RELATIVE VALUE ANALYSIS

by

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ABSTRACT

This study is an empirical analysis of a technical stock selection technique. A random sample is studied for the period 1956 - 1967. Portfolios are constructed and managed by building a model containing three variables--relative earnings, relative price-earnings and relative prices. The objective of the study is to gather evidence which will test the hypothesis that relative value analysis selects portfolios whose rate of appreciation is greater than that of a buy-and-hold strategy or that of the market.

The results of the analysis support the hypothesis. With few exceptions the strategies' rates of appreciation are up to several percentage points higher than the rate of appreciation of both the market and the buy-and-hold strategy. Secondly, portfolios which are constructed using both fundamental analysis and relative value analysis show even higher rates of appreciation.

As a consequence of the results obtained, a general conclusion and several specific conclusions are reached. The general conclusion formed is that relative value analysis is a profitable stock selection technique. More specifically, it is concluded that:

(a) Trends in stock prices do exist.

(b) Fundamental analysis serves an important function in selecting securities to maximize portfolio returns.

(c) The analyst who uses the relative value method will make buy and sell decisions which result in the selected portfolio outperforming the market.

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CHAPTER I

INTRODUCTION

Traditionally, each investor has had one or more of the following objectives:

(a) The preservation of capital.

(b) The preservation of the purchasing power of the dollar.

(c) To earn an adequate return on investment.

(d) To maximize return on investment.

Various techniques have been suggested in order that an investor may properly manage his portfolio and thereby achieve his objectives. Traditionally, first of all one evaluated stocks on the basis of either fundamental analysis or technical analysis.[1] More recently, however, the Theory of Random Walk[2] has been gaining acceptance as a good predictor of security price fluctuations. As a result, the Theory has been used to criticize the use of technical analysis, since the latter assumes the existence of trends in stock price movements.

The next step in the management of portfolios was to determine not only the percent distribution of securities in

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[1 & 2] Please refer to Section II of this chapter for definition of terms.

the portfolio but also how this distribution should vary over time.

In spite of the large amounts of effort which have been devoted to determining the best allocation of investor resources in order to meet specific objectives, there is a notable lack of investment techniques which are able to satisfy the investor whose aim it is to "outperform" the market.[3]

I. PURPOSE AND SCOPE OF THE STUDY

Purpose. This study was undertaken in order (1) to investigate the hypothesis that relative value analysis enables the investor to make buy and sell decisions which permit him to attain his objective of outperforming the market; (2) to offer support to the "trendist" school, which advocates that technical analysis of stock price data is a profitable technique; (3) to answer criticisms[4] of my previous study.[5]

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[3] "Outperforming" the market is an expression used to indicate that the selected portfolio exhibited a greater growth rate than the market, as measured by an Index.

[4] The conclusion reached in this study was that it was possible to consistently outperform the market. This conclusion was challenged on two grounds. First of all, the time period selected (1952-1963) was one of generally rising price-earnings ratios and therefore one did not need this technique to outperform the market. Secondly, the universe of firms from which the sample was selected was neither sufficiently large (it contained only 70 firms) nor was it representative of the stock market as a whole.

[5] David J. Nye, Relative Value Analysis, unpublished Bachelor of Commerce Thesis, University of British Columbia, Vancouver, May, 1965.

The scope of the study is very narrow but is nevertheless of considerable significance.

Importance of the Study. The study is important because, first of all, it explores a stock selection technique which has not been thoroughly investigated to date. It deals with the "art" of investing and attempts to contribute to the body of knowledge in this area. The word "art" is used rather than science for the following reason. Given that the stock market approaches the economist's ideal of a perfect market, i.e., excluding those who have both the means and the ability to maintain[6] a market in a particular security or act on the basis of "inside" information, the successful investor is more of a behaviorist than a scientist.

The past ten years has seen a notable increase in the use of quantitative techniques in security valuation and selection and they may safely be called progress. However, the most sophisticated model must still include one important variable--people and their expectations. Our present level of technology is improving in its ability to understand, measure and predict the actions of people but much remains to be done.

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[6]This, of course, excludes the "specialists" employed by the New York Stock Exchange, whose function it is to maintain an orderly market for the benefit of all investors. See Baumol, The Stock Market and Economic Efficiency.

A second reason why this study is important is the implication which successful technical analysis has with regard to the role of fundamental analysis of securities. If profits, i.e., appreciation in the price of a security, can be earned which are equal to or better than those attainable by the use of fundamental analysis in stock evaluation, then the allocation annually of substantial resources by the investment industry could be criticized on the grounds that the funds are not being put to their most productive use.

II. DEFINITION OF TERMS USED

Relative Earnings: Relative earnings are defined as the earnings per share of the firm for a 12-month period divided by the earnings per share of the market index for the same 12-month period. Mathematically, this may be expressed as:

$$X_{it} = \frac{e_{it}}{E_t}$$

Where:

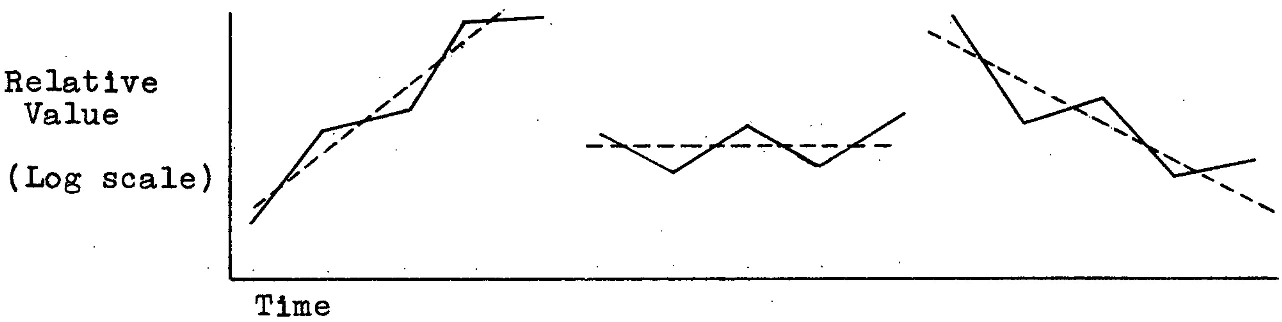
X_{it} = the 12-month relative earnings per share of the i th security at time t

e_{it} = the earnings per share of the i th company's stock for the 12-months ending at time t

E_t = the earnings per share of the market index for the 12 months ending at time t

Relative earnings may be computed for a series of time periods and plotted on semi-log graph paper, as shown in the following figures:

A	B	C
Superior Earnings Performance	Average Earnings Performance	Below Average Earnings Performance



As is shown in the figures, X may be trending upward, downward or horizontally, i.e., the slope of the relative earnings line may be positive, negative or zero. Consider the following, if we let:

$\dot{e} = \frac{de}{dt}$ = the rate of change of the firm's earnings with respect to time.

$\dot{E} = \frac{dE}{dt}$ = the rate of change of the Index's earnings with respect to time.

Then: Case (a) arises when (i) $\dot{e} > \dot{E}$

(ii) $-\dot{e} > -\dot{E}$

Case (b) arises when (i) $\dot{e} = \dot{E}$

(ii) $-\dot{e} = -\dot{E}$

Case (c) arises when (i) $\dot{e} < \dot{E}$

(ii) $-\dot{e} < -\dot{E}$

Then $\frac{dX}{dt}$ will be positive (Case (a)) when (i) $\dot{e} > \dot{E}$, (ii) $-\dot{e} > -\dot{E}$. Similarly, $\frac{dX}{dt}$ will be negative (Case (c)) when (i) $\dot{e} < \dot{E}$ or (ii) $-\dot{e} < -\dot{E}$ and $\frac{dX}{dt}$ will equal zero when (i) $\dot{e} = \dot{E}$ or (ii) $-\dot{e} = -\dot{E}$.

Relative Price: Relative price is defined as the price of a stock at a particular point in time divided by the price of a market index at that same point in time.

For purposes of this study, relative price is represented as Y_{it} and is equal to $\frac{p_{it}}{P_t}$ where:

p_{it} = the price of the i th security at time t

P_t = the price of the market index at time period t.

As indicated in the case of relative earnings, the slope of the relative price line $\frac{dY}{dt}$ may be either positive, negative or zero. By determining whether $\frac{dY}{dt}$ is greater than or less than zero, the analyst is able to measure whether a security is outperforming the market. Thus, if $\frac{dY}{dt}$ is > 0 for a security, then that security is outperforming the market.[7]

Relative Price-Earnings Ratio: The relative price-earnings ratio is defined as the price earnings ratio of the stock divided by the price-earnings ratio of the market index.

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[7]It should be noted that $\frac{dY}{dt}$ may be positive, although the change in p_{it} for the same period is negative and vice versa.

This may be expressed mathematically as:

$$Z_{it} = \frac{pe_{it}}{PE_t}$$

Where: pe_{it} = the price-earnings ratio of the i th security at time t

PE_t = the price-earnings ratio of the market index.

The slope of the relative price-earnings line (defined as $\frac{dZ}{dt}$) offers a reasonably good measure of investor "enthusiasm" for a security. If $\frac{dZ}{dt}$ is positive for a reasonably long period of time, e.g., 2-3 years, the slope of the line indicates that the security enjoyed investor popularity during that period of time. Conversely, if $\frac{dZ}{dt}$ is negative for a similar period, this is good evidence that the stock is out of favour and therefore should be avoided in most cases.

Relative Value Analysis: Relative value analysis is the use of the variables (relative earnings, relative price and relative price-earnings) to decide when a stock should be included and when it should be eliminated from a portfolio.

Fundamental Analysis: Fundamental analysis is the analysis and forecasting of economic, industry and firm factors in order to determine the intrinsic or theoretical value of a security at a specific point in time. The logic behind this method is that if the theoretical value is greater than market price, the stock should be purchased. Conversely, it should be sold if the intrinsic value is less than the market value. Opposed to this idea are the members of the technical analysis school.

Technical Analysis: Technical analysis is the study of security prices only in order to make an investment decision. This analysis is founded upon the belief that all factors affecting a security are reflected in its price eventually and one has only to interpret correctly the stock's price trend in order to predict accurately its price fluctuations.

This method of analysis assumes the existence of trends in successive price differences. Lately, this assumption has been severely criticized by some members of the academic community, who have proposed, as an alternative, the Theory of Random Walk.

Random Walk Theory: Briefly, the Random Walk Theory is defined here to mean that price changes of a stock are statistically independent of each other. Nothing can be learned about the future by looking at the stock's price series. Thus, buying a stock based on signals from a price chart will produce results no better than those from repeated flipping of a fair coin.

Growth Stock: Since the concept of a growth stock is quite important to the Theory of Relative Value Analysis, more space than usual is devoted to its definition.

As in other areas of finance, the theories of definition and valuation of growth common stocks have moved from a generally qualitative approach, such as that taken by Jenks, Kotler and Bernstein, to a quantitative method, such as that suggested by Burrell, Solomon, and, more recently, Mao.

Bernstein[8] makes an important distinction between a growth company and a growth stock. To him, a growth company is not one whose sales and earnings increase merely as a result of the firm's response to favourable external factors, such as population increase; rather, he sees true growth as being "inner directed." In other words, the management of a growth firm is the driving force. "The ability to create its own market is the strategic, the dominating, and the single most distinguishing characteristic of a true growth company."[9]

Opposed to this view is Kotler's somewhat loose definition of a growth situation. "... a growth stock is the stock of a company which has shown for a number of years and/or is showing annual percentage increases in net earnings which

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[8]P. Bernstein, "Growth Companies versus Growth Stocks," Harvard Business Review, vol. 34, No. 5 (September-October 1956), pp. 87-98.

[9]Ibid., p. 91.

substantially exceed the long run growth rate in the economy.[10]

To Jenks,[11] a growth company means "a company that will eventually be successful and that is now in or entering a phase of rapid development." He enumerates several characteristics of growth companies such as high multiples, low yields, plus several other technical price patterns. Although the article is easy to read, it is at a low level and does not represent a significant contribution to the theory.

Burrell,[12] writing in 1960, suggests that two important factors in valuing growth companies are the historical growth rate of the dividend and investor expectations. He suggests that a measure of investor expectations is the past relationship of dividends to market price with an additional indicator being current stock market levels.

The current price is the sum of the present values of two elements--the present value of the selling price at some future date plus the present value of an annuity of the

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[10] P. Kotler, "Elements in a Theory of Growth Stock Valuation," _Readings in Financial Analysis and Investment Management_, (Ed. E. M. Lerner). Homewood, Illinois, Richard D. Irwin, Inc., 1963, pp. 355.

[11] J. C. Jenks, "Investing in Growth Stocks," _ibid_., p. 325.

[12] O. K. Burrell, "A Mathematical Approach to Growth Stock Valuation," _ibid_., p. 338.

expected increasing dividends. Thus, Burrell must assume a growth rate in dividends, the duration of the growth, an appropriate capitalization rate and an appropriate discount rate.

According to Burrell, the current market price is then equal to:

$$P_o = \frac{D \quad (1+g)^n}{r_o \quad (1+r)^n} \qquad + \quad \sum_{i=1}^{n} D_o \left(\frac{1+g}{1+r}\right)^i$$

Where:

P_o = the proper price to pay for the stock today

D_o = the current dividend per share

r_o = the rate which an individual investor uses to capitalize a constant size income stream in perpetuity

r = the discount rate which an individual investor applies to a future dollar

The first term is an estimate of the present value of the stock's expected market price in years hence and the second term is an estimate of the present value of the expected dividend income over the period.

A more current definition of a growth company is that of Mao's: "... a company which has specific opportunities to invest funds at a perpetual after-tax annual return of r, where r is greater than the stockholder's required rate of return, y."[13] He summarizes two models of permanent growth

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[13]James C. T. Mao, Quantitative Analysis of Financial Decisions, unpublished manuscript, University of British Columbia, 1968. Chapter 10, p. 30.

as developed by Solomon[14] and Modigliani and Miller[15] and then develops a model which incorporates exponential growth of earnings, constant growth and declining growth.

It is important to note that these models recognize and incorporate the investment opportunities approach. Thus, in the final analysis, dividends are a function of earnings, which in turn are a function of the opportunity of the firm to invest its funds at a rate greater than y.

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[14] E. Solomon, The Theory of Financial Management, New York, Columbia University Press, 1963.

[15] F. Modigliani and M. Miller, "Dividend Policy Growth, and the Valuation of Shares," Journal of Business, vol. 34 (October 1961), pp. 421-449.

CHAPTER II

THE CONCEPT OF RELATIVE VALUE ANALYSIS

In Chapter I the hypothesis was stated, and in this chapter the underlying theory which led to the hypothesis will be given. In addition, the concept of risk and its relation to relative value will be discussed.

I. THE FIVE-STAGE THEORY

This theory states that, given a situation of increasing relative earnings, the market initially fails to respond favourably to the improved earnings, and then responds in an exaggerated manner. In other words, the theory rests upon the premise that human beings in general react in a manner which is not proportionate to the original stimuli. Benjamin F. Graham expressed essentially the same idea when he said:

> One thing badly needed by investors--and a quality they rarely seem to have--is a sense of financial history.[16]

> Yet the market tends to greet each upsurge as if it were the beginning of an endless growth and each decline in earnings as if it pressaged ultimate extinction.[17]

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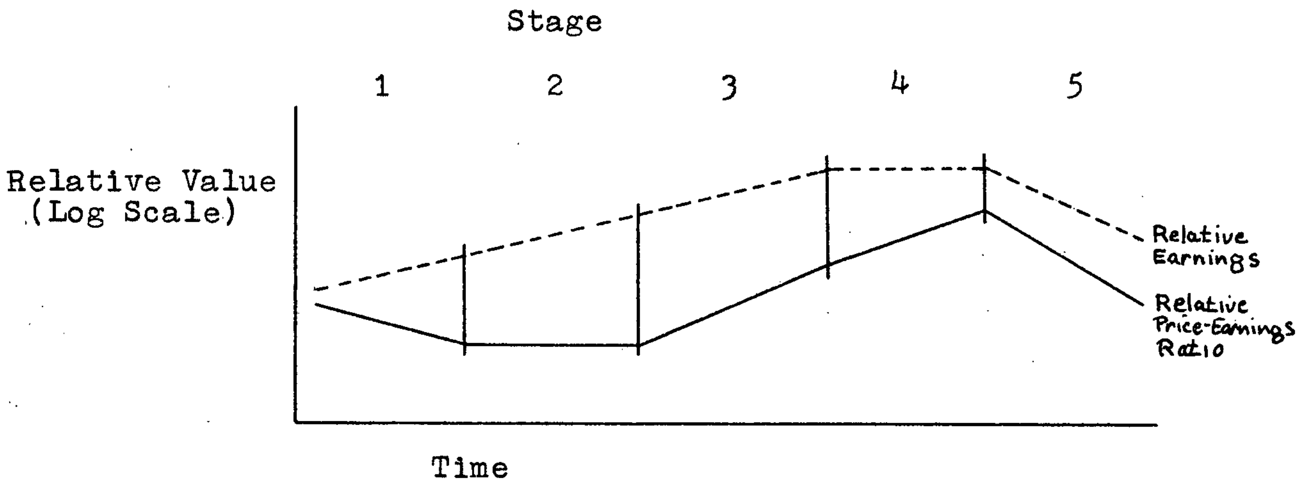
[16] B. Graham, The Intelligent Investor, 3rd edition, Harper & Row, New York (1965), p. 13.

[17] *Ibid.*, p. 14.

In order to better understand this idea, it is convenient to divide this process of under and over-reaction into five stages, as shown in Figure D.

FIGURE D

Graph of 5-Stage Cycle



During Stage 1, relative earnings are positively sloped while the relative price-earnings line has a negative slope. This means that the firm's earnings are growing at a greater rate than those of the index. At the same time, a falling relative price-earnings line indicates that the firm's multiple is rising at a slower rate than the multiple of the market is rising. On the other hand, it may indicate that the firm's multiple is falling at a rate greater than that of the market.

The impact of the relative earnings and relative price-earnings change upon relative price will depend upon their comparative movements. This idea may be expressed mathematically in the following manner. As indicated in Chapter I, we have:

X_{it} = the relative earnings per share of the i th security at time t

Y_{it} = the relative price of the i th security at time t

Z_{it} = the relative price earnings ratio of the i th security at time t.

Values of X, Y and Z for a number of time periods may be calculated. Having calculated these values and plotted them on semi-logarithmic graph paper as in Figure D, it is possible to regress these values against time and obtain an equation of the form $y = a + bx$. In our case we would obtain the following:

(1) $X = a + \alpha T$

(2) $Y = b + \beta T$

(3) $Z = c + \gamma T$

Where:

a, b and c are constants.

α = the slope of the relative earnings line.

β = the slope of the relative price line.

γ = the slope of the relative price=earnings line.

Then if:

(a) $\alpha = \gamma$; $\beta = 0$

(b) $\alpha > \gamma$; $\beta > 0$

(c) $\alpha < \gamma$; $\beta < 0$

Stage 1 has been called a "base building" stage since the market largely ignores the improving earnings picture of the firm. This is an interesting occurrence from a behavioural point of view and bears further discussion. Proceeding from the position that an earnings increase is "better" from the point of view of an investor seeking an outlet for his funds, why should β be negative? In other words, why should investors be willing to pay relatively less for increasing earnings? The possibility that they are not aware of the increase can safely be ignored. Another possibility is that they are aware of the situation but believe the increase to be only a temporary reversal of a long term downtrend as evidenced by previous data. This could be case (c) mentioned previously, where $\alpha < \gamma$ and the security price is being adjusted downwards on the basis of revised <u>long term</u> expectations.

In case (a), $\alpha = \gamma$, i.e., the slope of line X is equal to the slope of line Z. Under these conditions, investors appear to believe that earnings will return to a "normal" level and therefore, the adjustment in β reflects these expectations.

In case (b) it could be argued that some investors believe the increase to be a fundamental improvement and, on this basis, the falling relative price earnings ratio does not completely offset the rising relative earnings. In this situation there are enough "believers" to more than offset the "disbelievers" and as a result β is positive. Expectations are for continued improved earnings but they are by no means unanimous.

By focussing on relative earnings, we have excluded many other explanatory variables and they must now be mentioned. According to King,[18] market and industry factors are very important in the explanation of security price changes. However, by dividing firm data by market data, we have <u>excluded</u> the market and industry impact. This brings us once more to the firm and the impact that other variables such as dividends, cash flow and leverage will have on security prices.

Rather than attempting to determine the influence of each of these variables as stock price determinants, they are merely mentioned and the discussion will continue to be centered on earnings as an important variable in determining stock price changes.

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[18]Benjamin F. King, "Market and Industry Factors in Stock Price Behaviour," <u>Journal of Finance</u>, vol. 39, No. 1, Part II (January 1966), p. 139.

A good discussion of the concept of changed earnings and investor expectations is provided by Whitbeck and Kisor.[19] In their paper they point out that during a business cycle the multiples of many firms behave in a contra-cyclical manner. Thus, as earnings rise, the multiple falls and vice versa. Obviously investors have some notion of "normal" earnings for the firm and the occurrence of a Stage 1 merely indicates that they expect the firm to continue to behave in a cyclical fashion. As earnings rise, their expectations do not change and, as a result, the price of the security remains fairly constant and the multiple contracts. When earnings fall because of a decline in economic activity, investors expect them to increase when the economy resumes its growth. Therefore, security prices are maintained and the multiple expands.

Stage 2 is defined as one in which relative earnings continue to increase (i.e. $\alpha > 0$) but the relative price earnings ratio remains constant ($\gamma = 0$). This stage in the cycle indicates that investors are revising their expectations about the future earnings of the firm and as a result are willing to pay relatively more for each share in the belief

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[19] V. S. Whitbeck and M. Kisor, Jr., "A New Tool in Investment Decision Making." Reprinted in <u>Frontiers of Investment Analysis</u> (Ed. E. Bruce Fredrikson). Scranton, Pennsylvania, International Textbook Company, 1965, pp. 335-350.

that relative earnings will continue to expand. As will be indicated later, this stage does not play a significant role in the concept mainly because of definitional inadequacies.

During Stage 3 relative earnings continues to expand and, in addition, the relative price earnings ratio expands, resulting in relative price substantially outperforming the market. During this stage in the cycle investors appear to have confirmed their belief that this is a growth firm. They believe that this above average earnings growth is going to continue[20] and are, therefore, willing to pay more for each share.

Stage 3 is interesting to consider with reference to the Whitbeck and Kisor theory and the behaviouralist approach. It would appear that investors view the firm from a cyclical point of view until Stage 3 occurs. At this time, the over-reaction takes place and the value of the relative price earnings ratio increases to a level which is not justifiable when subjected to rational analysis.

Stage 4 is defined as having stable relative earnings, while relative price earnings continue to increase. Thus, a stock in this stage will continue to outperform the market as in Stages 2 and 3 (and possibly in Stage 1 also) but the above average performance is due solely to an increasing multiple

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[20]It is difficult to determine what is each investor's time horizon for this expected growth.

and is thus a potentially dangerous situation from the stockholder's point of view. The reasoning behind the occurrence of Stage 4 is as follows. As relative earnings increase through Stages 1, 2 and 3, an increasing number of investors become aware of the superior earnings gains being reported by the firm. As this number becomes sufficiently large, demand for the stock is initially less than supply, then equal, and finally exceeds supply. Stage 4 reflects the situation of continuing excess demand resulting in the expanding multiple.

What is the reason for this apparent excess demand? It is suggested that it is again the result of favourable investor expectations. If expectations are influenced by historical data (and it would appear that they are) then investors, noting the uptrend in earnings, continue to purchase the stock in the expectation that this uptrend will continue.

Stage 5 occurs when both relative earnings and relative price earnings slope downward, resulting in a substantial decline in share value to those holding the security during this stage. As is inevitable, almost all firms experience an earnings decline at some time or another. When such a relative earnings decline is experienced, concurrent with it is a relative price earnings ratio decline--again the result of revised expectations based on new information.

In all probability it is unrealistic to think that investors never expect a firm's earnings to decline since the mathematical implications of a high growth rate compounded for even a large finite time period are absurd. The question then becomes: why would the security continue to be purchased during Stage 4? Obviously the answer is timing. Investors expect earnings to decline eventually but not in the relatively near future. When the decline occurs, the downward sloping relative price earnings ratio implies that the downtrend in relative earnings is expected to continue for a time. Otherwise, relative price earnings would increase, based on the belief that the relative earnings decline was only temporary and that the growth would quickly be resumed.[21]

This is the very broad framework within which we shall be dealing and it is expected that there will be exceptions which are not explained by this theory. Indeed, it would be very naive to claim that security price movements can be explained by only one variable. However, we shall attempt to attain the previously mentioned objective by means of relative value analysis.

One of the points to be noted here is that relative value analysis is not, nor is it claimed to be, a method by which stocks may be valued. Rather, it focuses on the problem

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[21]Some support is lent to this by B. Graham's previously mentioned statement (see footnote 17).

that, given a certain security and its price, will it be a profitable investment, i.e., will it outperform the market? We are not really concerned with whether the stock is undervalued or overvalued but only whether that security, if purchased, will rise in price more than the market or fall in price less than the market.

II. RISK

No discussion of investments is complete without considering risk, and in this section risk will be studied within the context of relative values.

For our purposes the investor may be said to face two kinds of risk.

First of all, he faces what we will call <u>internal risk</u> and this risk is defined as the chance of the firm failing.[22] The use of the word "chance" implies some known probability of the firm failing. However, when one considers that <u>internal risk</u> includes both business risk and financial risk,[23] the derivation of a probability function is a complex, but not impossible, task. Internal risk is reduced to the extent

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[22]"Failure" is defined as the condition where the firm has insufficient resources to meet obligations as they fall due, or, in Donaldson's words, there exists a situation of "cash insolvency."

[23]"Business risk" and "financial risk" have the traditional meaning.

possible by limiting stocks eligible for inclusion in the sample to those listed on the New York Stock Exchange as at June 30, 1955.[24] No attempt will be made in this project to quantify internal risk. To the extent that all firms must meet these minimum standards, the probability of failure is maximized at a certain level with the larger firms presumably having a probability lower than this maximum.

In addition to internal risk, the investor also faces external risk, which is defined as the risk of a decline in the value of the firm due to all factors other than internal risk. The two types of risk may be measured by the variance in the market price of the security. In some work to be discussed in a later chapter it is evident that security price fluctuations are a function of market, industry and firm factors. Thus, even though an investor may purchase an interest in a firm, not only because of its demonstrated earning power but also because of its opportunities for profitable investment, he nevertheless is exposed to the risk of a decline in the value of his shares due to forces external to the firm. He thus faces, for example, the following anomaly; let us assume

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[24] N.Y.S.E. listing requirements are summarized in Appendix I in addition to the requirements of the smaller, regional exchanges. It is interesting to note that the first N.Y.S.E. Listing Committee was formed in 1866 and, over the years, standards have gradually been raised. However, since no existing contract is bound by later agreements, the improved standards applied only to the latest agreements and, legally, a previously listed firm was not bound by the new standards.

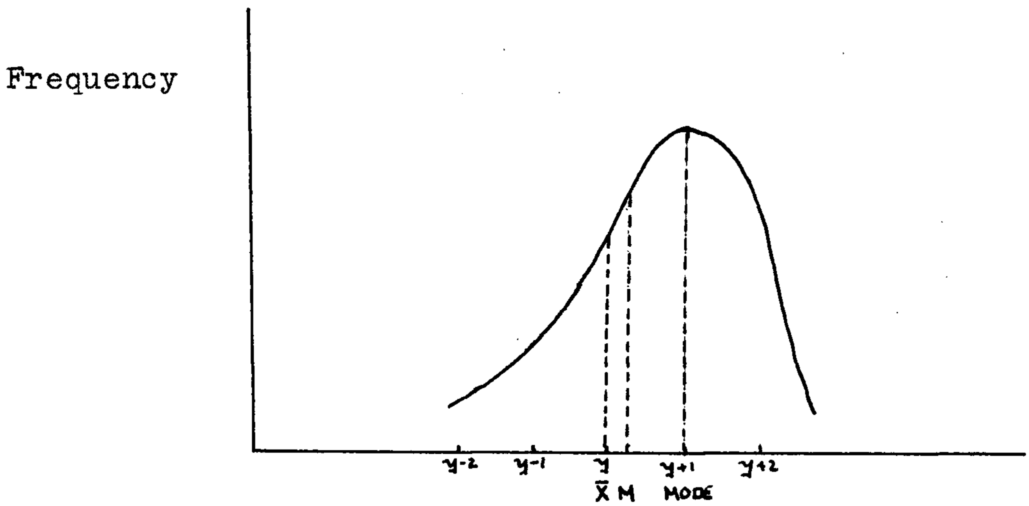
that our investor is one of those gifted individuals who is truly able to forecast the future better than most as a result of his excellent analytic ability, both with respect to economic factors and human behavior. Our investor has bought an interest in a firm based on his forecast of profitable investment opportunities available to the firm. Subsequently a government department announces a policy change which is expected to adversely affect this particular industry. This announcement results in changed expectations of present and potential investors and, consequently, the supply of this security exceeds demand, resulting in a new, lower price. However, to what extent would investors' expectations be revised as a result of logical analysis? This is an extremely difficult question on which to obtain empirical evidence but, intuitively, we would say not enough.

Our investor has assessed the situation, however, and is of the opinion that factors exist which will mitigate the effects of the policy change. Although the investor's analysis may be correct, he will suffer a loss or have his profits reduced, should he be forced to liquidate his holdings before market price has adjusted to the "theoretical" or "intrinsic" price.

Recognizing this external risk, the investor's objective is not to minimize variance but to choose a security whose expected distribution of future relative prices is negatively skewed, (see Figure E) during the time he owns shares in that company.

FIGURE E

Expected Distribution of Relative Price
in period t+1 given a level of y in year t



As Markowitz pointed out, variance is not a true measure of risk since this implies that deviations on <u>both</u> sides of the regression line are equally undesirable. However, positive deviations are infinitely more desirable than negative ones. Thus, a measure such as the semi-variance is more meaningful but it is also a much more complex programming problem.

In this chapter the concept of relative value has been given and, in addition, the notion of risk has been related to the subject of the study.

In the next chapter the literature on relative value will be reviewed and the subsequent chapter will explain the development of the model.

CHAPTER III

RELATIVE VALUE - REVIEW OF THE LITERATURE

Although the use of relative values is an intuitively appealing concept, a review of the literature indicates that very few writers have dealt with the subject explicitly. Most investors realize that performance must be measured against some standard and one need only refer, for example, to the prospectus of any mutual fund or investment advisory service to see the use of relative value. However, the interesting point to note is that very few investors appear to have investigated the relationship between the security and the standard in order to determine if a meaningful and consistent pattern exists.

The earliest reference this writer was able to find was that of Rose,[25] where relative values were used more as a descriptive tool, rather than an analytical selection technique.

Rose

Rose's main purpose was to study the rates of return achieved by financial institutions on their securities portfolio and his relative value technique is shown in an

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[25]Dwight C. Rose, A Scientific Approach to Investment Management. New York, Harper & Brothers, 1928.

appendix entitled "Relation of Stock Price Trends in Each Major Industry to the Price Trend of all Stocks." As the heading implies, Rose summed the market value of the stocks in a particular group, divided this sum by the market[26] and plotted this ratio on semi-log charts for the period January 1, 1918--December 31, 1927. In addition, he employed a scale on the vertical axis which indicated the percent variation of the group from all the stocks. One must suppose that the charts were useful to Rose as indicators of past performance of a group, e.g., his automobile group included:

1. General Motors
2. Willys Overland
3. Studebaker Corp.
4. White Motor
5. Packard Motor Car
6. Chrysler Motors
7. Chandler Cleveland Preference
8. Mack Truck
9. Pierce Arrow
10. Hupp Motor Car

However, the validity of his index would be open to question because of the weighting system he used. Apparently it did not occur to him that he should study further the relationships involved to see if anything meaningful could be uncovered. By 'meaningful' I have in mind a selection technique able to choose stocks which will outperform the market.

Rhea

The discovery of an apparently meaningful relationship

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[26]Rose did not indicate, however, which stock market index he used.

was made by Robert Rhea, who reported in a 1933 issue of Barron's[27] that he had had some success in using a relative value technique.

Rhea began with the simple observation that during the course of a stock market cycle some issues fluctuate more widely than others in relation to the Dow-Jones Industrial Average. A stock's volatility was measured by its index number and Rhea assumed that a stock which historically had been volatile would continue to exhibit volatility.

The security price was related to the market through the use of an appreciation index number derived as follows:

(1) Determine the percentage change in the security for time period t

(2) Determine the percentage change in the industry index for time period t

(3) Divide (1) by (2); then (3) indicates the gain which would have been made on an investment in the security relative to the gain recorded by the index.

Example	\$ Change ($t_1 - t_o$)	% Change ($t_1 - t_o$)	
Security	105.75 - 94.25	12.00	(1)
Index	24.65 - 20.90	17.94	(2)

$$(1) \div (2) = \frac{12.00}{17.94} = .668$$

Thus, this security did relatively poorer than the index

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[27] R. H. Rhea, "Stock Habits - A Simple Method to Follow Issues that Fluctuate More Widely than the Averages," Barron's, New York, May 8, 1933, p. 1.

since, for every $1.00 appreciation of the index, the stock went up only $ .668. A similar figure can be derived for declines in the index.

Rhea then determined these index numbers for the 10 declines and 10 rallies[28] which had occurred in the 15-month period January 1, 1932--May, 1933. He averaged these figures to arrive at one advance index and one decline index for each stock. These stocks were then divided into three groups of 15 each, according to the following criteria.

Group I - "lively" stocks--those that move with the market but with greater variation in advances than declines.

Characteristics:

(a) absence of dividend payers
(b) "heavy" leverage

Group II - Price variation "approximates" that of the average.

Characteristics:

(a) mostly dividend payers

Group III - Price variation is usually less than that of the average.

Characteristics:

(a) includes many higher priced stocks
(b) dividend payers are in the majority

The results of his test were as follows: Group I showed a $3.00 gain for every $1.00 gain recorded by Group III.

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[28]A peak was said to have occurred if the index declined more than 10% from a given level. This decline was said to have continued until the index had risen 10% from a given level.

Although Rhea's method is not at all rigorous and analytical, it does take the first step in using relative values as a technique to aid in the selection of above average securities as well as describing their historical performance.

Kourday

Although it is possible that many investors may have used relative values to aid their investment decision making, apparently none of them felt inclined to report it, since the next article did not appear until the early 1960's. At that time M. Kourday published an article[29] in the Financial Analysts Journal. His purpose in writing was to publicize a hypothesis which he had developed and tested during his career as a security analyst. He feels that relative value analysis is a useful technique for comparing securities with one another, selecting the best one, i.e., the stock which will outperform the averages, and determining when an issue should be eliminated from a portfolio. However, he also states that this analysis should not be the sole decision-making tool but that it is best used in conjunction with fundamental analysis.

His basic premise is that security prices are a function of earnings. As earnings increase, so should the stock's

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[29]M. Kourday, "Relative Values - A Method for Outperforming the Market," Financial Analysts Journal, vol. 19, No. 6 (November-December 1963), p. 35.

price. Similarly, as relative earnings increase, relative price should also rise. Thus, the share price of the company whose earnings are growing faster than average should show better than average price performance. This correlation also exists for the opposite situation, so Kourday claims. A relative earnings decline should be reflected in a relative price decline. Where a correlation does not exist, then there exists an opportunity to buy or sell. For example, if relative earnings are in an uptrend but relative price has not shown any growth,[30] the stock should be purchased. However, Kourday's apparently flagrant claim of a good and consistent correlation over time is qualified by additional hypotheses disguised as "facts" in other sections of his paper. Thus,"As is well known, even though there may be a time lag in earnings reports, the relative market performance of a stock can reflect a material change."[31]

The relative price-earnings ratio is a measure of the under or overvaluation of a security. Thus, the amount by which the relative price-earnings ratio is greater than 100% provides a measure of the premium which investors are paying. This may be compared to the analysts' and others' forecasts of share price to determine whether the premium, in their

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[30]The reader will remember that this has been defined as Stage 1.

[31]M. Kourday, <u>op. cit</u>., p. 36.

opinion, is justified.

The user of Kourday's method is required to forecast not only the earnings of the security under study but also the earnings of the Dow-Jones Industrial Average. For example, in September the analyst should forecast the last quarter earnings and also the following year's earnings.[32] From this data, and using current prices, he may determine relative values and evaluate the security from a relative standpoint.

Kourday's paper is very interesting to read; however, he is not able to provide any significant empirical support for his hypothesis other than a few examples mentioned in the text of his paper. In addition, his reasoning proceeds from basic assumptions, for which there is not empirical evidence. However, as mentioned earlier, the theory is intuitively appealing and deserving of additional thought and testing.

Whitbeck and Kisor

Whitbeck and Kisor[33] utilized the relative earnings concept in their empirical work which was reported in 1963. They addressed themselves to the problem of how much to pay

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[32]The accuracy of their forecasts is in considerable doubt. See the article by J. G. Cragg and B. C. Malkiel.

[33]V. S. Whitbeck and M. Kisor, Jr., "A New Tool Investment Decision-making," Financial Analysts Journal, vol. 19, No. 3 (May-June 1963), pp. 55-62.

for a given stock and determining what the proper multiple should be.

By plotting earnings per share over time on a logarithmic scale and fitting a least squares line, they determine the slope of the line, i.e., the average annual growth in earnings per share and the variability of the earnings as measured by the standard deviation.

Proceeding from the assertion that relative earnings and relative price earnings move in a contracyclical fashion, they infer that the market has a concept of "normal" or "cyclical average" earnings for the firm in question. This "normal" earnings level is defined as "... that level of net income which would prevail currently if the economy as a whole were experiencing mid-cyclical business conditions.[34]

Working in the belief that there should be a relationship between the projected rate of earnings growth and the p-e ratio, the authors conducted the following test. For 135 stocks of "general investment interest," the expected earnings growth rate was plotted along the x axis and the "normalized" p-e ratio along the y axis. The "normalized" p-e ratio was determined by dividing current (6/8/62) price by "normalized" earnings. A regression line was then fitted, which yielded

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[34]Whitbeck and Kisor, ibid., p. 58.

the following:[35]

$$Y = 9.3 + 1.5X$$

In this case investors were apparently willing to pay 9.3X earnings for a firm with no projected growth and, after that, each percentage point of growth was worth an additional 1.5 multiples.

The second part of their empirical work consisted of regressing what they consider to be the three principal factors of common stock valuation, growth, stability and payout upon the p-e ratio. Their analysis produced the following equation:[36]

$$y = 8.2 + 1.5x_1 + 6.7x_2 - 0.2x_3$$

Where:

y = price-earnings ratio

x_1 = growth rate

x_2 = payout

x_3 = standard deviation of earnings

Having determined the "proper" multiple with which to multiply "normal" earnings, Whitbeck and Kisor are able to arrive at the theoretical price of the security. This price may be expressed as a ratio of current actual price and they

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[35]Since no value for r^2 was given we can only assume that it was not significant.

[36]See footnote 35 for comment regarding the r^2 value.

thus have an index with which to measure under or overvaluation.

Given this theoretical price, their hypothesis then becomes "... the market price of the stock will seek this level faster than the theoretical price itself will change ..."[37] Why? Because "... changes in market psychology come, by and large, in a slow and orderly fashion ..."[38]

To test their hypothesis, they divided the stocks into the following groups:

(a) Undervalued group:

$$\frac{\text{Market Price}}{\text{Theoretical Price}} \quad .85$$

(b) Overvalued group:

$$\frac{\text{Market Price}}{\text{Theoretical Price}} \quad 1.15$$

The results of the study are shown in Table 1.

TABLE 1

	Undervalued Group		S & P 500		Overvalued Group	
Date	3 months' Change	Cumulative Change	3 months' Change	Cumulative Change	3 months' Change	Cumulative Change
	%	%	%	%	%	%
9/23/60	11.9	11.9	6.6	6.6	5.7	5.7
12/23/60	16.8	30.7	12.3	19.7	8.3	14.5
3/24/61	3.0	34.6	1.0	20.9	(1.4)	12.9
6/23/61	3.2	38.9	2.4	23.8	2.1	15.3

[37] Whitbeck and Kisor, <u>op. cit.</u>, p. 60.

[38] <u>Ibid</u>.

Thus, for each three-month period, the undervalued group outperformed the S. & P. 500, which in turn outperformed the overvalued group.

Smilen and Safian

In 1964 K. B. Smilen and K. Safian[39] discussed their concept and use of relative earnings. They enthusiastically support the concept of relative earnings but argue that not all companies should be related to one stock market average. To relate a cyclical firm and a growth firm to the same average is unfair. Thus they originated their concept of the Dual Market Principle.

They developed the Cyclical Average of the Dual Market Principle, which is composed of a representative group of 23 prime cyclical securities whose earnings are closely related to the level of economic activity. For example, automobile firms would be compared to the Cyclical Average.

Their Dual Market Principle's Growth Average is composed of 25 securities which the authors consider to have varying degrees of growth potential. In the case of firms whose earnings are dependent upon external factors, these are classified as satellite firms and are also related to the Growth Average.

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[39] K. B. Smilen and K. Safian, "Relative Earnings - A Fresh Perspective," Financial Analysts Journal (September-October 1964), vol. 20: No. 5, pp. 104-107.

Apparently believing that their terms have been adequately defined, they then proceed, aided and abetted by hindsight, to analyze a firm's earnings and make their investment decisions.

In addition to the absence of empirical support, the reasoning of Smilen and Safian is faulty. In this writer's opinion they have failed to justify the use of two averages with which to compare firms. We will proceed from the basic premise that the <u>fundamental purpose of relative values is to enable the user to compare all firms</u>. Smilen and Safian, using their method, are unable to do this. To compare a cyclical firm to a cyclical average is to study a subset of the universe. It carries with it an implied assumption that the user has already decided that he will include a firm from a particular industry in his portfolio and that his decision now is which one to include. Obviously, this is putting the cart before the horse. It is possible to conceive of a situation where a firm's earnings may be increasing relative to the Cyclical Average but decreasing relative to the market. In this case they would make an incorrect investment decision.

<u>Russell</u>

In the book <u>A Treasury of Wall Street Wisdom</u>,[40]

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[40] H. D. Schultz, <u>A Treasury of Wall Street Wisdom</u> (Ed. Samson Coslow), Palisades Park, New Jersey, Investors Press, Inc., 1966, pp. 90-92.

Richard Russell has a short selection on the use of relative strength as an investment selection criterion. Basically, his suggested procedure is as follows:

(a) Select those groups of stocks which have shown the best relative strength, i.e., those groups which are now starting to increase after long declines.

(b) From these groups pick the stocks with the best relative strength.

(c) From these stocks pick those with the best actual technical patterns.

Those stocks selected should be sold when any of the following situations occur:

(a) The stocks no longer outperform the market.

(b) When the relative strength line reverses.

(c) When the general market registers a sell signal.

Although Russell's technique sounds very logical and profitable, it lacks rigorous definition and empirical support. However, the present study attempts to correct several of these deficiencies.

Levy's Study

Perhaps the most interesting relative value study reported to date is that of Levy's[41] which offers strong

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[41]Robert A. Levy, "Relative Strength as a Criterion for Investment Selection," Journal of Finance, vol. 22, No. 4, (December, 1967), pp. 595-610.

support in favour of the Trendists. Although he has not refuted the Random Walk Theory, he has been able to effect a possible reconciliation between the two opposing camps. Generally speaking, Levy found that a serial correlation study of performance ranks rather than successive first differences detected the existence of trends over the long term (26 weeks) but not over the short term (4 weeks). Thus the "co-movement" of stock prices, found by King,[42] could in fact conceal existing dependencies in successive price changes. Then, Levy suggests that, by using ranks which measure relative strength, the veil of the general market movements may be parted and the underlying structure analyzed.

In order to investigate his hypothesis, Levy randomly chose 200 stocks and studied them over the 260-week period October 24, 1960--October 15, 1965. He constructed ratios designed to measure historical strength and future performance. Thus he used historical strength as a means for selection at time period t and subsequent ratios as a measure of his investment results.

After calculating the following three price ratios:

(a) C/A 26 - current week's price divided by the average of 26 previous weeks' price and including the current week.

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[42] Benjamin F. King, "Market and Industry Factors in Stock Price Behaviour," Journal of Business, vol. 39, No. 1, Part II (January 1966), pp. 139-190.

(b) 4/C - current week's price divided into the price 4 weeks subsequent to the current price.

(c) 26/C - current week's price divided into the price 26 weeks subsequent to the current week.

The ranks listed below were determined for the 200 stocks:

(1) Relative Strength Ranks - the highest ratio was given a rank of 000.

(2) Volatility Ranks - the coefficient of variation defined as $\frac{\sigma}{\bar{x}}$ was used as a ranking criterion with the highest ratio receiving the lowest rank (000).

(3) Market Ranks - each week the C/A 26 ratios were summed and the totals ranked.

(4) Divergence Ranks - the difference between the average of the C/A 26 ratios of the 20 strongest stocks and the average of the 200 stocks was determined and ranked with the largest divergence receiving a rank of 001. Similarly, the long term weak divergence ranks were determined.

Levy's results were extremely enlightening. Traditionally, it has been maintained that historic relative strength tends to continue for a period of time. Although the 4/C ranks and ratios did not support this hypothesis, the 26/C ranks and ratios showed that the 10% historically strong stocks gained on average 9.6% while the 10% weakest gained on average only 2.9%. In addition, he found good correlation between past performance groupings and 26-week future performance groupings, as shown in Table 2.

TABLE 2

C/A 26 Relative Strength Rank Group Number	Group Performance 26/C Average Group Ratios	Indicator Based Upon 26/C Average Group Ranks
1	1	1
2	2	2
3	3	3
4	7	6
5	4	4
6	8	8
7	5	5
8	6	7
9	9	9
10	10	10
r' =	.87	.92

Further investigation of his preliminary results led Levy to the following conclusions:

(a) Selection of stocks which historically had been both relatively strong and relatively volatile resulted in profits greater than those possible by random selection.

(b) Following from the results first mentioned, he found that superior performance could be achieved by purchasing stocks in a market which historically had been strong. Thus continuation of relative strength appears to apply to the general case (the market) as well as to individual securities.

Nye-Study No. 1

The next study to be considered is the author's B.Comm. graduating essay. The Five-Stage Theory described earlier was for its validity and for its ability to outperform the market.

The sample consisted of 30 U.S. industrials listed on the N.Y.S.E. These companies were being followed by the

Research Department of Eastman Dillon, Union Securities and Company, a large United States investment house based in New York, and included firms from almost all major industries.

For purposes of this test, a given stage was said to have occurred if the trend of relative earnings or relative price earnings was established for a minimum of four quarters. Although this was an arbitrary figure it was felt that to take any period less than that might not permit a trend to be clearly enough established, whereas to postpone the decisions past four quarters might result in lost investment opportunities.

In order to test for validity, a particular stage was first of all chosen and then the movements of relative earnings and relative price earnings were studied, both before and after the occurrence of the given stage. The results of this test will be given in the next section.

The next portion of the study was devoted to testing the ability of the system to achieve above average investment results. First of all, thirty charts were randomly chosen and coded from the original seventy. Following this, quarterly plottings of relative earnings, relative price earnings and relative price were made from the coded charts onto new charts by an impartial participant in the study. It should be pointed out that the author did not know the name of the company whose figures were on the chart nor was he aware of the year for which the figures were being plotted. After each plotting, the charts were given to the author, who studied them, made buy or sell decisions if indicated and returned the charts to the participant, who then made another quarterly plotting. In this way, seven years of data were plotted on the charts and studied.

Buy and sell decisions were based on the following criteria. A stock was purchased at the end of Stage 1 and held in the portfolio until Stage 5 had occurred. One further question was also studied. This was the effect, if any, of the level of relative price-earnings at the time of purchase on the performance of the portfolio. Thus two tests were carried out. The first one consisted of buying on a Stage 1 with relative price-earnings at any level, while the second test consisted of buying after Stage 1 only if relative price-earnings were less than 100.

Results:

First of all, the results of the test of the validity of the 5-stage hypothesis are summarized in Table 3 on the page following.

TABLE 3

Results of Testing the Validity of the Five-Stage Hypothesis

	% Frequency					
	R.P.E.			R.E.		
	Increased	Stable	Decreased	Increased	Stable	Decreased
Stage 1						
(a) Prior	78	4	18	17	12	71
(b) After	80	10	10	40	22	38
Stage 2						
(a) Prior	29	4	67	67	4	29
(b) After	96		4	36	28	36
Stage 3						
(a) Prior	15	30	55	67	12	21
(b) After	27	7	66	30	7	60
Stage 4						
(a) Prior	20	10	70	87	3	10
(b) After	13	4	83	56	11	33
Stage 5						
(a) Prior	57	3	40	60	13	27
(b) After	57	5	38	52	5	43

Stage 1 was most often preceded by increasing relative price earnings (R.P.E.) and decreasing relative earnings (R.E.). This tendency was very strong, as the figures indicate. After Stage 1, R.P.E. increased 80% of the time, while the R.E. movements indicated no strong trend in either direction. Stage 2 of the charts studied was most often preceded by declining R.P.E. and increasing R.E. Following Stage 2, 21 out of 22 cases showed an R.P.E. increase while R.E. was more or less evenly distributed between increasing, stable, and decreasing movements. Stage 3 was preceded by declining or stable R.P.E. 85% of the time. During this period R.E. was increasing with a frequency of 67%. After Stage 3 R.P.E. and R.E. decreased 66% and 60% of the time, respectively. Movements prior to Stage 4 indicate that declining R.P.E. and increasing R.E. were predominant. This was also the case for movements of R.P.E. and R.E. after Stage 4. Movements of R.P.E. prior to Stage 5 were roughly divided between increases and decreases, whereas R.E. either increased or remained stable 73% of the time. After Stage 5 had occurred, R.P.E. increases were made slightly more than half the time and R.E. movements were fairly evenly mixed between increases and decreases.

The theory of Markov chains was used to generate a transitional matrix. Thus, given a certain movement of R.E.

and R.P.E., it was possible to determine the probability of another stage following the given stage. These probabilities are summarized in Table 4.

TABLE 4

Transition Matrix

		To Stage				
		1	2	3	4	5
From Stage	1	.02	.05	.35	.50	.08
	2	.00	.00	.36	.59	.05
	3	.34	.00	.00	.31	.35
	4	.45	.00	.09	.09	.37
	5	.38	.00	.14	.43	.05

Source: These figures were calculated using the data from the sample charts.

Given Stage 1, Table 4 shows that there was a probability of .85 that either Stage 3 or 4 would follow. This would indicate that an investment at the end of Stage 1 would have a very good chance of outperforming the market, while the chance of substantial loss, i.e., Stage 5, was only 8%. Given Stage 2, the results are even more interesting since the probability of gain through either Stage 3 or 4 was .95. After the development of Stage 3 there was almost an even chance of either Stage 1, 4 or 5. Again, given Stage 4, the most likely outcome was either Stage 1 or 5. In Stage 5, the probability was .81 for either Stage 1 or 4. These figures indicate that given Stage 1 or 2, the probability of above average results was very high. However, an investment at the end of Stage 3 had a greatly reduced probability of substantial gain since the chance of Stage 4 was only .31. This probability was further reduced at the end of Stage 4, when the combined probability of either Stage 3 or 4 was only .18. As might be expected, an investment at the end of Stage 5 had a good chance of performing as well as or better than the market.

The results of the 2 tests are shown on the following page in Table 5. In calculating the results of the test, two different bases were used. First of all, the results were calculated on a per share basis. Thus whenever a buy signal was indicated, one share of that stock was purchased. The second basis of calculation was that of dollar commitments. In this case, when a buy signal was given, it was assumed that $1,000 worth of stock was purchased. In addition, as long as the stock was held, it was assumed that $1,000 was invested in the stock every January 1st. Also of note was the fact that all gains shown represent capital appreciation in the value of the stock plus stock dividends but exclude all cash dividends.

TABLE 5

Study No. 1 - Test Results 1952-1963

	Per Share		Dollar Commitment		
	Test #1	Test #2	Test #1	Test #2	Market
1952	5.7		11.2		11.7
1953	4.5		8.7		(0.3)
1954	93.2		41.2		43.8
1955	55.0	104.0	47.0	91.0	20.2
1956	28.2	19.3	36.0	36.1	3.6
1957	33.5	(11.1)	12.1	(7.4)	(12.2)
1958	84.2	93.2	69.0	58.9	34.0
1959	55.0	42.2	52.0	36.1	15.2
1960	12.6	36.6	14.0	42.7	10.0
1961	51.3	62.5	27.0	55.1	17.4
1962	(22.6)	22.0	35.0	30.1	(10.0)
1963	23.0	42.7	30.0	31.1	16.5
Average Annual Gain	35.3	45.7	31.9	41.5	12.5

Of the 27 stocks purchased at one time or another during the period 1952-1963, 22 or 85% of the purchases appreciated in value. Considering the test #1 results first, these show that the portfolio outperformed the market in 10 out of 12 years. Using the arithmetic mean of the annual results, the average gain per year was 35.3% while the market's average gain per year was only 12.5%. Calculated on a dollar commitment basis, the results show that the portfolio again outperformed the average in 10 out of 12 years but the mean gain was 31.9% or 3.4% less than the per share method.

The test #2 results show that both methods of calculation resulted in the portfolio outperforming the market in nine out of nine years. In both cases the average yearly gain was better under test #2 than under test #1. The per share basis showed a gain of 45.7% vs. 35.3% while the dollar commitment basis showed a 41.5% vs. a 31.9% gain.

<u>Conclusions:</u>

The results of the sequence validity test showed that a 1-3-5 series of movements was more likely. However, the point to be noted was that an investment made at the end of Stage 1 had a high probability of outperforming the market since a move into either Stage 2, 3 or 4 will usually result in above average returns. This type of analysis permitted an introduction of a qualitative judgment concerning risk since, once the stock had moved through Stages 1 to 4, the relative price earnings was quite high and as a result the stock price was vulnerable. It may be argued that the reason for this was that the market was discounting expected future earnings increases. Thus, if they did not materialize, the best that could be hoped for was an average market performance. On the other hand, the company's failure to achieve the expected earnings would likely result in a multiple contraction with its resulting capital losses. Similarly, we may say that an investment made at Stage 3 is subject to a higher risk than the same investment made at Stage 1. Good results may occur after Stage 5 because the relative price-earnings ratio usually increases. However, the risk still exists that this increase may be offset by a further decline in relative earnings.

On the basis of the results of tests #1 and #2, it was shown that relative value analysis was a useful technique in achieving an above average investment performance. It appeared that, regardless of the method of investing, i.e., either on a per share basis or a dollar commitment basis, or the level of the relative price earnings, this technique permitted the investor to outperform the market.

The study is, however, open to several major criticisms. These are the time period during which it was undertaken, possible biases in the sample and imprecise definition of variables.

In the first place, the time period of the study was one in which stock prices underwent a substantial re-evaluation by investors. Large amounts of funds were committed to the stock market by the public, not only individually but through other channels such as Mutual Funds and Pension Funds. As a result, the price-earnings ratio of the S. & P. Composite rose from 10.3 in 1953 to 22.7 in 1961 but by 1963 had declined to 17.8. Thus, one could argue that the success of the relative value technique was due in a large part to this multiple expansion.

Secondly, the test sample was biased in that it contained only firms which one investment house perceived to be growth firms. They were historically successful companies, which had been operating for many years and had a proven earnings record. Therefore it is quite possible that one could have achieved the same results by merely randomly selecting a portfolio from the group of 70 securities.

Thirdly, the slope of the relative earnings and relative price-earnings lines was determined visually and as a result it was not possible to increase the selectivity of the buy criteria and determine its impact.

Nye-Study No. 2

In order to further investigate the relationship between earnings and the p-e multiple, a second study was undertaken.

This study was conducted in an effort to determine the predictive significance of two variables in forecasting quarterly changes in security prices. For each of 35 companies the change in price of the stock at the end of quarter t + 1 was predicted at the end of the t th quarter. The change in price was measured in dollars and, if accurately predicted, then price in period t plus the change in price from t to t + 1 should give a reasonably good estimate of what the price of the security will be at the end of period t + 1.

This prediction of change in price involved forecasting the earnings per share of the company in question. This forecast was for the percentage change in earnings per share from period t to t + 1 and was made at the end of period t. This prediction thus implied an additional forecast--that of the earnings per share at the end of period t. For example, at September 30 the percentage change in earnings per share from the end of September to December 31 must be forecasted. This means that the earnings per share for the period ending September 30 must be known but, since the firm will not yet have published this data, an estimate must be made.

Relevant to this discussion is the work done by Green and Segall.[43] In their article they developed and tested six different models to forecast earnings per share. The six models were:

(a) Annual 1: $EPS_{t+1} = EPS_t$

(b) Annual 2: $EPS_{t+1} = EPS_t + (EPS_t - EPS_{t-1})$

(c) Annual 3: $EPS_{t+1} = EPS_t \pm \left(\frac{EPS_t - EPS_{t-1}}{EPS_{t-1}} \right)$

(d) Interim 1: $EPS_t = 4\ (1\text{st Quarter } EPS_t)$

(e) Interim 2: $EPS_{t+1} = EPS_t \pm \left(\frac{I\ Q.\ EPS_{t+1} - I\ Q.\ EPS_t}{I\ Q.\ EPS_t} \right) I\ Q.EPS_t$

(f) Interim 3: Regress I Q. EPS_t on previous five quarters.

They concluded that forecasts using first quarter interim reports are not clearly superior to those using only annual data. However, they stated that some knowledge (i.e., three months' earnings) is better than a twelve-month forecast. Also they found that, in companies with relatively large changes in earnings per share, the Interim 3 model provided the best results.

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[43] D. Green and J. Segall, "The Predictive Power of First Quarter Earnings Report," Journal of Business, vol. 40, No. 1 (January, 1967), pp. 44-55.

Hypothesis:

The hypothesis offered was that the change in price of common stocks is a function of two variables--the percentage change in earnings per share of the stock and the level of its relative price-earnings ratio.[1] Thus, for a given security:

$$_{i}P_{t+1} = f\left(\frac{_{i}EPS_{t+1} - {_{i}EPS_{t}}}{_{i}EPS_{t}} \; ; \; \frac{_{i}PE_{t}}{_{I}PE_{t}} \right)$$

Where: $_{i}P_{t+1}$ = the change in price of the i th security from period t to t+1

$\frac{_{i}EPS_{t+1} - {_{i}EPS_{t}}}{_{i}EPS_{t}}$ = the percentage change in earnings per share of the i th security from period t to t+1

$\frac{_{i}PE_{t}}{_{I}PE_{t}}$ = the price-earnings ratio of the security at the end of period t divided by the price-earnings ratio of the Dow-Jones Industrial Average at the end of period t.[2]

The data used was quarterly price and earnings per share as at March 31, June 30, September 30, and December 31 for the period 1960 to the second quarter of 1967, for a total of 30 observations for each of the 35 companies studied.[3] Similarly, data for the Dow-Jones was used for the same dates.[4]

To test the hypothesis, the computer was used to run a multiple regression on the data of each of the 35 companies.

Thus an equation of the form shown below was obtained for each company's stock.

$$ {}_iX_o t{+}1 \;=\; {}_iA + {}_ib_1\ {}_iX_1\ t{+}1 \;+\; {}_ib_2\ {}_iX_2\ t $$

Where:

${}_iX_o$ (t+1) = the dollar change in price from period t to t+1 of the i th security.

${}_iX_1$ (t+1) = the percentage change in earnings per share of the i th security from period t to t+1

${}_iX_2$ (t) = the level of the relative price-earnings ratio at period t.

${}_iA$ = a constant

${}_ib_i$ + ${}_ib_2$ = regression coefficients

The results of the regression are shown in Table 6.

TABLE 6

Results of Multiple Regression Analysis

Company	Constant	Regression Coefficients X_1	X_2	r^2
Abbot Laboratories	8.7	.45	-.06	.07
Allied Chemical Company	-7.8	.23	.07	.13
American Home Products	55.1	-1.7	-.3	.13
Armstrong Cork	6.2	.04	-.02	.003
Bristol Myers	- .66	.90	.004	.01
Celanese	-5.1	- .16	.13	.08
Chrysler Corporation	19.8	.32	-.27	.42
Cluett Peabody	22.6	.18	-.28	.24

TABLE 6 (continued)

Company	Constant	Regression Coefficients X_1	X_2	r^2
Corn Products	17.6	-.10	-.13	.10
Dupont	54.0	.81	-.41	.10
F.M.C. Corporation	30.0	.38	-.20	.15
General Cable	-7.0	-.03	.16	.19
General Motors	17.2	-.003	-.20	.06
Georgia Pacific	26.1	.19	-.23	.21
Honeywell Inc.	38.4	-.70	-.21	.13
Inland Steel	-105.7	-1.08	1.16	.02
International T. & T.	-2.82	-.39	.05	.03
Jones & Laughlin	-.17	-.06	.006	.06
Lockheed Aircraft	3.5	.16	-.04	.06
Magnavox	36.6	.08	-.27	.37
Monsanto Chemical	8.7	.56	-.09	.12
North American Aviation	3.47	.006	-.05	.02
Owen Illinois Glass	29.1	.63	-.27	.27
Pepsi	9.3	.12	-.06	.05
Polaroid	37.0	.21	-.10	.10
R.C.A.	19.6	.26	-.14	.14
R. J. Reynolds	9.0	.71	-.10	.09
Safeway	12.3	.29	-.14	.14
Smith, Kline & French	29.2	.80	-.21	.27
Standard Oil	4.8	.10	-.06	.04

TABLE 6 (continued)

Company	Constant	Regression Coefficients X_1	X_2	r^2
Sterling Drug	12.0	.44	-.08	.05
Textron	-2.5	-.02	.09	.12
U.S. Freight	37.0	.18	-.34	.28
Warner-Lambert	10.1	-.30	-.06	.06
Xerox	72.0	-.01	-.14	.17

In general, the results indicate that this model has no predictive value and therefore the hypothesis as it is presently formulated must be rejected.

Looking first at the correlation coefficients, no significant relationship was exhibited between P and X_1, P and X_2 or X_1 and X_2. The range of r for P and X_1 was -.25 to .59 with 15 negative signs and 20 positive signs. For P and X_2, the range was -.60 to .43 with 28 negative signs and 7 positive signs. Similarly, for X_1 and X_2, r ranged from -.39 to .72 with 15 negative signs and 20 positive signs. Thus, in some cases, the variables moved in opposite direction to each other while in the remaining cases they moved together in the same direction, but there was no consistency in the results. In addition, the highest r was .72 and almost all others were fairly close to zero, thus indicating that they varied randomly.

Looking at the r^2 values, the range was .003 to .42, indicating that the best "fit" explained only 42% of the total error.

Considering the F-ratios, at a 5% level of significance the value of F had to exceed 4.5. At values greater than 4.5 we can conclude that there is regression in the population and the improvement brought about by fitting the regression plane was not due to chance. The following list shows the frequency of F values greater than 4.5:

X_1 - 3 cases	r^2	X_2 - 5 cases	r^2
Chrysler	.42	Cluett-Peabody	.24
Owens-Illinois Glass	.27	General Cable	.19
Smith, Kline & French	.27	Georgia-Pacific	.21
		Magnavox	.37
		U.S. Freight	.28

Although the F-level did indicate an improvement, which was not due to chance, in a small number of cases the r^2 figures show that it was not a significant improvement.

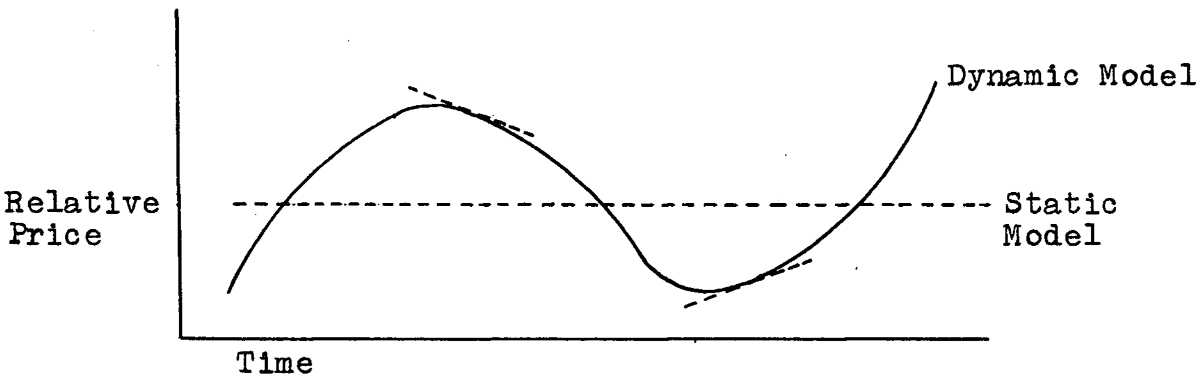
<u>Conclusion</u>:

As mentioned earlier, the results of testing the hypothesis show that it had no predictive value. In fact several cases resulted in the standard deviation of the predicted value being greater than the standard deviation of the mean.

Perhaps the reason the results compared so unfavourably with the results of Study No. 1 previously mentioned was that the former model is a static one whereas the latter is dynamic. Thus, because the variables have a wide movement over time, the "fit" of the line is not good. On the other hand, the relative value system appears to be able to permit the user to take advantage of these variations. Figure F illustrates this point.

FIGURE F

Relative Price - Dynamic and Static Models



Although the static regression model was not valuable as a predictive tool, these "stages" still occur and, if it is possible to develop a dynamic model, then the results might be improved.

CHAPTER IV

DEVELOPMENT OF THE MODEL AND THE TEST OF THE HYPOTHESIS

I. DEVELOPMENT OF THE MODEL

Description of the Test

As mentioned previously, the Study No. 2 results tended to support the relative value hypothesis. However, as a result of the earlier mentioned criticisms, the Study No. 1 test was first of all continued to the second quarter of 1967.

In addition, because of the possibility that the sample was biased in favour of successful companies whose shares were eagerly sought by investors, a new sample of firms was chosen. The test was then conducted from the second quarter of 1956 to the second quarter of 1967, providing a time span of more than ten years.

Selection of the Sample

The Standard & Poor's '500' index was chosen as representative of the market rather than the Dow-Jones Industrial Average in view of the recognized deficiencies of the latter.

In order that all selected portfolios could be compared with the market, only those stocks which were included in the

S. & P. index as at December 31, 1966 as listed in Standard & Poor's Trade and Securities Statistics, 1966 edition were eligible for inclusion in the sample.

Having thus defined the universe, a total of 50 firms was selected for the sample. The procedure was as follows: each stock in the Index was assigned a number from 001 to 500. A Random Number Table was then used to generate the sample. Once picked by a random number, the stock had to be listed on the N.Y.S.E. for the entire time period. In addition, it must have been included in Moody's Handbook of Common Stocks, Second Quarterly 1956 edition, and also the Third Quarterly 1967 edition. In addition, quarterly earnings figures had to be available. If the stock failed to meet any of these requirements it was excluded from the sample and another security was randomly chosen. On this basis the stocks of the following companies were included in the sample:

Abbot Laboratories, Inc.
Addressograph-Multigraph Corp.
Air Reduction Company, Inc.
Alpha Portland Cement Company
Aluminum Co. of America
Amerada Petroleum Corp.
American Airlines, Inc.
American Bakeries Company
American Potash & Chemical Corp.
American Smelting & Refining Co.
American Tobacco Company
Archer-Daniels-Midland Company
Beatrice Foods Company
Beckman Instruments, Inc.
Beneficial Finance Company
Bucyrus-Erie Company
Dupont (E.I.) De Nemours and Co.
Foremost Dairies
General Foods Corporation
General Instrument Corporation
General Portland Cement Company
Goodrich (B.F.) Company
Grant (W.T.) Company
Gulf Oil Corporation
Hudson's Bay Mining & Smelting Company, Ltd.
Keebler Company
Lockheed Aircraft Corporation
May Department Stores Company
Merck & Company, Inc.
Motorola, Incorporated
North American Aviation, Inc.

Burlington Industries, Inc.
Burroughs Corporation
Case (J.I.) Company
Chemetron Corporation
Colgate-Palmolive Company
Columbia Broadcasting System, Inc.
Consolidated Edison Company of New York, Inc.
Distillers Corporation-Seagrams, Ltd.
Dr. Pepper Company
Peabody Coal Company
Penn-Dixie Cement Corporation
Proctor & Gamble Company
Scovill Manufacturing Company
Sears, Roebuck and Company
Shell Oil Company
Superior Oil Company
Westinghouse Electric Corporation
Whirlpool Corporation
Wrigley (Wm.) Jr. Company

These firms comprise 24 industries with one or two firms from each industry. However, there is a concentration of Food Producing and Processing firms (8) and Petroleum companies (4). The sample includes firms in both cyclical industries (Building Materials) and relatively non-cyclical industries (Food). In addition, growth industries are represented (Office Equipment, Electronics) while stable or declining industries (Coal) are also a part of the sample.

Since the firm's stock had to be listed in both 1956 and 1967, the sample is biased towards firms which have been able to remain in business during that time. Thus the risk of a complete loss of capital through bankruptcy has been avoided. However, the sample does not contain only successful firms since the raw data shows that many of them, although they were able to maintain their listings, experienced deficit earnings. In some cases these deficits occurred as frequently as six out of the eleven years and their share prices suffered accordingly.

Data

Quarterly data for the period 1955 to the second quarter 1967 for the S. & P. Composite was obtained from Standard & Poor's Trade and Securities Statistics (1966 edition). The data included 12-month earnings for the quarters ending March 31, June 30, September 30 and December 30. The level of the Index was also obtained for corresponding periods, as was its price-earnings ratio.

Quarterly prices for each stock were obtained from Barron's and were checked from that source to ensure that no recording errors had been made. In addition, random checks were made using the Wall Street Journal to guard against possible printing errors. It was decided that the most representative price was the average of the Bid and Ask prices and consequently this was determined in each case and recorded. Although it is possible that either the Bid or Ask price would have been acceptable since most of the stocks were actively traded, some securities did exhibit a fairly wide spread and therefore it was decided to use the arithmetic mean. All prices were adjusted for stock splits and stock dividends.

Quarterly earnings data was obtained from various issues of Moody's Handbook of Common Stocks and was adjusted for stock splits and stock dividends. Eight of the companies had a fiscal year different from the calendar year and this presented certain data problems. The eight companies were:

Addressograph-Multigraph Corporation
Beatrice Foods Company
Case (J.I.) Company
Distillers Corporation-Seagrams, Ltd.
General Instrument Corporation
Grant (W.T.) Company
May Department Stores Company
Sears, Roebuck and Company

In the case where the firm's quarter ended on either January 31 or February 28, the data was compared to the Index's March 31 data. Thus an implied forecast of the Index and its earnings was made. It could be argued that a better method would have been to relate the data to the December 31 figures. However, the main objection to this is that it would have increased considerably the complexity of the computer programming while providing only doubtful returns.

Methodology

The first step was to define the variables. These were Relative Earnings, Relative Price and the Relative Price-Earnings Ratio. From the raw data the computer was instructed to calculate the three relative values for each quarterly time period. The logarithmic values of these relative figures were then determined and these values were then regressed against time on a four-quarter basis.

To repeat from Chapter I, let:

e_{it} = the 12-month earnings per share of the i th security at time period t

p_{it} = the price of the i th security at time period t

pe_{it} = the price-earnings ratio of the i th security at time period t

E_t = the 12-month earnings per share of the S. & P. Index at time period t

P_t = the price of the Index at the end of time period t

PE_t = the price-earnings ratio of the Index at time period t

Then:

$\frac{e_{it}}{E_t} = X_{it}$ = the 12-month relative earnings per share of the i th security at time period t

$\frac{p_{it}}{P_t} = Y_{it}$ = the relative price of the i th security at time period t

$\frac{pe_{it}}{PE_t} = Z_{it}$ = the relative price-earnings ratio of the i th security at time period t

For the time period $1 \leq t \leq 4$, log X_{it}, log X_{it+1}, log X_{it+2} and log X_{it+3} were regressed to obtain an equation of the form $X_i = a_i + \alpha_i T$ where α_i is the slope of the relative earnings line of the i th security for $1 \leq t \leq 4$ and T is time.

Similarly:

$Y_i = b_i + \beta_i T$ where β_i equals the slope of Y_i for $1 \leq t \leq 4$

$Z_i = c_i + \gamma_i T$ where γ_i equals the slope of Z_i for $1 \leq t \leq 4$

Values of α_i, β_i and γ_i were similarly computed for the time periods $2 \leq t \leq 5$, $3 \leq t \leq 6$, ..., $42 \leq t \leq 45$.

The buy and sell criteria were then defined so as to evaluate a total of 120 Strategies and to attempt to determine the optimum combination of buy and sell decisions in order to meet the two objectives of outperforming the market and maximizing return.

The criteria were as follows: As before, two tests were conducted--one where the level of Z_i was ignored and the other where $Z_i \leq 1.0$, i.e., the multiple of the i th security had to be equal to or less than the multiple of the S. & P. Index. The criteria are then:

Strategy No.	Buy if				Sell if
	α_i	β_i	γ_i	Z_i	β_i
1	$>.10$	>0	<0	$\leq 1.0 \leq$	$<-.05$
2	"	"	"	"	$<-.10$
3	"	"	"	"	$<-.15$
4	$>.15$	"	"	"	$<-.05$
5	"	"	"	"	$<-.10$
6	"	"	"	"	$<-.15$
7	$>.20$	"	"	"	$<-.05$
8	"	"	"	"	$<-.10$
9	"	"	"	"	$<-.15$
10	$>.25$	"	"	"	$<-.05$
11	"	"	"	"	$<-.10$
12	"	"	"	"	$<-.15$
13	$>.30$	"	"	"	$<-.05$
14	"	"	"	"	$<-.10$
15	"	"	"	"	$<-.15$
16	$>.10$	"	"	<1.0	$<-.05$
17	"	"	"	"	$<-.10$
18	"	"	"	"	$<-.15$
19	$>.15$	"	"	"	$<-.05$
20	"	"	"	"	$<-.10$
21	"	"	"	"	$<-.15$
22	$>.20$	"	"	"	$<-.05$
23	"	"	"	"	$<-.10$
24	"	"	"	"	$<-.15$
25	$>.25$	"	"	"	$<-.05$
26	"	"	"	"	$<-.10$
27	"	"	"	"	$<-.15$
28	$>.30$	"	"	"	$<-.05$
29	"	"	"	"	$<-.10$
30	"	"	"	"	$<-.15$

The remaining 30 Strategies were formulated in a similar manner except for a data time lag, the reason for which will be discussed first. As the data is presently formulated, Firm data for time period t is compared with Index data for time period t. However, this involves the use of hindsight. For example, on September 30, 1966 the latest earnings which would be available would be those for the period ending June 30, 1966. Thus the variables were re-defined to account for this time lag.

Let:

$X_{it} = \dfrac{e_{it-1}}{E_{t-1}}$ = the 12-month relative earnings per share of the i th security at time period t

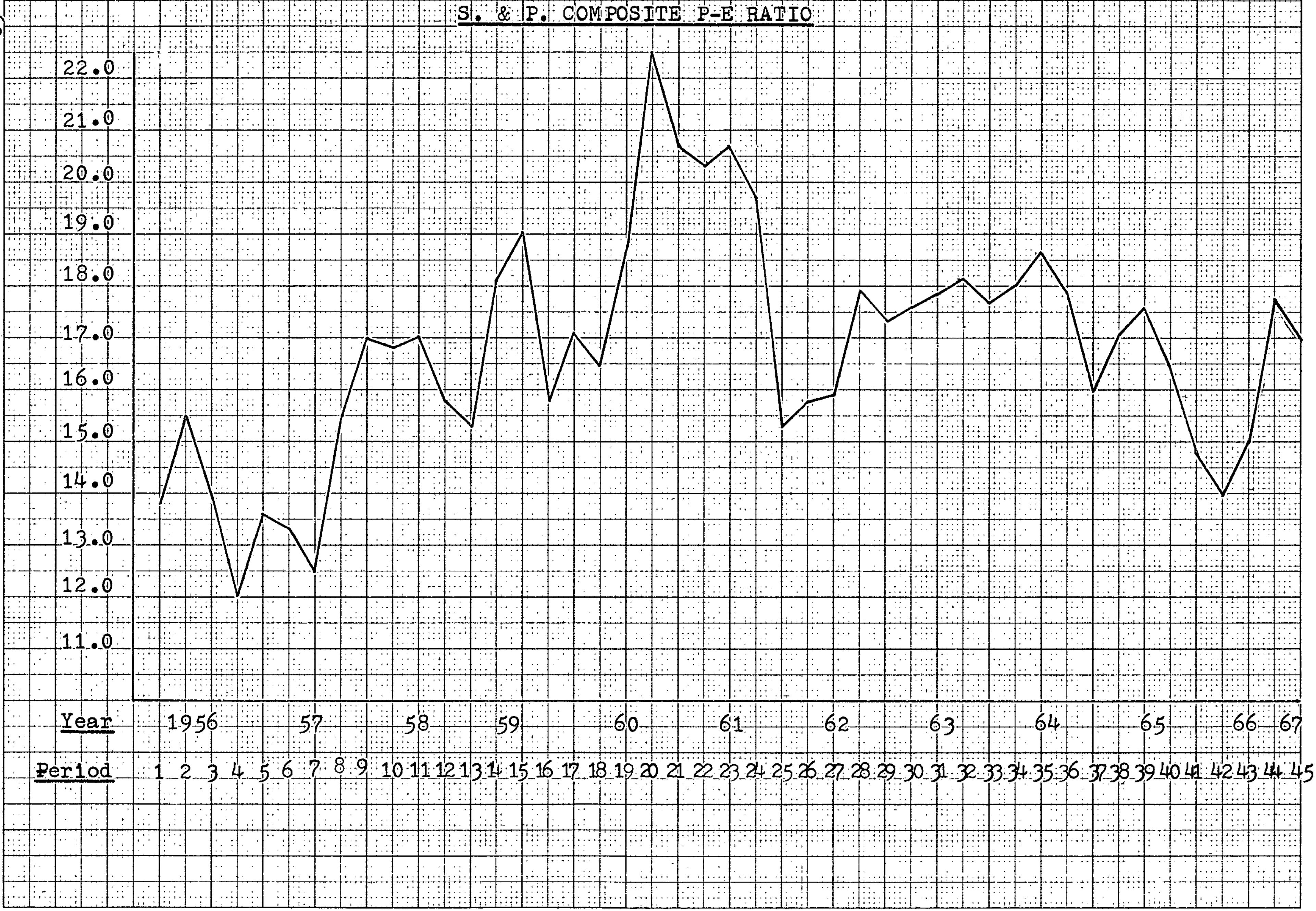
$Y_{it} = \dfrac{p_{it}}{P_t}$ = the relative price of the i th security at time period t

$Z_{it} = \dfrac{\dfrac{p_{it}}{e_{it-1}}}{\dfrac{P_t}{E_{t-1}}}$ = the relative price-earnings ratio of the i th security at the end of time period t

With the variables re-defined as above, the same combinations of criteria were applied and these additional 30 Strategies may be considered as more realistic than the previous 30.

For the next series of tests the time period was shortened from 1956, second quarter--1967, second quarter, to 1958, second quarter--1967, second quarter, and reference to Figure G will indicate the reason why.

FIGURE G


S. & P. COMPOSITE P-E RATIO
22.0
21.0
20.0
19.0
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
Year
1956 57 58 59 60 61 62 63 64 65 66 67
Period
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

One of the criticisms of the previous study was that it was conducted during a period of generally rising p-e ratios. Under such circumstances favourable results might be expected, regardless of the ability of the model to select superior securities. The present tests have been conducted during a time when the S. & P. Composite multiple rose from 13.77 to 17.01. An analysis of the computer output for the initial series of tests (see Table 7) shows that, on average, 43% of all purchases were made in the first 20% of the total test period. Therefore, it was decided to conduct the test during a period when the beginning and ending p-e ratio was the same.

When a buy signal occurred, the computer was instructed to purchase $1,000 worth of the security and to hold it until a sell signal occurred.

TABLE 7

Percent of Total Number of Purchases
Executed During Initial 20% of Sample Time Period

Strategy No.	%
1	41
2	46
3	53
4	36
5	36
6	38
7	48
8	48
9	48
10	45
11	45
12	45
13	40
14	40
15	40
16	41
17	45
18	51
19	34
20	34
21	37
22	45
23	45
24	45
25	45
26	45
27	45
28	40
29	40
30	40

II. RESULTS OF THE TEST

Table 8 shows the results of updating the original test to the second quarter of 1967. Test #1 again supports the relative value method since, in each year, the technique outperformed the market quite substantially and the average annual gain was greatly superior to that of the market.

The average annual gain of test #2 was slightly lower than that of the market, even though the technique outperformed the market in two of the four time periods. These results were not superior and do not lend support to the relative value technique.

TABLE 8

Summary of Results of Study #1
(1964-1967, 2nd Quarter)

	TEST #1		TEST #2		MARKET	
	Qtly.Avge. %	Annual %	Qtly.Avge. %	Annual %	Qtly.Avge. %	Annual %
1964	1.6	6.6	(1.14)	(4.48)	0.10	0.4
1965	8.7	39.6	3.2	13.4	1.7	7.0
1966	(0.11)	(0.52)	(5.96)	(21.79)	(4.79)	(17.83)
1967*	11.5	24.3	8.2	17.1	5.1	10.4
Average Annual Change		16.5		0.15		0.5

* First two quarters only.

In an earlier chapter the method of calculating the results of the first study was indicated and one point should be emphasized. By assuming that $1,000 was invested in the security every January 1, the entire portfolio was sold on December 31 and repurchased the following day--an obviously absurd assumption. In addition, the results were based on a portfolio which at no time contained a cash reserve. Thus the results were attainable only by assuming a continuous 100% commitment in stocks.

When building the present model, the arbitrary nature of the previous assumptions was kept in mind and avoided. But, as is true in any situation, the elimination of some problems creates new ones. From an overall point of view, however, the present test is more realistic: the portfolio was not turned over at year end; and provision was made for determining cash balances at any point in time.

Ideally, the results should be measured in such a way that a meaningful indicator of performance is used and, at the same time, some measure of risk is indicated.

The computer output for the first series of tests listed the amount of external funds required for purchases at any particular time period, the amount of cash on hand, and the market value of securities held. The cash account was viewed as a Current Account and, as such, was not considered to earn Interest. As will be shown later, this had a detrimental effect on results since, in certain cases, substantial cash balances resulted from the sale of securities and the unavailability of additional investment opportunities.

Since the prime objective of this test is to outperform the market through appreciation in the value of the portfolio, dividends were not included in calculating the results. In addition, since dividends are not considered in measuring the performance of the Index, the comparability of results is enhanced.

In attempting to arrive at a meaningful measure of performance in the current study, the method used in Study No. 1 was discarded for reasons mentioned previously. Next, a form of price index was considered. It was hoped that a base-weighted aggregative index, such as that developed by Paasche, would be useful. His modified formula was of the form:

$$\text{Index} = \frac{p_1 \; Q_1}{p_0 \; Q_0} \times 10$$

Where:

p_1 = the current market price of the security

Q_1 = the number of shares currently outstanding of the security

p_0 = the average price in the base period of the security

Q_0 = the number of shares outstanding of the security in the base period

An index of this type would have yielded a measure of performance and, in addition, could have been regressed against time in order to measure the variance and thus obtain a measure of risk. Upon study, however, it became apparent that the index would not have resulted in a meaningful measure of performance. The following example will explain why. Consider a portfolio which was started in time period t. During the next five periods various securities were purchased and sold. In period t + 6 security x is purchased, and at time period t + 7 we wish to measure the performance of the portfolio from period t + 6 to t + 7. By using the index formula the portfolio performance for t + 6 to t + 7 would be influenced by the price change of security x from period t to t + 7 and is therefore an unsatisfactory measure.

The measure finally decided upon yields a figure which reflects realistically the result of each strategy. The approach taken is that of an investment project, i.e., for an amount of $x required in time period t = 1, the Strategy yielded an amount of $y in time period t = 45. By using the ratio $\frac{y}{x} = p$, one is able to determine the compound rate of interest for which $1 in t = 1 is equal to $p in t = 45.

The amount of $x for each strategy was determined by discounting the funds required in period t over and above the cash on hand in period t. Thus in each strategy the investor's beginning equity was $0 but through trading he was able to generate cash for his subsequent investment opportunities. Any additional funds required were assumed to be available at no out-of-pocket cost, e.g., Savings Account deposits, estate funds or "rich benevolent Uncles."

Table 10 (see Appendix II) shows the results of this first effort.

The first point to note is that the system achieved an annual growth rate greater than that of the market, with the exception of Strategy 13. In addition, the growth rate of the buy and hold strategy was less than those of all the other strategies (again with the exception of Strategy 13).

In terms of maximizing returns, Strategy 3 yielded the largest dollar amount ($54,253) while Strategy 5 yielded the highest growth rate (12.018%).

Studying the strategies themselves, there is one result which was to be expected and another rather unexpected one. Considering the former first, as the selling criteria (β_i) was varied from $< -.05$ to $< -.10$, the results, as measured by the growth rate (Column D) and $ returns (Column A minus Column B) improve. The reason for this is that short-term declines from an upward sloping trend line caused the stocks to be sold (in the $< -.05$ case) with two effects; (a) an above average security was eliminated from the portfolio, and (b) commission costs were unduly large. When β_1 was changed to $<-.10$, these two effects were eliminated and the results improved, except for Strategies 16 and 17.

By reducing β to $<-.15$, the predominant effect was that of an improved growth rate. For values of α of $>.10$ and $>.15$ and Z at any level, $\beta<-.10$ was the dominant strategy. However, for all other combinations the compound annual percentage increase was larger and, in most cases, the difference was not negligible. Graphs A-D clearly show the effect.

Considering next the results obtained by varying α_1, it was found that the optimum slope was $>.15$. Improved results were obtained when α_1 was increased from $>.10$ to $>.15$. However, at values of $>.20$, $>.25$ and $>.30$, the results were poorer. It is suggested that this was due to firms being selected whose earnings experienced wide variations, such as cyclical companies. As a result, their price performance dominated that of the one or two firms whose R.E. was growing at a similar substantial rate but were of better quality, i.e., were more consistent. In addition, the wider price fluctuations meant that the security was sold more frequently than the $>.10$ and $>.15$ cases. Thus greater amounts of cash were held (see Column E) and this adversely affected performance.

It will be remembered that for Strategies 16-30 an additional constraint of R.P.E. less than 1.0 was imposed. Comparing Strategies 1-9 with Strategies 16-24 shows that the growth rate for the latter was greater than the former in six out of nine cases. For the remaining strategies there was no difference. This is a particularly interesting result in the light of returns obtained in Study No. 1. Based on this, one would have expected Strategies 16-30 to exhibit a clear superiority but such was not the case. The previous study's sample was comprised of <u>well researched</u> firms which hindsight had shown to be highly successful and which Eastman-Dillon considered would continue to be successful. On the other hand, the present sample was randomly chosen from many industries. As a result the former was biased in favour of superior firms and the results appear to have been inflated as a result of this bias. Evidence for this suggestion is contained in Table 9, which shows the results of the original study calculated on the same basis as the present study. The results are substantially better than any of the results from this study and present exciting implications regarding the role of Fundamental Analysis.

TABLE 9

Compound Growth Rate of the Original Study on a Dollar Commitment Basis

	TEST 1		TEST 2		DOW-JONES	
Year	$Gain	Portfolio Value*	$Gain	Portfolio Value	$Gain	Value
1952	112	1,112	-	-	117	1,117
1953	87	1,199	-	-	(3)	1,114
1954	412	1,611	-	-	438	1,552
1955	470	2,081	910	1,910	202	1,754
1956	360	2,441	361	2,271	36	1,740
1957	121	2,562	(74)	2,197	(122)	1,668
1958	690	3,252	589	2,786	340	2,008
1959	520	3,772	361	3,147	152	2,160
1960	140	3,912	427	3,574	100	2,260
1961	270	4,182	551	4,125	174	2,434
1962	350	4,532	301	4,426	(100)	2,334
1963	300	4,832	311	4,737	165	2,499
Compound Growth Rate**	25.32%		30.32%		7.84%	

* Based on $1,000 original investment and $1,000 invested every January 1.

** Compounded annually.

Table 11 summarizes the results obtained when the test was conducted during a time period when the market's beginning and ending multiple was the same. The results of Table 10 may be influenced by the fact that the S. & P. Composite multiple increased from 1956 to 1967. As the reader will remember, this was a major criticism of my first study and its validity had to be investigated.

The results vindicate the earlier findings and dissolve the multiple expansion argument. There are several interesting points to note in these results. Firstly, the technique outperformed the market (except for Strategies 31, 34, 37, 40 and 43) even though the beginning and ending multiple of the market was the same. For those Strategies which failed to outperform the market, the sell criterion ($\beta < -.05$) had a similar but more dramatic impact on the growth rate mentioned earlier for Table 10.

Considering the results in the first half of Table 11, those obtained when α_i was greater than .15 dominated the cases of α_i $>.10$, $>.20$, $>.25$ and $>.30$. As will be shown later (see Tables 12 and 13), this Strategy was dominant in every case.

As in the case of Table 10, the results of the second half of Table 11 were somewhat disappointing in that only 7 out of the 15 strategies showed higher growth rates than their twin strategy in the first half. Also, no patterns or trends appeared which could be considered particularly significant.

Tables 12 and 13 contain the results of lagging the data. As discussed earlier, it was conducted in order to assess the impact on performance if the investor applies this technique under actual conditions.

The results are extremely interesting in that, in many cases, the investor was able to maintain his above average performance, while in cases where the growth rate was lower (10 cases out of 15), only one strategy exhibited a decline of greater than 1% in the annual growth rate.

In the case of Table 12, 17 out of 30 strategies showed no variation in the annual growth rate while the results reported in Table 13 showed an increase in this figure to 24. Of the six which did change, four of these showed an increase in the annual growth rate while only two declined.

As before, the growth rate was maximized when α_1 was $>.15$ for the first half of Table 12. These strategies were also dominant for the constant multiple case, as shown in the first half of Table 13.

When the additional constraint of relative price-earnings, being less than 1.0, was added, the results in Tables 12 and 13 did not vary from those shown in Tables 10 and 11, respectively.

CHAPTER V

CONCLUSIONS

Preliminary Note and Suggestions for Further Study

In the results reported in Chapter IV an unexpected discovery was made. Earlier in the study it was suggested that, if the hypothesis was accepted, then the role of fundamental analysis would be open to question. Although the hypothesis was accepted, it is concluded that fundamental analysis has a <u>greater</u> rather than a <u>lesser</u> role since the rates of appreciation of the portfolios chosen from the analyzed sample were substantially greater than those of the random sample. Proceeding from this fact, the next step would be to analyze, say, a group of 50 stocks. These firms would be those which, in the opinion of the analyst, have expanding opportunities for investment.

A second sample of 50 stocks would be randomly chosen, as was done in the present study. The buy and sell criteria as formulated would then be used to construct portfolios from the two samples and it is expected that portfolios from sample one would outperform those of sample two and also the market.

It should be noted that the test could not be conducted using historical data since sample one may be constructed with the benefit of hindsight. Thus, the starting period would be at the time the analyst makes his forecast and would continue into the future for as long as is desired.

Since the area of risk was not incorporated in the model, a further study along the following lines might be undertaken.

Firstly, divide the slope of R.E. and R.P.E. into the following classes -

R.E. Slope (α_i)	R.P.E. Slope (γ_i)
0 -- .04999	(.04999) -- 0
.05 -- .09999	(.09999) -- (.05)
.10 -- .14999	(.14999) -- (.10)
.15 -- .19999	(.19999) -- (.15)
.20 -- .24999	(.24999) -- (.20)
.25 -- .29999	(.29999) -- (.25)
$>$ .30	$<$ (.30000)

Using the buy criteria defined earlier in the model, portfolios would be constructed from all possible combinations of the above classes. In addition, all possible time periods, i.e., starting and ending dates, would be considered. The one major variation, however, would be that the structure of the portfolio would not change over time. Thus, once the portfolio had been selected, its change in value could be compared with that of a market index over time and both performance and risk evaluated.

GENERAL CONCLUSION

The results of this research vindicate the conclusion reported in Study #1 that relative value analysis is a profitable stock selection technique.

Specific Conclusions

As indicated in Chapter I, the purpose of the study was:

(1) To investigate the hypothesis that relative value analysis enables the investor to make buy and sell decisions which permit him to attain his objective of outperforming the market.

(2) To offer support to the "trendist" school, which supports the idea that technical analysis of stock price data is a profitable technique.

(3) To answer criticisms of my previous study.

Considering the third purpose, the results reported in Chapter IV indicate that these criticisms may have been justified. The compound growth rate of the portfolios, although lower than that reported for Study No. 1, was nevertheless higher than that of the market. However, to what extent the lower growth rates are the result of the sample being random rather than the lack of a general multiple expansion is not known. But, considering the results when $t = 45$ (when there was an overall multiple expansion) with the results of Study No. 1, one is led to suspect that the non-randomness of the Study No. 1 sample inflated the results more than the multiple expansion.

The second conclusion of this study is that stock price trends do exist and therefore, as Levy found, it is possible to profitably exploit these trends.

Based on conclusion (2) is the third conclusion: that the stock market analyst who uses relative value analysis is able to detect and exploit these trends; and that relative value analysis renders acceptable the theory that the investor can make buy and sell decisions which result in the selected portfolio outperforming the market.

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APPENDIX I

LISTING REQUIREMENTS

<u>New York Stock Exchange</u>

a) Net earnings after taxes must equal at least \$1,000,000 annually over a three-year period.

b) Net tangible assets must be at least \$10,000,000.

c) There must be at least 500,000 shares outstanding distributed among 1,500 stockholders, each of which must hold at least 100 shares.

<u>American Stock Exchange</u>

a) Net earnings after taxes must be at least \$150,000 for the past fiscal year and average at least \$100,000 for the past three years.

b) Net tangible assets must be at least \$1,000,000.

c) There must be at least 200,000 shares outstanding distributed among 750 shareholders, of whom at least 500 must each hold 100 shares or more. Also, the stock must have an aggregate market value of \$2,000,000 outstanding and \$1,000,000 of publicly held shares.

<u>Midwest Stock Exchange</u>

a) The company must have an ability to show net earnings of at least \$100,000.

b) Net tangible assets must be at least \$2,000,000.

c) There must be at least 250,000 shares outstanding distributed among 1,000 shareholders.

<u>Pacific Coast Exchange</u>

a) The company must have demonstrated earning power of $100,000 annually

OR

b) Total assets of at least $1,000,000.

c) At least 250,000 shares must be outstanding, excluding family or concentrated holdings, distributed among 750 shareholders.

Source: Cooke, Gilbert W., <u>The Stock Market</u>, Simmons-Boardman Publishing Corporation, New York, 1964, pp. 214-215.

APPENDIX II

TABLES 10 - 13

TABLE 10

RESULTS OF STRATEGIES WITH NO DATA LAG AND t = 45

Strategy Number	Ending Value of Portfolio ($)* A	Funds Required ($)** B	Profit Ratio $C = \frac{A}{B}$	Annual Growth Rate(%)*** D	Average Cash Balance (% of A) E
1	$ 39,120	$ 15,138	2.5843	8.52	32.9
2	60,416	19,400	3.1143	10.22	7.2
3	81,397	27.144	2.9987	9.88	2.0
4	24,133	7,089	3.4046	11.06	30.9
5	44,122	11,415	3.8654	12.02	8.1
6	45,517	13,610	3.6827	11.74	3.5
7	13,127	5,304	2.4749	8.13	43.9
8	23,878	8,372	2.8522	9.42	16.6
9	28,361	9,767	2.9037	9.58	8.1
10	5,707	2,739	2.0833	9.56	43.9
11	12,990	4,400	2.9524	9.73	18.2
12	16,913	5,825	2.9034	9.67	6.3
13	3,598	1,884	1.9092	5.79	30.4
14	10,427	3,503	2.9253	9.64	22.3
15	14,123	4,912	2.8751	9.48	7.3

* Less commission of 1% on purchases and sales.

** Discounted at a rate of 6%

*** Compounded quarterly.

TABLE 10 (continued)

Strategy Number	Ending Value of Portfolio ($) A	Funds Required ($) B	Profit Ratio C	Annual Growth Rate(%) D	Average Cash Balance E
16	$ 22,229	$ 8,515	2.611	8.60	34.5
17	28,930	10,762	2.688	8.88	12.6
18	31,673	14,541	2.178	8.98	5.6
19	10,619	2,912	3.647	11.66	60.2
20	14,732	4,064	3.625	11.60	16.2
21	18,022	5,070	3.555	11.42	12.5
22	6,838	2,898	2.360	7.70	63.4
23	7,614	2,898	2.627	8.66	46.0
24	9,363	2,898	3.231	10.55	31.3
25	1,933	942	2.052	6.43	77.1
26	3,549	942	3.767	11.96	47.2
27	4,803	942	5.098	14.70	41.7
28	1,920	942	2.038	6.37	78.2
29	3,688	942	3.915	12.29	50.3
30	4,754	942	5.046	14.25	43.0

Standard & Poor's Composite 6.53%

Buy and Hold, i.e., $1,000 worth each
of 50 stocks 7.85%

TABLE 11

RESULTS OF STRATEGIES WITH NO DATA LAG AND t = 37

Strategy Number	Ending Value of Portfolio ($)* A	Funds Required ($)** B	Profit Ratio C	Annual Growth Rate(%)*** D	Average Cash Balance (% of A) E
31	$ 23,706	$ 13,187	1.798	6.38	22.9
32	47,108	16,484	2.878	11.50	7.2
33	63,384	19,818	3.198	12.72	3.6
34	15,739	6,795	2.316	11.78	21.9
35	33,653	9,773	3.443	13.56	9.7
36	37,870	10,967	3.453	13.60	6.2
37	9,028	5,803	1.556	4.80	40.9
38	19,832	6,558	3.024	12.46	19.0
39	22,595	7,009	3.224	12.86	12.8
40	4,353	3,954	1.100	1.01	42.5
41	12,419	4,661	2.664	10.73	15.3
42	15,108	5,346	2.826	11.38	5.9
43	3,275	2,923	1.120	1.43	43.5
44	9,928	3,662	2.711	10.93	18.1
45	12,611	4,347	2.901	11.67	7.3

* See Table 10.

** " " "

*** " " "

TABLE 11 (continued)

Strategy Number	Ending Value of Portfolio ($) A	Funds Required ($) B	Profit Ratio C	Annual Growth Rate(%) D	Average Cash Balance E
46	$ 13,933	$ 8,535	1.632	5.32	25.2
47	24,075	10,161	2.369	9.46	11.1
48	35,519	12,763	2.783	11.21	3.2
49	6,354	2,877	2.209	8.65	24.8
50	13,745	3,968	3.464	13.64	1.1
51	15,516	3,968	3.910	14.49	9.5
52	3,560	1,900	1.874	6.84	48.3
53	5,945	1,900	3.129	12.53	3.2
54	7,371	1,900	3.879	14.88	2.7
55	730	1,175	.621	-	-
56	3,307	1,615	2.048	7.82	1.1
57	4,734	1,615	2.931	11.78	2.0
58	730	1,155	.632	-	-
59	3,267	1,508	2.166	8.43	10.6
60	4,687	1,508	3.108	12.42	2.0

Standard & Poor's Composite 6.98%

Buy and Hold 10.52%

TABLE 12

RESULTS OF STRATEGIES WITH DATA LAG AND t = 45

Strategy Number	Ending Value of Portfolio ($)*	Funds Required ($)**	Profit Ratio	Annual Growth Rate(%)***	Average Cash Balance (% of A)
	A	B	C	D	E
61	$ 39,035	$ 15,239	2.526	8.31	32.8
62	60,082	21,070	2.852	9.41	6.6
63	81,107	27,076	2.996	9.88	1.8
64	24,048	7,182	3.348	10.88	33.3
65	44,024	11,499	3.829	13.02	6.9
66	49,831	13,552	3.677	11.73	3.4
67	13,127	5,304	2.475	8.13	43.9
68	23,644	8,318	2.843	9.38	15.3
69	28,070	9,486	2.959	9.08	7.6
70	5,546	2,698	2.056	6.44	41.0
71	12,756	4,346	2.9351	9.61	15.6
72	16,623	5,770	2.881	11.01	4.7
73	3,386	1,680	2.016	6.27	40.7
74	10,014	3,431	2.919	9.62	18.7
75	13,833	4,841	2.857	9.43	5.0

* See Table 10.

** " " "

*** " " "

TABLE 12 (continued)

Strategy Number	Ending Value of Portfolio ($) A	Funds Required ($) B	Profit Ratio C	Annual Growth Rate(%) D	Average Cash Balance (% of A) E
76	$ 22,229	$ 8,515	2.611	8.60	34.5
77	28,930	10,762	2.688	8.88	12.6
78	31,673	14,541	2.178	8.98	5.6
79	10,619	2,912	3.647	11.66	60.2
80	14,732	4,064	3.625	11.60	16.2
81	18,022	5,070	3.555	11.42	12.5
82	6,838	2,898	2.360	7.70	63.4
83	7,614	2,898	2.627	8.66	46.0
84	9,363	2,898	3.231	10.55	31.3
85	1,933	942	2.052	6.43	77.1
86	3,549	942	3.767	11.96	47.2
87	4,803	942	5.098	14.70	41.7
88	1,920	942	2.038	6.37	78.2
89	3,688	942	3.915	12.29	50.3
90	4,754	942	5.046	14.25	43.0

TABLE 13

RESULTS OF STRATEGIES WITH DATA LAG AND t = 37

Strategy Number	Ending Value of Portfolio ($)* A	Funds Required ($)** B	Profit Ratio C	Annual Growth Rate(%)*** D	Average Cash Balance (% of A) E
91	$ 23,624	$ 13,187	1.792	6.34	23.0
92	47,006	16,354	2.874	11.57	7.2
93	63,282	19,701	3.212	12.78	3.6
94	15,657	6,794	2.304	9.12	22.1
95	33,552	9,713	3.454	13.60	9.8
96	37,768	10,822	3.490	13.72	6.2
97	9,028	5,803	1.556	4.80	
98	19,832	6,558	3.024	12.46	19.0
99	22,595	7,009	3.224	12.86	12.8
100	4,353	3,954	1.100	1.01	42.5
101	12,419	4,661	2.664	10.73	15.3
102	15,108	5,346	2.826	11.38	5.9
103	3,275	2,923	1.120	1.43	43.5
104	9,928	3,662	2.711	10.93	18.1
105	12,611	4,347	2.901	11.67	7.3

* See Table 10.

** " " "

*** " " "

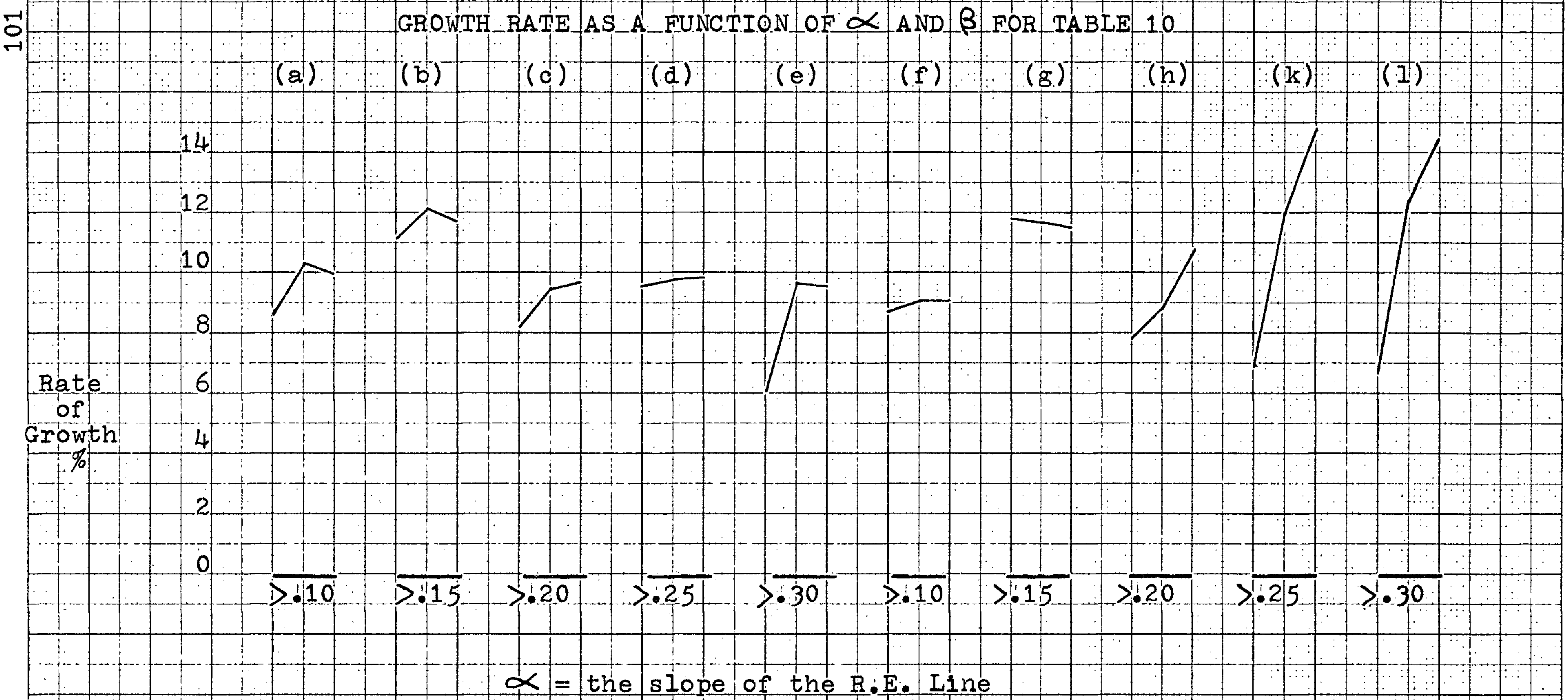
TABLE 13 (continued)

Strategy Number	Ending Value of Portfolio ($) A	Funds Required ($) B	Profit Ratio C	Annual Growth Rate(%) D	Average Cash Balance (% of A) E
106	$ 13,933	$ 8,535	1.632	5.32	25.2
107	24,075	10,161	2.369	9.46	11.1
108	35,519	12,763	2.783	11.21	3.2
109	6,354	2,877	2.209	8.65	24.8
110	13,745	3,968	3.464	13.64	1.1
111	15,516	3,968	3.910	14.49	9.5
112	3,560	1,900	1.874	6.84	48.3
113	5,945	1,900	3.129	12.53	3.2
114	7,371	1,900	3.879	14.88	2.7
115	730	1,175	.621	-	-
116	3,307	1,615	2.048	7.82	1.1
117	4,734	1,615	2.931	11.78	2.0
118	730	1,155	.632	-	-
119	3,267	1,508	2.166	8.43	10.6
120	4,687	1,508	3.108	12.42	2.0

APPENDIX III

GRAPHS A - D

GRAPH A

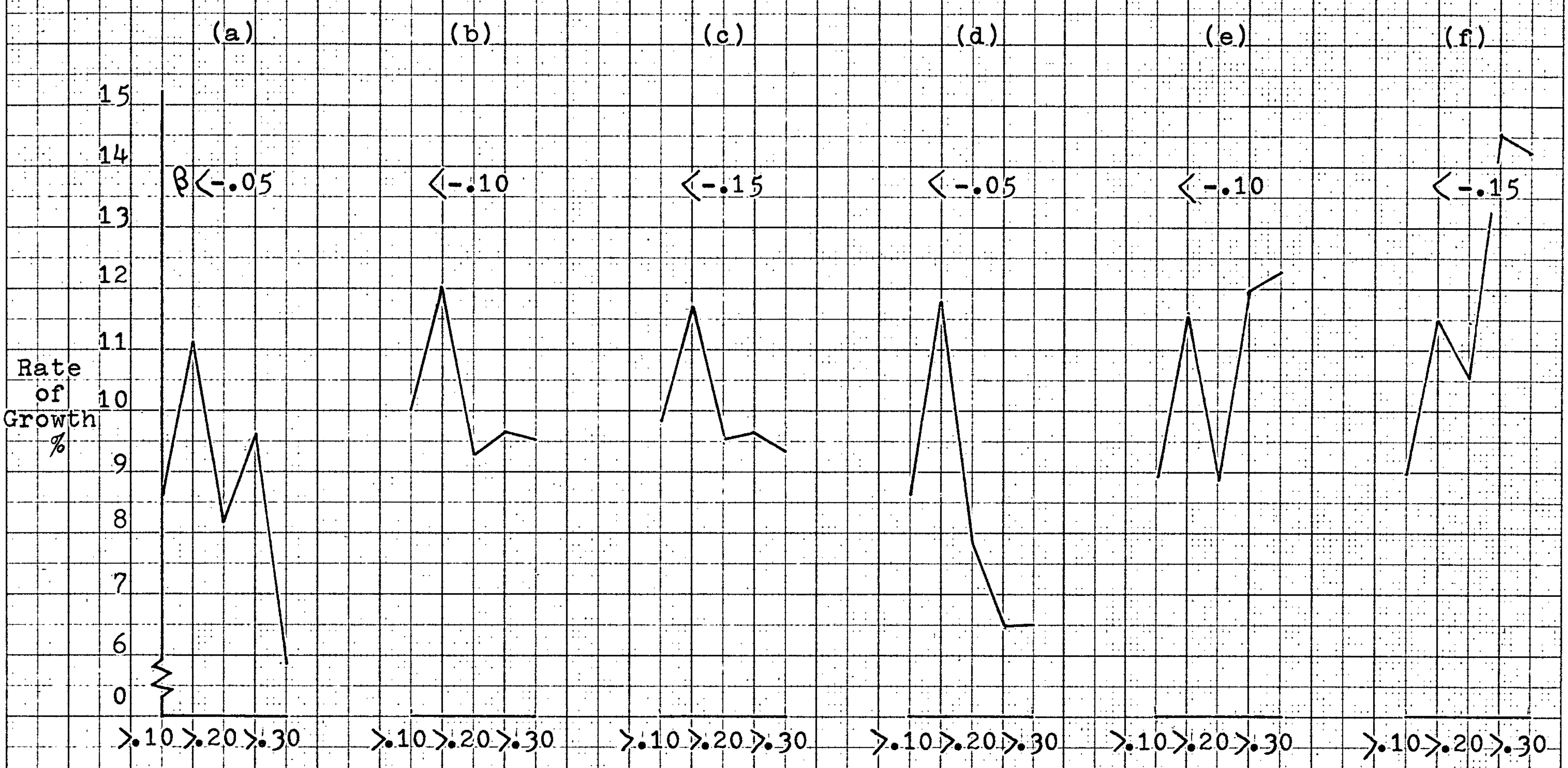


GRAPH A and subsequent graphs are read as follows:

Case (a): The 1st point represents the rate of growth when $\alpha > .10$ and $\beta < -.05$
" 2nd " " " " " " " " $< -.10$
" 3rd " " " " " " " " $< -.15$

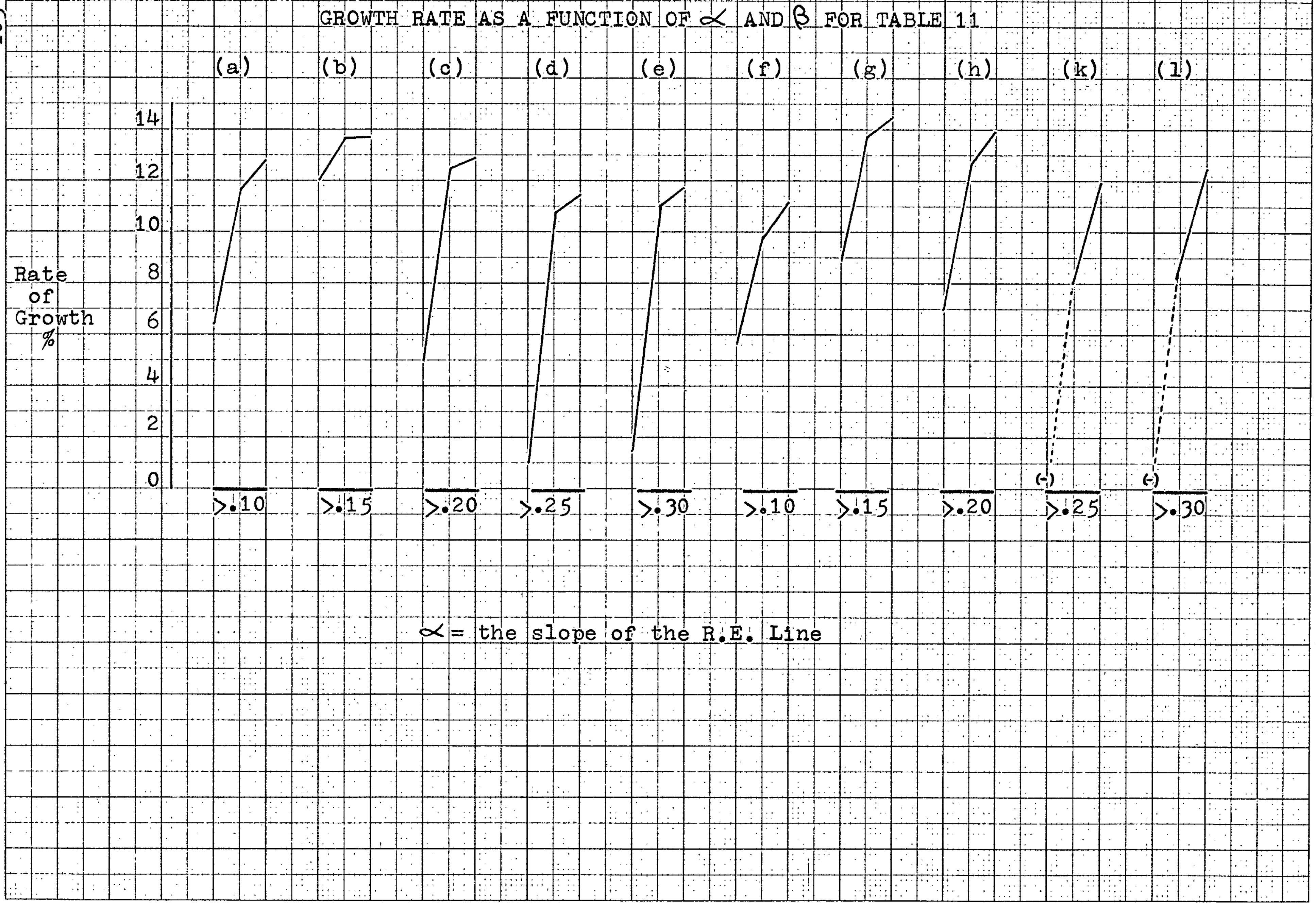
Similarly for Case (b), ..., (l).

GRAPH A (continued)

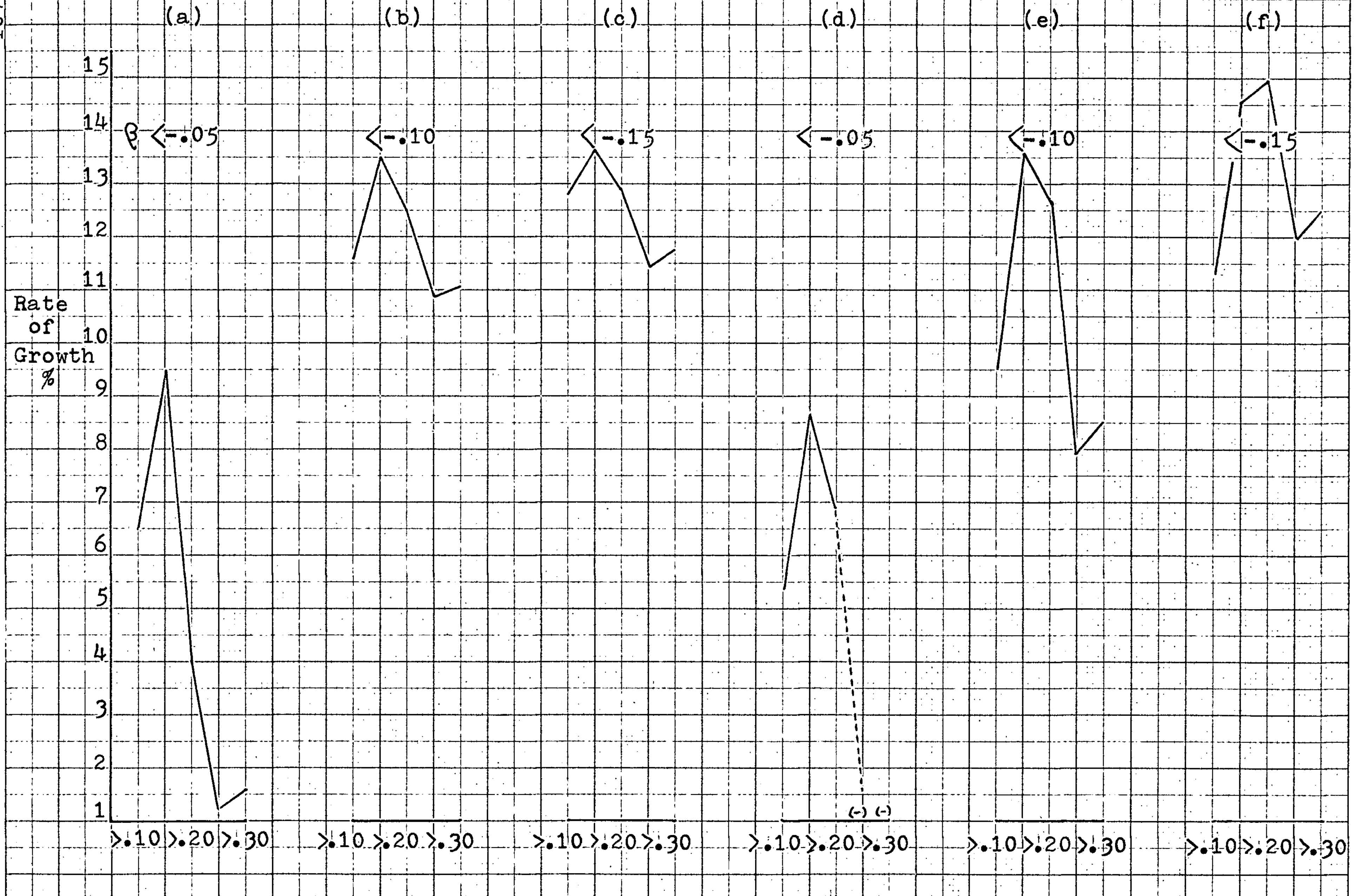


α = the slope of the R.E. Line

GRAPH B

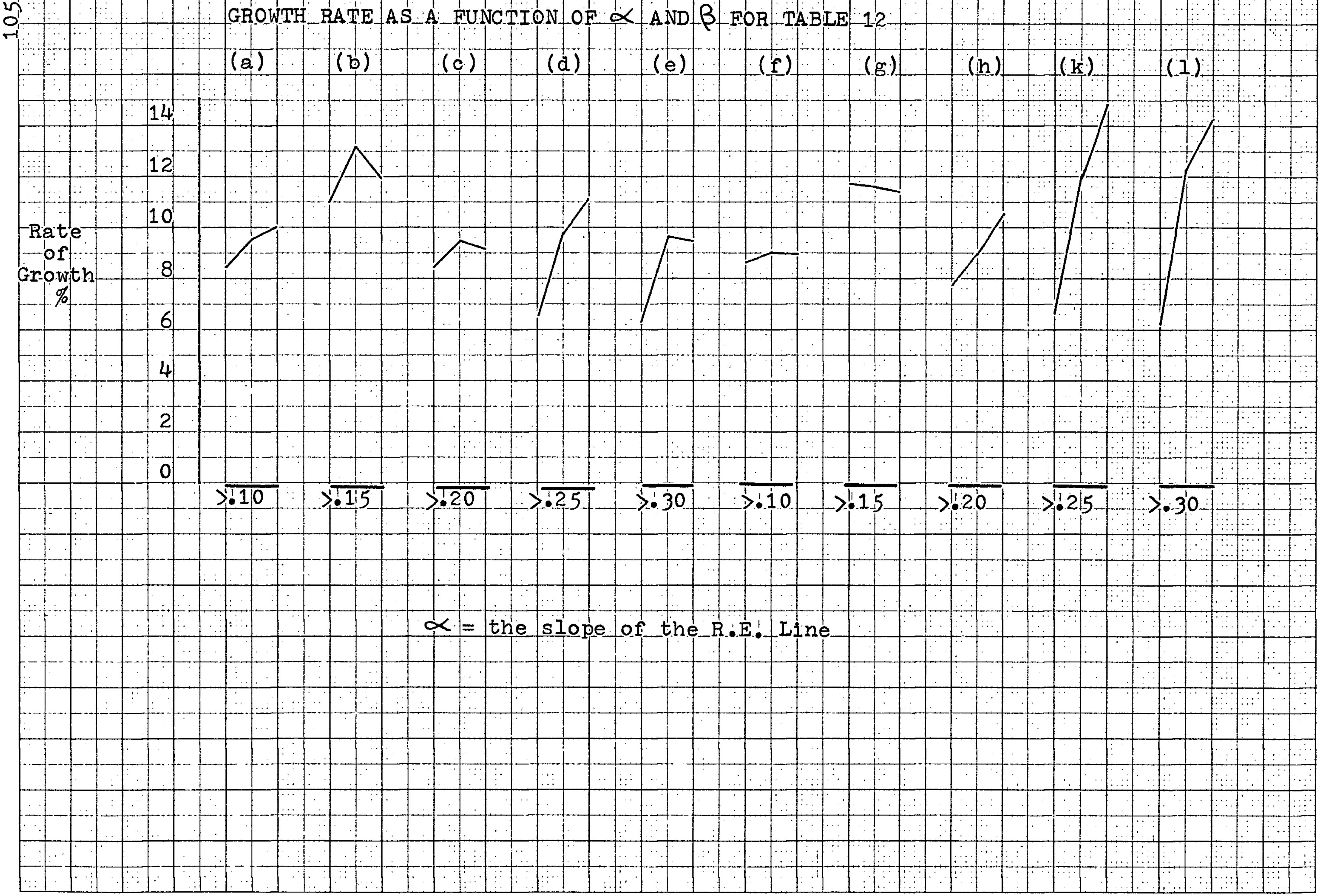

GROWTH RATE AS A FUNCTION OF α AND β FOR TABLE 11
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(k)
(l)
Rate of Growth %
14
12
10
8
6
4
2
0
(-)
(-)
$\overline{>.10}$
$\overline{>.15}$
$\overline{>.20}$
$\overline{>.25}$
$\overline{>.30}$
$\overline{>.10}$
$\overline{>.15}$
$\overline{>.20}$
$\overline{>.25}$
$\overline{>.30}$
α = the slope of the R.E. Line

GRAPH B (continued)

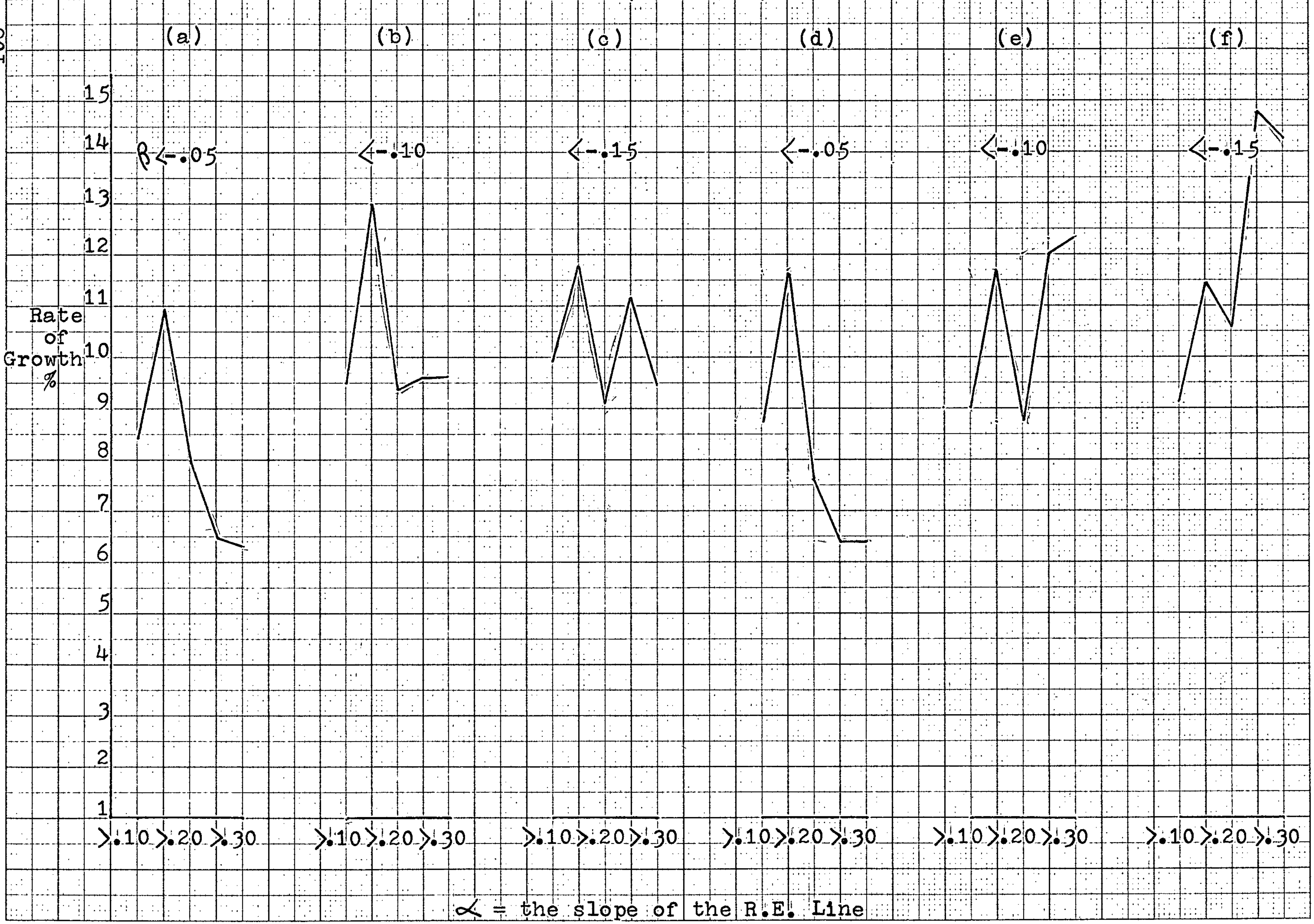


α = the slope of the R.E. Line

GRAPH C

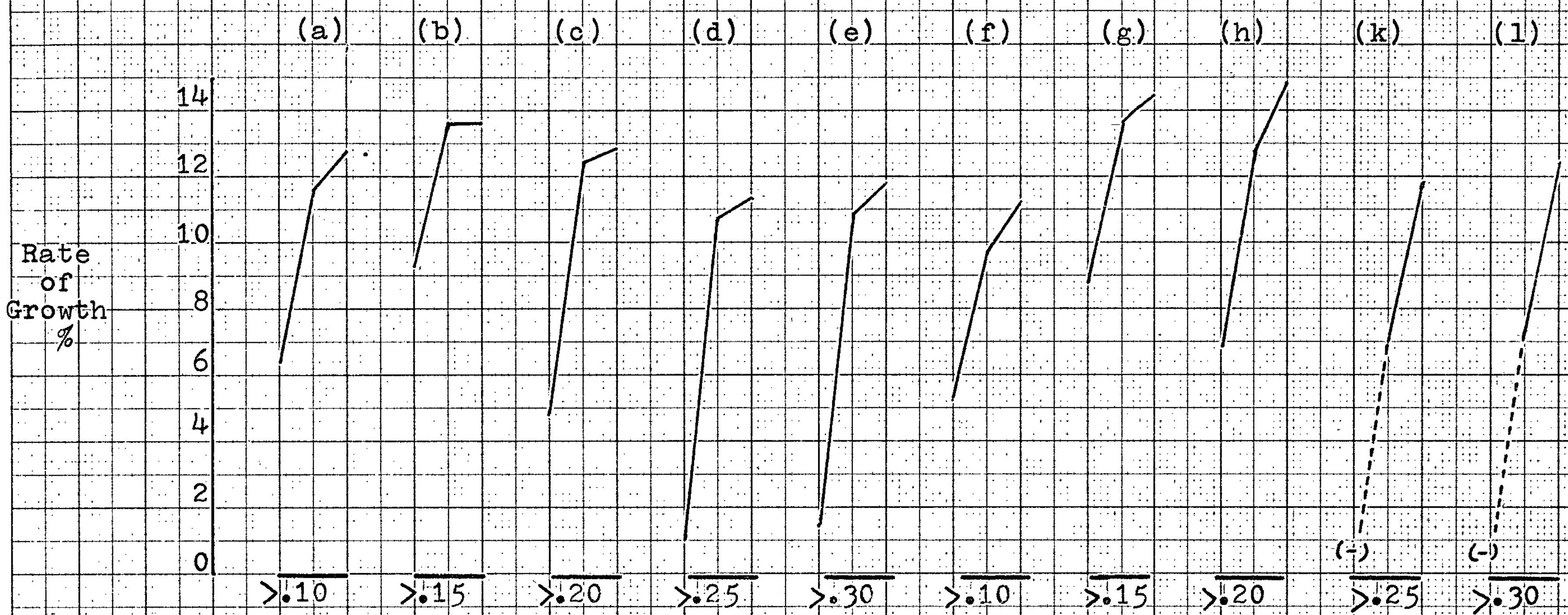

GROWTH RATE AS A FUNCTION OF ∝ AND β FOR TABLE 12
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(k)
(l)
14
12
10
8
6
4
2
0
Rate of Growth %
β.10
β.15
β.20
β.25
β.30
β.10
β.15
β.20
β.25
β.30
∝ = the slope of the R.E. Line

GRAPH C (continued)



GRAPH D

GROWTH RATE AS A FUNCTION OF α AND β FOR TABLE 13



α = the slope of the R. E. Line

GRAPH D (continued)