MEASURES OF THE QUALITY OF REAL PROPERTY ASSESSMENTS: AN EXAMINATION OF THEIR VALIDITY

by

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ABSTRACT

The tax on real property continues to be a major factor in the fiscal structure of municipal governments. Therefore, justice dictates that the impact of this tax be distributed equitably, however "equitable" may be defined by the legislative authority. This is an administrative problem.

In many jurisdictions the administration is directed to distribute the tax impact on an ad valorem basis of current market value. Statistical measures have been devised in order to measure the uniformity of the assessment of current market values. The random variable is defined as the ratio of assessed value to actual sale price, and the sample is the result of property sales which occur under conditions which are circumscribed by the definition of market value. The estimated parameters are then used to make direct statistical inferences regarding the level and uniformity of assessment. These parameters are then compared to pre-selected standard parameters in order to judge the relative uniformity of the assessment roll.

First, market generated sales do not produce a random sample of the assessment roll. Therefore, direct statistical inferences from the sample do not apply to the assessment roll.

Second, it is demonstrated that the standard parameters are implicitly based on an assumption that the universe of assessment-sale price ratios is normally distributed. The results of empirical investigation upon the Vancouver assessment roll indicate that the normality assumption cannot be justified. Therefore, the pre-selected standards are useless as measures by which to judge the relative uniformity of an assessment roll.

Two alternative measures of assessment quality are proposed. First, a binomial model based on correct and incorrect valuation is examined. Though this has some practicable possibilities, it cannot, without some important value judgements, be used to measure degrees of quality or uniformity.

A second alternative is proposed in order to overcome the above limitation. The alternative suggests that a good assessment roll have assessment-market value ratios which are normally distributed, and that the actual distribution of assessment-sale price ratios

be tested for "goodness-of-fit" to the ideal distribution. The test parameters can be transformed to probability levels which would measure the quality or uniformity of the assessment roll. This measure may indicate that a highly uniform assessment roll on which most of the properties are under-assessed is of lower quality than an assessment roll on which more properties are correctly assessed, but less uniformly assessed. Though it is unlikely that uniformity of assessment can be achieved without correct valuation, it cannot be held that an assessment roll on which more properties are correctly assessed is of higher quality of assessments than a roll on which most properties are under-assessed, if the uniformity of assessment on the former roll is not at least as high as the uniformity of assessment on the latter roll.

Since the measures of assessment quality and uniformity examined are not satisfactory, it is concluded that statistical analysis in assessment administration may be more usefully applied to the analysis of the causes of assessment errors than to the measurement of the number and size of the errors.

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CHAPTER I

INTRODUCTION

A. Purpose of the Study

The real property tax is but one tax in our complex fiscal system. In Canada, it is one of the taxes on wealth. Therefore, arguments for or against the real property tax should not be based on those criteria which are used in the evaluation of the entire tax structure.[1] Many writers have summarized those criteria,[2] and there is little point to pursuing them here. On the grounds of justice, the rule for a specific tax is equal treatment of equals, and the definition of "equals," or "like circumstances," is a value judgment to be made by the appropriate legislative authority.

Criticism, favourable or unfavourable, of the real estate tax will fall, therefore, on the administration of the tax, and will be actuated by its success, or lack of

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[1]U. K. Hicks, Public Finance (2nd ed. rev.; London: James Nisbet and Company Ltd., 1955), pp. 115-119; Adam Smith, Wealth of Nations, Book V, Chapter II, Part II, para. 3.

[2]For example, John F. Due, Government Finance: An Economic Analysis (3rd ed. rev.; Homewood, Illinois: Richard D. Irwin, Inc., 1963), pp. 102-121.

success, in achieving "equal treatment of equals."[3] After the real property tax has been adopted as a source of public revenue, and given the relevant interpretation of "like circumstances," a discussion of the tax centres about the selection of a tax base, the definition of the basis of assessment, and the administration of the tax which together will assure "equal treatment of equals."

More will be said about the tax base and the basis of assessment in Chapter II. The remarks there will be directed toward the implication of their selection and definition upon the administration of the tax. It is with the administration of the tax that this paper is concerned. Given a tax base and a basis of assessment which are consistent with "equality" in the distribution of the impact[4] of the tax, and which do not block the administration from effecting "equal treatment of equals," then the equitable distribution of the tax impact will rest upon the identification of assessable property and the uniformity of its assessment.[5]

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[3]The administration of a tax involves the identification of the liability to tax, measurement of the amount of liability, and the collection of the tax.

[4]Hicks, op. cit., p. 138. Hicks has used "formal incidence" to differentiate "impact" from "effective incidence" usually called "incidence."

[5]"Uniformity of assessment" requires that every

Specifically, the purpose here is to consider absolute uniformity of assessment, a practicable standard of assessment uniformity, and to relate statistically, or otherwise, actual uniformity of assessment to the practicable standard; to examine the assumptions upon which are based current standards of assessment uniformity; to suggest an alternative statistical approach which requires fewer assumptions about the underlying data.

B. Fiscal Importance of the Real Property Tax

There are at least two reasons why uniformity of assessment should be of concern. The first, and in the view taken here most important, is justice. Even if the revenue produced and the impact generated by a tax is almost insignificant the just administration of the tax is a necessity. If the yield of the real property tax is too small to compensate for the expenses involved in the equitable distribution of the impact, then that source of public revenue should be replaced by another source.[6]

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assessable property be included on the list at its "basis of assessment." Throughout this paper liability to assessment is kept distinct from liability to tax.

[6]Report of a Study Group of the Royal Institute of Public Administration, New Sources of Local Revenue (London: George Allen and Unwin Ltd., 1956), pp. 78-94; "Report of the Committee on State Equalization of Local Property Tax Assessments," National Tax Association, Proceedings of the 51st Annual Conference, p. 344.

The second reason is that the revenue produced and the impact generated by the real estate tax is not insignificant. From the information included in Table I it is self-evident that the tax on real property is, according to several measures, an important part of our fiscal system.

TABLE I

SELECTED FISCAL STATISTICS[a]
(Millions of Current Dollars)

Fiscal year ended nearest to December 31:	1954	1961
Gross National Product	24,041	36,844
All tax revenue	5,397	8,777
All property tax revenue[b]	659	1,400
Municipal general revenue[c]	885	1,718
Municipal tax revenue[d]	767	1,540
Municipal property tax revenue[b]	651	1,391

[a]Gross National Product: National Accounts Income and Expenditure (D.B.S., 1954 and 1961), Table 1; Taxation and Revenue Statistics: A Consolidation of Public Finance Statistics of Municipal, Provincial Governments and the Government of Canada (D.B.S., 1954 and 1961), Table 1.

[b]Annual levies except business taxes. Included are taxes on personal property, but it is an accepted fiscal fact that personal property tax revenue in Canada is an insignificant portion of the total figure (J. Stefan Dupre, "The Property Tax in Canada," National Tax Association, Proceedings of the 51st Annual Conference [1958], p. 77 and Table 3).

[c]Does not include transfer payments from other governments.

[d]Includes all municipal purpose property tax revenue and all school purpose property taxes levied by the municipality or on behalf of the municipality by a school board.

Tax revenues alone give the three levels of government in Canada a large measure of power to influence the allocation of productive factors. The degree of influence, as measured by the ratio of all tax revenue to gross national product, increased only slightly from 1954 to 1961, while the share of control arising out of property tax revenue increased from 12 per cent in 1954 to 16 per cent in 1961. Virtually all of the property tax revenue is collected at the local level of government.

At the local level the property tax accounts for over 80 per cent of municipal general revenue, and, of course, even a greater percentage of municipal tax revenue.

In 1961 the property tax revenue of the municipalities was 3.8 per cent of gross national product; therefore the collection of the property tax and the spending of the revenue produced can have an influence on the pattern of expenditures for goods and services. But of more direct importance is the following fact. Municipal governments are almost entirely dependent upon property tax revenue as the source of their share of funds for public expenditure.

This fact, coupled with the realization that the municipal property tax revenue is not, over the last few years, a decreasing portion of all tax revenue, is the most important fiscal aspect of the real property tax. Here

there is a pragmatic reason for ensuring an equal distribution of the impact of the real property tax within the municipality.[7]

C. Method

This section outlines the structure of the remainder of this paper. Before proceeding with the objectives of the study some concepts which will provide the foundation for the statistical discussions to follow will be explained and defined.

The statistical measure of uniformity currently used in British Columbia will be analyzed. The emphasis will be directed towards the basic assumptions and not towards the procedure. The measure of uniformity adopted by the Province of British Columbia is used because the Assessment Equalization Act[8] and the administration of the Assessment Commissioner under this Act are representative of

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[7]C. Ward Macy, "The Property Tax in the Fiscal System," National Tax Association, Proceedings of the 51st Annual Conference, 1958, pp. 57-76. Macy has concluded that the "property tax as a source of revenue for the support of local government must continue to play a significant part in the fiscal scene" despite the fact that "it has been maligned on both theoretical and pragmatic grounds." He also states that the future success of the property tax rests upon the improvement of its administration.

[8]British Columbia, Revised Statutes (1960), c. 20.

some of the current and past provincial and state policies of local assessment supervision.[9]

For the purpose of an empirical comparison of the currently used measure of uniformity with measures to be suggested later in the paper, a study of the current measure will be conducted using the City of Vancouver assessment roll as an example.

Alternative measures of assessment uniformity will be suggested, and from an abstract point of view their advantages over and disadvantages to the current measure will be examined.

The suggested alternative measures will be studied empirically using the City of Vancouver assessment roll as an example.

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[9]Two provinces with similar policies are Ontario, The Assessment Act, and Alberta, Alberta Municipalities Assessment and Equalization Act. Ronald B. Welch, "State Tax Department Responsibility for Local Assessments: Administrative Review," National Tax Association, Proceedings of the 34th National Conference, 1941. Welch indicates that before 1941 the following states attempted equalization among individuals (uniformity of assessment) as well as between districts. The states are: Kansas, Arizona, Kentucky, Montana, South Carolina, South Dakota, Washington, Alabama, Arkansas, Idaho, Maine, Michigan, Minnesota, Nebraska, New Hampshire, Oregon, Utah, and West Virginia. See also: National Association of Tax Administrators, Guide for Assessment Sales Ratio Studies, A Report of the Committee on Sales Ratio Data (Chicago: Federation of Tax Administrators, June 1954).

The results of the empirical studies will be compared in light of whether or not the assumptions underlying the current measure are justified. The implications of the results of this study on the development of measures and standards of assessment uniformity will be discussed.

CHAPTER II

PRELIMINARY CONCEPTS

One cannot over-emphasize the importance of some basic concepts to the problem of assessment uniformity. The explanations and definitions which follow may in places seem trite, tautological, or trivial, but a clear understanding of the application of statistics to problems of uniformity in the field of real property assessment requires more than a mastery of statistical technique. The questions that should be kept in mind are the following: what is being measured; why is it being measured; and how is it being measured.

A. Basis of Assessment

The basis of assessment is the measure used in determining the liability to the property tax.[1] Together with the physical description of the tax base, the exemptions, and the reliefs, the basis of assessment determines the theoretical distribution of the impact of the tax. At some times and in some places the basis of assessment has

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[1]Report of The Royal Commission on Finance and Municipal Taxation in New Brunswick (Fredericton: 1963), p. 206.

been a physical characteristic of the property.[2] Cannan reports that in 1250 C.E. each man of Romney Marsh had to maintain that portion of the seawalls which equaled the proportion that his land area measured of the land protected by the seawalls of Romney Marsh.[3] In 1287 C.E. the principle was extended to Sussex, but the responsibility was apportioned by land value rather than by land area. Later in Romney Marsh apportionment by land value replaced apportionment by land area.[4] But it would appear that by 1430 C.E. the apportionment of the levy according to value was generally accepted.[5]

However, as recently as 1964 F. H. Finnis has suggested the use of square foot area of land and buildings as the basis of assessment.[6] He is led to this unfortunate

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[2]Edwin Cannan, The History of Local Rates in England (2nd ed., London: P.A. King and Son, 1927), pp. 10-11.

[3]Ibid., p. 11.

[4]Ibid., pp. 21-22.

[5]Ibid., p. 15.

[6]F. H. Finnis, "Measuring the Local Tax Base," Canadian Tax Journal, Vol. XII, No. 1 (January-February, 1964), p. 48.

suggestion because he observes that a tax liability based on the value of real property does not reflect ability to pay or benefits received.[7] Regardless of the accuracy of his observation, the conclusion is wrong. The criteria he has used apply to the evaluation of the entire tax structure, and not to one particular tax. Other than the above suggestion the author knows of no serious recent suggestions or arguments which support a physically oriented basis of assessment.[8] It is generally agreed that value is the best basis of assessment.

"The basis of assessment affects the distribution of the burden (impact) of the tax rate because there are various kinds of value which may be attributed to real property."[9] At this point the tax experts part company. There are two basic types of value, and there are many different concepts of value. To further complicate the issue there is no general agreement on the definitions of

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[7] Ibid., pp. 47-48. Finnis thinks physical area is no worse in respect of ability to pay and benefit received, and much easier to measure than value.

[8] The use of cost manuals by assessors seems to imply a physically oriented basis of assessment, but the cost manuals are supposedly based on an equation between cost and value.

[9] Report of the Royal Commission on Finance and Municipal Taxation in New Brunswick, op. cit., p. 207.

the value concepts.[10]

The two types of value are annual value and capital value, the capital value being a lump sum present equivalent of the future annual returns. Each of the two types of value can be further subdivided into a value in current use and a value in exchange, and these two value concepts can each be subdivided into value to the owner and market value.[11] Further, each concept through mutable definitions and varying terminology has conveyed a different meaning in different places.[12] It should be emphasized that value in the above discussion is to be distinguished from cost. The latter term refers to historical cost. The only equivalence between cost and value is under long-run stationary equilibrium.[13]

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[10]See, for example, James C. Bonbright, The Valuation of Property, a Treatise on the Appraisal of Property for Different Legal Purposes (New York: McGraw-Hill Book Company Inc., 1937), Vol. I, Part I and Vol. I, Part III, Chapter XVII, p. 3 passim, 451-510.

[11]"Value in current use" is not to be confused with the classical economic concept of "use value." This latter concept is more closely related to "value to the owner." "Value in exchange" is not to be confused with the classical economic concept of "exchange value." This latter concept is more closely related to "market value."

[12]Bonbright, loc. cit.; Paul F. Wendt, Real Estate Appraisal, A Critical Analysis of Theory and Practice (New York: Henry Holt and Companym 1956), Chapters 1 and 2, p. 1 passim.

[13]Alfred Marshall, Principles of Economics (8th ed.; London: Macmillan and Co. Ltd., 1962), p. 305.

A satisfactory analysis and discussion of the above concepts of value would be a subject for both extensive and intensive research beyond the resources available for one study. With minor exceptions the real property tax in North America is a tax on capital value.[14] It is not the purpose here to debate the relative advantages of capital and annual value. The application of statistical method to the measurement of the accuracy with which assessors estimate capital value is the only interest.

As noted above there are several different concepts of capital value. In North America assessors are directed to base the assessments on market value;[15] however this may be defined.[16] This instruction may be given in various

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[14] Op. cit. New Sources of Local Revenue, p. 68; Harley Leist Lutz, Public Finance (4th ed.; New York: D. Appleton-Century Company Inc., 1947), p. 372; Frederic H. Finnis, Real Property Assessment in Canada (Canadian Tax Papers No. 30; Toronto: Canadian Tax Foundation, 1962), p. 9.

[15] Report of the Committee on State Equalization of Local Property Tax Assessments. National Tax Association, Proceedings of the 51st Annual Conference, 1958, p. 318; Finnis, op. cit., p. 16.

[16] There are numerous definitions of market value but there is an imperfect consensus as to what the definition should connote. It should express the following ideas: the buyer and seller are not under duress to buy or sell respectively; the knowledge that the property is available for purchase is to be made available to the market; the market participants--those who have at least some little intention of buying or selling--have some reasonable price

terminologies.[17]

There are instances, however, and until more recent years quite common, where the assessor is instructed to base the assessments on market value in some based period. Unless current market value is stipulated as the basis of assessment the quality of assessment cannot be evaluated with the use of observed market transactions. Instead, a series of independent valuations will be required as comparisons to the municipally assessed values. Since there is no reason to expect a single independent valuer to produce more uniform appraisals than the municipal assessor each property used in the evaluation of the quality of the assessment roll would have to be valued by at least two independent appraisers. This could prove to be a costly procedure.

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expectations, some knowledge, in the market place; that the exchange value realized will be the result of current reaction of demand and supply in the market place.

With the above ideas in mind, market value could be defined as the price an interest in land might reasonably be expected to realize when sold by a willing seller, after the property has had adequate exposure to the market, to a buyer, both seller and buyer being subject to the current conditions of the expected market for the property. Refer also to Bonbright, Wendt, loc. cit.

[17]Finnis, loc. cit. He points out that the Supreme Court has indicated that "actual value," "real value," and "fair value" are synonymous with "market value;" Report of the Committee on State Equalization of Local Property Tax Assessments, loc. cit.; Re Assessment Equalization Act: Rowan's Appeal (1962), 40 WWR 627, SCBC. On a case stated by the Assessment Appeal Board, Wootton, J. said that "actual value," "fair market value," and "actual cash value" are "relatively the same thing."

Also, if market value in a base year is used, there is no reason on the basis of equity or convenience to support market value as the basis of assessment over some other capital value concept.

Most important is the fact that current market value as the basis of assessment affords the only opportunity for the taxpayer to know that he is paying not more than his share of the property tax burden.[18] Current market value provides the required objective measure of value.

B. Market Price as Evidence of Market Value

Here, too, it is not the purpose to debate the disadvantages and advantages of the use of market value as the measure of capital value in real estate taxation. The endeavour is to establish statistical measures and standards by which to judge the assessor's success in estimating the designated value.

Therefore, an objective measure of market value is needed to evaluate the assessor's performance. The required evidence is actual sale prices, sale prices established in transactions which conform to the conditions

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[18] Harold F. Meyer, "Market Value--Beacon Light of Appraisal for Taxation," The Appraisal Journal, Vol. XXXIII, No. 4 (October, 1965), p. 570, 571-572.

of the market value definition. The immediate problem is to establish the relationship between market prices[19] established in transactions and, for the same properties, market values as they should appear on the assessment roll. There is a legal approach and an economic approach to this question.

First consider the economic approach.[20] At this point another value concept is introduced. This is value to the owner. Value to the owner is the value in exchange of a particular interest in a particular property to a particular individual at a particular time. It is an estimate of the present value to him of all the future utility he estimates that he will derive from the control over the interest in the property. The estimate of the future utility and its present value are conditioned by the alternatives open to the individual. All alternatives are relevant, but the more directly comparable alternatives

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[19]"Market prices" will be used to designate "sales prices established in transactions which conform to the conditions of the market value definition."

[20]Ralph Turvey, The Economics of Real Property: An Analysis of Property Values and Patterns of Use (London: George Allen & Unwin, Ltd., 1957), pp. 8-21. Turvey has produced a very clear, concise, and consistent analysis of the determination of real estate prices. The discussion in the text of this study follows his work.

are more relevant. For example, to an investor, the future income from an apartment building is more directly comparable to the future income from a second apartment building than is the future income from a government bond; the price for which he can obtain the second apartment building is more relevant to his estimate of the present value of the future utility to him of the first apartment building, than is the price for which he can obtain the government bond.

Turvey has used Floor price to denote the value of an interest to the present owner and Ceiling price to denote the value of an interest to someone other than the present owner. Theoretically every owner has a Floor price and everyone else has a Ceiling price for every interest which he does not already own.[21] In practice, of course, only owners with at least some little intention of selling will have a Floor price for the interest, and only others with at least some little intention of buying the interest will have a Ceiling price. It is these latter Floor prices and Ceiling prices which are the determinants of market values.

If, for a particular interest, there is at least one Ceiling price in excess of the Floor price, a range of

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[21]Turvey, op. cit., p. 19.

mutual gain, then there will be a transaction and a resulting market price. The actual price will be between Ceiling price and Floor price. If there is more than one Ceiling price, then the resulting market price will be between the two highest Ceiling prices. The relative position of the market price in the range of mutual gain will depend on the bargaining position and skill of the buyer and seller.

In any case, the resulting market price will be observed by prospective sellers and potential buyers of similar interests.

> Since the different potential purchasers of a particular property will to some extent consider the same alternatives, the spread of their ceiling prices will not be large. . . .
>
> It follows that in the case of properties for which there are substitutes, the price which will be realized is determinate within fairly narrow limits, and can be forecast (as market value) fairly well by an expert.[22]

Turvey then concludes that the market prices are fairly determinate (determinate within narrow limits) for property for which there is a "large market" and there is adequate market exposure.[23]

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[22]Turvey, op. cit., p. 19.

[23]Ibid., p. 20.

This conclusion means that sale prices resulting from transactions which occur within conditions specified by the definition of market value are acceptable, within narrow limits, as evidence of market value. The limits will be more narrow for low and medium value range residential properties than for commercial and industrial properties, because the former are, as is common knowledge, more actively traded than the latter.

A brief glance at the judicial view of the relation between prices determined in the market and market value may be useful. There are long lines of conflicting cases, and a detailed examination does not resolve the conflict, but merely confirms its existence.

Wootton, J. indicated that a sale price is not necessarily the best test of market value. He held that the conditions surrounding the sale must be considered.[24] In other words, for a sale price to be a market price, the transaction must occur within the conditions of market value. The above decision follows the statement of Atkin, L. J. in Norwich Assessment Committee vs. Pointer.[25] He there said that in arriving at market rent (rental value)

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[24] Loc. cit.: Re Rowan's Appeal.

[25] Norwich Assessment Committee vs. Pointer, 1922, 2 KB, p. 471, Court of Appeal.

"any evidence which is relevant to that question is in law admissible, and it must depend on the circumstances of the case. . . ." More directly to the point Lord Buckmaster[26] and Scott, L.J.[27] have held that the actual rent being paid by the occupier is relevant only so far as the conditions of the actual tenancy conform to the statutory conditions required of the estimate of gross value.[28] Bonbright quotes from the decisions of two cases which

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[26] Assessment Committee of the Metropolitan Borough of Poplar vs. Roberts (1922) 38 T.L.R. 499, House of Lords (see esp. p. 103).

[27] Robinson Brothers (Brewers) Ltd. vs. Houghton and Chester-le-Street Assessment Committee (1937), 2 K.B. 445 (see esp. p. 469).

[28] "Gross value" for the purpose of rating specifies an annual tenancy, the tenant be responsible for tenants' rates and taxes, and the landlord be responsible for repairs and maintenance. See the following legislation:

Great Britain, Statutes at Large 25 & 26 Vict. (1862), c. 103, "Union Assessment Committee Act," sec. 15(g); Great Britain, Statutes at Large 32 & 33 Vict. (1869), c. 67, "The Valuation (Metropolis) Act," sec. 4; Great Britain, Statutes at Large 15 & 16 Geo. 5 (1925), c. 90, "Rating and Valuation Act," sec. 68.

It is clear that the English definition of the basis of assessment relates to market rent in a hypothetical market, in other words a value in exchange, but the point to observe is that the Courts have held that actual rent is evidence of the hypothetical market rent only so far as the conditions surrounding a particular tenancy conform to the conditions of the hypothetical market.

"belittle the distinction" between market value and current sale price.[29]

The Committee on State Equalization of Local Property Tax Assessments in its report to The National Tax Association contends that sale price at market conditions is "highly persuasive of the market value of the property."[30] Further on they emphasize the point by saying that the sale produces "conclusive evidence of market value."[31]

C. Valuer's Margin of Error

With the above weight of evidence, economic and judicial, it can be concluded that an assessor's estimate of market value should lie within narrow limits of the observed market price. The question is: "How narrow are the limits?"

The setting of the limits is an exercise in value judgment. There is no sharp delimitation between a correct

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[29]Bonbright, op. cit., p. 464. He quotes from Kentucky River Coal Corp. vs. Knott County, 254 Ky. 882, 54 S.W. (2d) 377 (1932) (Ky. at 828, S.W. at 379); Atlantic States Coal Corp. vs. Letcher County, 246 Ky. 549, 55 S.W. (2d) 409 (1932) (Ky. at 551-552, S.W. at 409-410).

[30]Report of the Committee on State Equalization of Local Property Assessments, National Tax Association, Proceedings of 51st Annual Conference, 1958, p. 337.

[31]Ibid., p. 342.

and incorrect valuation. Byrne, J. in Bell Hotels (1935) Ltd. vs. Motion and Another[32] gives some indication of where the limits might lie. In that case a valuer estimated for a client "that a sale (of his public house) might be effected at 20,000 pounds, but . . . if a near offer were forthcoming, it should be seriously considered."[33] The owner sold the public house for 17,500 pounds. Shortly after this sale, the purchaser resold the public house for 25,000 pounds. Byrne, J. found the valuer negligent and allowed a claim of 5,000 pounds, but the court reasoned that the valuer should be allowed a ten per cent margin on the market price.

Limits of plus and minus ten per cent of market price should be the most generous limits for the estimate of market value on most types of property, because it is recognized that the valuation of licensed premises is more vexatious than the valuation of most other types of

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[32]Bell Hotels (1935) Ltd. vs. Motion and Another, The Estates Gazette, May 17, 1952. The decision revolved around whether or not the valuers were negligent, but the way in which damages were assessed is of interest to our problem here.

[33]A "public house" is one type of "licensed premises." See D. M. Lawrance, W. H. Rees, and W. Britton, Modern Methods of Valuation of Land, Houses and Buildings (5th ed.; London: The Estates Gazette Limited, (1962)), p. 433.

property.[34] Maclean, J. in Dodds and Dodds (plaintiffs) vs. Millman was not nearly so generous to the defendant in estimating the amount of the claim.[35] The defendant, the vendor's agent, produced a valuation of a small apartment building and caused this valuation to come into the hands of the plaintiffs, the purchasers. The agent's estimate of market value was $42,500, and it was accompanied by a projected "operating statement." Later, the plaintiffs found the revenue and expense estimates to be grossly inaccurate and they brought action against the vendor and his agent for fraud, or at least negligence. On the basis of expert witness the Court found negligence against the agent, and awarded the purchasers $8,500 in damages. The expert witness estimated that the market value at the time of purchase would have been $35,000. Maclean, J. allowed the agent no margin of permissible discrepancy. This last case is difficult, if not impossible to apply. In Bell vs. Motions[36] there was a second sale, and, therefore, an objective measure of the error. In Dodds vs. Millman[37] the Court but weighed the testimony of expert witness, and accepted the fact that the expert valuation

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[34]Lawrance, Rees, and Britton, loc. cit.

[35]Dodds and Dodds vs. Millman 45 D.L.R. (2d), 1964, p. 472.

[36]Supra, pp. 22-23

[37]Supra, p. 23.

may have been greater, as well as less than, the market price realized upon sale at the date of the purchase.

D. The Tax Base

There are several additional concepts which relate to the use of sale price, first, by the assessor in producing the valuation list, and second, by the auditor[38] of the assessment roll in evaluating the performance of the assessor.

The tax base is, in conjunction with the basis of assessment, the prescription of the property to be assessed.[39]

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[38] In several places throughout this paper the words "audit" and "auditor" are used. This reflects the author's opinion that just as the Municipal Accounts are each year subject to an external audit, so should the assessment roll be subject to an annual external audit to examine the uniformity of assessment, the identification of taxable properties, and the accuracy of the description of those properties and their ownership.

[39] Some authorities define the tax base as "the prescription of the property to be taxed." (Report of The Royal Commission on Finance and Municipal Taxation in New Brunswick, Fredericton, 1963). The phrase "of the property to be taxed" includes the basis of assessment, but the same phrase does not leave the way open for a clear distinction between assessed property exempt from taxation and specific property which is a part of the general genus of property to be assessed, but which by statute is exempt from assessment.

The Vancouver Charter[40] defines improvements to include "buildings, structures, machinery and other things so affixed to land as to make them in law a part thereof"[41] but for the purpose of City taxation (as contrasted to school taxation) the practice has been to interpret the definition of improvements to have the same meaning as their definition in the Municipal Act.[42]

The Municipal Act[43] requires that "land and improvements shall be assessed,[44] and except for school purposes defines improvements to exclude "fixtures . . . if so erected or affixed by a tenant would, as between landlord and tenant, be removable by the tenant."[45]

The Vancouver City Charter requires that land values and improvement values be shown separately on the assessment roll,[46] but each parcel . . . be estimated at its actual

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[40] British Columbia Statutes, 1957, c. 42.

[41] Ibid., sec. 2.

[42] Interview with C. Dowling, Deputy Assessment Commissioner, City of Vancouver, July 16, 1965.

[43] British Columbia Statutes, 1957, c. 42.

[44] Ibid., sec. 330 (1).

[45] Ibid., sec. 2.

[46] British Columbia Statutes, 1953, c. 55, sec. 341.

value."[47] The most obvious method to use in carrying out these instructions is to estimate the market value of the property and then with a bit of intellectual exercise separate the "whole" into its "parts," land and improvements. Fisher has an excellent comment on this separation. "First, the services rendered are the joint product of land and improvements . . . , the two unite to form a compound, not a mixture."[48] And for an indication of the difficulty and futility of this separation we can refer to Turvey. He has produced a clear and concise argument proving that the separation of land and building has no meaning except in long-run static equilibrium, and as such is useless in economic analysis except when the building is new and in its highest and best use.[49]

Where land and improvements are taxed at the same effective rate per cent the separation of the "whole" into the "parts"[50] has no effect on the equitable distribution of the tax impact. But where the legislature in its wisdom has decided to tax land and improvements at different effective rates per cent, then the separation is of great

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[47] Ibid., sec. 342.

[48] E. M. Fisher, Urban Real Estate Markets: Characteristics and Financing (New York: National Bureau of Economic Research, 1951), p. 5.

[49] Turvey, op. cit., pp. 22-24.

[50] This is by no means the same process as estimating the "parts" in order to construct the "whole."

moment. In the latter situation, and this is the situation in British Columbia, an audit of the assessment roll does not produce a measure of the equality, or inequality, in the distribution of tax impact. In order to do that the auditor would require sales of buildings apart from the land, and sales of land pulled out from under the buildings. Turvey has so aptly pointed out that "no ordinary building is ever sold floating in air."[51] Therefore, in this circumstance an audit of the valuation list using market prices can but measure the performance of the assessor in estimating the value of the "whole."

E. Sales Prices and Real Estate Finance

The phrase "current conditions of the expected market" used in the definition of market value[52] applies to the financing of real estate interests. As the words "expected market" imply, for a particular property there is a typical expected purchaser. People with high incomes are more likely to buy high valued housing than are people

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[51] Turvey, op. cit., p. 24.

[52] Supra, p. 14.

with low incomes.[53] Given a particular property and, therefore, a general expectation of a typical personal covenant, there will be available to the potential purchaser of the property mortgage financing pitched upon by the "current conditions of the expected market." At a given time, if for a particular property the financing used is different from market expectations, then the sale price resulting from a transaction will not be a direct indication of market price.

The effect of the above concept can be illustrated with an example. Assume, for the purpose of discussion, that there is a house with a known market price of $20,000, that the "expected" mortgage loan will have a loan-value ratio of 0.60, and that the "expected" interest rate is 7 per cent. There are many ways in which the actual

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[53] P. H. White, "Prologue to An Analysis of the Residential Mortgage Market in Vancouver," Conference of the Association of Canadian Business Schools, Proceedings, 1965 (footnote 24 of White's paper indicates a strong correlation between income and value of housings); Alvin E. Coons and Bert L. Glaze, Housing Market Analysis and the Growth of Nonfarm Home Ownership (Bureau of Business Research Monograph Number 115; Columbus: Bureau of Business Research, College of Commerce and Administration, The Ohio State University, 1963), pp. 29-30 and 132.

transaction may deviate from the expected transaction, and we shall explore a few cases and their effects.[54]

Case 1. The market price will be constructed as follows:

	Purchaser's equity	$ 4,000
20 year, 5%	-- 1st Mortgage Loan (institutional lender)	12,000
10 year, 10%	-- 2nd Mortgage Loan (vendor	4,000
	Market Price	$20,000

But the practice is to write the second mortgage at, or near, 5 per cent and bonus the face value to yield 10 per cent. More often than not a rule of thumb is used to calculate the bonus,[55] but a more accurate calculation can be provided.[56]

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[54] In practice one starts with an observation of the resultant effects and works back to market price. The interest rates in the examples are not those of the current market in a particular city.

[55] The bonus is, by the rule of thumb, usually set between one-quarter and one-third of the cash amount of the loan.

[56] Instead of transferring the title and the purchaser granting a mortgage to the vendor, the vendor could grant a right to purchase to the buyer.

Monthly payment to amortize $4,000 in 10 years at 10%	$52.42
The present value of $52.42 per month for 10 years discounted at 5%	$ 5,000

The sale price would be constructed as follows:

	Purchaser's equity	$ 4,000
20 year, 5%	-- 1st Mortgage Loan (institutional lender)	12,000
10 year, 5%	-- 2nd Mortgage Loan (vendor)	5,000
	Sale price	$21,000

Case 2. The market price will be constructed as follows:

	Purchaser's equity	$ 4,000
20 year, 6%	-- 1st Mortgage Loan[57] (vendor)	16,000
	Market price	$20,000

A $16,000 loan for 20 years at 6% is equivalent to a $12,000 loan for 20 years at 5% plus a $4,000 loan for 20 years at 8 $\frac{13}{16}$%.

The following calculation will illustrate this point.

Monthly payment to amortize:

$12,000 in 20 years at 5%	$ 78.86
4,000 in 20 years at 8 $\frac{13}{16}$%	35.10
$16,000 in 20 years at 6%	$113.96

[57]Instead of transferring the title and the purchaser granting a mortgage to the vendor, the vendor could grant a right to purchase to the buyer.

In this case the mortgage would be written at 5%, and the face amount bonused to yield 6%.

Monthly payment to amortize:

$16,000 in 20 years at 6%	$113.96
The present value of $113.96 per month for 20 years discounted at 5%	$17,100

The sale price would be constructed as follows.

	Purchaser's equity	$ 4,000
20 year, 5% --	1st Mortgage Loan (vendor)	$17,000
	Sale price	$21,000

Case 3. In some instances there may be against the title an unfavourable mortgage for which the purchaser cannot obtain a release. In other words, he must "take over" payment on a mortgage the terms of which are not as good as might be expected. To illustrate the effect of this point, assume that our example property has a 12-year, 6 1/2% $10,000 mortgage loan against the title, and that there are 9 years of the mortgage term remaining. The principal outstanding at the end of 3 years is $8,170. Under the assumed present market conditions the purchaser could expect a $12,000 mortgage at 5%. In order to obtain the additional $3,830 he will have to pay 9 1/2% on a second mortgage loan.[58]

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[58] In two years the purchaser could obtain a release on the first mortgage and refinance, but he could at that time still have three years before he could release the second mortgage.

Monthly payment to amortize:		
$12,000 in 9 years at 6 1/2%		$137.92
$ 8,170 in 9 years at 6 1/2%	$ 99.74	
$ 3,830 in 9 years at 9 1/2%	62.62	
	$163.36	

The purchaser's foregone opportunity, the excess of expected actual payment over the market expectation is $25.44 per month for 9 years. The present value of this opportunity loss at 9 1/2% is $1,850. The sale price would be constructed as follows.

"Take over" of 1st mortgage	$8,170
Cash to vendor	9,980
Sale price	$18,150

Case 4. If the vendor has a superior credit standing to that of the expected purchaser, then the vendor may be able to obtain a more favourable mortgage loan before selling the property than could the purchaser after buying the property. Suppose the vendor obtained a loan of $12,000 for 20 years at 4 1/2%, while the typical purchaser of the property might expect a loan of $12,000 for 20 years at 5%.

Monthly payment to amortize:	
$12,000 in 20 years at 5%	$78.86
$12,000 in 20 years at 4 1/2%	75.65
Monthly saving	$ 3.21

The gain to the purchaser is $3.21 per month, and the present value of the monthly savings over 20 years discounted at 5% is $490. The sale price would be constructed as follows.[59]

"Take over" of 1st mortgage	$12,000
Cash to vendor	8,490
Sale price	$20,490

The above illustrations are not meant to be indicative of current market conditions nor are they a complete compilation of real estate financing practices. They were presented only as some examples of transactions which do not conform to the conditions as set out in the definition of market value, and, therefore, which might record sale prices different from market prices. The difference may be small or large depending on the circumstances surrounding the specific transaction.

F. Sales Prices and Assessment Uniformity

That a periodic tax on durable goods may be capitalized, has long been recognized.[60] But almost in

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[59]There is of course some speculation as to whether or not the additional $490 is sufficient to cover the "cost" of the contingent liability of the vendor.

[60]Edwin R. A. Seligman, Shifting and Incidence of Taxation (5th ed. rev.; New York: Columbia University Press, 1926), pp. 174-183, 328-329.

all cases the durable good has been considered an investment in the cash income producing sense.[61] That approach is reasonable when discussing commercial and investment real estate, but not so reasonable when discussing owner occupied housing. Some economists consider owner-occupied housing a durable consumer good.[62]

First consider the effects of over and under assessment on the sale prices of commercial and investment real estate. Groves states that "capitalization can exist only to the extent that shifting does not," and stipulates the conditions under which capitalization will occur.[63] These

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[61]Hicks, op. cit., p. 164; Carl Shoup, "Capitalization and Shifting of the Property Tax," in Harold M. Groves, ed., Viewpoints on Public Finance ([reprinted from Property Taxes, Chap. 13, Tax Institute, New York; New York: Henry Holt and Company, 1947]), pp. 107-115; Harold M. Groves, Financing Government (3rd ed.; New York: Henry Holt and Company, 1950), pp. 126-128; Edwin R. A. Seligman, Shifting and Incidence of Taxation, loc. cit.

[62]Alvin E. Coons and Bert L. Glaze, Housing Market Analysis and the Growth of Nonfarm Home Ownership (Bureau of Business Research: Monograph Number 115; Columbus: Bureau of Business Research, College of Commerce and Administration, The Ohio State University, 1963). In this monograph the entire analysis is based on the belief that housing stock is a consumer durable. See pp. 16-18; Richard F. Muth, "The Demand for Non-Farm Housing," in Arnold C. Harberger, ed., The Demand for Durable Goods (Chicago: University of Chicago Press, 1960). Housing as a consumer durable is implicit in this study.

[63]Groves, op. cit., pp. 126-127.

conditions apply to income producing goods for which the future tax patterns can be anticipated, the total supply is relatively fixed, and which are taxed unequally in relation to alternative investments. The complexities of this problem are made quite clear by Shoup.[64] He first capitalizes a change in tax at current interest rates, but then goes on to recognize that a change in the tax may cause interest rates to change and hence offset to some degree the capitalization. He also argues that if the tax pattern is anticipated then the capitalization would be gradual, the present value of future changes in tax gradually affecting the value of the good as the tax change becomes more imminent. The situation for the present purpose is not so complex because this study is not concerned with a change in tax on one class of properties relative to another, but with the effect of non-uniform assessment of one property relative to another similar property.

Therefore, one can safely assume in the following illustration that the discount rate does not change. The maximum effect will be indicated if we assume that all properties except one are uniformly assessed, there are

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[64] Shoup, op. cit., pp. 107-115.

several properties similar in all respects,[65] and future assessments on the properties will bear the same relative relationships among each other.

Suppose that five similar properties recently sold under market conditions for $100,000 each, that each was assessed at $100,000, and that each is occupied under an annual tenancy at a net annual rent of $10,000. The property tax rate is 2% of assessed value. A sixth similar property is assessed at $130,000. The tax will be $600 more per year on the sixth property than on the other five, and the net rental value will be $600 less. The sixth property would sell for $94,000. Since the rate per cent of tax is small, a relative over assessment of 30% produces a sale price difference which in a real situation would be well within an allowable margin of error.[66] The assessment-market price ratio of 1.30 is but slightly smaller than the assessment-sale price ratio of 1.38. If the assessment on the sixth property were

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[65] "Similar in all respects" does not mean that all are on the same lot.

[66] Supra, p. 23. We would expect that where five out of six properties similar in all respects sold under market conditions, that an assessor would do much better than assumed in this example.

$110,000 the ratios would be 1.10 and 1.12 respectively. If on the other hand the assessment on the sixth property were $90,000 the ratios would be .90 and .88 respectively.

Nowturn to the consideration of the effect of over and under assessment on the sale prices of owner occupied housing. Even if owner occupied housing is viewed from an investment point of view and if sale price and rental data were available for single family dwellings the analysis could proceed no further. Coons and Glaze found that, "home ownership and rental tenure are not perfect substitutes," the latter being inferior in the consumers' minds to the former.[67] Therefore, occupants are willing to pay more per month for owner-occupied housing than for the same quantity and quality of rental housing.

The same study concludes that housing is a consumer durable,[68] and that housing expenditure is a budget item.[69] For illustrative purpose only, using a "budget approach" the effect on sale price of under and over assessment is demonstrated below.

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[67]Coons and Glaze, op. cit., p. 130.

[68]Ibid., pp. 132-136.

[69]Ibid., p. 132.

Assume that five houses similar in all respects recently sold for $20,000. The sale price consisted of $8,000 purchaser's equity and a $12,000 institutional first mortgage loan for 20 years at 5%. The properties are assessed at $20,000, and the rate of tax is 2%. The annual cost of occupation to the owners will be as follows:

Debt service -- 12 months at $78.86	$ 946.32
Taxes	400.00
Heating, maintenance	600.00
	$1946.32

A sixth property similar in all respects to the other five is assessed at $26,000. The taxes will be $520 per year, and if the purchaser is to pay no more to occupy this property than the other five he will reduce the debt service by $120 per year, or by $10 per month. But payments of $68.86 per month will in 20 years and at 5% amortize a loan of only $10,450. The sales price will be constructed as follows.

	Purchaser's equity	$ 8,000
20 year, 5% --	First Mortgage	10,450
	Sale price	$18,450

The sale price is within a 10% margin of $20,000. The assessment-market price ratio is 1.30, while the assessment-sale price ratio is 1.41. If the property were assessed at $22,000 the sale price would be $19,500.

The assessment-market value ratio and the assessment-sale price ratio would then be respectively 1.10 and 1.13.

The illustrations indicate only very generally what might be the maximum effect of over and under assessment on sale prices and assessment sale price ratios. As one would expect, the difference between assessment-market value ratio and assessment-sale price ratio decreases as the assessed value of the property approaches the assessed value of the similar properties.

The above examples are much too simple for practical application, but in reality the problem is too complex for an accurate practicable solution. The point to remember is that the difference between market value and sale price will increase as the difference between the assessed value of the subject property and the assessed value of comparable properties increases; as the dispersion of the assessment-market price ratios of the comparable properties decreases; as the degree of similarity among the subject and comparables increases; as market knowledge through active trading increases; and as the tax rate increases.

G. Uniformity of Assessment

For absolute uniformity of assessment to prevail all properties within a fiscal jurisdiction must be

assessed at the same proportion of market value. Since market value lies somewhere within a range of mutual gain,[70] reason suggests that a valuer be allowed a range of tolerance in his estimates of market value. With this in mind one can hardly expect an assessor to achieve absolute uniformity in his valuation list, but for the purpose of this section assume that absolute uniformity is possible.

The courts have handed down some interesting judgments on the problem of uniformity of assessment, but the conflicting judgments help little in establishing a practical statistical measure of uniformity or quality. One line of British cases agrees with the Divisional Court in Norwich Assessment Committee vs. Pointer,[71] a leading case in a line of similar cases. It was there held that while evidence of the assessment of similar properties is not the best evidence as to the statutory definition of value, it nevertheless can be considered in arriving at an assessed value. In the case before the court the bench did accept the assessment of similar properties as evidence for valuation. In other words,

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[70] Supra, p. 19.

[71] Norwich Assessment Committee vs. Pointer, 1922, 2 K.B. 49 (Divisional Court), 471 (Court of Appeal).

uniformity of assessment takes precedence over correct valuation.

On the other hand, Scrutton, L.J. in Ladies Hosiery and Underwear Ltd. vs. West Middlesex Assessment Committee,[72] was there of the opinion "that the assessing authority should not sacrifice correctness in order to ensure uniformity." In other words, the duty of the assessor is to value at the statutory basis of assessment.

Bonbright discusses several American cases, but here also there are lines of conflicting judgements.[73] Hellerstein reaches the same conclusions.[74]

Some Canadian courts apply the principle of uniformity of assessment, but they hedge their judgements with statements about correct valuations. Other Canadian courts do the reverse. The Supreme Court of Nova Scotia

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[72]Ladies Hosiery and Underwear Ltd. vs. West Middlesex Assessment Committee, 1932, 2 K.B., 679.

[73]Bonbright, op. cit., c. XVII.

[74]Jerome R. Hellerstein, "The Appeal Machinery in Property Taxation," National Tax Association, Proceedings of the 51st Annual Conference, 1958, pp. 429-455. Included in his paper is an excellent discussion of statute and case law in the United States with regard to valuation to the statutory basis of assessment as compared to valuation to the "tone of the list," the accepted level of assessment. He makes it quite clear that the situation is a morass of conflicting principles. "Tone of the list" is a British term synonymous with "level of assessment."

in Re Irving Oil Co. Assessment[75] held that under the Assessment Act[76] the valuation to be arrived at is market value. They then go on to say that the above figure should be checked by comparing it with the valuation of other similar properties.

In the Appellate Division of the Supreme Court of Alberta, Ewing, J.A. comments on real estate assessments in a case which is not directly concerned with municipal assessments.[77] A commissioner appointed under the Alberta Succession Duty Act[78] to arrive at market value for the purpose of levying succession duty used the municipal assessment.

Ewing J.S. said:

> While the municipality is bound to assess at the fair market value this provision is subject to another provision, viz., that the assessment is uniform with that of similarly situated properties. Thus the municipality is, in the result, compelled only to see that the assessments are uniform . . . . It is . . . notorious that the assessment often bears little relation to 'value' of the property.[79]

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[75]Re Irving Oil Co. Assessment, 1948, 2 D.L.R., 774.

[76]Nova Scotia, Statutes, 1938, c. 2.

[77]Re Withycombe Estate, 1944, 1 W.W.R, 385 (Appellate Division).

[78]Alberta, Revised Statutes, 1942, c. 57.

[79]Re Withycombe Estate, op. cit., p. 397.

In order to derive some standard by which to judge the quality of a valuation list it must be decided whether an assessment roll on which all properties appear at a uniform proportion of the statutory basis of assessment is of as good a quality as an assessment roll on which all the properties are valued at the statutory basis of assessment. Both rolls would produce the equitable distribution of the impact of the tax, but can they, in theory, be of a different quality?

The view taken here is that the second assessment roll is the better of the two, and, therefore, the quality of the valuation list and a measure of its uniformity will be based upon the proportion of the properties on the roll which are assessed at, or within a given margin of the statutory basis of assessment. This view is supported by an interpretation of Scrutton's remark[80] and by Ewing's comment.[81] There is some possibility that Scrutton believed that correct assessments would achieve uniformity. Ewing's comment can be interpreted to mean that the court can but give the

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[80] Supra, p. 42.

[81] Supra, p. 44.

principle of uniformity precedence over correct valuation when there are so few instances of valuation at, or near, the statutory basis of assessment.

A perfectly uniform assessment roll will then be absolutely uniform with all properties on the list valued at their market price.[82] A perfectly uniform assessment roll is an unrealistic objective. An assessment roll, where all the properties in the fiscal jurisdiction are valued within 10 per cent of the basis of assessment, is a practicable goal. If an assessment is within the limits, then it is correct. If it is outside the limits, then it is not correct. An assessment at 50 per cent above the basis of assessment is more grievous than an assessment at 15 per cent above the basis of assessment. Still, the factor carrying most weight in the evaluation of an assessment roll should be the proportion of correct assessments. For, except by the most fortuitous coincidence, uniformity is the result of accurate valuation.

H. Market Activity and Random Sampling

Some proponents of assessment-sales ratio studies assume that in a given period of time market activity

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[82] Or at the required statutory proportion of market value.

will generate a random sample of the assessment roll. They then make statistical inferences about the roll using their "sample." The Committee on Sales Ratio Data have made this assumption.[83] They then suggest that in order to get a reliable measure of the general level of assessment the sales be stratified by classifications of property because the classifications are not proportionately represented by sales.[84] The suggestion hardly exhibits faith in the assumption. The United States Department of Commerce describes a method used to eliminate the bias, relative to the assessment roll, caused because of varying turnover rates among different classes of property.[85] Because of the heterogeneity of parcels of real estate no reasonable amount of stratification is going to remove the effect of the bias.

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[83]Committee on Sales Ratio Data, Guide for Assessment-Sales Ratio Studies, A Report to the National Association of Tax Administrators (Chicago: Federation of Tax Administrators, 1954), pp. 5 and 57-60.

[84]Ibid., p. 24.

[85]United States Department of Commerce, Bureau of the Census, Taxable Property Values, Census of Governments, 1962, p. 10.

Corbert has constructed an elaborate geometrical model to illustrate that market activity does not generate a random sample.[86] He then adds, "The first and most obvious solution which might be adopted involves the designation of two separate universes."[87] The Committee on State Equalization of Local Property Tax Assessments examines the advantages and disadvantages of random sampling of the assessment roll as compared to the uses of a market generated sample, and they reach the same conclusion.[88]

That the market activity does not generate a random sample of the assessment roll cannot be too heavily emphasized. The distribution of assessment-sales price ratios so generated is a separate universe. Its parameters may be measured directly, or its parameters may be inferred from a random sample of sales. But from those results alone no inference, in the statistical sense, can be made about the other universe, the parcels on the assessment roll which did not sell on the market during the relevant period.

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[86] Leslie E. Corbert, "An Appraisal of Sales Ratio Studies," National Tax Association, Proceedings of the 51st Annual Conference, 1958, pp. 107-114.

[87] Ibid., p. 116.

[88] Op. cit., National Tax Association, Proceedings of the 51st Annual Conference, 1958, pp. 335-344.

I. Uniformity of Assessment and Equalization

Assessment equalization is the process of assuring comparability of general levels of assessment among assessment jurisdictions.[89] An assessment jurisdiction is the geographical area of the assessment roll. Without going into the economic welfare aspects of equalization we can state the two most common purposes of equalization. Equalization is necessary for the equitable impact of a property tax where fiscal jurisdiction boundaries are not co-terminous with assessment jurisdiction boundaries, or where several assessment jurisdictions constitute a larger fiscal jurisdiction. A fiscal jurisdiction is the area over which a public body has the power to levy rates, or to cause another public body to levy rates to produce revenue on its behalf. Municipalities, cities, towns, school boards, water boards, counties, townships, states, and provinces are fiscal jurisdictions.

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[89]The terms "internal equalization," "valuation equalization," or "intra-jurisdictional equalization" mean "uniformity of assessment" in some cases and equalization among classes of property within an assessment jurisdiction in other cases. The terms used for "equalization" as it is used in the text are: "external equalization"; "apportionment equalization"; and "interjurisdictional equalization."

The second purpose of equalization is to measure the fiscal capacity of jurisdictions where they receive grants, based on their taxable property value, from superior fiscal jurisdictions, or where their debt capacity is limited by statute to some measure of the fiscal capacity. A more comprehensive list of purposes is provided by Schwinden.[90]

Equalization, per se, is not within the subject matter realm of this study.[91] In 1886, Mathew, J. recognized the difference between equalization and uniformity of assessment.[92] Commenting on adjusting the total of assessed values on a valuation list without adjusting individual assessments on the list, so that the total of individual parcels was not in agreement with the grand total he said:

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[90]James Schwinden, "Real Property Assessment Policy and Practice in Minnesota" (unpublished Ph.D. dissertation, Department of Agriculture, University of Minnesota, 1961), pp. 22-24.

[91]Equalization is concerned with the level of assessment, the measure of central tendency, while uniformity of assessment is concerned with the measure of dispersion of individual assessment-sales ratios about the central tendency.

[92]R. vs. The Justices of General Assessment Sessions for the Metropolis, 1886, Q.B. 17, p. 394.

> But the result of increasing total values leaving the particular assessments untouched would only be to increase the sum to be contributed by the parish (to a Metropolitan government) and to compel the parish authorities to make higher rates.

Equalization was practised in the second quarter of the Sixteenth Century. Under the Poor-Law Rates established by Richard II in 1388 as amended by Henry VII in 1495 and by Henry VIII in 1530-31 and in 1535-36, the latter amendment required that, "the officers of each hundred and corporate town were apparently intended to exercise a general supervision, and to distribute the 'overplus' of the collections in wealthy parishes among the poorer parishes."[93]

However, since the accuracy of the measure of the level of assessment on a roll depends upon the variations among assessment-market value ratio of properties on the roll, the success of an equalization programme is dependent on the success of achieving uniformity within the assessment jurisdiction.[94]

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[93]Edwin Cannan, The History of Local Rates in England (2nd ed., London: P. L. King and Son, 1927), pp. 54-57, esp. p. 57.

[94]James Schwinden, op. cit., pp. 36-45. Schwinden after an elaborate mathematical deduction concludes that equalization depends on assessment uniformity; Edwin R. A. Seligman, Essays in Taxation (6th ed., London: Macmillan and Company Ltd., 1909), p. 26. Seligman says that where

Where the equalizing body has, de facto and not just de jure, authority over the level of assessment on each valuation list, equalization is automatic in the sense that a high measure of uniformity implies that the stipulated level of assessment is maintained.

Where the equalizing body has no responsibility for the level of assessment on each valuation list[95] they, in order to measure and compare fiscal capacities, measure the level of assessment on each roll. Some measure of the assessment-sales ratio central tendency is usually used. The Committee on Sales Ratio Data prefer a

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assessment rolls are inaccurate "boards of equalization are of no avail"; National Tax Association, op. cit., Proceedings of the 51st Annual Conference, 1958, pp. 317-321; M. S. Kendrick, Public Finance, Principles and Problems (New York: Houghton-Mifflin, 1951), p. 224. "The principal need in the administration of the property tax as an equitable original assessment of real estate"; H. M. Groves, Financing Government (3rd ed.; New York: Henry Holt and Company, 1950), p. 59. After discussing equalization he states, "Much more serious, usually, are the inequalities among taxpayers in the same district"; J. P. Jensen, Government Finance (New York: Lomas, Y. Crowell Company, 1937), p. 259. "The only way to equalize assessments is to make them equal in the first place"; William G. Murray, "Improvement in Real Estate Taxation through Assessment-Sales Studies," National Tax Journal, Vol. 5 (1952), p. 86. "Two facts are clear: first, assessment inequality is a major problem; and second, the place to begin is in the local district--internal equalization should come before external equalization.

[95] "Valuation list" is the British term for "assessment roll."

weighted median over the unweighted median, and the unweighted median over the arithmetic mean.[96] Other bodies prefer the weighted mean.[97] The Committee on State Equalization of Property Tax Assessments prefers the weighted mean of a random sample of the assessment roll.[98] The accuracy of any measure will depend on the assessment uniformity, because the greater the variability of individual members in a population the less accurate is the estimate of any measure of central tendency derived from only a portion of the universe of assessment-market value ratios. Uniformity of assessment is even more important when the equalizing board uses market generated sales to measure the level of assessment, because, as pointed out in the previous section, these sales do not constitute an unbiased random sample required for statistical inference. If there is a high measure of uniformity then the effect of the sampling bias on the estimate will be minimized.

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[96]National Association of Tax Administrators, op. cit., pp. 22-26.

[97]United States Bureau of the Census, Taxable Property Values, loc. cit.

[98]National Tax Association, Proceedings of the 51st Annual Conference, 1958. A measure based on a random sample requires the equalizing body to carry out independent appraisals.

J. Summary

Even with the narrow approach taken in the above discussion, the concepts considered can be brought to bear upon the many problems associated with the legislation and the administration of the real property tax. The problem examined in this paper, measuring quality of assessment, is only one of the administrative problems. Therefore if will be useful to summarize, from a point of view to be used in the following chapters, the concepts as they bear upon the measurement of assessment uniformity.

1. Assessment officers are instructed to appraise at market value.
2. Property transactions which occur within the conditions circumscribed by the definition of market value produce market prices. These market prices are the ultimate evidence of market value, and can be estimated within a reasonable margin of accuracy by appraisers.
3. The opinion here, and this is a value judgement, is that a valuer's estimate of market value should be within ten per cent either side of the market price generated by a sale which occurs under the conditions circumscribed by the definition of market value.

4. Because land and improvements may be assessed and taxed as separate items, the measure of uniformity of assessment is not a measure of the uniformity of the tax impact distribution.
5. The definition of market value implies that the financial arrangements accompanying the transaction be those which would not be reasonably expected to distort the sale price from market price in the current market. Before they are used as evidence of market value, prices from sales involving unusual financing should be adjusted to remove the effects of the unexpected financial arrangements. An alternative would be to reject entirely the sale price as evidence of market value. This latter alternative is preferable when the effect of the financing on the sale price is not exactly known.
6. Over and under assessment may affect the sale price of a property. Through what mechanism, capitalization or "budgetizing," or to what extent the effect takes place is not altogether evident. But the results of the process at its extreme do not alter significantly the observed assessment-sales price ratio from the assessment-market price ratio.

7. The courts have not made clear whether uniformity of assessment takes precedence over correct valuation. The interpretation here is that since overall correct valuation will produce uniformity of assessment, the quality of an assessment roll should be judged upon the proportion of assessments within ten per cent of market price. In other words, correct valuation rather than uniform error is the criterion by which to evaluate the quality of an assessor's valuation list. Yet, some recognition must be given to uniform assessment at some other than the statutory level.
8. In any given time period properties traded on the market do not constitute a random sample of the assessment roll. The assessment-market price ratios so provided, therefore, cannot be used to draw direct statistical inferences about the assessment roll.
9. Correct valuation of all properties in all assessment jurisdictions creates equalization. Effective equalization is still possible where assessment uniformity prevails within each jurisdiction while the level of assessment varies among jurisdictions. Since uniformity of assessment rests upon correct valuation, there can be no substitute for accurate appraisal as the goal of the assessor.

CHAPTER III

UNIFORMITY OF ASSESSMENT IN BRITISH COLUMBIA

The stage having been set, an empirical examination of current measures and standards of assessment uniformity can be started. Measures suggested by the Assessment Commissioner of the Province of British Columbia will be used.[1] These measures are similar to those suggested by the National Association of Tax Administrators,[2] and used elsewhere.[3] But before proceeding to the study of those measures the provisions of the Assessment

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[1]Province of British Columbia, Appraisal Manual, pp. 308-315.

[2]Committee on Sales Ratio Data, Guide for Assessment-Sales Ratio Studies, Report to National Association of Tax Administrators (Chicago: Federation of Tax Administrators, 1954), pp. 52-56.

[3]United States Bureau of the Census, Taxable Property Values, 1962, op. cit., p. 13; J. Edward Rountrey, "Equalization at Market Value," The Appraisal Journal, Vol. 24 (1956), pp. 219-222; Dudley W. Johnson, "A note on Local Administration of the Property Tax: A Case Study of Bethlehem, Pennsylvania," National Tax Journal, Vol. XI (1958), p. 269; Roswell G. Townsend, "Inequalities of Residential Property Taxation in Metropolitan Boston," National Tax Journal, Vol. IV (1951), p. 364.

Equalization Act[4] which relate directly to this study will be considered.

A. Assessment Equalization Act

"There shall be an Assessment Commissioner"[5] who is required, amongst other things, to "give advice and assistance to (Municipal) Assessors for the purpose of securing uniformity in land and improvements within the Province."[6] There can be little doubt that the Legislature intended to assure uniformity of land assessment and improvement assessment, but as argued in Chapter II[7] only the assessed value of the "whole" can be subject of an objective audit.

The Commissioner shall equalize the level of assessment[8] for "real-property taxation only"[9] for the purpose of school tax, and may direct an Assessor to equalize assessments among classes of property, or individual parcels, within the Municipality.[10] In other

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[4]British Columbia, Revised Statutes, 1960, c. 18.

[5]Ibid., sec. 3.

[6]Ibid., sec. 7(b).

[7]Supra, pp. 27-29.

[8]Assessment Equalization Act, op. cit., sec. 8(3).

[9]Ibid., sec. 8(5).

[10]Ibid., sec. 8(4).

words, equalization is to be achieved through Province-wide uniformity of assessment, but this uniformity is, as far as the legislation under consideration is concerned, to apply only for rates levied to raise school revenues. No direct provision has been made for the Provincial Assessment Commissioner to ensure that municipal general revenue rates will be levied on uniform assessment, and it does not appear that the Commissioner can, in the sense of measuring levels of assessment, equalize among classes of property, or individual parcels, for the general purpose levy. But, where a municipality selects the same basis of assessment for the general purpose levy as that used under the school levy, any discrepancy between school purpose and general purpose assessments would be untenable.[11] Since it is the practice in British Columbia for the municipality to use only the one basis of assessment, the Provincial Assessment Commissioner's de jure authority over the uniformity of school purpose assessment roll becomes de facto authority over the uniformity of the general revenue assessment roll.

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[11] Some discrepancy would arise because of different tax base definitions, but these would occur only in respect of farm properties and landlord and tenant equipment and fixtures in business premises.

B. Collecting Sales Data for Equalization

The Assessment Commissioner is interested primarily in equalization among jurisdictions, and for that purpose he needs assessment-sales ratio data to measure levels of assessment. He and the Assessors can use the same data to measure assessment uniformity.

Sales information is collected by the Land Registry Offices.[12] A real Property Transfer Record, Form A.C.2, must be completed with each voluntary transfer of title, grant of right to purchase or assignment of right to purchase. The questionnaire is included as Exhibit I in Appendix A. The A.C. 2 is then sent to the Assessment Commissioner who examines all sales rejecting those which he decides will not represent market price.[13] The Assessor is then sent a Request for Sales Analysis Records, Form A.C. 7 (Exhibit II), for those sales which are deemed to have taken place within the conditions circumscribed by

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[12]British Columbia, Revised Statutes, 1960, c. 208, "Land Registry Act." In British Columbia a modified Torren's system of land registration is used. All interests in land must be registered in order to protect the claim of the owner.

[13]The Assessor is also sent a Notification of Forms A.C. 2 Rejected, Form A.C. 19. The sales reported on this form are, at the discretion of the Assessor, used for assessment sales ratio studies and appraisal material at the local level only. The Assessor being closer to the local market can more easily adjust sales prices to market prices.

the definition of market value.[14] The Assessor then completes a Sales Analysis Record, Form A.C.3, sends one copy to the Provincial Assessment Commissioner, and keeps one copy for his own assessment-sales ratio studies and appraisal records. The Commissioner on the basis of his own investigation, the information on the A.C.3 Form, or advice from the Assessor may reject the transaction as not being representative of market price. The Assessor, of course, may make use of the transaction for his own information.

The Sales Analysis Records of transactions not rejected are used by the Commissioner for the purpose of measuring the level of assessment for the equalization. What measure he uses is not apparent to us, but in the Appraisal Manual he does recommend the median as being the best measure of central tendency.[15] Measuring the

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[14] See page 145 in Appendix A for the details of the editing procedure. Appendix A is an excerpt from the Province of British Columbia, Appraisal Manual, Assessment Equalization Act, pp. 301-321. This portion of the Manual is entitled, "Statistical Analysis and Control."

[15] Appraisal Manual, op. cit., p. 308. The level of assessment of a jurisdiction or particular class of properties may be measured. In fact, the jurisdiction level may be built up from the levels of assessment of any number of classifications of property within the jurisdiction. For an example of this method, see Committee on Sales Ratio Data, op. cit., p. 24.

level of assessment is not directly our purpose, but because the accuracy of this measure depends upon uniformity a short comment on this topic will be made.[16]

If the assessments are highly uniform, and if the deviations of assessed value from the median are random, and, therefore, distributed evenly on both sides with no bias to high or low value properties, then the median is a satisfactory measure of the level of assessment. When the above conditions are not satisfied the simple arithmetic mean or the mode are not much more accurate. More accurate measures of the level of assessment can be made with ratio estimates,[17] and they do represent the weighted arithmetic mean. The weighted mean does account for any bias between low and high values properties.

C. Measuring Assessment Uniformity

While measures of the level of assessment are based upon central tendency, the measures of uniformity of assessment are based upon dispersion of individual ratios about the measure of central tendency.

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[16]Some measure of the level of assessment is of indirect interest, because a measure of the uniformity of assessment will depend upon the variation of the assessment-market price ratios from this level.

[17]William G. Cochran, Sampling Techniques (2nd ed.; New York: John Wiley and Sons Inc., 1963), pp. 154-156.

The procedures followed by different bodies are the same, while the measures of uniformity used vary, but only slightly.[18] After the assessment-sales ratio data have been collected and edited, the ratios to be used are arranged into a frequency distribution. The central tendency is then determined, be it unweighted median, unweighted arithmetic mean, or mode. Next the dispersion is measured.

For the purpose of the discussion, and unless otherwise indicated the designated symbols will be defined as follows:

M - measure of central tendency selected

f_i - the frequency of the ith interval

x_i - mid-point of the ith interval

Q_i - the ith quartile

d - the mean deviation

s - the standard deviation

R_d - Russell "Index of Assessment Inequality"[19]

q - semi-quartile deviation

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[18]For procedure and methods see Appraisal Manual, op. cit., pp. 308-321; Committee on Sales Ratio Data, op. cit., pp. 21-29 and 52-56.

[19]Rountrey, op. cit., p. 221 has attributed the development in 1939 of the "Index" to Russell; United States Bureau of the Census, Taxable Property Values, 1962, p. 13. This study uses this measure and has used M as the median.

$$d = \frac{\sum_i \left| f_i(x_i - M) \right|}{\sum_i f_i}$$

$$q = (Q_3 - Q_1)/2$$

$$s = \left(\frac{\sum_i f_i(x_i - M)^2}{\sum_i f_i} \right)^{1/2}$$

$$R_d = 100(d/M)$$

Other measures similar to R_d are:

$$R_q = 100(q/M)$$

$$R_s = 100(s/M)$$

The first measure, R_d, is called the coefficient of dispersion and the last measure is the coefficient of variation or the relative standard deviation.[20] The supposed advantage of R_d, R_q, and R_s over d, q, and s respectively as measures of assessment uniformity is that

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[20] Appraisal Manual, op. cit., p. 314.

they allow assessment uniformity comparisons among valuation lists which have different average levels of assessment. The basic principle is that an assessment of 150% market value on a roll which has an average assessment level of 100% is relatively no more grievous than an assessment of 75% market value on a roll where the assessment level is 50% market value.

The advantage, unless an assumption about the shape of the frequency distributions is made, is only apparent, and not real. For example, assume M, the simple arithmetic mean, is the same for two distributions so that the size of R_d is dependent only upon d. Figure I shows two distributions which have the same mean and same mean deviation but are skewed in opposite directions.[21] R_d would be the same for both.[22]

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[21]Frederick E. Croxton, Elementary Statistics with Applications in Medicine and the Biological Sciences (New York: Dover Publications, Inc., 1959), p. 74. The relative positions of the mean, mode, and median of a skewed frequency distribution are presented.

[22]In this example, though the distribution of the tax impacts is different as between A and B, no case can be made for a difference in quality as between A and B. An examination of Dr. Russell's results in Appendix B will indicate the weakness of this type of measure of assessment uniformity.

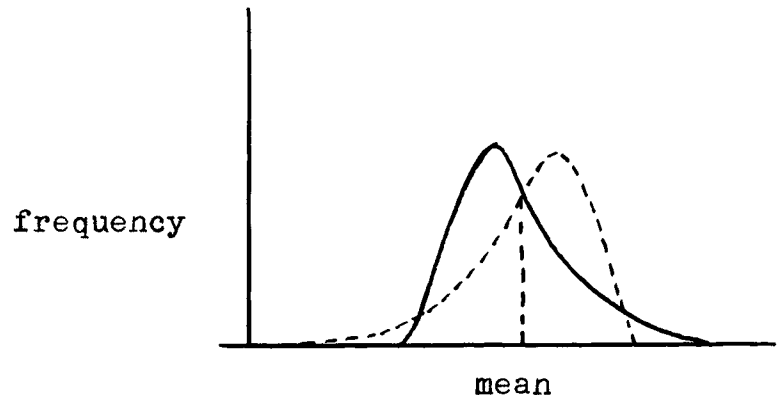

FIGURE 1

TWO SKEWED DISTRIBUTIONS

The distributions illustrated in Figure 1 will also have equal standard deviations and equal semi-quartile deviations. Figure 2 presents the nature of two distributions which are symmetrical and which have the same mode, mean, and median. Only by remote coincidence will any two of the three measures of dispersion be equal for both distributions. But, it is more likely that any one measure of dispersion will give the same value to the "Index of Assessment Inequality" for each distribution.

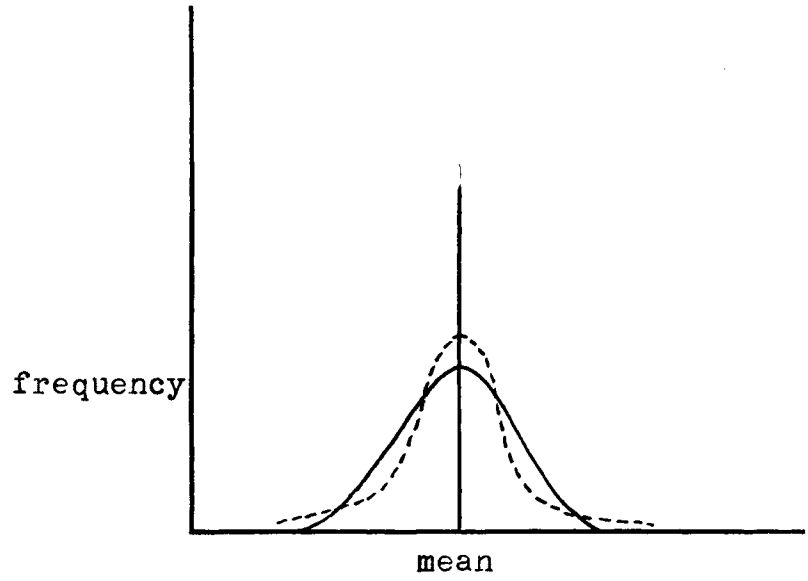

FIGURE 2

TWO SYMMETRICAL DISTRIBUTIONS

Without foundation R_d, R_q, and R_s are used as measures of assessment uniformity. In other words, in order to measure uniformity of assessment and compare the results with a selected minimum standard of uniformity or with the results from previous years of from other jurisdictions, it is necessary to make some assumption about the underlying form of the frequency distribution of assessment-sales price ratios. Further, for the analysis to be valid the actual distribution of assessment-sales ratio must fit the assumed distribution on which the measure of uniformity is based. When neither of these conditions is satisfied R_d, R_q, or R_s, or alternatively

d, q, or s, may be calculated but the results have little practical interpretation.

D. The Normality Assumption

Though Dr. Russell did not make his assumptions explicit he did suggest in order to more accurately judge the uniformity of assessment that the percentage of the sample assessment-sale price ratios within plus and minus 20 per cent of the average ratio from the average ratio be considered along with the "Index of Assessment Inequality."[23] Under the normal distribution the mean and the coefficient of dispersion fully specify the proportion of the distribution within the range of plus or minus 20 per cent of the mean from the mean.

It is evident then that Russell did not assume that the distribution of assessment-sale price ratios would be statistically normal. If assessment-market price ratios were normally distributed it would not be possible for an assessment roll with a low coefficient of dispersion to have a smaller proportion of ratios within plus and minus 20 per cent of the average ratio from the average ratio than an assessment roll with a higher coefficient

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[23]John H. Russell, "Inequality of Real Estate Assessments within Political Subdivisions," The Commonwealth Magazine (Virginia), 1939, p. 18.

of dispersion. Therefore, under normality conditions only the "Index of Assessment Inequality" is required to observe the uniformity of assessment.

That distributions of assessment-market price ratios are not normal is evidenced by Russell's results included in Appendix B.[24] These results are inconsistent with a normal distribution of assessment-market price ratios.

It is eminently clear that Dr. Russell did not assume the normality of distributions of assessment-sale price ratios, but as will be exhibited in the following paragraphs this assumption has been made in his name by other workers in the field.

The National Association of Tax Administrators[25] have not made explicit or implicit their assumptions.

The Province of British Columbia Assessment Commissioner has not made his assumptions explicit, but there are a few passages in the Appraisal Manual which indicate his assumptions. Whether or not he made the assumptions realizing their implications is not apparent.

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[24] Ibid., p. 18.

[25] Committee on Sales Ratio Data, op. cit.

The Commissioner prefers the use of the standard deviation over that of the mean deviation.[26] Since Russell's index, the coefficient of dispersion, is expressed in terms of the mean deviation, the Assessment Commissioner, in order to relate the coefficient of variation to Russell's suggested maximum value for his index, expressed the quantitative relations between mean deviation and standard deviation, and between the coefficient of dispersion and coefficient of variation.

> Comparisons of the relationships between the various absolute measures are as follows:--
>
> Range: Coarse calculation only; no definite relationship to other measures.
> Quartile deviation 0.6745 standard deviation 0.8453 average deviation.
> Average (mean) deviation 0.7979 standard deviation.
> Standard deviation 1.2533 average deviation 1.4826 quartile deviation.
>
> The coefficient of dispersion is the "index of assessment inequality" referred to by the late Dr. John H. Russell, former Director of Research, Virginia Department of Taxation. His recommendation was that a coefficient of dispersion of "20 should be considered a goal desirable of achievement and reasonably attainable," and that anything below this is to be considered as an excellent degree of equalization or uniformity. Conversely, he stated "an index as high as 45 should be judged cause for gravest concern."[27]

Accepting Dr. Russell's attainable limit of 20, and knowing that the average deviation on which it

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[26] Appraisal Manual, op. cit., p. 312.

[27] Ibid., p. 314.

> is calculated is 0.7979 standard deviation, then the corresponding coefficient of variation would be 20 ÷ 0.7979 or 25 per cent. Or, expressed another way, as the standard deviation is 1.25 average deviations, then the coefficient of variation will be 1.25 times the equivalent coefficient of dispersion. Thus a coefficient of variation of 25 should be "desirable of achievement and reasonably attainable" and a coefficient of variation of 56 should be "cause for the gravest concern." With an average assessment level of 50 per cent a coefficient of variation of 25 would mean that about 67 per cent of the items should have ratios between 37.5 and 62.5 per cent.[28]

First, in order to derive the relation between s and d the function of the frequency distribution must be known.

Second, in the first quotation above he says that the standard deviation equals 1.2533 times the average deviation. Demonstrated below is the derivation of that equality. The calculations are based on an assumption of a normal distribution.

For convenience only the normal distribution used is centred at the origin. This causes the mean to equal zero. A transposition of coordinates does not alter the fundamental relations between the parameters of a function, but it often can help emphasize the results of the calculations, rather than the calculations being emphasized.

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[28] Ibid., p. 314.

For any frequency distribution

$$d = \frac{\sum_{i=1}^{n} f_i(x_i - M)}{\sum_{i=1}^{n} f_i}$$

where M is the mean. M is to equal zero.

If N = total number of elements in all intervals, then

$$\sum_{i=1}^{n} f_i = N$$

Now write

$$d = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \sum_{i=1}^{n} (f_i/N)x_i$$

$$d = \sum_{i=1}^{n} p_i x_i \text{ , where}$$

$$p_i = f_i/N$$

Now, p_i (i = 1, 2, ... n) is the relative frequency distribution and d, the mean deviation, is expressed in terms of p_i and x_i. p_i (i = 1, 2, ... n) is also called the density function.

The normal distribution is a density function of a given form where n tends to infinity and the width of the class interval tends to zero forming a continuous distribution as compared to the discrete frequency function.

The normal density function is $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$, where σ is the standard deviation.

Therefore, d_N, the mean deviation for the centred normal distribution is

$$d_N = \int_{-\infty}^{\infty} xy\,dx = \int_{-\infty}^{0} xy\,dx + \int_{0}^{\infty} xy\,dx$$

since $y = f(x)$ is symmetrical about $f(0)$

$$d_N = 2\int_{0}^{\infty} xy\,dx$$

$$d_N = 2\int_{0}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}\,dx$$

$$d_N = -2 \left[\frac{\sqrt{\cdot}}{\sqrt{2\pi}} e^{-x^2/2\sqrt{}^2} \right]_0^{\infty}$$

$$= \sqrt{\frac{2}{\pi}} \sqrt{}$$

or $\sqrt{} = \sqrt{\frac{\pi}{2}} \; d_N$

$\sqrt{\pi} = 1.772453851$

$\sqrt{2} = 1.414214$

$\sqrt{\frac{\pi}{2}} = 1.253313749$

Therefore, $\sqrt{} = 1.2533 d_N$

Third, the Assessment Commissioner says, "With an average assessment level of 50 per cent a coefficient of variation of 25 would mean that about 67 per cent of the items should have ratios between 37.5 and 62.5 per cent."[29]

In other words $R_d = 25.$

Therefore, $\sqrt{}/M = 0.25$

$M = 0.5$

and $\sqrt{} = 0.125.$

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[29] Appraisal Manual, loc. cit.

The Commissioner in effect has said that

$$\int_{.375}^{.625} \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{0.125} \cdot e^{-(x-.5)^2/2(.125)^2} dx \doteq 0.67$$

If the integral of the normal density function is evaluated between the indicated limits the result would again demonstrate that the normal distribution has been assumed.

A much more simple approach is to standardize the normal distribution ($M = .5$, $\sqrt{\ } = .125$) to the normal distribution ($M = 0.0$, $\sqrt{\ } = 1.0$), then determine from already tabulated values of the integral whether or not 67 per cent of the ratios would lie between 37.5 and 62.5 per cent of market value.

Let z be the standardized normal deviate.

$$z = \frac{x - M}{\sqrt{\ }}$$

$$z = \frac{x - .5}{.125}$$

Now, find in tables of the standardized Normal Distribution the values of z, centred about the mean, and between which 67 per cent of the area lies. These values are minus 0.984 to plus .984.

$$x = .125 + 0.5$$

Therefore the limits between which 67 per cent of the assessment-sales ratios will lie are 37.7 per cent and 62.3 per cent.

There can be little doubt that the assumption underlying the use, in British Columbia at least, of the coefficient of variation for measuring uniformity of assessment is that the assessment-sales ratios are distributed normally.

Is this assumption justified? Deductive reasoning would not lead to an affirmative answer. An assessment-sales ratio cannot be less than zero, but there is no upper limit. Therefore, this observation would indicate a distribution skewed to the right. Also, the "capitalization effect" or "budget effect"[30] might tend to skew the distribution to the right. Where properties are under-assessed there is not likely to be as large a divergence between market price with incorrect assessment and market price with correct assessment as where properties are over-assessed. It is reasonable to suppose that purchasers of under-assessed properties, in the expectations of increased future assessments, will not "budgetize" the tax saving, while purchasers of over-assessed properties

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[30] Supra, pp. 35-41.

in the expectation of not being successful in reducing the future assessment or the cost involved in doing so, will "budgetize" a larger portion of the additional taxes. As illustrated in Chapter II[31] the "capitalization effect" and "budget effect" tend to accentuate the difference, as measured by the ratio of assessed value to sale price, between the assessment level of an incorrectly assessed property and the average level of assessment. When the property is over-assessed, the assessment-sale price ratio is greater than the assessment-market price ratio when the property is correctly assessed; when the property is under-assessed, the assessment-market price ratio is smaller than the assessment-market ratio when the property is correctly assessed. Because it might be expected that the "budget effect" occurs more often and to a greater extent on over-assessed properties, the net result should be a distribution of assessment-sales ratios skewed to the right. Assuming for the moment that the distribution of assessment-market price ratios is normal then the resultant effect on the distribution of assessment-sales price ratios is as illustrated in Figure 3.

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[31] Supra, pp. 38, 40.

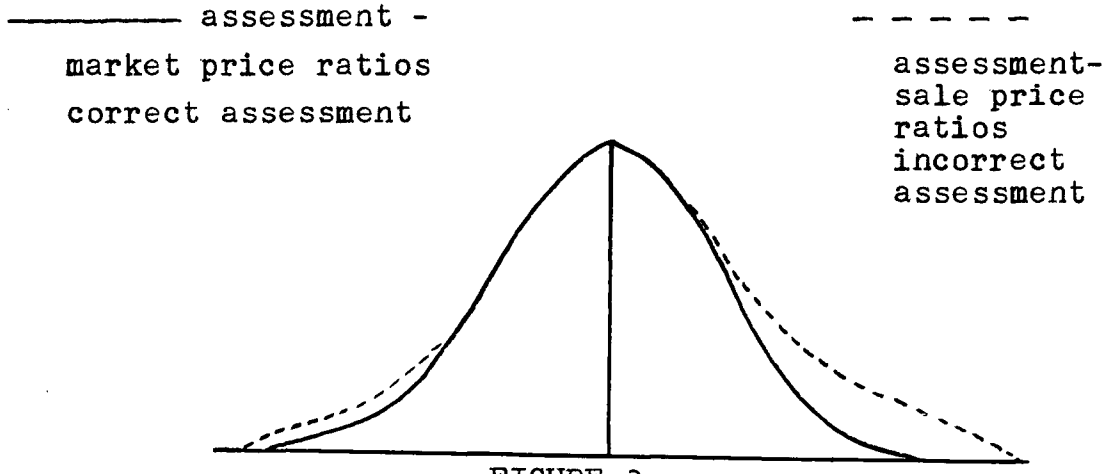

FIGURE 3

"BUDGET EFFECT" ON DISTRIBUTION OF ASSESSMENT-SALE PRICE RATIOS

E. Testing the Normality Assumption

The normality assumption can be tested empirically. This will be done below. The method and procedure are outlined here. November 1, 1964 to June 30, 1965 single and two-family residential property sale prices were collected from the Vancouver Multiple Listing Service records.[32] Only City of Vancouver sales were used. The

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[32]The transfer data collected by the Assessment Commissioner is confidential and, therefore, not available for this study.

The period November 1, 1964 to June 30, 1965 was selected because the assessment roll is "brought in" early in the calendar year. The sales in the designated period should be "timely" in respect of the assessed value. The

sales were edited to remove any transactions which did not occur under conditions of market value. Because the properties were sold by an agent the transactions will automatically be at arms-length. Therefore, the editing process was not required to reject the non-bona fide and non-voluntary transfers from the study.[33] The assessed values obtained from the City of Vancouver 1965 valuation

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ratio of listing price to selling price may be used as an indicator of market price trend. For the sales occurring during each month of the study period the ratio of total listing price to total sales price was calculated before the data were edited. The evidence below is not conclusive, but the lack of a sustained change in the ratio does support the contention that market prices were not changing rapidly over the period November 1, 1964 to June 30, 1965.

Month of Sale	Total of Listing Price / Total of Sales Price
November	1.069
December	1.071
January	1.062
February	1.060
March	1.067
April	1.072
May	1.071
June	1.071

[33]The following transfers were not included in multiple listing sales.

Changes of executors or trustees.
Transmissions by death or foreclosure.
Quit-claims.
Transfers to a beneficiary under a will.
Transfers between relatives or associated corporations.
Transfers to any church, lodge, school or charitable, benevolent, fraternal, or government organization.

list. The assessment sales ratios were calculated and arranged into a frequency distribution.[34] Then the frequency distribution was tested for "goodness-of-fit" to a normal distribution. The Chi Square Distribution will be used for the test.[35]

Transfers the circumstances of which are unusual and as a result render the transaction useless for sales analysis.
Transfers agreed upon some time prior to the year in which they were recorded.

The following sales were edited out of the study.

Transfers involving unusual financial provisions --cash to vendor less 25% of sale price or take over mortgage less than 40% of sale price.
Transfers involving trades.

[34]The class intervals of the "fitted" frequency distribution are nine in number and have the following limits and mid-points.

Class Interval	Mid-point
- .13085	.065425
0.13085 - .23085	.18085
0.23085 - .33085	.28085
0.33085 - .43085	.38085
0.43085 - .53085	.48085
0.53085 - .63085	.58085
0.63085 - .73085	.68085
0.73085 - .83085	.78085
0.83085 +	.905425

Since the assessed values are recorded on the roll to the nearest dollar, accuracy in the calculation of the assessment-sale price ratio should be to the nearest ten thousandth.

[35]The objective of this paper is not to measure the uniformity of a particular assessment roll. This result is merely a by-product of the analysis of a statistical measure of uniformity and quality of assessment. Therefore, the selection of the data is subsidiary to the observation of the conclusions about uniformity of assessment which are drawn from the application of a statistical method to the data.

Sampling theory and the theoretical background to the use of χ^2 density function are not of direct interest in the study, but in order that the results may be properly interpreted some aspects of these topics should be considered here.

Included in the statistical analysis will be all sale transactions which have resulted directly in a market price during the study period.[36] The assessment-sales price ratios will be treated as a sample from an infinite universe that would be generated if the forces that gave rise to the sample were to operate indefinitely without change in character.

There are three alternative approaches to this point. First, all of the assessment-market price ratios could be treated as a finite universe, and all of the available ratios could be included in the analysis. No

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[36]There are in effect three universes of assessed value-market price ratios. One universe consists of assessed value-market value ratios of properties not sold on multiple listing. They may or may not have sold during the study period. The second universe consists of assessed value-market price ratios of properties that sold on multiple listing during the study period but which were edited out of the sample. The third universe consists of assessed value-market price ratios which are generated under current market conditions through multiple listing sales. A sample from the last universe is not representative of either the first or second population. Forces such as location or type of property which generate the third universe may be different from those which generate the first or second universe.

sampling would be involved. Second, using the infinite population model, a random sample could be drawn from the available assessment-sales price ratios. Third, the available ratios could be viewed as a finite population from which a random sample is then drawn. The third alternative is the sampling model analogous to the first alternative. The second and third alternatives are useful where the volume of data available is so great that a greater degree of accuracy can be expected from the more detailed attention given to a sample than could be given to all the data available.

If:

f_j denotes the observed frequency in a class interval

F_j denotes the expected or theoretical frequency in a class interval

$j = 1, 2, \ldots n-1, n.$

$F_j \geq 5$ for all j

$$\sum_{j=1}^{n} f_j \geq 50^*$$

---

* $\sum f_j$ will imply $\sum_{j=1}^{n} f_j$ in the remainder of this paper.

then the non-parametric statistic

$$X^2 = \sum_{j=1}^{n} \frac{(f_j - F_j)^2}{\sum F_j}$$

will be distributed as χ^2 with m degrees of freedom.[37] The number of degrees of freedom is equal to the number of squared normal variates in the sum deriving the value X^2 less the number k, of linear constraints on that sum. In other words

$$m = n - k.$$

In this study n = 9 and k = 3. There are nine class interval and three linear constraints. In order to "fit"

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[37] A random variable has a χ^2 distribution with m degrees of freedom if it has the same distribution as the sum of the squares of m independent standardized normal variates. Where f_j varies randomly from F_j, as it should in an ideally uniform assessment roll:

$(f_j - F_j)$ is a normal variate

$(f_j - F_j)/F_j$ is a standardized normal variate

$(f_j - F_j)^2/F_j$ is a squared standardized normal variate.

Therefore, the probability integral of the χ^2 distribution can be used to measure the probability levels of X^2.

The density function of the χ^2 distribution is given by

$$g(\chi^2, m) = \frac{1}{2^{m/2}\Gamma(m/2)} \cdot (\chi^2)^{\frac{(m-2)}{2}} \cdot e^{-(\chi^2/2)}$$

Values of the probability integral are tabulated.

the data to a normal distribution M and s are calculated from the data, and Σf_j must equal ΣF_j, therefore decreasing the degrees of freedom by three from nine.

In order to apply this "goodness-of-fit" test, F_j is determined from the probability integral of the normal density function with mean, μ, equal to the estimator M and the standard deviation, σ, equal to $\hat{s}$, its unbiased estimator.[38]

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[38] Previously, three alternatives to the sampling model used in this paper were discussed. Since in the first alternative no sampling is involved

$$\mu = M$$
$$\sigma = s.$$

Where sampling is used, such as in the second and third alternatives, as well as in the model used in the text:

M is an estimation of μ

s is used to derive the estimator, $\hat{s}$, of σ

. Also, in sampling models where M is the estimator of μ, the limits within which μ will with a given probability be found are of interest. M, the sample mean, can be treated as a single member of an infinite population of sample means. The distribution of sample means will have a mean equal to μ, estimated by M, and a standard deviation

$\sigma_m = \sigma/\sqrt{\Sigma f_j}$, estimated by $\hat{s}_m = \hat{s}/\sqrt{\Sigma f_j}$ in the case of alternative two and in the sampling model of this paper; and in the case of alternative three, by

$$\hat{s}_m = \frac{\hat{s}}{\sqrt{\Sigma f_j}} \cdot \sqrt{\frac{N - \Sigma f_j}{N}}$$

N is the number of members in the finite population.

σ_m and $\hat{s}_m$ are often defined respectively as the standard error of the mean and the estimated standard error of the mean. Because the distribution of sample means is often normally distributed, even when the population distribution is not normal, $\hat{s}_m$ is used to calculate the confidence of limits for M as an estimator of μ.

$$\hat{s} = \frac{s\sqrt{\sum f_j}}{\sqrt{(\sum f_j) - 1}}$$

$$F_j = \left[\sum f_j\right]\left[\int_{\alpha_{j-1}}^{\alpha_j} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-\mu)^2/2\sigma^2} \cdot dx\right]$$

where α_j is the upper limit of class interval j.

In practice the probability integrals of the standardized normal distribution are used. If z represents the standardized normal variate, then

$$z = \frac{x - \mu}{\sigma}$$

$$z_{\alpha_j} = \frac{\alpha_j - \mu}{\sigma}$$

$$z_{\alpha_{j-1}} = \frac{\alpha_{j-1} - \mu}{\sigma}$$

and $dz = \frac{1}{\sigma} dx$

Therefore, $F_j = \left[\sum f_j\right]\left[\int_{z_{\alpha_{j-1}}}^{z_{\alpha_j}} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \cdot dz\right]$

Values of $\int_{-\infty}^{z_{\alpha_j}} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \cdot dz$

are tabulated and presented in Appendix C.

$$\text{If } A_j = \int_{-\infty}^{z_{\alpha_j}} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \, .dz$$

$$\text{then } F_j = \left[\sum f_j\right] \left[A_j - A_{j-1}\right]$$

where A_j and A_{j-1} are read from the prepared tables of the standardized normal distribution.[39]

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[39]Some tables contain evaluations of

$$B_j = \int_{0}^{z_{\alpha_j}} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \, dz$$

where $z_{\alpha_j} \geq 0$

Since the standardized normal distribution is symmetrical

$A_j = B_j + 0.5$ when $\alpha_j > \mu$

$A_j = -B_j + 0.5$ when $\alpha_j < \mu$

$A_j = B_j$ when $\alpha_j = \mu$

F. The Universe of Assessment-Market Price Ratios

The parameters of a particular population are to be estimated. They are most often used to measure uniformity of assessment, but here they are also used to test the "goodness-of-fit" to a normal distribution. This universe has been described in general terms as the assessment-market price ratios which would be generated by multiple listing sales of single and two family residential dwellings in the City of Vancouver if the same forces which produced the sample in the selected time period were to continue indefinitely without change in character. A more detailed operational description is required at this point.

First, the multiple listing sales are concentrated more in the lower price ranges than are all of the Vancouver City residential property sales occurring during the same time period. Because, as compared to the higher price ranges, there are more houses in the lower ranges of value; there is in the lower ranges of value less variation among houses of amenities produced; and there is a higher turnover rate in the lower ranges of value, the assessment-sales price ratios generated by the multiple listing sales can be expected to be more uniform than the ratios generated by all residential sales in Vancouver City.

There are a number of multiple listing sales which involve bonused vendor financing.[40] There are, also, a number of sales which involve better than normal financial arrangements in that a higher than expected loan to value ratio is accompanied by the interest rate expected on a prime or near prime first mortgage. Because these characteristics of a transaction may inflate the sale price above the market price, the assessment-sales price ratios resulting from these transactions will be eliminated from the sample and universe. Only rarely does a purchaser take over from the vendor unfavourable financial arrangements which would tend to effect a sale price somewhat less than market value, therefore there will be no further concern with this improbable possibility.

A two step procedure was used to eliminate from the sample assessment-sale price ratios deflated below the assessment-market price ratios. First, all sales in which the cash payment to the vendor was less than 25 per

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[40] Vendor financing involved a grant of a first or second mortgage from the purchaser to the vendor or a grant of a right to purchase or a sub-right to purchase from the vendor to the purchaser. Vendor financing would offer no difficulty if the nominal yield on the debt were equal to the effective yield at which the vendor could sell the paper. But almost without exception the practice has been to finance nominally at or near the yield rate at which prime first mortgage loans are available.

cent of the sale price were eliminated. Then, for all the assessed value-sale price ratios remaining the underlying sales contract was examined.

The first step is discussed first. Elimination based on the 25 per cent rule of thumb appears to be arbitrary, but the rule does have a good foundation. Until very recently, and with a few exceptions,[41] institutional first mortgage loans were limited to 66 2/3 per cent of the lending value of the property. Therefore, a cash payment of less than 33 1/3 per cent of sale price reflects vendor financing in one of two ways. First, the seller may finance all of the purchase price save only the downpayment of less than 33 1/3 per cent. This is equivalent to an institutional first mortgage loan

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[41]Loans under the National Housing Act may be as 95 per cent of the lending value.

"Package" 75 per cent loan-value ratio first mortgage loans have for a few years been available on superior personal and real covenants. The first 66 2/3 per cent is advanced by a mortgage lending institution and the top 18 1/3 per cent is advanced by a consumer finance type of institution. These latter small loan companies are not restricted on mortgage loans in the same way that are the other institutions.

American life insurance companies may, if their legal reserve in Canada exceeds their Canadian policy reserve, invest part of the excess in that portion of a mortgage loan which exceeds the 66 2/3 per cent limit. The actual maximum loan to value ratio is limited by the state of incorporation.

plus a vendor second mortgage.[42] Second, the purchaser may take over payments on existing debt as part of the purchase price. Where the total of the existing debt plus the downpayment is less than the sale price there is vendor financing. Seldom will the total of existing debt plus vendor financing be less than the 66 2/3 per cent available from a mortgage lending institution. Therefore, the loan from the vendor is, in at least part, equivalent to junior financing.

The 25 per cent rule may reject from the sample assessment-sale price ratios which result from transactions in which the vendor financing is written at the effective market yield for the junior security. As this is seldom done, few, if any of these ratios will be eliminated.

The purchaser may take over payments on debt which is in excess of 66 2/3 per cent of market value. When the interest rate is less than the expected effective yield, then the sale price may exceed market price. Three situations in which this can occur are as follows:

1. The value of the property has decreased at a faster rate than the debt outstanding on a conventional

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[42]The downpayment may be greater than 33 1/3 per cent of sale price and the vendor may finance the balance, but this is equivalent to an institutional first mortgage loan.

first mortgage loan has been amortized.

2. A house which was built within the last few years may have a National Housing Act mortgage registered against the title. If the loan-value ratio of the existing mortgage exceeds 66 2/3 per cent then the financing is more favourable than might be expected.

3. The house purchaser, who is vendor financed, may in a short time after his purchase decide to sell his interest in the property. The vendor financing may become advantageous to the new purchaser. For example, A buys Black Acre valued at $17,000.

The sale price is constructed as follows:

Take over first mortgage (7%)	$10,000
7%, 15 year, Bonused vendor second mortgage	8,000
Cash to vendor	2,000
	$20,000

Suppose A sells Black Acre to B. B will have the advantage of a high loan-value ratio at a lower than expected interest rate.

The sale price would be constructed as follows:

Take over first mortgage (7%)	$10,000
Take over second mortgage (7%)	8,000
Cash to A	2,000
Sale Price	$20,000

Yet, B could probably buy back the second mortgage for $5,000. Therefore, the market price would still be $17,000.

The 25 per cent rule will in a quick and convenient fashion eliminate most of assessment-sale price ratios generated by these transactions. Those that are missed will be eliminated in the second step, the detailed examination of the sales contracts.

A second reason for a detailed examination of the sales contracts is found partly in the explanation of why a 25 per cent rule was selected rather than a 33 1/3 per cent rule. Why, when the regular maximum institutional conventional loan to value ratio is 66 2/3 per cent is a 25 per cent rule used and not a 33 1/3 per cent rule? "Lending value" is not defined in the legislation, and the pressure of competition may induce lenders to arrive at a lending value in excess of market price. Therefore, vendor financing may not become prevalent until the property has been financed up to 70 or 75 per cent by a conventional first mortgage loan. A 33 1/3 per cent rule would be too restrictive, and would not entirely eliminate the need for a detailed examination of sale contracts. For example, assume a $10,000 sale price; $5,000 outstanding on a first mortgage at 7% and 18 years remaining term; and a vendor second mortgage loan of $1,500 at 7 1/2% for 10 years. Suppose the vendor could sell the mortgage paper to yield 14%.

The sale price would be constructed as follows:

7 1/2%, 10 year,	Take over first mortgage (7%)	$ 5,000
	Vendor second mortgage	1,500
	Cash to Vendor	3,500
		$10,000

The market price is constructed as follows:

14%, 10 year,	Take over first mortgage (7%)	$ 5,000
	Vendor second mortgage	1,140
	Cash to Vendor	3,500
	Market price	$ 9,640

An assessed value of $4,500 could result in the following ratios.

Assessed value-sale price	.450
Assessed value-market price	.467

The discrepancy in this example is not large, but the particular circumstances will control the size of the difference.

There is a third reason for the detailed examination of the sales contracts. There are a number of possible cases where the cash payment to the vendor may be as great as 90 per cent or more of the sale price and still the transaction could involve bonused vendor financing. The purchaser may obtain a substantial conventional first mortgage loan or National Housing Act first mortgage loan. The cash payment will equal the amount of the mortgage loan plus the purchaser's equity. Where this total is

less than the sale price, vendor junior financing is involved.[43]

One final point remains to be considered. Theoretically it is possible to reduce, as was done in the example above, every transaction price to its equivalent market price. Therefore, rather than classifying transactions involving a cash payment of less than 25 per cent of sale price as being inefficacious for generating assessment-market price ratios, should not all sale prices be reduced to their market price equivalents? When the adjustments can be made accurately this is a better alternative to using a decision rule. But in order to arrive at an accurate result the property sold would have to be inspected and visualized at its date of sale. Buyers of junior paper do not always inspect the property. They rely on the law of averages to protect their capital and income. Where dispersions are being measured the errors do not "average out." Further, the mortgage market, since it is more closely related to the money and bond markets, is more volatile than the market for fees. Therefore, it cannot, without some inquiry, be considered

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[43]This case should not be confused with the case where the purchaser's equity is more than 75 per cent of the sale price and the vendor's financing less than 25 per cent of the sale price.

stable over the same period as that used for the study, and the adjustments would have to be based on the state of the market on the date of the transaction.

It is the opinion here that the 25 per cent rule and the further examination of sales contracts to eliminate assessment-sales ratios reflecting bonused vendor financing and unusually advantageous financing to the purchaser will produce results as reliable as those arrived at by approximate adjustments of sales prices to market price equivalents.[44]

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[44] Of the 1,318 property sales supplied by the multiple listing service 139 were rejected because the properties could not be positively identified. Of the 1,179 sales remaining in the sample, 647 were rejected with the application of the "25 per cent rule." A sample of 50 was selected randomly from the 647 sales eliminated by the "25 per cent rule." For each one of these 50 transactions the sales contract was examined. Not one of the 50 properties traded in these sales would, on the basis of the sales contract, have been included in the final sample. This is a good indication of the effectiveness of the 25 per cent rule of thumb in eliminating sales which do not provide a direct basis for the calculation of assessment-market price ratios.

Also, of the 532 sales remaining after the application of the rule of thumb, 29 were rejected from the sample on the examination of the sales contract. These 29 included sales which involved property trades, car trades, personal property, illegal suites, addition of improvements after assessment date, a bonused vendor mortgage, and other questionable vendor financing. Of the 503 remaining three were rejected after an examination of the assessment records indicated improvement addition between November 1964 and the date of sale.

The universe, the parameters of which are being estimated, is, in the strict statistical sense, not representative of the assessment roll; all properties sold; all single and two family residential properties sold; and to the extent that bonused vendor financing is more prevalent among properties of certain types or in given areas is not representative of all single and two family houses sold on multiple listing.[45]

G. Statistical Tabulations

The results of the statistical tabulations of the data are exhibited in Table II.

1. Class interval limits

The limits of the class intervals are specified to midway between the ten thousandth. As the assessment-market price ratios were to be rounded to the nearest ten thousandth, this specification of the class limits aids in the identification of the class into which a particular ratio will fall.

2. Actual frequency

The values in these columns are the number of assessment-market price ratios falling between the specified limit.

[45] It might be possible for the Assessment Commissioner to work with samples and a population representing all of the properties sold in a City or Municipality.

TABLE II

ASSESSMENT-MARKET PRICE RATIOS

FREQUENCY TABULATIONS AND ESTIMATION OF PARAMETERS

Class Interval	Section A: Class Interval Limits	Section A: Mid-Point	Section A: Frequency Observed	Section B: Class Interval Limits	Section B: Mid-Point	Section B: Frequency Observed	Section C: Class Interval Limits	Section C: Mid-Point	Section C: Frequency Observed	Section C: Frequency Expected
1	-.14995	.074975	1	-.13395	.066975	1	-.13085	.065425	1	0
2	.14995-.24995	.199950	3	.13395-.23395	.183950	2	.13085-.23085	.18085	2	1
3	.24995-.34995	.299950	15	.23395-.33395	.283950	13	.23085-.33085	.28085	12	20
4	.34995-.44995	.399950	148	.33395-.43395	.383950	116	.33085-.43085	.38085	110	120
5	.44995-.54995	.499950	242	.43395-.53395	.483950	260	.43085-.53085	.48085	265	218
6	.54995-.64995	.699950	78	.53395-.63395	.583950	88	.53085-.63085	.58085	87	120
7	.64995-.74995	.799950	8	.63395-.73395	.683950	14	.63085-.73085	.68085	16	20
8	.74995-.84995	.899950	3	.73395-.83395	.783950	4	.73085-.83085	.78085	5	1
9	.84995+		2	.83395+		2	.83085+		2	0
μ(=M)		.48400			.48085			.48052		
$\hat{S}$					.087121					
sk					.103028					
γ_2					3.795996					
M_d					.475300					
M_o					.47187					
β_1					.244605					
β_2					6.795996					
X^2								21.95		
D								.061		

3. Expected frequency

The values in this column are the number of assessment-market price ratios which the "fitted normal distribution" with μ (=M) and standard deviation σ (= $\hat{s}$) would place between the specified limits. The numbers are equal to the value of normal probability integral between the limits multiplied by the total number of observations.

4. Parameters of the frequency distributions

The method of moments has been used in the calculations of the parameters of the three frequency distributions.[46] Since the discrete frequency distribution is constructed from an underlying continuous distribution Sheppard's correction has been applied to the second and fourth moments.[47] The adjusted moments can be denoted as m_1, m_2, m_3, and m_4.

(a) The mean, μ, as estimated by M = m_1 has been discussed previously.

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[46] The median, M_d, was selected as the point midway between the 250th and 251st largest ratios.

[47] W. F. Sheppard, "The Calculation of the Moments of a Frequency Distribution," Biometrika, 1907, and "On the Calculation of the Most Probable Values of Frequency Constants for Data Arranged According to Equi-Distant Divisions of a Scale," London Mathematical Society, Proceedings, 1898.

(b) The standard deviation, $\sqrt{\ }$, as estimated by $\hat{s}$ has been discussed previously.

$$\hat{s} = s\sqrt{\frac{\sum f_i}{(\sum f_i) - 1}}$$

$$s = m_2$$

(c) The skewness is estimated with the following formula.

$$sk = \frac{\left(\sqrt{\beta_1}\right)\left(\beta_2 + 3\right)}{2(5\beta_2 - 6\beta_1 - 9)}$$

where $\beta_1 = m_3^2/m_2^3$

and $\beta_2 = m_4/m_2^2$

The sign of the skewness is given by the sign of the mean minus the median. If the distribution is skewed toward the higher ratios then the median will be less than the mean, and, therefore, the skewness would be positive.

(d) The Kurtosis measures the excess of the peak of the frequency distribution over that of the normal distribution with the same mean and standard deviation. The value

$$\gamma_2 = \beta_2 - 3$$

is used for a measure of kurtosis. This measure should be used with care because some configurations of data may give a positive value for γ_2 when the kurtosis is slightly negative.

(e) The position of the mode, M_o, is in most instances dependent on the selection of class interval limits. A method of estimating the mode which overcomes this limitation uses the modal divergence, δ.

$$\delta = \hat{s} \cdot sk$$
$$M_o = M - \delta$$

5. Sections A, B, and C

The calculation of the moments of a frequency distribution is based on the assumption that the distribution of the items within the class intervals is linear and uniform about the class interval mid-points. To the extent that this is not true the selection of the class interval limits affects the values of the moments.[48] In Table II the class interval limits of Section A were pre-selected. Since the statutory level of assessment is .5, a frequency distribution which has .5 as the mid-point of the mid-class would be the most obvious choice. Subsequent calculations produced a mean of 0.484. The

[48]The first moment will not be affected where the non-linearity and uniformity are similar on both sides of the mid-point of the middle class interval, and if the first moment is equal to this pre-selected mid-point.

calculations were repeated using 0.484 as the mid-point of the middle class interval. This resulted in the tabulation of Section B. The mean in Section B was found to be 0.48085. Therefore the calculations were repeated using 0.48085 as the mid-point of the middle class interval. This produced Section C and a mean of 0.48052 which is insignificantly different from 0.48085, the mean of Section B. The process, therefore, was stopped at this point.

In most applications of "goodness of fit tests" the process stops at the selection of class interval limits in Section B. These limits and the parameters, as estimated in Section A, of the distribution are then used to derive the expected frequencies.[49]

The reason for extending the process here will be made clear in Chapter IV. It rests on the view that uniformity of assessment and correct valuation, though interdependent, are not the same thing. In Chapter III the concern is with uniformity of assessment, therefore, a frequency distribution centred about the mean of the assessment-market price ratios should be more useful in

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[49] In some instances the original class interval limits are used in calculating the expected frequencies. Where the mean and middle interval mid-point are not equal then the calculation is inconvenient and it produces an asymmetrical distribution of expected frequencies.

testing measures of uniformity than a distribution not centred about the mean assessment-market price ratio.

5. The Chi Square Test

The class interval limits as set out in Section C and centred about the mean as calculated in Section B, and the parameters as estimated in Section B were used to derive the expected frequencies to be used in testing the actual frequencies for "goodness-of-fit" to the normal distribution. Because class intervals one and two had expected frequencies less than five they were combined with class interval three. Similarly intervals eight and nine were combined with interval seven. This reduces the degrees of freedom from six to two. Also, the power of the test is reduced. The reduction will not be significant because most of the actual frequencies in class intervals one, two, and nine are small. The value of chi square at the 0.001 probability level is 13.82. Therefore, there is less than one chance in one thousand that random effects could have produced the not normally distributed sample.

The Kolmogorov-Smirnov test may be more powerful than the χ^2 test in this case. Where the underlying theoretical distribution is continuous and where the χ^2 test requires the grouping of class intervals, the Kolmogorov-Smirnov test of "goodness-of-fit" is usually superior to the χ^2 test.

The Kolmogorov-Smirnov test is based upon the statistic D, where

$$D = \max \left| F_N - S_N \right|$$

F_N is the cumulative relative frequency at interval N.

S_N is the cumulative relative frequency at interval N.

In Section C it is found that D = 31/500 at class interval five.

The sampling characteristic of D is known where the theoretical cumulative frequency distribution is fully specified. But where the parameters of the theoretical distribution are estimated from a sample, the sampling distribution of D is not known. Massey has indicated that where the test is applied in such cases the critical value for D will be less than the critical value for the fully specified case.[50] Therefore, when the null-hypothesis, that the distribution of assessment-market price ratios are normally distributed, is rejected on the basis of the known sampling distribution of D, the conclusion will be conservative and safe. The critical value for D at the .05 probability level is 0.0608.

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[50] F. J. Massey, "The Kolmogorov-Smirnov test for goodness of fit," Journal of the American Statistical Association, 1951, pp. 68-78.

The above observations should be expected because the kurtosis and skewness are high. E. A. Pearson has shown for samples of 500 drawn from normal populations that the sampling distribution of β_2 has a probability level of 0.01 at $\beta_2 = 3.60$.[51] For a normal population $\beta_2 = 3.00$. The value of β_2 for the sample of 500 assessment-market price ratios is 6.796. He has also shown for samples of 500 drawn from a normal population that the sampling distribution of β_1 has a probability level of 0.02 at $\beta_1 = 0.065$. The value of the sample of assessment-market price ratios is 0.245.

7. Conclusion

It is clear that even for properties more homogeneous than the entire assessment roll the universe of assessment-market price ratios is not normally distributed. Therefore, measures of uniformity of assessment based upon a normality assumption should be highly suspect.

In considering uniformity of assessment for the particular universe under consideration only 265 sample assessment-market price ratios are within plus and minus 0.05 of the mean assessment ratio. In other words, 53 per cent of the sample lies between 0.48085 - 0.05 = 0.43085 and 0.48085 + 0.05 = 0.53085.

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[51] E. A. Pearson, "A Further Development of Tests of Normality," Biometrika, 1930, p. 239 ff.

Since the mean assessment-market price ratio is less than 0.50, less than 53 per cent of the sample ratios will lie between plus and minus 10 per cent of the mean ratio from the average assessment-market price ratio. In other words, less than 53 per cent of the sample will lie between

$$0.48085 - (0.1)(0.48085) = .43276$$

and $$0.48085 + (0.1)(0.48085) = .52894.$$

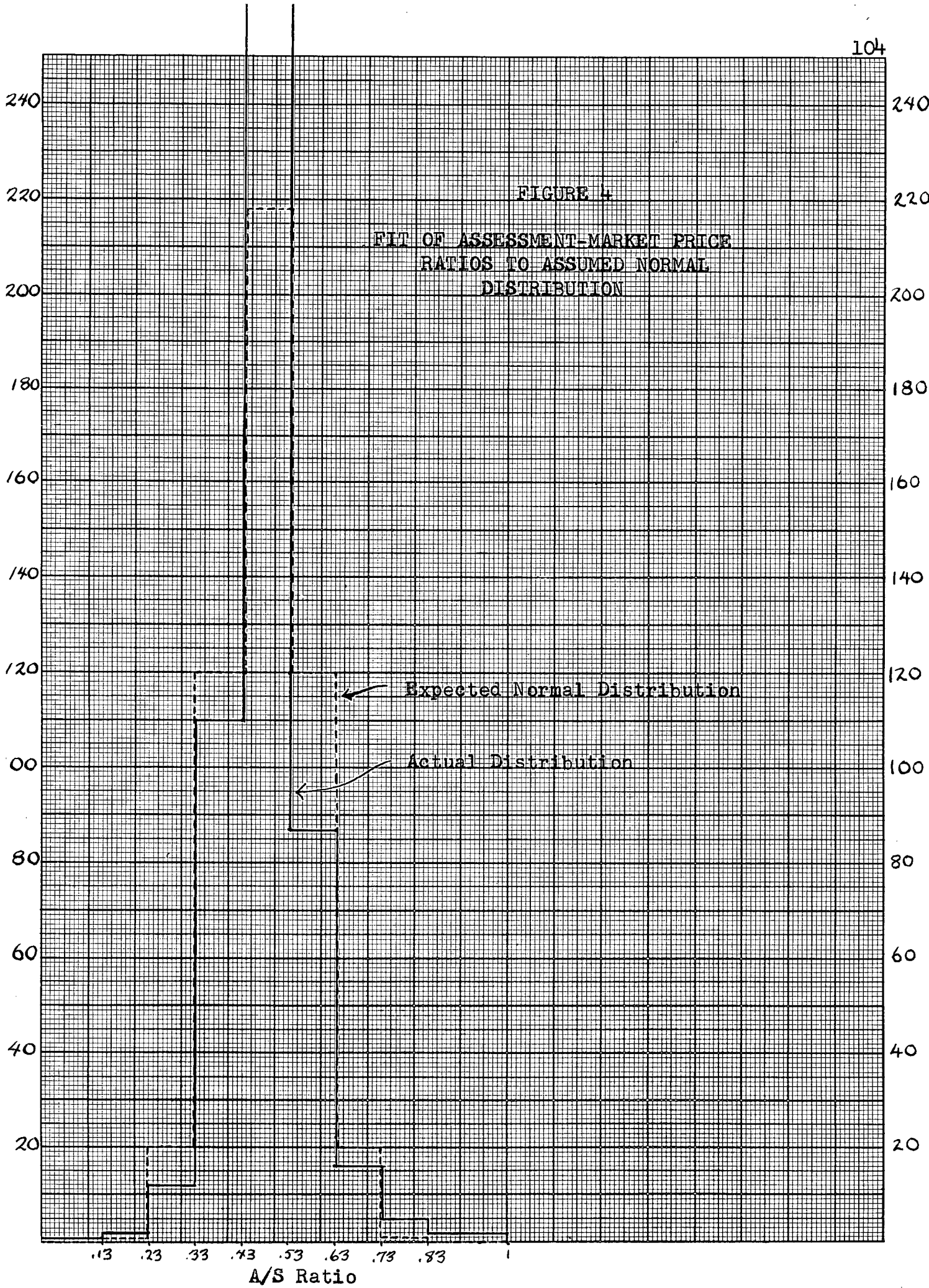

FIGURE 4

FIT OF ASSESSMENT-MARKET PRICE RATIOS TO ASSUMED NORMAL DISTRIBUTION

CHAPTER IV

ALTERNATIVE MEASURES OF ASSESSMENT UNIFORMITY AND QUALITY

A measure of uniformity or quality, if it is to be a useful administrative tool, should be capable of comparison to a standard measure. In other words, that the standard deviation of a distribution of assessment-market price ratios is .125 means very little unless that measure is related to some preset standard. Where measures of dispersion are to be meaningfully compared some assumption must be made about the underlying distribution. The normality assumption, as discussed in ChapterIII, is not always justifiable where distributions of assessment-market price ratios are concerned.

Schwinden has used a linear correlation model.[1] He has regressed assessed value on market price. This does away with a measure of dispersion between the assessment-market price ratios of different valued properties, but the uniformity is also dependent upon the dispersion of the assessed values about the regression line at all of the values on the abscissa. Hence, the measure of uniformity can be based upon the standard error of estimate

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[1]Schwinden, op. cit., pp. 57-60.

or the coefficient of correlation. This use of these measures assumes that the distributions of both the independent and the dependent variable are normal.[2] There can be some doubt about whether or not market values and assessed values are normally distributed. This assumption will not be tested here.

A. An Alternative Measure

An alternative measure of assessment quality and uniformity is proposed here. This measure rests upon one assumption and one value judgement.

The assumption is that an assessment can be judged correct or incorrect.

The value judgement is required to determine whether any given assessment is correct or incorrect. As argued previously, a ten per cent maximum margin of error should be permitted. In British Columbia an assessment lying between 45 and 55 per cent of market price would be correct.

In effect then, it is assumed that the assessments form a binomial distribution of correct and incorrect valuations. The higher the proportion of correct

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[2]Mordecai Ezekiel and Karl A. Fox, Methods of Correlation and Regression Analysis (3rd ed., New York, John Wiley & Sons Inc., 1959), pp. 279-281.

assessments then the greater the uniformity. If P is the proportion of correct assessment, Q is the proportion of incorrect assessments and σ_p is the standard deviation of the population, then

$$\sigma_p = \sqrt{P \cdot Q}, \text{ and } Q + P = 1.0$$

If $P \geq .5$

$Q \leq .5$

then as P increases Q decreases, and the standard deviation decreases. This is consistent with the view that uniformity of assessment is dependent upon correct valuation.

B. Standards of Assessment Uniformity Using the Alternative Measure

Assume four assessment rolls, A, B, C and D, and that their assessment-market value ratios are distributed as in Table III.

TABLE III

HYPOTHETICAL POPULATIONS OF ASSESSMENT-MARKET VALUE RATIOS A, B, C, D

	Population Assessment-Market Value Limits	A	B	C	D
		Proportion of Population between Limits			
(a)	.45 - .55	.90	.80	.90	.80
(b)	.40 - .60	.95	.95	.93	.95
(c)	.35 - .65	.97	.97	.95	.99

Where underlying continuous distributions are assumed it is possible to calculate the standard deviations from the above information. The standard deviations can then be compared. For example, if the underlying distribution is normal, and if only line (a) above were available, then the standard deviations of A and C would be equal and would be less than the standard deviations of B and D which are equal. Therefore A and C are more uniform than B and D. If in fact A were normally distributed with mean $\mu = 0.5$ then

$$\sigma_A = \frac{.55 - .5}{1.645} = .0304 ,$$

and the proportion of assessment-market value ratios between the limits of .40 and .60 would be .999 or 99.9 per cent.

That type of comparison cannot be made when the underlying distribution is the proposed binomial distribution. But since the major criterion for uniformity of assessment is correct valuation, then with only the information in line (a) above available, A and C are equal and more uniform than are B and D which are equally uniform. The secondary criteria provided in lines (b) and (c) above can be used to make a more fine measurement.

If the populations are listed vertically in order of decreasing uniformity, then:

Using information in line (a) only the ranking would be

A, C
B, D;

Using information in lines (a) and (b) only the ranking would be

A
C
B, D;

Using information in lines (a), (b), and (c) the ranking would be

A
C
D
B.

In effect, lines (b) and (c) are reporting the values of P for subsidiary binomial distributions of the

four assessment rolls. The binomial distribution of correct and incorrect valuations remains as the primary criterion by which to evaluate the uniformity of the assessment roll.

Suppose there are three assessment rolls, E, F, and G, and that their assessment-market value ratios are distributed as follows.

TABLE IV

HYPOTHETICAL POPULATIONS OF ASSESSMENT-MARKET VALUE RATIOS
E, F, G

	Population Assessment-Market Value Limits	E	F	G
		Proportion of Population between Limits		
(e)	.45 - .55	.70	.60	.60
(f)	.40 - .60	.70	.75	.70
(g)	.35 - .65	.70	.80	.90

If the same system of ordering good to bad assessment uniformity is used as was used for populations A, B, C and D, then considering lines (e), (f), and (g) the ranking would be as follows:

E
F
G.

There can be some doubt as to whether or not the assessment quality of F is greater than that of G. Also, there

is doubt as to whether or not the assessment quality of E is greater than that of F or G.

Except in extreme cases it would appear that the selecting of the better ordering would be a matter of judgement. Consider two extreme examples shown in Table V.

TABLE V

HYPOTHETICAL POPULATIONS OF ASSESSMENT-MARKET VALUE RATIOS
E_1 and F_2: E_2 and F_2: E_3 and F_3

Population Assessment-Market Value Limits		E_1	F_1	E_2	F_2	E_3	F_3
		Proportion of Population between Limits					
$(e)_1$	.45 - .55	.20	.05	.20	.05	.20	.05
$(f)_1$	.40 - .60	.20	.90	.20	.90	.20	.20
$(g)_1$	.35 - .65	.50	.95	.95	.95	.50	.95

It is reasonable to argue that the assessment quality of: F_1 is better than that of E_1; F_2 is better than that of E_2; and F_3 is better than that of E_3. E_1 and E_3 are equal in assessment quality; F_1 and F_2 are equal in assessment quality. Therefore the ranking of the six assessment rolls would be as follows.

F_1 or F_2

E_2

F_3

E_1 or E_3.

There is another point to consider. Suppose Populations G and H are distributed as in Table VI below.

TABLE VI

HYPOTHETICAL POPULATIONS OF ASSESSMENT-MARKET VALUE RATIOS
G, H

	Population Assessment-Market Value Limits	G	H
		Proportion of Population between Limits	
(g)	.45 - .55	.90	.0045
(h)	.40 - .60	.95	.0055
(i)	.35 - .65	.99	.006
(j)	.15 - .25	.0045	.90
(k)	.10 - .30	.0055	.95
(l)	.05 - .35	.006	.99

Roll H surely is not less uniform than roll G.[3] But where the assessor is directed to assess at a specific proportion of market value the quality of an assessment roll has two dimensions. One dimension is uniformity.

[3]If the full range of ratios is considered the distribution of assessment-market value ratios of roll H is more likely skewed than that of roll G.

The second dimension is correctness of valuation. The achievement of the first dimension rests extensively on the achievement of the second dimension, because, whether or not an assessor who is directed to assess at 50 per cent of market value, is attempting to keep an assessment roll up to date, only the wildest coincidence would result in all properties being assessed at 20 per cent of market value.[4] Therefore uniformity of assessment at any level rests on correct valuations.

Where the assessor is not directed to assess at any particular level of market value, but the selection of the level is left to him, or to chance, uniformity of assessment will still depend on the achievement of correct valuations.

In order to use the binomial model to measure only uniformity it is first necessary to determine the average of the individual assessment-market price ratios.

In order to measure the quality of an assessment roll the average of the individual ratios is not needed.

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[4] If the assessor arrives at the correct market value and then multiplies the assessment by 0.2 then all properties can be assessed at 20 per cent of market value. But an assessor who correctly estimates market value, and then when he is directed to multiply by 0.5, does multiply by some other factor, is surely impeachable.

Quality of assessment is dependent upon uniformity of assessment. Uniformity of assessment is to a great extent dependent upon correct valuations. Therefore, from this approach, quality of assessment is indirectly dependent upon correct valuation. Then, considering the second dimension, assessment quality is directly dependent upon correct valuation. Therefore a binomial model based on correct and incorrect valuation can be used to measure directly assessment quality and to measure indirectly uniformity of assessment.

C. Binomial Sampling

In the above discussion the binomial model was discussed in terms of a population. The population could be all the assessment-market value ratios assessment roll or it could be a universe of sales generated assessment-market price ratios. Remembering that market sales generate a sample of assessment-market price ratios from a universe which would be generated by the same forces that generated the sample if those forces were to continue indefinitely without change in character, the universe of sales generated assessment-market price ratios, unless the context requires otherwise, will be used throughout the rest of this chapter.

The parameters of the population are:

P - the proportion of all properties which would sell under market forces producing direct evidence of market value and were correctly assessed.
Q - the proportion of all properties which would sell under market forces producing direct evidence of market value and were incorrectly assessed.
$\sqrt{P.Q}$ - the standard deviation of the population proportions of correct valuations.

The parameters of the sample are:

p - the proportion of all properties which actually sold producing direct evidence of market value and were correctly assessed.
q - the proportion of all properties which actually sold producing direct evidence of market value and were incorrectly assessed.
s p.q - the standard deviation of the sample proportions of correct assessments.
n - the number of elements in the sample.

If $n > 50$ then the sample proportions, p, will be normally distributed with mean PP and standard deviation

$$\sqrt{p_e} = \sqrt{P.Q} / \sqrt{n}$$

p is often referred to as the standard error of a proportion.

Where P is not known and is estimated by p then

$$\hat{s}\,p = \sqrt{\frac{p.q}{n-1}}$$

is the unbiased estimator of p when the universe is infinite, and

$$\hat{s}p = \sqrt{\frac{p.q}{n-1}} \cdot \sqrt{\frac{N-n}{N}}$$

is the unbiased estimator of $\sqrt{p_e}$ when the universe is finite and of size N.[5]

[5] In this paper the universe is infinite.

The problem here is to estimate, given a sample proportion of correct valuations, the smallest reasonably expected proportion of correct assessments in the population. Remembering that the sample proportions, p, are normally distributed with mean P_m (= $\mathbb{P}$), the sampling distribution can be standardized with the following formula.

$$z = \frac{p - \mathbb{P}}{\sqrt{p_e}}$$

Since $\mathbb{P}$ is not known $\sqrt{p_e}$ is replaced by $\hat{s}p$ and $\mathbb{P}$ is replaced by p_m.

$$z = \frac{p - p_m}{\hat{s}p}$$

$$z = \frac{p - p_m}{\sqrt{\frac{p.q}{n-1}}}$$

A reasonable level of confidence for the estimate of the lower limit of p_m is to be selected. Ninety-five per cent would be a reasonable level of confidence. Therefore, denote the value of p_m at the 95 per cent confidence level by $\mathbb{P}_{mx}$ (x = .95).

$$z_{.95} = \frac{p - p_{m.95}}{\sqrt{\frac{p.q}{n-1}}}$$

$$p_{m_{.95}} = p - z_{.95}\sqrt{\frac{p \cdot q}{n-1}}$$

For example, suppose that out of 901 properties that sold the assessed values of 720 of them were between 45 per cent and 55 per cent of market price. The results are summarized below:

$$\begin{aligned} p &= .8 \\ q &= .2 \\ p \cdot q &= .16 \\ z_{.95} &= 1.645 \\ \frac{p \cdot q}{n-1} &= .0133 \\ (z_{.95})\left(\sqrt{\frac{p \cdot q}{n-1}}\right) &= 0.022 \\ p_{m_{.95}} &= .778 \end{aligned}$$

In effect the results indicate that if the proportion of correct valuations is only 77.8 per cent, then there are only five chances in one hundred that the sample would have more than 720 correct assessments.

On the other hand, it may be judged that the minimum standard of assessment requires 90 per cent of properties sold to have assessed values within 45 per cent to 55 per cent of market price. What is the minimum value of p which would meet this standard at the 95 per cent confidence level?

$$z_{.95} = \frac{p - P}{\sqrt{p_e}}$$

$$z_{.95} = \frac{p - P}{\sqrt{\frac{P.Q}{n}}}$$

$$z_{.95} = -1.645$$

$$P = .9$$
$$Q = .1$$
$$P.Q = .09$$
$$p = .010$$
$$p.z_{.95} = -0.016$$
$$p = 0.884$$

In other words, if there were 901 properties sold, then 796 would be correctly valued in order to meet the required standard.

It might be suggested that the sampling approach is superfluous, and that all the ratios actually generated by sales be treated as the universe. But the number of ratios, the number of sales, may vary from year to year, and from locality to locality, and the sampling model by correcting for n provides the adjustments for retaining comparability among the years and localities. For example, suppose that market activity generated only 400 reliable assessment-sales price ratios. Then

$$\sigma_{p_e} = \sqrt{\frac{P.Q}{n}} = \frac{.1 \times .9}{400} = 1.5 \times 10^{-2}$$

$$z_{.95}p = 0.025$$

$$p = 0.875$$

Therefore the standard would indicate that 349 correct valuations out of 400 assessments is equivalent to 796 correct valuations out of 900 assessments. Note that 349/400 is not equal to 796/900.

If only 100 ratios are available then p = 0.851, and 85 correct valuations could satisfy the standard of P = 0.90. Note here that (85/100) < (349/400) < (796/900).

D. Quality of the Population Actually Sampled

The sample has 242 correct valuations. Only the proportion 0.484* of the assessments are between 45 and 55 per cent of market value.[6] Therefore the estimated standard error of the proportion estimate is 0.02235, and the 95 per cent confidence limits for p_m, the universe proportion of correct valuations are a minimum of 0.439 and a maximum of 0.529. This result can be stated differently. If the population does in fact have 90 per cent of the properties correctly valued then there are less than 5 chances in one million that the sample would have only 48.4 per cent correctly assessed properties.

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[6] Only a coincidence has produced p = .484 and M = .484.

*242/500 = .484.

There are 93.6 per cent of the assessments between 35 and 65 per cent of market value.

Only 51.2 per cent of the ratios are between the limits of plus and minus 10 per cent of the mean from the mean. This does not indicate a high measure of uniformity.

How high is the quality of the assessment roll? First, for the reasons stated previously it is not likely to be as high as the universe being measured. The population represented here consists of assessment-market price ratios of properties which form a market more homogeneous than all of the properties represented on the entire assessment roll. Second, as compared to a standard of 90 per cent correct valuations, the assessment quality of the universe measured here is not high. The degree of the lack of quality is not completely measurable against the above standard. For example, would a universe with 65 per cent correct valuations but with only 80 per cent of the assessment-market price ratios between 35 and 65 per cent of assessed value be better or worse, as compared to the standard, than the universe measured here?

E. Additional Alternative Measures of Uniformity and Quality

One might suppose that a grading system would be desirable. Such a system is examined, and its limitations are noted here.

The standard requires that 90 per cent of the assessment-market price ratio be between 0.45 and 0.55. If the deviations from perfect assessment, assessment at 50 per cent of market value, are random, then the assessment-market price ratios will be normally distributed. Therefore, a possible standard by which to evaluate the quality of an assessment roll would be a normal distribution with μ, the mean, equal to 0.5, and σ, the standard deviation, equal to 0.0304.[7]

The actual frequency distribution of ratios could then be tested for "goodness-of-fit" to the expected distribution of ratios under the fully specified normal distribution. The χ^2 test or the Kolmogorov-Smirnov test can be used. The test statistic can then be translated into a probability level. The lower the probability level the lower the quality of the measured universe. The χ^2 test will in all cases have the same number of degrees of freedom, and the translation of the Kolmogorov-Smirnov statistic into a probability level will adjust for varying sample sizes. The use of the statistic χ^2 in this case reduces the degrees of freedom in the χ^2 distribution by one, instead of three, because μ and σ are specified and the only linear

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[7]There is no suggestion that the actual distribution of ratios is normal.

constraint is $\Sigma f_i = \Sigma F_i$. Table VII below summarizes the calculation of X^2 and D. The values in the column headed "Expected Frequency" are the number of expected observations in the class interval under the fully specified normal distribution.

All class intervals except four, five and six have zero expected frequency. Therefore, the relevant χ^2 test has 2 degrees of freedom. X^2 equal to 1053.33 is well beyond the critical χ^2 value of 13.82 at the 0.001 probability level.

The D value is 0.284. This is well in excess of 1.63/500, the critical D value at the 0.01 probability level.

The first limitation is that a universe of assessment-market price ratios might have more than 90 per cent correct valuations, and, therefore, a sample from such a universe could result in a low probability level. This limitation is minor, because a visual examination of the frequency distribution will make evident the high quality of this universe.

The second limitation is more significant. It exists because the grading system measures quality of assessment primarily. A sample from a universe in which all properties are assessed at 25 per cent of market value

TABLE VII

FIT OF OBSERVED FREQUENCIES TO STANDARD OF CORRECT VALUATION

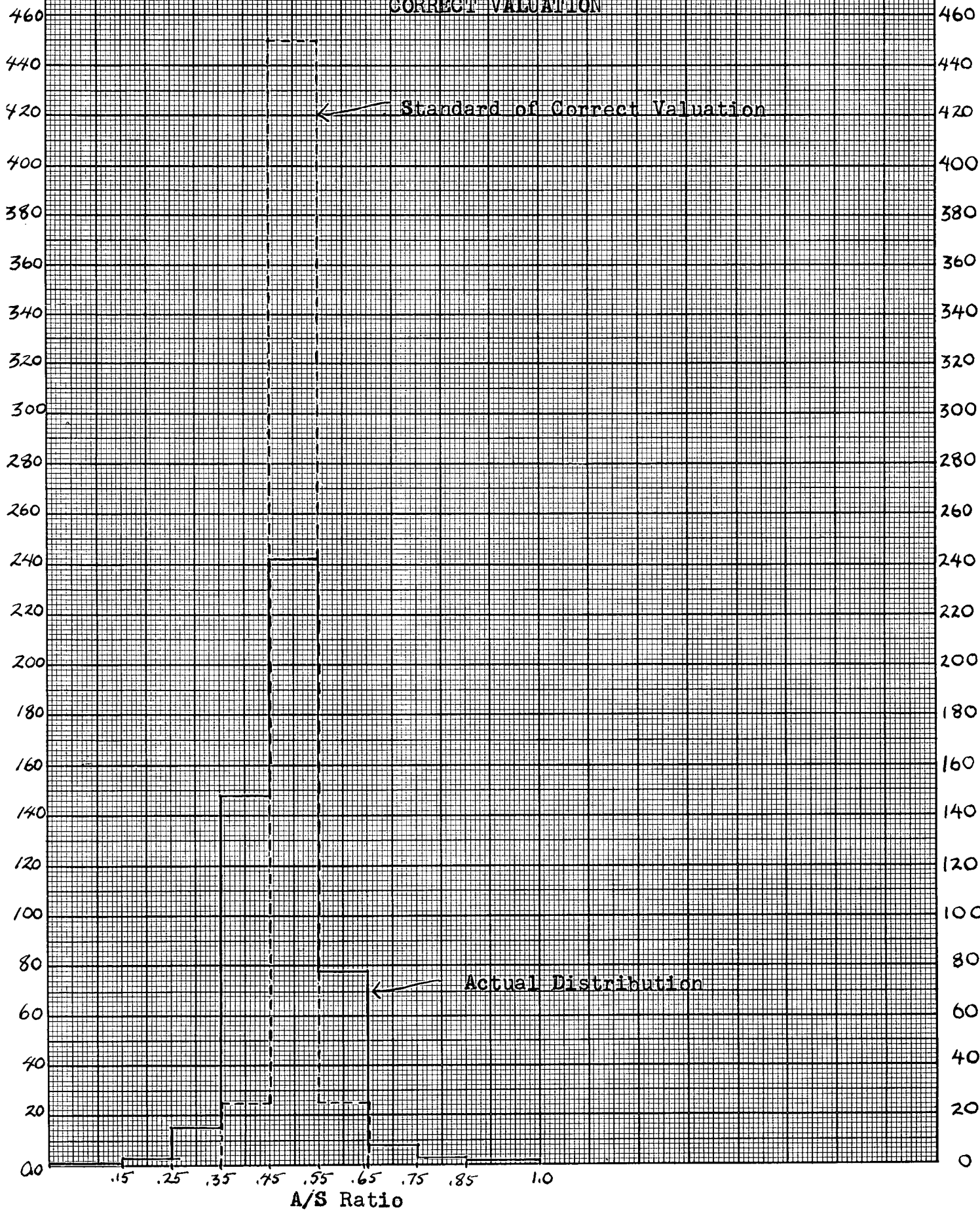
Class interval	Class interval limits	Observed frequency	Cumulative relative observed frequency, S_N	Expected frequency	Cumulative relative expected frequency, F_N	$\lvert F_N - S_N \rvert$
1	0.00000-.14995	1	1/500	0	0/500	1/500
2	.14995-.24995	3	4/500	0	0/500	4/500
3	.24995-.34995	15	19/500	0	0/500	19/500
4	.34995-.44995	148	167/500	25	25/500	142/500
5	.44995-.54995	242	409/500	450	475/500	66/500
6	.54995-.64995	78	487/500	25	500/500	13/500
7	.64995-.75995	8	495/500	0	500/500	5/500
8	.74995-.84995	3	498/500	0	500/500	2/500
9	.84995-1.000	2	500/500	0	500/500	0/500

μ = .5 $\quad$ X^2 = 1053.33

$\sqrt{}$ = .0304 $\quad$ D = 142/500

FIGURE 5

FIT OF OBSERVED FREQUENCIES TO STANDARD OF CORRECT VALUATION



would result in a low probability level. This measure makes no allowance for extreme cases such as in the above example, but again a visual examination of the data will reveal the cause of the low probability level. Where the assessment-market price ratios are tightly clustered about a mean ratio not far removed from 0.5 then a visual examination will not suffice. The selection of class intervals may obscure the nature of the underlying actual frequency distribution.

To some extent the latter limitation may be overcome. In addition to the above quality grading a uniformity grading can be introduced. If a universe of assessment-market price ratios is highly uniform then the ratios will be clustered tightly around and normally distributed about the mean ratio. In accordance with the measure of quality, 90 per cent of the ratios should be between the limits (M - .10M) and (M + .10M).

Table VIII below summarizes the calculation of the "goodness-of-fit" of the observed frequencies to the expected frequencies of a normally distributed population with mean 0.48085 and a standard deviation of 0.029234. X^2 equals 1042.45. The expected frequencies in all class intervals except four, five, and six were zero, therefore only the three class intervals were used for the

TABLE VIII

FIT OF OBSERVED FREQUENCIES TO STANDARD OF UNIFORM ASSESSMENT

Class interval	Class interval limits	Observed frequency	Cumulative relative observed frequency S_N	Expected frequency	Cumulative expected relative frequency	$\lvert F_N - S_N \rvert$
1	0.00000-.14422	1	1/500	0	0/500	1/500
2	.14422-.24040	3	4/500	0	0/500	4/500
3	.24040-.33658	20	24/500	0	0/500	24/500
4	.33658-.43276	106	130/500	25	25/500	105/500
5	.43276-.52894	256	386/500	450	475/500	89/500
6	.52894-.62512	87	473/500	25	500/500	27/500
7	.62512-.72130	20	493/500	0	500/500	7/500
8	.72130-.81748	5	498/500	0	500/500	2/500
9	.81748-1.0000	2	500/500	0	500/500	0/500

μ = .48085 $\quad X^2$ = 1042.40

σ = 0.029234 $\quad D$ = 105/500

calculation of X^2. Because there are two linear constraints

$$\Sigma f_i = \Sigma F_i$$

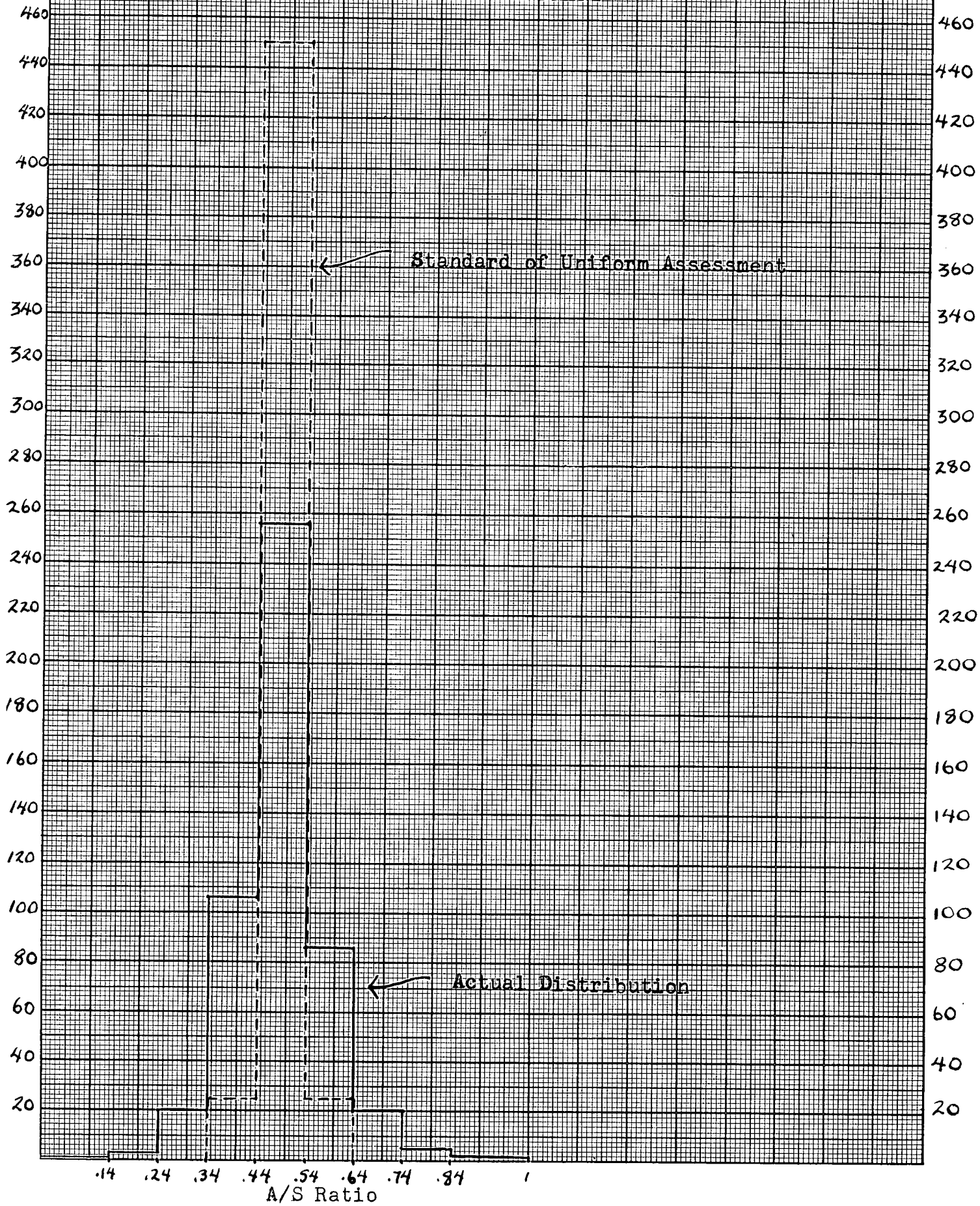
$$M = \mu,$$

there is only one degree of freedom. The probability level is below 0.001, as the critical value of χ^2 at one degree of freedom is 10.83.

The value of D is .210, and it is well above the critical value of 1.63/500 at the 0.01 probability level. The D value for the uniformity test is in this case lower than the D value for the "correctness of valuation" test, and, therefore, the use of both tests gives recognizance to both uniformity and quality of assessment.

FIGURE 6

FIT OF OBSERVED FREQUENCIES TO STANDARD OF UNIFORM ASSESSMENT



CHAPTER V

CONCLUSIONS

The first, and most obvious, conclusion is that unless an assessment roll is of the highest quality it is very unlikely that the assessment-market price ratios will be normally distributed. Therefore, a standard of assessment uniformity or quality based upon the assumption that the distribution of ratios is normal is without practicable meaning. If an assessment roll has a coefficient of variation less than the maximum allowable under the standard, but does not have normally distributed assessment-market price ratios, then the standard cannot be used to infer that the uniformity or quality of the roll is acceptable.

Not one of the χ^2 test, the Kolmogorov-Smirnov test, and the test of the sample moments indicated that the particular universe of Vancouver City assessment-market price ratios is normally distributed. Neither, therefore, is it likely that the assessment-market price ratios available to the assessor and Assessment Commissioner are normally distributed. Regardless of the size of the maximum allowable coefficient of variation, any statement that the Vancouver assessment roll is, according to that standard, uniform cannot be accepted.

The coefficient of variation measured here is denoted by V.

$$V = (\sqrt{V}/M)100 \quad \text{, where}$$

$\sqrt{V}$ is estimated by $\hat{s}$

$$V = (.087121/.48085)100$$

$$V = 18.1 \text{ per cent.}$$

The Assessment Commissioner states that a coefficient of 18.8 should be attainable and less than 18.8 indicates good results.[1] A coefficient of 18.8 per cent and a mean of 0.48085 imply a standard deviation of 0.0903980. Fitting the observed distribution to a normal distribution ($\mu = .48085, \sqrt{V} = .0903980$) yields $X^2 = 28.37$ at 3 degrees of freedom and D = 0.07. The value of χ^2 (3 degrees of freedom) at the 0.001 probability level is 16.27. The value of D, the Kolmogorov-Smirnov statistic, at the .05 probability level is .0608. Therefore, the Assessment Commissioner has a paradox. The coefficient of variation indicates uniform assessment, but according to the underlying assumptions of normality the assessments are not uniform. That the actual distribution does not "fit" the assumed distribution can be seen in Table IX.

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[1] Appraisal Manual, op. cit., p. 315.

TABLE IX

ACTUAL AND EXPECTED FREQUENCIES

		Actual frequency	Expected frequency
1	-.13085	1	0
2	.13085-.23085	2	1
3	.23085-.33085	12	23
4	.33085-.43085	110	121
5	.43085-.53085	265	210
6	.53085-.63085	87	121
7	.63085-.73085	16	23
8	.73085-.83085	5	1
9	.83085 +	2	0

It would appear from the arguments in Chapter IV that no single measure of quality or uniformity of assessment can replace judicious use of several alternative measures. The measures used must not imply unwarranted assumptions about the distribution of the ratios. The binomial models for assessment quality and assessment uniformity and the Chi square and Kolmogorov-Smirnov tests for assessment quality and uniformity used here might not be the only measures applicable to the problem. They do possess at least one important quality. None of the measures presented in Chapter IV requires an invalid assumption about the actual distribution of assessment-market price ratios.

According to these measures the quality and uniformity of the City of Vancouver Assessment roll are very low. The standards used may be high, but as stated previously, if "equal treatment of equals" cannot be achieved then the real property tax as a source of public revenue should be replaced.[2]

Further Study

The measurement of assessment uniformity and quality is not the only use for assessment-market price ratios. The overall objective of the analysis is to improve assessment quality and uniformity. The data used in measurement of these can also be used to determine the properties or characteristics of properties which are concomitant with over and under assessment. Oliver Oldman and Henry Aaron have attempted to analyze the pattern of assessment deviation from uniformity.[3] They have classified properties into nine areas, eleven property types, and ten price ranges for the purpose of assessment and measuring uniformity within groupings. The average ratio was used to measure the assessment level and the standard deviation was

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[2] Supra, p. 3.

[3] Oliver Oldman and Henry Aaron, "Assessment-Sales Ratios under the Boston Property Tax," National Tax Journal, 1965, pp. 36-49.

used to measure assessment uniformity. In order to analyze the influence of property type and location on the level of assessment they carried out a two way analysis of variance with multiple entry in each cell or grouping. The results of their study are not significant here, because, first, the use of the standard deviation to compare the uniformity within different groups assumes similar distributions of assessment-market price ratios within the groups; second, their variance analysis assumes that each group is an independent random sample and represents a normal population.[4] This condition does not prevail in Vancouver, nor was any indication given of the validity of the assumption for Boston.

A related method can be applied without the assumption of an underlying normal distribution. Contingency tabulations may be used. Properties can be classified by location, type, price range, age, type of construction, size, use, state of repair, and environmental amenities. A system may be used individually or as part of complex system consisting of two or more characteristics. The properties can also be classified by type of assessment

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[4]Frederick C. Mills, Statistical Methods (3rd ed.; New York: Henry Holt and Company, 1955), p. 571. He also points out that when the assumptions are not valid, the analysis may be used but that the results are not completely reliable.

correct, over, and under assessment. Cross tabulations on the type of assessment against the property characteristics and χ^2 tests applied to these tabulations will indicate whether or not the characteristics tallied are concomitant with over or under assessment. Where there is a significant relationship between correct or incorrect assessment the classification affected may be further subdivided by the same or other characteristics and the tabulation and test repeated. This will give a more detailed indication of the incidence of assessment errors. Where no significant relationship between property characteristic and correct or incorrect assessment is indicated several groups of properties may be combined. The original classification may have been too fine to give suggestive results.

Since uniform and high quality assessment have not been achieved further statistical research should be concentrated on the development in detail of methods and techniques which can be used to help the assessor isolate the sources of his errors.

The results of Chapter III and Chapter IV indicate that as in all other problems associated with valuation of property the measurement of assessment quality and uniformity requires the exercise of judgement in a real world.

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APPENDIX A

APPENDIX A

Included in Appendix A is the Statistical Analysis and Control section of the Province of British Columbia Appraisal Manual. Except for some examples of applications the entire section is presented below.

Also included are:

Exhibit I - Form A.C.2 - Real Property Transfer Record

Exhibit II - Form A.C. 7 - Request for Sales Analysis Records

Exhibit III - Form A.C. 3 - Sales Analysis Record.

STATISTICAL ANALYSIS AND CONTROL

Introduction

Assessment of real property creates the base on which is erected the taxable structure that produces the greatest part of the funds necessary for the operation of the jurisdiction. That structure is composed of a multitude of individual taxpayers, each of whom forms a part of the whole with an effect in proportion to his size. Valuation techniques determine the proper sizes of the individual tax liabilities, and statistical techniques are employed to check the effects of the valuation techniques and assist in ensuring that the individual valuations and consequent shares of tax liability are just, uniform, and equitable. "Just" as conforming to relevant statutory requirements; "uniform" as bearing a fair relationship to other comparable properties within the jurisdiction; and "equitable" in being valued similarly to comparable properties in all other jurisdictions in the Province.

Assembly and Analysis

In order to conduct a statistical check of valuation results, it is essential to collect or assemble the information which, when analysed, will produce useful information. There are three basic methods of checking assessed values--viz., (1) appraisal verified by another staff appraiser or by a private appraiser, (2) study of the income yield, and (3) assessment-sales analysis. Each method has certain inherent advantages and disadvantages which must be weighed before deciding on the method to be followed in a particular instance. The first two methods are, in fact, valuations, and the subsequent analysis is a comparison of values determined by different individuals or by means of different methods.

Assessment-sales ratio studies, wherein the assessed values are shown as percentages of the sale prices, form the most common vehicle for determining assessment inequalities.

Editing and Screening of Sales

The selection of sales items for study purposes requires very careful editing to ensure that only bona fide arm's

length transactions are considered. The sale value, or market value, declared must be the value agreed upon between a true vendor and a true purchaser. The best value obtainable is that arrived at by agreement between two parties, neither of whom is under pressure to buy or sell. What is required is the true market value, which may be defined as "the price which the property would bring in an open market on a free, not forced, sale between a willing buyer and a willing and able seller." By Statute "actual value" is the basis of real-property assessment, and in determining "actual value" the assessor may, among other things, give consideration to "the price that such land and improvements might reasonably be expected to bring if offered for sale in the open market by a solvent owner."

Every attempt must be made to avoid using sales information based upon unrepresentative sales. Sales may be considered to be unrepresentative for many reasons, which may be summarized into the statement that "the price does not indicate the price at which the property could be sold by someone who took due time and pains to secure the best market." A sale price may be unrepresentative also because it is abnormally high. Subsequent investigation may show that the purchaser had a special interest in the particular property, such as being the owner of an adjacent parcel. No sale should be discarded solely because the price is apparently too high or too low. Such action should be taken only after investigation indicates that the price is not representative. A market in transition may produce either high or low prices, which will indicate the beginning of a new value phase. Such trends are important, and irresponsible editing can destroy the evidence of such trends.

In British Columbia the obtaining of information about sale properties is much easier than in many jurisdictions. Six Land Registry Offices administered by the Province consolidate all land title information in their respective geographical locations. Title searches are thus facilitated. Each assessor is further assisted by being given a copy of every title change of ownership, complete with the market value of the property as declared to the Land Registry Office. As the fees collected by the Land Registry Offices for registration are levied on and ad valorem basis, it may be accepted that the declared values are reasonably close to the market value. When properly edited these "yellowback" copies of titles may be used in assessment-sales ratio studies, but as supplied these "yellowbacks" are unscreened for bona fide sales.

Further aid to the assessors is provided by the Office of the Assessment Commissioner, which, in co-operation with the Department of the Attorney-General and the Land Registry Offices, instituted a system of sales reporting whereby the Land Registry Offices demand the filing of a questionnaire titled "Real Property Transfer Record" and designated as Form A.C. 2. This questionnaire is required to be filed for each transaction where production of a tax certificate is required to accompany the application for title change. This requirement automatically excludes the following types of non-bona fide transfers:

(a) Changes of executors or trustees.

(b) Transmissions by death or foreclosure.

(c) Quit-claims.

(d) Transfers to a beneficiary under a will.

In order to exclude other non-bona fide transfers, specific exemption from filing the questionnaire is made for applications to register mortgages, options to purchase, leases, life estates, Crown grants, and tax sales. In this manner, then, questionnaires are received covering only sales which are predominantly bona fide transactions. Further editing is done by the Office of the Assessment Commissioner to eliminate transfers which, from information given in the questionnaire, are not bona fide sales. These would include family or related parties transfers, trades, and extremely low down-payment transactions, and others, as shown below:

(a) Transfers agreed upon some time prior to the year in which they were recorded.

(b) Transfers involving unusual financial provisions.

(c) Transfers of partly finished improvements.

(d) Transfers including personal property, the value of which cannot be separated from the total consideration.

(e) Transfers involving trades.

(f) Transfers between relatives or associated corporations.

(g) Transfers under financial duress.

(h) Transfers to any church, lodge, school, or charitable, benevolent, fraternal, or government organization.

(i) Transfers on which the value stated is an opinion rather than an actual exchange price.

(j) Transfers the circumstances of which are unusual and as a result render the transaction useless for sales analysis.

After this editing process, the sales remaining are reported to the assessor concerned by means of the "Sales Analysis Record," designated Form A.C. 3. Because of this progressive screening-out of non-bona fide transfers, the assessor receives Forms A.C. 3 for only a portion of the transfers reported to him on "yellowbacks." In addition to the Forms A.C. 3, the assessor also receives a form listing the Forms A.C. 2 rejected by the Office of the Assessment Commissioner. Any transfers reported on "yellowbacks" which are not accounted for by a Form A.C. 3 or on the Notification of Forms A.C. 2 Rejected (Form A.C. 19) should be only transfers such as those described above, for which no tax certificate is required to be filed. The assessor may be assured, then, that the sales not reported on Forms A.C. 3 are either suspect or definitely not bona fide transactions. As the editing by the Office of the Assessment Commissioner is primarily to obtain a hard core of bona fide sales for Province-wide equalization of assessments, it does result in the arbitrary rejection of some transfers which the assessor could, through local knowledge, salvage and use for both valuation and study purposes when the volume obtained by means of Forms A.C. 3 is too small.

Whenever possible the edited sales information shown on the Forms A.C. 3 should be confirmed by the assessor through his confidential inquiry with the purchaser, vendor, his solicitor or agent. The assessor must satisfy himself that the sales information is reasonably accurate before using it in the valuation process, or approving it for assessment-sales ratio studies. The assessor completes the Form A.C. 3 showing the relative assessed values, dimensions, type of property, and economic sub-district, and also, if required, comments as to the suitability of the sale for analysis.

From this information the Office of the Assessment Commissioner is able to make a further edit, then for each of the resulting bona fide sales the assessment-sales ratios are calculated, and by means of mechanical tabulation facilities assessment-sales ratio studies are prepared as required. Printed listings of these bona fide sales are available for reference and work-paper use.

Assessment-Sales Ratio Analysis

Having assembled the data required for study, the next step must be to prepare it in a manner suitable for analysis. The style of the Form A.C. 3 is such that many different combinations of information may be made for study purposes.

Because every jurisdiction includes areas or zones of differing activity, the formation or designation of sub-districts has been fostered by the Office of the Assessment Commissioner. Sales within these sub-districts may be studied by one sub-district alone or by larger areas formed by combining sales within two or more sub-districts.

Two appraisers may have slightly different approaches to valuation, and if they confine their valuation activities to specific geographic areas, the degree of divergence between their valuations may be determined by studying sales of similar properties occurring in their respective districts. This matter of sub-districts is very important, and much thought and study put into the location of sub-district boundaries will pay off in more informative evidences of value changes. The boundaries need not be permanent, but may be varied as the economic need for changes becomes obvious. While studies of comparative areas over a long period are preferable to short-term studies, if the basic characteristics of the area have changed during the long period, then the results of such a study will be misleading and erroneous. Recognition of the need for change must be made by altering sub-district boundaries, with advice to the Office of the Assessment Commissioner whenever such changes are made.

It may be that certain sub-district boundaries will apply for land valuation, whereas smaller sub-districts would be better for improvements valuation. There are two methods of handling this situation--(1) small sub-district boundaries suitable for improvements study and valuation

may be formed, and larger sub-districts comprising the required number of smaller sub-districts may be formed for land study and valuation, and (2) certain types or classes of improvements may be found in two or more areas in the jurisdiction, and for study and valuation purposes may be considered similar. Such similar groups may be given the same sub-district number wherever they are located in the jurisdiction. Super-imposed on these may then be laid boundaries for land study and valuation purposes. As provision exists for 3-digit sub-district numbers, thus the larger land sub-districts could be designated as 100, 200, and 300, and within those could be improvements sub-districts designated as 1, 2, 3, 4, etc. Thus the complete sub-district number could read 102, representing land sub-district 100 and improvement sub-district 2. In like manner, sub-district 302 would represent land sub-district 300, but also improvement sub-district 2, the improvements therein being similar to those represented by the 2 in land sub-district 102.

A further analysis breakdown that should not be overlooked is the separation of corner lots, and waterfront lots, from the inside lots. There may be a sizeable variation in prices paid for waterfront properties as compared with those paid for inside lots, and similarly a difference may exist for corner properties. By grouping these various types together indiscriminately, the influences of any special group are lost, but by separate analysis a proper valuation adjustment may be made where it is necessary.

Another precaution to take in preparing to analyse the assembled information is to be sure that the properties being studied conform in actual use to the predominant land use in the immediate neighbourhood. For example, if the predominant land use is residential and the actual use of a particular property is, say, commercial, then the subject property should not be studied with properties having a residential actual use. Study of the sale of a neighbourhood corner store will not yield much information to help in valuing residential properties, but it may be useful when compared with other sales of similar non-conforming properties. In so far as possible, like must be compared with like.

This is true, too, with the assessed values. Before studying assessment-sales ratios, the assessor must assure himself that the assessment reflects the value of the

property sold. If vacant land was sold, an assessment which includes the assessed value of an improvement is obviously incorrect. It is not so obvious, however, that lack of comparison exists in other instances. If a water-main, sewer, or road surfacing exists at the time of assessment, but did not at the date of sale, then the assessment does not reflect the property as sold. In a similar manner, improved properties must have assessed values for the improvements as they were at the time of sale in order to be most effective in a study. If a furnace, porch, basement room, or garage has been added and assessed since the date of the sale, then the transaction is not good for analysis. The Office of the Assessment Commissioner must reject such items because of insufficient information, but with care the assessor can salvage such items for his use. The effect of the physical change may be either very minor or substantial, but he must acknowledge it. He may adjust the assessed value in many instances, for study purposes only, by deducting the relative assessed value of the furnace, porch, basement room, garage, or whatever extra has been added in an attempt to obtain an assessment based on the property as it was at the date of sale. This method should be resorted to only if the number of sales is meagre.

Statistical analysis must be approached with great care when the volume of items is small. When numbers are few and the concentration of ratios is widespread, no factual statistical deductions are valid. In particular, no conclusion should be drawn from a single item, and no property should be revalued based upon a single sale of that property. That practice would destroy rather than create uniformity because the sale price for a given property is the end result of negotiation between two parties and may be high or low, according to the relative persuasive powers of the two parties. When the volume of items being analysed is small, each item has an appreciable effect on the result, but when the volume is large, then the individual influences are tempered and, as a consequence, extreme items, either high or low, are overshadowed by the bulk.

The question of valuation based upon the price paid for the property being valued was the subject of a Supreme Court decision by the Honourable Mr. Justice Wootton in the case of Robert J. Rowan v. City of Vancouver in 1962, and your attention is drawn to that decision.

Calculation of Statistical Measures

1. Measures of Absolute Dispersion

(a) Mean, Median, Mode. For relative ease of calculation of statistical measures, it is recommended that the assessment-sales ratios calculated and assembled as described above be in the form of an array in ascending order. If only a few items are involved they may be set up in numerical order, such as 20, 42, 42, 44, 44, 44, 45, 45, 46, 49, 50.

The arithmetic mean of these items is the sum of each divided by the number of items, or 471 ÷ 11 = 42.8 per cent.

The median is that value in the middle, which has the same number of items below it as above it; in the above instance it is 44 per cent.

The mode is that value which is most numerous. In the above there are three items with 44 per cent ratios; thus 44 per cent is the mode.

The use of the median is recommended as it is the measure of the middle item and is thus typical and is not greatly affected by extremely high or low ratios. Where a large number of items are to be studied, it is easier to enter the ratios in the form of a frequency distribution. This consists of the tallying of ratios in predetermined percentage intervals or groups, the resultant number of items falling in a percentage interval being the frequency of occurrence of ratios within that interval, and the whole being the distribution of ratios between the various intervals. A basic assumption is made that the number of items within an interval is spread evenly from the low to high limits of that interval. The size of the interval as used by the Office of the Assessment Commissioner has been 10 percentage points, as 30-39 per cent, 40-49 per cent, etc. A typical frequency distribution on this basis follows:

EXAMPLE 1.--ASSESSMENT-SALES RATIO ANALYSIS, VACANT LAND SALES

Type of Property	Median	Frequencies of Assessment Ratios											
		0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	100 and over	Total
Residential vacant--													
Inside lots-	50	--	--	2	8	10	12	3	2	1	1	1	40
Corner lots-	37.5	--	1	3	6	4	3	--	--	--	--	--	17
Waterfront lots-------	25	1	2	6	1	1	1	--	--	--	--	--	12

To calculate the median ratio for the above inside lots, for example, divide the total number by 2, thus 40 ÷ 2 = 20, and count from the lowest ratio until that number has been reached; thus in the interval 20-29 there are 2 items, and 8 in the interval 30-39, and 10 in the interval 40-49. Adding, 2 + 8 + 10 = 20, indicating that the median ratio is 50 per cent. This may also be determined by counting down from the highest ratios, thus 1 + 1 + 1 + 2 + 3 + 12 = 20, again arriving at 50 per cent. Whenever there is doubt about the accuracy of a calculated median, it may be checked in this manner.

To calculate the median ratio for the corner-lot frequency distribution, again divide the total by 2 (17 ÷ 2 = 8.5), and count from the lower end (10-19) until 8.5 is reached. In this case 1 in the 10-19 interval plus 3 in the 20-29 interval plus 6 in the 30-39 interval is too much. Up to the 30-39 interval there are 4 items; another 4.5 are needed. The basic assumption is that the items in an interval are evenly spread within that interval. Therefore, calculate 4.5 items of the group of 6, multiply by the size of the interval (10 in this case) and add it to the low limit of the interval (30 per cent here). Thus 4.5 ÷ 6 is .75, multiplied by 10 gives 7.5, which when added to the low limit of 30 gives the calculated median of 37.5 per cent. This also may be checked from the top end, but in this example 1.5 ÷ 6 or 2.5 per cent must be subtracted from the 40-per-cent limit. The answer is still 37.5 per cent.

Another measure that may be determined is the range, being the difference between the lowest ratio and the highest. Thus in the examples above the ranges would be:

(1) For inside lots, from 20 to over 100, or 80-plus percentage points.
(2) For corner lots, from 10 to 60, or 50 percentage points.
(3) For waterfront lots, from 0 to 60, or 60 percentage points.

It is obvious that one estreme item can affect the range unduly, and for that reason the range is not considered to be a satisfactory measure.

(b) Decile, Percentile, Quartile. Deciles, percentiles, and quartiles are other statistical measures. Deciles divide the number of items into 10 equal parts, percentiles divide the number into 100 equal parts, and quartiles divide the number of items into 4 equal parts. These are usually used in such a manner that the number of items receiving intensive study are those remaining after a deduction of those at the two ends of the array or frequency distribution. Thus the ranges between the 1st and 9th deciles or the 10th and 90th percentiles are the same thing, omitting the upper and lower 10 per cent. The range between the 1st and 3rd quartiles includes only 50 per cent of the total items.

Comparisons of these various measures calculated for one frequency distribution follow:

EXAMPLE 2.--ASSESSMENT-SALES RATIO ANALYSIS

0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99	Over 100	Total
---	---	4	6	15	30	18	15	12	---	---	100

	Per Cent
10th percentile or 1st decile	40
1st quartile	50
Mode	55.4
Median	58.3

	Per Cent
Arithmetic mean (common average) ------	59.5
3rd quartile ----------	71.3
90th percentile or 9th decile ---------	81.7

	Percentage Points
10 to 90 percentile range (or 1-9 decile range) ----------	41.7
1st to 3rd quartile range ----------	21.3
Range (from 20 to 90 per cent) --------	70

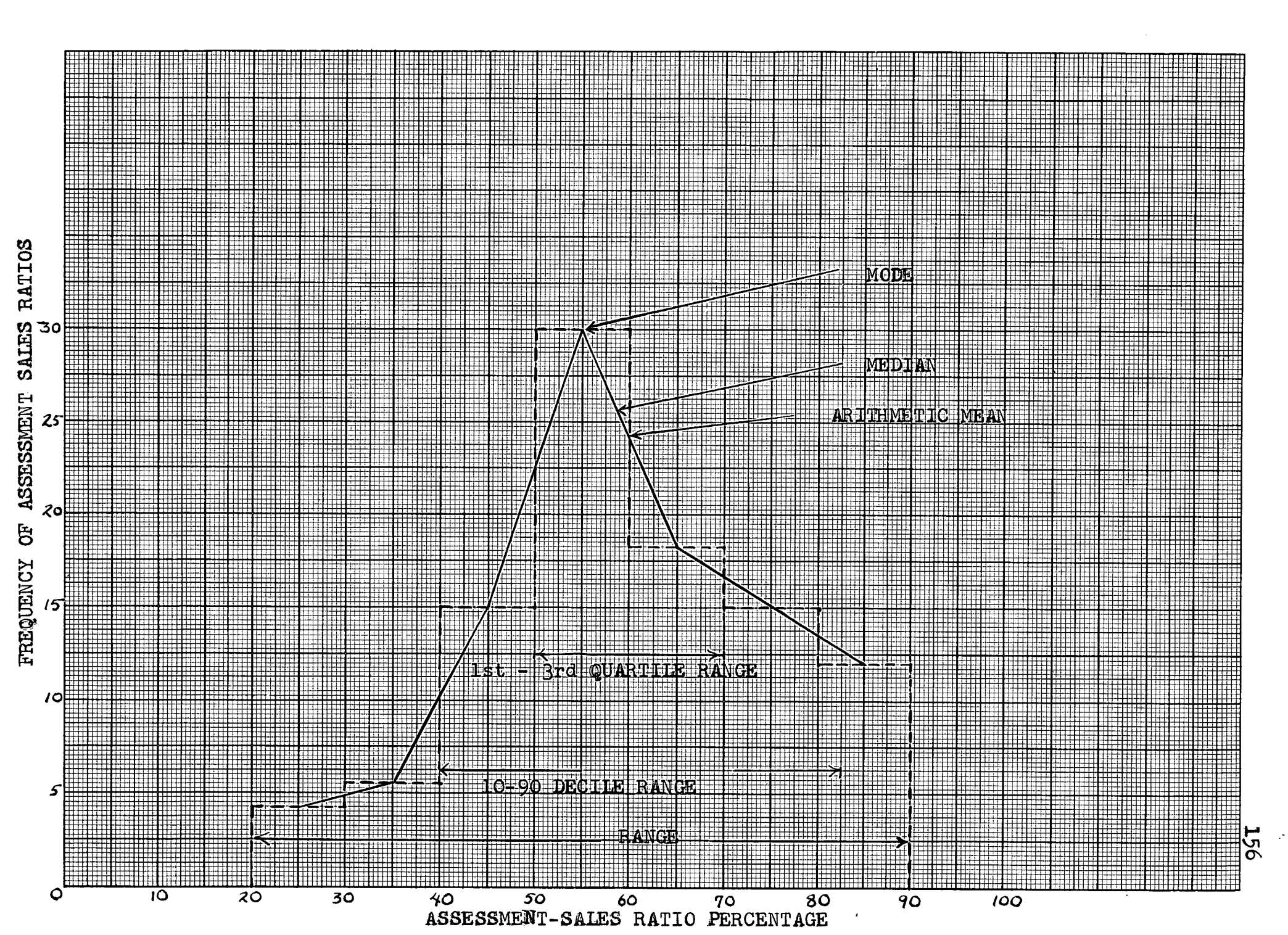
The frequency distribution may also be plotted on squared paper to derive a visual impression of the quality of assessment.

The above items are shown plotted on the curve in the illustration on the next page.

(c) Quartile Deviation, Mean Deviation, Standard Deviation. Reference to a measure of central tendency, such as the median, as the typical or average value is of limited value without further knowledge of the degree of scatter about that measure. While the range, deciles, percentiles, and quartiles serve as indicators of the degree of dispersion of items, there are other more useful and definite measures. These are the quartile deviation or the semi-interquartile range, the mean deviation, and the standard deviation.

As mentioned above, the range is simply the difference between the lowest and highest ratios and is not an indicator of the degree of concentration or central tendency of the items. Its value depends solely upon the extreme ratios and is of very limited use.

(i) The quartile deviation or the semi-interquartile range is a more indicative measure. It is calculated by subtracting the 1st from the 3rd quartile and dividing the result by 2. In the foregoing example the 1st quartile was 50 per cent and the 3rd quartile was 71.3 per cent, the difference between these two being 71.3 - 50 = 21.3. Dividing this by 2 gives the quartile deviation of 21.3 ÷ 2 = 10.65. If the distribution were perfectly symmetrical, the median would lie half-way between the quartiles, and thus one-half of the distance between the quartiles may be used as a measure of the average distance of each quartile from the


FREQUENCY OF ASSESSMENT SALES RATIOS
0
5
10
15
20
25
30
MODE
MEDIAN
ARITHMETIC MEAN
1st - 3rd QUARTILE RANGE
10-90 DECILE RANGE
RANGE
10
20
30
40
50
60
70
80
90
100
ASSESSMENT-SALES RATIO PERCENTAGE

median. As the dispersion of a frequency distribution is increased, the distance between the quartiles becomes greater. Hence the smaller the value of the quartile deviation, the more tightly concentrated are the items in the frequency distribution and the greater the degree of uniformity in the assessments. A disadvantage of this measure is that it is dependent upon only the central 50 per cent of the cases, while some other measures give consideration to a larger percentage of the cases and are thus considered to be more informative.

(ii) The mean deviation, or average deviation, is a measure having a value dependent upon the value of every item in the series. This measure consists of the average of the deviations of each item from their arithmetic mean, or median. Where the number of items is small, these deviations may be calculated individually, then averaged, as is done in the so-called "Russell Formula" calculations, but where the volume is great this becomes a chore. If the items are grouped in the form of a frequency distribution, as in the earlier example, the number of calculations may be reduced by means of the following procedure:

(1) Select as an arbitrary origin the mid-point of the interval containing the median.
(2) Obtain the deviations of the mid-points of each interval from the arbitrary origin.
(3) Multiply these interval deviations by the number of items in the interval.
(4) Total all the products derived in step (3) and divide by the total number of items. The resulting value is the average deviation in class intervals, except for an adjustment described below.

EXAMPLE 3

Interval (Per Cent)	Number of items	Deviation from arbitrary origin in intervals	Frequency X of deviation
0- 9	---	---	---
10-19	---	---	---
20-29	4	3	12
30-39	6	2	12
40-49	15	1	15
50-59	30	0	0

EXAMPLE 3 (continued)

Interval (Per Cent)	Number of items	Deviation from arbitrary origin in intervals	Frequency X of deviation
60-69	18	1	18
70-79	15	2	30
80-89	12	3	36
90-99	--	--	--
100 and over	--	--	--
Totals	100	--	123

As the median for the above, 58.3 per cent, is 3.3 percentage points higher than the assumed median (55 per cent), or 0.33 class intervals, an appropriate adjustment must be made to compensate. Assuming the items in each interval are all located at the mid-point, we see that in the above case 55 (30 + 15 + 6 + 4) items are understated and 45 (18 + 15 + 12) items are overstated. The mean deviation, or average deviation, then will be calculated in this manner.

(Total frequency X deviation from arbitrary origin) divided by (the total number of items) plus an adjustment of (the difference in class intervals between the assumed and actual medians) multiplied by (the difference between the under- and over-stated items), all divided by (the total number of items), thus--

$$\frac{123}{100} + 0.33 \times \frac{55 - 45}{100} = 1.263 \text{ class intervals.}$$

To convert to actual units it is necessary to multiply this figure by the size of the class interval, in this case 10, giving a mean deviation, or average deviation, of 12.63.

For a frequency distribution that is symmetrical and characterized by a normal curve, a measurement of one average deviation on each side of the median will include 57.5 per cent of the items. For moderately skewed distributions this will be approximately true.

(iii) A more easily calculated and more useful measure is the standard deviation, which is a special form of average deviation from the mean. The calculation of the standard deviation is as follows:

(1) Select as an arbitrary origin the mid-point of the interval containing the mean.
(2) Obtain the deviations of the mid-points of each interval from the arbitrary origin, showing minus signs for intervals below and plus signs for intervals above that origin.
(3) Multiply these interval deviations by the number of items in the interval.
(4) Multiply the products in step (3) again by the same interval deviations, thus obtaining the frequency (number) times the deviation squared.
(5) Total the products in step (3), recognizing the minus quantities.
(6) Total the products in step (4), all of which will be positive.
(7) Divide the total in step (6) by the total number of items.
(8) Divide the total in step (5) by the total number of items, and square the result.
(9) Take the square root of the difference between the results of steps (7) and (8).
(10) Multiply by the size of the class interval.

Class interval	Number of items	Deviation from arbitrary origin, in intervals	Frequency X deviation (Col. 2 X Col. 3)	Frequency X deviation (Col. 4 X Col. 3)
Col. 1	f Col. 2	d' Col. 3	fd' Col. 4	$f(d')^2$ Col. 5
0-9 ----------	--	--	(3)	(4)
10-19 ----------	--	--	--	--
20-29 ----------	4	-3	-12	36
30-39 ----------	6	-2 (2)	-12	24
40-49 ----------	15	-1	-15	15
50-59 ----------	30	0 (1)	0	0
60-69 ----------	18	1	18	18
70-79 ----------	15	2 (2)	30	60
80-89 ----------	12	3	36	108
90-99 ----------	--	--	--	---
100 and over ---	--	--	--	---
Totals -----	100	--	45(5)	261(6)

The figures (1) to (6) in parentheses above represent the steps outlined above by number.

Step (7): 261 ÷ 100 = 2.61.
Step (8): 45 ÷ 100 = .45.
.45 X .45 = .2025.
Step (9): 2.61 - .2025 = 2.4075.
Square root of 2.4075 = 1.55.
Step (10): 1.55 X 10 = 15.5.
Thus the standard deviation is 15.5.

The standard deviation measures the dispersion of a series: the greater the spread of the series, the greater the value of the standard deviation. As a measure of assessment uniformity, the smaller the value of the standard deviation, the greater the degree of uniformity.

For a normal distribution, if a distance equal to one standard deviation is measured off on both sides of the arithmetic mean, approximately 68 per cent of the values will be included within the limits thus calculated. Plus and minus 2 standard deviations includes about 95 per cent, and plus and minus 3 includes almost all of the values (99 per cent).

As the standard deviation is measured about the mean, it would be appropriate to describe how to calculate the mean. From the table above, the assumed mean is 55 per cent, but this requires adjustment to derive the true mean. The adjustment factor is simply the interval size multiplied by the total from step (5) divided by the total number of items. Thus it is necessary to add 10 X (45 ÷ 100), or 4.5, to the assumed mean of 55, giving a true mean of 59.5 per cent.

Both the average deviation and the standard deviation are affected by the value of every item, but greater emphasis is placed on the extremes in the standard deviation than in the mean deviation.

Comparisons of the relationships between the various absolute measures are as follows:

Range: Coarse calculation only; no definite relationship to other measures.
Quartile deviation = 0.6745 standard deviation = 0.8453 average deviation.
Average (mean) deviation = 0.7979 standard deviation.

Standard deviation = 1.2533 average deviation = 1.4826 quartile deviation.

2. Measures of Relative Dispersion

To this point all the measures discussed have been measures of absolute dispersion, expressed in terms of units of the problem--namely, percentage points. In addition, a measure of dispersion should be compared to the size of the average about which it is being measured. This then gives a measure of relative dispersion.

(a) Coefficient of variation is calculated by relating the standard deviation to the arithmetic mean and expressing it as a percentage. Thus, for the example given above, the mean is 59.5 per cent and the standard deviation is 15.5, so

$\frac{15.5}{59.5} \times 100 = 26.0$ per cent is the coefficient of variation.

This measure also is referred to by some authorities as the relative standard deviation.

(b) Coefficient of dispersion is the average deviation of a series divided by the mean of the series. In the example the average deviation is 12.63, the mean is 59.5, so the coefficient of dispersion is

$\frac{12.63}{59.5} \times 100 = 21.2$ per cent. The smaller the value of the coefficient of dispersion, the greater the degree of uniformity. The coefficient of dispersion is the "index of assessment inequality" referred to by the late Dr. John H. Russell, former Director of Research, Virginia Department of Taxation. His recommendation was that a coefficient of dispersion of "20 should be considered a goal desirable of achievement and reasonably attainable," and that anything below this is to be considered as an excellent degree of equalization or uniformity. Conversely, he stated "an index as high as 45 should be judged cause for gravest concern."

Accepting Dr. Russell's attainable limit of 20, and knowing that the average deviation on which it is calculated is 0.7979 standard deviation, then the corresponding coefficient of variation would be $20 \div 0.7979$ or 25 per cent.

Or, expressed another way, as the standard deviation is 1.25 average deviations, then the coefficient of variation will be 1.25 times the equivalent coefficient of dispersion. Thus a coefficient of variation of 25 should be "desirable of achievement and reasonably attainable" and a coefficient of variation of 56 should be "cause for the gravest concern." With an average assessment level of 50 per cent a coefficient of variation of 25 would mean that about 67 per cent of the items should have ratios between 37.5 and 62.5 per cent. With a coefficient of variation smaller than 25 the degree of uniformity would be higher. For example, if the coefficient of variation was 16, then 67 per cent of the items would fall between 42 and 58 per cent. The ultimate, which is impossible to achieve, is a coefficient of zero, indicating no deviation, and a perfect assessment in which every assessed value is at the same percentage of sales value as every other assessed value.

Dr. Russell's limit of 20 was determined in 1939; more recent developments and improved techniques are believed to have such an effect that the limit of 20 should now be 15, and that a coefficient of dispersion of 30 would indicate need for a general reassessment. These revised limits converted to the coefficient of variation would be 18.8 and 37.6 per cent, or 18 and 37. Thus 18 should be attainable, less than 18 indicates good results, more than 37 is unacceptable, and coefficients of variation of from 18 to 37 may be acceptable according to existing circumstances.

Special Comments

1. Information must be adequately screened or edited.
2. Sale prices must be confirmed, if at all possible.
3. Assessed values must be for the conditions and extent of the property as at date of sale.
4. The degree of uniformity for cheap properties (either land only, or for poor-quality old improvements) will be less than for good- and middle-value properties. Uniformity will suffer more again in high-value properties where whimsey enters into the sale prices.
5. Actual value cannot be determined solely on the evidence of a single sale.

EXHIBIT I

For Land Registry Office Use Only

APPLICATION No(s).

Date Received

REAL PROPERTY TRANSFER RECORD

FORM A.C. 2

This form shall accompany every application for the registration of title in all cases where a tax certificate is required under the provisions of section 130 of the "Land Registry Act," except applications to register mortgages, options to purchase, leases, life estates, Crown grants, tax sales, and additional applications required by virtue of section 129 (1) of the Act.

1. LEGAL DESCRIPTION OF LAND (Insert lot, block, section, district, plan.)

(If insufficient space, use reverse side of form.)

2. Vendor's name.....

Purchaser's name and address.....

(If insufficient space, use reverse side of form.)

3. Date of sale - - ....., 19.....

4. What was the full sale price agreed upon by the Vendor and Purchaser? - - $.....

5. What was the actual cash payment received by the Vendor? - - - - $.....

Did the Purchaser assume any encumbrances or other financial obligations?
Yes. ☐ No. ☐

If yes, state amount owing on:—

(a) First mortgage - - - - $.....

(b) Second mortgage - - - $.....

(c) Agreement for sale - - - $.....

(d) Other (specify):
..... $.....

(e) Other (specify):
..... $.....

6. Is this a sale of—

(a) Vacant land only? - - - - - - ☐

Or

(b) Land and improvements (i.e., land with buildings or structures)? - - - - - ☐

7. Did the full sale price include any property other than land and buildings?
Yes. ☐ No. ☐

If yes, itemize the value of the real and personal property.

Real { Land - - - - / Main buildings - - / Other buildings - - } $.....

(List items.)

Personal { 1. ..... $..... / 2. ..... $..... / 3. ..... $..... / 4. ..... $..... }

To be completed by Land Registry Office.

Municipality or Provincial Assessment District:

8. Did the transfer include a trade of any real or personal property?
Yes. ☐ No. ☐

If yes, state type of property traded, and estimate the market value.

.....

.....

.....

.....

.....

9. Are the Vendor and Purchaser related or associated corporations?
Yes. ☐ No. ☐

10. Is this a forced or distressed sale?
Yes. ☐ No. ☐

11. Is this transaction simply a conveyance issued upon completion of payments under an agreement for sale?
Yes. ☐ No. ☐

12. Did the transfer include the assignment of any land held under lease from the Crown?
Yes. ☐ No. ☐

If yes, state type, acreage, lease number.

.....

.....

13. State any other details of the sale which would indicate that the sale price agreed upon is not indicative of a fair market value.

.....

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.....

.....

14. I, the undersigned, do hereby certify that the information given is complete and correct in all respects.

(Signature).....

* *Vendor.*
* *Purchaser.*
* *Solicitor for Vendor or Purchaser.*
* *Agent for Vendor or Purchaser.*
(* Strike out the words not applicable.)

Date....., 19.....

All information submitted herein by the applicant will be treated as strictly confidential.

200M 6 (8)

EXHIBIT II

REQUEST FOR SALES ANALYSIS RECORDS

.....,19...

To the Assessor,.....

In accordance with Regulations passed pursuant to the "Assessment Equalization Act," you are requested to complete Sales Analysis Records, Form A. C. 3 for each sale of real property represented in the following numbered copies of Certificates of Title and Agreements-for-sale, etc. Completed Sales Analysis Records, Forms A. C. 3 shall be forwarded within one month of your receipt of this request to the Assessment Commissioner, Parliament Buildings, Victoria, B. C.

Certificates of Title			Agreements-For-Sale, etc.			

FORM A. C. 7

K. E. B. WILDMAN,
ASSESSMENT COMMISSIONER

EXHIBIT III

SALES ANALYSIS RECORD

CONTROL NO. (1) ☐☐☐ ☐☐☐☐☐

(2) Date of sale.....Municipality or assessment district.....(3) School District No.....

Market value (full sale price).....(4) Sub-district No.....

LEGAL DESCRIPTION

No.	Lot and Subdivision	Block	Section	District Lot	Division of Land District	Plan No.	Map No.
1.							
2.							
3.							
4.							
5.							

ASSESSED VALUES OF PROPERTY TRANSFERRED

No.	Land Registry Application Nos.	Assessment Roll Nos.	Acreage or Dimensions	ASSESSED VALUES OF — Land	Main Building or Residence	Other Buildings	Other Improvements	Totals
1.								
2.								
3.								
4.								
5.								
	Summary							

If you do not consider this sale suitable for analysis, indicate reason briefly.....

(5) TYPE OF PROPERTY

Predominant Land Use	Actual Use	Special Lot Characteristics	(6) Manual Class of Main Building
RESIDENTIAL 1 ☐	**For All Properties Other than "Agricultural" and "Small Holdings"**	Corner lot 01 ☐	
COMMERCIAL 2 ☐	Vacant land - 01 ☐; One-family dwelling - 02 ☐; Two-family dwelling - 03 ☐; Row house - 04 ☐; Conversion - 05 ☐; Apartment - 06 ☐; Shack - 07 ☐; Domestic garage - 08 ☐; Auto court - 09 ☐; Motel - 10 ☐; Hotel - 11 ☐; Hall - 12 ☐; Store - 13 ☐; Store and living-quarters or offices - 14 ☐; Office building - 15 ☐; Bank building - 16 ☐; Theatre building - 17 ☐; Theatre drive-in - 18 ☐; Gas and service station - 19 ☐; Commercial garage, workshop - 20 ☐; Cold storage - 21 ☐; Storage and warehousing— Open - 22 ☐; Closed - 23 ☐; Marine facilities - 24 ☐; Other commercial (specify) - 25 ☐	Water-front 02 ☐	
INDUSTRIAL 3 ☐	Sand, gravel, clay pit, or stone quarry 26 ☐; Manufacturing (specify) - 27 ☐	Water-lot 03 ☐	
AGRICULTURAL 4 ☐	Other industrial (specify) - 28 ☐		
SMALL HOLDING 5 ☐	**For All "Agricultural" and "Small Holdings"** *(Entry required in each of A, B, and C below.)*		
OTHER (SPECIFY) 6 ☐	A. Extent of farm buildings: Vacant, no buildings - 4— ☐; Residence and outbuildings - 5— ☐; Residence, no outbuildings - 6— ☐; Outbuildings, no residence - 7— ☐		
	B. Type of operation: Grain and forage - —1 ☐; Vegetable and truck - —2 ☐; Tree-fruits - —3 ☐; Small fruits - —4 ☐; Cattle - —5 ☐; Dairy - —6 ☐; Poultry - —7 ☐; Mixed - —8 ☐; Other (specify) - —9 ☐		
	C. Is land classified as "farm land"? Yes ☐ No ☐		

Age-group
Before 1910:
Cheap - 1— ☐
Average - 2— ☐
Good - 3— ☐
1910-1930:
Cheap - 4— ☐
Average - 5— ☐
Good - 6— ☐
After 1930:
Cheap - 7— ☐
Average - 8— ☐
Good - 9— ☐

Structure
Frame - —1 ☐
Brick-wood - —2 ☐
Brick-concrete —3 ☐
Brick-steel - —4 ☐
Steel - —5 ☐
Concrete - —6 ☐
Log - —7 ☐

For use by Assessment Commissioner.

Code (5)	Code (6)	Roll No.	(7) Land Assessed Value	(8) Improvements Assessed Value

ASSESSMENT SALES RATIO CALCULATION

(12) Assessed Value / (13) Market Value = ... = % (14)

Group $ —$ =(15)

Market Value of Land/Unit.....=(16)

Residual Land Value

Received and checked ☐☐☐☐

USE	REJECT

Mail original to office of Assessment Commissioner, Parliament Buildings, Victoria, B.C. Form A.C. 3—100M sets - M.B.F. 10-60

APPENDIX B

APPENDIX B

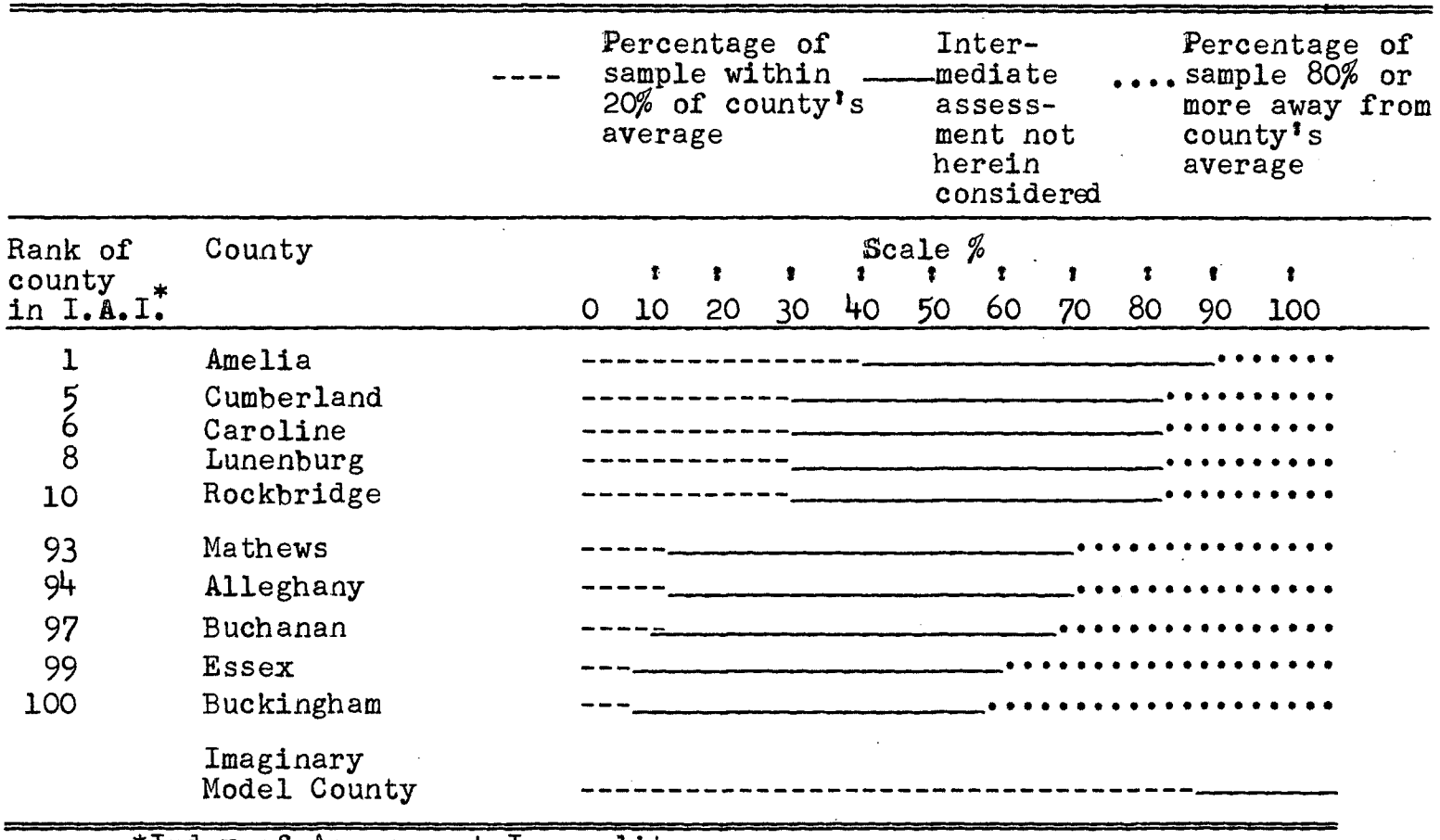
In 1939, while he was Director of Research with the Virginia Department of Taxation Dr. John H. Russell published in "The Commonwealth" an organ of the Virginia State Chamber of Commerce some results of assessment-sale price ratio studies. The article from which the results exhibited below were taken was entitled "Inequality of Real Estate Assessments within Political Subdivisions." An examination of the data indicates some relation between the "Index of Assessment Inequality" and the proportion of assessments within a given range of the mean, but it is clear that the coorelation is not perfect. Therefore, Russell could not have assumed that assessment-sales ratios are normally distributed.

County or city	Index of Assessment Inequality	Percentage of "Good" assessments (within 20% of average)	Percentage of "Very Bad" assessments (80% or more away from average)
Roanoke City	28	47.2	3.4
Buckingham County	75	12.6	43.0
Amelia County	36	40.8	9.2
Essex County	75	11.2	41.1
Richmond City	30	46.2	7.4
Hampton City	66	24.2	29.3
Richmond City, 1936	30	46.2	7.4
Richmond City, 1939	25	49.1	3.8

Eighty per cent is arbitrarily taken as the point of separation of bad and "very bad" assessments. Actually, assessments missing the average by 45 per cent will generally be considered as very bad.

PROPORTIONS OF "GOOD" AND "VERY BAD" ASSESSMENTS IN TEN SELECTED VIRGINIA COUNTIES

1936



Rank of county in I.A.I.*	County
1	Amelia
5	Cumberland
6	Caroline
8	Lunenburg
10	Rockbridge
93	Mathews
94	Alleghany
97	Buchanan
99	Essex
100	Buckingham
	Imaginary Model County

*Index of Assessment Inequality.

APPENDIX C

APPENDIX C

The limits of the frequency intervals used in this study and the values of M and $\hat{S}$ calculated from the data produce values of z_{α_j}, (j = 1, ..., 9), the corresponding frequency interval limits of the standardized normal distribution, which are not available in the tables prepared for researchers. It was, therefore, necessary to evaluate the probability integrals. This Appendix briefly sets out the theory, procedure and results of these calculations.

The density function of the normal distribution is

$$f(\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

If $z = (x-\mu)/\sigma$, then

$$f(o,1) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

f(0,1) can be expanded in a Maclaurin series.

$$f(o,1) = \frac{1}{\sqrt{2\pi}} \left[\sum_{n=1}^{\infty} U_n \right]$$

where $U_n = \dfrac{(-1)^{(3n-1)} \; z^{(2n-2)}}{(2n-2)!}$

Since $\lim_{n\to\infty} \left| \dfrac{U_{n+1}}{U_n} \right| = 0$, U_n converges absolutely. The probability integral of the normal distribution is to be calculated.

$$F(z) = \int_{-z}^{z} f(0,1) = \int_{-z}^{z} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} dz$$

Since $|U_n|$ converges there is some value of n beyond which $|U_n|$ decreases monotonically. Therefore, $\sum_{n=1}^{\infty} U_n$ can be integrated term by term to give the following result.

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-z}^{z} \left(\sum_{n=1}^{\infty} U_n \right) dz = z\sqrt{\frac{2}{\pi}} \sum_{n=1}^{\infty} V_n$$

where $V_n = \frac{1}{2z} \int_{-z}^{z} U_n dz$

$$V_n = \frac{(-1)^{(3n-1)} \cdot z^{(2n-2)}}{2^{(n-1)} \cdot (2n-1) \cdot (n-1)!}$$

Since $\lim_{n \to \infty} \left| \frac{V_n / z^{(2n-2)}}{V_{n+1} / z^{2n}} \right| = \infty$

$\sum_{n=1}^{\infty} V_n$ converges to F(z) for $|z^2| < \infty$

Of course the summation to infinity is not performed, but it was determined for the range of values of the argument that F* (z) produced an accurate result.

$$F^*(z) = z\sqrt{\frac{2}{\pi}} \sum_{n=1}^{29} V_n$$

On page 84 of the text it is noted that

$$A_j = \int_{-\infty}^{z_{\alpha_j}} f(0,1)dz$$

whereas

$$F(z_{\alpha_j}) = \int_{-z_{\alpha_j}}^{z_{\alpha_j}} f(0,1)dz$$

Therefore in order to determine A_j a further calculation was performed.

$$A_j = \frac{F^*(|z_{\alpha_j}|)}{2} + .5 \quad , \quad j \geq 5$$

$$A_j = -\frac{F^*(|z_{\alpha_j}|)}{2} + .5 \quad , \quad j < 5$$

$$|A_j - A_{j-1}| = \frac{F^*(|z_{\alpha_j}|) - F^*(|z_{\alpha_j}|)}{2} \quad , \quad j \geq 5$$

$$|A_j - A_{j-1}| = \frac{F^*(|z_{\alpha_j}|) - F^*(|z_{\alpha_j}|)}{2} \quad , \quad j < 5$$

$$|A_j - A_{j-1}| = \frac{F^*(|z_{\alpha_j}|) + F^*(|z_{\alpha_{j-1}}|)}{2}, \quad j = 5$$

The results are tabulated below.

	$M = 0.48085$		$\hat{S} = 0.087121$		$\sum F_j = 500$	
j	α_j	z_{α_j}	$F^*(z_{\alpha_j})$	A_j	$A_j - A_{j-1}$	F_j
1	.13085	-4.0174	.9999	0.0000	0.0000	0
2	.23085	-2.8695	.9959	.0020	.0020	1
3	.33085	-1.7217	.9150	.0425	.0405	20
4	.43085	-0.5739	.4343	.2828	.2403	120
5	.53085	0.5739	.4336	.7168	.4340	218
6	.63085	1.7217	.9148	.9574	.2406	120
7	.73085	2.8695	.9958	.9980	.0406	20
8	.83085	4.0174	.9999	1.0000	.0021	1
9	∞		1.0000	1.0000	0.0000	0

Notes. (i) α_j is upper limit of interval j.

(ii) Theoretical upper limit of interval 9 is infinity.

(iii) Since the calculation of $F^*(z_{\alpha_j})$ involves the summation $\sum_{n=1}^{29} V_n$, where

$$V_n = \frac{(-1)^{(3n-1)} \cdot z^{(2n-2)}}{2^{n-1} \cdot (2n-1) \cdot (n-1)}$$

the computer must handle some very large and very small numbers. The formation of V_{30} caused an overflow in the denominator (and a truncation to zero in the numerator where $j = 4, 5$). The result of this technical difficulty caused some instability which accounts for the inconsistency of the fourth decimal place in the column headed $F^*(z_{\alpha_j})$. Because the error is so slight it had no effect on the final results.