ C_{ij} - E(C_{ij}) \right]^2 \]
APPENDIX IV (cont'd)

PROGRAMME XLAMC

FLOW CHART FOR SUCCESSIVE APPROXIMATION OF LAMBDA

\[ C_{ij} = \left( \sum_{j=1}^{n} C_{ij} \right)^2 \]

\[ \sum_{j=1}^{n} \left[ C_{ij} - \frac{\sum_{j=1}^{n} C_{ij}}{N} \right]^2 \]

\[ R = \sqrt{1 - \frac{\sum_{j=1}^{n} \left[ C_{ij} - E(C_i) \right]^2}{\sum_{j=1}^{n} \left[ C_{ij} - \frac{\sum_{j=1}^{n} C_{ij}}{N} \right]^2}} \]

Plot \( \lambda, R \)
APPENDIX IV (cont'd)

PROGRAMME XIAMC

FLOW CHART FOR SUCCESSIVE APPROXIMATION OF LAMBDA

\[ \mu \]

\[ \text{is } \] (\( R > R_l \) ) \rightarrow \( R_l = R \)

\[ \text{R max} = R \]
\[ \lambda \text{ max} = \lambda \]
\[ E \text{ max} = E(C_i) \]
\[ C \text{ max} = C_{ij} \]

\[ \text{increment } \lambda \]

\[ \text{is the initial value of } \lambda \text{ } \] than upper limit of \( \lambda ? \)

Print
\[ R \text{ max} \]
\[ \lambda \text{ max} \]
\[ E \text{ max} \]
\[ C \text{ max} \]

Stop

\[ i = i + 1 \]

\[ \text{is } i \leq K ? \]
APPENDIX V  Programme XLAMC : Fortran

0 * SIBFC XLAMC
1 *  DIMENSION S(25),C(50,25),T(50,25),EC(50,25),XEC1(50)
2 *  CALL PLOTS
3 *  1 FORMAT (12F6.2)
4 *  2 FORMAT (2I6)
5 *  3 FORMAT (8F10.4)
6 *  4 FORMAT (3F6.3)
7 *  READ (5,2) K,N

12 *  READ (5,3) (S(J),J=1,N)
17 *  READ (5,1) (C(I,J),J=1,N),I=1,K)
30 *  READ (5,1) (T(I,J),J=1,N),I=1,K)
41 *  READ (5,4) XLAMO,DLAM,XLAM1
42 *  6 FORMAT (6X,20H NEIGHBORHOOD NUMBER I3)

44 *  DO 70 I=1,K
45 *  WRITE (6,6) I
46 *  CALL AXIS (.0,.0,1HR,+.1,.90,.0,.1)
47 *  CALL AXIS (.0,.0,6HLAMBDA,-6,.9,0,.5,5,5)
50 *  SUMC=0.
51 *  DO 10 J=1,N
52 *  SUMC=SUMC+C(I,J)
53 *  10 CONTINUE
55 *  XLAM=XLAMO

56 *  RL=0.
57 *  IPEN=3
60 *  15 SUMST=0.
61 *  DO 20 J=1,N
62 *  SUMST=SUMST+S(J)/T(I,J)**XLAM
63 *  20 CONTINUE
65 *  XN=N
66 *  SUMCEC=0.
67 *  SUMCCN=0.
70 *  DO 25 J=1,N
71 *  EC(I,J)=(S(J)/T(I,J)**XLAM)*SUMC/SUMST
72 *  CEC=(C(I,J)-EC(I,J))**2
73 *  SUMCEC=SUMCEC+CEC
74 *  CCN=(C(I,J)-SUMC/XN)**2
75 *  SUMCCN=SUMCCN+CCN
76 *  25 CONTINUE
APPENDIX V (cont'd.)

Programme XLAMC: Fortran

100 * SR=1.-SUMCEC/SUMCCN
101 * IF (SR)33,30,30
102 * CALL SRNEG(XLAM)
103 * GO TO 36
104 * R=SQRT(SR)
105 * GO TO 36
106 * IF(R-R1)35,50,50
107 * XLAM=XLAM+DLAM
108 * IF(XLAM1-XLAM)60,15,15
109 * R1=R
110 * XLAMM=XLAM
111 * DO 51 J=1,N
112 * XEC(J)=EC(I,J)
113 * RMAX=R1
114 * XLAMM=XLAM
115 * DO 51 J=1,N
116 * XLAMM=XLAM
117 * CONTINUE

121 * GO TO 35
122 * WRITE(6,9) RMAX, XLAMM, (XEC(J),C(I,J),T(I,J),J=1,N)
123 * CALL SYMBOL (2.*XLAMM-1.,10.*RMAX,0.21,4,0.,-1)
124 * CALL PLOT (11.,0.,-3)
125 * CONTINUE
126 * CALL PLOTD
127 * STOP
128 * END

0 * $IBFTC AUX
1 * SUBROUTINE SRNEG(XLAM)
2 * WRITE (6,11) XLAM
3 * 11 FORMAT (/6X,32H SR IS UNDEFINED AT LAMBDA EQUAL F6.3)
4 * XLAM=XLAM+0.5
5 * RETURN
6 * END