A STUDY OF THE RELATIONSHIP BETWEEN PERCEPTUAL TRAINING AND ARITHMETIC COMPUTATION

by

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ABSTRACT

An analysis of the literature showed that many conclusions about the relationship between perceptual skills and reading were based on either the measurement of certain reading skills which did not depend upon the perceptual skills tested, or training programs which were not matched with the perceptual skills being studied. This, together with the fact that research in the field of mathematics has found that many computational mistakes are made because of mistaken symbols, led the author to define a perceptual skill, the search mechanism, which was specifically determined by the method of working algorithms.

A pilot study was performed to establish testing procedures. The results of this study indicated that there was a relationship between the search mechanism and arithmetic computation.

The experiment consisted of a treatment group and a control group. All subjects were given pre- and post-tests on each of four measures; the search mechanism, vertical span, horizontal span and arithmetic computation. The treatment group was given training in the search mechanism. The following statistical results were established: there was a

significant difference between the control and experimental groups on a measure of change of search ability; there were no significant differences between the control and experimental groups on measures of vertical span, horizontal span and arithmetic computation. Using the post-test on the control group only, it was established that: search ability was correlated with vertical span; search ability was not correlated with horizontal span; vertical span was correlated with horizontal span; search mechanism, with the effects of vertical span and horizontal span removed, was correlated with arithmetic computation. This latter finding means that the variance in arithmetic scores accounted for by search mechanism, vertical span, and horizontal span was significantly different from that accounted for by vertical span and horizontal span alone.

Two possible conclusions were suggested. The first was that the training period was too short for transfer from the search skill to algorithmic performance to take place. The second was that the increase in the search procedure test could be explained by vertical span being used with increasing efficiency within the new context of search procedure testing. Because a low correlation was obtained between arithmetic and vertical span the achieved stability of the arithmetic scores was to be expected.

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CHAPTER I

DESCRIPTION OF THE STUDY

I. INTRODUCTION

There are many procedures which affect the acquisition and maintenance of computational or algorithmic skills. One of the purposes of research is to find those procedures or combination of procedures which most affect the acquisition of skills. Various attempts have been made in the realm of arithmetic research.

One such procedure is drill. A student was believed to acquire the solution procedure when required to make a given response to a particular pattern or problem situation.[1,2,3] Once drill was generally accepted it then became important to increase the effectiveness of the drill procedure. Motivational techniques were employed with drill procedures being accompanied by feedback materials in the form of charts or graphs of programs.[4,5]

It was Brownell and Chazal, however, who first emphasized the effects of drill as a teaching procedure. They observed that the previous studies were related to improving, fixing, maintaining, and rehabilitating computational skills;

and that the variables in these studies were time, accuracy and both time and accuracy which measured the result of the computation. That is, these measures did not show the procedures used in obtaining the answers.[6] In their study performed on grade three students they found that even after one month of drill, 5 minutes each day, on the items taught in grades one and two that 24.4 percent of the responses were obtained by guessing, 22 percent by counting, 14.1 percent by indirect solution and only 39.5 percent by memorization which was the desired result.[7] They concluded, "...drill is exceedingly valuable for increasing, fixing, maintaining, and rehabilitating efficiency <u>otherwise</u> <u>developed</u> [emphasis not in the original]."[8] This implied that drill was not as effective as it might have been considered because such a procedure neglects the initial acquisition of skills.

It is the purpose of this study to identify a set of perceptual skills which either precede or are contemporaneous with the initial acquisition of algorithmic skills. It will be argued that there are certain implications of studies done on algorithmic performance which indicate a dependence of the student upon skills analogous to some reading skills. Further, the studies cited on the relationship between reading and arithmetic do not account for the abilities or skills needed in algorithmic performance. Finally, there is a specific perceptual skill, search procedure, which should be considered as a potentially significant factor in the acquisition of arithmetic skills.

As the search procedure has not been identified in the research examined its development will follow upon some implications of related research done in the fields of arithmetic and reading.

Perception must be interpreted carefully for, as stated in the Encyclopedia of Educational Research,

> "Perception" like many other terms has a number of meanings and connotations....This lack of restrictedness is to be found even in the usages given the term by those who study perception.[9]

Alport's book presents evidence of this fact by tracing the historical development of theories of perception.[10] However, he notes that perception is more than just the awareness of a symbol but "...also involves to some degree, an understanding awareness, a 'meaning' or a 'recognition' of those objects."[11] Because of these various meanings and connotations it is necessary to introduce a set of definitions, not all of which pertain to perception, which will be used throughout this thesis.

II. DEFINITIONS

Perception

Perception will have been said to take place if the subject can copy a visual symbol presented to him. For example, if a student is asked to write the tenth digit appearing in a sequence and he succeeds then perception is said to have taken place.

Perceptual Skill

Perceptual skill (syn. perceptual behaviour) is a skill which allows a more complex stimulus to be perceived in a given period of time or allows a given stimulus to be perceived more rapidly. Examples of these skills in reading are scan and eye movement patterns.

Algorithm

An algorithm is a procedure which, by correct, sequential application to a problem, is believed to lead to a solution. Within the context of this study arithmetic algorithms will refer to the following methods:

```
addition           13          subtraction      527
                   27                         - 391
                   63                         -----
                 + 42                           136
                 ----
                  145

                                                 577
multiplication    256          division     23)13271
                x  42                          115
                -----                          ---
                  512                           177
                 1024                           161
                -----                           ---
                10752                            161
                                                 161
                                                 ---
```

Fixation

Fixation is that period of rest between successive eye movements when the eye focuses upon a set of stimuli to be perceived. In reading there is a successive series of eye movements each followed by pauses during which the perception of symbols takes place.

Span

Span is the width or length of symbolic material which can be perceived in one fixation of the eye. That is the number of randomly ordered digits in either a single horizontal number or in a single vertical column.

Search Procedure

Search procedure (syn. search mechanism, search ability) is that procedure used to perceive a specific bit of required material within a complex stimulus. In arithmetic terms it is the procedure used to select the digits required for the next step of an algorithm.

Algorithmic Performance

Algorithmic Performance (syn. arithmetic performance, arithmetic ability, ability to work an algorithm, computational ability) is defined in terms of the test given. In the case of this study the meaning of the above words will be restricted to algorithms of the whole numbers and the speed and accuracy of performance on the tests administered (see Appendix II).

Causal Relation

A causal relation will be said to exist if a change in one skill is accompanied by a change in another. That is, if a change produced in search procedure is accompanied by a change in arithmetic ability then it will be said that there is a causal relation between search procedure and arithmetic ability.

III. SIGNIFICANCE OF THE STUDY

It will be argued that some of the anomalies in the results of several research studies on algorithmic performance may be accounted for by the consideration of the subjects perceptual skills--that these behaviours are similar to some of those considered in reading research--that the literature cited on the relationship of reading to arithmetic does not consider explicitly perceptual skills. It is upon these arguments that the significance of the study rests.

Algorithmic Performance

Several studies which have tried to identify specific difficulties in algorithmic performance are related here. Within some, perceptual difficulties are implicit. Within others, perceptual difficulties are explicitly considered. None of the cited studies attempt to experimentally define these perceptual difficulties or their relationship to algorithmic performance.

Flournoy considered higher decade addition and bridging in column addition.[12] It was found that students were prone to count when numbers were arranged in a column. This study indicates that the symbolic arrangement of the problem influenced the child's method of performance.

Lessinger investigated discrepancies between arithmetic achievement (low) and reading (high) on mental age tests. He found that because of faulty reading of symbols by students in grades three through eight, incorrect operations were used and a loss of 6.1 months in M.A. score resulted. After the students were trained in reading skills this loss was reduced to .7 months.[13] The reading program consisted of training the students in "...the ability to focus attention upon key words and on technical signs, the ability to focus combinations in one eye pause and to read unimportant parts hurriedly, etc."[14] These skills are close to what are defined above as search procedure and span together with a new skill of identifying the important and unimportant sections of what was read. It does not however identify any skills specific to arithmetic performance.

The method of algorithmic performance, for example, adding upward or downward, has been examined in order to find the most efficient way. These studies are of interest primarily because some of the implications of these studies appear to have been overlooked.

Brownell cited a Scotish study which showed that a group taught downward addition and then upward addition were faster adding up and down but classes first taught upward addition were superior on accuracy.[15] Since the actual difference between the groups was small, he concluded that the selection of methods used "...may be decided also on the basis of considerations other than speed and accuracy."[16] A small difference could be expected because both groups had the same experiences with the exception of order.

Buckingham[17] found that of 493 student teachers, 303 preferred to add up and 190 preferred the opposite. He suggested four possibilities for preferring downward addition: first, in downward addition the eye moves in the direction of ordinary reading; second, figures are commonly written downward; third, in downward addition the eye successively fixes toward the point at which the answer is written; fourth, if the number of the columns are increased then more complex eye movements are needed in upward addition.[18] Buckingham,[19] in a later study which showed that grade two and three children who were taught downward addition achieved better results than those who were taught upward addition, made no attempt to experimentally relate these findings to the reasons which he had proposed in his previous article.

A study by Cole on the same subject revealed more subtle differences in behaviour. Subjects adding downward did so less rapidly but more accurately than those adding upward.[20] It was also found that those counting objects to the left were slower but more accurate than those counting to the right.[21]

These studies indicate that there is some scanning process similar to that taught in reading which may make a difference in the performance of algorithms. However, the following studies which relate reading ability to arithmetic ability apparently neglected such perceptual skills.

Reading and Arithmetic

There are many studies dealing with the relation of reading skills and the ability to solve word problems. Typical studies have been done by Wilson,[22] who suggested that acting out word problems gave greater meaning to the words read; Stevens,[23] who showed that tests of problem analysis seem to have higher correlations with problem solving than do tests of general reading or of fundamental operations; Johnson,[24] Call[25] and Pribnow,[26] who demonstrated the positive relation between vocabulary and problem solving ability.

Those studies which attempted various methods of improving problem solving prescribed activities which would emphasize the relation between the words of the problem and the corresponding arithmetic operations. They do not however, attempt to train perceptual skills. Other studies which simply found a relationship between arithmetic and reading performance may have been accounting for that part of each common to general intelligence. It is possibly this which leads Ballow to conclude,

> That reading achievement would not be related to computational with the effects of intelligence controlled is reasonable....The subject need only read numbers and operation signs.[27]

This last statement does not agree completely with Lessinger's findings cited above, nor with a study done by Gilmary.

She studied the effects of remedial instruction in both reading and arithmetic and found that when I.Q. was used as a co-variate the former students made greater gains.[28]

Earp's examination of experimental studies dealing with teaching of reading skills in mathematics did not facilitate the development of teaching methods for teachers "...who continue to assert that most children have difficulty in reading content material."[29] After citing Lessinger's study he concludes, "The teacher of mathematics at any level should also be a teacher of reading."[30]

The above studies have been cited to show that there is a need for continued research into the relationship between reading and arithmetic. However, future research must account for particular perceptual difficulties which seem to have been indicated. It is conceivable that many of the tests of reading and arithmetic abilities which have been used are so general as to mask the specific skills which need to be developed. It is for these reasons that this study is significant.

It has been established that some of the perceptual skills needed may be similar to those considered in the teaching of reading. It will now be necessary to examine the research related to those perceptual problems.

IV. RELATED RESEARCH

In the discussion which follows studies typical of research into the perceptual skills and their associated training methods will be examined. It will be shown that although the efficacy of perceptual training as a method of improving reading performance has been questioned, many of the conflicting research results may be explained in terms of a lack of congruence between the skill being taught and the test used to establish skill performance. This would imply that perceptual skills specific to the task desired must be developed.

Reading-Perception Relationship

Glass correlated rate of reading with various academic, personality, and perceptual measures. In his multiple correlation analysis he found that the three highest beta weights were assigned to vocabulary (.431), rate of perception (.221) and compulsiveness (.214).[31] The validity of these results could be questioned in that a study done by M. Santoro[32] showed that visual perception tests had a correlation of .438 with the Otis Quick Scoring Mental Ability tests. Glass' findings could then be interpreted as possibly just measuring some part of general intelligence which is influenced by perception and reading rate.

A study done by Gates, however, contradicts this interpretation. Three perceptual tests were given and compared to reading skills. The tests required students to determine whether pairs of geometrical figures, digits, and words, were different in some small respect. He found that the test with the greatest relation to the reading skills was the word differentiation test. The correlations between the word differentiation test and spelling, pronunciation, and silent reading were .544, .555, and .69 respectively with the influence of intelligence removed.[33] He stated:

> These figures imply that the perceptive factor, irrespective of intelligence, is more closely associated with reading and spelling than all the other functions embraced in "intelligence" as measured.[34]

There is evidence, then, that perceptual skills are related to reading skills. In addition Gates' study indicated that the perception of words rather than geometrical or numerical differences is linked with reading skills. Thus perceptual skills may be quite specific to the behaviours tested. This is borne out by analysis of attempts to improve reading through perceptual training.

Perceptual Training for Reading Skills

The present position of reading teachers with respect to perceptual training may be summed up by Witty. He conducted a remedial reading program for college students which consisted of reading with the assistance of a reading accelerator (a machine which forced the eye move more quickly over the printed page), materials of interest to the students, speed reading practice, tachistoscopic practice (ten minutes per session), vocabulary building, practice on reading skills, frequent testings, practice in reading different kinds of materials, frequent conferences, and help in reading textbooks.[35] Seventeen years after this program he stated,

> ...at more advanced levels, pupils may derive some value from the enhanced interest resulting from the introduction of pacing devices. That such devices are necessary [emphasis not in the original] for conducting a successful program has yet to be demonstrated.[36]

It should be noted that he did not state that pacing devices were useless but that they were just not "necessary." The implication is that there may be alternative methods for developing the same skills. His own remedial program, for example, contained several methods of improving reading skills.

It will be argued that although there may be several ways of training for perceptual skills, the acquisition of these skills is of importance. The following studies present conflicting evidence for the importance of perceptual training in the acquisition of reading skills.

Grob used forced speeded oral reading rather than a pacing device because,

> by being pushed rapidly ahead the student's attention is being focused, and this increases the accuracy of his perception.[37]

Freeburn[38] examined the effects of perceptual span and perceptual speed upon reading ability. The group was trained by systematically lengthening the phrases which the students were required to perceive and by decreasing the exposure times for the phrases during the reading period. He found that the span and speed had a correlation of .758 but that these perceptual skills and the results on reading were not significantly related.[39]

Two studies done by Amble and Muehl on phrase training by means of a tachistoscope had varying results. The first showed that the differences between the control and experimental groups on a measure of comprehension and on the Iowa Test of Basic Skills were significant at the .05 and .025 levels respectively. There was no difference between the groups in either rate of reading or vocabulary acquisition.[40] The results of the second study, however, indicated a two and one-half year gain on the Rate and Comprehension subtest of the Iowa Test of Basic Skills.[41] This latter study then conflicts with both the first study and that of Freeburn.

The Frostig Visual Perceptual Training Program[42] was used by Jacobs[43] in his study on perception in the primary grades. The program consists of copying geometrical figures designed to give the student skills in perceiving vertical, horizontal and sloped lines, and specific organized patterns. He found that although all grades improved, the higher grades improved the most. This was an unexpected result in that it was suggested by Jacobs that the earlier grades would have shown the greatest increase because the training was done at an earlier period of skill acquisition.

A study, done by N. Santore, showed that it was possible to direct the eyes from right to left using the Controlled Reader with Guided Slot.[44] The gains on reading achievement tests were markedly superior to those attained when the other methods of, Controlled Reader with no Guided Slot, Shadow scope, Rate-o-meter and timed reading were used. These results were maintained in the retention test given several months later. It is interesting that one particular form of reading rate training should be superior to all the others.

Some of these apparent conflicts may be resolved by a more careful consideration of the above studies. Freeburn's study and the first study of Amble and Muehl agree that little or no improvement in reading resulted from perceptual training. However, the second Amble and Muehl study contradicts that result perhaps because of the more appropriate matching of material to the students abilities to read. That is, the material matched the performance behaviours of the students.

Jacob's findings, that older children gained more than the younger, may be interpreted as supporting Bonsall and Dornbush who state,

> ...it appears that as the child matures, his reading ability is dominated by different functions; the exact course of these remains to be defined.[45]

In other words the Frostig materials may better match the mental functions of the older child.

Santore's findings that only one kind of device, namely the Controlled Reader with Guided Slot, produced significant improvement in reading achievement further supports the present argument. There is generaly agreement that in reading a page of material the eye must go from the top left hand corner to the bottom right hand corner of the page, and only the reader with guided slot trained this particular skill. Santore concludes that the modification of performance skills were the most important effects.[46]

From these and previous considerations it can be concluded that perceptual skills, analogous to those found in reading, may be a significant factor in arithmetic computation, that there is a relation between perceptual skills and reading achievement, and that these perceptual skills must be specifically related to the behaviours required. As no analysis of the perceptual skills required in arithmetic performance was found the following section will present a discussion of the search procedure.

V. DISCUSSION OF THE PROBLEM

Arithmetic performance depends upon at least three abilities. The student must know the steps of the algorithm and be able to perform them in the proper sequence. He must either know or determine the basic number facts. He must perceive the symbols involved in the algorithm. It is this latter ability which is of interest in this study. A discussion of the differences between the perceptual skills needed for word recognition and those needed for numeral recognition follows.

An aid to the recognition of a word is its shape. For example, the word "big" is of different overall shape than the word "bad." However, the numeral "279" is of the same overall shape as "384" and, in fact, so are all three digit numerals. Therefore, the student must be able to identify each specific digit within the numeral in order to be able to read the numeral. In reading the word "idemnification" all the information is present in the order in which it is used even if the word is broken down into syllables. On the other hand, in reading the numeral "378,521" it is first necessary to identify the number of digits and then, using this information, give to each digit the proper place value associated with it.

As well as lacking an identifying shape and requiring rescanning, the usual forms used for algorithmic working require even more complex skills. The horizontal writing of the individual numerals together with the fact that the performance of the algorithm proceeds from right to left, might suggest that horizontal scanning and horizontal span should be considered in the perceptual skills required. Likewise, the placement of the individual numerals in the usual vertical positions suitable for the algorithms may suggest that vertical scanning and vertical span should be considered.

However, the required eye movements in algorithmic performance, especially for multiplication and division obviate against any continuous scanning process, such as that required for reading a page of a novel. Further, the student must pick from the several numerals those digits with which he is to operate. He must then retain the result while searching for the next digit or digits to combine with his result, or transfer this result to either its permanent or temporary position required by the algorithmic procedure. This combination of scanning and selection is called the search procedure.

VI. STATEMENT OF THE PROBLEM

The ability to work algorithms may be increased by training in perceptual skills. Three have been identified: vertical span, horizontal span, and search procedure. A preliminary study, as described in Appendix IV, was performed for two purposes: to establish standard test procedures; to indicate the relative importance of the three perceptual skills identified. On the basis of a three week training period in which each of three groups was trained in one of the perceptual skills, the search procedure training seemed to produce the greatest improvements in arithmetic achievements. Therefore, the primary problem is, to what extent will a period of training in the search procedure produce gains in algorithmic performance?

It is quite conceivable that the changes so produced could be due to the increases of horizontal and vertical span being developed during the search procedure training session. Hence, two further questions are: Does vertical span increase? and Does horizontal span increase?

Finally, this study is concerned with relationships between the various perceptual skills and arithmetic ability. Therefore, a test for a linear relationship between arithmetic and each of the three perceptual skills as well as a test for a linear relationship between algorithmic performance and search procedure with the effects of vertical and horizontal span removed will be made.

VII. QUESTIONS TO BE ANSWERED

The questions to be answered by this study are the following:

Over a period of training does the search procedure improve?

Over this same period do vertical and horizontal spans improve?

Over this period of training does the speed and accuracy of algorithmic performance improve?

Is the search mechanism correlated with vertical and horizontal span?

Which of the perceptual skills is more highly correlated with algorithmic performance?

Is the correlation between the search procedure and algorithmic performance significantly different from the multiple correlation using algorithmic performance as the dependent variable and horizontal span, vertical span and search procedure as the independent variables.

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CHAPTER II

RESEARCH DESIGN

I. EXPERIMENTAL DESIGN

Selection of Classes and Assignment of Treatments

Two schools were selected from the Vancouver, British Columbia School System. They were considered to be a random choice as far as treatment effects were concerned. Each school had two grade seven classes, both of which were taught by the same teacher. In order to control the effects of ability, the teachers were asked to select their class which performed the best in arithmetic. The experimental treatment was randomly assigned to one of the two high ability groups and to the low ability group in the other school. Thus in one school the higher ability group was the experimental and the lower ability the control and in the other the order was reversed.

The Hawthorne effects were controlled in the following ways: first, both experimental and control groups were told they were part of an experiment; second, each group in the same school was taught by the same teacher; third, each group was taught their regular classwork with the overhead projector being used in place of the chalk-board (this latter being a novel situation for both groups).

Method of Training

The subject material during this period consisted of geometry and a review of some of the general concepts covered during the year. The teachers were asked to select topics which did not involve much calculation. The control group was given no special training. The experimental group received a five to ten minute training session each scheduled hour of arithmetic over a period of three weeks while continuing their regular work.

Training for the search mechanism was accomplished by projecting an 11 x 11 matrix of digits onto a screen with an overhead projector. The teacher read a sequence of digits from a given row, column, or diagonal and as the students identified the sequence they recited along with the teacher. A complete description of the method of training together with examples appears in Appendix I.

Data Acquisition

There were pre- and post-tests given on vertical and horizontal span, search procedure, and arithmetic speed and accuracy. The Form I to Form II correlations for the tests were .76, .62, .81, and .65. A complete description of the test construction and administration appears in Appendix II.

To counter any systematic effects of the two parallel forms of the arithmetic test the forms were crossed over the experimental-controls groups as can be seen from Table I.

TABLE I

PLAN OF THE EXPERIMENTAL DESIGN AND ADMINISTRATION OF ARITHMETIC TESTS

School			
Group		I	II
Experimental	Ability level Pre-test Post-test	Higher Form I Form II	Lower Form II Form I
Control	Ability level Pre-test Post-test	Lower Form II Form I	Higher Form I Form II

The perceptual tests were given as a group in one period. In order to compensate for any specific learning associated with any of the tests the order of administration was changed from pre-test to post-test.

II. STATISTICAL DESIGN

This study is in two parts. The first is designed to establish whether or not there is a causal relationship between training in the search procedure on the one hand and vertical span, horizontal span, and arithmetic computational ability on the other. It will be referred to as Analysis of

Variance. The second part is designed to test for the existence of linear relationships between the various perceptual skills and computational ability. It will be referred to as Correlational Analysis. Each of these headings refers to the statistical tests made in each. In both sections questions raised in Chapter I will be stated in null hypothesis form and a description of the appropriate tests will be given.

Analysis of Variance Hypotheses

There is no significant difference between the control and the experimental groups on the measure of the change in search ability.

There is no significant difference between the control and experimental groups on the measure of change in vertical span.

There is no significant difference between the control and the experimental groups on the measure of change in horizontal span.

There is no significant difference between the control and the experimental groups on the measure of change in arithmetic computational ability.

For convenience the tests of perceptual skills and arithmetic are numbered as follows: (1) search procedure, (2) vertical span, (3) horizontal span, and (4) algorithmic performance. The groups are labeled (E) experimental and (C) control.

The symbolic null hypotheses are:

H_{0_1} : E1 - C1 = 0 H_{0_3} : E3 - C3 = 0

H_{0_2} : E2 - C2 = 0 H_{0_4} : E4 - C4 = 0

The experimental and control groups both consisted of two classes of students. Since it was not possible to randomly select students to assign to each of the groups a nested design with classes randomly chosen was employed.[1] As the treatment effects were random the term D in the expression for the expected mean squares of the treatment effect (A in Table II) approaches 1. Therefore, the denominator for the F_{ratio} is the pooled mean squares of the effects of the classes (B in Table II).

TABLE II*

SOURCES OF VARIATION, DEGREES OF FREEDOM, AND EXPECTED MEAN SQUARES USED FOR THE ANALYSIS

Source of variation	df	E(MS)
A	p-1	$\sigma_E^2 + nD_q\sigma_\beta^2 + nq\sigma_\alpha^2$
B (pooled)	p(q-1)	$\sigma_E^2 + n_\beta^2$
Experimental error (within cells)	pq(n-1)	σ_E^2

*Reproduced from Winer, op. cit., p. 185.

The tests of search mechanism, horizontal and vertical span and arithmetic computation were independent measures and therefore the analysis outlined above was used to test the difference between the experimental and control groups on each measure (hypotheses one, two, three, and four). The level of significance was set at .05 for all tests. The $F_{probabilities}$ were calculated by a standard program on the University of British Columbia computer.

Correlational Analysis Hypotheses

There is no significant correlation between the tests of the search procedure and vertical span.

There is no significant correlation between the tests of the search procedure and horizontal span.

There is no significant correlation between the tests of horizontal and vertical span.

There is no significant difference between the correlation of the search procedure with arithmetic ability and the multiple correlation found by using algorithmic ability and the dependent variable and search procedure, vertical span and horizontal span and the independent variables.

Using the same notation as in the previous section the hypotheses in symbolic form are as follows:

$H_{0_5} : r_{C_{12}} = 0$ $H_{0_7} : r_{C_{23}} = 0$

$H_{0_6} : r_{C_{13}} = 0$ $H_{0_8} : r_{C_{1(23)4}} = 0$

The correlations relating to hypotheses five through eight were calculated only on the final results of the control group. The final tests were used because the students were capable of familiarity with the testing procedure. The experimental group results were excluded in case the treatment had any effect which would bias the correlations.

The formula[2]

$$t = \frac{r_{XY} \; N-2}{1-r^2_{xy}}$$

was used to transform the correlations into an approximate t-score with N-2 degrees of freedom. This was then used to test the null correlational hypotheses. The level of significance was set at .05 for all tests. The $F_{probabilities}$ were computed by a standard program on the University of British Columbia computer.

Hypothesis eight was tested in the following way. A multiple correlation was calculated for vertical and horizontal span and the search measures onto arithmetic scores. A second correlation was calculated ignoring the measure of search mechanism. The significance of the difference in the variances was calculated by a computer program using the method of Bottenberg and Ward.[3] The technique involves the comparison of the variance accounted for by one multiple correlation and the variance accounted for by another with some independent variables excepting the variable of interest.

III. ASSUMPTIONS AND LIMITATIONS

Study Limitations

The study has limitations resulting from the following considerations.

An optimum training program may not have been developed because an easily followed program would be more likely accepted in the classroom.

The angle which the training and testing material subtended was not a constant for all children, but, by keeping the seating arrangement the same from testing to testing the angle was kept constant for each student.

The angle at which the training material and perceptual tests were presented bore no relation to the angle at which the students perceived their written arithmetic tests.

The lighting conditions on a child's desk were quite different from those of a projection screen.

Because the sequences were presented aurally to, and then sought out visually by the subject there may have been a confounding of the perceptual processes. That is, the child does not hear the number he is to look for when he is working an arithmetic algorithm. Thus the training was not completely analogous to the search process used in computation.

The time period may have been somewhat too short to detect even a relatively low order change. Also the training was limited to one procedure. Perhaps some combination of training procedures for the three perceptual skills could produce greater change in computation.

The type of testing material was chosen to indicate some change in computational ability. No attempt was made to test other aspects such as the solution of problems requiring extensive reading and transferral of information and numbers. For example, word problems often require rewriting on a piece of work paper either the problem or an associated algorithm or both.

Statistical Assumptions

Another set of assumptions made involve the statistics chosen for the study. The first is that the statistical error terms are independent. It is clear that as the students could not be chosen randomly there could be a systematic variance due to being in the same class for a year. It was for this reason that the nested design was chosen. It is therefore assumed that the errors are independent.

The second is the assumption of normality. A histogram was generated on all the scores and, as no gross departures from normality occurred, the assumption was accepted.

The third assumption is that the variances are homogeneous. As the test for significance involved an F_{ratio} which is robust with respect to violation of this assumption when sample sizes are equal,[4] no test was made.

Footnotes - Chapter II

[1]B.J. Winer, Statistical Principles in Experimental Design, (New York: McGraw-Hill Book Company, Inc., 1962), p. 184.

[2]William L. Hays, Statistics, (New York: Holt, Rinehart and Winston, 1963), p. 529.

[3]R.A. Bottenberg and J.H. Ward, Applied Linear Multiple Regression, U.S. Office of Technical Services Technical Report PRL-TDR-63-6 (Washington: Department of Commerce, 1963).

[4]Rodger E. Kirk, Experimental Design: Procedures for the Behavioral Sciences, (California: Brooks/Cole Publishing Company, 1968), p. 61.

CHAPTER III

RESULTS OF THE STUDY

As in the preceding chapter it will be convenient to present the analysis of data in two sections, one dealing with questions to be tested by analysis of variance and the other with correlational analysis. These two sections respectively correspond to questions which attempt to establish a causal relationship and those which describe.

I. ANALYSIS OF VARIANCE HYPOTHESES

The means and standard deviations on the pre- and post-test of each measure are presented in Tables III and IV respectively. Groups I and III are from one school; groups II and IV are from the other. Table V contains the means of the difference scores which were used to calculate the change over the period of the experiment in each group. It is these means which are used in the analysis of variance.

Tests of Significance Used

In each of the four hypotheses treated by analysis of variance a nested design was used (see Chapter II, section V, Analysis of Data). Therefore the F_{ratio} was calculated using the mean squares associated with the treatment in the numerator and the mean squares associated with the class nested within the treatment for the denominator.

The degrees of freedom associated with the numerator and denominator were 1 and 2 respectively.

As the alpha level chosen for each test was .05 the F_{ratio} required for significance was 18.5. For the detailed calculations see Appendix III.

TABLE III

MEANS OF ALL TEST ADMINISTRATIONS

Group		Control		Experimental	
Test		I	II	III	IV
Search mechanism	pre	7.04	9.93	6.67	11.81
	post	7.93	10.44	11.74	15.85
Vertical span	pre	10.78	11.96	10.07	11.78
	post	11.07	11.56	10.33	11.70
Horizontal span	pre	12.19	15.33	10.04	13.81
	post	12.62	11.96	12.25	11.48
Arithmetic computation	pre	15.26	17.96	12.22	19.19
	post	17.26	17.30	14.25	18.74

TABLE IV

STANDARD DEVIATIONS OF ALL

TEST ADMINISTRATIONS

Group		Control		Experimental	
Test		I	II	III	IV
Search mechanism	pre	5.61	5.13	4.42	5.80
	post	6.18	5.85	5.51	4.60
Vertical span	pre	4.63	3.77	3.40	3.38
	post	3.90	3.92	3.25	4.00
Horizontal span	pre	4.62	5.02	3.60	5.64
	post	5.12	5.09	3.64	4.82
Arithmetic computation	pre	5.22	5.75	5.77	4.52
	post	5.73	4.88	5.69	4.52

TABLE V

MEANS OF THE DIFFERENCE SCORES FOR THE FOUR

GROUPS ON EACH OF THE FOUR MEASURES

Group						
Test	I	II	Com-bined	III	IV	Com-bined
Search mechanism	-0.94	-0.23	-0.77	-5.26	-4.38	-4.8
Vertical span	-0.31	0.42	0.06	-0.27	0.15	-0.06
Horizontal span	-0.46	3.2	1.35	-2.3	2.5	0.96
Arithmetic computation	-2.1	0.69	-0.69	-2.2	0.46	-0.85

Note: As the differences were calculated by taking the post-test scores from the pre-test scores a negative value indicated an improvement.

Hypothesis I

Hypothesis I was that there would be no significant difference between the experimental and control groups on a measure of change in search ability. The summary of the analysis of variance is in Table VI. On the basis of these results the null hypothesis was rejected. Because the means of the experimental groups were significantly more negative than those of the control it was concluded that the experimental groups improved more on the search mechanism than did the control.

TABLE VI

SUMMARY OF ANALYSIS OF VARIANCE FOR HYPOTHESIS I

Source	ss	df	MS
Treatment	428.09	1	428.09
Classes within treatment	11.40	2	5.70
Cell	1567.27	100	15.67
Total	2006.76	103	

$F_{ratio} = 75.10$ $F_{prob.(1,2)} 75.10 = .0225$

Hypothesis II

Hypothesis II was that the experimental group would not be significantly different from the control group on a measure of change in vertical span. The summary of the analysis of variance is to be found in Table VII.

TABLE VII

SUMMARY OF ANALYSIS OF VARIANCE FOR HYPOTHESIS II

Source	ss	df	MS
Treatment	0.35	1	0.35
Classes within treatment	4.46	2	2.23
Cell	713.19	100	7.13
Total	718.00	103	

$F_{ratio} = .1568$ $F_{prob(1,2)} .1568 = .7229$

On the basis of these results the null hypothesis was not rejected. There was no significant difference between the control and the experimental groups on a measure of vertical span.

Hypothesis III

Hypothesis III was that the experimental group would not be significantly different from the control group on a measure of change of horizontal span. Table VIII is a summary of the analysis of variance for this hypothesis. On the basis of these results the null hypothesis was not rejected. That is, there was no significant difference between the control groups and the experimental groups on a measure of change in horizontal span.

TABLE VIII

SUMMARY OF ANALYSIS OF VARIANCE FOR HYPOTHESIS III

Source	ss	df	MS
Treatment	42.34	1	42.34
Classes within Treatment	474.58	2	237.79
Cell	1180.54	100	11.81
Total	1696.46	103	

$F_{ratio} = .178$ $F_{prob(1,2)}.178 = .7079$

Hypothesis IV

Hypothesis IV stated that there would be no significant difference between the control and experimental groups on a measure of change in arithmetic computation. Table IX summaries the Analysis of Variance for this hypothesis. These results show that the null hypothesis was not to be rejected. There was no significant difference between the control and experimental groups on this measure.

TABLE IX

SUMMARY OF ANALYSIS OF VARIANCE FOR HYPOTHESIS IV

Source	ss	df	MS
Treatment	.61	1	.61
Classes within treatment	188.62	2	24.31
Cell	1129.23	100	11.29
Total	1318.46	103	

F_{ratio} = .00636 $F_{prob(1,2)}$ .00636 = .9013

It should be noted that the F_{ratios} of hypotheses II, III, and IV are less than 1.0. F_{ratios} less than one may indicate that the assumptions underlying the F distribution are being violated. However, the probability that an F_{ratio} would be below the ones calculated is just 1-$F_{prob.}$ and in this case is above the level of significance established for the experiment.

For example the lowest F_{ratio} is that associated with hypothesis IV (.00636). The calculated $F_{prob.}$ is .901. Therefore the probability of getting an F_{ratio} below this is 1.0 - .901 = .099 which is above the .05 level. The hypothesis, that the assumptions underlying the F distribution are being violated, is rejected.

II. CORRELATIONAL ANALYSIS

The correlations for hypothesis V, VI and VII, together with the generated t-scores and probabilities, are presented in Table X. Hypothesis VIII will be accorded a section itself because of the analysis used. For the equation used to transform the correlations to t-scores see Chapter II, section V, Analysis of Data.

TABLE X

TABLE OF CORRELATIONS AND SIGNIFICANCE LEVELS

Hypothesis	Correlation	t-score	Probability
$H_{0_5} : r_{12} = 0$	0.3697	7.915	0.0069
$H_{0_6} : r_{13} = 0$	0.2460	3.219	0.075
$H_{0_7} : r_{23} = 0$	0.7604	68.553	0.000

Note: 1,2 and 3 denote search mechanism, vertical span and horizontal span respectively. Only the post-test scores were used in the calculations of the correlations because it was desired that the subjects become acquainted with the testing procedures.

Product Moment Correlations

The correlations reported were calculated by a standard computer program. The probabilities were calculated by the same program as was used for the $F_{probabilities}$. The level of significance reported is a one-tailed probability.

Hypothesis V stated that the search mechanism would not be correlated with vertical span. A significance level of .0069 was achieved and therefore the null hypothesis was rejected.

Hypothesis VI stated that the search mechanism would not be correlated with horizontal span. The level of significance was .075 and therefore the null hypothesis was accepted.

Hypothesis VII was that the vertical span would not be correlated with the horizontal span. This null hypothesis was rejected with a confidence level beyond .01.

Multiple Correlation

The final null hypothesis to be tested was that search mechanism would not be related to arithmetic computation with the effects of horizontal and vertical span removed. Specifically, in this case a multiple correlation was calculated with search procedure, horizontal span and vertical span being the independent variables and arithmetic computation being the dependent variable. This correlation was .325. Then a second correlation with horizontal and vertical span was calculated. It was .103. An F_{ratio} between the variances accounted for the two equations was then calculated. The result was 5.186 with a probability of .027. The null hypothesis was therefore rejected. That is, the amount of additional variance accounted for by the search mechanism score was significant.

III. INTERPRETATION OF RESULTS

Introduction

This study is divided into two parts: causal questions and relational questions; or, questions derived from past experimentation and questions to assist future experimentation. As these questions are answered by analysis of variance and correlational analysis, this section will treat these separately. This section is divided into two parts: Analysis of Variance and Correlational Analysis.

Analysis of Variance Hypothesis

Null hypothesis I was rejected. There is, then, a way in which the teacher, using easily obtainable materials, can successfully train for search ability as defined in this study (see Chapter I, section II, Definitions). However, null hypothesis IV, that such training would also have no effect on arithmetic computation as defined, was not rejected. Therefore, it is to be concluded that this perceptual training program was unsuccessful in increasing the ability to work algorithms. A causal link was not established.

As null hypotheses II and III, which stated that vertical and horizontal span would not be changed by the training period, were not rejected it could be concluded that training in the search skills has no effect on vertical and horizontal span. More will be said in the section on the correlational analysis.

The conclusion that search training has no effect upon arithmetic computation must be approached with caution. The author constructed the tests for a specific purpose and their content defines what was meant by arithmetic computation. Examination of the tests will reveal that the problems were of a most elementary type. Also, the tests were built on the assumption that the questions were a measure of skill and speed as evidenced by the pre-experiment test validation. However, there was an indication that this was not a test of speed for the experimental groups.

One of the teachers mentioned that about half of his students were finished the tests before the end of the assigned time limit. There is circumstantial evidence then, that this may not have been a test of speed and that the students were able to go back and made corrections of mistakes. Thus the test was not as speeded as expected.

Again, the arithmetic tests were of a specific type and the evidence of the pilot study indicates that there may be more complex test situations in which such a skill as the search mechanism might be of importance.

Correlational Analysis Hypotheses

The following discussion compares the correlations of vertical span to search, and horizontal span to search, and then associates these with the results of the analysis of variance.

It was shown that the search mechanism was significantly correlated with arithmetic computation. It was also shown that the search training program did not produce any change in arithmetic computation. One might conclude that these were two phenomena which did co-exist but had no causal relationship. However, it was also shown that vertical span was correlated with search mechanism and hence an alternative conclusion may be proposed: that the increase in search ability was caused by the use of vertical span upon a new task, namely the performance on the search procedure test. This conclusion is further strengthened by noting that the multiple correlation of vertical and horizontal span to arithmetric computation was .1.

Hence it could be concluded that there was no new skill taught. Since there is little correlation between vertical span and the performance of algorithms an increase in the use of span on the search procedure test should not necessarily increase arithmetic ability. This, in fact, was the result of the analysis of variance. Further, because the significant correlation between search mechanism and arithmetic computation was calculated on the control group only, it is possible to suggest that the search procedure is an already established mechanism and that the training period was too short to establish a detectable increment in algorithmic performance.

CHAPTER IV

CONCLUSIONS AND HYPOTHESES FOR FURTHER RESEARCH

I. CONCLUSIONS

Part of this study involved an attempt to determine whether or not there exists a causal relationship between search ability and arithmetic computation. No such relationship was found. Nevertheless, the result that search mechanism accounted for a significant proportion of the variance of the arithmetic score must be explained.

It has already been noted that the search mechanism was significantly correlated with vertical span. The author suggests that those students who had an already well developed vertical span were more likely to learn to perform well on the search procedure test. Therefore, rather than developing a new skill, the training period developed a new task performance (search procedure test) based upon an old or already developed skill (vertical span). That is, it may be concluded that the improvement in the search mechanism was not the result of the development of the search ability. Moreover, because the correlation between vertical span and arithmetic computation was very low then one should expect no increase in arithmetic computation. The study findings bear this out.

The results of the pilot study together with the results of this study suggest two further conclusions. The first is that the search mechanism may be a factor in more complex arithmetic testing situations, and the second, that the training for this mechanism may best be done at an earlier stage.

II. FURTHER RESEARCH

This study indicates several possibilities for future research. These questions may be divided into two areas: educational psychology and mathematics education.

Educational Psychology

Perception is, in the framework of earlier duscussions, a central issue of psychology. Therefore, there are several specific questions, related to the search mechanism, which are of concern to psychologists.

Is search mechanism a compound perceptual skill or is it a unique skill independent of vertical span, horizontal span, the ability to distinguish items from a "noisy" background, and the ability to perceive shapes?

If the former is the case can the search mechanism be best taught by training designed for that skill or can it be best developed by training those abilities of which it is composed? Specifically this study indicated that those students scored highest on a measure of vertical span were also those who did well in the search mechanism. Therefore, can the search mechanism be increased by training in vertical span? A more general question would be are there components or other lower level perceptual skills which form the search procedure?

Analysis of the training method shows that other perceptual skills were involved in the training plan. They include auditory discrimination, aural to visual translation, and the aural-visual alternation which provided interest and relief from the possible monotony of one kind of stimulus.

Mathematics Education

Although the results of this study indicate that arithmetic calculation was not increased by the search training, it should not be concluded that training in perceptual skills for an increase in computational ability is of no value. The reasons for this are clear.

As this is the first time that the search procedure has been defined it cannot be assumed that the training plan was the best that could be designed. During the administration of the tests the author became aware that in order to have an effect in an educational situation one must train in perceptual skills which exactly mirror the behaviour required of the child by that situation. For example, it is useless to say that controlled readers are of no value in helping the child attain speed and comprehension in reading when only one or two actually train the child in the required perceptual skill, namely making the eye travel in a general diagonal fashion down the page.

It is here perhaps that the present training procedure is deficient: it has not accurately paralleled what the child is required to do when he works problems by himself. First the child hears few things which aid him to search for the required numbers to multiply together in a multiplication problem. For example, the student while working the question

$$\begin{array}{r} 273 \\ \underline{\times 96} \\ 8 \end{array}$$

and having placed the eight hears no voice saying, "Look for the seven."

Future researchers must refine the training procedures, and look for answers to the following questions.

Is the search mechanism the only perceptual skill related to mathematics? There is a possibility that some perceptual skills are linked with certain geometric abilities. For example, the perception of spatial relationships may be a necessary skill.

Is the search mechanism only useful on specific kinds of mathematical problems? This study indicates that the search mechanism may be more useful in complex situations or at least in situations where the visual pattern of an algorithm is presented for the first time.

Is there a particular age at which the perceptual skills should be taught? Again, this study indicates that either the training period was too short to make any changes in the training mechanism or that it would be best to train students at an earlier age.

It is to be hoped that future research will not be hampered because of the failure of this study to establish a significant link between a perceptual skill and arithmetic computation. Caution must be taken though for even if the search mechanism or any other perceptual skill can be analyzed into individual components it may still be most efficatious both from educational and psychological viewpoints to consider these skills as a single unit.

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A P P E N D I X

APPENDIX I

TRAINING METHODS

In this study only training in the search mechanism was presented. However, the pilot study was made in order to suggest which of the three perceptual skills, search mechanism, vertical span or horizontal span would produce the greatest change in arithmetic skills. Therefore the procedures used for the three training methods are presented below.

Vertical Span

The teacher was supplied with an overhead transparency of a 16 by 16 matrix of digits (black on white) and an adjustable paper mask. The large matrix was used to provide the teacher with a large number of digits which he could randomly select by moving the mask.

During the training period the following procedure was administered for five to ten minutes daily. The teacher covered the matrix with the mask and selected the width of numeral desired by opening the mask being careful to keep the numeral hidden with his hand. The students were required to write the numeral they had seen and were then shown the correct answer. As the period of training progressed the teacher would select larger numerals and shorten exposure times.

The students were encouraged to keep a record of their progress by collecting each day's result.

Horizontal Span

The method of training for horizontal span was identical to that for vertical span except for the positioning of the mask.

Search Mechanism

Using the same type of matrix as mentioned above, the teacher was supplied with a mask which covered all but a 12 by 12 digit matrix. This allowed the selection of a different matrix each day by simply moving the mask.

The teacher would select a row, column or diagonal and begin to read the digits beginning at either end of the row, column or diagonal. The students were to find the sequence and begin to read orally with the teacher. This provided the slower students time to search and find the sequence.

At the beginning of the training period the teacher read either rows from left to right or columns from top to bottom. As the period progressed he would reverse direction more frequently and read more rapidly.

At the end of each five to ten minute session the teacher would give a short test (as described in Appendix II) the student being encouraged to keep a record of the results.

APPENDIX II

DESCRIPTION OF TESTS, PRESENTATION AND SCORING

Horizontal and Vertical Span

Only the horizontal span tests are described because the vertical span tests are the same except for the digits being arranged in columns.

Material and content: Slides, of standard 35 mm color film, were taken of five 2, 3, 4, 5, 6 and 7 digit rows. This gave a total of thirty slides. Because of the experience of the pilot study the 7 digit numbers were omitted when the tests were administered in the final study.

The numbers were photographed from the matrix supplied to the teachers in order to keep the format of the numbers presented constant.

The rows of digits were black on a white field and were masked with matt black construction paper. They were projected from a position that would allow the 6 digit row to almost fill a 50" by 50" screen at the front of the room.

A standard slide projector was used to project the slides. In order to control the duration of the visual presentation a camera with a focal plane shutter, back and lense removed, was placed in front of the slide projector lense.

The shutter speeds of the camera were 1, 1/2, 1/4, 1/8, 1/15, 1/30, 1/60, 1/125, 1/500, and 1/1000 of a second. The list of numbers and the speeds of presentation are given in Table XI.

Presentation: The sets of five slides for 2, 3, 4, 5, 6 and 7 digit numbers were arranged in the order named in Table XI. The numerals of length 2 and 3 were presented at 1/4, 1/8, 1/15, 1/30, and 1/60 of a second; of length 4 and 5 at 1/2, 1/4, 1/8, 1/15, and 1/30; of length 6 and 7 at 1, 1/2, 1/4, 1/8, 1/15. These speeds were selected because of the experience gained in the preliminary study. Before each presentation the students were warned to watch the screen.

Scoring: The number of correct answers for each of the tests was considered to be the child's score. The maximum score possible under this method would be 30 (25 in the final administration). The order of the items was changed for the post-test. Test-retest correlations were calculated for vertical span and horizontal span and were .76 and .62 respectively.

Search Mechanism Test

Materials and content: The materials consisted of an overhead projector transparency of an 11 x 11 matrix of digits written in black. See Tables XII and XIII.

TABLE XI

ITEMS COMPRISING THE VERTICAL AND

HORIZONTAL SPAN TESTS

Number of Digits	Duration	Vertical Test	Horizontal Test
	.25	25	63
	.125	74	87
2	.066	59	36
	.033	41	17
	.017	67	94
	.25	942	713
	.125	281	138
3	.066	572	326
	.033	418	879
	.017	374	956
	.5	1253	2958
	.25	2356	5817
4	.125	2547	7432
	.066	1937	2637
	.033	4136	3846
	.5	57294	43138
	.25	41896	13956
5	.125	87281	74129
	.066	63654	38474
	.033	32191	72581
	1.0	937863	494558
	.5	341815	562713
6	.25	896964	352158
	.125	234395	326235
	.066	374594	145367

TABLE XII

ARRAY PRESENTED FOR THE SEARCH MECHANISM TEST WITH THE STARTING POINTS OF THE SEQUENCES USED

9		6/12				20			5			
1	7	1	3	9	5	6	9	3	8	1	5	4
1	8	6	9	8	7	9	4	2	4	9	7	
	3	4	5	3	2	6	2	3	5	1	9	11
	3	6	1	4	9	4	5	5	8	9	7	
13	5	1	9	1	4	5	3	6	7	3	6	
	2	5	8	6	2	1	4	9	2	7	5	
	4	3	7	3	5	2	1	5	8	8	4	
2	3	8	3	6	4	5	8	5	1	6	7	3
	6	4	5	5	7	3	1	1	9	3	1	
	7	8	4	4	5	9	5	6	7	6	3	17
16	4	3	8	7	2	6	4	9	3	9	8	
18		8			7/15					19		14

Note: This table should be used as follows: To find thirteenth sequence used locate the number 13 on the perimeter of the array and read the sequence beginning at that point; 51914. The correct response is 53. Further, sequence 18 is a diagonal sequence read from the lower left hand corner; 48565. The correct response is 13.

TABLE XIII

SEQUENCES AND THE CORRECT RESPONSES USED
FOR THE SEARCH MECHANISM TEST

Number	Sequence	Response	Number	Sequence	Response
1	86987	94	11	91532	62
2	38364	58	12	39519	67
3	76158	54	13	51914	53
4	51839	65	14	86951	14
5	84587	28	15	69352	15
6	16431	53	16	43872	64
7	25745	24	17	36765	95
8	38483	51	18	48565	13
9	76544	11	19	83174	56
10	59553	15	20	94253	41

The test consisted of calling out a sequence of 5 digits, the first being located at one of the boundaries of the matrix. Thus, a sequence beginning at element 11,4 of the matrix (the eleventh row, the fourth column), would be read vertically from bottom to top, whereas the sequence beginning at 5,11 would proceed horizontally from right to left.

The students responded to the sequence of five digits by giving the sixth and seventh digits, in order, of the sequence.

<u>Presentation</u>: In order to administer the search mechanism test identically to all groups and consistently from beginning to end, a tape recorder was used. Two sequences from left to right, two from right to left, two from top to bottom, two from bottom to top and two on the diagonals were presented

at 7 second intervals. An additional ten sequences, two from each of described directions were read in random order. The sequences were changed for the post-test and the correlation between pre- and post-test was .81.

Arithmetic Tests

Initial selection of items: In order to develop a suitable test for speed and accuracy in arithmetic computation an initial selection of forty addition, subtraction, multiplication and division questions were administered over an hour period to the grade Seven classes in a Burnaby school. The tests were marked and items whose p value was less than .15 and greater than .85 were deleted. The initial and final values of the mean, standard deviation and KR20 were 26.88, 7.48, and 0.88 and 23.34, 7.37, 0.88 respectively. Few of the students finished within an hour and therefore the test was considered to be a good test of speed and accuracy. A correlation matrix was then calculated for the remaining items. Items were selected for the two parallel forms of the final test on the basis of the operations involved, content and correlation. There were several questions which could not be matched on the basis of content. These were assigned to one or other of the tests using p values to bring the means close together. Items of parallel content were then constructed and assigned to the alternate test.

The final tests had six items for each of the four operations: three of each six were written horizontally and three vertically. For example, compare questions 1 and 7 of test I at the end of this appendix. The tests were written in the author's handwriting because the perceptual tests as well as the training materials were so written. The tests had identical format with the items from each of the eight categories randomly ordered.

The final tests: The final tests are included at the end of this Appendix. Because they were used as pre- and post-tests for an experiment and the forms were alternated over the experimental group (see Table I) only the following administrations of the test could be used for the calculation of the KR20: form I, pre-test group II, pre-test group III and post-test group I; form II, pre-test group I, pre-test group IV and post-test group II. Note that only the pre-tests from the experimental groups were used in case there was any systematic effect of the treatment. The mean, standard deviation and KR20 for forms I and II were 16.58, 5.5, 0.88 and 17.91, 3.78, 0.73 respectively. The KR20 of form II was low when compared with that of form I and that of the initial pool of items (.88). This may be attributed to two out of the four groups used for the analysis being from schools whose students were more homogeneous and therefore achieving a lower test variance.

The test was further broken into subtests in order to see if there were any systematic differences between the four arithmetic operations, and between the vertically and horizontally written questions. The summary of the analyses of the addition, subtraction, multiplication and division subtests are in Table XIV. In Table XV the vertical and horizontal subtest analyses are summarized. Because the measures of consistency (KR20) for these subtests were so poor no attempt was made to apply any a posteriori tests of significance to the differences between the experimental and control groups on these measures.

TABLE XIV

SUMMARY OF THE ANALYSIS OF THE FOUR ARITHMETIC OPERATION SUBTESTS

Subtest	Addition		Subtraction		Multiplication		Division	
Form	I	II	I	II	I	II	I	II
Mean	2.01	4.31	4.94	5.08	4.20	4.66	3.50	3.86
SD.	1.73	1.21	1.47	1.09	1.52	1.15	1.95	1.82
KR20	0.67	0.22	0.72	0.44	0.56	0.29	0.76	0.71

TABLE XV

SUMMARY OF THE ANALYSES OF THE VERTICAL AND HORIZONTAL SUBTESTS

Subtest	Vertical		Horizontal	
Form	I	II	I	II
Mean	8.59	8.89	7.89	9.03
SD	2.76	2.23	3.0	1.91
KR20	0.76	0.60	0.79	0.47

Another measure of reliability of concern in the study was the inter-form reliability. Again, the post-tests of the experimental group could not be used because of any systematic variance induced by the treatment. A correlation of the control subjects performance of the two forms was found to be .65. This is quite low for parallel forms of a test. It can be attributed to the difference between the performance of the group used to establish parallel items and that of the experimental group. As was noted above, the initial form of the test was given to the two classes which formed the whole grade seven population of a school which came from the same kind of economic area as the schools used for the experiment. However comparison of the results showed that the initial group of students averaged about 65 percent whereas the experimental group averaged 70 percent.

TEST I

PAGE I

1.
```
 938
 765
 476
 832
 516
 318
```

2. 265 x 92 =

3. 4138 - 509 =

4. 42)7365

5.
```
 4001
 -292
```

6. 1376 ÷ 71 =

7. 56 + 42 + 81 + 39 + 62 + 17 + 83 =

8.
```
 583
 x29
```

9. 37)7821

10. 13726 - 4927 =

11. 905 x 71 =

12. 48 + 35 + 76 + 58 + 62 + 27 + 16 =

TEST I

PAGE II

13. 9675 ÷ 18 =

14.
```
 8221
 -597
```

15.
```
512
890
 31
  6
765
418
  9
```

16.
```
 583
 x29
```

17. $63\overline{)2976}$

18. 2005 - 947 =

19.
```
 527
x602
```

20.
```
243
 78
541
965
  9
 56
  8
```

21. 438 x 96 =

22. 1865 ÷ 32 =

23.
```
 25965
 -4103
```

24. 413 + 876 + 531 + 29 + 871 + 13 + 9 =

TEST II

PAGE I

1.
```
 578
 362
 985
 413
 794
 185
 431
```

2. 256 x 901 =

3. 9763 - 407 =

4. 87)5963

5.
```
 7003
 -698
```

6. 2876 ÷ 56 =

7. 54 + 31 + 28 + 97 + 65 + 83 + 21 =

8.
```
 385
 x92
```

9. 29)6821

10. 13876 - 5487 =

11. 806 x 82 =

12. 45 + 63 + 67 + 82 + 75 + 26 + 61 =

TEST II

PAGE II

13. 8374 ÷ 95 =

14.
```
 17182
-9413
```

15.
```
433
 19
876
 54
321
757
```

16.
```
 341
x87
```

17. 82)7136

18. 5003 - 949 =

19.
```
 928
x95
```

20.
```
324
 87
415
596
  8
 65
  8
```

21. 403 x 96 =

22. 2173 ÷ 36 =

23.
```
 7863
-928
```

34. 314 + 156 + 832 + 854 + 138 + 765 + 143 =

APPENDIX III

STATISTICAL ANALYSIS OF TEST RESULTS

As outlined in Chapter II, section V, the statistical design was hierarchical with classes nested within treatment groups. A design of this kind is to be considered with caution for although the nuisance variable of class effect is parcelled out there is a loss of degrees of freedom, which, unless the number of classes is large, can be excessive.

The calculations are summarized below and follow the outline of Kirk who uses the following notation:

p levels of a_i,	where p = 2
q levels of b_j,	where q = 4
$q_{(i)}$ levels of b_j nested within A,	where $q_{(i)} = 2$
n levels of s_m,	where n = 26

"A" denotes the treatment groups, "B" the classes, "S" the subjects and [] a sum of squares.

Hypothesis I

The experimental group was significantly different from the control group on a measure of the search mechanism. The test results may be found in Table XVI. The detailed calculations follow.

TABLE XVI

SEARCH MECHANISM TEST RESULTS

Group I				Group II				Group III				Group IV			
x_1	x_2	d	d^2	x_1	x_2	d	d^2	x_1	x_2	d	d^2	x_1	x_2	d	d^2
5	3	2	4	16	19	-3	9	2	11	-9	81	11	16	-5	25
4	3	1	1	1	1	0	0	2	3	-1	1	18	20	-2	4
16	17	-1	1	*15	15	0	0	10	15	-5	25	17	20	-3	9
3	3	0	0	13	17	-4	16	5	17	-12	144	13	17	-4	16
3	1	2	4	4	4	0	0	7	16	-9	81	12	16	-4	16
11	16	-5	25	13	10	3	9	7	12	-5	25	4	18	-14	196
9	11	-2	4	8	16	-8	64	4	0	4	16	16	19	-2	4
2	2	0	0	13	15	-2	4	12	10	2	4	18	20	-2	4
6	11	-5	25	7	4	3	9	7	16	-9	81	10	16	-6	36
4	3	1	1	3	2	1	1	13	15	-2	4	11	16	-5	25
17	19	-2	4	4	3	1	1	17	18	-1	1	15	15	0	0
16	19	-3	9	9	7	2	4	3	9	-6	36	17	18	-1	1
3	10	-7	49	11	18	-7	49	6	16	-10	100	17	16	1	1
1	1	0	0	18	12	6	36	11	9	2	4	3	17	-14	196
1	3	-2	4	14	12	2	4	5	17	-12	144	14	17	-3	9
4	3	1	1	8	6	2	4	4	9	-5	25	12	19	-7	49
14	9	5	25	17	19	-2	4	5	15	-10	100	5	11	-6	36
5	10	-5	25	7	12	-5	25	1	12	-11	121	1	7	-6	36
9	12	-3	9	15	12	3	9	3	8	-5	25	16	15	1	1
14	6	8	64	5	7	-2	4	13	17	-4	16	*20	18	2	4
0	3	-3	9	13	11	2	4	9	9	0	0	7	18	-11	121
16	16	0	0	7	6	1	1	7	10	-3	9	16	20	-4	16
10	10	0	0	5	4	1	1	12	16	-4	16	16	19	3	9
6	6	0	0	18	13	5	25	0	12	-12	144	18	17	1	1
11	16	-5	25	11	15	-4	16	5	7	-2	4	3	7	-4	16
0	1	-1	1	8	14	-6	36	10	18	-8	64	12	20	-8	64
				10	15	-5	25					14	17	-3	9

*These scores were randomly selected for deletion to equalize group group size.

TABLE XVII

SUMMARY OF GROUP RESULTS

Group Treatment		I	II	III	IV	Totals
Experimental	Σd			-137	-114	-251
	Σd^2			1271	900	2171
Control	Σd	24	-6			-40
	Σd^2	290	360			650

$$\sum_{1}^{n} ABS = 1291$$

$$\sum_{1}^{n} (ABS)^2 = [ABS] = 2171 + 650 = 2821$$

$$\frac{(\sum_{1}^{n} ABS)^2}{npq_{(i)}} = [X] = \frac{(-291)^2}{26x2x2} = \frac{84681}{104} = 814.24$$

$$\frac{\sum_{1}^{p} (\sum_{1}^{q} A)^2}{nq_{(i)}} = [A] = \frac{(-251)^2 + (-40)^2}{26 \text{ x } 2} = \frac{64601}{52} = 1242.33$$

$$\frac{\sum_{1}^{p} \sum_{1}^{q} (AB)^2}{n} = [AB] = \frac{(-24)^2 + (-16)^2 + (-237)^2 + (-114)^2}{26}$$

$$= \frac{32597}{26} = 1253.73.$$

$$SS_{total} = [ABS] - [X[= 2821 - 814.24 = 2005.76$$

$$SS_{A} = [A] - [X] = 1242.33 - 814.24 = 428.09$$

$$SS_{B.w.A} = [AB] - [A] = 1253.73 - 1242.22 = 11.40$$

$$SS_{w.A} = [ABS] - [AB] = 2821 - 1253.73 = 1567.27$$

$$F_{ratio} = \frac{428.09}{5.70} = 75.10 \qquad F_{prob(1,2)} (75.10) = .9652$$

The alpha level of the study was .05 and $1 - F_{prob} = .0347$. Therefore the null hypothesis was rejected. In other words the treatment groups was significantly different at the .0347 level of confidence.

Hypothesis II

The experimental group was significantly different from the control group on a measure of vertical span. The test results may be found in Table XVIII. The detailed calculations follow.

$$\sum_{1}^{n} ABS = 0$$

$$\sum_{1}^{n} (ABS)^2 = [ABS] = 718$$

$$\frac{(\sum_{1}^{n} ABS)^2}{npq_{(i)}} = [X] = \frac{-3 + 3}{26x2x2} = 0$$

$$\frac{\sum_{1}^{p}(\sum_{1}^{q} A)^2}{nq_{(i)}} = [A] = \frac{(-3)^2 + (3)^2}{26 \ x \ 2} = \frac{18}{52} = .35$$

$$\frac{\sum_{1}^{p} \sum_{1}^{q} (AB)^2}{n} = [AB] = \frac{(-7)^2 + (4)^2 + (-8)^2 + (11)^2}{26}$$

$$SS_{total} = [ABS] - [X] = 718 - 0 = 718$$

$$SS_{A} = [A] - [X] = .35 - 0 = .35$$

$$SS_{B.w.A} = [AB] - [A] = 4.81 - .35 = 4.46$$

$$SS_{w.A} = [ABS] - [AB] = 718 - 4.81 = 713.19$$

$$F_{ratio} = \frac{.35}{2.23} = .1568 \qquad F_{prob(1,2)}(.1568) = .21$$

The alpha level of the study was .05 and 1 - (F_{prob}) = .79 Therefore the null hypothesis was accepted. That is, there was no significant difference between the experimental and the control groups on a measure of vertical span.

TABLE XVIII

VERTICAL SPAN TEST RESULTS

Control								Experimental							
Group I				Group II				Group III				Group IV			
x_1	x_2	d	d^2	x_1	x_2	d	d^2	x_1	x_2	d	d^2	x_1	x_2	d	d^2
12	13	-1	1	11	11	0	0	7	9	-2	4	11	15	-4	16
15	14	1	1	7	8	-1	1	8	11	-3	9	11	16	-5	25
10	14	-4	10	*17	12	5	25	11	11	0	0	11	10	1	1
6	8	-2	4	16	15	1	1	12	13	-1	1	14	14	0	0
8	9	-1	1	13	12	1	1	13	14	-1	1	6	5	1	1
13	15	-2	4	14	11	3	9	13	13	0	0	11	12	-1	1
7	9	-2	4	12	15	-3	9	8	10	-2	4	13	11	2	4
9	10	-1	1	15	10	5	25	14	11	3	9	11	9	2	4
8	12	-4	16	11	11	0	0	7	9	-2	4	16	19	-3	9
15	6	9	81	8	12	-4	16	10	6	4	16	12	12	0	0
18	17	1	1	9	10	-1	1	9	11	-2	4	11	12	-1	1
15	13	2	4	14	14	0	0	15	13	2	4	16	17	-1	1
8	12	-4	16	13	12	1	1	9	11	-2	4	13	5	8	64
2	2	0	0	15	18	-3	9	9	9	0	0	10	9	1	1
14	13	1	1	6	1	5	25	8	10	-2	4	12	15	-3	9
18	15	3	9	13	14	-1	1	12	10	2	4	11	13	-2	4
11	12	-1	1	15	16	-1	1	10	13	-3	9	11	9	2	4
16	13	3	9	14	12	2	4	12	13	-1	1	12	8	4	15
12	15	-3	9	10	10	0	0	13	9	4	16	15	14	1	1
12	14	-2	4	10	11	-1	1	10	14	-4	16	*10	11	-1	1
12	9	3	9	17	15	2	4	3	5	-2	4	12	11	1	1
7	11	-4	16	13	10	3	9	13	11	2	4	8	12	-4	16
10	9	1	1	12	12	-1	1	11	14	-3	9	15	14	1	1
16	14	2	4	17	15	2	4	9	5	4	16	13	14	-1	1
13	11	2	4	11	10	1	1	15	12	3	9	13	12	1	1
4	9	-5	25	12	12	0	0	11	12	-1	1	17	13	4	16
				15	14	1	1					14	14	0	0

*These subjects were deleted at random to give equal group size.

TABLE XIX

SUMMARY OF GROUP RESULTS

Group / Treatment		I	II	III	IV	Total
Experimental	Σd			-7	4	-3
	Σd^2			153	198	351
Control	Σd	-8	11			3
	Σd^2	242	125			367

Although the F_{ratio} is below 1, which could indicate that the assumptions of the $F_{distribution}$ were violated, the probability shows that the calculated ratio was not significantly different from 1.

Hypothesis III

The experimental group was significantly different from the control group on a measure of horizontal span. The test results may be found in Table XX. The detailed calculations follow.

$$\sum_{1}^{n} ABS = 5 + 71 = 76$$

$$\sum_{1}^{n} (ABS)^2 = [ABS] = 915 + 837 = 1752$$

$$\frac{\left(\sum_{1}^{n} ABS\right)^2}{npq_{(i)}} = [X] = \frac{(76)^2}{26x2x2} = 5776 = 55.54$$

$$\frac{\sum_{1}^{p} \left(\sum_{1}^{q} A\right)^2}{nq_{(i)}} = [A] = \frac{(7)^2 + (7)^2}{26 \text{ x } 2} = \frac{5090}{52} = 97.88$$

$$\frac{\sum_{1}^{p} \sum_{1}^{q} (AB)^2}{n} = [AB] = \frac{(-60)^2 + (65)^2 + (-12)^2 + (83)^2}{26} =$$

$$= \frac{14858}{26} = 571.46$$

TABLE XX

HORIZONTAL SPAN TEST RESULTS

Group I				Group II				Group III				Group IV			
x_1	x_2	d	d^2	x_1	x_2	d	d^2	x_1	x_2	d	d^2	x_1	x_2	d	d^2
11	17	-6	36	14	11	3	9	7	14	7	49	15	12	3	9
16	12	4	16	6	7	-1	1	8	14	-6	36	16	13	3	9
13	15	-2	4	*18	8	10	100	11	12	-1	1	16	13	3	9
9	7	2	4	20	18	2	4	10	13	-3	9	17	15	2	4
14	11	3	9	15	14	1	1	14	16	-2	4	3	1	2	4
14	18	-4	16	13	10	3	9	16	14	2	4	10	7	3	9
10	6	4	16	15	14	1	1	9	12	-3	9	18	16	2	4
9	11	-2	4	15	14	1	1	12	13	-1	1	12	13	-1	1
12	12	0	0	13	14	-1	1	11	12	-1	1	20	17	3	9
15	8	7	49	17	9	8	64	10	13	-3	9	17	7	10	100
20	21	-1	1	13	15	-2	4	8	15	-7	49	15	13	2	4
15	18	-3	9	17	12	5	25	17	15	2	4	20	17	3	9
10	12	-2	4	14	6	8	64	9	12	-3	9	15	2	13	169
0	1	-1	1	20	11	9	81	9	13	-4	16	9	10	-1	1
16	15	1	1	5	1	4	16	7	10	-3	9	20	12	8	64
16	19	-3	9	18	18	0	0	12	17	-5	25	19	15	4	16
13	13	0	0	18	21	-3	9	12	14	-2	4	13	11	2	4
18	16	2	4	19	10	9	81	11	8	3	9	0	6	-6	36
13	18	-5	25	12	9	3	9	10	8	2	4	20	13	7	49
16	17	1	1	14	12	2	4	9	16	-7	49	*15	14	1	1
11	13	-2	4	21	16	5	25	5	8	-3	9	12	12	0	0
13	8	5	25	17	13	4	16	14	13	1	1	10	13	-3	9
11	12	-1	1	17	10	7	49	7	15	-8	64	15	14	1	1
13	14	-1	1	21	19	2	4	6	6	0	0	18	15	3	9
15	14	1	1	18	14	4	16	14	15	-1	1	12	12	0	0
6	13	-7	49	17	10	7	49	13	13	0	0	18	19	-1	1
				21	19	2	4					14	11	3	9

*These subjects were deleted at random to give equal group size.

TABLE XXI

SUMMARY OF GROUP RESULTS

Group Treatment		I	II	III	IV	Total
Experimental	Σd			-69	65	5
	Σd^2			376	539	915
Control	Σd	-12	83			71
	Σd^2	290	547			837

$SS_{total} = [ABS] - [X] = 1752 - 55.54 = 1696.46$

$SS_{A} = [A] - [X] = 97.88 - 55.54 = 42.34$

$SS_{B.w.A.} = [AB] - [A] = 571.46 - 97.88 = 473.58$

$SS_{w.A.} = [ABS] - [AB] = 1752 - 571.46 - 1180.54$

$F_{ratio} = \frac{42.34}{237.79} = .178 \qquad F_{prob(1,2)} .178 = .221$

As the alpha level of the study was .05 and 1 - (F_{prob}) = .779, the null hypothesis was accepted. There was not significant difference between the experimental and the control groups on a measure of horizontal span.

The F_{ratio} was not significantly below one and therefore it was concluded that the assumptions of the F_{ratio} were not violated.

<u>Hypothesis IV</u>

The experimental group was significantly different from the control group on a measure of arithmetic computation. The test results may be found in Table XXII. The detailed calculations follow.

$\sum_{1}^{n} ABS = -44 + -36 = -80$

$\sum_{1}^{n} (ABS)^2 = [ABS] = 604 + 776 = 1380$

TABLE XXII

ARITHMETIC COMPUTATION TEST RESULTS

x_1	x_2	d	d^2	x_1	s_2	d	d^2	x_1	x_2	d	d^2	x_1	x_2	d	d^2
17	12	5	25	17	18	-1	1	14	18	-4	16	18	18	0	0
23	18	5	25	12	15	-3	9	6	10	-4	16	20	20	0	0
19	22	-3	9	21	18	3	9	3	6	-3	9	18	14	4	16
12	16	-4	16	23	17	6	36	12	21	-9	81	15	20	-5	25
9	16	-7	49	6	12	-6	36	12	11	1	1	22	20	2	4
12	18	-6	36	10	17	-7	49	15	6	9	81	18	16	2	4
18	17	1	1	21	20	1	1	14	17	-3	99	16	22	-6	36
18	23	-5	25	23	24	-1	1	5	10	-5	25	22	20	2	4
18	18	0	0	17	12	5	28	21	22	-1	1	22	19	3	9
13	19	-6	36	18	15	3	9	2	1	1	1	23	22	1	1
20	24	-4	16	22	21	1	1	18	18	0	0	17	21	-4	16
14	19	-5	25	15	15	0	0	10	17	-7	49	19	20	-1	1
18	19	-1	1	23	16	7	49	12	13	-1	1	22	24	-2	4
21	22	-1	1	23	22	1	1	11	17	-6	36	20	18	2	4
12	12	0	0	18	12	6	36	11	16	-5	25	23	19	4	16
13	10	3	9	23	19	4	16	17	14	3	9	24	23	1	1
18	20	-2	4	14	16	-2	4	13	14	-1	1	20	17	3	9
22	23	-1	1	22	23	-1	1	13	14	-1	1	16	15	1	1
16	23	-7	49	15	14	1	1	12	15	-3	9	20	21	-1	1
11	11	0	0	19	18	1	1	19	18	1	1	18	19	1	1
13	17	-4	16	24	21	3	9	14	8	6	36	22	22	0	0
12	22	-10	100	18	16	2	4	11	15	-4	16	18	18	0	0
6	55	1	1	17	20	-3	9	9	14	-5	25	23	20	3	9
20	21	-1	1	23	22	1	1	24	22	2	4	21	19	2	4
16	21	-5	25	22	22	0	0	11	18	-7	49	19	15	4	16
21	20	1	1	19	20	-1	1	21	21	0	0	21	23	-2	4
				21	20	1	1					19	20	-1	1

TABLE XXIII

SUMMARY OF GROUP RESULTS

Group Treatment		I	II	III	IV	Totals
Experimental	d			-56	-12	-44
	d^2			418	186	604
Control	d	-54	18			-36
	d^2	474	302			776

$$\frac{\left(\sum_{1}^{n} ABS\right)^2}{npq_{(i)}} = [X] = \frac{(-80)^2}{26x2x2} = \frac{6400}{104} = 61.54$$

$$\frac{\sum_{1}^{p}\left(\sum_{1}^{q} A\right)^2}{nq_{(i)}} = [A] = \frac{(-44)^2 + (-36)^2}{26 \times 3} = \frac{3232}{52} = 62.15$$

$$\frac{\sum_{1}^{p}\sum_{1}^{q}(AB)^2}{n} = [AB] = \frac{(-56)^2 + (12)^2 + (-54)^2 + (18)^2}{26}$$

$$= \frac{6520}{26} = 250.77$$

$$SS_{total} = [ABS[- [X] = 13.80 - 61.54 = 1318.46$$

$$SS_{A} = [A] - [X] = 62.15 - 61.54 = .61$$

$$SS_{B.w.A} = [AB] - [A] = 250.77 - 62.15 = 188.62$$

$$SS_{w.A} = [ABS] - [AB] = 1380 - 250.77 = 1129.23$$

$$F_{ratio} = \frac{.61}{94.31} = .00646 \qquad F_{prob(1,2)}\,.00646 = .087$$

The alpha level for this test was set at .05 and as 1 - (F_{prob}) = .913, the null hypothesis is accepted. There was no significant difference between the experimental and the control groups on a measure of computational ability.

As the F_{prob} was not significantly different from one the assumptions underlying the F_{ratio} were taken to be not violated.

APPENDIX IV

PILOT STUDY

There were two major reasons for conducting a pilot study. The author wished to establish which of the three methods of training, vertical span, horizontal span, or search training would be most likely to be significant in demonstrating that perceptual training is a factor in arithmetic computation. It was also important to standardize the method of perceptual test presentation.

An open area classroom was available in which there were approximately sixty grade five students enrolled. The class was randomly divided into three groups each of which were given pre- and post-tests on vertical span, horizontal span, search ability and the arithmetic sections of forms A and B of the Canadian Test of Basic Skills. Each of the three different training methods was assigned to one of the groups. Because of the expected differences in testing administration between pre- and post-test it was decided to do all the testing in one session. All three groups were scattered over the testing room and a note was made of where each student sat. The students were kept at the same seats for each testing session. Thus, although there were considerable differences between pre- and post-tests, these differences were kept constant for each of the training groups. For the final system of testing and the tests see Appendix II.

After a training period of three weeks it was found that fifteen students in each group had scores for all the test administrations. The summary of the differences between pre- and post-tests will be found in Table XXIV.

TABLE XXIV

SUMMARY OF PILOT STUDY RESULTS

Test Groups		Arith-metic$_1$	Arith-metic$_2$	Vertical	Hori-zontal	Search
Horizontal	x	18	12	-14	-49	40
training	x^2	1174	898	162	487	270
Vertical	x	1	10	-6	40	26
training	x^2	791	1460	84	400	246
Search	x	35	93	-24	-38	58
training	x^2	459	2654	238	400	372
F ratio		.28	1.63	.54	.16	3.23

A simple F_{ratio} was calculated for each of the tests. No levels of significance are reported because of the differences in the testing procedures between pre- and post-tests. However, the trend is clear. The table shows that highest ratios were for the search test and the arithmetic tests. Moreover, the search trained group had the greatest difference scores on all but the horizontal test. It was on this basis that the search training was chosen for the final study.