AN AIR MASS CLIMATOLOGY OF CANADA DURING THE EARLY NINETEENTH CENTURY

An Analysis of the Weather Records of Certain Hudson's Bay Company Forts

by

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Abstract

The post journals of certain Hudson's Bay Company forts were examined for evidence that air mass frequencies during the first half of the nineteenth century were markedly different from those of a modern (1955-1959) period. The "partial collective" technique of Bryson was used to determine the modern frequencies and to provide the basis of the conditional probability structure employed to estimate the historic air mass frequencies. There is evidence from each station for which analysis was performed of a greatly increased presence of "Arctic" air, probably as a consequence of a weakened zonal atmospheric circulation and a decrease in the eastward penetration of "Pacific" air.

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CHAPTER I

INTRODUCTION

A paper by Bryson (1966) introduced some interesting perspectives into the study of the Canadian climate, making clear for the first time the manner in which the climate of the central portion of the country may be interpreted in terms of the seasonal predominances of and interactions between "Arctic", "Pacific" and "United States" airflows. The relationship between Bryson's schema and the recognized structure of the mid-latitude general circulation of the atmosphere promotes speculation concerning the altered disposition of the three airflow types during a period with a rather different general circulation regime. It is generally conceded (Lamb, 1963) that the strength of the mid-latitude westerly vortex was considerably less than that of the present during the first half of the nineteenth century--this thesis represents an attempt to deduce the consequences of such a weakened zonal circulation in terms of Bryson's airflow model and to test the validity of these deductions using a previously untapped source of "meteorological" data for nineteenth century Canada.

The data are contained in the post journals of the forts of the Hudson's Bay Company and were abstracted for the period 1824-1851. These journals are essentially a record of the daily life and transactions of the forts and, as a rule, contain some reference to the state of the weather. Thus, apart from their unique interest to the historian, the journals contain material of use to the climatologist concerned with recent secular climatic variation.

The observations are mostly of a qualitative nature, comments on wind direction, precipitation type and the like, although occasional more comprehensive meteorological journals are to be found. But to obtain useful results from data of this kind it is necessary to resort to rather unusual analytic procedures, developed within the framework of a careful experimental design in order to test a very specific research hypothesis.

It is probably useful to provide a brief statement of this research hypothesis at this early point in the discussion. Recalling that the aim of the study is to attempt to develop insights into the Canadian climate of the nineteenth century, consideration of the results of other researchers leads to the conclusion that the investigation can be performed to greatest effect by instituting tests of the following general research hypothesis

> That the strength of the zonal winter circulation of the atmosphere was weak relative to the present during the historic period. That this weakness was reflected in a decrease in the frequency of Pacific air at stations to the east of the Cordillera.

A discussion which leads to the framing of this hypothesis is contained in the subsequent chapter. The remainder of the thesis consists of the development of suitable analytic procedures and the testing of the hypothesis on the data obtained from selected Hudson's Bay Company forts.

CHAPTER II

AIR MASSES AND CLIMATIC CHANGE

Air mass climatology is no longer fashionable. Most climatologists would probably share Hare's (1960, p. 356) sentiments

> Instead of applying rules of thumb to the crude air mass and frontal concepts of yesterday, we now attempt to apply the baroclinic and barotropic theorems to the entire three-dimensional field of the atmospheric elements. The concept of air masses was a brilliant approximation necessary in an age when upper air measurement was extremely difficult.

Hare is at pains to stress that modern climatology has not simply attached more euphonic or otherwise more impressive labels to older concepts, but rather is attempting to apply the theory and associated laws of the relevant branches of physics towards an understanding of the atmosphere. A crudely stated air mass climatology, with air of homogeneous and distinctive character located over a well-defined source region and clashing with air of a different character at frontal "battle-grounds" cannot be part of this analysis (although it is an engaging and successful teaching device, as is witnessed by its use in Strahler (1966) and Trewartha (1954), military analogies and all). The suggestion that air mass climatology is not an adequate model (or "approximation") in the context of contemporary knowledge of the atmospheric circulation and modern sophistication in the measurement of atmospheric phenomena forms a convenient springboard for this section, since it leads to the conclusion that some modification of air mass analysis might be an appropriate technique when measurement is only of the crudest and most inconsistent kind. This leads to an examination of modern approaches to air mass analysis and their relevance to schemes of recent climatic change on the North American continent. Thus justification and background are provided for the mathematical model developed and used in subsequent sections.

It would appear to be a reasonable general principle that the coarser the data available, the cruder, although not necessarily the simpler, the analytic model that should be used. The data used in this thesis are particularly coarse, consisting mostly of qualitative assessments of the state of the weather. Therefore the model used must be crude, although not necessarily without mathematical or conceptual elegance. We have learned above that air mass analysis is "crude" and involves the use of "rules of thumb"; this in itself is hardly sufficient justification for its inclusion in this study, these comments stress that the essentially two-dimensional approach of conventional air mass analysis does not adequately reflect the state of the three-dimensional system that is the atmosphere. But the data of this study were compiled by unskilled observers, restricted to impressions of temperature, wind and precipitation conditions in the lower layers of the atmosphere. The only observations directly representing processes operating in the third dimension are those of cloud formation, type and movement. Since the data used are mostly two-dimensional it would not be realistic to reject the use of air mass analysis on the grounds that it is inherently two-dimensional. But the analysis may be made more realistic by the incorporation of contemporary knowledge of the general circulation and the distribution of meteorological elements.

Realism may be introduced in two respects; firstly by considering the air mass concept in relation to present knowledge of the atmospheric circulation over the North American continent and secondly by realising that the concept of the air mass is essentially a statistical one. These improvements, largely due to Bryson (1966), lead to a conceptually attractive mode of analysis.

In its simplest form, air mass analysis assumes that air of a homogeneous character develops in the "source regions", above the "polar" and "tropical" oceans and continents. These air masses then meet in the zone of the mid-latitude westerlies at temperature and humidity discontinuities known as "fronts", along which the depressions of mid-latitudes form. Bryson (1966) develops this simple scheme in two ways, firstly by introducing the concept of the "climatic complex" and secondly by stressing the significance of the physiographic context of North America.

The simplistic view of the air mass treats it as a homogeneous body of air stretching from source region to front--the climatic complex involves the more limited conclusion that, at any particular station, air coming from a specific direction will tend to be associated with a relatively narrow range of temperatures, with particular cloud conditions, with certain precipitation types and amounts etc. This air may be tracked to its source region by any of a number of techniques (discussed later in this chapter) and an air mass climatology so constructed. Bryson offers a quite convincing discussion of the climatic complex as the determinant of the position of the northern boundary of the Canadian Boreal forest, stressing that it is the <u>complex</u> rather than its individual elements which determine the location of this boundary.

The air mass determinations for North America of Brunnschweiler (1952) are those included in the familiar elementary climatology and physical geography texts. Firmly based on the European pattern, these determinations do not correspond well with the physiography and known airflow characteristics of North America. By means of a rather tedious trajectory analysis, Bryson was able to demonstrate that the contribution of Pacific air to the Canadian Prairies and even to some parts of Ontario is much greater than that shown by Brunnschweiler.

Bryson (1966, pp. 230-234) chose to delimit four source regions for Canada--the Pacific, the Arctic, the United States and (relatively small) the Atlantic. Figure 1 summarises Bryson's analysis; note in particular the broad wedge of Pacific air penetrating the prairies and central Canada, and the zone of contact between Pacific and Arctic air, prominent in later discussion as the "Arctic Front". Conventional analysis tends to group together the Arctic and Pacific air in the category "continental Polar", obscuring the reality of the Pacific intrusion.

More explicit acknowledgement of the statistical content of the air mass concept has been made. Discussion of the climatic complex above was phrased in such terms as "associated", "tend" and "range", illustrating the nature of this change. Bryson (1966) has observed that frequency distributions of daily temperatures are often multimodal or of a very irregular shape, leading to the hypothesis that the frequency distributions represent the composite of several distributions,

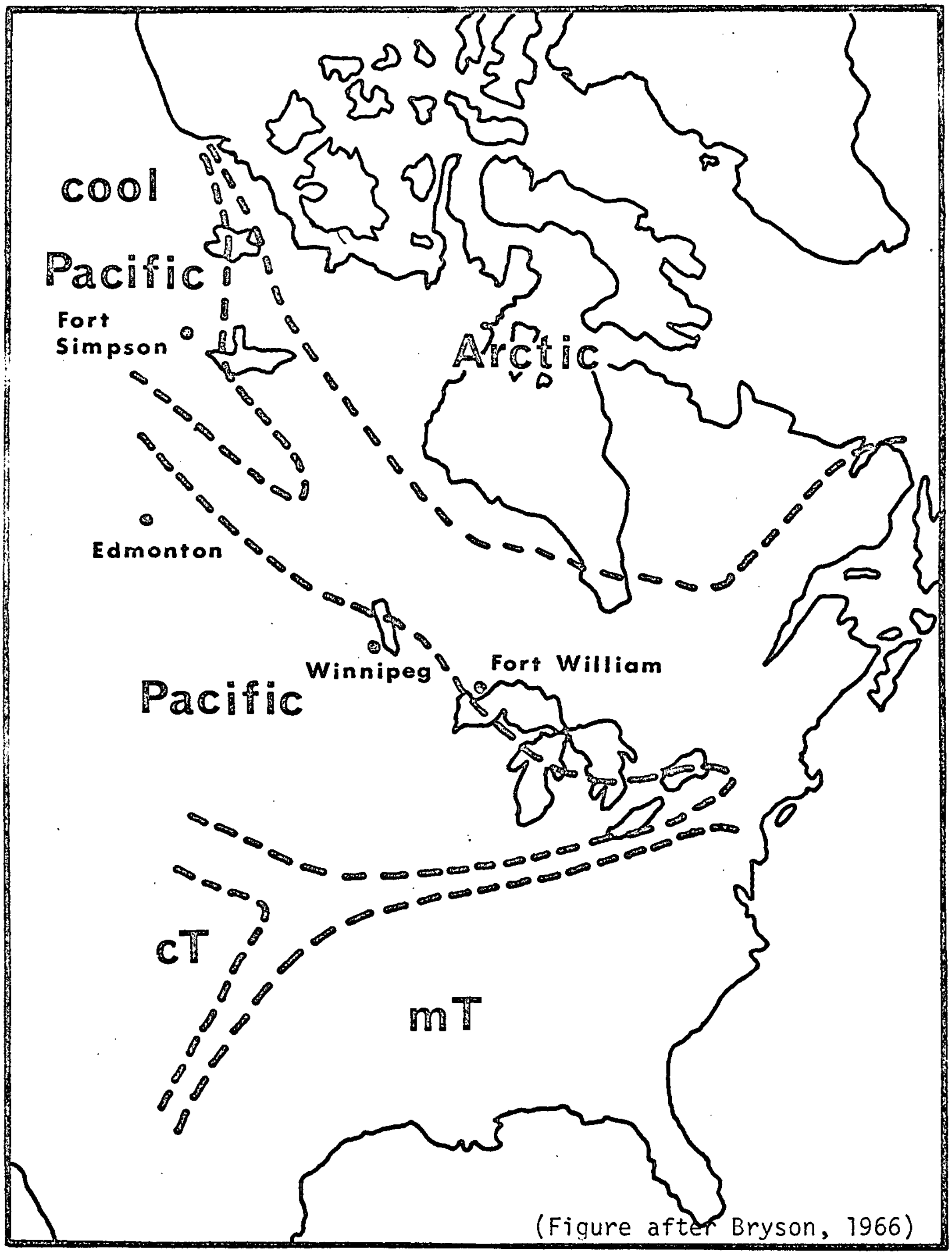

FIGURE 1

AIR MASSES OF NORTH AMERICA. AREAS CONTAINED WITHIN THE HEAVY BROKEN LINES EXPERIENCE THE NAMED AIR MASS 50% OF THE TIME

mT = maritime Tropical cT = continental Tropical each representing the contribution of a particular air mass.

Frequency distributions derived for air mass source regions were unimodal normal type curves, leading Bryson to the conclusion that the form of the frequency of temperature at a station outside of the source region represents a composite of several such curves. The consistency with which he was able to identify the separate components throughout the North American continent supports this conclusion. A detailed discussion of the techniques appropriate to the identification of the various components will be given in a subsequent section; here the aim is to stress the consciously statistical approach employed. The use of this approach implies that the air mass climatology of a station should be discussed in terms of the relative frequency of the various air masses, rather than in terms of the dominance of any one particular air mass.

A further implication is that the mean temperature of an air mass, as expressed by the mean temperature of the appropriate component of the composite distribution, may exhibit a systematic spatial variation. It is clear that, in the Northern hemisphere, temperature should decline systematically from mid-latitudes northward to the pole in response to differing insolation, quite apart from any effects of the air mass distribution.*

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*One rather simple-minded attempt to incorporate this effect into the analysis was made by the author (Minns 1968) in a study to establish the relationship between the boundaries of continuous and discontinuous permafrost and air mass distribution in Canada. The data were divided into groups within and without the permafrost boundaries and an analysis of covariance was then performed, removing the latitudinal influence by linear regression. Analysis of variance on the adjusted means revealed

The use of these techniques provides Bryson with a comprehensive series of maps illustrating the seasonal predominances of the various source regions and the locations of the major fronts. It will be recalled, however, that the aim of this study is to unify in some quantifiable manner series of weather observations from the nineteenth century. Recall also the uneven quality of these records. Neither streamline nor trajectory analysis yield an immediately obvious means of completing this task, at least in any way lending itself readily to adequate mathematical formulation.

Bryson's third technique is that of "partial collectives", alluded to above in the discussion of the form of the histogram of frequency of temperature. The aim of this technique is to break the histogram into its component distributions and to regard these as contributions from separate air masses. This technique is much better suited to the purposes of this study and will be discussed below.

Computational aspects and validation of the technique will be discussed in later sections; the present object is to point out the implications of a successful application of the technique. The form of the histogram of temperature frequency may be regarded as a reflection of the probability density function of temperature at the station under consideration. Similarly, the individual normal distributions isolated by the technique of partial collectives may be regarded as a reflection

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significant differences between the groups, which might with some justification be regarded as evidence of an air mass effect.

of the probability density of temperature in a particular air mass.*
It is then possible to estimate probability of the occurrence of any air mass given a particular temperature.** Since the conditional probability of this temperature given wind, precipitation and cloud conditions may also be estimated, it is possible to derive an estimate of the probability of occurrence of a particular air mass, given the prevailing meteorological conditions. Making certain assumptions of independence and constancy, it is possible to apply these results in order to obtain estimates of air mass frequency for certain critical locations during the nineteenth century.

The especial virtue of the technique described is that it may be applied to records of uneven quality, although, clearly, the less complete the record, the less precise the estimate of air mass frequency.

It has been shown that if the data of this study are approached in terms of the air mass concept, they may be unified in some meaningful way by the application of the technique of partial collectives. The intent of the early part of this chapter has been to show that modern objections to the use of air mass analysis are largely not meaningful, in the context of this study and its data sources. Bryson's analyses were discussed in order to demonstrate that the air mass

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*i.e., the conditional probability of temperature T, given the presence of air mass A ($P(T|A)$).

**a more detailed mathematical treatment is given at the beginning of the next chapter.

can co-exist with contemporary emphasis on the three dimensional structure of the atmosphere in general, and the westerly vortex in particular. To this moment a rather negative case has been made for not rejecting the use of air mass analysis. A more positive approach will follow, since discussion of the nature of recent climatic change leads to the conclusion that analysis of air mass frequencies should provide a particularly sensitive index of the state of the general circulation, so long as the analysis is based on a careful experimental design.

It is convenient to introduce this discussion with the following brief quotation from Sawyer (1966, p. 226)

> The causes of such persistent anomalies in the circulation are not known, but the fact that it is quite common for the large-scale circulation to revert to a particular abnormal form several times in one season (despite intervening more normal periods) does suggest that the more spectacular and persistent anomalies of the seasons are more than accidental vagaries of a rather unstable system.

An important implication of this statement is that the atmospheric system contains within it sufficient variability to take on, at least for a short time, circulation patterns which seem to have predominated during periods of "climatic change". A further implication is that the causes of such shifts in the circulation patterns are unknown. A brief scanning of the nine papers contained in "Theories of changes of climate" (Part iii, UNESCO 1963, pp. 277-380) is evidence enough of this. For the purposes of this study, such questions are regarded as metaphysical; discussion is restricted to the apparent <u>form</u> of the recent climatic variation. The first of these two implications is the focus of the early discussion since it leads to a critical and open-minded consideration of the form and structure of climatic change. Thus prepared it is possible to embark upon a brief survey of what is known of climatic fluctuation during the nineteenth and twentieth centuries and to use the benefits of what has largely been a European experience to construct an adequate and appropriate experimental design for the present study.

One of the most refreshing developments in the study of climatic change is due to Curry (1962). He illustrates that large fluctuations from the mean are not at all unlikely in series of random variables having a number of simple probability distributions. He therefore concludes that climatic series might encompass similar large fluctuations about the mean, without any profound change in the controlling parameters. His models serve only as very crude approximations to climatological realities, but the approach is an interesting one, insofar as it is a contrast to the conventional deterministic approach of the meteorologist. However, Sawyer's words, quoted above, warn against ascribing climatic fluctuation to "accidental vagaries", stressing that the atmosphere appears to alternate between a number of distinct states, rather than the random fluctuation envisaged by Curry. Nevertheless, both authors seek to comprehend climatic fluctuation in terms of the action of contemporary processes, Curry favouring the suggestion that observed secular variation may lie within the expected variability of the process producing the climatic series, Sawyer preferring to analyse the probable response of the atmosphere to changes in external energy supply.

As a result of this analysis, Sawyer stresses that a climatic variation should not be expected to take the form of north-south shifts of the familiar climatic belts, but rather alterations in the intensity of east-west precipitation and temperature gradients, in response to changes in the strength of the atmospheric circulation. In support of this contention he shows that the poleward transport of heat by the general circulation varies in the ratio 7:1 between winter and summer, yet the poleward shift of the subtropical high is only of the order of six degrees of latitude.

Dzeerdzeevskii (1963, p. 285) states that daily synoptic analyses may be grouped into four basic circulation patterns for the period 1899-1954, transitional types being present only 2 per cent of the time. The four patterns are as follows

1. Well formed polar anticyclone; zonal circulation in high latitudes; two to three intrusions of southern cyclones into high latitudes. Over the greater part of the hemisphere the zonal transfers are preserved.

2. "Violation of zonality"--a single intrusion of arctic air masses over a hemisphere; zonal flows are preserved in all other sectors.

3. Two to four simultaneous intrusions over a hemisphere; this is the group of meridional circulation types.

4. Development of cyclonic activity in high latitudes and over the Arctic Ocean. Inflows of "southern cyclones" reach far to the north and often cross the north pole region.

(Dzeerdzeevskii, 1963, p. 225)

Other workers, notably Willett (1950) and Butzer (1957), have preferred to talk in terms of the so-called "index-cycle"; essentially considering fluctuation between Dzeerdzeevskii's state 1, strong zonal flow, and Dzeerdzeevskii's state 3, disturbed zonal flow. A brief review of the use of zonal index models is contained in Barry (1967, pp. 110-112) and will not be repeated here, except to echo Barry's conclusion (p. 112) that this concept, originally developed for synoptic purposes, appears better suited to a discussion of palaeoclimatology. It is generally held that strong zonal flow (high index) is associated with milder weather in mid-latitudes, while disturbed zonal flow (low index) is associated with more extreme conditions, especially in continental interiors (Butzer, 1959; Willett, 1950).

Figure 2 illustrates Mitchell's (1963) determinations of world temperature trends since 1840. Note the general warming trend in the annual values and the pronounced warming trend in the winter values, although there is some evidence of a decline since 1940. Mitchell (p. 163) states that, for the period of more reliable data (1890-1960), the warming trend to 1940 and the subsequent cooling are both statistically significant. It is necessary to recall Sawyer's warning that climatic change is unlikely to occur as a uniform increase or decrease of temperature over the entire earth as might be implied by the previous discussion of Mitchell's results. Mitchell's maps of the twenty year change 1900-1919 to 1920-1939 indicate, for North America, some cooling within the zone shown by Bryson (see figure 1) to be dominated by Arctic air, and warming over the remainder of the continent. This would be consistent with a hypothesis of increasing zonal flow and hence an increasing incursion of Pacific air into the continental interior. This hypothesis is supported by the work of Lamb (Lamb and Johnson, 1959; Lamb, 1963; Lamb, Lewis and

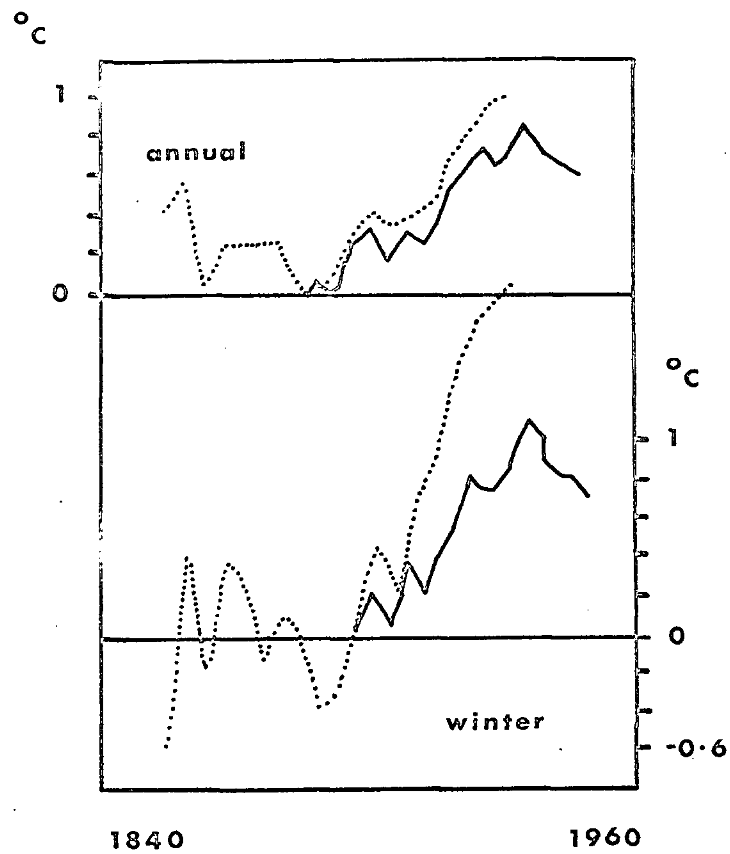

FIGURE 2

TRENDS OF WORLD TEMPERATURE SHOWN FOR SUCCESSIVE PENTADS RELATIVE TO THE 1880-1884 PENTAD. SOLID CURVES REPRESENT RELIABLE WEIGHTED AVERAGES, BROKEN CURVES ARE WILLETT'S 1950 DETERMINATIONS, BASED ON LESS EVIDENCE

(Figure and text after Mitchell, 1963)

Woodruffe, 1966).

Lamb (1963, p. 128) reviews the evidence for what is commonly called the Little Ice Age, which he dates at approximately A.D. 1430-1850. He cites expansion of the Arctic ice pack, colder sea temperatures in the North Atlantic, glacier advance in Europe, Asia Minor and North America and many other features as evidence for the existence of this phase. A subsequent meteorological analysis of the available evidence (pp. 129-140) leads to the following conclusion:

> Increases of strength of the zonal circulation have been found in January and July over widely separated parts of the globe, apparently being quite general from around the middle of the last century and culminating around 1930 in the Northern Hemisphere and 1900-1910 in the Southern Hemisphere . . . The increases from circa 1800 to 1930 in the strength of the zonal circulation over the North Atlantic in January amounts to between 5 and 10 per cent, but in general is probably less than this.

Lamb suggests that this increased westerly flow was associated with a northward shift of the wind belts in the northern hemisphere.

The consequences of a westerly flow weak relative to the present should include a decrease in the amount of air reaching localities to the east of the Cordillera. Figure 3 shows Bryson's (1967) conjecture of the air mass pattern during times of low and high zonal index. The low index pattern should bear some relationship to the patterns prevalent during the nineteenth century, although perhaps in a less extreme form. Subsequent analysis should therefore be designed so as to test for the existence of this kind of low-index pattern during the nineteenth century.

The stations for which the analysis was performed were chosen for their location on or close to the winter position of the "Arctic

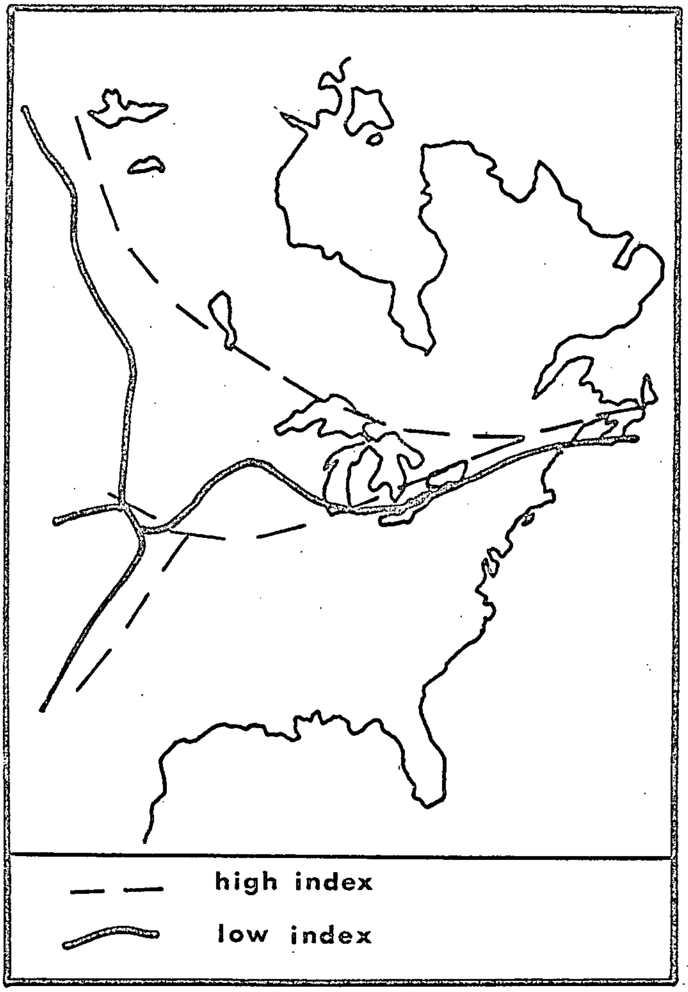

FIGURE 3

SCHEMATIC DRAWING SHOWING THE EASTWARD PENETRATION OF PACIFIC AIR DURING TIMES OF HIGH AND LOW ZONAL INDEX

(Figure after Bryson and Wendland, 1967)

Front" between Arctic and Pacific air (see figures 1 and 3). The stations chosen were Fort Simpson, Edmonton, Winnipeg and Fort William, each marked on figure 1.

The initial intention was that the analysis should be performed for three stations only, Fort Simpson, Winnipeg and Fort William. The stations were chosen carefully so as to test the general hypothesis of a zonal circulation weak relative to some modern control period; each station having its own particular relevance to this hypothesis.

Fort Simpson was included in order to investigate Bryson's (1966, p. 267) assertion that the Arctic Front is

> . . . topographically anchored year round at the northern end of the Cordillera near Aklavik, but swings north and south with the seasons in the continental interior where the terrain is relatively flat.

Presumably this result may be extended by analogy to periods of weakened or strengthened zonal circulation. Air mass frequencies for Fort Simpson might thus be expected to be relatively more stable than those at stations in the continental interior.

Winnipeg, lying (figure 1) on the northern edge of significant flow from the Pacific would appear to be a particularly suitable station to use in the construction of some measure of the degree of penetration of the westerlies during the historic period. Unfortunately, data for Winnipeg were relatively scarce and of generally poor quality, so data for Edmonton were added to the analysis as a further measure of the strength of the westerlies.

Fort William was included as a further check on the depth of penetration of the Pacific air and, assuming a decrease in the frequency of air from that origin, to determine the extent to which it was replaced by air of Arctic or of United States origin.

The next chapter outlines the framework of the analysis devised for testing the general hypothesis of a weakened zonal circulation during the historic period.

CHAPTER III

THE MODEL

The use of the term "model" implies only that the framework of the analysis will be exposed and that basic assumptions will be stated and discussed. Questions which may be more justly considered as computational are delayed until the final chapter.

It is first of all necessary to present more precisely some of the material discussed in the previous chapter. The suggestion was made that the form of the histogram of temperature frequency for any one month at some location would represent contributions from two or more air mass source regions.

Suppose there to be two significant sources, A and B, then

$$\text{freq}(T) = A(T) + B(T)$$

where: freq(T) = frequency of observations of temperature T

A(T) = frequency of air mass A at temperature T

B(T) = frequency of air mass B.

Bryson (1966, p. 237) points out that, in an air mass source region, the histogram of temperature frequency closely approximates the form of the density of a normally distributed random variable, i.e.

$$f(T) = (2\pi b_3)^{-\frac{1}{2}} \exp\left(-(T-b_2)^2/2b_3\right)$$

where: f(T) = probability density of temperature in the source region

b_2 = the mean of this density, i.e., the mean temperature in the source region

b_3 = the variance of the distribution.

It therefore seems not unreasonable to postulate that the contribution from any source in another location should retain a density function of this kind, although with some change in parameter values. Confirmation of this postulate will be provided in the following chapter--an example of a curve from a source region is figure 4. This curve is from the sub-Arctic station Baker Lake for the month of February during the years 1950-1959. Bryson's streamline map for February (Bryson, 1966, p. 255) shows this station to be influenced primarily by a north westerly outflow from an anti-cyclone centered over the western Arctic, but also by a more southerly variety of Arctic air. The bimodal form of the curve is indicative of a situation of this kind.

Given that such a histogram does represent a composite of several, say two, normal distributions, then it is possible to write

$$\mathrm{freq}(T) = b_{A1} \exp\left[-[T-b_{A2}]^2/2b_{A3}\right] + b_{B1} \exp\left[-[T-b_{B2}]^2/2b_{B3}\right]$$

where: $\mathrm{freq}(T)$ = as before

b_{A1} = the frequency of the mean of the distribution of observations from air mass A

b_{A2} = the mean temperature of air mass A, <u>at this location</u>

b_{A3} = the variance of temperature for air mass A at this location.

Generalisations to situations with three or more source regions are obvious.

Given equation 1, adopting a "relative frequency" definition of probability* and making one rather gross assumption, it is possible to

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*The terminology and elementary manipulations of probability

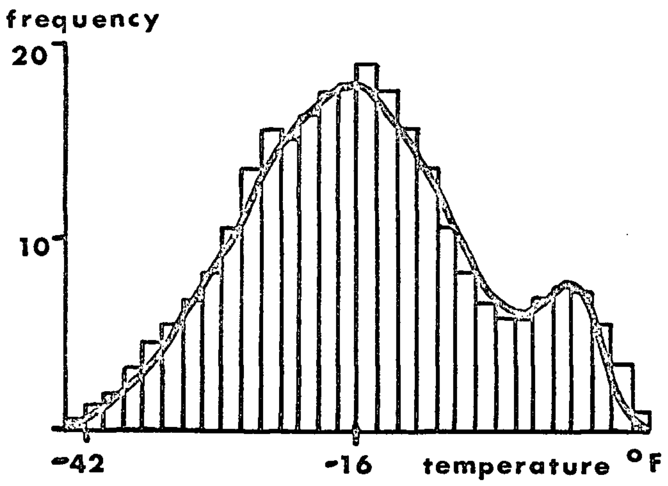

FIGURE 4

TEMPERATURE-FREQUENCY HISTOGRAM FOR BAKER LAKE, FEBRUARY 1950-1959. SOLID LINE IS THE FITTED CURVE write the probability of air from source A on any particular day, given that the temperature is T

$$P(A|T) = A(T)/freq(T)$$

where: $A(T)$ and $freq(T)$ are defined as before

$P(A|T)$ = the conditional probability of air from source A given that the temperature is T.

The "gross assumption" noted is that the day to day probabilities are independent of one another, that the probability does not vary from day to day according to the temperature and air mass source recorded for the previous day, or for the previous few days.

In view of the very marked persistence effects evident in climatic series it is necessary to undertake some discussion of this assumption. First recall the definition of conditional probability

$$P(A|T) = P(A \cap T)/P(T)$$

where: $\cap$ = the symbol for set intersection

$P(A \cap T)$ = the probability that the temperature will be T <u>and</u> that the air mass will be A

$P(T)$ = the probability that the temperature will be T.

i.e., the conditional probability is the quotient of the probability that the air will both have temperature T <u>and</u> come from source A, and the probability that the air will have temperature T. Now it is clear that both of these probabilities will vary from day to day, that the probability of temperature T will increase if the temperature of the

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used here may be found in any introductory text; see for example Feller, W., <u>An Introduction to Probability Theory and its Applications</u>, Wiley, New York, 1957.

previous day was close to T and that the probability that the air mass source will be A will increase if A was the source on the previous day; indeed this is one of the principal justifications of the air mass concept. But it is also clear that these probabilities will tend to vary together, thus minimising the effect of their variability on their quotient.

A numerical example may help to clarify this statement. Suppose that the air can take on one of two temperatures, T and T_o, and that the following holds

$P(T|T)$ = the probability of temperature T if the previous day's temperature was T = 0.8

$P(T|T_o) = 0.2$.

Now hypothesise that $P(A|T)$ is a constant (that is the assumption stated above), and that this constant is 0.5. Then, from the definition of conditional probability, $P(A \cap T)$ must equal 0.4 if the previous day's temperature was T, and 0.2 if the previous day's temperature was T_o, i.e., as the probability of T increases so does that of $A \cap T$. It is intuitively obvious that a relationship of this kind exists outside the scope of this simplified example, and it seems not unreasonable to propose that the conditional probability of an air mass, given the temperature, is sufficiently stable to permit meaningful analysis.

A second plea for this assumption is a pragmatic one, stated here by Arbib (1964, Preface, no page number)

> We apply (original italics) mathematics to derive far-reaching (sic) conclusions from clearly stated premises. We can test the adequacy of a model of the brain (or climate) by expressing it in mathematical form and using mathematical tools to prove general theorems. In the light of any discrepancies we find between

these theorems and experience we may return to our premises and reformulate them, thus gaining a deeper understanding of the workings of the brain.

To this point, an estimate has been made of the conditional probability $P(A|T)$. Our present purpose is to relate this estimate to other meteorological phenomena, in order to facilitate estimates of air mass frequencies without consistent records of temperature. The procedure is most easily demonstrated by the following much simplified example.

Suppose we have the following distribution of temperature (in 5° classes)

T	P(T)
30	0.10
35	0.40
40	0.20
45	0.25
50	0.05

and

T	P(A|T)	P(B|T)
30	1.0	0.0
35	1.0	0.0
40	0.7	0.3
45	0.2	0.8
50	0.0	1.0

Clearly, air mass A, the colder of the two, is the predominant contributor to this distribution.

Let us further suppose that records of the frequency of winds from the various quarters have been kept. By associating these with records of temperature, the following values for the probability of temperature T on a day with a north wind have been derived

T	P(T|N)
30	0.15
35	0.60
40	0.15
45	0.05
50	0.05

It is clear that the north wind is strongly associated with air mass A--to make this notion more precise note that

$$P(A|T|N) = P(A|T).P(T|N)$$

where: $P(A|T|N)$ = the probability of air from A given temperature T, with the probability of T conditioned by the occurrence of the north wind.

For the example

T	P(A|T|N)	P(B|T|N)
30	0.150	0.000
35	0.600	0.000
40	0.105	0.045
45	0.010	0.040
50	0.000	0.050

The final step is to calculate the probability of air from A, given a north wind, $P(A|N)$, which is given by

$$P(A|N) = \Sigma\ P(A|T|N) = 0.865$$

and similarly

$$P(B|N) = 0.135$$

Once again, generalisations to cases with more than two air masses and where meteorological elements other than wind direction are measured are obvious.

The records of the Hudson's Bay Company include a number of post journals which have fairly continuous series of climatic observations for the first half of the nineteenth century. Sometimes these are measures of temperature, but more often they are of wind direction, amount of cloud cover, the occurrence and type of precipitation and other similar quantities. It is suggested that the application of similar procedures to those described above should lead to a useful assessment of the relative frequencies of the various air masses at these posts. But this application requires one further assumption; that the conditional probability of any air mass source, given any particular wind, precipitation or cloud conditions remains relatively constant in the long term. Under this assumption, and those discussed above, and assuming that the frequency distribution can be successfully decomposed into the contributions of the various source regions, an estimate of air mass frequency for the historic period can be made. The problem of fitting the appropriate curve and of the form the estimate should take are discussed in the following chapter; this chapter concludes with consideration of the implications of the assumptions of constancy.

Consider the example of Arctic air and the north wind. An increase in the frequency of Arctic air would presumably represent an increase in the frequency of the north wind--there appears to be no reason to suppose that the degree of association between the north wind and Arctic air should alter radically without some sort of catastrophic alteration in the major determinants of the atmospheric circulation, the disposition of incoming solar radiation, the orbit of the earth or the distribution of the continents. The argument previously applied for the short term may be applied again; referring once more to the definition of conditional probability,

$$P(A|N) = P(A \cap N)/P(N)$$

where all quantities are defined as above. $P(A \cap N)$ and $P(N)$ should vary together in time, hence their quotient should remain relatively stable.

A pragmatic argument was offered as a justification for the assumption in the short term case; this assumption may be regarded as a reflection of the wish of meteorologists to comprehend recent climatic fluctuation in terms of the action of contemporary processes. Recall the discussion of the work of Sawyer and Curry in the previous chapter, both of whom stress that we should seek the mechanisms of recent climatic change within observed contemporary pattern and process.

In addition, it is clear that if the assumption is not valid, then the error will be such as to bias the values of air mass frequency calculated for the historic period towards those of the present. Therefore any change detected may be regarded as doubly significant in view of the probable "inertia" or conservatism in the estimation process.

If the records were of an even or homogeneous character, the analysis might be performed by the use of more conventional techniques --calculation of mean temperatures, construction of wind roses, determination of precipitation amounts, etc., but the technique outlined here may be applied to records of quite uneven quality--although the less adequate the record, obviously the less precise the estimate of air mass frequency. With limited data it is only to be expected that the analysis will be subject to limiting assumptions or conditions such as those stated above.

CHAPTER IV

THE ANALYSIS

The analysis follows the order in which the model was developed in the previous chapter.

1) Contemporary data a) fitting the composite curves
   b) establishment of the relevant conditional probabilities
2) Historic data - determination of probable air mass frequencies.

The organisation of both sections will be the same--firstly a discussion of the data used, their relevance to the problem and their source, and secondly an outline of the techniques and facilities used in computation, and finally presentation and discussion of the results obtained. The chapter will conclude with a summary of the results and a discussion of their significance to the hypothesised model of climatic change.

It will be recalled from the second chapter that four stations were used in this analysis, Fort Simpson, Edmonton, Winnipeg and Fort William and that winter months were to be the focus of the analysis. The months of January, February, March and April were considered, since the Arctic Front is located along the line of the four stations during this period, and in order to investigate any delay in the development of the spring circulation pattern. It was intended initially that analysis of contemporary conditions should be performed on data for the period 1950-1959 but difficulties in obtaining some of the relevant information caused this period to be reduced to the pentad 1955-1959, inclusive. A problem which arose, probably as a result of this reduction, will be discussed later in this chapter.

The analysis requires series of daily temperature for the fitting of the composite curves and series of cloud, precipitation and wind observations for the establishment of the relevant conditional probabilities. Table 1 (page 31) lists the sources used. The cloud, precipitation and wind series, Table I, were modified so as to make them as compatible as possible with the historic records. Maximum temperature was chosen following Bryson (1966, p. 235) since he obtained significant results using this quantity and because he suggests that maximum temperature is an especially characteristic quality of any particular type of air.

Bryson (1966, p. 236) outlines a technique for the fitting of a composite curve consisting of the sum of several normal distributions to a histogram of temperature frequency. This graphical approach seemed rather unsatisfactory, leading Bryson to investigate the development of a least squares fitting routine. Since, given the equation of the distribution, estimates of the parameters may be made by conventional curve fitting procedures this approach has been followed here.

Computation was performed in the Computing Centre of the University of British Columbia, using that centre's IBM 7044 facility and its successor, an IBM 360/67. The program used in the fitting of

TABLE I

SOURCES OF MODERN DATA

Element	Characteristic	Units	Source
Temperature	Daily maximum	°F	Monthly Record
Precipitation	Occurrence & type	1/2	Monthly Record
Wind	Direction	Bearing	Table 14
Cloud	Amount	1/2/3	Table 14

Sources: Monthly Record, published monthly by the Meteorological Branch, Department of Transport, Canada.

Table 14, Weather Summary, Synoptic Hours also available from the Department of Transport.

Units: Precipitation - trace or none = 1; measurable amount = 2.

Cloud - 0,1,2 tenths = 1; 3,4,5,6,7 tenths = 2; 8,9,10 tenths = 3.

the curves was obtained through the SHARE program library--routine SDA 3094; Least squares estimation of non-linear parameters, familiarly known as NLIN2.

Input for NLIN2 consisted of temperature frequency histograms, calculated separately for the five year period for each month at each station. Bryson (1966, p. 235) used a 2°F class interval in the calculation of similar histograms; experiments by the writer confirmed that, over the ten year period, this class interval gave relatively smooth curves and clearly differentiated peaks. However, for the five year period used in this study, a 3°F class interval yielded more regular curves.

The equation for distributions comprising the composite of several normal distributions was given in the previous chapter. This equation includes the sums of several exponentials; such functions are notoriously difficult to fit,* therefore, subsequent to the calculation of the histogram, the frequencies were subjected to a "center-weighted" filter of the form

$$y_i' = (y_{i-1} + 2y_i + y_{i+1})/4$$

where: y_i^1 = the adjusted ith frequency

y_{i-1} = the observed i-1th frequency

y_i = the observed ith frequency

y_{i+1} = the observed i+1th frequency

The purpose of this filter is to minimize the effect of random error

*The system library program for least-squares curve fitting at the University of B.C. was unable to fit this function.

on the form of the histogram and thus to facilitate the fitting of the curves.

In effect, the filter "smooths" the curves and removes all but the major peaks. Justification for the use of the filter is twofold; firstly, and primarily, Bryson obtained significant and consistent results through its use and secondly it would be almost impossible to fit the curves without prior smoothing, in view of the nature of the function involved. It should also be remembered that one application of the least-squares procedure is the smoothing of curves (McCalla, 1967, chapter 7), therefore it can be argued with some conviction that a least-squares solution of the best fitting curve for the observed histogram should not differ considerably from the least-squares solution for the filtered histogram.

The algorithm for NLIN2 is presented by Marquardt, 1963; a readable account of the <u>differential correlation</u> technique on which it is based is given in McCalla (1967, pp. 255-261). No discussion of this technique will be made here. Suffice it to point out that input for the routine consists of the temperature frequency histogram and initial, fairly accurate, estimates of the values of the parameters of the individual normal distributions. The program contains a plotting option which makes it particularly suitable for terminal operation in those cases where it was difficult to establish a suitable fit. The program converged to a solution on only about 50 per cent of the occasions on which it was used, since poor estimates of the parameters will lead to a rapid propagation of error through the program.

But before NLIN2 is used it is necessary to decide on the number of normal distributions represented in the histogram. To this end, the frequency of winds from each direction for each temperature class was calculated. Comparison of these results with the histogram of temperature frequencies usually reveals that a certain range of temperatures is associated with a certain range of wind directions. Two or three groups of predominant wind directions are usually apparent. With justification these groups may be regarded as representing separate air masses and the curves fitted accordingly. Inter-month consistency is probably the most important criterion of the validity of this analysis.

From the analysis of chapter II, it would appear that the curves for each station should reflect the presence of Pacific, Arctic and United States air. Consider first the curves for Edmonton. For each month, three components have been fitted; from their temperature and wind associations these have been identified as Arctic, United States and Pacific, as indicated in table II (page 35). The interconsistencies in the monthly results are an encouraging index of the appropriateness of the analysis.

Table III (page 36) presents the temperatures and source regions for Winnipeg. Once again westerly, southerly and northerly components indicate the presence of air from each of the three source regions. Two differences are to be seen between these figures and those for Edmonton; firstly, whereas the Pacific air was the warmest at Edmonton, for Winnipeg the US air becomes the warmest. Two factors probably

TABLE II

AIR MASS TEMPERATURES AND SOURCES FOR EDMONTON

JANUARY				FEBRUARY			
#	Mean T	Winds	Source	#	Mean T	Winds	Source
1	-1	N	Arctic	1	10	N	Arctic
2	13	S	US	2	28	S	US
3	34	W	Pac	3	41	W	Pac
MARCH				APRIL			
#	Mean T	Winds	Source	#	Mean T	Winds	Source
1	16	N	Arctic	1	30	N	Arctic
2	29	S	US	2	43	S	US
3	42	W	Pac	3	57	W	Pac

(Mean T is mean maximum daily temperature in Fahrenheit degrees)

TABLE III

AIR MASS TEMPERATURES AND SOURCES FOR WINNIPEG

JANUARY				FEBRUARY			
#	Mean T	Winds	Source	#	Mean T	Winds	Source
1	2	N&W	Arc/Pa	1	1	N	Arctic
2	22	S	US	2	17	W	Pac
				3	32	S	US
MARCH				**APRIL**			
#	Mean T	Winds	Source	#	Mean T	Winds	Source
1	18	N	Arctic	1	45	N	Arctic
2	32	W	Pac	2	64	W	Pac
3	45	S	US	3	78	S	US

account for this change, the prolonged travel and modification of the Pacific air on its track across the Prairies and the inhomegeneity of the United States air; Winnipeg experiencing a mixture of Gulf and interior U.S. air, whereas Edmonton would probably be restricted to the interior variety. The second change is that for January, it was impossible to distinguish between Arctic and Pacific air. This is an indication of the cause of the tendency to attribute large frequencies to the classification "continental Polar". Data for a longer period and the use of a smaller class interval in the calculation of the histogram would probably make it possible to separate the two kinds of air.

As table IV (page 38) shows, the histograms for Fort William appear to comprise two components, one associated with westerly, north-westerly and northerly winds, the other with south-westerly, southerly and south-easterly winds. The wide range of wind-directions associated with the former implies that it represents the sum of Pacific and Arctic flows, whereas the latter appears to represent primarily United States (Gulf) air. However, a marked decrease in the frequency of south-west winds in April, a month in which Bryson (1966, p. 258) notes a very considerable weakening in the eastward penetration of the westerlies, leads to the suspicion that the Pacific air is also represented in the warmer of the two distributions. Indeed, Bryson's maps of the circulation in the first four months of the year indicate that both Arctic and Pacific air reach this location in a predominantly westerly flow, Arctic mostly from the north-west and Pacific mostly from

TABLE IV

AIR MASS TEMPERATURES AND SOURCES FOR FORT WILLIAM

JANUARY				FEBRUARY			
#	Mean T	Winds	Source	#	Mean T	Winds	Source
1	8	NW	Arc/Pa	1	15	NW	Arc/Pa
2	26	S	US	2	29	S	US
MARCH				**APRIL**			
#	Mean T	Winds	Source	#	Mean T	Winds	Source
1	24	NW	Arc/Pa	1	46	NW	Arc/Pa
2	34	S	US	2	67	S	US

the south-west. Therefore, the results from Fort William must be treated with caution.

The predominant winds at Fort Simpson are from the north-west and south-east; the fact that there is no difference in temperature associations between them implies some sort of topographic control of the flow of the wind. Confirmation is provided by the location (see figure 5) on the Mackenzie River between the Horn Mountains to the north-east and the Mackenzie Mountains and the Cameron Hills to the south-west. At this point the Mackenzie flows from south-east to north-east, in the direction of the predominant winds. It appears reasonable to suppose that the predominant air mass, associated with the south-east and north-west winds is an Arctic type entering the valley either to the south or to the north of the Horn Mountains (see figure 5). This air is designated "Mid Arctic" in table V (page 41), to distinguish it from a colder variety which Bryson (1966, p. 254) shows as developing over the north of the District of Mackenzie. This air is here designated "Full Arctic" and is apparent in the tabulations for January and March, primarily in association with calms and northerly winds. Fort Simpson is situated at the confluence of the Liard and Mackenzie rivers, the Liard flowing into the Mackenzie from the south-west. Bryson (1966, p. 238) distinguishes a type of air which he designates "Yukon Pacific", crossing the Cordillera through the gap made by the valley of the Liard River. It is this kind of air which must be represented by the warmest component present in the histograms of temperature frequency, associated with westerly, south-westerly

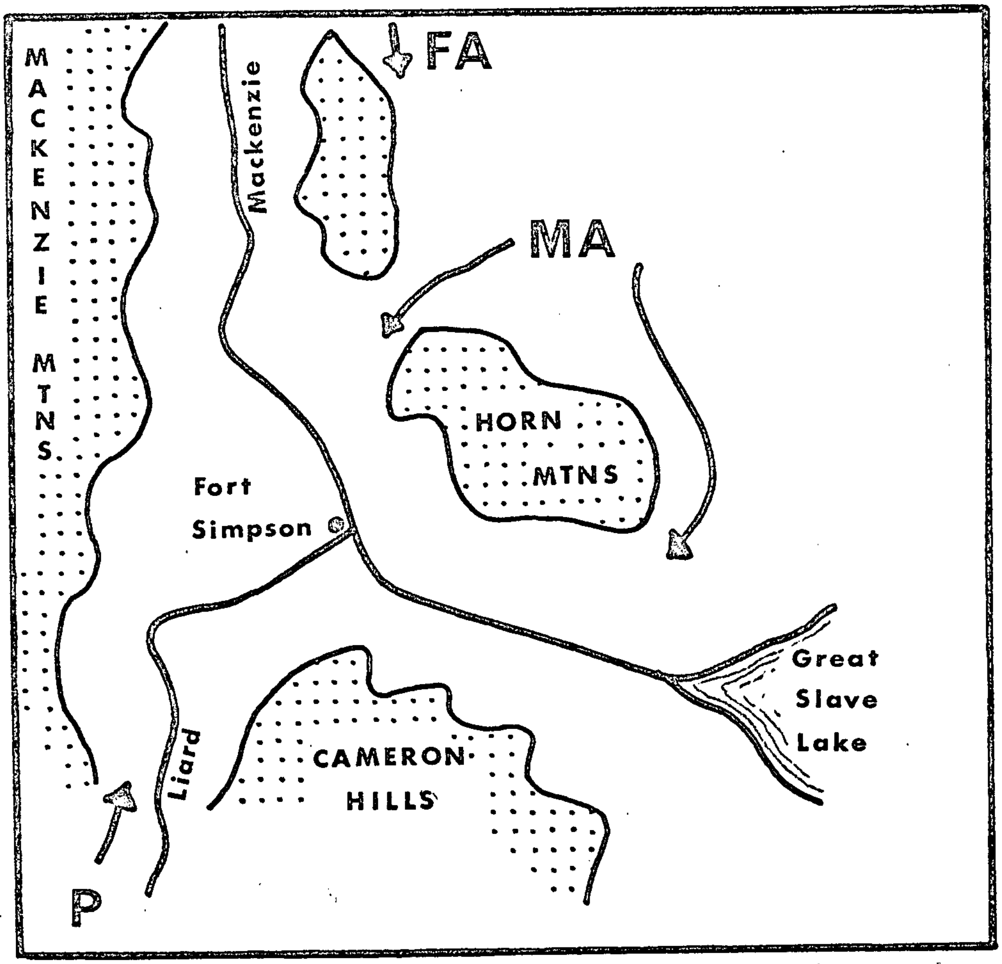

FIGURE 5

SKETCH OF THE LOCATION OF FORT SIMPSON
AND POSSIBLE AIR MASS SOURCES

FA = full Arctic
MA = mid Arctic
P = Pacific

Stippled areas
are land above
1000 feet

TABLE V

AIR MASS TEMPERATURES AND SOURCES FOR FORT SIMPSON

JANUARY

#	Mean T	Winds	Source
1	-24	N&C	F. Ar
2	-5	NW&SE	M. Ar
3	10	W	Pac

FEBRUARY

#	Mean T	Winds	Source
1	-3	NW&SE	Arctic
2	19	W	Pac

MARCH

#	Mean T	Winds	Source
1	0	N&C	F. Ar
2	22	NW&SE	M. Ar
3	40	W	Pac

APRIL

#	Mean T	Winds	Source
1	35	NW&SE	Arctic
2	44	W	Pac

(F. Ar = full Arctic, M. Ar = mid Arctic) and north-westerly winds.

In summary, the curves fitted for Edmonton, Winnipeg and Fort William accord moderately well with the Arctic/Pacific/United States model proposed in the second chapter, and are therefore appropriate to this analysis of changes in the relative frequencies of these components. The curve for Fort Simpson can be given a realistic physical interpretation, but topographic control of the airflow hinders its use in the subsequent analysis. Nevertheless, all stations were included in the tests for climatic change to be described in the following paragraphs.

The next stage of the analysis is the establishment of the conditional probabilities required to implement the model developed in previous chapter. The following table lists these probabilities and their estimators.

PROBABILITY	ESTIMATOR
$P(A\|T)$	$fA(T)/f(T)$
$P(T\|W)$	$fW(T)/f(W)$
$P(A\|W)$	$\Sigma\ P(A\|T).P(T\|W)$

where: $P(A|T)$ = the probability of air mass A given temperature T

$P(T|W)$ = the probability of temperature T given wind W

$P(A|W)$ = the probability of air mass A given wind W fA(T) = the frequency of air mass A at temperature T

f(T) = the frequency of occurrence of temperature T

fW(T) = the frequency of wind W at temperature T

f(W) = the frequency of wind W.

These probabilities are listed in Appendix A for each of the four stations and for each of the four months at each of the stations.

Under the assumptions discussed in the previous chapter, these estimated probabilities may be used to produce an estimate of air mass frequencies during the historic period.

The writer spent the major portion of the summer of 1968 examining the records of the Hudson's Bay Company in the Public Archives of Canada, Ottawa. In particular, the post journals of Fort Simpson, Edmonton, Winnipeg and Fort William were inspected to determine the availability of regular series of climatic observations. The records are generally fairly continuous after the union of the Hudson's Bay Company and the North-West Company in 1821, so attention was concentrated on the period 1820-1851. Table VI (page 44) lists the availability of daily records of climate for each five year period between 1820 and 1850. The table shows clearly that Fort Simpson has the most complete record, and that the records of Edmonton and Winnipeg span the earlier part of the period and those of Fort William the latter part.

The quality of the records is most uneven; at times there being a complete meteorological journal (e.g., Fort Simpson 1839), but more

TABLE VI

THE AVAILABILITY OF DAILY RECORDS OF CLIMATE

PERIOD	Fort Simpson	Edmonton	Winnipeg	Fort William
1820-1824	No	Yes	Yes	No
1825-1829	Yes	Yes	Yes	No
1830-1834	Yes	Yes	No	Yes
1835-1839	Yes	No	No	Yes
1840-1845	Yes	No	No	No
---	--	--	--	--
1849-1851	No	No	No	Yes

often a simple comment on the state of the wind and weather. There may be rich material in these records for those whose interests lie in the standardization and homogenization of historic temperature series, but for this study the quantitative records of temperature are too isolated, the units of measurement too uncertain and the exposure of the instrument too obviously faulty to warrant their inclusion.

Despite the uneven quality of the records it was found that the daily observations could be categorised into one or more of the following classes.

TEMPERATURE	PRECIPITATION	CLOUD	WIND
Cold	Fine	Clear	Direction
Mild	Rain	Half	
Hot	Snow	Overcast	

These classes correspond to those qualities for which the conditional probability of the occurrence of the various air masses was calculated. Therefore, bearing in mind the discussion of chapter III, for each day of the historic period with a meterorological observation it is possible to estimate the probability of occurrence of the various air masses.

Individually these estimates can provide little information as to the climate of the nineteenth century, and consequently it is necessary to aggregate them in some meaningful way. By assumption, and of necessity, these estimates are considered to be independent of one another, i.e., their joint probability is given by the product of the individual probabilities. It is therefore theoretically possible to determine precisely the probability distribution of the proportion of days dominated by each air mass. But such a determination would require an astronomical amount of calculation, so a more convenient approximate solution must be sought.

A method of determining confidence limits for estimates of the relative proportions of the various air masses in the historic period was found by application of the Central Limit Theorem.

Since the mathematical details of this discussion are at a considerably more advanced level than those in other parts of this thesis, the discussion has been relegated to appendix C. The result of this discussion is that the following expression gives an approximate confidence limit for the relative frequency of an air mass during a period of length n days

$$P\left(p - z_{\alpha/2}\left(\frac{p(1-p)}{n}\right)^{\frac{1}{2}} < Sn/n < p + z_{\alpha/2}\left(\frac{p(1-p)}{n}\right)^{\frac{1}{2}}\right) = \alpha$$

where: $p = (p_1 + p_2 + \dots + p_n)/n$

p_i = the probability of air mass A on the ith day

$z_{\alpha/2}$ = the standard normal variate at $\alpha/2$

S_n = the number of days on which A occurs.

Setting $\alpha = 0.05$, i.e., constructing 95 per cent confidence limits,

$$P\left(p - 1.96\left(\frac{p(1-p)}{n}\right)^{\frac{1}{2}} < Sn/n < p + 1.96\left(\frac{p(1-p)}{n}\right)^{\frac{1}{2}}\right) = 0.05$$

Significance of a change from the 1955-59 control period was measured as the probability of obtaining a value for the relative frequency of air mass A as extreme as the present, using the confidence limits described above. Note that the width of the confidence band depends primarily on the size of n, the length of the record.

Before 1955-1959 can be used as a control period it is necessary to establish the nature of the climate during that time. In general, averages of temperature are slightly below the long-term average temperatures published by the Department of Transport for January, February, and March and slightly above for April. Large positive deviations from the normals are evident in January of 1958, apparently associated with a strong eastward penetration of Pacific air. In contrast, large negative deviations from the normals are recorded in January of 1959, especially at Fort Simpson and Edmonton, probably representing an increased presence of Arctic air, cutting off the westerly flow to Edmonton. But the temperatures are sufficiently close to the normals to permit valid comparison with the nineteenth century data, and since such deviations as exist are generally towards a colder regime, such as that hypothesised for the early nineteenth century, indications of a climate significantly colder than that of the control period would be doubly significant. The results of the analysis are presented in figures 6 to 9 and are tabulated in appendix B. Five per cent confidence bands are included on each diagram.

The results of the analysis for Fort Simpson are shown diagrammatically in figure 6. The most prominent feature of this diagram is the increased frequency of Arctic air in comparison with the 1955-1959 control period. With the exception of February 1824-1829 and April

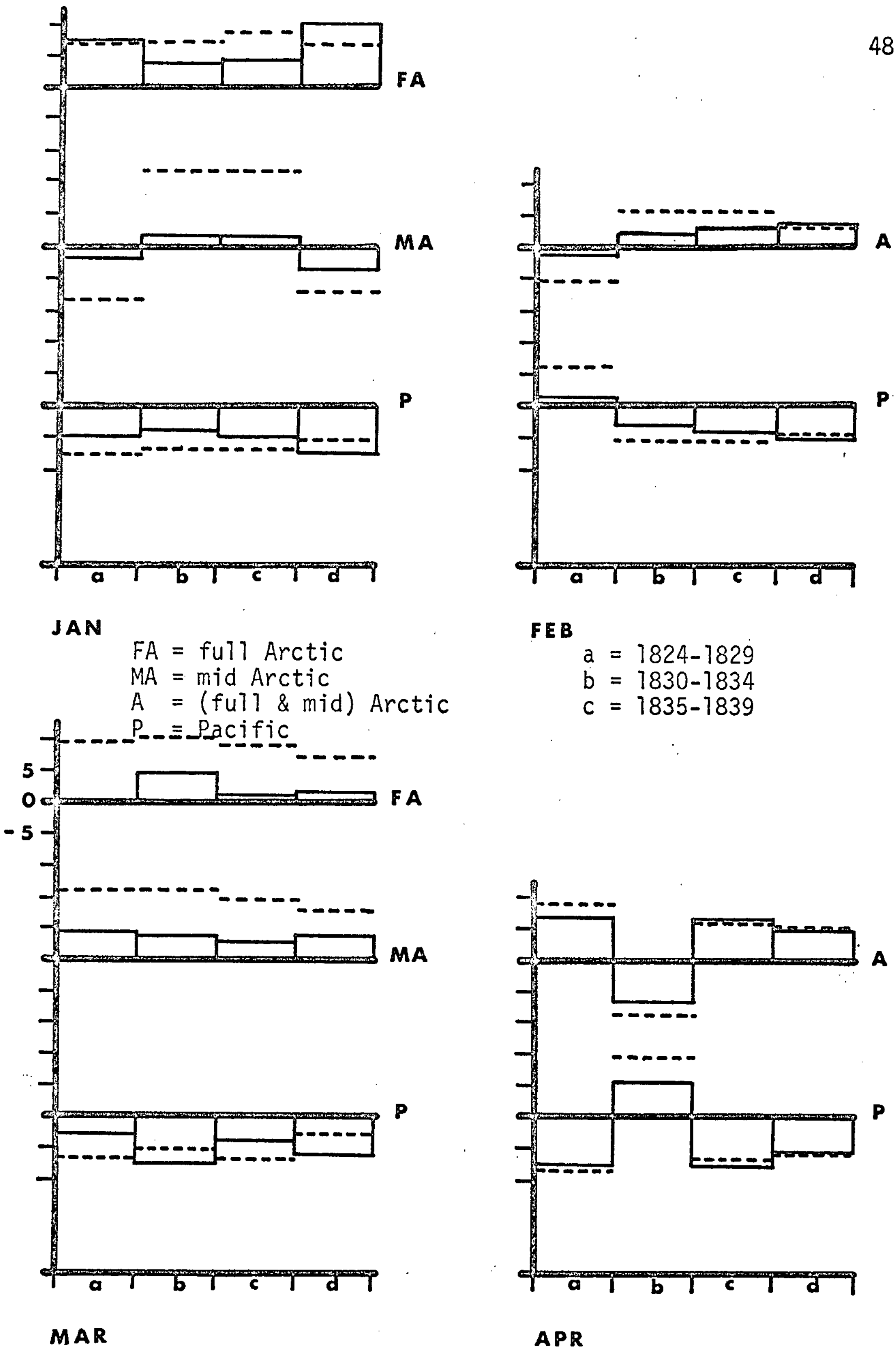

FIGURE 6

FORT SIMPSON: DEVIATIONS FROM 1955-59
AIR MASS FREQUENCIES (%)

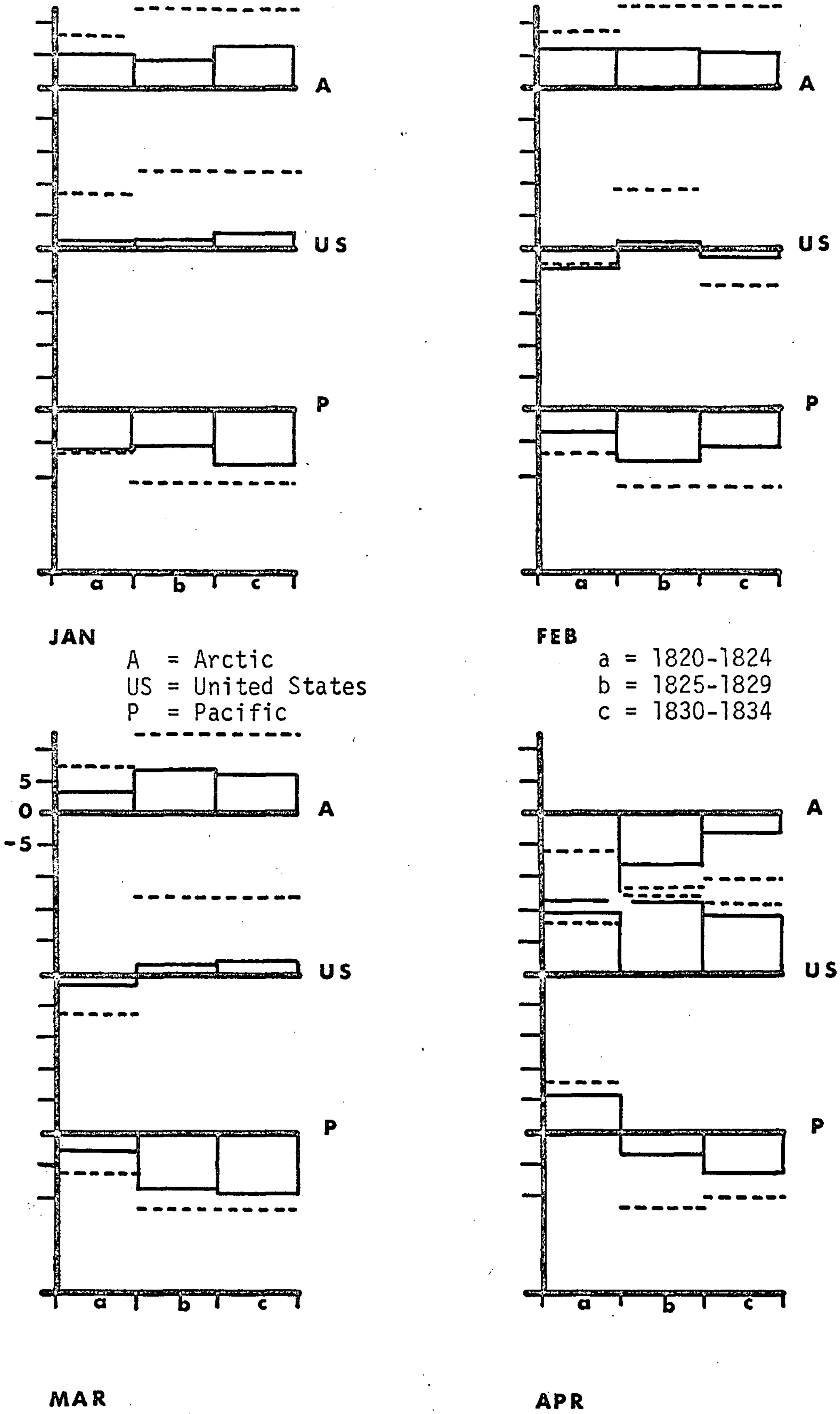

FIGURE 7

EDMONTON: DEVIATIONS FROM 1955-59
AIR MASS FREQUENCIES (%)

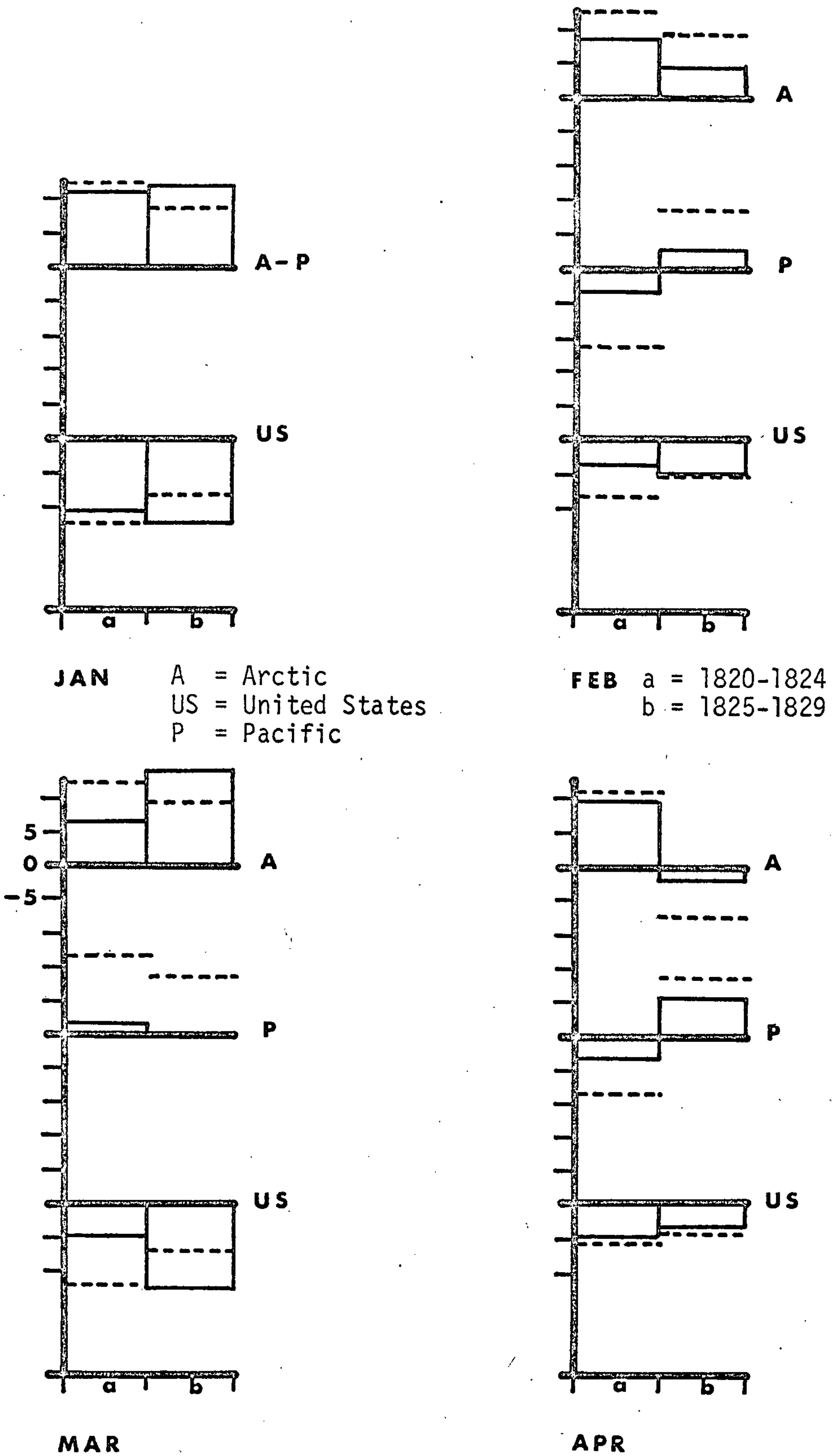

FIGURE 8

WINNIPEG: DEVIATIONS FROM 1955-59
AIR MASS FREQUENCIES (%)

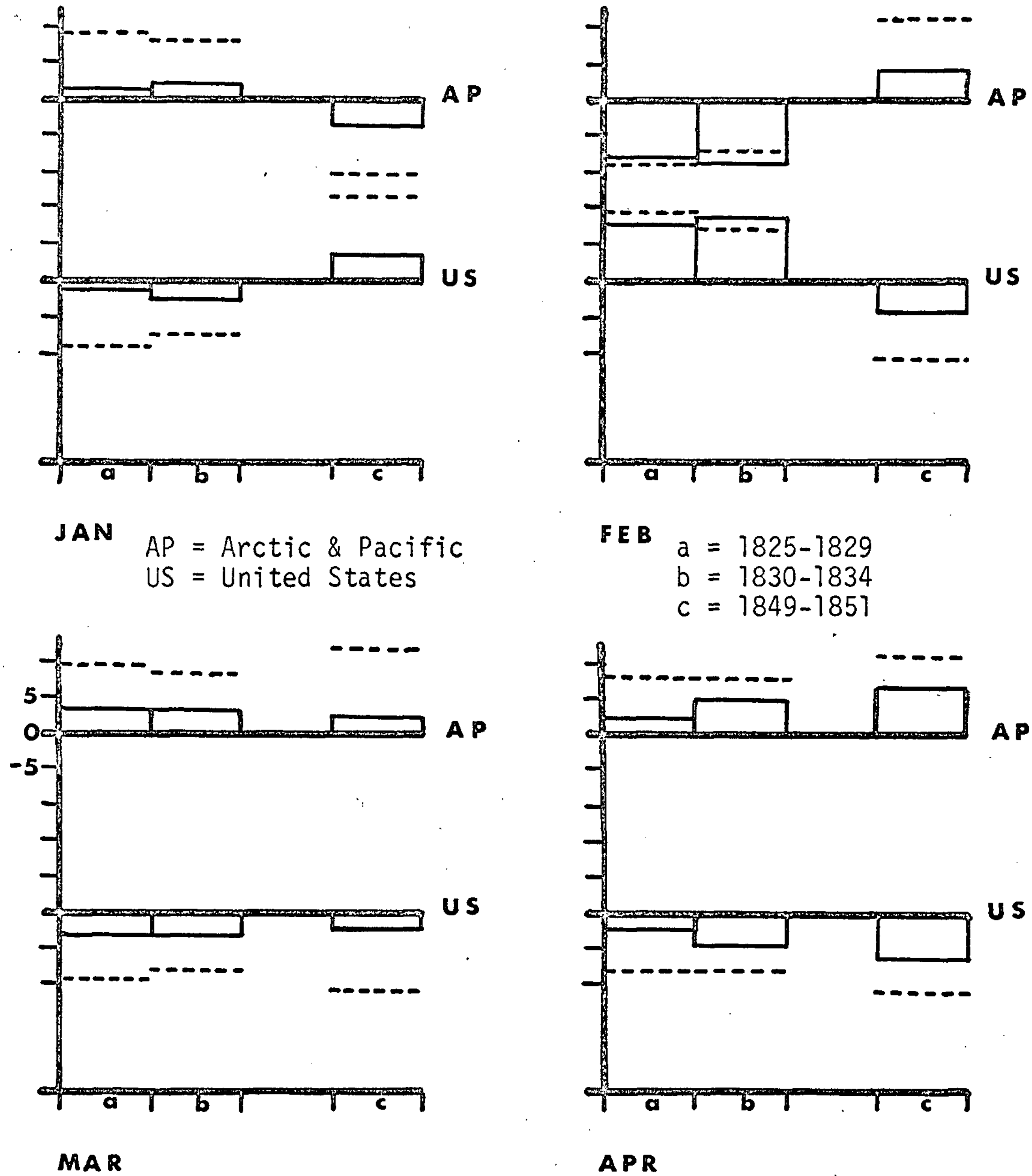

FIGURE 9

FORT WILLIAM: DEVIATIONS FROM 1955-59
AIR MASS FREQUENCIES (%)

1830-1834, the frequency of this air (the sum of "Mid" and "Full" Arctic) appears to have increased markedly, at the expense of the warmer Pacific air.

Similar results were obtained for Edmonton (see figure 7). Arctic air appears to have been of greater frequency during 1820-1834 than during the control period, largely at the expense of air of Pacific origin, at least during the months of January, February and March. In contrast, the frequency of southern, United States, air appears to have been greater during April.

At Winnipeg too (see figure 8), Arctic air appears to have been more prominent than during the control period, especially during the first three months of the year. However, unlike Edmonton, this increase seems to have been largely at the expense of United States air. There is no evidence of any marked decline in the frequency of air from the west.

The results for Fort William were disappointing. There is no obvious consistency or pattern in the fluctuations indicated, a surprising result in view of the relative uniformity of the results for the other stations. Since the record for Fort William is a very full one, at least during the decade 1830-1839, it was possible to investigate the reason for these surprising results. Figure 10 shows wind roses for the control period and the two pentads 1830-1834 and 1835-1839.

These wind roses indicate a quite remarkable change in wind regime; during the control period south-westerly and westerly winds

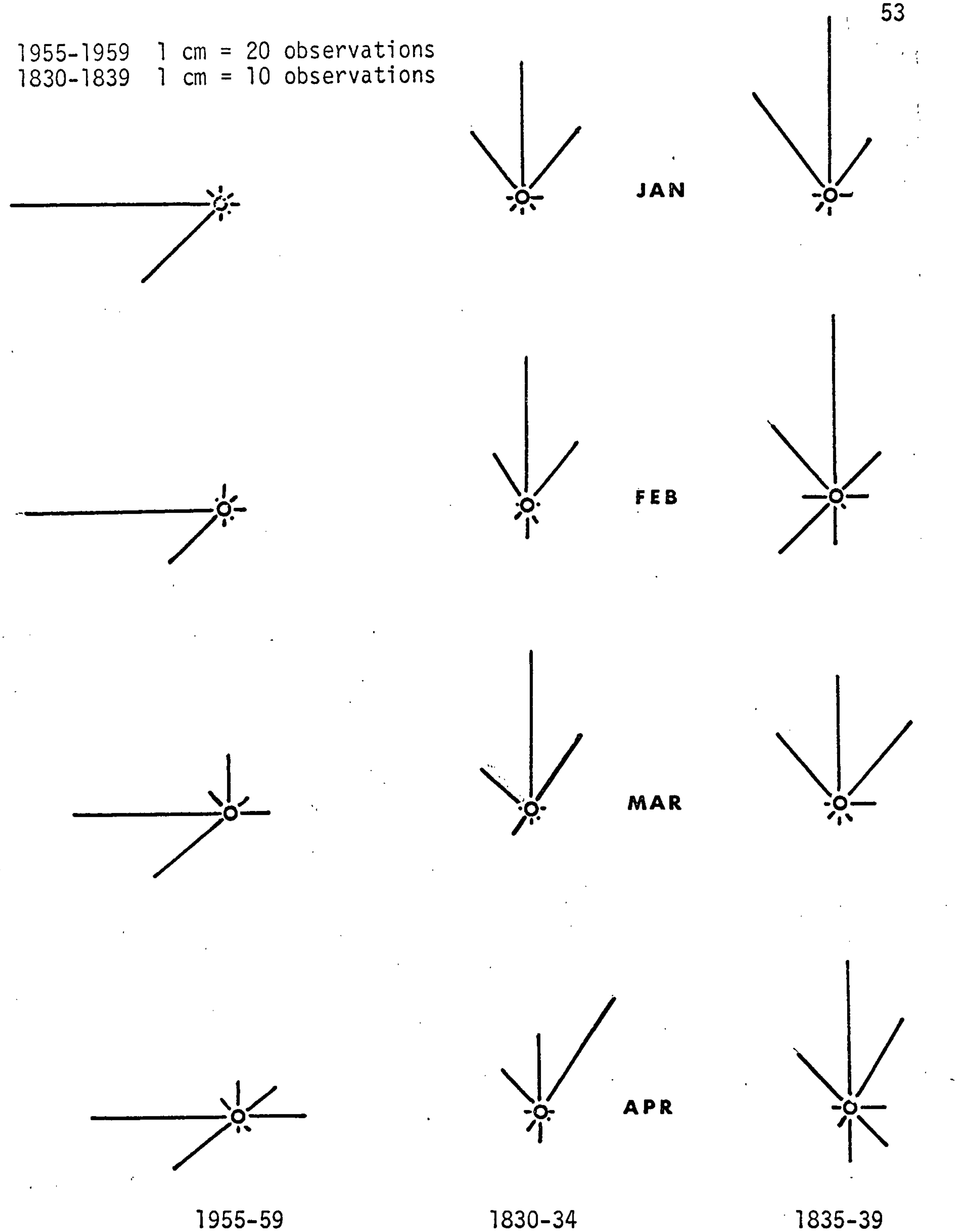

FIGURE 10

WIND ROSES FOR FORT WILLIAM

predominated in each of the four months, but during both the historic pentads winds are primarily from the north-east, north and north-west. Presumably this reflects an increase in air of Arctic origin.

But why was this change not detected by the regular analysis? The answer to this question must lie in the comparative rarity of north and north-east winds during the control period, leading to very poor estimates of the relationship between Arctic air and northerly winds. Furthermore, it will be recalled that it was not possible to distinguish between the Pacific and Arctic airstreams in the decomposition of the temperature frequency histogram. Similar problems did not arise at other stations since no equivalent shifts in the dominant wind direction took place.

There is considerable evidence of an increase in the relative frequency of Arctic air at all stations in the period 1820-1850. This is consistent with the discussion of chapter II where it was suggested that the nineteenth century was a period during which the zonal atmospheric circulation was markedly weaker than that of the present time. A consequence would be a southward extension of Arctic air and a decreased penetration of Pacific air into the Prairies. The analyses for Fort Simpson, Edmonton and Fort William are entirely in accord with this hypothesis, but there is no evidence of any decline in the frequency of westerly wind at Winnipeg.

First of all it was necessary to ascertain whether this result was not due to any inappropriateness in the analysis. Inspection of that part of the record with an adequate number of wind observations showed no apparent decline in the frequency of winds from the west and south-west and, unlike Fort William, the relationships between the dominant wind directions and air mass type appear to have been well established. Therefore it is not possible to conclude that there was any marked decrease in the frequency of Pacific air at Winnipeg during the historic period. The model of climatic change must take account of this.

To this end, note Lamb's (1963, p. 134) suggestion that there was a southward displacement of the major wind systems in the northern hemisphere during the early nineteenth century, in association with a decline in the strength of the westerly vortex. Bryson's (1966, p. 249) map of air mass dominances shows that the Pacific air present at Edmonton arrives by means of a relatively northern track through the Cordillera. A southward shift of the westerlies of only a few degrees would lead to a very marked decline in the frequency of Pacific air at this location. Similar reasoning may be applied to the decline in the frequency of Pacific air at Fort Simpson. Winnipeg receives Pacific air from both the northerly track and from the main core of the westerlies, and would therefore not respond to the same degree to a southward shift of the wind belts. An example of a situation of this kind arose in January 1959, with average temperatures at Fort Simpson and Edmonton more than 10°F below the long term normals, whereas the Winnipeg mean was only some 5°F below the normal.

It was pointed out in chapter III that the process for estimating the historic air mass frequencies probably possesses a strong bias towards the control period--what might be called an "inertia". In view of this factor we must conclude that the decline in the strength of the westerlies at this station has not been proven, rather than that it did not take place. The data for the other stations supports the hypothesis of a weakened westerly vortex; it is particularly unfortunate that the data for Winnipeg were the least full of any of the stations, since that station had been intended to provide the prime index of the state of the westerlies.

The results were calculated separately for each pentad in order to demonstrate that the changed climatic regime is characteristic of the entire period, rather than any isolated group of years within that period. Similarly, the regularity of the results obtained for each of the four months are further evidence of a uniformly different climatic regime during the historic period.

It will be recalled that a general hypothesis of a weakened westerly vortex was presented in chapter II; we have seen above that the evidence for Winnipeg precludes an unqualified acceptance of this hypothesis, although the results for the other stations are as expected. It was also suggested that the frequencies recorded for Fort Simpson might be relatively more stable than those for the other stations in view of the topographic anchoring of the Arctic Front in this area. This has been shown not to be the case, although possibly as a result of specific local conditions, notably the presence of the Liard gap.

The above are such as conclusions as may be drawn with respect to the existence of a secular climatic fluctuation in Canada during the early part of the last century; what conclusions may be drawn as to the practability of the technique employed?

Firstly, it should be noted that the five year period used for the calculation of the modern results was probably too short for the accurate establishment of some of the relevant conditional probabilities. The problems encountered in the analysis for Fort William is a case in point. A longer period might permit a more precise separation of air masses which share relatively similar air mass characteristics.

A considerable amount of time is necessary to establish the computational procedures for the analysis, although once these have been developed the analysis proceeds relatively smoothly. It would probably not be worthwhile to use the analytic tools of this study unless the data was of a similar kind and quality. Wherever regular observations are available, simple procedures such as averaging and graphing would probably yield at least as much information to the skilled interpreter. However, such records were not available for this study; the data could only be analysed by some specialist technique such as that developed here. In view of the lack of other records for the same period and the lack of previous Canadian studies of this period, it is suggested that the results obtained here are interesting and are not without climatological significance.

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APPENDICES

APPENDIX A

LIST OF CONDITIONAL PROBABILITIES

FORT SIMPSON

WIND

		N	NE	E	SE	S	SW	W	NW	C
Jan.	F. Arctic	0.15	--	0.17	0.09	0.10	--	0.10	0.09	0.38
	M. Arctic	0.64	--	0.38	0.65	0.75	--	0.79	0.68	0.53
	Pacific	0.21	--	0.45	0.26	0.15	--	0.11	0.23	0.09
Feb.	Arctic	1.00	1.00	0.92	0.91	0.98	--	0.68	0.93	0.87
	Pacific	0.00	0.00	0.08	0.09	0.02	--	0.32	0.07	0.13
Mar.	F. Arctic	0.67	0.13	0.43	0.30	0.16	0.00	0.48	0.37	0.31
	M. Arctic	0.33	0.87	0.55	0.52	0.74	0.01	0.42	0.56	0.54
	Pacific	0.00	0.00	0.02	0.18	0.10	0.99	0.10	0.07	0.15
Apr.	Arctic	0.94	0.99	0.78	0.74	0.39	--	0.86	0.81	0.80
	Pacific	0.06	0.01	0.22	0.26	0.61	--	0.14	0.19	0.20

CLOUD					PRECIPITATION		
		1	2	3	FINE	RAIN	SNOW
Jan.	F. Arctic	0.27	0.25	0.05	0.18	0.00	0.12
	M. Arctic	0.64	0.63	0.66	0.65	0.00	0.64
	Pacific	0.09	0.12	0.29	0.17	1.00	0.24
Feb.	Arctic	0.90	0.94	0.93	0.90	0.00	0.95
	Pacific	0.10	0.06	0.07	0.10	1.00	0.05
Mar.	F. Arctic	0.31	0.38	0.38	0.35	0.00	0.41
	M. Arctic	0.55	0.47	0.50	0.50	0.05	0.54
	Pacific	0.14	0.15	0.12	0.15	0.95	0.05
Apr.	Arctic	0.76	0.90	0.89	0.80	0.81	0.99
	Pacific	0.24	0.10	0.11	0.20	0.19	0.01

EDMONTON

WIND

		N	NE	E	SE	S	SW	W	NW	C
Jan.	Arctic	0.44	0.31	0.21	0.52	0.18	0.03	0.16	0.33	0.36
	U.S.	0.42	0.68	0.54	0.44	0.47	0.44	0.24	0.34	0.63
	Pacific	0.14	0.01	0.25	0.04	0.35	0.53	0.60	0.33	0.01
Feb.	Arctic	0.77	0.92	0.84	1.00	0.56	0.18	0.43	0.56	1.00
	U.S.	0.03	0.07	0.04	0.00	0.16	0.07	0.01	0.02	0.00
	Pacific	0.20	0.01	0.12	0.00	0.28	0.75	0.56	0.42	0.00
Mar.	Arctic	0.41	0.54	0.50	0.27	0.16	0.13	0.15	0.34	0.27
	U.S.	0.27	0.23	0.35	0.41	0.32	0.01	0.17	0.23	0.51
	Pacific	0.32	0.23	0.15	0.32	0.52	0.86	0.68	0.43	0.22
Apr.	Arctic	0.52	0.55	0.35	0.23	0.04	0.06	0.29	0.13	0.01
	U.S.	0.35	0.41	0.40	0.39	0.53	0.28	0.41	0.57	0.28
	Pacific	0.13	0.04	0.25	0.38	0.43	0.66	0.30	0.30	0.71

CLOUD					PRECIPITATION		
		1	2	3	FINE	RAIN	SNOW
Jan.	Arctic	0.29	0.34	0.22	0.25	0.00	0.27
	U.S.	0.37	0.31	0.50	0.41	0.02	0.52
	Pacific	0.34	0.35	0.28	0.34	0.98	0.21
Feb.	Arctic	0.74	0.55	0.55	0.56	0.01	0.86
	U.S.	0.07	0.07	0.09	0.10	0.01	0.10
	Pacific	0.19	0.38	0.36	0.34	0.98	0.04
Mar.	Arctic	0.35	0.25	0.29	0.22	0.03	0.52
	U.S.	0.21	0.21	0.34	0.25	0.08	0.41
	Pacific	0.44	0.54	0.37	0.53	0.89	0.07
Apr.	Arctic	0.16	0.19	0.37	0.20	0.38	0.96
	U.S.	0.46	0.48	0.40	0.47	0.47	0.04
	Pacific	0.38	0.23	0.23	0.33	0.14	0.00

WINNIPEG

WIND

		N	NE	E	SE	S	SW	W	NW	C
Jan.	Arc/Pac	0.67	0.86	0.38	0.20	0.33	0.99	0.86	0.86	--
	U.S.	0.33	0.14	0.62	0.80	0.67	0.01	0.14	0.14	--
Feb.	Arctic	0.51	0.20	0.13	0.05	0.21	0.32	0.38	0.56	--
	Pacific	0.45	0.79	0.80	0.81	0.61	0.51	0.43	0.43	--
	U.S.	0.04	0.01	0.07	0.14	0.18	0.17	0.19	0.01	--
Mar.	Arctic	0.61	0.82	0.33	0.19	0.33	0.44	0.70	0.80	0.02
	Pacific	0.32	0.18	0.26	0.41	0.31	0.51	0.19	0.17	0.39
	U.S.	0.07	0.00	0.41	0.40	0.36	0.05	0.11	0.03	0.59
Apr.	Arctic	0.82	0.65	0.55	0.74	0.45	0.39	0.67	0.76	0.73
	Pacific	0.17	0.15	0.32	0.15	0.31	0.61	0.32	0.23	0.27
	U.S.	0.01	0.20	0.13	0.11	0.24	0.00	0.01	0.01	0.00

CLOUD					PRECIPITATION		
		1	2	3	FINE	RAIN	SNOW
Jan.	Arc/Pac	0.75	0.53	0.48	0.66	0.00	0.49
	U.S.	0.25	0.47	0.52	0.34	1.00	0.51
Feb.	Arctic	0.45	0.51	0.16	0.40	0.00	0.27
	Pacific	0.48	0.45	0.62	0.48	0.00	0.61
	U.S.	0.07	0.04	0.22	0.12	1.00	0.12
Mar.	Arctic	0.61	0.31	0.45	0.42	0.03	0.65
	Pacific	0.20	0.30	0.35	0.27	0.44	0.31
	U.S.	0.19	0.39	0.20	0.31	0.53	0.04
Apr.	Arctic	0.68	0.78	0.85	0.80	0.81	1.00
	Pacific	0.26	0.15	0.11	0.16	0.16	0.00
	U.S.	0.06	0.07	0.04	0.04	0.03	0.00

FORT WILLIAM

WIND

		N	NE	E	SE	S	SW	W	NW	C
Jan.	Arc/Pac	0.57	0.22	0.26	0.05	0.24	0.50	0.60	0.50	0.22
	U.S.	0.43	0.78	0.74	0.95	0.76	0.50	0.40	0.50	0.78
Feb.	Arc/Pac	0.29	0.33	0.08	0.06	0.21	0.46	0.51	0.81	0.42
	U.S.	0.71	0.67	0.92	0.94	0.79	0.54	0.49	0.19	0.58
Mar.	Arc/Pac	0.33	0.26	0.26	--	0.00	0.16	0.25	0.44	0.19
	U.S.	0.67	0.74	0.74	--	1.00	0.84	0.75	0.56	0.81
Apr.	Arc/Pac	1.00	0.84	0.89	--	0.29	0.66	0.85	0.99	0.62
	U.S.	0.00	0.16	0.11	--	0.71	0.34	0.15	0.01	0.38

CLOUD					PRECIPITATION		
		1	2	3	FINE	RAIN	SNOW
Jan.	Arc/Pac	0.66	0.55	0.33	0.58	0.00	0.35
	U.S.	0.34	0.45	0.67	0.42	1.00	0.65
Feb.	Arc/Pac	0.60	0.42	0.33	0.53	0.05	0.33
	U.S.	0.40	0.58	0.67	0.47	0.95	0.67
Mar.	Arc/Pac	0.25	0.20	0.26	0.22	0.10	0.33
	U.S.	0.75	0.80	0.74	0.78	0.90	0.67
Apr.	Arc/Pac	0.82	0.85	0.84	0.79	0.93	1.00
	U.S.	0.18	0.15	0.16	0.21	0.07	0.00

APPENDIX B

PERCENTAGE DEVIATIONS FROM 1955-59 AIR MASS FREQUENCIES

FORT SIMPSON

	JANUARY			FEBRUARY			MARCH			APRIL		
	F.A.	M.A.	P	A		P	F.A.	M.A.	P	A		P
1824-29	+7.12	-2.13	-5.00	-1.17		+1.16	-0.66	+4.12	-3.48	+7.27		-7.27
1830-34	+3.28	+1.03	-4.31	+2.80		-2.80	+4.39	+3.27	-7.66	-5.86		+5.86
1835-39	+3.50	+1.24	-4.75	+2.98		-2.98	+0.72	+2.95	-3.67	+7.38		-7.38
1840-44	+10.07	-3.50	-7.56	+3.20		-3.20	+1.94	+3.14	-6.10	+5.19		-5.19

EDMONTON

	A	U.S.	P	A	U.S.	P	A	U.S.	P	A	U.S.	P
1820-24	+5.12	+1.20	-6.32	+6.47	-3.45	-3.02	+3.85	-1.35	-2.50	-14.81	+9.24	+5.65
1825-29	+4.27	+1.64	-5.83	+6.58	+0.67	-7.25	+7.22	+1.38	-8.50	-8.18	+11.18	-3.22
1830-34	+6.13	+2.54	-8.67	+6.46	-0.61	-5.85	+6.51	+2.14	-8.67	-3.74	+9.60	-5.87

WINNIPEG

	A/P		U.S.	A	P	U.S.	A	P	U.S.	A	P	U.S.
1820-24	+10.35		-10.35	+6.78	-3.90	-3.88	+6.07	-0.70	-5.37	+9.65	-4.64	-5.01
1825-29	+11.81		-11.81	+3.56	+2.24	-5.80	+13.33	+0.38	-13.72	-2.08	+5.23	-3.15

FORT WILLIAM

	A/P		U.S.	A/P		U.S.	A/P		U.S.	A/P		U.S.
1830-34	+0.90		-0.90	-7.88		+7.88	+3.74		-3.74	+1.92		-1.92
1835-39	+2.07		-2.07	-8.20		+8.20	+3.40		-3.40	+4.92		-4.92
1849-51	-4.42		+4.42	+3.41		-3.41	+2.15		-2.15	+6.10		-6.10

APPENDIX C

CONFIDENCE LIMITS FOR AIR MASS FREQUENCY

<u>Confidence limits for air mass frequencies</u>

Consider the following fairly general form of the Central Limit Theorem.

<u>THEOREM</u> If $X_1, X_2, \ldots, X_n$ are a sequence of mutually independent random variables with finite expectations and variances, and if

$$Sn = X_1 + X_2 + \ldots + X_n$$
$$Mn = \mu_1 + \mu_2 + \ldots + \mu_n$$
$$Bn^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2$$

where: μ_i = the expected value of X_i

σ_i^2 = the variance of X_i

then $$P\left(\frac{Sn - Mn}{\sqrt{n}\, Bn} < a\right) \rightarrow N(a) \text{ as } n \rightarrow \infty$$

so long as the following conditions hold

1) The random variables X_i are uniformly bounded
2) $Bn \rightarrow \infty$ as $n \rightarrow \infty$

where: a = some fixed real constant

$N(a)$ = the standard normal distribution function at a, i.e.,

$$N(a) = \int_{-\infty}^{a} (2\Pi)^{-\frac{1}{2}} \exp\,(-\, t^2/2)\, dt$$

This rather imprecise statement of the theorem follows the discussion of Feller (1957, chapter X). For a very precise statement of the theorem and a proof reference might be made to Lamperti (1966, pp. 69 ff). The proof is not a simple one and will not be included here.

Confidence limits for air mass frequencies

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$$Mn = \mu_1 + \mu_2 + \ldots + \mu_n$$

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so long as the following conditions hold

1) The random variables X_i are uniformly bounded

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The frequency of air mass A during some period of length n days may be treated in the following manner.

Define the random variables X_i

$$X_i = 1 \text{ if the air mass present was A}$$
$$= 0 \text{ if the air mass present was not A.}$$

The X_i may be considered as a sequence of Bernoulli trials and we may define

$$P(X_i = 1) = P_i$$

hence the expectation of X_i

$$E(X_i) = \mu_i = P_i$$

and the variance

$$\text{var } (X_i) = \sigma_i^2 = p_i \ (1-p_i)$$

These random variables satisfy the conditions of the theorem since

1) $E(X_i)$ and var (X_i) are finite
2) The trials are independent (by assumption)
3) For all X_i, the uniform bound $0 \leq X_i \leq 1$ exists
4) Σ_i var $(X_i) \to \infty$, as $n \to \infty$

To make use of the Central Limit Theorem we require estimates of Mn and Bn. The most convenient estimate for Mn follows directly from its definition:

$$Mn = \mu_1 + \mu_2 + \ldots + \mu_n$$
$$Mn = p_1 + p_2 + \ldots + p_n$$

The pooled variance could be estimated in a similar manner, but it was found more convenient to make use of the following lemma:

LEMMA If X_i are a sequence of mutually independent random variables, defined as above, and Sn is the partial sum of the first n members of the sequence

then $\qquad var(Sn) = max$

if $\qquad p_1 = p_2 = \ldots = p_n = p = \Sigma\, p_i/n$

for any constant value of p.

PROOF The variance of X_i is given by

$$var(X_i) = p_i(1-p_i)$$

Then the variance of Sn

$$var(Sn) = \Sigma\, p_i(1-p_i)$$

in view of the assumption of independence.

The problem is clearly to maximise $\Sigma_i p_i(1-p_i)$ subject to the constraint $\Sigma p_i/n = p$. To this end, form the function

$$G\,(p_1,p_2,\ldots,p_n) = \Sigma\, p_i - \Sigma\, p_i^{\,2} + \lambda\, \Sigma\,(p_i - np)$$

where: λ = some appropriately chosen real constant or Lagrange multiplier.

Then var(Sn) will have an extremum where

$$\frac{\partial G}{\partial p_i} = 0 \text{ for all } i$$

Now $\qquad \dfrac{\partial G}{\partial p_i} = 1 - 2p_i + \lambda = 0$

implies $\qquad p_i = \dfrac{1 + \lambda}{2} = \text{const.}$

Therefore the extremum occurs where

$$p_1 = p_2 = \ldots = p_n = \Sigma_i\, p_i/n$$

It is easy to check that the extremum is a maximum, and then the lemma is proved.

The significance of this lemma lies in the fact that if $p_1 = p_2 = \ldots = p_n$ then the distribution of Sn is the familiar binomial distribution with variance

$$\mathrm{var}(Sn) = np(1-p)$$

Therefore we have an upper bound for the variance of Sn in the case where the p_i are not equal.

The preceding discussion may be summarised in the following expression:

$$\lim_{n\to\infty} P(z_{-\alpha/2} < (Sn - np)/(np(1-p))^{\frac{1}{2}} < z_{\alpha/2}) \leq \alpha$$

using the Central Limit Theorem and the expression for the variance derived above.

$$\text{where:} \quad z_{\alpha/2} = \int_{-\infty}^{\alpha/2} (2\pi)^{-\frac{1}{2}} e^{-t^2/2}\, dt$$

Manipulation of this expression yields

$$P(p - z_{\alpha/2} \left(\frac{p(1-p)}{n}\right)^{\frac{1}{2}} < \frac{Sn}{n} < p + z_{\alpha/2} \left(\frac{p(1-p)}{n}\right)^{\frac{1}{2}}) \leq \alpha$$

for sufficiently large n.

We may use this as a confidence limit for Sn/n, the relative frequency of air mass A. Significance of deviations from the modern values was measured as the probability of obtaining a value as extreme as that of the control period, given the estimates of the p_i obtained by previous analysis.

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