AN EVALUATION OF A COMPUTER-ADMINISTERED CHALLENGING TEACHING STRATEGY

by

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ABSTRACT

This study was motivated by the belief that teaching a student in a challenging way would increase his ability to apply what he had learned to new, though related, problems. A specific challenging teaching strategy was chosen, which attempted to challenge all students appropriately, and to give the minimum amount of help. It was administered by the computer, which considerably facilitated the use of such an individualised strategy.

The evaluation was done by comparing the effects of the challenging teaching strategy with those of a linear program, also computer-administered. A linear program was considered to exemplify an unchallenging approach. Both programs taught elementary base five arithmetic to Grade Six students, the students being assigned to the programs at random. The effects of the two strategies were then measured by means of a post-test. This aimed at evaluating both the grasp of the basic material and the ability to extrapolate from it to solve new problems in the same general subject area.

The results of the post-test showed that both strategies succeeded in teaching the basic material equally well, so that neither strategy gave the student an advantage in this respect. However, the challenged group of students showed far greater ability to extrapolate from the material than did the linear program group, with an average score over

45% better. This was significant at the .007 level.

These results suggest that further investigation of the merits and application of a challenging teaching strategy should be eminently worthwhile.

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CHAPTER I

THE PROBLEM

I. BACKGROUND

Today it is more important than ever before that education should not only provide the student with a solid base of information and skills, but should also make him capable of adapting what he has learned to new situations and new problems, as they arise in a rapidly changing world. Much of the information and many of the skills he learns during his schooldays will be obsolete long before the end of his working life, and his education should make it possible for him to cope with this. Hence any teaching strategy that fosters this ability has evident educational value.

Piaget's model of intellectual development provides considerable guidance in the design of such a strategy.[1] In the Piagetian model, the ability to adapt to a new situation depends on the complexity of the intellectual structure that has already been developed. The development of this intellectual structure is stimulated by demands being made upon it; such development is consolidated by practice in

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[1]John H. Flavell, The Developmental Psychology of Jean Piaget (New York: Van Nostrand Reinhold, 1963)

the use of the skills it makes possible. Simon's information-processing model of problem-solving leads to similar conclusions.[2]

It follows, then, that a teaching strategy which facilitates a student's adaptation of what he has learned to new situations needs to have two principal characteristics. First of all, it must make demands on a student's abilities and must make him think for himself; it should not give the solution to any difficulty that may arise before the difficulty has arisen. The demands must be sufficiently taxing as to require some thought on his part, but must not be so extreme as to be impossible for him to meet. The appropriateness of the demands made on each individual student is crucial. Secondly, such a teaching strategy must consolidate the progress that a student has made, by giving him practice in the skills he has just learned. Without such reinforcement, the development that has occurred may be only temporary. An approach with these two characteristics will be called a challenging teaching strategy.

The essentially tutorial nature of this approach makes its implementation in the classroom extremely difficult. However, computer-assisted instruction does make such an individualised approach feasible, and as it is a method whose

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[2]Herbert A. Simon, The Sciences of the Artificial (Cambridge,Mass: M.I.T. Press, 1969)

costs should eventually come within the range of the educational budget, its use has practical significance. It is also a much easier way of investigating the merits of teaching strategies than is trying them out in the classroom, as, unlike even the best of teachers, it is consistent in its treatment of different students. Furthermore, the evaluation of a teaching strategy is not bedevilled by imponderables such as teacher-student interactions. In fact, Stolurow argues that the principal use of computer-assisted instruction at the present time should be in educational research.[3] Hence the aim of this study is to use a challenging teaching strategy to teach a small amount of mathematics to a group of students, by means of computer-assisted instruction, and to see what effect the strategy has on their ability to adapt what they have learned to new problems in the same subject area.

Statement of the problem

Does teaching a student in a challenging way increase his ability to adapt what he has learned to new, though related problems ?

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[3]Lawrence M. Stolurow, "Some Factors in the Design of Systems for Computer-Assisted Instruction," *Computer-Assisted Instruction: A Book of Readings*, ed. Richard C. Atkinson and H. A. Wilson. (New York: Academic Press, 1969), p. 91.

II. REVIEW OF THE LITERATURE

Piaget describes intellectual development in terms of two interwoven processes, which he calls assimilation and accommodation.[4] Just as the ability that an organism has to assimilate food is governed by the digestive powers it possesses, so the assimilation of intellectual stimuli by a person is limited by the intellectual structure that has been developed. The assimilation of an intellectual stimulus, in other words its recognition and understanding, does not depend on whether the person has already assimilated one exactly like it on a previous occasion, but on whether the stimulus is sufficiently close to earlier ones successfully assimilated by the person. The intellectual structure is capable of adapting, in a small way, to the special characteristics of a novel stimulus, and this adaptation Piaget calls accommodation. This accommodation corresponds to gradual modifications in the intellectual structure, and this is how intellectual development occurs. All such modifications need to be reinforced by frequent use, if the development is to be maintained.[5] Thus intellectual growth results from demands being made on the present capabilities of a student, provided that the challenge they represent is not beyond his powers; the growth is consolidated by practice

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[4] Flavell, _op_. _cit_., pp. 46-50, 237-249.

[5] _Ibid_., p. 57.

in its use.

Simon describes the cognitive powers a person possesses in a different way, drawing an analogy with the computer.[6] He discusses the problem in terms of short-term and long-term memory, the short-term memory being that part of the mind that is dealing with the immediate situation, whilst the long-term memory contains the accumulated stores of experience. The relationship between the two can be compared with that between the core of a computer and its file storage. The memory stores its knowledge in units which Simon calls 'chunks', and the current environmental situation causes the relevant chunks to be transferred from storage in long-term memory into the short-term memory.[7] Here they can be put to work; the process is rather like drawing on a library of computer programs. The limiting factors in this process are firstly, the small number of chunks, or programs, that the short-term memory can handle simultaneously, and secondly, the sophistication of the programs themselves. Program development only occurs when the need is felt for it, and so a person's ability to tackle a new problem successfully depends on the experience he has previously had. New programs will only be developed, or existing programs combined into more versatile ones, when the person finds it

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[6]Simon, op. cit., pp. 33-34. [7]Ibid., p. 34.

necessary. Hence the educator must provide a challenging environment in which the appropriate needs are felt, if any development is to occur.

Pask adopts a challenging approach in the development of his teaching-machine programs.[8] His is a cybernetician's point of view, and to him the relationship between teacher and taught is one in which a teaching-machine and student are coupled to form a single system. At the beginning of the learning process, the material to be taught is contained entirely within the machine; the instructional aim is to transfer this from the machine to the student in as efficient a way as possible. The machine continually attempts to challenge the student, in what Pask describes as an 'intellectual donkey and carrot race'.[9] By giving him hints and help where necessary, it functions as an extension of the student's brain, enabling him to perform tasks that he would not otherwise be capable of performing. The emphasis is on challenging the student and keeping him working at his optimum level. The instruction is prepared by analysing the material taught into components; lack of mastery of any one of these will lead to errors in certain problems related to

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[8]Gordon Pask, "Theory and Practice of Adaptive Teaching Systems," _Teaching Machines and Programmed Learning II_, ed. Robert Glaser. (National Education Association of the United States, 1965) pp. 213-266.

[9]_Ibid_., pp. 231-2.

the topic. These components Pask refers to as error factors.[10] Once the error factors in the material have been identified, it is possible to design questions for the machine to ask the student, the answers to which will demonstrate whether or not the student is in need of help with specific error factors. Thus, as teaching proceeds, the machine can keep track of the student's mastery of the material, and can correct such error factors as need correction. The system has the additional capability of having more sophisticated information at its disposal, in the form of interrelationships that may weel exist between the error factors. These can be investigated in an initial stage of program development, and such relevant facts as 'removing error factor 2 helps to remove error factor 5, but not vice versa' can be taken into account in the system's final decision structure.[11] It would also be possible to refine and update the decision structure as necessary, using information gained from students learning the material in this way.[12]

Smallwood's work on appropriate decision structures for teaching machines suggests how a challenging teaching strategy might be refined in the light of experience with

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[10]Pask, op. cit., p. 226.

[11]Ibid., p. 228. [12]Ibid., p. 229.

students.[13] His study was essentially an exploratory one, with the principal aim of investigating the feasibility of a particular approach to the problem. He drew an analogy with the way in which a human tutor might behave. Initially, when the tutor is confronted with a new student, he will adopt a definite approach, which may or may not suit the student involved. The tutor will search for ways of explaining things, until he meets with success. As he gains experience with teaching this subject matter to successive students, he becomes more and more efficient at finding the most appropriate approach as quickly as possible. Smallwood's system is designed to learn in the same kind of way, by experience with students. He summarises the basic configuration of his system in the following way:

1. The decomposition of the subject matter into a set of concepts that the educator would like to teach to the student.
2. A set of test questions for each concept, that adequately tests the student's understanding of the concept.
3. An array of information blocks for each concept that can be presented to the student in some order - to be determined by the teaching machine - and thus provide a course of instruction to the student on the concept.
4. A model that can be used to estimate the probability that a given student with a particular past history will respond to a given block or test question with a particular answer.
5. A decision criterion upon which to base the decisions mentioned in 3.[14]

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[13] Richard D. Smallwood, A Decision Structure for Teaching Machines, (Cambridge, Mass: M.I.T. Press, 1962).

[14] Ibid., p. 27.

He took a short topic, a miniature geometry, and used his program to teach it to twenty M.I.T. students. He discusses several possible models for estimating the probability of success at any point in the program, and explains the rationale for the one he finally chose to use. The effect of the procedure was to teach each student as fast as possible, subject to the restriction that his expected number of errors be below an arbitrary maximum. With a sample of only twenty students, he was limited in the conclusions he was able to draw. For the first five students the program used a set of a priori probabilities for its decisions, corresponding to the initial approach decided upon by the tutor. For the sixth and subsequent students the experience with earlier students was taken into account. The machine was using its experience just as a tutor would do. Smallwood was able to show that, even with such a small number of students, the a priori probabilities were modified. Thus the structure he had devised was capable of adaptation in the light of experience. His principal aim was to demonstrate this, and, as he points out, more questions were raised than were answered.[15] The most important question is whether or not an approach such as this is worthwhile; will a structure of this kind really tend to an optimum teaching strategy or not.[16] The changes that were made in the decision

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[15]Smallwood, op. cit., p. 2. [16]Ibid., pp. 106-7.

criteria were not necessarily changes for the better, let alone a foolproof procedure for arriving at the best of all possible systems.[17]

Stolurow lays particular emphasis on the use of the computer for educational research, stressing the usefulness of the replicability it permits.[18] Using a computer, it is possible to evaluate alternative instructional strategies, with the ultimate aim of developing a meaningful and useful theory of teaching. A teaching system with which he has been closely associated is the SOCRATES system, at the University of Illinois. This system has three levels, only two of which are relevant here.[19] They are:

1. Pretutorial: at this level the system has to decide how to initiate the teaching process, given certain information about the student. The problem is to decide just what information might be relevant; aptitude scores, personality test scores, reading rate, and knowledge of prerequisite material are all possibilities. Neither Pask nor Smallwood attempts to tackle this problem. Both of them start all students in the same way, and then adapt to their subsequent needs.

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[17]Smallwood, op. cit., p. 103.

[18]Stolurow, op. cit., pp. 65-93. [19]Ibid., pp. 72-73.

2. Tutorial: at this level the problems are the same as those of Pask and Smallwood, to both of whom Stolurow refers. The system reacts to the responses the student makes, and attempts to provide the student with the most appropriate instruction.

The third level is the administrative one.

The problem that Stolurow is attempting to solve is that of finding the best way of using all the information available about a student in order to optimise on the teaching strategy used with him. He concludes this article by suggesting that the main contribution of computer-assisted instruction is to enable us to investigate these very problems, and to make our understanding of the conditions for learning more precise.

CHAPTER II

DESIGN OF THE STUDY

I. INTRODUCTION

The rationale for using a challenging teaching strategy was that it would generate greater ability to solve new problems in the same subject area than an unchallenging strategy would. Such problems will be called extrapolation questions. Therefore the validity of the argument can be tested by comparing the ability to solve extrapolation questions shown by two groups of students, one taught the material by means of a challenging teaching strategy, and the other by means of an unchallenging one.

The unchallenging strategy chosen was a linear program. Of necessity, such programs lead students step by step through the material, at the difficulty level of the least able among them. They must cater for all possible errors and misconceptions, though hardly any student will need all the help provided.

The subject matter chosen was half an hour to an hour's instruction in elementary base five arithmetic, and two computer programs were written to teach the material. One used a challenging strategy and the other taught by means of a linear program. 29 Grade Six students were brought to the university, to work through one or other of these programs, at a teletypewriter connected to the U.B.C. 360/67. The students were assigned to the programs at random, using a table of random numbers. Immediately after completing his program each student wrote a post-test. Some of the post-test questions covered the specific material taught, and these will be called straightforward questions. The other questions were extrapolation questions.

The entire process took from one hour to one and a half hours for each student, so that it was possible for three students to complete both program and test in the morning, and for two students to complete them in the afternoon.

For each student there were two post-test scores, one obtained on the straightforward items, and the other obtained on the extrapolation items. The mean scores of the challenged group and the linear program group were compared, using two-sample t-tests.

II. DEFINITION OF TERMS

(a) challenging teaching strategy: a teaching strategy that attempts to make demands on a student, and to make him think for himself, giving hints and help only when necessary. It seeks to consolidate the student's progress by giving him practice in the skills he has just acquired.

(b) <u>reinforcing questions</u>: those questions that the student is asked during the teaching of the material, when he has responded successfully to a new challenge. They give him practice in the new skills he has just worked out for himself.

(c) <u>straightforward questions</u>: those post-test questions which measure grasp of the specific skills taught - all these questions are similar to ones that students have been taught how to solve, and have had practice in solving.

(d) <u>extrapolation questions</u>: those post-test questions which involve the skills the student has been taught, but which are different from any he has solved hitherto. They require him to use what he has learned in a new way.

III. FORMATION OF THE GROUPS

The population chosen was Grade Six students, from Vancouver schools. Grade Six students could be expected to have sufficient background for elementary base five arithmetic. However, they would not normally have encountered it, as number bases are usually taught in Grade Seven. This was certainly true of all the schools involved, both in the main study and in the pilot stages. Due to transportation and administrative difficulties, the sample ultimately had to be confined to all the Grade Six students from a single school.

The students were assigned to the two programs at random, using a table of random numbers, so that the groups could be assumed to be of equivalent ability. The only restriction was that the proportion of boys and girls in each group should be approximately equal. There were nine girls and six boys in the challenged group, and eight girls and six boys in the linear program group.

IV. DEVELOPMENT OF MATERIALS

Content

The specific topics covered by both the challenging and the linear program were:

1. Quick review of place value in base ten.
2. Given a set of objects thus * * * .... * * * , how to express the number of objects in different bases. All bases are less than ten, and no numeral has more than two digits.
3. Discussion of the symbols required in base five, and base five counting.
4. Development of base five addition facts.
5. The use of a base five addition table.
6. Addition of two two-digit base five numerals, with no carrying required.
7. Addition of two two-digit base five numerals, with carrying required.
8. Multiplication by two of two-digit base five numerals.

The challenging teaching strategy

The content taught consists of eight sections, and the challenging teaching strategy was used on all but the first and fifth sections. Both these sections were taught to the students in the challenged group in the same way as they were taught to the students in the linear program group. The first section was taught in this way because it was review, and contained no new material. Teaching it in this way had the added advantage of getting the student used to the teletypewriter without his having to solve challenging questions at the same time. The fifth section was taught in this way as it involved a purely technical skill.

For all the sections taught using the challenging teaching strategy, the following material was prepared:

An initial challenge: This was for all students. It presented a minimal amount of information and a question on the content of the section, so that if a student was capable of working it out for himself he had the chance to do so.

A hint: This was only for the students who failed to respond correctly to the initial challenge. It consisted of some additional information and a question intended to guide the student along the right lines.

A second challenge: This was for those students who had failed with the initial challenge, but had successfully answered the 'hint' question. It was similar to the initial challenge, but with different numbers.

Reinforcing questions: These were questions for those students who had responded correctly to either the initial or the second challenge. They were similar to the challenging questions, but with different numbers.

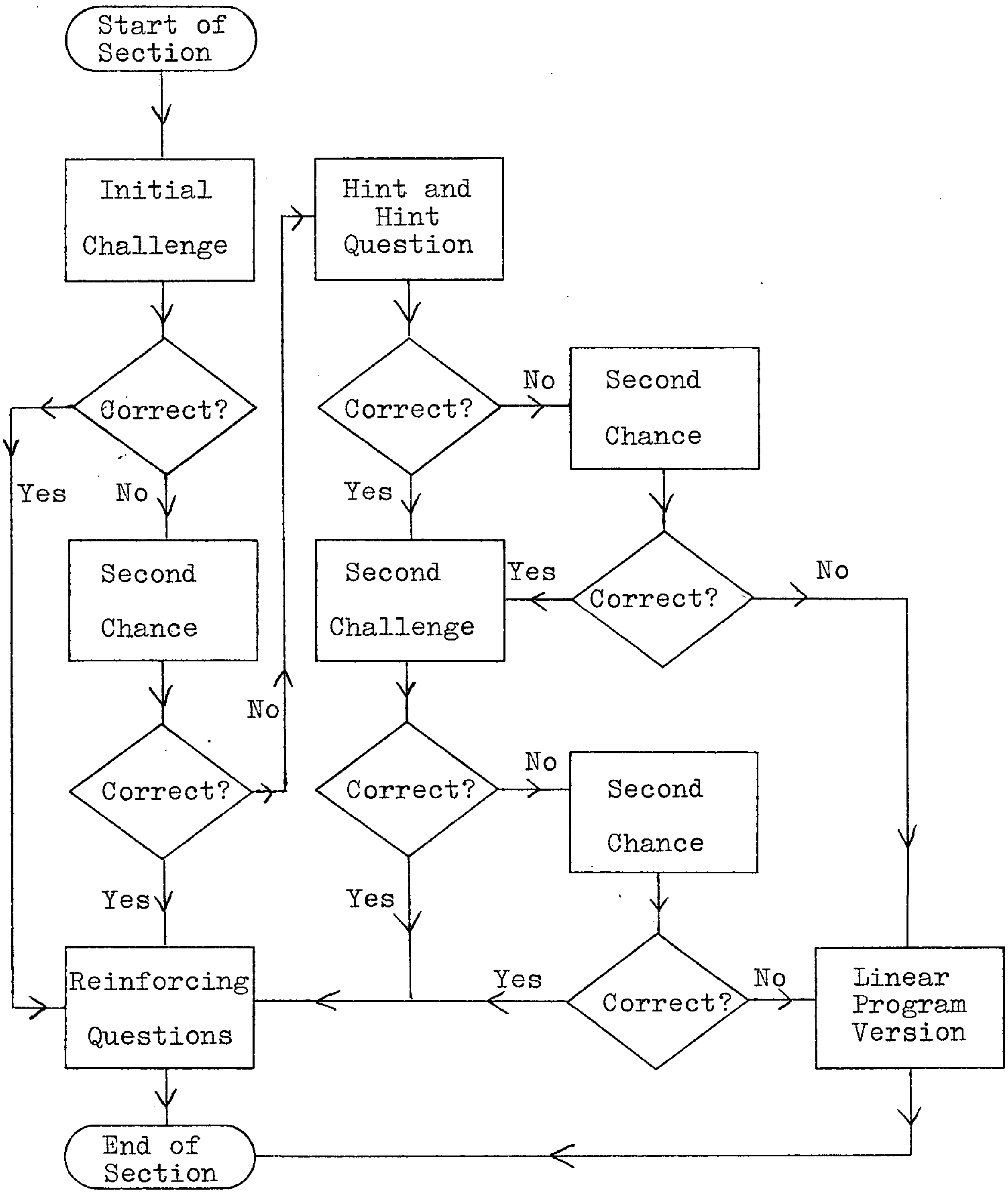
A linear program version of the material: This was for those students who failed to give the correct answer to both the initial challenge and the 'hint' question, or who answered the 'hint' question correctly but could not so answer the second challenge. It was the same version of the material as was taught to all students in the linear program group.

The strategy for each section can be summarised thus. Present all students with an initial challenge, and give the successful students some reinforcing questions before going on to the next section.Give those students that are not successful with the initial challenge some help, and a question to set them thinking along the correct lines. If a student cannot answer this question correctly, refer him to the linear program version of the material for the section. Give those students that do respond correctly to the hint question a second challenge. Present the successful students with some reinforcing questions before going on to the next section, and the unsuccessful ones with the linear program version of the material.

The student was given a second chance with all questions he was asked, except when he was referred to the

FIGURE 1.

FLOW-CHART OF THE CHALLENGING STRATEGY



linear program; this has been shown to lead to success for a large number of students, without any additional help.[1] Each student's path through the material was recorded, as was the time taken for him to complete the program.

The challenging strategy is illustrated in flow-chart form in Figure 1. Details of the initial challenge, hint, second challenge, reinforcing questions, and the linear program version of the material, for each section, can be found in Appendix B. Tables V and VI in Appendix D contain the time each student took to complete the program and the paths of the students through the material, respectively.

The linear program

This program was tested, in book form, in a pilot study. First of all, four Grade Six students, whose mathematical ability was below average, worked through the program, one after the other. Each student was told to call attention to anything he did not understand. In this way ambiguities were clarified, and over-difficult frames simplified. After each student had finished, the modifications suggested by his experience with the program were made before the next student began. By the time the third and fourth students worked through the program, no

[1] John J. Schurdak, "An Approach to the Use of Computers in the Instructional Process, and an Evaluation," American Educational Research Journal, 4: 72, 1967.

further alterations were necessary. The program was then tested on 137 Grade Six students, 80 from Coquitlam schools and 57 from a Vancouver school. These students also wrote the post-test, immediately on completing the program. The results of this test demonstrated that the program was teaching the material satisfactorily. The entire program can be found in Appendix A.

<u>The post-test</u>

This was validated and item-analysed in conjunction with the pilot test of the linear program. A reliability coefficient (Kuder-Richardson Formula 20) of 0.92 was obtained. The test contained 12 straightforward questions and 30 extrapolation ones.

The straightforward questions tested the specific content taught. The extrapolation questions fall into the following categories:

1. Addition in base five of
   (a) three two-digit numerals
   (b) two three-digit numerals
2. Multiplication in base five of
   (a) three-digit numerals by two
   (b) two-digit numerals by numbers greater than two
3. Counting
   (a) in base five beyond 30
   (b) in base four

4. Using a base eight addition table to
   (a) add two two-digit numerals
   (b) multiply two-digit numerals by two
5. The symbols that are used in base six.
6. Deduction of the base being used.
7. Conversion from one base to another.
8. Development of a base four addition table.
9. Subtraction in base five.
10. Development of a base five multiplication table.

The complete post-test can be found in Appendix C.

V. STATISTICAL ANALYSIS

Data

For each student two sub-scores on the post-test were obtained. The first was his score on the 12 straightforward questions and the second was his score on the 30 extrapolation questions. The time that each student took to complete his program was recorded, as was the path taken through the material by each student in the challenged group. All this data can be found in Appendix D.

Statement of hypotheses

Two things were expected to happen, namely:

1. Both teaching strategies would result in the same mastery of the specific skills taught, so that both groups would solve the straightforward questions equally well.

2. The challenged group would be much better at solving extrapolation questions than the linear program group.

Stated as null hypotheses, these were:

1. There is no significant difference in the ability to solve straightforward questions between the challenged group and the linear program group.
2. There is no significant difference in the ability to solve extrapolation questions between the two groups.

It was expected that the first hypothesis would be accepted and the second one rejected.

Statistical treatment of the data

Both the straightforward sub-scores and the extrapolation sub-scores were analysed in the same way. The following statistics were calculated in both cases:

	Linear Program Group	Challenged Group
Mean Score	$\overline{X}_1$	$\overline{X}_2$
Variance	s_1^2	s_2^2
Number in Group	N_1	N_2

The two-sample t-value was then computed thus:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\left[\left(\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}\right)\left(\frac{1}{N_1} + \frac{1}{N_2}\right)\right]^{\frac{1}{2}}}$$

and compared with the tabulated value for $N_1 + N_2 - 2$ degrees of freedom.

The means of the straightforward scores were compared using a two-tailed test, as what was being tested was whether there was any difference between the groups. The means of the extrapolation scores were compared using a one-tailed test, as the issue here was whether the challenged group was better than the linear program group.

CHAPTER III

ANALYSIS OF THE RESULTS

I. TESTING OF HYPOTHESES

The straightforward scores

The hypothesis concerning the straightforward scores was that there would be no statistically significant difference between the linear program group and the challenged group. The table below summarises the results obtained on the twelve straightforward questions. Each question was worth one mark, so the maximum score possible was twelve.

	Linear Program Group	Challenged Group
Mean Score	9.93	10.67
Number in Group	14	15

The t-value obtained was 1.35, which, with 27 degrees of freedom and a two-tailed test, is significant only at the .186 level. It is reasonable to conclude that there was no statistically significant difference between the groups as regards performance on the straightforward questions. This confirms the hypothesis.

The extrapolation scores

In the case of the extrapolation scores, it was expected that the null hypothesis of no significant difference between the groups would be rejected, and that the challenged group would obtain appreciably higher scores than the linear program group. The table below summarises the results obtained on the thirty extrapolation questions. Each question was worth one mark, except for two questions which were worth two marks each, so the maximum score possible was thirty-two.

	Linear Program Group	Challenged Group
Mean Score	16.07	23.33
Number in Group	14	15

The t-value obtained was 2.62, which, with 27 degrees of freedom and a one-tailed test, is significant at the .007 level. Thus the challenged group performed significantly better on the extrapolation questions, as was expected.

II. CONCLUSIONS

There was no statistically significant difference between the linear program group and the challenged group as regards their performance on the straightforward questions. That both groups averaged scores of over 80% on these questions showed that the basic subject matter had been successfully taught; that the groups were so close together demonstrated that both the linear program and the challenging program had done their teaching equally well. Evidently, teaching these students this material in a challenging way was neither a handicap nor a help to them vis-a-vis mastery of the specific subject matter taught.

While the linear program group averaged a score of 16.07 on the extrapolation questions, the challenged group averaged 23.33, more than 45% better. Evidently, teaching these students this material in a challenging way was a considerable advantage to them as regards solving the extrapolation questions.

III. ANALYSIS OF ADDITIONAL DATA

The time taken to complete the programs

The students in the challenged group completed the program more quickly than did the students in the linear program group, as can be seen from Table V in Appendix D. The average time for the challenged group was 36.9 minutes, while the linear program group averaged 60.7 minutes.

The paths taken through the challenging program

Table VI in Appendix D summarises the routes taken by the challenged group. The following points are of interest:

1. In each section some students were able to respond correctly to the initial challenging question, indicating

that the challenges were not too difficult.

2. In all but the fourth section some students were able to respond correctly to the second challenging question, after having been given a hint, so that the hints appear to have been relevant and helpful.
3. In each section at least one student needed the linear program version of the material, showing that the challenging questions really were challenges, and were not so easy that every student could do them immediately.
4. All students responded successfully to at least one initial challenging question, demonstrating that each student found something he could work out for himself, even though he might have needed considerable help with other parts of the material.
5. The students needed help at different points, so that no single presentation of the material would have been appropriate for all students. Each individual obtained his own presentation of the material, and this varied considerably from student to student.

CHAPTER IV

IMPLICATIONS OF THE STUDY

I. INTRODUCTION

The extrapolation questions were intended to test a student's ability to extend the specific material he had learned to solving new problems in the same general subject area. If it is accepted that the questions did indeed perform this function, then the considerable difference between the challenged group and the linear program group provides strong support for the general hypothesis discussed in Chapter I. Teaching a student in a challenging way makes him better at adapting his knowledge to new situations and new problems than does an unchallenging teaching strategy.

In addition, it is reasonable to claim that this study has demonstrated the feasibility of using a computer to administer a challenging teaching strategy of the kind chosen.

Nevertheless, one swallow does not make a summer, and one study alone does not confirm a theory. Clearly, therefore, there is a need for further studies confirming the results obtained, and extending them beyond the particular circumstances of this study.

II. THE NEED FOR PARALLEL STUDIES

This study had some unavoidable limitations, and all of these suggest interesting parallel studies.

A wider sample

As was mentioned in Chapter II, the students involved all came from a single school. This school generally has students of above average ability, so that the participants were probably not a representative sample of the Grade Six population. It would be very interesting to see if similar results were obtained with less able students. It is quite possible that a challenging teaching strategy is less suitable for such students, especially in the form used in the study.

Removal of Hawthorne effects

There was only one teletypewriter available for this study, which meant that the students had to work through their programs in succession, instead of simultaneously. Consequently there is a real possibility that some of the later students heard about what they were going to do from their predecessors, since all of them were from the same school. This could have affected their performances on the questions, and hence the results obtained. Obviously, then, there is a need for further studies that circumvent this problem, either by using a large number of teletypewriters, or by using students from different schools, some distance apart.

Teaching of other material

The topic chosen for this study was base five arithmetic. The choice of this material made it possible to formulate all the questions asked, in the course of either program, in such a way that the answers were always numeric, never verbal. In the absence of a language specifically designed for computer-assisted instruction, numeric answers simplified the programming considerably.

There is no reason to suppose that there was any interaction between the choice of topic and the challenging teaching strategy. There would appear to be nothing about base five arithmetic that would make teaching it in this way singularly appropriate. However, obtaining similar results from parallel studies, using different material, would enhance the significance of the results.

III. FURTHER DEVELOPMENT OF THE STRATEGY

Although this study demonstrated some of the flexibility of the computer, it did not take full advantage of its potential. The challenging strategy adapted to the responses of the individual student within the confines of each section of the material, but it did not take his performance in earlier sections into account. Nor were the decision rules of the strategy susceptible to change; for example, the initial challenge was always presented at the beginning of a section, regardless of whether a student had failed with all previous initial challenges, and a second chance was always given, though never a third. The strategy did not modify itself in the light of its accumulated experience with successive students.

Such a teaching strategy can be considered to be in the first stage of development, characterised by two principal properties:

1. A student's performance in earlier sections does not affect the presentation of the material he receives in later sections.
2. The decision structure is immutable; the teaching strategy does not learn from its experience what approach is likely to succeed, and what is not, and so never changes.

These properties suggest two further stages of development which could be undertaken.

The second stage of development would make the strategy more flexible by removing the restriction imposed by the first property. The information on the basis of which decisions are made about the most appropriate next step for a given student, would be extended to include his responses to the questions of earlier sections, as well as his performance in the current section. The decision structure would still be fixed, as, for a given set of responses, the most appropriate next step decided upon would be the same for the hundredth student as for the first. However, it would be making use of considerably more information. It would be adapting to the individual student to a far greater extent than before. By keeping track of his difficulties, the teaching strategy would be able to offer the most appropriate challenges in later sections, and could also cause a student to repeat a section, if his inadequate grasp of it was resulting in an inability to perform successfully in other sections.

The main difficulty in implementing the second stage of development would be determining just what use should be made of the additional information available. Pask's concept of error factors, and his ideas about finding relationships between them, as discussed in Chapter I, could be very useful in this context. By recording the paths of students through the material, when taught by the strategy in its first stage, relationships between performance at different points in the material could be investigated. For instance, it might be found that failure to respond correctly to the initial challenge in Section 2 meant that there was a 95% chance of failing with the initial challenge in Section 4. The teacher-programmer would then have to make a qualitative judgement as to how to incorporate this relationship in the teaching strategy; whether the decision structure should never offer the initial challenge in Section 4 to a student who failed with the initial challenge of Section 2, whether it should offer it to such students as meet certain other similar requirements, or whether it should always be offered. Whatever the teacher-programmer decided to do in such circumstances, each student with the same response pattern would get identical treatment, even though, in practice, the decision turned out to be unsuccessful.

The third stage of development of the teaching strategy would be one in which the decision structure was no longer fixed, so removing the restriction imposed by the second property of the first stage. No longer would unsuccessful decision rules remain inviolate; once shown to be mistaken, they would be changed. In other words, the strategy would learn from its experience.

Unsuccessful rules could be eliminated manually from time to time, on the basis of student records, but the ultimate aim would be to design a strategy that learnt continuously from its experience. Such a strategy would test the effectiveness of alternative decision rules by trying them out, in much the same way that a good teacher would. Smallwood's system, as discussed in Chapter I, was a very simple version of such an approach.

The principal problem with this would be that, just as in the case of the good human teacher the strategy is seeking to emulate, a great deal of experience is required before a really satisfactory strategy can be evolved. As a result, this approach will only be really feasible when computer-assisted instruction is used extensively, and on a regular basis.

IV. SUMMARY

This study has shown that a challenging teaching strategy merits further investigation. Suggestions have been made for parallel studies, whose success would reinforce the conclusions drawn here, and for development of the teaching strategy until it was capable of systematically replicating many of the important characteristics of a good teacher. The writer considers that there is tremendous scope for useful research in this area, and that the educational benefits accruing from it would be well worth the time and effort expended.

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APPENDIX A

THE LINEAR PROGRAM

FRAME NUMBER	FRAME	CORRECT ANSWER
1.	In the number 26, which of these does the 2 mean ? 2 20 200 2000	20
2.	In 26 the 2 means 20. What does the 6 mean ?	6
3.	So 26 = 20 + 6 which we can write as 26 = 2 x 10 + 6 So the 2 in 26 tells you there are 2 ....'s.	10
4.	Look at the number 84. 84 = 8 x .... + 4	10
5.	So 84 = 8 x 10 + 4 You can see how important 10 is in our counting system. Our counting system is based on it, and we say that we count in base 10. How many fingers have you got (including thumbs) ?	10
6.	A lot of people count on their fingers. That probably explains why we count in base	10

FRAME NUMBER	FRAME	CORRECT ANSWER
7.	Before someone thought of number bases they had to write numbers like this. ////////////////////////// Would this take longer than our usual way of writing numbers ? 1. yes 2. no Type 1 or 2, whichever answer you think is correct.	1
8.	Then a clever person invented a short code to save a lot of time. He decided to count in tens and see how many groups of ten he could make. Here is that number again. ////////////////////////// How many groups of ten could he make ?	2
9.	How many would be left over ?	6
10.	Because he could make 2 groups of ten and then had 6 left over his code for the number was 26. In his code the 2 in 26 means 2 x	10

FRAME NUMBER	FRAME	CORRECT ANSWER
11.	Everybody who knew his code knew that when he wrote 57 the 5 meant 5 x	10
12.	Of course, if you didn't know his code you wouldn't know what he was talking about. You might be used to a different code. Suppose, instead of counting in tens, you decided to use a code based on eight. Then you would make as many groups of ..... as you could.	8
13.	Here is that number again. //////////////////////////// How many groups of eight can you make ?	3
14.	How many are left over ?	2
15.	Because you could make 3 groups of 8 and there were 2 left over, you would write 32 in this code. Say this to yourself as three-two. Don't say thirty-two because it does not mean that. In this code the 3 in 32 means 3 ....'s.	8
16.	When you use base 8 code you make as many groups of .... as you can.	8

FRAME NUMBER	FRAME	CORRECT ANSWER
17.	Here is another number. ////////////////////// What is it in base 8 code ?	26
18.	The base 8 code number 26 tells you that there were 2 groups of	8
19.	If we were using base five code we would make as many groups of .... as we could.	5
20.	Write this number in base 5 code. /////////////////////	41
21.	Write this number in base 5 code. ////////////////////////	44
22.	Instead of talking about base 5 code we will just say base 5 from now on. If 42 means 4 x 5 + 2 we are using base	5
23.	If 42 means 4 x 10 + 2 we are using base	10
24.	If 32 means 3 x 8 + 2 we are counting in base	8

FRAME NUMBER	FRAME	CORRECT ANSWER
25.	If we are counting in base 5, 43 means 4 x .... + 3	5
26.	If we are counting in base 8, 43 means 4 x .... + 3	8
27.	Do 43 in base 5 and 43 in base 8 mean the same ? 1. yes 2. no Type 1 or 2, whichever answer you think is correct.	2
28.	So in order to tell the difference between the two 43's we need to know what bases are being used. Suppose you were watching someone counting some things, and to help himself he was arranging them like this. * * * * * * * * * * * * * * * * * * * * * * * What number base would you guess he was using ?	10

FRAME NUMBER	FRAME	CORRECT ANSWER
29.	Base 10 would be the obvious one to assume because he has arranged as many of them as possible in groups of	10
30.	How many complete groups of ten are there in question 28 ?	2
31.	How many are left over ?	3
32.	So the number of things is 2 x 10 + 3 which is written in base 10 as	23
33.	Here are the same things, arranged differently. * * * * * * * * * * * * * * * * * * * * * * * What base do you think this person is using ?	6
34.	It looks as though he is using base 6 because he has arranged as many of them as possible in groups of	6
35.	How many are left over ?	5

FRAME NUMBER	FRAME	CORRECT ANSWER
36.	How many complete groups of 6 are there ?	3
37.	The number of things is 3 x 6 + 5 , so that a base six person would write: there are ..... things.	35
38.	In 35 in base 6 the 3 means 3 ....'s.	6
39.	Here are the same stars, not arranged. * Suppose you were used to counting in base 5. The first thing you would do would be to make as many complete groups of .... as you could.	5
40.	To help you count, mark off the stars in fives, like this. * * * * */* * * * */* and so on. With the stars in question 39, how many complete groups of five can you make ?	4
41.	How many are left over ?	3

FRAME NUMBER	FRAME	CORRECT ANSWER
42.	So a base five person might arrange the stars like this: * * * * * * * * * * * * * * * * * * * * * * * in 4 groups of 5 with 3 left over. He would write: there are .... stars.	43
43.	In base 5 the 4 in 43 means 4 ....'s.	5
44.	Here is another collection of stars. * * * * * * * * * * * * * * * A base 7 person would write: there are .... stars.	21
45.	The 2 in 21 in base 7 tells you that you were able to make 2 groups of	7
46.	Here are the same stars. * * * * * * * * * * * * * * * Write the number of stars in base 8.	17
47.	It is 17 because when you have made 1 group of 8 there are .... left over.	7

FRAME NUMBER	FRAME	CORRECT ANSWER
48.	So 17 in base 8 and 21 in base 7 both mean the same thing, namely the number of stars. Do 17 and 21 normally mean the same thing ? 1. yes 2. no	2
49.	So it is important to know what base is being used. Now we will work in base 5 for a while and see how this changes our arithmetic. When we count in base 5 we make as many groups of .... as we can.	5
50.	Here are some more stars. * * * * * * * * * * * * * * What is the number of stars in base 5 ?	24
51.	Counting can be illustrated this way. * * * * * * * * * * * * * * * You can write each of these as base 5 numbers. What is * * * in base 5 ?	3
52.	What is * * * * * in base 5 ?	10
53.	What is * in base 5 ?	1

FRAME NUMBER | FRAME | CORRECT ANSWER

54. So our counting so far is

*   * *   * * *   * * * *   * * * * *

1   3   10

There are two spaces here.

What goes in the first one ?   2

55. What goes in the second space ?   4

56. So we have

* 1

* * 2

* * * 3

* * * * 4

* * * * * 10

* * * * * *

* * * * * * *

* * * * * * * *

* * * * * * * * *

* * * * * * * * * *

Let's fill in some of these spaces.

The first space is opposite * * * * * *

What is the base 5 number for this ?   11

57. The third space is opposite * * * * * * * *

What is the base 5 number for this ?   13

FRAME NUMBER	FRAME	CORRECT ANSWER
58.	The fifth space is opposite * * * * * * * * * * What is the base 5 number for this ?	20
59.	So our counting is 1 2 3 4 10 11 .. 13 .. 20 What comes after the 11 ?	12
60.	What comes after the 13 ?	14
61.	So base 5 counting looks like this. 1 2 3 4 10 11 12 13 14 20 ... When we count in base 5 do we use the symbol 6 ? 1. yes 2. no	2
62.	When we count in base 5 do we use the symbol 5 ? 1. yes 2. no	2
63.	So when we count in base 5 we only use the symbols 0,1,2,3, and 4. In base 5 we only use those symbols that are less than	5

FRAME NUMBER	FRAME	CORRECT ANSWER
64.	Here is a base 5 question: 3 + 4	
	We can write this as	
	* * * + * * * * = * * * * * * *	
	3 + 4 = ...	
	What is the answer ?	
	Remember, this is base 5.	12
65.	In base 5,	
	* * * * * * * = * * * * */* *	
	which is 12 .	
	So, in base 5, 3 + 4 = 12	
	Here is another question in base 5.	
	* * + * * * * = * * * * * *	
	2 + 4 = ...	
	What is the answer ?	11
66.	So, in base 5, 2 + 4 = 11	
	Try these base 5 questions.	
	Draw stars to help you if you like.	
	1 + 1 = ...	2
67.	1 + 2 = ...	3
68.	1 + 3 = ...	4

FRAME NUMBER	FRAME	CORRECT ANSWER
69.	Remember, you are using base 5.	
	1 + 4 = ...	10
70.	The answer to question 69 cannot be 5, since we don't use the symbol 5 in base 5 code.	
	Our counting went 1 2 3 4 10 11 ...	
	In stars the question 1 + 4 can be written	
	* + * * * * = * * * * *	
	1 + 4 = ...	
	What is * * * * * in base 5 ?	10
71.	So we have	
	1 + 1 = 2	
	1 + 2 = 3	
	1 + 3 = 4	
	1 + 4 = 10	
	Now try some more.	
	2 + 1 = ...	3
72.	2 + 2 = ...	4
73.	2 + 3 = ...	10

FRAME NUMBER	FRAME	CORRECT ANSWER
74.	2 + 4 = ...	11
75.	So we have 2 + 1 = 3 2 + 2 = 4 2 + 3 = 10 2 + 4 = 11 Now try these. Remember, all this is in base 5. 3 + 1 = ...	4
76.	3 + 2 = ...	10
77.	3 + 3 = ...	11
78.	4 + 1 = ...	10
79.	4 + 2 = ...	11
80.	4 + 4 = ...	13
81.	Now we can summarise these base 5 addition facts in a table. In front of you is a blue folder, and inside it is a base 5 addition table. Take it out, so you can use it.	

FRAME NUMBER	FRAME	CORRECT ANSWER
	To show how it works, you will find the answer to 2 + 3 Look down the left-hand column to 2, and put your finger there. Keep that finger where it is and look across the top row to 3 and put another finger there. Move the 2 finger across, and the 3 finger down, until they meet, which should be at 10 This tells you that 2 + 3 = 10 Now use the table to answer 2 + 4 = ...	11
82.	Use the table to answer 4 + 1 = ...	10
83.	Use the table to answer 3 + 3 = ...	11
84.	Use the table to answer 4 + 3 = ...	12
85.	Use the table to answer 1 + 3 = ...	4

FRAME NUMBER	FRAME	CORRECT ANSWER
86.	This table can help you to do harder addition problems in base 5. Use the table whenever you like. Look at this question. It is a base 5 question. 41 +32 The first thing to do is add 1 and 2. What is 1 + 2 in base 5 ?	3
87.	So the first step is 41 +32 3 Next we add 4 and 3. What is that in base 5 ?	12
88.	So the answer to the question is 41 +32 123 Is this the same answer as you would get in base 10 ? 1. yes 2. no	2

FRAME NUMBER	FRAME	CORRECT ANSWER
89.	The answer is different because numbers like 41 mean different things in base 5 and base 10. In base 5 the 4 in 41 means 4 ....'s.	5
90.	In base 10 the 4 in 41 means 4 ....'s.	10
91.	In base 8 the 4 in 41 would mean 4 ....'s.	8
92.	Here is another base 5 question. $\begin{array}{r} 32 \\ +22 \\ \hline \end{array}$ Which is the first thing to do ? 1. 3 + 2 2. 2 + 2	2
93.	You always add the right-hand column first. What is 2 + 2 in base 5 ? Don't forget, you can use the table whenever you like.	4
94.	So the question begins like this. $\begin{array}{r} 32 \\ +22 \\ \hline 4 \end{array}$	

FRAME NUMBER	FRAME	CORRECT ANSWER
	The next step is to add 3 and 2.	
	What is this in base 5 ?	10
95.	So the question is $\begin{array}{r} 32 \\ +22 \\ \hline 104 \end{array}$ Now try these base 5 questions. Use the table whenever you like.	
	$\begin{array}{r} 21 \\ +42 \\ \hline \end{array}$	113
96.	$\begin{array}{r} 40 \\ +34 \\ \hline \end{array}$	124
97.	$\begin{array}{r} 43 \\ +31 \\ \hline \end{array}$	124
98.	$\begin{array}{r} 32 \\ +32 \\ \hline \end{array}$	114
99.	$\begin{array}{r} 30 \\ +20 \\ \hline \end{array}$	100

FRAME NUMBER	FRAME	CORRECT ANSWER
100.	This one needs some care. 23 +14 The first thing to do is to add 3 and 4. What is that in base 5 ?	12
101.	So you have to write down 2 and carry 1. You have not had to do any carrying before in base 5, but it works just the same way as usual. So what is the answer to the question ? 23 +14	42
102.	Here are some more questions in which you will have to do some carrying. They are all base 5 questions. 13 +24	42
103.	14 +24	43
104.	14 +2	21

FRAME NUMBER	FRAME	CORRECT ANSWER
105.	23 +12	40
106.	13 +23	41
107.	Some of these questions involve carrying, and some do not. They are all base 5 questions. 20 +34	104
108.	12 +24	41
109.	23 +34	112
110.	43 +44	142
111.	Most of these answers are different from the ones you would usually get because this is base .... arithmetic.	5
112.	When we count in base 5 we arrange as many things as we can in groups of	5

FRAME NUMBER	FRAME	CORRECT ANSWER
113.	For instance, 34 in base 5 means that there were .... groups of 5 and 4 left over.	3
114.	Which of these is the same as 2 x 41 ? 1. 2 + 41 2. 41 x 41 3. 41 + 41	3
115.	So the multiplication question 41 <u>x2</u> and the addition question 41 <u>+41</u> mean the same and so will have the same answer. So you can find the answer to 41 <u>x2</u> by working out 41 <u>+41</u> What is the answer to this question ? Remember, base 5.	132

FRAME NUMBER	FRAME	CORRECT ANSWER
116.	Here is another base 5 multiplication question. 32 <u>x2</u> Write it as an addition question on a piece of paper if you like. You only have an addition table, not a multiplication table, so addition is probably easier for you. What is the answer to the question ?	114
117.	Try this one. 23 <u>x2</u>	101
118.	Try this one. 33 <u>x2</u>	121
119.	Try this one. 24 <u>x2</u>	103
120.	So now you know how to count in base 5 and how to do some addition and multiplication. This is the end of the lesson. Go and tell the teacher you have finished. Goodbye.	

APPENDIX B

THE CHALLENGING PROGRAM

The material covered by the programs has 8 sections. The students in the challenged group were taught in the following way:

Section 1: Linear program, frames 1-11.

Section 2: Challenging program.

Section 3: Challenging program.

Section 4: Challenging program.

Section 5: Linear program, frames 81-85.

Section 6: Challenging program.

Section 7: Challenging program.

Section 8: Challenging program.

Section 1 is a review of place-value in base ten. Section 5 is the section which teaches the use of a base 5 addition table. The challenging program functions on sections 2,3,4,6,7, and 8 of the content taught. Summarised below, for each of these sections, are:

1. The initial challenging question.
2. The hint that will be given, if necessary.
3. The second challenging question (similar to the first, but with different numbers).
4. Details of the reinforcing questions.
5. The linear program to which the student will be referred, if necessary.

<u>Section 2: Expressing a number of objects in different bases, all numerals being less than 3 digits long, and all bases being less than ten.</u>

1. <u>Initial challenge</u>: There is nothing special about ten. We are just used to using it as a base. Eight would have done just as well. Try writing the number of stars as a base eight number.

   * * * * * * * * * * * * * * * * * * *

2. <u>Hint</u>: When you count in base ten you make as many groups of ten as you can. So, when you count in base eight, you make as many groups of .... as you can.

3. <u>Second challenge</u>: Now have a try at this question. Write this number as a base eight number.

   * * * * * * * * * * * * *

4. <u>Reinforcing questions</u>: Writing numbers * * *   * * * in different bases, until 5 are correct, or 8 have been tried.

5. <u>Linear program</u>: Let's have a think about this. Here is a number. //////////////////////////

   How many groups of eight can you make ?

   Continue with linear program, frames 14-50.

Section 3: The symbols used in base 5, and base 5 counting.

1. Initial challenge: Counting can be illustrated this way.

   *   * *   * * *   * * * *   * * * * *   * * * * * *

   and in base ten we would write

   1   2   3   4   5   6

   to describe this. In base 5 counting we begin 1 2 3 4 ....

   What comes next ?

2. Hint: When you use base five numbers you arrange as many things as possible in groups of five. What is * * * * * as a base five number ?

3. Second challenge: Now we have 1 2 3 4 10 .... so far for our base five counting. What comes next ?

4. Reinforcing questions:

   (a) Here is some base five counting, with some gaps.

   1 2 3 4 10 11 12 .. 14 .. What goes in the first gap ?

   (b) What goes in the second gap ?

   (c) In base ten we use the symbols 0,1,2,3,4,5,6,7,8,9.

   What symbols do we use in base five ?

   1. 0,1,2,3,4,5,6,7,8,9

   2. 0,1,2,3,4,5

   3. 0,1,2,3,4

5. Linear program: Now you saw that counting could be illustrated with stars. Continue with linear program, frames 56-63.

Section 4: Base 5 addition of single digit numbers.

1. Initial challenge: Here is a base 5 sum. 3 + 4
   In base ten the answer would be 7.
   What is the answer in base 5 ?

2. Hint: You can write this sum as
   * * * + * * * *
   3 + 4
   and the answer in stars is * * * * * * *
   What is * * * * * * * as a base five number ?

3. Second challenge: So the answer to the base five sum
   3 + 4 is 12. What is the answer to this base five
   sum ? 2 + 3

4. Reinforcing questions: Similar questions until 5 are
   correct or 8 have been tried.

5. Linear program: Look at this sum 3 + 4 again.
   * * * + * * * * = * * * * * * *
   3 + 4 = ...
   What is the answer ? Remember, this is base five.
   Continue with linear program, frames 65-80.

<u>Section 6: Addition of two two-digit base five numerals, with no 'carrying' required.</u>

1. <u>Initial challenge</u>: Here is a base five addition sum. What is the answer ? You can use the table to help you.

   $$\begin{array}{r} 41 \\ \underline{+32} \end{array}$$

2. <u>Hint</u>: Perhaps you forgot that this was base five. In base five 3 + 4 is not 7, but .....

3. <u>Second challenge</u>: This makes the answer to the question 123, as you were told. Now try this one. Remember, base 5.

   $$\begin{array}{r} 21 \\ \underline{+33} \end{array}$$

4. <u>Reinforcing questions</u>: Similar questions until 3 are correct or 5 have been tried.

5. <u>Linear program</u>: Let's have a closer look at that earlier question.

   $$\begin{array}{r} 41 \\ \underline{+32} \end{array}$$

   The first thing to do is add 1 and 2.

   What is 1 + 2 in base five ?

   Continue with linear program, frames 87-99.

<u>Section 7: Addition of two two-digit numerals in base five, with 'carrying' required.</u>

1. <u>Initial challenge</u>: Be careful with this one.

    23
    <u>+14</u>

2. <u>Hint</u>: This was the first question in which you had some carrying to do. This one requires carrying too. See if you can do it.

    12
    <u>+14</u>

3. <u>Second challenge</u>: Now try this one.

    24
    <u>+14</u>

4. <u>Reinforcing questions</u>: Similar questions, some with 'carrying' and some without, until 5 are correct or 8 have been tried.

5. <u>Linear program</u>: Let's have another look at that earlier question.

    23
    <u>+14</u>

    The first thing to do is to add 3 and 4.

    What is that in base five ?

    Continue with linear program, frames 101-113.

Section 8: Multiplication by two of two-digit base five numerals.

1. Initial challenge: You have just done quite a lot of base five addition. Now here is a multiplication question, still in base five.

   41
   x2

2. Hint: There is an easy way for you to do these questions, by changing them to addition questions. Which of these means the same as 41 x 2 ?

   1. 41 x 41
   2. 41 + 41
   3. 41 + 2

3. Second challenge: So 41 x 2 and 41 + 41 mean the same and so have the same answer. Now try this one. Write it as an addition question on a piece of paper, if you like, and then do it.

   31
   x2

4. Reinforcing questions: Similar questions until 3 are correct or 5 have been tried.

5. Linear program: Let's have another think about 41 x 2 Continue with linear program, frames 115-120.

APPENDIX C

THE POST-TEST

All questions were worth 1 mark, except for Nos. 38 and 41, which were worth 2 marks each. Base five and base eight addition tables were provided. Included in this appendix is a complete copy of the test, and copies of the two addition tables provided.

The questions cover the following content:

<u>The 12 straightforward questions</u>

1. Recognition of the base being used. (No. 1)
2. Expression of a number of objects in different bases. (Nos. 2-6)
3. Base five counting. (No. 7)
4. Addition of two two-digit base five numerals. (Nos. 8-10)
5. Multiplication by two of two-digit base five numerals. (Nos. 11,12)

<u>The 30 extrapolation questions</u>

1. Addition in base five of three two-digit numerals, and of two three-digit numerals. (Nos. 13-17)
2. Multiplication in base five of three-digit numerals by two, and of two-digit numerals by numbers greater than two. (Nos. 18-21)
3. Counting in base five beyond 30, and counting in base four. (Nos. 22,23)
4. Using a base eight addition table to add two two-digit base eight numerals, and to multiply two-digit base eight numerals by two. (Nos. 24-29)

5. The symbols that are used in base six. (No. 30)

6. Deduction of the base being used. (Nos. 31-34,37)

7. Conversion from one base to another. (Nos. 35,36,42)

8. Development of a base four addition table. (No. 38)

9. Subtraction in base five. (Nos. 39,40)

10. Development of a base five multiplication table. (No. 41)

NAME:

1.	If 27 means 2 x 9 + 7 , what number base is being used ?
2.	Write the number of stars in base 7. * * * * * * * * * * * * * * * * * *
3.	Write the number of stars in base 6. *
4.	Write the number of squares in base 8. □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □
5.	Write this sum as a base 5 sum. * * * * + * * * * * * * = * * * * * * * * * * * ______ + __________ = ________________
6.	Write this sum as a base 4 sum. * * * * * + * * * * * * = * * * * * * * * * * * ________ + _________ = ________________
7.	Count in base 5 from 1 to 20. (both 1 and 20 are base 5 numbers)

All the questions on this page are base 5 questions.

You may use your base 5 addition table whenever you like.

8.	Add in base 5.	10 42	15.	Add in base 5.	321 240
9.	Add in base 5.	13 14	16.	Add in base 5.	24 33 42
10.	Add in base 5.	32 24	17.	Add in base 5.	432 324
11.	Multiply in base 5.	32 x2	18.	Multiply in base 5.	321 x 2
12.	Multiply in base 5.	24 x2	19.	Multiply in base 5.	234 x 2
13.	Add in base 5.	21 31 40	20.	Multiply in base 5.	21 x4
14.	Add in base 5.	412 231	21.	Multiply in base 5.	34 x3

22.	Count in base 4 from 1 to 12. (both 1 and 12 are base 4 numbers)
23.	Count in base 5 from 32 to 44. (both 32 and 44 are base 5 numbers)

The next six questions (numbers 24 to 29) are base 8 questions.

Use the base 8 table provided whenever you like.

24.	Add in base 8.	24 73	27.	Multiply in base 8.	63 x2
25.	Add in base 8.	36 24	28.	Multiply in base 8.	35 x2
26.	Add in base 8.	47 52	29.	Multiply in base 8.	57 x2

30.	If you were counting in base 6, what symbols would you use ?

31.	What base would a person be using if he wrote: I have 14 toes ? (in fact he has the same number of toes as everyone else)
32.	What base is this person counting in ? . . . . . 33, 34, 35, 36, 40, 41, ...
33.	What base is being used here ? 4 + 3 = 10
34.	Here is an addition problem: 34 + 62 Could this be a base five sum ? Why ?
35.	If Ann writes: "I have 18 dollars", when she is counting in base ten, then if she were using base five she would write: "I have ________ dollars".
36.	Pete and Bill have the same number of books. Pete counts his in base five and writes that he has 43 books. Bill counts his in base ten and writes that he has ________ books.

37. What is the smallest possible base a person could be using if he wrote down the sum 35 + 23 ?

38. Here is part of a base four addition table. Fill in the spaces.

+	1	2	3
1			
2			
3			

39. Subtract in base 5.

$$\begin{array}{r} 33 \\ \underline{-4} \\ \underline{} \end{array}$$

40. Subtract in base 5.

$$\begin{array}{r} 23 \\ \underline{-14} \\ \underline{} \end{array}$$

41. Here is part of a base five multiplication table. Fill in the spaces.

x	1	2	3	4
1				
2				
3				
4				

42. If I have 23 dollars in base 6, how many do I have in base 5 ?

BASE FIVE ADDITION TABLE

+	1	2	3	4
1	2	3	4	10
2	3	4	10	11
3	4	10	11	12
4	10	11	12	13

BASE EIGHT ADDITION TABLE

+	1	2	3	4	5	6	7
1	2	3	4	5	6	7	10
2	3	4	5	6	7	10	11
3	4	5	6	7	10	11	12
4	5	6	7	10	11	12	13
5	6	7	10	11	12	13	14
6	7	10	11	12	13	14	15
7	10	11	12	13	14	15	16

APPENDIX D

THE EXPERIMENTAL DATA

TABLE I

STRAIGHTFORWARD SCORES FOR THE LINEAR PROGRAM GROUP

SCORES FOR	STUDENT NUMBER													
EACH ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1.	1	1	1	1	1	0	1	0	0	1	0	1	1	1
2.	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4.	1	1	1	1	1	1	1	1	1	0	1	1	1	1
5.	1	1	1	0	0	0	1	1	0	0	1	0	1	1
6.	1	1	1	1	0	1	0	0	0	0	0	1	1	1
7.	1	1	1	1	1	1	1	0	1	1	1	1	0	1
8.	1	1	1	1	1	1	1	1	0	1	1	1	1	1
9.	1	1	1	1	1	0	1	1	1	1	1	1	1	1
10.	1	1	1	1	1	0	1	1	1	1	1	1	1	1
11.	1	1	0	1	1	1	1	1	0	1	1	1	1	1
12.	1	1	0	1	1	1	1	1	1	0	0	0	1	0
TOTAL SCORE	12	12	10	11	10	8	11	9	7	8	9	10	11	11

TABLE II

STRAIGHTFORWARD SCORES FOR THE CHALLENGED GROUP

SCORES FOR	STUDENT NUMBER														
EACH ITEM	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1.	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1
2.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
5.	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
6.	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1
7.	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1
8.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9.	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
10.	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1
11.	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1
12.	1	1	1	1	1	1	1	0	1	1	0	1	0	1	1
TOTAL SCORE	12	12	11	12	12	9	8	11	11	12	8	11	10	11	10

TABLE III

EXTRAPOLATION SCORES FOR THE LINEAR PROGRAM GROUP

SCORES FOR EACH ITEM	STUDENT NUMBER 1	2	3	4	5	6	7	8	9	10	11	12	13	14
13.	1	1	0	0	1	1	1	1	0	1	1	1	1	1
14.	1	1	1	1	1	1	1	0	0	1	1	1	1	1
15.	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16.	1	1	0	0	0	0	0	0	0	1	1	0	0	0
17.	1	1	1	1	0	0	0	1	1	1	1	1	1	1
18.	1	1	0	1	1	1	1	1	0	1	0	1	1	1
19.	1	1	0	0	1	1	1	0	0	1	1	0	1	0
20.	1	1	0	1	1	1	1	1	0	1	1	0	1	0
21.	1	1	0	0	0	0	1	0	0	1	0	0	0	1
22.	1	1	0	0	0	0	0	0	0	0	1	1	0	0
23.	1	1	0	0	1	1	1	0	0	1	1	1	0	0
24.	1	1	1	1	1	1	1	0	1	1	1	0	1	1
25.	1	1	1	1	1	1	0	0	1	1	1	0	1	0
26.	1	1	1	1	0	1	1	0	1	0	1	0	1	1
27.	1	1	0	0	1	1	1	0	0	1	1	0	0	1
28.	1	0	0	0	0	1	0	0	0	1	0	0	0	1
29.	1	0	0	0	1	1	0	0	0	1	0	0	1	1
30.	1	1	0	0	1	1	1	0	0	1	0	1	0	0
31.	1	1	0	0	1	0	1	0	0	0	0	1	0	1
32.	1	1	0	0	1	1	1	0	0	0	1	1	1	0
33.	1	1	0	0	0	1	1	0	0	0	0	0	1	0
34.	1	1	0	0	1	0	1	0	0	0	0	0	0	0
35.	1	1	0	0	1	1	1	0	0	0	0	1	1	0
36.	1	1	0	0	1	0	1	0	0	0	0	0	1	0
37.	1	1	0	1	0	0	1	0	0	0	0	0	0	0
38.	2	2	0	0	0	0	0	0	0	0	0	0	0	0
39.	1	1	0	0	0	1	0	0	0	0	1	0	1	0
40.	1	0	0	0	0	1	0	0	0	0	1	0	1	0
41.	2	2	0	0	2	2	2	0	0	2	2	1	0	0
42.	1	1	0	0	0	0	1	0	0	0	0	0	0	0
TOTAL SCORE	32	29	6	9	19	21	22	5	5	18	18	12	17	12

TABLE IV

EXTRAPOLATION SCORES FOR THE CHALLENGED GROUP

SCORES FOR EACH ITEM	STUDENT NUMBER 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
13.	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1
14.	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
15.	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
16.	1	0	1	1	1	1	1	1	1	1	0	0	1	1	0
17.	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1
18.	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
19.	1	0	1	1	1	1	1	1	1	1	0	1	0	1	1
20.	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
21.	1	0	1	0	0	0	0	0	0	0	0	1	1	0	1
22.	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
23.	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1
24.	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
25.	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1
26.	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
27.	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1
28.	1	1	1	0	1	0	1	1	1	1	0	0	1	1	1
29.	1	1	1	1	1	0	0	1	1	1	0	1	1	1	1
30.	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1
31.	1	1	0	1	0	0	0	1	1	1	0	1	0	0	1
32.	1	1	1	1	1	0	0	1	1	1	0	1	1	1	1
33.	0	1	1	1	0	0	0	1	1	0	1	1	1	0	1
34.	1	0	1	0	0	0	0	1	0	0	0	0	0	1	1
35.	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1
36.	0	1	1	1	1	0	0	1	0	0	0	1	1	0	0
37.	0	1	1	1	1	0	0	1	1	0	0	0	1	1	1
38.	1	2	2	0	2	2	2	2	2	2	0	2	2	2	1
39.	0	1	1	1	1	0	0	1	0	0	0	0	1	1	1
40.	0	1	1	1	1	0	0	1	0	1	1	0	1	0	1
41.	2	2	0	2	0	2	2	2	0	2	0	0	2	2	2
42.	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0
TOTAL SCORE	25	27	29	27	25	15	21	31	24	24	6	20	27	21	28

TABLE V

TIME TAKEN TO COMPLETE PROGRAM

LINEAR PROGRAM GROUP		CHALLENGED GROUP	
STUDENT NUMBER	TIME TAKEN (MINUTES)	STUDENT NUMBER	TIME TAKEN (MINUTES)
1.	45	1.	50
2.	50	2.	35
3.	65	3.	40
4.	77	4.	21
5.	49	5.	36
6.	58	6.	35
7.	59	7.	33
8.	75	8.	22
9.	59	9.	63
10.	62	10.	31
11.	64	11.	54
12.	60	12.	49
13.	70	13.	25
14.	57	14.	31
		15.	28

TABLE VI

PATHS THROUGH CHALLENGING PROGRAM

STUDENT	LEVEL AT WHICH SECTION WAS COMPLETED					
NUMBER	SECTION 2	SECTION 3	SECTION 4	SECTION 6	SECTION 7	SECTION 8
1.	1	1	3	3	1	2
2.	1	1	1	1	1	2
3.	2	1	1	1	1	1
4.	1	1	1	1	1	1
5.	2	3	1	1	1	1
6.	1	2	1	1	2	1
7.	1	2	1	1	2	1
8.	1	1	1	1	1	1
9.	3	3	3	1	1	2
10.	2	1	1	1	1	2
11.	2	1	1	2	3	1
12.	3	1	1	1	1	2
13.	1	1	1	1	1	1
14.	1	1	1	1	1	3
15.	2	3	1	1	1	1

Level 1: Initial challenging question correct.

Level 2: Second challenging question correct (after hint was given).

Level 3: Linear program required.